STATISTICAL QUALITY CHARACTERIZATION OF SELECTIVE LASER MELTING (SLM) FOR ADDITIVE MANUFACTURING

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Western Carolina University in partial fulfillment of the
requirements for the degree of Master of Science in Technology

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DEDICATION

To Mom and Dad:

Thank you- those are the simplest words I have that express how grateful I am for the advice and direction you gave me that led to this thesis. Without your influence it wouldn't have been possible, and even in your passing you teach me. Thank you.

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ABSTRACT

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Selective Laser Melting (SLM) is a relatively new technology to the rapid prototyping field. The process is unique in that it can produce rapid prototyped metal parts vs. conventional polymer three dimensional printing. The benefit of Selective Laser Melting is that metal parts can be made that would not be capable of being created through traditional machining techniques. Selective Laser Melting is also advantageous because the production of similar parts compared to traditional machining requires less skill and training, which equates to cost reduction.

The College of Engineering and Technology has acquired an EOS M290 Selective Laser Melting machine that has not yet been validated. Validating the SLM machine will benefit future research by establishing the machine capability. Future research will depend on this information as a basis for further development of rapid prototyping techniques. This will also support the Center for Rapid Product Realization for clients by increasing the quality of parts produced and increasing the opportunities available by broadening the scope of their current capabilities. Both of these will promote and support further economic development for the region as well as the university.

The purpose of the study will be to use statistical quality methodology to characterize SLM printing with regards to machine capability. This study will use Analysis of Variance (ANOVA) to determine how part placement on the build plate affects machine capability. Ultimately, the results of the statistical analysis may provide EOS M290 users a better understanding of how build plate location affects part geometry capability related to height and diameter.

CHAPTER 1: STATEMENT OF PROBLEM

1.1 Background and Need for the Study

Selective Laser Melting (SLM) is a relatively new technology to the rapid prototyping field. The process is unique in that it can produce rapid prototyped metal parts vs. conventional polymer three dimensional printed parts. The benefit of Selective Laser Melting is that metal parts can be made that would not be capable of being created through traditional machining techniques. Selective Laser Melting is also advantageous because the production of similar parts compared to traditional machining requires less skill and training regarding machine operation, which equates to cost reduction.

The College of Engineering and Technology has acquired an EOS M290 Selective Laser Melting machine that has not yet been validated. Validation of the SLM machine will benefit future research by establishing the machine capability. Future research will depend on this information as a basis for further development of rapid prototyping techniques. This will also support the Center for Rapid Product Realization for clients by increasing the quality of parts produced and increasing the opportunities available by broadening the scope of their current capabilities. Both of these will promote and support further economic development for the region as well as the university.

1.2 Goals of the Study

The overall goal of the study is to determine how plate placement affects machine capability regarding height and diameter measurements of the EOS M290 SLM machine.

- Is there a difference in height and diameter measurements as part location on the build plate moves farther from center?

- Is there a difference in height and diameter measurements as part location on the build plate moves around the center?
- Is there a difference in variation between part diameters and part heights?

1.3 Objectives of the Study

The goals of this study were met by comparing part geometry from different locations on the build plate. The objectives were as follows:

- Determine part geometry with measurement features that are created normal to the Z axis, and normal to the X-Y axes.
- Determine machine parameter settings to be used for the build.
- Develop a measurement strategy for the coordinate measurement machine that accurately reflects the part measurement surfaces as opposed to individual points on the surfaces.
- Develop a plan for statistical analysis that encompasses data distribution, variance, and analysis of variance.

1.4 Significance of the Study

The study helped to characterize the process capability of an EOS M290 SLM machine with respect to build plate location for 316L stainless steel. Distribution and variance testing were performed, as well as running ANOVA statistical General Linear Models for two-way main effects and interaction of plate bands and sectors for three different build plates. The purpose of this testing was to give EOS M290 users a better understanding of how build plate location affects part geometry capability related to height and diameter.

1.5 Definitions and Key Terms

Additive Manufacturing (AM) - a process of joining materials to make objects from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies.

Synonyms: additive fabrication, additive processes, additive techniques, additive layer manufacturing, layer manufacturing, and freeform fabrication. (ASTM International)

Alpha Value- The significance level used for making the determination to accept or reject the null hypothesis.

Analysis of Variance (ANOVA)- A statistical analysis tool used to make inferences about populations means associated with various treatments. (Mendenhall & Sincich, Statistics for Engineering and the Sciences, 1992)

Anderson-Darling Test- A statistical test used to test if a sample of data came from a population with a specific distribution. (NIST/Sematech, 2012)

Bartlett's Test- A statistical test used to test if *k* samples have equal variances. (NIST/Sematech, 2012)

Design of Experiments (DOE) - statistically based techniques to organize experimentation to obtain the maximum amount of information at the minimum cost and time expenditure. (Beauregard, Mikulak, & Olson, 1992)

EOS GmbH™- Electro Optical Systems is the manufacturer of the M290 Selective Laser Melting machine. Referred to throughout this study as EOS.

General Linear Model (GLM)- A regression analysis that has the ability to accommodate distinctions on quantitative variables representing continuous measures and categorical distinctions representing groups or experimental conditions. (Rutherford, 2011)

Laser Sintering (LS)- a powder bed fusion process used to produce objects from powdered materials using one or more lasers to selectively fuse or melt the particles at the surface, layer by layer, in an enclosed chamber. (ASTM International)

Minitab™- Statistical testing software. Referred to throughout this study as Minitab, and Minitab 17.

P Value- The probability (assuming the null hypothesis is true) of observing a value of the test statistic that is at least as contradictory to the null hypothesis, and supportive of the alternative hypothesis. (Mendenhall & Sincich, Statistics for Engineering and the Sciences, 1992)

PTC Creo™- A solid three dimensional modeling software package. Referred to throughout this study as PTC Creo, Creo, and Creo 2.0.

Selective Laser Sintering (SLS) - denotes the LS process and machines from 3D Systems Corporation. (ASTM International)

Tukey Test- A statistical test that considers all possible pairwise differences of means at the same time. (NIST/Sematech, 2012)

Variance- Differences between parts of a production process. The values associated with variation are symmetrically distributed around a central value, and the probability associated with occurrence decreases as the value moves away from the mean. (Barone & Franco, 2012)

Zeiss Calypso™- Software package for use with the Zeiss Contura HTG coordinate measurement machine. Referred to throughout this study as Calypso.

1.6 Delimitations of the Study

The purpose of the study is to present a statistical analysis of variance for parts build in the EOS M290 at different locations on the build plate, regarding height and diameter. The constraints and restrictions of this study are as follows:

- Machine Settings- Each build was completed in an EOS M290 selective laser melting machine. The machine settings used for the study were the default parameter set from EOS for 316L stainless steel, printed in 20 µm layers.

- Material Type- All test samples were printed using 316L stainless steel powder.
- Replicates- Three plates containing 160 parts each were built for this study. The build plates were not considered replicates due to variation introduced in the machine setup. This variation could not be controlled or quantified so the three builds were used to analyze trends.
- CMM- A Zeiss Contura HTG coordinate measuring machine was used to collect all height and diameter data. The same measurement strategy and probes were used on each build plate.

CHAPTER 2: LITERATURE REVIEW

The purpose of this study was to characterize the process capability of an EOS M290 SLM machine with respect to build plate location for 316L stainless steel, regarding measurements for part height and part diameter. Chapter II presents the history and development of rapid prototyping along with information specific to the SLM process, an explanation of measurement and metrology, and a description of the statistical methods for data analysis.

2.1 History and Development of Rapid Prototyping

Rapid prototyping and manufacturing have historically fallen into one of two categories; subtractive manufacturing and additive manufacturing. Subtractive manufacturing techniques include process that remove material from a stock workpiece to create the user's desired geometry, while additive manufacturing techniques consist of methods where parts are created through the addition of material. Subtractive manufacturing is a more traditional approach to part production and is accomplished through the use of cutting tools in machines such as mills and lathes. Additive manufacturing is accomplished through methods such as fused deposition modeling (FDM), ultra-violet polyjet (UV polyjet), and selective laser melting (SLM) (ASTM International, 2016). This study focuses on the SLM process as it relates to metal. Traditionally additive manufacturing has been used for rapid prototyping, idea development, and test fit of parts. The industry is shifting from the ideology of rapid prototyping to rapid production, and the metal SLM process is a relatively new technology that will help accommodate that shift.

The field of additive manufacturing is currently experiencing a trend of expansion. The cost of the equipment is going down and becoming simpler to use which is causing an increase in its use in both commercial and consumer markets. This also has expanded the scope for the application of additive manufacturing technology. In 2014 a survey was conducted by Wohlers Associates INC. that includes 111 companies that produce and sell industrial grade additive manufacturing systems to indicate the area of the industry that each company services. The results of this survey are shown in figure 2.1. The "Other" category consists of industries such as oil and gas, non-consumer sporting goods, commercial marine products and others that do not fit into the named categories (Wohlers Associates, INC., 2014).

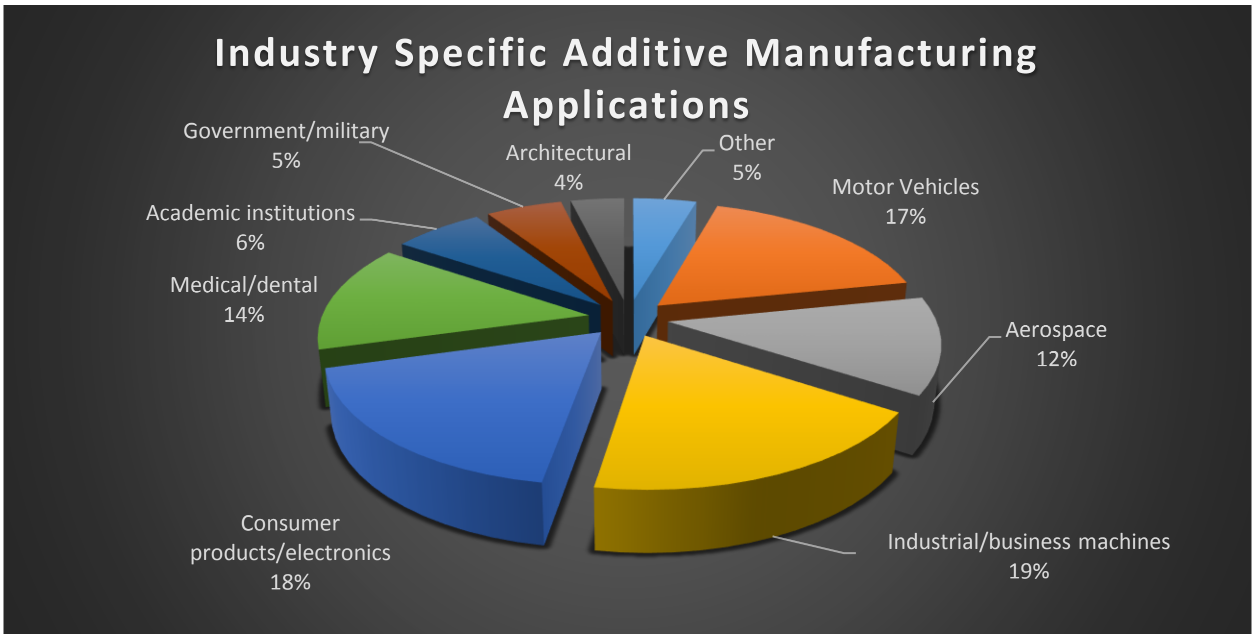

Figure 2.1: Industry Specific Additive Manufacturing Application

Industrial and business machines, consumer products and electronics, and motor vehicles represent around fifty percent of the current state of the industry for additive manufacturing applications. These include both products produced through additive manufacturing and systems produced for additive manufacturing.

There is also a wide range for the use of additive manufacturing within the specific industrial areas. The most popular of which is functional parts, which is shown in figure 2.2 (Wohlers Associates, INC., 2014). These are parts intended for use in the final stage of product development and is where the industry has recently expanded the most. Historically the primary use of additive manufacturing has been primarily for rapid prototyping and test parts. One of the most rapidly expanding areas for the application of SLM is moving from the historical idea of rapid prototyping to the idea of rapid production (Bak, 2003).

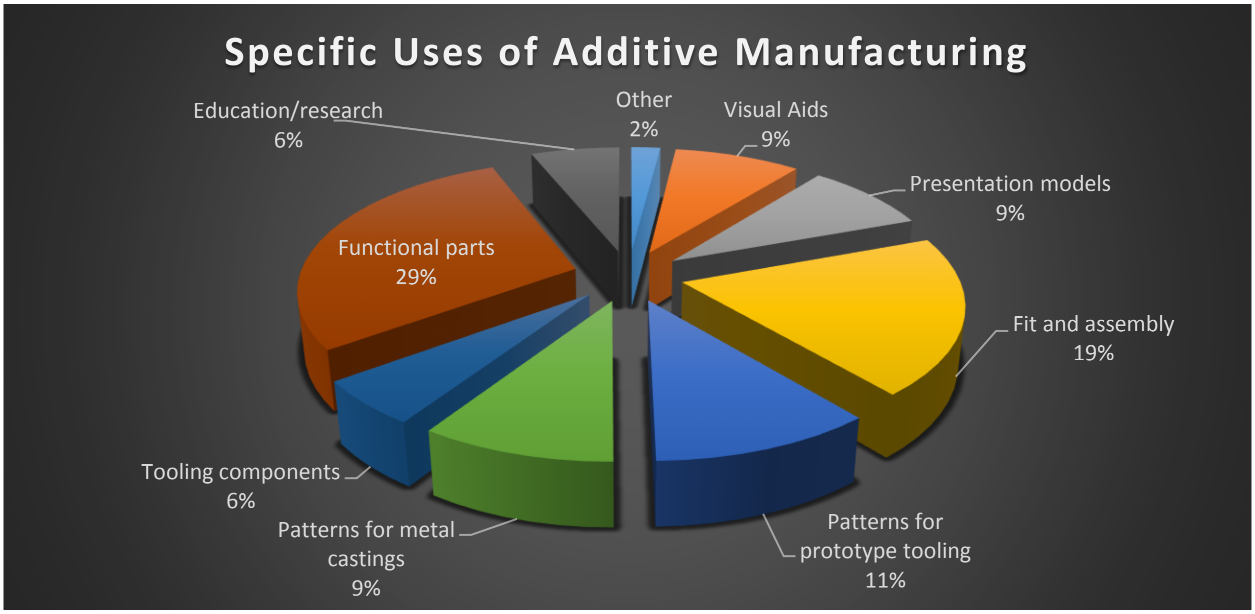

Figure 2.2: Specific Uses of Additive Manufacturing

2.1.1 History of Selective Laser Melting (SLM)

Selective Laser Melting (SLM) is a relatively new technology to the rapid prototyping field. The process is unique in that it can produce metal parts vs. conventional polymer additive manufacturing. Current SLM technology is based on Selective Laser Sintering (SLS) research beginning in the 1980's. The terms SLS and SLM are used interchangeably in industry, however the difference should be clarified. Sintering refers to a powder bed process in which the build material is heated to a point where atomic diffusion occurs, but the material does not reach a liquid state. Early sintering processes used materials such as acrylonitrile butadiene styrene (ABS), which is an amorphous solid (Lou & Grosvenor, 2012). This characterization means that the material is a supercooled liquid and cannot be melted. The sintering process heats these materials to the glass transition temperature, but are not melted because a phase change does not occur. As the SLM process was developed, crystalline materials were used which do undergo a phase change during the melting process (Lou & Grosvenor, 2012).

This study will be conducted using an EOS M290 Selective Laser Melting machine. EOS GmbH Electro Optical Systems was founded in 1989 by Dr. Hans J. Langer and Dr. Hans Steinbichler. The foundation of EOS is also in polymer machines, however metal sintering began to be developed in the early 1990's. In 1994 EOS presented the EOSINT M160, which was their first prototype of a commercial metal SLM machine. Development within the company throughout the next fifteen years, concentrating on process control and material research led to the introduction of the EOS M280 in 2010. The M290 was an improvement that was based on the M280. (EOS, n.d.)

2.2 Selective Laser Melting (SLM) Overview

The SLM process is an additive manufacturing powder bed fusion production process in which parts are built in layers. Part geometry is predetermined and imported into pre-processing software as a three dimensional solid model. The purpose of pre-processing the files is to determine the build plate placement, machine scan settings, and the geometry and scan paths for each individual build layer. The thickness of each layer is dependent on the build material. The EOS M290 operates in a four step process that repeats with each scan cycle. The machine deposits a layer of metal powder onto the build plate. For this study 316L stainless steel will be used, which will result in a 20 μm (.02 mm) layer thickness. This is the suggested layer thickness from EOS for 316L stainless steel. After depositing the layer of powder, a 400 watt fiber laser scans the cross section of each part, melting the topmost layer and fusing it to the layer below. When the scanning of each part is complete, the build plate drops by a single layer.

The machine then deposits a new layer of metal powder and the process repeats until each part on the plate has been completed. Some of the factors that are inherent to the part processing and operation of the machine may impact part geometry.

Research has shown that parameters of similar SLM processes do impact the quality of the parts produced. A 2007 study from DIPI Laboratory in France considered material powder, laser specifications, and the laser scan strategy to be important factors as variables that affect part geometry. This goal of this study was to determine the optimal settings for each of the factors that results in the highest accuracy with respect to nominal part dimensions of small parts (Yadroitsev, Bertrand, & Smurov, 2007). The details of these factors that were analyzed are as follows-

- *Powder: composition, size distribution, shape, optical and heat transfer properties, thickness of deposited layer for each cycle of fabrication.*
- *Laser: power, spot size, beam spatial distribution, scanning velocity and application of protective gas atmosphere.*
- *Strategy of manufacturing: decomposition of each plane to be sintered on a number of elementary elements (vectors), definition of orientation and distance between them, definition of relative positions of elementary elements in two consecutive planes.*

  (Yadroitsev, Bertrand, & Smurov, 2007)

This study concluded that each of the factors are important regarding machine repeatability and part variation, and determined optimal settings for each that contribute towards the highest quality part creation. One factor that was not analyzed in this research was how position on the build plate impacts part geometry.

Build plate location may have an impact on part geometry. This is important to characterize because from a rapid production viewpoint, the distribution and variation of dimensions from parts created using this process impact the subsequent manufacturing processes and machine capability. For high volume production this will equate to cost savings and reduction of scrap and rework parts. This thesis will analyze the impact build plate location has on part geometry, while keeping the machine process parameters constant. Variation due to process parameters is important to minimize to understand how build plate location affects part geometry, and the factors that could introduce measurement variation regarding strategies and methods are also important to identify.

2.3 Measurement

Measurement strategy is important because the resolution and variation of the measurement device can contribute to false results regarding statistical analysis. Two types of methods are available- contact measurement and non-contact measurement. Contact measurement requires the use of a machine with a probe that makes contact with the part geometry to determine measurement values. Other methods of contact measurement include manual based measurement methods such as calipers and micrometers. These are typically less accurate than machine based methods because human error is introduced as variation. Non-contact measurement is an optical based method where an image is processed and pixel counts are used to determine measurement values. Typically non-contact measurement has a higher level of precision, but is limited. Non-contact measurement cannot record values in the Z direction, only in X and Y. Parts build for this study will be left on the build plate, which eliminate the ability to use backlighting for image capture. Backlighting parts on the build plate is not possible and height data will need to be collected. Due to these limitations contact measurement methods will be used.

A Zeiss Contura HTG coordinate measuring machine (CMM) is available for data collection, which is shown in figure 2.3.

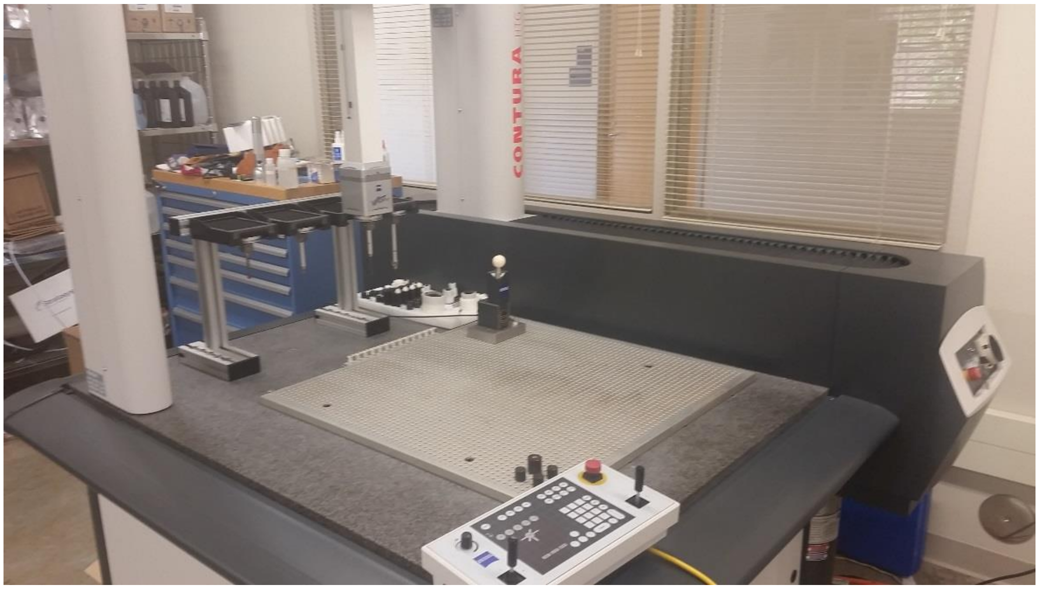

Figure 2.3: Zeiss Contura HTG CMM

The CMM uses a probe to collect data that is used as measurements for the surface geometry of a part. The probe carriage moves in the X, Y, and Z axes and is capable of recording values in those directions. When the probe comes into contact with the part with a predetermined amount of force, a data point is collected. This data can be single point measurements or scans along the surface. Single point measurements are accurate, however surface scans are preferred because it is a more accurate representation of the geometry. A three dimensional model is imported into the software used to run the CMM, and the machine collects a predetermined number of data points along the surface of the part being measured. The software then filters and averages these points and the resulting output is a single value that represents the measured feature to a higher degree of resolution than a single point value. Variation in measurements due to the machine can also be introduced by changing the location of the part within the measurement volume of the machine. One way to reduce this variation for parts that are being measured repeatedly is to place the part in the same place within the machine measurement volume. The CMM software is preloaded with different types of features to build a measurement strategy. These features include two dimensional objects such as planes, distance between points, circles, and lines; and also include three dimensional objects such as cones, cylinders, and cubes. Defining these features in the software to represent the geometry being measured is the preferred way of operating the CMM. Once data is collected a statistical analysis can be performed to understand how characterize the data. This study will use statistical analysis to perform hypothesis testing regarding distribution, variance, and analysis of variance (ANOVA) to obtain a better understanding of how build plate placement affects part geometry, regarding heights and diameters.

2.4 Statistical Analysis

Variation between parts is inherent to any production technique, regarding both additive and subtractive manufacturing methods. This study will determine if build plate location in the EOS M290 has an impact on that variation through statistical analysis. The statistical analysis for this study will be performed to determine if significant differences in part diameter and part height were present with respect to bands, sectors, and the interaction of bands by sector, which will ultimately be done by using a two way analysis of variance (ANOVA). Build plate layout regarding bands and sectors is discussed further in Chapter III.

The validity of results from a two way ANOVA requires assumptions to be made concerning the data that is being analyzed. These assumptions include the data is normally distributed, the samples are independent, the variance between samples must be equal, and the grouping of data being analyzed have an equal sample size (Jones, n.d.). Statistical testing will be done to show that these assumptions are true regarding height and diameter data collected from three build plates. Distribution of data is tested through normal probability plots and histograms using the Anderson-Darling test statistic, and variance is tested through a Bartlett's test.

To determine if the data collected fits a normal distribution histograms and normal probability plots will be used, using the Anderson-Darling test statistic to make a decision. The Anderson Darling test applies more weight to the tails of the distribution which will be important for this study. Variation introduced in the machine setup process could contribute to variation in parts built. The variation in machine setup cannot be controlled, resulting in an inability to identify assignable causes for outlier elimination. For the purposes of this study, outliers will not be eliminated. The equation for the Anderson-Darling test statistic is as follows, where A^2 is the test statistic, N is the number of samples, F is the cumulative distribution function, and Y_i is the data point (NIST/Sematech, 2012).

$$A^2 = -N - S \qquad (2.1)$$

where

$$S = \sum_{i=1}^{N} \frac{2i-1}{N} \left[\ln F(Y_i) + \ln\left(1 - F(Y_{N+1-i})\right)\right] \qquad (2.2)$$

Variance testing will be performed using the Bartlett's test statistic. Build plates for this study will be divided into bands and sectors and this test will show if the variance between each is equal or unequal. Variance will be compared across sectors and bands separately, with no comparison between them. Build plate layout regarding bands and sectors is discussed further in Chapter III. The equations for the Bartlett test statistic are as follows, where s_i^2 is the variance of the ith group, N is the total sample size, N_i is the sample size of the ith group, k is the number of groups, and s_p^2 is the pooled variance (NIST/Sematech, 2012).

$$T = \frac{(N-k)\ln s_p^2 - \sum_{i=1}^{k}(N_i - 1)\ln s_i^2}{1 + \left(1/\left(3(k-1)\right)\right)\left(\left(\sum_{i=1}^{k} 1/(N_i - 1)\right) - 1/(N-k)\right)} \qquad (2.3)$$

$$s_p^2 = \sum_{i=1}^{k} (N_i - 1) s_i^2 / (N-k) \qquad (2.4)$$

After the statistical testing to analyze the assumptions required for a two way ANOVA is performed, the two way ANOVA can be run on the data. A two way ANOVA is a statistical tool that allows the user to analyze the data in such a way that conclusions can be drawn regarding population means (Mendenhall & Sincich, Statistics for Engineering and the Sciences, 1992). This analysis is performed to associate the data with the treatments related to the study. With respect to this study, the analysis will be performed on height and diameter data, with treatments being bands and sectors on the build plate. The formulae for performing a two factor ANOVA as well as the structure and calculations for a summary ANOVA table are shown below, where SSA is the sum of squares for factor A, SSB is the sum of squares for factor B, SSAB is the sum of squares for the interaction of factors A and B, SSE is the sum of squares for the error, and SST is the total sum of squares. The variables *a* and *b* represent the total number of levels for factors A and B, while *r* represents the number of replicates. x_{ijk} is the observation corresponding to the k^{th} replicate taken from treatment *i* of factor A and treatment *j* of factor B, $\bar{x}_{i\cdot}$ is the sample mean of the observations in treatment *i* in factor A, $\bar{x}_{\cdot j}$ is the sample mean of the observations in treatment *j* in factor B, $\bar{x}_{ij}$ is the sample mean of the observations corresponding to treatment *i* of factor A and treatment *j* of factor B, and $\bar{\bar{x}}$ is the overall sample mean of n_T observations (Department of Mathematics, Sinclair Community College).

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\left(x_{ijk} - \bar{\bar{x}}\right)^2 \quad (2.5)$$

$$SSA = br\sum_{i=1}^{a}(\bar{x}_{i\cdot} - \bar{\bar{x}})^2 \quad (2.6)$$

$$SSB = ar\sum_{j=1}^{b}\left(\bar{x}_{\cdot j} - \bar{\bar{x}}\right)^2 \quad (2.7)$$

$$SSAB = r\sum_{i=1}^{a}\sum_{j=1}^{b}\left(\bar{x}_{ij} - \bar{x}_{i\cdot} - \bar{x}_{\cdot j} + \bar{\bar{x}}\right)^2 \quad (2.8)$$

$$SSE = SST - SSA - SSB - SSAB \quad (2.9)$$

Table 2.1: Two Way ANOVA (General Form)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	a-1	SSA	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	
Band	b-1	SSB	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	
*Sector*Band*	(a-1)(b-1)	SSAB	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	
Error	ab(r-1)	SSE	$MSA = \frac{SSE}{ab(r-1)}$		
Total	$n_T - 1$	SST			

Contour plots of z-scores will also be generated for this study. Z-scores are a statistical analysis that is performed to normalize data in such a way that the scale of the scoring is how many standard deviations the data are from the sample mean. This is useful tool for comparison because all of the data is represented on the same scale. The formula for calculating z-scores is as follows, where X is the individual data point from the sample, μ_x is the sample mean, and σ_x is the sample standard deviation (Chou, 1975).

$$Z = \frac{X-\mu_x}{\sigma_x} \quad (2.10)$$

The z-scores will be plotted on a contour plot to serve as a method of graphical analysis to observe trends.

2.5 Summary

Chapter II presents the history and development of rapid prototyping along with information specific to the SLM process, an explanation of measurement and metrology, and a description of the statistical methods for data analysis. Using statistical methodology to determine the impact of plate location on part geometry for the EOS M290 will allow users to have a better understanding of how to increase machine capability and productivity. The research identified in this chapter will provide the foundation and justification for the methodology discussed in Chapter III.

