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AN INVESTIGATION OF THE REAL-PROBLEM-SOLVING CURRICULUM IN THE COLLEGE GENERAL EDUCATION MATHEMATICS COURSE

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AN INVESTIGATION OF THE REAL-PROBLEM-SOLVING CURRICULUM IN THE COLLEGE GENERAL EDUCATION MATHEMATICS COURSE

by

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Most colleges offer a general education mathematics course for students whose background and interest are minimal and whose expected use of mathematics is limited to fairly fundamental topics. A review of the literature and the experiences of instructors revealed that the teaching of this course is generally mechanistic and skill-oriented. The students gain little ability to solve real-life problems. The real-problem-solving approach to the acquisition of mathematical and problem-solving skills is seen as a potential basis for a more effective general education course. This approach is exemplified by the Unified Sciences and Mathematics for Elementary Schools (USMES) program which provides a curriculum, teaching materials, and a guiding philosophy featuring real-problem solving and inquiry.

The real-problem-solving curriculum was tried out in one of two sections of a general education college mathematics course while the traditional approach was used in the other section. This investigation, in the form of a case study, examined the experiences, insights, and frustrations of the instructor in planning and carrying out this approach. Teacher-made and standard tests and questionnaires were used to measure comparative gains in mathematical knowledge, changes in attitudes toward mathematics and classroom procedures, and changes in problem-solving ability.

The instructor and the students had difficulty in adjusting to the emphasis on student-directed learning and real-problem solving; however, all groups eventually completed investigations of real problems. The experimental class showed a greater improvement in attitudes towards mathematics and scores on tests of mathematical skills than the control section, but not significantly. Students in the experimental class showed better problem-solving abilities at the end of the semester. Approximately as many students expressed a preference for the activity-oriented approach as expressed a preference for traditional teaching. Unexpected outcomes included difficulties which students had in working together in small groups and difficulties which the instructor had in presenting the problem-solving challenge.

The major conclusion of the study was that the real-problem-solving approach, with modifications depending on the instructor, goals of the course, and the students, can be considered as one of several viable alternatives to the traditionally taught, general education mathematics course.

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CHAPTER I

REAL-PROBLEM SOLVING AND THE COLLEGE GENERAL EDUCATION MATHEMATICS COURSE

The Problem

Most two- and four-year colleges offer a semester or a two-semester course in arithmetic, elementary algebra, and a variety of other topics for students whose background and interest in school mathematics are minimal and whose expected use of mathematics is limited to fairly fundamental topics. This course is usually a "liberal arts" mathematics course offered in partial fulfillment of the general education requirement and is the only mathematics course taken by students who do not expect to need any additional mathematics. The students come to the course with a great range of abilities and goals, but all too often they have not mastered arithmetic operations, have only a vague idea of the use of formulas and algebraic operations, and either have never studied geometry or have barely grasped the deductive reasoning of such a course. Ultimately, they carry with them years of frustration in not being able to solve mathematical problems. Many will claim a dislike for mathematics and will admit to having a bad case of what is commonly called mathematics anxiety. Finally, many of these students do not expect to use any mathematics in their potential careers or major fields and would willingly settle for skills in consumer mathematics.

The faculty, in choosing topics to be taught in the liberal arts mathematics course, frequently determines very specific skills which are to be required of all students: arithmetic, elementary algebra and graphing, some intuitive geometry, perhaps some probability and statistics together with some business mathematics. Some textbooks written for this group contain a rehash of poorly learned arithmetic of junior high school, while others have offered essentially the same list of topics: sets and logic, numeration systems, modular arithmetic, the structure of the real number system (approached axiomatically), geometric concepts, measurement, some probability and statistics, and perhaps a little mathematics for consumers. These topics may fascinate mathematicians who enjoy the abstraction and structure inherent in a study of modular arithmetic or truth tables, but the student who is struggling with the addition of fractions or the physical representation of a negative number will be lost and frequently bored in a class where such is the "bill of fare." To these students the topics have little or no significance. These students see little applicability for the topics presented, frequently can't answer questions correctly, and feel frustrated by the whole experience.

Some textbooks have appeared in the past ten years which have shown some movement toward a choice of topics more relevant to "real-life applications," with a greater emphasis on everyday experiences. Nevertheless, the topics to be studied in the liberal arts mathematics courses are still selected by the instructor, usually closely following a textbook, and all students in the class are expected to learn the same material.

The method of teaching leaves as much to be desired as does the choice of topics. Most of the teaching is based on lecturing and assigning homework problems. Frequently the instructor goes over a few examples which the students are to copy down and use as a basis for doing exercises. There is little classroom involvement of the students. A few manage to do all the homework; most either do not or cannot. Then the students are given a test on these problems as the only (or at least the "best") way of finding out how much they know. Those students who can "play the game" of memorizing for tests earn the high grades but they still may not be able to use the algorithms and skills learned to solve any important problems. Thus, all too often the liberal arts mathematics course is characterized by what Alfred North Whitehead called "inert ideas":

> In training a child to activity of thought, above all things we must be aware of what I will call "inert ideas"--that is to say, ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combinations.
>
> In the history of education, the most striking phenomenon is that schools of learning, which at one epoch are alive with a ferment of genius, in a succeeding generation exhibit merely pedantry and routine. Education with inert ideas is not only useless; it is, above all things, harmful. (Whitehead, 1956, p. 13)

As I look back upon my 20 years of teaching, mostly in college freshman and sophomore courses (first as a teaching assistant while in graduate school), I can see that for years I taught in the same way that I had been taught and had learned while I was in graduate school. In my classroom I presented a new topic with a reasonably sound mathematical explanation of why a theorem or formula was correct or why a technique to solve a problem worked, without worrying about why the topic or the mathematical fact had any importance to the student. It has taken me most of those 20 years to realize that teaching the quadratic formula or the Egyptian numeration system, important as these ideas may be in the proper context, made no sense at all to my students, and were nothing more than "inert ideas" as I was presenting them. I had to conclude finally that the hodgepodge of topics taught in a standard liberal arts mathematics course had little meaning, no coherence, and certainly no lasting value for most of the students. I could not be content with my role in this unacceptable process, I could not continue to inflict this kind of experience on my students, and I needed to begin a search for some sort of solution. Inasmuch as I held mathematics to be important not only as a discipline but also as a means for making reasonable decisions in everyday life, I sought a way to provide an opportunity for students to be able to use mathematics in as wide a realm as possible--in the social and natural sciences, in consumer and career-related areas, as well as in any other life problems which could be subjected to mathematical analysis.

General Goals of Education and Mathematics

The question of how to change the teaching of mathematics cannot be answered properly without a consideration of some generally acceptable broad educational goals. It is fairly easy to justify the teaching of certain topics in algebra and trigonometry for students who have committed themselves to the study of engineering or one of the sciences, but it is not nearly so clear what topics and abilities are important for students pursuing careers in fields such as communications, the arts, or the social sciences. Students in these disciplines often do not recognize the need for specific skills to solve pertinent problems in subsequent courses, and hence they may lack motivation for learning. What then are the more general educational goals for a liberal arts or a technical education which may be at least partly achieved in a mathematics course? What combinations of mathematical topics and instructional strategies should be emphasized? In short, of what importance is instruction in mathematics for the college student who must satisfy a general education requirement?

A statement entitled "The Essentials of Education," sponsored by 12 professional educational organizations including the National Council of Teachers of Mathematics and the Association for Supervision and Curriculum Development, focused on the following:

> Educators agree that the overarching goal of education is to develop informed, thinking citizens capable of participating in both domestic and world affairs. The development of such citizens depends not only upon education for citizenship, but also upon other essentials of education shared by all subjects. . . .
>
> More specifically, the essentials of education include the ability to use language, to think, and to communicate effectively; to use mathematical knowledge and methods to solve problems; to reason logically; to use abstractions and symbols with power and ease; to apply and to understand scientific knowledge and methods; to make use of technology and to understand its limitations; . . . (NCTM et al., 1978)

Education should prepare persons to cope with the changes, conflicts, and complexities of the future, when the greatest challenges will be people- and process-oriented. This is a theme suggested by Geneva Gay as part of her call for revitalization in the 1980's. She listed the following goals:

> . . . self-understanding, the capability to cope with change, critical thinking and decision making abilities, interpersonal relations and communications, the capacity for self-education, and tolerance for ambiguity. (1981, p. 80)

Harry Broudy stated that general education must develop reasoning and thinking skills which go far beyond the learning of specific facts. What use do we make of our education, outside of our own field of specialization? He argued that while we can recall relatively little of the specific facts of literature, science, and social sciences, ". . . most of what is studied formally functions tacitly rather than explicitly in post-school life" (1982, p. 578). He continued:

> . . . for even though the details are no longer recallable, they furnish a repertoire of images and concepts with which we think, imagine, and feel. They give richness to our response by a wealth of associative resources. . . . (p. 578)

Broudy suggested that a possible general education curriculum for K-12 (and presumably, this would also hold true for general education courses in liberal arts colleges) would include symbolics for information (linguistic, mathematical, imagic), basic concepts of mathematics and sciences, developmental strands (of the cosmos, institutions, and culture), problem-solving, and exemplars in the arts and sciences (p. 577). All five of these strands are important; indeed it is "only when subject matter is repeatedly encountered in a wide variety of materials that it becomes part of the very structure of mind" (p. 577).

Mathematics, then, plays an important role in such a proposal, in terms of development of problem solving, learning of basic concepts of mathematics, and experiences with symbolism. George Polya, a pioneer in the heuristics of problem-solving in the mathematical arena, remarked on the singular role that mathematics education plays:

> This is the great opportunity of mathematics: mathematics is the <u>only</u> high school subject in which the teacher can propose and the students can solve problems on a scientific level. This is so because mathematics is so much simpler than the other sciences. Because of this simplicity, the individual, just as the human race, can arrive so much earlier to a clear view in mathematics than in the other sciences. (cited in Kralik & Reys, 1980, p. 2)

The setting out of possible broad educational goals does not include any specific computational skills which students must have. Such a focus, all too common in today's schools, emphasizes functional skills. This has the effect of preparing students for narrowly defined, vocational roles and the passing of standardized tests, rather than educating students to meet the goals listed above. This does not mean that computation skills are to be ignored; it simply means that they should be put in proper perspective. Many authors (Broudy, 1982; Gay, 1981; Simon, 1980) point out that mastery of basic skills and vocational competency are not sufficient to prepare students for adequate participation in society or for maximum personal fulfillment. The question then is how curriculum and instructional strategies in mathematics can be formed to help students achieve the broader goals?

The literature contains many articles which call for major teaching changes in order to accomplish these far-ranging goals. Geneva Gay suggested that:

> Far more learning situations should be created wherein students are simultaneously exposed to multiple, conflicting stimuli wherein several, equally appropriate and attractive options are available; wherein problems without apparent solutions are presented . . . and wherein no directions are provided except those that are offered by the students themselves. (1981, p. 83)

Several programs in mathematics--particularly the Minnesota School Mathematics and Science Teaching Project (MINNEMAST), 1961-1970; the Madison Project, 1961-1969; and the Unified Science and Mathematics for Elementary Schools

(USMES), 1968-1979--have emphasized a discovery-activity approach. These programs were guided by a concern for what students are interested in, an attempt to minimize rote-learning, and an emphasis on the solutions of problems rather than the learning of specific facts and skills.

It might be hoped that these more open approaches to the teaching of mathematics would have paved the way for a general change in the curriculum, but this has not happened. The evaluations of the programs have indicated mixed success (NACOME, 1975, pp. 30-31) and the prevailing mode of teaching still emphasizes the traditional approach.

Goals in Mathematics Education

Most statements about the goals of mathematics education in recent years have included as one of the most important that of problem solving. But this has not always been the case and frequently the fundamental aims include a wide variety of emphases. The 1963 Cambridge Conference on School Mathematics called for the acquisition of subject matter in grades K to 12 that would be equivalent to two years of college and a semester each of modern algebra and probability theory. Subject matter was emphasized. The 1967 Cambridge Conference on the Goals for the Correlation of Science and Mathematics sought to balance the curriculum changes which were moving at that time toward the so-called "new" mathematics of the 1960's. The 1967 conference report stated that the main goals of the educational system are not only to "bring every child to a level of literacy and competence which will enable him to become a productive member of our society" (Cambridge Conference, 1969, p. 4), but also to help the student to be able to think through specific problems as well as to master techniques. In order to be able to deal with the political and technological problems of the world in which they live, students will need to be able to find solutions through "deep scientific thinking." The report made this very forceful statement:

> Each child must be convinced that his thinking is worthwhile. This is basically a matter of respect. The child must learn to respect his own thinking. To do so, the child must see that his own thinking can improve his ability to cope with the world. His teachers must respect his thinking, and they must have curricular materials which will call forth in him a response honestly worthy of respect. (p. 6)

Herbert Fremont, in his <u>Teaching Secondary Mathematics Through Applications</u>, held that mathematics teaching should develop student awareness in five different directions:

1. mathematics helps us to understand our environment
2. mathematics is the language of science
3. mathematics and society are interdependent
4. mathematics is an abstract system of ideas, and
5. mathematics is the study of patterns. (1979, pp. 3-6)

Fremont argued that mathematics is not two-dimensional--subject matter and problem solving--but rather involves also symbolic language, applications, and an abstract system of definitions, theorems, and proofs. Mathematics is multifaceted and the teacher should attempt to help the student to understand all of its aspects. As Fremont pointed out:

> Emphasizing the interaction of mathematics with the culture, as well as its role as a tool for, and the language of, science does not mean that we shall mask its true identity and nature. Whatever else it may be, mathematics is still an abstract system of ideas and must be seen as such by our students. Thus, as students use mathematics to solve problems, we shall have to be prepared to indicate clearly how the mathematics "thing" and the physical "thing" are not one and the same. (p. 4)

The task of teaching mathematics is thus made more difficult and complicated if one is to try to accomplish all these goals as well as to help students prepare for competency tests in the basic skills and to develop problem-solving abilities.

The National Council of Teachers Agenda for Action expanded on the goals of problem-solving and basic skills, and in so doing further complicated the job of the teacher. This report recommended that mathematics programs should give students experience in formulating key questions, defining problems and goals, discovering patterns and similarities, searching data, trying out possible strategies, transferring skills, and being able to apply previously learned knowledge to mathematics and other disciplines. "Mathematics teachers should create classroom environments in which problem solving can flourish" (1980, p. 4). In addition, the report reflected the almost universally accepted "agreement among parents, educators, and mathematicians on the need for teaching basic skills with greater effectiveness" (p. 5). Such skills as estimation and approximation, constructing and interpreting charts and graphs, appropriate computational skills, measurement, and computer literacy are also to be included in an instructional program.

Among the many and varied goals, teachers must choose their own workable set of goals, which may differ depending on the particular course and group of students. Because of the difficulty of accomplishing all of the goals set out by educators, it is necessary to make some choices, which will be based on personal values and the goals of the institution in which the teaching takes place.

Guiding Assumptions and Values

Every curriculum should be developed in the context of a set of values and a set of convictions about how people learn. These assumptions form a framework which can clarify the goals of the curriculum planner. They serve as a basis for deciding which of competing strategies in teaching should be used and help to explain why some elements of curriculum are included and others are excluded.

Man is seen as a dynamic being acting holistically in relationship to the environment. Persons are not perceived as being separate from the world, as objects defined in terms of membership in classifications or according to personality attributes, but rather they are seen as acting within the environment and with other persons. Macdonald, Wolfson, and Zaret (1973) expressed these ideas as follows:

> Man experiences holistically; his physiological, intellectual, social, and emotional development occurs and is experienced totally rather than discretely. . . .

> "Learning" emerges in the flow and continuity of man's total experiencing and growing; growth is not a static process, nor can there be static outcomes of "learning." (pp. 8, 9)

Learning does not take place in discrete, disconnected experiences but rather must involve inner and outer experiences which are interconnected and must emphasize equally experiences from the past, present, and future.

The person is more important than the subject matter being taught. Indeed, to assume that all persons must learn some fixed set of skills or subject matter is to assume a mechanistic point of view. In this view, schools are seen as factories in which the raw products (students) are to be shaped. This violates the holistic outlook and fragments the educational process. Macdonald et al. wrote:

> Traditional emphasis on the so-called "basic skills" for making it in our society (reading, writing, arithmetic) implies that these skills can be separated out both from the larger cultural context in which they are to become operative and from the personal-holistic context of human development . . . the prevailing approach has not been meeting its aims. . . . Yet the fragmented focus on isolated "learning of skills" is continued in new waves of intensive "teaching" that can only be terrorizing to . . . children and self defeating in the long run. (pp. 20, 21)

The role of education should first be one of developing a student's self-confidence, self-learning ability, problem-solving and decision-making skills and self-understanding. Schooling should be liberating instead of controlling and allow for a wide variety of experiences, appropriate to each individual. "The desirable environment is one which promotes growth and fosters inner freedom" (Harman, 1972, p. 23).

A sense of how people learn comes from John Dewey who argued for the creation of challenging environments within which students would interact in such a way as to produce autonomous individuals prepared to live creatively in a participative democratic state. Education is seen in terms of the cognitive-development theory of psychology, which holds that the child develops through a "reorganization of psychological structures resulting from organism-environment interactions" (Kohlberg & Mayer, 1972, p. 457) through which the child moves from stage to stage in development. It emphasizes liberty, development, and democracy. For such learning the teacher needs to be supportive and try to stimulate interest while being non-authoritarian.

From these basic assumptions follow some specific statements concerning teaching:

1. One of the main concerns in teaching should be for what the students are interested in, what they wish to learn or know, and what sort of problems they wish to solve.

2. Successful experiences will help develop self-confidence and excitement about the process of learning.

3. Activities in the classroom should help the personal growth of each student and strengthen the student's self-concept.

4. Experiences in the classroom should help the student learn to work in a group cooperatively, sharing and respecting the opinions and ideas of others.

5. The student should be able to choose from among several options for learning and take responsibility for the consequences of those decisions.

6. The teacher should serve as a collaborator and resource person, and as such should continue personal learning and growth.

7. Education should emphasize transactions in learning--which occur when the students do something to the facts and the facts, in turn, do something to the students--by rendering the experience more intelligible and by helping the students see the relationships between themselves and their environment.

From these guiding assumptions and statements, it follows that the particular goals for teaching mathematics will go beyond that of merely teaching mathematical concepts. In order to assist in the development of "informed, thinking citizens," the experiences a student has in a mathematics class must develop the ability ". . . to use mathematical knowledge and methods to solve problems; to reason logically; to use abstractions and symbols with power and ease . . ." (NCTM et al., 1978). The choice of topics and teaching strategies should help students develop the problem-solving and analytical thinking abilities necessary for informed, confident, and capable citizens.

Student-directed Learning

Development of autonomy will not take place in a teacher-directed learning situation. Usually the general education mathematics course and most college mathematics courses are taught traditionally where subject matter is imposed from above and the student receives the subject matter passively. With this approach the students are considered less important than what is being taught. They are not asked or even allowed to take responsibility for their own learning. Since one of the aims of education should be to help students learn to respect their own thinking, they should be taught in such a way that they can come to see that their own thinking can improve their ability to cope with the world. Malcolm Knowles listed four reasons why a course should emphasize self-directed learning:

> . . . there is convincing evidence that people who take the initiative in learning learn more things, and learn better, than do people who sit at the feet of teachers passively waiting to be taught . . . that self-directed learning is more in tune with our natural processes of psychological development . . . many of the new developments in education . . . put a heavy responsibility on the learners to take a good deal of initiative in their own learning. . . . When a person leaves schooling he or she must not only have a foundation of knowledge acquired in the course of learning to inquire, but more importantly, also have the ability to go on acquiring new knowledge easily and skillfully for the rest of his or her life. (1975, pp. 14-16)

Hence, one of the problems to be examined by the inquiry is whether or not it is possible to develop a method of teaching based on a commitment to individualization of instruction and the development of the autonomy of the learner, a method which at the same time is also effective in meeting the various goals of a good mathematics course.

What Is Being Done about the Problem

Some educators are aware of the problems as outlined, and the literature contains a number of attempts to provide a different direction for the general education mathematics course. Barbara Lederman (1977) has surveyed the community college basic mathematics courses, and she pointed out:

> Educators believe that the community college should offer at least one year of mathematics which is general in appeal, challenging in content and useful in everyday life.
>
> In view of these conclusions, two philosophies have risen. One emphasizes the mastery of skills, the other, the importance of critical and analytical thinking and understanding. Both schools, however, want a mathematics course to be a course in, and not merely about, mathematics. The tendency is for basic mathematics courses to combine both schools of thought through stressing the use of mathematics as well as its logic and meaning and to no longer require memorization of formulae and complicated theory.
>
> While most educators agree that there is a need for a basic mathematics course for the non-technical, non-mathematical student, disagreement still exists over what topics to offer, how much of a topic to teach, to whom and for whom to offer and create new mathematics courses. (p. 31)

John Brevitt (1975) investigated the offering of four different syllabi to freshmen at Western Kentucky University following a cultural approach, a historical approach, a utilitarian approach, and the traditional modern approach. Other authors (Billie, 1976; Duncan, 1971; Steger & Willging, 1976) wrote of attempts to try out different approaches.

However, a common characteristic of all of these attempts at innovation is that teaching remains relatively teacher-centered and the textbook usually dictates what materials are to be covered.

An alternative approach to learning mathematics--the real-life problem-solving approach--has had some recent successes on the elementary and junior high school levels. Called the Unified Sciences and Mathematics for Elementary Schools (USMES) project, it had its genesis in John Dewey's Laboratory School at the University of Chicago, where children learned about the world they lived in by being involved in projects or problems rather than passively following information given by the teacher. This project was one of the indirect results of the 1967 Conference on the Goals for the Correlation of Science and Mathematics and was headed by Dr. Earle Loman, a participant of the conference, from its inception in 1970 to its formal conclusion in 1978.

USMES endeavors to help students develop the ability to make informed decisions on matters of particular importance in their own lives through the repeated exposure to comprehensive real-problem-solving situations. A "real problem" means more than the usual use of the term, that of a problem with the boundaries carefully stated, controlled, and made-up, with a correct solution to be found by applying previously taught methods:

> . . . problems which are real in the above ways may have many educational uses, but they will not prepare a person to cope with the complicated and many-faceted problem situations met in real life. . . . in USMES a "real-problem" connotes a practical, immediate impediment to a good, safe, or pleasurable living. Is the cafeteria service lengthy, crowded, or wasteful? Is the classroom a good place to learn? Do people have difficulty crossing the street near the school? What are the best products to buy with our budget? Such problems challenge children and have a range of attributes met only in problems that are real in this sense. (Loman, Beck, & Arbetter, 1975, p. 54)

Dealing with problems involves identifying the variables, seeing if they can be quantified, making observations, setting up a model, discussing with others, and making value judgments. "The cognitive skills needed are flexible and integrative, not specific and hierarchical" (Loman et al., 1975, p. 55).

An USMES unit gives the students a challenge to solve a practical problem which may lead to an improvement in the situation being investigated. These problems should be manageable for elementary students and should require students to use concepts and methods from science, mathematics, social sciences, and the language arts. The "practicality of the problem not only motivates the students, but provides a criterion (try it and see if it works) for judging the correctness of hypotheses and conclusions" (Loman et al., 1975, p. 56).

In working through a unit, the teacher is expected to be a guide but not a director, to ask stimulating questions but not to supply the answers. The program has been characterized by the motto "a large amount of freedom of choice based on a small amount of directed learning." The teacher presents the children with a challenge or several challenges, which may arise from discussing issues in a broader context. The class as a whole must be willing to accept the challenge. The students are then expected to spend from 45 to 60 hours in investigation. They work in groups and each group may work on a different aspect of the problem depending on interest and ability. The approach puts more of a burden on the teacher but provides more flexibility for children (USMES, 1976b, pp. 6-8).

The USMES program provides a potential model for the college-level liberal arts mathematics course. Students can be given the opportunity to explore questions of immediate importance to them and the mathematics which is learned will be acquired within the context of solving a meaningful problem. Since these students are in a terminal course, there is no clear need for learning particular mathematical skills to be used in a later course. However, the gap between the description of the USMES project by its chief proponents and the actual use of its ideas in a college classroom is wide indeed. How easily will college students who have been through 12 years of traditional teaching be able to change to a new system of learning? The class schedule of two or three 50-minute meetings a week is considerably different from the schedule in the elementary schools; how will this affect instruction? Materials appropriate for college students will need to be substituted for USMES materials. In what way can this be done?

Purpose of the Study

Earlier in this chapter the problems currently found in the traditional teaching of a general education college mathematics course were outlined. It is noted that a variety of changes in curricula have been tried with some success, but all were characterized by a teacher-oriented rather than a student-centered approach. None of these experimental courses involved the students in inquiries of direct importance to them. Thus, the problem of this study is to develop a general education mathematics course for college freshmen featuring the real-problem-solving approach based on the guidelines and concepts of the USMES program and emphasizing self-directed learning.

A search of the literature did not reveal any other attempts at using the USMES approach on a college level. Furthermore, Dr. Earle Loman indicated in a personal communication that he was not aware of any use of an USMES approach in a college liberal arts mathematics course anywhere in the country and he felt that such an approach would be appropriate. Because of the potential it had for helping students to learn in a meaningful way, a clear need existed to find out what happens in the classroom where it is used.

In the Winter of 1979, when the study was begun, the author was teaching two sections of the second-semester, general education mathematics course at Bennett College, a small, liberal arts, Methodist college for women in Greensboro, North Carolina. A modified problem-solving approach was tried in one of the two classes and the second class was taught in the traditional way. At the beginning of the course each student was asked how she would solve a problem of a "best buy" and was given a 20-question attitude inventory concerning her feeling toward mathematics. The students in the experimental class were asked to work through an USMES-type "challenge" in small groups, to hand in notebooks, and to keep a record of their impressions of their attempts to solve the problems they were investigating. The author kept a personal journal of his own impressions and a narrative of what was happening in the class. At the end of the semester the attitude inventory was again given, the students in each class were asked to evaluate their experiences, and each student was given a teacher-made test on mathematical skills.

Guiding Questions of this Inquiry

The following questions served to focus the efforts of the inquiry:

1. What happens in the classroom where the real-problem-solving approach is being used?

2. What changes in attitudes take place during a semester course?

3. What changes in the ability of students to take responsibility for their own learning take place?

4. What mathematical ideas and skills do the students learn?

5. What are the unanticipated outcomes--both negative and positive--resulting from the use of the materials?

6. What experiences, insights, frustrations, and feelings of accomplishment does the teacher have in working with students in this kind of classroom situation?

Two further questions which have been previously discussed are listed below:

7. Can a method of teaching based on a commitment to individualized instruction and the development of the autonomy of the learner be devised which at the same time is effective in meeting the various goals of a good mathematics course?

8. Can college students who have been through 12 years of traditional teaching change to a new system of learning?

Plan of the Study

Chapter I offers an introduction to the problems of teaching the college general education mathematics course in a meaningful and humanistic way. It discusses some general goals in mathematics education and describes the real-problem-solving approach to teaching mathematics. The chapter also outlines the broad aspects of the dissertation problem and the questions to be considered.

Chapter II considers the college general education mathematics course and alternatives to the traditional methods of teaching--for example, the prototype project (USMES) and other attempts at the real-problem-solving approach. The chapter concludes with a rationale for the naturalistic method of research used in the investigation.

Chapter III details the teaching and learning activities in the classroom and the procedures used in the study.

Chapter IV includes the observations and findings of the inquiry.

Chapter V presents a summary and evaluation of the findings, some answers to the guiding questions, and recommendations for future attempts to use the real-problem-solving approach in the mathematics classroom.

CHAPTER II

THE REAL-PROBLEM-SOLVING CURRICULUM AND THE RESEARCH METHODOLOGY

The sections in this chapter give substantial background from the literature to the two main themes of the investigation, namely, the college general education mathematics course and the real-problem-solving curriculum. In addition, the research techniques used to arrive at conclusions about the questions being explored in this study are examined. The first section focuses on the general education mathematics course. Viewpoints of mathematics educators, recommendations of an officially recognized panel of mathematicians, surveys of the status of the course in the 1970's, and recent attempts to make the course more effective, significant, and enjoyable, are presented. The second section deals with problem solving, and presents an in-depth look at the real-problem-solving curriculum and the Unified Sciences and Mathematics for Elementary Schools (USMES) project. The third section considers why the various attempts to improve the course have been generally unsuccessful. Finally, the naturalistic method of research which was used in the investigation is explored.

The College General Education Mathematics Course

The content and the teaching of the usual liberal arts course offered to freshmen at many colleges and universities leaves much to be desired. Recommendations with regard to the general education mathematics course and the status of this course in the 1970's are summarized below. The theoretical and recommended teaching practices are explored here, along with attempts to narrow the differences between theory and practice.

Theoretical Considerations and Recommendations

Henry Adler (1965), writing in The American Mathematical Monthly at the time he was secretary of the Mathematical Association of America, pointed out that official recommendations had been made previously for appropriate mathematics courses for prospective teachers, research mathematicians, and users of mathematics (those going into the physical sciences, engineering, biology and management), and there was "much evidence that something should also be done for the large mass of students, such as those majoring in the humanities and some of the social sciences, who do not require a mathematics course as part of their training in the major. . ." (p. 41). A beginning course in mathematics, which in 1965 would most likely have been taught with emphasis on "definition-proof-theorem" and on the general underlying principles, was not suitable, he argued, for the many students in the humanities and social sciences, in the same way a technical course in musical composition was not appropriate for the potential student of mathematics.

