HUREWICZ THEOREM FOR EXTENSION DIMENSION

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Abstract. We prove a new selection theorem for multivalued mappings of C-space. Using this theorem we prove extension dimensional version of Hurewicz theorem for a closed mapping \( f : X \to Y \) of \( k \)-space \( X \) onto para-compact C-space \( Y \): if for finite CW-complex \( M \) we have \( e\text{-dim}Y \leq \lceil M \rceil \) and for every point \( y \in Y \) and every compactum \( Z \) with \( e\text{-dim}Z \leq \lceil M \rceil \) we have \( e\text{-dim}(f^{-1}(y) \times Z) \leq \lceil L \rceil \) for some CW-complex \( L \), then \( e\text{-dim}X \leq \lceil L \rceil \).

1. Introduction

The classical Hurewicz theorem states that for a mapping of finite-dimensional compacta \( f : X \to Y \) we have

\[
\text{dim}X \leq \text{dim}Y + \text{dim}f, \text{ where } \text{dim}f = \max\{\text{dim}(f^{-1}(y)) \mid y \in Y\}.
\]

There are several approaches to extension dimensional generalization of Hurewicz theorem \([3],[1],[7],[8],[9]\).

Using the idea from \([3]\) we improve Theorem 7.6 from \([1]\):

**Theorem 3.1.** Let \( f : X \to Y \) be a closed mapping of a \( k \)-space \( X \) onto para-compact C-space \( Y \). Suppose that \( e\text{-dim}Y \leq \lceil M \rceil \) for a finite CW-complex \( M \). If for every point \( y \in Y \) and for every compactum \( Z \) with \( e\text{-dim}Z \leq \lceil M \rceil \) we have \( e\text{-dim}(f^{-1}(y) \times Z) \leq \lceil L \rceil \) for some CW-complex \( L \), then \( e\text{-dim}X \leq \lceil L \rceil \).

The notion of extension dimension was introduced by Dranishnikov \([4]\): for a CW-complex \( L \) a space \( X \) is said to have extension dimension \( \leq \lceil L \rceil \) (notation: \( e\text{-dim}X \leq \lceil L \rceil \)) if any mapping of its closed subspace \( A \subset X \) into \( L \) admits an extension to the whole space \( X \).

To prove Theorem 3.1 we need an extension dimensional version of Uspenskij’s selection theorem \([11]\). In section \([3]\) we prove Theorem 2.8 on selections of multivalued mappings of C-space. Then Theorem 2.9 helps us to prove Theorem 2.9 — a needed version of Uspenskij’s theorem.

Filtrations of multivalued maps are proved to be very useful for construction of continuous selections \([10],[1]\). And we state our selection theorems in terms

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of filtrations. Note that Valov \cite{12} used filtrations to prove a selection theorem for mappings of finite C-spaces.

Let us recall some definitions and introduce our notations. A space $X$ is called a \textit{k-space} if $U \subset X$ is open in $X$ whenever $U \cap C$ is relatively open in $C$ for every compact subset $C$ of $X$. The \textit{graph} of a multivalued mapping $F: X \to Y$ is the subset $\Gamma_F = \{(x, y) \in X \times Y : y \in F(x)\}$ of the product $X \times Y$.

We denote by $\text{cov}X$ the collection of all coverings of the space $X$. For a cover $\omega$ of a space $X$ and for a subset $A \subseteq X$ let $\text{St}(A, \omega)$ denote the star of the set $A$ with respect to $\omega$. We say that a subset $A \subset X$ refines a cover $\omega \in \text{cov}X$ if $A$ is contained in some element of $\omega$. A covering $\omega' \in \text{cov}X$ strongly star refines a covering $\omega \in \text{cov}X$ if for any element $W \in \omega'$ the set $\text{St}(W, \omega')$ refines $\omega$.

\textbf{Definition 1.1.} A topological space $X$ is called \textit{C-space} if for each sequence $\{\omega_i\}_{i \geq 1}$ of open covers of $X$, there is an open cover $\Sigma$ of $X$ of the form $\bigcup_{i=1}^{\infty} \sigma_i$ such that for each $i \geq 1$, $\sigma_i$ is a pairwise disjoint collection which refines $\omega_i$.

If the space $X$ is paracompact, we can choose the cover $\Sigma$ to be locally finite and every collection $\sigma_i$ to be discrete.