CHAPTER 3: METHODOLOGY

This chapter presents the methodology used in this study for determining part geometry, build plate placement, build parameters, statistical tests, and analysis. The statistical tests and analysis used in this study were The statistical tests used for this study were Anderson-Darling tests for distribution, Bartlett's tests for variance, running independent General Linear Models to determine if there is a significant difference between bands and sectors on the build plate, and Tukey post-hoc testing to determine which bands and sectors are significant, each regarding height and diameter measurements.

3.1 Part Geometry, Build Plate Layout, and Build Parameters

Part geometry was determined based on how the EOS M290 creates features during the build. This required geometry where diameter and height data could be collected independently because height feature creation is dependent on deposition of powder layers while X-Y geometry is dependent on laser head accuracy. The final part was determined to be a rectangular base measuring 5 mm x 5 mm x 10 mm, with a 3 mm diameter pin on top with a height of 5 mm. Height measurements were taken on the top of the pin and diameter measurements were taken along the full height of the pin. The purpose of the rectangular base was to provide stability for the measured feature and allowed the machine to establish consistent build layers prior to creating the pin. A model of the pin can be seen below in figure 3.1.

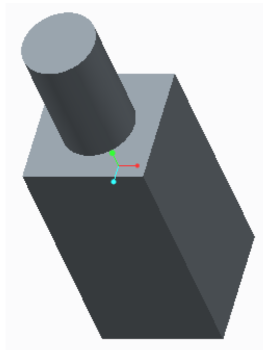

Figure 3.1: Part Model

Build plate layout was also determined based on how the EOS M290 creates parts. The build plate was divided into sections that reflect rotation of part placement and a distance from center, resulting in eight sectors and four bands. A drawing for the layout is shown below in figure 3.2. At the intersection of each sector and band is a zone which contains five parts. This allowed for 160 parts to be placed on each build plate for analysis. Each part was rotated such that the sides of the part base would come into contact with the re-coater blade of the machine at 45 degrees. This was done to reduce variation introduced by the re-coater blade coming into contact with a parallel surface and allowed for each side of the base to be created in an identical manner. The parts were created and assembled in the build plate orientation using PTC Creo Parametric 2.0, then exported as a single model for the builds. Three builds were completed using identical machine settings. The machine settings used were the default parameter set for 316L stainless steel in 20 micron layers on an EOS M290 DMLS machine. For the purposes of this study the builds are not considered replicates due to pre-build variation in the setup process that could not be controlled.

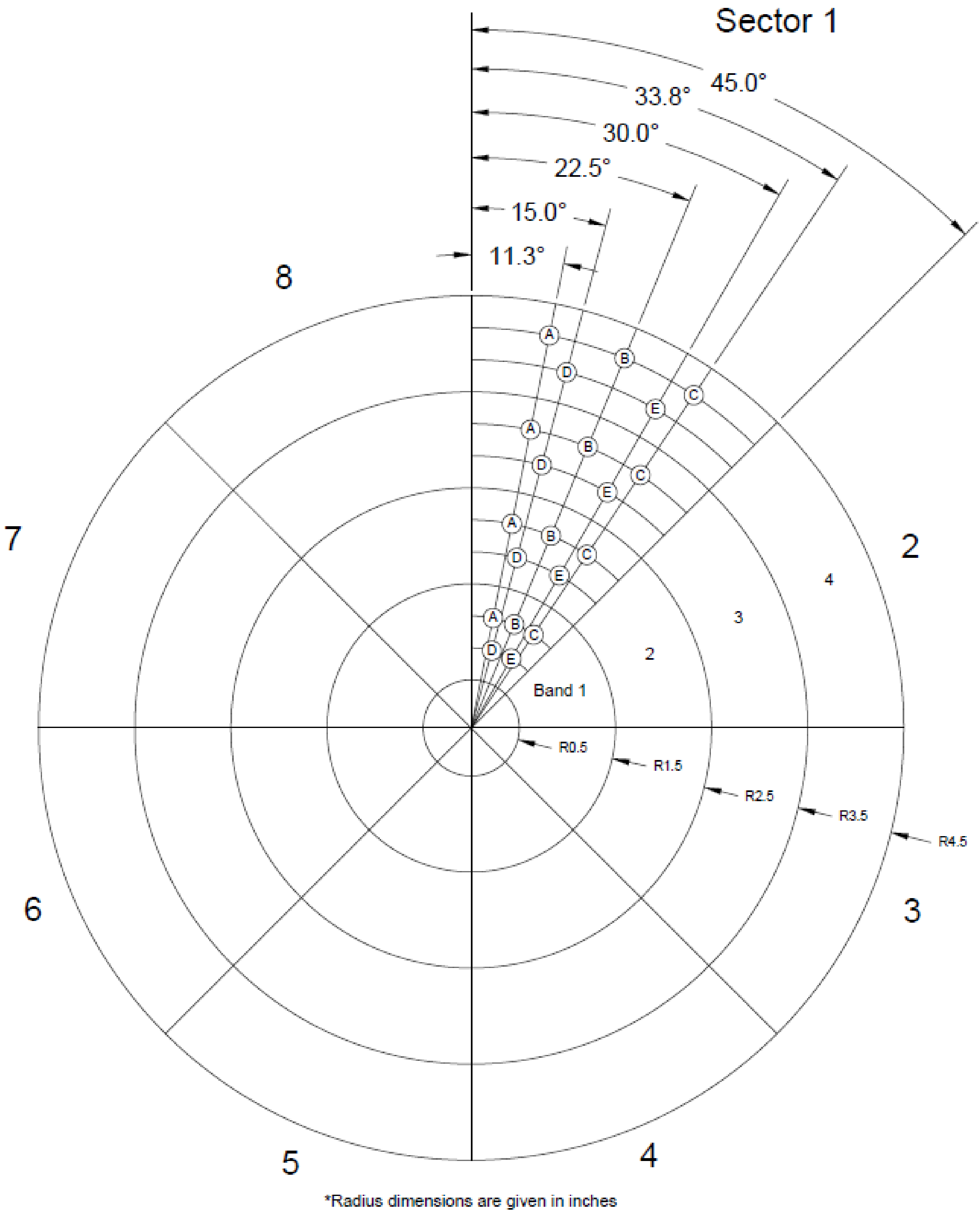

Figure 3.2: Build Plate Layout

3.2 Measurement Strategy

This section details the measurement strategies used to obtain data for statistical analysis of height and diameter. Measurements were taking using a Zeiss Contura HTG CMM with Calypso software. A program was written to collect measurements that directly referenced the part model to ensure that each part was measured using the same strategy with a reliable alignment (0,0,0).

3.2.1 Coordinate Measuring Machine (CMM) Alignment

Each build plate was measured using a Zeiss Contura HTG CMM, and reference datums were printed on each build plate in order to minimize variation introduced in the measurement process. These datums acted as reference surfaces for alignment of the CMM that were independent of the build plate. As shown below in figure 3.3, establishing an X,Y and Z (0,0,0) that was independent of the build plate was important because it minimized variation introduced from the build plate by having a point located by printed features as a reference for measuring other printed features. Each build plate was measured at the same location within the measurement volume of the machine and touched off independently.

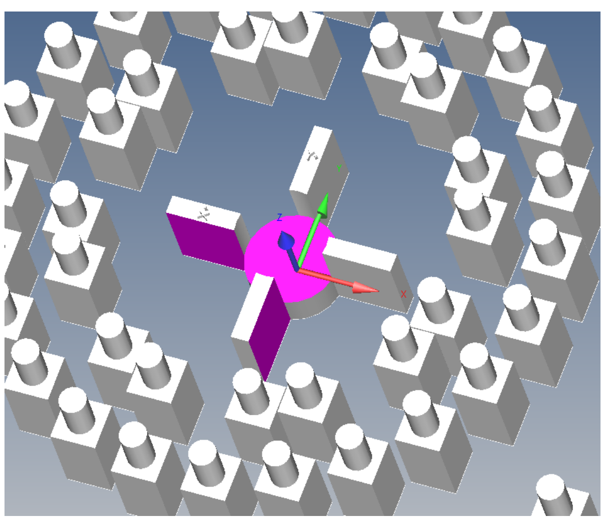

Figure 3.3: CMM Alignment (0,0,0)

3.2.2 Height and Diameter Measurement Strategies

Height and diameter measurements were taken using a scan of the surface as opposed to identifying single points on the surface. This resulted in approximately 120 data points for part height and 100 data points for part diameter for each sample. The software averaged these points to determine a single value that represented the measurement. This was done so that the single measured value would be an accurate representation of the surface of the part. Height measurements were determined by a scan path of four concentric circles with a horizontal scan on the top surface of each pin. Diameter measurements were determined by a helical path that revolved twice around the vertical surface of each pin. Each strategy can be seen below in figure 3.4.

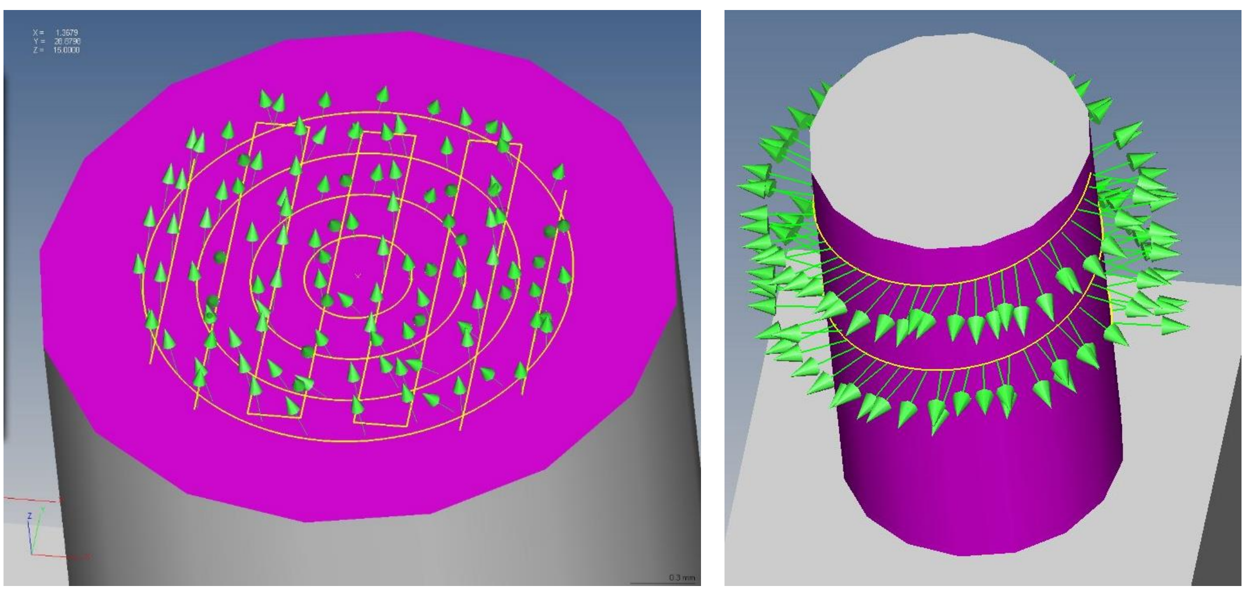

Figure 3.4: Height and Diameter Measurement Strategies

3.3 Statistical Testing and Analysis

Statistical testing was done on three build plates individually, the plates were not treated as replicates since the individual plates were independent builds of parts performed at different times. The purpose of the testing was to determine if significant differences in part diameter and part height were present with respect to bands, sectors, and the interaction of bands by sector. Analysis of Variance was used to perform the analysis, and separate two-way ANOVA runs were generated for diameter and height. Ultimately, the results of the statistical analysis may provide EOS M290 users a better understanding of how build plate location affects part geometry capability related to height and diameter. All statistical tests were run using Minitab 17.

Data distribution was analyzed using histograms and normal probability plots. The purpose of this was to determine if the data followed a normal distribution, which was necessary for further statistical testing. The purpose of plotting the data was to give a visual representation of how well the fit was to a normal distribution, however determination on accepting or rejecting the null hypothesis that the data follows a normal distribution was through Anderson-Darling testing. This test was used as a fit test because more weight is placed on the tails of the distribution. Outliers were not eliminated from the study because understanding how variation in the build setup could not be defined or controlled. Therefore, assignable causes could not be used as justification for outlier elimination. The equation for the Anderson-Darling test statistic is as follows, where A^2 is the test statistic, N is the number of samples, F is the cumulative distribution function, and Y_i is the data point (NIST/Sematech, 2012).

$$A^2 = -N - S \qquad (3.1)$$

where

$$S = \sum_{i=1}^{N} \frac{2i-1}{N}\left[\ln F(Y_i) + \ln\big(1 - F(Y_{N+1-i})\big)\right] \qquad (3.2)$$

Variance testing was done individually on bands and on sectors for height data and diameter data. The purpose of this was to understand if the variance in height and diameter measurements were equal across bands and across sectors. Determination of accepting or rejecting the null hypothesis that the variance is equal for the set of data being analyzed was done through a Bartlett's test. The equations for the Bartlett test statistic are as follows, where s_i^2 is the variance of the *i*th group, *N* is the total sample size, N_i is the sample size of the *i*th group, *k* is the number of groups, and s_p^2 is the pooled variance (NIST/Sematech, 2012).

$$T = \frac{(N-k)\ln s_p^2 - \sum_{i=1}^{k}(N_i-1)\ln s_i^2}{1+\left(1/\left(3(k-1)\right)\right)\left(\left(\sum_{i=1}^{k}1/(N_i-1)\right)-1/(N-k)\right)} \quad (3.3)$$

$$s_p^2 = \sum_{i=1}^{k}(N_i-1)s_i^2/(N-k) \quad (3.4)$$

The General Linear Model function in Minitab was used to perform a two way ANOVA to determine if the means of individual sectors for height and diameter were statistically different when compared to other sectors. The same test was done to compare the means of each band for height and diameter. The GLM also determined if interaction between sectors and bands had a statistically significant impact on the height and diameter of the parts measured in that zone. If the interaction was shown to be not significant then it was removed from the model and added to the error term. The formulae for performing a two factor ANOVA as well as the structure and calculations for a summary ANOVA table are shown below, where SSA is the sum of squares for factor A, SSB is the sum of squares for factor B, SSAB is the sum of squares for the interaction of factors A and B, SSE is the sum of squares for the error, and SST is the total sum of squares. The variables *a* and *b* represent the total number of levels for factors A and B, while *r* represents the number of replicates. x_{ijk} is the observation corresponding to the k^{th} replicate taken from treatment *i* of factor A and treatment *j* of factor B, $\bar{x}_{i.}$ is the sample mean of the observations in treatment *i* in factor A, $\bar{x}_{.j}$ is the sample mean of the observations in treatment *j* in factor B, $\bar{x}_{ij}$ is the sample mean of the observations corresponding to treatment *i* of factor A and treatment j of factor B, and $\bar{\bar{x}}$ is the overall sample mean of n_T observations (Department of Mathematics, Sinclair Community College).

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}\left(x_{ijk} - \bar{\bar{x}}\right)^2 \tag{3.5}$$

$$SSA = br\sum_{i=1}^{a}(\bar{x}_{i\cdot} - \bar{\bar{x}})^2 \tag{3.6}$$

$$SSB = ar\sum_{j=1}^{b}\left(\bar{x}_{\cdot j} - \bar{\bar{x}}\right)^2 \tag{3.7}$$

$$SSAB = r\sum_{i=1}^{a}\sum_{j=1}^{b}\left(\bar{x}_{ij} - \bar{x}_{i\cdot} - \bar{x}_{\cdot j} + \bar{\bar{x}}\right)^2 \tag{3.8}$$

$$SSE = SST - SSA - SSB - SSAB \tag{3.9}$$

Table 3.1: Two Way ANOVA (General Form)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	a-1	SSA	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	
Band	b-1	SSB	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	
*Sector*Band*	(a-1)(b-1)	SSAB	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	
Error	ab(r-1)	SSE	$MSA = \frac{SSE}{ab(r-1)}$		
Total	$n_T - 1$	SST			

The GLM produced a regression equation but for the purposes of this study was not used. This is important regarding two way ANOVA results because the equation is used to predict a response given the factors used in the study as input parameters. These equations were not used due to the fact that the prediction of the actual diameter of a three millimeter part that is 15 millimeters tall is not practical regarding this application, and only categorical comparisons of band, sectors, and interactions were studied. However, regression equations from this study with variable coefficients and full analysis are in appendices A-C for reference. The general form of the regression equation is as follows, where a is a constant, b_1 is the weight applied to factorX_1,

b_2 is the weight applied to factor X_2 through a number of factors k (Quirk, 2016). The factors were determined by Tukey post hoc testing, which defined the individual bands, sectors and interactions that were significantly different.

$$Y = a + b_1 X_1 + b_2 X_2 + \cdots + b_k X_k \qquad (3.10)$$

The two way ANOVA determined if sectors, bands or interactions between sectors and bands had a statistically significant impact on part creation; but did not reveal which sectors, bands or interactions were significant. A Tukey post-hoc pairwise comparison was performed on the data to qualify this. The build plates were not considered replicates for this study, however comparison of individual sectors and bands to each other allowed for trends to be observed across plates.

CHAPTER 4: RESULTS AND ANALYSIS

The goal of the study was to characterize the process capability of an EOS M290 SLM machine with respect to build plate location for 316L stainless steel. ANOVA statistical General Linear Models were run for two-way main effects and interaction of plate bands and sectors for three different build plates. Chapter IV presents data and analysis of height and diameter measurements collected with a Zeiss Contura HTG coordinate measuring machine. Data was collected from three build plates that were created with the EOS M290 direct metal laser sintering system, and the analysis was conducted using MiniTab statistical software. Each build plate was created with identical machine settings; however for the purposes of this study parts on different plates are not considered replicates. Replicates were not considered since some factors and parameters in the machine setup could not be controlled or repeated for each build and could contribute to variation between plates. Ultimately, the results of the statistical analysis may provide EOS M290 users a better understanding of how build plate location affects part geometry capability related to height and diameter. For all statistical testing, α=.05 has been used for the decision on retaining or rejecting the null hypothesis. The following sections are separated into groups by plate number, containing subsections of height and diameter with corresponding statistical testing for data distribution, variance, General Linear Model results and multiple comparisons. The null and alternate hypotheses are shown at the beginning of each section, when applicable.

4.1 Plate One

4.1.1 Height

4.1.1.1 Data Distribution

H_0: The height data is normally distributed

H_a: The height data is not normally distributed

The normal probability plot and histogram for the height data of plate one can be seen in figures 4.1 and 4.2. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of .271 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

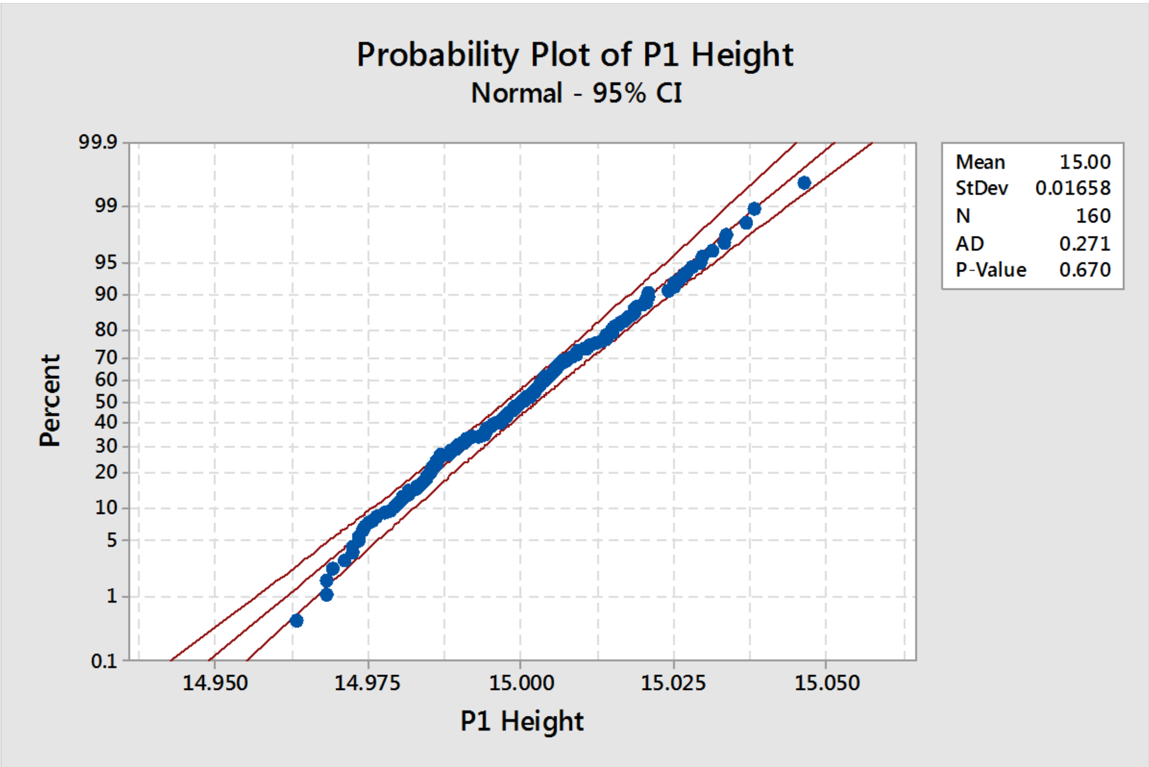

Figure 4.1: Plate 1 Height Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. The Anderson-

Darling testing was used because it applies more weight to the tails of a distribution when determining if the data likely represents a normal distribution.

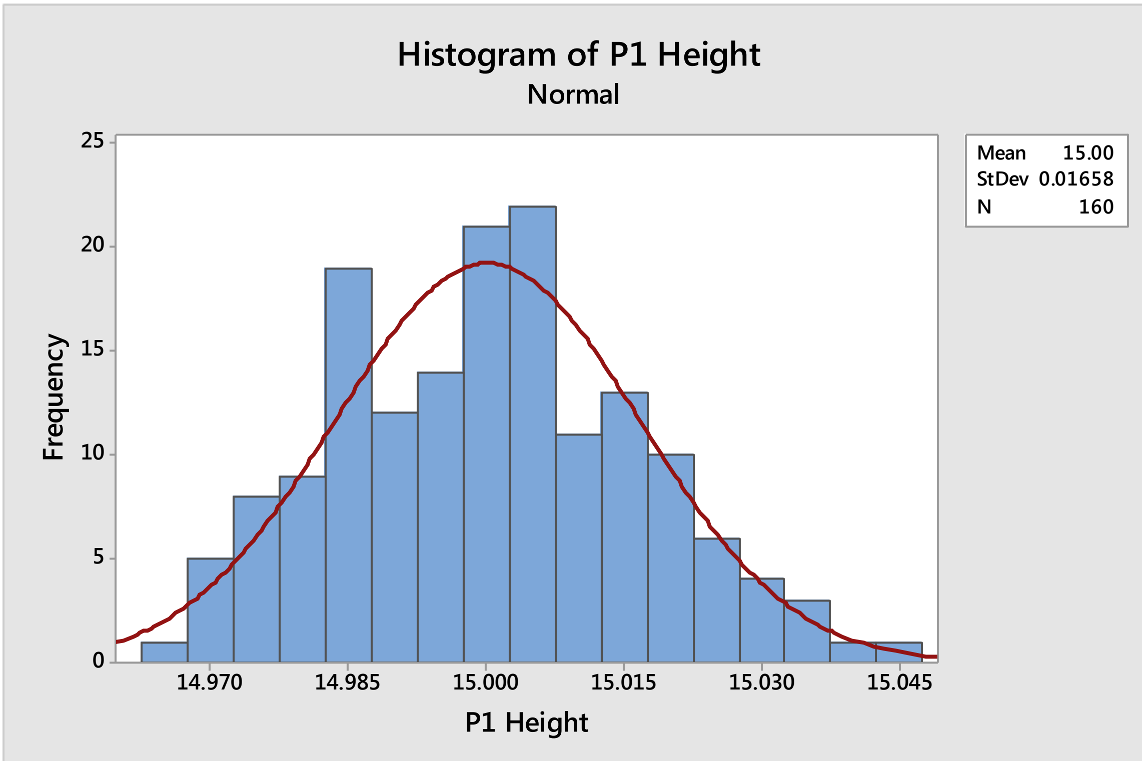

Figure 4.2: Plate 1 Height Histogram

4.1.1.2 Sector Variance

H$_0$: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between sectors are equal

H$_a$: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if height variance was equal across sectors for the plate. The null hypothesis was retained because the resulting P-Value of .065 is greater than the alpha value of .05. Figure 4.3 shows the comparison plot for height vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance are assumed to be equal.

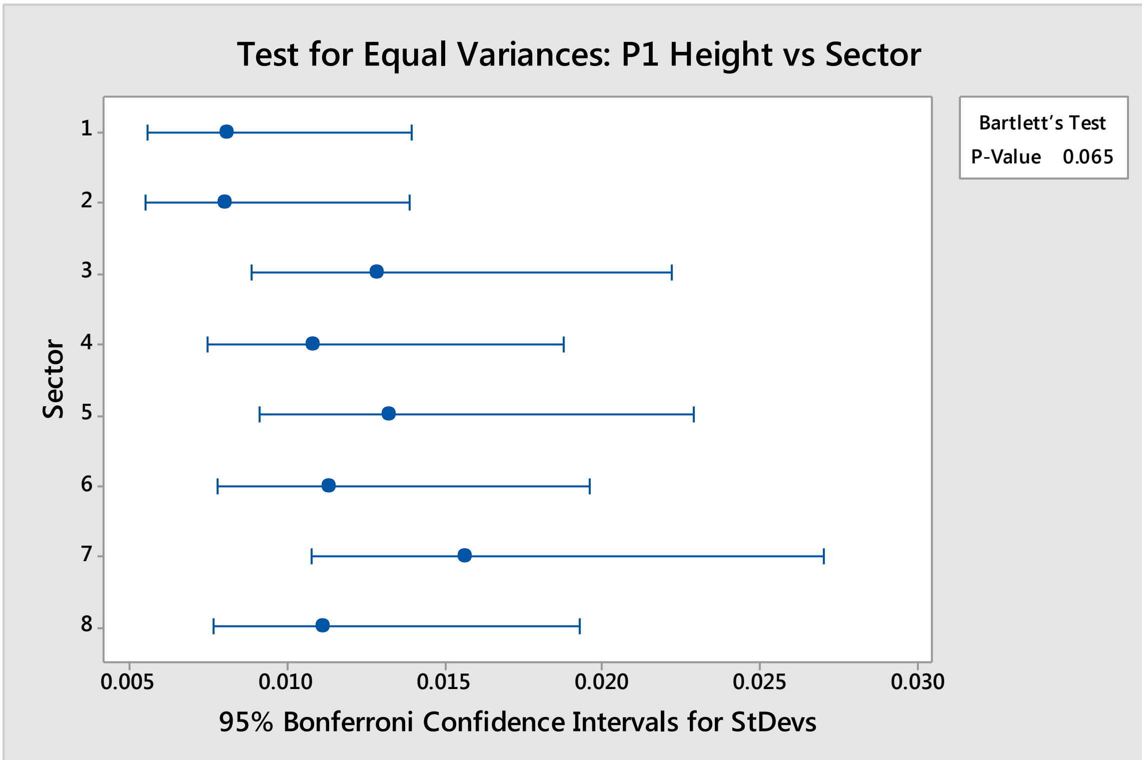

Figure 4.3: Plate 1 Variance Comparison Plot for Height vs. Sector

4.1.1.3 Band Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between bands are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if height variance was equal across bands for the plate. The null hypothesis was retained because the resulting P-Value of .305 is greater than the alpha value of .05. Figure 4.4 shows the comparison plot for height vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, demonstrating that the variance between them can be assumed to be equal.

