Brand positioning under lexicographic choice rules

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Abstract:
This paper discusses a brand positioning model in which two brands of a product are to be positioned in a price-quality space under a new behavioral assumption. This assumption asserts that customers determine the highest-quality product within their reservation price and purchase it, provided its quality does not fall short of a minimum standard. The model also includes producers' costs that are incurred for delivering a certain quality. We first delineate reaction functions for the optimal location of one brand, give a location of its competitor. We then show that Nash equilibria do not exist as long as price and quality are both variable. Finally, we consider a two phase model: in the first phase, the duopolists sequentially choose their quality levels under the assumption that both competitors know that in the second phase, a Nash equilibrium in prices follows. Single-variable mathematical programming formulations are presented to solve the problem. A numerical example is also given to illustrate the working of the model.

Keywords: Brand positioning; Competitive location model; Nash equilibria; Stackelberg solution

Article:

1. Introduction

The objective of brand positioning models is to optimally (re-)design a product in a competitive market. Such models have been studied extensively in economics and marketing. Excellent recent surveys from an economist's point of view can be found in Ireland (1987) and Anderson et al. (1992). Comprehensive surveys of brand positioning models in the marketing literature are provided by Shocker and Srinivasan (1979) and Urban and Hauser (1980), Schmalensee and Thisse (1988) and Moorthy (1993).

It is customary to distinguish between horizontal and vertical brand positioning models, see, e.g., Gabszewicz and Thisse (1980). Models in which customers make purchasing decisions on the basis of the proximity of their (hypothetical) ideal brand to that of the existing brands, such as in the classic Hotelling model (Hotelling, 1929), belong to the class of horizontal models. On the other hand, customers in vertical models agree on the ranking of desirability of brands. They then purchase different brands depending on their ability and willingness to pay. Typically, vertical models involve only a small number of variables, which is hardly surprising as consumers are assumed to agree on their evaluations of the brands. Yet, contrary to what it may seem, horizontal and vertical brand positioning models are not diametrically opposed: Cremer, H. and Thisse (1991) have shown horizontal models to be a special case of vertical models. Research on both models is reported by Waterson (1989). The model in this study belongs to the class of vertical models.

Each brand positioning model will have to make a variety of assumptions and include a number of key elements, such as the number of brands to be located on the market, the decision makers' i.e., producers' objectives such as profit or market share maximization, the process of locating or relocating brands, and the distribution of customers. The most important element of any brand positioning is, however, the decision criterion by which customers decide which brand to purchase. This is precisely where our study differs from the existing work, as we employ a lexicographic decision rule for customer behavior.

The remainder of this paper is organized as follows. In Section 2 we outline our basic model. Section 3 delineates rules for the positioning of a brand given that another brand is already located and solves the two-brand problem with sequential positioning. Section 4 examines the existence of Nash equilibria, and in Section 5, we investigate the case in which duopolists sequentially choose a quality level first, and then realize a price equilibrium. Section 6 describes a numerical example to illustrate our model and finally, Section 7 summarizes the paper.

2. The model

In this paper we consider two brands of a product, which are sold by duopolists. Whenever no confusion can arise, we refer to the competitors as well as their brands as A and B. Each brand is characterized by exactly two attributes, viz., price and quality. Prices and qualities of the two brands will be denoted by pA, pB, qA, and qB, respectively. Given finite prices, we can assume without loss of generality that prices are normalized between 0 and 1. Assigning one dimension to each of the two attributes, each product can be represented by one (price, quality) pair. This does, however, require a number of assumptions. One assumption is that customers are aware of the correct prices of the products. This is quite realistic in case of higher priced items, such as furniture or automobiles, in which case a limited number of vendors exists and price information can easily be obtained by a few phone calls. Considering the relatively high expected value of such information, consumers are likely to engage in some market research. A similar argument applies to consumers' perception of quality. Determining the quality of a product is likely, provided the price of the product justifies it. The assumption that all customers agree on the same ranking of quality can be justified by considering the rankings and composite "quality" scores provided by publications such as "Consumer Reports", see, e.g., Anderson et al. (1992, footnote, p. 66).

One of the key features of our model is the behavioral assumption regarding the decision making behavior of consumers. There are many different approaches documented in the literature that model consumer decision making. Gabszewicz and Thisse (1979) consider a situation in which consumers have identical tastes, but different levels of income. Most brand positioning models with vertical differentiation define scalar utility functions for customers that are a function of customer's income or wealth, perceived product quality, product price and, in case of stochastic models, a random element to account for the heterogeneity in consumers' tastes. A typical example is the paper by Mussa and Rosen (1978). Assuming that customers are rational and completely informed about the parameters that enter their utility function, they will choose the product that maximizes their utility.

Our model attempts to capture consumer behavior as prevalent in case of big-ticket items such as houses, vehicles, and furniture. In these cases, a typical consumer will first attempt to obtain some information about what is available by either scanning pertinent publications, friends' advice, or relying on personal observations such as open house visits or test drives. Considering the features price and quality, we hypothesize that consumers tend to first consider their budget and establish a reservation price, i.e., an upper bound on the amount they are able or prepared to pay. This amount will include the money at hand as well as potential credit, installment plans, etc. Even though in most decision situations, a customer's reservation price will not be rigid, items that are too expensive are considered to be out of reach and are not considered further.

Once potential customers have decided on a maximum amount of money they can spend on an item, they will try to find the brand with the highest quality, provided its price does not exceed their reservation price. This is the product that the customer will ultimately purchase, provided it meets or exceeds the quality standards expected by the customer. The fact that customers may reject all brands as either too expensive or too low in quality implies that we only consider nonessential products. Finally, if both brands are acceptable as far as their prices are concerned and are of equal quality, consumers will choose the cheaper of the two brands.

Throughout the paper, we assume that the duopolists A and B do not produce absolutely identical products, i.e., locate at the same point. This assumption is common in the literature, see, e.g., Prescott and Visscher (1977). From a practical point of view, prices among competitors tend to be very similar, yet different (thus allowing claims of being less expensive than one's competitor). Furthermore, even if prices were to be identical, it is highly unlikely that the qualities of two brands are equal.

Formally, we can write the above decision making procedure as follows:

Step 1: Set a reservation price.
Step 2: Among all brands with prices not exceeding the reservation price and having an acceptable quality, find the brand(s) with the maximum quality.
Step 3: Do brands as specified in the previous step exist?
If yes: Go to step 4.
If no: Do not buy anything.
Step 4: Among the chosen brands, purchase the one with the lowest price.

Note the sequential, rather than simultaneous, incorporation of price and quality in the decision-making process of consumers. The utility function implied by this process can be written in a more compact form by a utility vector. For that purpose, denote by $p_i(q_i)$ the price (quality) of product i, and let p^* be a customer's reservation price. Furthermore, define the function LAQ(p_i) that assigns a Lowest Acceptable Quality to each brand with price p_i. All consumers agree on the LAQ function in the sense that all would demand a quality of at least LAQ(p) if they were to spend p dollars on a product. We may reasonably assume that the LAQ function is increasing in p as consumers tend to expect higher-priced goods be of higher quality. We also assume that the LAQ function is twice continuously differentiable. The utility a customer with reservation price p^* associates with product i can then be expressed as the vector

$$U_i = \begin{pmatrix} f(p_i) \\ g(q_i|p_i) \\ p^* - p_i \end{pmatrix}$$

where $f(p_i)=1$, if $p_i \leq p^*$ and $-\infty$ otherwise, and $g(q_i|p_i)=q_i$, if $q_i \geq \text{LAQ}(p_i)$ and $-\infty$ otherwise. In simple terms, the three components of the utility vector are the acceptability of prices, the acceptability of quality (as well as its actual quality index), and a tie-breaking rule.