Norman Locksley (1970) in his article entitled "Why Should a Nice Liberal Arts Student Study Math?" suggested three reasons for taking such a course:

> Everyone needs a certain body of mathematical information just to function effectively in today's world . . . the essence of mathematics is the power of logical thought.
>
> Mathematics is the only exact language of change. . . . What does one use to keep up? Mathematics is the tool and language of discovery.
>
> . . . Because mathematics is there. . . . if we believe that a liberal arts education should include that which is universal and classical as well as that which is relevant and applicable--well, isn't mathematics both? (pp. 622-623)

What then should such a course entail? Adler (1965) suggested that the course should give the students an appreciation of mathematics "as an <u>exciting</u>, <u>valuable</u>, <u>vital</u> and <u>human</u> creation . . ." (p. 62). Moreover, the course should emphasize mathematical ideas, and Adler suggested such topics as the infinity of primes, the fundamental theorem of arithmetic, strange algebras, and the representation of numbers. He felt that the main change should be in the way the course is taught and suggested avoiding excessive emphasis on manipulative skills, requiring formal proofs of those statements which are obviously true, and unnecessary terminology. Jack Forbes (1972), reflecting a different concern, cautioned against merely replacing meaningless drill with meaningless emphasis on structure and rigor:

> Some "deadwood" has been stripped from the curriculum; some remains. However, some has merely been replaced by "deadlier-wood." When meaningless computational drill is merely replaced by meaningless recitation of

> multisyllabic words or meaningless reproduction of memorized proofs it is difficult to see the gain. Furthermore, some quite "livewood" may have been lost in the rapid movement toward "structure" and "rigor" at the possible expense of intuition and understanding. It is certainly important to be able to prove a theorem. It is even more important to "believe it", to "know what it means", and to "know why you should want to know it." (p. 480)

Partly in response to Adler's call for a definitive statement about the content and proper instructional strategies in a college-level liberal arts course, a panel was established by the Committee on the Undergraduate Program in Mathematics (CUPM), a standing committee of the Mathematical Association of America which considered various issues about curriculum in undergraduate mathematics programs and courses. The panel was directed to consider the nature of a course for non-mathematics majors with weak backgrounds in mathematics. It produced a report entitled "A Course in Basic Mathematics for Colleges," (CUPM, 1971), which considered the difficulties of providing a single mathematics course to meet the needs of those for whom remediation is recommended, of potential elementary school teachers who need computational skills and an understanding of certain concepts (sets, number systems, numeration, geometry), and of those who need some mathematics but who have a dislike for the subject. The key recommendation was summarized in the following two sentences:

> We propose the replacement of some of the currently existing basic mathematics courses by a single flexible one-year course, A Course in Basic Mathematics for Colleges . . . , together with an accompanying mathematics

> laboratory. <u>The main aim of this course will be to provide the students with enough mathematical literacy for adequate participation in the daily life of our present society</u>. (CUPM, 1971, p. 259)

The CUPM panel envisioned that this course would provide remedial help for students in the suggested mathematics laboratory and would cover sufficient standard topics to allow the students to continue in the usual algebra-trigonometry sequence if they decided to do so. Thus the proposed course should include material on simple algebraic formulas, handling simple algebraic expressions, the distributive property (common factor), setting up and solving linear equations in one or two variables, the beginning of graphing, and the rudiments of plane geometry. In addition, nonlinear relationships, the computer, and statistics and probability could be included. Not all of these topics need be covered; others could be substituted.

Inherent in the recommendations was emphasis on a new approach appropriate to students who had been "turned off" by high school mathematics. The panel recommended using flow-charting and computer-related ideas as a focus around which the various topics would be taught. The panel emphasized that mathematical ideas should be related to the everyday lives of the students:

> Topics of everyday concern, such as how bills are prepared by computer, calculation of interest in installment buying, quick estimation, analyses of statistics appearing in the press, and various job-related algebraic and geometric problems, are mainstays of the syllabus.

> We have tried to make the proposed course coherent; that is, after a topic is introduced, it should be used in other parts of the course and not left dangling. The students must be actively involved throughout the course and should be encouraged to formulate problems on their own, based on their experience. Full advantage should be taken of playful impulses of the human mind; interesting tricks and seemingly magic ways of solving problems are to be exploited. (CUPM, 1971, p. 260)

The differences in the recommended topics of Adler, those of interest particularly to mathematicians but accessible to non-mathematicians, and the topics of everyday concern recommended by the CUPM panel, were quite striking. But the CUPM recommendations in 1971 were the result of a small but growing concern for real or "everyday problems" in mathematics classes, together with a de-emphasis on proof, a concern which had developed throughout the 1960's. One of the leading critics of the emphasis on abstract mathematics was Morris Kline, who forcefully argued for the kinds of reforms suggested by the CUPM report.

In the Spring of 1958, when he addressed the Thirty-sixth Annual Meeting of the National Council of Teachers of Mathematics, Kline (1958) argued the "modernist approach" was ill-advised for several reasons. First, the ability to appreciate rigor must be developed, not assumed, and second, it ignored the basic reason for the existence of mathematics and the chief motivation for the study of mathematics, which, he claimed, was the investigation of nature. He argued that:

> Mathematics is significant and vital because it is the chief instrument for the study of the physical world. On the other hand, the pure mathematics which modernists wish to present is a pointless mathematics, a manipulation of meaningless symbols which can appeal only to an esoteric group. (1958, p. 424)

Finally the modernist approach failed to present the life and spirit of mathematics, the creative, inventive process which derives ideas from real problems, formulates relevant questions, derives a potential conclusion intuitively, and proves the argument deductively as the last step.

Throughout the 1960's and 1970's Kline presented his argument. He was the author of a textbook, Mathematics for Liberal Arts (1967), specifically written for the general education course. This text, which required the same sort of sophisticated reading skills and interest in liberal arts education as a student would need to tackle a music appreciation textbook, featured a historical approach to mathematical topics. In the preface, Kline wrote:

> I believe as firmly as I have in the past that a mathematics course addressed to liberal arts students must present the scientific and humanistic import of the subject. Whereas mathematics proper makes little appeal and seems even less pointed to most of these students, the subject becomes highly significant to them when it is presented in a cultural context. In fact, the branches of elementary mathematics were created primarily to serve extra-mathematical needs and interests. . . .
>
> The historical approach has been retained [from his previous text, Mathematics: A Cultural Approach] because it is intrinsically interesting, provides motivation for the introduction of various topics, and gives coherence to the body of material. Each topic or branch of mathematics dealt with is shown to be a response to human interests, and the cultural import of the technical development is presented. I adhered to the

> principle that the level of rigor should be suited to the mathematical age of the student rather than to the age of mathematics. (1967, p. v)

Topics in the textbook included logic, number, the nature and uses of Euclidean geometry, mathematics and painting in the Renaissance, formulas in applications, trigonometric analysis of musical sounds, the statistical approach to the social and biological sciences, and the nature and values of mathematics. It would take an instructor of considerable breadth and a somewhat more sophisticated student than normally populates a freshman mathematics class to be able to handle such a textbook, but the text clearly presented a view of mathematics as a powerful force in the development of our civilization.

Six years later in his controversial book, Why Johnny Can't Add, Kline (1973) called for a broad rather than a deep education for the large percentage of students who do not specialize in mathematics:

> It should be a truly liberal arts education wherein students not only get to know what a subject is about but also what role it plays in our culture and our society. (p. 145)

Mathematics should be taught in its proper relationship to other human interests as part of a "broad cultural mathematics curriculum." Furthermore, motivation for the non-mathematician must come from a study of real, largely physical problems; it should not be motivated in mathematical terms. Going well beyond the proposal of the CUPM panel, Kline argued that most mathematics courses in high school and the first-level college course should emphasize motivation through applications:

> The motivation must be presented along with the topic taught. It will not do to assure students that they will some day appreciate the value of the mathematics they are asked to learn. If a subject has any value, then as Whitehead points out, the student must be able to appreciate its importance immediately. . . .
>
> Would real problems meet the interests of young people? They live in the real world and, like all human beings, either have some curiosity about the real phenomena or can be far more readily aroused to take an interest in them than in abstract mathematics. Hence there is an excellent prospect that the genuine motivation will also be the one that interests students, and, indeed, some limited experience has shown this to be the case. (pp. 150, 151)

As a summary to Why Johnny Can't Add Kline listed five pedagogical principles: (1) to teach students to pursue knowledge as a part of a liberal arts education, to think and not to follow; (2) to introduce concepts in the historical order; (3) to replace deductive proof (important mathematically but a disaster pedagogically) with an intuitive argument with pictures, heuristics, analogies, and physical arguments; (4) to emphasize the development of critical thinking; and (5) to use a deductive proof as a final step with the level of rigor suited to the students' levels of development. His principles set a high standard for the teaching of mathematics, perhaps unrealistically high. An examination of the goals of Adler, the CUPM recommendations, and Kline reveals how far apart these were from the actual choice of topics and teaching which took place in the 1970's.

Status of College General Education Mathematics Courses in the 1970's

Several authors and study groups undertook to assess the status of the college general education mathematics courses found throughout the United States at various times in the 1970's. In addition, they also presented their own recommendations for what should be taught and what the method of teaching ought to be.

Writing in The American Mathematical Monthly, Edwin Spanier (1970) reported results of his interviews with students enrolled in various mathematics courses, not just the liberal arts courses. He found that the students

> exhibited narrowness of training, lack of mathematical taste, lack of intellectual curiosity and no sense of responsibility for their own education. . . . [This is] the fault of graduate schools, both through their requirements on entering students and through the training that they give to future college teachers. (p. 754)

Here were mentioned two themes to be repeated in later reports, namely the narrow training of college teachers and the lack of any sense for self-directed learning on the part of the students. Dorothy Bleyer, in her 1977 dissertation on the attitudes of students toward mathematics, reported that a majority (58%) of the students held attitudes toward mathematics which were significantly negative. Responses given by students in her investigation indicated that women had "observably different" attitudes than men and were more negative (1978, pp. 5972-73).

Douglas Aichelle (1971) described an investigation of the effects of a "terminal mathematics course" taught in a traditional manner on the change of attitudes toward mathematics. The course was required for graduation for students who did not take a more advanced course. The percentage of students indicating agreement with the statement "Most work in mathematics is the memorization of information" increased from 20% for the pretest to 43.1% on the posttest. A second significant change in attitude, again toward a more negative viewpoint, occurred with the statement "Almost all present mathematics was known at least a century ago"; only 9.3% agreed at the beginning while 27.7% agreed at the end of the course. The students had gained some strong impressions about mathematics which were not intended by the instructor. Aichelle summarized:

> Many of the students believed that there is usually some rule to follow in solving problems, but that mathematics did not help them think according to strict rules. It was anticipated that more students would conceive of mathematics as a field for creative people to enter with emphasis placed on originality and a good sense of logic. Findings from this part of the Attitude Scale suggest that the creative aspects of mathematics were not promoted or explored to the desired extent. (p. 204)

Aichelle called for some changes in content, methodology and activities, and concluded that the "results of this research indicate certain areas of responsibility for the instructional staff . . ." (p. 205).

Another investigator (Shevokas, 1975) found a deterioration in attitudes in a traditionally taught class, while others (Olson, 1976; C. Smith, 1975) found no significant changes in attitudes.

Four summaries appeared in 1973, 1974, 1977, and 1978, all of them in the Two-Year College Mathematics Journal. Unlike The American Mathematics Monthly, which covers almost exclusively problems of curriculum and instruction for courses for the mathematics major, and the Mathematics Teacher, which focuses on curriculum and instruction in secondary schools, the Two-Year College Mathematics Journal regularly brings to the attention of its readers the problems and successes of college courses in mathematics for non-mathematics students.

The first article, by S. K. Stein (1973), focused on the "captured student," a phrase which "was chosen to convey the mood of failure, fear, frustration, and perhaps the hatred that such a student frequently brings to his mathematics class" (p. 62). Stein's survey, carried out in 1971, involved two-year colleges "in part because the problem of mathematics for the captured student is acknowledged there frankly and clearly, and in part because four-year colleges and universities contain, whether they admit it or not, two-year colleges" (p. 62). William Mitchell (1974) surveyed over 2000 two-year college catalogs for the period 1970-1973. Barbara Lederman's survey (1977), "The Community College Basic Mathematics Course," already mentioned in Chapter I, was based on a search of the literature of the 1960's and on surveys which she conducted. Finally, Charles Freisen's brief report, entitled "A Look at General Education Mathematics Programs" (1978), was based on replies from 86 schools to a questionnaire.

One of the common themes of the four surveys was that there was a clear need for a challenging general education mathematics course. Stein (1973) found that such a course was needed to meet the needs of the "incredible variety of students' ages, abilities and background . . ." (p. 63). He continued:

> . . . is there a need to meet the captured student <u>not halfway</u>, but exactly where he is, and to help him go as far as he is able in the direction that he chooses or is compelled to follow? I believe it does have that responsibility. (p. 63)

Lederman drew the following conclusion from her survey:

> There is, however, still dissatisfaction with the mathematics being offered in today's community colleges. The students entering the community college today still lack many basic skills in mathematics [because of minimal high school preparation or a sizeable absence from school] and little motivation to learn those skills. They still fail to see benefits to be gained from knowing basic mathematics skills. Thus, need yet exists for a good basic mathematics course that teaches the basic skills while demonstrating that those skills are applicable to today's community college students. (p. 31)

There was general agreement that general education mathematics "should be different from other mathematics offerings" (Mitchell, 1974, p. 35). This course must meet the needs of students who are poorly motivated, come with a weak background in mathematics and "are usually convinced of their mediocrity in this area" (Mitchell, p. 35).

The four summaries indicated that there had been little change in course content throughout the 1970's despite the recommendations of the CUPM report and other advocates of a move toward relevancy to real life. Mitchell found that roughly two-thirds of the institutions which he surveyed offered essentially the same topics: mathematical systems, introduction of logic, proofs and arguments, sets and operations, sets and logic, counting numbers, Cartesian products and functions, relations, total orderings, probability, and elementary geometry (p. 33). This list, of course, reflected the continuing influence of the "modern mathematics" emphasis of the 1960's as well as the attempt to satisfy simultaneously the needs of students in elementary education (sets, classification, logic, number systems, and systems of numeration) and the needs of students in humanities and social sciences (functions, probability). Other topics taught in the other one-third of the classes included review of arithmetic, business mathematics, statistics and finite mathematics.

Lederman (1977) listed the following topics found most often in her survey: arithmetic concepts, algebraic and graphing concepts including solving word problems using algebra and solving systems of linear equations, geometric concepts, measurement including the metric system, and statistics and probability. Some of the courses, she noted, included some right triangle geometry and applications of various topics in business and personal interactions. She observed that the topics in these courses fairly closely followed the recommendations of CUPM, which she characterized as follows:

> While stressing the basic arithmetic and algebraic skills, this course is designed to offer these same topics with a new, interesting approach, well suited to the community college. (p. 33)

Some of the more abstract concepts were thus being replaced by the more traditional topics of ninth-grade algebra as well as by an inclusion of arithmetic skills.

Friesen, whose survey covered the middle part of the 1970's, found a notable increase in the number of different, distinct topics included in general education college mathematics courses. The various conflicting trends were all represented in his list--high school algebra, abstract mathematics, applied mathematics, and historically oriented mathematical topics. For the first time computers and programming were found. The most popular topics included elementary algebra (86%), graphs (74%), probability and statistics (73%), and set theory and logic (73% and 68%). History of mathematics surprisingly was included in 5% of the courses and number bases in 51%. Hence, according to Friesen's report, there was little indication that CUPM's 1971 recommendation for teaching topics of everyday concern had been followed by 1978. His findings seemed to contradict those of Lederman, who, as noted above, had concluded that the topics usually found in the course were consistent with the CUPM suggestions.

Textbooks slowly began to appear which reflected the recommendations of the CUPM panel. By the end of the 1970's a number of the remnants of the "modern mathematics" era, particularly sets, truth tables, numeration systems, the structure of the various number systems, and mathematical systems (e.g., clock arithmetic) were deemphasized or dropped from books. Titles such as Contemporary Mathematics (Meserve & Sobel, 1972) and Mathematics and the Modern World (Triola, 1973) gave way to such titles as Mathematics, an Everyday Experience (Miller & Heeren, 1976). In the March 1977 issue of the Two-Year College Mathematics Journal, for example, seven different textbooks for a college general education course were advertised, offering the instructor a choice in emphasis. Among these were two books by Miller and Heeren, Mathematical Ideas (1978), reflecting the abstract emphasis, and Mathematics, an Everyday Experience (1976), reflecting the emphasis on relevancy. Mathematics as a Second Language (Lake & Newmark, 1977) was advertised as follows:

> To reflect changing approaches to teaching liberal arts as mathematics, the authors provide more practical applications and expanded historical coverage in this edition. New topics include: abstract mathematical systems, genetics, and percentiles. A new chapter on the computer has also been added. (inside back cover)

What was remarkable about this statement was the emphasis on historical topics and the promotion of the addition of topics which were being dropped from other texts at the same time. The other textbooks exemplified the variety available, as the titles indicate: The Power of Relevant Mathematics: Basic Concepts (Whipkey, Whipkey, & Jarocky, 1977), Mathematics: Tools and Models (Hunkins & Pirnot, 1977), The Nature of Modern Mathematics (K. Smith, 1976), and The Whole Craft of Number (Campbell, 1976). The latter book emphasizes a historical development of mathematics.[1]

Mitchell criticized the textbooks as containing a collection of unrelated topics which were not particularly relevant to the general student. His remarks in 1974 still had relevance to the textbook of 1979. After commenting on the fact that textbooks provided for both the liberal arts student and the prospective elementary teacher with conflicting topics, he wrote:

> A second characteristic of current texts is a compulsion to stimulate relevance, to attempt to convince the general student that each of the six to eight mathematical topics covered in the one term course is of personal practical value, and therefore should properly develop some measurable skill. A third characteristic of general education texts is the attempt to maximize the independence of chapters, to provide the greatest flexibility in using the text. (p. 34)

Another concern each of the four writers had was that of implementation and the background and commitment of the instructor. Stein argued:

> . . . the math department has the responsibility to implement [the requirement to provide a course in mathematics for non-mathematics majors] in an imaginative

[1]An English major in the experimental class used this book for outside reading and reported favorably on the presentation and content of the ideas in it.

> and kindly way. In particular, I believe that there is a place for a mini-course whose only object is make people feel better about mathematics. (p. 63)

He called for a variety of teaching strategies, particularly small-group learning, mathematics laboratories, and individualized learning. The reliance on lectures, with resulting boredom (as evidenced by the statistic that only 30% of the students in a classroom pay attention at any given time) and passive learning demand alternatives. It is important, Stein argued, to select instructors who are concerned about the problems and are capable of changing methods of teaching.

Mitchell echoed the recommendation for committed faculty. He wrote:

> The design and presentation of activities which do not depend heavily on facile computation, yet properly relate the role of computation to the mathematical process, require both a mature teacher and a knowledgeable mathematician. Therefore, the general education course should usually be taught by senior, not junior faculty. (p. 35)

Mitchell recommended in addition that the general education course should not be content-oriented. Because of the profusion of topics and the variety of students, it was unlikely, argued Mitchell, that a single topic would "be relevant to all. The decision of what to teach cannot therefore be made solely or even principally on the basis of content or student interest" (p. 35). Flowcharting and algorithmic processes were to be encouraged. This, claimed Mitchell, "cannot be said to adequately represent mathematics in the broad sense, but it does represent that aspect of mathematics which is nearest the experience of the student, and it provides understanding of the mechanization of mathematical problems" (p. 36). Other possibilities involved building the general education course around a theme, using whatever mathematics was necessary to explore the mathematical models inherent in the theme, or emphasizing an appreciation of what a mathematician does, by demonstrating the mental activities undertaken by mathematicians, both pure and applied.

Lederman emphasized the need to teach in such a way as to give students new experiences and exposure to new ideas, even if the standard skill had not yet been mastered. She wrote:

> Even courses which on the surface appear to satisfy course objective requirements may in reality be the same old material under a new name or taught using a new artificial approach. Students recognize the poor camouflage and are still dissatisfied with mathematics. (p. 32)

She noted that some innovative courses did exist, but "few are good enough. It remains for us to incorporate skills lists . . . into creative and useful mathematics presentations. New approaches are needed" (p. 32).

Friesen found that by the mid-1970's the lecture method continued to be used by a large majority of the colleges responding--95%. Individualized instruction, the use of programmed instruction, and, for the first time, computer-assisted instruction, were also listed (51%, 31%, and 11% respectively). The idea proposed in the CUPM report, that of using minicourses, had been adopted by 22% of the colleges.

Several types of mathematics courses were available, indicating a greater diversity in the way the courses were organized. Over half of the colleges offered a survey course (breadth, not depth) while 24% featured the "block and gap type of course where several topics were studied at length and others ignored" (p. 219). The plea not to use those instructors with the least experience, those students just out of graduate school with a narrow specialization, apparently was being followed. None of the colleges reported assigning the general education course to the instructor with the least seniority; in 69% of the colleges all instructors were involved and in 15% of the cases the instructor was someone who showed an interest (p. 221).

A major report on the status of mathematics education, specifically an overview and analysis of school-level mathematics in the United States, with objectives, current practices, and attainments, was prepared by a committee of the Conference Board of the Mathematical Sciences in 1975. The committee was officially called the National Advisory Committee on Mathematics Education (NACOME) and the report was dubbed the NACOME Report. While not specifically addressing the college-level mathematics courses, the report dealt with the same issues and trends which affected the college liberal arts course in mathematics. It considered in detail two of the growing concerns in the mathematical community which had largely been ignored in the four general education summaries above--namely the "back to basics" movement and the revived interest in problem solving in all of its aspects. The report also cautioned that the good features of the "new math" curricula of the previous decade should be retained and used wisely; in particular, it provided a counterpoint to the arguments of Morris Kline.

The NACOME Report further stated that the most convincing arguments for the "New Math" curricula, including such unifying concepts as sets, functions, and the algebraic field properties,

> . . . rest on the role that these ideas can play in organizing other mathematical facts and methods--in demonstrating connections between apparently different mathematical tools so that acquisition, retention, and transfer can be enhanced. Set theoretic ideas offer another vehicle for explaining, illustrating, and practicing basic mathematical concepts and skills. (NACOME, 1975, p. 16)

Below are given four of Kline's criticisms of teaching mathematics as a structured discipline together with replies given in the NACOME Report.

1) Novel content such as set theory, symbolic logic, etc., is inappropriate for school curricula--

> Experience with these new ideas over several years of classroom implementation has now led to a much better perspective in both texts and teaching . . . curriculum development projects . . . are generating promising approaches for introducing the concepts and new curriculum structure which capitalizes on their unifying power. (p. 16)

2) Rigorous deductive logical presentation of ideas is contrary to all historical experience of mathematical discovery--

> . . . the rationale for this emphasis [in deductive reasoning] is not based on naive misunderstanding of mathematical methods. It is instead grounded in the hypothesis that students perceiving the structure of mathematical ideas will become more effective learners and users of the subject. . . . Most of our students seek from mathematics a collection of well established concepts and methods that can be applied to problems outside of mathematics. It is not at all clear to us how organization of these tools into logical structure will be a barrier to learning--though in many cases <u>initial</u> presentation might well be guided by psychological rather than logical organization. (pp. 17-18)

3) The emphasizing of abstract ideas is not properly grounded in the concrete experiences of the physical origins for mathematics--

> A curriculum organized by broad structural concepts and processes is predicated on the ability of learners to make appropriate abstractions and generalizations. . . . Current school mathematics curricula can probably be improved by more creative interplay of concrete experience and abstract ideas. . . . Reaction against abstraction at any level denies the very real contributions that its process and product make to mathematics and would be a step backward in developing improved school programs. (pp. 18-19)

4) The new mathematics employs "too much" meaningless terminology and symbolism--

> . . . historically the precise symbolic language of mathematics has played an important role in the development and communication of ideas. The most effective instructional balance of informal and rigorous expression is a function of mathematical topic, student aptitude, and student experience . . . in a world of increasing complexity weighted down by a flood of concrete and detailed information, understanding and skill with processes of abstraction and symbolic representation may be as important in the ordinary citizen's equipment for life as were the purely numerical skills [which should not be neglected] in an earlier era. (p. 20)

The <u>NACOME Report</u>, in spite of its defense of some of the best aspects of the "new math", called for an eclectic approach. The dichotomization of curricular issues--old vs. new, skills vs. concepts, concrete vs. abstract--would not solve any of the problems in mathematics education. Neither the mechanistic approach to teaching little bits of skills in a highly sequential manner nor allowing students to explore only those topics of interest to them in a completely unorganized way would result in meaningful learning. Instead, the report urged:

> Teachers should be eclectic pragmatists, selecting those methods and materials which seem to work best at a particular time for a particular student or group of students working with a particular concept. . . . Perhaps the optimum, judicious mix of all these modes is what the conscientious teacher seeks. (p. 80)

The next researcher looked specifically at the college general education course and echoed the call for the instructor to have an opportunity to choose.

John Brevitt (1975), in his dissertation on four different syllabi for the college-level, general education mathematics course, gave a pessimistic view. He pointed out that in his experience, little progress had been made in changing the course outline to one of greater relevance for the student: "The student who finds adding two fractions a mouth-twisting chore is asked to appreciate the esoteric depths of Hilbert and Cantor, in much emasculated form, of course" (p. 4). He was particularly concerned about "inactive indifference" of the students towards their mathematics and the relationship between the students and the instructor and questioned whether if only two or three students appreciate the course and work to gain the goals of the course, whether continuing the present procedure was justified (p. 4). Brevitt acknowledged that the reasons for this unacceptable situation were not completely due to the uninspired student; part of the blame had to be placed on the instructor: "Some of the difficulty lay with those assigned to teach such a course; the typical graduate-schooled mathematician was not noted for his breadth" (p. 21). This observation, of course, was essentially the same as that made by Spanier in 1970; little had changed in the intervening five years. Indeed, Brevitt reached the conclusion that neither the students' attitudes nor their abilities were being quantitatively improved by the usual general education course.

Two other dissertations also verified the claim of considerable disparity between the characteristics of effective instruction in college mathematics espoused by various writers and the actual situation in the classroom as perceived by the instructors. Alan Allen (1975) surveyed articles which appeared in the <u>American Mathematical Monthly</u>, the <u>Mathematics Gazette</u>, <u>School Science and Mathematics</u>, <u>The Mathematics Teacher</u>, and <u>Mathematics Magazine</u> from 1894 to 1971. He found little support in these articles (which mainly dealt with what <u>should</u> be taking place in the classroom) for the use of the axiomatic approach, highly formalized exposition, and student exercises to be worked out without any help from the instructor. Instead, most writers of the articles surveyed encouraged "appealing to student intuition, proceeding from concrete to abstract, balancing intuition and rigor, motivating material to be studied, and proceeding from particular to general" (p. 7781). On the other hand, Norman Fletcher (1979) surveyed instructors of mathematics courses for nonmathematicians as to differences in the content of such a course among two-year institutions, four-year private colleges, and four-year public institutions. He found that his hypothesis that ". . . there is not a significant relationship between the content as perceived as more desirable by the selected mathematics instructors and the content of the Committee on the Undergraduate Program in Mathematics (CUPM) sequence . . . was rejected" (p. 3845). He found that instructors in the two-year colleges and women were much more strongly committed to following these recommendations than the comparison groups respectively; those in the four-year public colleges were mildly interested in doing so. Furthermore, the list of course topics actually being taught tended to contradict his conclusion since the same abstract topics found in the 1960's remained, mixed with a few more applied topics: review of basic algebra, set theory, logic and truth tables, mathematics modeling and problem solving (a topic beginning to appear on the scene in its own right), introduction to statistics, introduction to probability, and properties of and operations on various sets of numbers.

Attempted Changes in Content, Teaching Procedures, and Organization

Despite reports and recommendations of noted mathematicians, mathematics educators, and mathematics organizations and a variety of writings in mathematics journals urging innovations and relevance, the general education, college-level mathematics course was generally characterized in the late 1970's by an emphasis on topics of interest to mathematicians, a lecture-oriented approach to teaching by instructors whose primary interest was in mathematics per se, and a general disinterest on the part of the students. Nevertheless, a number of efforts to try out alternatives were being made in the classroom to overcome these problems and to improve attitudes or achievement or both. These are summarized below.

Thirty journal articles or dissertations, which related specifically to the general education college mathematics course, were surveyed. Nine of the reports featured experimental courses focusing on one or two topics around which the usual skills would be taught. Duncan (1971) investigated the effects of integrating number theory, topology, and probability on achievement and attitudes; he found mixed results depending on the level of entering aptitude scores and the topic taught. Kipp (1975) investigated mixing the topics of elementary algebra with elementary probability; the experimental group showed somewhat higher gains in positive attitude and retention on achievement scores. A comparison between a computer-oriented Monte Carlo approach, a manual Monte Carlo approach, and a traditional approach to the teaching of probability and statistics showed that there were significantly higher achievement scores for the Monte Carlo approaches and a positive gain in attitudes toward mathematics for students in the noncomputer Monte Carlo group (Shevokas, 1975). A course to develop basic mathematical skills through the teaching of elementary probability and statistics showed a slight improvement in the mastery of basic skills and also in the understanding of probability and statistics with the experimental group (Lee, 1975).

Allen Holmes, Walter Sanders, and John LeDuc (1977) developed a course for the general education student emphasizing statistical inference and avoiding the direct teaching of arithmetic and of combinatorics before covering inference. They argued that teaching the background topics first "makes it difficult to get to the general education portion of the subject in the time alloted most general education courses" (pp. 223-224). The course used activities involving a "sampling paddle" with holes and colored wooden balls; graphing; and simulation with coins, dice, and spinners. The authors made this forceful argument for their approach:

> We believe that for the general education student the ideas of statistical inference and the resulting decision rules are of prime importance. This belief is based on the assumption that general education courses are included in the curriculum in order to help students

> to gain an understanding of their own essence, of their relationship to others, of the world around them, and of how man goes about knowing. . . .
>
> The approach [to teaching statistical inference without the traditional work in descriptive statistics and probability] is best understood by looking at it in three parts. The first part involves very concrete experiences with sampling and an intuitive introduction to confidence intervals. The second part develops the concepts of probability via Monte Carlo Simulations. . . . The third part takes a look at traditional statistical treatments. (p. 224)

The authors pointed out that their approach trades weaknesses in arithmetic and algebra for deficiencies in writing the required reports of the simulations. Thus the course requires "clear and logical exposition" and it brings the real world to interact with the student and some good mathematics.

Two investigations about the use of computer programming as part of the general education course showed mixed results. Computer-augmented instruction had a significant positive effect on the mathematical attitudes of disadvantaged students but not upon their achievement in one investigation (DeLoatch, 1978). In another class the attitudes toward mathematics were not significantly improved (LeCuyer, 1977). LeCuyer, specifically following the CUPM recommendations to teach basic college mathematics using computers, wrote and used a textbook about the computer language APL and included some rather stiff topics: sets, logic, vectors and matrices, introductory calculus, and probability and statistics. He found that the experimental approach did not detract from conventional learning, and, in addition, that the students learned some programming and enjoyed using the computers.

David Travis (1977) developed and taught a "one semester pure general education course in appreciation of mathematics for students whose major field did not require any more mathematics skills than they already had (same as appreciation of art). The students were Art, Music and Speech-Theater-Drama majors" (p. 6987). The topics were relatively short and non-sequential, and they included some recreational-mathematics topics. One of the goals of the course was to demonstrate some of the strategies used by mathematicians to solve problems. The students did not solve textbook problems. They read chapters from a variety of textbooks. No attempt was made to assess gains in achievement test scores but it was determined that attitudes toward mathematics increased positively in most categories tested.

