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Vietoris topology on partial maps with compact domains

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ABSTRACT

The space $\mathcal{P}_K$ of partial maps with compact domains (identified with their graphs) forms a subspace of the hyperspace of nonempty compact subsets of a product space endowed with the Vietoris topology. Various completeness properties of $\mathcal{P}_K$, including Čech-completeness, sieve completeness, strong Choquetness, and (hereditary) Baireness, are investigated. Some new results on the hyperspace $K(X)$ of compact subsets of a Hausdorff X with the Vietoris topology are obtained; in particular, it is shown that there is a strongly Choquet X, with 1st category $K(X)$.

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1. Introduction

Given the Hausdorff spaces X, Y, and a continuous function $f : X \to Y$, its graph $\Gamma(f)$ is a closed subset of $X \times Y$; moreover, if X is compact, then so is $\Gamma(f)$. As a consequence, it is natural to view the space $C(X, Y)$ of continuous functions $f : X \to Y$ as a subspace of the hyperspace $CL(X \times Y)$ of nonempty closed (resp. compact) subsets of $X \times Y$, which in turn allows one to endow $C(X, Y)$ with various hyperspace topologies inherited from $CL(X \times Y)$. This is a well-known approach leading to new topologies on $C(X, Y)$ [36,5], as well as allowing to study classical function space topologies in a new light [39,24].

More generally, one can consider the space of all partial maps $f \in C(B, Y)$ for all nonempty closed (resp. compact) $B \subseteq X$ as sitting in $CL(X \times Y)$ endowed with the so-called Vietoris topology, as was first done by *Zaremba* [42], and *Kuratowski* [27,28] (see also [20,26]). Other topologies have been also studied in this context [17,18,11,12]; in particular, the *generalized compact-open topology* on partial maps received some recent attention [4,23,24]. Note that spaces of partial maps arise naturally in differential equations [38,10], mathematical economics [4], or in dynamic programming models [30,40]; moreover, since the classical function space topologies (e.g. pointwise, compact-open, uniform, resp.) are defined for functions with the same domain, appropriate partial map space topologies are essential for useful applications in these areas.

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It is the purpose of this paper to investigate various completeness properties of partial maps with compact domains endowed with the Vietoris topology, and explore the relationship to various hyperspace and function space topologies. Proving completeness properties for the compact-open topology on $C(X,Y)$ frequently calls for some extension theorem (Tietze, Dugundji), which then requires restrictions on Y (cf. [35]); we will show how spaces of partial maps can be used to obtain these results for a considerably more general Y.

In what follows, let X and Y be Hausdorff spaces. If X is Tychonoff, cX will denote a fixed Hausdorff compactification of X. We will write B^c, $\operatorname{int} B$, and $\overline{B}$ for the complement, interior, and closure, respectively, of $B \subseteq X$. Denote by $CL(X)$ the family of nonempty closed subsets of X, and by $K(X)$ the family of nonempty compact subsets of X. If $K \in K(X \times Y)$, write $K(x) = \{y \in Y\colon (x,y) \in K\}$. The symbol π_X, π_Y will denote the projection map from $X \times Y$ onto X, Y, respectively. For any $B \in K(X)$, and a topological space Y, $C(B,Y)$ will stand for the space of continuous functions from B to Y. Denote by

$$\mathcal{P}_K = \mathcal{P}_K(X,Y) = \bigcup \bigl\{C(B,Y)\colon B \in K(X)\bigr\}$$

the family of all partial maps with compact domains. We will identify a partial map f with its graph $\Gamma(f) \in K(X \times Y)$. The *Vietoris topology* τ_V on $K(X)$ has subbase elements of the form

$$U^- = \bigl\{A \in K(X)\colon A \cap U \neq \emptyset\bigr\} \quad \text{and} \quad U^+ = \bigl\{A \in K(X)\colon A \subseteq U\bigr\}$$

where $\emptyset \neq U \subseteq X$ is open, so a base for τ_V consists of

$$\langle U_0, \dots, U_n\rangle = \Bigl(\bigcup_{i \leqslant n} U_i\Bigr)^+ \cap \bigcap_{i \leqslant n} U_i^-$$

where $U_i \subseteq X$ are open. We will use the same notation τ_V for the Vietoris topology on $K(X)$, as well as on $K(X \times Y)$, and any of its subspaces. We will consider two topologies on $C(X,Y)$, the *uniform topology* τ_U, and the *compact-open topology* τ_{CO} [19,35].

Proposition 1.1. *X, Y and $K(X)$ embed as closed subspaces in $(\mathcal{P}_K(X,Y), \tau_V)$.*

Proposition 1.2.

(i) *$\mathcal{P}_K(X,Y) \subseteq K(X \times Y)$; moreover, $\mathcal{P}_K(X,Y) \subseteq K(cX \times cY)$, if X, Y are Tychonoff.*
(ii) *If X is dense-in-itself, then $\mathcal{P}_K(X,Y)$ is dense in $(K(X \times Y), \tau_V)$.*
(iii) *If X is regular and Y has a G_δ-diagonal, then $\mathcal{P}_K(X,Y)$ is a G_δ-subset of $(K(X \times Y), \tau_V)$.*

Proof. (i) and (ii) are easy to see, only (iii) needs some explanation: let $\{G_n\}_n$ be a sequence of open sets in $Y \times Y$ such that the diagonal $\Delta = \{(y,y)\colon y \in Y\} = \bigcap_{n \in \omega} G_n$. We will show that the set

$$\mathcal{G}_n = \bigl\{K \in K(X \times Y)\colon \forall x \in \pi_X(K)\ \exists \text{ open } V_x \text{ with } K(x) \times K(x) \subseteq V_x \times V_x \subseteq G_n\bigr\}$$

is τ_V-open in $K(X \times Y)$ for every $n \in \omega$: take some $K \in \mathcal{G}_n$ and $x \in \pi_X(K)$. Then there is a Y-open V_x such that $K(x) \times K(x) \subseteq V_x \times V_x \subseteq G_n$. We can find an open neighborhood U_x of x such that $K(z) \subseteq V_x$ for every $z \in U_x$ (otherwise, there is a net $\{z_\lambda\}$ converging to x, and some $y_\lambda \in K(z_\lambda) \setminus V_x$ for each λ, hence, $(z_\lambda, y_\lambda) \in K$ has a cluster point $(x,y) \in K$, such that $y \notin V_x$ contradicting $y \in K(x) \subseteq V_x$).