Customers will now assign a utility vector to each brand and then make their choice by choosing the product with the lexicographically maximal utility vector. This choice is equivalent to the selection process described in the above four-step procedure. The idea of employing lexicographic or hierarchical decision rules in consumer choice is by no means new. In fact among the first studies to propose this was Hausner (1954) who hypothesized that if the set of preferences of a customer satisfies all of the axioms for von Neumann and Morgenstern utility except for the Archimedean axiom, then those preferences can be represented not by a scalar but a vector; a view that was further supported by Georgescu-Roegen (1954) and Chipman (1960). In voting theory, Taylor (1970) was among the first to use such a lexicographic utility function. In marketing, a similar hierarchical rule is used in analyzing consumer behavior; see for example, Urban and Hauser (1980), pg. 92 for the consumer decision process for purchase of a deodorant. A comprehensive survey of decision making models that use lexciographic choice rules is given in Fishburn (1974).

Assuming that the total demand is finite, let the demand be normalized to one and define $r(p)$ as a density function that specifies the number of customers with reservation price p. For simplicity, assume that $r(p) > 0$ everywhere on the domain. With the above behavioral assumption, a monopolist charging a price p' would capture all customers with reservation prices higher than p', i.e., the demand at price p' is then

$$R(p') = \int_{p}^{1} r(p)\mathrm{d}p$$

So far, we have discussed assumptions concerning consumer behavior. As far as producers are concerned, we assume that they are profit maximizers. Formally, we define π(p,q) as the profit associated with a brand of quality q that a monopolist can sell for a price p.

We assume that cost functions are identical for all producers, which may be the result of the same technologies being available to all producers. Furthermore, fixed and variable costs are both assumed to be nonnegative and increasing functions of quality. Regarding fixed costs, this can be justified by the fact that high quality machinery is considerably more expensive than equipment that manufactures lower-quality goods. As far as variable costs are concerned, higher quality requires more stringent quality control testing procedures, again resulting in higher variable costs. Let now $F(q)$ and $V(q)$ denote the fixed and the variable costs as a function of the quality q, and assume that $r(p)$, $F(q)$, and $V(q)$ are twice continuously differentiable on their respective domains. Then a producer facing some demand D will generate profit $\pi(p,q) = pD - V(q)D - F(q)$. If this producer were a monopolist, then $D = R(p)$. We can now define the *Maximum Possible Quality* function MPQ(p) which, for any given price p', is MPQ(p') = $\max\{q{:}\pi(p',q) \geq 0\}$. This function denotes the highest quality that a monopolist can produce at a given price, without sustaining a loss. Clearly, the shape of the MPQ curve depends on the demand curve $R(p')$, and the cost functions $V(q)$ and $F(q)$. While in this paper we make no assumptions whatsoever regarding $R(p)$, it appears reasonable to approximate the demand function by an appropriately scaled income curve. We also require that the best quality a producer can offer for a given price is unique and that the MPQ function has a finite number of inflection points; this assumption is made to exclude pathological cases. These (mild) assumptions are necessary in our analysis below.

Although our model deals with competitive rather than monopolistic situations, the MPQ curve is still useful in our analysis. In summary, the lowest acceptable quality function models the concerns of the consumers whereas the maximum possible quality function models the interests of the producers. Juxtaposing both interests, if we let $p_{\max}$ represent the highest price at which MPQ(p) ≥ LAQ(p), we obtain a feasible set $S=\{(p,q){:}\ 0 \leq p \leq p_{\max}$ & [LAQ(p) ≤ q ≤ MPQ(p)]$\}$. Given the above assumptions regarding LAQ(p) and MPQ(p), the set S has a unique leftmost point $(\underline{p}, \underline{q})$ defined as the lowest price at which a producer can manufacture the product with a quality that provides nonnegative profit and is acceptable to consumers. In the following we assume that S is nonsingular (i.e., it contains more than one point) and connected. A sufficient condition for nonsingularity is that there exists at least one price at which a monopolist can profitably market the product at a quality strictly better than the lowest quality acceptable to the market at that price. If S is not connected, then, by virtue of our assumptions regarding LAQ and MPQ functions, it comprises of a finite number of connected pieces, and we can consider each connected subset individually. The LAQ and MPQ functions and the set S are shown in Fig. 1.

For ease of reference, we summarize the assumptions made in our model.

The model: Two brands locate in price–quality space with coincidental location prohibited.
The customers: Customers have perfect information regarding prices and quality, and they apply a lexicographic decision rule for choosing a brand.
The demand: The total demand is normalized to one and the demand density function $r(p)$ is positive on its domain and twice continuously differentiable.
The producers: Both producers maximize their respective profit. The cost functions are identical for both producers and the variable and fixed cost functions $V(q)$ and $F(q)$ are both increasing functions of q and twice continuously differentiable.
The feasible set: The feasible set S is bounded by the LAQ (which all consumers agree upon and which is increasing in p and twice continuously differentiable) and the MPQ whose slope is less than infinity. The set S is assumed to be a nonsingular, connected set.

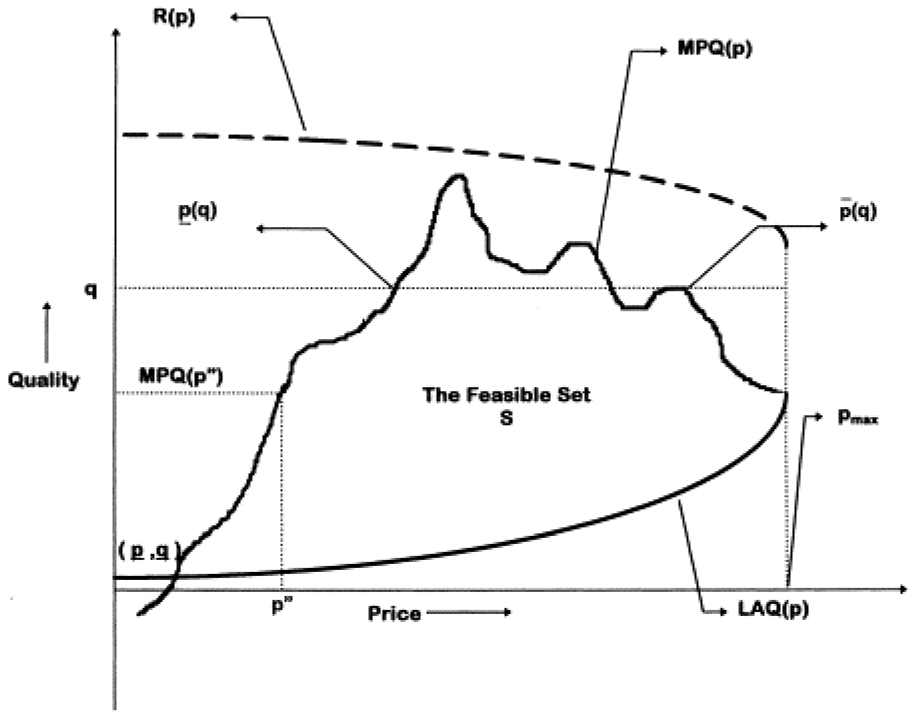

Fig. 1. Feasible set.

3. Competitive behavior

In this section, we discuss the behavior of a producer, given that his competitor has already located in (p,q) space; these results will be used in later sections. If no confusion can arise, we use the terms brand, competitor, and duopolist interchangeably. Furthermore, our discussion in this and the succeeding sections will result in a number of mathematical optimization problems that will have to be solved numerically, thus requiring fixed precision arithmetic. In particular, any numerical algorithm based on fixed precision arithmetic will recognize two numbers a and b to be different, if and only if $|a - b| \geq \varepsilon$. For example, in the implementation of ANSI C on an 8-bit microcomputer, this constant ε is called FLT_EPSILON and is equal to 10^{-5}; see, e.g., Kernighan and Ritchie (1988). This is the context in which the constant ε is used throughout this paper. Assume now that one producer has located his brand A at (p_A,q_A). Furthermore, define $\underline{p}(q_A)$ and $\overline{p}(q_A)$ as the leftmost and rightmost points of intersection of the horizontal line $q = q_A$ with the set S. The fact that this intersection may consist of multiple line segments and singular points necessitates a more involved analysis than would otherwise be required. We are now able to examine competitor B's possibilities to locate. The following discussion makes extensive use of Fig. 2.

