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AN ANALYSIS OF MATHEMATICAL SKILLS OF EIGHTH-
GRADE PUPILS IN CERTAIN SCHOOLS OF
GREENSBORO, NORTH CAROLINA

by

FRANCES RHYNE LAEL

4221

A thesis submitted to
the Faculty of
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Approved by

Franklin H. McNutt

Adviser

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The writer also wishes to express her gratitude to the teachers who administered the tests necessary for this study.

151412

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CHAPTER I

INTRODUCTION

Justification for the Problem

Mathematics teachers are facing a job steadily increasing in difficulty, but unassailably important. In a world where quantitative situations must be met and handled accurately and with precision, mathematical literacy is equally as important as the ability to read and write. Hildreth maintains that mathematics is of the utmost importance in our struggle for life.

> No normal person gets along very well in a civilized society without some arithmetic computation and reasoning skill . . . A limited knowledge of arithmetic is of survival value in practical affairs and justifies the inclusion of arithmetic practice in schools. The modern aim in arithmetic teaching is prompt and accurate solution of practical problems such as the average person encounters in his daily affairs.[1]

In our everyday social activities we have wide need of a knowledge of numbers. We cannot engage in a serious conversation without the necessity of understanding numerical relations. It is almost impossible to listen to a speaker unless the listener is able to visualize in terms of large numbers. Morrison gives another need for mathematics when he says:

> To keep up with the progress of science is to read books--not the juveniles of popular science, but the substantial volumes of reputable scientists written for educated readers. Very few of these can be read unless the reader is in command of elementary mathematics, not as a professional mathematician but as an educated person.
>
> Mathematics is often called a tool subject. In one sense that

1. Gertrude R. Hildreth, Learning the Three R's. Philadelphia: Educational Publishers Incorporated, 1936. pp. 156-157.

is true. In a much wider and more fundamental sense, mathematics is interpretation of the world, the supreme science in itself.[2]

Simon suggests that mathematics be taught for cultural reasons.

> . . . your teaching of arithmetic . . . may merely train your class in a number of processes which will let them pass an examination at the end of the term. That is "useful." It may also help them to manage their savings accounts better or get a job on graduation. That is useful too--and this time without quotation marks. But if you can develop in them an understanding of number relations, if you can teach them to visualize distances and quantities, to appreciate imaginatively the meaning of "ten million" or "one thousandth of an inch," then you are training them culturally: they will forever after be more sensitive, more appreciative, more understanding, even though they may do no better on a formal examination.[3]

The road to competency in mathematics is a long one and should be continuous. It should have its foundations well laid in the early grades and the progress should be-level by level-in unbroken sequence. Mathematics is a "vertical" subject. Understanding comes from a building-up of principles and concepts; each is a vital block in the total structure of mathematical understanding. If one block is missing, the concept is gone. A child who has the misfortune of being absent at a critical time or of being placed under the guidance of an ineffectual arithmetic teacher experiences disastrous results in later mathematics classes. On the other hand, such a study as history is a "horizontal" subject as compared with arithmetic, a "vertical" one. A child may miss a few weeks, even a year, of history, and little or no irreparable damage is done; but, conversely, interruptions in the flow of understanding in mathematics will affect that child's future ability to comprehend

2. Henry C. Morrison, <u>The Curriculum of the Common School</u>. Chicago: The University of Chicago Press, 1940. p. 73.

3. Henry W. Simon, <u>Preface to Thinking</u>. New York: Oxford, 1938. p. 34.

that subject.[4]

Realizing the need for a better foundation in the fundamentals of education, the 1941 General Assembly of the State of North Carolina made provision for a twelfth year of public-school training. Professional school people and parents believed that the greatest good might be achieved if the twelfth year were distributed all along the way from grade one through grade twelve. It was felt that, when the twelve-year program was in full operation, students should cover one-twelfth of their school curriculum in each year rather than one-eleventh in each of eleven years. To bring about the curricular changes so that the additional year would be diffused throughout the curriculum, the revision of the program for the eighth grade was the starting point.[5] Interested lay people helped to formulate objectives and reasons for so doing.

> . . . This twelfth year should receive its impetus in the primary and grammar grades. Every boy and girl ought to have the fundamentals about which the public most complains. The criticism is that our boys and girls are not becoming skilled in those fundamentals: English, arithmetic, grammar, and spelling . . . Do not use the twelfth year in expanding the curriculum; concentrate, rather than spread wide, in the lower grades where fundamentals should be had if they are ever had at all.--Mr. W. A. Dees, President, North Carolina School Board Association.[6]

Another North Carolinian said:

> Probably the twelfth year should be added in the elementary school where it will reach the greatest number . . . I would suggest the logical insertion between the sixth and ninth grades to

4. Franklin H. McNutt, Supervision: Evaluation and Improvement of Instruction. Class Lecture, Woman's College of the University of North Carolina, Greensboro, North Carolina, October, 1945.

5. North Carolina State Superintendent of Public Instruction, A Suggested Twelve-Year Program for the North Carolina Schools. Raleigh, North Carolina: The Author, 1942. p. 14.

6. Ibid., pp. 9-10.

help keep in school that big army of youth which otherwise drops out, often to become maladjusted adults.--Mrs. R. J. Pearse, representing the North Carolina Congress of Parents and Teachers.[7]

In the revised program of studies for the eighth grade it was decided that students should not be allowed to earn credits for high school graduation, but rather that the program for this grade should be planned on the basis of a last year of elementary school. Careful consideration was given to the grades above and below, since what was done in this grade had bearing upon the work throughout the entire school.[8] The North Carolina state bulletin indicated the purpose of mathematics in the seventh and eighth grades by stating:

> The purpose of mathematical study in grades VII-VIII is to bring the students to an understanding of the meaning and the significance of the concept of number, measurement, equation, formula, graph, table, function, operation, as being well-established methods of studying quantitative relationships, and to develop skills in using these concepts for solving the problems of everyday living.[9]

The teaching of arithmetic in grade school has always been open to sharp criticism. We repeatedly hear the cry that our children cannot deal with numbers as effectively as their parents and grandparents could.

Hildreth upholds this by saying:

> Junior high school pupils prove unable to compute long division problems, a process supposedly learned in the fourth grade. They are "weak" in fractions, supposed traditionally to have been mastered in the fifth grade, and for decimals and percentage, one baffled teacher exclaimed, "they haven't the slightest notion what it's all about." The pupils can do a little computation or problem-solving in parrot fashion if they are told the rules to follow or the principles that apply, but as for independent problem-solving

---

7. Ibid., p. 16.

8. Ibid., p. 15.

9. Ibid., p. 135.

that is quite out of the question.[10]

Twenty-one recent courses of study were examined by The Research Service of the Silver Burdett Company to find the common grade placement of the main topics of arithmetic. Without exception, all of the schools reported that the main functions of their eighth-grade arithmetic were maintenance and review of the arithmetic taught in the earlier grades. Decimals, fractions, percentage, graphs, and measurement are all introduced in a preceding grade, but all are reintroduced and studied in the eighth grade. Most problem-solving in this grade is done to apply and maintain the skills which were learned earlier.[11]

Being aware of the necessity of fundamental number knowledge and the purposes of maintenance and repair for eighth-grade arithmetic, the writer has made this study to analyze the skills of eighth-grade students and to locate their weaknesses in hope of better preparing them for their future mathematical needs.

Statement of the Problem

The title of this thesis is: "An Analysis of Mathematical Skills of Eighth-Grade Pupils in Certain Schools in Greensboro, North Carolina."

In developing this major topic the following sub-problems should be solved:

1. What are the fundamental difficulties in the mechanics of arithmetic?

2. What fundamental difficulties appear in problem-solving?

---

10. Hildreth, op. cit., pp. 445-446.

11. The Supervisor's Exchange: A Research Service, Volume IV, Number 2. New York: Silver Burdett Company, January, 1946. pp. 42-46.

3. What recommendations can be made?

Scope of the Problem

This study was limited to:

1. The eighth-grade arithmetic classes in certain schools of Greensboro, North Carolina.

2. The North Carolina state curriculum in use at the time this study was made.

Method

In order to avoid duplication of any previous work and to locate related material, the writer has consulted the following reference works:

Palfrey, Thomas R. and Colman, Henry E. Guide to Bibliographies and Theses in the United States and Canada. Second Edition. Chicago: American Library Association, 1940. 54 pp.

United States Library of Congress. Catalogue Division. List of American Doctoral Dissertations. Washington, D. C.

Doctoral Dissertations Accepted by American Universities. New York: The H. W. Wilson Company, 1934-1946.

United States Office of Education. Library. Bibliography of Research Studies in Education. Washington, D. C.: Government Printing Office, 1929-1940.

Good, Carter Victor, editor. "Doctors' Theses under Way in Education." Journal of Educational Research (January issues, 1931-1946).

Bibliographic Index: A Cumulative Bibliography of Bibliographies. New York: The H. W. Wilson Company, 1938-1946.

Education Index: A Cumulative Author and Subject Index to a Selected List of Education Periodicals, Books and Pamphlets. New York: The H. W. Wilson Company, 1929-1946.

In making the above survey of literature, the author found no studies which were closely enough related to this study to be of any real value in analyzing the mathematical skills of eighth-grade pupils.

The above-mentioned analysis was undertaken in Greensboro, North Carolina. It was decided to give a test to certain eighth grades during the first week of school, to determine the needs of these students in mathematics. In order to find a test that would measure the ability of these students, a careful examination of the North Carolina State Bulletin, A Suggested Twelve-Year Program, was made by the writer for accurate information on the state course of study, and, more specifically, on the purposes of eighth-grade arithmetic as proposed therein. The test considered best for evaluating these students in arithmetic was: The Metropolitan Achievement Tests, Advanced Arithmetic Test: Form A (Revised).

This test was constructed in two parts. The first part contained sixty examples for testing the pupils' ability in the mechanics of arithmetic. The second part contained forty problems to be used in measuring their ability to solve problems.

Each paper was scored according to the grade and age equivalent for both the test on fundamentals and that on problem-solving. These scores were averaged, to discover an average arithmetic age and grade equivalent in the tested group.

The tests were given to approximately 290 pupils in three different schools scattered over Greensboro. Before the tests were given, permission was obtained from the principal of each school to be used for this testing. Eighth-grade teachers who were in sympathy with the study were selected to administer the tests, and in return each teacher received the scores of his pupils and an analysis of the same.

CHAPTER II

THE FUNDAMENTAL DIFFICULTIES WHICH APPEAR IN THE MECHANICS OF ARITHMETIC

Introduction

This study is based on the results of the Metropolitan Achievement Tests, Advanced Arithmetic Test: Form A (Revised) given to 288 students during their first week in the eighth grade. The tests were administered in the following schools: the Charles B. Aycock School, the Central Junior High School, and the Curry Demonstration School. These schools were selected in different sections of Greensboro, North Carolina; the pupils were typical of those found in an average urban school.

The Charles B. Aycock School serves an area inhabited by many families employed in a large textile mill, as well as a section inhabited by the owners of the city's industrial concerns, that is to say, business and professional people of the community. Immediately surrounding the school are to be found the homes of average, middle-class families. In this school, students represent homes of all the economic levels.

Central Junior High School is the largest junior high school in Greensboro and is "fed" by elementary schools scattered over the city. As the name implies, it is located in a central, downtown position. The students of this school are neither better nor worse in family background and scholastic ability than those of other junior high schools in the Greensboro area. The homes represented here, for the most part, are from the average American middle class.

Curry Demonstration School is located on the campus of Woman's College of the University of North Carolina. Students of this school come from a district surrounding the campus. Some few of the pupils are "faculty children," but the majority come from a middle- or a low-income class of homes, due to the section of town in which the district lies.

The three schools used for testing were selected because they are representative of the types of students found in junior high schools in any city of like size and composition.

The principals and teachers of these schools were highly cooperative, else this study would not have been possible. Each teacher administered the tests to his students during the regular class period. On the first occasion, the students were allowed thirty-five minutes in which to complete as many examples as possible on the test in fundamentals. The classes were instructed, before the test began, that there were some examples in which correct answers were expected of all, and others which only the very best students could do. If they were unable to work an example, they were to leave it and go on to the next, going back later, if the time permitted, to any examples thus omitted. The students worked uninterruptedly for thirty-five minutes, whereupon everyone was asked to stop and the tests were collected. On the following day, each child received his own paper; and the portion of the test dealing with arithmetic problems was administered. The same instructions were repeated; at the end of thirty-five minutes the papers were collected again.

Each test was scored according to the directions with the key, raw scores being translated in terms of grade and age equivalent for both the test on fundamentals and that on problem-solving. These were added to obtain an average arithmetic grade and age equivalent.

A Graphic Representation--Fundamentals

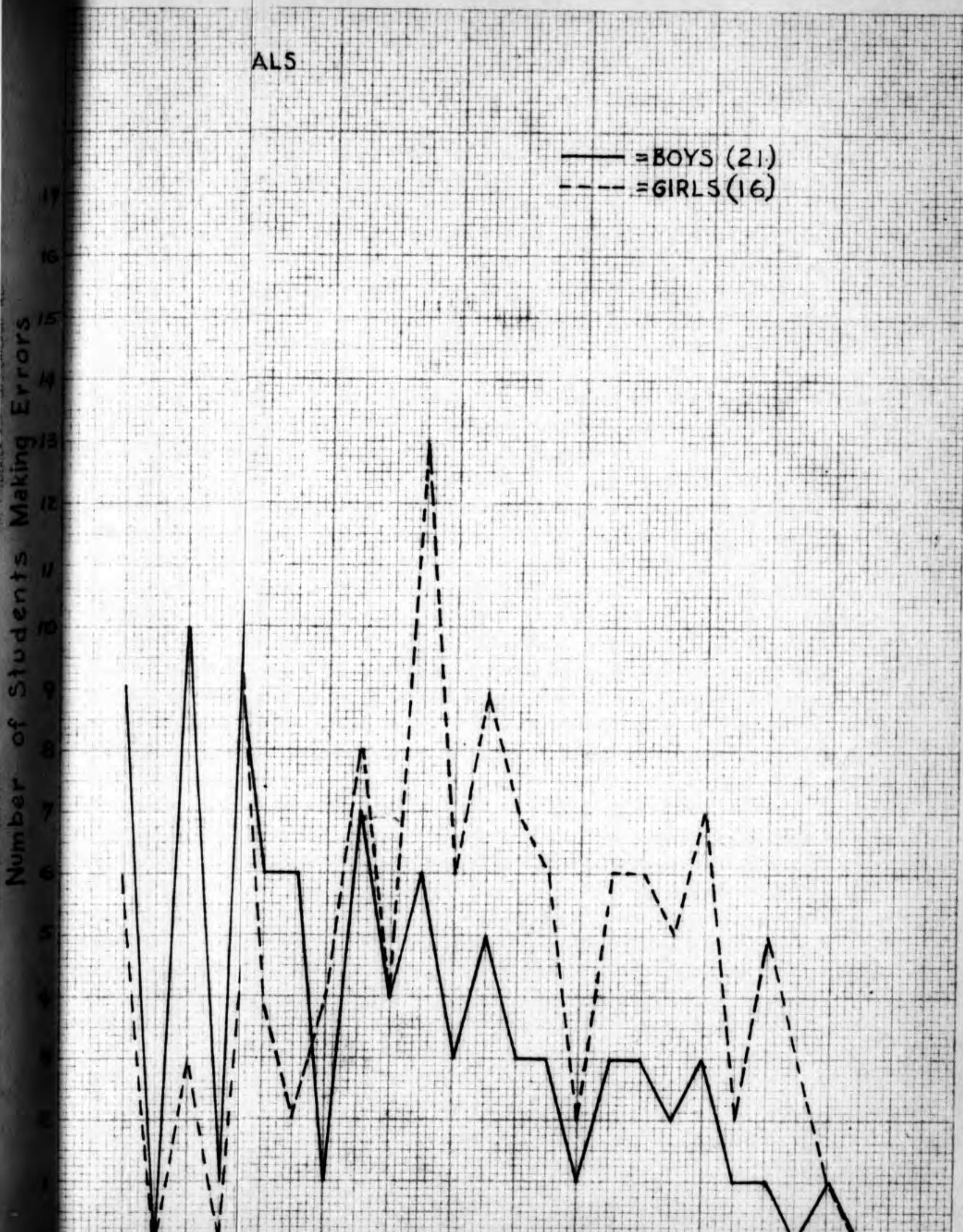
The test papers for each class were divided into groups, boys and girls separately. The examples solved incorrectly on every paper were recorded, with the results translated into a graph for each class. Figures One through Eight, found on the following pages, are graphic representations of the errors of the students in the various classes. The graphs compare the ability of the boys and girls on the examples in the test. Figure Nine represents the errors made by all of the boys and all of the girls, as well as the total number of mistakes for the entire group of students tested.

Generalizations from the Test

The primary purpose of administering this test was not to compare the schools which participated in the study, but rather to locate the points of strength and weakness in the arithmetical abilities of eighth-grade pupils.

In examining the tests as a group, it was found that there was no marked difference between the ability of the boys and that of the girls to solve arithmetic examples. Their errors were parallel, each group finding the same problems difficult as did the group of the opposite sex. The girls were equally as competent as the boys in obtaining the correct solutions for the examples. The test results did not indicate a single example consistently worked more easily by boys than by girls. This was also true in the reverse. The results from these tests indicate that the sex of the child has little or no bearing on his ability to comprehend mathematics. This confounds the belief prevalent among students who would attempt to "alibi" for their lack of understanding on the grounds of sex inequalities in learning ability.

This conclusion--i.e., of the irrelevancy of sex to the problem--may draw some challenge. In this instance, however, if the direction of the curves of these graphs is compared, rather than the number of errors made by each sex in any group, it will be evident that the points of greatest perpendicular rise in each case occurred, in the main, at the same intervals for both sexes. Inasmuch as the number of girls and boys used in the test differed, the use of numbers of errors is not as important a comparison as the direction of each curve. In other words, certain work on these tests, which gave trouble to one sex, gave trouble to the other in approximately similar emphasis. It would have been virtually impossible to have found study groups with equal numbers of boys and girls, and equally as impossible to balance the numbers in any fashion without making one group more selective than another, or creating unfair test situations; hence the groups were considered exactly as they were found.



ALS
=BOYS (21)
=GIRLS (16)
Number of Students Making Errors
1 2 3 4 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60

FIGURE 1

ERRORS MADE BY 8-A CENTRAL SCHOOL

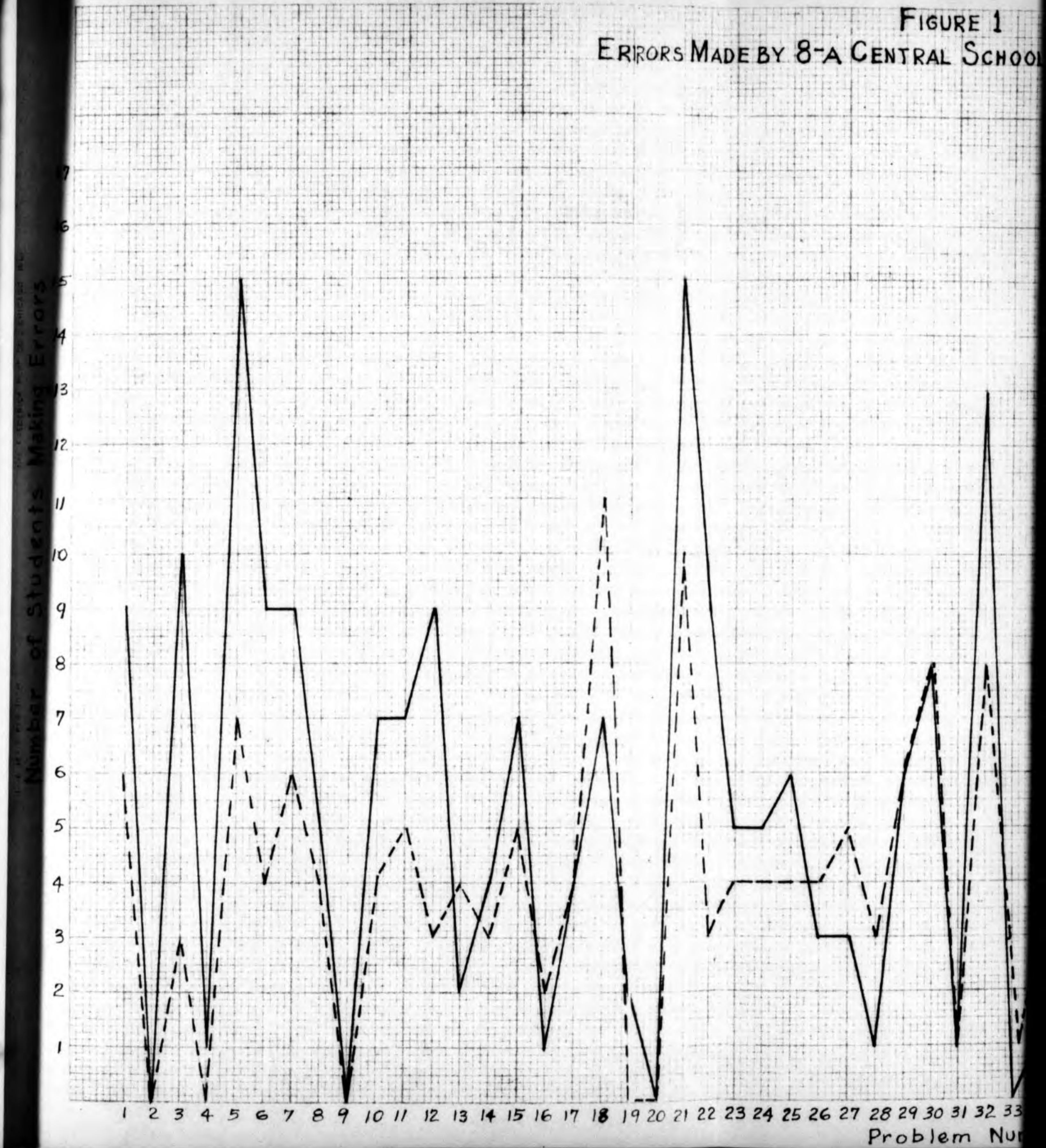
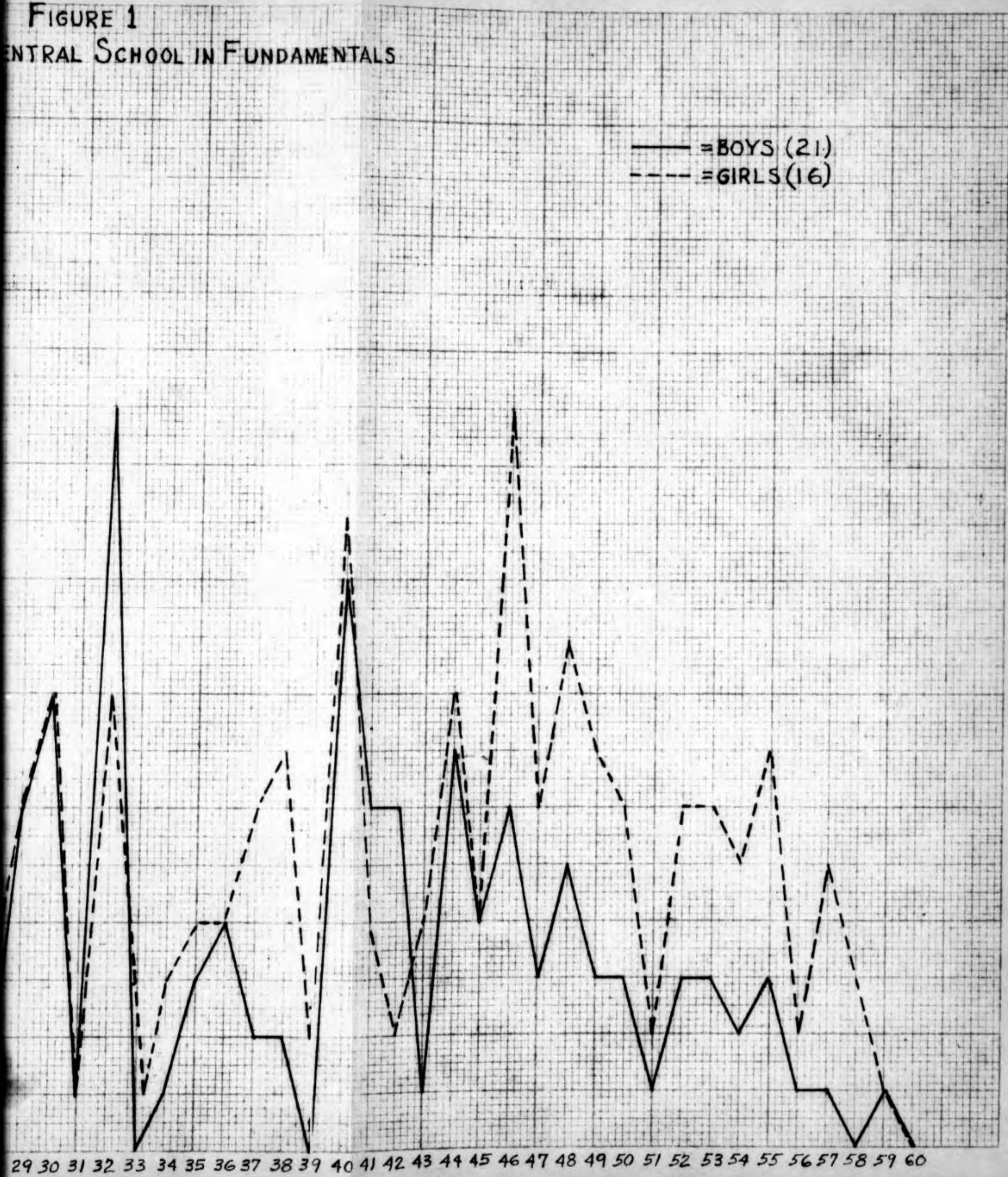


FIGURE 1

ENTRAL SCHOOL IN FUNDAMENTALS



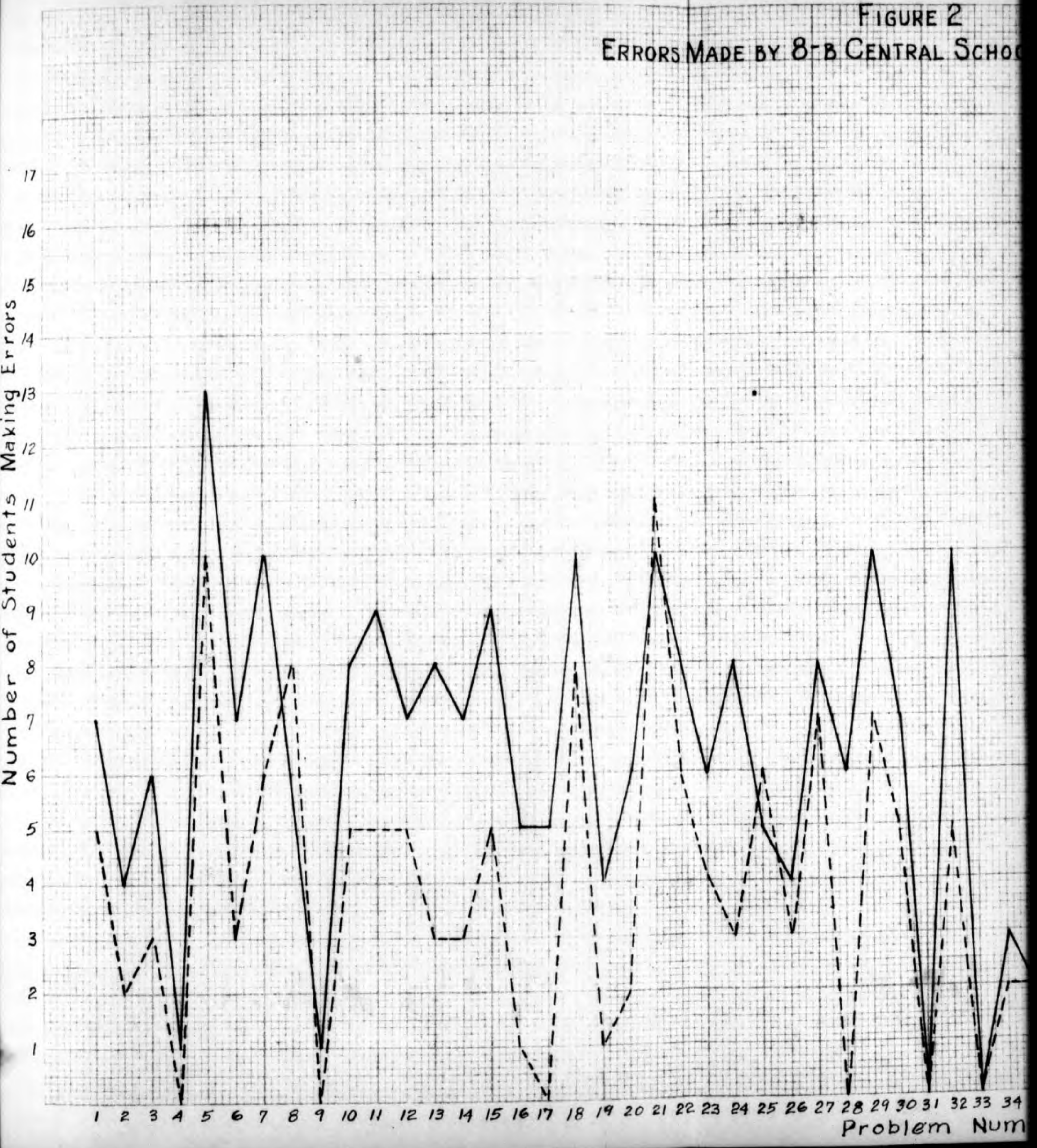


FIGURE 2
ERRORS MADE BY 8-B CENTRAL SCHOO
Number of Students Making Errors
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
Problem Num