\textbf{Definition 1.2.} A multivalued mapping $F: X \to Y$ is said to be \textit{strongly lower semicontinuous} (briefly, strongly l.s.c.) if for any point $x \in X$ and any compact set $K \subset F(x)$ there exists a neighborhood $V$ of $x$ such that $K \subset F(z)$ for every $z \in V$.

\textbf{Definition 1.3.} Let $L$ be a CW-complex. A pair of spaces $V \subset U$ is said to be $[L]$-\textit{connected} (resp., $[L]_c$-\textit{connected}) if for every paracompact space $X$ (resp., compact metric space $X$) of extension dimension $\text{e-dim}X \leq [L]$ and for every closed subspace $A \subset X$ any mapping of $A$ into $V$ can be extended to a mapping of $X$ into $U$.

An increasing sequence of subspaces $Z_0 \subset Z_1 \subset \cdots \subset Z$ is called a \textit{filtration} of space $Z$. A sequence of multivalued mappings $\{F_k : X \to Y\}$ is called a \textit{filtration of multivalued mapping} $F : X \to Y$ if $\{F_k(x)\}$ is a filtration of $F(x)$ for any $x \in X$.

\textbf{Definition 1.4.} A filtration of multivalued mappings $\{G_i : X \to Y\}$ is said to be \textit{fiberwise $[L]_c$-connected} if for any point $x \in X$ and any $i$ the pair $G_i(x) \subset G_{i+1}(x)$ is $[L]_c$-connected.

\section{Selection theorems}

The following notion of stably $[L]$-connected filtration of multivalued mappings provides a key property of the filtration for our construction of continuous selections.

\footnote{We consider only increasing filtrations indexed by a segment of the integral series.}
**Definition 2.1.** A pair $F \subset H$ of multivalued mappings from $X$ to $Y$ is called **stably $[L]$-connected** if every point $x \in X$ has a neighborhood $O_x$ such that the pair $F(O_x) \subset \cap_{z \in O_x} H(z)$ is $[L]$-connected.

We say that the pair $F \subset H$ is called **stably $[L]$-connected with respect to a covering $\omega \in \text{cov}X$**, if for any $W \in \omega$ the pair $F(W) \subset \cap_{x \in W} H(x)$ is $[L]$-connected.

A filtration $\{F_i\}$ of multivalued mappings is called **stably $[L]$-connected** if every pair $F_i \subset F_{i+1}$ is stably $[L]$-connected.

Clearly, any stably $[L]$-connected pair of multivalued maps of a space $X$ is stably $[L]$-connected with respect to some covering of $X$.

We denote by $Q$ the Hilbert cube. We identify a space $Y$ with the subspace $Y \times \{0\}$ of the product $Y \times Q$ and denote by $\text{pr}_Y$ the projection of $Y \times Q$ onto $Y$.

**Definition 2.2.** For a subspace $Z \subset Y \times Q$ we say that $Y$ **projectively contains** $Z$. We say that a multivalued mapping $F: X \to Y$ **projectively contains** a multivalued mapping $G: X \to Y \times Q$ if for any point $x \in X$ the set $\text{pr}_Y \circ G(x)$ is contained in $F(x)$.

**Lemma 2.3.** Let $L$ be a finite CW-complex. If a topological space $Y$ contains a compactum $K$ of extension dimension $\text{e-dim} K \leq [L]$ such that the pair $K \subset Y$ is $[L]_c$-connected, then $Y$ projectively contains a compactum $K'$ of extension dimension $\text{e-dim} K' \leq [L]$ such that $K$ lies in $K'$ and the pair $K \subset K'$ is $[L]$-connected.

**Proof.** There exists $AE([L])$-compactum $K'$ of extension dimension $\text{e-dim} K' \leq [L]$ containing the given compactum $K$ [4]. Clearly, the pair $K \subset K'$ is $[L]$-connected. Since $\text{e-dim} K' \leq [L]$, there exists a mapping $p: K' \to Y$ extending the inclusion of $K$ into $Y$.

It is easy to see that there exists a mapping $q: K' \to Q$ such that $q^{-1}(0) = K$ and $q$ is an embedding on $K' \setminus K$. Now define an embedding $j: K' \to Y \times Q$ as $j = p \times q$. Since $q^{-1}(0) = K$, the mapping $j$ coincide with $p$ on $K$ which is inclusion on $K$.

