An Increase in Per Unit Cost: The Response from a Sales Maximizer and a Profit Maximizer

Lydia Gan*,a, Shuntian Yaob and Ke Lic

aDepartment of Economics, Finance, and Decision Sciences, School of Business, University of North Carolina at Pembroke, Pembroke, NC 28372, USA; bDivision of Economics, School of Humanities and Social Sciences, Nanyang Technological University, Singapore; cGraduate School of Business, Nihon University, Japan

Abstract: This paper investigates the decision making of firms with two different objectives – profit maximization and sales maximization, and compares their responses when encountering a higher per unit production cost or if per unit tax is imposed. The discussion in this paper is a follow up of Li and Yao [1], except in this case, a more comprehensive literature review is provided, and a very restricted result previously proposed has been substantially expanded to a few more general propositions.

Keywords: Profit maximization, per unit cost increase, sales maximization, JEL classification: D21, M21.

I. INTRODUCTION

In a recent paper by Li and Yao [1], the authors mainly focused on pointing out the theoretical flaws in Moschandreas [2, 3], in which the author proposed a proposition, arguing that when a per unit cost is incurred, the producer who maximizes sales always reduces the output more substantially than the producer who maximizes profits. In addition to pointing that Moschandreas' proposition is only valid for a very special case – when the cost functions are quadratic functions, a simple counter example was constructed by Li and Yao [1].

In this paper, we establish a much more general result than that shown in Li and Yao [1]. We intend to prove that, as long as the increase in per unit cost is *not too substantial*, a sales maximizer will show a more sensitive response than a profit maximize, *no matter what cost function* he has. As a result, a per unit tax introduced into a sales maximization industry may cause a much bigger negative welfare effect than if it were introduced into a profit maximization industry.

II. LITERATURE REVIEW

While Baumol [4] indicated that maximization of the growth rate of sales is a motivational assumption for an oligopolistic firm, Baldwin [5] found that profit maximization closely approximates the actual motive of the typical large firm. Yet others suggested that corporate recognition of public responsibility can be another managerial motivation.

Past studies suggested that the type of management control may affect the behavior of the firms. While Monsen

and Downs [6] observed that the interest of firms that are controlled by management does not necessarily coincide with owners' preferences, Boudreaux [7] and Palmer [8] proved that managers in management-controlled firms prefer activities that lead to lower risk than those resulting from owners' decisions in owner-controlled firms, given that both managers and owners are having the identical expected utility-maximization. Furthermore, Amihud and Kamin [9] found that revenue maximizing behavior is more prevalent among management-controlled firms than among owner-controlled ones, and more prevalent among oligopolistic firms than among particularly competitive firms. Moreover, the market power of firms was found to have a greater influence on the firms' behavior than their type of control.

The constrained revenue maximization hypothesis has been explored in a variety of forms, among them are both single product and multi-product technologies with and without the embellishments[10-13]. One result is that given the resource base and cost structure/technology, both the sales maximizer and the profit maximizer will yield the same optimal output. Panik [14] showed that given resource, costs, and technology, both profit maximizers and profit-constrained sales maximizers may use different combination of inputs. Specifically, if both types of firms have the same maximal activity mix, then given the technology constraints, the shadow price for the input of the revenue maximizer will exceed that of the profit maximizer by its market price.

Zabojnik [15] analyzed the effects of providing managers with incentives to maximize sales in addition to profits on managerial efforts and human capital investment. According to Baker, Jensen, and Murphy [16], the salaries of top executives increased by 3% when the firm's sales increased by 10%. Zabojnik's study revealed that the optimal incentive scheme for the manager in the presence of human capital investment will impact positively on sales and profits. For instance, by emphasizing sales, the worker's bargaining

*Address correspondence to this author at the Department of Economics, Finance, and Decision Sciences, School of Business, University of North Carolina at Pembroke, Pembroke, NC 28372, USA; Tel: (910) 775-4265; Fax: (910) 521-6750; E-mail: lydia.gan@uncp.edu

power will increase and hence he is more likely to acquire human capital that will eventually benefit the firm.

