A Study of the Literature of the History of Mathematics

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Although men have been drawn to the study of mathematics from the earliest times, the study of the history of mathematics has not had a similar attracting power. Interest in this branch of learning is a comparatively recent development. There is reason to believe that as late as 1870, mathematicians still regarded the pursuit of this study as being of little or no value. Rather they considered that any old results which would be likely to assume a permanent place in the progress of mathematical learning could be found in improved form in new treatises.[1] Thus they confined themselves to the reading of current mathematical discovery. Even today there are those who believe that technical papers on mathematical research represent the only meaningful history of mathematics to the mathematician.[2]

There came a change in this traditional point of view, however, at the end of the nineteenth century as mathematicians came to recognize mathematics "as an increasing variable instead of as a fixed constant."[3] Now the study and the teaching of the history of mathematics assumed a greater value. Its importance to the mathematician

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[1] G. A. Miller, Historical Introduction to Mathematical Literature (New York: The Macmillan Company, 1916), p. 4.

[2] Eric Temple Bell, The Development of Mathematics (New York: McGraw-Hill Book Company, Inc., 1945), p. x.

[3] Miller, p. 5.

was particularly stressed. George Sarton was evidently concerned about the public image of the mathematician, for he advocated the study of the history of mathematics, saying that it "will not make better mathematicians but gentler ones, it will enrich their minds, mellow their hearts, and bring out their finer qualities."[4] Not only did Sarton emphasize the importance of this study to the mathematician, but he also considered it possible for only a select group to engage in such study. To him, mathematics more than any other science, was capable of being understood by the specially initiated only.[5] With such ideas being prevalent, it is curious to note the direction which the majority of English language works on the history of mathematics have taken.

Almost without exception such works are aimed not at the mathematically proficient, but rather at students with an elementary knowledge or at the masses of the reading public. This tendency is present in both the histories of the early part of the century and in those of more recent date. Among the older historians, W.W. Rouse Ball offers a short and popular account of the main facts of the history of mathematics for those people who either cannot or will not give time for a thorough study of the subject,[6] while Florian Cajori states the purpose of his similar history as being "for the use of readers who can-

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[4] George Sarton, The Study of the History of Mathematics (Cambridge: Harvard University Press, 1936), p. 28.

[5] Ibid., p. 4.

[6] W.W. Rouse Ball, A Short Account of the History of Mathematics (London: Macmillan and Co., Limited, 1908), p. v.

not devote themselves to an intensive study of the history of mathematics."[7] David Eugene Smith considers his work an introductory text and has written it in the hopes of arousing the student's interest in the subject so that he will be inspired to pursue the subject further.[8]

In general the more recent writers, such as Eric Temple Bell, Howard Eves, Morris Kline, and Lancelot Hogben, have also pitched their approaches at a low enough level so that all the mathematics involved may be understood by a layman. Bell has written two books on the history of mathematics which have popular appeal: Men of Mathematics and Mathematics: Queen and Servant of Science. In the former he admits to using two criteria in his selection of the mathematicians to be considered: the importance of a man's work to modern mathematics and the personal appeal of the man's life and character. Not only does appeal play an equal part with importance, but it assumes the major part in cases where two mathematicians have made comparatively equal contributions.[9] In this book, as in the second, the mathematics involved should be understood by anyone who has had a high school mathematics course. Bell expresses the hope that the

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[7] Florian Cajori, The History of Mathematics (New York: The Macmillan Company, 1926), p. v.

[8] David Eugene Smith, History of Mathematics, Volume 1 (Boston: Ginn and Company, 1923), p. v.

[9] Eric Temple Bell, Men of Mathematics (New York: Simon and Schuster, 1937), p. 3.

latter book will have an appeal to young people with no more than a high school mathematics education, who want to see what lies beyond, and non-mathematicians who can remember enough of their elementary mathematics to want to know more about it and about the spirit which lies behind it.[10]

Both Eves and Kline have written books planned as texts for college courses. Yet their texts are not aimed at the mathematics major. For Eves is planning an introductory undergraduate course restricted primarily to elementary mathematics,[11] while Kline states that his book, Mathematics: A Cultural Approach, is planned for students taking terminal courses in liberal arts colleges and not planning to go on with mathematics, for teachers of elementary mathematics, and for high school students, especially those whom it might inspire to go on with mathematics.[12] Kline has also written two books, Mathematics in Western Culture and Mathematics and the Physical World, in the field of history of mathematics which are restricted largely to elementary mathematics and are geared to the person with little technical knowledge.

Lancelot Hogben perhaps represents the farthest limits of this

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[10] Eric Temple Bell, Mathematics: Queen and Servant of Science (New York: McGraw-Hill Book Company, Inc., 1951), p. vi.

[11] Howard Eves, An Introduction to the History of Mathematics (New York: Holt, Rinehart and Winston, 1953), p. 1.

[12] Morris Kline, Mathematics: A Cultural Approach (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1962), p. v.

tendency toward popular appeal. He confesses that he wrote one work for the fun of it as "a private citizen interested in education"[13] and tells us that he wrote the second "to make it easier for a wider reading public to get the best out of a large number of available contemporaneous sources."[14]

Among all the various works on the history of mathematics, only two stand out as not having been written primarily for those with little knowledge of mathematics. Eric Temple Bell, although he has produced popular accounts, is also the author of one work, The Development of Mathematics, which he claims to have written for the "cultivators" of mathematics, rather than for those with only a superficial interest or no interest at all.[15] This aim becomes quite obvious when one considers the increasing difficulty and abstraction of the mathematical concepts involved in the chapters treating developments from the eighteenth century on. And a reviewer has said about Dirk J. Struik's A Concise History of Mathematics that "students, researchers, historians --- specialists and laymen alike --- will find it extremely useful and interesting."[16] Thus neither author has found it necessary to make a

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[13] Lancelot Hogben, Mathematics for the Million (New York: W.W. Norton & Company, Inc., 1937), p. xi.

[14] Lancelot Hogben, Mathematics in the Making (Garden City, New Jersey: Doubleday & Company, Inc., 1960), p. 4.

[15] Bell, Development, p. v.

[16] Dirk J. Struik, A Concise History of Mathematics (New York: Dover Publications, Inc., 1948), inside front cover.

conscious effort to be appealing to the reader.

Thus we see how the trend has varied since the early years of the century when the history of mathematics was thought to be an exclusive subject. It is good that the importance of at least some knowledge of the subject has been recognized. However, the danger has lain in too great an effort to attract the masses and correspondingly too great a neglect of the more sophisticated type of work which would challenge the professional. For it is as true now as it was in 1900 that the mathematician needs to be acquainted with the background of his field. The history of mathematics in its striving for mass acceptance must not totally dissociate itself from the mathematician. Let us examine the direction which the search for mass appeal has led works on the history of mathematics to take.

The earlier writers make frequent use of the anecdote in order to make their histories more palatable to the ordinary reader. Both Rouse Ball and Florian Cajori introduce stories about various mathematicians which, to be sure, are interesting, but have very little to do with the history of mathematics. Thales, being one of the earliest mathematicians (c. 600 B.C.), is the most frequent victim of the anecdote. Rouse Ball tells of him that

> once when transporting some salt which was loaded on mules, one of the animals slipping in the stream got its load wet and so caused some of the salt to be dissolved, and finding its burden thus lightened it rolled over at the next ford to which it came; to break it of this trick Thales loaded it with rags and sponges which, by absorbing the water, made the load heavier and soon effectually cured it of its troublesome habit. 17

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17 Ball, p. 14.