**Definition 2.4.** We say that a filtration $F_0 \subset F_1 \subset \ldots$ of multivalued mappings from $X$ to $Y$ **projectively contains** a filtration $G_0 \subset G_1 \subset \ldots$ of multivalued mappings from $X$ to $Y \times Q$ if for any point $x \in X$ and any $n$ the set $\text{pr}_Y \circ G_n(x)$ is contained in $F_n(x)$.

**Theorem 2.5.** For a finite CW-complex $L$ any fiberwise $[L]_c$-connected filtration of strongly l.s.c. multivalued mappings of paracompact space $X$ to a topological space $Y$ projectively contains stably $[L]$-connected filtration of compactvalued mappings.
Proof. For a given fiberwise \([L]_c\)-connected filtration \(F_0 \subset F_1 \subset \ldots\) of strongly l.s.c. multivalued mappings we construct stably \([L]\)-connected filtration \(G_0 \subset G_1 \subset \ldots\) of compact-valued mappings \(G_n : X \to Y \times Q^n\) as follows: successively for every \(n \geq 0\) we construct a covering \(\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n} \in \text{cov}X\) and a family of subcompacta \(\{K^n_\lambda\}_{\lambda \in \Lambda_n}\) of \(Y \times Q^n\), and define the mapping \(G_n\) by the formula

\[
G_n(x) = \bigcup \{K^n_\lambda \mid x \in W^n_\lambda\}.
\]

First, we construct \(G_0\), i.e. the covering \(\omega_0\) and the family \(\{K^0_\lambda\}_{\lambda \in \Lambda_0}\). Since \(F_0\) is strongly l.s.c., there exists a locally finite open covering \(\omega_{-1} = \{W^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}} \in \text{cov}X\) and a family \(\{M^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}}\) of points in \(Y\) such that \(W^{-1}_\lambda \times M^{-1}_\lambda \subset \Gamma_{F_0}\) for any \(\lambda \in \Lambda_{-1}\). Denote by \(H_0\) a multivalued mapping taking a point \(x \in X\) to the set \(H_0(x) = \bigcup \{M^{-1}_\lambda \mid x \in W^{-1}_\lambda\}\). Note that \(H_0(x)\) is contained in \(F_0(x)\) and consists of finitely many points. By Lemma 2.3 for any \(x \in X\) there exists a compactum \(\hat{H}_0(x) \subset F_1(x) \times Q\) of extension dimension \(\text{e-dim}\hat{H}_0(x) \leq [L]\) such that the pair \(H_0(x) \subset \hat{H}_0(x)\) is \([L]\)-connected. Since \(F_1\) is strongly l.s.c., any point \(x \in X\) has a neighborhood \(O_0(x)\) such that the product \(O_0(x) \times \hat{H}_0(x)\) is contained in \(\Gamma_{F_1} \times Q\). Since \(X\) is paracompact, we can choose neighborhoods \(O_0(x)\) in such a way that the covering \(O_0 = \{O_0(x)\}_{x \in X}\) strongly star refines \(\omega_{-1}\). Let \(\omega_0 = \{W^0_\lambda\}_{\lambda \in \Lambda_0}\) be a locally finite open cover of \(X\) refining \(O_0\). For every \(\lambda \in \Lambda_0\) we fix a point \(x_\lambda\) such that \(W^0_\lambda \subset O_0(x_\lambda)\) and put \(M^0_\lambda = H_0(x_\lambda)\). For every \(\lambda \in \Lambda_0\) we fix \(\alpha(\lambda) \in \Lambda_{-1}\) such that \(\text{St}(W^0_\lambda, O_0) \subset W^{\alpha(\lambda)}_{\lambda}\) and put \(K^0_\lambda = M^{\alpha(\lambda)}_\lambda\).