Applying the Cournot model for a duopolistic market in the automobile industry, Tabeta and Wang [17] provided an explanation for the huge trade deficit of the U.S. in relation to Japan based on an analysis of different strategic behaviors between U.S. firms and Japanese firms. There are several differences between Japanese and U.S. firms that result in the Japanese firm typically being more inclined to be a revenue maximizer, while the U.S. firm is more likely to be profit oriented. They showed that a revenue-maximizer (as in Japan) should obtain a larger market share and a higher profit than a profit-maximizer (like many in the U.S.). Blinder [18] too concluded that as long as it has a secure source of capital, a revenue-maximizer is at a distinct advantage when competing with profit-maximizers. Tabeta and Wang [17] further argued that if U.S. firms adopt a revenue maximizing strategy, an improvement in the trade deficit will result if they are in a matured market but mutual loss will be the consequence if they are in a declining market. In a declining market, the cultural-institutional equilibrium led to a bitter trade dispute; nevertheless, if both Japanese and U.S. firms adopted the profit maximizing strategy it would be an effective way of balancing trade.

III. THE BACKGROUND

In order to compare the decision-making of a profit-maximizing firm with a sales-maximizing firm, Baumol suggested three main differences between the behaviors of these two types of firms in his well-known book *Economic Theory and Operation Analysis* [19-21]. In the discussion, Baumol assumes that the sales maximizer is to choose an output to maximize the revenues while he is subject to some minimum profit constraint [21]. Baumol's theoretical results have been very influential, having been quoted by many management textbooks which are currently in use.

These results are later quoted by Moschandreas [2, 3] as three predictions:

Prediction 1. Compared with a profit maximizer, a sales maximizer produces more and charges a lower price for his product.

Prediction 2. A sales maximizer will tend to spend more on advertising and other sales promotional activities than a profit maximizer.

Prediction 3. A sales maximizer will increase price and reduce production in response to an increase in fixed costs or lump sum taxes.

As regards to Prediction 3, and according to Baumol, sale maximizers will react to an increase in fixed costs or lump sum taxes while profit maximizers will not simply because the profit maximizing criterion MC = MR is not affected by an increase in fixed costs or lump sum taxes. However, Baumol did not provide any answer to the effect of an increase of per unit cost on the decision of a sales revenue maximizer nor that of a profit maximizer. In fact, answering this question is a bit more difficult than establishing the three predictions mentioned above. Moschandreas [2, 3] did try to establish a proposition as given below:[1]

*Prediction 4. When an increase in variable costs or per unit tax is imposed, a sales maximizer will reduce output and increase price **by a greater amount** than a profit maximizer will.*

Unfortunately this prediction is not correct in general. A counter-example was given by Li and Yao [1] who pointed out that this prediction is only valid for a very restricted case, i.e., when the cost functions are in quadratic form.

The contributions of Li and Yao [1] are somewhat limited in that there was no **general conclusion** as to under what circumstances the result of "Prediction 4" applies. In this paper we will conduct a deeper analysis in order to derive some general results particularly in theory and applications.

IV. SOME GENERAL RESULTS

For simplicity, we only consider a monopoly firm or a firm in the perfect competition in our current model. We do not consider oligopolistic competition in this paper because the interaction among firms in such market makes the argument slightly more complicated. We leave it for future extension of this study.

We have seen that Prediction 4 in general is false. To compare the decision making of a profit maximizer and that of a sales maximizer in response to the increase of per unit cost, we need to make a more careful analysis. One important point we can make is that, whether the conclusion of Prediction 4 is true will depend on the minimum profit level *m* the firm has to attain while maximizing the sales. We will establish a general result, and then concentrate on some special cases.

For simplicity we consider the case in which a firm has a continuous and strictly concave profit function $\pi = \pi(q)$ defined for all $q \in [0, \infty)$. Let q^* be the unique maximum of $\pi(q)$ such that $\pi(q^*) > 0$. It is reasonable to assume that for some $Q > q^*$: $\pi(Q) = 0$ and $\pi(q) < 0$ for $q > Q$. We will compare the individual response of a profit maximizer and a sales maximizer to the introduction of a per unit tax *t*.