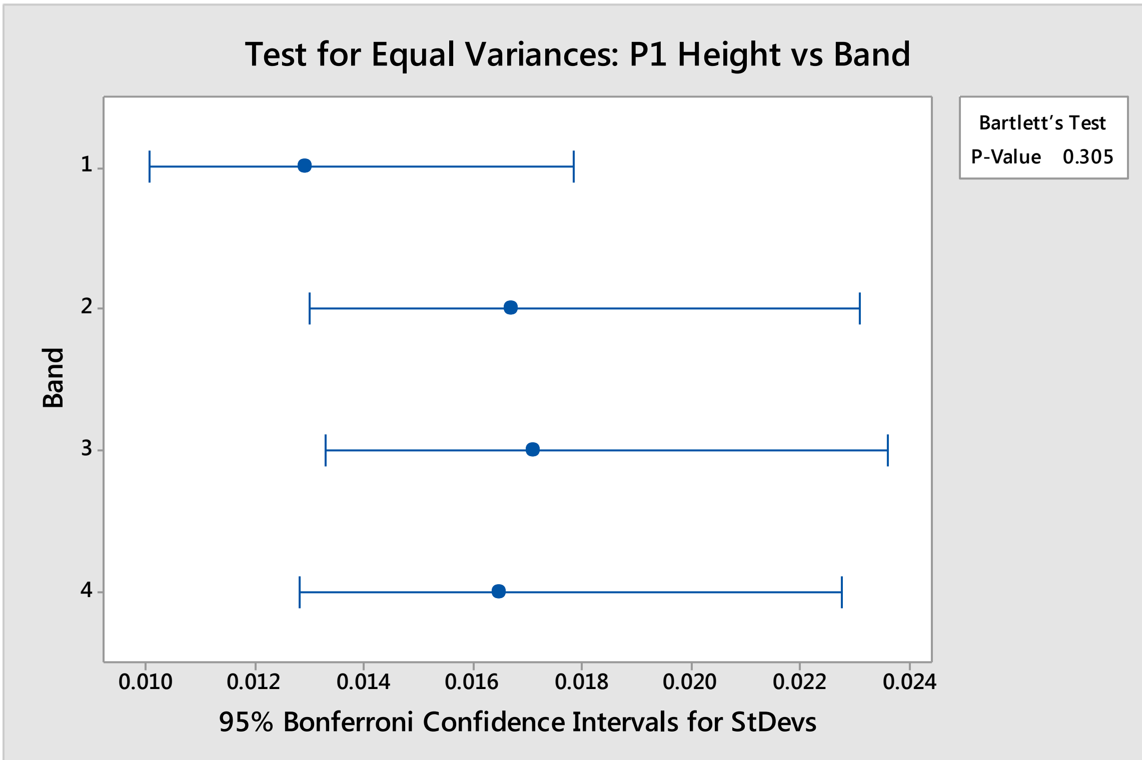

Figure 4.4: Plate 1 Variance Comparison Plot for Height vs. Band

4.1.1.4 General Linear Model

H_0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (i,j)

H_a: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (i,j)

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding variation of height. The analysis of height measurements for plate one indicate that each was significant. The General Linear Model produces a regression equation, but will not be used in this study since only categorical comparisons were made. The ultimate goal of the General Linear Model was to determine if the main effects and interaction between sectors and bands is statistically significant through a two way ANOVA. The summary ANOVA can be seen in table 4.1.

Table 4.1: Plate 1 Height Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.023077	0.003297	33.38	0.000
Band	3	0.004357	0.001452	14.70	0.000
*Sector*Band*	21	0.003643	0.000173	1.76	0.030
Error	128	0.012641	0.000099		
Total	159	0.043719			

The results of the ANOVA show that sectors ($p<.05$), bands ($p<.05$), and sector/band interaction ($p<.05$) were significant for plate one with respect to height.

4.1.1.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant. The results of the individual band and sector comparisons are shown in tables 4.2 and 4.3.

Table 4.2: Plate 1 Height Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P1 Height, Term = Sector				
Grouping Information Using the Tukey Method and 95% Confidence				
Sector	N	Mean	Grouping	
7	20	15.0174	A	
8	20	15.0120	A	
1	20	15.0089	A	
6	20	15.0085	A	
2	20	14.9911		B
4	20	14.9900		B
5	20	14.9898		B
3	20	14.9835		B
Means that do not share a letter are significantly different.				

Table 4.3: Plate 1 Height Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P1 Height, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
4	20	15.0174	A
3	20	15.0120	A
2	20	15.0089	B
1	20	15.0085	B
Means that do not share a letter are significantly different.			

The comparison for both sectors and bands show that one half of plate one is significantly different than the other half. The means for sectors one, six, seven and eight are equal, while the means for sectors two, three, four and five are equal; however the sector groups are significantly different from each other. Similarly, the means for bands four and three are equal as are the means for bands one and two.

Figure 4.5 shows the significant interactions between sectors and Figure 4.6 shows the significant interactions between bands. The interaction is considered significant if the interval of the comparison does not contain a zero. Regarding height, the significantly different sectors were 2-1, 3-1, 4-1, 5-1, 6-2, 7-2, 8-2, 6-3, 7-3, 8-3, 6-4, 7-4, 8-4, 6-5, 7-5, and 8-5. A trend can be noticed that most comparisons containing sectors six, seven and eight are different. The band comparisons that are significantly different are 3-1, 4-1, 3-2, and 4-2. These interactions can also be seen in figure 4.7, which shows the means for each sector and the means for each band.

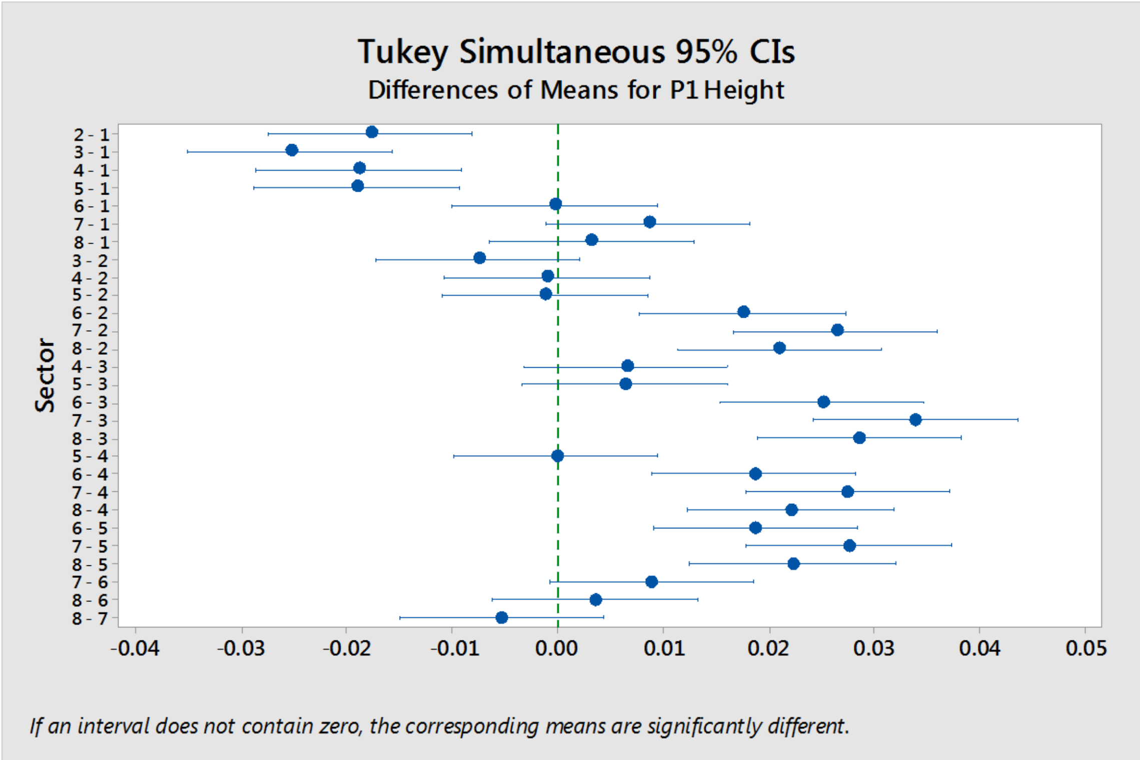

Figure 4.5: Plate 1 Height Sector Interaction Comparison

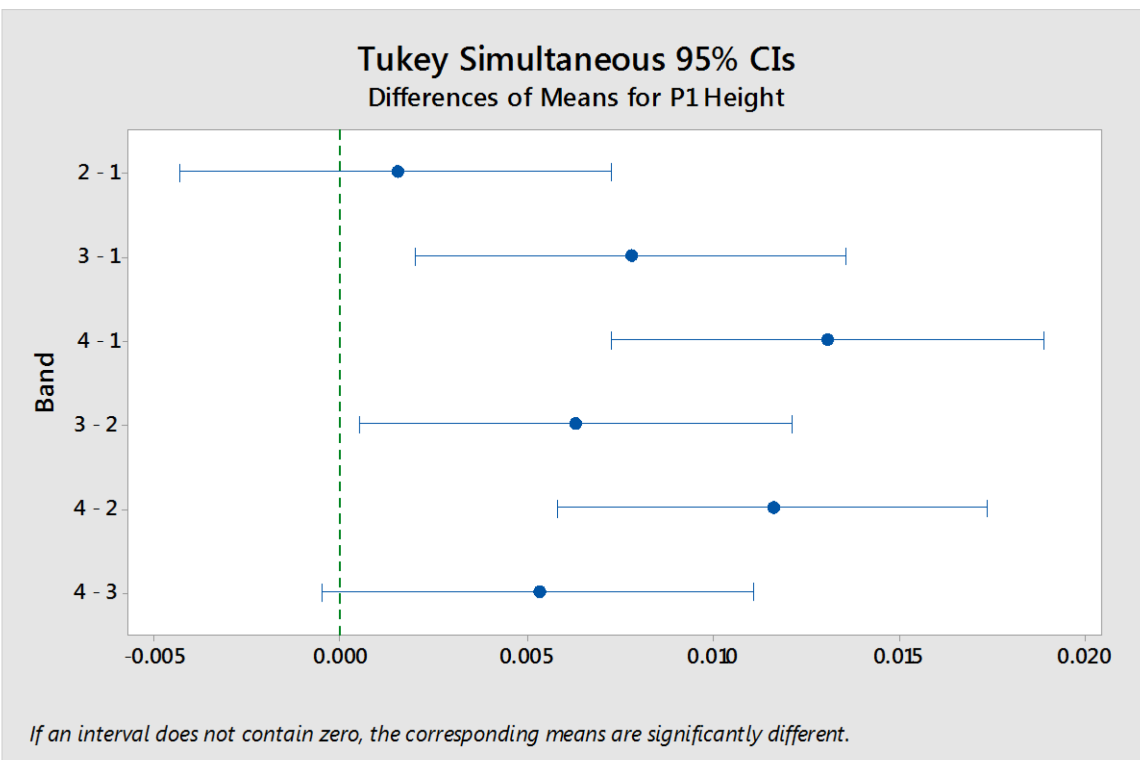

Figure 4.6: Plate 1 Height Band Interaction Comparison

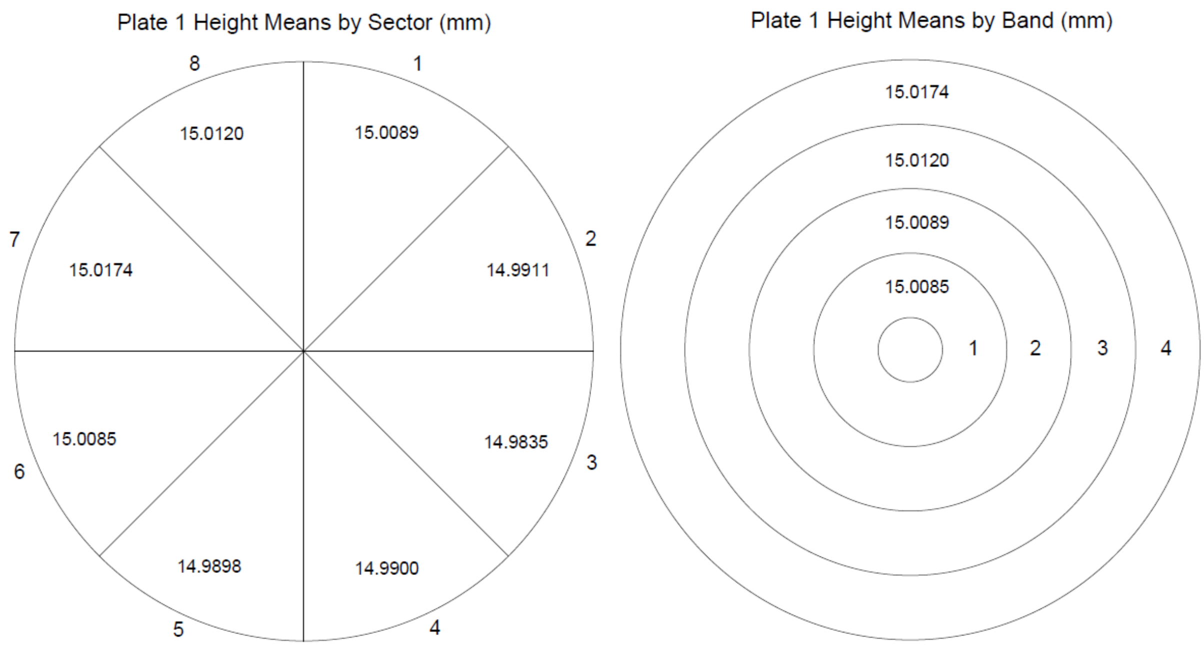

Figure 4.7: Plate 1 Height Means by Sector and Band

4.1.2 Diameter

4.1.2.1 Data Distribution

H_0: The diameter data is normally distributed

H_a: The diameter data is not normally distributed

The normal probability plot and histogram for the diameter data of plate one can be seen in figures 4.7 and 4.8. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of .3 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

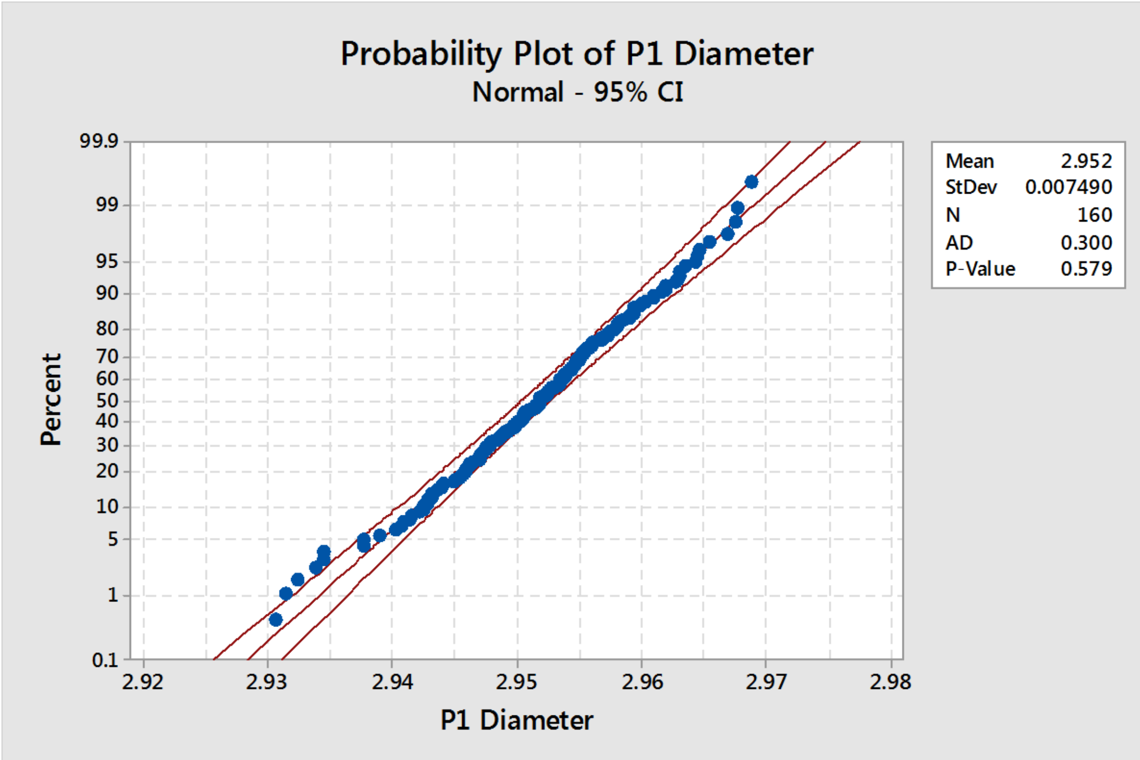

Figure 4.7: Plate 1 Diameter Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. Anderson-Darling testing was used because it applies more weight to the tails of a distribution which allows the conclusion to be drawn that the data does follow a normal distribution.

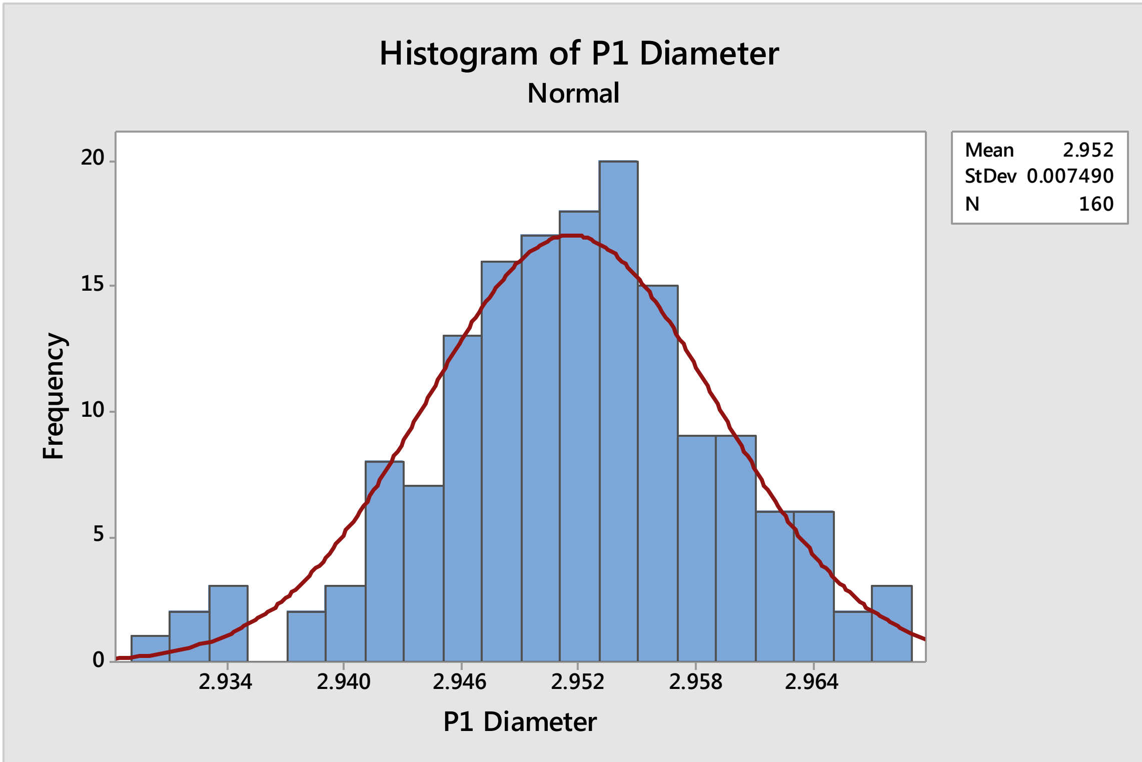

Figure 4.8: Plate 1 Diameter Histogram

4.1.2.2 Sector Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between sectors are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if diameter variance was equal across sectors for the plate. The null hypothesis was retained because the resulting P-Value of .533 is greater than the alpha value of .05. Figure 4.9 shows the comparison plot for diameter vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance between are assumed to be equal.

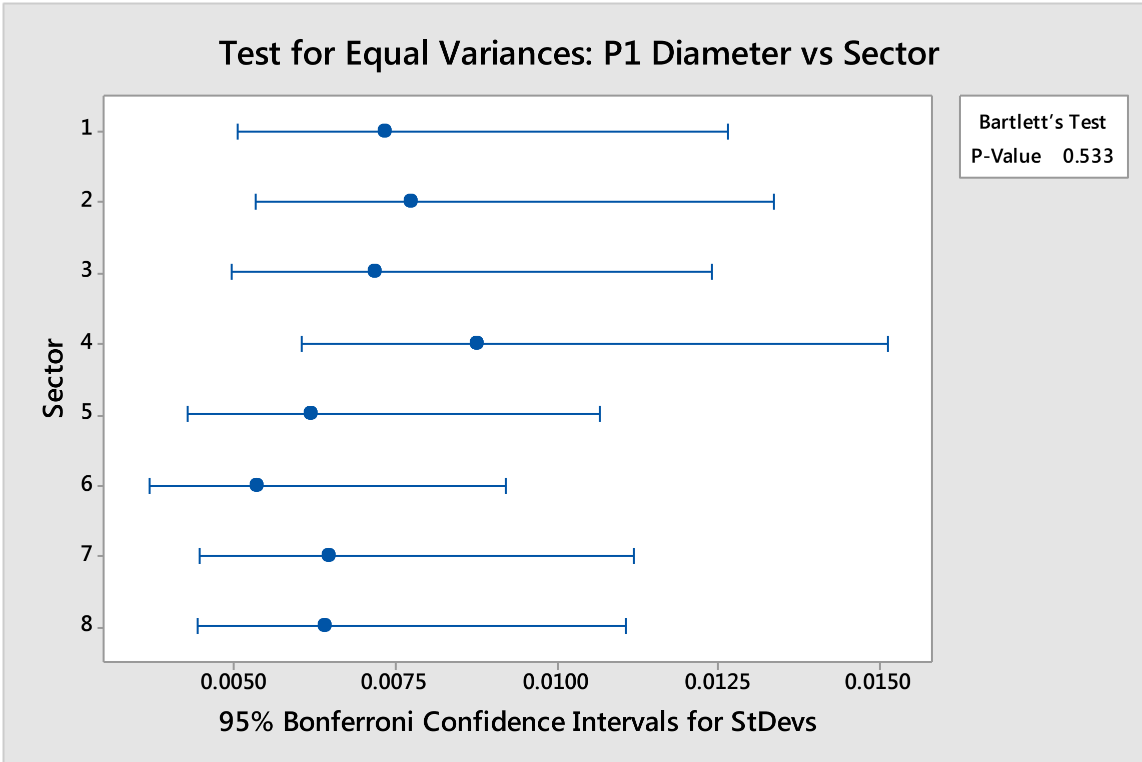

Figure 4.9: Plate 1 Variance Comparison Plot for Diameter vs. Sector

4.1.2.3 Band Variance

H$_0$: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between bands are equal

H$_a$: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if diameter variance was equal across bands for the plate. The null hypothesis was retained because the resulting P-Value of .336 is greater than the alpha value of .05. Figure 4.10 shows the comparison plot for diameter vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, results show that the variances can be assumed to be equal.

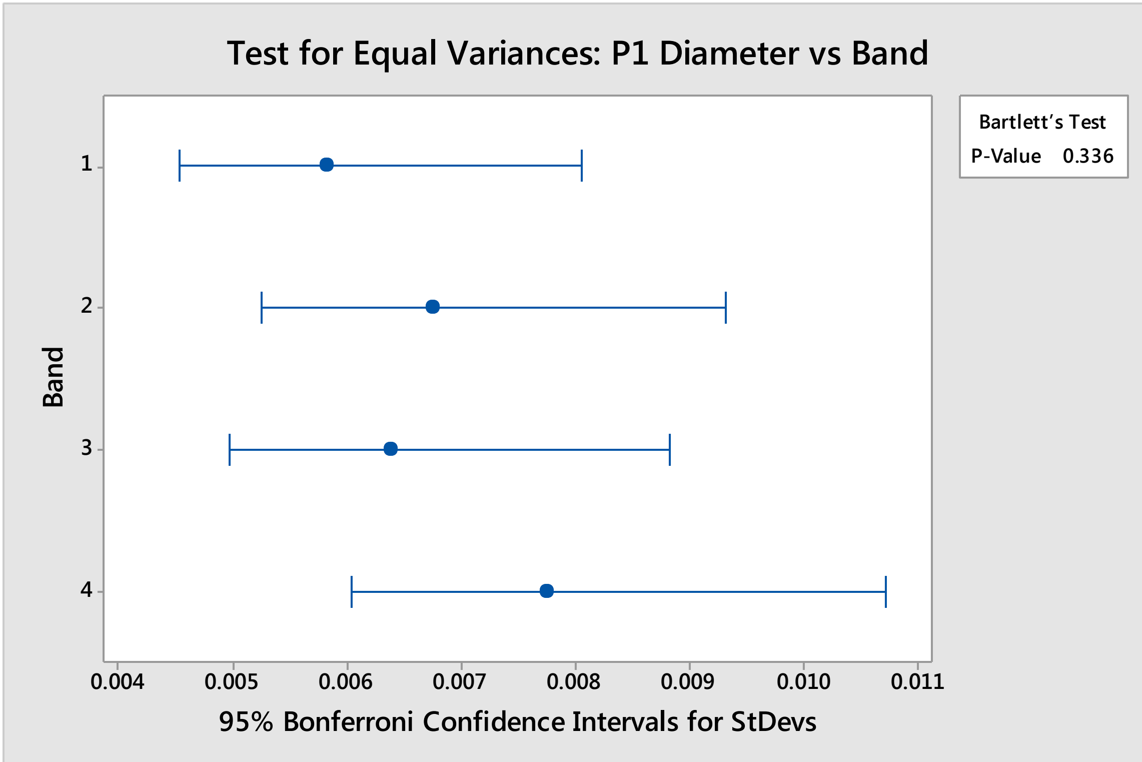

Figure 4.10: Plate 1 Variance Comparison Plot for Diameter vs. Band

4.1.2.4 General Linear Model

H_0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (*i*,*j*)

H_a: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (*i*,*j*)

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding mean diameters of measured parts. The analysis of diameter measurements for plate one indicate that bands and sectors individually were significant ($p<.05$), but the interaction term was not ($p>.05$). This can be seen in table 4.4.

Table 4.4: Plate 1 Diameter Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.001475	0.000211	5.79	0.000
Band	3	0.001876	0.000625	17.19	0.000
*Sector*Band*	21	0.000913	0.000043	1.19	0.267
Error	128	0.004656	0.000036		
Total	159	0.008919			

Since the p-value of the interaction term is larger than the alpha value of .05, the interaction was removed. A second General Linear Model was run with the interaction added to the error term. The interactions were added to the error as “lack of fit,” but the total error is determined by combining the “lack of fit” data with the “pure error.” The p values appear to remain unchanged because the data analysis in Minitab extends to three significant figures. The change is evident in the recalculation of the F-value. The summary ANOVA for the second General Linear Model can be seen in table 4.5.

Table 4.5: Plate 1 Diameter Summary ANOVA (Interaction Removed)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.001475	0.000211	5.64	0.000
Band	3	0.001876	0.000625	16.74	0.000
Error	149	0.005568	0.000037		
Lack of Fit	21	0.000913	0.000043	1.19	0.267
Pure Error	128	0.004656	0.000036		
Total	159	0.008919			

4.1.2.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant with respect to diameter. The results of the individual band and sector comparisons are shown in tables 4.6 and 4.7. Only the main effects are shown because the interactions were insignificant.

Table 4.6: Plate 1 Diameter Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P1 Diameter, Term = Sector			
Grouping Information Using the Tukey Method and 95% Confidence			
Sector	N	Mean	Grouping
6	20	2.9551	A
5	20	2.9550	A
7	20	2.9539	A B
4	20	2.9537	A B C
3	20	2.9511	A B C D
2	20	2.9484	B C D
8	20	2.9480	C D
1	20	2.9475	D
Means that do not share a letter are significantly different.			

Table 4.7: Plate 1 Diameter Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P1 Diameter, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
1	20	2.9557	A
2	20	2.9530	A B
3	20	2.9513	B
4	20	2.9463	C
Means that do not share a letter are significantly different.			

The results of the Tukey testing show that the difference in means for diameter measurements on plate one are approximately divided in to two halves regarding sectors- meaning that the halves are independently similar, but when compared to each other they are significantly different. There is more consistency and less variation when compared to the height measurements. The difference in means regarding bands show that there is a significant difference as the part location moves from the inside of the plate to the outside.

4.1.3 Plate One Z-Scores

Figures 4.11 and 4.12 show contour plots of the z-scores for the height and diameter measurements taken from plate one, respectively. This is a visual representation of the multiple comparisons and Tukey testing results. The height measurements are consistently larger than the plate average towards the top left of the plate and consistently smaller towards the bottom right of the plate. There is an evident divide between the two across the center of the plate, from bottom left to top right.

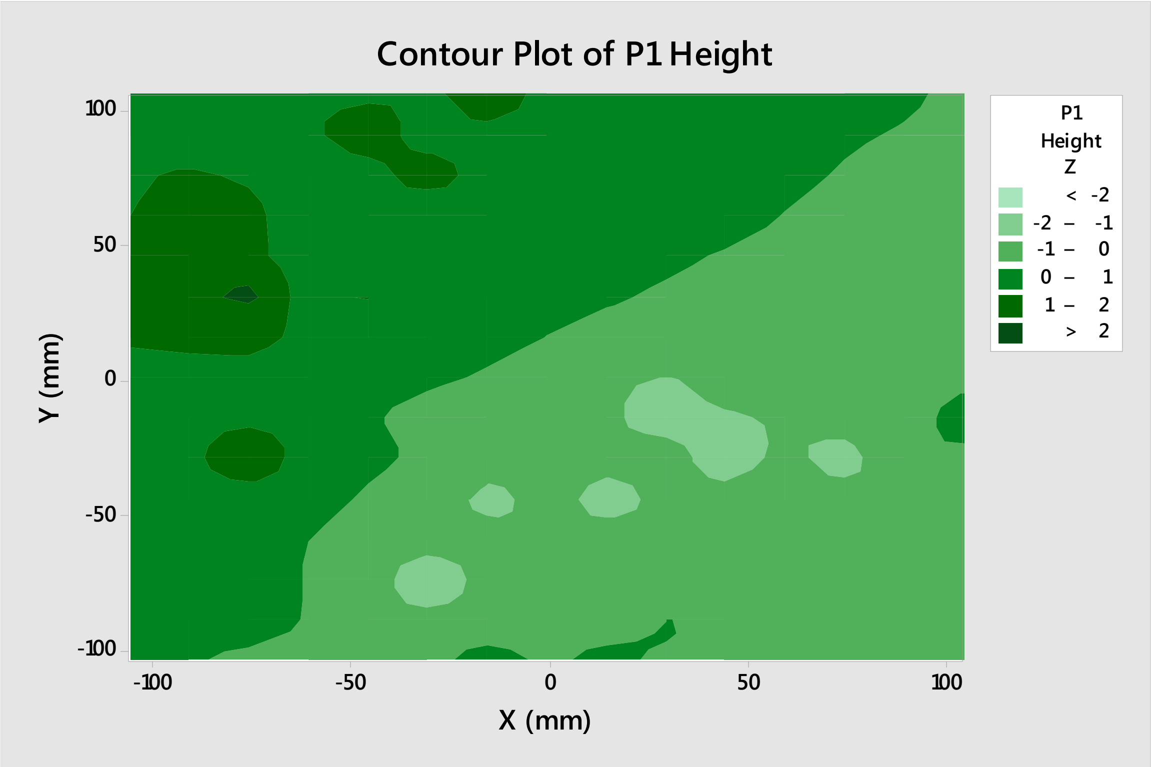

Figure 4.11: Contour Plot of Plate 1 Height

The diameter measurements show similar results but the dividing line across the plate is not the same as the line dividing the plate for height data. The diameter measurements are larger towards the bottom left corner of the plate and smaller towards the top right.