Inductive step of our construction is similar to the first step. Suppose that a covering \(\omega_{n-1} = \{W^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}} \in \text{cov}X\) and a family \(\{M^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}}\) of compacta in \(Y \times Q^{n-1}\) are already constructed such that \(\text{e-dim}M^{n-1}_\lambda \leq [L]\) and the product \(W^{n-1}_\lambda \times M^{n-1}_\lambda\) is contained in \(\Gamma_{F_{n-1}} \times Q^n\) for any \(\lambda \in \Lambda_{n-1}\). Denote by \(H_n\) a multivalued mapping taking a point \(x \in X\) to the compactum \(H_n(x) = \bigcup \{M^{n-1}_\lambda \mid x \in W^{n-1}_\lambda\}\). Note that \(H_n(x)\) is contained in \(F_n(x) \times Q^n\) and has extension dimension \(\text{e-dim}H_n(x) \leq [L]\). By Lemma 2.3 for any \(x \in X\) there exists a compactum \(\hat{H}_n(x) \subset F_{n+1}(x) \times Q^{n+1}\) of extension dimension \(\text{e-dim}\hat{H}_n(x) \leq [L]\) such that the pair \(H_n(x) \subset \hat{H}_n(x)\) is \([L]\)-connected. Since \(F_{n+1}\) is strongly l.s.c., any point \(x \in X\) has a neighborhood \(O_n(x)\) such that the product \(O_n(x) \times \hat{H}_n(x)\) is contained in \(\Gamma_{F_{n+1}} \times Q^{n+1}\). Since \(X\) is paracompact, we can choose neighborhoods \(O_n(x)\) in such a way that the covering \(O_n = \{O_n(x)\}_{x \in X}\) strongly star refines \(\omega_{n-1}\). Let \(\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n}\) be a locally finite open cover of \(X\) refining \(O_n\). For every \(\lambda \in \Lambda_n\) we fix \(x_\lambda\) such that \(W^n_\lambda \subset O_n(x_\lambda)\) and put \(M^n_\lambda = \hat{H}_n(x_\lambda)\). For every \(\lambda \in \Lambda_n\) we fix \(\alpha(\lambda) \in \Lambda_{n-1}\) such that \(\text{St}(W^n_\lambda, O_n) \subset W^{\alpha(\lambda)}_{\lambda}\) and put \(K^n_\lambda = M^{\alpha(\lambda)}_\lambda\).

To show that the pair \(G_{n+1} \subset G_n\) is stably \([L]\)-connected, we prove that the pair \(G_{n+1}(\{W^n_\lambda\} \subset \bigcap \{G_n(x) \mid x \in W^n_\lambda\}\) is \([L]\)-connected for any \(W^n_\lambda \in \omega_n\). By the construction of \(G_n\), the set \(K^n_\lambda\) is contained in \(\bigcap \{G_n(x) \mid x \in W^n_\lambda\}\). We
know that the pair \( H_{n-1}(x_{\alpha(\lambda)}) \subset \hat{H}_{n-1}(x_{\alpha(\lambda)}) = M^{n-1}_{\alpha(\lambda)} = K^n_\lambda \) is \([L]\)-connected. Therefore it is enough to show the following inclusion:

\[
G_{n-1}(W^n_{\lambda}) = \bigcup \{ K^{n-1}_\beta \mid W^n_\lambda \cap W^{\beta n-1}_\beta \neq \emptyset \} \subset \bigcup \{ M^{n-2}_\nu \mid x_{\alpha(\lambda)} \in W^{\nu n-2}_\nu \} = H_{n-1}(x_{\alpha(\lambda)})
\]

which follows from the fact that \( W^n_\lambda \cap W^{\beta n-1}_\beta \neq \emptyset \) implies \( x_{\alpha(\lambda)} \in W^{\nu n-2}_\nu \) (note that \( M^{n-2}_{\alpha(\beta)} = K^{n-1}_\lambda \)). By the choice of \( \alpha(\lambda) \) we have \( W^n_\lambda \subset O_{n-1}(x_{\alpha(\lambda)}) \). Then \( W^n_\lambda \cap W^{\beta n-1}_\beta \neq \emptyset \) implies \( O_{n-1}(x_{\alpha(\lambda)}) \cap W^{\nu n-1}_\nu \neq \emptyset \) and \( x_{\alpha(\lambda)} \in O_{n-1}(x_{\alpha(\lambda)}) \subset St(W^{\nu n-1}_\nu, O_{n-1}) \subset W^{n-2}_\alpha \).

**Definition 2.6.** For a space \( Z \) a pair of spaces \( V \subset U \) is said to be **\( Z \)-connected** if for every closed subspace \( A \subset Z \) any mapping of \( A \) into \( V \) can be extended to a mapping of \( Z \) into \( U \).

**Definition 2.7.** A pair \( F \subset H \) of multivalued mappings from \( X \) to \( Y \) is called **\( stably Z \)-connected** if every point \( x \in X \) has a neighborhood \( O_x \) such that the pair \( F(O_x) \subset \cap_{z \in O_x} H(z) \) is \( Z \)-connected.