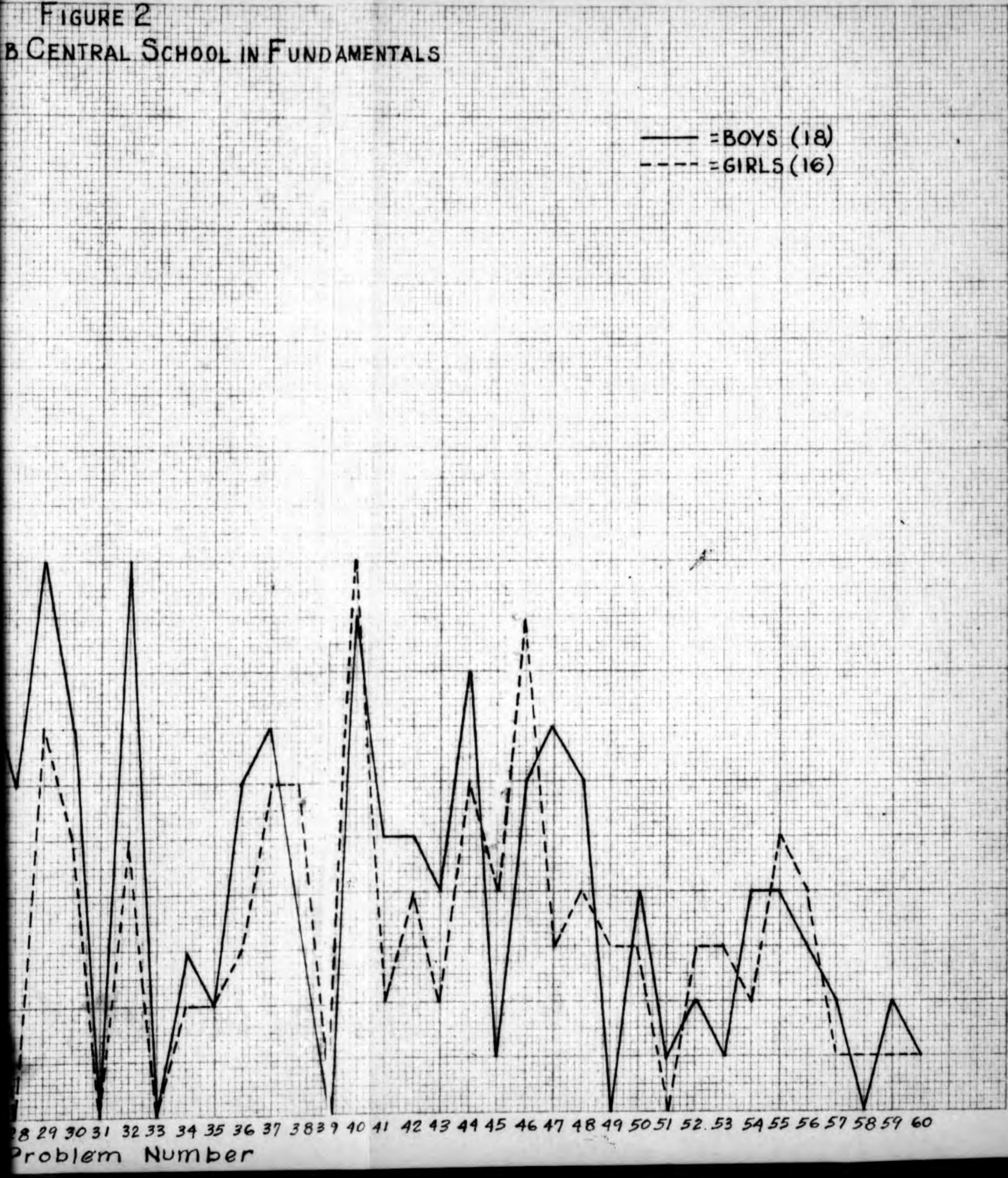


FIGURE 2
B CENTRAL SCHOOL IN FUNDAMENTALS
=BOYS (18)
=GIRLS (16)
28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
Problem Number

FIGURE 3

ERRORS MADE BY 8-C CENTRAL SCHOO

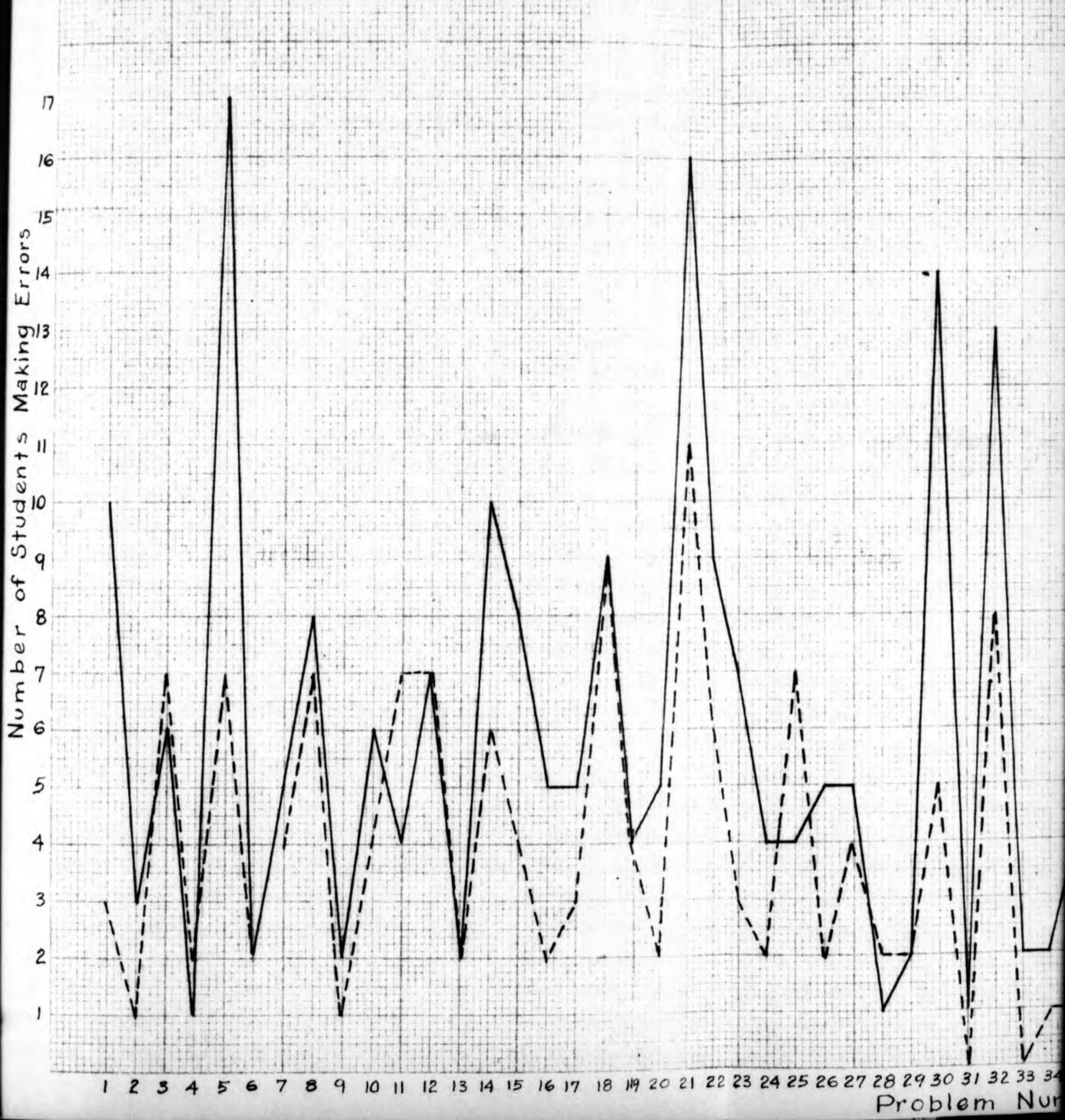
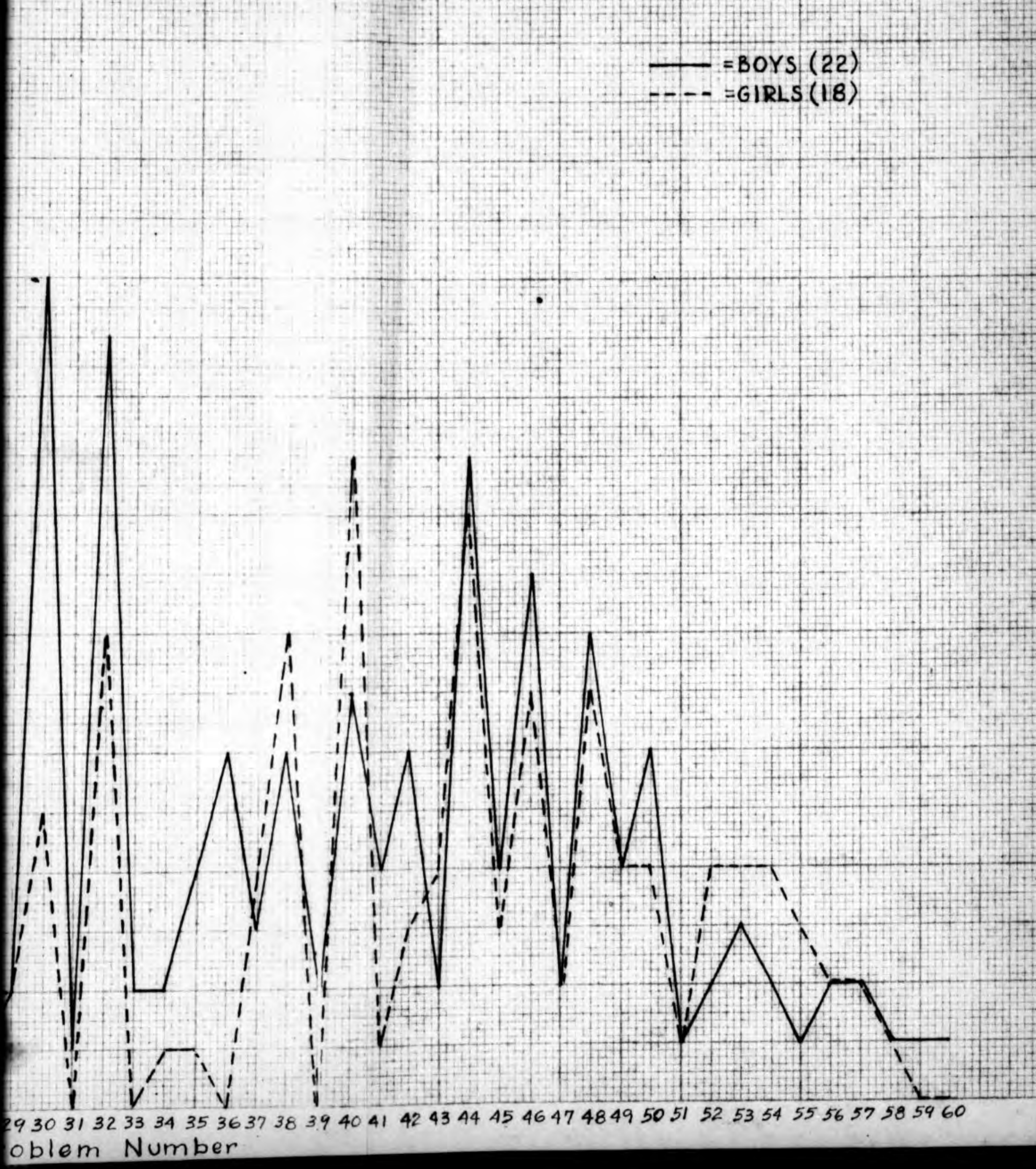


FIGURE 3

CENTRAL SCHOOL IN FUNDAMENTALS



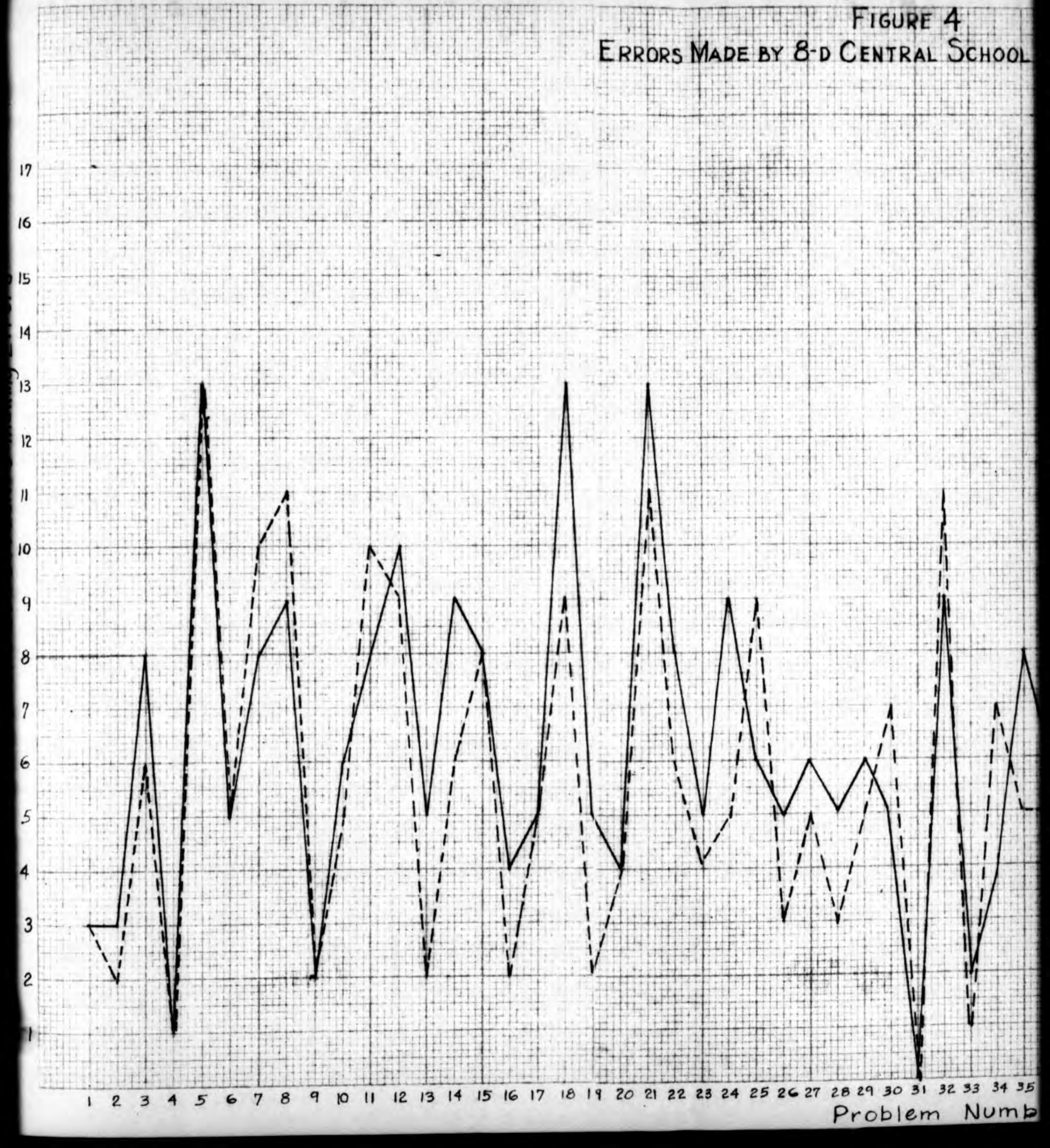
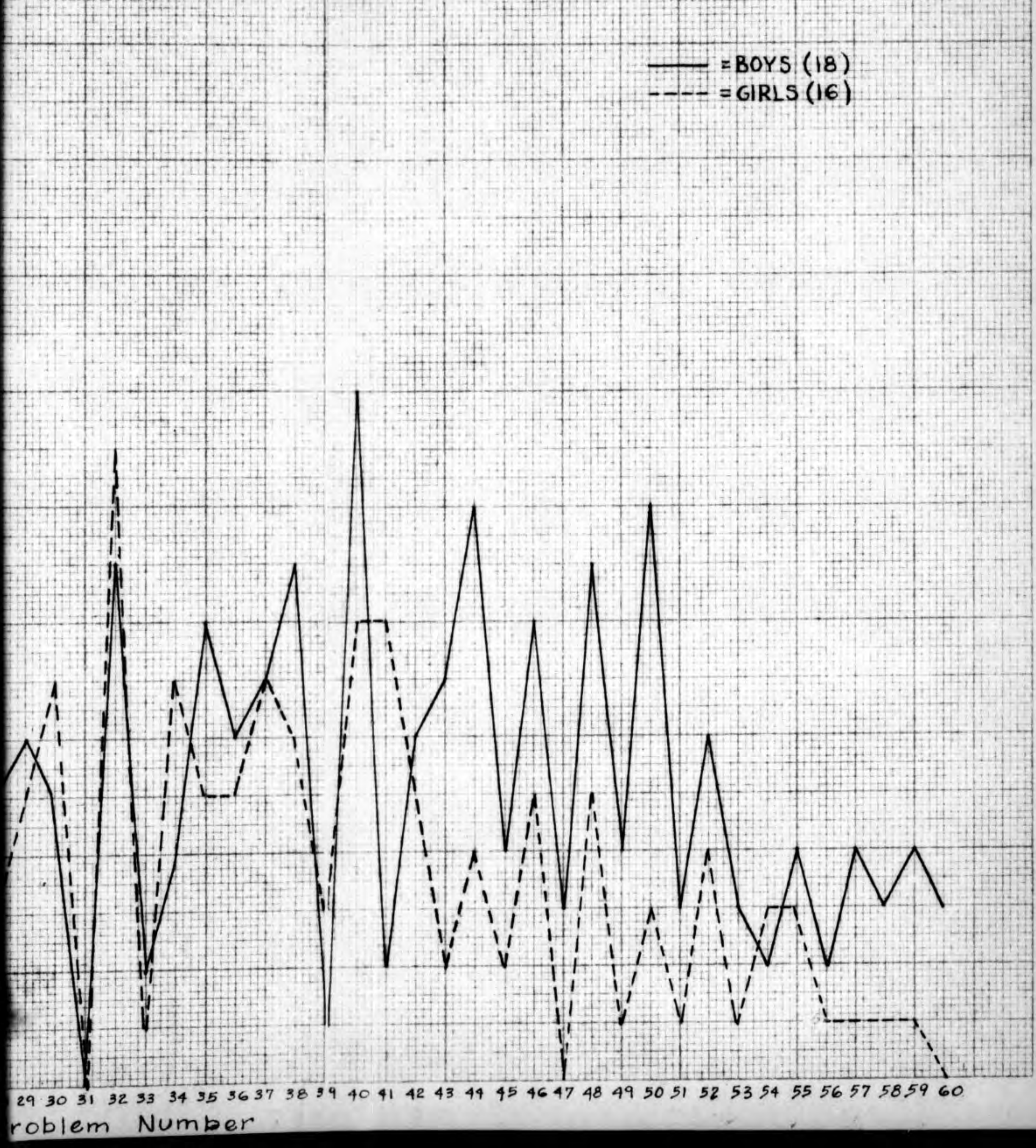


FIGURE 4
ERRORS MADE BY 8-D CENTRAL SCHOOL
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Problem Numb

FIGURE 4
CENTRAL SCHOOL IN FUNDAMENTALS



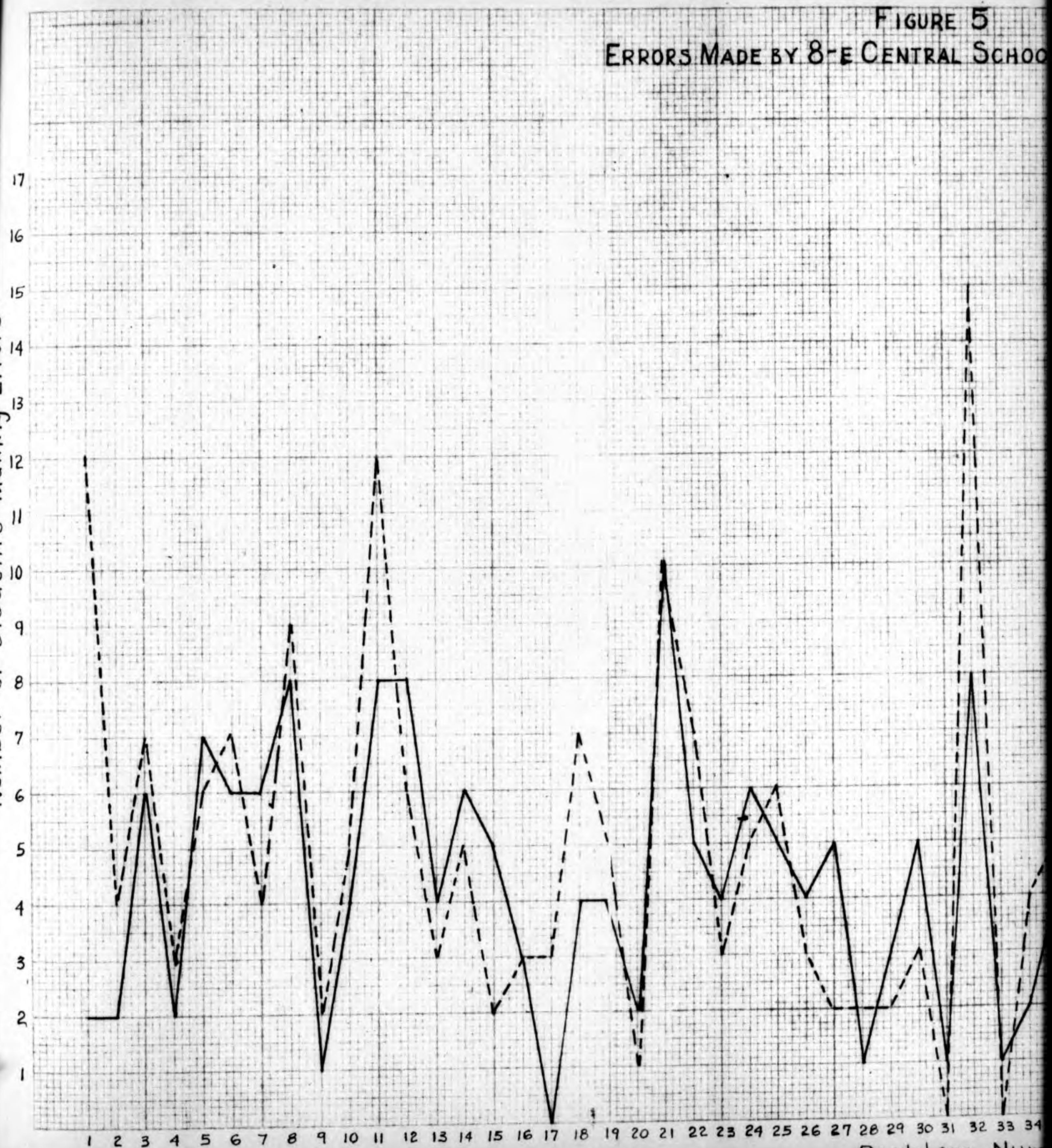


FIGURE 5
ERRORS MADE BY 8-E CENTRAL SCHOO
Number of Students Making Errors
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
Problem Num

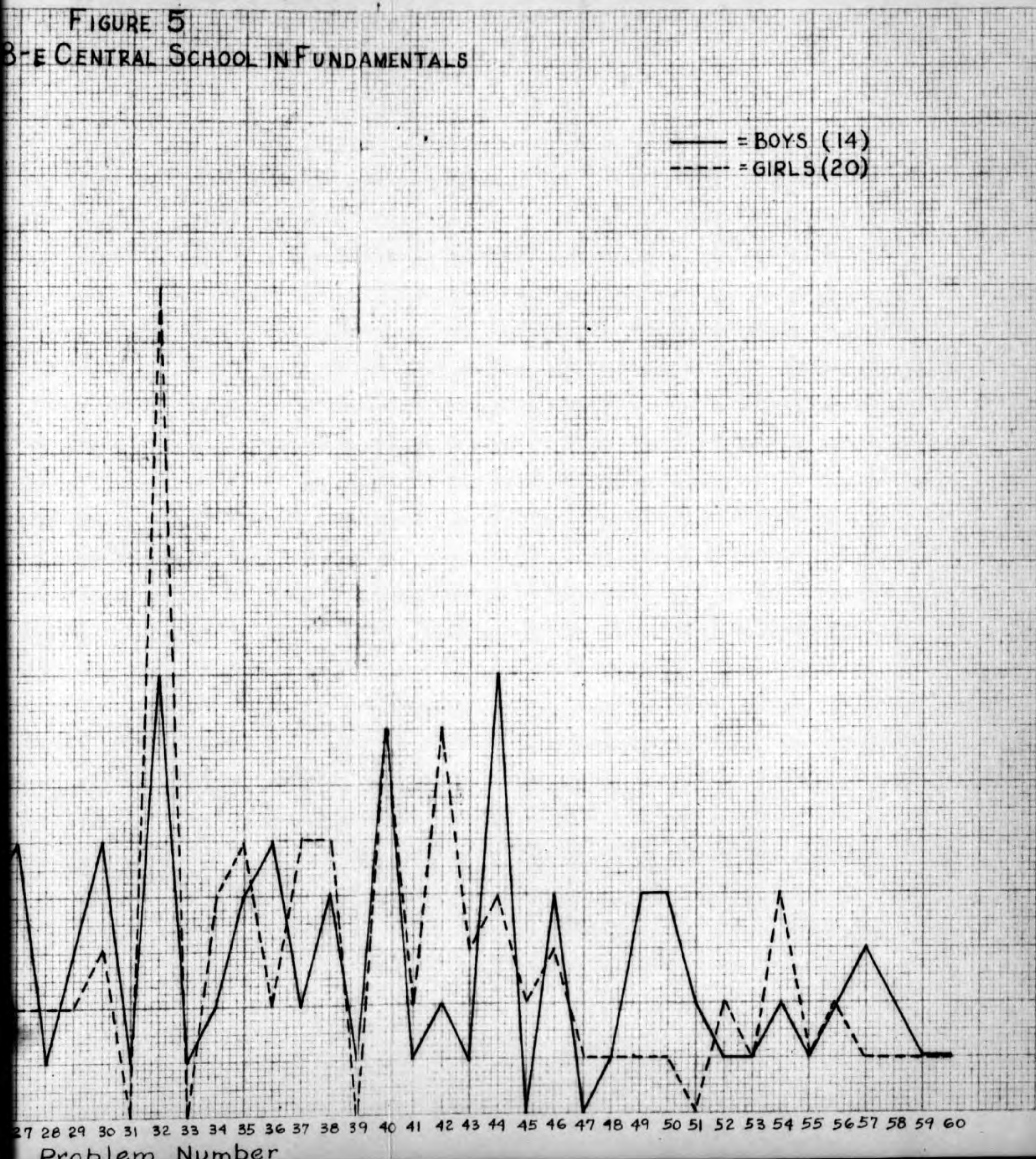


FIGURE 5
8-E CENTRAL SCHOOL IN FUNDAMENTALS
= BOYS (14)
= GIRLS (20)
27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
Problem Number