After the introduction of per unit tax *t*, the new profit function is

$$\bar{\pi}(q) = \pi(q) - tq \qquad (1)$$

$\bar{\pi}(q)$ is obviously continuous and strictly concave. As a result, $\bar{\pi}(q)$ has a unique maximum $\bar{q}$, and of course $\bar{\pi}(\bar{q}) < \pi(q^*)$.

Let *m* > 0 be the minimum profit requirement for a sales maximizer. It is reasonable to assume that $m \le \bar{\pi}(\bar{q})$, otherwise the sales maximizer can never fulfill the minimum profit requirement.

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[1] Please refer to Moschandreas [2], pp. $279 - 280$; and Moschandreas [3], pp. 202-203.

According to our assumptions, $\pi = \pi(q)$ must be strictly decreasing in the interval $[q^*, \infty]$, and there exists a unique Q^* in this interval such that $\pi(Q^*) = m$, Note that q^* and Q^* will be the respective output of a profit maximizer and a sales maximizer before per unit tax is introduced. And note that the value of Q^* depends on m, and since $\pi(q)$ is strictly increasing and continuous in $[q^*, \infty]$, $Q^* = Q^*(m)$ is well-defined and is continuous in m.

After per unit tax is introduced, as mentioned above, a profit maximizer will produce quantity $\overline{q}$. Note that $\overline{\pi}(q)$ is strictly decreasing and eventually becomes negative. There must exist a unique $\overline{Q} > \overline{q}$ such that $\overline{\pi}(\overline{Q}) = \pi(\overline{Q}) - t\overline{Q} \cdot = m$ This $\overline{Q}$ is the quantity produced by the sales maximizer after per unit tax is introduced. Again $\overline{Q}$ depends on m, and $\overline{Q} = \overline{Q}(m)$ is well-defined and continuous. To compare the values of $q^*, \overline{q}, Q^*, \overline{Q}$, we have:

Lemma. We always have $\overline{q} < q^*$ and $\overline{Q}(m) < Q^*(m)$ for every m, i.e. when the per unit cost increases or the per unit tax is introduced, both the profit maximizer and the sales maximizer will reduce their outputs.

Proof. Because the profit function $\pi = \pi(q)$ is strictly concave, the derivative $\pi'(q)$ is decreasing. Note that q^* is determined by $\pi'(q^*) = 0$ and $\overline{q}$ is determined by $\overline{\pi}'(\overline{q}) = \pi'(\overline{q}) - t = 0$ or $\pi'(\overline{q}) = t$. Thus, $\pi'(\overline{q}) > \pi'(q^*)$ implies $\overline{q} < q^*$.

To see that $\overline{Q}(m) < Q^*(m)$, we consider two cases. If $\overline{Q}(m) \leq q^*$, then $\overline{Q}(m) < Q^*$ is trivial. If $\overline{Q}(m) > q^*$, then $\pi(\overline{Q}(m)) = m + c\overline{Q}(m) > m = \pi(Q^*)$ and that $\pi(q)$ is decreasing in $[q^*, \infty]$ and this implies $\overline{Q}(m) < Q^*$. QED.

We will establish a general result:

Proposition 1. Assume that the profit function $\pi(q)$ is continuous and strictly concave. When per unit tax t is introduced and the profit maximizer reduces her output from q* to $\overline{q}$, there always exists an interval $(m, \overline{\pi}(\overline{q})]$, such
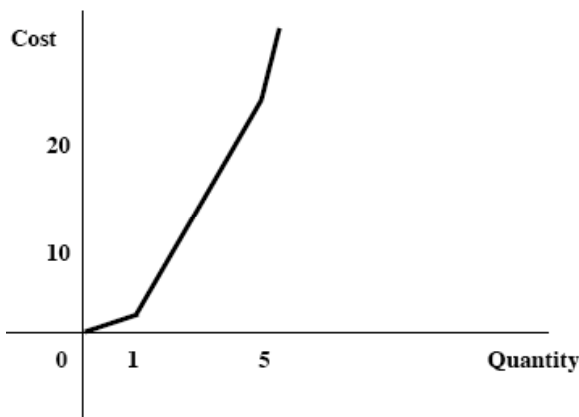

Fig. (1). Cost function.

that when $m \in (m, \overline{\pi}(\overline{q})]$, the sales maximizer will reduce output and increase price by an amount greater than (q*-$\overline{q}$).