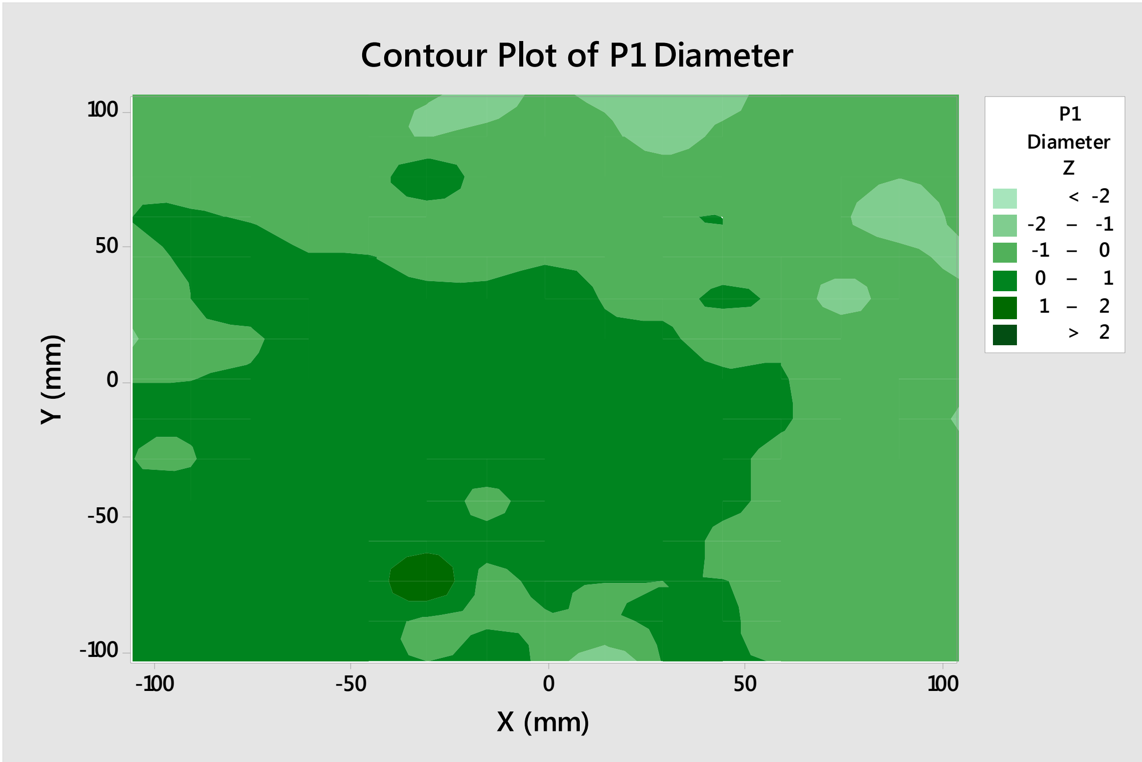

Figure 4.12: Contour Plot of Plate 1 Diameter

4.2 Plate Two

4.2.1 Height

4.2.1.1 Data Distribution

H_0: The height data is normally distributed

H_a: The height data is not normally distributed

The normal probability plot and histogram for the height data of plate two can be seen in figures 4.13 and 4.14. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of 1.314 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

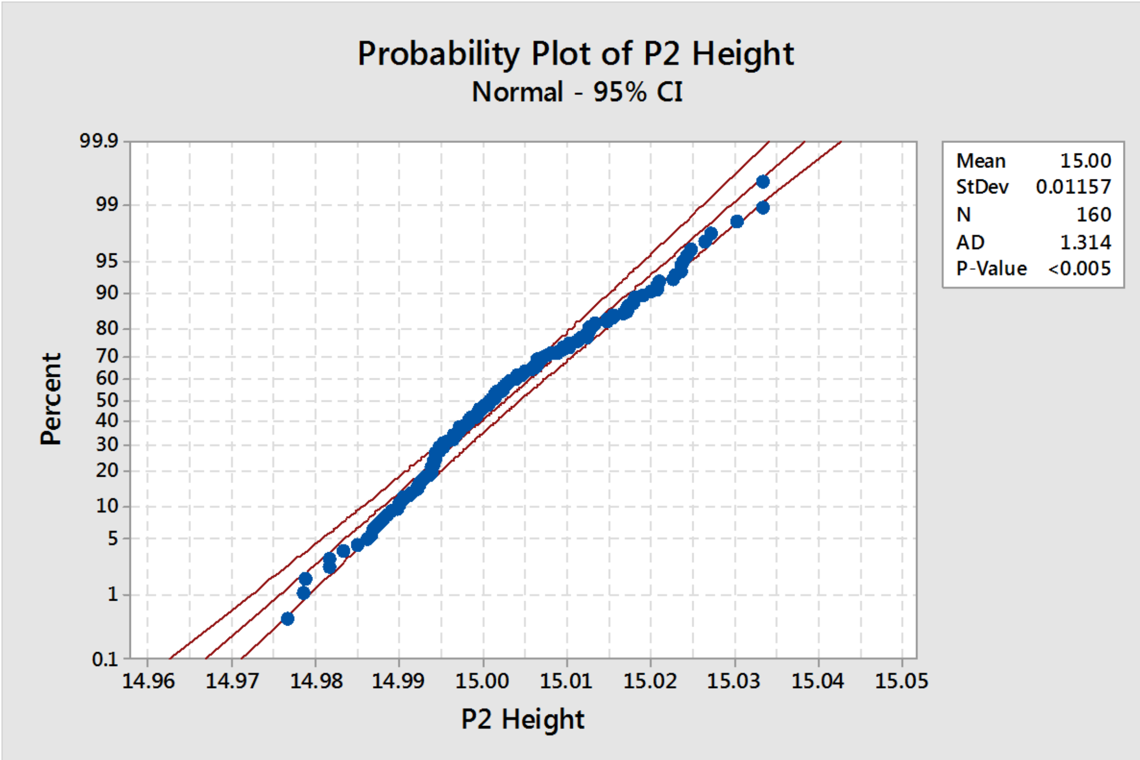

Figure 4.13: Plate 2 Height Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. Anderson-Darling testing was used because it applies more weight to the tails of a distribution which allows the conclusion to be drawn that the data does follow a normal distribution.

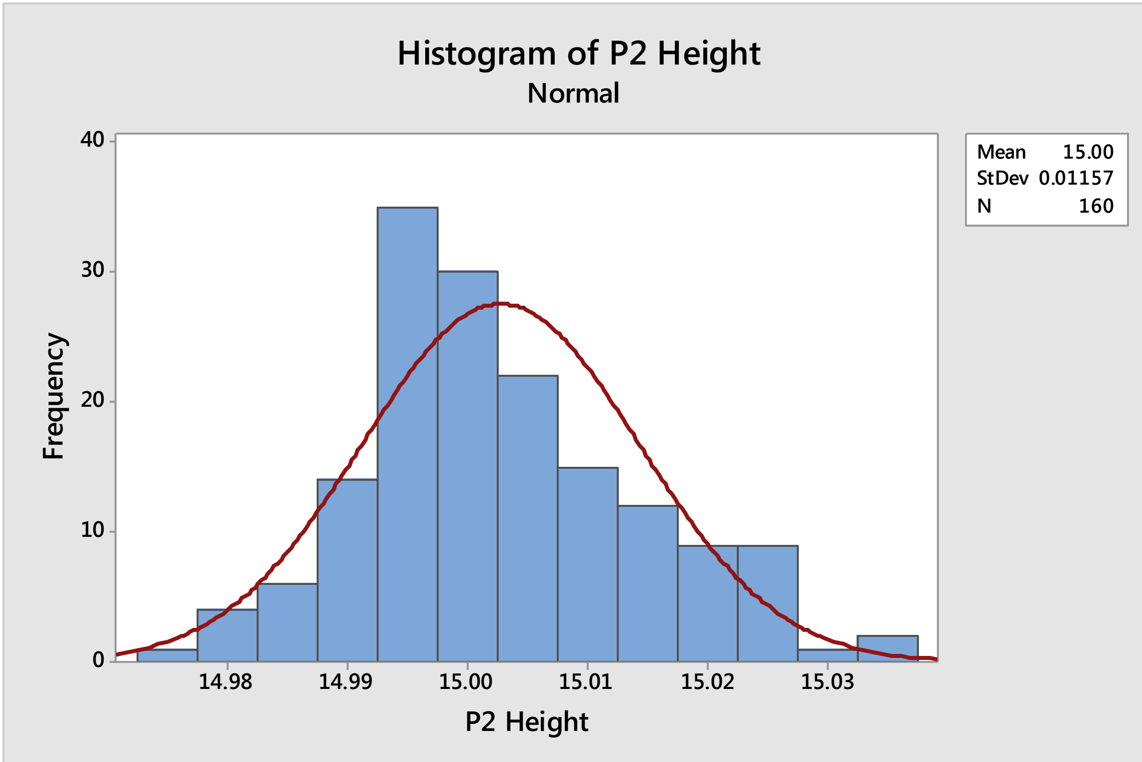

Figure 4.14: Plate 2 Height Histogram

4.2.1.2 Sector Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between sectors are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if height variance was equal across sectors for plate two. The null hypothesis was retained because the resulting P-Value of .427 is greater than the alpha value of .05. Figure 4.15 shows the comparison plot for height vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance are assumed to be equal.

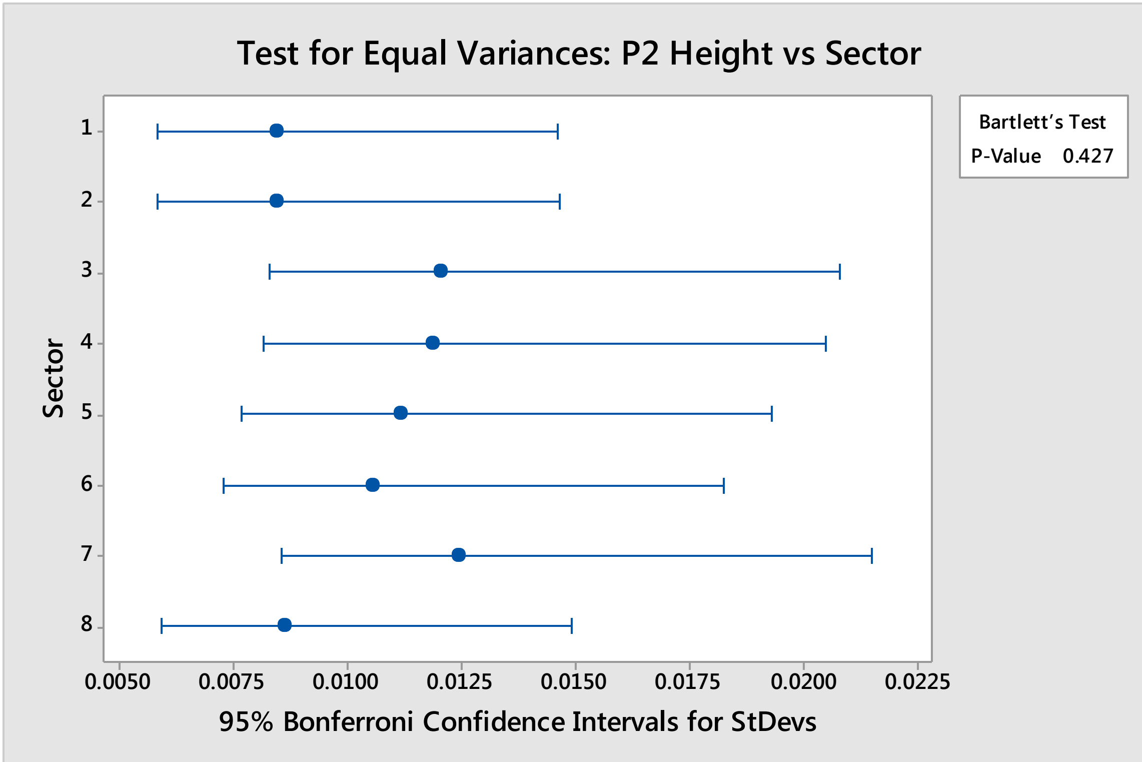

Figure 4.15: Plate 2 Variance Comparison Plot for Height vs. Sector

4.2.1.3 Band Variance

H$_0$: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between bands are equal

H$_a$: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if height variance was equal across bands for plate two. The null hypothesis was retained because the resulting P-Value of .856 is greater than the alpha value of .05. Figure 4.16 shows the comparison plot for height vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, demonstrating that the variance between them can be assumed to be equal.

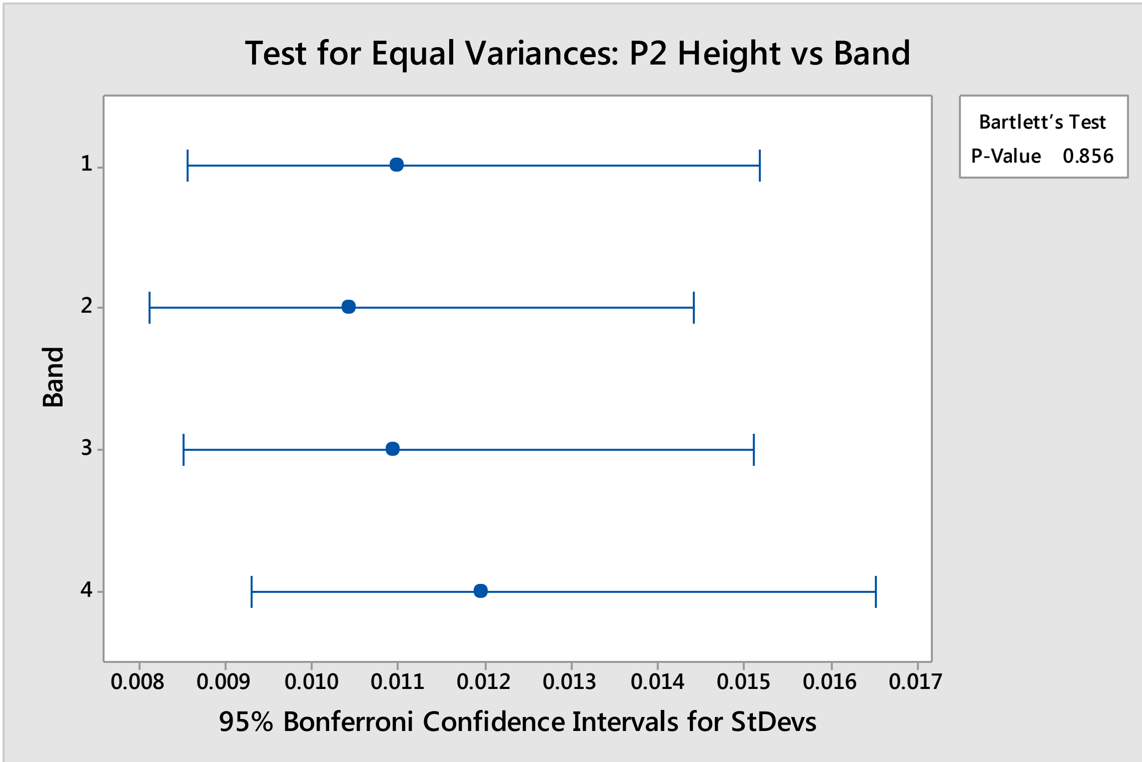

Figure 4.16: Plate 2 Variance Comparison Plot for Height vs. Band

4.2.1.4 General Linear Model

H_0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (*i*,*j*)

H_a: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (*i*,*j*)

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding mean heights of measured parts. The analysis of height measurements for plate two indicate that bands and sectors individually were significant ($p<.05$), but the interaction term was not ($p>.05$). This can be seen in table 4.8.

Table 4.8: Plate 2 Height Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.004265	0.000609	6.18	0.000
Band	3	0.002091	0.000697	7.07	0.000
*Sector*Band*	21	0.002318	0.000110	1.12	0.336
Error	128	0.012611	0.000099		
Total	159	0.021284			

Since the p-value of the interaction term is larger than the alpha value of .05, the interaction was removed. A second General Linear Model was run with the interaction added to the error term. The interactions were added to the error as "lack of fit," but the total error is determined by combining the "lack of fit" data with the "pure error." The p values appear to remain unchanged because the data analysis in Minitab extends to three significant figures, however the change is evident in the recalculation of the F-value. The summary ANOVA for the second General Linear Model can be seen in table 4.9.

Table 4.9: Plate 2 Height Summary ANOVA (Interaction Removed)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.004265	0.000609	6.18	0.000
Band	3	0.002091	0.000697	6.96	0.000
Error	149	0.149280	0.000100		
Lack of Fit	21	0.002318	0.000110	1.12	0.336
Pure Error	128	0.012611	0.000099		
Total	159	0.021284			

4.2.1.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant. The results of the individual band and sector comparisons are shown in tables 4.10 and 4.11.

Table 4.10: Plate 2 Height Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P2 Height, Term = Sector			
Grouping Information Using the Tukey Method and 95% Confidence			
Sector	N	Mean	Grouping
6	20	15.0126	A
7	20	15.0082	A B
8	20	15.0044	A B C
1	20	15.0020	B C
4	20	15.0010	B C
5	20	15.0000	B C
3	20	14.9977	C
2	20	14.9960	C
Means that do not share a letter are significantly different.			

Table 4.11: Plate 2 Height Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P2 Height, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
4	20	15.0082	A
3	20	15.0036	A B
2	20	15.0002	B
1	20	14.9989	B
Means that do not share a letter are significantly different.			

The comparison for both sectors and bands show that sector six and band four are significantly different than the other sectors and bands. The height means for plate two show that there is a large grouping of sectors one, two, three, four, five and eight whose respective means are not significantly different. The grouping of bands is similar to that of plate one.

4.2.2 Diameter

4.2.2.1 Data Distribution

H_0: The diameter data is normally distributed

H_a: The diameter data is not normally distributed

The normal probability plot and histogram for the diameter data of plate two can be seen in figures 4.17 and 4.18. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of .538 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

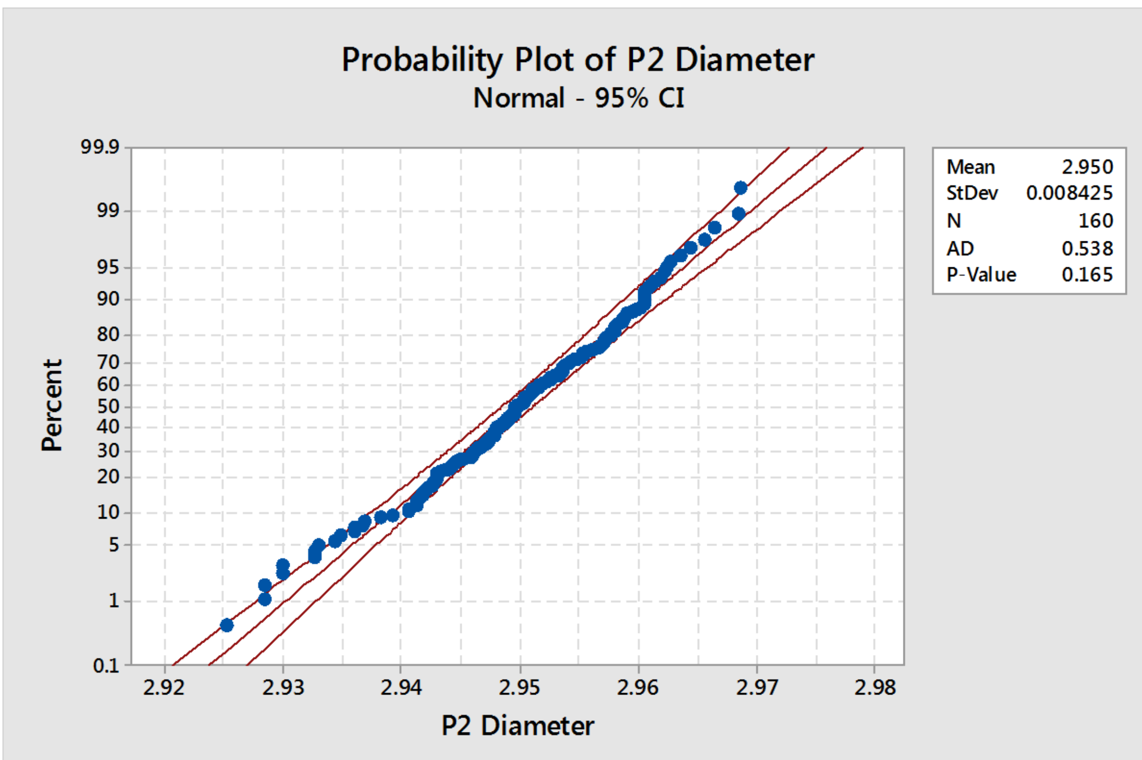

Figure 4.17: Plate 2 Diameter Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. Anderson-Darling testing was used because it applies more weight to the tails of a distribution which allows the conclusion to be drawn that the data does follow a normal distribution.

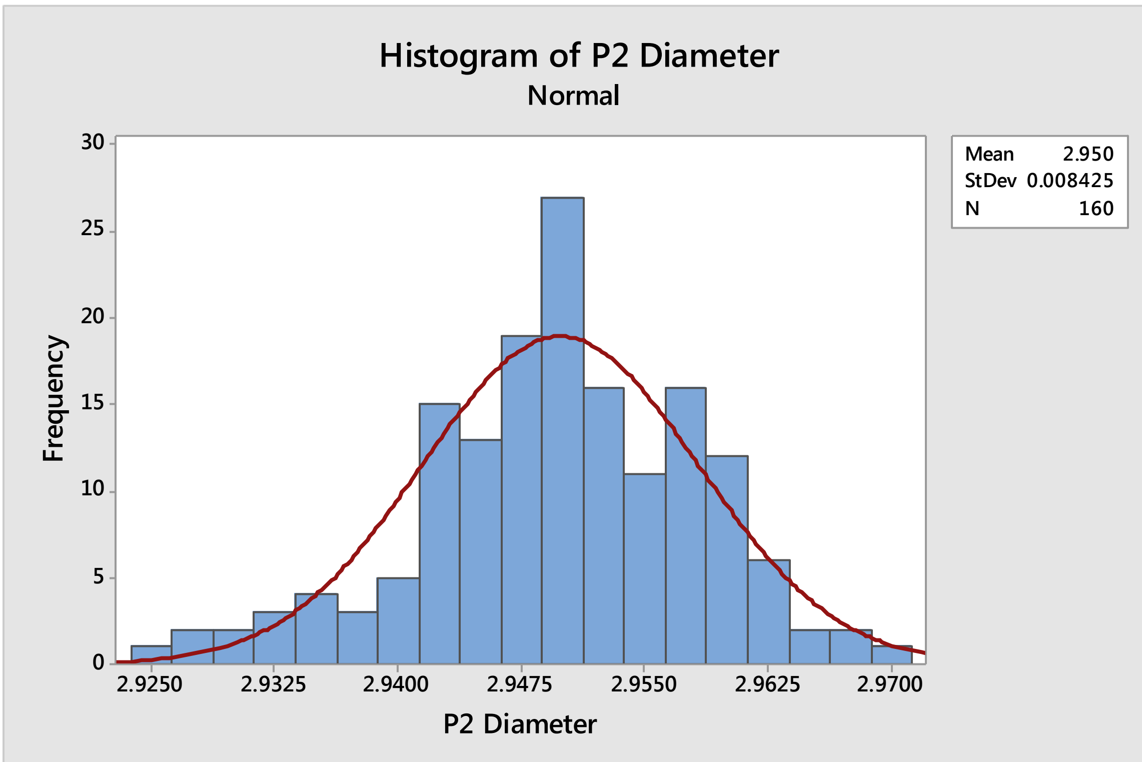

Figure 4.18: Plate 2 Diameter Histogram

4.2.2.2 Sector Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between sectors are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if diameter variance was equal across sectors for the plate. The null hypothesis was retained because the resulting P-Value of .461 is greater than the alpha value of .05. Figure 4.19 shows the comparison plot for diameter vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance between them are assumed to be equal.

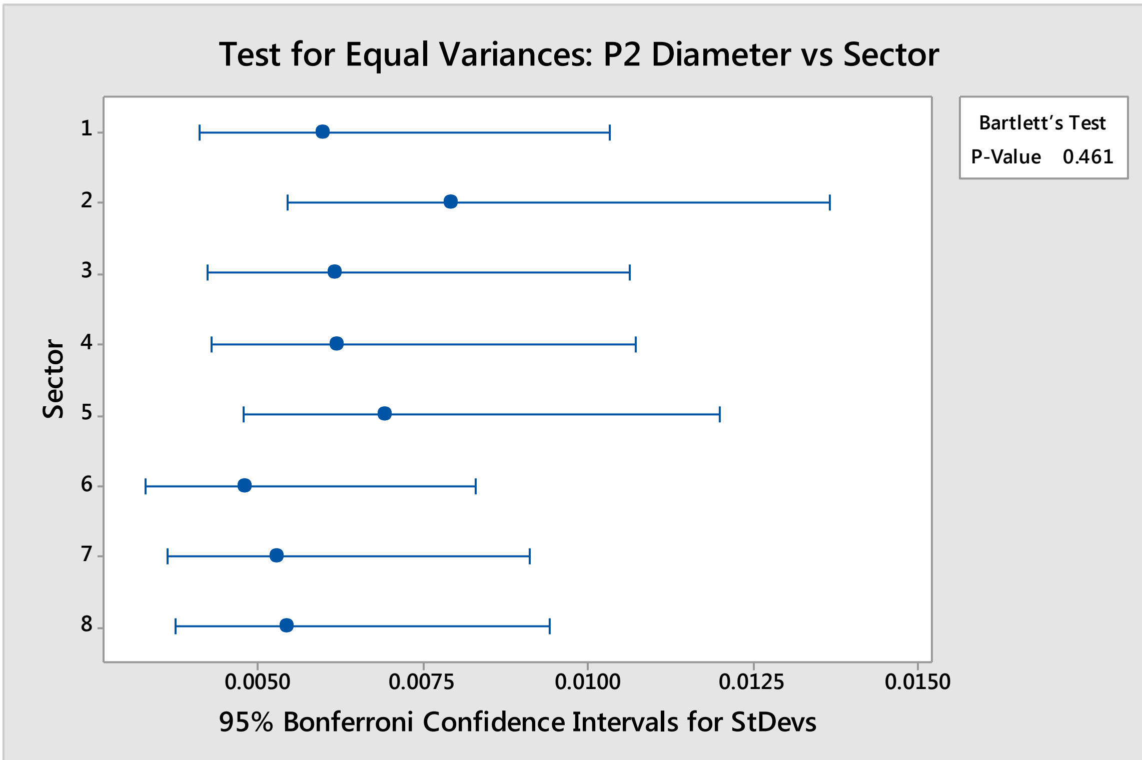

Figure 4.19: Plate 2 Variance Comparison Plot for Diameter vs. Sector

4.2.2.3 Band Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between bands are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if diameter variance was equal across bands for the plate. The null hypothesis was rejected because the resulting P-Value of .002 is less than the alpha value of .05. Figure 4.20 shows the comparison plot for diameter vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, results show that the alternate hypothesis was assumed meaning there is at least one pair of diameter variances that are significantly different.