FIGURE 6

ERRORS MADE BY 8TH GRADE CURRY

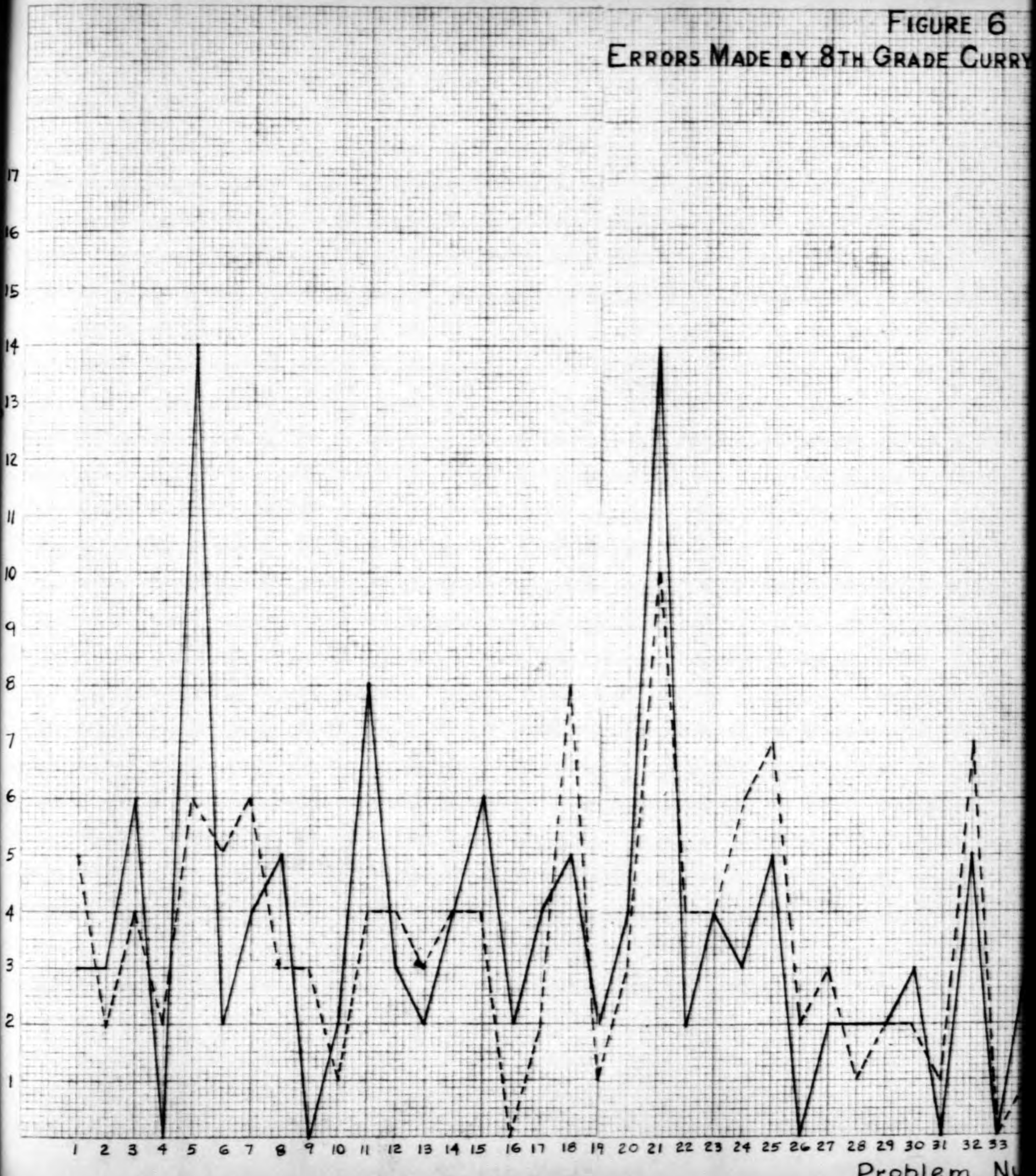


FIGURE 6

TH GRADE CURRY SCHOOLS IN FUNDAMENTALS

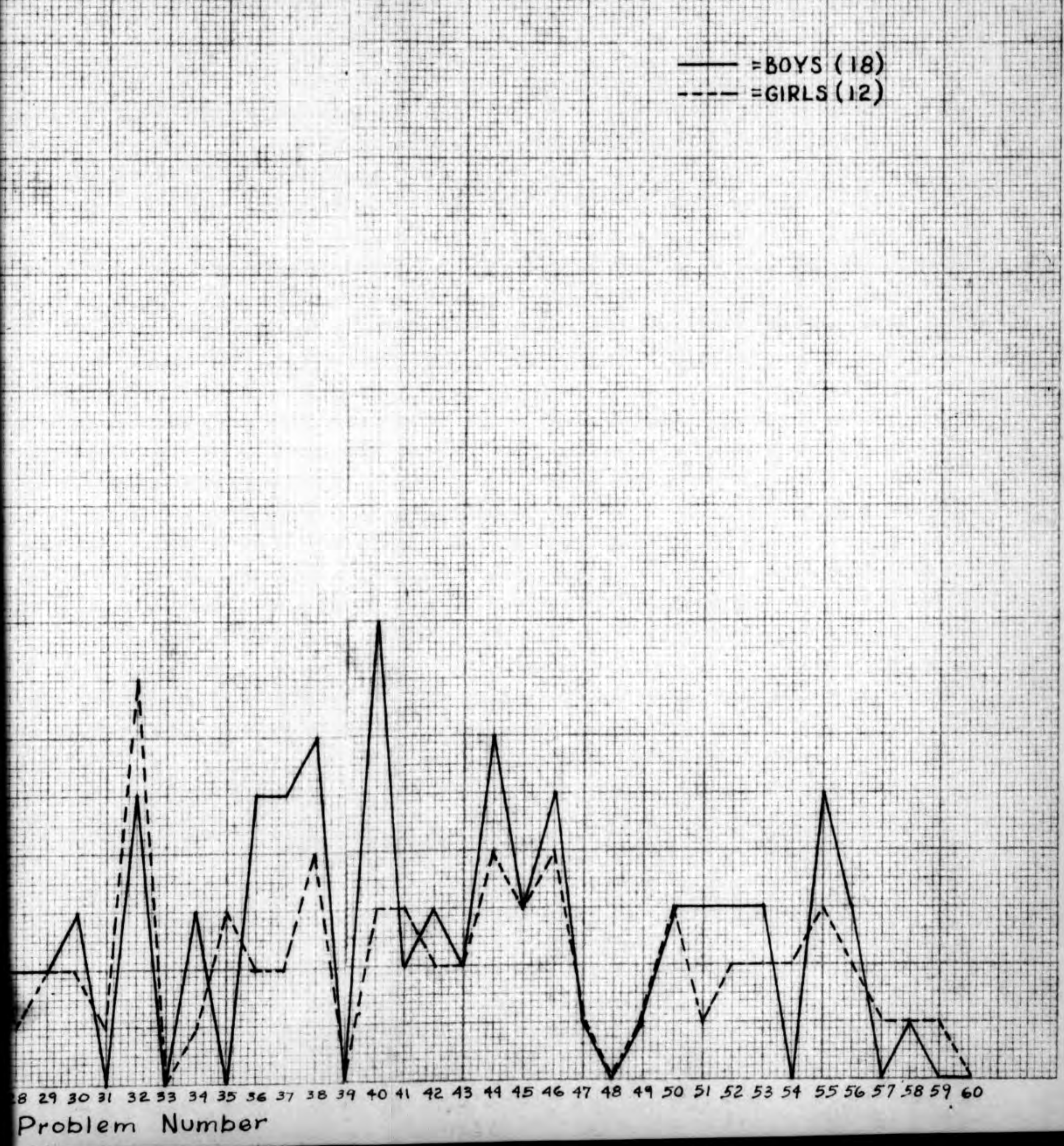


FIGURE 7
ERRORS MADE BY 8-A AYCOCK SCHOOL

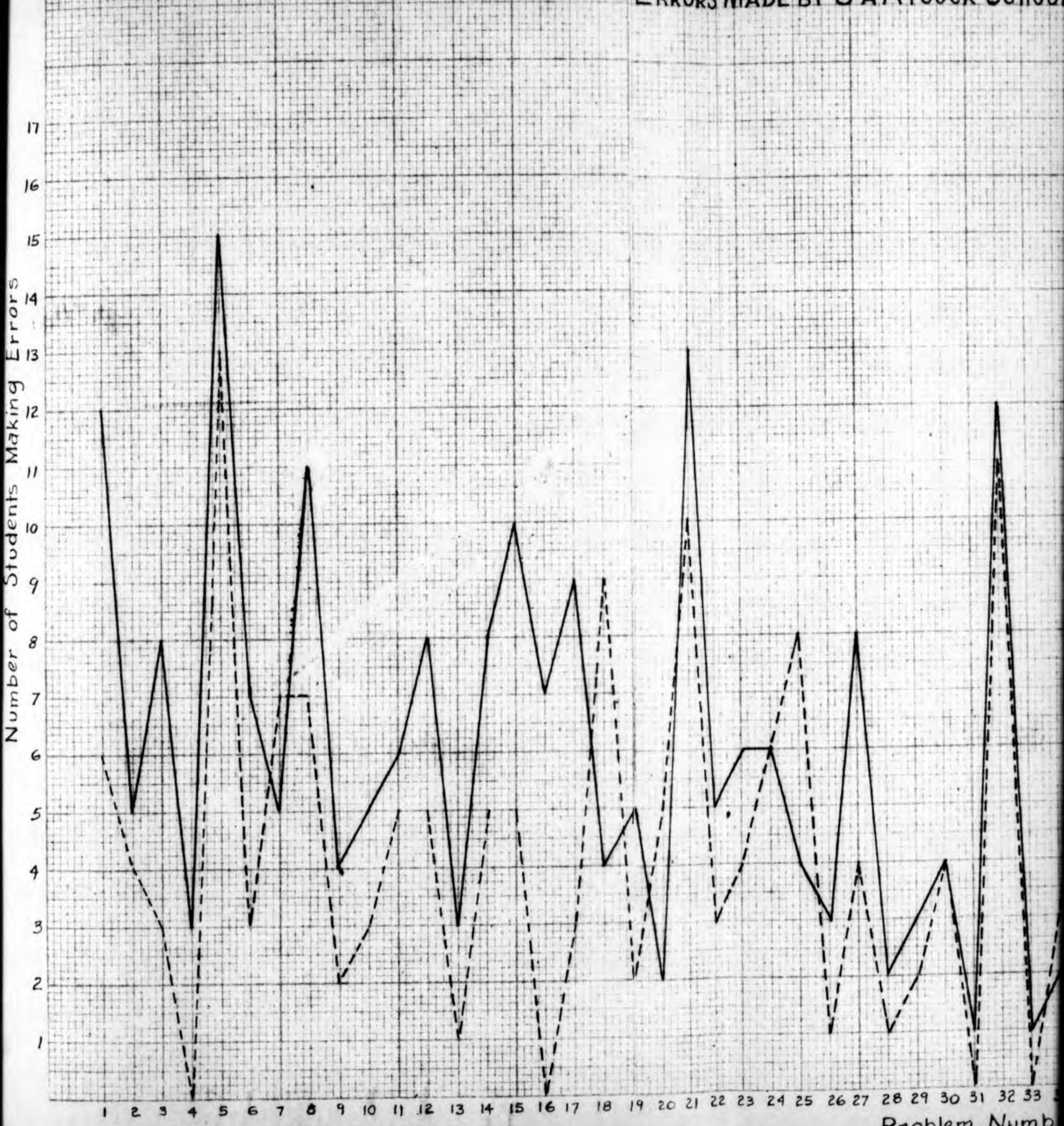
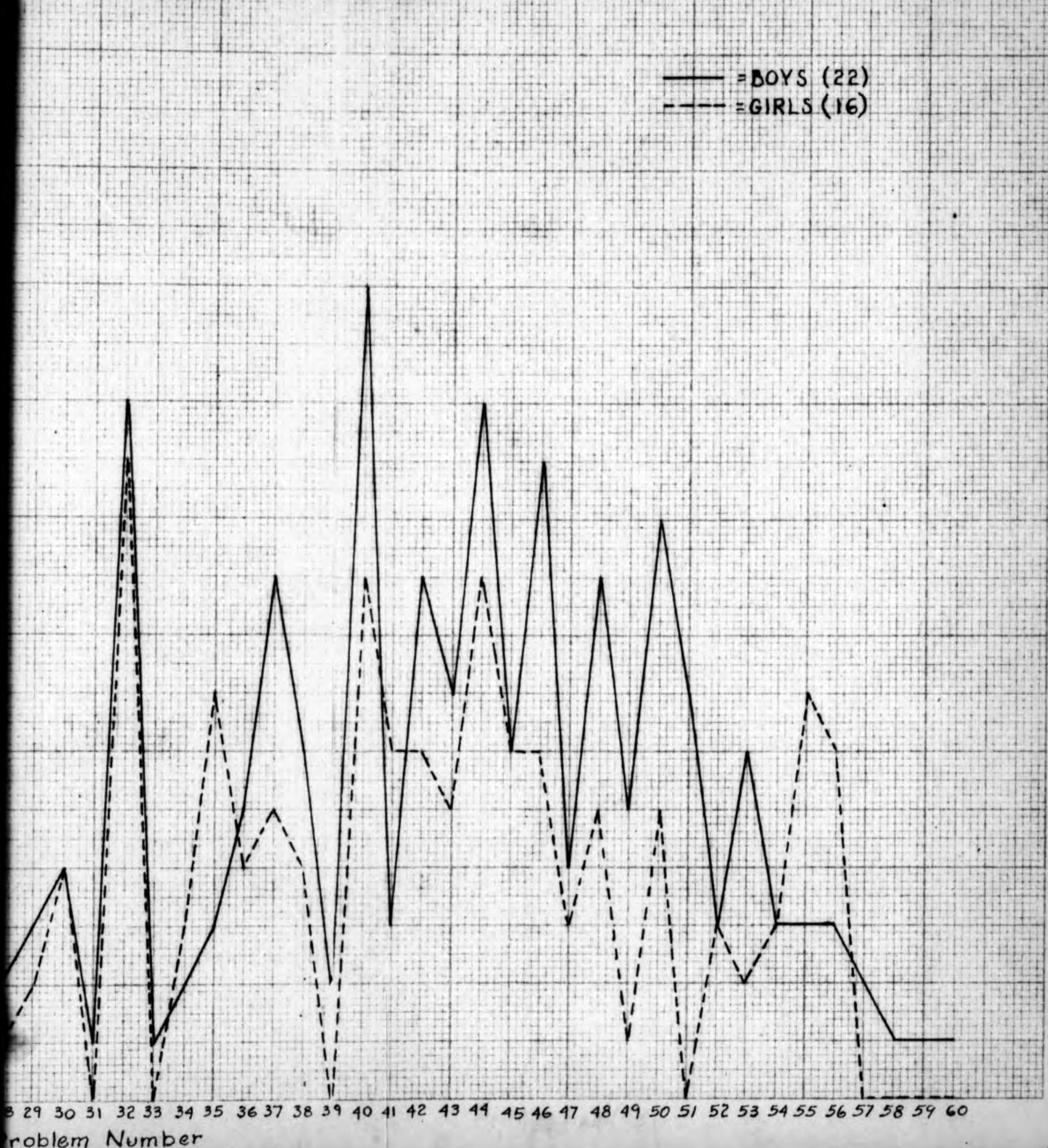


FIGURE 7

AYCOCK SCHOOL IN FUNDAMENTALS



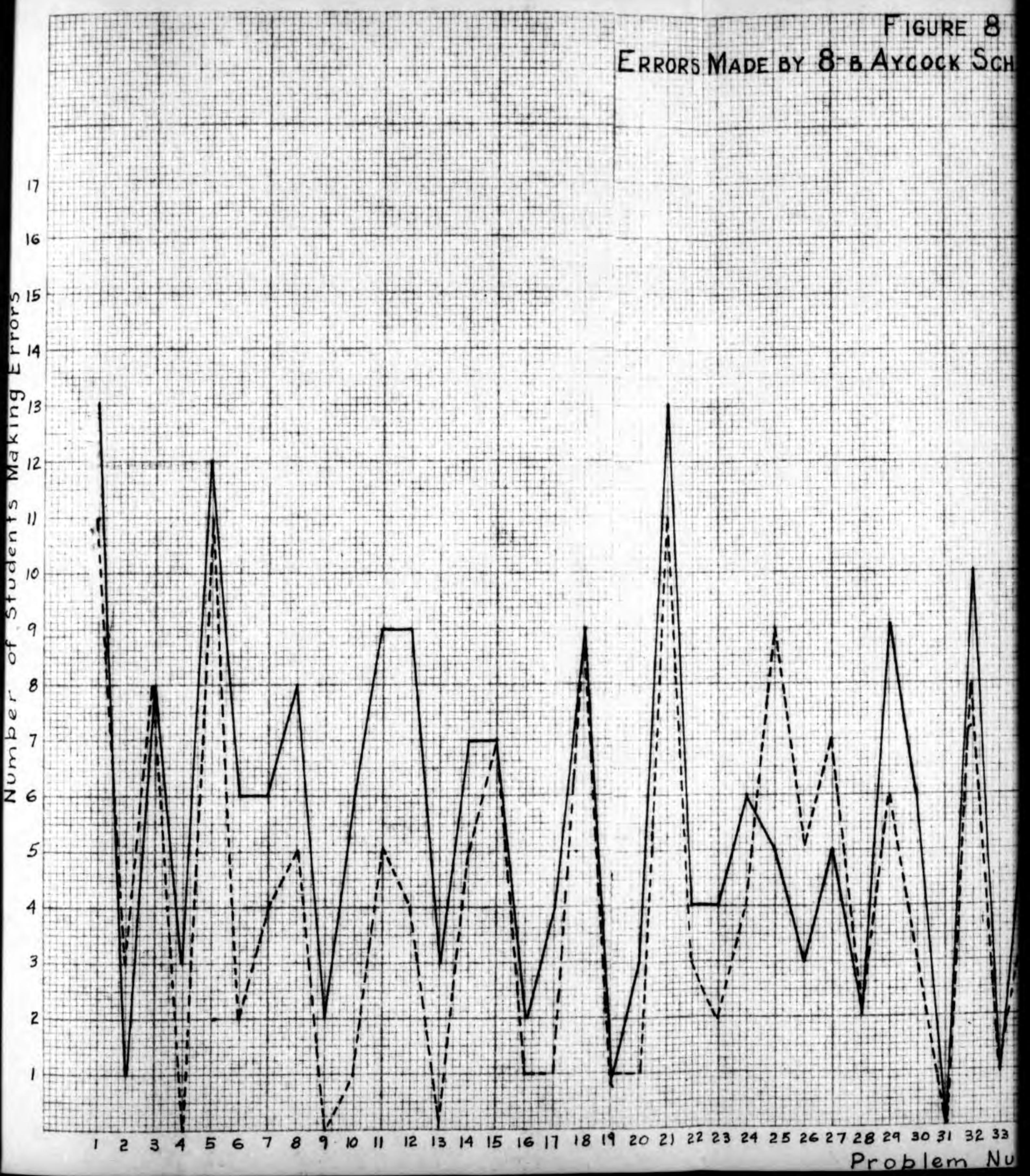
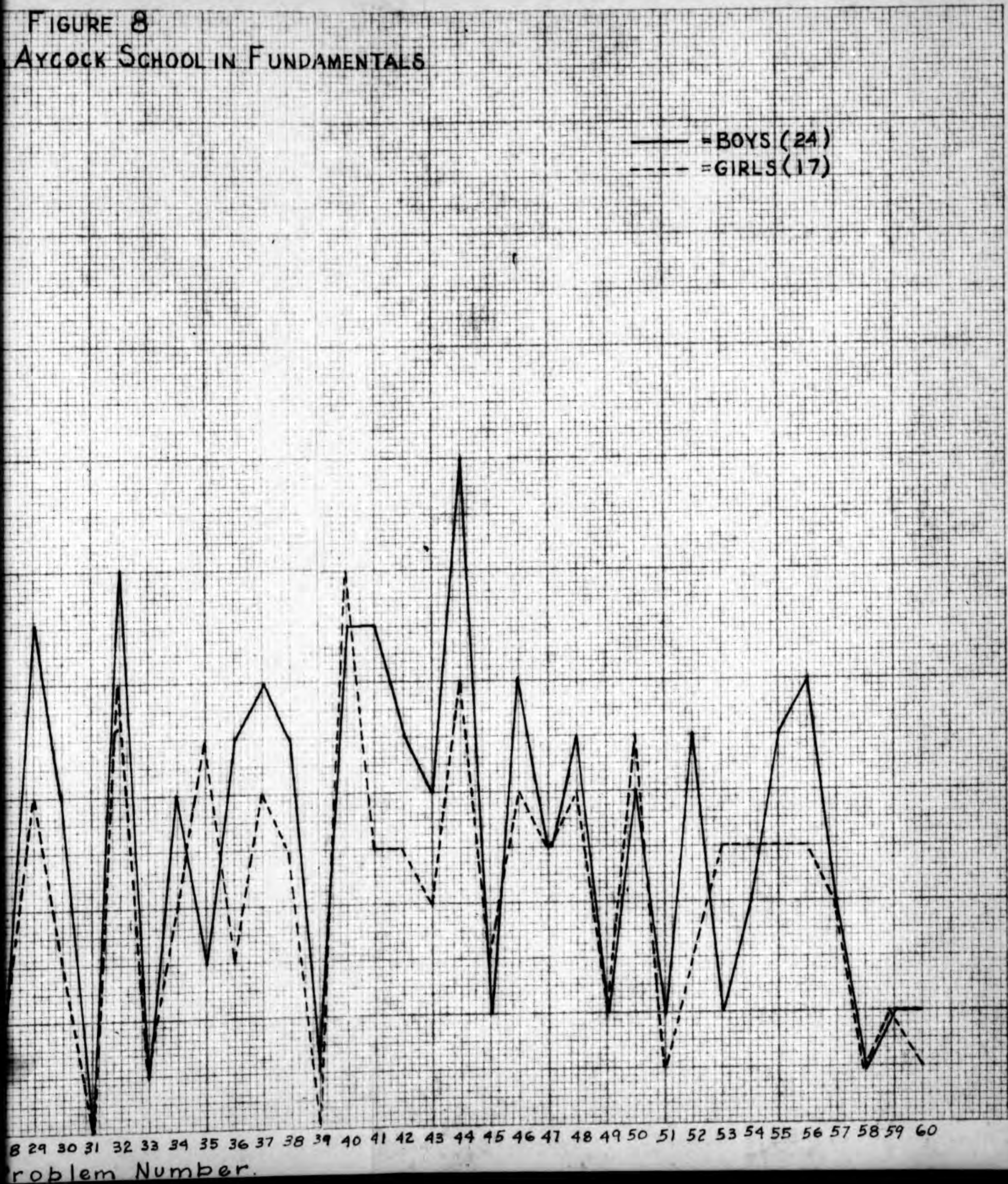


FIGURE 8
ERRORS MADE BY 8-B AYCOCK SCH
Number of Students Making Errors
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33
Problem Nu

FIGURE 8

AYCOCK SCHOOL IN FUNDAMENTALS



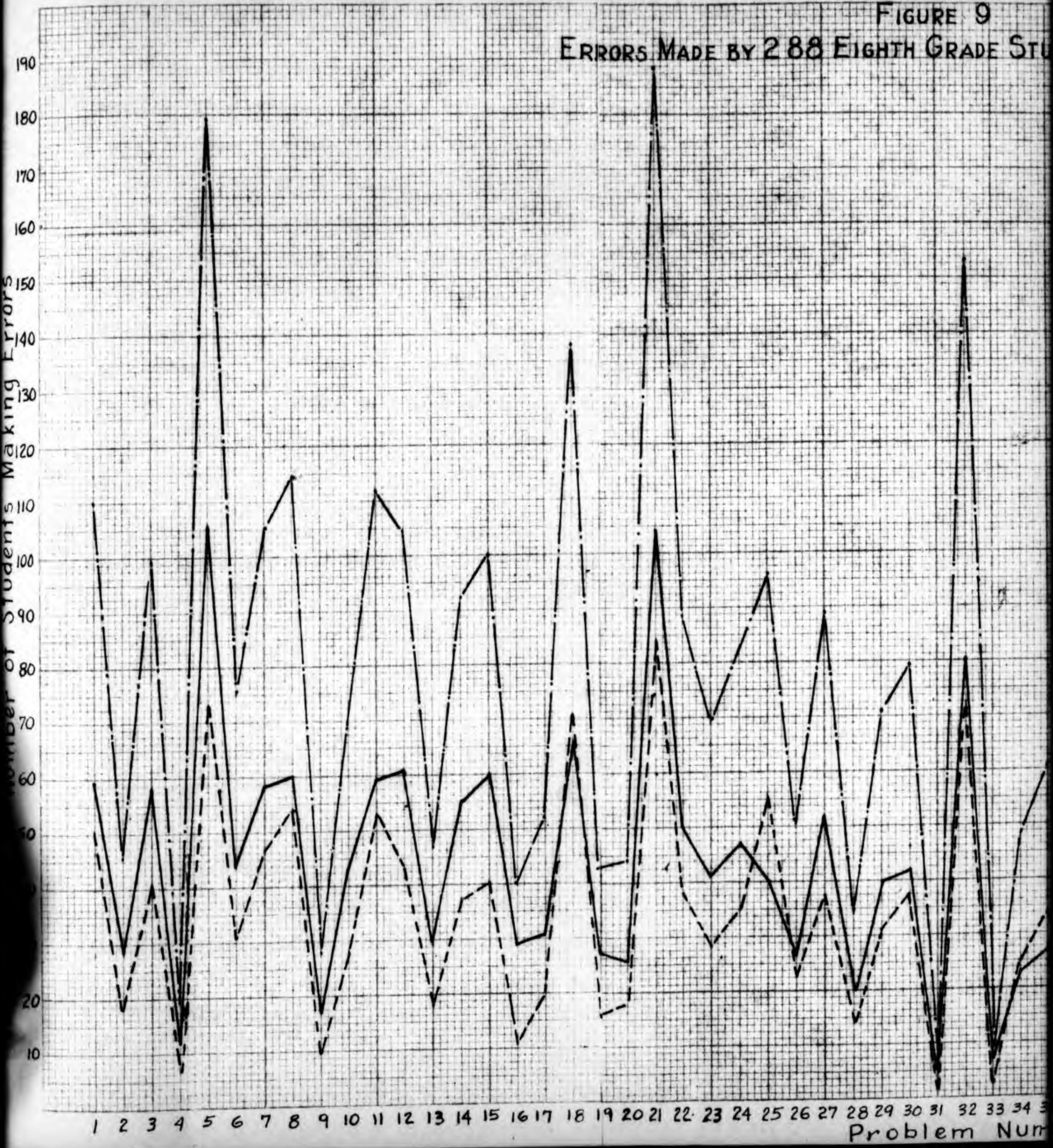


FIGURE 9
ERRORS MADE BY 288 EIGHTH GRADE STU
Number of Students Making Errors
190
180
170
160
150
140
130
120
110
100
90
80
70
60
20
10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
Problem Num

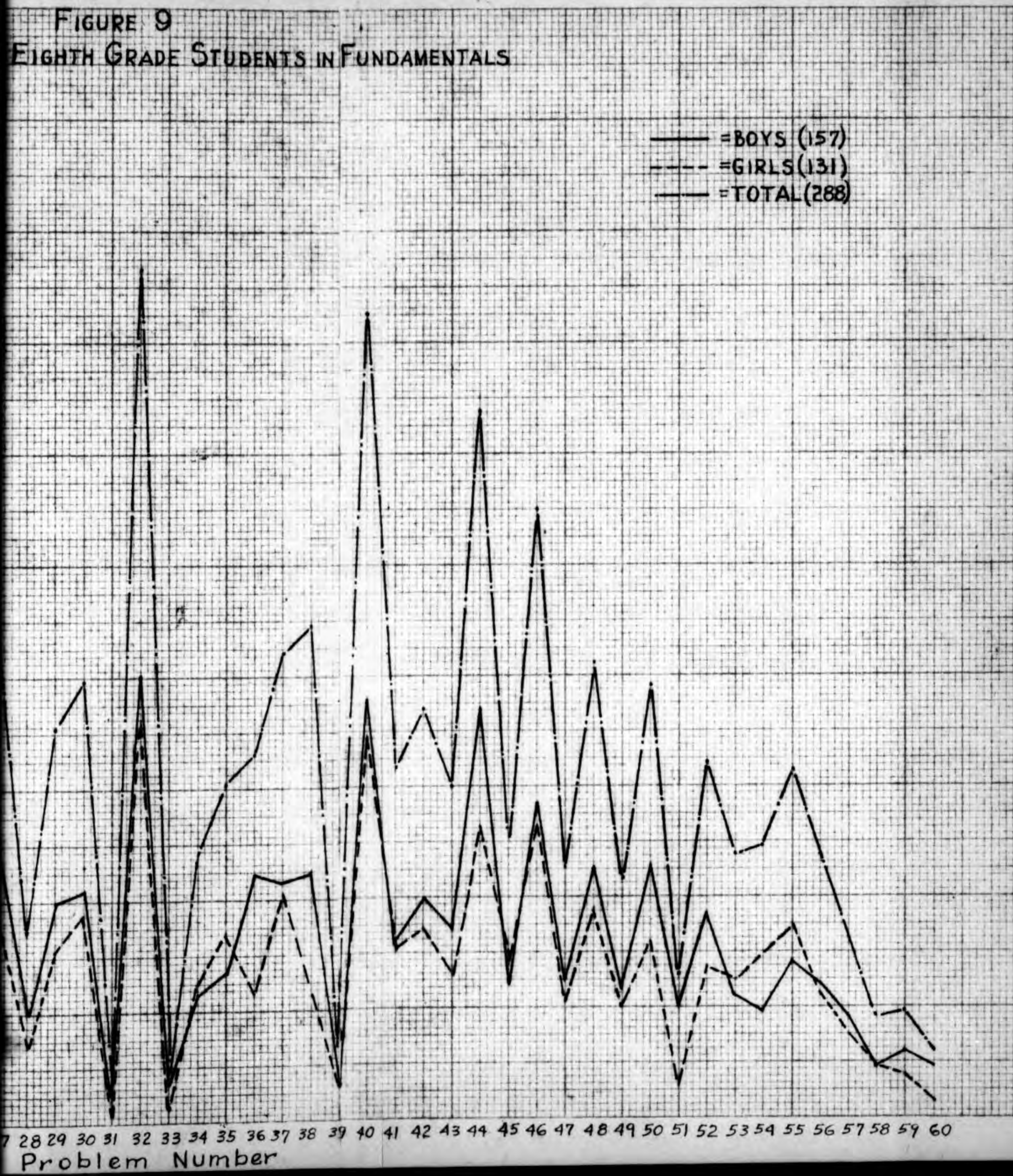


FIGURE 9
EIGHTH GRADE STUDENTS IN FUNDAMENTALS
=BOYS (157)
=GIRLS (131)
=TOTAL (288)
28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
Problem Number

The schools selected for this study were typical junior high schools. The pupils were the average to be found in eighth grades. There was not a noticeable difference in the results by schools or by classes. There was, of course, a slight variation, but this was sufficiently small to be considered negligible. This would suggest that the schools examined are placing similar emphasis on mathematics from the standpoint of fundamental processes. There was not enough difference in the results of the individual classes to show that any one teacher was doing a piece of work superior to that of the others, particularly in view of the fact that the year had just begun. Table I shows the mean of each class in the schools tested.