Certainly such a tale has no bearing on either the development of mathematics or on Thales' contributions to the subject. Not only is it irrelevant, but it may not even be accurate as the same story has often been told to illustrate the ingenuity of other figures. Florian Cajori also advocated the use of the anecdote to make the subject more interesting to students.[18] He has his own tale to tell of Thales --- a more popular tale, found in several of the histories. He says that Thales

> while contemplating the stars during an evening walk, ... fell into a ditch. The good old woman attending him exclaimed, "How canst thou know what is doing in the heavens, when thou seest not what is at thy feet?"[19]

Here again there is no attempt to relate the incident to the history of mathematics. Cajori is merely telling the tale for the sake of telling a good story. Cajori follows the same procedure of extensive use of the anecdote in his A History of Elementary Mathematics, which is even more anecdotal than his History of Mathematics.

Perhaps more shocking to all of us, however, is David Eugene Smith's admission that he has used the anecdote "to relieve the monotony of mere historical statement."[20] We find it hard to believe that any historian could consider his subject to be monotonous and still continue to pursue it. Whether or not he does hold this belief, it is certainly true that he makes more frequent and flagrant use of the

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18 Cajori, p. 2.

19 Ibid., p. 16.

20 Smith, p. v.

anecdote than any other of the early writers. He too has his favorite tale about Thales which he takes from Plutarch:

> Solon went ... to Thales at Miletus, and wondered that Thales took no care to get him a wife and children. To this Thales made no answer for the present, but a few days after produced a stranger to pretend that he had left Athens 10 days earlier and Solon inquiring what news there was, the man replied... "None but a young man's funeral...the son...of an honorable man, the most virtuous of the citizens, who was not then at home, but has been traveling a long time." ... [After questioning the servant, Solon discovers it was his own son. Then] Thales took his hand and, with a smile, said, "These things, Solon, keep me from marriage and rearing children, which are too great for even your constancy to support; however, be not concerned at the report for it is a fiction."[21]

Smith goes even farther afield in his treatment of Chinese mathematics about which little of a concrete nature is known. He relates that the Chinese emperor, Chóu-Kung (c. 1100 B.C.), had the "habit of rushing several times from his bath, holding his long, wet hair in his hand, to consult with his officials." Chóu-Kung also "had a wrist like a swivel, on which his hand could turn completely round."[22] Smith never is very clear about what relation Chou-Kung has to mathematics. But these are odd and interesting facts! In addition Smith shows by the topics he discusses that he is trying to be as appealing as possible. In the first pages of Volume 1 of his <u>History of Mathematics</u>, which is a chronological treatment, he goes back as far as the birth of the solar system and treats of cosmic figures as the beginning of the history of mathematics. He continues by discussing such topics as mathe-

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21 <u>Ibid</u>., pp. 65-6.

22 <u>Ibid</u>., p. 31.

matical figures in nature, speculating upon how the leaves of the fern came to adopt the principle of the Golden Section in their arrangement on the stalk.[23] Volume 11 shows similar evidence of an attempt at popular appeal in its devotion of a chapter to the subject of mathematical recreations, interesting problems, magic squares, and number puzzles.

Eric Temple Bell represents the first historian able to resist the temptation to try to make Thales more human by telling some anecdote about him. Actually this fact is representative of Bell's entire approach. Although his Men of Mathematics is anecdotal, we must realize that Bell is here trying to portray the lives and personalities of mathematicians. His book is essentially more biographical than historical and thus warrants the inclusion of such material as the political adventures of Jacobi. Bell's catering to the non-mathematician in Mathematics: Queen and Servant of Science is represented by the assertion of his intention to employ "wider viastas than would satisfy a peering professional" in his approach, rather than to tackle the "Minutiae of modern mathematics."[24] This is not to imply that Bell does not use the anecdote. He does do so, although he seems at times to do it reluctantly. Unlike the earlier writers, however, Bell's anecdotes either have a relevancy to or illustrate a point about the history being narrated. In this respect he is representative of

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23 Ibid., p. 24.

24 Bell, Queen, p. 15.

the general trend in histories of mathematics after his time.

The same attitude may be found in the works of Lancelot Hogben. Despite the fact that he claims that the "asides and soliloquies [in Mathematics for the Million]...are put in to sweeten the pill,"[25] we find here as in Bell's histories that the "asides and soliloquies bear definite relevance to the text. In spite of their purpose, they do not break in and interrupt the flow of thought as in Ball, Cajori, and Smith. The essence of the popular appeal of Hogben's Mathematics in the Making is visible at a glance through its pages. The proportion of text to pictures, diagrams, and puzzles is small. And all these are in full color. They represent the author's belief in "exploiting visual aids to an extent as yet seriously undertaken by no text books"[26] in his attempt to speed up the process of assimilation of material on the part of the reader.

Morris Kline in his several books on the subject has adopted special approaches to the history of mathematics. Kline believes the amateur to be more interested in understanding ideas than in learning complicated technique and symbolism. Thus he concentrates on the evolution of mathematical ideas in their relations to culture and to science. Continuing to use Thales as an example of a type of treatment which we also find associated with other mathematicians, we discover Kline repeating in two of his books, Mathematics in Western Culture (p. 24) and Mathematics and the Physical World (p. 139), the

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25 Hogben, Million, p. xi.

26 Hogben, Making, p. 4.

same old legend of Thales' fall into the ditch which Florian Cajori has related. And in Mathematics: A Cultural Approach, Kline tells still another Thales story which is also found in Rouse Ball:

> In a year when olives promised to be plentiful, Thales shrewdly covered all the oil presses to be found in Miletus and in Chios. When the olives were ripe for pressing, Thales was in a position to rent out the presses at his own price.[27]

In each case, however, Kline integrates the legend with the text, using it to dispel a misconception or illustrate a characteristic of mathematicians. Thus he is a part of the recent trend.

Howard Eves is in both the new and the old tradition in his treatment of the anecdote. He uses it as an integral part of the text in some cases and in others as merely digression. Our old friend Thales is a favorite with Eves, and he offers us a number of anecdotes about Thales simply for the charm of them. Not only does he tell us the stories of the oil presses, of the mule in the stream, and of the fall into the ditch, but he adds several of his own, among them how "when asked what was the strangest thing he had ever seen, he answered, 'An aged tyrant.'"[28] However, his own special brand of popular appeal consists of the stimulating problem studies which he affixes to each chapter. He has not done this with the object of teaching mathematics, but instead with the purpose of giving the student the opportunity to work the same type of problem in the same manner as some famous mathe-

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27 Kline, p. 44.