Put in another way, proposition 1 states: If the minimum profit requirement upon a sales maximizer is sufficiently high, then his reaction to the introduction of per unit tax will be more substantial than that of a profit maximizer.

Proof. What we want to show is that in some interval of *m*:

$$Q^*(m) - \overline{Q}(m) > q^* - \overline{q} \qquad (2)$$

For $m = \overline{m} = \overline{\pi}(\overline{q})$ we have: $\overline{Q}(m^*) = \overline{q}$, $Q^*(m^*) > q^*$ and therefore

$$Q^*(\overline{m}) - \overline{Q}(\overline{m}) > q^* - \overline{q} \qquad (3)$$

Note that $Q^*(m) - \overline{Q}(m)$ depends on m continuously. Thus there exists a neighborhood of $\overline{m}$ in which proposition 1 remains true. In particular, there is an interval $(m, \overline{m}] = (m, \overline{\pi}(\overline{q})]$ in which proposition 1 holds. QED.

Example 1. Assume the inverse demand function for the firm being

$$p = 5 - 0.01q.$$

where q is the quantity produced by the firm and p is the market clearing price. Assume that this firm has the piece-wise linear cost function originally, as shown in Fig. (**1**).

$$\text{TC} = \begin{cases} q; & q \in [0,1) \\ 4.75q - 3.75; & q \in [1,5) \\ 7.5q - 17.5; & q \in [5,\infty) \end{cases} \qquad (4)$$

This was the counter-example in which Li and Yao [1] showed that Moschandreas's "Prediction 4" is in general not correct. They assumed that the minimum profit required for the firm was *m*=2, comparing the decision changes between a profit maximizer and a sales maximizer when per unit cost was increased by 0.5. They showed that the profit maximizer reduced his output by 4, and yet the sales maximizer reduced her output by 0.97 only.

However, for this same example, with per unit cost increased by the same amount of 0.5, if we confine the minimum profit values of $m \in (3.341, 3.49)$ other than maintaining *m*=2, then the result in Moschandreas's "Prediction 4" would have been correct. (see Fig. **2**). Therefore, in general which type of producer adjusts more in terms of output will depend on the magnitude of the minimum profit he or she wants to guarantee.

By the way, for some special forms of profit functions, the conclusion of Proposition 1 is true for all values of *m*, i.e. for all $m \in (-\infty, \overline{m}]$. All quadratic profit functions are in this category.

Proposition 2. Assume that the firm has a linear inverse demand function $p = A - Bq$ and a quadratic cost function $TC = C + Dq + Eq^2$ ($A > 0, B > 0, D > 0, C \geq 0, E \geq 0$ are all constants), then when per unit cost increases or per unit tax
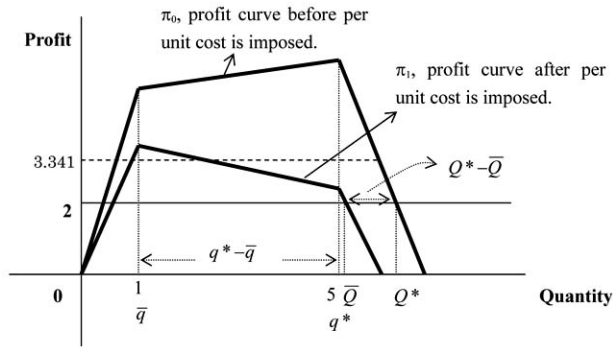
Fig. (2). The decision makings of a profit maximizer and a sales maximizer before and after the imposition of per unit cost (example 1).

Notes: When profit = 2, note that $q*-\overline{q} > Q*-\overline{Q}$, where

$q*$ = Output produced by a profit maximizer before the imposition of a per unit cost

$\overline{q}$ = Output produced by a profit maximizer after the imposition of a per unit cost

$Q*$ = Output produced by a sales maximizer before the imposition of a per unit cost $\overline{Q}$ = Output produced by a sales maximizer after the imposition of a per unit cost

is imposed, a sales maximizer will reduce output and increase price **by a greater amount** than a profit maximizer.