Table I

A Measure of Central Tendency for the Fundamentals of Arithmetic

School	Section	Mean
Central	8-a	7.959
Central	8-b	7.891
Central	8-c	8.122
Central	8-d	7.441
Central	8-e	7.723
Curry	8	8.446
Aycock	8-a	7.994
Aycock	8-b	8.082

Examples Found in the Test[1]

To facilitate the understanding of this study, Figures Ten, Eleven and Twelve were included. They are the pages of examples which the eighth-grade pupils were asked to solve for this report. The example

1. Metropolitan Achievement Tests, Advanced Arithmetic Test: Form A (Revised), 1932. Part I, "Arithmetic Fundamentals," pp. 2-4.

numbers in Tables II and III correspond to the example numbers in the test.

Figure 10. Examples Which Appeared on the Test

Metropolitan: Adv. Arith.: A

TEST 1. ARITHMETIC FUNDAMENTALS

Directions. Work each example and write the answer in the box near it. If you have to copy your answer, be sure to copy it correctly. *Reduce all improper fractions to mixed numbers, and all fractions to lowest terms.*

1. ***Add***

$$\begin{array}{r} 29699 \\ 88687 \\ 99899 \\ 87739 \\ 42478 \\ \hline \end{array}$$

2. ***Subtract***

$$\begin{array}{r} 763811 \\ 464762 \\ \hline \end{array}$$

3. ***Multiply***

$$\begin{array}{r} 5978 \\ 87 \\ \hline \end{array}$$

4. $38\,\overline{)\,76114}$

5. $78\,\overline{)\,11638}$

6. $\frac{1}{10} + \frac{1}{10} =$

7. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6} =$

8. ***Add***

$$\begin{array}{r} 6\frac{3}{4} \\ 17\frac{1}{2} \\ 8\frac{3}{5} \\ \hline \end{array}$$

9. ***Subtract***

$$\begin{array}{r} 9\frac{3}{5} \\ 4\frac{1}{5} \\ \hline \end{array}$$

10. $\frac{5}{6} - \frac{1}{3} =$

11. ***Subtract***

$$\begin{array}{r} 6\frac{1}{6} \\ \frac{2}{5} \\ \hline \end{array}$$

12. $4\frac{5}{8} - \frac{2}{3} =$

13. $8 \times 2\frac{3}{4} =$

14. $\frac{3}{8} \times 10\frac{2}{3} \times \frac{4}{5} =$

15. $10 \times 2\frac{2}{5} \times 1\frac{7}{8} =$

16. $1\frac{3}{5} \div 1\frac{3}{5} =$

17. $\frac{7}{10} \div \frac{3}{4} =$

18. $\frac{1}{7} \div 7 =$

19. $3\frac{3}{4} \div \frac{3}{4} =$

20. ***Add***

$$\begin{array}{r} .35 \\ .224 \\ .0542 \\ 325.042 \\ .9 \\ .0025 \\ \hline \end{array}$$

(Go right on to the next page.)

Figure 10. Examples Which Appeared on the Test

METROPOLITAN ACHIEVEMENT TESTS

ADVANCED ARITHMETIC TEST: FORM A

(*Revised*)

By RICHARD D. ALLEN, PH.D.
Assistant Superintendent of Schools,
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HAROLD H. BIXLER, PH.D.
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and FREDERICK B. GRAHAM, PD.M.
Assistant Superintendent of Schools, New York City

Adv. Arith.

A

(*Revised*)

For Grades 7 and 8

Name.............................. Date..............19...

Grade.......Age....yrs....mos. Teacher.....................

School.............. City............... State..............

TEST	SCORE	GRADE EQUIVALENT	AGE EQUIVALENT
1. Arithmetic Fundamentals			
2. Arithmetic Problems			
Average Arithmetic			

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PRINTED IN U.S.A. MAT: ADV. ARITH.: A-20

Figure 11. Examples Which Appeared on the Test

Metropolitan: Adv. Arith.: A

21. $.45 + \frac{2}{3} + .83\frac{1}{3} =$ [ ]

22. $8.7 + 20 + .325 + .05 =$ [ ]

23. $\$81 - \$3.62 =$ [$]

24. $8.7 - .645 =$ [ ]

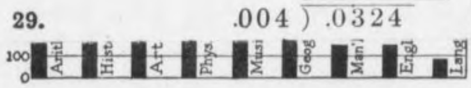
25. $\frac{1}{4} - .03 =$ [ ]

26. $5 \times .3 =$ [ ]

27. $100 \times 55.4 =$ [ ]

28. $9 \overline{) .72}$ [ ]

29. $.004 \overline{) .0324}$ [ ]



31. Which subject was liked best? .... [ ]

32. About how many pupils liked physical training best? ..... [ ]

33. Which was liked by most pupils, geography or music? .... [ ]

34. 25% of 16 = [ ]

35. $33\frac{1}{3}$% of 90 = [ ]

36. 8% of 20 = [ ]

37. 150% of 380 = [ ]

38. 27 is 18% of —— [ ]

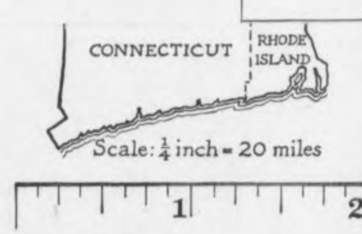
39. 1 ft. = [in.]

40. 1 pk. = [qt.]

41. *Add*

2 ft.	4 in.
3 ft.	8 in.
4 ft.	6 in.

[ ]



44. What is the distance in miles across the northern border of both Connecticut and Rhode Island? .... [mi.]

[3]

(Go right on to the next page.)

Figure 11. Examples Which Appeared on the Test

Metropolitan: Adv. Arith.: A

21. $.45 + \frac{2}{3} + .83\frac{1}{3} =$ ☐

22. $8.7 + 20 + .325 + .05 =$ ☐

23. $\$81 - \$3.62 =$ $ ☐

24. $8.7 - .645 =$ ☐

25. $\frac{1}{4} - .03 =$ ☐

26. $5 \times .3 =$ ☐

27. $100 \times 55.4 =$ ☐

28. $9 \overline{)\,.72}$

29. $.004 \overline{)\,.0324}$

30. $1.25 \overline{)\,87.5}$

31. Which subject was liked best? .... ☐

32. About how many pupils liked physical training best? ..... ☐

33. Which was liked by most pupils, geography or music? .... ☐

34. 25% of 16 = ☐

35. $33\frac{1}{3}$% of 90 = ☐

36. 8% of 20 = ☐

37. 150% of 380 = ☐

38. 27 is 18% of —— ☐

39. 1 ft. = ☐ in.

40. 1 pk. = ☐ qt.

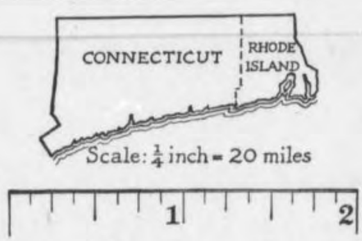
41. *Add*

2 ft. 4 in.
3 ft. 8 in.
4 ft. 6 in.

☐

42. How much must be added to 6 lb. 9 oz. to make 14 lb.? .... ☐

43. 4 ft. 4 in. = ☐ ft.



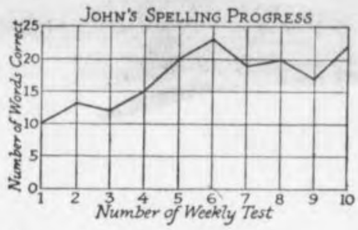
44. What is the distance in miles across the northern border of both Connecticut and Rhode Island? .... ☐ mi.

[3]

(*Go right on to the next page.*)

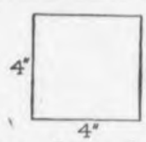
Figure 12. Examples Which Appeared on the Test

The graph below shows the number of words spelled by John Marks on each of 10 weekly spelling tests of 25 words.



45. On which two weeks did John spell correctly the same number of words? . . . .

46. The largest gain in one week was —— . . . .



47. The above figure is called a —— . . . . . .

48. Its perimeter is —— . in.

figure is —— . . . . . .

51. Line *AC* is called the —— . . . .

52. Find the average of 5, 7, 9, 11, 13, 15. . . . .

53. Selling price = \$2,500
Rate of commission = 18%
Commission = —— . . \$

54. Principal = \$800
Time = 1 yr. 3 mo.
Rate = $4\frac{1}{2}$%
Interest = —— . . . \$

55. Marked price = \$425
Discounts = 15% and 20%
Net price = —— . . \$

56. Principal = \$570
Time = 30 da.
Rate = 6%
Amount = —— . . . \$

57. Assessed value = \$12,000
Tax rate = \$2.32 per

60. $\frac{2}{3} n + 8 = 10$ . . . . $n =$

STOP!

No. RIGHT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Score	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63

No. RIGHT	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Score	64	66	67	68	69	70	71	72	73	74	75	77	79	80	81	82	83	84	85	86	87	89	90	91	92	93

No. attempted . . . . .
No. wrong . . . . .
No. right . . . . .

Score, Test 1 . . . . . .

Competency in Fundamentals

In the analysis of the papers of the students, and the subsequent comparison of these with the graph of errors for all the pupils, it was very easy to locate the examples that seemed to give little trouble in their solutions. The following examples gave comparatively little difficulty and it was evident that these eighth-grade students were capable of solving them.

Table II

Types of Mechanical Competency

Process	Example Number	Percentage of Errors	Explanation
Subtraction of whole numbers	2	16.3	"Borrowing" was necessary
Long division	4	10.0	The quotient had no remainder
Subtraction of mixed numbers	9	9.4	No "borrowing"
Multiplication of a small mixed number and a whole number	13	16.3	The product was a whole number
Division of small mixed numbers	16	13.8	The quotient was a whole integer
Divisions of fractions	17	18.1	These fractions are in popular use
Division of a mixed number and a fraction	19	14.4	
Addition of decimal fractions and whole numbers	20	14.6	The addends were small
Translation of a bar graph	31	2.8	Selecting the largest item represented
Translation of a bar graph	33	3.5	Selecting the better-liked subject between two bars
Changing of one foot to inches	39	4.5	
Naming the hypotenuse of a right triangle	51	9.0	

In analyzing the weaknesses of eighth-grade boys and girls, it is important to know the processes which they were able to perform. To summarize the results of the above examples--these typical eighth-grade students were reasonably capable of carrying out the following:

1. They were able to select from a bar graph the highest item represented and the lowest, as well as the relative positions of the other topics.

2. Students at this grade level apparently understood the principle of subtraction. They were able to borrow without difficulty, when whole numbers were involved. The subtraction of mixed numbers presented them no problem as long as borrowing was unnecessary.

3. A majority of the students successfully solved addition of small whole numbers combined with simple decimal fractions.

4. They had little trouble on the test with multiplication of small, whole, and mixed numbers.

5. They were also capable of handling long division of small integers without a remainder, as well as the division of mixed numbers and of common and decimal fractions.

The students showed satisfactory ability in the fundamental operations. When the numbers were small and the fractions were those in common use, they had little difficulty in dealing with them.

Fundamentals Presenting Difficulties

The graph in Figure Nine shows at a glance that certain examples were difficult for a large group of the students. It is worthwhile to examine these examples and locate the reasons for the difficulties. Table III indicates the examples that were solved incorrectly by a large number of the boys and girls.

Table III

Types of Mechanical Difficulty

Process	Example Number	Percentage of Errors	Explanation
Addition	1	38.1	Five-column example using large integers
Multiplication	3	34.2	Two and four numbers in multiplier and multiplicand
Long division	5	62.2	The quotient has a remainder
Addition of mixed numbers	8	36.6	The fractions did not have common denominators
Subtraction of a fraction and a mixed number	11	38.8	The denominators of the fractions were not common
Subtraction of a mixed number and a fraction	12	26.1	They were not in a column
Multiplication of a mixed number and two common fractions	14	31.9	
Multiplication of a whole and two mixed numbers	15	34.8	
Division of a fraction by a whole number	18	47.6	The quotient was not a fraction in popular use
Addition of decimal fraction, common fraction, and decimal and common fraction	21	65.6	This example was solved incorrectly by the largest number
Subtraction of a decimal and a common fraction	25	32.2	

Process	Example Number	Percentage of Errors	Explanation
Multiplication of a whole and a mixed number	27	30.4	
Translation of a bar graph	32	53.1	Estimating the number represented by the bar graph
Third use of percentage	38	30.4	
Changing of one peck to quarts	40	50.4	
Estimating distance on a scale map	44	44.1	
Interpretation of a broken-line graph	46	38.1	Selecting the period of time with the largest gain
Finding the perimeter of a square	48	28.5	Confused the formula for area with one for perimeter
Finding the area of a triangle	50	27.0	The formula for finding the area of a parallelogram was used rather than the one for a triangle
Averaging of whole numbers	52	22.9	
Successive discount	55	22.2	

From the examination of the above problems, the writer concludes that eighth-grade students have difficulty in solving examples involving the following types of computation:

1. Long division with a remainder in the quotient, as well as the division of a fraction by a whole number.

2. Multiplication of whole numbers, mixed numbers, common and decimal fractions, particularly when two or more of these are combined in one example.

3. Addition of large numbers, mixed numbers, or a combination of these.

4. Subtraction with fractions and mixed numbers involved.

5. Estimation of the amounts represented by bars and lines of the graphs in the test, also estimation of the distance on a scale map.

6. The third use of percentage--finding a number when a percentage of it is known--knowing "what" to do rather than "how" to do it.

7. Conversion of one peck to quarts.

8. Formulas in measurements, and solutions related thereto.

9. Following through two or more steps, particularly the averaging of numbers and the deducting of successive discounts.

Some of the boys and girls did not attempt the last examples. This may have been due to a lack of time or to inability to solve them. More students possibly would have had, or did have, difficulties with the last examples, but the test results did not indicate this, inasmuch as those omitted were not considered wrong.

By way of summation: the important facts obtained from analyzing the examples which eighth-grade students work with ease and those needing some help are: they understand and are able to use the four fundamental operations as long as the integers are small and the fractions are those found in everyday use; but they have difficulty dealing with large numbers, fractions, and mixed numbers, especially if the fractions are not familiar or meaningful to them. Furthermore, graphs, percentage, dry measure, areas, averages, and discounts need more attention.

Contributing Factors

The work habits shown in the students' arithmetic were examined to determine the causes of some of the errors in solving the examples.

More errors on the test were due to number combinations than to any other single factor. Some of the errors resulted from carelessness; for example, particular number combinations caused difficulty only once during the entire test, while certain students missed number combinations every time they occurred.

Many boys and girls had the correct solution for a given example, but set the decimal point in the wrong place or left it out entirely. According to the test results, eighth-grade pupils need more understanding of the decimal system and the value of the zero.

Various types of errors were made in the use of fractions. Although the directions at the beginning of the test instructed the students to reduce all fractions to the lowest terms, students either forgot to do this or they reduced them incorrectly. Changing common fractions to decimal fractions and changing mixed numbers to improper fractions were sources of difficulty for many of the pupils. In the division of fractions, many of the students inverted the dividend instead of the divisor. In some cases they failed to invert either, merely multiplying the fractions as they appeared in the example. Failure to get a common denominator before adding and subtracting fractions resulted in errors for some; and pupils added and subtracted denominators as well as numerators.

Errors apparently caused by hurrying were: failure to subtract or add the whole integer in mixed numbers, failure to carry, incorrectly copying the example, poor construction of figures, improper alignment, and failure to borrow.

It is impossible to list all of the difficulties of eighth-grade students in arithmetic fundamentals, but the types of errors numbered above were found in the test often enough to make it apparent that they are definitely recurrent weak spots in the arithmetic of this grade level.

CHAPTER III

THE FUNDAMENTAL DIFFICULTIES WHICH APPEAR IN THE SOLUTION OF PROBLEMS

The second part of the test administered to the eighth-grade students involved the solution of problems.[1] These problems included several of each of the more important types from the standpoints of process and of content.[2] The problems on this part varied in difficulty. Some were simple problems with only one step necessary for solution; others were more complicated, making it necessary for the pupil to select the pertinent facts and carry them through several steps before the final solution was obtained.

Generalizations from the Test

The general results from the test on problem-solving were similar to those on mechanics, some of which are:

1. The boys and girls were equally competent in solving the problems.

2. There was little difference in the results of the schools, the scores of the various classes indicating that the classroom situations with respect to students and teachers of all the groups were similar. Table IV compares the means of the classes used for testing.

---

1. Metropolitan Achievement Tests, Advanced Arithmetic Tests: Form A (Revised), 1932. "Arithmetic Problems," pp. 5-7.

2. Richard D. Allen and others, Metropolitan Achievement Tests Supervisor's Manual. New York: World Book Company, 1941. p. 68.

Table IV

A Measure of Central Tendency for Problem-Solving

School	Section	Mean
Central	8-a	8.338
Central	8-b	8.550
Central	8-c	8.467
Central	8-d	7.679
Central	8-e	7.855
Curry	8	8.410
Aycock	8-a	7.957
Aycock	8-b	7.943

Improvement in Problem-Solving over the Fundamental Processes

A study of the results of the test revealed that a majority of the students had a higher rating for the test on problem-solving than for the one on fundamentals. Some of the students showed only one month higher in score in their grade equivalents on the second test; others scored as much as two or more years higher on the solving of problems than on the mechanics of arithmetic. Table V shows the percentage of students who ranked stronger in the solving of problems than in the fundamentals, the percentage making the same score on both tests, and the percentage ranking higher on fundamental processes than on the solution of problems.

Table V

Comparison between Problem-Solving Abilities and Facility with Fundamental Processes

School and Section	Percentage of Students More Skilled in Problem-Solving	Percentage of Students Showing No Difference between Problem-Solving and Fundamental Processes	Percentage of Students Less Skilled in Problem-Solving
Central 8-a	67.6	10.8	21.6
Central 8-b	64.86	2.94	32.2
Central 8-c	70.0	5.0	75.0
Central 8-d	53.0	8.8	38.2
Central 8-e	61.9	2.9	35.2
Curry 8	53.4	13.3	33.3
Aycock 8-a	44.9	7.9	47.2
Aycock 8-b	46.4	0.0	53.6
Entire Group	57.7	6.2	36.1

The above table shows that the majority of the boys and girls were better prepared to solve problems than to work examples. Apparent reasons for this are:

1. The problems were meaningful, while the examples presented isolated number situations.

2. The integers used in the problems were smaller than the ones used in the examples.

3. The fractions in the problems were more familiar than those in the examples.

4. There were not as many problems to be solved; therefore

the pupil did not feel as rushed during the test on problem-solving as he did during the one on fundamentals.

5. Following a trend in recent years toward emphasis on the need of ability to solve practical problems, alert teachers have stressed problem-solving in their classroom procedure, thereby strengthening the ability of boys and girls to cope with meaningful number situations.

It will be noted from Table V that the students tested in the Aycock School preserved an approximate equality between the numbers more skilled in problem-solving and those less skilled in problem-solving (e.g., more skilled in fundamental processes). This may indicate a deviation from the present trend toward heavy emphasis on problem-solving, in favor of an instruction more evenly divided between problems and fundamentals; or it may be merely a specific tendency of a particular group of children. It has no implications in the writer's problem of ascertaining difficulties and correcting them except as the types of these errors may be of value in this study. It has been indicated previously in the study that no comparison of schools or teachers was desired, the purpose of the investigation being discovery of types of errors most glaring and recommendations for lessening or totally remedying these.

Problems Found in the Test

Figures Thirteen, Fourteen, and Fifteen are samples of the problems solved by the boys and girls who were tested. The problem numbers correspond to the problem numbers in Tables VI and VII.

A Graphic Representation--Problem-Solving

The scores for Part Two of the test were recorded in the same manner as the scores for Part One. The scores are represented by a graph for each class, indicating the number of the problem and the number of students unable to solve it correctly. These graphs may be found in Figures Sixteen through Twenty-three. Figure Twenty-four shows the number of problems included in the test, and the number of boys and the number of girls, as well as the total number of students, who made errors in attempting to solve the problems.

Competency in Problem-Solving

After examining the papers of the students and comparing them with the graph in Figure 24, on page 51, the writer found it obvious that certain types of problems presented little difficulty for eighth-grade students. Table VI, on page 52, shows the problems solved correctly by the majority of students, as well as the number of steps necessary to obtain the solution, and the operations used in solving the problem.

Figure 13. Problems Which Appeared on the Test

Metropolitan: Adv. Arith.: A

TEST 2. ARITHMETIC PROBLEMS

Directions. Work each problem and write the answer in the box after the problem. Do your work in the margin at the right of the page.

1. My mother bought a gallon can of maple syrup, which she put into pint jars. How many pint jars did she use? . . . . [ ] 1

2. A bill for pencils for our school store was \$4.32. If the pencils were 3¢ each, how many should we have received? . . . . [ ] 2

3. Apples are selling at 4 for 10¢. How much should I pay for 2 dozen? . . . . [¢] 3

4. Emil's father got 18 baskets of berries. He said he would give a quarter of them to his brother and a quarter to a neighbor and keep the rest. How many baskets was he going to keep? . . . . [ ] 4

5. Nellie practices her music lessons $\frac{3}{4}$ hour every day. How many hours of practicing does she do in 7 days? . . . . [hr.] 5

6. Mr. Nelson bought a table for \$12.75. He paid \$1.50 to have it carted to his shop. He spent \$1.69 to repaint it. He sold it for \$20.00. How much did he gain on it? [\$] 6

7. Nan was told to buy 8 pounds of nuts for a party. If they come in 8-ounce boxes, how she buy all together? . . . . [ ] 7

11. Mr. Lane bought a new flagpole for his front yard. It is $28\frac{7}{8}$ feet long. If he puts it $4\frac{1}{2}$ feet into the ground, how many feet of the pole will be above ground? . . . . [ft.] 11

12. If Bert puts 25¢ in the bank each week, how much money will he save in a year? . . . . . . [\$] 12

13. After each sale of dress goods, the clerk marked on a little tag the amount of material left. If $2\frac{7}{8}$ yards were sold from a piece containing $10\frac{3}{4}$ yards, what should the clerk write on the tag? . . . . [yd.] 13

[5]

(Go right on to the next page.)

Figure 14. Problems Which Appeared on the Test

Metropolitan: Adv. Arith.: A

14. Bob worked 4½ hours on Monday, 3¾ hours on Tuesday, and 4 hours on Thursday. At 40¢ an hour, how much did he earn? . . . . . $ 14

15. How many pieces, ⅞ yard in length, could you cut from 3½ yards of linen? 15

16. What will ice cream for 256 persons cost if you allow 1 quart of ice cream for 8 persons and pay $2.00 a gallon for it? . . . . $ 16

17. To go to Morristown, which is 30.8 miles from New York City, you pass through South Orange, which is 14.9 miles from New York City. How far is it from South Orange to Morristown? . . . . mi. 17

18. Nancy had the following marks in her mid-term tests: 92, 68, 84, 74, and 100. What was her average mark? 18

19. Newton is 5¾ miles north of camp, and Andover is 6½ miles north of Newton. How far is it from camp to Andover and back? mi. 19

20. My father earns $240 a month. We spend ¼ of it for rent, ⅝ of it for other expenses, and save the rest. How much money do we save a month? . . . . $ 20

21. Max wants to make a yard for his rabbits, which will measure 6 feet 4 inches by 4 feet 8 inches. How much will the wire netting to enclose it cost at 10¢ a foot? . . . . $ 21

much will it cost to scrape and varnish the floor at 12¢ a square foot? . . . . $ 24

25. Stone Bros. have failed in business and state that they can pay only 70 cents on the dollar. They owe Dan's father $10,500. How much should he receive? . . . . $ 25

26. Lena's sister earns $32 a week. Every Monday she puts $4 in the bank. What per cent of her money does she save? . . . . . % 26

27. Our ball team played 12 games and won 9. What per cent of their games did they win? % 27

(Go right on to the next page.)

Figure 14. Problems Which Appeared on the Test

14. Bob worked $4\frac{1}{2}$ hours on Monday, $3\frac{3}{4}$ hours on Tuesday, and 4 hours on Thursday. At 40¢ an hour, how much did he earn? ..... $ 14

15. How many pieces, $\frac{7}{8}$ yard in length, could you cut from $3\frac{1}{2}$ yards of linen? 15

16. What will ice cream for 256 persons cost if you allow 1 quart of ice cream for 8 persons and pay $2.00 a gallon for it? .... $ 16

17. To go to Morristown, which is 30.8 miles from New York City, you pass through South Orange, which is 14.9 miles from New York City. How far is it from South Orange to Morristown? .... mi. 17

18. Nancy had the following marks in her mid-term tests: 92, 68, 84, 74, and 100. What was her average mark? 18

19. Newton is $5\frac{3}{4}$ miles north of camp, and Andover is $6\frac{1}{2}$ miles north of Newton. How far is it from camp to Andover and back? mi. 19

20. My father earns $240 a month. We spend $\frac{1}{4}$ of it for rent, $\frac{5}{8}$ of it for other expenses, and save the rest. How much money do we save a month? .... $ 20

21. Max wants to make a yard for his rabbits, which will measure 6 feet 4 inches by 4 feet 8 inches. How much will the wire netting to enclose it cost at 10¢ a foot? .... $ 21

22. Mr. Jones took in $3537 in the 27 business days of July. How much did he average for one day? .... $ 22

23. Our butcher had on his counter a cut of meat that weighed 12 pounds. From this cut, the roast we bought weighed 4 pounds 12 ounces. How much meat was left? .... lb. oz. 23

24. My bedroom is 12 feet by 16 feet. How much will it cost to scrape and varnish the floor at 12¢ a square foot? .... $ 24

25. Stone Bros. have failed in business and state that they can pay only 70 cents on the dollar. They owe Dan's father $10,500. How much should he receive? .... $ 25

26. Lena's sister earns $32 a week. Every Monday she puts $4 in the bank. What per cent of her money does she save? ..... % 26

27. Our ball team played 12 games and won 9. What per cent of their games did they win? % 27

(Go right on to the next page.)

Figure 14. Problems Which Appeared on the Test

TEST 2. ARITHMETIC PROBLEMS

Directions. Work each problem and write the answer in the box after the problem. Do your work in the margin at the right of the page.

1.	My mother bought a gallon can of maple syrup, which she put into pint jars. How many pint jars did she use?		1
2.	A bill for pencils for our school store was $4.32. If the pencils were 3¢ each, how many should we have received?		2
3.	Apples are selling at 4 for 10¢. How much should I pay for 2 dozen?	¢	3
4.	Emil's father got 18 baskets of berries. He said he would give a quarter of them to his brother and a quarter to a neighbor and keep the rest. How many baskets was he going to keep?		4
5.	Nellie practices her music lessons $\frac{3}{4}$ hour every day. How many hours of practicing does she do in 7 days?	hr.	5
6.	Mr. Nelson bought a table for $12.75. He paid $1.50 to have it carted to his shop. He spent $1.69 to repaint it. He sold it for $20.00. How much did he gain on it?	$	6
7.	Nan was told to buy 8 pounds of nuts for a party. If they come in 8-ounce boxes, how many boxes should she buy?		7
8.	Mother sent Mary to the store for $\frac{1}{2}$ gallon of vinegar. How many quarts did she get?	qt.	8
9.	John paid 25¢ for his lettuce seed. He sold 78 heads of lettuce for 5¢ a head. How much did he make on his lettuce?	$	9
10.	Mrs. Hall bought a $2\frac{3}{4}$-yard piece of cotton cloth and a $1\frac{1}{2}$-yard piece. How much did she buy all together?	yd.	10
11.	Mr. Lane bought a new flagpole for his front yard. It is $28\frac{7}{8}$ feet long. If he puts it $4\frac{1}{2}$ feet into the ground, how many feet of the pole will be above ground?	ft.	11
12.	If Bert puts 25¢ in the bank each week, how much money will he save in a year?	$	12
13.	After each sale of dress goods, the clerk marked on a little tag the amount of material left. If $2\frac{7}{8}$ yards were sold from a piece containing $10\frac{3}{4}$ yards, what should the clerk write on the tag?	yd.	13

(Go right on to the next page.)