28 Eves, p. 52.

matical figure did. Thus, for example, he feels that in solving a quadratic equation by the Greek geometrical method, the student accomplishes two things. He increases his understanding of and appreciation for Greek mathematics.[29] Thus we find the student asked to

> write 574 ... in (a) Egyptian hieroglyphics, (b) Roman numerals, (c) Attic Greek numerals, (d) Babylonian cunieform, (e) Traditional Chinese-Japanese, (f) Alphabetic Greek, (g) Mayan numerals.[30]

James R. Newman in collecting the various selections to appear in his four volume World of Mathematics, searched through both the popular literature on mathematics and the technical and scholarly literature in order to find examples which the reader could both understand and enjoy. It was his purpose to illustrate the wide range of mathematics --- its many facets and ideas. He wanted to present mathematics as a tool, a language, an art, an end in itself, and a "fulfillment of the passion for perfection," as a source of satire, humor, and controversy.[31] As a result of such treatment, it is only natural that he should present selections which would appeal to all classes of people. For the more serious student there are excerpts from the actual writing of such famous mathematicians as Descartes, Archimedes, Newton, Euler, Dedekind, Daniel Bernoulli, Boole, Laplace,

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29 Ibid., p. 1.

30 Ibid., p. 26.

31 James R. Newman, ed., The World of Mathematics, 4 vols., (New York: Simon and Schuster, 1956), p. vii.

Poincare, and many others. For the less serious reader there are the sections on "Mathematics in Literature" which includes an excerpt from Jonathan Swift's Gulliver's Travels called "Cycloid Pudding" and on "Amusements, Puzzles, and Prizes" which includes "What the Tortoise Said to Achilles and Other Riddles" by Lewis Carroll and two humorous essays by Stephen Leacock.

The author who has perhaps stretched the function of history the farthest is Edna Kramer. In her Main Stream of Mathematics, she is not content with merely relating mathematical anecdotes, but she puts her imagination to work on a myth and recreates her own version of the incident. The result is a pleasant story, but not history.

In considering the manifest desire for mass appeal present in the most prominent representatives of the literature of the history of mathematics, we have noted, particularly with relation to the use of anecdotes, a changing trend in the history of mathematics. We have already formed an arbitrary division of the authors into earlier writers and more recent writers. It is more than a different approach to popular appeal that has caused us to make such a difference. We will find that the histories of Rouse Ball, Florian Cajori, and D.E. Smith differ from those of such writers as Eric Temple Bell, Howard Eves, Morris Kline, and Lancelot Hogben in ways more basic and significant.

While the earlier writers from the tone of their comments seem to be offering their histories as a service to the uninitiated who might become interested if the effort were made to interest them, the more recent historians seem to have written their works under the influence of a strong belief in the necessity for a knowledge of mathematics and its history for anyone who lives in our modern world. This difference is reflected in the author's attitude as expressed or demonstrated. Florian Cajori is representative of the earlier writers in his belief that the study of the history of mathematics is both interesting and instructive in that it tells us what we have and shows us how to proceed, shows us our own errors, warns against hasty conclusions, shows the importance of a good notation to progress, discourages over-specialization, saves the student time and energy in trying to solve problems already solved or insolvable.[32] If these are the only reasons for studying the subject, there is certainly no necessity for everyone to do so. Such knowledge would be of value to the potential mathematician, certainly. But the layman has no need for learning how to go about making a mathematical discovery or for being warned against wasting time on problems which he will probably never even think about trying to solve. Both Cajori and D.E. Smith also think the history of mathematics a necessary part of the preparation of a mathematics teacher. Neither of the two, however, makes an attempt to convince us that the history of mathematics should be studied by everyone.

In contrast, we find the more recent authors emphasizing the importance of at least a basic knowledge of mathematics and of its history, which they consider inseparable from it. Morris Kline states that the purpose of Mathematics and the Physical World is "to display the role of mathematics in the study of nature" and by following "the

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[32] Cajori, p. 1.

gradual development of mathematical power and the increasing absorption of mathematics in the scientific enterprise...[to] learn how and why mathematics has become the essence of scientific theories."[33] He is not content merely to achieve these purposes, however. Before beginning with the task he has laid down for himself, Kline devotes a chapter of his book to proving by example and logical argument that "mathematical reasoning can produce knowledge which guesswork, intuition, and experience cannot produce or can produce only inaccurately"[34] and that a study of the development of that type of reasoning and its consequences is therefore indispensible for the man who would call himself educated. To demonstrate further the relevance of mathematics to many fields, Kline in his various books covers the role which mathematics has played in art, biology, the social sciences, economics, literature, philosophy, religion, music, and many other branches of knowledge in addition to its most obvious role in the physical sciences. Thus the history of mathematics becomes a vital subject for study because of the "extent to which mathematics has molded our civilization and culture."[35]

Lancelot Hogben considers mathematics a means of communication and its history "a facet of the history of the technique of human

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[33] Morris Kline, Mathematics and the Physical World (New York: Thomas Y. Crowell, 1959), p. viii.

[34] Ibid., p. 7.

[35] Kline, Approach, p. v.

communications."[36] Thus to him the study of the "grammar of size" becomes as important as the study of any other grammar. Hogben's view of the history of mathematics is every bit as broad as Kline's for as he also sees it, it is "a mirror of civilization, interlocking with man's common culture, his inventions, his economic arrangements, his religious beliefs."[37] This broad view of the history of mathematics is typical of recent historians. By contrast the earlier historians were prone to see mathematical history as a separate entity, which fact shows why they did not consider its study a necessity. Since the modern historians largely followed George Sarton in regarding the history of mathematics as "the kernel of the history of culture,"[38] they naturally stress its universal appeal and its special significance to all men.

The difference between early and late histories is not merely one of the feeling of the authors toward their subject, for each author's feelings are reflected in the approach which he takes to his subject. Thus we find a marked difference in approach between the two groups of historians.

Rouse Ball gives us the clue to the earlier method of presentation when he describes his A Short Account of the History of Mathematics as "a historical summary of the development of mathematics, illus-

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36 Hogben, Making, p. 4.

37 Hogben, Million, p. 34.

38 Sarton, p. 4.

trated by the lives and discoveries of those to whom the progress of science is mainly due."[39] The earlier historians present us with a history of mathematicians rather than a history of mathematics. This emphasis on the mathematicians instead of the mathematics is further highlighted by the use of dark print for the names of the major mathematicians and of italics for the names of less important figures in the histories of Ball and Cajori. Smith divides the chapters of Vol. 1 of his History of Mathematics into sections and subsections, the subsections usually being titled by a mathematicians name. In all cases the historical presentation is that of naming a mathematician and listing his accomplishments. Smith's Vol. 11 is an exception to the foregoing rule, being a presentation by mathematical fields. Even in this type of approach, however, he manages to give the mathematician a place of prominence above that of the mathematics developed. Cajori's A History of Mathematics shows particular emphasis on mathematicians in the nineteenth century. In its treatment of recent mathematics, his work is so chockful of names as to be overcrowded and to become tedious, unrewarding reading.

There is very little actual mathematics found in the earlier history books, and that which is present is not such as will be likely to prove either useful or enlightening to the modern reader. For example, Rouse Ball devotes six pages to the presentation of various historical methods for performing the elementary operations of multiplication and division. Cajori in A History of Elementary Mathematics

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[39] Ball, p. v.

offers seven pages of the same type of material. D.E. Smith, however, exceeds both of them in the second volume of his History of Mathematics, devoting nineteen pages to illustrating approximately eighteen methods of multiplication followed by nineteen formulas for multiplication used by various mathematicians, six pages to showing several historical forms of multiplication tables, thirteen pages to demonstrating approximately eighteen methods of division used by different mathematicians in the early history of mathematics, six pages to recounting the history of different values of π, which men obtained them, and to what number of places it was determined in each instance, and fifteen pages to listing historical forms of writing equations used in different countries and by different writers. No wonder Smith feels he needs to use anecdotes to relieve the monotony!