John Brevitt (1975), as previously noted, studied the relative contributions of four different syllabi to quantitative thinking ability and positive attitudes toward mathematics. He taught four different sections of a college-level general mathematics course using four different approaches. The utilitarian approach featured mathematics applied to the solution of useful problems, the cultural approach exhibited mathematics as a result of deductive reasoning and creative thinking, the historical approach showed the influence of mathematics in past civilizations and its place today, and the modern approach consisted of a selection of topics from a standard text: sets, logic, functions, probability and statistics, analytic geometry, computers, and numeration systems. One of Brevitt's observations deserves emphasis here for it points to one of the difficulties faced by instructors and curriculum planners today:

> . . . we note [from the literature] considerable expenditure of experimental thought has been devoted to adjusting teaching techniques, but that the topics taught have varied little since the initial pre- and post-war introduction of a general education mathematics course. . . . All the suggestions for change we have quoted cannot be crammed into a single one-semester course, but perhaps a variety of approaches can achieve similar results. (p. 26)

Brevitt found that no one presentation resulted in a significantly greater or lesser gain in quantitative thinking ability. However, he found some useful results when considering the <u>major</u> of the students as a dependent variable. Social science majors preferred the historical syllabus; the social science majors exposed to the modern syllabus and the liberal arts majors exposed to the historical approach made the least gains in attitudes. The cultural approach was significantly more effective in improving attitudes than any of the others.

One of Brevitt's main conclusions was that, because of such a variety of possible topics and ways of teaching, the instructor should be given considerable latitude:

> What is suggested is this: allow the instructor to choose his own text and his own approach to the students in general mathematics. The course is terminal for almost all students. So long as the instructor does an honest job of selecting appropriate and valuable material and presenting this material to the very best of his ability, and of giving the preparation for this class the same attention he would for classes enrolling those more mathematically apt, he will be successful. If there is particular selection of material which the instructor feels is within the grasp of students at this level of maturity and which he feels is important to their knowledge, understanding, appreciation, and growth, and if he feels strongly enough about such material that he is able to present it with enthusiasm and conviction, keeping in mind both the material and student are worthy of respect and concern, and if the instructor is willing to vary his approach from lecture to small-group to laboratory until he finds a style or combination of styles that serves his purpose best--if, in other words, he is willing to give this group of "captured students" (to use Stein's phrase) the same intelligent attention he gives the more immediately responsive student who elects and enjoys mathematics--then as an instructor he will find these students more than willing to respond favorably toward mathematics. (pp. 56-57)

Two dissertations considered the effects of small-group instruction versus large-group instruction. One of these found a positive impact on achievement in favor of small groups (Chang, 1978), and one found no significant differences between small groups being given help by the instructor, by another instructor, or no help at all (Greenwood 1978). Seven dissertations investigated various kinds of individualized teaching in comparison with traditional teaching and team teaching (Baker, 1979; Frazier, 1975; Greenwood, 1978; Herring, 1976; Kerrigan, 1976; M. Smith, 1976; Williams, 1976). These gave varying and inconsistent results, although all of them focused on increasing the learning of basic skills and involved the usual list of topics. Two investigations involved the experimental use of hand-held calculators (Dyce, 1978; Gooden, 1978). Neither reported any significant improvement in achievement or any loss of proficiency with paper-and-pencil skills due to the use of calculators. Finally, one investigation (Olson, 1976) explored the question of whether the frequency of quizzes and the importance of quiz results in the grading procedure made any difference in achievement scores and attitude scores. He found no significant difference.

Three articles described variations in class organization and grading procedures, which sought to create more meaningful and enjoyable classes for students. Horner (1974) described a fairly successful alternative approach featuring in-class study from a standard textbook with no lecture, i.e., "nonlab, nonprogrammed, and nonlecture." This procedure was based on a number of operating principles:

> Students must manage the bulk of their learning processes. Students must be able to render a substantial amount of self-evaluation of their progress. . . .
> Performance objectives are useful. . . .
> Self-pacing is not necessarily an academic virtue. . . .
> Standard expository texts must be used. . . .
> There must be an "expectation" that the students will do well in succeeding courses where the lecture may be dominant. (p. 40)

Without citing any objective evidence, Horner claimed that this approach had been successful. The format of the class provided advantages to the students in the way of flexibility, and materials developed in connection with the class allowed for considerable enrichment. In addition, the success in "shifting the primary responsibility for learning from the instructor to a willing student represent [ed] a growth and experience worth perhaps more than mathematics itself" (p. 43).

W. Miller (1974) found that using a contract method of grading resulted in significantly better achievement scores in a class for underachievers. Steger and Willging (1976) described a clever way to set up a system of offering three or four modules, so that students could choose among 10 to 12 different topics, each consisting of 10 lessons. In this way one of the greatest problems of the liberal arts mathematics course, that of meeting the wide variety of needs and interests of the students, could be partly overcome. Furthermore, it allowed teachers to teach topics toward which they felt the most inclined:

> In determining which topics should be offered, we postulated that teachers would rather teach topics they are interested in and students would rather learn about topics of their own choosing. (p. 24)

The only Aptitude-Treatment-Interaction investigation (Baldwin, 1977) considered whether or not field-independent and field-dependent students would be more successful in a group setting or individual study. Neither group performed significantly better with respect to treatment, but field-independent students did achieve significantly higher than field-dependent students, as one might expect. Finally, Crumpton (1978) found not surprisingly that mathematics anxiety decreased as the students' mathematical competence increased, but the fact that counselling and other direct efforts to reduce mathematics anxiety did not help was unexpected.

Activity-based approaches to the teaching of the general education freshman mathematics course were reported by several investigators. C. Smith (1975) developed some activities for teaching mathematics to high-risk students. Mathematics achievement was significantly higher in the experimental group than in the control group but attitude was not. The activity-based teaching of statistical inference by Holmes, Sanders, and LeDuc (1977) has already been reported. A major institutionalized activity-based program involving eventually 27 predominantly black colleges, called the Thirteen-College Curriculum Program due to the number of institutions which participated in its beginning, included not only a variety of experiences in mathematics but also the whole range of freshman level courses. Bennett College was involved in this program (1968-1972) and the author observed the activities as a teacher in a "traditional class" and participated in a summer workshop.

The Thirteen-College Curriculum was developed in cooperation with the Institute for Services to Education by the original consortium of 13 colleges. It included new curricular materials for freshman courses in English, mathematics, social science, physical science, and biology and for part of the sophomore year in philosophy and humanities. It was begun in 1967 and lasted formally until the mid-1970's. Its purpose was stated as follows:

> The program is designed to reduce the attrition rate of entering freshmen through well thought-out, new curricular materials, new teaching styles, and new faculty arrangements for instruction. In addition, the program seeks to alter the educational pattern of the institutions involved by changing blocks of courses rather than by developing single courses. In this sense, the Thirteen-College Curriculum Program is viewed not only as a curriculum program with a consistent set of academic goals for the separate courses, but also as a vehicle to produce new and pertinent educational changes within the consortium institutions. (Institute for Services to Education, 1970, p. i)

Selected freshmen at each of the institutions took courses taught only by instructors in the program, using specially prepared materials and teaching styles consistent with the emphasis on discovery. A variety of booklets, emphasizing an activity approach, resulted from a series of summer workshops for the program's instructors; the workshops gave the participating faculty a chance to share ideas about innovative teaching techniques.

The booklets in mathematics included the topics of trigonometry, finite geometry, sets and logic, tools and concepts (arithmetic progressions, mathematical induction, logarithms and slide rule), probability and statistics, base numeration systems and introduction to computer programming, graphing, and functions. In addition there were a student "textbook" and an "illustrative materials" booklet consisting of sections taken from the other booklets. The preface of the latter booklet, entitled Mathematics, Thirteen-College Curriculum Program, Illustrative Materials, gave a good sense of the activity-oriented, discovery-approach which the program was trying to implement:

> We feel that the most important part of any teaching idea in mathematics is the beginning. How can we get students immediately involved and doing mathematics? . . .
>
> What makes a good beginning? There is no pat answer, and we certainly do not want to give the impression that we have answered that question. We have looked over our best materials that involved students and have gotten them excited about mathematics. We have discerned four very general features:
>
> First, the best units begin with real life situations and try to draw from these situations problems and questions that indicate the mathematics hidden there: life expectancy, the number of leaves on a tree . . . buying and selling, . . .
>
> Second, we have searched for ways to begin classes so that very soon students are doing something. We have avoided like the plague situations where the teacher must explain a great deal before the student can actually do anything: make constructions, calculations, draw pictures, move around, or whatever. . . .
>
> Third, we have sought out investigations which would allow the student to do the discovering. . . . What are the theorems [needed] . . . ? It would be easy enough to teach [concepts] that would explain the system, but how much more interesting it is for both the teacher and the student to begin with the problem and see what the students come up with. . . .
>
> Fourth, we hope to have enough materials that will eventually lead students to some of the most exciting, and accessible mathematics, such as topology, probability, number theory, computer programming, abstract algebra, and others. Why shouldn't students be confronted with these most delightful and challenging questions that have plagued mathematicians and have led them to the development of the rich areas of mathematics? . . .
>
> Finally, we recognize that students everywhere need practice in the skills of mathematics and that these can best be achieved when students are doing something they want to do and need to learn. (Institute for Services to Education, 1973, pp. i, ii)

The usual topics in mathematics were covered but the approach was clearly different in emphasis from the traditional lecture-expository approach.

At Bennett College the program lasted until 1972 when it was phased out. The intention of fully involving instructors in the regular mathematics program was not fulfilled; furthermore, it was found that students in the program did not have the necessary prerequisites to be successful in more advanced courses in mathematics in the regular college program.

The Thirteen-College Program was the subject of two dissertations. Dorothea Smith (1976) investigated the effectiveness of the materials and methods in mathematics on attitude changes toward mathematics, self-concept, and achievement in mathematics in the program by looking at four separate "generations" (1967-1970). She concluded that there was "no significant difference in self-concept between the ISE and non-ISE group," (p. 4435) but there were significant differences in attitude and achievement. Annette Billie (1976) found in addition that the teachers were enthusiastic about the method of teaching, the students felt that the materials were "more germane to their interests and needs" (p. 1442), and the program resulted in lower attrition, more positive attitudes, students' becoming active in their own learning, and teachers' becoming innovative in developing curriculum and being sensitive to the needs of the students.

In its recommendations for the college general education mathematics course, the Committee on the Undergraduate Program cited the Thirteen-College Curriculum Program as a worthy example of the kinds of innovations which could be made to increase the relevancy and effectiveness of the course:

> The Thirteen-College Curriculum Program has made very successful use of this technique [pointing out to students the intellectual similarity between dealing with puzzles and with solving ordinary mathematical problems] and has found it a very useful and efficient way to lead students into a study of mathematics that the students rejected originally. This material contains stimulating problems that have been used successfully by the participants in the Thirteen-College Curriculum Program to engage the attention and reasoning of students taking courses at approximately the level of Mathematics E [the recommended Basic Mathematics Course]. (CUPM, 1971, pp. 278-279)

The Thirteen-College Curriculum Program deserved a better fate for it truly tried to involve students in questioning and exploring. However, it failed to expand beyond the group of 27 predominately black colleges and it continued to focus on the same mathematical topics found in most other liberal arts mathematics courses around the country. In its implementation its emphasis was on solving questions which were fundamentally mathematical in nature, rather than on solving problems relating to the everyday lives of the students.

Problem Solving and the Real-Problem-Solving Curriculum

In reviewing the literature about the college general education mathematics course of the 1970's, one is struck by the almost complete lack of mention of problem solving in any of the expository articles, descriptions of innovations in the classroom, and the research reported in the various dissertations. The CUPM (1971) recommendations called for relevancy, choosing topics of interest to the student, improving skills of translating word problems into mathematical formulas, estimating, and the ability to apply algorithms. Where is problem solving? Buried in its 58 pages among the listings of content material, the descriptions of the suggested mathematics laboratory, and the sample exercises was the following paragraph:

> The following points should be considered in the selection and presentation of examples and exercises: (1) The majority of problems should describe familiar situations in which variables represent quantities that could reasonably be expected to be unknown. (2) To develop the ability to translate English into mathematics, it may be necessary to begin with problems that students are able to solve without the aid of an equation or other model. However, at several points in the course, students will have the background necessary to construct models for problems that they will not be able to solve completely. If full advantage is taken of this situation, then applications can be used to motivate the need for being able to solve equations and otherwise manipulate models to produce solutions. (3) Students should frequently be asked to estimate an answer to an exercise prior to writing the equation or formulating a model, and to explain the process by which the estimation was made. . . . (4) Students should be encouraged to describe problems which they have actually encountered and which they would like to be able to solve. The experiences of students with charge accounts,

> savings accounts, tax problems, other college courses, . . . , and situations that occur in playing cards and other games can be excellent sources of problems at various times within the course. (p. 279)

Item (4) closely agrees with the rationale of the real-problem curriculum of the Unified Sciences and Mathematics for Elementary Schoool (USMES) program, but this recommendation was not carried out during the 1970's.

Articles about problem solving in college-level mathematics dealt almost exclusively with problems of a strictly mathematical nature. Alan Schoenfeld, perhaps the leading writer and researcher on problem solving in mathematics, for example, discussed in detail four strategies of problem solving he had used, but applied these strategies to indefinite integration. John Lucas (1974) wrote about "The Teaching of Heuristic Problem-Solving Strategies in Elementary Calculus," Johnson and Harding (1979) wrote about the relationship of computing courses and mathematical problem-solving ability at Cambridge University, and Katchalski, Klamkin, and Lieu (1981) described problem-solving techniques in calculus and number theory. Three other articles about teaching heuristics in problem solving by Schoenfeld appeared in print--one in the Journal for Research in Mathematics Education (1979), one in The American Mathematical Monthly (1978), and one in the 1980 Yearbook of the National Council of Teachers of Mathematics (1980a) which was devoted entirely to problem solving in school mathematics. However, these various articles provided no help for the teacher wishing to consider the relevance and uses of mathematics rather than the solution of some esoteric problems in mathematics per se.

Status of Problem Solving in the Late 1970's

While the literature on the college level did not reflect the growing interest in problem solving found elsewhere in the mathematics community, much work was being done on the elementary and secondary school level as well as among psychologists, particularly those interested in information-processing theories of cognition. The National Council of Supervisors of Mathematics at its 1976 annual meeting made this remarkable proposition:

> Learning to solve problems is the principle reason for studying mathematics. (1977, p. 19)

Edwin White summarized his perception of this movement:

> Beginning in the late 1960s and continuing into the 1970s, a small but growing movement has supported the notion that the primary purpose of education is to develop problem-solving or decision-making abilities. This would appear to be a logical pragmatic goal, since if we analyze our own lives and activities, the common denominator is a daily need to make decisions and solve problems. (1978, p. 183)

Several surveys in the late 1970's and in the 1980's further indicated the greatly increasing interest in problem solving in precollege education, despite the emphasis of back-to-basics at the same time from outside pressures.

Carpenter, Corbitt, Kepner, Lindquist, and Reys (1980) in studying the results of the mathematics portion of the second National Assessment of Educational Progress (NAEP) found that "respondents demonstrated a lack of the most basic problem-solving skills" (p. 338). They found that students, with reasonable proficiency in computational skills, tried various sorts of calculations without analyzing the problem and devising a plan. The NAEP results

> . . . provide some insights into students' problem-solving abilities, but they offer no evidence regarding effective ways to teach problem solving. These results do suggest, however, that mathematical knowledge and skills level do not automatically transfer to problem situations. Although the specific question of instruction is not addressed by the assessment, we believe that in order to improve students' problem-solving abilities <u>specific attention must be given to teaching problem-solving strategies</u>. (p. 430)

Teacher attitudes about problem solving and its relationship to basic skills were also assessed, for as Carpenter et al. observed, teaching basic skills apart from problem solving does not automatically insure the ability to solve problems. Denmark and Kepner (1980) reported on a survey sponsored by the Instructional Affairs Committee (IAC) of the National Council of Teachers of Mathematics to answer the question "What is NCTM's role in the back-to-basics movement?" They found that most teachers agree that "teaching basic skills should not be isolated from development of motivation, mathematical appreciation, and maturity" (p. 117). Of the sample of persons involved with mathematics education, 86% agreed with that statement. However, a majority of those who responded (54%) felt that the basic skills should be mastered before concepts and applications were taught. In a separate question only 14% felt that concepts and applications should be introduced before developing mastery of basic skills. As Treadway observed:

> This survey reflects what is evident in most of our classrooms--we emphasize the skills and topics to be covered first and find that we have little time left for problem solving and applications. We wonder why the students have so little interest in the topics that we choose and why the students question the relevance of the subject material they are asked to study. (1981, p. 4)

The National Council of Teachers of Mathematics also sponsored an extensive survey of the opinions of various sectors of the mathematics-education community toward the various recommendations in its Agenda for Action (1980). The project, called Priorities in School Mathematics (PRISM), interviewed persons from nine different populations including parents, supervisors, readers of the Mathematics Teacher, readers of The American Mathematics Monthly, and instructors of mathematics classes in two-year colleges. Among the findings discussed in two reports by Osborne and Kasten (1980) and the members of the PRISM Project in its executive summary of the PRISM Project (NCTM, 1981) were the following which were particularly relevant to real-problem solving and the college liberal arts course in mathematics:

(1) From all populations in the survey, there is strong support for problem solving in the classroom.

(2) Instructors from two-year colleges agree with the need to emphasize strategies for problem solving in

instruction, particularly the ability to translate problems into number and algebraic sentences, to search for patterns, and to solve a simpler problem before extending the strategy to the original problem.

(3) No extreme position concerning teaching methodology for problem solving was preferred. However, Osborne and Kasten observed:

> Although "projects that involve real-life problem situations" was a methodology statement highly ranked by all professional samples, the older the students taught by a sample, the lower the ranking. That is, the methodology involving real-life situations was most important to the AT [Arithmetic Teacher] sample, less important to the MT [Mathematics Teacher] and two-year college samples, and least important to the college-level mathematics sample. (1980, p. 57)

(4) Problems should be used as the vehicle to develop and introduce mathematics topics. But Osborne and Kasten questioned this result:

> . . . we know of little evidence that the indicated preference for introducing ideas through problems is typical of current classroom practice. Perhaps the appropriate question is, "If this is a widely held belief about the role of problem solving in teaching mathematics, then what situational variables in the school setting or in the mathematics classroom restrict using problems as vehicles for the development and introduction of mathematical concepts?" (p. 57)

(5) The professional samples gave a low ranking to giving long problems taking more than a single class session to solve (e.g., a USMES style of problem) (Osborne & Kasten, 1980, pp. 56-57).

(6) Support was "minimal" (59%) for offering interdisciplinary work in problem solving (NCTM, 1981, p. 13).

(7) Resources for teaching problem-solving which were most strongly supported included resource guides to real-life problems, materials for modeling problems and solutions, and supplementary materials with additional problems.

(8) Strategies for teaching problem solving which were most strongly supported included projects involving real-life situations for individuals or teams, problem assignments, and using problems to introduce mathematical topics.

Status of the Teaching of Problem Solving

The results of the various surveys indicated widespread interest in enhancing the problem-solving skills of students but also indicated an uncertainty as to how to carry this out, since problem solving has not been described in terms of traditional curricular design terms (Osborne & Kasten, 1980, p. 53). Jeremy Kilpatrick, one of the leading researchers in problem solving, and E. G. Begle, a leading mathematician and mathematics educator, assessed the research literature in problem solving in the 1970's and found it lacking and yielding insufficient results to be able to provide truly satisfactory answers. Kilpatrick wrote in 1978:

> Problem solving is perennially a popular topic for research in mathematics education, probably because most people recognize both the centrality of problem solving as a goal in mathematics instruction and the difficulty of teaching other people how to solve mathematical problems. . . .
>
> Anyone masochistic enough to sift through the pile of research literature on problem solving in mathematics will likely come away depressed, partly because of the numerous flaws and limitations of the research studies themselves, but primarily because of the routine exercises used as "problems" in so many studies. (p. 189)

Begle cautioned that "simplistic efforts to improve our students' problem-solving abilities will not be enough. The task is much more complex than that" (1979, p. 144). Furthermore, Begle stated that coming up with a certain collection of strategies used successfully by students in solving problems will be quite difficult. "In fact, there are enough indications that problem-solving strategies are both problem- and student-specific often enough to suggest that hopes of finding one (or few) strategies which should be taught to all (or most) students are far too simplistic" (p. 145).

Other researchers have more recently taken a more positive view. Mary Kantowski (1981), Frank Lester (1980), and Marilyn Suydam (1980, 1982) have reported their findings from the research literature that characteristics of good problem solvers can be identified, the general nature of problem solving can be examined, and problem-solving strategies can be taught. Suydam summarized her findings as follows:

> When heuristics are specifically taught, they are then used more and students achieve correct solutions more frequently. Training in a variety of heuristics is necessary so that students have a repertoire from which they can draw as they meet the wide variety of problems that exist; different mathematical content evokes different strategies. Various strategies are used in different stages in solving problems. Thus training on both integration (the capacity to integrate remaining components in a sequence of operations required for problem solution) and evaluation (capacity to judge whether the solution is correct) seems to be required for solving problems. . . . (1980, pp. 43, 44)

Kilpatrick, while generally agreeing with Suydam's assessment for explicit attention to teaching problem solving, remained more cautious:

> The research literature has sometimes been interpreted as suggesting that in mathematics instruction explicit attention to problem-solving methods is wasted. A more careful review, however, ought to yield a different interpretation. It appears that techniques which attempt to "program" the solver to follow a fixed sequence of steps are not very effective. Further, improvement in problem-solving abilities does not occur overnight. And one should not expect transfer to problems that are quite different from those used in instruction. Experience and some research suggest, however, that certain heuristic procedures which will improve mathematical problem-solving performance can be learned--provided the teacher illustrates how the procedures work; gives ample opportunity for discussion, practice, and reflection; and supports and encourages the learner's efforts. The evidence on this issue is far from definitive, but it is encouraging enough to warrant consideration. (1978, p. 191)

Investigations by Kantowski (1975, 1977), Maracucci (1980), Schoenfeld (1979, 1980b) and Webb (1979) have given credence to these conclusions. In many of these surveys and research results little is said about finding problems that are interesting or are important to the student. Problem solving takes on many forms, and while some advances are being made in dealing with classroom problem solving, it is not clear how much of this will transfer to solving problems outside of a mathematics class, which should ultimately be our goal. It is not the intention here to consider all of the various aspects of problem solving. Only a brief summary of the situation at the end of the 1970's when this study was begun will be given here, for to give an in-depth examination would detract from the emphasis on real-problem solving.

The Many Facets and Definitions of Problems and Problem Solving

In considering effective ways to help students become good problem solvers, the many facets and interpretations of problem solving should be examined. It can and has been interpreted as a routine endeavor, trouble shooting, solving exercises in textbooks, working puzzles, and solving creatively difficult problems. Problem solving can be considered as a goal, as a process, or as a skill; clarifying the differences between the three interpretations can help us understand pedagogical ramifications. Nicholas Branca (1980) pointed out that in stating the main reason for teaching mathematics as the assistance it gives in solving problems, we are emphasizing the goal:

> When problem solving is considered as a goal, it is independent of specific problems, or procedures or methods, and of mathematical content. The important consideration here is that learning how to solve problems is the primary reason for studying mathematics. This point of view influences the tone of the entire mathematics curriculum and has important implications for classroom practice. (pp. 3-4)

When interpreted as a process, the emphasis is on the methods and strategies used to consider the given facts of the problem, developing some sort of plan of attack and substrategies, and understanding what is needed or desired in terms of output or a solution. The new emphasis on problem solving today is focusing on the interpretation of problem solving as a process. Finally, as Branca pointed out:

> In interpreting problem solving as a basic skill, one is forced to consider specifics of problem content, problem types, and solution methods. The focus is on the essentials of problem solving that all students must learn, and difficult choices need to be made regarding the problems and techniques to be used. (p. 6)

There is a danger in looking at problem solving strictly as a basic skill, teaching specific and disconnected methods for specific kinds of problems as in a list of minimal competencies for basic skills in calculation.

In any discussion of problem solving, a clear definition of a problem is required, so that there is no misunderstanding between the author and reader. Henderson and Pingry (1953) have suggested a set of very useful criteria which have been adopted by others (Kinsella, 1970, p. 242; Lester, 1980, p. 30). Henderson and Pingry listed the following necessary conditions for the existence of a "problem-for-a-particular-individual":

> 1. Individual has a clearly defined goal of which he is consciously aware and whose attainment he desires.
> 2. Blocking of the path toward the goal occurs, and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block.
> 3. Deliberation takes place. The individual becomes aware of the problem, defines it more or less clearly, identifies various possible hypotheses (solutions), and tests these for feasibility.

It follows from these conditions that whether or not a situation is a problem is subjective--it depends on the individual. Lester (1980) observed that the following are consequences of the definition of a problem:

> . . . in order for a situation to be a problem for an individual, the person must: (1) be <u>aware</u> of the situation, (2) be <u>interested</u> in resolving the situation, (3) be <u>unable to proceed directly</u> to a solution, and (4) make a <u>deliberate attempt</u> to find a solution. A <u>mathematical problem</u> then, is simply a problem for which the solution involves the use of mathematical skills, concepts, or processes. (p. 30)

Textbook "problems," the most frequently encountered kind, can be enigmas for students "who do not know what the symbols mean, or see no relationships whatsoever [since] the 'problems' have no chance of becoming problems" (Henderson & Pingry, p. 231). For students who can work a "problem" in a list of homework problems automatically, using well-known algorithms, e.g., solving a linear equation, there is no blocking and the problem is simply an exercise. These do not develop problem-solving abilities; rather they allow a student drill on skills and algorithms. But for a person who does not know the algorithm, the problem becomes a challenging one, requiring a variety of approaches.

Henderson and Pingry emphasized that the first condition involves desiring the attainment of the solution:

> Whether textbook "problems" are problems can be determined by examining them in terms of the three necessary conditions. . . . When this is done, the only conclusion is that it all depends on the student's reaction. If he accepts the "problem" as his own (that is, if his ego becomes involved), then the solution of the "problem" has met the first conditions for a problem.
>
> It is important to note that it really makes no difference whether the student poses the "problem" (question) for himself or whether the teacher or textbook poses it. The crucial factor is the extent to which the student's ego becomes involved in the problem. (pp. 231, 232)

All too often the textbook problem is one for which the student response is "who cares?" The question "How is .1 (base ten) represented in base sixteen?" may be interesting to the teacher, but it will not be exciting for many students. How much better for the teacher and the student if the problem is generated by the student and the solution to the problem is strongly desired by the student.

> Problem solving generally involves the
>
> coordination of knowledge, previous experience, intuition, and various analytical and visual abilities in an effort to determine a workable outcome to a situation regarded as a problem; furthermore a procedure for completely determining the outcome is not already known. (Lester, 1980, p. 32)

The entire subject of problem solving is so complex that to be able to understand it better, several authors have broken it down into categories and examined it one aspect at a time. Claire Hill (1979) in her helpful annotated bibliography, Problem Solving, Learning and Teaching, separated the entries into 11 groupings, ranging from problem solving as using associations to problem solving as a goal and as solving exercises (see Appendix A). Mary Kantowski (1981) has suggested separating problem solving into three distinct categories: verbal, non-routine, and real. Three steps seem to be most often used to solve verbal problems: recognizing the structure of verbal problems, selecting an appropriate algorithm, and correctly applying the algorithm (Knifong & Holtan, 1976; Meyer, 1978). "Skill in computational processes is necessary for solving problems . . . but having these skills does not guarantee successful problem-solving" (Kantowski, 1981, p. 114). Certain standard techniques such as trial and error, drawing diagrams, organizing data into a table and trying simpler problems are all important in solving the non-routine problems. The four steps suggested by Polya are well-known: understand the problem, plan the steps, carry out the plan, and examine the solution. Other authors have suggested variations and extensions (Lester, 1980; Schoenfeld, 1980a, pp. 9-10).

The concept of a real problem is discussed in Chapter I. It is characterized as having no given boundaries; the correct solution cannot be found by applying a previously taught algorithm. A problem is considered real if it satisfies the following conditions:

> (1) the problem applies to some aspect of student life in the school or community, (2) a solution is needed and not presently known, at least for the particular case in question, (3) the students must consider the entire situation with all the accompanying variables and complexities, and (4) the problem is such that the work done by the students can lead to some improvement in the situation. (USMES, 1976b, p. 5)

Little research has been done on real-problem solving; indeed Kantowski's article mentioned only the work done in the USMES project.

Another useful way of looking at problem solving is to categorize the relationship between the three major parts of the procedure: the question (the input, what is given); the procedure, method or process; and the solution (what is to be found, the output). The following chart summarizes the seven possibilities.

		question	procedure	solution
1	typical exercise	given	given	
2	geometry proofs	given		given
3	forgotten computer program		given	given
4	tests, puzzles	given		
5	practice on algorithms		given	
6	unusual situation			given
7	completely open			

In many situations, particularly textbook problems, the questions are given and the appropriate method or algorithm has been explained, so that the only part left unknown is the answer or solution (type 1). A standard "supply-the-proof" question in a high school geometry course begins with the questions (hypothesis) and the solution (conclusion) and asks for the procedure or proof (type 2). An example of type 3 is the computer program, written six months ago following the procedures of good programming so that it runs properly (solution), but being poorly documented, the purpose of the program (question) has been forgotten. Type 4 follows the format of a typical exam question where the question is given but the appropriate method and solution must be determined. Types 5 and 6 are unusual, but may be occasionally given as variations to the other kinds of problems.

The only one of these forms which approximates problems found in the "real world" is the last variety where the question must be formulated by the student as must the method of solution and the conclusion. It is this latter kind of problem which is featured by USMES. It is to be hoped that the increased ability to solve the more closed and restrictive of these kinds of problems will develop the necessary cognitive skills to solve some of the more open kinds. But this will probably not happen without the student's being exposed at some time to the more open problems. As Loman et al. (1975) pointed out, the open-ended problem has a distinct character; arranging experiences to help students solve this kind of problem should be the ultimate goal of our teaching. They made the following distinction:

> There are a variety of uses of the phrase "real problem" which are not relevant to the need for practical problem-solving skills. Sometimes "real problem" is used to denote a fragment of the whole situation at hand with boundaries carefully drawn and controlled. Often it means a made-up problem which however is consistent with physical law and the general societal structure. An "application" of a mathematical formula or a science concept is an attempt to relate such abstracts to reality, but it eliminates the need to <u>search</u> for the knowledge and methods which may apply.
>
> Problems which are real in the above ways may have many educational uses, but they will not prepare a person to cope with the complicated and many-faceted problem situations met in real life. (p. 54)

A detailed examination of the real-problem-solving curriculum and the USMES program as an important background to this study of the use of the concepts and instructional strategies of the USMES program in a college general education mathematics course is described more fully below.