Regularity of X implies that there is an open neighborhood H_x of x such that $H_x \subseteq \overline{H_x} \subseteq U_x$. Compactness of $\pi_X(K)$ implies that there is a $k \in \omega$ with $\pi_X(K) \subseteq \bigcup_{i \leqslant k} H_i = H$, where for each $i \leqslant k$, $H_i = H_{x_i}$ for some $x_i \in \pi_X(K)$. We will also write U_i for U_{x_i}, and V_i for V_{x_i}. For every $i \leqslant k$ put

$$\mathcal{L}_i = \bigl[U_i \times V_i \cup (X \setminus \overline{H}_i) \times Y\bigr]^+ \cap (H \times Y)^+.$$

Then $\bigcap_{i \leqslant k} \mathcal{L}_i$ is a τ_V-open neighborhood of K such that $\bigcap_{i \leqslant k} \mathcal{L}_i \subseteq \mathcal{G}_n$: indeed, let $L \in \bigcap_{i \leqslant k} \mathcal{L}_i$, and $x \in \pi_X(L)$. Then $x \in H$, hence $x \in H_i$ for some $i \leqslant k$; moreover, $L(x) \subseteq V_i$, and $V_i \times V_i \subseteq G_n$, so $L \in \mathcal{G}_n$.

Clearly, $\mathcal{P}_K(X,Y) \subseteq \bigcap_n \mathcal{G}_n$, since if $K \in \mathcal{P}_K(X,Y)$, then $K(x)$ is a singleton; on the other side, if $K \in \bigcap_n \mathcal{G}_n$, then for each $x \in \pi_X(K)$, $K(x) \times K(x) \subseteq \bigcap_n G_n = \Delta$, so $K(x)$ is a singleton. This means that K is a compact graph of a function f with compact domain $\pi_X(K)$, which implies that f is continuous, whence $K = \Gamma(f) \in \mathcal{P}_K(X,Y)$. □

Corollary 1.3. *Let X be regular, Y have a G_δ-diagonal, and P be a topological property that is closed- and G_δ-hereditary. Consider the following properties*:

(i) *$(K(X \times Y), \tau_V)$ has property P;*
(ii) *$(\mathcal{P}_K(X,Y), \tau_V)$ has property P;*
(iii) *X, Y have property P.*

Then (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii).

We will now explore some properties of the function

$$\eta : \big(K(X), \tau_V\big) \times \big(C(X,Y), \tau_U\big) \to \big(\mathcal{P}_K(X,Y), \tau_V\big) \quad \text{defined via } \eta(B,f) = \Gamma(f\restriction_B).$$

Proposition 1.4. *Let X, Y be Tychonoff spaces. Then η is continuous.*

Proof. Let $\mathcal{U}$ and $\mathcal{V}$ be uniformities on X and Y, respectively, and $(B,f) \in K(X) \times C(X,Y)$. Take an open set O in $X \times Y$ such that $\Gamma(f\restriction_B) \in O^-$; we can assume that $O = O_1 \times O_2$, where O_1 is X-open, and O_2 is Y-open. If $(x, f(x)) \in O_1 \times O_2$, $x \in B$, and $V \in \mathcal{V}$ is such that $V \circ V[f(x)] \subseteq O_2$, then continuity of f at x implies that there is an X-open U with $x \in U$, $U \subseteq O_1$ and for every $z \in U$, $f(z) \in V[f(x)]$. It is easy to verify that the set

$$\mathcal{H} = U^- \times \big\{g \in C(X,Y)\colon\ g(v) \in V\big[f(v)\big] \text{ for every } v \in X\big\}$$

is a neighborhood of (B,f), and $\Gamma(h\restriction_C) \in O^-$ for each $(C,h) \in \mathcal{H}$; thus, $\eta^{-1}(O^-)$ is open.

Now let $G \subseteq X \times Y$ be open such that $\Gamma(f\restriction_B) \in G^+$. Let $U \in \mathcal{U}$ and $V \in \mathcal{V}$ be such that $U \times V[\Gamma(f\restriction_B)] \subseteq G$, further, $V_1 \in \mathcal{V}$ be symmetric with $V_1 \circ V_1 \subseteq V$. Using uniform continuity of $f\restriction_B$ and compactness of B, we can find a symmetric $U_1 \in \mathcal{U}, U_1 \subseteq U$ so that whenever $x \in B$ and $(z,x) \in U_1$, $(f(z), f(x)) \in V_1$. If $H \in \mathcal{U}$ is open symmetric such that $H \subseteq U_1$, then for every

$$(C,h) \in H(B)^+ \times \big\{g \in C(X,Y)\colon\ \big(g(z), f(z)\big) \in V_1 \text{ for every } z \in X\big\}$$

we have $\eta(C,h) \in G^+$; thus, $\eta^{-1}(G^+)$ is open. □

Recall, that a map $\psi : X \to Y$ is *feebly open* [22], provided $\operatorname{int}\psi(U) \neq \emptyset$ for each nonempty open $U \subseteq X$.