Mention has already been made of the fact that the earlier historians tended to regard the history of mathematics as a separate entity. This tendency has led them to see one mathematical discovery proceeding from another with very little outside influence. However, there is evidence in these early works of a shallow attempt at a broader viewpoint, particularly a sociological point of view.

Florian Cajori expresses the belief that the history of mathematics is also important as a part of the history of civilization, that mathematical progress is closely allied to human and intellectual progress.[40] Thus Cajori attempts to show how interrelations between various cultures may provide a stimulus to mathematical developments. Unfortunately,

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[40] Cajori, p. 3.

his attempts do not go beneath the surface. In both his History of Mathematics [p. 15] and his History of Elementary Mathematics [p. 46] he finds great significance for the future development of Greek mathematics from the fact that in the seventh century B. C. extensive commercial relations sprang up between Greece and Egypt and resulted in an interchange of ideas. However, Dirk J. Struik, a firm proponent of the sociological theory of mathematical development, in an article on the subject of the sociological approach states that "to establish the existence of mercantile relations is not of itself sufficient to prove the assured growth of mathematics beyond a certain elementary level."[41] As an example he notes the fact that both Greece and Babylonia had contacts with Egyptian mathematics but approached mathematics differently and left contributions of widely different value to us.[42] Obviously the true sociological approach then, which would penetrate beyond surface appearances and obvious conclusions, would consist in showing what specific cultural factors of Greece and Babylonia caused those two races to develop along separate lines.

Further examples of Cajorie's vaguely sociological interpretations are to be found in both of his books. In one he refers to certain origins of Hindu mathematics in Greek mathematics and states that these relations are both interesting and difficult to trace.[43] But he makes

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41 Dirk J. Struik, "On the Sociology of Mathematics," Mathematics: Our Great Heritage, ed. William L. Schaaf (New York: Harper & Brothers, 1948), p. 85.

42 Ibid., p. 84.

43 Florian Cajori, A History of Elementary Mathematics (New York: The Macmillan Company, 1917), p. 94.

no attempt to trace them for us, merely leaving us with that statement. In another case he finds that the revolutionary changes in the sixteenth century world which resulted from the Renaissance led England to greatness and Germany to degradation.[44] There follows no attempt to prove his statement or to show why these changes should have had different effects in the two countries named.

D.E. Smith also gives some attention to an investigation of influences on mathematical progress outside the field of mathematics. He feels that "linguistic and racial influences tend to develop tastes in mathematics as they do in arts and in letters and certain centuries stand out with interesting prominence."[45] Thus he announces his intention to place emphasis on racial groups who were especially productive during certain periods. In this connection he raises the question as to why mathematical achievement was concentrated in Greece during the period of time from 1000 B.C. to 300 B.C.[46] Yet he never attempts really to understand why or to answer the question. Smith also attempts to use comfortable, general, and conventional causes to explain extraordinary development at certain times. He is prone to accept any condition of an age as a major influence on that era's progress. In this respect, he often proves to be contradictory. For example, he states the theory that

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44 Ibid., p. 146.

45 Smith, p. iv.

46 Ibid., p. 96.

"Philosophy, letters, mathematics, art and all the finer products of the mind require peaceful surroundings for their development [and proceeds to apply it]. It is for this reason that mathematics at this time [1000-300 B.C.] flourished best on the protected islands of the Agean Sea, on the Greek peninsula and in the Greek towns of Southern Italy."[47]

Smith finds these locations ideal for development because they were safe from invasion yet open to commerce and intellectual communication so that peace without stagnation was assured. Hence Smith establishes peace as a stimulus to mathematical growth. However, several chapters later he discovers that in a new age such influences as the beginnings of World War 1 as seen in the founding and rising of the military machine of Prussia, the turning back of the Turks by Austria, and the Thirty Years War provide the spur toward the development of mathematics.[48] Here his theory has changed to one of struggle being necessary to stimulate progress. Whereas the rise of the Prussian military machine is in Chapter 1X seen as a factor leading toward discovery, in Chapter 1V the rise of the Roman military power is credited with causing the suppression of intellectual activity and the consequent stagnation of mathematical invention.[49] Smith makes no attempt to differentiate between these various conditions and show why like circumstances may cause different results or unlike cause similar.

There is also present in Smith the barest beginnings of a cultural

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47 Ibid., p. 54.

48 Ibid., p. 358.

49 Ibid., p. 120.

approach which he, however, never chooses to follow through. He considers the cultural events of an age but makes no real attempt to tie them to the mathematical advances. In his discussion of the Renaissance, he does mention the literary works of such writers as Dante, Petrarch, and Boccaccio, but shows no reciprocal influences between their works and that of the mathematicians. When he covers perspective, it is as a topic in mathematics, rather than as an area of development which shows the cultural interaction of art and mathematics.

That there has been a change in the type of product turned out by the mathematical historian in recent years has not gone unnoticed by the historians themselves. As Lancelot Hogben sees it:

> During the past twenty years, the work of later scholars, in particular Otto Beugebauer and Joseph Needham, has given us good reason to revise much of what we learned from the writings of D.E. Smith, Cajori, Rouse Ball and others of the same vintage.[50]

And to be sure, the more recent writers are much more likely to acknowledge their indebtedness to the valuable researches of Neugebauer than to the massise volumes of Moritz Cantor as the earlier writers did. This shifting of loyalty has been accompanied by a corresponding shift in emphasis within the histories themselves. The essence of this second shift is to be found in the words of Morris Kline, who tells that, in his opinion, the "extent to which the creative faculties of men are exercised in mathematics could be determined only by an examination of the creations themselves."[51] It is important to note that Kline has said

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[50] Hogben, Making, p. 4.

[51] Morris Kline, Mathematics in Western Culture (New York: Oxford University Press, 1953), p. 4.

"creations" rather than "creators." The two words have been put into their proper perspective by the later historians. As Eric Temple Bell has put it, "mathematics overshadows its creators; we are interested in mathematics."[52] It is true that history is made by men but "it is especially true of mathematics that, while the creative work is done by the individual, the results are the fruition of centuries of thought and development."[53] Thus it is that the more recent historians concern themselves with an examination of the development of important mathematical ideas and concepts.

It is ironic that on the title page of Florian Cajori's A History of Mathematics there appears the following quotation:

> I am sure that no subject loses
> more than mathematics by any attempt
> to dissociate it from its history.
> J. W. L. Glaisher

Although Cajori and the other writers of his generation did not dissociate mathematics from its history, they certainly did dissociate the history from mathematics. This is certainly not true of the more recent historians. Howard Eves expresses what seems to be the modern viewpoint --- that the history of a subject cannot be fully appreicated without a certain degree of understanding of the subject itself.[54] Thus it is that he makes use of the problem studies at the ends of the chapters --- studies which increase historical as well as technical know-

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52 Bell, Development, p. 115.