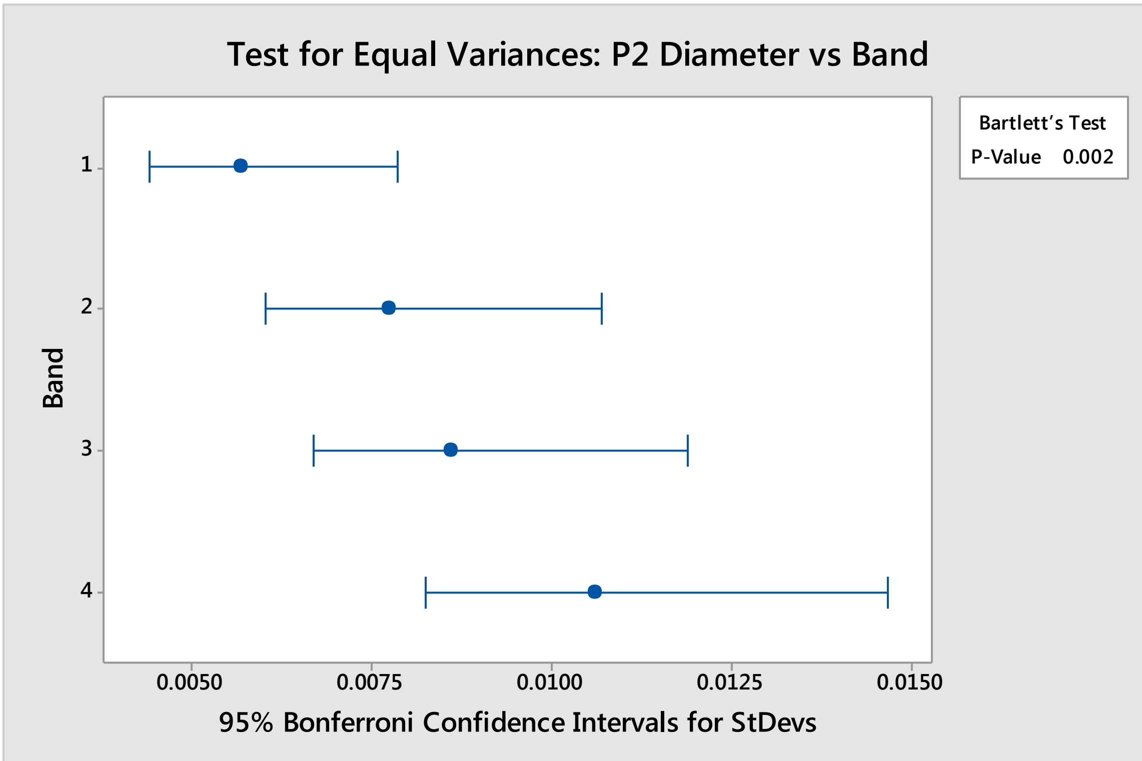

Figure 4.20: Plate 2 Variance Comparison Plot for Diameter vs. Band

4.2.2.4 General Linear Model

H_0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (i,j)

H_a: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (i,j)

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding mean diameters of measured parts. The analysis of diameter measurements for plate two indicate that bands and sectors individually were significant ($p<.05$), but the interaction term was not ($p>.05$). This can be seen in table 4.12.

Table 4.12: Plate 2 Diameter Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.005516	0.000788	22.91	0.000
Band	3	0.000384	0.000128	3.72	0.013
*Sector*Band*	21	0.000983	0.000047	1.36	0.150
Error	128	0.004403	0.000034		
Total	159	0.011285			

Since the p-value of the interaction term is larger than the alpha value of .05, the interaction was removed. A second General Linear Model was run with the interaction added to the error term. The interactions were added to the error as "lack of fit," but the total error is determined by combining the "lack of fit" data with the "pure error." The p values appear to remain unchanged because the data analysis in Minitab extends to three significant figures. The change is evident in the recalculation of the F-value. The summary ANOVA for the second General Linear Model can be seen in table 4.13.

Table 4.13: Plate 2 Diameter Summary ANOVA (Interaction Removed)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.005516	0.000788	21.80	0.000
Band	3	0.000384	0.000128	3.54	0.016
Error	149	0.005386	0.000036		
Lack of Fit	21	0.000983	0.000047	1.36	0.150
Pure Error	128	0.004403	0.000034		
Total	159	0.011285			

4.2.2.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant with respect to diameter. The results of the individual band and sector comparisons are shown in tables 4.14 and 4.15. Only the main effects are shown because the interactions were insignificant.

Table 4.14: Plate 2 Diameter Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P2 Diameter, Term = Sector			
Grouping Information Using the Tukey Method and 95% Confidence			
Sector	N	Mean	Grouping
6	20	2.9562	A
4	20	2.9555	A
7	20	2.9553	A
5	20	2.9546	A
3	20	2.9470	B
8	20	2.9467	B
1	20	0.9432	B C
2	20	2.9404	C
Means that do not share a letter are significantly different.			

Table 4.15: Plate 2 Diameter Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P2 Diameter, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
1	20	2.9518	A
2	20	2.9508	A B
3	20	2.9491	A B
4	20	2.9478	B
Means that do not share a letter are significantly different.			

The results of the Tukey testing show that the difference in means for diameter measurements on plate two are approximately divided in to two halves regarding sectors- meaning that the halves are independently similar, but when compared to each other they are significantly different. There is more consistency and less variation when compared to the height measurements. The difference in means regarding bands show that there is a significant difference as the part location moves from the inside of the plate to the outside.

4.2.3 Plate Two Z-Scores

Figures 4.21 and 4.22 show contour plots of the z-scores for the height and diameter measurements taken from plate two, respectively. This is a visual representation of the multiple comparisons and Tukey testing results. The height measurements are consistently larger than the plate average towards the left side of the plate and consistently smaller towards the right side of the plate. There is an evident divide between the two vertically across the center of the plate.

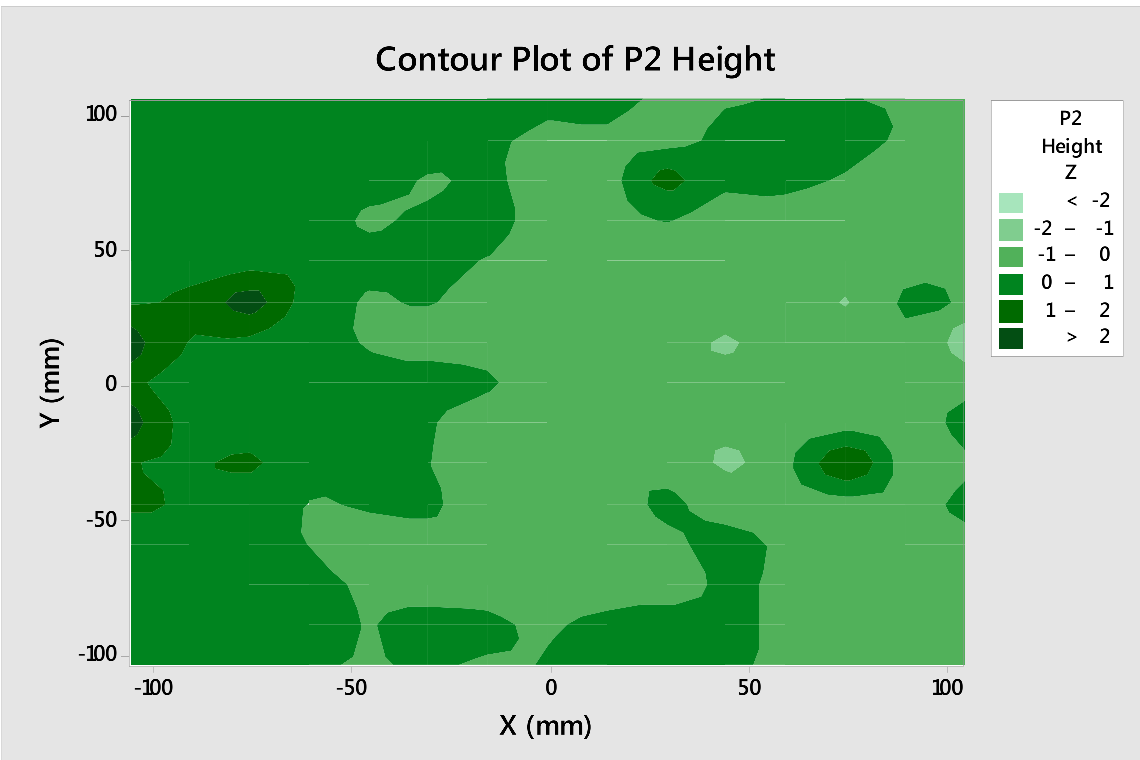

Figure 4.21: Contour Plot of Plate 2 Height

The diameter measurements show similar results but the dividing line across the plate is not the same as the line dividing the plate for height data. The diameter measurements are larger towards the bottom left corner of the plate and smaller towards the top right. There is a clear separation between the two sides which is confirmation of the multiple comparisons and Tukey testing.

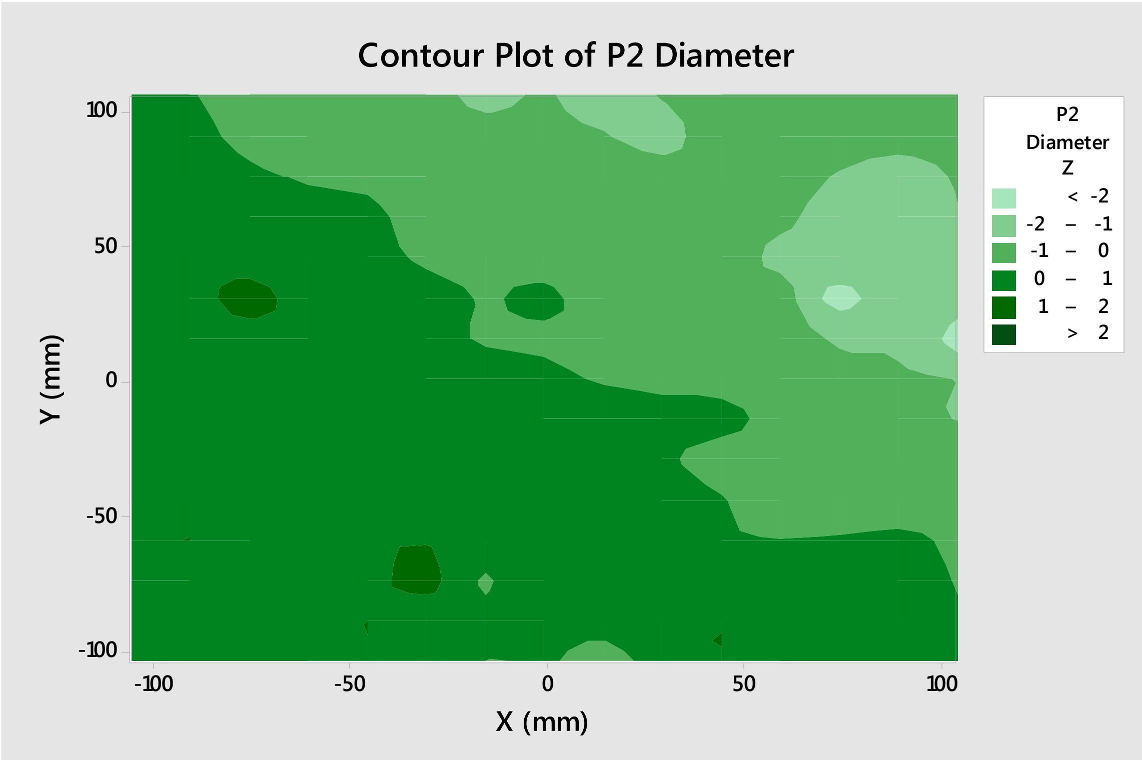

Figure 4.22: Contour Plot of Plate 2 Diameter

4.3 Plate Three

4.3.1 Height

4.3.1.1 Data Distribution

H_0: The height data is normally distributed

H_a: The height data is not normally distributed

The normal probability plot and histogram for the height data of plate three can be seen in figures 4.23 and 4.24. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of .673 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

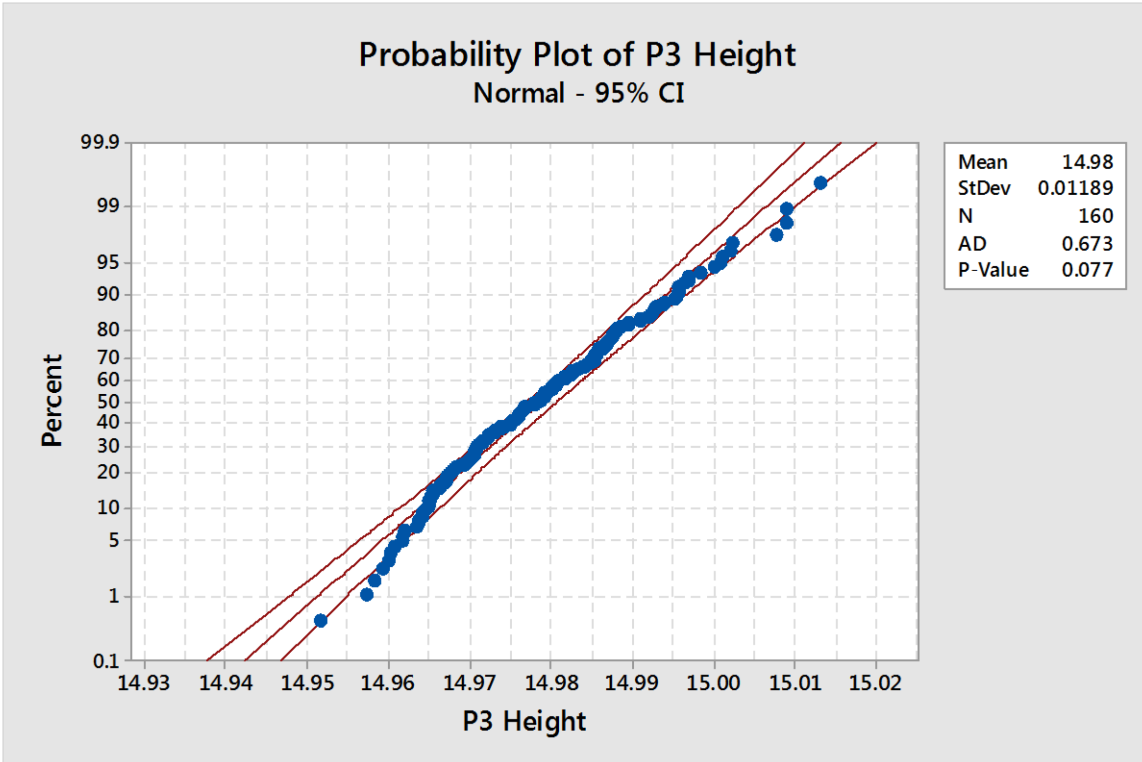

Figure 4.23: Plate 3 Height Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. Anderson-Darling testing was used because it applies more weight to the tails of a distribution which allows the conclusion to be drawn that the data does follow a normal distribution.

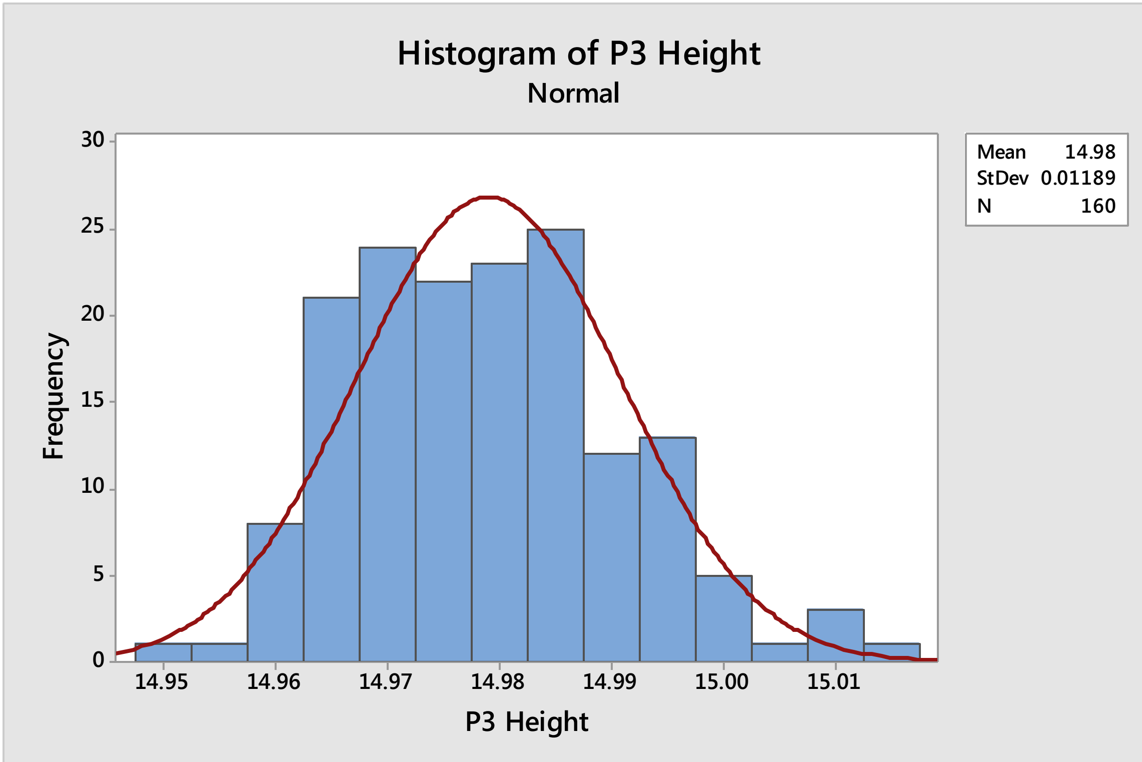

Figure 4.24: Plate 3 Height Histogram

4.3.1.2 Sector Variance

H$_0$: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between sectors are equal

H$_a$: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if height variance was equal across sectors for the plate. The null hypothesis was retained because the resulting P-Value of .096 is greater than the alpha value of .05. Figure 4.25 shows the comparison plot for height vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance are assumed to be equal.

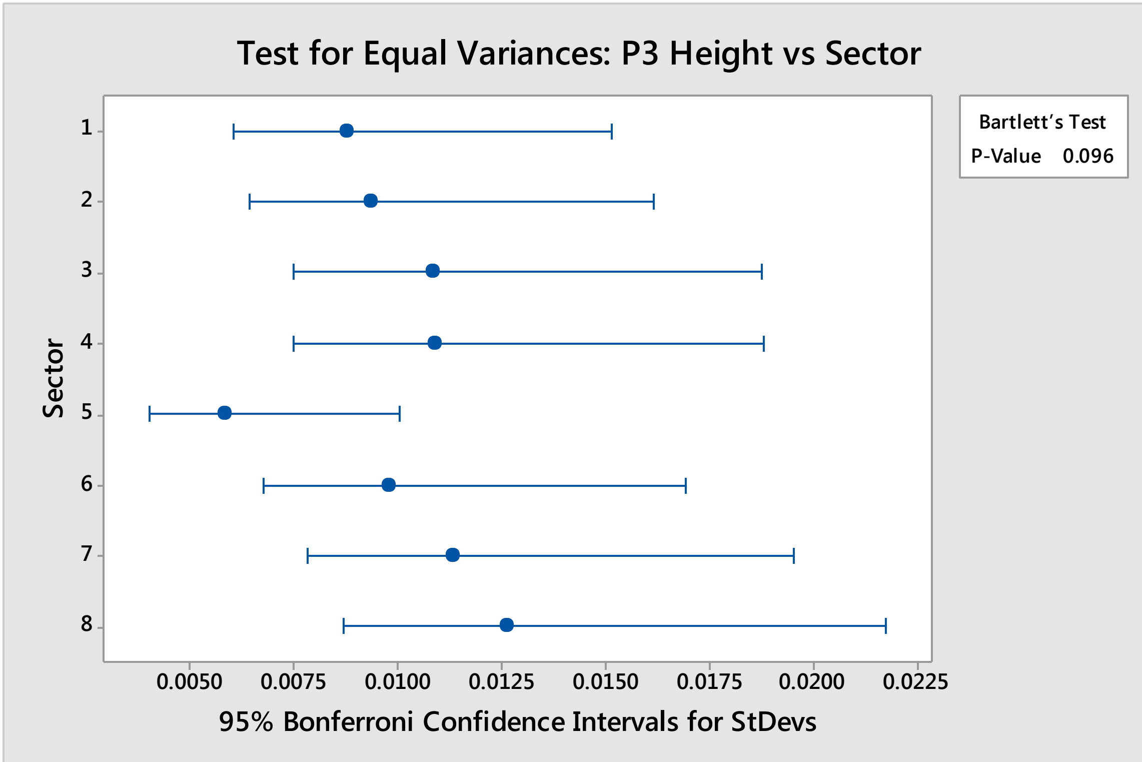

Figure 4.25: Plate 3 Variance Comparison Plot for Height vs. Sector

4.3.1.3 Band Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All height variance between bands are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if height variance was equal across bands for the plate. The null hypothesis was retained because the resulting P-Value of .380 is greater than the alpha value of .05. Figure 4.26 shows the comparison plot for height vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, demonstrating that the variance between them can be assumed to be equal.

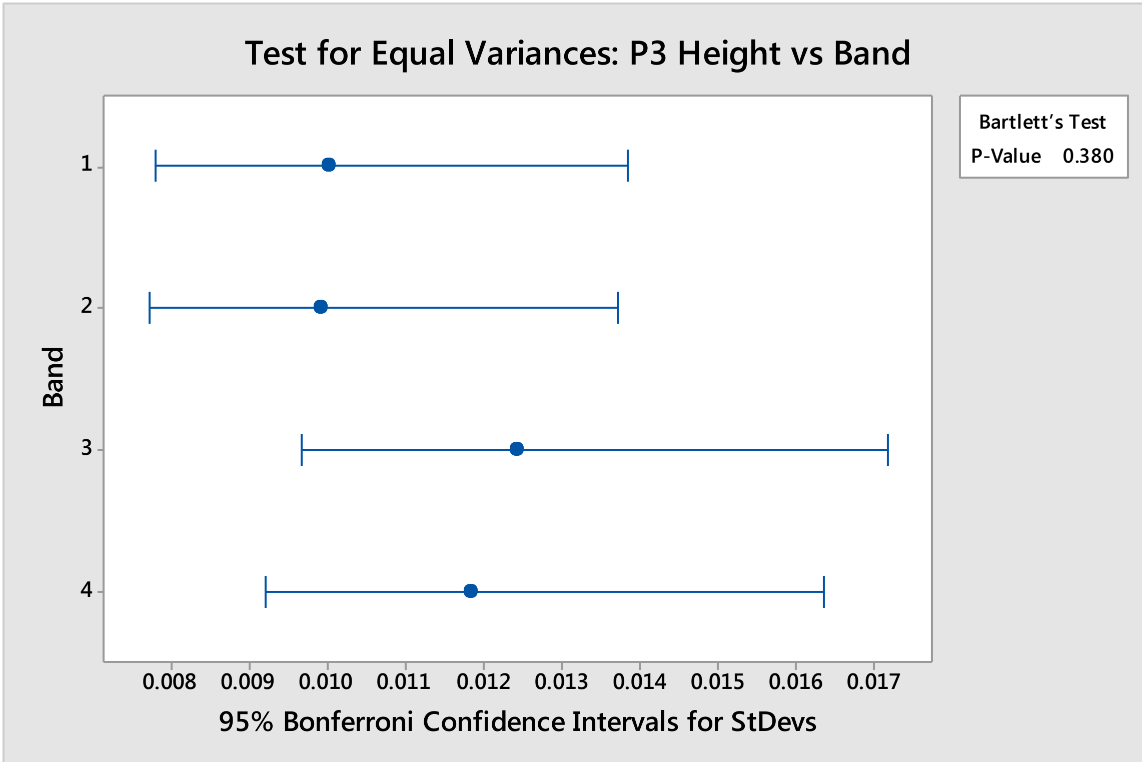

Figure 4.26: Plate 3 Variance Comparison Plot for Height vs. Band

4.3.1.4 General Linear Model

H0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (*i*,*j*)

Ha: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (*i*,*j*)

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding mean heights of measured parts. The analysis of height measurements for plate three indicate that bands and sectors individually were significant ($p<.05$), but the interaction term was not ($p>.05$). This can be seen in table 4.16.

Table 4.16: Plate 3 Height Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.006923	0.000989	12.16	0.000
Band	3	0.003217	0.001072	13.18	0.000
*Sector*Band*	21	0.001907	0.000091	1.12	0.340
Error	128	0.010413	0.000081		
Total	159	0.022460			

Since the p-value of the interaction term is larger than the alpha value of .05, the interaction was removed. A second General Linear Model was run with the interaction added to the error term. The interactions were added to the error as "lack of fit," but the total error is determined by combining the "lack of fit" data with the "pure error." The p values appear to remain unchanged because the data analysis in Minitab extends to three significant figures, however the change is evident in the recalculation of the F-value. The summary ANOVA for the second General Linear Model can be seen in table 4.17.

Table 4.17: Plate 3 Diameter Summary ANOVA (Interaction Removed)

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.006923	0.000989	11.96	0.000
Band	3	0.003217	0.001072	12.97	0.000
Error	149	0.012320	0.000083		
Lack of Fit	21	0.001907	0.000091	1.12	0.340
Pure Error	128	0.010413	0.000081		
Total	159	0.022460			

4.3.1.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant. The results of the individual band and sector comparisons are shown in tables 4.18 and 4.19.

Table 4.18: Plate 3 Height Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P3 Height, Term = Sector			
Grouping Information Using the Tukey Method and 95% Confidence			
Sector	N	Mean	Grouping
7	20	14.9880	A
8	20	14.9845	A B
6	20	14.9842	A B
1	20	14.9839	A B
4	20	14.9759	B C
3	20	14.9743	C
2	20	4.9728	C
5	20	14.9683	C
Means that do not share a letter are significantly different.			

The comparison for both sectors show that the grouping of height measurements for plate three are very similar to plate one in that the means of the sector height measurements at the top left of the plate are similar, the means of the sector height measurements at the bottom right of the plate are similar, but the two groupings are statistically significantly different from each other. The band Tukey test results can be seen in table 4.19. These results show that the grouping of bands change as the measurement locations moves from the outside of the plate to the inside.

Table 4.19: Plate 3 Height Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P3 Height, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
4	20	15.0174	A
3	20	15.0120	A B
2	20	15.0089	B C
1	20	15.0085	C
Means that do not share a letter are significantly different.			

4.3.2 Diameter

4.3.2.1 Data Distribution

The normal probability plot and histogram for the diameter data of plate three can be seen in figures 4.27 and 4.28. These data show to be normally distributed and the null hypothesis is retained because the Anderson-Darling value of .893 (shown as AD on the normal probability plot) is greater than the alpha value of .05. The histogram is a visual representation of the distribution of the data compared to a normal distribution curve.

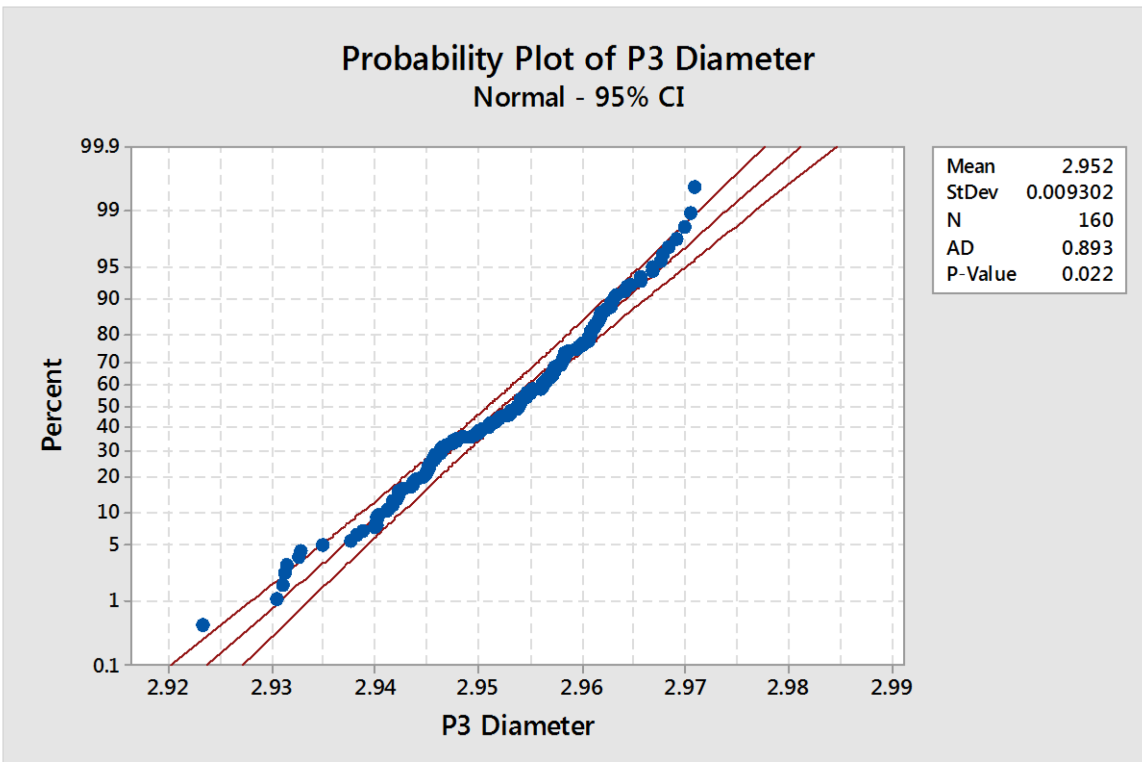

Figure 4.27: Plate 3 Diameter Normal Probability Plot

Data that is exactly normally distributed will follow a straight line on a probability plot. These data show that the distribution is approaching normal with possible outliers. Anderson-Darling testing was used because it applies more weight to the tails of a distribution which allows the conclusion to be drawn that the data does follow a normal distribution.