Proposition 1.5. *Let either X be paracompact, and Y locally convex, completely metrizable, or X be normal, and $Y = \mathbb{R}$. Then η is feebly open.*

Proof. Let d be a compatible metric on Y, and take a basic open set in $(K(X), \tau_V) \times (C(X,Y), \tau_U)$:

$$\mathcal{H} = \langle U_0, \ldots, U_n\rangle \times \big\{g \in C(X,Y)\colon\ d\big(g(x), f(x)\big) < \epsilon,\ \forall x \in X\big\},$$

where $\emptyset \neq U_i \subseteq X$ are open ($i \leqslant n$), $\epsilon > 0$ and $f \in C(X,Y)$. For every $i \leqslant n$, take $x_i \in U_i$ and, without loss of generality, assume that the x_i's are distinct; then $\Gamma(f\restriction_{\{x_0,\ldots,x_n\}}) \in \eta(\mathcal{H})$. For each $i \leqslant n$, let H_i be a convex neighborhood of $f(x_i)$ such that

$$H_i \subseteq \left\{y \in Y\colon\ d\big(f(x_i), y\big) < \frac{\epsilon}{4}\right\},$$

and by continuity of f, let V_i be an open neighborhood of x_i such that $\overline{V_i} \subseteq U_i$, $f(z) \in H_i$ for every $z \in \overline{V_i}$, and the family $\{\overline{V_i}\colon\ i \leqslant n\}$ is pairwise disjoint. The set

$$\mathcal{L} = \bigcap_{i \leqslant n} (V_i \times H_i)^- \cap \left(\bigcup_{i \leqslant n} V_i \times H_i\right)^+$$

is a τ_V-neighborhood of $\Gamma(f\restriction_{\{x_0,\ldots,x_n\}})$ such that $\mathcal{L} \cap \mathcal{P}_K \subseteq \eta(\mathcal{H})$: indeed, let $g \in \mathcal{L} \cap \mathcal{P}_K$, and $B = \operatorname{dom} g$. Then $B \in \langle U_0, \ldots, U_n\rangle$ and, using the appropriate extension theorem (an application of the Michael Selection Theorem [6, p. 92, Corollary 7.5.], and Tietze's Theorem, respectively), we can extend g to $g^* \in C(X,Y)$ so that $d(g^*(x), f(x)) < \epsilon$; thus $g = \eta(B, g^*) \in \eta(\mathcal{H})$. □

2. Completeness properties of $K(X)$

From now on, cX is a fixed Hausdorff compactification of a Tychonoff space X. Recall, that X is *Čech-complete* [19], if X is G_δ in its compactification cX. We will say that X is a *p-space* [3,21], provided there is a *feathering* for X, i.e. there is a sequence $\{\mathcal{V}_m\}_m$ of open covers of X in cX such that $\bigcap_m \operatorname{st}(x, \mathcal{V}_m) \subseteq X$ for all $x \in X$, where $\operatorname{st}(x, \mathcal{V}_m) = \bigcup\{V \in \mathcal{V}_m\colon\ x \in V\}$. Analogously, we can define *cp-spaces*, if we require $\bigcap_m(\bigcup\{V \in \mathcal{V}_m\colon\ K \cap V \neq \emptyset\}) \subseteq X$ for all $K \in K(X)$. A *cp*-space is clearly a *p*-space; on the other hand, a paracompact *p*-space, or a Čech-complete space is a *cp*-space. A space is *sieve complete* provided it is the continuous open image of a Čech-complete space [41], so Čech-complete spaces are sieve complete; on the other hand, paracompact sieve complete spaces are Čech-complete [32]. A space X is a *Baire space*, provided countable collections of dense open subsets of X have a dense intersection [22,25]; equivalently, if nonempty open subsets of X are of 2nd category (i.e. not of 1st category, which would be a countable union of nowhere dense sets); X is called *hereditarily Baire* if every nonempty closed subspace is Baire.

Theorem 2.1. *The following are equivalent*:

(i) $(K(X), \tau_V)$ *is completely metrizable* (*Čech-complete, sieve complete, resp.*);
(ii) X *is completely metrizable* (*Čech-complete, sieve complete, resp.*).

Proof. Since all these properties are closed-hereditary, (i) ⇒ (ii) follows as X sits in $K(X)$ as a closed subset.

(ii) ⇒ (i). See [29] for complete metrizability, [43], or [13, Theorem 4], for Čech-completeness. As for sieve completeness, let $f : Z \to X$ be an open continuous mapping from a Čech-complete space Z onto X. Define $F : (K(Z), \tau_V) \to (K(X), \tau_V)$ as $F(K) = f(K)$ for each $K \in K(X)$. Then F is continuous and, since f is compact-covering [19, Problem 5.5.11(e)], F is onto.

Also, F is an open mapping, since if $\mathbf{U} = \langle U_0, \dots, U_n\rangle \in \tau_V(Z)$, then $F(\mathbf{U}) = \langle f(U_0), \dots, f(U_n)\rangle = \mathbf{V}$. Indeed, clearly $F(\mathbf{U}) \subset \mathbf{V}$, on the other hand, if $K \in \mathbf{V}$, we can find some $x_i \in K \cap f(U_i)$ and a corresponding $z_i \in U_i$ with $f(z_i) = x_i$ for each $i \leqslant n$. Now, $U = \bigcup_{i\leqslant n} U_i \subseteq Z$ is Čech-complete and $f\restriction_U$ is a continuous open mapping of U into $\bigcup_{i\leqslant n} f(U_i)$ and hence compact-covering. It means that for some Z-compact $L_0 \subseteq U$, $f(L_0) = K$ and therefore $L = L_0 \cup \{z_0, \dots, z_n\} \in \mathbf{U}$ and $F(L) = K$. □

Proposition 2.2. *If X is a cp-space, then $(K(X), \tau_V)$ is a p-space.*

Proof. See [13, Theorem 3]. □

In the *strong Choquet game* [25] two players, α and β, take turns in choosing objects in the topological space X with an open base $\mathcal{B}$: β starts by picking (x_0, V_0) from

$$\mathcal{E} = \mathcal{E}(X, \mathcal{B}) = \big\{(x, V) \in X \times \mathcal{B}\colon\ x \in V\big\},$$