53 Kline, Western Culture, p. xi.

54 Eves, p. 1.

ledge. In his treatment of Egyptian and Babylonian mathematics, he refers to tablets recently discovered to give examples of the types of problems worked and an indication of the extent of their mathematical knowledge. He devotes an entire chapter to the three famous problems of antiquity --- the trisection of the angle, the quadrature of the circle, and the duplication of the cube --- showing historical attempts to solve them both with and without the Platonic restrictions to ruler and compass. Incidental to this extensive treatment is the illustration of how many mathematical discoveries and even new branches of mathematics were discovered through attempts to obtain the solution to their problems.

Eric Temple Bell also seems to be trying to give a broad basis for the understanding of the basic principles and concepts of mathematics in his historical works. Although his Men of Mathematics is primarily biographical, Bell does attempt with each mathematicisn to give the reader some understanding of the actual mathematical work done. He does this in such a way as to insure that anyone with a high school mathematics course would be able ro comprehend the mathematics, although, of necessity since we are dealing with modern mathematics, material far beyond that level is considered. In Mathematics: Queen and Servant of Science, Bell finds it necessary to assure us that his book is not meant to be a substitute for a text or treatise on any subject of pure or applied mathematics, but it does include moderately detailed explanations of enough of the ideas of modern mathematics so that he hopes "that mathematical amateurs ...will sense enough of the spirit of modern mathematics to make them want to go on to more detailed accounts."[55] Actually, this work seems to be more a presentation and explanation of ideas of mathematics accompanied by the history of the development of the ideas rather than the opposite. The Development of Mathematics contains fewer examples and more history than Bell's earlier work. However, we are introduced to such modern developments as group theory, linear associative algebras, lattice theory, theory of quadratic forms, and vector and tensor analysis --- not by mention of name only, but by explanation of underlying theory.

By working with ratio and proportion, we can establish Lancelot Hogben's Mathematics for the Million as more of a mathematics book than a history book. Although he uses historical background material, we can see by taking the chapter on geometry as an example that mathematics is his principle interest. In this seventy-page chapter, he has devoted ten pages to principles and theorems, fifty pages to the working of various examples, and only the remaining pages to history. This relative emphasis is typical of most of the chapters in the book. He achieves a complete association of mathematics with its history as advocated by Glaisher.[56] His Mathematics in the Making is, however, more historically oriented than the first book. He makes a more consistent attempt at establishing a chronology of events. Even so, it contains extensive examples of mathematical work with sample problems and puzzles to be worked by the reader.

Morris Kline in all three of his books includes a great number

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55 Bell, Queen, p. vii.

56 Supra, p. 23.

of diagrams, examples, sample problems, derivations, and applications. Kline presents his books in protest against the method of teaching mathematics as a series of techniques, for he believes that mathematics taken apart from its intellectual setting in our culture and presented as techniques is nothing but a distortion.[57] Thus in all three books he has presented mathematical ideas in roughly chronological order. Although he includes numerous examples and in Mathematics: A Cultural Approach even provides problems to be worked by the students between the different sections of the chapters, his primary emphasis is on ideas and general methods, rather than on technique and rigorous proof which he considers to be of little value to the non-professional. His plea is, "Let us cease teaching scales to students who do not intend to play mathematical sonatas."[58] In each of his books he follows roughly the same plan --- de-emphasize technique, present ideas and examples in their historical context.

We have seen how the earlier histories of mathematics contained rude attempts at a sociological approach and hints of the beginnings of awareness of the cultural significance of mathematics. These are the first indications of any attempt to discover the place of the history of mathematics in the framework of general history. Among the recent writers there has been a tendency to take certain specialized approaches to the subject.

Eric Temple Bell was one of the first writers to attempt to re-

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57 Kline, Western Culture, p. x.

58 Kline, Approach, p. v.

late anything more than a purely mathematical account. In his earlier works he takes something of an aesthetic approach, finding mathematics an art. In Men of Mathematics he concerns himself with "the things which the great mathematicians have considered worthy of loving understanding for their intrinsic beauty."[59] Thus he sees mathematics as an art whose beauty inspires the artist (or mathematician) to create. At the same time he sees mathematical development as having the "ability to create human values."[60] He states the same thesis in Mathematics: Queen and Servant of Science where he finds mathematical creators to be inspired to creation by the art rather than the utility of mathematics. However, in this book, as the title would suggest, Bell is chiefly interested in establishing the reciprocal indebtedness to and influence on one another of mathematics and the physical sciences. It is in The Development of Mathematics, however, that Bell takes the widest view. In writing this book, he attempted to satisfy numerous requests for a history which would give a survey of decisive epochs, an account of general development, some technical hints, an investigation of why certain areas attract interest while others are ignored, a treatment of the social implications of mathematics, and a study of "what part civilization with its neuroses, its wars and its national jealousies, has played in mathematics."[61] In addition, he covers the role of mathe-

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59 Bell, Men, p. 4.

60 Ibid., p. 15.

61 Bell, Development, pp. vi-vii.

matics in man's attempt to give a rational description of nature, the practical influences on mathematical development, and the place which the aesthetic appeal of mathematics has played in its development. Since his main goal is to produce a history of the subject --- which history he sees in terms of general trends toward abstraction and generality --- and it is only his secondary aim to consider these various other aspects of the subject, he provides the most complete study which we have of the history of mathematics among our twentieth century historians.

Others of the recent writers have adopted specialized views of the subject. Chief among them is Dirk J. Struik, who deals especially with the influence which certain forms of social organization have had on the origin and growth of mathematical concepts and methods. To him the greatest handicap which the fact that his work is a concise history and thus limited in length gives him is the restriction placed on him with regard to making a full exploration of "the general cultural and sociological atmosphere in which the mathematics of a period matured --- or was stifled."[62] Despite his definite preference for a sociological approach, Struik also realizes that it can be carried too far, for he recognizes that "scientific process often follows the courses suggested by its internal logical structure, the frame built by results already obtained."[63] At the same time, he acknowledges the influence of logical, artistic, and personal factors on mathematics.

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62 Struik, Concise, p. xi.

63 Struik, "Sociology," p. 86.

Struik feels that "though the social roots of mathematics may have become obscured in modern times, they are fairly obvious during the early section of man's history."[64] Thus he gives a rather detailed delineation of the social factors involved in early mathematical development. It is interesting to look at Struik's analysis of the sociological elements involved in the great mathematical activity in Greece in comparison with D.E. Smith's analysis of the same topic. We can immediately perceive the difference in depth of approach. Struik, in contrast with Smith, sees Greek development taking place in a period of struggle and activity. His analysis follows, somewhat condensed:

> The activities of the "sea-raiders" ... were originally accompanied by great cultural losses.... When stable relations were again established... the stage was set for an entirely new type of civilization, the civilization of Greece.
>
> The towns which arose ... were trading towns in which the old-time feudal landlords had to fight a losing battle with an independent, politically conscious merchant class. During the Seventh and Sixth Centuries B.C. this merchant class won ascendancy and had to fight its own battles with the small traders and artisans, the demos. The result was the rise of the Greek polis, the self governing city-state, a new social experiment entirely different from the early city states of ... Oriental countries....
>
> This new social order created a new type of man. The merchant trader had never enjoyed so much independence, but he knew that this independence was a result of a constant and bittle struggle. The static outlook of the Orient could never be his. He lived in a period of geographical discoveries... he recognized no absolute monarch or power vested in a static deity ... The absence of any well established religion ... stimulated ... the growth of rationalism and the scientific outlook.[65]

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64 Struik, Concise, p. 9.