Figure 15. Problems Which Appeared on the Test

28. Our kitchen is 15 feet by 12 feet. Linoleum costs \$1.75 a square yard. If it costs 10¢ a square yard to lay it, what will it cost to cover our kitchen floor? . . . . \$ ______ 28

29. My father wishes to lay a cement walk, 18 inches wide, from our kitchen door to the garage, a distance of 28 feet. What will the walk cost at 25¢ a square foot? . . . . . . \$ ______ 29

30. Our luncheon napkins are 9 inches on a side. How many can we cut from 1 square yard? ______ 30

31. How many feet of fencing are needed to enclose a lot 30 yd. 1 ft. by 20 yd. 2 ft.? . . ______ ft. 31

32. To make concrete, cement and sand should be mixed in the ratio of 1 bag of cement to 3 of sand. How much cement must I use if I use 12 bags of sand? . . . . ______ 32

33. If I own property assessed at \$8500 in a city where the tax rate is \$2.54 per \$100, how much is my tax? . . . . \$ ______ 33

34. Mr. Brown is told that, on a \$1200 auto, he may have discounts of 10% and 10%, or one discount of 20%. How much would the auto cost him if he took the better offer? \$ ______ 34

35. Find the amount of \$400 loaned for 1 year, 1 month, 15 days, at 4% . . . . \$ ______ 35

36. How many square yards are in the area of a field shaped like a right triangle, having a base of 180 feet and an altitude of 120 feet? ______ sq. yd. 36

37. How much interest will John's father have to pay on a loan of \$760 for 60 days at 6%? \$ ______ 37

38. The school paper offers $16\frac{2}{3}$% commission to pupils who secure advertisements. Sam wants to earn \$30 this term. How many dollars' worth of advertising must he get? ______ 38

39. General Motors stock was selling at $47\frac{1}{8}$. How much would I pay for 25 shares if, besides the cost of the shares, I had to pay $17\frac{1}{2}$¢ a share brokerage? . . . . \$ ______ 39

40. Tony pays 50¢ for each 200 pounds of ice. If he sells it at 15¢ for a 25-pound piece, how much does he make on 200 pounds? . . ______ ¢ 40

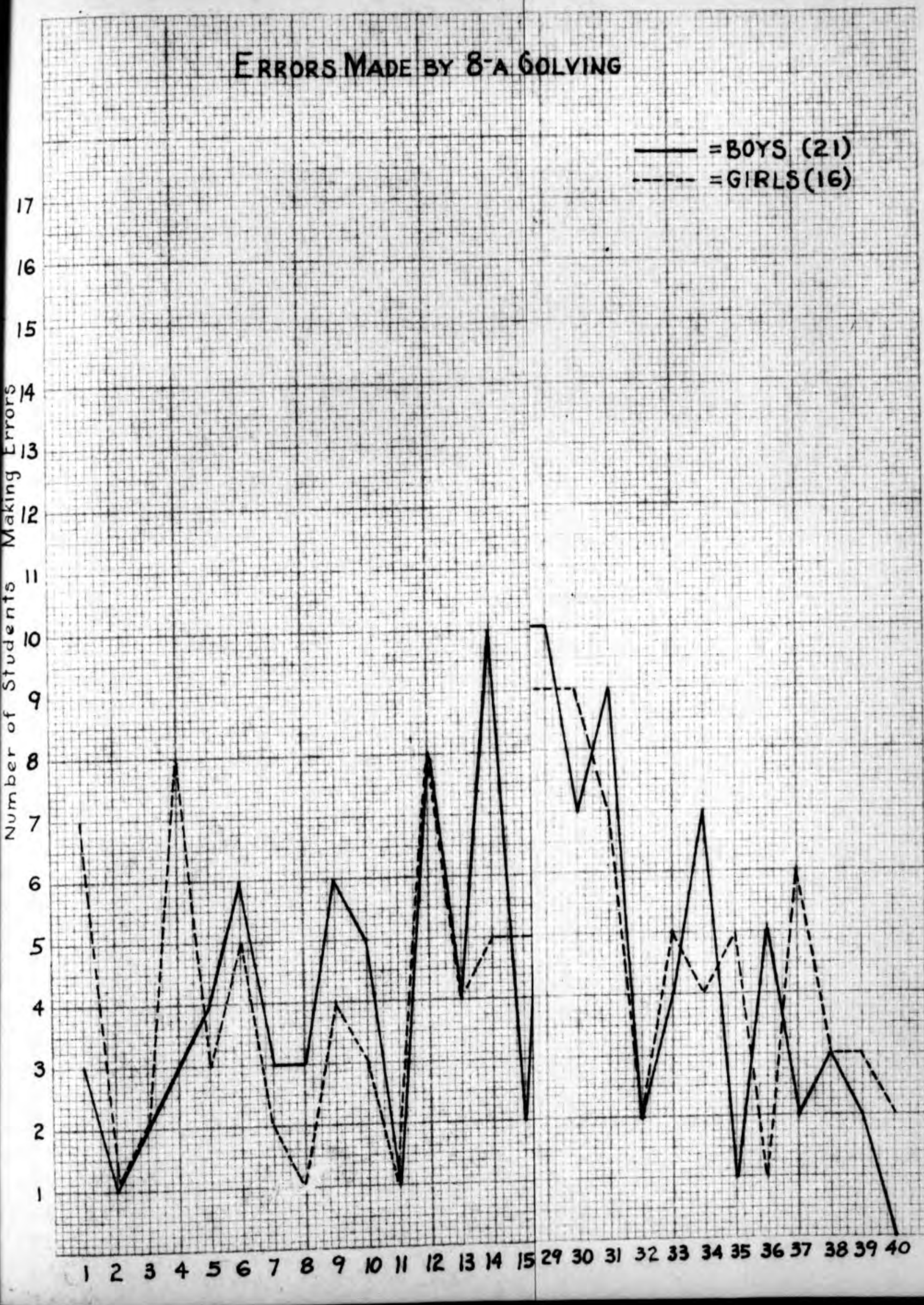
STOP!

No. right	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Score	35	37	38	40	42	43	44	46	47	49	50	52	54	55	57	59	60	62	64	65	67	68	70	72	73	75	76	78	80	81	83	84	86	88

No. right	35	36	37	38	39	40
Score	89	91	92	94	96	98

No. attempted. . . . .
No. wrong. . . . .
No. right. . . . .

Score, Test 2. . . . . .



ERRORS MADE BY 8-A SOLVING
— =BOYS (21)
---- =GIRLS (16)
Number of Students Making Errors
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 29 30 31 32 33 34 35 36 37 38 39 40

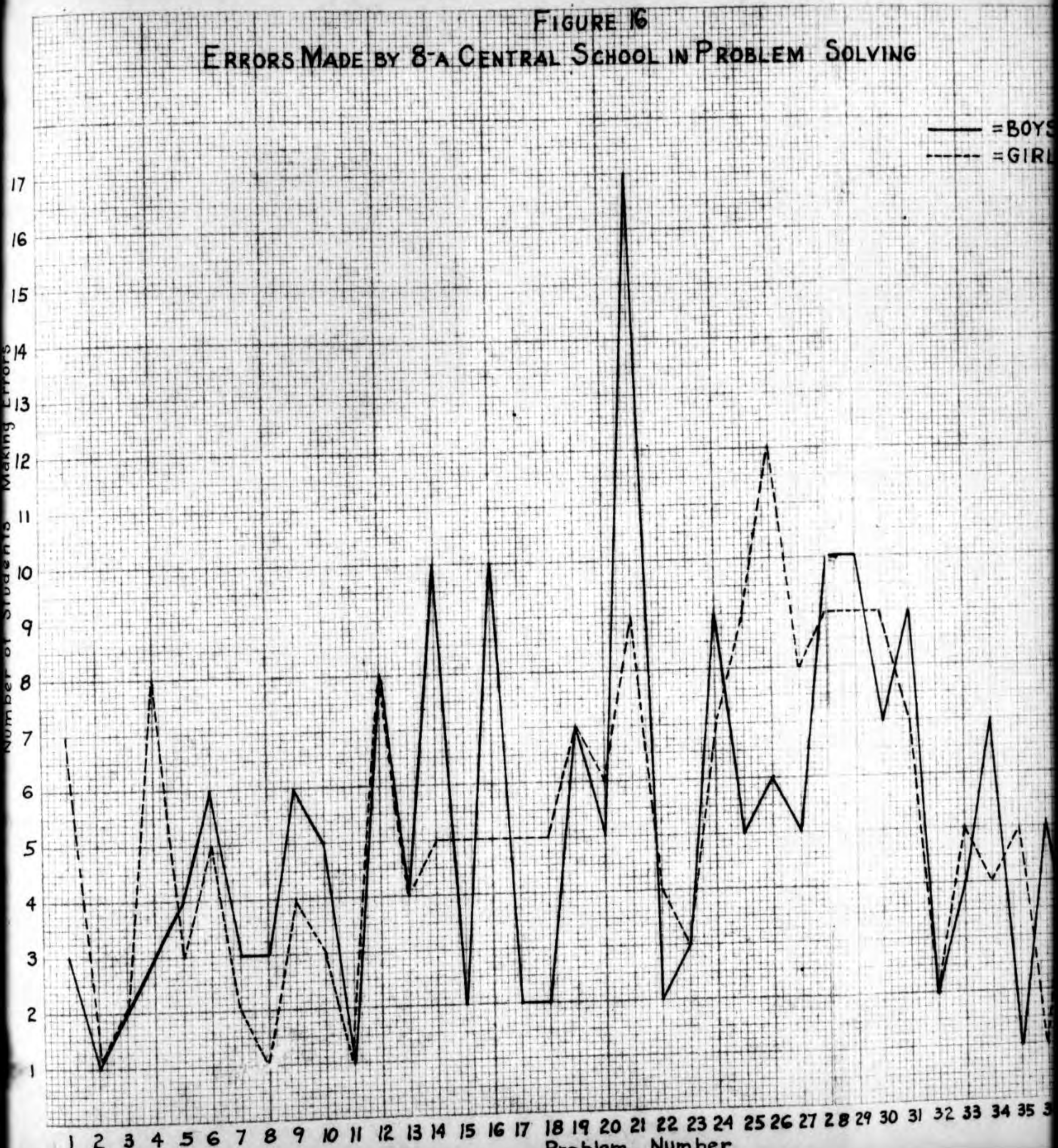
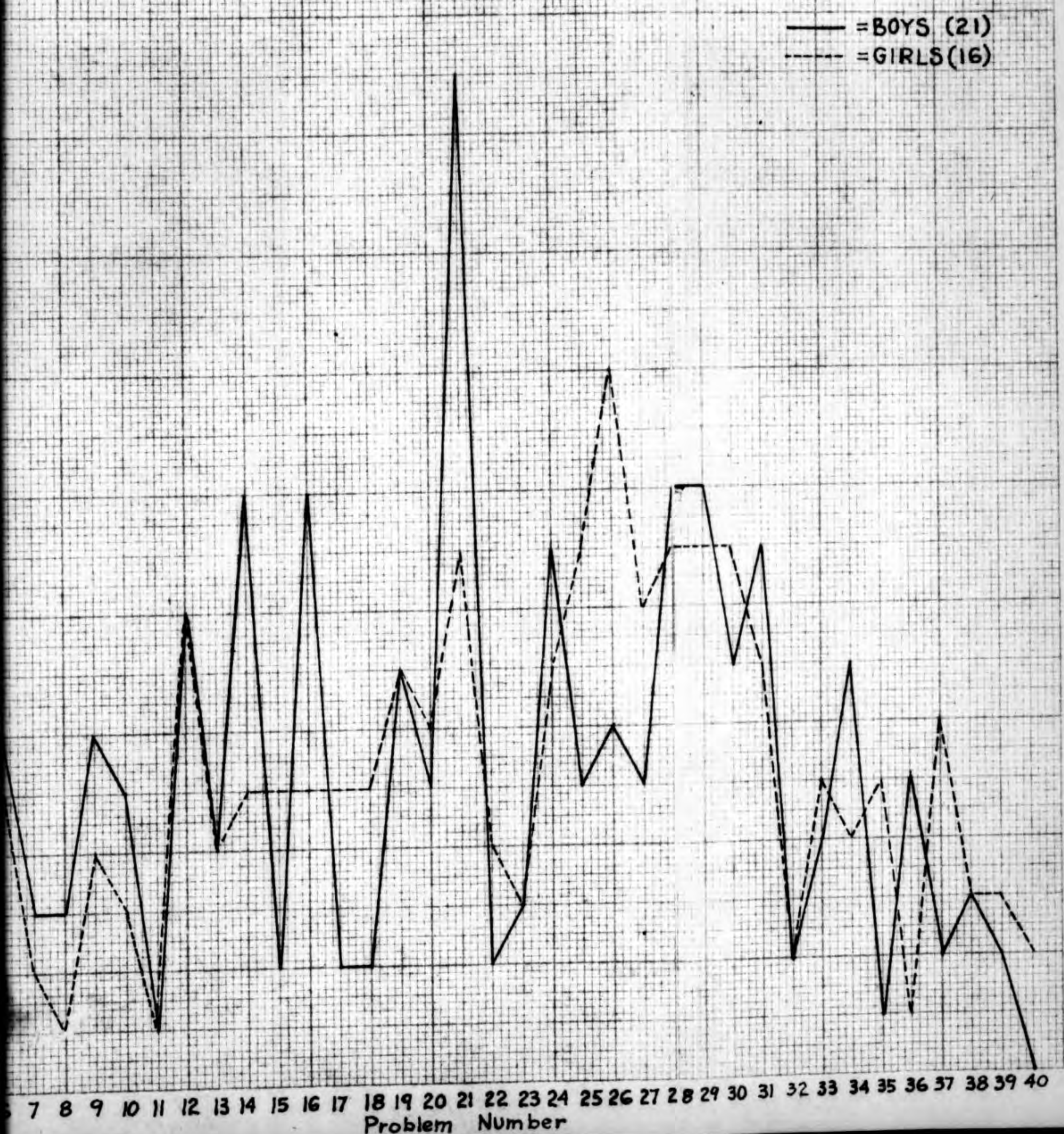


FIGURE 16
ERRORS MADE BY 8-A CENTRAL SCHOOL IN PROBLEM SOLVING
=BOYS
=GIRL
Number of Students Making Errors
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Problem Number

FIGURE 16

ERRORS MADE BY 8-A CENTRAL SCHOOL IN PROBLEM SOLVING



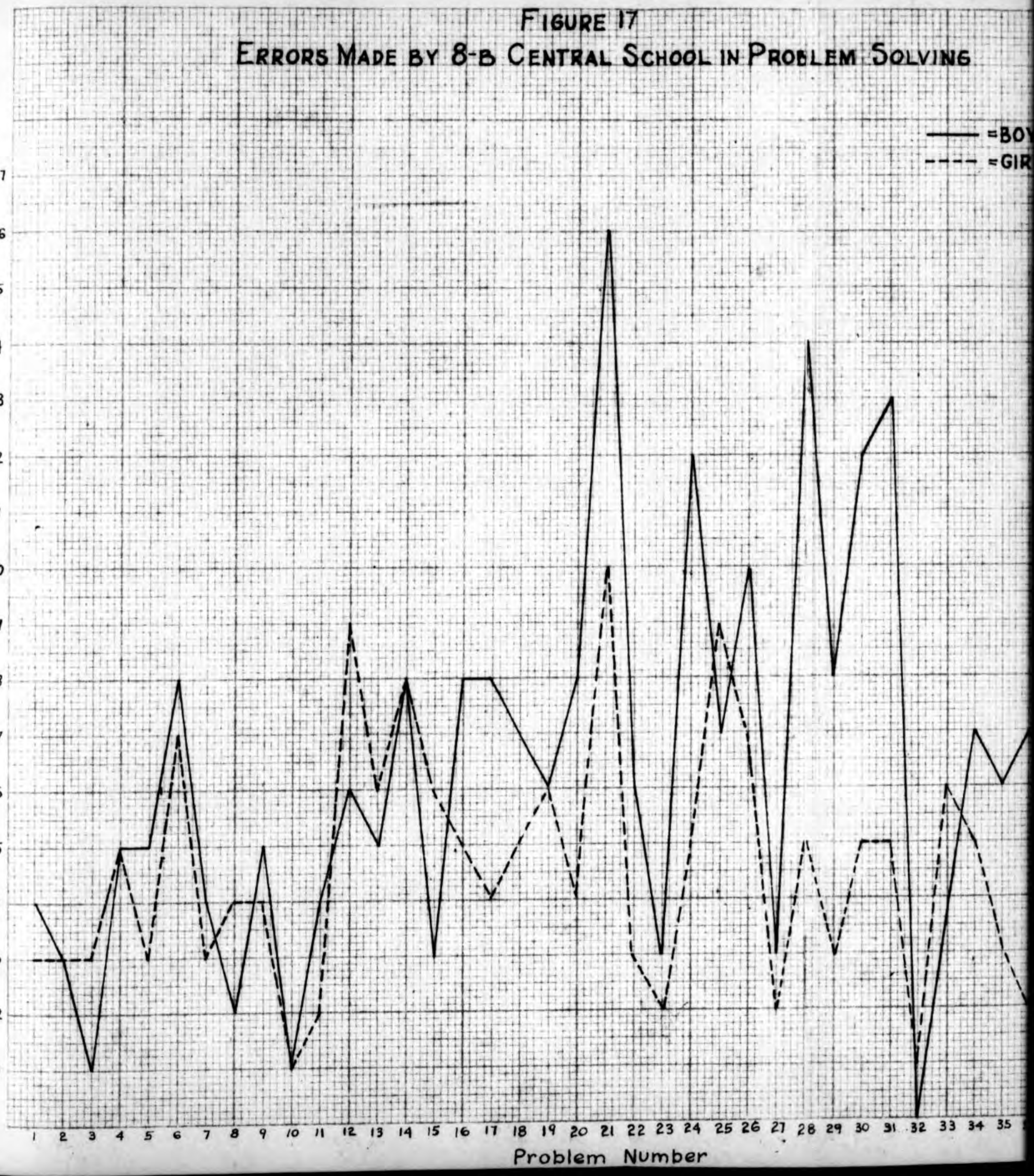


FIGURE 17
ERRORS MADE BY 8-B CENTRAL SCHOOL IN PROBLEM SOLVING
=BOY
=GIR
Number of Students Making Errors
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Problem Number

FIGURE 17

ERRORS MADE BY 8-B CENTRAL SCHOOL IN PROBLEM SOLVING

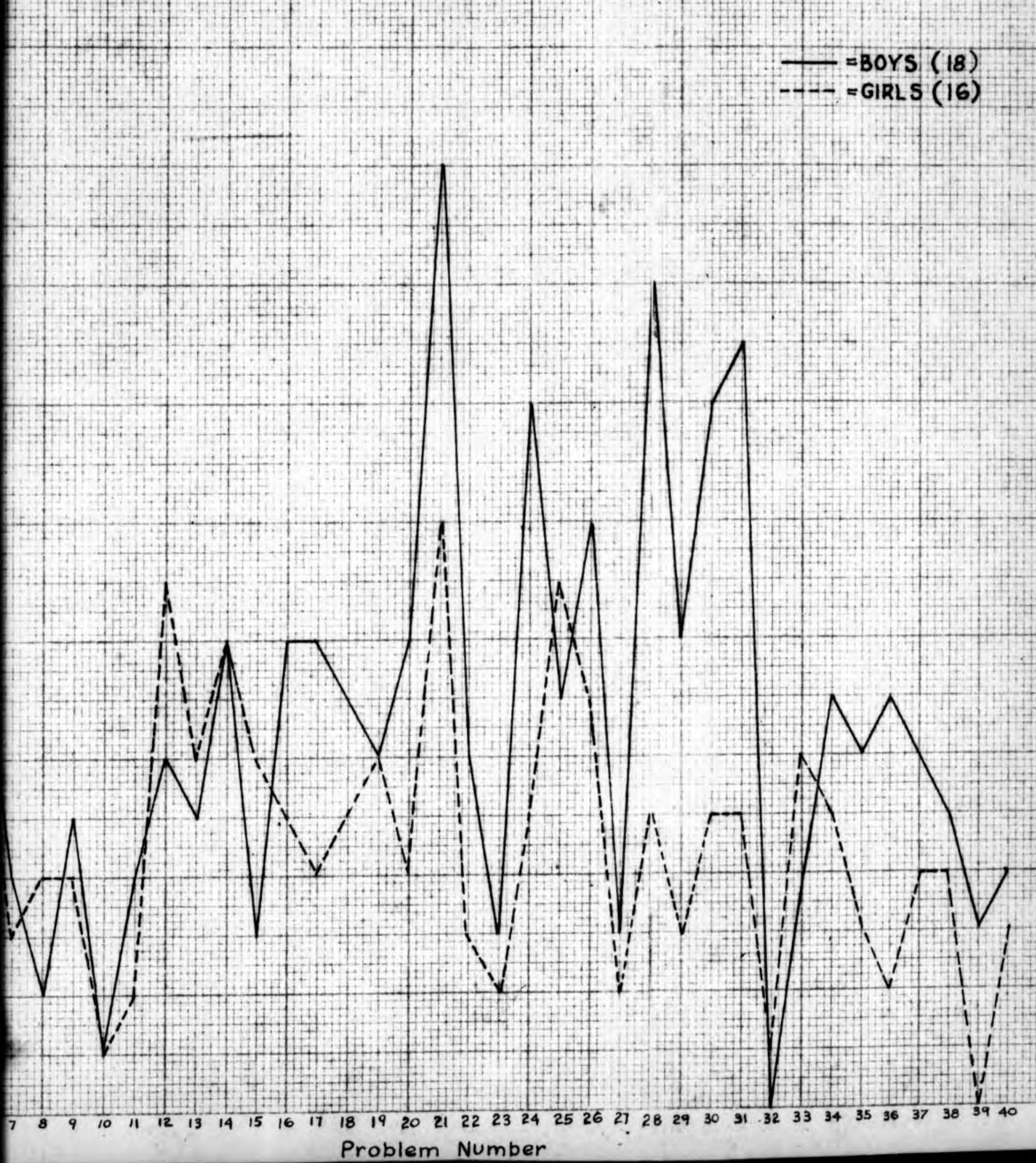
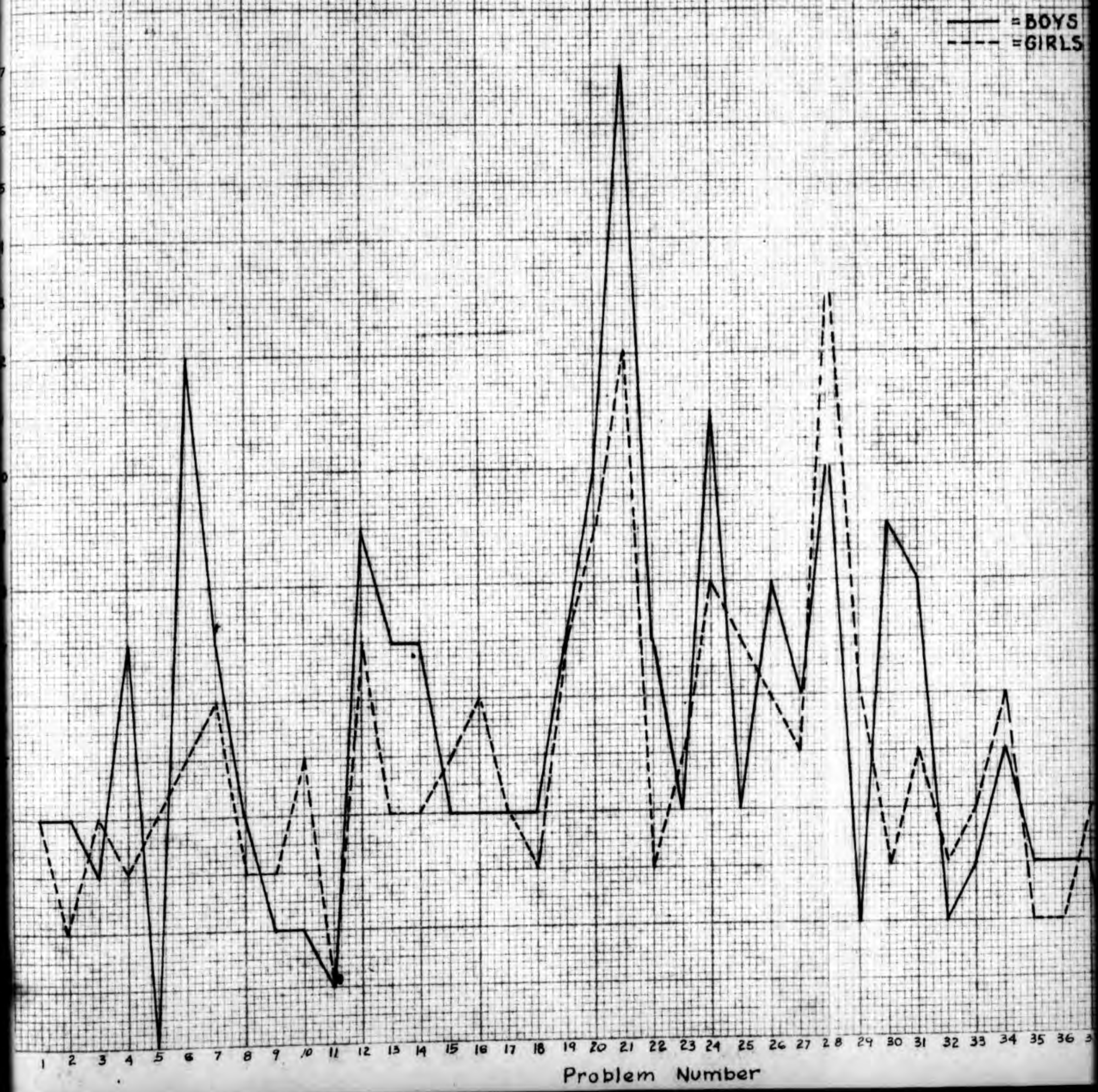


FIGURE 18

ERRORS MADE BY 8-C CENTRAL SCHOOL IN PROBLEM SOLVING



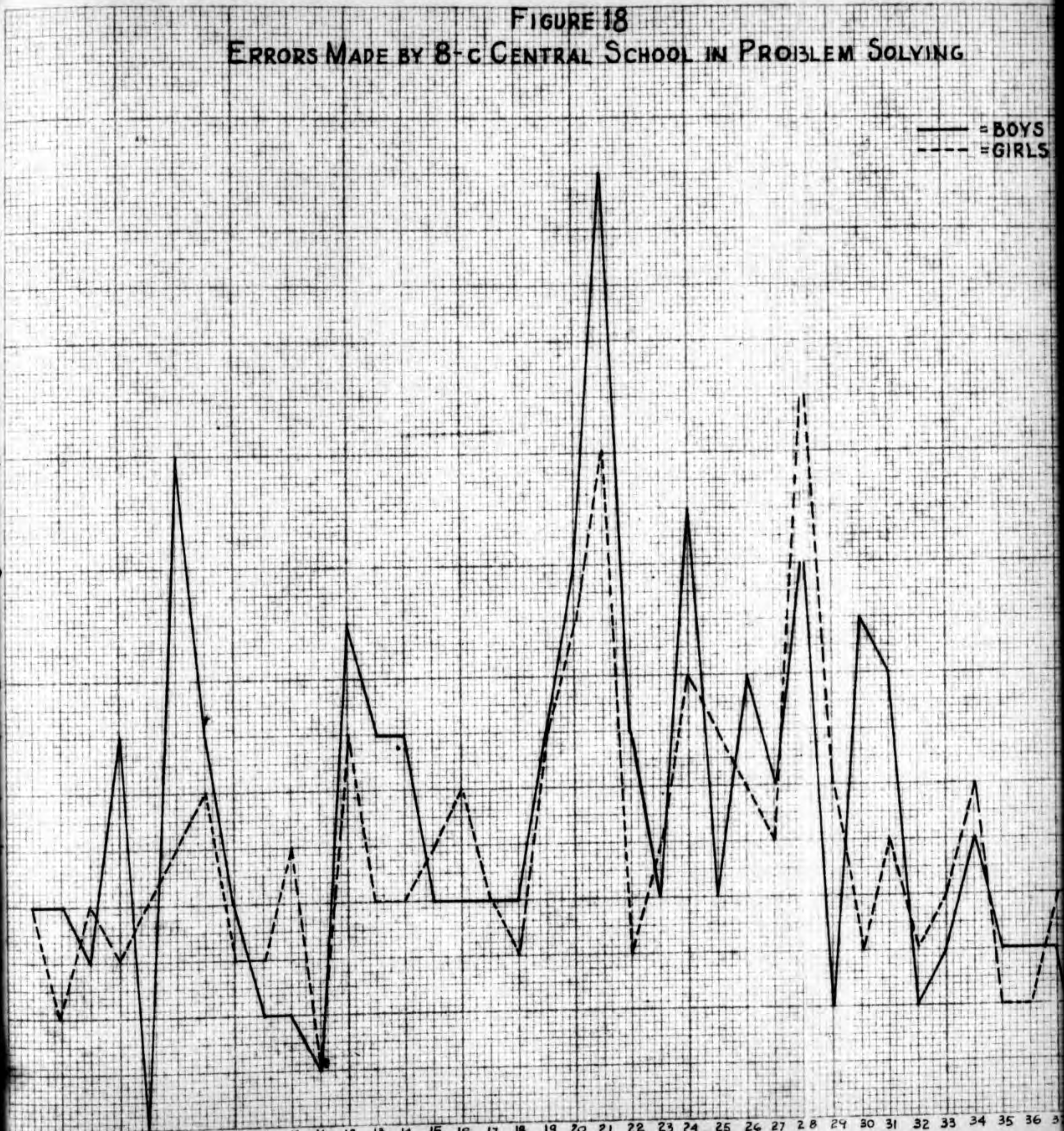


FIGURE 18
ERRORS MADE BY 8-C CENTRAL SCHOOL IN PROBLEM SOLVING
= BOYS
= GIRLS
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
Problem Number