If B locates strictly to the northeast of A, i.e., not including the axes, then brands A and B are not comparable: B has a higher price, but also a higher quality than A. The reverse is true should B locate to the southwest of A (not including the axes). Again, A and B are not comparable. In area (quadrant) II, i.e., the northwest of A including the axes, B has the same or a lower price than A but a higher or the same quality than A. This means that brand A is dominated by brand B, and consequently As sales will drop to zero and its profit is $-F(q)$. Similarly, if B locates to the southeast of A in area IV (including the axes), A dominates B and B will have a zero market share. Hence, for any given location of brand A, competitor B will locate in one of the areas I, II, or III.

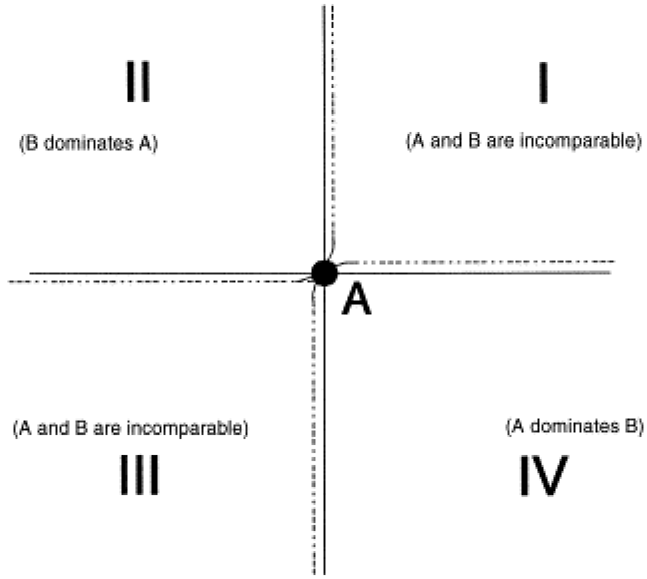

Fig. 2. Competitive behavior.

Facility B's choices are further clarified by the consequences of its decisions. First, consider market shares. If B locates in quadrant I, then B captures all demand to its right whereas A is left with the customers in the vertical strip between its own location and that of B. The customers to the left of A are not served. In case B locates in the second quadrant (including the axes), then A is cut out and B, now a monopolist, captures the entire market to the right of its own location. Customers to the left of B remain unserved. Finally, if B locates in the third quadrant, it captures all customers in the vertical strip between its own location and that of A, whereas A serves the entire upper (i.e., right) end of the market, and the market to the left of B remains unserved. Note that as long as B does not move from one quadrant to another, its market share does not change if it only increases or decreases its quality while retaining the same price. This implies that wherever B decides to locate, it can decrease its costs without changing its sales (i.e., increase its profit), by decreasing the quality as much as is possible while staying in the same quadrant.

The above remarks allow us to characterize sets that include B's profit-maximizing solution. We consider optimal solutions in each quadrant, one at a time. Any optimal solution in quadrant I will be referred to as Solution 1, Solution 2 is in quadrant II, and Solution 3 is in quadrant III. Duopolist B will then choose the best of these solutions, some of which may not exist.

Solution 1. For any given location of duopolist A at (p_A,q_A), one can show that competitor B will choose a location at some point (p_B, q_B) with $q_B = q_A + \varepsilon$ with some $\varepsilon > 0$ as discussed above. The reason is that for any price p_B, competitor B can reduce its quality and thus its cost without sacrificing revenue. Moving on the horizontal line $q_B = q_A + \varepsilon$ from any given point to the right does not change B's costs, but increases its price while its sales decrease. Then B's optimal location is at some point (p_B, q_B) with $q_B = (q_A + \varepsilon)$ and $p_B \in]p_A; \overline{p}(qA + \varepsilon)]$, or, for prices higher than $\overline{p}(qA + \varepsilon)$, on the LAQ itself. In particular, B's optimal location can be determined by solving the following two single-variable mathematical programming problems, and choosing the one with the larger profit. They are

$$\begin{aligned} P_{\mathrm{IB}}^{\alpha} \max \pi_{\mathrm{IB}}^{\alpha} &= R(p_{\mathrm{B}})[p_{\mathrm{B}} - V(q_{\mathrm{A}} + \varepsilon)] \\ &\quad - F(q_{\mathrm{A}} + \varepsilon) \\ \text{s.t.}\, p_{\mathrm{B}} &\in \{]p_{\mathrm{A}}; \overline{p}(q_{\mathrm{A}} + \varepsilon)] \cap S\} \end{aligned}$$

and

$$
\begin{aligned}
P^b_{\mathrm{IB}}\ \max \pi^b_{\mathrm{IB}} &= R(p_{\mathrm{B}})[p_{\mathrm{B}} - V(\mathrm{LAQ}(p_{\mathrm{B}}))] \\
&\quad -F(\mathrm{LAQ}(p_{\mathrm{B}}) \\
\text{s.t.}\, p_{\mathrm{B}} &\in \{]p_{\mathrm{A}}; \overline{p}(q_{\mathrm{A}} + \varepsilon)]p_{\max}\}
\end{aligned}
$$

Since all optimization problems proposed in this paper are akin to $\mathrm{P}^a_{\mathrm{IB}}$ and $\mathrm{P}^b_{\mathrm{IB}}$ we will briefly allude to the general solution procedure for solving them. Note that each of these two problems is a single-variable optimization problem where the feasible region is given by line segment(s). Therefore the solution procedure involves (i) finding the roots of the single variable equations corresponding to the first- and second-order conditions on the objective function to find the unconstrained global maxima and (ii) checking if any of them lie in the feasible region. If so, then the optimal solution is given by the unconstrained global maxima; if not, then it is given by one of the endpoints of the line segment(s) corresponding to the feasible region. Therefore the complexity of this task depends on the functions $r(p)$, $V(q)$, $F(q)$, and LAQ(p) and may require the use of numerical methods. Also note that one or both of the feasible sets in $\mathrm{P}^a_{\mathrm{IB}}$ and $\mathrm{P}^b_{\mathrm{IB}}$ may be empty. For example, the feasible set of $\mathrm{P}^a_{\mathrm{IB}}$ is empty if A is located on the LAQ, and the feasible set of $\mathrm{P}^b_{\mathrm{IB}}$ is empty, if A is located at ($p_{\max}$; MPQ($p_{\max}$). The optimal profit of B is then $\pi_{\mathrm{IB}} = \max\{\pi^a_{\mathrm{IB}}; \pi^b_{\mathrm{IB}}\}$, and its price at optimum is the corresponding value of p_{B}.

Solution 2. Anywhere in the second quadrant including the axes, B cuts out A, and there is no reason why B should not accomplish this as cheaply as possible. In other words, B will choose $q_{\mathrm{B}} = q_{\mathrm{A}}$ and select a price somewhere between p_{A} and $p(q_{\mathrm{A}})$. Note that as B cuts out A, competitor B is indeed a monopolist, and thus the MPQ curve applies. Also note that as B moves to the left on $q_{\mathrm{B}} = q_{\mathrm{A}}$, its price decreases whereas its sales increase. Duopolist B's optimal location and the resulting optimal profit are determined by solving the single-variable nonlinear programming problem

$$
\begin{aligned}
\mathrm{P}_{\mathrm{IIB}}\ \max\ \ \pi_{\mathrm{IIB}} &= R(p_{\mathrm{B}})[p_{\mathrm{B}} - V(q_{\mathrm{A}})] - F(q_{\mathrm{A}}) \\
\text{s.t.}\ \ p_{\mathrm{B}} &\in \left\{\left[\underline{p}(q_{\mathrm{A}}); p_{\mathrm{A}}\right] \cap S\right\}
\end{aligned}
$$

Similar to our previous discussion, the feasible region of $\mathrm{P}_{\mathrm{IIB}}$ is empty if $p_{\mathrm{A}} - \underline{p}(q_{\mathrm{A}})$.