65 Ibid., pp. 39-41.

Morris Kline, on the other hand, considers the most important aspect of the history of mathematics to be its cultural relationships. He has written two books which present the thesis that "mathematics has been a major cultural force in Western civilization."[66] These two books, Mathematics in Western Culture, and Mathematics: A Cultural Approach, are dissimilar only in certain minor respects. Beyond the first two chapters of each, practically one to one correspondence can be set up between the chapters of the two books. Toward the end there is a difference in some of the topics of modern mathematics treated by each. The second book which is set up in text book form seems almost to be a reworking of the first in greater depth. Both articulate the same ideas about the cultural dependence and influence of mathematics. Kline states that the philosophy behind his approach is that "knowledge is not additive but an organic whole and that mathematics is an inseparable part of that whole," and thus he aims to demonstrate the "interrelations of the various branches of knowledge."[67] Kline believes that the vitality of the mathematics produced by any civilization is largely dependent upon its cultural life, while the absence of mathematical development may give an indication of the nature of the civilization lacking it. He uses Greece as an example of the former and Rome as an example of the latter.[68] In various chapters of the two works, Kline shows that

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[66] Kline, Western Culture, p. ix.

[67] Kline, Approach, p. vi.

[68] Kline, Western Culture, p. 11.

interrelations of mathematical progress with the directions of philosophical thoughts, with the destruction and construction of religious doctrines, with the content of economic and political thought, with styles in painting, music, architecture, and literature, with the resolution of questions about the nature of man and the universe, with developments in the fields of physics, biology, and chemistry, and with research in the social sciences. The Renaissance is an especially rich field for discussion in the type of approach Kline is using, and thus a large portion of each book is devoted to advances made or fostered during the Renaissance, especially to those proceeding from the work of Galileo in developing a new scientific methodology. Illustrative of Kline's technique is his discussion of the relevance of the evolution of perspective in painting to the mathematics of the Renaissance. He calls the Renaissance painter "the most accomplished and also the most original mathematician" of the fifteenth century.[69]

Lancelot Hogben's special approach is through the metaphor of mathematics as a means of communication. He sees mathematics as undergoing the same influences as a language in <u>its</u> changes and development. Thus he states his intention of narrarating "how the grammar of measurement and counting has evolved under the pressure of man's changing social achievement."[70] He finds that "mathematical brainwork...depends on our biological and cultural inherit-

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[69] Kline, <u>Approach</u>, p. 203.

[70] Hogben, <u>Million</u>, p. 26.

ance, our social and physical environment."[71] and is shaped by the "material needs and the intellectual climate of earlier times."[72] The metaphor which he employs becomes an excuse for exploring influences from a variety of sources.

In one additional and significant respect, the earlier historians differ from the more recent ones. This distinction lies in their way of defining mathematics and the nature of mathematical achievement. D.E. Smith states the case for the early writers. He considers that by accepting the beginning of the consistent application of deductive reasoning to the proving of mathematical concepts as the beginning of the science of mathematics, the historian accepts a limitation which would prohibit his going back beyond Thales in time to discuss the earliest steps in the development of mathematics (as he defines it). He announces his intention of going back to (what he considers) the very genesis of mathematics before the period when mathematics can be said to exist according to the restrictive definition.[73] This we may be willing to accept until we discover that his idea is that the genesis of mathematics is coincident with the genesis of the solar system. So it is that his definition of mathematics leads him to his idea of "mathematical truths which have no beginning in time and which shall have no end..../Thus to him the history of mathematics is/

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71 Ibid., p. 36.

72 Hogben, Making, p. 8.

73 Smith, p. 2.

a record of the discovery of existing laws in this science and the invention of better symbols as needed from time to time for their expression."[74] Setting himself up with such definitions, Smith gives himself great freedom in the subjects which he will choose to call mathematics in his discussion. Rouse Ball and Florian Cajori share similar ideas about the nature of mathematical truth. Cajori, in particular, says that the mathematician may be proud that his science is an exact science,[75] thus implying that mathematics is capable of giving true descriptions.

The more recent writers, from E.T. Bell on, are unanimous in their belief that mathematics does not exist without deductive reasoning and that mathematics as a science therefore does not exist before Thales. They are also in agreement on the fact that mathematics is an "arbitrary creation of the mathematician."[76] Morris Kline in all three of his works continually reiterates the statement that "mathematics...is a human creation in every respect."[77] Yet he sees in this statement a paradox. For although mathematics contains no truths, it does give us great power over nature. The extent of the paradox is increased when we realize that truth makes no difference. Kline tells us that as far as theories of the structure of the universe are concerned, it makes no difference whether Ptolemy with his geocentric

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[74] Ibid., p. 3.

[75] Cajori, History of Mathematics, p. 1.

[76] Bell, Development, p. 330.

[77] Kline, Approach, p. 665.

theory and its epicycles and deferents or Kepler with his heliocentric theory is right. Either is capable of giving accurate mathematical results. We use the heliocentric theory simply because it has the advantage of simplicity.[78]

Even those adopting the conception of mathematics as an invention with no claim to absolute truth differ from one another in their ideas of what mathematical invention is. Bell prefers to think of mathematics as a game in which the rules "may be any that we please, provided that they do not lead to flat contradictions."[79] and as a game in which no one stops to consider whether the rules are true, but only whether they are adhered to. Hogben disagrees with this point of view quite decidedly, saying that the terming of mathematics as a game only gives a personal attitude and does not tell anything about what meaning a mathematical statement will have for the people as a whole. In his mind, mathematics is the language of size, and its rules are the rules of grammar, not of a game. They are not eternal rules either, but "conveniences without whose aid truths about the sorts of things in the world can be communicated from one person to another."[80]

Despite the fact that the two groups of mathematical historians differ from one another in so many ways, there is one feature which histories from both groups share. They have certain weaknesses, characteristic of their own group to be sure, which lessen their value

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[78] Kline, Physical World, p. 127.

[79] Bell, Queen, p. 22.

[80] Hogben, Million, pp. 27-8.

as general histories of mathematics.

Rouse Ball announces that because his work is only a short account, he has had to limit himself to leading events and to bypass many figures who have had little or no influence.[81] Yet in his book he wastes space by dealing with mathematicians of that very description. For example, he talks of Anaximenes, Mamercus, and Mandryatus whose importance lies in the fact that they were pupils of Thales; of Plato's pupils Leodamus, Neocleides, and Amyclas and their pupils Leon, Theudius, Cyzicenus, Thasus, Hermotinus, and Philippus as if the identity of one's teacher or teacher's teacher were to make him a great mathematician; and Conon and Dostheus, successors of Euclid at Alexandria, and Zeuxippus and Nicotelus, also lecturers there, whose chief claim to mathematical fame is the fact that Archimedes thought highly of them. Ball is guilty of this fault not only in dealing with early ages when a scarcity of outstanding mathematicians might excuse it, but he also finds it necessary to devote fourteen pages to mathematicians of slight note in the period from 1637 to 1675, one of the most prolific in the history of mathematics. Again, he devotes ten pages to eighteenth century mathematicians who, he says, "barely escape mediocrity."[82] Ball's lack of balance is not his only fault. Men who have made studies of the relative merits of various histories of mathematics have found him to be unreliable, misleading, and obsolescent.[83]

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81 Ball, p. vi.