FIGURE 18

ERRORS MADE BY 8-C CENTRAL SCHOOL IN PROBLEM SOLVING

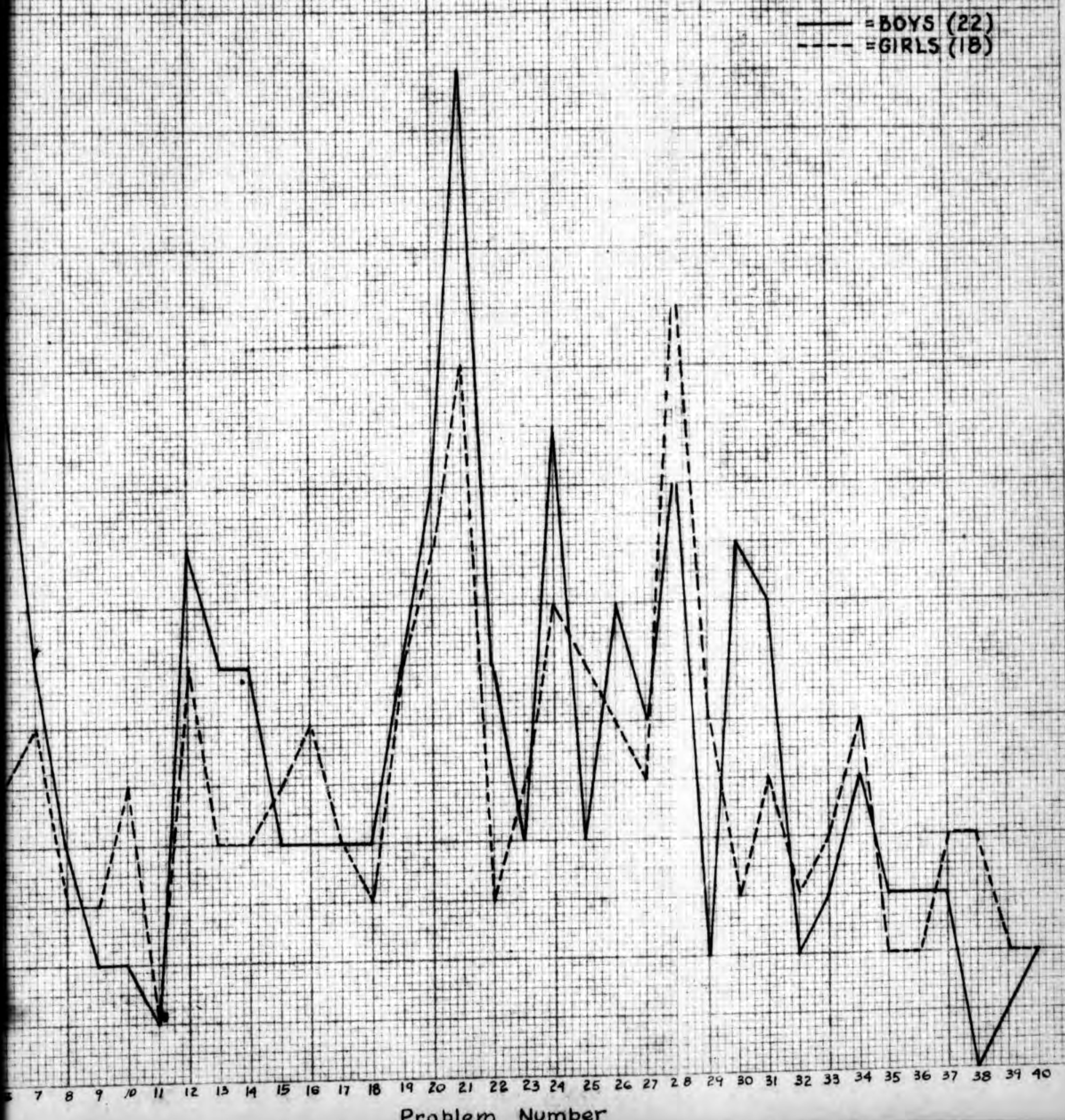


FIGURE 19

ERRORS MADE BY 8-D CENTRAL SCHOOL IN PROBLEM SOLVING

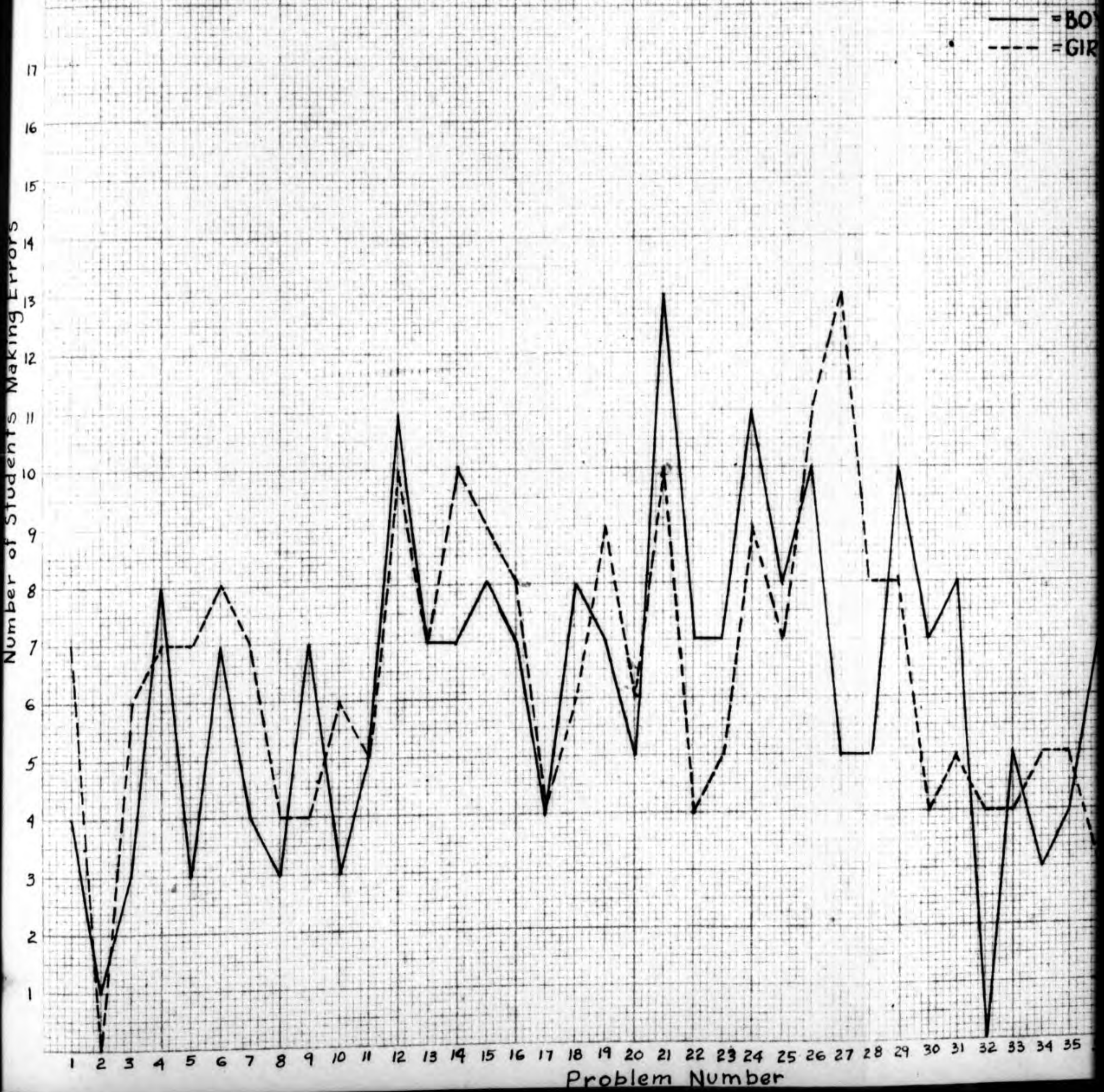


FIGURE 19

ERRORS MADE BY 8-D CENTRAL SCHOOL IN PROBLEM SOLVING

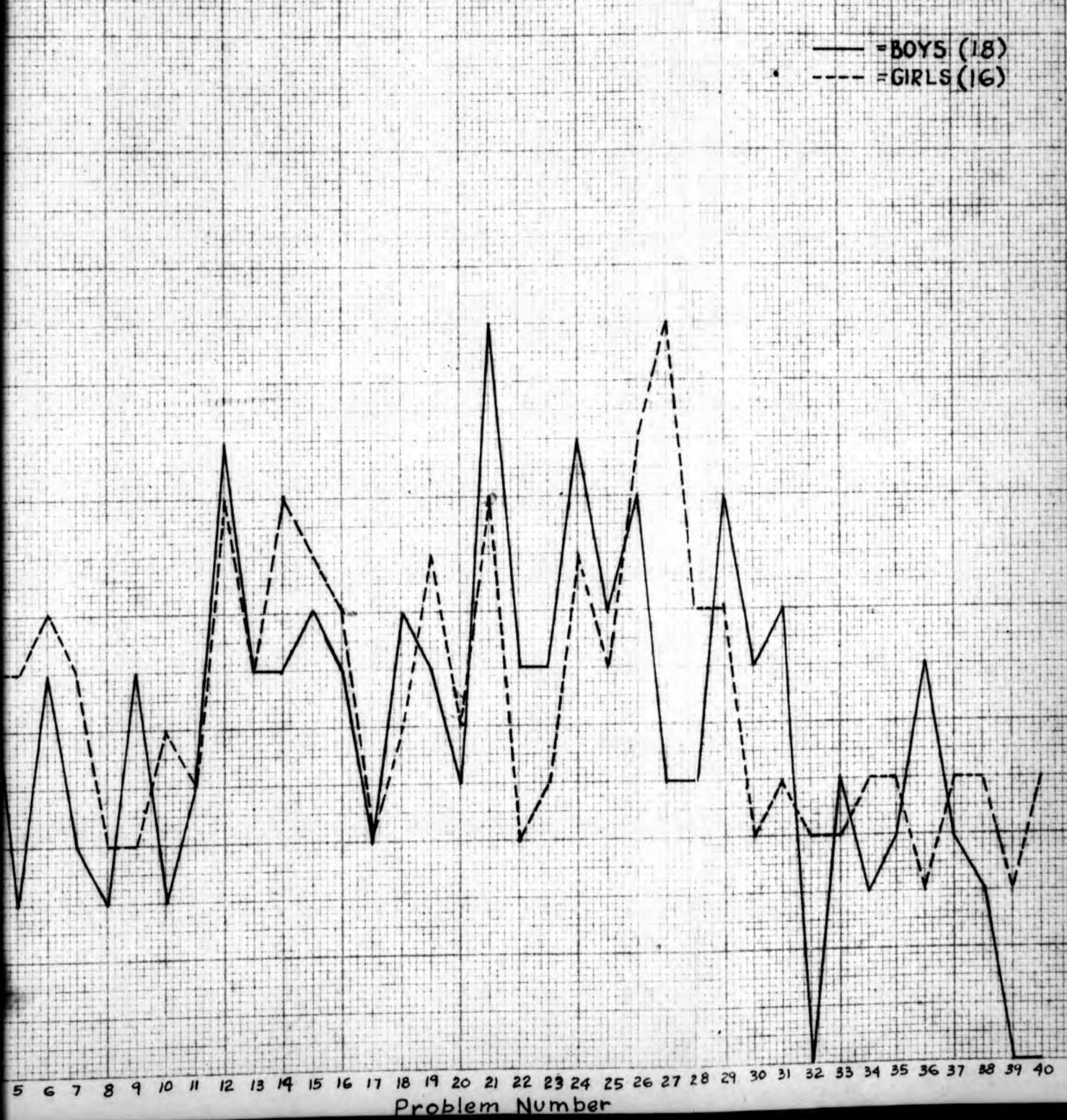


FIGURE 20

ERRORS MADE BY 8-E CENTRAL SCHOOL IN PROBLEM SOLVING

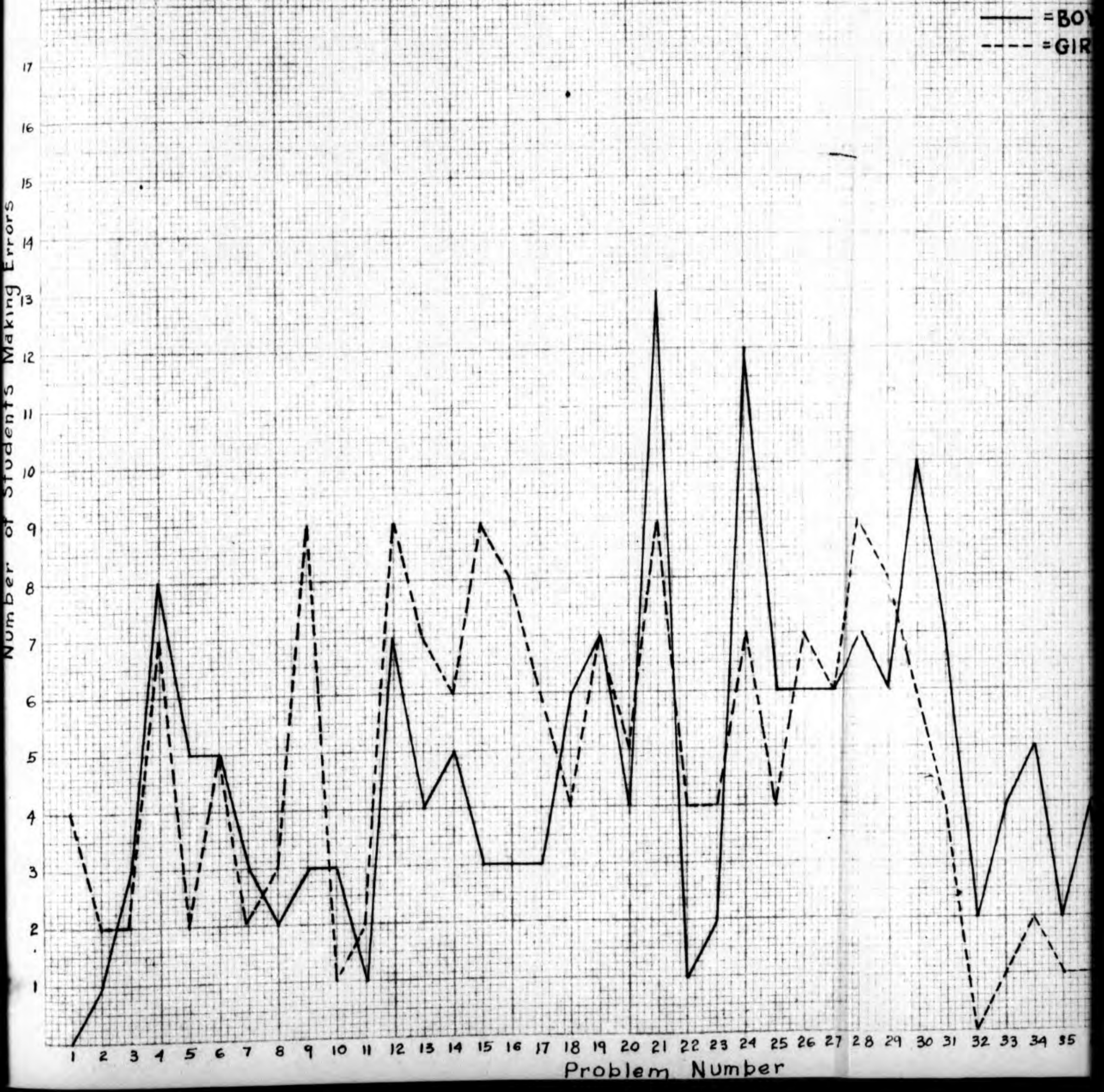
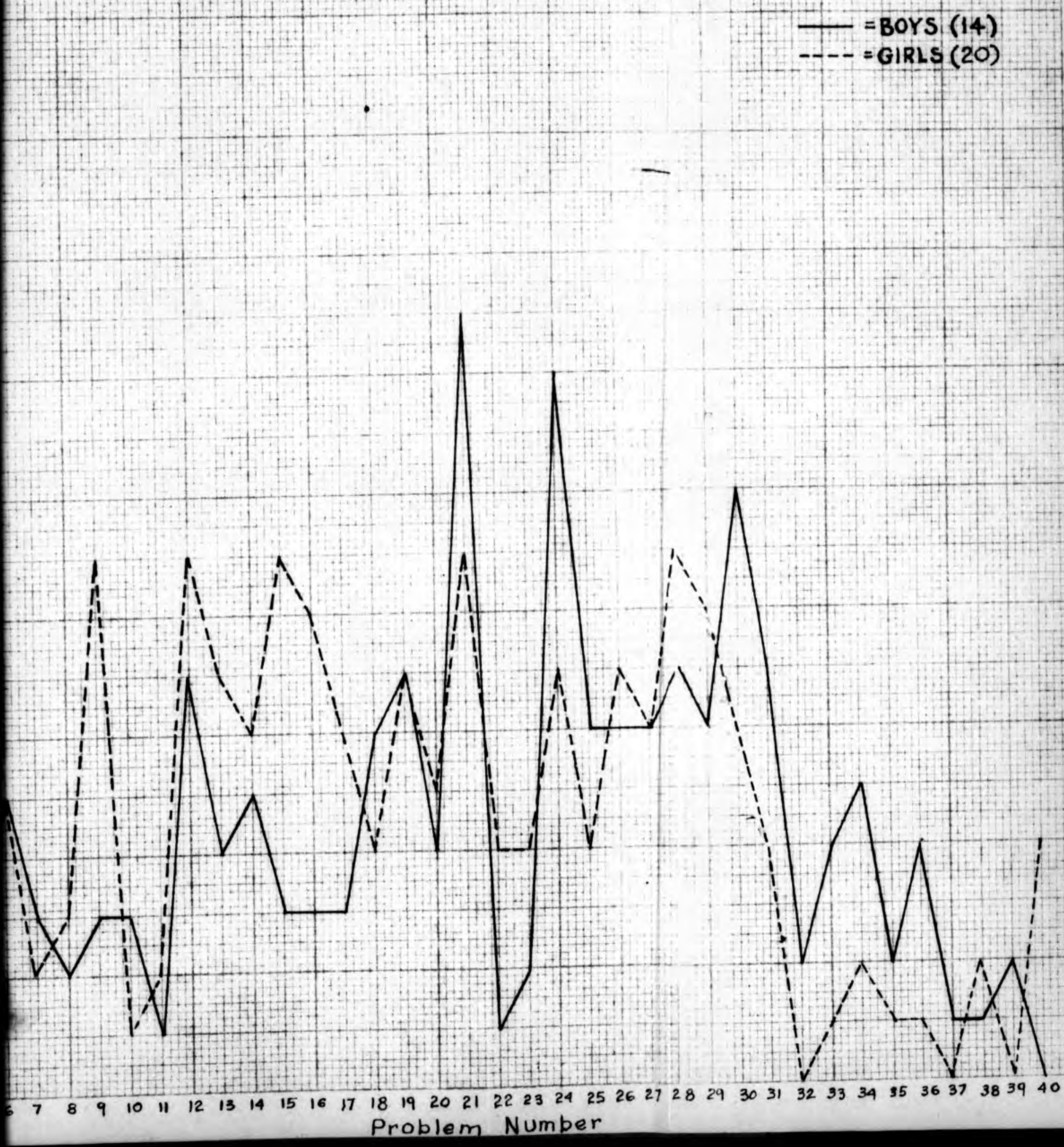


FIGURE 20

ERRORS MADE BY 8-E CENTRAL SCHOOL IN PROBLEM SOLVING



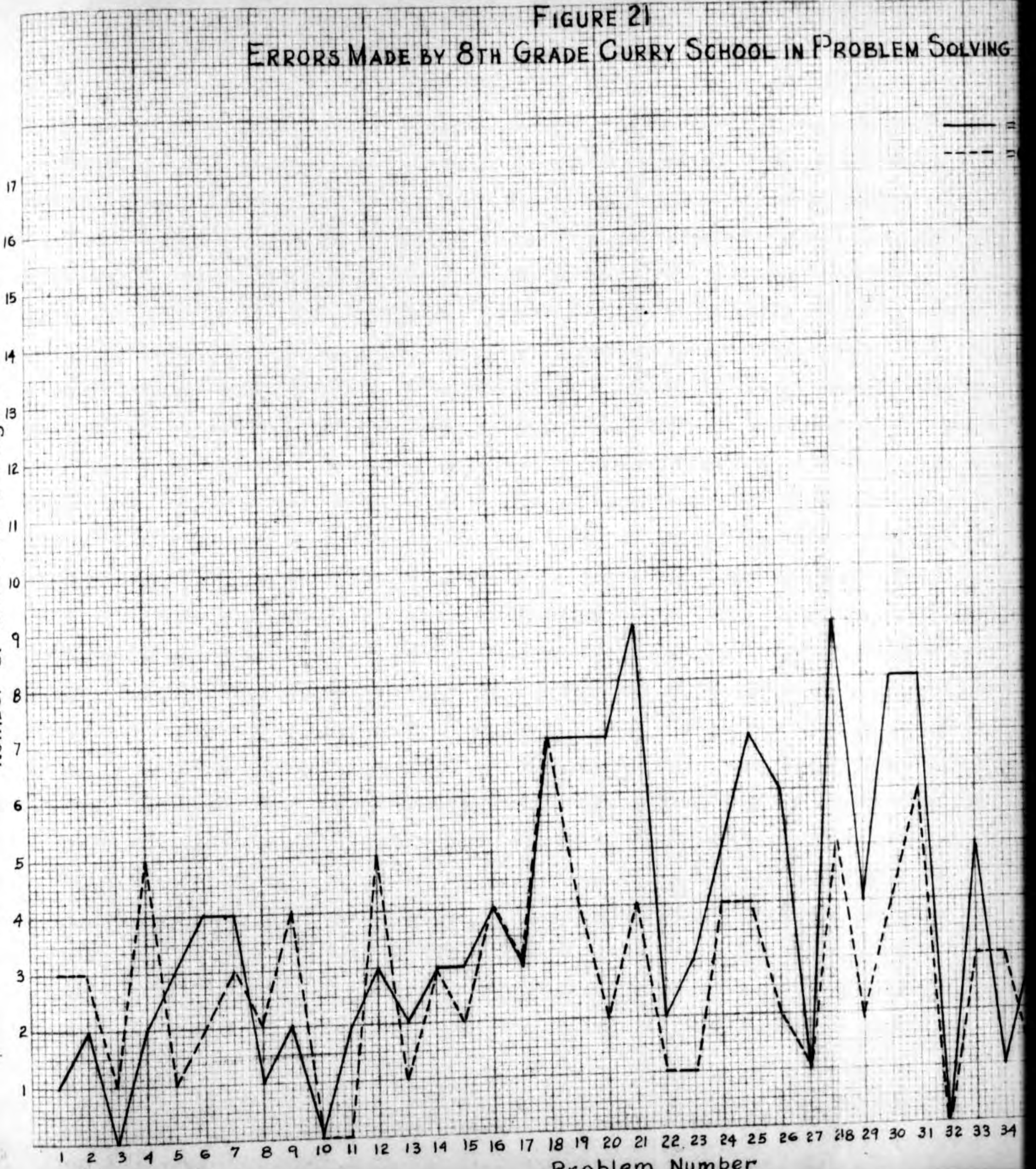


FIGURE 21
ERRORS MADE BY 8TH GRADE CURRY SCHOOL IN PROBLEM SOLVING
Number of Students Making Errors
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34
Problem Number

FIGURE 21

ERRORS MADE BY 8TH GRADE CURRY SCHOOL IN PROBLEM SOLVING

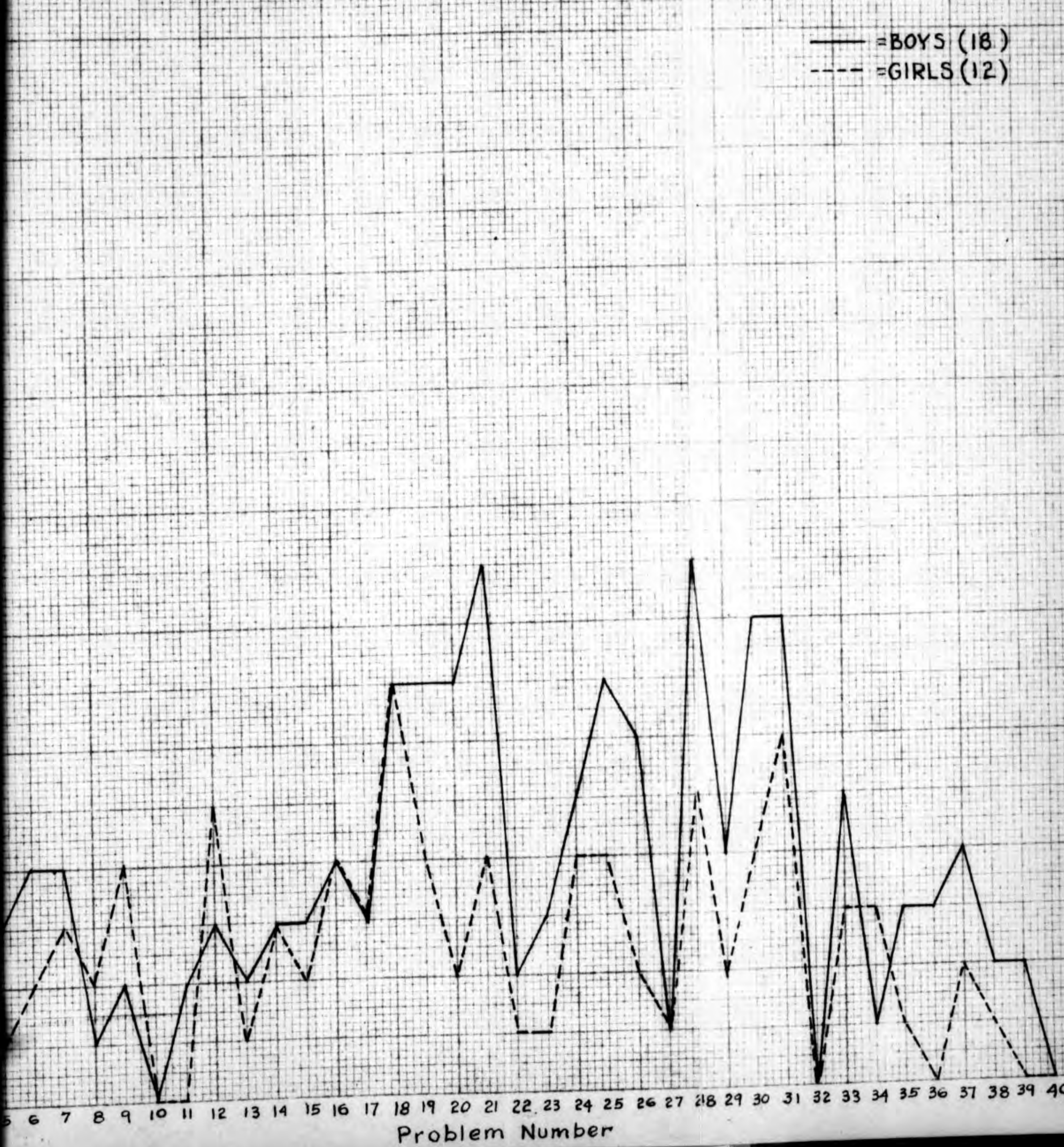
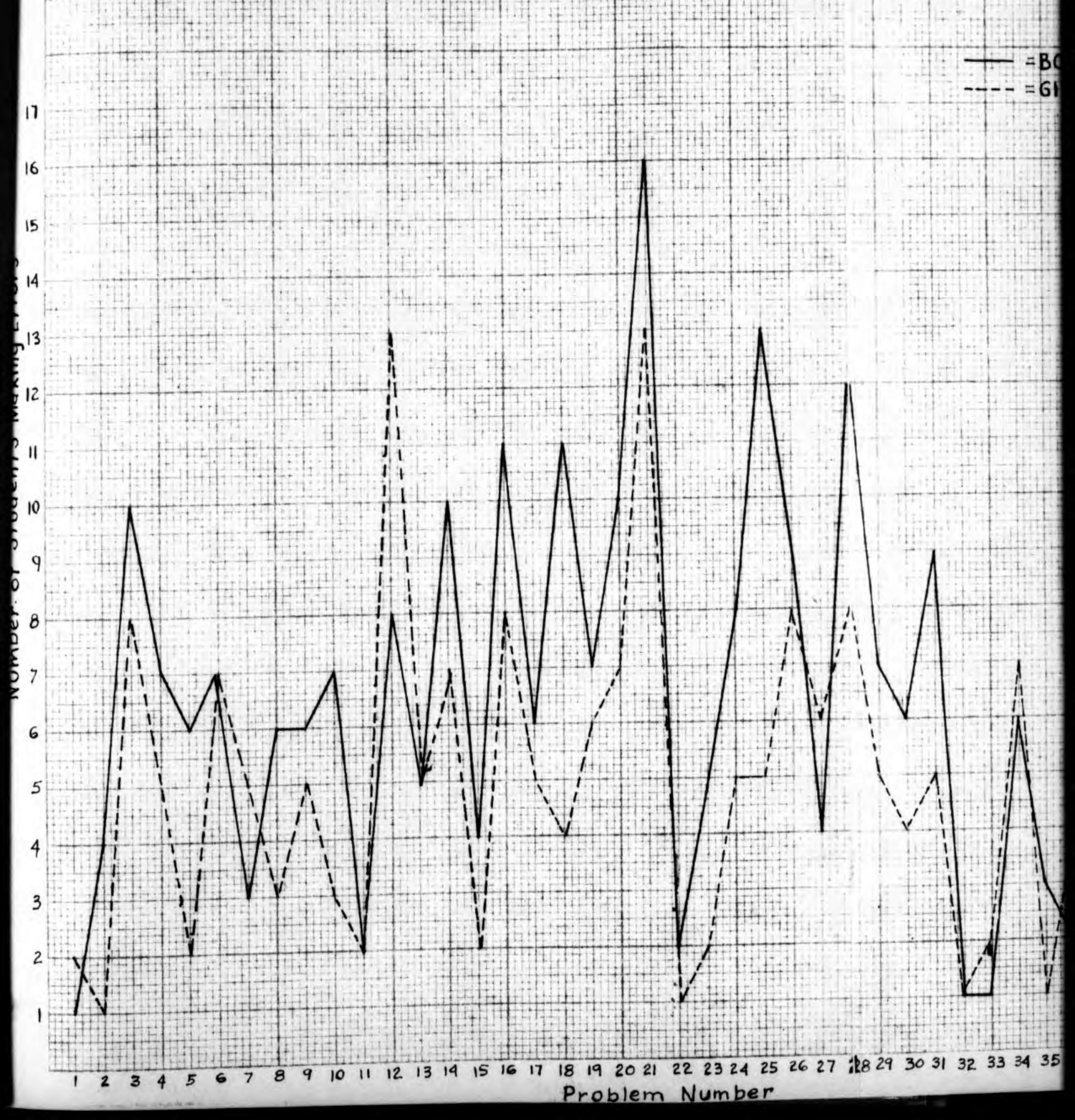