The Real-Problem-Solving Curriculum

Since the time of John Dewey there has been an interest in making education relevant and meaningful to the student.

In Dewey's Laboratory School at the University of Chicago, children learned about the work of man and his occupations by being involved in projects or problems rather than passively following information given by the teacher. Dewey argued that children would learn to think if they were actively trying to solve problems of genuine interest to them. Dewey summarized his philosophy in his book, Experience and Education (1938).

Basic to his own philosophy is a single "permanent frame of reference--namely the organic connection between education and personal experience; or that the new philosophy of education is committed to some kind of empirical and experimental philosophy" (p. 25). Truly educative experiences need to be agreeable for the student (arouse curiosity, strengthen initiative), to be based on some realization of the student's past experiences and to "live fruitfully and creatively in subsequent experiences" (p. 28). This idea Dewey called the principle of Continuity of Experience. Furthermore, experience not only changes the person inwardly (formation of attitudes, new knowledge) but it also changes the objective conditions under which experiences take place. Equal importance then should be assigned to both the internal and external factors which interact with each other. This was Dewey's second principle of experience. Finally, Dewey argued that the principles of continuity and interaction themselves intertwine and provide a basis on which to judge the measure of an experience.

What are some of the ramifications of these principles? Traditional education is deficient because it emphasizes the external conditions of learning rather than the kinds of internal experiences the students have. The subject matter as viewed in traditional education consists of skills, facts, and cultural heritage from the past which should be taught to all students, who may need to know the material for future courses, jobs, and responsibilities in the future. But preparation, for Dewey, is a "treacherous" idea if it is believed that acquisition now of unrelated facts and truths will be useful at some time in the future or that a student will be able to make effective use of these facts in new and different situations and considerations. Such a false assumption violates the principle of continuity, which implies that education must deal with present meaning and understanding as well as with the truths of the past and preparation for the future. At the same time Dewey warned that we should not make the opposite mistake which progressives frequently make of emphasizing only activities of the present.

The routine lecture, discussion, and homework sequence of activities found in the traditional classroom spawns boredom and leads to what Dewey called "miseducative experiences." But he cautioned against making the opposite mistake of allowing only those experiences which are completely agreeable for the student, determined by the wishes and desires of the students which may not be worthwhile or "educative." The trouble with the former experiences is that they take into account so little of the internal factors of an experience, thus violating the principle of interaction from the "internal side." On the other hand, educational theories which "subordinate objective conditions to those which reside in the individuals" (p. 58) fail to take into account the importance of external ways (equipment, books, manipulative materials and social situations) as well as the expertise gained from the experience of the past. Dewey observed:

> [The teacher] must survey the capacities and needs of the particular set of individuals with whom he is dealing and must at the same time arrange the conditions which provide the subject-matter or content for experiences that satisfy the needs and develop these capacities. The planning must be flexible enough to permit free play for individuality of experience and yet firm enough to give direction towards continuous development of power. (Dewey, p. 58)

There was a third major issue which Dewey discussed, namely that of individual freedom versus external control. First of all, Dewey argued for democratic and humane arrangements in setting up an educational framework:

> Does not the principle of regard for individual freedom and for decency and kindliness of human relations come back in the end to the conviction that these things are tributary to a higher quality of experience on the part of a greater number than are methods of repression and coercion or force? (p. 34)

A goal of education should be the development of the ability to exercise self-discipline and self-learning. Neither an authoritarian approach nor a complete lack of control will lead to this end. Freedom is not an end in itself and the freedom of the teacher is as important as the freedom of the students. For Dewey, the planning of the teacher must be such that the teacher becomes a collaborator and a leader, not a boss.

Finally, we consider Dewey's concept of how subject matter is a part of a theory of experience. Important objective conditions have been linked with the learning of subject-matter, and include observation, memory, information derived from books and teachers, imagination and problem-solving; but these must be involved in integrated and meaningful activities. The problem of instruction is that of finding material and classroom experiences which develop previous experiences "into a fuller and richer and also more organized form, a form that gradually approximates that in which subject-matter is presented to the skilled, mature person" (pp. 73-74). The starting point of instruction should be determined in terms of what the child already knows, and the orderly development of concepts should be made in terms of careful consideration of facts and ideas. But basic ideas cannot be picked up in a confused, cursory manner (p. 79).

When classroom experiences lead to unfamiliar ideas, problems arise; these problems stimulate thinking and often provide motivation for the learners to pursue solutions. The educator must consider two aspects <u>equally</u>:

> First, that the problem grows out of the conditions of the experience being held in the present, and that it is within the range of capacity of students; and,

> secondly, that it is such that arouses in the learner an active quest for information and for production of ideas. (p. 79)

Experiences should involve the organizing principles of analysis and synthesis. Using the scientific method as a prototype of a way to investigate and organize ideas is highly recommended, although this should not be done with young children. Through this method, with proper attention to the organization of facts and ideas, truly successful educational experiences may be obtained.

In a 1918 article entitled "The Project Method" in the Teacher's College Record, William Kilpatrick argued in a similar vein that education should be considered as life itself and not simply as a preparation for life or for some course to be taken later. Seven years later Kilpatrick wrote in his book Foundations of Method: Informal Talks on Teaching that the practice of teaching narrowly conceived bits and pieces of knowledge and skills to children was ineffective and ill-advised. Believing that children at some time could effectively unify the isolated components of their education was doing a great disservice to the children (White, 1978, p. 184).

During the Depression, partly due to the instability of the economic and social situation, the development of the problem-solving approach to the teaching of mathematics was minimal. However, in 1932 and 1933 the Progressive Education Association helped to bring together 30 secondary schools throughout the nation to develop and evaluate alternatives to the usual practice of "subject-matter acquisition." The project was called the Eight-Year Study, and was characterized as follows:

> These schools sought to plan their curricula around the problems arising from the personal-social interactions of the individual in the various aspects of living. Using the categories of personal living, immediate personal-social relationships, social-civic relationships, and economic relationships, the schools attempted to determine basic adolescent needs in a given situation, and to design units of study to meet these needs in such a way as to develop the characteristics of personality needed for effective functioning in a democratic society. This may be called the adolescent-needs approach to core course organizations. (Giles, McCutchen, & Zechiel, 1942, p. 44)

The interest in problem-solving goals grew during the 1940's and 1950's, but the traditional approach continued to dominate. Several states moved to a "social-functions curriculum" which emphasized the teaching of problem solving encompassing several subject fields (White, 1978, pp. 186-187). A number of educational leaders, including Harold Alberty, Roland Faunce, and Morris Kline, called for integrating the teaching of mathematics with other subjects. At the same time the essentialists, those who felt "that our democratic way of life would be continued only if each and every child comprehended the uniqueness of all separate subjects in the same manner as did the scientists, mathematicians, and scholars who represented those disciplines" (White, 1978, p. 187), argued for their differing educational point of view.

The success of the launching of Sputnik by the Russians in 1957 provided the catalyst for the essentialists to push their curriculum ideas, leading to the so-called reforms of the 1960's. Many of these reforms, such as the School Mathematics Study Group (SMSG) and the University of Maryland Mathematics Project (UMMaP), did develop materials which were less drill-oriented, but they emphasized the deductive approach to the teaching of the material. While some of these projects began to provide opportunities for explorations and integration of some mathematics and science topics, they were mainly content-oriented.

The strong advocacy of Morris Kline in the late 1950's and the 1960's for the use of real problems to motivate the learning of mathematics and to provide a source of experiences in problem solving has been outlined earlier in this chapter. In 1961 Professor Robert B. Davis spearheaded the Madison Project and Professor Paul Rosenbloom directed the MINNEMAST Project, both of which attempted to put into practice mathematics teaching following philosophies similar to those stated by Morris Kline. During the 1960's the development of the teaching of mathematics was greatly influenced by three gatherings in Cambridge, Massachusetts, of mathematicians and mathematics educators. The dominant message of the first two conferences was that of essentialists, rather than that of Kline, Davis, and Rosenbloom, with problem solving relegated to a place of minor importance, and defined in a restricted way, namely, the posing of word problems which were made up by the textbook writer and which indicated the writer's perception of the children's needs. The justification for the recommended approach was based on the conviction that children would learn best if the ideas were introduced with an emphasis on the unifying structure of the discipline.

> If the high school mathematics program was to incorporate new content in a curriculum organized around powerful but abstract structuring concepts and processes, preparation in elementary and junior high school had to change too. At the same time [Jerome] Bruner forecast improved acquisition and transfer through focus on the structure of disciplines, he re-emphasized concern for psychological issues like readiness, intuitive versus analytical thinking and concrete versus formal experience in learning. These curricular and psychological forces had a strong impact on thinking about goals for mathematics instruction K-8. (NACOME, 1975, p. 3)

These reports lacked any consideration of what students might be interested in learning, any attempt at integrating mathematics with science in a meaningful way, any effort at making the learning of mathematics enjoyable, and any notion of the conception of real comprehensive problem solving (Webb & Ost, 1978, p. 199).

The third conference, the 1967 Cambridge Conference on the Goals for the Correlation of Science and Mathematics, sought to bring a balance to the directions of curriculum changes. One main theme of the conference was the consideration of the teaching of mathematics and science in an integrated fashion. The report for the Conference argued that combined teaching gives greater strength and an opportunity to look at problems from two different points of view. It warned against assuming that the relatively few children who could learn abstractions would be able to transfer the general principle to important specific cases. Finally, "the very use of concrete examples and new concepts and motivations emanating from the science side increases the flexibility of the organization of mathematics instructions" (Cambridge Conference, 1969, p. 8). Indeed, abstractions can be learned by most students better through experiences with their concrete representations.

The report admitted that a mathematics course emphasizing solving of real problems would not be easy to achieve. It would require a major change in the style of school instruction, involving semi-individualized and small group learning. It would allow teachers and curriculum planners to ask some important questions, e.g., why do we spend so much time on discrete situations and not on measurement of continuous quantities? What is most important for the child?

> We believe that the primary goal of science education is an understanding of scientific methodology, and that the goal of mathematics education is a familiarity with logical reasoning, particularly as it concerns quantitative reasoning. We specifically do not include any particular bit of scientific knowledge or any particular mathematical technique. . . . Only a sample of [the various topics] could be taught to even the most brilliant children and by an approach depending completely on memorization. Since we hope that most of the effort will be spent on teaching the scientific

> method, we must expect that children will acquire only a small sample of scientific knowledge. Moreover, we see no reason why one child's sample should be the same as another's. (Cambridge Conference, 1967, pp. 9-10)

While the conference report focused on the ability to solve real-life problems rather than on the learning of specific topics, it did devote an entire chapter to the discussion of some of the more specific educational objectives of the integrated curriculum. Conceptual models should play a central role. The student is to learn how to pose a question in order to solve a problem of relevance to him or her and then to find sufficiently simple models which underlie the complex situation. Ability to manipulate the model mathematically allows for prediction and control of the behavior of the real objects. In order to obtain solutions, observation, reason, and factual knowledge need to be used. Certain attitudes are needed for modeling, among them are included a feeling for the relative significance of the different aspects of an observed situation, a facility for reducing the problem to a simplified abstraction, an ability to estimate before and after doing calculations, and a facility to make computations, both mentally and with calculators. It is also expected that the student should learn certain topics in order to be able to solve real-life problems most effectively. Among these topics are included inequalities, the order structure of real numbers, real number addition and multiplication, the laws of number operations, the algorithms, functions and graphs, intuitive geometry, probability and statistics, and deductive thinking. Beginning ideas in each of these areas can be begun in the early grades and learned more deeply as needed by the student.

In 1973 there were four mathematics conferences, each of which included in its report a statement calling for combining mathematics and other disciplines, for teaching mathematics as an exciting, enjoyable subject, and for providing experiences which directly involve students. These conferences were the Orono Conference on the Middle School Mathematics Curriculum, the Cape Ann Conference on Junior High Mathematics, the Snowmass Conference on the K-12 Curriculum and the Estes Park Conference on Real Problem Solving in adolescent education. Hence, while the interest in real-problem solving in the school curriculum was being discussed by a large number of participants in the early 1970's, the lack of similar interests and activities with real-problem solving on the college level, as we have observed, is all the more surprising. Furthermore, during the same decade the USMES project developed into a major curricular program with over 20 booklets (study guides), a series of how-to booklets, background papers, workshops and implementation in elementary schools throughout the United States.

<u>Unified Sciences and Mathematics for Elementary Schools (USMES)</u>

The Unified Sciences and Mathematics for Elementary Schools (USMES) Program was an outgrowth of the recommendations of the 1967 Cambridge Conference and was directed by one of its participants, Dr. Earle Loman. It flourished for approximately 10 years starting in 1970, was coordinated by the staff of the Educational Development Center in Newton, Massachusetts, and was funded by the National Science Foundation. We have already considered the main principles of USMES in Chapter I and the definition of the open-ended, real problem earlier in this chapter. We will look at the USMES program in some detail here.

A real problem originates from an experience in the real world and "connotes a practical, immediate impediment to good, safe and pleasurable living" (Loman et al., p. 54). Such problems are selected by the students or the entire class and arise from difficulties faced by the students--what is the best way to learn, what is the best buy with a limited amount of money, or can cafeteria service be improved? Dealing with problems involves identifying the variables, seeing if they can be quantified, making observations, setting up models, talking with others and making value judgments.

> A real problem is then ill-defined, complex, and many-faceted. Tackling such a problem successfully requires more than the ability to handle the range of specific things that will need to be done. The more important task is to make sense out of the vagueness, relate the many facets, and pull together the strategies which will ameliorate the situation. Thus, the cognitive skills needed are flexible and integrative, not specific and hierarchical. (Loman et al., pp. 54-55)

The basic assumption that "experience is the best teacher for the attainment of proficiency in the complexities of problem solving and decision making . . ." (Loman, n.d., p. 1) is, according to Loman, based on research done by Gagne. Gagne (1971) came to the conclusion that "cognitive strategies" must be developed differently than "intellectual skills." In order to help a student become a "good thinker," the teacher must provide many opportunities for the student to "encounter, formulate and solve problems of many varieties" (p. 522). USMES was developed as a curriculum to provide opportunities for the student to become a "good thinker."

The project was used most successfully in the elementary school. The teacher may present several challenges which arise from discussing issues in a broader context. The class, or small groups, accept the challenge and spend up to 60 hours in investigation, working out solutions, and testing the proposed solutions. If all goes well, student interest and motivation to learn are enhanced. Students may study mathematics skills as are needed to solve the problems and presumably learn these skills more meaningfully. There is less time needed for reteaching and reinforcing required skills. However, the USMES Guide made the following proviso:

> Because real problem solving activities cannot possibly cover all the skills and concepts in the major subject areas, other curricula as well as other learning modes (such as "lecture method," "individual study topics," or programmed instruction) need to be used in conjunction with USMES in an optimal education program. However, the other instruction will be enhanced by the skills, motivation, and understanding provided by real problem solving, and in some cases, work on an USMES challenge provides the context within which the skills and concepts of the major areas find application. (1976b, p. 30)

A variety of resource materials were used. Twenty-six problem units were produced for grades 1 through 8. Teacher manuals for each of these units gave suggestions for classroom and field activities in order to carry out the investigations. "How To" cards and "How To" booklets for the students discussed how to collect data, simplify data, measure, graph, make electrical devices, and build measuring devices. These were very helpful as resources. An integral part of the program was a design lab in a designated area of the school, where building materials and tools were to be kept to be used by the students in exploring their ideas. A list of the resource materials and a summary of the units are given in Appendix B.

There were eight stated teacher responsibilities. Teachers must first introduce the challenge in a meaningful way which allows the children to relate to it and open up general avenues of approach. They must be coordinators, collaborators, and assistants in the projects. They must help the children get involved in the challenge and provide opportunity for them to work on it two to three times a week. The teachers must make materials and tools available. They must let the children make their own mistakes and find their own way and be ready to point out sources of help for specific information. The teachers need to provide frequent opportunities for group reports and student exchange of ideas in class discussions, letting the children improve the ideas of the class. They should ask appropriate questions to stimulate student thinking, and to increase the depth of investigation and analysis of data. Finally, they should make sure that the groups are of sufficient size that the activities do not become fragmented or stalled (USMES, 1976b, pp. 7-8).

In addition to developing an elementary school real-problem-solving curriculum, USMES undertook the widespread diffusion of the program through workshops to inform teachers and administrators of its rationale and classroom procedures. The following description is taken from L. Davidman's evaluation of USMES (1976):

> In 1971-72, after a few development workshops which were not part of its Resource Team Workshop program, USMES/EDC committed itself to a series of two-week training programs which would diffuse its resource materials and philosophy on a nationwide basis. Primarily, this would be accomplished by (1) training teachers to be USMES classroom implementers and local change agents simultaneously, and (2) by setting up Regional Centers throughout the nation to carry on the diffusion efforts after USMES/EDC phased itself out in 1978. Where possible, regional USMES centers would be manned by university personnel and first generation USMES teachers. These teams, like USMES/EDC, would seek National Science Foundation funds and other sources of income to further USMES/EDC's broad goals as well as their own regional needs. USMES/EDC would cooperate in this post-1978 effort by publishing an occasional journal of articles relating to USMES implementation and diffusion. (p. 148)

In order to provide assistance for those who were workshop leaders, a book, <u>Preparing People for USMES, an Implementation Resource Book</u>, was published in 1975. It outlined the purpose of USMES informational meetings, namely, to engage in dissemination activities because of "the critical ways in which USMES differs from other curriculum programs," to inform personnel in target schools and associated school districts about the advantages of using USMES in the classroom, and to allow participants to experience real-problem solving by working on an USMES unit challenge at an adult level (p. 5). The entire list of key elements for an USMES Information Meeting is given in Appendix C. The workshops were an important part of the overall USMES project, but, according to Davidman, there were insufficient pre-workshop preparation of participants and a lack of careful selection of teachers who really wanted to participate in the workshops and in USMES.

The USMES project staff and a team of evaluators from Boston University undertook to assess the effectiveness of the program through the use of standardized tests, specially designed instruments involving a situational problem-solving exercise, in-depth interviews of teachers and administrative personnel, and various observations of interactions and activities within the classroom. Three questions were investigated in the process of evaluation. First was the question of whether or not USMES students attain higher or lower scores on basic scholastic skills in mathematics and reading. The Paragraph Meaning and Arithmetic computation subtests of the Stanford Achievement Test were given to control and experimental groups. The results showed that the USMES program did not hinder acquisition of basic skills (USMES, 1976b, pp. 34-35).

George Stalker (1978) found that in a reasonably high percentage of sessions involving USMES-type challenges, skills in one or more of the categories of mathematics, language arts, science, or social sciences were used and discussed. For example, in 67.1% of the sessions, some kind of mathematics skill was used. In summary:

> The correlation of skills practice with class discussion of the relation of recent work to solving the challenge had a positive correlation at the .01 level. In other words, the more USMES strategy was followed, the more basic skills were learned. (Loman, n.d., p. 10)

A second question considered whether or not USMES classes attained a higher ability to solve real-life problems than did students in regular classes. Shapiro (1973) developed a Notebook Problem which involved observing students' attempts to determine the best buy among a variety of notebooks. This is described in some detail in Chapter IV. Shann (1974) developed a Playground Design Problem and a Picnic Problem which were used in subsequent years. The USMES Student Study of 1976-77 (Arbetter, 1978) reported on the use of a Pencil Problem test which was administered to small groups of students. Children were selected from USMES and non-USMES groups at random and were presented with the current situational problem-solving test. For the Notebook Problem and the Playground Problem, the students were given an hour to try to find a solution. The Pencil Problem test "had a tightly organized administration and scoring protocol and could be administered to three to five students in 15 minutes. This made it practical as a diagnostic instrument" (Loman, n.d., p. 5).

The results of the Notebook Problem indicated that USMES students achieved significantly higher levels on the use of quantifiable variables and objective reasons for decisions (Shapiro, 1973). Problems in test design made the results in 1972-73 and 1973-74 unreliable, but the data showed no significant differences. The USMES Student Study found that students with more than 20 hours of experience in USMES-type projects scored significantly higher on the Pencil Problem than non-USMES students. The school which showed the most positive effects of USMES was the one whose faculty and principal also indicated the strongest support of the USMES program (Arbetter, 1978). Views of teachers indicated their impressions that the USMES program had taught problem-solving skills and effectively changed student behavior in that area (Loman, n.d., p. 8).

A third question dealt with the kinds of classroom organization, verbal interaction, interpersonal relations, and group dynamics which are typical in USMES classes. Trained observers visited 30 USMES classrooms and 30 non-USMES classrooms nine times during the year and noted the activities and interactions. USMES classes used small groups more often and changed the format of the class more often than non-USMES groups. Children in the USMES classes added their own ideas to discussions in large group meetings much more frequently than the students in non-USMES classes, while the non-USMES students gave only specific responses to specific questions more often. The USMES classes involved much more child-child interaction and in general showed more flexibility in the activities which took place. Finally, student responses to questionnaires about their feelings about USMES classes indicated that a large proportion felt that USMES was fun and interesting, 4% felt that USMES classes were boring, and 50% said that USMES was hard work (USMES, 1976b, pp. 35-37). The 1976-77 USMES Student Study used "A Questionnaire on Attitudes Toward Real Problem Solving" (QARPS) in which questions were classified in three categories: those dealing with attitudes toward producing effective results; those dealing with attitudes toward working with others in small groups; and those dealing with attitudes toward specific activities. Students who had spent more than 20 hours on USMES projects showed significantly stronger attitudes in each of these categories than students with no USMES experience (Arbetter, 1978).

In contrast to the fairly rosy picture painted by the USMES evaluation reports and the Loman article is the more detached assessment in the NACOME report:

> Evaluation completed during the 1974-75 school year indicates that these problem-solving experiences are attractive to students and teachers, who see many important mathematical ideas developed by the units. However, the extreme diversity in patterns of trial usage make it difficult to judge the potential of USMES curricula. If anything, USMES teachers have replaced regularly scheduled science time, not mathematics. Progress toward the goal of enhanced student problem solving ability is very difficult to assess. (NACOME, 1975, p. 27)

A major evaluation of the USMES program was undertaken by Leonard Davidman as a doctoral research project while he was a member of the Department of Elementary Education at San Jose State University. He used tape recordings of classroom lessons and interviews with 156 students, with teachers, and with administrators at one of the elementary schools in the California Suburban School District. Further, he made daily observations in one USMES classroom (9 hours a week from October 1974 to May 1975), interviewed teachers in five other USMES classrooms, used attitude inventories, carefully looked over publications written by personnel of USMES, surveyed 90 teachers and administrators involved elsewhere with USMES projects, and looked over literature from other developmental curriculum projects (Davidman, 1978, p. 397).

From this wide variety of information Davidman presented six findings and several recommendations about the USMES program. These are summarized below:

(1) The vast majority of the students did not seem to be coming into contact with problematical situations, partly due to the tendency of the teachers to become involved in the identification and the direction of the problem-solving effort. In many cases the students were not allowed to experience short-term frustration.

(2) The findings of

> . . . self-report evidence gathered by the Krutchfield/ Covington Attitude Inventory strongly suggests that the effective environments in three out of the six USMES

> classes were detrimental to the positive development of problem-solving skills and attitudes related to sharing, trusting, risk-taking, question-asking and respectful listening. (p. 400)

The teachers did not seem to help students build better human relationships in the classrooms. About 50% of the students remained reluctant to ask questions, felt that there had to be a correct answer before they would volunteer, and expected other children to reject their suggestions.

(3) About half of the students felt that they lacked the basic skills to start even the first steps in solving a problem, that they had missed some important idea, and they got rattled and confused when trying to think.

(4) The USMES developers "chose to work with the assumption that educational research had already validated certain aspects of their real-problem solving concepts" (p. 402). This attitude, according to Davidman, was manifest in the USMES decisions not to develop fully some of the material needed to help participating teachers understand the process of teaching, to make workshops "one-recipe affairs," and to try to disseminate rapidly the units and give the impression that the units were part of an established curriculum.

(5) There was too little reflective thinking on the part of the participants and too little feedback from administrators.

(6) A number of hidden learnings emerged: e.g., cooperation can lead to achievement of goals but so can competitive procedures; competition may be rigged; teachers may compete

rather than cooperate; and students can learn about large complex problems and at the same time not be mature enough to identify their own problems. Davidman concluded:

> It is difficult to assess the significance of the contradictory character of the USMES/CSSD's [California Suburban School District's] latent curriculum. This difficulty notwithstanding, I believe that it is wise to assume that the explicit, central objectives of a curriculum project will be more readily achieved when the explicit and implicit aspects of a curriculum reinforce each other. (1978, p. 405)

Davidman recommended that the staff of USMES (1) provide information and define questions to help teachers work with developing human relations, (2) give teachers suggestions in order to avoid pitfalls which occur when the class leaves the classroom and goes into the community, (3) develop a social-studies/math unit, and (4) stress the importance of "developing a proper affective climate to facilitate the emergence of real-problem skills" (1978, p. 406). For educators there are seven recommendations including the following two: "Be cautious about the claims you make regarding problem solving education. . . . [B]uild eclecticism into your design because it is conceivable, even probable, that differing types of problem-solving experiences reinforce each other" (1978, p. 407). Finally, Davidman reminded his readers that those activities and experiences which help the student grow personally and gain self-confidence, i.e., those experiences which "make teachers effective change agents," are not always the same activities which help students solve problems more effectively (1978, p. 408).

Difficulties in Trying to Improve the General Education College Mathematics Course

Most of the teaching in the college-level general education mathematics courses has been characterized by the use of the lecture approach and the teaching of disconnected mathematical topics. While a number of instructors have tried changes in curricula or teaching approaches with some success, few if any of these experimental courses have involved students in inquiries of direct importance to themselves or encouraged them to take responsibility for their own learning. While attempts in some elementary schools to use the solving of real problems as a framework to help students develop problem-solving skills and learn mathematics in the context of the immediate need for mathematics have been partly successful, similar attempts have not been made in the college mathematics course. Changes to improve the situation have not been, on the whole, successful.

One of the most obvious reasons is the narrow training of most of the teachers of this course. College mathematics teachers usually have earned their degrees in the mathematics departments of graduate schools in this county. Mathematics graduate courses are noted for extreme specialization, are taught in isolation, and are frequently characterized by the theorem-proof approach with little attempt to motivate. When the students in these courses become college teachers, they frequently don't know much about the history of mathematical ideas or how these ideas relate to the world outside of mathematics. The mathematics teacher, who has been enthralled with the tight logical connection between mathematical statements and by the beauty of abstract mathematical ideas, finds a tremendous gap in this thinking and the naive thinking of his students. Furthermore, the teacher has little or no training in other fields such as science and social science and is unable to foster an interdisciplinary approach.

Teachers who come from such a background find it very difficult to change their style of teaching. As Malcolm Knowles (1975), in describing his experiences in becoming a collaborator with students trying to take responsibility for their own learning, pointed out:

> In the first place, my concept has changed from that of teacher to that of facilitator of learning. This may seem to be a simple and perhaps even superficial change. But I found it to be fundamental and terribly difficult. It required that I focus on what was happening rather than on what I was doing. It required that I divest myself of the protective shield of an authority figure and expose myself as me--an authentic human being, with feelings, hopes, aspirations, insecurities, worries, strengths, and weaknesses. It required that I be clear about precisely what resources I did and did not have that might be useful to the learners, and that I make the resources I did have accessible to them on their terms. (pp. 33-34)

An instructor trying to implement a student-centered approach needs to become more involved with meeting the needs of students and to deal with personal feelings of frustration, uncertainties, and satisfactions. Many teachers, however, prefer to maintain the comfortable role as "authority figure."

Most colleges have requirements which may conflict with the goals of a class featuring alternative teaching methods and student involvement in solving real-life problems. Catalog course descriptions must be followed, grades need to be determined, and attendance policies are to be enforced. Teaching in an alternative manner frequently takes more preparation time and requires more involvement with students, particularly when individual students are working on different problems. The teacher must cope with students who are not used to an alternative method of learning and feel disoriented and frustrated when asked to take responsibility for their own learning. Finally, the teacher who really puts the needs of his students above the subject matter taught must be prepared to deal with different styles of learning and with varied concerns.

Another problem which has to be dealt with is the frequent poor background of students in mathematics and the less-than-enthusiastic attitudes which the students bring to the classroom. The task of finding real problems of a general interest which require in some way the use of mathematics for their solutions is not at all easy. Performance of this task requires getting to know the students in the classroom and perhaps outside of the classroom and trying to see their environment and their problems from their own point of view.

Teachers can easily be confused by the conflicting learning theories, e.g., the sequence theory development of Gagne with its emphasis on the need to learn a subordinate skill before learning a higher skill and the discovery learning approach of Bruner with its emphasis on the overall structure of knowledge. It is so much easier to adopt one approach and ignore the claims of the other, when in reality a blend of several approaches may produce the most effective learning for individual students.

Finally, mathematics is multidimensional and teachers are called upon to try to give their students a wide variety of learning experiences and to meet a long list of educational goals. They are asked to teach mathematics so as to display its historical, cultural, applied and abstract aspects; to teach problem-solving and basic skills. Major curricular changes in the past have had mixed success, due in part to misdirected planning, mistaken pedagogy, or in many cases teachers' inexperience in using the new curriculum. But there must be a procedure which generates some interest and enthusiasm for mathematics and which involves the students in what is really important, that of trying to solve some problems from real-life situations. The difficulty is to find out what that procedure is.