82 Ibid., p. 369

83 Miller, p. 282 and Sarton, p. 45.

Florian Cajori shares in many of the weaknesses of Ball, weaknesses which seem characteristic of the early histories. He too places false emphasis on mathematicians whom he finds of little merit. In his History of Mathematics he names many of the same mathematicians whom Ball has named and for no better reason. He also gives Neocleides, Leodamus, Amyclas of Heraclea, Cyzicenus of Athens, and Philippus without mentioning any of their accomplishments, as well as Conon, Dositheus, and Zeuxippus as successors of Euclid at Alexandria. The similarities of treatment found in many sections of Cajori's histories and of Ball's A Short Account of the History of Mathematics, coupled with the obvious and expressed admiration of both men for the massive history of Moritz Cantor, leads one to suspect that perhaps both borrowed rather heavily from that historian. Indeed, they often give him credit for many of their statements. It is from Cantor that they have obtained the basis for the assumption, which both make in their works, that the Egyptians were familiar with the so-called Pythagorean theorem, at least for the case of the 3:4:5 ratio, as early as 2000 B.C. when they supposedly used it in constructing right angle corners in their pyramids. G.A. Miller in his discussion of mathematical myths, tells us that the researches of Otto Neugebauer have revealed this to be a legend which proceeds from the early historians' misinterpretation of Cantor.[84] Since there has already been written a small volume devoted solely to the correcting of the thousands of errors in Cantor's history,[85]

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[84] G.A. Miller, "Mathematical Myths," National Mathematics Magazine, X11 (May, 1938), p. 390.

[85] Ibid., p. 388.

we can see that it might be dangerous for a historian to base his statements on that work. Many of the errors found in Ball and Cajori proceed from this source.

Again in D.E. Smith, we find repeated the same errors and weaknesses present in Ball and Cajori. He also incorporates faults of his own. His plan in writing his two volume history was to give first a general chronological survey and then to break the history of mathematics down into a discussion of the evolution of certain topics. He hoped by taking this approach to break away from the tendency of mathematical histories to be nothing more than a chronological recital of facts.[86] His approach, however, involves certain defects. In both volumes there is the loss of all sense of coherent chronology and of the relation of different developments to one another and to periods of time. This is the result of the breakdown by country in Volume 1 and the division according to the branch of mathematics in Volume 11. As far as Volume 1 is concerned, he fears that the reader may not find the mere statement of the fields in which a particular mathematician was interested very illuminating. Yet at the same time, if he corrected this situation, he would be defeating the purpose of Volume 11 by rendering it nothing more than a rehashing of Volume 1. On the other hand, in Volume 11 Smith feels that he must include the names of men whose only contributions were the writing of texts which established symbols and terms which now are widely used.[88]

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86 Smith, p. iii.

87 Ibid., p. iv.

88 David Eugene Smith, History of Mathematics, Volume 11 (Boston: Ginn and Company, 1925), p. v.

Perhaps we would be able to bear with these difficulties were it not for the fact that we are also asked to accept as mathematicians such men as Marcus Terentius Varro ("his one extant work certainly has no great merit" - Vol. 1, p 121), Balbus (" his contributions were unimportant" - Vol. 1, p. 125), Quintus Serenus (his "works...merely show the debased state of learning..." - Vol. 1, p. 132), Cassiodorus ("nothing could better show the debased state of learning than [his] feeble attempt at scholarship" - Vol. 1, p. 180), Francois de Foix (he "contributed nothing to the general theory of geometry" - Vol. 1, p. 309), and Kircher (his "mathematical works...are not to be taken too seriously" - Vol. 1, p. 422). When Smith finally reaches the eighteenth century, he states his intent "to limit the study chiefly to a consideration of those mathematicians whose achievements were so noteworthy that everyone who is interested in mathematics should be informed concerning them."[89] Yet he discusses Nicholas Saunderson, not because of his achievements or contributions which were not great, but because he "labored under difficulties"[90] and Charles Hutton, noted for his "perserverence rather than...his scientific ability."[91]

As we have noted before, Smith allows himself great freedom in the topics which he discusses and finds justification for doing so in the definition of mathematics which he has adopted. Thus he calls the recognition which certain animals seem to have of relative quantities

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89 Smith, Vol. 1, pp. 445-6.

90 Ibid., p. 454.

91 Ibid., p. 458.

of objects and sizes of groups "the first uncertain steps in the development of an arithmetic."[92] At the end of each chapter, Smith has affixed a list of topics for discussion to highlight the major points covered within the chapter. A partial list from the first chapter gives some indication of the wide range of subjects which Smith considers to come under the heading of mathematics:

> 1. Geometric forms that were in existence before the advent of life on the planet.
> ...
> 3. Geometric forms that appear prominently in the vegetable world and in the bodily structure of certain animals.
> 4. Geometric forms that appear prominently in the products of the labor of the lower animals....
> 5. The question of animal counting or pseudo-counting as discussed by psychologists.
> ...
> 14. Various stages of the geometric ornamentation in Cyprus, Crete, and the mainland of Greece.[93]

None of these topics really has anything to do with the history of mathematics.

Although all of the earlier historians used the anecdote, none did so to such an extent as Smith. That he has carried the practice to an extreme is shown by the fact that George Sarton makes special comment on it in a review of the book. He finds "the inclusion of fanciful portraits,..a serious mistake for which [he] can find no justification."[94] Sarton has commented on the works of all of the earlier historians, but with none of the others is he so harsh.

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92 *Ibid.*, p. 5.

93 *Ibid.*, p. 19.

94 G.A. Miller, "Correcting Errors in the History of Mathematics," *School Science and Mathematics*, XXXV (December, 1935), p. 980.

Because of their many inaccuracies, their discussion of subjects not truly pertinent to the history of mathematics, and their recognition of mathematicians on the basis of tradition rather than on sound judgement of contributions, we find that the early historians do not provide us with a history of mathematics adequate to suit the needs of the person with a serious mathematical interest. We have seen how the trend moved from the history of mathematics for the mathematician only to the history of mathematics for everyone. The consequence has been a relative dearth of histories which hold interest for the mathematically literate also. Works by the more recent historians fail in this respect just as surely as those by the earlier writers. The modern historian's reasons for failure to be adequate, however, differ vastly from those of Ball, Cajori, and Smith. Their failure cannot be attributed to such obvious faults as we have shown to be the case with the latter writers. They fail because they are not complete. And they are not complete because they do not try to be. The recent writers are not attempting to write comprehensive histories of mathematics.

For example, James R. Newman's World of Mathematics does not profess to be a history of mathematics although it does cover many of the possible facets of mathematics and does contain a section devoted specifically to history. Newman confesses prejudice in making his selection of topics to include, slighting those areas which he does not care for --- number theory, chemistry, algebra, and economics --- and giving extensive coverage to fields he especially enjoys --- arithmetic, physics, geometry, probability, mathematics of infinity, and logic.[95] That he does so is evident from a survey of the table of contents.

[95] Newman, p. viii.

Newman is incomplete for the very reason that he is prejudiced. His feelings cause him to slight important branches of mathematics. He covers well what he does cover, but does not fulfill the need for a comprehensive history of mathematics.