We say that the pair \( F \subset H \) is called **\( stably Z \)-connected with respect to a covering** \( \omega \in \text{cov} X \), if for any \( W \in \omega \) the pair \( F(W) \subset \cap_{x \in W} H(x) \) is \( Z \)-connected.

An filtration \( \{ F_i \} \) of multivalued mappings is called **\( stably Z \)-connected** if every pair \( F_i \subset F_{i+1} \) is stably \( Z \)-connected.

**Theorem 2.8.** Let \( F: X \to Y \) be a multivalued mapping of paracompact \( C \)-space \( X \) to a topological space \( Y \). If \( F \) admits infinite \( stably X \)-connected filtration of multivalued mappings, then \( F \) has a singlevalued continuous selection.

_Proof_. Let \( \{ F_i \}_{i=1}^{\infty} \) be the given filtration of \( F \). Let \( \{ \omega_i \}_{i=1}^{\infty} \) be a sequence of coverings of \( X \) such that \( \omega_{i+1} \) refines \( \omega_i \) and the pair \( F_i \subset F_{i+1} \) is stably \( X \)-connected with respect to the covering \( \omega_i \). Since \( X \) is paracompact \( C \)-space, there exists a locally finite closed cover \( \Sigma \) of \( X \) of the form \( \Sigma = \cup_{i=0}^\infty \sigma_i \) such that \( \sigma_i \) is discrete collection refining \( \omega_i \). Define \( \Sigma_n = \cup_{i=0}^n \sigma_i \). We will construct a continuous selection \( f \) of \( F \) extending it successively over the sets \( \Sigma_n \).

First, we construct \( f_0: \Sigma_0 \to Y \). We define \( f_0 \) separately on every element of \( \sigma_0 \): take a point \( p \in F_{-1}(s) \) and put \( f_0(s) = p \). Since \( s \) refines \( \omega_0 \), then \( p \in F_0(x) \) for any \( x \in s \) and therefore \( f_0 \) is a selection of \( F_0|\Sigma_0 \).

Suppose that we already constructed \( f_n \) — a continuous selection of \( F_n|\Sigma_n \). Let us define \( f_{n+1} \) on arbitrary element \( Z \) of discrete collection \( \sigma_{n+1} \). Since \( \Sigma \) is locally finite, the set \( A = Z \cap \Sigma_n \) is closed in \( X \). Since \( f_n \) is a selection of \( F_n \), then \( f_n(A) \) is contained in \( F_n(Z) \). Since the pair \( F_n(Z) \subset \cap_{x \in Z} F_{n+1}(x) \) is
X-connected, we can extend $f_n|_A$ to a mapping $f'_n : Z \to \cap_{x \in Z} F_{n+1}(x)$. Clearly, $f'_n$ is a selection of $F_{n+1}|_Z$. We define $f_{n+1}$ on the set $Z$ as $f'_n$.

Finally, we define $f$ to be equal to $f_n$ on the set $\Sigma_n$. 

**Theorem 2.9.** Let $L$ be a finite CW-complex and $F : X \to Y$ be a multivalued mapping of paracompact C-space $X$ of extension dimension $e\text{-dim}X \leq [L]$ to a topological space $Y$. If $F$ admits infinite fiberwise $[L]_c$-connected filtration of strongly l.s.c. multivalued mappings, then $F$ has a singlevalued continuous selection.

**Proof.** By Theorem 2.5, the mapping $F' : X \to Y \times Q$ defined as $F'(x) = F(x) \times Q$ contains a stably $[L]$-connected filtration of multivalued mappings. By Theorem 2.8, $F'$ has a singlevalued continuous selection $f'$. Then the mapping $f = \text{pr}_Y \circ f'$ is a singlevalued continuous selection of $F$. 

3. **Hurewicz theorem**

The proof of the following theorem is similar to the proof of Theorem 2.4 from [3].

**Theorem 3.1.** Let $f : X \to Y$ be a closed mapping of $k$-space $X$ onto paracompact C-space $Y$. Suppose that $e\text{-dim}Y \leq [M]$ for a finite CW-complex $M$. If for every point $y \in Y$ and for every compactum $Z$ with $e\text{-dim}Z \leq [M]$ we have $e\text{-dim}(f^{-1}(y) \times Z) \leq [L]$ for some CW-complex $L$, then $e\text{-dim}X \leq [L]$.