Research Methodology

The choice of the appropriate research methodology must ultimately depend on the answer to the question of how and where educators can get reliable and valid information in order to make decisions about curriculum and classroom procedures. The traditional "mechanistic mode" of scientific inquiry depends not only on random sampling (or an alternative) but also on a reasonably large number of subjects placed in control and experimental groups in order to satisfy statistical assumptions. Furthermore, it assumes that the research instruments used, such as attitude inventories and standardized tests, measure the constructs which they are intended to measure. In this study satisfying the first two assumptions (random sample and sufficiently large number of subjects) were impossible, and satisfying the assumption about the validity of the instruments can be challenged. It was clear, therefore, that alternative methods of inquiry should be considered. Indeed, as Frank Lester and Donald Kerr (1979) pointed out:

> Experimental studies force the experimenter to adapt the phenomenon being studied to the research methodology rather than require the experimenter to adapt the research methodology to fit the phenomenon. Although this approach may enhance internal validity (i.e., reliability), there is considerable danger that external validity (i.e., generalizability) is destroyed. It seems appropriate to place more emphasis on non-experimental research that does not require adapting the phenomenon in order to enhance the potential for generalizing results. (p. 230)

The intention was that the curriculum builder should undertake formative evaluation, separate from the situation, and try to find out what was happening in the classroom, what the students were doing and learning, and how they felt about themselves in the particular context. This would require two roles, potentially conflicting, namely, those of implementer in the classroom and of evaluator of the situation. The observer could not just come into the classroom and observe what was happening since he was also involved in setting the conditions in which the students operated. As desirable as goal-free observation might be, some hoped-for goals were acknowledged, nevertheless. The goals were that the students would have improved attitudes toward mathematics, that they would learn basic skills in mathematics at least as well as in a traditional approach, that they would develop some ability to solve real-life problems, and that they would increase their ability to be responsible for their own learning. On this issue Michael Scriven (1967) has written:

> Now any curriculum builder is automatically engaged in formation evaluation, except on a very strict interpretation of "evaluation." He is presumably doing what he is doing because he judges that the material being presented in the existing curriculum is unsatisfactory. So, as he proceeds to construct new material, he is constantly evaluating his own material as better than that which is already current. Unless entirely ignorant of one's shortcomings as a judge of one's own work, he is also presumably engaged in field-testing the work while it is being developed, and in so doing he gets feedback on the basis of which he again produces revisions; this is of course formative evaluation. If the field-testing is elaborate, it may amount to summative evaluation of the early forms of the new curriculum. He is usually involved with colleagues. . . who comment on the material as they see it--again, this is evaluation, and it produces changes which are allegedly for the better. (p. 43)

This experimenter could not operate apart from the phenomenon he investigated; accordingly, it is possible that his own involvement enhanced the judgments which he made.

Beyond these two considerations there is a third reason, more compelling for an alternative methodology, namely that the mechanistic mode of inquiry assumes that the world and its inhabitants are objects to be manipulated and tested for specific characteristics, an assumption which contradicts a holistic view of man and his environment.

> Logic, as a method of inquiry, depends upon the abstraction and isolation of phenomena through the predication of attributes, and the ordering and classifying of them. . . . (Carini, 1975, p. 3)

Discussing an alternative paradigm, which she called phenomenological inquiry, Carini made a significant observation:

> The basic phenomenological process of immersion in direct observation of a small number of cases over extended periods of time within their natural setting goes against the grain of persons accustomed to conceiving of research in terms of empirical data, gathered objectively (i.e., independently of any given observer and any given setting), and thus available to normative statistical treatment and replication. On the other hand, with greater or lesser degrees of awareness, it is generally recognized that virtually all the major breakthroughs in thought have occurred not from exhaustive, empirical studies replicated on large and carefully stratified samples, but from intensive observation and reflection upon a few cases. (pp. 5, 6)

In her article Carini compared logical inquiry and phemonenological inquiry in terms of five ideas: character of observer, relationship of the observer and the phenomenon, nature of the inquiry, function of the inquiry, and the methods of the inquiry. In each case logical inquiry violates the holistic view of man and his world. In logical inquiry the observer is presumed interchangeable with any other observer and his subjects can be observed separately and various characteristics of the subjects can be measured. In phenomenological inquiry "the observer is part of the datum and his or her point of view is central to the inquiry" (p. 19). In phenomenological inquiry a fundamental unity between the observer and the environment is assumed, while in logical inquiry the observer describes the phenomenon in terms of its object properties; the object is isolated and depersonalized. In the logical approach the phenomenon is assumed to be knowable and objectifiable, "through the predication, analysis, and summation of its parts" (p. 10) with one unchanging meaning, while phenomenological inquiry "increasingly thickens the meaning of the phenomenon as it reveals the multiplicity of internal reciprocities that constitute the phenomenon's integrity" (p. 11).

The function of logical inquiry is to organize knowledge within a system of classification and to place the phenomenon in a "chain of causal events in order to control and predict it" (p. 11). On the other hand, the function of phenomenological inquiry is to seek increased meaning of "unconcealment" of the phenomenon by revealing the coherence, durability, and integrity of it. The inquiry itself--the observations, recordings, organization, and research--must reveal the multiple meanings of the phenomenon. The observer becomes immersed in the setting and becomes a participant. The strategy, the questions asked, and the observations made are refined and altered as the observer begins to see the nature of the phenomenon and to understand relationships. Observation and recording inform each other and the organization intensifies the observer's participation.

Within the framework of phenomenological inquiry, a combination of specific methods was used in this study, so that the weaknesses of some methods were compensated by the strengths of the others. This research followed therefore the guidelines of "between-method triangulation":

> When a hypothesis can survive confrontation of a series of complementary methods of testing, it contains a degree of validity unattainable by one tested within the constricted framework of a single method. Findings from this latter approach must always be subject to the suspicion that they are method-bound: Will the comparison totter when exposed to an equally prudent but different testing method? There must be a multiple operationalism. . . . It is through triangulation of data procured from different measurement classes that the investigator can most effectively strip away the plausibility of rival explanations for his comparison. (Webb, Campbell, Schwartz, & Sechrest, 1971, p. 174)

In this study direct observation, interviews, questionnaires, attitudinal scales, results of topic tests in mathematical skills, analysis of student work, student journals, and various unobtrusive measures were all used, within the framework of the case study methodology.

"Case-study is the examination of an instance in action. The choice of the 'instance' is significant in this definition, because it implies a goal of generalization" (MacDonald & Walker, 1977, p. 181). The essential feature of a case study is its boundaries. In this study the boundaries have already been indicated--specifically, it is the exploration of a real-problem-solving approach, whose features have been developed by USMES, to the teaching of a college-level liberal arts mathematics course in a classroom, subject to the usual restrictions of a college setting. The larger questions such as open education or interdisciplinary curriculum have not been explored. The focus is on what is happening in the classroom in an ordinary mathematics course in a single semester, and how the students feel about their learning experiences. An understanding of this particular case, with its idiosyncracies and its regularities, its complexities and its totality, within the stated boundaries, was sought. As McDonald and Walker (1977) pointed out:

> Case-study is the way of the artist, who achieves greatness when, through the portrayal of a single instance locked in time and circumstance, he communicates enduring truths about the human condition. For both scientist and artist, content and intent emerge in form. (p. 162)

Although case study is more compatible with the particular situation considered here and with the values of the investigator than is the traditional experimental approach, certain limitations of the case-study methodology should not be overlooked. MacDonald and Walker listed several. In the first place, since case studies are public documents it is necessary to mask the identities of the persons involved without altering the implications of what was reported. Second:

> Case-study methods rely heavily on human instruments, about which only limited knowledge can be obtained and

> whose private expectations, desires and interests may bias the study in unanticipated and unacknowledged ways. (p. 187)

Third, any method involving direct observation and interviewing has the problems of reactivity which arise from the subjects knowing that they are part of an inquiry. Hence, unobtrusive methods must be used to reduce threats to external and internal validity. While the students knew they were part of an experimental method of teaching, they did not realize that their notebooks and reports of investigations would be used as documentation in the research. Fourth, case studies cannot be full accounts, due to limitations of time and space; they are always partial accounts. The researcher must make selections, decide what to observe and what to disregard, what to include and what to leave out of the report. "Educational case-studies are almost always conducted under constraints of time and resources and therefore reliability and validity pose considerable problems" (MacDonald & Walker, 1973, p. 187).

The use of test data and statistical analysis was one of several sources of information in the case study. The following statement from Parlett and Hamilton (1977) summed up their value:

> Finally there are published or custom-built tests of attitude, personality and achievement. Such tests enjoy no privileged status within the study. Test scores cannot be considered in isolation; they form merely one selection of the data profile. Interest lies not so much in relating different test scores, but in accounting for them in the study's findings as a whole. (p. 17)

In this investigation, then, the Aiken Attitudinal Scale, teacher-made topic tests, and a situational problem-solving test similar to the Notebook Problem test, were all used and statistical analyses were performed; the results were used to help round out the entire picture.

Sophisticated statistical procedures were deemed unnecessary in view of the nature of the attitudinal questions and the uncertain validity of the teacher-made tests. There are, however, two reasons for employing such procedures: first, in the context of this naturalistic inquiry, they are a possible means for supporting judgments arising from other observations--in short, a cross-checking; second, those who might be interested in this research would find objective numerical data that permit statistical analysis useful.

This research plan was designed to be flexible; indeed, the methods and instruments used, the questions asked, and the evaluation strategies employed were modified somewhat during the semester. As Denzin (1978) observed:

> No investigation should be viewed in a static fashion. Researchers must be ready to alter lines of action, change methods, reconceptualize problems, and even start over again if necessary. They must continually evaluate their methods, assess the quality of incoming data, and note the relevance of the data to the theory. (p. 304)

Finally, one of the questions being considered here is how an instructor, accustomed to the traditional lecture-discussion approach to teaching, can adapt to a different style of teaching, with its frustrations, uncertainties, and reassessment of what is being tried. This becomes very subjective, but the observations are important. Indeed, these personal thoughts and reflections may be the most important part of the research for another instructor with the same reservations about his or her teaching, but who may desire to try an approach which asks the students to become engaged in solving important and useful problems and thereby seeing more clearly the vitality of mathematics.

CHAPTER III

THE PHYSICAL AND HUMAN CONTEXT OF THE INQUIRY

This chapter deals with the students and the college where the experimental instruction took place, the nature of the general education freshman mathematics course, the questionnaires and tests given before and at the close of the semester, and the instruction and activities which took place during the semester in the real-problem-solving curriculum. A discussion of the content and instruction of the comparison class is also included.

Background at Bennett College

The College

Bennett College is a liberal arts college for women and is affiliated with the United Methodist Church. Its enrollment in 1979 was approximately 600 students. The college awards the Bachelor of Arts, Bachelor of Science, and Bachelor of Arts and Sciences in Interdisciplinary Studies degrees. It is accredited by the Southern Association of Colleges and Secondary Schools and the North Carolina State Department of Education. Among a number of organizations of higher education, the college holds membership in the University Senate of the United Methodist Church, the American Association of Colleges, and the United Negro College Fund.

Originally founded as a coeducational normal school, the institution's first sessions were held in the basement of a Methodist Episcopal Church in Greensboro, North Carolina, in 1873. It was founded through the inspiration of newly emancipated slaves who bought the land on which the college now stands. As a result of an appeal of its founders for assistance, the Freedmen's Aid Bureau and the Southern Methodist Society of the Methodist Episcopal Church assumed responsibility for the support of the school. It became a college in 1886 and graduated men and women who assumed positions of leadership in all walks of life, particularly as ministers and teachers.

Reorganization of the college was undertaken in 1926 at which time it became a senior college for women. Since that time more than 4000 women have graduated and Bennett continues to focus on the particular needs of black women. The physical plant in 1926 consisted of nine buildings and occupied 38 acres. From 151 high school students and 10 college students in 1926, the enrollment increased steadily and by 1930 there was a college population of 130 students. By the 1977-78 academic year the enrollment peaked at 626 students and then declined somewhat to below 600 in the succeeding years. The college assets were listed in 1978 at $11,128,590.

The college is located in Greensboro, North Carolina, a city of approximately 150,000 (in 1978), with a large college-oriented population since four other institutions of higher education are also located in the city: North Carolina A & T State University, Greensboro College, Guilford College, and the University of North Carolina at Greensboro. These colleges participate in a consortium which features cross-registration in classes; the three private schools (Bennett, Greensboro, and Guilford) cooperate in other ways, including some joint majors and a combined summer school program. The white enrollment in the student body at Bennett comes from the college's participation in the Greensboro Regional Consortium and has not yet exceeded 5% of the student enrollment. The majority of its students come from the southeastern United States and other regions with large numbers of lower- and lower-middle-class families in special need of the kinds of services and educational programs being offered at Bennett. The college is especially proud of its record in working with underadvantaged students and of their successes after college.

The basic philosophy which provides a framework for the college is

> its belief in a need to provide for students a unique and flexible program of instruction, supplemented by rich experiences in group participation and community involvement, designed to meet the needs of an ever changing society. Additionally, it is felt that its program must encourage the development of critical and analytical thinking, this being necessary for students to continue to educate themselves in the years after college. . . . It recognizes the importance of each student, and as a church related college insists that education be related to humanitarian as well as utilitarian ends. (Bennett College Catalog, 1978, p. 8)

The college catalog lists 10 objectives (see Appendix D) among which are the college's expectations that its students will become liberally educated while at the same time adequately prepared for successful pursuit of careers and graduate study. Further, the college seeks to provide learning experiences which will develop problem-solving abilities and analytical thinking.

<u>Student Characteristics</u>

In 1978 Bennett was primarily a dormitory college with almost all students in the usual age range of 17-22 for college students. The average age was 19.5 years. Nine out of ten students received financial aid in the form of scholarships, private loans, or Basic Education Opportunity Grants, etc. The median range of family income was $8000 to $9898. Of the students, 95% were seeking degrees in over 30 programs.

As an indication of the relatively weak academic background of the students, Table 1 shows the mean, standard deviation and frequency distribution of the total Scholastic Aptitude Test scores and of the mathematics scores on the Scholastic Aptitude Test for Fall 1977, Fall 1978, and Fall 1979.

Mathematics Courses

In the 1978-79 academic year the Department of Mathematics offered a major in mathematics with four options:

Table 1

Comparison of SAT Scores for Entering

Freshmen in 1977, 1978, and 1979

Group	Number[a]	Mean	St. Dev.
SAT Total			
Fall 1977	165	633.4	111.1
Fall 1978	200	641.1	131.2
Fall 1979	132	625.9	124.6
SAT Mathematics			
Fall 1977	165	325.3	56.0
Fall 1978	200	331.5	64.7
Fall 1979	132	325.0	70.7

Frequency Distributions

Percentages

	1977	1978	1979
	SAT Totals		
Above 1100	.0	1.0	.8
1000-1090	.0	1.5	.8
900-990	1.2	2.0	2.3
800-890	7.3	7.5	6.8
700-790	18.2	17.0	16.7
600-690	33.9	30.0	24.2
500-590	31.5	35.5	35.6
400-490	7.9	5.5	12.9
	SAT Mathematics		
550-590	.0	.5	.8
500-540	1.2	2.0	1.5
450-490	2.4	5.0	3.8
400-440	7.9	6.0	11.3
350-390	17.0	23.0	14.3
300-340	43.0	34.0	30.1
250-290	24.8	26.0	28.6
200-240	3.6	3.5	9.8

[a]Represents only those students for which data were available.

pure mathematics, mathematics and teaching, mathematics with a computer science concentration, and mathematics and engineering in a dual-degree program with nearby North Carolina A & T State University. Five different mathematics courses were available to freshmen--a developmental course named Survey of Basic Mathematics (Math 001-002), a general education course called Modern Mathematics (Math 101-102), a standard algebra and trigonometry course entitled Fundamentals of Mathematics (Math 103-104), Elementary Analysis (Math 105-106), and Calculus (Math 221-222). Students were placed into these courses mainly on the basis of the SAT mathematics scores, although some consideration was given to courses taken in high school. The guidelines for placement in the Fall of 1978 were as follows: below 300--Survey of Basic Mathematics, 300-440--Modern Mathematics or Algebra and Trigonometry, 450-600--Elementary Analysis, and above 600 with four years of high school mathematics--Calculus. Modern Mathematics was required for students in the humanities, education, and social sciences, while the Algebra and Trigonometry course was taken by students in business and the sciences. Students who completed Math 001 or 002 were eligible to take either Math 101 or Math 103. Table 2 shows the enrollments and percentages for three years preceding and including 1979.

Modern Mathematics, therefore, was the title of the general education, freshman-level mathematics course. The

Table 2

Enrollments of Students in Freshman Mathematics

Classes in 1977, 1978, and 1979

Course/Description	Fall 77	Fall 78	Fall 79
Math 001/002 SAT below 300 Remedial	64	47	65
Math 101/102[a] SAT 300 or above Non-science Students	76	103	40
Math 103/104[a] SAT 300 or above Science Students	80	94	50
Math 105/106[a] SAT above 450 Mathematics Students	16	12	10
Totals	232	236	165

[a]These classes also include freshmen and sophomores who have completed prerequisites.

title indicated its historical nature, since the title had not changed since the 1960's and as of 1978, neither had its content. The College Catalog description was as follows:

> Mathematics 101-102 Modern Mathematics
> A course in the basic principles of mathematics designed to meet the needs of students in general education. Emphasis is placed on discovery. Some of the topics explored are: sets, logic, number systems, open sentences and graphs, intuitive geometry and measurement, statistics and computer programming. (Bennett College Catalog, 1978, p. 61)

Despite the statement about discovery, much of the teaching in the course in the ten years prior to 1978 had followed the lecture-discussion mode. Indeed, many of the topics such as sets, logic and number systems (integers, rationals, irrationals, real numbers, etc.) were usually presented in a straightforward manner and were highlighted by fairly abstract discussions of definitions, theorems, truth tables, the field properties, and the comparative properties of the various subsets of the real numbers. Little attempt was made to show how these ideas could be used to solve problems from everyday life.

Prior to 1978 the textbook for the course was Contemporary Mathematics by Meserve and Sobel; it comprised 14 chapters: Part 1: sets, logic, mathematical systems, systems of numeration, the set of integers, the set of rational numbers and the set of real numbers; and Part 2: sentences in one variable, sentences in two variables, nonmetric geometry, metric geometry, coordinate geometry, an introduction to statistics, and an introduction to probability. A brief unit on computers was also included in the course. The topics for the two different semesters followed the two parts of the textbook.

Beginning in the Fall of 1978 a different textbook was used: Mathematics, An Everyday Experience by C. V. Miller and V. E. Heeren. The emphasis of the book was more on how mathematics could be used in solving everyday problems such as consumer problems, minimum and maximum problems and those involving simple linear and quadratic equations, statistics and probability, and introduction to computers. The overall theme was to view mathematics not as mathematicians view their subject--from definitions and postulates to theorems through deductive reasoning--but rather to see how mathematics is used as a tool. The textbook was intended, after all, for non-science students whose future use of mathematics in their major fields and daily lives would not require a knowledge of how the formulas or the proofs were obtained.

One of the difficulties in planning a course for non-science students in general is that the needs of humanities students, of social science students, and of students in education are somewhat different. Students in the social sciences particularly need the mathematics which leads to a thorough understanding of statistics and probability including some elementary algebra and graphing; students in education need to learn about sets, logic, numeration systems, and nondecimal bases, as well as the structure of the various kinds of numbers (integers, rationals, real numbers, etc.). These latter topics tend to be more abstract and of little intrinsic interest to the other students. The students in humanities benefit from a study of how mathematics has been an influence in the development of civilization as described, for example, in Morris Kline's book, Mathematics in Western Civilization. At a small college, such as Bennett, it was not possible in 1979 to offer two or three different courses because of scheduling difficulties and too small an enrollment to justify setting up separate courses. Hence all students were placed in the one course.

The first semester of Modern Mathematics I (Math 101) covered roughly the first half of Mathematics, an Everyday Experience. The course outline is given in Appendix E-1. A brief review of arithmetic operations, percentages, and the use of formulas began the course. A unit on number sequences, including arithmetic, geometric and powers of 10, was followed by a unit on consumer mathematics, featuring simple interest, compound interest and annuities as an application of geometric sequences. A short section on logic preceded two units on geometry: the geometry of shapes and the geometry of measures. The final section dealt with graphing simple equations and inequalities. The course met twice a week for lecture and discussion and once a week for computer-assisted instruction (CAI) in the computer center.

The college catalog described the computer-assisted portion of the instruction as follows:

> The computer center houses a medium size computer and a mini-computer programmed to reinforce basic skills in English, mathematics, and reading. These courses are taught jointly by an instructor and the computer.
>
> The CAI laboratory is open to students twelve hours per day. It provides students with:
>
> 1. Conversational and individual assistance and drills to remove deficiencies of pre-college training.
> 2. Flexibility to pace yourself in the completion of general education requirements.
> 3. A psychologically effective learning atmosphere for the student who feels hindered by classroom competition. (1978, p. 35)

The teaching modules on the computer were written by instructors at Bennett so that the choice of topics, the sequence, and the level of presentation would be consistent with the courses at Bennett. The units, using the "Author" program of the Hewlett Packard system, were combined into groups of one or more to form lessons. Each unit involved some instructional explanation with frequent questions for student responses, two to three examples, several practice problems in which hints and suggestions were given for wrong answers (three chances allowed), and some drill questions with only the correct answer given. An instructor was present in the computer-assisted instruction (CAI) lab to answer questions and to give assistance.

The lessons required for Math 101 consisted of fractions, decimals, negative numbers, evaluating algebraic expressions, the properties of the real number system, linear equations, and linear inequalities. The subject matter was rigidly determined for the students and was presented in a straightforward "example-explanation-formula-practice" approach.

Some branching for students who answered a question on the first try was available, but this was the only flexibility in the programs.

Two-thirds of the grade was determined by work in the classroom and one-third of the grade came from the results of short tests taken on the computer after each lesson was finished.

The students who successfully completed Modern Mathematics I moved on to the second semester of the course, Mathematics 102 (Modern Mathematics II), which was the setting for the experimental real-problem-solving approach of instruction under consideration in this inquiry. Most of the students had taken Math 101 in the previous semester (Fall 1978) (although some failed Math 102 during the first semester and so had to repeat it and others did not take any mathematics during the previous semester). All of the students in Math 102, having previously passed Math 101, had a reasonable knowledge of arithmetic, could evaluate simple expressions (but could not always set up an equation in a given situation), could solve given linear equations, and had some background in plotting points and drawing simple graphs of predetermined equations. The students varied greatly in their ability to work these kinds of problems in different settings and situations. If they were given contrived information, they could follow fairly routine procedures to find answers to problems. But they generally had difficulty in setting up equations and had no idea how to derive equations of lines or set up graphs from realistic situations. In other words, the mathematics they had learned was not useful in solving realistic problems.

The Experimental and Comparison Sections of Modern Mathematics II

In the Spring of 1979 I was assigned to teach two sections of Mathematics 102. No attempt was made to assign individuals to a particular section of Math 102 on the basis of background, abilities or interest, for to do so would have been difficult at a small college such as Bennett where only one or two sections of each course are offered. Hence the students registered for one of two sections according to the sections which fit best in their overall schedule of classes.

One of the sections of Math 102 was chosen arbitrarily to be the experimental section and the other to be the comparison section. Table 3 shows the comparison of the entering SAT-Math scores and SAT-Total scores for the two sections. The scores for five of the students in the comparison section were missing.

As can be seen from the data, the quantitative SAT scores of the experimental class were slightly higher than those of the comparison section, both in terms of the mean (but not the median) and in terms of the frequency distribution. Indeed, 25% of the students in the experimental class scored above or equal to 350, while only 19.4% of the students

Table 3

SAT Scores of Students in the Experimental and Comparison Sections

	SAT TOTAL Comparison	SAT TOTAL Experimental	SAT MATHEMATICS Comparison	SAT MATHEMATICS Experimental
Number with known scores[a]	31	36	31	36
Mean	604.8	624.7	310.0	331.1
Median	590	625	310	310
Standard Dev.	62.7	93.8	42.0	52.3

Frequency Distributions
Percentages

	Comparison	Experimental
SAT Total		
1000-1090	0.0	2.8
900-990	0.0	0.0
800-890	3.1	0.0
700-790	9.7	8.3
600-690	35.5	50.0
500-590	51.6	36.1
400-490	0.0	0.0
SAT Mathematics		
Above 500	0.0	0.0
450-490	0.0	0.0
400-440	0.0	8.3
350-390	19.4	16.4
300-340	54.8	44.4
250-290	16.1	19.4
200-240	9.7	11.1

[a]Each class had 36 students but five of the SAT scores for comparison students were missing.

in the comparison section did so. On the other hand, a smaller percentage of students in the comparison group scored under 300 (25.8% comparison versus 30.5% experimental). Under the assumption that the samples were drawn independently from the same population, a _t_ test[2] can be used to test the hypothesis that the means are not significantly different. Here $t = 1.801$ with $n_1 + n_2 - 2 = 65$ degrees of freedom. The probability is .9618, so that the difference between the means is significant at the 5% level but it is not significant at the 2½% level.

The regular Math 102 class (comparison group) took up the usual topics for the second semester. (The course outline is given in Appendix E-2.) The work begun in the previous semester on simple graphing was continued to include graphing parabolas, ellipses, circles, and hyperbolas. This was extended to include the graphing of linear inequalities and the use of inequalities to solve minimum and maximum problems (linear programming). Basic concepts in probability, including multiplying and adding probabilities, were combined with a fairly extensive treatment of statistics, including the use of the normal curve in reaching conclusions about statistical situations. Finally a unit on computers led to

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[2]The formula used was

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}}$$

Calculations were performed on Texas Instrument 58 Calculator with its Statistical Module.

the students learning enough of the concepts of programming and simple statements in the BASIC programming language so that they could write simple programs of at least 10 lines.

The comparison group followed the traditional teaching approach. New material was presented by the instructor, a reading assignment and homework assignments were made from the textbook, and the homework problems were discussed in class the next class period. Subsequently other problems were handed in to be graded and periodically tests were given. The grade was determined by averaging homework, two tests, a project in statistics, and the final examination.

College-required Course Outlines

The content and method of instruction for the experimental section of Modern Mathematics II is described in detail below, but it should first be pointed out that Bennett College puts certain restrictions on the latitude and flexibility an instructor has in determining how a course is taught and what topics are to be covered. First of all, the course had to follow the catalog description, which is fairly general. Hence it was necessary to be sure the students studied "open sentences and graphs, . . . statistics and computer programming" (Bennett College, 1978, p. 61). Secondly, a formal letter grade to represent the quality of the student's work was to be determined in a systematic and fair manner. The topics and grading in the CAI lab were determined by the members of the Mathematics Department (see Appendix E-3), so there was no flexibility in that part of the course. Finally, each instructor was required to submit a formal course outline giving a list of competencies, the method of implementation, the units to be covered, and the method for determining the grade. Hence, it was necessary to develop a course outline which would satisfy these restrictions while at the same time following the general features of a real-problem curriculum and emphasizing student-directed learning. Fortunately, the topics of open sentences, graphing, and statistics lent themselves to this approach more than other topics such as geometry and number systems would have done. Furthermore, the use of CAI materials allowed the development of specific skills in a separate setting. One of the major tasks, then, in developing the course outline, was to try to integrate the emphasis on investigations with the "basic skills" topics of the CAI and the more general topics of the catalog description.

Preliminary Tests and Questionnaires

Both of the classes were given five kinds of inventories during the first week of classes in order to assess the entering qualifications of the students in terms of knowledge of mathematical concepts, problem-solving abilities, and attitudes toward mathematics. The results of these questions would provide some follow-up analysis when compared with similar tests given at the end of the course.

Fourteen questions on five topics (algebraic expressions, linear open sentences, quadratic equations, graphing, and statistics) constituted a pretest on standard mathematical knowledge (see Appendix E-4). It was assumed that the students had seen this material before but probably had not mastered the skills. The tests were teacher-made and except for the two questions on statistics, the material had been covered in the previous course. They consisted of such simple questions as "evaluate $4x+2x^2/3+3$ when x is 6" and "solve for x: $5x-3=2x+1$" as well as more difficult questions involving the use of the quadratic formula and finding a frequency distribution for a set of numbers.

A second set of questions given was that of the Aiken Attitude Scale, which consisted of 20 questions concerning a student's feelings about mathematics and the learning of mathematics (Aiken, 1972). (See Appendix E-5.) The test was analyzed by using a Likert method of summated ratings, with a weighted scale of 1 to 5 depending on a student's response in the direction of dislike-like of mathematics. The choices given in this survey were strongly disagree, disagree, undecided, agree and strongly agree. Concerning the reliability and validity of this instrument, its author, Lewis Aiken, noted:

> Consistent with the findings of other investigations, the results show that the reliability and validity of this scale vary somewhat with grade level, being generally more reliable and valid in high school and college. The reasons for this are that not only do attitudes become more stable with maturity, but the degree

> of self-insight and conscientiousness with which students can express their attitudes increases with age. (1972, p. 30)

The intent of giving this inventory and the follow-up repeat of the same questions at the end of the course was to see whether or not the kind of instructional strategy, i.e., traditional or real-problem solving, makes any significant improvement in attitudes toward mathematics. A t test was used for statistical analysis.

In addition, four questions to determine student perceptions of how a mathematics course should or might be taught and how the individual learns best were asked for two reasons: (1) in order to lead the respondent to think about some alternatives, and (2) to give the instructor some insight into the thinking of the individuals and of the class as a whole. The questions were as follows:

> Write a few sentences concerning how you feel about your previous mathematics courses, in particular, Math 101 or a similar "general education" course, in response to each of these questions.
>
> 1. Do you feel that the instructor ONLY should choose the topics in mathematics for the students to learn or should the students have some say in what topics are studied? Explain.
>
> 2. How do you learn best--by following a teacher's lecture, reading the text, doing the homework problems, becoming involved in an activity of interest to you, or by some other means? Explain.
>
> 3. Do you feel frustrated and lost in learning if the instructor doesn't indicate what pages in the textbook you should read and which questions should be answered?
>
> 4. What previous experiences in mathematics classes have led to your like or dislike of mathematics?

Each student was asked to give a brief biographical sketch (Appendix E-6) in terms of mathematics background and interest in mathematics. The questions involved previous mathematics courses taken in high school and in college, and the student's major or special interest. Two other questions required open-ended short answers:

> If you could choose any topic in mathematics which you might find interesting and/or useful, what would the topic be? Why?
> What sort of uses for mathematics do you expect to have in the near future or in your major field?

These questions seemed to puzzle students somewhat since they had done little thinking about the issues. Generally they answered in terms of the mathematics encountered previously in high school or in terms of consumer mathematics, not in terms of mathematics needed for academic subjects in their major fields.

Finally, a simple question concerning approaches a student might make using quantitative considerations in solving a problem was given early in the course so as to assess prior abilities in this area. The following question was based on the style of the "notebook" questions in the USMES evaluation:

> Write a page indicating how you would solve the following. You should answer the questions:
> (a) What information do you need?
> (b) How would you obtain it?
> (c) What would you do with the information?
> (d) On what basis would you arrive at a conclusion?
> In a grocery store you can buy eggs in four sizes: small, medium, large and extra large. Which is the best buy?

Student responses to these questions were evaluated according to a schema involving (a) whether or not any of the student's reasons for selection were stated in terms which were measurable (weight, size-volume, cost, etc.) and (b) the highest level of warrant associated with the selection (personal opinion, some quantitative considerations, test suggested vaguely, or a test suggested in detail). This method of evaluation was used by USMES in its report on The Notebook Problem (Shapiro, 1973). A comparison of pretest scores between the experimental and comparison group is discussed in Chapter IV.