Morris Kline is also incomplete. But he, too, is not aiming in any of his works to give a history of mathematics. He rather seems to be taking an historical approach to an appreciation of mathematics. The two cultural approaches do take on a roughly chronological order, but this order is not always perceivable. Since the aim is to give a cultural view point, there are, of course, many chapters which do not deal directly with mathematics but rather with such subjects as "Religion in the Age of Reason" and "Reason in Literature and Aesthetics." Each book being mainly concerned with ideas, there is little factual material involved. He does not treat all of the important developments of mathematics, but only those which are readily understandable to the majority of readers and those which serve to illustrate his thesis. In Mathematics in the Physical World, Kline takes a slightly different point of view in showing the relevance of mathematics to the study of nature. Less attention is given to chronology in this book than in the other two. In all of Kline's books, one finds that many of the chapters are logically independent of one another and may be considered out of order. It is no criticism to say that these works do not give a comprehensive view of mathematical history, for they are not intended to do so. But they do not give us the required, adequate history of mathematics we are looking for.

Other examples of works which fail to fit the requirements we have set because they do not cover the history of mathematics completely are E.T. Bell's Men of Mathematics and his Mathematics: Queen and Servant of Science. Of course, Men of Mathematics by its own biographical nature denies that it is a history of mathematics. However, the book contains other disqualifications. A review referring to this book finds that it has numerous errors and declares that the "popular recent writings of E.T. Bell tend to emphasize various mathematical myths instead of to reduce their number."[96] Bell, thus, in his earliest work shows evidence of carrying over some of the faults of the earlier historians. In Mathematics: Queen and Servant of Science, he confesses that he has not covered all of the famous mathematical developments since the beginning of mathematics, but then he denies that he is writing a history of mathematics.[97] His main concern is with modern mathematics, and he rarely considers the history of a development prior to 1637. Actually he is so concerned with mathematics that the presentation of mathematical ideas and principles often takes precedence over the history. His purpose as demonstrated is to give an understanding of the basic ideas of modern mathematics, not of their history.

As we have already seen, the two books of Lancelot Hogben also emphasize mathematics over history. He believes in presenting the mathematical idea in its historical context. This is more evident in Mathematics for the Million which is avowedly a mathematics text.

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96 Mathematics Gazette, XX1 (1937), pp. 311-312 quoted in Miller, "Myths," p. 388.

97 Bell, Queen, p. viii.

Mathematics in the Making is more historically inclined, but Hogben declares that it is "not an authoritative guide to the history of mathematics."[98] Actually the percentage of text found in the book as compared to the percentage of pictures, diagrams, problems, and examples is so small as to prohibit its being a complete guide in any case.

Howard Eves' An Introduction to the History of Mathematics presents a strange mixture of the faults of the older writers and the virtues of the recent ones. As with the early historians, he is prone to place the man above the mathematics at times and to include discussions of those figures of minor importance always found in the earlier histories, such as Bede, Alcuin, and Gerbert, who were merely translators. Yet Eves was also influenced by the recent scholarship of such men as Otto Neugebauer as shown by his expressed admiration for the man's work in the mathematics of antiquity and his use of the results of that work in the book. He also adheres to the inventive theory of mathematics. On the other hand, he can at times be just as tedious as Smith. He relates a chronology of every bit as detailed as that of Smith. Eves is concerned with presenting the development of elementary mathematics, and thus gives very little coverage of the nineteenth and twentieth centuries.

There are two books which may be said to come closest to fulfilling our requirements for an adequate history of mathematics that will be of benefit to the mathematically literate portions of our population.

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98 Hogben, Making, p. 4.

Dirk. J. Struik's A Concise History of Mathematics is one of these. In his book, Struik covers the main developments and figures of mathematics through the nineteenth century. Because he is limited as to space, he cannot present all the inventions or dwell upon all the influences of mathematical history. This limitation is especially noticeable in the nineteenth century treatment, where he is unable to give us as complete an idea of the concepts of modern mathematics as might be desirable. Restriction as to space has had one positive affect --- the elimination of all of those unimportant mathematicians found in earlier books. Because of its brevity, we are inclined to classify Struik's book as an outline history.

However, The Development of Mathematics by E.T. Bell is the best example of a real history of mathematics to be found among those histories which we have considered. Bell denies that this book is a history of mathematics, just as he has done with his other books. In this case, we must disagree with him. Although Bell is definitely in the modern tradition and does not follow the example of Moritz Cantor, he is dazzled by Cantor's accomplishment. He cites the fact that it took Cantor thirty-six hundred pages to write an outline history of mathematics to 1799 and expresses the belief that to do the same thing with the history of mathematics after 1800 would take seventeen thousand pages.[99] Thus he regards the writing of a complete history of mathematics as a tremendous task, complex and impossible to perform. However, we must be realistic and admit that, although

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[99] Bell, Men, p. 7.

such a work as Bell envisions would be an invaluable reference work, it would not be the sort of book one would read to learn about the history of mathematics. Bell's The Development of Mathematics is such a work. It is sufficiently detailed to include all important fields and developments, yet it is not just a chronological recitation of factual material. Bell has had the purpose of offering a "broad account of the general development of mathematics, with particular reference to the main concepts and methods that have... survived."[100]

Bell does more than avoid the faults of the earlier historians; he also corrects them. He announces his intention to refrain from incorporating the "wilderness of trivialities that might be mistaken for mathematics or its history."[101] Thus he makes a point of correcting errors or myths which appear in the earlier histories. He shuns the use of anecdotes as not being proper material for a historical work. And most important of all he does not crowd his pages with the hosts of unimportant and insignificant mathematical figures who take up space in the histories of Ball, Cajori, and Smith. He announces his intention to depart from the traditional point of view which always had included the names of Gerbert, Bede, Alcuin, Psellus, Adelard of Bath, and Robert of Chester in a treatment of mediaeval mathematics. Bell says that these names could be dropped forever from the

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100 Bell, Development, pp. v-vi.

101 Ibid., p. v.

the history of mathematics with no loss.[102] He thus makes a distinction between the real history and the traditional or official history of the middle ages.

In writing his history, Bell has conceived the idea of wedding the talents of the mathematician and the historian in the production of a work that aspires to be a competent history of mathematics. He, as an historian, has consulted with professionals who know by personal experience what mathematical inventions is and which inventions are to be considered vital to include.[103] We can readily see how advice from mathematicians would be especially valuable in the task of evaluation, particularly in the evaluation of twentieth century developments. Perhaps this sort of approach is just what is needed if we are to continue to produce adequate histories of mathematics. Modern mathematics has become too complicated for the historian to be able to understand and evaluate all of it. No one man could do so. On the other hand, the mathematician is not liable to be familiar enough with the past ages of history, not only of mathematics but of civilization, to treat these periods adequately.

Bell's history is now almost twenty years old, and the mathematics produced during that time is sufficient in quantity and importance to indicate a necessity for a revaluation of twentieth century mathematics at least, if not that of the previous ages. Mathematics is not static and neither is its history. Each new development may alter the shade of all those that have gone before. Such a growing and

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102 Ibid., p. 88.

103 Ibid., pp. vii-viii.

changing field of study requires a continuous flow of new literature to assure its vitality. However, Bell's book, revolutionary in its time, continues to stand alone as the only representative of a more sophisticated historical literature. Meanwhile, the pressure to return the history of mathematics to the mathematician, at least to some degree, grows more insistent.

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