Solution 3. In the third quadrant, competitor B's best choice is again the lowest cost, i.e., quality, solution. This means that B will locate somewhere on the LAQ curve. In doing so, B's sales are $R(p_{\mathrm{B}}) - R(p_{\mathrm{A}})$. Note that for any given price $\mathrm{P}_{\mathrm{IIB}} \in [\underline{p}(q_{\mathrm{A}}); p_{\mathrm{A}}]$, Solution 2 does not necessarily dominate the corresponding solution on the LAQ curve, as Solution 2 has higher sales, but also higher costs due to its higher quality. Brand B's optimal solution in quadrant III is found by solving the single-variable nonlinear programming problem

$$
\begin{aligned}
\mathrm{P}_{\mathrm{IIIB}}\ \max\ \ \pi_{\mathrm{IIIB}} &= [R(p_{\mathrm{B}}) - R(p_{\mathrm{A}})][p_{\mathrm{B}} - V(\mathrm{LAQ}(p_{\mathrm{B}}))] - F(\mathrm{LAQ}(p_{\mathrm{B}})) \\
\text{s.t.}\ \ p_{\mathrm{B}} &\in [\underline{p}; p_{\mathrm{A}}[
\end{aligned}
$$

The feasible region of $\mathrm{P}_{\mathrm{IIIB}}$ is empty, if A is located at $(\underline{p},\underline{q})$.

In summary, B's reaction function, i.e., its optimal response given a choice of price and quality by its opponent, is shown as bold lines in Fig. 3. Hence, for any given location of A, duopolist B will determine Solutions 1–3. Its optimal response to As location is then the profit-maximizing of its three solutions. Note, however, that while any of the individual problems $\mathrm{P}^a_{\mathrm{IB}}$, $\mathrm{P}^b_{\mathrm{IB}}$, $\mathrm{P}_{\mathrm{IIB}}$, and $\mathrm{P}_{\mathrm{IIIB}}$ may have empty feasible regions, at least one of these problems is guaranteed to have a nonempty feasible region. The reason is that otherwise the set S would be singular, which violates one of our assumptions. As an example, consider the location of brand A in Fig. 3. Here, all individual optimization problems have nonempty feasible regions. It is also apparent that the discontinuity of the reaction function is the major difficulty encountered in this model.

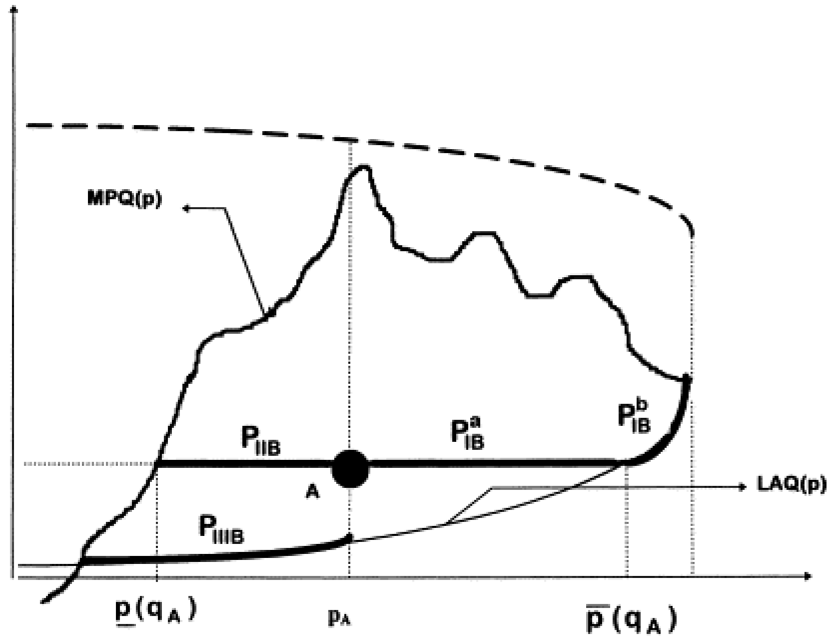

Fig. 3. Reaction function for B.

Also note that whereas in Solutions 1 and 2 brand B is guaranteed a positive profit, brand A is not. In Solution 3 the situation is reversed. The reason is that in Solutions 1 and 2 competitor B realizes a monopolistic profit for a given quality, while in Solution 3 duopolist A can act as a monopolist given q_{A}. This means that in each of these three solutions the MPQ curve applies to one of the competitors and not to the other.

4. Nash equilibria

In this section we will investigate whether or not stable locational arrangements exist, given the behavioral assumptions outlined in the previous section. For that purpose, first define a concept of stability, called a Nash equilibrium, first proposed by Nash (1950); see Tirole (1995) for other details. Define $\pi_{\mathrm{A}}(p_{\mathrm{A}},q_{\mathrm{A}};\, p_{\mathrm{B}},q_{\mathrm{B}})$ as competitor As profit, given that A locates at $(p_{\mathrm{A}},q_{\mathrm{A}})$ and B locates at $(p_{\mathrm{B}},q_{\mathrm{B}})$. Locational Nash equilibria are then defined as follows.

Definition 1. A pair of locations $(p_{\mathrm{A}}^* q_A^* p_B^* q_B^*)$ is called a locational Nash equilibrium, if

$$\begin{aligned}
\pi_{\mathrm{A}}(p_{\mathrm{A}}^*, q_{\mathrm{A}}^*; p_{\mathrm{B}}^*, q_{\mathrm{B}}^*) &\geq \pi_{\mathrm{A}}(p_{\mathrm{A}}, q_{\mathrm{A}}; p_{\mathrm{B}}^*, q_{\mathrm{B}}^*)\\
&\quad \forall (p_A, q_A) \in S,\\
p_{\mathrm{B}}(p_{\mathrm{A}}^*, q_{\mathrm{A}}^*; p_{\mathrm{B}}^*, q_{\mathrm{B}}^*) &\geq \pi_B(p_{\mathrm{A}}^*, q_{\mathrm{A}}^*; p_{\mathrm{B}}, q_{\mathrm{B}})\\
&\quad \forall (p_{\mathrm{B}}, q_{\mathrm{B}}) \in S,
\end{aligned}$$

Simply, a locational Nash equilibrium is a pair of locations where neither duopolist has an incentive to relocate given that his competitor does not move. Below, we employ the competitive behavior derived in the previous section to establish conditions for the existence of Nash equilibria.

We will now prove that if a Nash equilibrium exists, then both facilities must locate on the LAQ curve. In order to do so, we first show that at least one of the two facilities must locate on the LAQ curve. Suppose this were not true and both facilities are located in the open set S\LAQ. Without loss of generality assume that A locates to the left of B, i.e., $p_{\mathrm{A}} < p_{\mathrm{B}}$. We now distinguish between two cases.

Case 1: $q_{\mathrm{A}} \geq q_{\mathrm{B}}$. In this case, facility B is dominated by A, its sales are zero and its profit equals $\pi_{\mathrm{B}} = -F(q_{\mathrm{B}})$. Competitor B could decrease its losses by choosing a lower quality, e.g., relocate down onto the LAQ curve. Hence, if a Nash equilibrium were to exist, this case cannot apply.