FIGURE 22
ERRORS MADE BY 8-A AYCOCK SCHOOL IN PROBLEM SOLVING



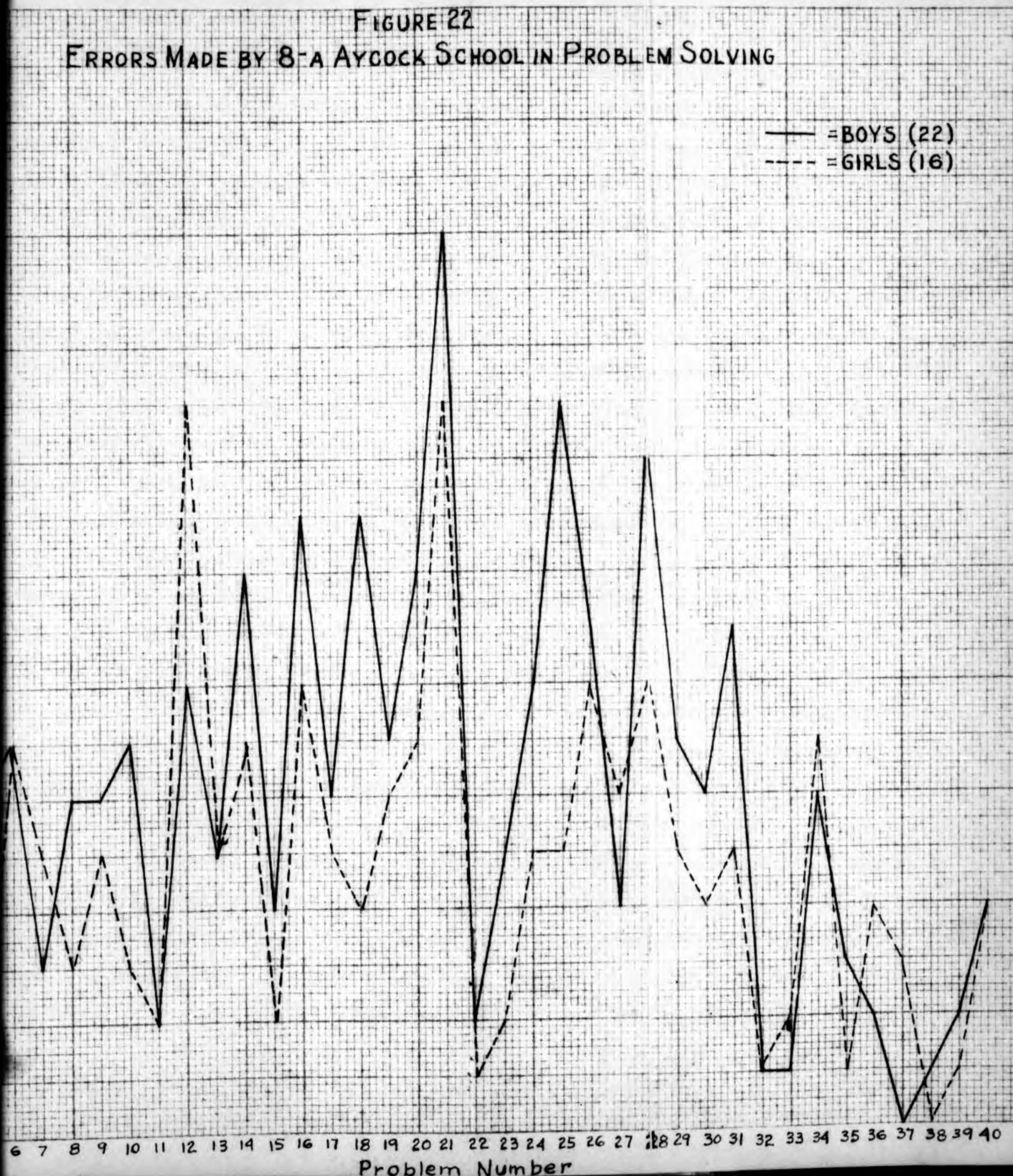


FIGURE 22
ERRORS MADE BY 8-A AYCOCK SCHOOL IN PROBLEM SOLVING
=BOYS (22)
=GIRLS (16)
6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
Problem Number

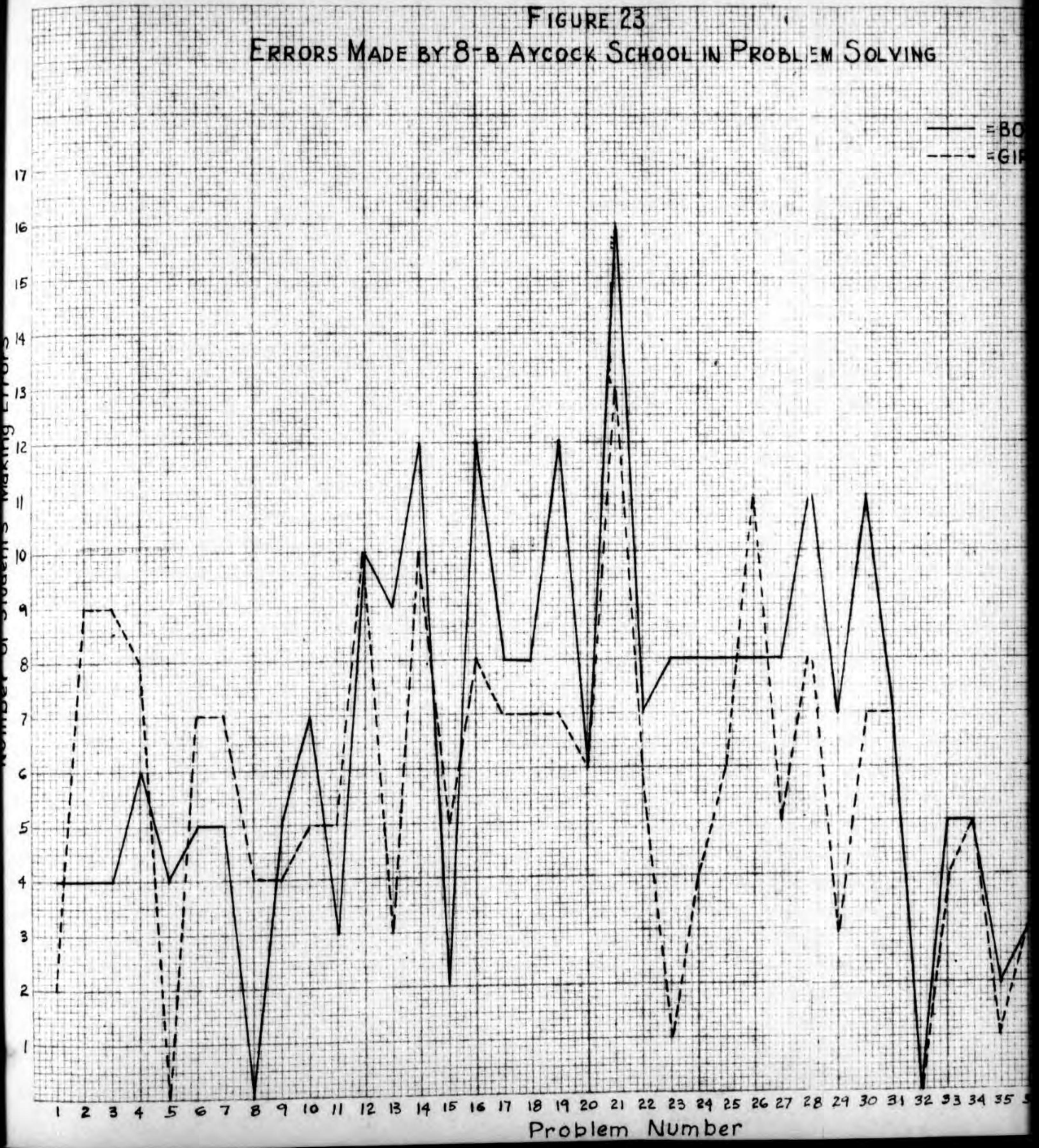


FIGURE 23
ERRORS MADE BY 8-B AYCOCK SCHOOL IN PROBLEM SOLVING
=BO
=GIR
Number of Students Making Errors
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Problem Number

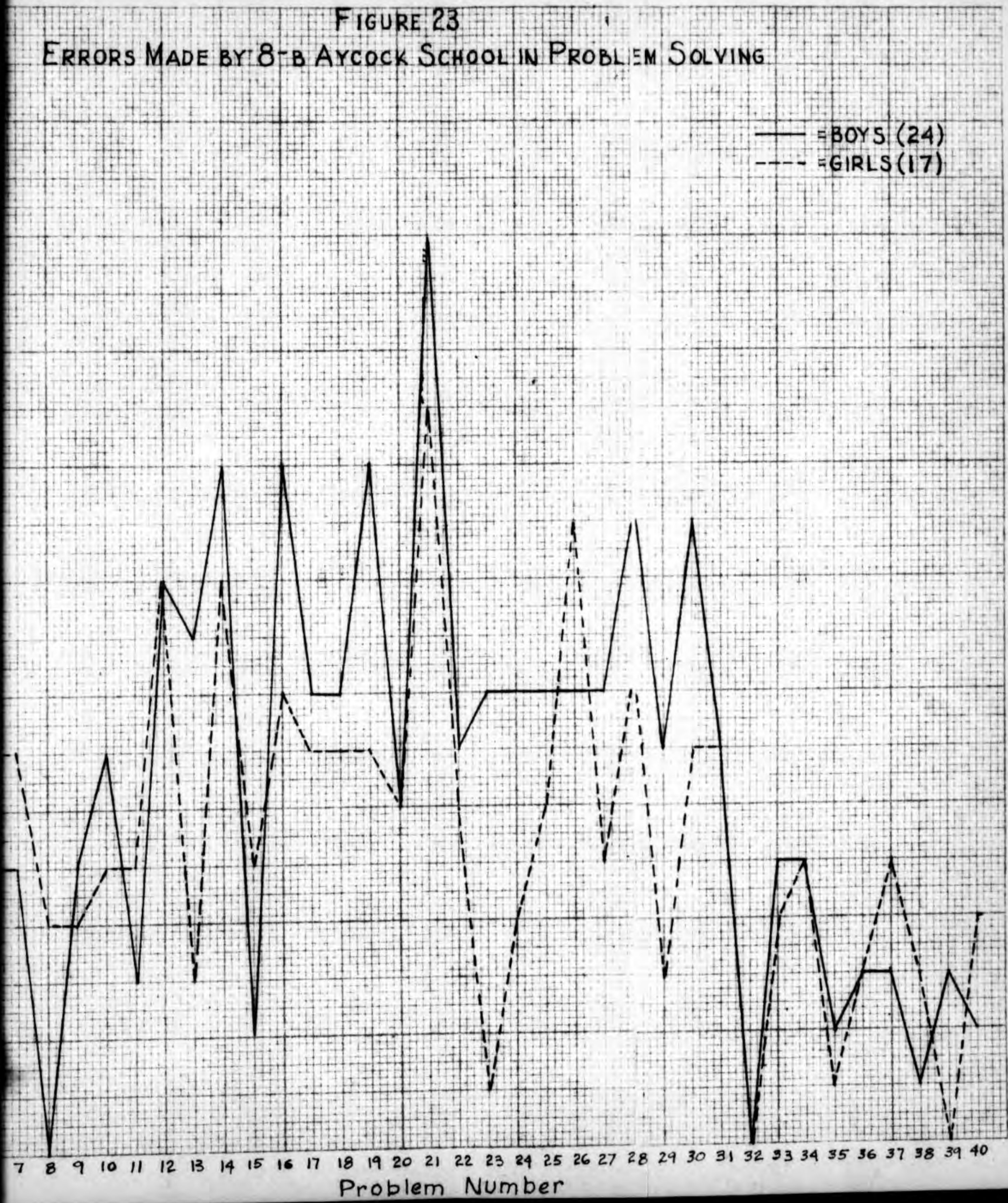


FIGURE 23
ERRORS MADE BY 8-B AYCOCK SCHOOL IN PROBLEM SOLVING
=BOYS (24)
=GIRLS (17)
7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
Problem Number

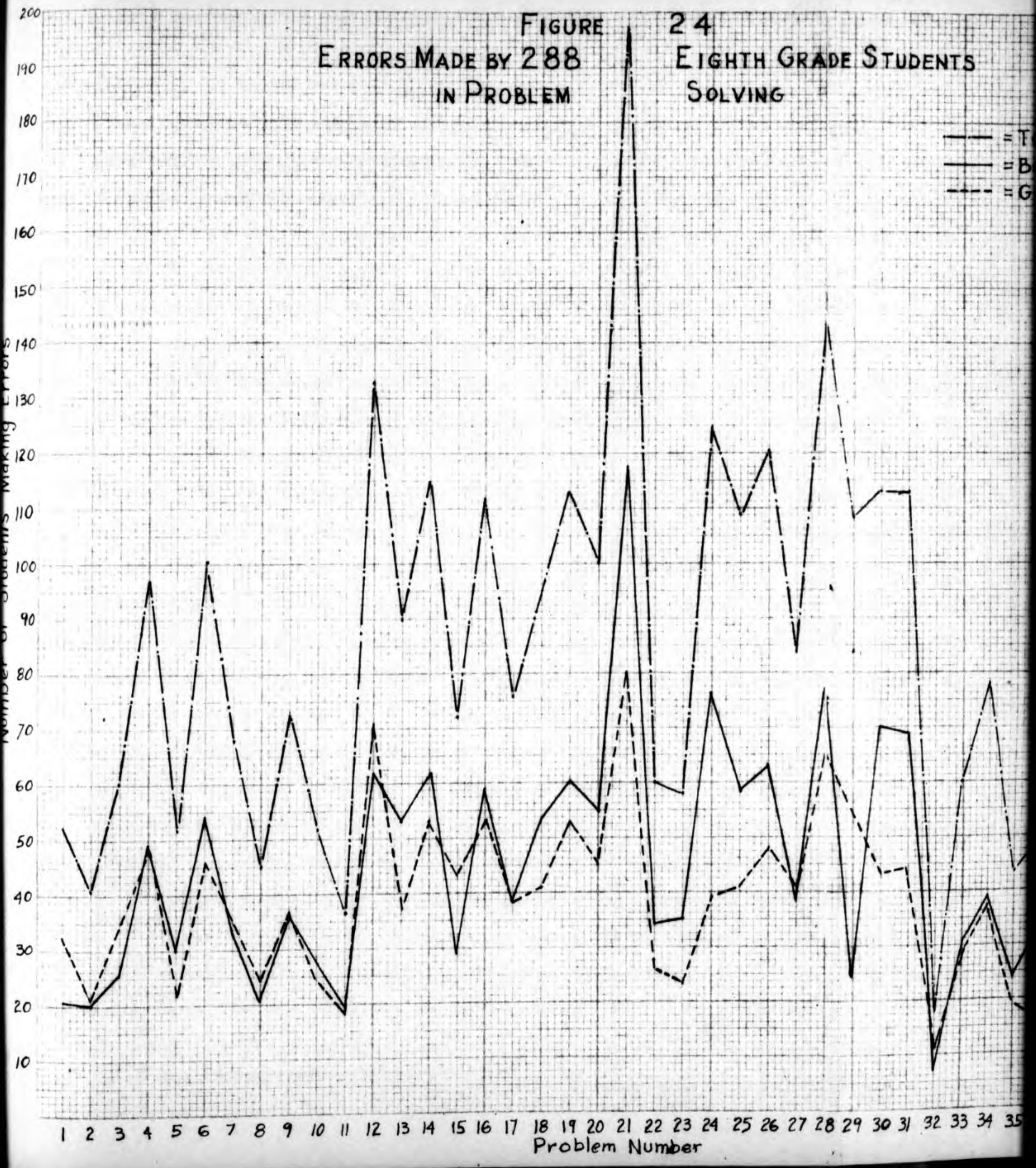


FIGURE 24
ERRORS MADE BY 288 EIGHTH GRADE STUDENTS IN PROBLEM SOLVING
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Problem Number

FIGURE 24

ERRORS MADE BY 288 EIGHTH GRADE STUDENTS IN PROBLEM SOLVING

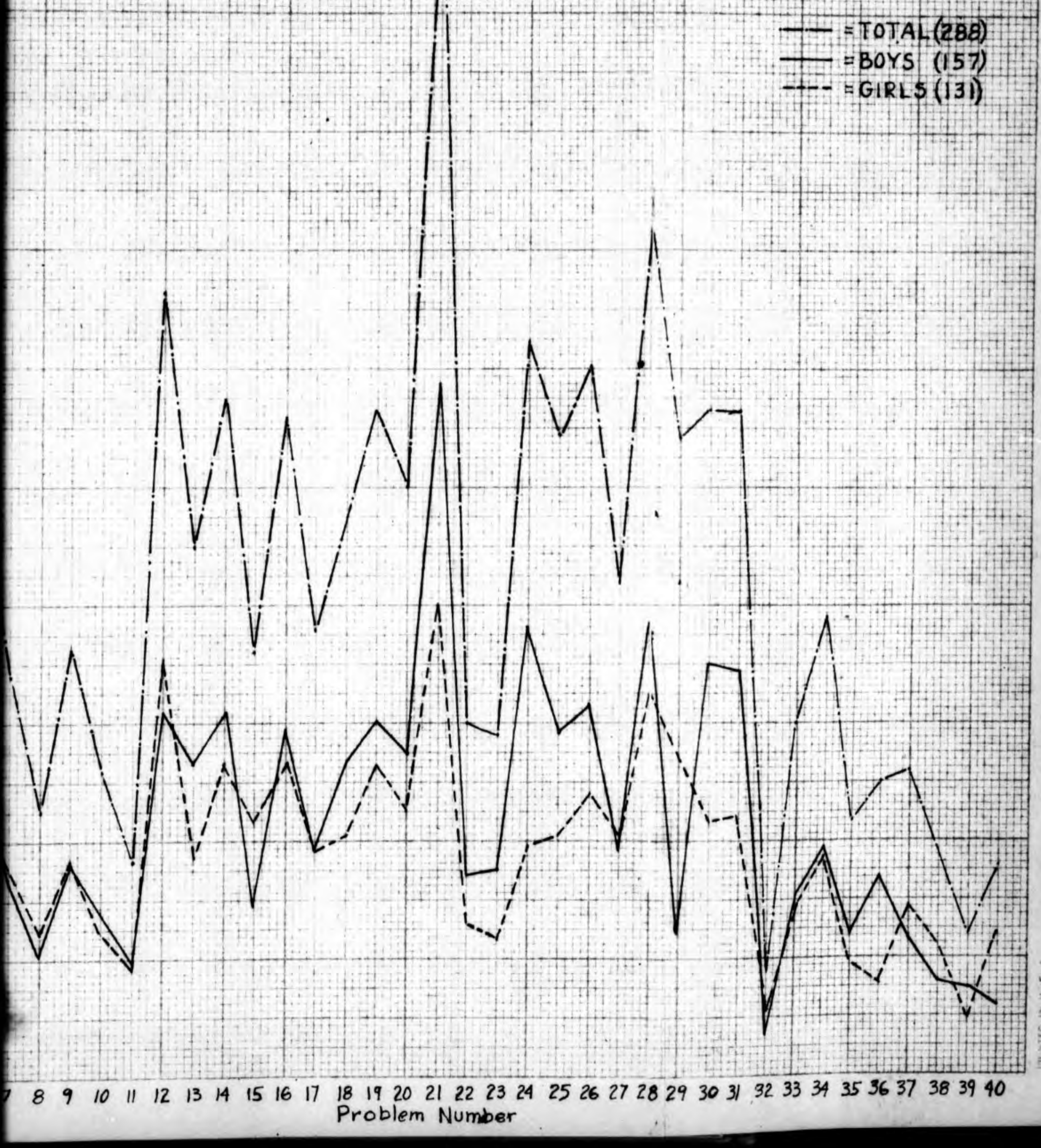


Table VI

Types of Problems Solved with Competency

Process	Problem Number	Percentage of Errors	Number of Steps	Explanation
Division	1	18.4	1	It was necessary to know the number of pints in a gallon.
Division	2	14.2	1	Small amounts of money were used in this problem.
Multiplication	5	18.1	1	A small fraction and a whole number were in this.
Multiplication	8	15.6	1	It was necessary to know the quarts in a gallon.
Addition	10	18.1	1	Mixed numbers were used in this problem.
Subtraction	11	12.9	1	Mixed numbers were used here.
Division	22	20.8	1	There was no remainder in this division of whole numbers.
Subtraction	23	20.1	1	It was necessary to borrow in this problem, using pounds and ounces.
Multiplication	35	14.9	2	Simple interest is concerned here.
Division	38	13.2	1	Third use of percentage is important here.
Division, multiplication, and subtraction	39	8.3	3	Whole and mixed numbers were used in this problem of buying stocks.

In helping boys and girls to become more proficient in mathematics, it is necessary to know what they are capable of doing. The preceding table lists the problems which presented little difficulty for the pupils. This table substantiates the following summation as permissible:

1. They were able to solve problems that contained one step in the solution.

2. Division of small integers in problems was not difficult for these students.

3. Multiplication of familiar fractions caused little trouble with the problems.

4. Subtraction of small, mixed, and whole numbers, when used in a problem, were solved correctly by a large group of the pupils.

5. Addition problems caused no difficulty when the numbers were small.

6. Problems using simple words were solved more readily.

Difficulties in Problem-Solving

Problem-solving was more difficult to analyze than work with examples in arithmetic. Problems use the fundamentals of arithmetic, including also questions of their own to be solved. The student must know what is being asked, which facts to use, and what to do. The chances for error are much higher in the solution of problems than in the solution of examples. The purpose of Table VII is to show the problems that the students were unable to do, and to indicate the causes of the difficulties.

Table VII

Types of Difficulty in Problem-Solving

Process	Problem Number	Percentage of Errors	Number of Steps	Explanation
Multiplication and subtraction	4	33.7	2	Whole numbers and fractions were used in this problem.
Addition and subtraction	6	34.8	3	Whole numbers with decimal fractions were used.
Multiplication and subtraction	9	25.2	2	Decimals were used with whole numbers.
Multiplication	12	46.3	2	It was necessary to know the number of weeks in a year.
Subtraction	13	31.2	1	Mixed numbers were used in this problem.
Multiplication and addition	14	40.0	2	Mixed numbers and decimal fractions were necessary for this.
Division	15	25.0	1	The fraction and the mixed number made this difficult.
Division and multiplication	16	38.9	3	The student must know the number of quarts in a gallon.
Subtraction	17	26.2	1	Whole numbers and decimals were used in this.
Addition and division	18	32.6	2	Averaging whole numbers was necessary here.

Process	Problem Number	Percentage of Errors	Number of Steps	Explanation
Addition and multi-plication	19	39.3	2	Mixed numbers were used in this prob-lem.
Multipli-cation, addition, and sub-traction	20	34.8	3	Fractions and whole numbers were used in this.
Addition and multi-plication	21	67.5	3	Finding the perim-eter of a rectan-gle was important.
Multipli-cation	24	43.5	2	Finding the area of a rectangle was the problem.
Multipli-cation	25	38.0	1	The problem was difficult to un-derstand.
Division	26	41.1	1	Second use of per cent was involved.
Division	27	29.2	1	Second use of per cent was also used here.
Multipli-cation and division	28	49.7	3	Converting area from square feet to square yards was the problem.
Multipli-cation and division	29	37.5	3	Converting area from square inches to square feet was the problem.
Multipli-cation and division	30	46.7	2	Converting area from square inches to square yards was involved here.
Multipli-cation and addition	31	46.7	2	Finding the perim-eter of a rectan-gle was necessary.

Process	Problem Number	Percentage of Errors	Number of Steps	Explanation
Multiplication and division	33	20.8	2	The vocabulary was specialized in this problem.
Multiplication and subtraction	34	20.2	4	Successive discount was used.
Multiplication	37	18.1	2	Simple interest was involved.

Table VII reveals that, among these eighth-grade students, the recorded difficulties allow the following observations:

1. Multiplication caused more trouble than any other operation in problem-solving.

2. Division was difficult when combined with other functions, particularly if the numbers involved were ones that are not in common use.

3. Some of the problems used words and terms that were not understood by many of the students.

4. Problems with two or more steps were very confusing to the pupils. The students appeared able to work problems when only one process was required; but, if asked to carry the work through several steps, they became baffled.

5. The students had difficulty in selecting the pertinent facts in a problem.

6. Addition and subtraction were troublesome when combined with other operations.

Summary of Problem-Solving Ability

The errors revealed by this phase of the test were caused, in some instances, by carelessness and by deficiencies in mechanical ability. The fact, as shown earlier in this chapter, that the students were more proficient in problem-solving than in the fundamentals, indicates that the mechanical deficiencies did not hinder them greatly. Some of the errors were, doubtless, caused in this way, although probably not the bulk of them.

The interpretation of the raw scores, according to the standard norms provided by the authors, shows that, for the most part, the students tested were above the standard level for eighth-grade students. The test results did not indicate that the pupils were exceptional, but that they were average students performing with greater facility than was expected on this test. The results were good; the students were capable of solving the problems that average eighth-grade students should be able to solve. Teachers, however, are always searching for ways of maintaining skills already developed and improving weak spots in learning. To help the boys and girls of this group, the following are indicated as points of weakness in their ability to solve problems:

1. There is a great need for insight. More understanding of the problems would have cut down the errors.

2. The students' work did not indicate a logical approach to problem-solving.

3. Many of the solutions offered by the pupils were not reasonable for the problems in which they were used.

4. A stronger mechanical background of fundamentals could possibly have eliminated more of these errors.

CHAPTER IV

CHANGES ESSENTIAL TO IMPROVEMENT

The purposes of this chapter are: to indicate basic principles in the teaching of arithmetic, to point out the weaknesses in certain methods of teaching which are in common use today, and to recommend other methods of instruction to help eliminate arithmetical difficulties. Before considering the methods of teaching arithmetic, it would be well to review the difficulties of eighth-grade pupils as indicated in Chapters Two and Three of this study.

Areas needing improvement in mechanics:

1. number combinations
2. understanding the zero
3. understanding the decimal system
4. fractions

Areas needing improvement in problem-solving:

1. insight
2. logical approach to problem-solving
3. reasonable answers

In solving arithmetic problems and examples, pupils use habits formed during drill exercises, such as those used in fixing the number combinations, and the insight which they obtained during the explanation of "how" and "why" a problem was solved in the way that it was. Both habit and insight are necessary components of arithmetic competence; they are not separate and distinct, but rather two aspects of the ability to solve situations involving the use of numbers. The writer has separated them arbitrarily to simplify the report, but it should be understood that they work together for the same end--pupils competent in mathematics.

A Basic Principle in Mechanics

In teaching the fundamentals of arithmetic, it is well to keep in mind that residual learning is of basic importance. This is so because a large percentage of the material presented is quickly forgotten and the remainder of the impression prevails with a relatively small loss as the time interval increases.

The rate of forgetting material that has been learned is illustrated by Ebbinghaus's "Curve of Forgetting" in Figure Twenty-five,[1] which resulted from a study made on students' learning nonsense syllables to show how quickly they were forgotten. The syllables learned for that study can be compared with the learning of number combinations in arithmetic or arbitrary facts in any subject. Pillsbury explains the curve by saying:

> . . . Ebbinghaus found . . . a rather rapid forgetting at first, and a relatively slow rate in the longer intervals . . . The comparative rapidity of forgetting during the first few days suggested . . . that there might very likely be two factors to take into consideration; first, the tendency to perseveration or the memory after-image which diminishes very rapidly and may be regarded as disappearing in the first two days or so, and the associative tendency, which . . . is to be conceived as increasing in strength for two days or more and then decreasing in strength very slowly.[2]

In all vocations, experience is a "great teacher." Employers

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1. W. B. Pillsbury, The Fundamentals of Psychology. New York: The MacMillan Company, 1916. p. 361.

2. Ibid., p. 360.

of the ability to solve situations involving the use of numbers. The writer has separated them arbitrarily to simplify the report, but it should be understood that they work together for the same end--pupils competent in mathematics.