and α responds by $U_0 \in \mathcal{B}$ with $x_0 \in U_0 \subseteq V_0$. The next choice of β is some couple $(x_1, V_1) \in \mathcal{E}$ with $V_1 \subseteq U_0$ and again α picks U_1 with $x_1 \in U_1 \subseteq V_1$ etc. Player α wins the run $(x_0, V_0), U_0, \dots, (x_n, V_n), U_n, \dots$ provided $\bigcap_n U_n = \bigcap_n V_n \neq \emptyset$, otherwise β wins. A *winning strategy* for α (resp. β) is a function $\sigma : \mathcal{E}^{<\omega} \to \mathcal{B}$ (resp. $\sigma : \mathcal{B}^{<\omega} \to \mathcal{E}$) such that α (resp. β) wins every run of the game compatible with σ, i.e. such that $U_n = \sigma((x_0, V_0), \dots, (x_n, V_n))$ (resp. $(x_{n+1}, V_{n+1}) = \sigma(U_0, \dots, U_n)$) for all n. The space X is called a *strong Choquet space* [25], if α has a winning strategy in $Ch(X)$. Čech complete spaces are strongly Choquet [37], and so are sieve complete spaces; moreover, a metrizable space is strongly Choquet iff it is completely metrizable [25]. We will say that $Ch(X)$ is β-favorable, if β has a winning strategy in $Ch(X)$. It is known that if X is a 1st countable regular space, and $Ch(X)$ is β-favorable, then X contains a closed copy of the rationals, and so X is not hereditarily Baire; moreover, a Moore space is hereditarily Baire iff $Ch(X)$ is not β-favorable [14].

A regular space X is a *Moore space* [21], if there is a sequence $\{\mathcal{V}_n\colon\ n \in \omega\}$ of open covers of X such that for each $x \in X$, $\{\mathrm{st}(x, \mathcal{V}_n)\colon\ n \in \omega\}$ is a base of neighborhoods at x.

Theorem 2.3.

(i) *Let X, Y be regular. If $Ch(K(X \times Y))$ is β-favorable, so is $Ch(K(X) \times K(Y))$.*
(ii) *Let X, Y be Moore spaces. If $(K(X), \tau_V) \times (K(Y), \tau_V)$ is hereditarily Baire, then so is $(K(X \times Y), \tau_V)$.*

Proof. (i) Denote $S = K(X \times Y)$ and $T = K(X) \times K(Y)$. Let σ_S be a winning strategy for β in $Ch(S)$. We will define a winning strategy σ_T for β in $Ch(T)$: if $(B_S, \mathcal{B}_S)$ is β's choice in $Ch(S)$ (at some step), where $\mathcal{B}_S = \langle U_0 \times V_0, \dots, U_k \times V_k\rangle$, let

$$B_T = \big(\pi_X(B_S), \pi_Y(B_S)\big), \qquad \mathcal{B}_T = \langle U_0, \dots, U_k\rangle \times \langle V_0, \dots, V_k\rangle,$$

and $(B_T, \mathcal{B}_T)$ be β's corresponding step in $Ch(T)$. If α's response in $Ch(T)$ is $\mathcal{A}_T = \langle W'_0, \dots, W'_m\rangle \times \langle Z'_0, \dots, Z'_n\rangle$, then $B_T \in \mathcal{A}_T$, so

$$\pi_X(B_S) \subseteq \bigcup_i W'_i \subseteq \bigcup_p U_p \quad \text{and} \quad \forall p\ \exists i \quad \text{with } W'_i \subseteq U_p,$$

$$\pi_Y(B_S) \subseteq \bigcup_j Z'_j \subseteq \bigcup_p V_p \quad \text{and} \quad \forall p\ \exists j \quad \text{with } Z'_j \subseteq V_p.$$

Considering only the intersections $W'_i \cap U_p$ and $Z'_j \cap V_p$ that hit $\pi_X(B_S)$ and $\pi_Y(B_S)$, respectively, we can assume that

$$\forall i\ \exists p \quad \text{with } W'_i \subseteq U_p \quad \text{and} \quad \forall j\ \exists p \quad \text{with } Z'_j \subseteq V_p.$$

If $a \in \pi_X(B_S)$, denote $U_a = \bigcap\{W'_i\colon\ a \in W'_i\}$, and if $b \in \pi_Y(B_S)$, denote $V_b = \bigcap\{Z'_j\colon\ b \in Z'_j\}$. By regularity, for each $(a, b) \in B_S$ we can find open U'_a, and V'_b containing a, b, respectively, such that $\overline{U'_a \times V'_b} \subseteq U_a \times V_b$. By compactness of B_S, there is a finite subcover of $\{U'_a \times V'_b\colon\ (a, b) \in B_S\}$ covering B_S; enumerate this subcover as $\{W_0 \times Z_0, \dots, W_r \times Z_r\}$. We can also assure that for all $p \leqslant k$ there is $s \leqslant r$ with $W_s \times Z_s \subseteq U_p \times V_p$, so if we denote $\mathcal{A}_S = \langle W_0 \times Z_0, \ldots, W_r \times Z_r \rangle$, then $B_S \in \mathcal{A}_S \subseteq \mathcal{B}_S$, and $\mathcal{A}_S$ can, and will, be α's next step in $Ch(S)$.

Now, if $(B_T^{(0)}, \mathcal{B}_T^{(0)}), \mathcal{A}_T^{(0)}, \ldots, (B_T^{(k)}, \mathcal{B}_T^{(k)}), \mathcal{A}_T^{(k)}, \ldots$ is a run in $Ch(T)$ compatible with σ_T, then $(B_S^{(0)}, \mathcal{B}_S^{(0)})$, $\mathcal{A}_S^{(0)}, \ldots,$ $(B_S^{(k)}, \mathcal{B}_S^{(k)}), \mathcal{A}_S^{(k)}, \ldots$ is a run in $Ch(S)$ compatible with σ_S. Moreover, if there exists some $\emptyset \neq (K_1, K_2) \in \bigcap_{k\in\omega} \mathcal{A}_T^{(k)}$ then, by compactness of $K_1 \times K_2$, we have

$$\emptyset \neq K_1 \times K_2 \cap \bigcap_{k\in\omega} \left(\bigcup_{s\leqslant r_k} \overline{W_s \times Z_s} \right) \in \bigcap_{k\in\omega} \mathcal{A}_S^{(k)},$$

which is a contradiction.