Case 2: $q_A < q_B$. In this situation, competitor A captures the market between the two facilities. It can decrease its costs and thus increase its profit by relocating downwards towards the LAQ curve. Again, should a Nash equilibrium exist, this case cannot apply.

The above two cases prove that at least one facility must locate on the LAQ curve in a locational Nash equilibrium. Below, we show that not only one, but both facilities must locate on the LAQ curve. Again, we distinguish between two cases.

Case 3: Facility B is located on the LAQ curve, but A is not.

According to the competitive behavior discussed in the previous section, we can assert that facility A will locate due left to B where A chooses $q_A = q_B$, or below, where A selects $q_A < q_B$. In the former case, B is cut out and could either improve its profit by marginally increasing its quality thus capturing the entire market to its right, or sliding down the LAQ curve and decrease its cost. In the latter case, A could decrease its cost by moving downward towards the LAQ without changing its revenue. Hence, if Case 3 applies, the situation is not at equilibrium.

Case 4: Facility A is located on the LAQ curve, but B is not.

By virtue of the fact that the LAQ curve is monotonically increasing, $q_B > q_A$. Competitor B could now relocate downwards onto the LAQ curve without changing its revenue, while decreasing its cost.

The above discussion implies the following lemma.

Lemma 1. *If a Nash equilibrium exists, then both competitors must locate on the LAQ curve.*

The result of Lemma 1 allows us to prove the following proposition.

Proposition 1. *Let A and B be profit-maximizing duopolists. Then, with the assumptions outlined above, a locational Nash equilibrium does not exist.*

Proof. By virtue of Lemma 1, a locational Nash equilibrium requires that competitors A and B are located on the LAQ curve. Again, let $p_A < p_B$. It is easy to see that A and B could not be minimally differentiated at equilibrium: if they were, A, whose present market share is virtually zero, could increase its profit by either moving downwards on the LAQ curve or leapfrogging to the right beyond its competitor B. Hence, if a Nash equilibrium were to exist, A and B must be spatially separated and both have positive market shares. Now, if either brand's profit were larger than that of its competitor, its opponent could relocate strictly to the northeast or west of the competitor with the higher profit, and that way enjoy that higher profit itself. This leaves the case in which the profits of the duopolists are equal. In such a case, brand B could move left to some point on the LAQ curve to the right of A. In doing so, competitor B decreases its costs while, at the same time, increases its sales. Hence, Bs profit at its new location must be larger than before. ■

Despite the nonexistence of Nash equilibria in the general case, two special cases should be mentioned. The first special case considers a model in which prices are fixed. As the applicability of fixed price models is fairly limited, we only present the main result which asserts that a Nash equilibrium can only exist if A has a higher profit on the LAQ function than it can achieve by cutting out B, i.e., by setting $q_A=q_B$.

Proposition 2. Given fixed prices $p_A < p_B$, a locational Nash equilibrium exists if and only if

$$[R(p_A)-R(p_B)][p_A-V(\mathrm{LAQ}(p_A))] - F(\mathrm{LAQ}(p_A) \geqslant R(p_A)[p_A-V(\mathrm{LAQ}(p_B))] - F(\mathrm{LAQ}(p_B)).$$

From an economic point of view, this means that as prices are fixed, both competitors will provide only the bare minimum quality acceptable to the public, i.e., they behave in the same fashion a monopolist would. Although they still compete in quality, they have no incentive to offer more than absolutely necessary. This is not really surprising, as the lack of price competition decreases the general level of competitiveness, leading to a solution that is unfavorable to consumers. A similar observation was recently made by Zhang (1995) in the context of price matching policies which also eliminated price competition and, in the end, provided less benefits to consumers.

Consider now the case in which the two qualities q_A and q_B are fixed. This case is realistic in the short run, as technologies, once chosen by the duopolists, are costly to change and thus can be assumed to be used for a certain period of time. As technologies, by and large, determine qualities, they may be assumed to be fixed for some time. The remaining decision variables are then the prices charged by the competitors for their brands.

If the two fixed qualities are equal, then price undercutting will ensue until one facility is located on the MPQ curve and the other just to its right. In this situation, the brand on the MPQ curve captures the entire market to its right whereas its opponent gets nothing, implying that neither brand can make a positive profit in this situation.

In case of different quality levels, we can assume without loss of generality that $q_A > q_B$. Then for any $p_B \geq p_A$, competitor B is dominated. Also note that regardless of B's location, A will always capture the entire market to its right. This leads to

Lemma 2. *Given fixed qualities $q_A > q_B$, competitor A can always occupy the monopolist's position and achieve monopolistic profits.*

Thus, given $q_A > q_B$, competitor A locates at the monopolist's price location at the quality level q_A. This optimal price, denoted by $p_A^*(q_A)$, can be found by solving the following single-variable optimization problem:

$$P_1 \quad \max \quad \pi_A(p_A, q_A) = R(p_A)[p_A - V(q_A)] - F(q_A)$$
$$\text{s.t. } p_A \in \left\{\left[\underline{p}(q_A); \overline{p}(q_A)\right] \cap S\right\}.$$

Any optimal solution to P_1 is either at a boundary point of S or in its interior. In case of a boundary point, the solution is either at $\overline{p}(q_A)$ or one of the intersections of the line $q = q_A$ with the MPQ curve. If an interior point is optimal, it must satisfy $d\pi_A(p_A,q_A)/dp_A = 0$, i.e., $r(p_A)[p_A - V(q_A)] - R(p_A) = 0$. For any given quality level q_A, the optimal monopolistic profit earned by A by locating at an optimal point $(p_A^*(q_A), q_A)$ will henceforth be denoted by $\pi_A^*(q_A)$.

Given one of the above optimal strategies adopted by A, then B's optimal response will be to locate to the left of A, at a price $p_B^*(q_B|q_A)$ that is given by solving the following conditional single-variable optimization problem:

$$P_1 \quad \max \quad \pi_A(p_A, q_A) = R(p_A)[p_A - V(q_A)] - F(q_A)$$
$$\text{s.t. } p_A \in \left\{\left[\underline{p}(q_A); \overline{p}(q_A)\right] \cap S\right\}.$$

Note that B's profit is zero at both $p_B = \underline{p}(q_B)$ and $p_B = p_A^*(q_A)$. Therefore, the problem P_2 can be simplified similar to the previous case by taking derivatives, resulting in B's optimal price $p_B^*(q_B|q_A)$, such that

$$r(p_B)[p_B - V(q_B)] - R(p_B) = R(p_A^*(q_A))$$

As a result, duopolist B's optimal location is at ($p^*_{\mathrm{B}}(q_{\mathrm{B}}|q_{\mathrm{A}})$:$q_{\mathrm{B}}$) with profit $\pi^*_{\mathrm{B}}(q_{\mathrm{B}}|q_{\mathrm{A}})$. It is worth noting that in case of non-unique solutions $p^*_{\mathrm{A}}(q_{\mathrm{A}})$, competitor B's optimal strategy may also be non-unique and the players may consider collusion in order to choose one of competitor A's optimal solution that is most beneficial to duopolist B. The above discussion allows us to state

Theorem 1. *Given fixed qualities $q_{\mathrm{A}} > q_{\mathrm{B}}$, a locational Nash equilibrium exists with the higher quality brand A charging the monopolist's price of $p^*_{\mathrm{A}}(q_{\mathrm{A}})$, as defined by P_1, with a monopolistic profit of $\pi^*_{\mathrm{A}}(q_{\mathrm{A}})$. The lower quality brand B charges a price of $p^*_B(q_{\mathrm{B}}|q_{\mathrm{A}})$ which is the solution of problem P_2. Competitor B's resulting profit is then $\pi^*_{\mathrm{B}}(q_{\mathrm{B}}|q_{\mathrm{A}})$.*

Thus Theorem 1 suggests that if the two duopolists are restricted to two different quality levels, the brand with higher quality can afford to ignore the price completion and can charge the monopolist's price and enjoy monopolistic profits. The lower quality brand charges a lower price than the other brand. Thus at equilibrium, the market has two differentiated products; one that serves the higher end of the market by selling a high quality and a higher price and another that serves the lower end by producing a cheaper product with a lower quality.