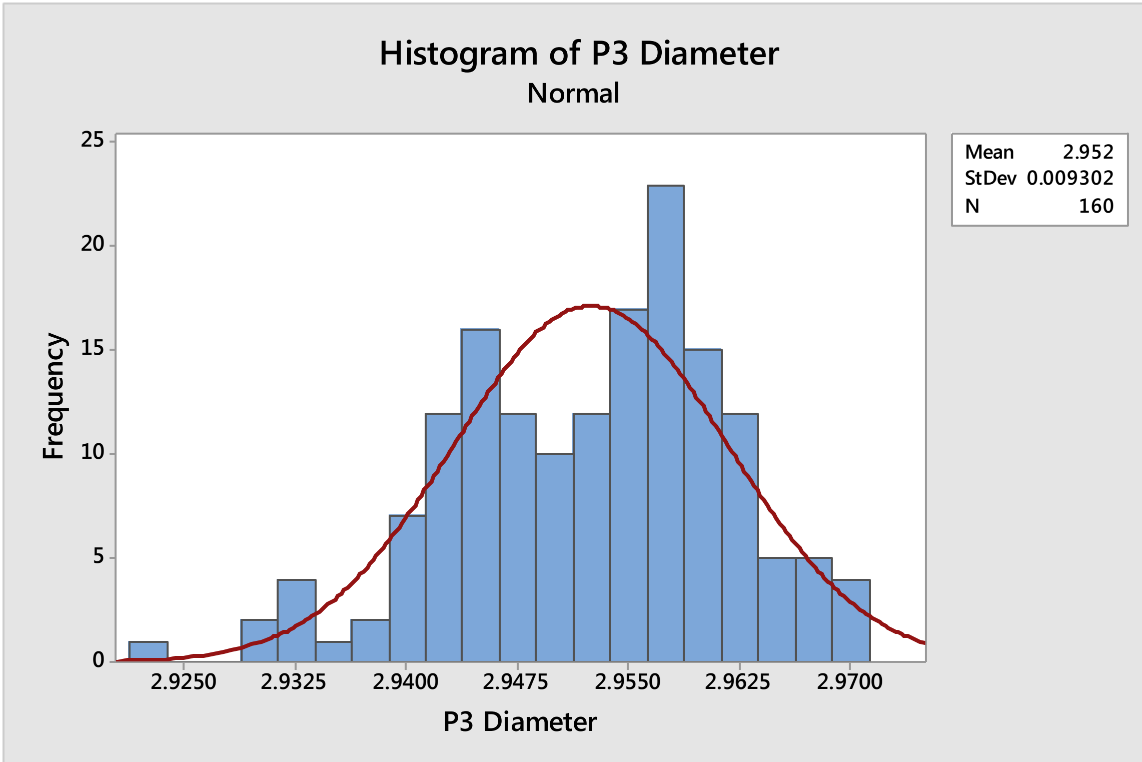

Figure 4.28: Plate 3 Diameter Histogram

4.3.2.2 Sector Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between sectors are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one sector variance is different

The Bartlett test was used to determine if diameter variance was equal across sectors for the plate. The null hypothesis was retained because the resulting P-Value of .161 is greater than the alpha value of .05. Figure 4.29 shows the comparison plot for diameter vs. sector. This is a visual representation showing how the interval for each sector overlaps with the intervals for other sectors, indicating that the variance between them are assumed to be equal.

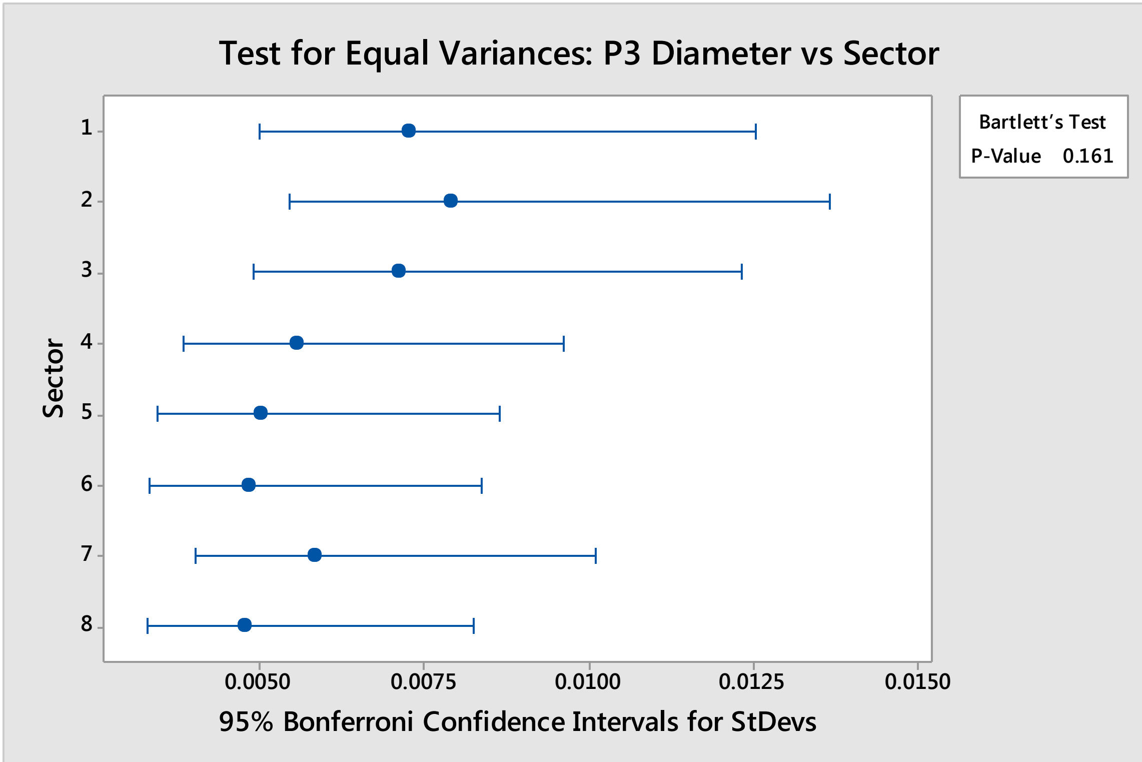

Figure 4.29: Plate 3 Variance Comparison Plot for Diameter vs. Sector

4.3.2.3 Band Variance

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

All diameter variance between bands are equal

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (i,j)

At least one band variance is different

The Bartlett test was used to determine if diameter variance was equal across bands for the plate. The null hypothesis was rejected because the resulting P-Value of .002 is less than the alpha value of .05. Figure 4.30 shows the comparison plot for diameter vs. band. This is a visual representation showing how the interval for each band overlaps with the intervals for other bands, results show that the alternate hypothesis was assumed meaning there is at least one pair of diameter variances that are significantly different.

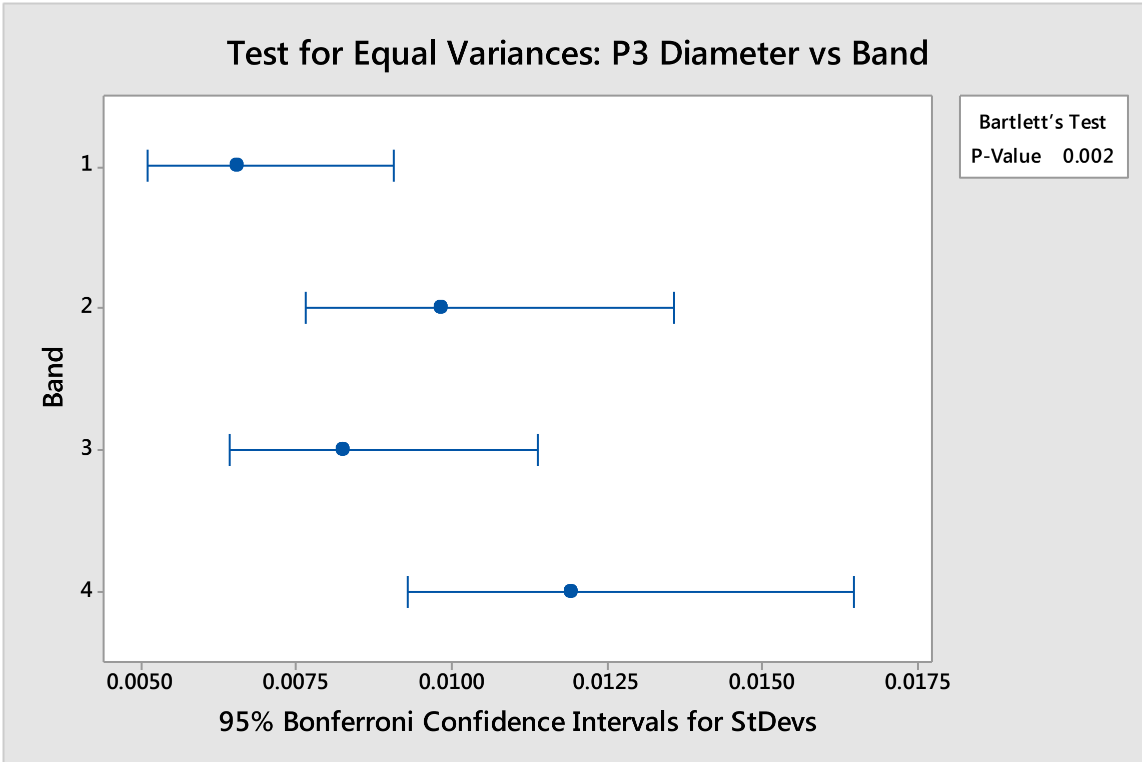

Figure 4.30: Plate 3 Variance Comparison Plot for Diameter vs. Band

4.3.2.4 General Linear Model

$$H_0: \bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)} \text{ for all } (i,j)$$

$$H_a: \bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)} \text{ for at least one pair } (i,j)$$

A two factor, multiple level General Linear Model was generated to determine if sectors, bands, or the interaction between the two were statistically significant regarding variation of diameter. The analysis of diameter measurements for plate three indicate that each was significant. The General Linear Model produces a regression equation, but will not be used in this study since only categorical comparisons were made. The ultimate goal of the General Linear Model was to determine if the main effects and interaction between sectors and bands is statistically significant through a two way ANOVA. The summary ANOVA can be seen in table 4.20.

Table 4.20: Plate 3 Diameter Summary ANOVA

Summary ANOVA					
Source	*DF*	*Adj SS*	*Adj MS*	*F-Value*	*P-Value*
Sector	7	0.007995	0.001142	33.69	0.000
Band	3	0.000119	0.000040	1.17	0.325
*Sector*Band*	21	0.001304	0.000062	1.83	0.022
Error	128	0.004339	0.000034		
Total	159	0.013757			

The results of the ANOVA show that sectors (p<.05), bands (p<.05), and sector/band interaction (p<.05) were significant for plate one with respect to diameter.

4.3.2.5 Multiple Comparison

A Tukey Pairwise comparison was run to determine which sectors have similar means, which bands have similar means, and which sector/band interactions are statistically significant. The results of the individual band and sector comparisons are shown in tables 4.21 and 4.22.

Table 4.21: Plate 3 Diameter Tukey Sector Comparison

Tukey Pairwise Comparisons: Response = P3 Diameter, Term = Sector				
Grouping Information Using the Tukey Method and 95% Confidence				
Sector	N	Mean	Grouping	
5	20	2.9611	A	
6	20	2.9604	A	
7	20	2.9583	A	
4	20	2.9568	A	
3	20	2.9489	B	
8	20	2.9475	B	C
1	20	2.9436	B	C
2	20	2.9427		C
Means that do not share a letter are significantly different.				

Table 4.22: Plate 3 Diameter Tukey Band Comparison

Tukey Pairwise Comparisons: Response = P3 Diameter, Term = Band			
Grouping Information Using the Tukey Method and 95% Confidence			
Band	N	Mean	Grouping
1	20	2.9532	A
2	20	2.9530	A
3	20	2.9524	A
4	20	2.9510	A
Means that do not share a letter are significantly different.			

The comparison for sectors and bands show that one half of plate one is significantly different than the other half. The means for sectors five, six, seven and four are equal. Sectors three, eight, one and two are similar, but sector three is significantly different than sector two. The means for all bands are equal.

Figure 4.31shows the significant interactions between sectors and Figure 4.32 shows the significant interactions between bands. The interaction is considered significant if the interval of the comparison does not contain a zero. Regarding height, the significantly different sectors were 4-1, 5-1, 6-1, 7-1, 3-2, 4-2, 5-2, 6-2, 7-2, 4-3, 5-3, 6-3, 7-3, 8-4, 8-5, 8-6, and 8-7. A trend can be noticed that most comparisons containing sectors four, five, six and seven are different. All band comparisons were found to be not statistically significant. These interactions can also be seen in figure 4.33, which shows the means for each sector and the means for each band.

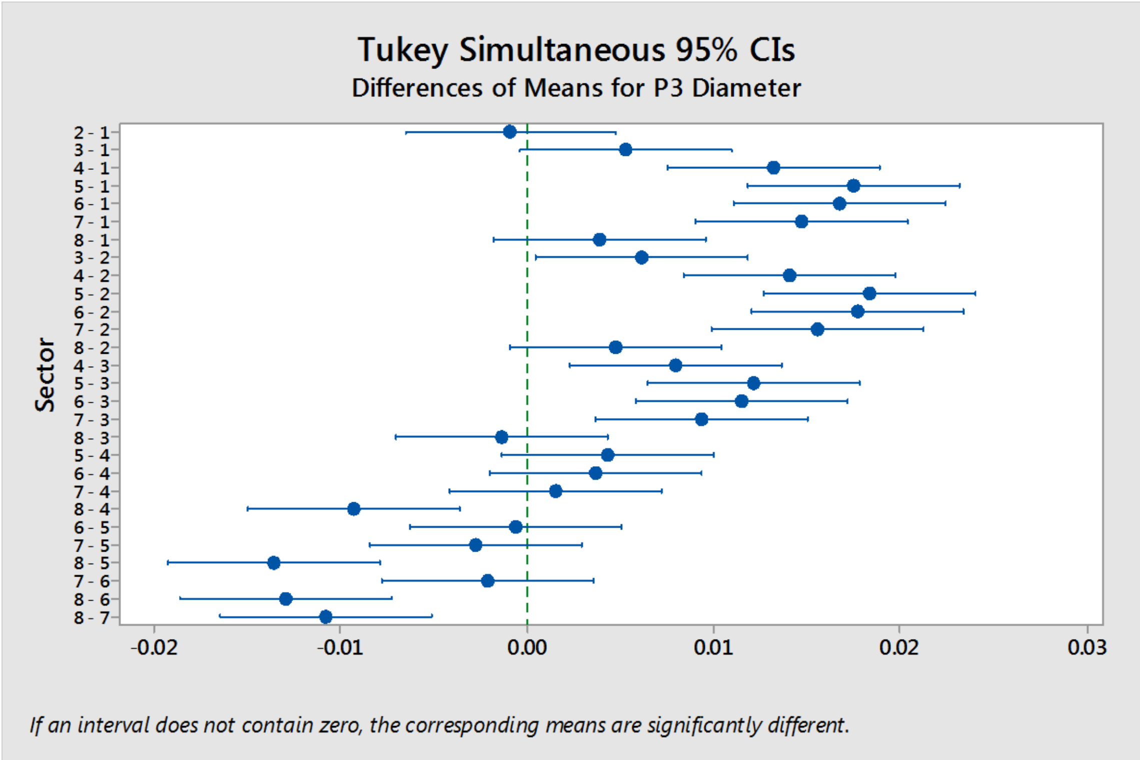

Figure 4.31: Plate 3 Diameter Sector Interaction Comparison

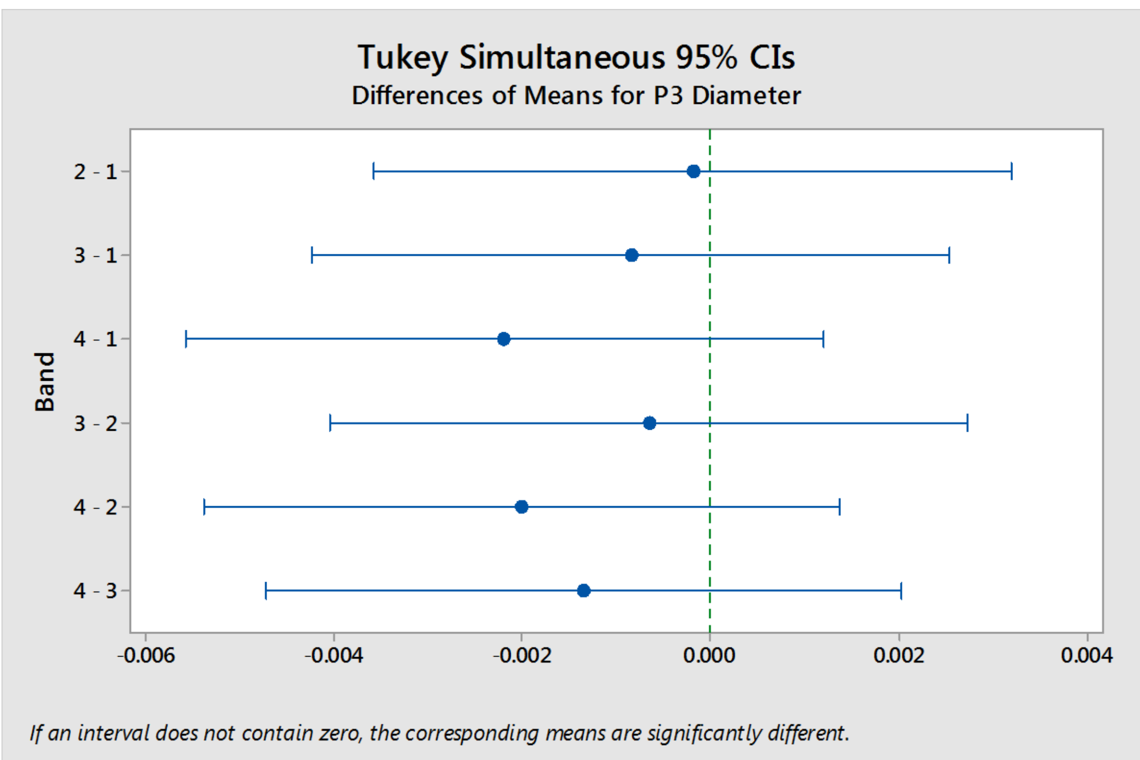

Figure 4.32: Plate 3 Diameter Band Interaction Comparison

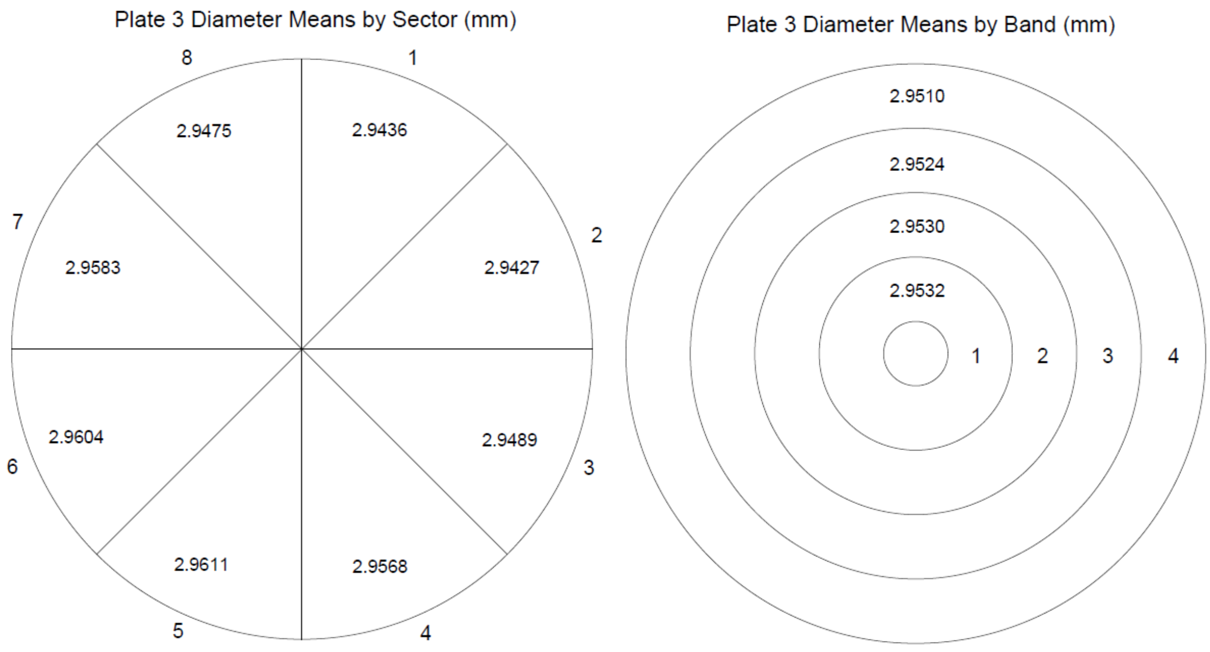

Figure 4.33: Plate 3 Diameter Means by Sector and Band

4.3.3 Plate Three Z-Scores

Figures 4.34 and 4.35 show contour plots of the z-scores for the height and diameter measurements taken from plate three, respectively. This is a visual representation of the multiple comparisons and Tukey testing results. The height measurements are consistently larger than the plate average towards the top left of the plate and consistently smaller towards the bottom right of the plate. There is an evident divide between the two across the center of the plate, from bottom left to top right.

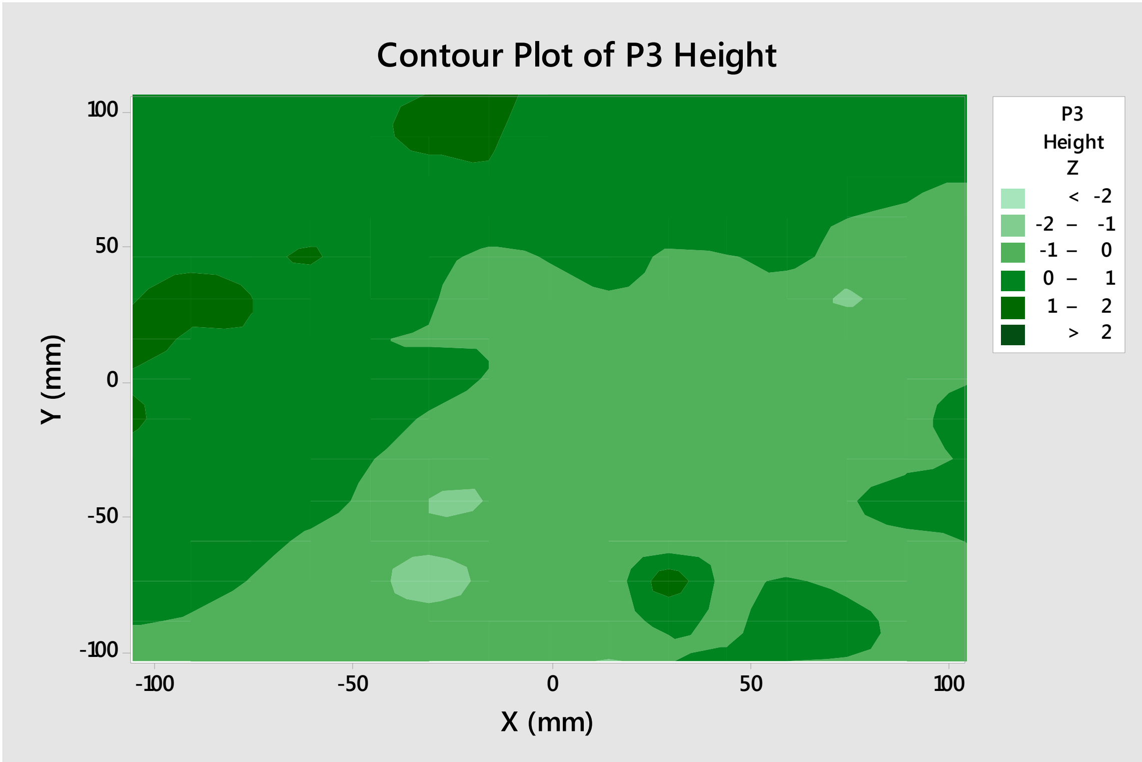

Figure 4.34: Contour Plot of Plate 3 Height

The diameter measurements show similar results but the dividing line across the plate is not the same as the line dividing the plate for height data. The diameter measurements are larger towards the bottom left corner of the plate and smaller towards the top right.

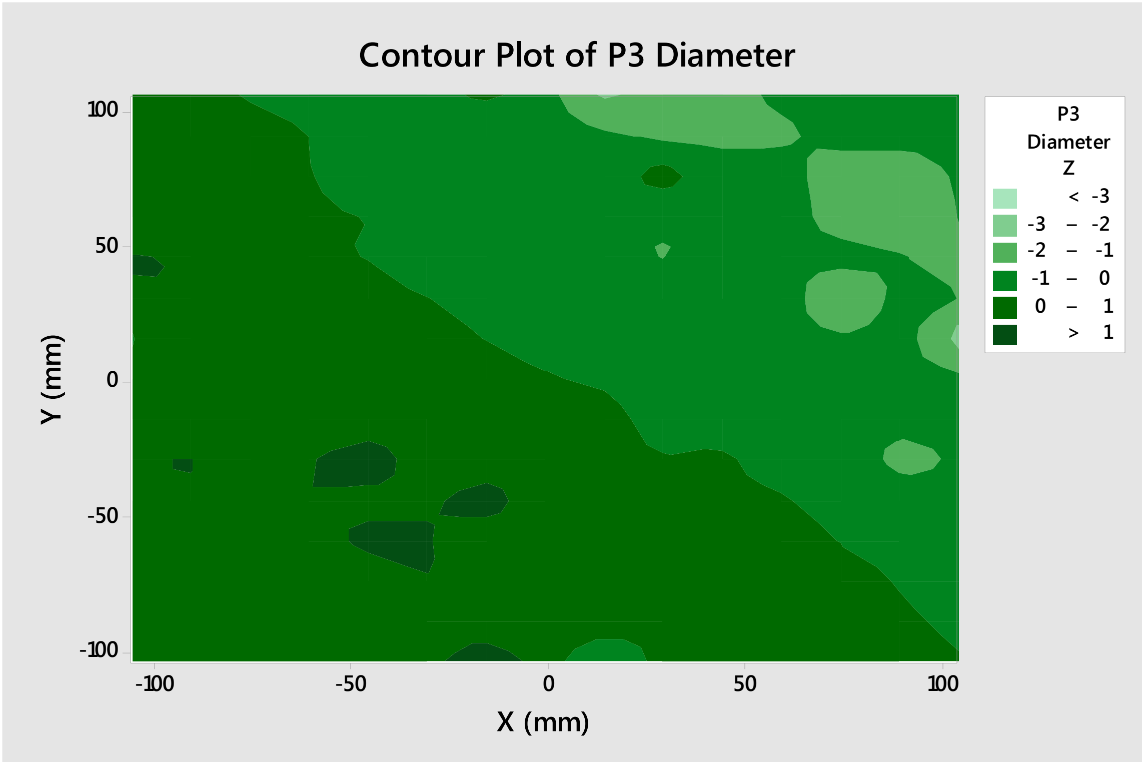

Figure 4.35: Contour Plot of Plate 3 Diameter

4.4 Summary

The goal of the study was to characterize the process capability of an EOS M290 SLM machine with respect to build plate location for 316L stainless steel. Distribution and variance testing were performed, as well as running ANOVA statistical General Linear Models for two-way main effects and interaction of plate bands and sectors for three different build plates. The purpose of this testing was to give EOS M290 users a better understanding of how build plate location affects part geometry capability related to height and diameter. The parts between plates were not considered replicates since some factors and parameters in the machine setup could not be controlled or repeated for each build and could contribute to variation between plates, however trends can be observed across the three plates. The discussion of these result, conclusions and recommendations for future work are presented in Chapter V.

CHAPTER 5: CONCLUSION AND FUTURE WORK

The goal of the study was to characterize the process capability of an EOS M290 SLM machine with respect to build plate location for 316L stainless steel. Parts were created using and EOS M290 SLM machine at specific points on three build plates. Height and diameter measurements were taken on each part using a Zeiss Contura HTG CMM. These data were used for statistical analysis. ANOVA statistical General Linear Models were run for two-way main effects and interaction of plate bands and sectors for each plate. This chapter presents the conclusions from the analysis in Chapter IV along with recommended future research.

Height and diameter measurements of 160 parts were taken from each of three build plates. These plates were not considered replicates because the means for heights and diameters were not equal for each plate. This variation could be due to pre-build variation that could not be controlled. The statistical tests performed were to test for a normal distribution, equal variance across sectors and equal variance across sectors for both height and diameter data. A General Linear Model was then run to determine if the means between sectors and the means between bands were significantly different, and also determined if the interaction between the two was significant. If the interaction was not significant it was removed from the model. A multiple comparison was then performed to determine which sectors or bands were different. Contour plots of z-scores were generated as a visual representation of variation. All statistical testing was performed using Minitab 17.