(ii) By a theorem of Mizokami [33], $K(X), K(Y), K(X \times Y)$, and hence, $K(X) \times K(Y)$ is a Moore space, so if $K(X) \times K(Y)$ is hereditarily Baire, then $Ch(K(X) \times K(Y))$ is not β-favorable, and neither is $Ch(K(X \times Y))$ by (1); thus, $K(X \times Y)$ is hereditarily Baire. □

The *Banach–Mazur game* $BM(X)$ is played analogously to the strong Choquet game, except, both α and β choose elements of $\mathcal{B}$. It is known that β has a winning strategy in $BM(X)$ iff X is not a Baire space [25]. A space is called *weakly α-favorable* iff α has a winning strategy in $BM(X)$ [22]. A space is *quasi-regular* [34] iff each nonempty open set contains the closure of a nonempty open set. We will use that Baire spaces are invariant of continuous, feebly open maps [22, Proposition 4.4(ii), and Theorem 4.7].

Theorem 2.4. *Let X, Y be quasi-regular. The following are equivalent*:

(i) $(K(X), \tau_V) \times (K(Y), \tau_V)$ *is Baire*;
(ii) $(K(X \times Y), \tau_V)$ *is Baire.*

Proof. (i) ⇒ (ii). Denote $S = K(X \times Y)$ and $T = K(X) \times K(Y)$. If S is not Baire, then β has a winning strategy σ_S in $BM(S)$. We will define a winning strategy σ_T for β in $BM(T)$: if $\mathcal{B}_S = \langle U_0 \times V_0, \ldots, U_k \times V_k \rangle$ is β's choice in $BM(S)$ (at some step), let $\mathcal{B}_T = \langle U_0, \ldots, U_k \rangle \times \langle V_0, \ldots, V_k \rangle$ be β's corresponding step in $BM(T)$. Let α's response in $BM(T)$ be $\mathcal{A}_T = \langle W'_0, \ldots, W'_m \rangle \times \langle Z'_0, \ldots, Z'_n \rangle$. Then

$$\bigcup_i W'_i \subseteq \bigcup_p U_p \quad \text{and} \quad \forall p\, \exists i \quad \text{with } W'_i \subseteq U_p,$$
$$\bigcup_j Z'_j \subseteq \bigcup_p V_p \quad \text{and} \quad \forall p\, \exists j \quad \text{with } Z'_j \subseteq V_p.$$

Considering the intersections $W'_i \cap U_p$ and $Z'_j \cap V_p$, if necessary, we can assume that

$$\forall i\, \exists p \quad \text{with } W'_i \subseteq U_p \quad \text{and} \quad \forall j\, \exists p \quad \text{with } Z'_j \subseteq V_p.$$

For each i, j find nonempty open $W''_i \subseteq W'_i$, and $Z''_j \subseteq Z'_j$ such that $\overline{W''_i \times Z''_j} \subseteq W'_i \times Z'_j$. Enumerate the collection $\{W''_i \times Z''_j\colon\ W''_i \times Z''_j \subseteq U_p \times V_p \text{ for some } p\}$ as $\{W_0 \times Z_0, \ldots, W_r \times Z_r\}$, and let α's next step in $BM(S)$ be $\mathcal{A}_S = \langle W_0 \times Z_0, \ldots, W_r \times Z_r \rangle$.

Now, let $\mathcal{B}_T^{(0)}, \mathcal{A}_T^{(0)}, \ldots, \mathcal{B}_T^{(k)}, \mathcal{A}_T^{(k)}, \ldots$ be a run in $BM(T)$ compatible with σ_T, and assume that there exists some $(K_1, K_2) \in \bigcap_{k\in\omega} \mathcal{A}_T^{(k)}$. Then $\mathcal{B}_S^{(0)}, \mathcal{A}_S^{(0)}, \ldots, \mathcal{B}_S^{(k)}, \mathcal{A}_S^{(k)}, \ldots$ is a run in $BM(S)$ compatible with σ_S, moreover, by compactness of $K_1 \times K_2$,

$$\emptyset \neq K_1 \times K_2 \cap \bigcap_{k\in\omega} \left(\bigcup_{s\leqslant r_k} \overline{W_s \times Z_s} \right) \in \bigcap_{k\in\omega} \mathcal{A}_S^{(k)},$$

which is a contradiction.

(ii) ⇒ (i). Define the mapping $\psi : K(X \times Y) \to K(X) \times K(Y)$ via

$$\psi(C) = \big(\pi_X(C), \pi_Y(C)\big),$$

which is clearly onto. Then ψ is continuous: let $K \in K(X \times Y)$, and $\mathcal{A} = \langle U_0, \ldots, U_n \rangle \times \langle V_0, \ldots, V_m \rangle$ be an open neighborhood of $\psi(K)$. Enumerate $\{U_i \times V_j\colon\ U_i \times V_j \cap K \neq \emptyset\}$ as $\{W_0, \ldots, W_k\}$. Then $K \in \langle W_0, \ldots, W_k \rangle \subset \psi^{-1}(\mathcal{A})$.

Also, ψ is feebly open: let $\mathcal{B} = \langle U_0 \times V_0, \ldots, U_n \times V_n \rangle$, and using quasi-regularity of X, Y, choose $U'_i \subseteq X$, $V'_i \subseteq Y$ open with $\overline{U'_i \times V'_i} \subseteq U_i \times V_i$ for each $i \leqslant n$. If $\mathcal{A} = \langle U'_0, \ldots, U'_n \rangle \times \langle V'_0, \ldots, V'_n \rangle$, then $\mathcal{A} \subseteq \psi(\mathcal{B})$. □

Proposition 2.5. *If $(K(X), \tau_V)$ is strongly Choquet, then so is X.*

Proof. Let σ_K be a winning strategy for α in $Ch(K(X))$. Let β's initial choice in $Ch(X)$ be (x_0, V_0), and $(\{x_0\}, V_0^+)$ be β's initial choice in $Ch(K(X))$. If $\mathcal{U}_0 = \sigma_K(\{x_0\}, V_0^+) = \langle A_0, \ldots, A_n\rangle$, then $x_0 \in \bigcap_{i\leqslant n} A_i \subseteq V_0$, and we can choose $U_0 = \sigma(x_0, V_0) = \bigcap_{i\leqslant n} A_0$ to be α's response in $Ch(X)$. Assuming that $U_{k-1} = \sigma((x_0, V_0), \ldots, (x_{k-1}, V_{k-1}))$ has been defined for $k \geqslant 1$, and (x_k, V_k) is β's next step in $Ch(X)$, let

$$\mathcal{U}_k = \sigma_K\big(\big(\{x_0\}, V_0^+\big), \ldots, \big(\{x_k\}, V_k^+\big)\big) = \langle B_0, \ldots, B_m\rangle.$$