5. The Stackelberg solution

In this section, we discuss a model in which a sequential game is followed by a Nash equilibrium. A two-person game in which players sequentially choose their strategies is customarily referred to as a Stackelberg game (named after the economist Stackelberg (1943)), in which the player to select and announce his strategy first is called the leader, and the player to move second is said to be in the follower position. In such a situation, the leader will choose his strategy on the basis of the follower's reaction function, while the follower decides on the basis of the leader's strategy. In the context of our brand positioning problem we observe that whereas prices can be adjusted at will virtually without cost, this is not true for quality. As pointed out in the previous section, quality is mostly determined by technology which, once chosen, is very costly to change. This describes our two-stage game: in the first stage, duopolist A, the Stackelberg leader, chooses a quality level knowing that its competitor B will choose its own quality level thereafter. With A's quality level known, competitor B will then determine his own quality. In phase two, the duopolists realize the Nash equilibrium in prices given their chosen qualities. We assume that both players know that this is the game they will play. As usual in such cases, player A uses a minimax strategy, whereas player B attempts to determine a conditional optimum. Among the first to describe such “Stackelberg-then-Nash” scenario appears to have been Lane (1980). A similar two-stage equilibrium concept is employed by Choi et al. (1990). Other related references are Shaked and Sutton (1982) and Neven (1987).

Suppose now that in the duopoly under consideration, the leader A has chosen some quality level q_{A}. The follower B now has two choices: either select a quality $q_{\mathrm{B}''} \geq q_{\mathrm{A}}$ (solution 4) or choose a quality $q_{\mathrm{B}''} < q_{\mathrm{B}}$ (solution 5). We discuss each of these solutions separately.

Solution 4. $q_{\mathrm{B}'} = q_{\mathrm{A}}$. If $q_{\mathrm{B}'} = q_{\mathrm{A}}$, then duopolist B chooses a price between $\underline{p}(q_{\mathrm{A}})$ and p_{A}. In this interval, B behaves like a monopolist. If $q_{\mathrm{B}'} > q_{\mathrm{A}}$ (recall that fixed and variable costs are assumed to increase with increasing quality), it is optimal for B to locate at a quality level $q_{\mathrm{B}'}$ that is only marginally better than q_{A}, or, for prices higher than $\overline{p}(q_{\mathrm{A}})$, on the LAQ itself. In other words, B acts like a monopolist as long as $q_{\mathrm{B}'} > q_{\mathrm{A}}$. Then, from our discussion in Section 3 concerning competitive behavior, we know that the higher-quality competitor B will charge the monopolist's price $p^*_{\mathrm{B}}(q_{\mathrm{B}'})$ (as determined by solving problems $\mathrm{P}^a_{\mathrm{IB}}$, $\mathrm{P}^b_{\mathrm{IB}}$ and $\mathrm{P}_{\mathrm{IIB}}$ and choosing any solution with the largest profit), and achieve a monopolistic profit of $\pi^*_{\mathrm{B}}(q_{\mathrm{B}'})$. Given this location by B, duopolist A will then charge a conditional price of $p^*_{\mathrm{A}}(q_{\mathrm{A}}|q_{\mathrm{B}'})$, which is obtained by solving optimization problem P_2, and achieve a profit of $\pi^*_{\mathrm{A}}(q_{\mathrm{A}}|q_{\mathrm{B}'})$.

Solution 5. $q_{\mathrm{B}''} < q_{\mathrm{A}}$. By virtue of the same argument as above, it is optimal for B in this case to locate to the left of A on the LAQ. Since A has the higher quality level in this case, it will charge a monopolist's price of $p^*_{\mathrm{A}}(q_{\mathrm{A}})$ and achieve the monopolist's profit of $\pi^*_{\mathrm{A}}(q_{\mathrm{A}})$. Given this location by A, competitor B's optimal price on the

LAQ is $p_{\mathrm{B}}^{*}(\mathrm{LAQ}|q_{\mathrm{A}})$, which is given by any optimal solution to the following optimization problem

$$
\begin{aligned}
P_3 \ \max \ & [R(p_{\mathrm{B}} - R(p_{\mathrm{A}}^{*}(q_{\mathrm{A}}))] \\
& [p_{\mathrm{B}} - V(\mathrm{LAQ}(p_{\mathrm{B}}))] - F(\mathrm{LAQ}(p_{\mathrm{B}})) \\
s.t. \ \ & p_{\mathrm{B}} \in [\underline{p}; p_{\mathrm{A}}^{*}(q_{\mathrm{A}})[\, .
\end{aligned}
$$

By locating at $(p_{\mathrm{B}}^{*}(\mathrm{LAQ}|q_{\mathrm{A}}).\ \mathrm{LAQ}\big(p_{\mathrm{B}}^{*}(\mathrm{LAQ}|q_{\mathrm{A}})\big)$, competitor B will be assumed to achieve a profit of $\pi_{\mathrm{B}}^{*}(\mathrm{LAQ}|q_{\mathrm{A}})$. These two solutions that can be realized are shown in Fig. 4(a) and (b).