Description, Planning, and Implementation of the Experimental Course

Preparation of the Course Outline

The first formal aspect of the experimental course came with the preparation of the course outline, as required by the college. This outline was to include the competencies, implementation, and the units to be covered in the course. The concept of competencies seemed to be inconsistent with the spirit of an open-ended teaching process; it connoted a teacher-oriented determination of what was to be learned by the students. The word "competencies" was changed to "objectives" which has much the same general meaning, but the word "objectives" tends to give the impression of what the teacher would like to have happen rather than what the student must learn. The objectives included the usual one found in the various descriptions of problem-solving courses, namely, that the student develop an ability to solve problems. This was not meant to be a behavioral objective--there was no statement of how success would be measured or determined--but rather simply a statement of the nature and spirit of the course. It was included in the course outline in order to let the students know what to expect in the course even though they would be given some choice in the matter.

Two other statements began with "Upon completion of the course, it is hoped . . ."[3] in order to indicate some non-behavioral objectives, some hoped-for goals, such as the student's having an increased confidence in her ability to think and to use mathematics, one of the possible intangible results of any mathematics course. In addition, certain skills were listed: those which were to be covered by the computer-assisted instruction materials and those which would probably be used in connection with the process of solving the challenges. The precise statement was:

> The student should be familiar with the basic concepts of mathematics such as number, equations and inequalities, functions, graphing deductive and inductive thinking, and statistics. (The specific skills acquired in these areas will be determined by the interests of the students and the requirements for sólving a chosen problem.)

The second main part of the course outline dealt with implementation. It was difficult to write this part before experiencing the process of teaching a class from a

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[3]The quoted phrases and sentences in this section were taken from the course outline--see Appendix E-7.

problem-solving approach. The USMES Guide (Unified Sciences and Mathematics for Elementary Schools, 1976) was used in much of this section. It was unfortunate to have to write such a definitive statement at the beginning of the course, but again, it was only fair to the students that they should know what to expect and what was expected of them from the beginning. A proviso was included reserving the right to modify procedures of the experiences during the semester indicated that it would be in the best interests of the students to do so.

The plan for the course outlined six steps. First, the instructor would present some "real and practical problems" which were intended to "apply to some aspect of (the) daily lives" of the students and would be of "some direct interest to the students." Each student would select a problem which she would pursue with others in small groups. Second, the students would decide themselves how "to proceed in solving the problem, to make observations, gather data, draw graphs, analyze the data, form and test . . . hypotheses, and take some final action or draw conclusions." In the process of undertaking the investigation, when the students discovered that certain facts or techniques were needed to move toward a conclusion, they would consult references, including the textbook and handouts from the instructor, and they could confer with him. When it seemed that several groups needed to go over similar topics, those topics or procedures would be discussed in a formal classroom setting. The instructor would, theoretically, throughout the semester, serve actively as a "coordinator and collaborator" so that each group could move toward some sort of conclusion and success on the investigations. The students would individually keep a record--a personal journal--of the work they were doing (graphs, models, reports, and mathematical topics). Finally, "work on the computer terminals (CAI) and a study of mathematical references--reading explanations from an assortment of textbooks, drill, practice, and testing on specific topics" were intended to allow the students to learn those topics which would be of importance to them.

The statement about implementation reflected the ambiguity which I sensed in the USMES program and within myself about the relationship between (1) creating activities which would interest and challenge the students, (2) setting up a formal process through which students would learn basic skills needed to solve the problems, and (3) creating the environment in which students could develop their problem-solving skills. How could these three thrusts be coordinated and brought together?

Finally, in the section on Units were listed those topics which could potentially be used as challenges, i.e., those which would be of some interest to the students. Two problems recurred as I worked out the plans for the course in anticipating student reactions and in trying to overcome my tendency to be the person in charge. The first was to find out which kinds of situations would interest the college-level students in my class, as distinguished from the challenges listed in the various USMES study guides, designed for elementary and junior-high school students. The second question which I pondered was how I could provide some sort of direction to the students in order to avoid confusion, while at the same time helping the students begin to develop responsibility for their own learning and to help them to make academic progress. I decided to list those skills which the <u>USMES Guide</u> indicated were frequently used in working the challenges as well as topics to be covered directly, such as computer programming and the CAI topics. Hence, the course outline (Appendix E-7) listed the following topics: use of free time, consumer research, more effective ways to learn/teach, ways to serve large numbers, and various mathematics concepts which might be needed in order to solve problems and which would be of interest to the student.

<u>Grading Procedure</u>

The second formal aspect of preparing for the class was to develop the procedure for determining grades. As has been previously noted, this was necessary in order to satisfy the requirements of the college. In preparing to work out a statement about grading, I was bothered by the idea that the teacher eventually determined a single letter grade to represent all the work a student had done, because this often creates an anxiety which interferes with the process of learning and it often causes the student to focus on doing only those activities which are needed to get a suitable letter grade rather than on the process of learning skills and concepts in order to solve problems. Using formal tests in which the desired products were determined by the teacher seemed inconsistent with the real-problem-solving philosophy because this would force the teacher to become the controller of the course, not the consultant and collaborator.

An article by Arthur Ellis and Janet Alleman-Brooks (1978) entitled "Informal Evaluation Strategies for Real Problem Solving" served as a guide in my planning. They noted that students frequently become confused or threatened in formal testing situations. Informal strategies would lessen the threat of tests and at the same time provide a suitable process to discover the progress that the students were making in solving real problems. They argued:

> Real problems have real outcomes and they therefore develop complexities too subtle to be captured adequately by formal measures alone.
>
> Why we evaluate is as basic an issue as that we evaluate. Most of us agree that evaluation of student progress is necessary, but we often have different reasons for reaching that agreement. In problem solving the processes by which choices are made and outcomes determined are equally as important as the products of student learning. Because of this, a pressing argument can be made for the diagnostic function of evaluation. Yet both diagnostic and summative classroom evaluation of problem solving is often neglected. One reason certainly is the difficulty in using standard evaluation approaches, normally pencil and paper tests, to assess the higher level outcomes sought by teachers using innovative methods. Some other reasons are:

> It is easier to write tests for preconceived teacher expectations than for real problems, which are open-ended. Problem solving is process oriented rather than product oriented and thus goes beyond learning that can be tested at the recall level. . . .
>
> Tests are often extremely artificial and totally foreign to real-life problem solving. They can also put pressure on students to compete for high scores, and as a result, sometimes have less than desirable outcomes, such as emotional strain or memorization for the moment with little retention beyond the test. (p. 240)

The article by Ellis and Alleman-Brooks suggested a number of alternative ways of evaluating informally, such as the use of artifacts (graphs, diagrams, and charts) produced by students, interviews, class observation, and reports written by students, but it did not address the issue of how letter grades were to be determined. One possible way to do so would be to list the activities that the students would be expected to demonstrate in order to solve a problem and to specify the evidence necessary to show that they had done the work. The emphasis would be on the effort that they made. However, it was also necessary to assess the quality of the work. Not every student could be expected to be motivated by the chance to engage in interesting problem solving. Furthermore, in any educational setting, such as a high school or college, where other teachers are using traditional grading procedures (homework, papers or tests with deadlines), students will work on short-term required tasks rather than working on long-range projects. Hence, it was necessary to develop a grading procedure which required the same sort of effort in mathematics that the students would be making in their other classes.

I found it difficult to set up a grading procedure in an experimental program when I had not yet had a chance to see how the program would work. Furthermore, I needed to face two other difficulties. The grade from the computer-assisted instruction tests would be numerical and the required final examination had to be included. How much should this count? How was it to be combined with work done throughout the semester? In order to be consistent with my attempt to let students have a part in the decisions of how the course would be run, and because I was interested in finding out how the students as a whole felt about these issues, I decided to let them respond to some alternative proposals for grading.

I prepared a (legal-sized) page outlining the arguments for informal evaluation and stating two grading alternatives. The first suggestion consisted of homework, two tests, topic tests, a project, and a final exam. Each of these was to be weighted numerically and a single numerical grade determined. The second alternative consisted of self-evaluation by the student and evaluation by the teacher through a "diary" written by the student of work done and progress made during the course. The statement read as follows:

> The student will keep a written document, i.e., journal or notebook, in which she will write up the work she has done, the attempts, the results, and impressions of that work. This written document will include any drill work done by the student in the process of learning a particular topic or skill.

Four activities were listed: problem-solving skills, specific mathematical topics, self-evaluation, and final examination. At the bottom of the statement the students were asked to write a page on "how I think the grade ought to be determined." The document included instructions to the students to indicate their feelings about the use of tests, what sort of evaluation would be "fair," and how a specific letter grade should be determined. (See Appendix E-8.)

The First Two Weeks of Class

The first two weeks turned out to be a crucial time in getting the experimental project underway successfully. On the first day of class, I passed out the sheets on which the students gave their names, backgrounds in mathematics, and the kinds of topics in mathematics which they would like to learn. I also passed out the two-page course outline and the sheet on grading, and we discussed the broad outlines and philosophy of what I wanted to try to do in the course. I pointed out that in theory, there should not be any particular topic which everyone in the class had to learn. For this class, learning how to solve problems and to take responsibility for one's own learning should be the major purpose of a "terminal" mathematics course. I went over the outline with the students and mentioned that the work in the course would be experimental in connection with a research project.

The students seemed interested and intrigued by the ideas I presented in class and on the outline, although some of the students indicated that they weren't quite sure what would be expected of them and this concerned them somewhat. I passed out the sheet on grading and discussed with the class my feelings that the usual numerical approach had limitations particularly in any attempt to make the mathematics course less threatening. I asked them how they felt about their previous mathematics courses and there was a lively exchange of ideas expressed. Several of the students pointed out that they did not particularly like mathematics, that when they took tests they got confused and they felt that the test did not adequately demonstrate how much they knew about the material on the test. I asked them to write a one-page summary of how they perceived grades should be determined as a response to the alternatives outlined.

To acquire a sense of the manner in which each student approached an open-ended problem, I passed out the sheet with the questions about eggs in a store, described earlier, and asked each student to write about how she would reach a conclusion. The assignment was intended to demonstrate the sort of questions we would be investigating.

During the last 20 minutes of the class I gave the first part of the pretest (see Appendix E-4 which is discussed earlier in this paper). I felt that it had been a reasonably good start; the students reacted positively and with curiosity as to what the course might entail. They probably did not really know what was in store for them. I felt that I had not spent enough time in conversation with them but the class had been somewhat hectic as students came in late during the first 15 minutes. I felt the press of time even on the first day--since there were only approximately 30 days in the semester and I wanted to get the students involved in the first project as soon as was possible. Hence, I gave the first part of the pretest on the first day.

On the second day of classes four new students came into the class. They had not participated in the discussion of the first day and so were not aware of the thrust of the course nor had they done the two homework assignments. During the first 20 minutes of the period I gave the second part of the pretest.

I then asked the students to consider how they would determine which brand of a product was the best buy for a specific use. I asked them how they could determine any relationship between cost and quality. I indicated that each of them would be working on an investigation of some sort and suggested that they might be interested in other kinds of questions, not involving consumer matters. As an example I wrote 15 hypothetical scores on a test for students being taught by one method and about that many for students being taught by an alternative method. "Can you tell which method is more effective?" I asked. They suggested (a) looking at the scores--no computations (the eyeball method), (b) finding a mean, and (c) drawing a graph. Following some of the ideas suggested in the USMES booklets, I asked them to consider how they would test for the quality of different brands of paper towels.

The discussion was lively; students responded and seemed to become involved. With 30 students, however, it was difficult for me to see exactly what was going on with all the students. I could not really tell how many were interested but were not talking and how many weren't interested at all.

One of the concerns I had was to make sure the students would be working on something concrete from one class to another, since my class met only on Monday and Friday. Further, I felt the need for students to have an adequate background with certain mathematical skills which might well be needed to carry out the investigations, so I suggested that they study a particular section of the textbook on plotting ordered pairs (points). Indeed, one of the suggestions made by a student earlier was to graph the two sets of data (on the hypothetical test), so that my suggesting some study on the graphing of coordinates seemed appropriate. Although I was concerned that I had given "too much" direction to the work that they were to do, I knew that the topic of graphing was one of the choices available to the students. Hence suggesting (not requiring) that they read the material and do the homework in the textbook seemed to give some direction to their study until the next class period.

I collected the one-page answers to my previous assignments on "suggested grading procedure" and "the buying of eggs."

Before the next class I considered the statements about grading that the students had turned in. These answers showed the same sort of ambiguity about grading procedures which I had been feeling. Some of the students were reluctant to have the method of grading changed from a familiar one to one which they weren't sure they understood. A common theme was that the tests put too much pressure on the individual so that the student did not show what she really knew because of the anxiety caused by the test-taking situation. For some, the idea of receiving a numerical score had the effect of letting them know exactly how well (or how poorly) they were doing. Several expressed the concern that the grading system, however it was worked out, should put sufficient pressure on them so that they would do enough work. They needed an outside push to "make them study"; this was one of the purposes of using tests in grading. I was intrigued that many of those students who got high grades in Mathematics 101 during the previous semester seemed to prefer Method I (emphasis on tests) because they had been successful with this approach during the previous course, while many of the students with lower grades indicated that they would like to see a change. Fourteen of the students preferred the second alternative (contracts), while 10 opted for the first alternative (testing).

The following two comments sum up the arguments which the majority of the students gave in favor of the second alternative:

> Student I:
> I personally feel we should get away from the old routine of taking examinations, which frightens everyone at times, and resort to alternative II. This will give everyone a chance to write more often, allowing us to be more expressive in our own individual ways. . . .
>
> Student II:
> In my opinion, alternative II is a practical and reasonable method that needs to be practiced at this particular time in this particular course. According to the syllabus, the Math 102 course will be taught in an experimental manner this semester and the objectives of the course are obviously different than those of last semester. Because of this fact, the use of topic tests and homework that we did last semester may be unnecessary.
>
> I think the grade ought to be determined by the quality of the students' work. It doesn't seem as though the correct answer should determine the grade, if we are gathering data to solve a problem that can be solved in so many different ways and render so many different types of answers. I don't think the grading system should be done away with. A test at midterm and a final seem to be the best way to determine the student's knowledge. But these tests shouldn't carry too much weight when it comes to assigning an A, B, or C grade. The quality and maybe the quantity of the student's homework, classwork and activities should determine most of the letter grade.

My next task then was to work out a definitive statement about grading. The complete statement is given in Appendix E-9. It specifies that a D was to be earned upon completing the five activities in the investigation and three topic tests passed, as well as the self-evaluation. A C-grade would be earned by completing in addition, three activities on a second investigation and a fourth topic test. A self-evaluation, two complete investigations, five topic tests, and one reading report would be sufficient to earn a B. An A could be earned by doing all the work required for a B, writing an additional reading report, and passing six topic tests. The class activities would constitute one-half the grade, the final examination would be one-sixth of the grade, and the CAI would be worth one-third of the grade.

The third day of class was a crucial one. I felt the need to get the students involved in some sort of project which would appeal to them. I realized that I would not be able to let a "group challenge" develop naturally as takes place in the usual structure of the USMES classroom, partly due to a lack of time (1 hour, twice a week) and partly due to the compartmentalization of the mathematics courses in any college. The first challenge would not arise from class discussions in social science or science, but would have to arise from artificial developments in the class. I chose to work with ideas from the Consumer Research Guide booklet prepared by USMES (1976a).

I started the class by discussing my proposed grading procedure and indicated that my attempt at arriving at a suitable grading process was an example of the sort of problem solving we all might need to do in our "daily lives" or in our careers. I pointed out that I had gone through certain steps: formulation of the problem, gathering data and information, and analyzing the data. I showed them a simple graph indicating the grade earned in the previous course versus the choice (I or II) of the grading-procedure alternatives and suggested that the written statements indicated that some students felt that they did better on tests while other students thought they could do better on other kinds of assignments. I asked the students if there was any reason to believe that one method really evaluates what a person knows or how well a person performs better than another method.

I next talked about the projects. I realized that the students were still trying to get a solid idea of what they were to be doing in the class. I presented some hypothetical data about the durability of several samples of two similar products and asked the class how they might decide which was the "better" product. Some of the students suggested that one measure--the average--would be a suitable way to summarize and compare the information given. I showed the students some of the How To booklets from USMES and indicated that there were other references available in the library which would be available for their use. Next I asked the students for some suggestions of consumer products which they might be interested in examining more closely so as to determine a "best buy"; three suggestions were made: cosmetics, pizza, and clothing. We discussed some of the possible questions that might be asked in trying to determine what a "best buy" was. Since time was short I asked the students to come back to class with a statement of the sort of work that they might want to do and suggested that they look over the first two sections concerning statistics in the textbook.

I realized that suggesting that the students study specific topics in statistics was contrary to the preferred method of letting the students discover on their own the need to learn some statistical ideas in an attempt to solve problems of their own choice, but I was concerned that the students would waste a lot of time, not knowing what to do or having some sense of the statistical "tools" which might be used in their work. Hence, they would find the whole process confusing. Clearly I was trying to work through some ambiguities which I was feeling about my role as a teacher, a director, or a facilitator. Since this was my first attempt in using this approach, I was not exactly sure what to expect and I was not confident of success.

At the next class period (the fourth session) two more students arrived. The class now totaled 35 students, which was a larger number than expected. This increased size would make supervising the individual work a bit more difficult. I asked the students to turn in the statements concerning the kinds of products that they might want to investigate. Most had not taken the time nor made the effort to consider the assignment, but some had given considerable thought. One group of four students had discussed the possibility of working together and of several potential topics for investigation. I decided to ask those who had not done the assignment to take 10 minutes to write out a statement. I next passed out the Aiken attitude survey together with my additional four open-ended questions. In addition, I gave each student a questionnaire concerning attitudes about the CAI operation on behalf of the chairperson of the Mathematics Department. I was glad that I had both of these questionnaires to pass out at the same time, for then the emphasis would be on questions involving the overall instruction in the freshman mathematics program at Bennett, rather than on questions of attitude toward mathematics and student involvement in learning in a particular mathematics course.

The students finished the questionnaires in less than 30 minutes, leaving 10 minutes to discuss the statements about group projects in class. Five topics were suggested, which I wrote on the blackboard: juvenile crime, cosmetics, deodorants, clothing, and TV viewing. For each topic I also wrote down the names of students according to interest indicated on the statements. Each student who had not expressed a preference was given the opportunity to choose one of the groups. Some students changed their minds, so that eventually there were approximately seven students in each of the five groups. Each group was asked to meet before the next class period to discuss a particular focus of the group and to develop a plan for carrying out the investigation.

While the groups met at the end of the class period, I passed out a sheet of 19 questions adapted from the USMES book Consumer Research (1976a). (See Appendix E-10.) I asked each group to look over the questions and to prepare a specific statement about what it would be trying to do for its investigation and how it planned to proceed. I felt that good progress had been made in getting the topics chosen and the groups organized, but I was aware that the students would need a good deal of assistance in order to carry out meaningful projects.

Group Dynamics and Progress on Investigations

During the next month I faced two problems in carrying out the aims of the course. The first problem was unexpected and involved the personal interrelationships of the individuals in the various groups. During the first week after organizing the groups, two students of one of the groups came to me and indicated that they were no longer interested in nor satisfied with what the other members of the group were doing, and they wanted to work together on a separate project as a pair. The original concept for the topic for their group came from four others--four "friends"; the two students who came to see me joined the group later. They felt that they did not fit in with the others although they acknowledged that they had chosen voluntarily to join the group. Their decision had been based solely on a preference for the topic. I indicated that I would not like to see too many different groups because the proliferation of topics would make it difficult for me to have time to help or to keep track of the development of the investigations. At the same time I wanted each person to be working on a project in which she was interested and with which she felt comfortable. I encouraged the two students to try once more to work with their original group, but if they were still unable to do so, we would talk about other possibilities later. Eventually, these two students remained with their original group and helped with one of the best reports from the class.

During the next two weeks in talking with members of the various groups in separate sessions, I learned that one or two of the members of each group were apparently not contributing very much to the work being done. Furthermore, in two of the groups there was a delicate problem of "leadership." Some of the groups seemed to be able to function without a specific person taking charge, while in other cases one of the members seemed to assume the role of leader and was generally accepted as the leader. I was told, however, that sometimes one or two of the members of the group were not happy with the decisions made nor with the tasks assigned.

A second, more pervasive problem which developed was that of frustration and uncertainty among the students. During the fourth week of classes one student told me that she thought the class should go back to the "old way" of teaching, that she felt she was not learning very much, that she did not understand what she was supposed to be doing, and, consequently, that she was not doing much work. In order to find a time that I could give assistance to students outside of class time on an individual or small group basis, I set aside the hour from 6 p.m. to 7 p.m. for conferences and consultation twice weekly. At one such session late in January, only one student showed up, but this gave me the opportunity to discuss how her group was progressing. She indicated that she felt the group "was not together." Her group had not met for a second time to decide on the specific question to be investigated or on the methods needed to gather information. They did not have a good sense of what was possible in terms of strategies or ways of organizing the information after it had been gathered.

As I reflected on her comments, I realized that this did not really surprise me. Clearly the process of having the student groups find a question in which they were really interested and of developing a method to solve the problem would indeed be frustrating to them; how could it be otherwise? I began to see that the difference between the college setting and the elementary school setting of the USMES classrooms in terms of the time spent in the classroom with the teacher present--the difference between several hours a day 5 days a week and 2 hours each week--was a crucial one. I sensed the need to talk with the groups frequently, to try to help them focus more and to give them more guidance. One paragraph from Self-Directed Learning by Malcolm Knowles (1976) was helpful to me at this point. He wrote:

> One problem concerns the issue of structure versus non-structure. Many students enter into a new learning situation feeling a deep need for the security of a clear structural plan--an outline, course syllabus, time schedule, and the like. They want teachers who know what they are doing, who are in charge. When they first hear me describe my perception of the role of a facilitator and resource to self-directed learners it seems so structureless to them that they become anxious. So I have learned to emphasize that we shall be working within a structure, but that it is a different kind of structure from what they have been used to--that it is a process structure; whereas they have been used to a content structure. And I assure them that I am in charge of the process, that I shall make decisions about procedures when they aren't in a position to make them with me, and that I know what I am doing. I assure them that I understand that they are being asked to take more responsibility for learning than they are used to, and that I shall help them learn to do it. (p. 37)

I did not regret the frustration that the students were having thus far, for perhaps it would help them see the importance of suggestions which I would make and of certain mathematical topics which I would like to introduce. Perhaps it would help them to gain some insights into the limitations of being dependent on an instructor in the process of learning.

These experiences led me to realize that I needed to move to a period of time in the course in which I gave more direction to the process and, to some extent, the content so that the student could grasp more clearly the strategies for carrying out the investigations. I realized that the frustrations that the students were having would not serve a good purpose if allowed to continue for too long a time. I needed to make procedures of the course more specific.

Hence, I decided to prepare some resource sheets on related mathematical topics, a list of suggested topics with references (Appendix E-11), an assignment sheet with due dates and a guideline to the outside reading reports (Appendix E-12), a checklist of completed activities (Appendix E-13) and guidelines for the entries to be made in the student notebooks (Appendix E-14).

As I made the shift, I realized that my basic problem remained: How does any instructor help a student become genuinely interested in a "real" problem so that the motivation for learning comes from her desire to become capable of solving the problem? At what point is the mistake made of giving the student too much direction so that she remains dependent on the instructor? I felt sufficiently unsure about my own role so that student complaints about my "not teaching and leaving everything up to them" were of considerable concern to me.

Routine Work During the Middle Part of the Semester

The next seven weeks were characterized by an increase in the amount of teacher direction and routine, although the investigations remained as the main thrust of the course. During this time, as previously mentioned, I prepared six resource sheets, as a substitute for the HOW TO booklets distributed by the USMES program. Although these HOW TO booklets were available for the students to study, they seemed too juvenile for use by college students. Unfortunately, my resource sheets were not done in the animated, attention-getting style of the HOW TO booklets; rather, they were typed and dittoed. Nevertheless, these sheets outlined the main ideas and computations which a student would need to know in the following areas:

#1 Choosing a sample and collecting data

#2 Classifying Data, Simplifying Data, and Standard Deviation

#3 Normal curve and its uses (3 pages)

#4 Graphs of First Degree Equations and First Degree Equations of lines

#5 Best fit lines

#6 Fundamental statements in BASIC

These are included in Appendix E-15. The students were not required to work on these topics, but since they were required to work on some topics (of their own choice), I pointed out to them that unless they had some definite ideas of the topics which they wanted to study, they should follow my discussion of the ideas on the resource sheets in class, read the textbook, and work some of the homework problems. I tried to relate the work being done in the investigations with the usual topics of probability and statistics, drawing and analyzing graphs, and of using the computer, so that the students could get an idea of how to use the mathematical topics and hence take more interest in the concepts, computational techniques, and interrelationships between mathematics and the real world.

During this period it was necessary to confront another problem, namely, the difficulty of trying to teach one topic to the majority of the class in a semi-individualized teaching environment. When one student asked me about a topic which was not being covered in class, I had to decide whether the question was of sufficient interest to others in the class to stop what we were talking about to discuss a separate concept or problem. Occasionally, the question had enough appeal to warrant such flexibility on my part. However, when I chose to do this, one or two students left early, bored and hoping to escape. Occasionally, a student would ask if she could work on the material on her own, without having to come to class, which was allowed only in rare cases, lest every student choose to request the same privilege.

Despite these difficulties, the major assignments of the course were completed by most of the students. The students took required topic tests once in February and once in March (Appendix E-16). They handed in notebooks, showing the work they had done and included self-evaluations (impressions) of their work so far (the day before Spring vacation). While only a bare majority of them selected chapters for their reading assignment by the first due date, most did hand in preliminary statements about their investigations according to the schedule. By the end of March, the class had covered most of the material on statistics, some of the material on probability, and some work on setting up linear formulas based on a best-line fit. The first work on computers was begun at the end of March.

Most of the material presented did not strike any particularly positive response, although several of the students indicated in their talks with me that they were beginning to see the relationships between the investigations and the material covered in class. Two "nonbook" situations elicited the most interest from the students. The first arose after my return from jury duty for three days. After having discussed some of my experiences in the courtroom with the students, I asked them to consider what the probability of being selected as one of the 12 members of a jury if the choice was made strictly at random from a pool of 35 potential jurors. Because of the immediacy of the question--it was a problem which I had been most recently very interested in solving and the students realized that they too could be in a similar situation in the future--and because of our recent discussion of probability, although on a fairly artificial basis, the students were interested in seeing the solution to the question. They had a chance to apply some recently learned theory to an immediate real-life problem. Several students (fortunately for the instructor) were able to use the concepts we had discussed in class to arrive at an answer. This entailed a rare class-wide participation in the discussion.

The class as a whole was also quite interested when we began the discussion of computers and programming. Several of the students, who had done required work reluctantly, entered into the discussion about computers with enthusiasm. These were the first to try to use the terminals to run a previously saved program to find the mean and standard deviation of a set of data. The possibility of using a computer to evaluate some of the data gathered in the investigations seemed to strike a responsive chord; the fact that working with computers was active rather than bookish seemed to bring about increased interest.

During the latter part of this seven-week period, the investigations moved to the data-gathering stage. Groups doing surveys of opinions of Bennett students determined samples of 20 to 30 students by a table of random numbers and a list of the names of students at Bennett. Members of other groups asked their neighbors in the dormitories to participate in tests of products. I realized by the middle of March that the idea of having the students do two investigations, the second based on the experience of the first, was not going to succeed. The members of my class would need the entire semester to finish one investigation in order to do a good job and to have time to write an acceptable report. The group dynamics continued to slow progress for two of the groups. Student A complained that since student B had taken charge and delegated assignments, she (student A) had only done some simple tasks and wanted the opportunity to do more. Student A also felt that student C felt the same way and that student D had not really contributed. Other groups, however, worked more harmoniously.

The middle period of the semester closed with most groups finishing the data-gathering phase and moving toward the next part of the investigations, that of analyzing the data and coming to some sort of conclusion.

Completing Investigations and Other Work: The Last Part of the Semester

The last month of the semester was characterized by "finishing up" the various assignments given at the beginning of the course. The material from the textbook on programming was finished early in the month and the rest of the period was used by the students in writing computer programs which ran satisfactorily (Appendix E-17). Several class periods were used for the students to work on their investigations, particularly in analyzing the data and writing up the conclusions. A sheet with suggestions on writing the conclusion was handed out (see Appendix E-18). The students had two more opportunities to pass topic tests and they were given additional chances outside of class. The students worked on their reading reports and handed them in on the last day of classes, and each student handed in her notebook for the second time. The notebook, theoretically, consisted of all work done during the semester. Each student was asked to include the "self-evaluation" of progress made in the course, but several of them did not make any effort to write such an evaluation. A discussion of the quality and nature of these various reports is given in Chapter IV.

The final examination was developed to take into consideration the various topics which the students had chosen to study during the semester. It consisted of five parts: four topic tests and two general questions involving problem-solving strategies and the analysis of questionnaires. Each student was able to choose the four topics with which she felt the most familiarity and competence from among the following: statistics, graphing, computer, probability, inequalities, linear equations, measurement, the normal curve, sets and logic, and consumer mathematics. The tests on linear equations, measurement and inequalities covered material which the students had studied in the computer-assisted laboratory. A copy of the final examination is given in Appendix E-19.

The first of the two general questions was identical to the problem pre-course question, so that a comparison of the students' answers before and after the experiences of the semester could be made. Hence, the question read:

> Suppose in a store you wish to find out which is the best buy: a dozen small eggs, a dozen medium eggs, a dozen large eggs or a dozen extra large eggs. You have as much time as you need to investigate. How would you go about deciding what the best buy is?

The question was intended to test the students' abilities to choose variables, to make observations, and to make decisions.

In order to answer the question completely, since no data were given, the students were expected to state the problem in quantitative terms and indicate what sort of data they would collect. In addition, the students were expected to discuss how the data would be analyzed and how the conclusion would be determined. Any sort of intelligent answer on any of these parts would indicate a development of problem-solving abilities during the semester. In addition to being part of the final exam and counting as part of the grade, the question was also intended to be used as part of the analysis of the progress the students had made in problem-solving ability. These results are contrasted to the progress of the students in the comparison section and are reported in Chapter IV.

The second general question on the final examination required the analysis of a hypothetical two-question, five-choice questionnaire, given to 50 students. The made-up data were given as part of the question, so that, unlike the first question, the students were simply asked to evaluate the data. The question was as follows:

> A questionnaire is given to 50 people at random with the following results:
> A score of 1 is assigned to a choice of strongly agree,
> 2 is assigned to a choice of agree,
> 3 is assigned to a choice of no opinion,
> 4 is assigned to a choice of disagree, and
> 5 is assigned to a choice of strongly disagree.