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In all vocations, experience is a "great teacher." Employers

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1. W. B. Pillsbury, The Fundamentals of Psychology. New York: The MacMillan Company, 1916. p. 361.

2. Ibid., p. 360.

generally prefer to hire people with some experience rather than those with little or none. Some people are willing to accept a job with a low salary just "to get some experience." This would indicate that, up to an optimum, the more experience an individual has in a given occupation, the better trained he will be for that job. This is not only true in business, but is also true of persons becoming skilled in a trade. Pupils of arithmetic are being trained in a skill: that of correctly solving problems and examples. They, too, learn by experience; and the more practice which they have, within limits, the more proficient they become at computing in arithmetic. Boys and girls learn a skill by doing rather than by hearing or watching. The real test of their ability comes when they use the skill independently; and the more they have used it, the better the results will be.

That students improve and profit from repeated experience is illustrated by an adaptation of the "Curve of Forgetting," in Figure Twenty-six.[3] This curve is based on the assumption that, each time a pupil is exposed to a fact or principle, the impression made is greater. The pupils will forget about three-fourths of the total amount (according to Ebbinghaus's curve in Figure Twenty-five); but, since the impression is higher, the residue will be greater. People learn by repeated experiences which build up a greater amount of learning. ". . . The more the repetitions are distributed over different days the fewer the repetitions required and the more thoroughly the mate-

3. Franklin H. McNutt, Supervision: Evaluation and Improvement of Instruction. Class Lecture, Woman's College of the University of North Carolina, Greensboro, North Carolina, October, 1945.

rial is mastered."[4] The graph in Figure Twenty-six also indicates ways of repeating the experiences and thus making the impression higher. The curves in Figures Twenty-five and Twenty-six have many implications for the instruction in arithmetic and explain some of the problems that teachers have encountered in their classrooms.

Instructors have often been satisfied with a particular lesson, feeling certain that the students understood the process explained in class. But when the pupils were asked about it on the following day, they remembered very little of the process that had been so carefully demonstrated in the previous lesson. Figure Twenty-five shows that about one-fourth of the impression is actually retained as a permanent part of the memory, and that students should not be expected to remember all that has been given to them in one class after a single exposure. This indicates that teachers must expose students to many experiences, each one making the impression higher; hence the true learning will be greater.

Some of the ways to heighten the impression in the teaching of arithmetic are as follows:[5]

1. Develop insight.
2. Stress purpose.
3. Increase functional use.
4. Arouse intention to recall--improves the memory 50 per cent or more.
5. Use of prolific associations.

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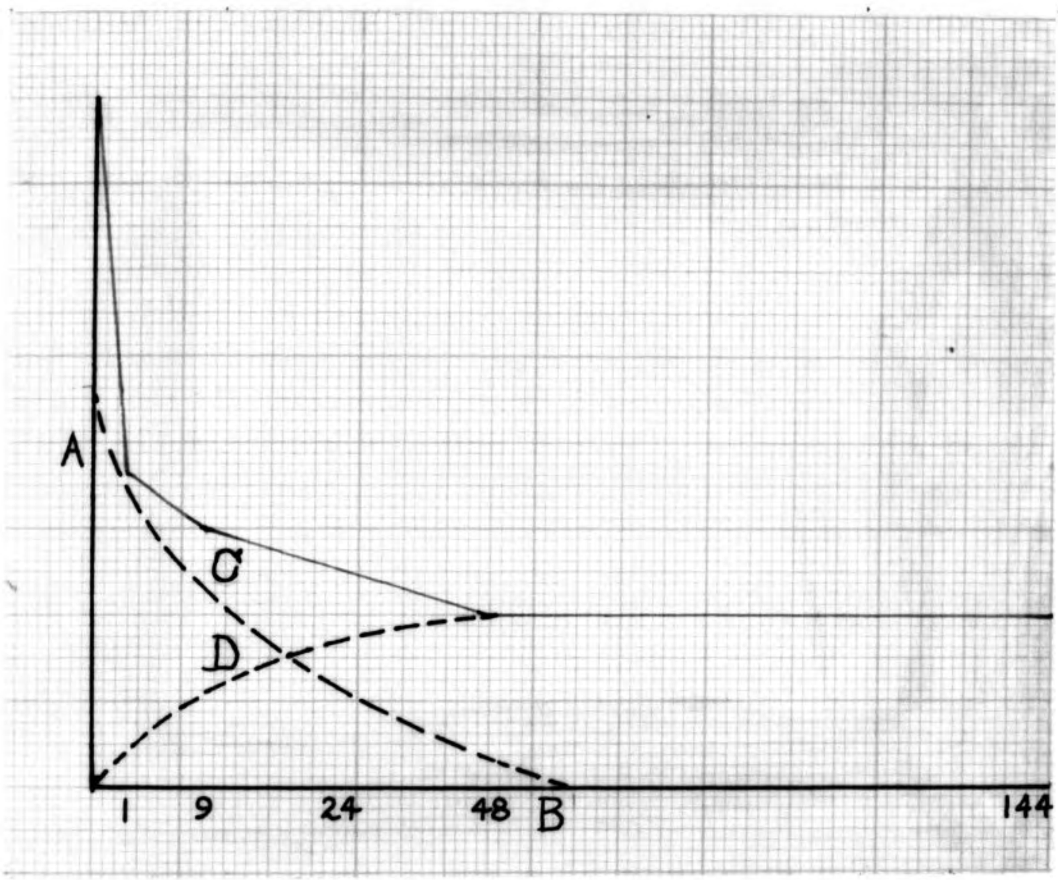
4. Pillsbury, op. cit., p. 354.

5. McNutt, loc. cit.

Figure 25. "The Curve of Forgetting"
(Ebbinghaus)



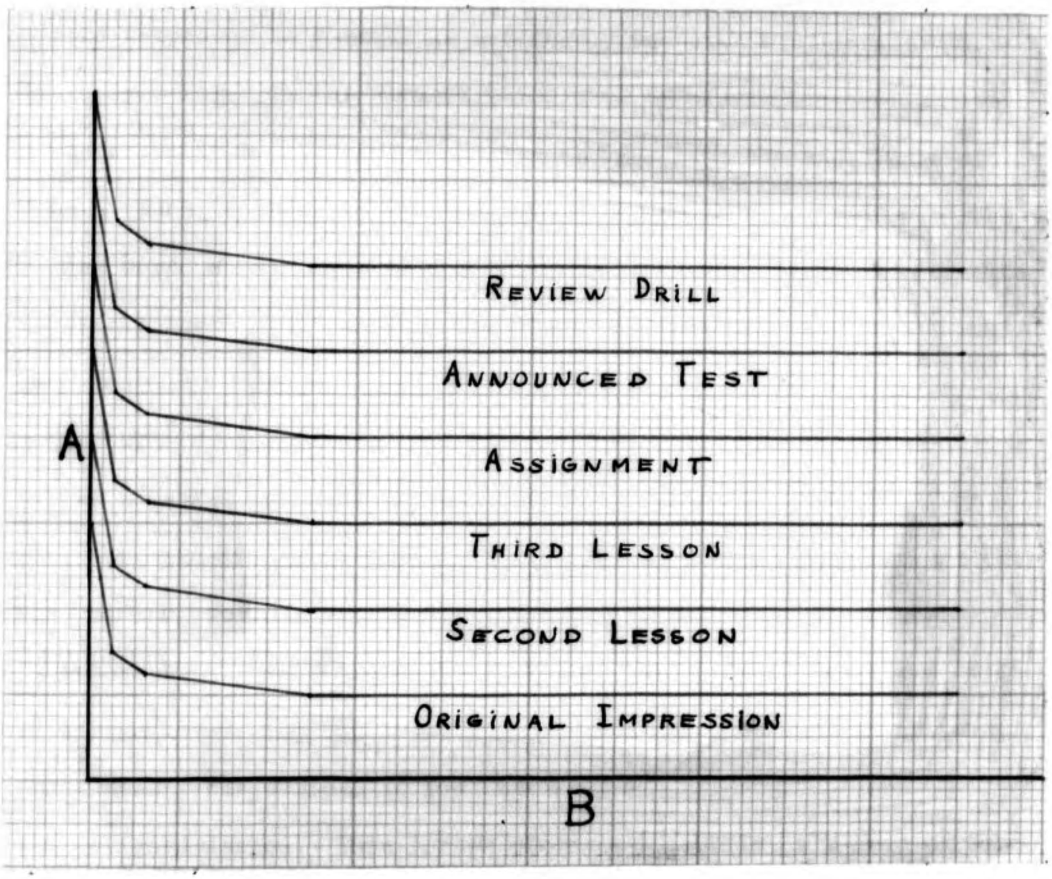
A = the height of the impression

B = the time element

C = the perseveration tendency--the tendency of a pattern to persist in activity after stimulation

D = the associative set--permanent change, the trace left by experience.

Figure 26. An Adaptation of "The Curve of Forgetting" (McNutt)



A = the height of the impression

B = the time element

<u>Common Practices That Are Inadequate</u>

The demonstration method of teaching arithmetic should be used to help the students get insight into the principles and processes, but as a stable method of instruction it is ineffective. The way to become competent in a skill is not merely to watch and listen, but to learn by doing. In the demonstration type of class, the pupils have no opportunity to "try for themselves"; they lack the experience of doing. Much interest is lost in this type of class, because the students become bored when others do the work at the board. This often results in disciplinary problems that would not have been present if the student had been allowed to work rather than to watch.

The demonstration-type class often results in a "rehashing" of the material assigned for outside work. This is due to the fact that the students are unable to solve the problems at home because they have had no previous experience under the guidance of the teacher. They have not tried to solve the problems while their memory was still at a high level. By the time they have reached a study hall or home, their memory has dropped so low that they are not able to carry out the assigned lesson. This forces students to obtain help from parents or older brothers and sisters, which often leads to false information and faulty work habits. Boys and girls copy papers from better students because they are unable to do the work assigned. The teacher using this method of instruction would doubtless meet so many questions each day on the previous day's assignment that the entire class period would be questions concerning that work. The class period would then be used for review and explanation of the assignment of the day before; whereas the explanation actually should <u>precede</u> the outside drill exercise. The assignment should be further use of a principle which has been discussed and used in class.

Inadequate review is a common fault among teachers. They feel that, after a fact or process has been discussed, explained, and drilled upon, they have fulfilled the requirements and may forget it until the time for the yearly review in preparation for the exam. Students, also, forget the principles and facts studied unless there is provided a systematic review to help maintain the information learned. Bond says:

> . . . the law of disuse . . . expresses the truth that knowledge gained or skills formed need further exercise from time to time for their maintenance. The recall of previously known material for the preparation for new learning will provide exercise for a large part of the maintenance program . . . There is need to see that all previous learning has sufficient exercise to insure its retention.[6]

The need for understanding in mathematics has been recognized as a vital part of the program, but it should not constitute the entire learning situation in that subject. Students should have acquired the ability to compute as well as the ability to do quantitative reasoning. Giving part credit for correct method used in problem-solving is simply a means for covering up the inaccuracy of the students in number combinations. Pupils possessed of the ability to reason are capable of learning the number facts. Even students with low ability are capable of learning the number combinations if they have been presented properly.[7]

"Ladder associations" have caused some of the difficulties

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6. Elias A. Bond, The Professional Treatment of the Subject of Arithmetic. New York: Bureau of Publications, Teacher's College, Columbia University, 1934. pp. 23-24.

7. cf. post, pp. 70-71.

in the fundamentals of arithmetic. Boys and girls are guilty of memorizing tables and number combinations in such fashion that when they want a number fact they must start back and run over a part of the table until they reach the desired combination. This is due to the fact that associations recur in the order learned. Students should not associate number combinations with a part of a table; but, rather, their responses should be automatic; they should immediately produce the answer. If the pupils have been taught facts by learning tables, they will experience difficulties in reversal associations. A person might know that eight times seven equals fifty-six, without knowing that seven times eight is identical in answer. Students clutter up their memory with unnecessary facts because of their failure to make reversal associations.

A Basic Principle in Problem-Solving

Boys and girls learn to solve problems in much the same way as they learn to work examples--they learn by doing. The practice which they get in solving problems is used to increase their ability, to prepare them for future needs, and to maintain the facts already learned. The ability attained in problem-solving is largely the result of insight, which is a chief factor in the development of this ability.

Today, world citizens feel a need for number knowledge. They do not want a rule to learn today and to forget tomorrow; they want to know why a process is done as it is. If the pupils are to retain the facts and processes learned in school, they must first understand them.

Pupils may gain insight if the arithmetic is within their experience and is appropriate to their stage of maturation. Figure Twenty-seven illustrates the idea that insight should always occur at the frontier of the pupil's experience. If it is too far ahead of his experience, the effort is wasted; and if the insight is on familiar material already understood by the pupil, he does not need the additional explanation. In this connection Morton believes that:

> Instruction in this subject should be organized according to a systematic plan which will gradually reveal to the pupils the meaning of number and the number system and which will lead him to see principles and relationships and to acquire insight.[8]

Some students have sufficient intelligence to gain insight into number relationships in spite of ineffectual teaching. But most students will never gain much from their study of arithmetic unless they have good teachers. Even gifted children will make far less progress with a poor teacher than with a good one. ". . . Other things being equal, the better the teaching the greater the degree of insight which the pupil achieves."[9]

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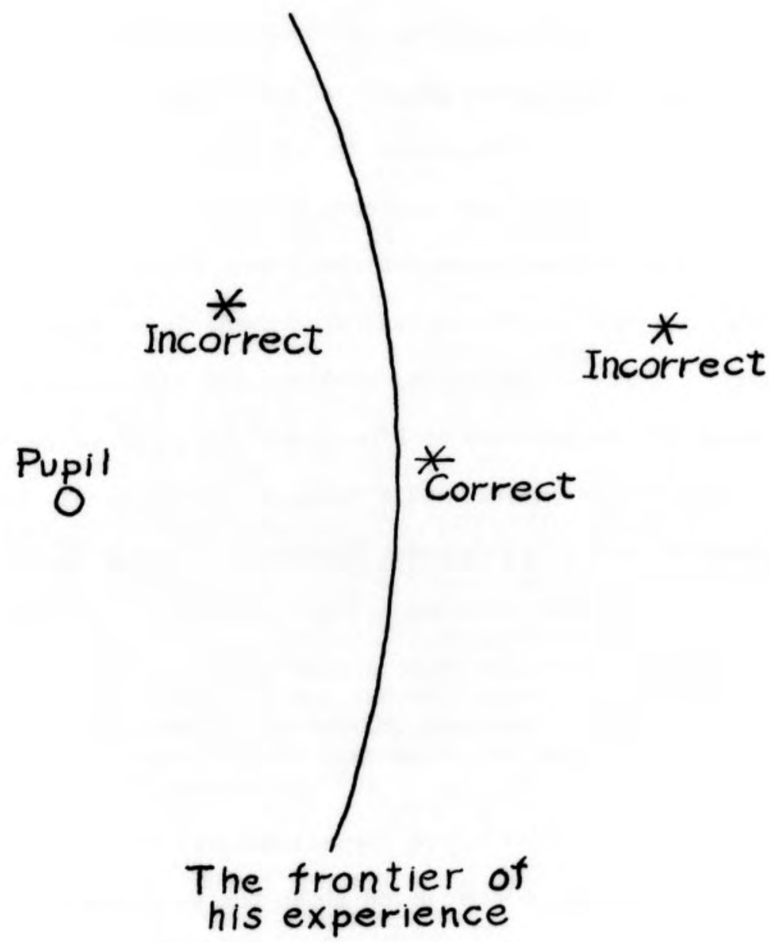
8. Robert L. Morton, Teaching Arithmetic in the Elementary School, Volume III. New York: Silver Burdett Company, 1939. p. 18.

9. Ibid., p. 19.

Figure 27

The Relationship of Individuals to Experience and Insight



(* = the point at which insight may occur.)

10. McNutt, loc. cit.

Common Practices That Are Inadequate

Students learn to solve problems by doing. Far too many teachers expect the students to learn by listening rather than by trying for themselves. The demonstration method is excellent for explaining the reasons and helping the students to understand the solutions, but it is not an effective means for developing problem-solving ability.

Problem-solving should not be taught by rules. The easy method for the teacher is to tell the students what to do and to give them examples similar to the ones used for demonstration. But no understanding is developed, with the result that the students are unable to solve a problem differing in any way from the original example. Teaching should be not by the rule, but should develop insight in quantitative situations. The rule may be given after insight is attained.

Another common, inadequate practice is that of stressing examples rather than problems. Morton believes that:

> . . . each new phase of a process should be introduced through the medium of a problem, and the skill which pupils acquire should be put to use promptly in solving problems. The four fundamental operations are not ends in themselves but only means to an end; the end is problem-solving.[11]

Students are individuals and should be treated as such. They have varying capacities and needs which must be considered in planning the work of the class. The procedure or method suited for one child is not a cure-all for the rest of the class. Some teachers use only one method of teaching the solution of problems and examples, and force that way on the entire group of students, whether it is beneficial to all of the students or not.

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11. Morton, op. cit., Volume I, p. 346.

Suggestions for Improved Methods

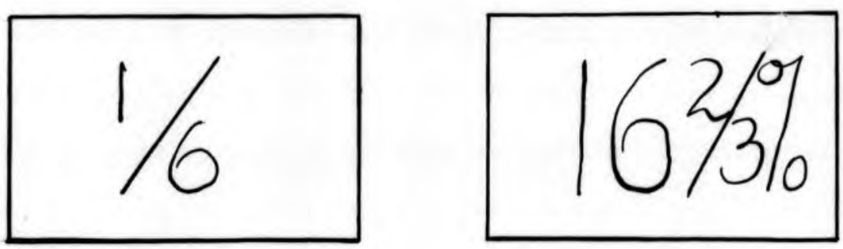
The method of instruction in arithmetic is the sum total of the approach used, the class organization, and everything involved in any way with the learning of the pupils. The following paragraphs include suggestions for improving methods in arithmetic over common, inadequate ones.

Systematic Review

One of the present-day inadequate practices is the lack of systematic review. This can be remedied by devoting five to ten minutes of each class period in rapid drill. The teacher may give, orally, ten to fifteen questions, with the pupils writing out the answers on paper. This review or drill should be on material that is supposedly mastered. It is given to help the students in maintaining the principles and facts as well as in repairing any weak spots in their ability to compute in arithmetic. The pupils should be informed each day what to expect in the rapid drill for the following day, as it gives them an opportunity to review the material that is to be tested. This is another way to heighten the impression of a particular process, which also increases the total amount learned.

Association Cards

The association cards are excellent for helping the pupils learn arbitrary facts. There is a vast amount of information that can be put on these cards other than the number combinations. Some of them suitable for eighth-grade pupils are: formulas, elements from tables of measurements, squares of integers up to twenty, common square roots, and aliquot parts. These cards need not be elaborate; they can be made out of a small piece of cardboard, two inches by four inches. On one side of the card is placed a number combination or fact and on the other the correct answer (See Figure Twenty-eight). Each pupil should have a set of these cards to use in studying the number facts. As the pupils run through the cards, they are able to isolate their difficulties and to concentrate upon them, putting aside the cards as they master the material.



obverse reverse

Figure 28
Association Card

The way in which the cards are constructed makes it possible for the pupils to study a set on the obverse and then on the reverse. In this way, when a child learns that one-sixth equals sixteen and two-thirds per cent he will also learn that sixteen and two-thirds per cent equals one-sixth, doing away with the "ladder associations" and insuring reversal associations.

Parents often want to help their child but do not know how to do it. The association cards offer a way for the parents to help by "calling out" the numbers on one side of the card and checking the pupil's response on the other side. This requires no ability to explain or to teach on the part of the parents. In this way the association cards utilize the home and strengthen the pupil at the same time. The cards may also be used to an advantage in connection with the systematic review. They give the pupils definite material to be studied before each review period and they simplify the teacher's assignment of that material. She needs only to announce the pack of cards that will be tested on the following day.

Individual Differences

There is not merely one good way of teaching arithmetic; rather, there are many. The method ideal for a given class or a specific pupil may not be the best for another. The purpose of the method used is to train students to compute and to solve problems in numerical situations. The ideal way is to induce pupils to reason quantitatively; but this requires a certain amount of native ability on the part of the students, some of whom lack that ability. A large majority of the students have the ability to comprehend and reason and should be instructed in such a way as to develop that ability. But if they are not capable of understanding and reasoning, then they must be taught by the method that they can use. Some pupils who are unable to do quantitative thinking can compute accurately, and, if given a rule, are able to solve problems. This is recommended only for those students who would not be able to deal with numerical situations unless they were told what to do. Pupils may be taught to count on their fingers, if it is the <u>only</u> way that they are capable of learning to add. This would be considered an atrocious method for students with more mental ability. The method of instruction used should be the one best suited to the individual in need of help. An effort should be made to break the "lockstep" of teaching all of the students in a class in the same manner.

"In-Class" and "Out-of-Class" Experiences

The term, "assignment," connotes a lesson that is to be done at home and then brought back to the class for discussion. This should not be the case. There is a certain amount of material to be covered in a course of study, a number of things must be done in order to accomplish the necessary work, and a limited amount of time is available. The pupils should be aware of this and should be made to realize that some assignments must be done under the guidance of the teacher while others may be done after school or as an "outside lesson." They should feel the need of an "in-class lesson" and an "out-of-class lesson." The in-class and out-of-class lessons should consist of the following types of plans:

Table VIII

Types of Lesson Plans

Out-of-class Lesson	In-class Lesson
1. Learning facts	1. Gaining insight
2. Drilling for mastery and maintenance	2. Initial practice
3. Gathering material	3. Added experience
4. Working on aspects of projects	4. Testing and evaluating
5. Gathering data for in-class experiences	5. Diagnosing
	6. Reviewing

In this type of organization, the pupils do not feel that they are doing something outside only because the assignment will be checked upon the following day, but they become aware of the need of the exercise in the total arithmetic program. As explained in Chapter One, arithmetic is a "vertical" subject, building up facts and processes as bricks build up to form a wall. If a "brick" is missing it is impossible to continue to build. When the pupils realize the importance of each principle in arithmetic they become willing to accept a part of the responsibility for learning as an out-of-class job.

Suggested Procedures

The mathematics class should be organized to give the pupils as much insight into the facts and processes as possible and to give them a sufficient number of experiences to enable them to compute and to solve the problems considered suitable for their particular grade level. This may be accomplished by furnishing the pupils with experience in the necessary numerical situations and by helping them to gain insight into the principles involved.

Insight.--"Arithmetic should be less of a challenge to the pupil's memory and more of a challenge to his intelligence."[12] The main objective in teaching this subject is to develop in the pupils the ability to do quantitative thinking, which is much more than the mere ability to compute. The ability to reason does not develop rapidly, but comes very slowly; and the pupil is in need of constant guidance toward this aim. The following suggestions are offered as steps of procedure for developing insight:

1. Arithmetic is an abstract subject, and it is helpful to use concrete and semi-concrete objects in explaining processes wher-

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12. Ibid., Volume II, p. 19.

ever possible.

2. The instructor should proceed slowly with the explanation, presenting it in a carefully graded series of steps. He must be certain that the pupils understand each step before the next one is attacked.

3. The easier phases of the topics should be presented first.

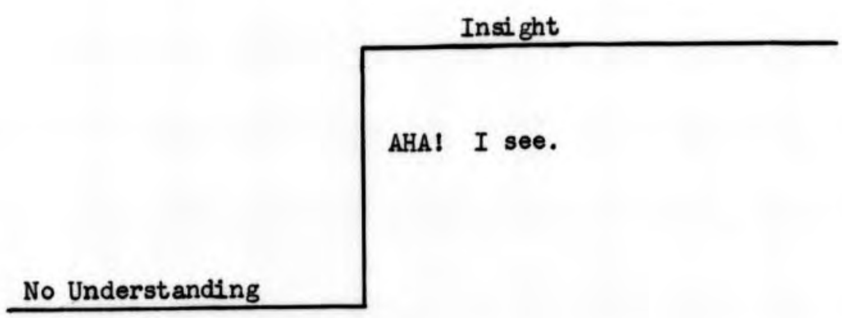
4. The relationships of the topic under discussion and others previously learned by the pupils should be made clear.

5. The principle should be explained and demonstrated in as many ways as seem necessary for the pupils to comprehend.

6. The explanation and demonstration of a process should continue until the pupils indicate by their responses that they understand. The moment at which this occurs differs with pupils. This may be called the "AHA, I see" moment. The way in which pupils arrive at an understanding is illustrated by Figure Twenty-nine.[13]

Figure 29

Gaining of Insight



13. McNutt, <u>loc</u>. <u>cit</u>.

7. After the pupils develop some understanding and have had some practice in the solving of the problems, the teacher should constantly ask them, "Why did you do it in that way?" When the pupils have been exposed to this question repeatedly, they will appreciate arithmetic as a mode of thinking which derives its rules from the principles of the number system.[14]

Drill.--". . . Drill does not make arithmetic meaningful but it is of value in fixing the skills and maintaining them at a high level of usefulness."[15] The term, "drill," does not necessarily mean the monotonous repetition of facts and principles, but, rather, practice on the phases of mechanics and problem-solving which appear to be difficult for the pupils. Drill should always be preceded by the explanation and should come only after the pupils thoroughly understand the process that is to be drilled upon. A logical procedure for drilling on certain facts or processes is given in the following steps:

1. The first drill on a particular fact or principle should be easy enough for all the pupils to do with little difficulty. This encourages them to try, and gives them a feeling of success.

2. The next exposure should be more difficult, stressing problems similar to the ones used in the explanation.

3. Out-of-class exercises should be given to check the pupil's ability to do the work independently.

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14. W. A. Brownell, "Psychological Considerations in the Learning and Teaching of Arithmetic," Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teacher's College, Columbia University, 1935. pp. 28-29.

15. Morton, op. cit., Volume II, p. 13.

4. The amount of drill necessary is dependent upon the degree of competence of the pupils and the difficulty of the principle. But after they understand the process, they should have the experience of solving varied problems involving the same principle. Pupils should be able to analyze the situation, to isolate the pertinent facts, and to decide upon and execute the correct method of solving the problems.

Laboratory Method

The laboratory method is recommended as an effective way of organizing the class. In this type of class the students work at their own rate of speed, concentrating and drilling on the spots which they find are weak.

The class period may consist of three parts: the assignment for the out-of-class exercise, the rapid drill, and the class lesson. The out-of-class lesson needs no explanation, just the page and problem number in the text or workbook. The rapid drill may take the first five or ten minutes of the class period for the maintenance of facts already learned. The remainder of the class period, when it is not being used for explanation, may be devoted to a laboratory-type work period.

The pupils should accept part of the responsibility for carrying out the class program. They may be used for "checkers" to mark errors on the papers as the pupils work, to score assignment papers, and to help backward pupils. This relieves the teacher for the more important tasks of giving insight, diagnosing difficulties, and evaluating the pupils' progress.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

In the preceding pages of this study, an effort has been made to locate the weaknesses of eighth-grade pupils in arithmetic and to suggest methods of instruction which will help to eliminate or to minimize these difficulties. In order to find the needs of this grade level, The Metropolitan Achievement Tests, Advanced Arithmetic: Form A (revised) was given to approximately 290 eighth-grade pupils in Greensboro, North Carolina. The results from the study indicated the following:

1. The schools tested were average urban schools.

2. The test was a valid measure of the pupils' ability.

3. The results of the test would be similar to the results of the same test if given to other schools of this type.

4. On the whole the pupils who were tested did very well.

5. A majority of the pupils scored higher on problem-solving than on fundamentals.

Conclusions

As a result of this study, the following conclusions can be drawn with a considerable degree of confidence:

1. Boys and girls in Greensboro, North Carolina, have equal arithmetical ability.

2. Pupils are competent in the fundamental operations when:

   a. the numbers are small.
   b. the fractions are those in common use.

3. There is difficulty in dealing with large integers and mixed numbers.

4. Arithmetic competency in varied number combinations is weak.

5. A need is revealed for a better understanding of the use of the zero and of the decimal system.

6. The problem-solving ability of the majority of the pupils is greater than their ability on fundamentals.

7. Insight is often lacking.

8. Checking answers for reasonableness can eliminate many errors.

9. Pupils are capable of solving one-step problems, but become baffled if two or more steps are necessary to obtain the answer.

Recommendations

On the basis of this study, certain recommendations may be made:

1. The results of the test indicate that the objectives for eighth-grade arithmetic should be: the maintenance and the repair of facts and principles already learned.

2. Teachers should seize every opportunity to develop in their pupils the ability to do quantitative reasoning.

3. The organization of the course should provide much drill in the mechanics.

4. "Lock-step" methods should be abandoned in favor of those suited to the individual capacities and abilities of the pupils.

5. In the interest of developing habits of thought that yield

insight, problems, rather than examples, should be stressed.

6. In assisting the individual pupil, the teacher should guide his reasoning rather than demonstrate the solution.

7. Inasmuch as pupils learn a skill by doing rather than by watching and listening, a laboratory method of class organization should be used.

8. For learning arbitrary facts, association cards are strongly recommended.

9. The traditional assignment should be abandoned in favor of an "out-of-class" exercise of a character that the pupil can perform without guidance.

10. The "recitation," insofar as it now connotes a rehash of an "assignment," should be abandoned in favor of appropriate "in-class" activities.

11. Inasmuch as there can be no superior instruction divorced from systematic review, the teacher should allocate a portion of the time each day for this purpose.

12. In order to free the teacher for important teaching duties, the pupils should share in the responsibility for carrying out mechanical aspects of the class program.

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