Proof. It is easy to derive the profit function

$$\pi = -C + (A - D)q - (B + E)q^2 \qquad (5)$$

For convenience we introduce the notations:

$$C = \gamma, \ A - D = \beta, \ B + E = \alpha \qquad (6)$$

Then we can write

$$\pi = -\alpha q^2 + \beta q - \gamma \qquad (7)$$

It is reasonable to assume that $\beta > 0$ and $\beta^2 > 4\alpha\gamma$ so that there exists a range of q for positive profits. It is easy to compute the optimal quantity for profit maximization:

$$q* = \frac{\beta}{2\alpha} \qquad (8)$$

Imagine that per unit cost increases by t, so the profit function is changed to $\overline{\pi}(q) = -\alpha q^2 + (\beta - t)q - \gamma$. Again it is reasonable to assume that $t < \beta$ so that the maximum profit is positive. It is easy to verify that the optimal quantity for profit maximization is:

$$\overline{q} = \frac{\beta - t}{2\alpha} \qquad (9)$$

and the maximum profit is:

$$\overline{m} = \overline{\pi}(\overline{q}) = \frac{(\beta - t)^2}{4\alpha} - \gamma \qquad (10)$$

Thus we have $q*-\overline{q} = \frac{t}{2\alpha}$. We need to verify for all $m \le \overline{m}$:

$$Q*(m) - \overline{Q}(m) > \frac{t}{2\alpha} \qquad (11)$$

Note that $Q*(m)$ and $\overline{Q}(m)$ are, respectively, the larger root of the equations:

$$-\alpha q^2 + \beta q - \gamma = m, \text{ and } -\alpha q^2 + (\beta - t)q - \gamma = m \qquad (12)$$

By directly solving quadratic equations:

$$Q*(m) = \frac{\beta + \sqrt{\beta^2 - 4\alpha(\gamma + m)}}{2\alpha}, \text{ and}$$

$$\overline{Q}(m) = \frac{\beta - t + \sqrt{(\beta - t)^2 - 4\alpha(\gamma + m)}}{2\alpha} \qquad (13)$$

We thus have:

$$Q*(m) - \overline{Q}(m) = \frac{t}{2\alpha} + \frac{\sqrt{\beta^2 - 4\alpha(\gamma + m)} - \sqrt{(\beta - t)^2 - 4\alpha(\gamma + m)}}{2\alpha} > \frac{t}{2\alpha} \qquad (14)$$

The inequality holds because

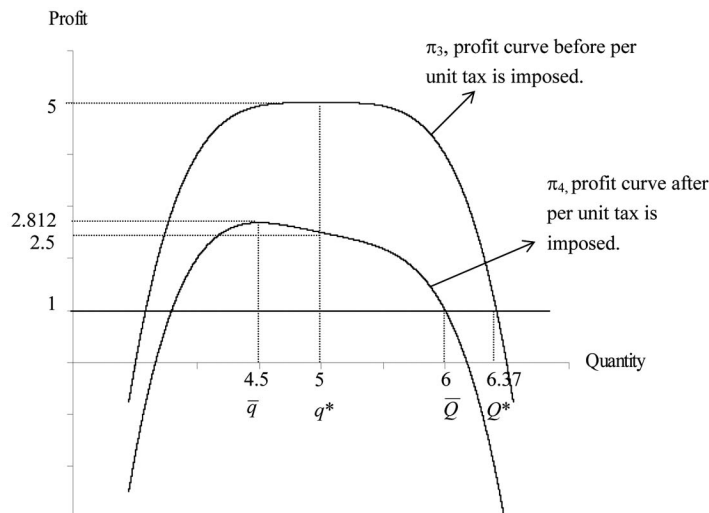$$\sqrt{\beta^2 - 4\alpha(\gamma + m)} - \sqrt{(\beta - t)^2 - 4\alpha(\gamma + m)} > 0 \qquad (15)$$

QED.

For a *general* "negative" result we have:

Proposition 3. Define d = $q*-\overline{q}$. If for the minimum profit level m, it holds that $\overline{\pi}(\overline{Q}(m)) > \pi(\overline{Q}(m) + d)$, then a sales maximizer reduces output and increases price **by a smaller amount** than the profit maximizer.