The frequency distributions of the two questions are given:

	Question 1: Boys do better in math than girls — score	frequency	Question 2: Boys do better in English than girls — score	frequency
Strongly agree	1	15	1	6
Agree	2	17	2	10
No opinion	3	3	3	10
Disagree	4	10	4	15
Strongly Disagree	5	5	5	9

Find the mean for each question. #1_______ #2________

How would you analyze the results of the survey as to the opinions of those questions?

These questions gave the students a chance to do some calculations and to give an interpretation of the results.

Description and Implementation of the Comparison Section

The other section of Modern Mathematics was taught to a class of 35 students in a fairly conventional way with little change from the procedures used in past semesters. The same material was taught to all students in the class with no choice of topics given. There was no attempt made to involve students in an investigation. Only one assignment outside of the textbook was given. The class followed the textbook closely, which allowed for some additional topics to be covered. The presentation was expository with frequent questions and discussion in order to involve the students; specific homework questions were assigned each class period--some to be done for practice only and some to be handed in to be graded. Two one-hour exams were given during the semester and each student was required to turn in a project involving statistics, but the data were given to the students prior to the completion of the project. Each student took a final examination.

The majority of the time was spent on statistics and the use of formulas and graphing. There was no coverage of probability, per se. Several weeks were spent in using graphs of linear inequalities to solve minimum-maximum problems (linear programming), which because of its utilitarian nature, seemed to appeal to a greater number of students than did the other topics. The section on computers was well received.

The final examination consisted of seven sections as follows: (a) 20 true/false questions covering all topics of the course, 2 questions on computers, 1 on quadratic graphs; (b) same as statistics topic test; (c) same as graphing topic test; (d) same as normal curve topic test; (e) BASIC program trace; (f) graphs of linear inequalities; and (g) same as the two general questions in the final examination for the experimental group. (See Appendix E-19.)

The students in the comparison section showed frequent boredom and occasional interest with the topics presented. Some worked homework assignments with determination while others did the required work with lack of enthusiasm. But there was no sense of frustration at not knowing what to do or what was expected. Neither was there much excitement or sense of accomplishment at having completed new tasks or at having discovered something about the real world which would be personally helpful to the student.

The Aiken attitude test and the four open-ended questions, which had been given to both classes at the beginning of the semester, were given to both groups as part of the final examination. An analysis of the comparisons of the results of these questionnaires is given in the next chapter.

CHAPTER IV

OBSERVATIONS, RESULTS, AND FINDINGS

This chapter provides additional perspective on the relative merits of the real-problem-solving approach to the teaching of general education college mathematics. Further, it examines changes in attitudes toward mathematics and learning and any increases in mathematical and problem-solving abilities as a result of experiences during the semester. Specifically, the chapter contains (a) an examination of some of the observations the students made in their notebooks, (b) a description in detail of a few of the group investigations, (c) an evaluation of the differences in the pretest and posttest results for mathematical skills, (d) an examination of the increase or decrease in problem-solving ability during the semester, and (e) a comparison of the changes in attitudes toward mathematics and toward the process of learning between the experimental and comparison sections.

A variety of methods were used to undertake such an examination. These various pretests and posttests, self-appraisals, and direct observations showed some definite trends but no significant results. A number of students completed the investigations successfully and felt a sense of accomplishment, while others found considerable difficulty in coming to a successful conclusion.

Assessment of attitudes about mathematics and the preferred process of instruction took two forms. The Aiken Attitudinal Test as a pretest and a posttest and four additional questions at the end of the semester took the form of a series of attitude statements with five allowable choices for an answer--strongly agree, agree, undecided, disagree, and strongly disagree. These were scored using a method of summated ratings. This form of questionnaire, of course, forced the respondent to narrow a potentially complex and multi-faceted answer into one of the five categories, but it also allowed rapid assessment of the overall attitudes of the student and of the class on a numerical basis. The answers were kept anonymous to encourage honest answers and most students readily answered these questions.

In addition to these Likert-type questions, several open-ended short-answer questions were given before classes started and again at the end of the semester. The intent was to allow the students an opportunity to express themselves more completely on the issues. The questions dealt with student-oriented learning versus teacher-directed instruction, a comparison of the traditional curriculum with the use of activities, and a comparison of Mathematics 101 with Mathematics 102. As with the Aiken Attitudinal inventory, the answers were kept anonymous, and although it was not possible to follow the change in attitudes of individuals, a good deal of information about the entire class was obtained. Two limitations, however, should be noted. First of all, despite the format of the questions which encouraged answers with explanations and rationalizations, many students preferred to answer simply "yes" or "no" with perhaps an additional phrase. Few took the time or effort to explain their answers fully. Second, no clear consensus developed. Roughly as many students argued for one point of view as argued for the other viewpoint. Nevertheless, the cumulative replies and some of the specific comments did give some insight into the issues involved.

Student Notebooks

The students in the experimental section were asked to keep a record of the work they did each day and to write down reflections they had on what they were doing in connection with the course. This journal, or notebook (a term which was more familiar to the students), was turned in twice during the semester and was to contain all the homework done, their reflections, work done on investigations and outside readings, and any other matters which the students wished to include. The notebooks were handed in just before spring vacation and again at the end of the semester. This section will summarize the contents of these notebooks. The other activities, such as the reading reports and the computer programs, were quite routine and will not be given any further comment.

Student Journals

The students were so unaccustomed to making journalistic entries that, by and large, they did not do so. None of the students made any day-to-day observations; the most useful entries were one-page comments on their feelings about the course to date. Two of these comments will be given below in an attempt to show the general attitudes the students had about the course. Hence, the use of journal entries as part of the evidence for this inquiry was unfortunately not possible.

The conflict between the activities of classroom and a research investigation explains, in part, why the students did not keep journals. The participants were involved primarily as students, not as subjects in a research study. When the role came into conflict, the student role had to predominate. I could not in good conscience ask the students to spend time that they didn't have to write out details of what they were doing in order to help with my investigation. It became too time consuming for them to do so. I encouraged them, but it soon became apparent that the students simply did not have the time and inclination to comply nor did they really understand what was expected. Here is a typical response which I wrote to one student in returning her notebook at midsemester:

> . . . Your notebook is well done and shows some good work, although it seems a bit sparse in working exercises. Your statement about the goals of your project was good and informative, although again somewhat

> brief. I expected to see more of an account (day-by-day) of the work you have done outside of the class and the text--when did your group meet, what was discussed, what did you do each time you went to lab, what problems you have had, and what your plans for topic tests are?

To another student I added an indication that I realized that the journal writing was a new experience and suggested she follow the directions about notebooks which I handed out previously (see Appendix E-14):

> . . . I expected you to give a day-by-day account of the work done in your group on your project, the thinking you have done in formulating questions, the time you have spent in outside reading, what other materials you have looked at, etc. You should indicate which topic tests you are planning to take and the work you have done each lab period.
>
> I realize that you did not understand what I expected on this notebook, so I suggest you go back and read the dittoed sheet on notebooks and grading carefully. I hope that your notebook will be much more complete, showing me all the work you have done. . . .

Clearly, the writing of a day-by-day journal was asking too much of the students, so I gave it up for the last part of the semester. I felt that it was sufficient for them to put enough time on their other assignments for the class. A few students did write out an evaluation of the class. Here are two with somewhat differing opinions. The first critique, written during the middle of the semester, indicates some of the thoughtfulness and searching that some of the students were doing:

> I really don't like the way the class is being guided, because I can't get into it. I would rather work through the book in the order of the book. Having assignments from the book. [sic] It is not that I don't understand the material that is being taught. But, it is just that one day we're doing one thing and the next day we are

> doing something completely different. I also don't like the group project we have. Because I can't do all the work and neither can anyone else. You can't really show your full potential because the project is not just done by you. I don't see how you can evaluate each person separately as distinct from the group. It seems like we have 3 different classes, #1 on the subjects from topic tests, #2 on the project and #3 on the lab.

The second critique gave a considered response and was written after the investigations were completed:

> I feel that this class was truly different than any other class that I have. This class has allowed me to work at my own pace. My notebook was kept up. I tried to keep mainly notes and only important findings in this notebook. Therefore you won't find an abundance of unnecessary clutter.
>
> I have enjoyed this class and feel that my math class as a whole doesn't really participate in class. I don't feel that this class was unsuccessful due to the teaching but due to the students. You can't achieve anything without putting forth some effort. I myself have learned quite a bit from the class that will be helpful to me later in life.

Daily Work

While the requirement to write a journal was deleted, I still expected to see notebooks representing the work students had done and were doing. The quality of these notebooks varied greatly--some were quite complete with a good deal of homework and some consisted of only three or four pages. For the notebooks at midsemester, most students showed several pages of homework and wrote brief descriptions concerning the investigations.

The notebooks handed in at the end of the semester showed no improvement in the quality or amount of material included. The students had spent their time preparing for topic tests, working on investigations, reading and writing book reports, and completing computer programs, rather than keeping up their notebooks. They were "after thoughts," turned in to satisfy this one annoying requirement. Several of the students commented that they felt the notebooks were "busy work"; clearly the notebooks were more for my benefit in order to have documentation for the work I was doing than for the benefit of the students. Hence, the hoped for self-evaluations were not completed. The best student evaluation of the course came from several open-ended questions to which the students responded on the last day of class. The results of these questions are discussed later in the chapter.

Group Investigations

In order to convey some sense of the activities, deliberations, consultations, and decisions involved with the investigations, the work of two groups and one individual is described as the investigations progressed from choosing a topic to writing a conclusion. In doing so, it is hoped that a greater idea may be given of the frustration and the feelings of accomplishment as well as a sense of what sort of skills and abilities the students developed.

Group on Juvenile Crime

The group on juvenile crime was formed, as were the other groups, during the second week of classes, and it was perhaps the strongest group of any. It consisted of seven individuals, who participated in varying degrees, and who seemed to work together without the difficulties some of the other groups had. The group was able to carry out its investigation with less assistance from the instructor in comparison to the other groups, but it ran into some difficulties in conceptualizing exactly what problem the members wished to consider and how they expected to go about reaching their conclusions. I met with the group once at the end of January to discuss with them their ideas. Early in February the group submitted the following questions to be used as part of a survey:

1. Juvenile crime is more prevalent in today's society. Do you agree, strongly agree, undecided, disagree, strongly disagree.
2. What is your feeling toward the source of juvenile crime? Do you feel it starts in the: Home Schools Community All
3. In what ethnic groups or nationality is the juvenile crime rate the highest: Whites Blacks Spanish Other
4. What age group is most likely to have the highest juvenile crime? Ages 6-10 11-15 16-18
5. In what juvenile sex group is the crime rate highest? Male Female
6. Do you feel the juvenile crime rate is increasing or decreasing? Why?
7. Do you feel juveniles that commit crimes should be judged as children or adults?
8. What are some ways to prevent juvenile crimes?
9. If a child commits one crime, do you feel they will continue to offend the law?
10. Counseling is an alternative to reform schools for juveniles? Yes No Why?

As part of my interaction with the group, I made several suggestions for changes in the questionnaire (see Appendix F-1).

Early in February several of the members of the group submitted a preliminary report. Below are given two of these:

> Our group is doing a survey on juvenile crime. We have met three times already. We have made up a set of questions that we have typed up and are supposingly [sic] getting ready to start the survey.
>
> Our survey deals with what age group of children will commit crimes and why they commit them. What influences children to commit crimes? Do children who commit one crime commit another?
>
> Topic: The psychological statistical aspects of juvenile crime. Purpose of topic: The purpose of this topic is to obtain statistics to justify the psychological reasons of juvenile crime. Procedure: We do not plan to conduct intense research on juvenile crime, but the main purpose of the group is to conduct surveys and obtain statistical data on the psychological reasons for juvenile crime. We do not plan to make our project a research of the factual data of juvenile crime, though we may have to include some of these facts to round out the paper, but we plan to make it pertain to the statistics of such crimes. We will conduct a survey of campus students to obtain their views of the subject matter and we plan to research the statistical facts relating to juvenile crime.

In response to these statements, I asked the group to be more specific in determining the statement of the problem that they were investigating, the plan for specific action, how the respondents would be chosen, how the data would be tallied, and the method for reaching the conclusion. I pointed out that, as far as I could tell, the students were not determining the "actual" facts concerning these questions, but rather how the population at Bennett felt about these questions by taking a sample of the students on campus and asking for opinions. The group proceeded to revise the questionnaire and change the focus of the project. The revised questionnaire (below) was later given to a sample of students on campus:

Survey on Juvenile Crime

The purpose of this survey is to find out how a group of college students would respond to the issue of juvenile crime. Please underline the answer you feel is most appropriate.

1. Juvenile crime is more prevalent in today's society than ten years ago.
   Do you: Agree Strongly Agree Undecided
   Disagree Strongly Disagree
2. Do you feel the source of juvenile crime starts in the: Home School Community
   All of the previous
3. In what ethnic group or nationality is the juvenile crime rate the highest: Whites Blacks Spanish
   Others
4. What age group is most likely to have the highest juvenile crime rate? Ages 6-10 11-15 16-18
5. In what juvenile sex group do you feel that the crime rate is highest? Males Females
6. The juvenile crime rate is: Increasing Decreasing
7. Juveniles that commit crimes should be judged as adults. Strongly Agree Agree Undecided
   Disagree Strongly Disagree
8. Some ways to prevent juvenile crime are:
   Counseling More youth activities Reform schools
9. If a child commits one crime, he will continue to offend the law.
   Strongly agree Agree Undecided Disagree
   Strongly Disagree
10. Counseling is an alternative to reform schools for juveniles.
    Strongly Agree Agree Undecided Disagree
    Strongly Disagree

THANK YOU FOR YOUR PARTICIPATION IN THIS SURVEY!

It is important to note that this particular investigation, unlike the others, did not really relate to solving an immediate real-life problem, the solution of which would bring about an immediate betterment in the lives of the persons involved. It was the only one of an academic nature and served, presumably, to satisfy some sort of curiosity. Nevertheless, the process seemed to be useful for those involved.

The next stage of this investigation was to determine the sample of students to be given the questionnaire. The members of the group used an alphabetical list of all students enrolled at Bennett and a table of random numbers to choose a sample of 42 persons. According to one of the descriptions, "It was conducted on a person-to-person basis, by three members of the group." The data which the group produced consisted of a frequency distribution, histograms, and a determination of means for each question. Each member of the group then wrote a separate conclusion in an attempt to analyze the results. It was in this step that the students had the most difficulty--in interpreting the numerical data such as a mean in terms of meaning in the real world. One of the students, who was particularly interested in the subject (see the second preliminary report on p. 175) found actual statistics from a textbook on the subject, which she tried to relate to the results of the questionnaire. Her attempt fell somewhat short.

Two of the conclusions reached by students are given here:

> I think that our survey on crime was very successful. We asked forty-two students at Bennett Colleee questions dealing with crime. All forty-two students responded. . . . We picked the numbers by random sampling.
>
> Although I don't think the group worked together very well, I feel that we worked together well enough to get the work done.
>
> The questions were mainly dealing with juvenile crime. How the young children commit crime and how they should be punished, and why people thought the young children committed crime.

On the whole the information gathered was very successful. It took a lot of time and effort to complete the survey.

In the analysis I found that most people feel that crime (juvenile) is a very serious problem in society. It begins in the home, then in schools and finally in the community. Most of the students acted very intelligently and interested in the survey despite those that thought of it to be ridiculous.

In regards to the project itself I have learned that it takes a lot . . . to conduct a survey properly. Also I learned how to use a sample-counter on getting the students who would be selected to be included in the survey.

I have enjoyed the time that was used to create the various questions used in the survey. The only disappointing occurrence was that each member of group did not work as much as others. Also, a project of this nature should be done by people who are interested in the subject.

Comments which I made to the group are found in Appendix F-2.

Group on Antiperspirants

A second group, which also consisted of seven members, worked on a "best buy" question dealing with antiperspirants. The group had considerable trouble with interpersonal relationships, as described in Chapter III. Despite the difficulties involved with leadership, the work done and the results given in the various reports showed a good deal of effort, thought, and development of the ability to analyze. The group met once during class to get organized and one time with me in late January. The statement of the problem to be investigated and their plan of action was prepared and turned in early February. They planned to carry out a survey of which brands were used most by Bennett students, to find out about the prices of the products, and to determine relative effectiveness. Various tasks were planned and the members of the group agreed to carry out one or more of the tasks. The original plan of the group was given as follows:

Statement

(1) 2 girls will conduct the survey and determine the two leading products.
  (a) need to survey 30 girls by random sample
  (b) need to determine the top brands--two leading brands

(2) 2 girls will price the two products, determine the validity of advertising claims, determine how much you pay (price per ounce)
  (a) need to go to two different stores (or even more) to find the two products
  (b) need to write down the prices, ounces and the advertising claims that they see about the products on TV, mags, etc.

(3) 4 girls need to test the effectiveness of the two products
  (a) two girls will undergo tests, using the two brands (extreme tests)
  (b) two girls will keep records on the test; one will record one product, the other will record the other product
  (c) the first two girls will undergo a test in which they wear the product all day long under normal conditions
  (d) the other two girls will record the results at the end of the day

(4) One girl will be the record keeper, she will obtain graph paper for the group, determine what sort of records and data should be obtained by the other girls, keep the total record of the group's progress, activities and findings, etc. All the girls will determine the results of the group's tests, findings, and results. We will have a meeting at the end of the study of antiperspirant.

(5) Two girls will write up the total summary of the test by answering the "Suggested uestions for Planning and Investigation" (ditto sheet), explain the group's activities, the evidence and the conclusion--which brand is better.

At the bottom of the two-page report were names of the students who agreed to do each of the tasks.

I responded to this first draft with some suggestions, particularly that the group share in distributing the questionnaires and that the process of measuring effectiveness should be thought out more carefully. I also pointed out that each of the members of the group was expected to analyze the data and write separate conclusions.

During the next month the members of the group worked on developing their own instruments or procedures for finding out the assigned information. The first student passed out a survey to students as determined by thoroughly mixing slips of paper numbered 1 to 603 (the number of students enrolled at Bennett) in a paper bag. Thirty-four of these slips of paper were pulled out without looking. This process, although cumbersome when compared to the use of a table of random numbers, indicated that these students understood the concept of a random sample quite well. The respondents, as determined by the numbers drawn, were asked the following questions:

1. Which brand of anti-perspirant do you use?
2. What amount of influence does the price have on your choice?
   Heavy, average, or no influence
3. What amount of influence does the effectiveness of the product have on your choice?
   Heavy, average, or no influence
4. What amount of influence does the advertising of the product have on your choice?
   Heavy, average, or no influence

After the survey was undertaken, frequency distributions and histograms were prepared. (It turned out that effectiveness had the greatest influence on the choice of a brand; advertisement, little influence; and price, only average influence.)

Finding the best price for the various brands was straightforward--prices at three stores for each brand were gathered and the average price per ounce determined. The students found considerable difference in the average price.

Testing the effectiveness of the antiperspirants took a good deal more inventiveness. Here is the way one of the members of the group described the experiment:

> We proceeded to test the effectiveness of the three brands, but then opted to use only two brands in order to accurately compare and contrast the results. For 8-9 days, two members used one of the two brands, "Secret" or "Ultra Ban"--both roll-ons. The days were divided into four segments, each segment totaling four hours, and they decided whether or not their brand was effectively keeping them dry, slightly dry, or wet. The experiment also required that the daily activities be jotted down in order to get a better picture of just how the user spent her day.
>
> Experimenter #1 used Secret and #2 used Ultra Ban. Both girls followed mainly normal daily schedules, so the comparison was easy. On an average, experiment[er] #1 [scored] 2.46 and experiment[er] #2 [scored] 2.61, based . . . on numerical values--dry=3, slightly dry=2, wet=1, very wet=0. The difference between these values is .15 which seems to lose some of its significance considering the extreme activities that <u>were</u> performed by both experimenters.

With limited time and facilities, this was, I thought, a commendable effort to gain some idea of the effectiveness. I pointed out to the group that to be able to say that you have an unbiased experiment, you need to (a) make sure the experimenter does not know what brand she is using in order to avoid any personal preferences, and (b) have each experimenter use both kinds to avoid any differences in individuals.

Each of the students of the group wrote a conclusion. The quality of these reports varied greatly, but for the most part they agreed on what the data showed. This group had had a difficult time getting started, but they were able to work out a reasonable combination of ways to analyze the product. The members, with one exception, expressed positive reactions after the project had been completed. Below are comments from two of the students in the group:

> Student #1. I really enjoyed this project once we got the ball rolling because the results began to get very interesting. I also enjoyed working as the record keeper, pulling together the results, and analyzing them. In my opinion, analyzing the results was a project within itself.
>
> Student #2. The group found that "Secret" roll-on which represents 35% of the thirty-four girls [was preferred over] Ultra Ban--17%. I feel that Secret is the number one and best buy.
>
> My conclusion is that Secret was the one that was the best due to circumstances that deal with different people's preferences, and the way [anti-perspirants are] advertised can influence a person, but not as heavy [sic] as effectiveness. The two girls that wore the two different deodorants came up with different results due to the difference of bodies and activities.
>
> I feel our group had its ups and downs but withstood all the misunderstanding and overall worked well as a group.

Other Investigations

Not all the investigations were as successful as the two just reported. All the other groups dissolved into smaller numbers of two or three students each. Three students decided to work completely independently. Over half of the remaining groups used only one questionnaire which was passed out without using random sampling, with no other attempts to gather other facts or information about the product involved. Often neighbors on the same floor in a dormitory were asked to respond to the questionnaire. The results of the questionnaire were interpreted incorrectly, and if graded on a scale of 1 to 5, the mean for each question would be computed incorrectly. In these cases the groups were asked to rewrite and resubmit the conclusions. Asking these students to write up credible conclusions without specifically teaching the skills needed to do so correctly led to confusion although the students understood the need for making the corrections.

One of the students, who had struggled through two semesters of remedial mathematics as well as Math 101, worked on her investigation by herself. She passed out a questionnaire about why persons bought certain kinds of clothing. She finally figured out how to compute the frequency distribution. In some comments addressed to me she wrote:

> After talking to you, I understood more what I was supposed to do. I hope you are as pleased with the project as I am.

> This time I asked the questions of only two people, and I wasn't confusing myself with a large number of people; and in general I felt a whole lot better about the project. I don't know who the people were, I just walked up to them and asked if they would mind answering some questions.
>
> In conclusion, I was very pleased, and enjoyed talking with these people. . . . I ran into some complications doing the project the way we discussed, so I "redecorated" by rearranging the questions and the responses. I really enjoyed doing this project, and I hope that I've gotten all the "bugs" out.

For this particular student, the project gave her a much more positive attitude toward her mathematics class, despite a naivete, a limited knowledge of the subject, and three attempts to write up her conclusions.

Several of the students decided to work on second investigations, partly motivated by the chance to earn a higher grade and partly because, having done one, they felt much more sure of undertaking a second. The quality of the second surveys was as good or better than the first ones.

Those students who expressed personal reactions to doing the investigations generally gave favorable responses, once having completed the task and having come to some sort of meaningful conclusion. But as will be seen later in this chapter, based on questions asked of the students two weeks after the investigations were turned in, about half of the students indicated a preference for the usual melange of classroom activities, e.g., homework, lectures, and following the textbook.

Acquisition of Standard Freshman Mathematical Skills

Let us now turn to a consideration of the effect of the real-problem-solving curriculum on the acquisition of mathematical skills. One of the difficulties of assessing the effects is that the process was modified in the course of the semester, and although direct teaching was used in both classes, the specific teaching of particular skills was used for a greater amount of time in the comparison section than in the experimental class. Did this make any difference?

Both groups were given the same pretest (see Appendix E-4) and similar, but not identical, topic tests as part of the final examination (see Appendix E-19). For the pretest each of the five subtopics (algebraic expressions, solving linear equations, quadratic equations, simple graphing, and basic statistics) was graded on the basis of 20 points, so that the total possible was 100 points. The first four topics had been covered in the previous course; the fifth topic involving mean, median, mode, and frequency distribution for a collection of single digit numbers was new to most students. The results of the test are given in Table 4.

The pretest showed a somewhat surprising result in that the means of the pretest scores--40.13 and 30.06--differed significantly. It seemed reasonable to expect that the usual student selection of sections of mathematics during registration, a selection based on a variety of factors,

Table 4

Comparison of Quantitative Pretest Scores

	Math 102-01 Experimental	Math 102-02 Comparison
	Quantitative Pretest	
Number who took test	33	28
Mean	40.13	30.06
Standard Deviation	20.59	17.90
	Means for the subsets (20 points)	
Algebraic Expressions	12.8	9.3
Linear Equations	8.8	7.4
Quadratic Equations	5.6	3.0
Graphing	10.5	9.3
Statistics	1.4	0.9

Difference of means $t = 2.0429$ $p < .05$

Frequency Distribution

	Experimental	Comparison
[0-10)	1	4
[10-20)	3	3
[20-30)	7	10
[30-40)	4	10
[40-49)	3	0
[50-59)	5	4
[60-69)	4	1
[70-79)	0	1
[80-89)	1	0
Minimum	8	0
Maximum	88	77
Correlation with SAT-Math Scores		
	.545	.509

would result in a random cross-section of students with reasonably equal backgrounds in terms of Math-SAT scores and previous knowledge of mathematical facts. After all, the students had all taken Mathematics 101 during some previous semester. But this was not the case. Not only were the Math-SAT scores significantly different between the two classes (as noted in Chapter III), so were the differences in the pretest scores. Indeed, the $\underline{t}$ test for making inferences about the difference of means using independent samples with unequal numbers indicated that the assumption that the means of the two populations were equal must be rejected at the 5% level ($\underline{t} = 2.0429$ and $P_{59}(\underline{t}) = .9772$).[4] Hence the students brought mathematical knowledge to the classroom which was at considerable variance. In making any analysis of progress during the semester, it was necessary to take this difference into consideration.

A breakdown into the five subtopics showed a consistent pattern--the comparison group scored lower than the experimental group in each of the five categories. Both groups showed little previous knowledge about quadratic equations or statistics: 23 of the 33 students in the comparison class scored zero on the quadratic equation while 16 of 28 students in the experimental class did not even try to answer the question. The results for the two questions on statistics showed

[4]All calculations were done on a Texas Instrument 58 Programmable calculator. Statistical values were determined by using the applied statistics module insert.

even worse scores. Here only 5 of the 28 students in the experimental class and 4 of the 33 students in the comparison section scored more than zero on the statistics question.

The original intention at the beginning of the semester was to have the students in Math 102-01 "discover" for themselves those concepts which they would need to study in order to find solutions to the problems they were trying to solve. The hypothesis was, as discussed in Chapter III, that this procedure would increase the motivation the students would have for learning specific mathematical concepts and hence increase the actual learning. The simplified model thus was to be:

Problem to be solved
↓
Attempts to reach a solution
↓
Consultation with teacher, research in textbook
↓
Learning needed mathematical skills
↓
Solution to problem

The process quickly broke down at Step 2 when it became apparent that most of the students had no idea of the kinds of ideas to consider, what data to collect, what to do with the data, what formulas to consider, etc. Hence the model of teaching procedure was changed somewhat to provide for greater teacher intervention as follows:

Problem to be solved
↓
Attempts to reach a conclusion--find out the difficulties involved in finding a solution
↓
Directed suggestions as to the mathematical facts and procedures needed
↓
Teaching of the topic to the class or to groups
↓
Solution to the problem

By making this modification it was hoped the frustration of the first approach would be lessened but the motivation resulting from an attempt to solve an interesting problem would be maintained.

In contrast, the teaching model for the comparison section was as follows:

Teacher choice of topic
↓
Directed teaching of the topic
↓
Test and application

Much more time was taken with homework, lecturing, class discussion and review of the specific mathematics topics in the comparison section than in the experimental, since the experimental class spent the equivalent of two weeks of class time in carrying out the investigations.

In order to assess the performance of the students, the four "topic" tests, which were part of the final examination, were scored on the basis of 25 points each in order to determine a total score of 100 possible points. The tests were similar to those which the students had been given during the semester; however, the students in the comparison section were given no choice as to which topics they were tested on while, consistent with the outline of the experimental course, the students in the experimental class were allowed to make a choice. The topics for the comparison section were statistics, graphing, evaluating the normal curve, and computers. The students in the experimental section were allowed to choose from among the same four topics (with identical questions) and, in addition, probability, linear equations, measurement, and inequalities. The results of the posttest scores are given in Table 5.

A _t_ test of the hypothesis that there was no difference between the means was performed. The value of _t_ equals 2.0043, and $P\ (\underline{t} < 2.043) = .977$ with $df = 60$. Hence, the hypothesis that there was no difference between the means must be rejected. The fact that this statistical test indicates that the experimental class did a better job on the posttest doesn't tell us much since they also did significantly better on the pretest. To get a clearer picture of the relationship between the variables, a linear regression analysis was made.

The linear model for each subject was the following:

$$z = a_0 + a_1x + a_2y + e$$

where x is the pretest score and y is a qualitative variable--0 for membership in the experimental group and 1 for membership in the comparison group. z, the covariable, is the posttest score. The summary of the analysis of the model is given in Table 6.

Table 5

Comparison of Posttest Score in

Mathematical Topics

	Math 102-01 Experimental	Math 102-02 Comparison
	Quantitative Posttest	
Number who took the test	30	32
Mean	72.8	63.8
Standard deviation	20.59	17.90
	Number of students taking the topic tests	
Statistics	30	32
Graphing	27	32
Computer	15	32
Normal Curve	6	32
Probability	8	
Measurement	7	
Linear Equations	20	
Inequalities	6	
	Frequency distribution of posttest scores	
40-49½	4	2
50-59½	3	8
60-69½	5	13
70-79½	4	8
80-89½	10	1
90-99½	4	1

Difference of means $\underline{t} = 2.043$ $p < .05$

Table 6

Analysis of Linear Regression

Pretest and Section Versus Posttest

Number		57
Coefficient of determination	R^2	.2597
Standard Error of Estimate		13.16

	constant	x pretest	y section	z posttest
Mean		35.62	.53	68.89
Coefficients	63.88	.25	-7.55	
Standardized Coef.	.0	.3631	-.2771	

Correlation Coefficients and *F*-ratios

	Corr. Coeff.	df	*F*
All 3 variables	.51	2,54	9.471*
z with x alone	.43	1,55	12.73*
z with y alone	-.37	1,55	8.69*
x with y alone	-.25		

*Significant at the 1% level

This data crunching, for what it is worth, shows, as would be expected, that the pretest scores contributed significantly to the posttest scores, but it also indicates that membership in the comparison section had a "negative" effect on the posttest scores. This latter result occurred despite the fact that a good deal more time was spent in the comparison class "teaching" the topics covered by the tests than in the experimental class. All three $\underline{F}$ tests show significant results at the usual 5% level, and, indeed, at the 1% level. The regression of z on x and y is statistically significant, since the probability of an $\underline{F}$ ratio as large as 9.471 is less than .01. The relation between z and a linear least squares combination of x and y could probably not have occurred by chance (Kerlinger & Pedhazur, 1973, p. 40). The two independent variables contribute a rather meager amount to the final results, namely only 26%. Furthermore, the data indicate that the pretest scores contribute approximately 19% to the posttest scores while the contribution of y is roughly 13½%.