5.1 Discussion of Results from Statistical Testing

The normal distribution testing revealed that the measurements taken for height and diameter fit a normal distribution for each plate. Anderson-Darling tests were used for this because it applies more weight to the tails of the distribution. The null hypothesis that the data fits a normal distribution was retained for all plates, for both height and diameter. This was important because assignable causes regarding pre-build variation could not be identified or controlled for outlier elimination. Determining that the data was normally distributed allowed for the use of parametric methods for statistical analysis.

The second set of statistical tests performed was to test for equal variance between each band, and separately between each sector. These tests were performed on height measurements and again on diameter measurements for each plate. This was done using the Bartlett test statistic. The results of the variance testing showed that all variance between sectors were equal, and all variance between bands were equal; with the exception of the diameter measurements between bands of plate two and plate three. The comparison plot for diameter vs. band on plate two shows that the variance between bands one and four is not equal because the intervals do not overlap. The Bartlett test revealed the same results for the diameter measurements on plate three regarding bands. The comparison plots for each can be seen below in figures 5.1 and 5.2.

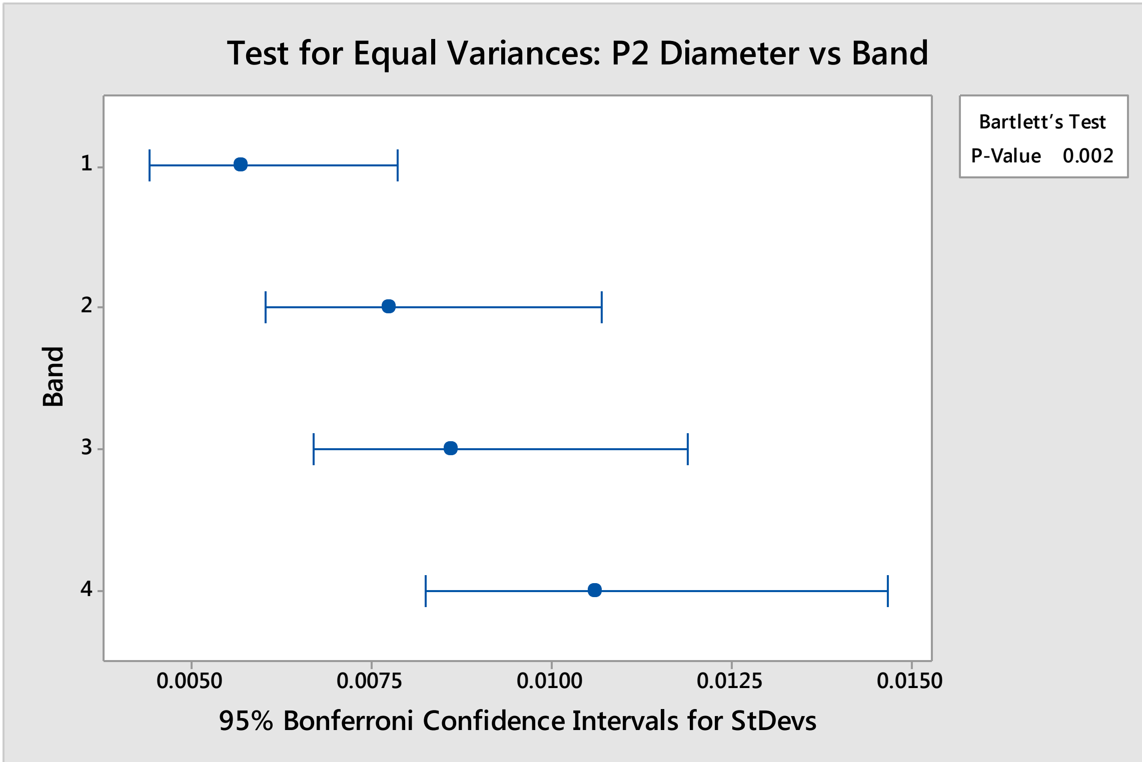

Figure 5.1: Plate 2 Variance Comparison Plot for Diameter vs. Band

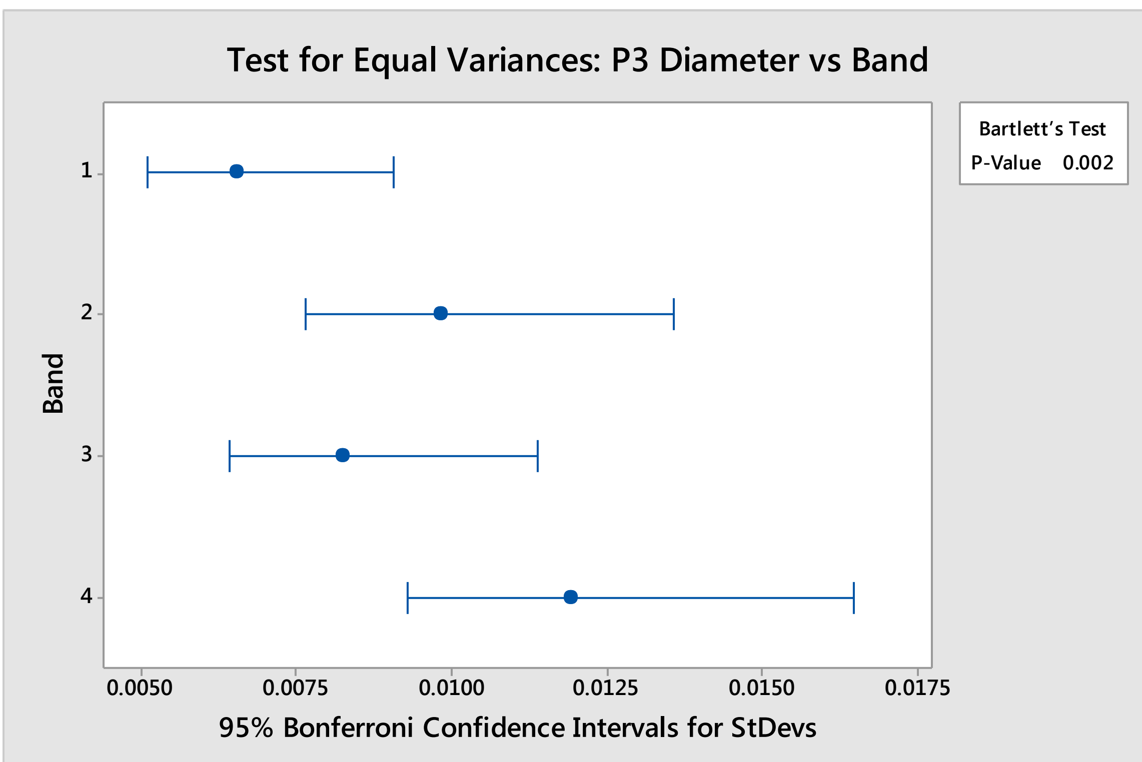

Figure 5.2: Plate 3 Variance Comparison Plot for Diameter vs. Band

For variance testing the null hypothesis was $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$; meaning the variance of diameter measurements for band one is equal to the variance of diameter measurements of band two, through band *k*. The alternate hypothesis was $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (*i*,*j*); meaning that at least one pair of variances is different. For each of these the null hypothesis was rejected and the alternate hypothesis was assumed.

A General Linear Model was then run to determine if there was a significant difference of means between each band, each sector, or if the interaction between bands and sectors was significant. This was done by a two way ANOVA. The results of the ANOVA's showed that there was a difference in means for bands and sectors on all three plates, with the exception of the diameter measurements on plate three. The GLM for these measurements showed that there was no difference between bands. The interaction between sectors and bands were significant on the height measurements for plate one and the diameter measurements on plate three. Table 5.1 shows the results of the hypothesis testing for distribution, variance and the General Linear Model. The null and alternate hypotheses for each test have been restated.

Distribution Testing Hypotheses

H_0: The data is normally distributed

H_a: The data is not normally distributed

Variance Testing Hypotheses

H_0: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$

H_a: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair (*i*,*j*)

General Linear Model Hypotheses

H_0: $\bar{x}_{sector(i)band(j)} = \bar{x}_{sector(i)band(j)}$ for all (*i*,*j*)

H_a: $\bar{x}_{sector(i)band(j)} \neq \bar{x}_{sector(i)band(j)}$ for at least one pair (*i*,*j*)

Table 5.1: Hypothesis Testing Summary

					GLM		
		Normal Distribution	Sector Variance	Band Variance	Sectors	Bands	Interaction
Plate 1	Height	Null Retained	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Alternate Assumed
	Diameter	Null Retained	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Null Retained
Plate 2	Height	Null Retained	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Null Retained
	Diameter	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Alternate Assumed	Null Retained
Plate 3	Height	Null Retained	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Null Retained
	Diameter	Null Retained	Null Retained	Alternate Assumed	Alternate Assumed	Null Retained	Alternate Assumed

Each plate showed significant differences between bands and sectors with the exception of the diameter measurements for bands on plate three, as shown in Table 5.1 above. This suggests that the repeatability of the machine is higher when parts are placed closely on the build plate. Interactions between sectors and bands trend toward insignificant. Overall the results suggest that there is no significant interaction between sectors and bands, assuming the critical dimensions of the part are either perpendicular or parallel to the build plate surface. This study does not include analysis of the interaction of X-Y vs. Z capability of the EOS M290.

5.2 Future Work

The future work recommended is based on knowledge obtained throughout conducting this study. Factors were discovered that require further exploration to qualify and quantify the impact they have on part production by the EOS M290.

For this study 3D solid models were created in Creo 2.0. These parts were then put into an assembly to control build plate placement, which resulted in a single file containing 160 parts for each individual build. The pre-processing software for the EOS M290 treated all 160 pieces as a single part, as well as performed the build as if the 160 pieces were a single part. Using the default settings for the 316L stainless steel parameter set, on each layer the laser scans the interior surfaces and then does a profile scan prior to moving to the next part on the build plate. Using the single assembly file in this study, the laser scanned the interior surfaces for all parts before performing the profile scan. This may have resulted in unequal cooling rates for each part, which may attribute to variation in geometry. It is recommended that each part be imported to the build pre-processing software, then define plate location individually. This would allow for more equal treatment for each part across the build plate. The thermal characteristics that are inherent to the process are also suggested for future study.

There are unknown thermal processes that occur in the build material during each scan cycle of the metal SLM process. The laser heats the material to a melting point and it is fused to the layer below it. During this process there is heat transfer to each previous layer that happens during each scan. This heat transfer is repeated and reduced with each successive build layer and results in a heating and cooling cycle that is dependent on scan cycle time and part size. This occurs in each part on the build plate, which could introduce warping and internal stresses.

Part size could also impact machine capability. This study was performed on parts with a 3 mm diameter and a total height of 15 mm. The impact of the thermal characteristics could affect dimensional precision. It is recommended that a Design of Experiments be conducted that includes part size as a factor in dimensional capability along with high level parameter settings in the build setup. These build parameters include settings such layer thickness, the number of times each build layer is scanned by the laser (single vs. double) and direction of part build order across the plate (front to back vs. back to front).

This study did not encompass assignable causes for the variation of parts due to plate placement. While many factors may impact part geometry, one possible cause of this could be a parallax effect that is created between the part on the build plate and the laser scan head as the parts are placed closer to the edge of the plate. Identifying the impact of this parallax effect would contribute towards increasing the capability of an EOS M290 for the users.

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APPENDIX A

The following is Minitab output for plate one height variance testing.

Test for Equal Variances: P1 Height versus Sector

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Sector   N      StDev            CI
     1  20  0.0080894  (0.0055646, 0.0139659)
     2  20  0.0080263  (0.0055212, 0.0138570)
     3  20  0.0128467  (0.0088372, 0.0221792)
     4  20  0.0108537  (0.0074662, 0.0187383)
     5  20  0.0132618  (0.0091227, 0.0228959)
     6  20  0.0113417  (0.0078019, 0.0195808)
     7  20  0.0156292  (0.0107512, 0.0269830)
     8  20  0.0111758  (0.0076878, 0.0192945)

Individual confidence level = 99.375%


Tests

              Test
Method    Statistic  P-Value
Bartlett      13.32    0.065
```

Test for Equal Variances: P1 Height versus Band

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Band   N      StDev            CI
   1  40  0.0129174  (0.0100436, 0.0178446)
   2  40  0.0167090  (0.0129917, 0.0230824)
   3  40  0.0170833  (0.0132827, 0.0235995)
   4  40  0.0164739  (0.0128089, 0.0227577)

Individual confidence level = 98.75%
```

Tests

Method	Test Statistic	P-Value
Bartlett	3.63	0.305

The following is Minitab output for the plate one height General Linear Model.

General Linear Model: P1 Height versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.023077	0.003297	33.38	0.000
Band	3	0.004357	0.001452	14.70	0.000
Sector*Band	21	0.003643	0.000173	1.76	0.030
Error	128	0.012641	0.000099		
Total	159	0.043719			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0099378	71.08%	64.08%	54.82%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	15.0001	0.0008	19092.60	0.000	
Sector					
1	0.00871	0.00208	4.19	0.000	1.75
2	-0.00909	0.00208	-4.38	0.000	1.75
3	-0.01663	0.00208	-8.00	0.000	1.75
4	-0.01017	0.00208	-4.89	0.000	1.75
5	-0.01032	0.00208	-4.96	0.000	1.75
6	0.00838	0.00208	4.03	0.000	1.75
7	0.01723	0.00208	8.29	0.000	1.75
Band					
1	-0.00558	0.00136	-4.10	0.000	1.50
2	-0.00411	0.00136	-3.02	0.003	1.50
3	0.00220	0.00136	1.61	0.109	1.50
Sector*Band					
1 1	0.00072	0.00360	0.20	0.842	2.63
1 2	0.00415	0.00360	1.15	0.251	2.63
1 3	-0.00037	0.00360	-0.10	0.919	2.63
2 1	0.00729	0.00360	2.02	0.045	2.62
2 2	-0.00133	0.00360	-0.37	0.712	2.62

```
  2 3         0.00152  0.00360      0.42    0.673  2.63
  3 1        -0.00414  0.00360     -1.15    0.253  2.63
  3 2        -0.00277  0.00360     -0.77    0.443  2.62
  3 3         0.00288  0.00360      0.80    0.425  2.63
  4 1         0.00647  0.00360      1.80    0.075  2.63
  4 2        -0.00398  0.00360     -1.11    0.271  2.62
  4 3        -0.00606  0.00360     -1.68    0.095  2.63
  5 1         0.00940  0.00360      2.61    0.010  2.63
  5 2        -0.00206  0.00360     -0.57    0.567  2.63
  5 3        -0.00526  0.00360     -1.46    0.146  2.63
  6 1        -0.00672  0.00360     -1.87    0.064  2.63
  6 2         0.00396  0.00360      1.10    0.273  2.63
  6 3         0.00621  0.00360      1.73    0.087  2.63
  7 1        -0.00992  0.00360     -2.75    0.007  2.63
  7 2        -0.00026  0.00360     -0.07    0.943  2.63
  7 3         0.00290  0.00360      0.81    0.422  2.62


Regression Equation

P1 Height = 15.0001 + 0.00871 Sector_1 - 0.00909 Sector_2 - 0.01663 Sector_3
            - 0.01017 Sector_4 - 0.01032 Sector_5 + 0.00838 Sector_6
+ 0.01723 Sector_7
            + 0.01189 Sector_8 - 0.00558 Band_1 - 0.00411 Band_2 + 0.00220 Band_3
            + 0.00749 Band_4 + 0.00072 Sector*Band_1 1 + 0.00415 Sector*Band_1 2
            - 0.00037 Sector*Band_1 3 - 0.00450 Sector*Band_1 4
+ 0.00729 Sector*Band_2 1
            - 0.00133 Sector*Band_2 2 + 0.00152 Sector*Band_2 3
- 0.00748 Sector*Band_2 4
            - 0.00414 Sector*Band_3 1 - 0.00277 Sector*Band_3 2
+ 0.00288 Sector*Band_3 3
            + 0.00403 Sector*Band_3 4 + 0.00647 Sector*Band_4 1
- 0.00398 Sector*Band_4 2
            - 0.00606 Sector*Band_4 3 + 0.00357 Sector*Band_4 4
+ 0.00940 Sector*Band_5 1
            - 0.00206 Sector*Band_5 2 - 0.00526 Sector*Band_5 3
- 0.00207 Sector*Band_5 4
            - 0.00672 Sector*Band_6 1 + 0.00396 Sector*Band_6 2
+ 0.00621 Sector*Band_6 3
            - 0.00345 Sector*Band_6 4 - 0.00992 Sector*Band_7 1
- 0.00026 Sector*Band_7 2
            + 0.00290 Sector*Band_7 3 + 0.00727 Sector*Band_7 4
- 0.00311 Sector*Band_8 1
            + 0.00230 Sector*Band_8 2 - 0.00183 Sector*Band_8 3
+ 0.00263 Sector*Band_8 4


Fits and Diagnostics for Unusual Observations

Obs  P1 Height      Fit    Resid  Std Resid
 25    14.9734  14.9928  -0.0194      -2.18  R
 60    14.9745  14.9950  -0.0206      -2.31  R
 63    15.0139  14.9909   0.0231       2.59  R
 90    15.0024  14.9837   0.0188       2.11  R
135    14.9999  15.0225  -0.0226      -2.54  R

R  Large residual
```

The following is Minitab output for plate one diameter variance testing.

Test for Equal Variances: P1 Diameter versus Sector

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Sector   N      StDev            CI
     1  20  0.0073221  (0.0050369, 0.0126413)
     2  20  0.0077491  (0.0053306, 0.0133785)
     3  20  0.0071860  (0.0049432, 0.0124063)
     4  20  0.0087678  (0.0060314, 0.0151372)
     5  20  0.0061814  (0.0042522, 0.0106719)
     6  20  0.0053359  (0.0036705, 0.0092121)
     7  20  0.0064767  (0.0044553, 0.0111816)
     8  20  0.0064032  (0.0044047, 0.0110548)

Individual confidence level = 99.375%


Tests

               Test
Method    Statistic  P-Value
Bartlett       6.06    0.533
```

Test for Equal Variances: P1 Diameter versus Band

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Band   N      StDev            CI
   1  40  0.0058254  (0.0045294, 0.0080474)
   2  40  0.0067498  (0.0052481, 0.0093244)
   3  40  0.0063943  (0.0049717, 0.0088333)
   4  40  0.0077598  (0.0060334, 0.0107197)

Individual confidence level = 98.75%


Tests

               Test
Method    Statistic  P-Value
Bartlett       3.38    0.336
```

The following is Minitab output for the plate one diameter General Linear Model.

General Linear Model: P1 Diameter versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.001475	0.000211	5.79	0.000
Band	3	0.001876	0.000625	17.19	0.000
Sector*Band	21	0.000913	0.000043	1.19	0.267
Error	128	0.004656	0.000036		
Total	159	0.008919			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0060309	47.80%	35.16%	18.44%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.95157	0.00048	6190.58	0.000	
Sector					
1	-0.00410	0.00126	-3.25	0.001	1.75
2	-0.00319	0.00126	-2.53	0.013	1.75
3	-0.00043	0.00126	-0.34	0.734	1.75
4	0.00209	0.00126	1.65	0.101	1.75
5	0.00341	0.00126	2.71	0.008	1.75
6	0.00348	0.00126	2.76	0.007	1.75
7	0.00234	0.00126	1.86	0.065	1.75
Band					
1	0.004136	0.000826	5.01	0.000	1.50
2	0.001381	0.000826	1.67	0.097	1.50
3	-0.000242	0.000826	-0.29	0.770	1.50
Sector*Band					
1 1	0.00178	0.00218	0.82	0.417	2.63
1 2	-0.00342	0.00218	-1.57	0.120	2.63
1 3	0.00399	0.00218	1.83	0.070	2.63
2 1	0.00186	0.00218	0.85	0.397	2.62
2 2	0.00113	0.00218	0.52	0.605	2.62
2 3	-0.00197	0.00218	-0.90	0.370	2.63
3 1	0.00088	0.00218	0.40	0.688	2.63
3 2	0.00434	0.00218	1.99	0.049	2.62
3 3	-0.00339	0.00218	-1.55	0.123	2.63
4 1	-0.00042	0.00218	-0.19	0.846	2.63
4 2	0.00387	0.00218	1.77	0.079	2.62
4 3	-0.00245	0.00218	-1.12	0.263	2.63
5 1	-0.00109	0.00218	-0.50	0.617	2.63
5 2	-0.00321	0.00218	-1.47	0.144	2.63
5 3	0.00088	0.00218	0.40	0.687	2.63

```
  6 1         -0.00292   0.00218    -1.34    0.183  2.63
  6 2         -0.00162   0.00218    -0.74    0.458  2.63
  6 3          0.00242   0.00218     1.11    0.270  2.63
  7 1         -0.00186   0.00218    -0.85    0.396  2.63
  7 2          0.00254   0.00218     1.16    0.248  2.63
  7 3         -0.00160   0.00218    -0.73    0.465  2.62
```

```
Regression Equation

P1 Diameter = 2.95157 - 0.00410 Sector_1 - 0.00319 Sector_2 - 0.00043 Sector_3
              + 0.00209 Sector_4 + 0.00341 Sector_5 + 0.00348 Sector_6
+ 0.00234 Sector_7
              - 0.00361 Sector_8 + 0.004136 Band_1 + 0.001381 Band_2 - 0.000242 Band_3
              - 0.005275 Band_4 + 0.00178 Sector*Band_1 1 - 0.00342 Sector*Band_1 2
              + 0.00399 Sector*Band_1 3 - 0.00235 Sector*Band_1 4
+ 0.00186 Sector*Band_2 1
              + 0.00113 Sector*Band_2 2 - 0.00197 Sector*Band_2 3
- 0.00103 Sector*Band_2 4
              + 0.00088 Sector*Band_3 1 + 0.00434 Sector*Band_3 2
- 0.00339 Sector*Band_3 3
              - 0.00183 Sector*Band_3 4 - 0.00042 Sector*Band_4 1
+ 0.00387 Sector*Band_4 2
              - 0.00245 Sector*Band_4 3 - 0.00099 Sector*Band_4 4
- 0.00109 Sector*Band_5 1
              - 0.00321 Sector*Band_5 2 + 0.00088 Sector*Band_5 3
+ 0.00343 Sector*Band_5 4
              - 0.00292 Sector*Band_6 1 - 0.00162 Sector*Band_6 2
+ 0.00242 Sector*Band_6 3
              + 0.00213 Sector*Band_6 4 - 0.00186 Sector*Band_7 1
+ 0.00254 Sector*Band_7 2
              - 0.00160 Sector*Band_7 3 + 0.00093 Sector*Band_7 4
+ 0.00179 Sector*Band_8 1
              - 0.00362 Sector*Band_8 2 + 0.00212 Sector*Band_8 3
- 0.00028 Sector*Band_8 4
```

Fits and Diagnostics for Unusual Observations

Obs	P1 Diameter	Fit	Resid	Std Resid	
63	2.94317	2.95737	-0.01420	-2.63	R
77	2.95979	2.94739	0.01240	2.30	R
78	2.93454	2.94739	-0.01285	-2.38	R
92	2.96760	2.95562	0.01198	2.22	R
131	2.93769	2.95207	-0.01438	-2.67	R

R Large residual

The following is Minitab output for the plate one diameter General Linear Model (interaction removed).

General Linear Model: P1 Diameter versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor Type Levels Values

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.001475	0.000211	5.64	0.000
Band	3	0.001876	0.000625	16.74	0.000
Error	149	0.005568	0.000037		
Lack-of-Fit	21	0.000913	0.000043	1.19	0.267
Pure Error	128	0.004656	0.000036		
Total	159	0.008919			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0061131	37.57%	33.38%	28.02%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.95157	0.00048	6107.33	0.000	
Sector					
1	-0.00410	0.00128	-3.20	0.002	1.75
2	-0.00319	0.00128	-2.49	0.014	1.75
3	-0.00043	0.00128	-0.34	0.737	1.75
4	0.00209	0.00128	1.63	0.105	1.75
5	0.00341	0.00128	2.67	0.008	1.75
6	0.00348	0.00128	2.72	0.007	1.75
7	0.00234	0.00128	1.83	0.069	1.75
Band					
1	0.004136	0.000837	4.94	0.000	1.50
2	0.001381	0.000837	1.65	0.101	1.50
3	-0.000242	0.000837	-0.29	0.773	1.50

Regression Equation

```
P1 Diameter = 2.95157 - 0.00410 Sector_1 - 0.00319 Sector_2 - 0.00043 Sector_3
              + 0.00209 Sector_4 + 0.00341 Sector_5 + 0.00348 Sector_6
+ 0.00234 Sector_7
              - 0.00361 Sector_8 + 0.004136 Band_1 + 0.001381 Band_2 - 0.000242 Band_3
              - 0.005275 Band_4
```

Fits and Diagnostics for Unusual Observations

Obs	P1 Diameter	Fit	Resid	Std Resid	
63	2.94317	2.95779	-0.01462	-2.48	R
67	2.96880	2.95504	0.01376	2.33	R
78	2.93454	2.94838	-0.01384	-2.35	R
92	2.96760	2.95474	0.01286	2.18	R
131	2.93769	2.95367	-0.01598	-2.71	R

R Large residual

APPENDIX B

The following is Minitab output for plate two height variance testing.

Test for Equal Variances: P2 Height versus Sector

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett's method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Sector   N      StDev            CI
     1  20  0.0084535  (0.0058151, 0.0145945)
     2  20  0.0084754  (0.0058302, 0.0146323)
     3  20  0.0120564  (0.0082935, 0.0208148)
     4  20  0.0118636  (0.0081609, 0.0204819)
     5  20  0.0111682  (0.0076826, 0.0192814)
     6  20  0.0105802  (0.0072781, 0.0182662)
     7  20  0.0124581  (0.0085699, 0.0215083)
     8  20  0.0086298  (0.0059364, 0.0148988)

Individual confidence level = 99.375%


Tests

               Test
Method    Statistic  P-Value
Bartlett       7.02    0.427
```

Test for Equal Variances: P2 Height versus Band

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett's method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Band   N      StDev            CI
   1  40  0.0109926  (0.0085470, 0.0151856)
   2  40  0.0104244  (0.0081052, 0.0144006)
   3  40  0.0109318  (0.0084998, 0.0151016)
   4  40  0.0119632  (0.0093017, 0.0165265)

Individual confidence level = 98.75%
```

Tests

Method	Test Statistic	P-Value
Bartlett	0.77	0.856

The following is Minitab output for the plate two height General Linear Model.

General Linear Model: P2 Height versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.004265	0.000609	6.18	0.000
Band	3	0.002091	0.000697	7.07	0.000
Sector*Band	21	0.002318	0.000110	1.12	0.336
Error	128	0.012611	0.000099		
Total	159	0.021284			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0099257	40.75%	26.40%	7.42%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	15.0027	0.0008	19119.13	0.000	
Sector					
1	-0.00072	0.00208	-0.35	0.728	1.75
2	-0.00679	0.00208	-3.27	0.001	1.75
3	-0.00507	0.00208	-2.44	0.016	1.75
4	-0.00173	0.00208	-0.83	0.407	1.75
5	-0.00273	0.00208	-1.31	0.191	1.75
6	0.00988	0.00208	4.76	0.000	1.75
7	0.00546	0.00208	2.63	0.010	1.75
Band					
1	-0.00384	0.00136	-2.83	0.005	1.50
2	-0.00254	0.00136	-1.87	0.064	1.50
3	0.00088	0.00136	0.65	0.518	1.50
Sector*Band					
1 1	0.00028	0.00360	0.08	0.938	2.63
1 2	0.00153	0.00360	0.43	0.671	2.63
1 3	-0.00014	0.00360	-0.04	0.968	2.63
2 1	0.00549	0.00360	1.53	0.129	2.62
2 2	-0.00218	0.00360	-0.61	0.546	2.62

2 3	0.00002	0.00360	0.00	0.996	2.63
3 1	-0.00114	0.00360	-0.32	0.753	2.63
3 2	-0.00174	0.00360	-0.49	0.628	2.62
3 3	0.00568	0.00360	1.58	0.117	2.63
4 1	-0.00263	0.00360	-0.73	0.467	2.63
4 2	0.00205	0.00360	0.57	0.570	2.62
4 3	-0.00126	0.00360	-0.35	0.726	2.63
5 1	0.00400	0.00360	1.11	0.269	2.63
5 2	-0.00155	0.00360	-0.43	0.667	2.63
5 3	-0.00527	0.00360	-1.46	0.146	2.63
6 1	-0.00027	0.00360	-0.08	0.940	2.63
6 2	0.00275	0.00360	0.76	0.447	2.63
6 3	-0.00594	0.00360	-1.65	0.101	2.63
7 1	-0.00528	0.00360	-1.47	0.144	2.63
7 2	-0.00646	0.00360	-1.80	0.075	2.63
7 3	0.01108	0.00360	3.08	0.003	2.62

Regression Equation

```
P2 Height = 15.0027 - 0.00072 Sector_1 - 0.00679 Sector_2 - 0.00507 Sector_3
            - 0.00173 Sector_4 - 0.00273 Sector_5 + 0.00988 Sector_6
+ 0.00546 Sector_7
            + 0.00170 Sector_8 - 0.00384 Band_1 - 0.00254 Band_2 + 0.00088 Band_3
            + 0.00550 Band_4 + 0.00028 Sector*Band_1 1 + 0.00153 Sector*Band_1 2
            - 0.00014 Sector*Band_1 3 - 0.00167 Sector*Band_1 4
+ 0.00549 Sector*Band_2 1
            - 0.00218 Sector*Band_2 2 + 0.00002 Sector*Band_2 3
- 0.00333 Sector*Band_2 4
            - 0.00114 Sector*Band_3 1 - 0.00174 Sector*Band_3 2
+ 0.00568 Sector*Band_3 3
            - 0.00280 Sector*Band_3 4 - 0.00263 Sector*Band_4 1
+ 0.00205 Sector*Band_4 2
            - 0.00126 Sector*Band_4 3 + 0.00184 Sector*Band_4 4
+ 0.00400 Sector*Band_5 1
            - 0.00155 Sector*Band_5 2 - 0.00527 Sector*Band_5 3
+ 0.00282 Sector*Band_5 4
            - 0.00027 Sector*Band_6 1 + 0.00275 Sector*Band_6 2
- 0.00594 Sector*Band_6 3
            + 0.00347 Sector*Band_6 4 - 0.00528 Sector*Band_7 1
- 0.00646 Sector*Band_7 2
            + 0.01108 Sector*Band_7 3 + 0.00067 Sector*Band_7 4
- 0.00045 Sector*Band_8 1
            + 0.00561 Sector*Band_8 2 - 0.00416 Sector*Band_8 3
- 0.00101 Sector*Band_8 4
```

Fits and Diagnostics for Unusual Observations

Obs	P2 Height	Fit	Resid	Std Resid	
50	15.0124	14.9934	0.0190	2.14	R
52	15.0245	15.0042	0.0202	2.28	R
62	15.0168	14.9946	0.0222	2.50	R
80	15.0266	15.0084	0.0182	2.05	R
84	15.0211	15.0002	0.0209	2.35	R
136	15.0334	15.0144	0.0190	2.14	R

R Large residual

The following is Minitab output for the plate two height General Linear Model (interaction removed).