Then $x_k \in \bigcap_{i\leqslant m} B_i \subseteq V_k$, and we can choose $U_k = \sigma((x_0, V_0), \ldots, (x_k, V_k)) = \bigcap_{i\leqslant m} B_i$ to be α's next step in $Ch(X)$. Let $(x_0, V_0), U_0, \ldots, (x_n, V_n), U_n, \ldots$ be a run of $Ch(X)$ compatible with σ. Then $(\{x_0\}, V_0^+), \mathcal{U}_0, \ldots, (\{x_n\}, V_n^+), \mathcal{U}_n, \ldots$ is a run of $Ch(K(X))$ compatible with σ_K, so there is some $\emptyset \neq K \in \bigcap_{n\in\omega} V_n^+$; thus, for any $x \in K$ we have $x \in \bigcap_{n\in\omega} V_n$, and α wins in $Ch(X)$. □

To show that the above implication cannot be reversed we first need

Proposition 2.6. *Let X be a dense-in-itself space where the compact subsets are finite, and Y be arbitrary. Then $(K(X\times Y), \tau_V)$ is of 1st category.*

Proof. For $n \geqslant 1$, consider $F_n = \{A \in K(X\times Y)\colon |\pi_X(A)| \leqslant n\}$. Then $K(X\times Y) = \bigcup_n F_n$, and we just need to show that each F_n is nowhere dense in $(K(X\times Y), \tau_V)$: let $A \in K(X\times Y)$ with $\pi_X(A) = \{p_0, \ldots, p_m\}$. Then $A = \bigcup_{k\leqslant m}\{p_k\}\times C_k$ for some $C_k \in K(Y)$. If $\mathcal{U} = \langle U_0, \ldots, U_r\rangle$ is a neighborhood of A, we can make sure that each U_i is a product $V_i \times W_i$ of open sets so that if $p_k \in V_i \cap V_j$ then $V_i = V_j$, and if $p_k \in V_i, p_l \in V_j$ for different k, l, then $V_i \cap V_j = \emptyset$. If $m+1 > n$ then $\mathcal{U} \cap F_n = \emptyset$, otherwise, take pairwise disjoint nonempty X-open subsets $G_0, \ldots, G_{n-m}$ of V_0, and observe that $\langle G_0 \times W_0, \ldots, G_{n-m}\times W_0, U_1, \ldots, U_r\rangle$ is a subset of $\mathcal{U}$ disjoint from F_n. □

McCoy [34] showed that if X is a *Bernstein set* (i.e. $X \subseteq \mathbb{R}$ such that both X and $\mathbb{R}\setminus X$ meets every dense-in-itself G_δ subset of $\mathbb{R}$), then $K(X)$ is of 1st category. It is known that X is hereditarily Baire, but not weakly α-favorable [22, Theorem 2.6], so the following is a new observation:

Example 2.7. There is a strongly Choquet X such that $(K(X), \tau_V)$ is of 1st category.

Proof. Let $X = \mathbb{R}$, with the topology having

$$\mathcal{B} = \{I \setminus C\colon\ I \text{ open interval},\ C \subseteq X \text{ countable}\}$$

as its base. Then X is clearly dense-in-itself and T_2. Further, if X has an infinite compact subset A with $\{a_i\colon i<\omega\} \subseteq A$, then the open cover $\{\{a_j\colon j \geqslant i\}^c\colon i \geqslant 1\}$ of A has no finite subcover, a contradiction. By Proposition 2.6, $K(X)$ is of 1st category (just take Y to be a singleton).

Also, X is strongly Choquet: indeed, let $\{C(i)\colon i<\omega\}$ be the enumeration of a countable $C \subseteq \mathbb{R}$. Inductively define a winning strategy σ for α in $Ch(X)$; let (x_0, B_0) be β's first step, where $x_0 \in B_0 = I_0 \setminus C_0 \in \mathcal{B}$. Choose an open interval J_0 with $x_0 \in J_0 \subseteq \overline{J_0} \subseteq I_0 \setminus \{C_0(0)\}$ that has at most half the length of I_0, and put $\sigma(x_0, B_0) = J_0 \setminus C_0$. Assume that $\sigma((x_0, B_0), \ldots, (x_{n-1}, B_{n-1}))$ has been defined for some $n \geqslant 1$, and $x_k \in B_k = I_k \setminus C_k \in \mathcal{B}$ for $k \leqslant n-1$. If (x_n, B_n) is β's next step, where $x_n \in B_n = I_n \setminus C_n \in \mathcal{B}$, choose an open interval J_n with $x_n \in J_n \subseteq \overline{J_n} \subseteq I_n \setminus \{C_k(i)\colon k, i \leqslant n\}$, which has at most half the length of I_n, and put $\sigma((x_0, B_0), \ldots, (x_n, B_n)) = J_n \setminus C_n$. Then there is a unique $x \in \bigcap_n I_n$, which will avoid all the C_n's; thus, $x \in \bigcap_n B_n$. □

3. Completeness properties of $\mathcal{P}_K$

An old result of Kuratowski [27] can be extended as follows:

Theorem 3.1. *The following are equivalent*:

(i) $(\mathcal{P}_K(X, Y), \tau_V)$ *is completely metrizable*;
(ii) X, Y *are completely metrizable.*

Proof. If X, Y are completely metrizable, then so is $X\times Y$ and $K(X\times Y)$, respectively (Theorem 2.1). The rest follows from Corollary 1.3. □

Theorem 3.2. *If Y has a G_δ-diagonal, the following are equivalent*:

(i) $(\mathcal{P}_K(X, Y), \tau_V)$ *is Čech-complete*;
(ii) X, Y *are Čech-complete.*

Proof. If X, Y are Čech-complete, then so is $X \times Y$ and $K(X \times Y)$, respectively (Theorem 2.1). The rest follows from Corollary 1.3. □