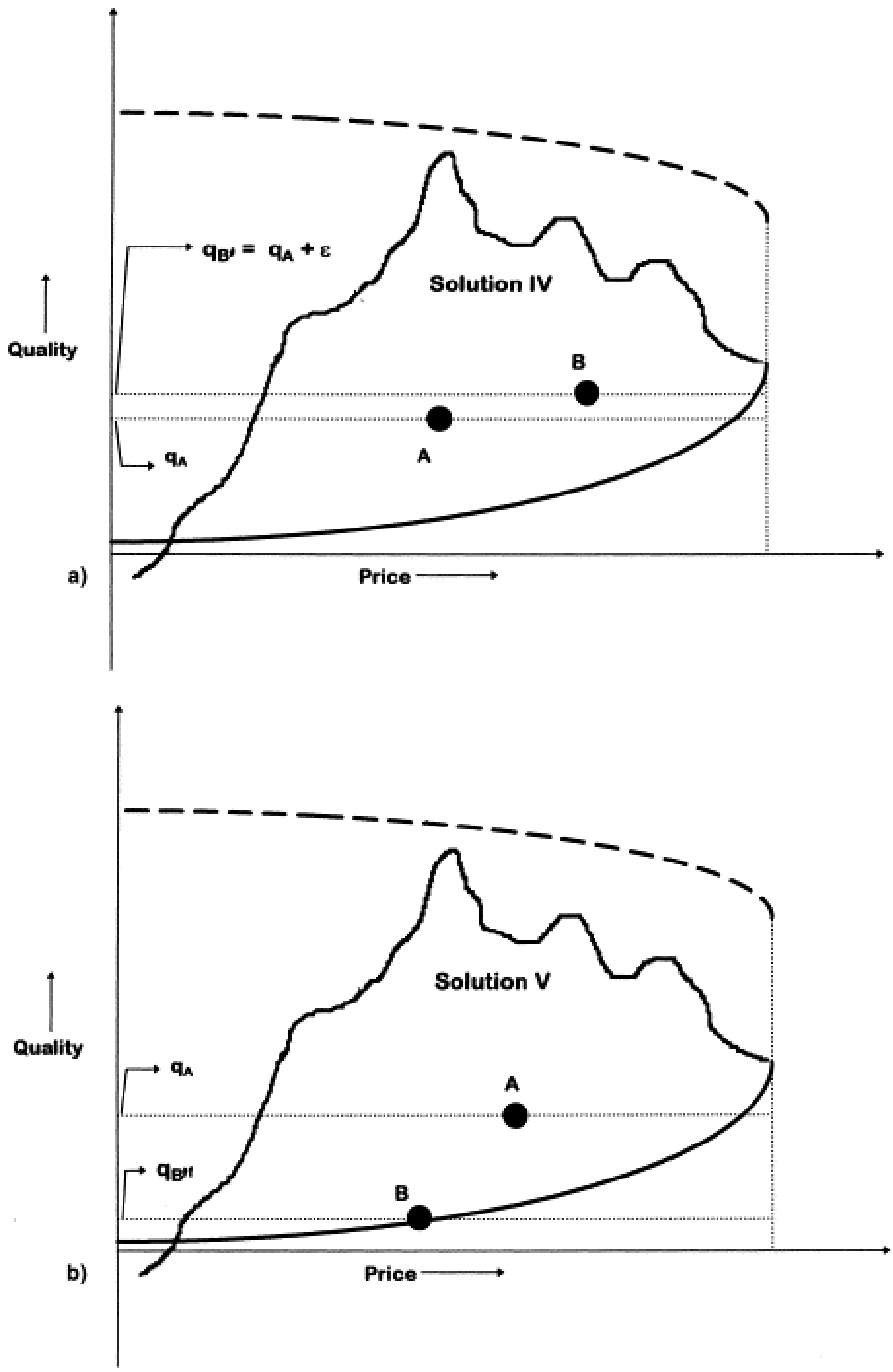

Fig. 4. Stackelberg solutions.

On the basis of the above discussion, it is clear that if the leader, A, locates at quality level $q = q_A$, and the follower, B, chooses Solution 4, then A's profit is $\pi_A^*(q_A|q_B)$. On the other hand, if B chooses Solution 5, then A is left with the monopolistic profit of $\pi_A^*(q_A)$. Of the two Solutions 4 and 5, duopolist B will choose the one with the higher profit.

In the following paragraphs, we will delineate the different quality levels at which B will choose one solution over the other. For that purpose, define Q' as the set of all quality levels q, such that if A chooses $q_A = q$ inside Q', then B prefers Solution 4 to Solution 5. In other words, $Q' = \{q | \pi_B^*(q) \geq \pi_B^*(\mathrm{LAQ}|q)\}$. If at this point A chooses a quality level $q_A \in Q'$, then it would guarantee itself a profit of $\pi^*_A(q_A|q_B)$. Thus A's optimal location in the set Q' is at a quality level $q_{A'}^*$ which is given by any optimal solution to:

$$P_4 \max \pi_A^*(q_A|q_{B'})$$
$$s.t.\ q_A \in Q'.$$

Given our assumptions about the continuity of the functions $V(q)$, $F(q)$ and $r(p)$, the set Q' and the quality level $q_{A'}^*$ can be determined in time that is polynomial in the degree of these functions. On the basis of the discussion above, it can be concluded that if A chooses to locate in the set Q', its optimal location is at $\left(p_A^*\left(q_{A'}^*|q_{B'}\right); q_{A'}^*\right)$, resulting in a guaranteed profit of $\pi_A^*(q_{A'}^*|q_{B'})$.

Define now $Q'' = \{q | \pi_B^*(\mathrm{LAQ}|q) \geq \pi_B^*(q)\}$ as the set of quality levels q in which competitor B prefers Solution 5 over Solution 4. If A locates at a quality level $q = q_A \in Q''$, then B will prefer Solution 5 on the LAQ. Thus if A locates at a quality level $q_A \in Q''$, it can guarantee itself the monopolistic profit of $\pi_A^*(q_A)$ by locating at the monopolist's location of $(p_A^*(q_A); q_A)$. We can then determine $q_{A'}^*$, A's optimal quality level in the set Q'', which is given by any optimal solution to:

$$P_5 \max \pi_A^*(q_A)$$
$$s.t.\ q_A \in Q''.$$

Employing the same argument as in the previous case, As optimal location with $q_A \in Q''$ is at $(p_A^*\left(q_{A''}^*\right); q_{A''}^*)$, which guarantees a profit of $\pi_A^*\left(q_{A''}^*\right)$. Competitor A's final step is to determine the larger of its potential profits in Q' at $(p_A^*\left(q_{A'}^*|q_{B'}\right); q_A^*)$, and in Q'' at $(p_A^*\left(q_{A''}^*\right); q_{A''}^*)$. The above process is summarized in the following procedure.

An algorithm for the Stackelberg brand positioning problem

Step 1: Determine the set Q', a corresponding optimal quality level $q_{A'}^*$ and A's profit at $(p_A^*\left(q_{A'}^*|q_{B'}\right); q_A^*)$ (Problem P_4).

Step 2: Determine the set Q'', a corresponding optimal quality level $q_{A''}^*$ and A's profit at $(p_A^*\left(q_{A''}^*\right); q_{A''}^*)$ (Problem P_5).

Step 3: Compare competitor A's profits computed in Steps 1 and 2 and select the maximum. Any corresponding quality determines A's optimal strategy.

In the analysis above, we have assumed that quality levels can be changed by fixed amounts of ε by both the duopolists. However, the quality of a product is largely dictated by the technology chosen to manufacture it. Since these technologies themselves can only be changed by discrete amounts, it seems reasonable to assume so can the quality of the products associated with them. One way to address this concern is to consider a model similar to that in Anderson et al. (1992). More specifically, it assumes that the entering brands can choose only one of K different, but fixed, quality levels, $q_1 < q_2 < \ldots < q_j \ldots < q_K$. Our analysis above can be easily extended for a similar discrete version of our model, one wherein the duopolists, A and B, have to choose from the K different, but fixed quality levels.

In particular, assume that the Stackelberg leader A chooses a quality level $q_A = q_j$. Again, if the duopolists are prohibited from choosing identical quality levels, follower B can either choose a quality level $q_{B'} < q_j$ (Solution 6) or choose a quality level $q_{B''} > q_j$ (Solution 7). A simple enumeration technique along the lines of the Stackelberg-then-Nash algorithm will then solve the problem.

6. A numerical example

In this section we will illustrate the working of our model with the aid of the following numerical example that highlights all the essential features of the model. Assume that the functions $r(p)$, LAQ(p) and the variable and fixed costs of quality are given as follows:

$$r(p) - 1 \text{ for } 0 \le p \le 1, \text{implying that}$$
$$R(p) - 1 - p \text{ for } 0 \le p \le 1,$$
$$\mathrm{LAQ}(p) = p \text{ for } 0 \le p \le 1,$$
$$F(q) = V(q) = q/100,$$
$$\varepsilon = 10^{-5}.$$

Then, by stipulating that $\pi(p,q) \ge 0$, we obtain

$$\mathrm{MPQ}(p) = [100p(1 - p)]/(2 - p).$$

By requiring that MPQ(p) ≥ LAQ(p), we obtain $p_{\max} = 98/99$ and the feasible set S is

$$S = \{0 \le p \le 98/99;\ p \le q \le [100p(1 - p)]/(2 - p)\}.$$

To illustrate the results of Section 3 on the competitive behavior of the duopolists, assume that A has located on the LAQ at p_A=q_A=0.5. It is easily verified that with S as shown above, $\underline{p}(q_A - 0.5) - 0.015$ and $\overline{p}(q_A - 0.5) =$ 0.5 in this case. Consider first solution I. As mentioned in this section, the feasible region of $\mathrm{P}_{\mathrm{IB}}^{a}$ is empty; while, $\mathrm{P}_{\mathrm{IB}}^{b}$ reduces to

$$\mathrm{P}_{\mathrm{IB}}^{b}\ \ \max\ (1 - p_B)(p_B - p_B / 100) - p_B / 100$$
$$\text{s.t.}\ 0.5 + 10^{-5} < p_B \le 98/99.$$

By checking the appropriate first and second order condition, it can be verified that the global unconstrained maximum is located at $p_B = 0.49999 < 0.5$. Hence, the optimal location for B in this solution is at $p_B = q_B = 0.5 + \varepsilon = 0.5 + 10^{-5}$; in other words, a point slightly to the northeast of A. It is readily verified that with this solution, B can cut out A completely and earn a profit of 0.2425.