**Proof.** Suppose $A \subset X$ is closed and $g : A \to L$ is a map. We are going to find a continuous extension $\overline{g} : X \to L$ of $g$. Let $K$ be the cone over $L$ with a vertex $v$. We denote by $C(X, K)$ the space of all continuous maps from $X$ to $K$ equipped with the compact-open topology. We define a multivalued map $F : Y \to C(X, K)$ as follows:

$$F(y) = \{h \in C(X, K) \mid h(f^{-1}(y)) \subset K \setminus \{v\} \text{ and } h|_A = g\}.$$ 

Claim. $F$ admits continuous singlevalued selection.

If $\varphi : Y \to C(X, K)$ is a continuous selection for $F$, then the mapping $h : X \to K$ defined by $h(x) = \varphi(f(x))(x)$ is continuous on every compact subset of $X$ and because $X$ is a $k$-space, $h$ is continuous. Since $\varphi(f(x)) \in F(f(x))$ for every $x \in X$, we have $h(X) \subset K \setminus \{v\}$. Now if $\pi : K \setminus \{v\} \to L$ denotes the natural retraction, then $\overline{g} = \pi \circ h : X \to L$ is the desired continuous extension of $h$.

**Proof of the claim.** We are going to apply Theorem 2.9 to infinite filtration $F \subset F \subset F \subset \ldots$. To do this, we have to show that $F$ is strongly l.s.c. and that the pair $F(y) \subset F(y)$ is $[M]_c$-connected for every point $y \in Y$.

First, we show that $F$ is strongly l.s.c. Let $y_0 \in Y$ and $P \subset F(y_0)$ be compact. We have to find a neighborhood $V$ of $y_0$ in $Y$ such that $P \subset F(y)$ for every $y \in V$. For every $x \in X$ define a subset $P(x) = \{h(x) \mid h \in P\}$ of $K$. Since $P \subset C(X, K)$ is compact and $X$ is a $k$-space, by the Ascoli theorem, each
$P(x)$ is compact and $P$ is evenly continuous. This easily implies that the set $W = \{ x \in X \mid P(x) \subset K \setminus \{v\} \}$ is open in $X$ and, obviously, $f^{-1}(y_0) \subset W$. Since $f$ is closed, there exists a neighborhood $V$ of $y_0$ in $Y$ with $f^{-1}(V) \subset W$. Then, according to the choice of $W$ and the definition of $F$, we have $P \subset F(y)$ for every $y \in V$.

Fix an arbitrary point $y \in Y$. Let us prove that the pair $F(y) \subset F(y)$ is $[M]_c$-connected. Consider a pair of compacta $B \subset Z$ where $e\dim Z \leq [M]$ and a mapping $\varphi: B \to F(y)$. Since $B \times X$ is a $k$-space (as a product of a compact space and a $k$-space), the map $\psi: B \times X \to K$ defined as $\psi(b, x) = \varphi(b)(x)$ is continuous. Extend $\psi$ to a set $Z \times A$ letting $\psi(z, a) = g(a)$. Clearly, $\psi$ takes the set $Z \times f^{-1}(y) \cap (Z \times A \cup B \times X)$ into $K \setminus \{v\} \cong L \times [0, 1)$. Since $e\dim(Z \times f^{-1}(y)) \leq [L]$, we can extend $\psi$ over the set $Z \times f^{-1}(y)$ to take it into $K \setminus \{v\}$. Finally extend $\psi$ over $Z \times X$ as a mapping into $AE$-space $K$. Now define an extension $\tilde{\varphi}: Z \to F(y)$ of the mapping $\varphi$ by the formula $\tilde{\varphi}(z)(x) = \psi(z, x)$.

**Corollary 3.2** (cf. Theorem 2.25 from [4]). Let $f: X \to Y$ be a mapping of finite-dimensional compacta where $e\dim Y = [M]$ for finite CW-complex $M$. If for some CW-complex $L$ we have $e\dim(f^{-1}(y) \times X) \leq [L]$ for every point $y \in Y$, then $e\dim X \leq [L]$.

**Proof.** By Theorem 6.3 from [4] for any compactum $Z$ with $e\dim Z \leq e\dim Y$ we have $e\dim(f^{-1}(y) \times Z) \leq [L]$. Thus, we can apply Theorem 3.1.

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