Theorem 3.3. *If X is regular and Y has a G_δ-diagonal, the following are equivalent*:

(i) $(\mathcal{P}_K(X,Y), \tau_V)$ *is sieve complete*;
(ii) X, Y *are sieve complete.*

Proof. If X, Y are sieve complete, so is $X \times Y$ and $K(X \times Y)$, respectively (Theorem 2.1). The remaining follows from Corollary 1.3 observing that sieve completeness is closed- and G_δ-hereditary [31, Remark 8.8]. □

Theorem 3.4. *Let X, Y be cp-spaces, and Y have a G_δ-diagonal. Then $(\mathcal{P}_K(X,Y), \tau_V)$ is a p-space.*

Proof. Since $X \times Y$ is a *cp*-space, if X, Y are, it follows by Proposition 2.2 that $K(X \times Y)$ is a p-space. Finally, by Proposition 1.2, $\mathcal{P}_K(X,Y)$ is a p-space, since being a p-space is a G_δ-hereditary property. □

Theorem 3.5. *Let X be a dense-in-itself space where the compact subsets are finite, and Y be arbitrary. Then $(\mathcal{P}_K(X,Y), \tau_V)$ is of* 1*st category.*

Proof. By Propositions 2.6 and 1.2, $\mathcal{P}_K(X,Y) \subseteq K(X \times Y)$ is of 1st category. □

Corollary 3.6. *There is a strongly Choquet space X, so that $(\mathcal{P}_K(X,Y), \tau_V)$ is of* 1*st category for any Y.*

Proof. See Example 2.7. □

Theorem 3.7. *Let X, Y be Moore spaces, and Y a Čech-complete space. Then the following are equivalent*:

(i) $(\mathcal{P}_K(X,Y), \tau_V)$ *is hereditarily Baire*;
(ii) $(K(X), \tau_V)$ *is hereditarily Baire.*

Proof. (ii) ⇒ (i). $K(X), K(Y)$ are Moore spaces [33], and $K(Y)$ is Čech-complete by Theorem 2.1, so $K(X) \times K(Y)$ is hereditarily Baire by [7, Corollary 2.2]. This in turn implies hereditary Baireness of $K(X \times Y)$ by Theorem 2.3(2). Then $\mathcal{P}_K(X,Y)$ is hereditarily Baire by Corollary 1.3.

(i) ⇒ (ii). $K(X)$ sits in $\mathcal{P}_K(X,Y)$ as a closed subspace. □

Note that the above theorem is of a different character than the previous theorems, since hereditary Baireness of X is only necessary for hereditary Baireness of $(\mathcal{P}_K(X,Y), \tau_V)$. Indeed, if we take the hereditarily Baire (separable) metric space X of [2], then $K(X)$ is not hereditarily Baire [9, Remark 4.3]; for another example, see [34]. Our next theorem gives a sufficient condition for hereditary Baireness of $(\mathcal{P}_K(X,Y), \tau_V)$. Recall, that X is *consonant* [15,16], provided the upper Kuratowski topology and the cocompact topology coincide on the hyperspace of closed subsets of X; Čech-complete spaces are consonant [16], but there are separable metrizable hereditarily Baire non-consonant spaces [1].

Corollary 3.8. *Let X, Y be Moore spaces, the separable closed subsets of X be consonant and Y be a Čech-complete space. Then $(\mathcal{P}_K(X,Y), \tau_V)$ is hereditarily Baire.*

Proof. See [9, Corollary 4.7], and our Theorem 3.7. □

The mapping $\psi : X \to Y$ is *feebly continuous*, provided $\mathrm{int}(\psi^{-1}(U)) \neq \emptyset$ for each open $U \subseteq Y$ with $\psi^{-1}(U) \neq \emptyset$; further, ψ is δ-*open* iff $\psi(A)$ is somewhere dense for each somewhere dense $A \subseteq X$. Baire spaces are invariant of feebly continuous, δ-open maps [22, Theorem 4.7].

Theorem 3.9. *Let X be a regular, dense-in-itself space. Let Y be a quasi-regular space with a G_δ-diagonal such that $(K(Y), \tau_V)$ is weakly α-favorable. The following are equivalent*:

(i) $(\mathcal{P}_K(X,Y), \tau_V)$ *is a Baire space*;
(ii) $(K(X), \tau_V)$ *is a Baire space.*

Proof. (ii) ⇒ (i). $K(X) \times K(Y)$ is a Baire space by [22, Theorem 5.1], and so is $K(X \times Y)$ by Theorem 2.4. By Proposition 1.2, $\mathcal{P}_K(X,Y)$ is a dense G_δ-subset of $K(X \times Y)$, and hence a Baire space.

(i) ⇒ (ii). Define $\psi : \mathcal{P}_K(X,Y) \to K(X)$ via $\psi(f) = \operatorname{dom} f$. Given a $K(X)$-basic open set $\langle U_0,\dots,U_n\rangle$, we have $\langle U_0 \times Y,\dots,U_n \times Y\rangle \cap \mathcal{P}_K(X,Y) \subseteq \psi^{-1}(\langle U_0,\dots,U_n\rangle)$, so ψ is feebly continuous. Furthermore, if $\mathcal{A}$ is dense in $\mathcal{U} = \langle U_0 \times V_0,\dots,U_n \times V_n\rangle \cap \mathcal{P}_K(X,Y)$, then $\psi(\mathcal{A})$ is dense in $\langle U_0,\dots,U_n\rangle$: indeed, let $\langle W_0,\dots,W_m\rangle \subseteq \langle U_0,\dots,U_n\rangle$. Then $\bigcup_j W_j \subseteq \bigcup_i U_i$ and $\forall i\ \exists j$: $W_j \subseteq U_i$. For all $i \leqslant n$, denote $\mathcal{U}_i = \{W_j \cap U_i\colon\ W_j \cap U_i \neq \emptyset\}$, and define

$$\mathcal{U}' = \bigcap_{i\leqslant n}\bigcap_{Z\in\mathcal{U}_i}(Z \times V_i)^- \cap \left(\bigcup_{i\leqslant n}\bigcup_{Z\in\mathcal{U}_i} Z \times V_i\right)^+.$$