General Linear Model: P2 Height versus Sector, Band

```
Method

Factor coding  (-1, 0, +1)


Factor Information

Factor  Type   Levels  Values
Sector  Fixed       8  1, 2, 3, 4, 5, 6, 7, 8
Band    Fixed       4  1, 2, 3, 4


Analysis of Variance

Source          DF    Adj SS    Adj MS  F-Value  P-Value
  Sector         7  0.004265  0.000609     6.08    0.000
  Band           3  0.002091  0.000697     6.96    0.000
Error          149  0.014928  0.000100
  Lack-of-Fit   21  0.002318  0.000110     1.12    0.336
  Pure Error   128  0.012611  0.000099
Total          159  0.021284


Model Summary

        S    R-sq  R-sq(adj)  R-sq(pred)
0.0100096  29.86%     25.15%      19.12%


Coefficients

Term          Coef  SE Coef   T-Value  P-Value   VIF
Constant   15.0027   0.0008  18959.02    0.000
Sector
  1       -0.00072  0.00209     -0.35    0.730  1.75
  2       -0.00679  0.00209     -3.24    0.001  1.75
  3       -0.00507  0.00209     -2.42    0.017  1.75
  4       -0.00173  0.00209     -0.83    0.410  1.75
  5       -0.00273  0.00209     -1.30    0.195  1.75
  6        0.00988  0.00209      4.72    0.000  1.75
  7        0.00546  0.00209      2.61    0.010  1.75
Band
  1       -0.00384  0.00137     -2.80    0.006  1.50
  2       -0.00254  0.00137     -1.86    0.065  1.50
  3        0.00088  0.00137      0.64    0.521  1.50


Regression Equation

P2 Height = 15.0027 - 0.00072 Sector_1 - 0.00679 Sector_2 - 0.00507 Sector_3
            - 0.00173 Sector_4 - 0.00273 Sector_5 + 0.00988 Sector_6
+ 0.00546 Sector_7
            + 0.00170 Sector_8 - 0.00384 Band_1 - 0.00254 Band_2 + 0.00088 Band_3
            + 0.00550 Band_4
```

```
Fits and Diagnostics for Unusual Observations

                                        Std
Obs  P2 Height      Fit   Resid  Resid
 25    15.0118  14.9921  0.0197   2.04  R
 52    15.0245  14.9986  0.0259   2.68  R
 62    15.0168  14.9972  0.0196   2.03  R
 80    15.0266  15.0065  0.0200   2.08  R
 84    15.0211  14.9962  0.0249   2.58  R
132    15.0305  15.0091  0.0214   2.21  R
136    15.0334  15.0137  0.0197   2.04  R
147    15.0238  15.0019  0.0219   2.27  R

R  Large residual
```

The following is Minitab output for plate two diameter variance testing.

Test for Equal Variances: P2 Diameter versus Sector

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett's method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Sector   N      StDev            CI
     1  20  0.0059812  (0.0041144, 0.0103262)
     2  20  0.0079143  (0.0054442, 0.0136636)
     3  20  0.0061605  (0.0042378, 0.0106358)
     4  20  0.0062134  (0.0042742, 0.0107271)
     5  20  0.0069398  (0.0047738, 0.0119811)
     6  20  0.0047993  (0.0033014, 0.0082857)
     7  20  0.0052783  (0.0036309, 0.0091128)
     8  20  0.0054450  (0.0037456, 0.0094005)

Individual confidence level = 99.375%


Tests

               Test
Method    Statistic  P-Value
Bartlett       6.70    0.461
```

Test for Equal Variances: P2 Diameter versus Band

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett's method is used. This method is accurate for normal data only.
```

95% Bonferroni Confidence Intervals for Standard Deviations

Band	N	StDev	CI
1	40	0.0056838	(0.0044193, 0.0078518)
2	40	0.0077538	(0.0060288, 0.0107114)
3	40	0.0086154	(0.0066987, 0.0119017)
4	40	0.0106237	(0.0082602, 0.0146759)

Individual confidence level = 98.75%

Tests

Method	Test Statistic	P-Value
Bartlett	14.75	0.002

The following is Minitab output for the plate two diameter General Linear Model.

General Linear Model: P2 Diameter versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.005516	0.000788	22.91	0.000
Band	3	0.000384	0.000128	3.72	0.013
Sector*Band	21	0.000983	0.000047	1.36	0.150
Error	128	0.004403	0.000034		
Total	159	0.011285			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0058647	60.99%	51.54%	39.04%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.94984	0.00046	6362.23	0.000	
Sector					
1	-0.00669	0.00123	-5.45	0.000	1.75
2	-0.00943	0.00123	-7.69	0.000	1.75
3	-0.00287	0.00123	-2.34	0.021	1.75
4	0.00565	0.00123	4.61	0.000	1.75
5	0.00476	0.00123	3.88	0.000	1.75
6	0.00630	0.00123	5.14	0.000	1.75

7	0.00544	0.00123	4.43	0.000	1.75
Band					
1	0.001925	0.000803	2.40	0.018	1.50
2	0.000950	0.000803	1.18	0.239	1.50
3	-0.000778	0.000803	-0.97	0.335	1.50
Sector*Band					
1 1	0.00381	0.00212	1.79	0.076	2.63
1 2	-0.00354	0.00212	-1.66	0.098	2.63
1 3	0.00081	0.00212	0.38	0.705	2.63
2 1	0.00362	0.00212	1.71	0.090	2.62
2 2	0.00496	0.00212	2.33	0.021	2.62
2 3	-0.00289	0.00212	-1.36	0.176	2.63
3 1	0.00247	0.00212	1.16	0.247	2.63
3 2	0.00117	0.00212	0.55	0.584	2.62
3 3	-0.00286	0.00212	-1.34	0.181	2.63
4 1	-0.00140	0.00212	-0.66	0.512	2.63
4 2	0.00011	0.00212	0.05	0.957	2.62
4 3	-0.00220	0.00212	-1.04	0.302	2.63
5 1	-0.00225	0.00212	-1.06	0.292	2.63
5 2	-0.00117	0.00212	-0.55	0.584	2.63
5 3	0.00168	0.00212	0.79	0.431	2.63
6 1	-0.00239	0.00212	-1.13	0.262	2.63
6 2	-0.00051	0.00212	-0.24	0.809	2.63
6 3	0.00181	0.00212	0.85	0.396	2.63
7 1	-0.00310	0.00212	-1.46	0.147	2.63
7 2	0.00160	0.00212	0.75	0.454	2.63
7 3	0.00042	0.00212	0.20	0.842	2.62

Regression Equation

```
P2 Diameter = 2.94984 - 0.00669 Sector_1 - 0.00943 Sector_2 - 0.00287 Sector_3
              + 0.00565 Sector_4 + 0.00476 Sector_5 + 0.00630 Sector_6
+ 0.00544 Sector_7
              - 0.00316 Sector_8 + 0.001925 Band_1 + 0.000950 Band_2 - 0.000778 Band_3
              - 0.002097 Band_4 + 0.00381 Sector*Band_1 1 - 0.00354 Sector*Band_1 2
              + 0.00081 Sector*Band_1 3 - 0.00108 Sector*Band_1 4
+ 0.00362 Sector*Band_2 1
              + 0.00496 Sector*Band_2 2 - 0.00289 Sector*Band_2 3
- 0.00569 Sector*Band_2 4
              + 0.00247 Sector*Band_3 1 + 0.00117 Sector*Band_3 2
- 0.00286 Sector*Band_3 3
              - 0.00078 Sector*Band_3 4 - 0.00140 Sector*Band_4 1
+ 0.00011 Sector*Band_4 2
              - 0.00220 Sector*Band_4 3 + 0.00348 Sector*Band_4 4
- 0.00225 Sector*Band_5 1
              - 0.00117 Sector*Band_5 2 + 0.00168 Sector*Band_5 3
+ 0.00173 Sector*Band_5 4
              - 0.00239 Sector*Band_6 1 - 0.00051 Sector*Band_6 2
+ 0.00181 Sector*Band_6 3
              + 0.00110 Sector*Band_6 4 - 0.00310 Sector*Band_7 1
+ 0.00160 Sector*Band_7 2
              + 0.00042 Sector*Band_7 3 + 0.00108 Sector*Band_7 4
- 0.00077 Sector*Band_8 1
              - 0.00262 Sector*Band_8 2 + 0.00323 Sector*Band_8 3
+ 0.00016 Sector*Band_8 4
```

Fits and Diagnostics for Unusual Observations

Obs	P2 Diameter	Fit	Resid	Std Resid	
39	2.94599	2.93262	0.01337	2.55	R
58	2.95800	2.94409	0.01391	2.65	R

67	2.96855	2.95656	0.01199	2.29	R
78	2.94523	2.95688	-0.01165	-2.22	R
94	2.93927	2.95550	-0.01623	-3.09	R
95	2.96875	2.95550	0.01325	2.53	R
99	2.94299	2.95424	-0.01125	-2.14	R
148	2.93304	2.94501	-0.01198	-2.28	R

R Large residual

The following is Minitab output for the plate two diameter General Linear Model (interaction removed).

General Linear Model: P2 Diameter versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.005516	0.000788	21.80	0.000
Band	3	0.000384	0.000128	3.54	0.016
Error	149	0.005386	0.000036		
Lack-of-Fit	21	0.000983	0.000047	1.36	0.150
Pure Error	128	0.004403	0.000034		
Total	159	0.011285			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0060120	52.28%	49.08%	44.97%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.94984	0.00048	6206.39	0.000	
Sector					
1	-0.00669	0.00126	-5.32	0.000	1.75
2	-0.00943	0.00126	-7.50	0.000	1.75
3	-0.00287	0.00126	-2.28	0.024	1.75
4	0.00565	0.00126	4.49	0.000	1.75
5	0.00476	0.00126	3.78	0.000	1.75
6	0.00630	0.00126	5.01	0.000	1.75
7	0.00544	0.00126	4.33	0.000	1.75
Band					
1	0.001925	0.000823	2.34	0.021	1.50
2	0.000950	0.000823	1.15	0.251	1.50
3	-0.000778	0.000823	-0.94	0.346	1.50

Regression Equation

```
P2 Diameter = 2.94984 - 0.00669 Sector_1 - 0.00943 Sector_2 - 0.00287 Sector_3
              + 0.00565 Sector_4 + 0.00476 Sector_5 + 0.00630 Sector_6
+ 0.00544 Sector_7
              - 0.00316 Sector_8 + 0.001925 Band_1 + 0.000950 Band_2 - 0.000778 Band_3
              - 0.002097 Band_4
```

Fits and Diagnostics for Unusual Observations

Obs	P2 Diameter	Fit	Resid	Std Resid	
36	2.92529	2.93831	-0.01303	-2.25	R
58	2.95800	2.94487	0.01313	2.26	R
67	2.96855	2.95644	0.01211	2.09	R
77	2.96563	2.95340	0.01223	2.11	R
94	2.93927	2.95382	-0.01455	-2.51	R
95	2.96875	2.95382	0.01493	2.57	R
97	2.96449	2.95250	0.01199	2.07	R
148	2.93304	2.94763	-0.01460	-2.52	R

R Large residual

APPENDIX C

The following is Minitab output for plate three height variance testing.

Test for Equal Variances: P3 Height versus Sector

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Sector   N      StDev            CI
     1  20  0.0087690  (0.0060321, 0.0151392)
     2  20  0.0093423  (0.0064265, 0.0161290)
     3  20  0.0108652  (0.0074741, 0.0187583)
     4  20  0.0108844  (0.0074873, 0.0187914)
     5  20  0.0058162  (0.0040010, 0.0100414)
     6  20  0.0098022  (0.0067429, 0.0169230)
     7  20  0.0113230  (0.0077891, 0.0195486)
     8  20  0.0126063  (0.0086718, 0.0217642)

Individual confidence level = 99.375%


Tests

               Test
Method    Statistic  P-Value
Bartlett      12.13    0.096
```

Test for Equal Variances: P3 Height versus Band

```
Method

Null hypothesis         All variances are equal
Alternative hypothesis  At least one variance is different
Significance level      α = 0.05

Bartlett’s method is used. This method is accurate for normal data only.


95% Bonferroni Confidence Intervals for Standard Deviations

Band   N      StDev            CI
   1  40  0.0100131  (0.0077854, 0.0138324)
   2  40  0.0099211  (0.0077139, 0.0137054)
   3  40  0.0124299  (0.0096646, 0.0171711)
   4  40  0.0118409  (0.0092066, 0.0163575)

Individual confidence level = 98.75%
```

Tests

Method	Test Statistic	P-Value
Bartlett	3.07	0.380

The following is Minitab output for the plate three height General Linear Model.

General Linear Model: P3 Height versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.006923	0.000989	12.16	0.000
Band	3	0.003217	0.001072	13.18	0.000
Sector*Band	21	0.001907	0.000091	1.12	0.340
Error	128	0.010413	0.000081		
Total	159	0.022460			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0090194	53.64%	42.41%	27.56%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	14.9790	0.0007	21007.12	0.000	
Sector					
1	0.00487	0.00189	2.58	0.011	1.75
2	-0.00615	0.00189	-3.26	0.001	1.75
3	-0.00473	0.00189	-2.51	0.013	1.75
4	-0.00307	0.00189	-1.63	0.106	1.75
5	-0.01067	0.00189	-5.65	0.000	1.75
6	0.00522	0.00189	2.77	0.006	1.75
7	0.00903	0.00189	4.79	0.000	1.75
Band					
1	-0.00543	0.00124	-4.40	0.000	1.50
2	-0.00289	0.00124	-2.34	0.021	1.50
3	0.00217	0.00124	1.76	0.081	1.50
Sector*Band					
1 1	-0.00253	0.00327	-0.77	0.440	2.63
1 2	0.00439	0.00327	1.34	0.181	2.63
1 3	-0.00273	0.00327	-0.84	0.404	2.63
2 1	-0.00099	0.00327	-0.30	0.762	2.62
2 2	0.00088	0.00327	0.27	0.788	2.62

2 3	0.00243	0.00327	0.74	0.459	2.63
3 1	0.00348	0.00327	1.07	0.289	2.63
3 2	-0.00202	0.00327	-0.62	0.538	2.62
3 3	-0.00499	0.00327	-1.53	0.129	2.63
4 1	0.00361	0.00327	1.11	0.271	2.63
4 2	-0.00048	0.00327	-0.15	0.882	2.62
4 3	-0.00140	0.00327	-0.43	0.668	2.63
5 1	0.00696	0.00327	2.13	0.035	2.63
5 2	-0.00066	0.00327	-0.20	0.841	2.63
5 3	-0.00472	0.00327	-1.45	0.151	2.63
6 1	-0.00262	0.00327	-0.80	0.424	2.63
6 2	0.00081	0.00327	0.25	0.805	2.63
6 3	0.00669	0.00327	2.05	0.043	2.63
7 1	-0.00165	0.00327	-0.51	0.614	2.63
7 2	-0.00400	0.00327	-1.23	0.223	2.63
7 3	0.00493	0.00327	1.51	0.134	2.62

Regression Equation

```
P3 Height = 14.9790 + 0.00487 Sector_1 - 0.00615 Sector_2 - 0.00473 Sector_3
            - 0.00307 Sector_4 - 0.01067 Sector_5 + 0.00522 Sector_6
+ 0.00903 Sector_7
            + 0.00548 Sector_8 - 0.00543 Band_1 - 0.00289 Band_2 + 0.00217 Band_3
            + 0.00615 Band_4 - 0.00253 Sector*Band_1 1 + 0.00439 Sector*Band_1 2
            - 0.00273 Sector*Band_1 3 + 0.00087 Sector*Band_1 4
- 0.00099 Sector*Band_2 1
            + 0.00088 Sector*Band_2 2 + 0.00243 Sector*Band_2 3
- 0.00232 Sector*Band_2 4
            + 0.00348 Sector*Band_3 1 - 0.00202 Sector*Band_3 2
- 0.00499 Sector*Band_3 3
            + 0.00353 Sector*Band_3 4 + 0.00361 Sector*Band_4 1
- 0.00048 Sector*Band_4 2
            - 0.00140 Sector*Band_4 3 - 0.00173 Sector*Band_4 4
+ 0.00696 Sector*Band_5 1
            - 0.00066 Sector*Band_5 2 - 0.00472 Sector*Band_5 3
- 0.00158 Sector*Band_5 4
            - 0.00262 Sector*Band_6 1 + 0.00081 Sector*Band_6 2
+ 0.00669 Sector*Band_6 3
            - 0.00488 Sector*Band_6 4 - 0.00165 Sector*Band_7 1
- 0.00400 Sector*Band_7 2
            + 0.00493 Sector*Band_7 3 + 0.00072 Sector*Band_7 4
- 0.00626 Sector*Band_8 1
            + 0.00108 Sector*Band_8 2 - 0.00020 Sector*Band_8 3
+ 0.00538 Sector*Band_8 4
```

Fits and Diagnostics for Unusual Observations

Obs	P3 Height	Fit	Resid	Std Resid	
7	15.0023	14.9854	0.0169	2.10	R
28	14.9876	14.9708	0.0167	2.08	R
41	14.9955	14.9723	0.0232	2.88	R
61	14.9922	14.9741	0.0181	2.25	R
64	14.9518	14.9741	-0.0223	-2.76	R
72	14.9966	14.9767	0.0199	2.47	R
137	14.9767	14.9949	-0.0182	-2.26	R
144	14.9912	14.9728	0.0184	2.28	R
160	15.0134	14.9960	0.0174	2.15	R

R Large residual

The following is Minitab output for the plate three height General Linear Model (interaction removed).

General Linear Model: P3 Height versus Sector, Band

```
Method

Factor coding  (-1, 0, +1)


Factor Information

Factor  Type   Levels  Values
Sector  Fixed       8  1, 2, 3, 4, 5, 6, 7, 8
Band    Fixed       4  1, 2, 3, 4


Analysis of Variance

Source          DF    Adj SS    Adj MS  F-Value  P-Value
  Sector         7  0.006923  0.000989    11.96    0.000
  Band           3  0.003217  0.001072    12.97    0.000
Error          149  0.012320  0.000083
  Lack-of-Fit   21  0.001907  0.000091     1.12    0.340
  Pure Error   128  0.010413  0.000081
Total          159  0.022460


Model Summary

        S    R-sq  R-sq(adj)  R-sq(pred)
0.0090931  45.15%     41.46%      36.75%


Coefficients

Term          Coef  SE Coef   T-Value  P-Value   VIF
Constant   14.9790   0.0007  20836.73    0.000
Sector
  1        0.00487  0.00190      2.56    0.011  1.75
  2       -0.00615  0.00190     -3.23    0.002  1.75
  3       -0.00473  0.00190     -2.49    0.014  1.75
  4       -0.00307  0.00190     -1.62    0.108  1.75
  5       -0.01067  0.00190     -5.61    0.000  1.75
  6        0.00522  0.00190      2.75    0.007  1.75
  7        0.00903  0.00190      4.75    0.000  1.75
Band
  1       -0.00543  0.00125     -4.36    0.000  1.50
  2       -0.00289  0.00125     -2.32    0.022  1.50
  3        0.00217  0.00125      1.74    0.084  1.50


Regression Equation

P3 Height = 14.9790 + 0.00487 Sector_1 - 0.00615 Sector_2 - 0.00473 Sector_3
            - 0.00307 Sector_4 - 0.01067 Sector_5 + 0.00522 Sector_6
+ 0.00903 Sector_7
            + 0.00548 Sector_8 - 0.00543 Band_1 - 0.00289 Band_2 + 0.00217 Band_3
            + 0.00615 Band_4
```

Fits and Diagnostics for Unusual Observations

Obs	P3 Height	Fit	Resid	Std Resid	
7	15.0023	14.9810	0.0213	2.43	R
28	14.9876	14.9700	0.0176	2.01	R
41	14.9955	14.9688	0.0267	3.04	R
61	14.9922	14.9705	0.0217	2.48	R
64	14.9518	14.9705	-0.0187	-2.13	R
72	14.9966	14.9781	0.0185	2.11	R
115	15.0091	14.9864	0.0228	2.59	R
134	15.0077	14.9902	0.0176	2.00	R
142	14.9603	14.9790	-0.0187	-2.13	R
160	15.0134	14.9906	0.0228	2.59	R

R Large residual

The following is Minitab output for plate three diameter variance testing.

Test for Equal Variances: P3 Diameter versus Sector

Method

Null hypothesis	All variances are equal
Alternative hypothesis	At least one variance is different
Significance level	$\alpha = 0.05$

Bartlett's method is used. This method is accurate for normal data only.

95% Bonferroni Confidence Intervals for Standard Deviations

Sector	N	StDev	CI
1	20	0.0072671	(0.0049990, 0.0125462)
2	20	0.0079198	(0.0054480, 0.0136731)
3	20	0.0071302	(0.0049049, 0.0123100)
4	20	0.0055756	(0.0038355, 0.0096260)
5	20	0.0050152	(0.0034499, 0.0086584)
6	20	0.0048532	(0.0033385, 0.0083788)
7	20	0.0058588	(0.0040302, 0.0101149)
8	20	0.0047716	(0.0032824, 0.0082379)

Individual confidence level = 99.375%

Tests

Method	Test Statistic	P-Value
Bartlett	10.52	0.161

Test for Equal Variances: P3 Diameter versus Band

Method

Null hypothesis	All variances are equal
Alternative hypothesis	At least one variance is different
Significance level	$\alpha = 0.05$

Bartlett's method is used. This method is accurate for normal data only.

95% Bonferroni Confidence Intervals for Standard Deviations

Band	N	StDev	CI
1	40	0.0065552	(0.0050968, 0.0090556)
2	40	0.0098169	(0.0076329, 0.0135614)
3	40	0.0082454	(0.0064110, 0.0113905)
4	40	0.0119319	(0.0092774, 0.0164832)

Individual confidence level = 98.75%

Tests

Method	Test Statistic	P-Value
Bartlett	14.53	0.002

The following is Minitab output for the plate three diameter General Linear Model.

General Linear Model: P3 Diameter versus Sector, Band

Method

Factor coding (-1, 0, +1)

Factor Information

Factor	Type	Levels	Values
Sector	Fixed	8	1, 2, 3, 4, 5, 6, 7, 8
Band	Fixed	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Sector	7	0.007995	0.001142	33.69	0.000
Band	3	0.000119	0.000040	1.17	0.325
Sector*Band	21	0.001304	0.000062	1.83	0.022
Error	128	0.004339	0.000034		
Total	159	0.013757			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0058225	68.46%	60.82%	50.71%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	2.95242	0.00046	6413.98	0.000	
Sector					
1	-0.00878	0.00122	-7.21	0.000	1.75
2	-0.00968	0.00122	-7.95	0.000	1.75
3	-0.00351	0.00122	-2.88	0.005	1.75
4	0.00438	0.00122	3.60	0.000	1.75
5	0.00864	0.00122	7.10	0.000	1.75

```
  6            0.00799   0.00122     6.56    0.000  1.75
  7            0.00588   0.00122     4.82    0.000  1.75
Band
  1           0.000806  0.000797     1.01    0.314  1.50
  2           0.000621  0.000797     0.78    0.437  1.50
  3          -0.000037  0.000797    -0.05    0.963  1.50
Sector*Band
  1 1          0.00381   0.00211     1.81    0.073  2.63
  1 2         -0.00240   0.00211    -1.14    0.258  2.63
  1 3          0.00383   0.00211     1.82    0.071  2.63
  2 1          0.00409   0.00211     1.94    0.055  2.62
  2 2          0.00040   0.00211     0.19    0.849  2.62
  2 3          0.00055   0.00211     0.26    0.796  2.63
  3 1          0.00115   0.00211     0.54    0.587  2.63
  3 2          0.00176   0.00211     0.84    0.405  2.62
  3 3         -0.00106   0.00211    -0.50    0.617  2.63
  4 1         -0.00219   0.00211    -1.04    0.302  2.63
  4 2          0.00389   0.00211     1.85    0.067  2.62
  4 3         -0.00078   0.00211    -0.37    0.711  2.63
  5 1         -0.00282   0.00211    -1.34    0.184  2.63
  5 2          0.00194   0.00211     0.92    0.359  2.63
  5 3         -0.00011   0.00211    -0.05    0.960  2.63
  6 1         -0.00275   0.00211    -1.30    0.195  2.63
  6 2         -0.00102   0.00211    -0.48    0.629  2.63
  6 3          0.00123   0.00211     0.58    0.561  2.63
  7 1         -0.00239   0.00211    -1.13    0.260  2.63
  7 2          0.00043   0.00211     0.20    0.839  2.63
  7 3         -0.00395   0.00211    -1.87    0.063  2.62


Regression Equation

P3 Diameter = 2.95242 - 0.00878 Sector_1 - 0.00968 Sector_2 - 0.00351 Sector_3
              + 0.00438 Sector_4 + 0.00864 Sector_5 + 0.00799 Sector_6
+ 0.00588 Sector_7
              - 0.00492 Sector_8 + 0.000806 Band_1 + 0.000621 Band_2 - 0.000037 Band_3
              - 0.001390 Band_4 + 0.00381 Sector*Band_1 1 - 0.00240 Sector*Band_1 2
              + 0.00383 Sector*Band_1 3 - 0.00525 Sector*Band_1 4
+ 0.00409 Sector*Band_2 1
              + 0.00040 Sector*Band_2 2 + 0.00055 Sector*Band_2 3
- 0.00504 Sector*Band_2 4
              + 0.00115 Sector*Band_3 1 + 0.00176 Sector*Band_3 2
- 0.00106 Sector*Band_3 3
              - 0.00186 Sector*Band_3 4 - 0.00219 Sector*Band_4 1
+ 0.00389 Sector*Band_4 2
              - 0.00078 Sector*Band_4 3 - 0.00092 Sector*Band_4 4
- 0.00282 Sector*Band_5 1
              + 0.00194 Sector*Band_5 2 - 0.00011 Sector*Band_5 3
+ 0.00098 Sector*Band_5 4
              - 0.00275 Sector*Band_6 1 - 0.00102 Sector*Band_6 2
+ 0.00123 Sector*Band_6 3
              + 0.00254 Sector*Band_6 4 - 0.00239 Sector*Band_7 1
+ 0.00043 Sector*Band_7 2
              - 0.00395 Sector*Band_7 3 + 0.00591 Sector*Band_7 4
+ 0.00109 Sector*Band_8 1
              - 0.00501 Sector*Band_8 2 + 0.00029 Sector*Band_8 3
+ 0.00363 Sector*Band_8 4


Fits and Diagnostics for Unusual Observations

Obs  P3 Diameter      Fit     Resid  Std Resid
 10      2.93136  2.94186  -0.01050      -2.02  R
```

```
 36      2.92339  2.93631  -0.01292       -2.48  R
 39      2.94788  2.93631   0.01158        2.22  R
 40      2.94697  2.93631   0.01066        2.05  R
 59      2.93134  2.94566  -0.01432       -2.75  R
106      2.97086  2.96001   0.01085        2.08  R
136      2.95190  2.96281  -0.01091       -2.10  R

R  Large residual
```