Proof. By definition $\overline{\pi}(\overline{Q}(m)) = \pi(Q*(m))$. Therefore $\overline{\pi}(\overline{Q}(m)) > \pi(\overline{Q}(m) + d)$ implies $\pi(Q*(m)) > \pi(\overline{Q}(m) + d)$. Because $\pi(q)$ is strictly decreasing in $[q*, \infty]$, we thus have $Q*(m) < \overline{Q}(m) + d$, which is equivalent to $Q*(m) - \overline{Q}(m) < q*-\overline{q}$. QED.

Let us give an example to illustrate the result of Proposition 3. This seems to be a more "natural" counter-example against Moschandreas' result.

Example 2. Consider a firm with profit function $\pi(q) = -(q - 5)^4 + 5$ (say, with inverse demand $p = 500 - q$, and $TC = q^4 - 20q^3 + 150q^2 + 620$). The optimal output for profit maximization is $q* = 5$. Imagine that the government imposes a per unit tax of 0.5. Then the new profit function is $\overline{\pi}(q) = -(q - 5)^4 - 0.5q + 5$, and the new optimal output for profit maximization is 4.5, i.e. $d = 0.5$. Now consider the decision of a sales maximizer. Assume that the minimum profit required is $m = 1$. It is easy to compute $\overline{Q} = 6$.

By direct calculation,

$$\pi(\overline{Q} + d) - \overline{\pi}(\overline{Q}) = \pi(6.5) - \overline{\pi}(6) = -0.0625 - 1 < 0 \qquad (16)$$

Thus by Proposition 3, the sales maximizer reduces output and increases price by a smaller amount than the profit maximizer. In fact, before per unit tax is imposed, the sales maximization output is 6.36778…; after the tax is imposed, the sales maximization output is 5.99035…. The change in output is 0.37.

V. WHAT'S THE WRONG WITH THE ARGUMENT FOR PREDICTION 4?

Moschandreas' [2, 3] argument for Prediction 4 was made by applying the idea of geometric transformation. It is argued that, after per unit tax is introduced, the total cost curve is shifted upward and pivoted to the left – or more precisely – rotated counter-clockwise. (And as a result, the profit curve is shifted downward and rotated clockwise.) This is generally incorrect. In most cases, the new cost curve cannot be obtained from the old one through a translation and a rotation, and neither can the new profit curve be obtained from the old one through a translation and a rotation. To see this, it is sufficient to show that the new curve is not congruent with the old one, because the transformation from the old to the new is not distance-reserved.

Let us revisit Example 1. The new cost curve C_1 and the old one C_0 are sketched in Fig. (**3**).

The two corner points on the old cost curve C_0 are (1, 1) and (5, 20), and the two corner points on the new cost curve C_1 are (1, 1.5) and (5, 22.5). Obviously the distance between the two corner points on the new cost curve is greater than that between the two corner points on the old cost curve.

Let us revisit Example 2. The profit curve π_3 before per unit tax introduced and π_4, the profit curve after per unit tax is introduced are shown in Fig. (**4**). Obviously π_4 cannot be obtained from π_3 through a translation and a pivot to the right.

CONCLUSION

Having discussed completely the responses of a profit maximizer and a sales maximizer to the introduction of per

Fig. (3). Cost functions before and after the imposition of per unit tax.

unit tax (or per unit cost increase), we establish general results as stated in Proposition 1 and Proposition 2. In addition, we point out that in other cases, Prediction 4 by Moschandreas [2, 3] can be false. As long as the increase in per unit cost is *not too substantial*, a sales maximizer will show a more sensitive response than a profit maximize, *no matter what cost function* he has. Since a per unit tax introduced into a sales maximization industry may cause a much bigger negative welfare effect than if it were introduced into a profit maximization industry, when the government considers an introduction of per unit sales tax into an industry, it pays to estimate the welfare effects accordingly, and on a case-by-case basis.

The goal of sales maximization can be explained by the management's desire to maintain a firm's competitive position, which depends largely on its size. Unlike the shareholders who are interested in profit, the management is interested in sales revenue, either because large sales revenue is a matter of prestige or because its remuneration is often related to the size of the firm's operations than to its profits.

Fig. (4). The decision makings of a profit maximizer and a sales maximizer before and after the imposition of per unit tax (example 2).

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