Along similar lines, it is readily seen that for Solutions 2 and 3, the problems $\mathrm{P}_{\mathrm{IIB}}$ and $\mathrm{P}_{\mathrm{IIIB}}$ reduce to

$$\mathrm{P}_{\mathrm{IIB}}\ \ \max\ (1 - p_B)(p_B - 0.5/100) - 0.5/100$$
$$\text{s.t.}\ 0.015 \le p_B < 0.5,$$
$$\mathrm{P}_{\mathrm{IIIB}}\ \max(0.5 - p_B)(p_B - p_B/100) - p_B/100$$
$$\text{s.t.}\ 0 \le p_B < 0.5.$$

The optimal solution to $\mathrm{P}_{\mathrm{IIB}}$ and $\mathrm{P}_{\mathrm{IIIB}}$ is at $p_B = 0.5 - 10^{-5}$ (i.e., slightly to the left of A on the LAQ) and $p_B =$ 0.2449, respectively. The resulting optimal profit to B is 0.2425 in Solution 2 and 0.0594 in Solution 3. Taken together, this implies that the optimal strategy for B is to choose either Solution 1 or Solution 2 and be minimally differentiated from A, thereby cutting out A and earning an optimal profit of 0.2425.

To illustrate the results of Section 4 on Nash equilibrium, assume that the duopolists are restricted to be on quality levels $q_A = 0.5$ and $q_B = 0.25$. It can be verified that with the given feasible set S, $\underline{p}(q_B - 0.25) - 0.005$ and $\overline{p}(q_B - 0.25) - 0.25$. Furthermore, by virtue of Lemma 2, competitor A behaves as a monopolist. Hence, As optimal location is determined by

$$P_1 \ \max \ (1 - p_A)(p_A - 0.5/100) - 0.5/100$$
$$\text{s.t.}\ p(q_A = 0.5) = 0.015 \le p_A \le p(q_A = 0.5) = 0.5$$

Solving P_1 along the same lines as before, it can be asserted that As optimal price is $p_A^*(q_A = 0.5) = 0.5$, with a resulting profit of $\pi_A^*(q_A = 0.5) = 0.2425$. Given this result, duopolist B's optimal location is decided by the conditional optimization problem P_2, which in this case reduces to

$$P_2 \ \max \ (1 - p_B - 0.5)(p_B - 0.25/100) - 0.25/100$$
$$\text{s.t.}\ 0.005 \le p_B \le 0.5.$$

Once again, it is readily verified that the optimal price for B, denoted by

$$p_B^*(q_B = 0.25 | q_A = 0.5) = 0.25125$$

with a resulting optimal profit of

$$\pi_B^*(q_B = 0.25 | q_A = 0.5) = 0.059376.$$

Finally, to solve for the Stackelberg solution for A, we begin by illustrating the concepts of Solutions 4 and 5 used therein. Hence, assume that the leader A wishes to locate at $q_A = 0.5$. If B chooses Solution 4, then it can act like a monopolist by locating at either $q_{B'} = 0.5 + 10^{-5}$, or, for prices higher than $\overline{p}(q_A = 0.5 + 10^{-5} = 0.5 + 10^{-5}$, on the LAQ itself. Therefore B's optimal location is determined by solving P_{IB}^a, P_{IB}^b and P_{IIB} and choosing the solution that gives higher profit. Doing so results in P_B^* $(q_{B'} = 0.5 + 10^{-5}) = 0.5 + 10^{-5}$ and $\pi_{B'}^*(q_{B'} = 0.5 + 10^{-5}) = 0.2425$. Given B's behavior, duopolist A's optimal location is found by solving the optimization problem P_2, which in this example is

$$\max(0.5 - p_A)(p_A - 0.5/100) - 0.5/100$$
$$\text{s.t.}\ 0 \le p_A \le 0.5.$$

Solving this problem results in A's optimal price $p_A^*(q_A|q_{B'}) = 0.2525$ with the resulting optimal profit $\pi_A^*(q_A|q_{B'}) = 0.0562$.

On the other hand, if B chooses Solution 5, then competitor A can act like a monopolist with $q_A=0.5$, and achieve the optimal monopolistic profit $\pi^*_A(q_A=0.5)=0.2425$ by locating at $p^*_A(q_A=0.5)=0.5$. Then, as discussed above, duopolist B will locate on the LAQ, at an optimal location determined by the solution of the problem

$$P_3 \ \max \ (0.5 - p_B)(p_B - p_B/100) - p_B/100$$
$$s.t.\ 0 \le p_B < 0.5.$$

Similar to solving P_{IIB}, we can easily verify that competitor B's optimal price is obtained by solving the problem P_3, resulting in

$$p_B^*(\text{LAQ} | q_A = 0.5) = 0.2449$$

with the associated optimal profit

$$\pi_{\mathrm{B}}^{*}(\mathrm{LAQ}|q_{\mathrm{A}}=0.5)=0.0594.$$

Thus, it can be claimed that if duopolist A locates at a quality level of q_{A}=0.5, then competitor B would choose Solution 4 over Solution 5. In other words, the quality level of 0.5 belongs to set Q'. Similar reasoning reveals that in our example, duopolist B would prefer Solution 4 over Solution 5 for all quality levels in the feasible set, i.e., in our example, $Q' = S$. By solving P_4, we can then see that the optimal Stackelberg location for A is at $\left(p_{\mathrm{A}}^{*}\left(q_{\mathrm{A}'}^{*}|q_{\mathrm{B}'}\right); q_{\mathrm{A}'}^{*}\right) = (0.2525, 0.5)$, earning a profit of $\pi_{\mathrm{A}}^{*}(q_{A'}^{*}|q_{\mathrm{B}'}) = 0.0625$. Finally, in this Stackelberg solution B would locate on the LAQ at $p_{\mathrm{B}} = q_{\mathrm{B}} = 0.5 + 10^{-5}$, earning a profit of 0.2425.

7. Conclusion

In this paper we have introduced a two-brand brand positioning model with a sequential choice rule, according to which consumers lexicographically maximize a utility function that incorporates brand quality and different reservation prices. We have demonstrated that the problem does not possess a Nash equilibrium when both competitors locate their brands simultaneously. However, if the two brands are restricted to have fixed prices, then we show that this eliminates all completion among the duopolists, resulting in an equilibrium where the consumers are delivered products with the lowest possible quality. In the case of (temporarily) fixed qualities of the two brands, it was shown that the higher-quality brand “dominates” the market in the sense that it can behave as a monopolist given its quality level. We also demonstrated that in this case, the lower-quality brand chooses a price-location that is different from that of the higher-quality brand, thus resulting in product differentiation. We then investigated a related model in which one of the duopolists serves as the leader in a sequential quality selection game, then its opponent chooses its quality level, and finally a price equilibrium (which is proved to exist) is realized. Single-variable mathematical programming problems were presented to solve the problem. An extension to the discrete case of finitely many quality levels was also discussed.

There are a number of possible extensions and variations of this analysis. For instance, empirical research may determine a value of ε that measures the smallest quality difference customers are able to perceive. Such a value will differ markedly from the constant ε employed in this paper in that some of the mathematical programming problems may no longer have feasible solutions. Following a different strand, one could also attempt to incorporate incomplete information of the players or even misinformation in the process. Another possibility is to consider more than two brands. One can expect such models to be considerably more complex than the model examined in this paper, as players now have to guard against *all* successors and/or consider *all* predecessors. Along similar lines, one may also allow free entry, in which case the number of brands will be determined endogenously.

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