Then $\mathcal{U}' \subseteq \mathcal{U}$, so there is $f \in \mathcal{A} \cap \mathcal{U}'$, hence, $\psi(f) \in \langle W_0,\dots,W_m\rangle$; thus, ψ is δ-open. □

Theorem 3.10. *$(\mathcal{P}_K(X,Y),\tau_V)$ is a Baire space in each of the following cases:*

(i) *X is a dense-in-itself, Tychonoff, 1st countable, consonant space, and Y is a quasi-regular sieve complete with a G_δ-diagonal;*
(ii) *X is a normal, consonant, 1st countable space, and $Y = \mathbb{R}$;*
(iii) *X is a paracompact, consonant, 1st countable space, and Y is locally convex, completely metrizable.*

Proof. (i) By [8, Proposition 5], $K(X)$ is a Baire space, and by Theorem 2.1, $K(Y)$ is sieve complete and hence weakly α-favorable. Then $\mathcal{P}_K(X,Y)$ is a Baire space by the previous theorem.

(ii) and (iii). By [8, Proposition 5], $K(X)$ is a Baire space, and since Y is completely metrizable, $(C(X,Y),\tau_U)$ is completely metrizable; thus, $(K(X),\tau_V) \times (C(X,Y),\tau_U)$ is a Baire space [22, Theorem 5.1]. It also follows from Propositions 1.4 and 1.5, that $\eta : (K(X),\tau_V) \times (C(X,Y),\tau_U) \to (\mathcal{P}_K(X,Y),\tau_V)$ is continuous and feebly open, and hence $\mathcal{P}_K(X,Y)$ is a Baire space. □

4. Completeness properties of $C(X,Y)$

The following theorem was also proved in [24] using the so-called *generalized compact-open topology* τ_C defined on the space

$$\mathcal{P} = \mathcal{P}(X,Y) = \bigcup\bigl\{C(B,Y)\colon\ B \in CL(X)\bigr\};$$

we give an alternative proof using our partial map space $\mathcal{P}_K(X,Y)$:

Theorem 4.1. *Let X be a hemicompact k-space, and Y be Čech-complete (sieve complete, cp-space, resp.) with a G_δ-diagonal. Then $(C(X,Y),\tau_{CO})$ is Čech-complete (sieve complete, p-space, resp.).*

Proof. Let $\{K_n\colon\ n \in \omega\}$ be a cofinal family in $K(X)$, and $Z = \bigoplus_n K_n$ the topological sum of the K_n's. Observe that τ_V and τ_{CO} coincide on $C(K_n,Y)$, moreover, $(C(K_n,Y),\tau_V)$ is closed in $(\mathcal{P}_K(K_n,Y),\tau_V)$ for all n. It follows by Theorem 3.2 (resp. 3.3, 3.4) that $(C(K_n,Y),\tau_{CO})$ is Čech-complete (sieve complete, p-space, resp.) for each n; thus, $\prod_{n\in\omega}(C(K_n,Y),\tau_{CO})$ is Čech-complete (sieve complete, p-space, resp.), and so is $(C(Z,Y),\tau_{CO})$, as it is homeomorphic to $\prod_{n\in\omega}(C(K_n,Y),\tau_{CO})$ [35, Corollary 2.4.7.]. Finally, notice that Z is hemicompact, locally compact and, since X is a k-space, the natural mapping $\psi : Z \to X$ is compact-covering and quotient. Consequently, the map $\psi^* : (C(X,Y),\tau_{CO}) \to (C(Z,Y),\tau_{CO})$, defined via

$$\psi^*(f) = f \circ \psi, \quad \text{for all } f \in C(X,Y),$$

is a closed embedding [35, Corollary 2.2.8(b), and Theorem 2.2.10]. □

In the final results we will explain why it is possible to obtain the above results for the compact-open topology on $C(X,Y)$ through $(\mathcal{P}_K,\tau_V)$, and $(\mathcal{P},\tau_C)$, respectively; the reason is that they coincide if X is compact. The advantage of our approach is that it is considerably less complicated to prove the completeness properties for $(\mathcal{P}_K,\tau_V)$. Recall [23,24], that τ_C has subbase elements of the form

$$\bigl\{f \in \mathcal{P}(X,Y)\colon\ U \cap \operatorname{dom} f \neq \emptyset\bigr\}, \quad \text{and} \quad \bigl\{f \in \mathcal{P}(X,Y)\colon\ f(K \cap \operatorname{dom} f) \subseteq I\bigr\},$$

where U is open in X, $K \in K(X)$ and I is an open (possibly empty) subset of Y.

Proposition 4.2. *Let X, Y be topological spaces. Then $\tau_C \subset \tau_V$ on $\mathcal{P}_K(X,Y)$.*

Theorem 4.3. *Let X, Y be Tychonoff spaces. The following are equivalent:*

(i) *X is compact;*
(ii) *$\tau_C = \tau_V$ on $\mathcal{P}_K(X,Y)$.*

Proof. (i) $\Rightarrow$ (ii). By Proposition 1.2(1), $(\mathcal{P}_K(X,Y),\tau_V)$ is a subspace of $(K(X\times cY),\tau_V)$. It is well known that the Vietoris topology τ_V coincides with the Fell topology on $K(X\times cY)$, since $X\times cY$ is compact [5]. By [24, Proposition 2.2], the generalized-compact open topology τ_C coincides with the Fell topology induced from $(CL(X\times cY)\ (=K(X\times cY))$ to $\mathcal{P}(X,Y)$ $(=\mathcal{P}_K(X,Y))$, since X is compact.

(ii) $\Rightarrow$ (i). Suppose X is not compact. Let $\{x_\sigma\}$ be a net in X without a cluster point in X. Choose $x\in X$, $y\in Y$, for every σ put $C_\sigma=\{x,x_\sigma\}$, and define $f_\sigma : C_\sigma\to Y$ to be identically equal to y. If $f:\{x\}\to Y$ is defined as $f(x)=y$, then $\{f_\sigma\}$ τ_C-converges to f, but fails to τ_V-converge to it. □

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