Looking more carefully at the difference between the results for the two sections, we see that since y is 1 for students in the comparison section and the coefficient -7.55 of y is negative, the mere fact that a student was in the comparison section would mean, according to this analysis, a decrease of more than 7½ points compared to a student in the experimental class with the same pretest score. For example, a student with a pretest score of 40 in the experimental class would, according to the regression equation, score 74 points while a student in the comparison section with the same pretest score would have a predicted score of only $66\frac{1}{2}$.

Obviously, this equation is a greatly simplified attempt to quantify what was taking place during the semester and there were many variables not accounted for in the analysis. It would be tempting to try to claim that the experimental class, because of increased motivation and interest in learning the subject matter, simply learned the material better. But the evidence is lacking since no attempt was made to assess the contribution of any other attributes. As a counterargument, it seems reasonable that students who have already done better in learning mathematical concepts, as the SAT-Math and the pretest scores indicated, would continue to do even better. It is also likely that being able to choose the topics to be tested upon contributed to the higher scores, but this may be due partially to allowing students to study concepts which they find useful and of interest, not necessarily because they find the topics easier. The results of the data and the arguments which have been given can be summarized as indicating that the students in the experimental class progressed at least as well in concept acquisition as the comparison students.

Evaluation of Problem-Solving Ability

In order to make some sort of assessment of the improvement in problem-solving ability during the semester and to compare such ability between the two classes, the Egg Problem was given to both classes as a pretest and as a posttest, and a questionnaire analysis was given to both classes at the end of the semester. The Egg Problem was an open problem: the student had to decide on strategies, needed data, and how to interpret the data. The questionnaire analysis was a closed problem: the question and the data were given and the student was asked simply to interpret the data.

The Egg Problem

The formulation and the assessment of the Egg Problem were patterned after the Notebook Problem used by the Unified Sciences and Mathematics for Elementary Schools (USMES) program in 1971-72 as part of its evaluation of its program. A brief review of the use of the Notebook Problem will help to explain the use of the Egg Problem.

Bernard Shapiro wrote:

> In addition to the data yielded by [teacher logs, classroom observations and standardized tests], however, there was a desire to observe the problem solving behavior of elementary school children in a situation which was standardized and structured but which provided the subjects with an opportunity to consider and test hypotheses with concrete materials.
>
> In order to accomplish this purpose, the Notebook Problem was devised. It consisted essentially of presenting the testee with three notebooks selected so as to differ from each other in terms of such dimensions as number of pages, number of lines per page, binding,

> price, space between lines, width of ruled margin, etc. and asking the testee to (a) select the most appropriate one for his class, and (b) indicate the reasons for his selection. (1973, p. 1)

The Notebook Problem was administered to five randomly-selected children from each of several experimental and control classrooms involved in USMES projects. Trained personnel evaluated the student's procedures and interviewed the students to find out the reasons they had for making decisions. The problem-solving protocols were assessed using a simple scheme:

> (a) whether or not any of the subject's reasons for selection were stated along dimensions that were measureable within the test and
> (b) the highest level of warrant associated with the given reasons for selection. The dimensions measurable within the test situation were (i) size-volume . . . , (ii) weight . . . , (iii) quantity . . . , and (iv) cost. . . . Three categories were developed for level of warrant. These were (i) reasons given . . . expressed simply as personal opinion, (ii) a test was suggested to assess the reason given, and (iii) a test was actually performed to test the reason given. These levels were considered a hierarchy in increasing order of appropriateness, and each protocol was assigned to the highest level present among the several that an individual subject may have used. (Shapiro, 1973, p. 4)

To assess the benefit of the USMES program in developing problem-solving skills chi-square tests were used to test the results of (a) and (b) separately as two-way classification tables, involving the pretest and the posttest for the same category and school, i.e., the experimental and the control groups were considered separately. In all schools tested the increase in scores in the experimental groups were statistically significant at the 5% level while none of the chi-square values for the control groups exceeded the theoretical values.

Using this evaluation procedure as a model, I proceeded to set up a related problem which could be given to all students. The problem was essentially how to decide which size of eggs--small, medium, large, and extra large--would be the best buy. At the beginning of the semester the students were given two days to consider the question and to write a response in as much detail as they felt was needed. The same question was included as part of the final examination and graded at the time generously; the main reason for making it part of the final examination was to be sure all students responded to it.

The pretest was not scored in any detail until the end of the semester, so that the pretest and the posttest could be scored at the same time with the same analysis. I had expected answers to indicate a mathematical approach, for example, finding out the weight of a dozen eggs and then dividing to find the cost per ounce. Most of the answers, however, were quite a bit more vague than that. The answers alluded to checking the price and the grade and then comparing (no statement of what to compare) or to determining what the eggs would be used for (so that the size, not the price, would be the determining factor).

My first attempt at setting up a method of evaluation, similar to that of the Notebook Problem, involved recognizing any suggestion for considering price, weight, or volume as quantitative measures, but I found that many of the students used the word "size" which I had originally not recognized since I felt that it was too vague. I later decided to include "size" as a legitimate response. Furthermore, in assessing the various levels of warrant, I originally accepted three levels: personal opinion (no attempt made to gather and evaluate data), vague idea of some sort of a test, and a specific test. I discovered that this three-way grading procedure did not allow a distinction between those answers which gave only a personal opinion and those which alluded to, but did not specify, a specific process, e.g., "would compare," or "would decide on how the eggs would be used." Hence, the final procedure of analyzing the levels of warrant included a hierarchy in increasing order of appropriateness including personal opinion, a vague allusion to quantitative analysis (comparison, number of eggs used), one specific operation, and a complete set of procedures in order to come to a conclusion. In assessing the number of measurable dimensions, I included not only my original three (price, weight, and volume) but also size, grade, and purpose if quantifiable (use in recipe, use for a family of four, etc.). I assessed both the pretests and the posttests twice over a period of two days in order to check for consistency of judgment, and in those cases that the analysis did not agree, I checked the protocols a third time.

As an indication of the wide variety of answers given by the students, several are included below. These were given at the beginning of the semester:

> I would evaluate all four sizes of eggs. I would need to obtain all the information needed about the eggs, the sizes, and costs. I would compare the information gotten on all sizes of eggs, then determine which is the best buy. I would say the best buy varies. I think the best buy would be according to what the eggs are being used for. You would use different sized eggs for different uses and purposes. After arriving at my conclusion I would then test my hypothesis to see if it is correct.

This protocol was given a rating of 3 quantities but a <u>vague</u> rating for the highest warrant. The words "compare" and "determine" do not convey any sort of specific problem-solving process. Nevertheless, when contrasted to the next response, the reply above had some merit:

> I would choose the larger eggs as the best buy because some times there are two egg yolks in them, so I would get two eggs instead of one.

A home economics major provided the following information:

> When you purchase eggs, one should consider many factors in order to get a good buy. First of all, all eggs offered for sale in most retail stores are graded. Because eggs also are sold on the basis of weight, the consumer must consider both grade and weight in relation to the price of eggs. Grade and size are independent factors since Grade A eggs have the quality whether they are small or large. It is usually assumed in recipes that the egg size is either medium or large. So, the best buy would be a medium grade A egg. In order to arrive at a conclusion, one must first gather information on eggs. Then [find out] about different types of eggs and try to determine the best buy. The information should be thought out and summarized.

I gave this reply a score of 2 (grade, weight) and S (she had a specific method and reason for her conclusion). The Egg Problem I gave was clearly an open-ended question, since the students first had to decide what was meant by "best" and also how they would go about determining which product satisfied the criteria. The home economics major had not mentioned price at all nor any consideration of getting the "most" for her money. For her, large eggs would not be satisfactory since they could not be used properly in recipes.

Essentially the same question was given at the end of the course, except that the students were asked to take not more than 20 minutes to answer the question. Some of the students made no attempt to respond to the question at all; many gave the same sort of vague suggestions of what quantities to consider but did not tell what to do with the information. Two showed that they had learned the steps for doing an investigation, but they left out the specific details. Here is one example:

> Set up a survey concerning the products being compared. Prepare questionnaires. Collect data. Compute data. Compare data. Compare data that has already been gathered to your data. Draw a conclusion based on raw data. Compute to see if conclusion is valid. Compare statistical facts.

As can be seen, this student outlined the general steps but did not indicate any specific quantities to be considered or any questions to be included on the questionnaire.

Some of the students who worked on investigations during the semester suggested that one way to find out the best buy would be to ask 30 shoppers (at random) what size they preferred; the most popular choice would determine the best buy--the democratic rationale. The next example shows how difficult it was for one student to be precise; she indicated some of the considerations which would go into her decision, but she could not put them in truly quantitative terms:

> When you go into the store to buy the best eggs, you compare each dozen eggs by the price. Then you decide which size would be logically the best buy; the small eggs are too small, the medium eggs are in between . . . , the extra large eggs are too big and the large eggs would <u>appear</u> and <u>seem</u> [underlining mine] to be the best buy, because of their size and their being the best size of eggs.

Perhaps the best response in pinning down a quantitative process for making a decision came from one of the students in the experimental class:

> I would take an example of each size of egg and test it to see which egg would give me more use at a low price. I would make a cake for example. Suppose the recipe called for two eggs. Now I would buy half a dozen small eggs at 24¢, half a dozen medium eggs at 34¢, and half a dozen large eggs at 45¢. The half dozen small eggs are 4¢ each and the large eggs are 8¢ each; the medium eggs are 5¢ each. In the cake recipe if I used two small eggs, it would cost 8¢. If I used two medium eggs, it would cost about 10¢. But since the large eggs at 8¢ can do, I would buy the large eggs. They are cheaper in the long run.

The summary data for the Egg Problem are given in Table 7. This table shows that the largest number of students in each of the categories considered <u>two</u> measurable items, most often the price and the size. The only change in frequencies of any importance occurred in the experimental group in the column for 3 measurable items (5 for the pretest

Table 7

Reasons for Decision on Egg Problem

	Number of Measurable Items				
	0	1	2	3	Total
Experimental--Beginning	2	5	14	5	26
Experimental--Conclusion	2	2	14	12	30
Comparison--Beginning	1	6	16	5	28
Comparison--Conclusion	6	7	14	5	32

	Highest Level of Warrant				
	Opinion	Vague	One Computation	Specific	Total
Experimental--Beginning	9	14	1	2	26
Experimental--Conclusion	3	18	5	4	30
Comparison--Beginning	6	19	3	0	28
Comparison--Conclusion	12	18	2	0	32

and 12 for the posttest). As with the Notebook Problem, chi-square contingency tests were made to compare the before and after results for the experimental and comparison groups separately. In addition, the results for the two groups before the semester started were compared and the results for the two groups at the end of the semester were compared. The specific statistical question considered in each of the four cases was whether there was a significant difference between what was observed and what was expected, where the expected frequencies were determined by taking each two-way classification table and multiplying each row total by each column total and dividing by the overall total (Freund, 1970, p. 254). Since some of the cells contained entries of four or less, Yates' correction formula was used for the computations (Gordon & Schaumberger, 1978, p. 165). Table 8 gives the results.

These results show that none of the relationships were significant at the 5% level. However, it can be seen that at the beginning of the semester the results for the two classes are almost identical for the measurable items while at the end of the semester the difference from the expected in the experimental versus comparison had increased greatly.

In terms of the level of warrant associated with the subject's reasons for making a decision, the summary data in Table 7 show a less clear result, although the number of "opinions only" decreases for the experimental section while

Table 8

Egg Problem: Chi-Square Test Results

	df	χ^2	Cumulative Probability
Measurable Items			
Beginning--Experimental vs. Comparison	3	.0593	.0037
Ending--Experimental vs. Comparison	3	4.8980	.8206
Experimental--Beginning vs. Ending	3	2.4157	.5093
Comparison--Beginning vs. Ending	3	2.1619	.4605
Level of Warrant			
Beginning--Experimental vs. Comparison	3	1.9290	.4127
Ending--Experimental vs. Comparison	3	6.9301	.9258
Experimental--Beginning vs. Ending	3	4.0519	.7440
Comparison--Beginning vs. Ending	2[a]	2.4228	.7021

[a]Since the cells in the last column for the comparison section for both the beginning of the semester and the ending of the semester contained 0, the expected frequency is 0, so that these cells were eliminated in the chi-square test.

the number of "opinions only" increases for the comparison group. Table 8 indicates that this difference between the posttest and pretest results for the experimental group, while not significant at the 5% level, is significant at the 10% level.

The Egg Problem was intended to assess some possible trends in the development of problem-solving ability. The evaluation procedures suffered from vagueness of the question, lack of quality control in assessing the results of the student responses, and a poor choice of timing for giving the posttest since students were expected to write out a response in 20 minutes at the same time they were finishing a final exam. Furthermore, any attempt to reduce such a task as problem-solving to such a simplistic statistical analysis is suspect. The replies of the students for the most part were disappointing in contrast to what should be expected of all students at the end of the semester's work. Nevertheless, the data do show that the process of using an investigation as part of the teaching process is helpful in making the students aware of at least some of the aspects of a problem-solving approach to a real-life problem.

Questionnaire Analysis

A second question on the final examination used to assess problem-solving ability dealt with the interpretation of given data in the form of frequency distributions for two hypothetical questions on a questionnaire. The students were asked simply to evaluate the data and analyze the results of the survey. This skill had been discussed in the comparison section, albeit in a fairly theoretical way.

The difficulty for the unaware student in this problem was that the mean was found by taking the sum of all 50 scores (for the first problem this included 15 ones, 17 twos, 3 threes, 4 tens and 5 fives) and then dividing the sum by the total number of scores (50). Analysis of the data followed from the observation that the middle score between 1 and 5 was 3, so that a score of less than 3 would be interpreted as favoring a general feeling of "agreement" while a mean above 3 would indicate a disposition toward "disagree." Many of the students in two classes found either the mean of the scores, 1, 2, 3, 4, 5 (hence getting 3 for both questions) or found the mean of the frequencies, hence getting a score of 50/5 or 10. Unfortunately, neither class did very well with the question. The students tended to jump to conclusions rather than analyzing the data carefully. Again, perhaps they were tired after completing the rest of the final examination; perhaps they would have done better if the question had been given during a class period and more time had been allowed for its completion.

For what it is worth, the question was graded on a 15-point basis as follows: up to 5 points for computing the mean of either the scores or the frequency; 10 points for computing the means of the scores correctly (2.46 and 3.22

respectively) and up to 5 additional points for interpreting the means even if computed incorrectly. The experimental group had a mean of 9.13 while the comparison group had a mean of 5.09. The difference in the scores indicates that the active learning required in the investigations led to a better understanding of the process. Most of the students in the experimental class went through this process in interpreting questionnaires used in investigations. The computations of the means apparently made more sense to them than to those in the comparison section who had previously worked a similar problem only as a homework problem.

Attitudes Toward Mathematics

To get a greater sense of the attitudes the students had toward mathematics, they were given the Aiken Attitudinal Test and an open-ended question early in the semester.

Previous Experiences in Mathematics Classes

The short-answer question concerning student attitudes about mathematics was "What previous experiences in math classes have led to your like or dislike of math?" There was a wide discrepancy of answers, but a majority in both sections told of experiences which had led to a general dislike of mathematics--17 of 28 in the comparison section and 18 of 27 in the experimental section.

A frequent reason given for disliking mathematics was having had a "poor" instructor; conversely, a frequently given reason for liking mathematics was a previous experience with a good, interesting, and helpful teacher. For example:

> All my experiences I've had with math were horrible. I realize some may be due to lack of interest but I have always hated it and never really tried <u>hard enough</u> to study it. Especially after my horrible Algebra teacher in high school. . . . (Comparison)

> I've always liked math, but my previous experience is having a good teacher for mathematics. (Experimental)

Here is a slightly different slant on what one student felt made a good math teacher:

> My dislike [came when I had] a boring teacher who never assigned homework and tried to make himself clear (by repeating the same explanation). My like [sic] is having a teacher who assigns homework and one whom I am not afraid to ask questions of like the math teacher I have now. (Comparison)

The next response described the most difficult experience given by any of the students:

> When I was a senior in high school I had a business math teacher who was really dumb. . . . If a student could not work a problem, she would make fun at her and send her to the blackboard so she could have something to play with and she did that to me. It really hurt. I lost respect for math teachers. I feel that they think they know everything and I lost respect for math at that time. . . . (Experimental)

Finally, here is an example of a student who mentioned both favorable and unfavorable experiences in previous mathematics courses:

> I have disliked math ever since I first entered school. . . . Back in Jr. High I hated my mathematics teacher and have been turned off toward math ever since. . . . Since I have been enrolled in your class I have learned a great deal from math. I'm beginning to understand and like it more and more. (Experimental)

Other students listed boring classes or boring topics--particularly high school geometry--as leading to a dislike for mathematics. A sizeable group of students indicated the difficulty of the subject led to their not liking the subject as evidenced by their getting bad grades, giving up, and not being able to get the "correct" answers. Approximately 25% of each class indicated <u>liking</u> mathematics: "It has always been exciting"; "I have always liked the challenge of solving mathematics problems"; or "I have enjoyed challenges in mathematics since 3rd grade and have always worked well with numbers."

<u>Aiken Attitudinal Test</u>

The Aiken Attitudinal Test was given at the beginning of the semester and at the end of the semester in an attempt to get a sense of the direction and magnitude in any change of attitude toward mathematics. The reliability for this test has been found to be .94 for the test-retest (Aiken & Drieger, 1961, p. 18). This test was graded on the scale of 1 through 5 with the smaller numerical value indicating attitudes of dislike for mathematics. A summary of the results is given in Table 9.

It might be expected that those students who have chosen fields not involving quantitative analysis such as mathematics and science would have a fairly negative attitude toward mathematics. We have already seen that many of the students

Table 9

Aiken Attitudinal Scores[a]

	Pretest Experimental	Pretest Comparison	Posttest Experimental	Posttest Comparison
		Results		
Number of Students	34	34	30	24
Mean	2.62	2.50	2.93	2.77
Standard Deviation	.865	.949	.851	.795
		Frequency Distribution--Percentages		
[1.0-1.5]	8.8	17.6	3.3	4.2
(1.5-2.0]	23.5	14.7	10.0	17.7
(2.0-2.5]	23.5	35.3	16.7	16.7
(2.5-3.0]	17.6	8.8	33.3	20.8
(3.0-3.5]	5.9	5.9	10.0	25.0
(3.5-4.0]	11.8	11.8	13.3	12.5
(4.0-4.5]	8.8	2.9	10.0	4.2
(4.5-5.0]	.0	2.9	3.3	.0

[a]The values were assigned these meanings: 1-Strongly Dislike 2-Dislike 3-Neutral 4-Like 5-Strongly Like

wrote of unpleasant experiences in previous mathematics courses. However, the results of the Aiken Attitudinal inventory at the beginning of the course indicated only a mild dislike. Since a score of 2 represents "dislike" and a score of 3 represents a "neutral" attitude, then the pretest means of 2.6 and 2.5 represent scores approximately halfway between "dislike" and "neutral." On the other hand, in terms of sheer numbers of students, a large majority of the students showed some degree of dislike for mathematics: 73.4% of the experimental class and 76.4% of the comparison class had a score of 3 or less.

Table 10 gives a summary of the Aiken Attitudinal scores and with only four categories, it shows more clearly that the results for the two groups for the pretest were quite similar. For both groups there was a slight shift toward a more favorable attitude toward mathematics as a result of the experiences the students had during the semester. On the posttest a slightly greater number of students in the experimental class showed either a dislike or a mild dislike than in the comparison group: 63.3% to 58.4%. Apparently, several of those students who really disliked mathematics in the comparison group found enough favorable experiences in the traditional classroom and hence came to feel somewhat better about mathematics in general. Another shift of note took place in the last category, "like mathematics," in which 8.8% in the pretest for the experimental group increased to

Table 10

Summary of Aiken Attitudinal Scores

Four Categories, Percentages

Category	Range of Scores	Pretest Experimental	Pretest Comparison	Posttest Experimental	Posttest Comparison
Dislike	1-2	32.3	32.3	13.3	20.9
Mild Dislike	2-3	41.1	44.1	50.0	37.5
Mild Like	3-4	17.7	17.7	23.3	37.5
Like	4-5	8.8	5.8	13.3	4.2

13.3% on the retest, while in the comparison section, the percentage for the highest level group decreased somewhat. Perhaps it can be speculated that some of the students who came to the class with a favorable attitude toward mathematics found the investigations enjoyable while students in the comparison section found the lecture-discussion approach unchallenging.

It should be pointed out that since the Aiken Attitudinal Inventories were anonymous, a correlation or comparison for individuals could not be made. A more careful analysis of how the two approaches affected individuals in the various categories could have been made if the students had been required to put their names on their replies. But this might have produced somewhat less candid replies and seemed inappropriate in a classroom setting in which students were to receive grades. Similarly, an analysis using a linear regression could not be made.

To discover more completely the effect of the real-problem-solving curriculum, a $\underline{t}$ test was performed to compare four different pairs of groupings (Glass & Stanley, 1970, p. 295). A summary is given in Table 11.

The results do not allow any significant advantages of one method to be claimed over the other (at the 5% level), but they do show some trends.

As would be expected by the results of the frequency distributions, there was very little difference between the

Table 11

Aiken Attitudinal Scores

t test, Difference of Means

Comparison	*t* value[a]	Degrees of Freedom	Cumulative Probability[a]
1) Experimental: pretest vs. posttest	1.427	62	.9208
2) Comparison: pretest vs. posttest	1.114	66	.8654
3) Pretest: experimental vs. comparison	.504	64	.6921
4) Posttest: experimental vs. comparison	1.166	50	.8755

[a]*t* values and cumulative probabilities were computed with a TI-58 programmable calculator using the applied statistics module insert.

two groups (line 3) at the beginning of the semester. The difference between the two means would be equal to or more than the observed value by chance approximately 30% of the time. However, when the test was taken at the end of the semester, as indicated in line 4, the probability that the difference of means would occur by chance decreased to approximately 12% of the time. While not a significant result, this would seem to indicate that quite a number of the students in the experimental class found that the various experiences during the semester gave them a somewhat more positive attitude toward mathematics. This increase may be due, of course, to maturation in general, to influences outside of the classroom which had a bearing on attitudes toward mathematics, to the introduction to some new experiences and concepts in mathematics (e.g., computer or statistics) not found in high school, or, perhaps, to a sympathetic teacher.

The first two lines in Table 11 give a clue to how much the real-problem-solving approach influenced the attitudes between the two sections. The change in means between the test and the retest with the experimental group would occur by chance only 8% of the time while the change between the means for the comparison group would occur by chance only 13% of the time. There is not much difference between the two values. Some experience in both classes led to an improvement in attitudes, but it is not at all clear that the opportunity to carry out real-problem solving had anything to do with the change.

Attitudes Toward Learning and Curriculum

The students' responses to other short-answer questions, designed to assess their attitudes toward other issues, are summarized in this section.

Use of Daily Assigned Homework Problems

One short-answer question, given to students in both sections at the beginning of the semester, tried to assess their feelings about being told or not being told exactly what homework problems to work. The question asked if the student felt frustrated and lost if the instructor did not indicate the precise pages in the textbook and the exact homework problems to be worked. Most students, instead of answering the questions in depth, gave very short replies--"no," "sometimes," "most of the time," or "yes." The results of the student responses are given in Table 12.

A slight majority of the students felt that they would be frustrated if the teacher didn't tell them what to do. Several of the students expressed the feeling that if the teacher didn't spell out exactly what they were to do, the instructor was lazy. Others argued that a college student should be able to make some decisions on her own.

Some sample opinions are given below:

> No, I don't feel lost and frustrated because I know I could find [the problems]. I feel lost and frustrated when the instructor continues to lecture about something that's of no interest to me. Because I am bored and just don't want to do [anything] (sic). (Comparison)

> Yes. Because an instructor is supposed to tell you what he wants to be done. (Comparison)

Table 12

Results for "Should Instructor Determine the Entire Assignment?"

Question: Pretest Both Sections

Do you feel frustrated and lost if the instructor doesn't indicate what pages in the textbook you should read and which questions should be answered?

	Percentages			
	No	Sometimes	Most of the time	Yes
Experimental	41.1	6.9	6.9	48.3
Comparison	20.0	6.7	.0	56.7

Experimental: 29 answers Comparison: 30 answers

Yes. I do feel this way. I feel the teacher is lazy and just [doesn't] want to do his job. (Comparison)

No, I do not. I simply look and read over the whole chapter on whatever we're doing. (Experimental)

Sometimes I feel lost when the instructor doesn't indicate which pages and assignments should be done. But I will take time to do some work myself so I can have a better understanding of the work. (Experimental)

Yes. I do feel frustrated and lost. It makes me feel uncomfortable when test time comes, because I am not sure if what I have read is correct [the material to be covered on the test]. (Experimental)

No, I do not feel frustrated or lost if the teacher does not indicate the page in the textbook. As a college student, I should be able to locate it in the table of contents or index. (Experimental)

Self-directed versus Teacher-directed Learning

The second issue examined was how much responsibility the student should assume for her own learning and how much direction the teacher should be expected to give. The questions, time, and section, and the categorized responses are given in Table 13.

For the first question, the answers generally fell into four categories: the instructor should choose all topics, the students should have some say, there should be an equal sharing in the choice of topics and activities, and the student should have the entire choice. Both groups felt that while the instructor should have most of the responsibility for choosing the topics, the students should have some choice. The only surprising result was that none of the students responding in the comparison section felt that a student

Table 13

Self-directed versus Teacher-directed Learning

1) Do you feel that the instructor ONLY should choose the topics in mathematics for the students to learn or should the students have some say in what topics are studied? Explain.

Both sections — Beginning of the semester

	Number	Percentages			
		Instructor Only	Both Equally	Student Some Say	Student Only
Experimental	32	28.1	18.8	43.8	9.4
Comparison	26	38.5	15.4	46.2	0.0

2) Do you feel that all students should be forced to learn the same material; if a survey is a part of the course should it be a group activity or done individually?

Experimental section — End of the semester

Number	Same Topics	Choice of Topics	No answer
30	6	5	19

3) Do you feel that in a course such as Math 102--the last math course for most students--the instructor should pick the topics to be learned (the same for all students) or should the students have some choice?

Comparison Section — End of the semester

Number	Same Topics	Choice of Topics	No answer
22	3	7	12

4) Do you feel that an instructor in college should have to assign daily homework, some of which must be handed in and graded, or should college students take responsibility for their own learning? Explain.

Comparison Section — End of the semester

Number	Instructor Homework	Student	No answer
24	8	2	14

should have complete control in the choice of topics she studied. Here are some typical responses made by the students:

> I think that both the students and the instructor should have a chance to choose a topic in math, because the students . . . should know what pertains to their major, which is very important to me. Therefore, they should suggest to the instructor that he [teaches] the math [needed] for that major first! Then, the instructor, if time permits, can go over topics he feels the students will need in the future. (Comparison)

> Both. Because students may not know what they need and choose [only] easy things. (Comparison)

> I think that a student should have some say into what should be studied because if a student is studying something she is interested in or enjoys, she is more likely to learn the topics better. (Experimental)

> I feel that the instructors along with the student should decide on what is to be taught, but the instructor would have the dominant hand in the final decision. I feel he knows what the students [are] required to obtain from the course. The students should have some suggestions for outside projects. (Experimental)

The follow up questions at the end of the semester were not exactly the same for the two sections, but there was sufficient similarity to allow a comparison. Only eleven of the students in the experimental section bothered to answer this question; of these, six gave answers generally favoring the same topics being taught to everyone. Some typical comments were:

> Students all need to be exposed to the same topics.

> Teach the same as last semester--did not really get into the course.

> Class should go back to the old way mathematics is supposed to be taught.

> Students should work all the problems of the text; they will need it later.

On the other hand, three of those who responded called for a combination--start by teaching in the usual way, covering certain set topics and then allow the students to work on an activity or an investigation during the latter part of the semester. These students felt that doing surveys without any prior explanation was too difficult. Finally, the following reply represented the viewpoint expressed by several others in the class:

> I feel that I have learned a great deal from the class and [I felt] free to talk to my instructor at any time. I would recommend this class to others.

The question given the comparison group (#3 in Table 13) was also given a meager response. Only 10 of the 24 respondents answered it. This would seem to be an indication that most students did not really care about the issue. Seven of the students called for a student choice while only three felt that the teacher should make all the choices, based on the alleged wisdom of the instructor. Hence a majority of those students in the comparison class, the class which allowed the students no choice concerning the topics to be learned, favored a choice, while a majority (a bare majority) of those students in the experimental class, the class which allowed a great deal of choice, preferred that the teacher make most of the decisions for the class.

The second follow-up question given to the comparison section (#4 in Table 13) asked about the student's perception about the necessity for required daily homework: "Do you feel that an instructor in college should have to assign daily homework, some of which must be handed in and graded, or should college students take responsibility for their own learning?" This question seemed to baffle the majority of the students since the question had never come up in this section of Mathematics 102; indeed only 10 of the 24 students answered it. Eight voted for the instructor and two for the student. It was no contest. The students were used to being told what to learn and how to learn it and they were most uneasy in being asked to take initiative. One student used the expression "forced to learn on our own" and she did not think that this was a good idea:

> We shouldn't be forced to learn on our own but [if] teachers want to feel that they have taught their students something, give them [the students] something to do daily, [in order] to make sure that they will interpret.

In other words, this student felt that teachers don't "teach" unless they tell the student exactly what to read and what questions to answer.

Another student questioned whether the students were mature enough to take initiative for their own learning:

> I think the instructor should assign homework daily, because some students need a little motivation.

Another student interpreted responsibility as doing what the teacher assigned:

> The teacher give [sic] assignments like always and the student takes the responsibility of doing it and passing it in.

In summary, the students rejected the concept of student initiative in the comparison section while they slightly favored the concept in the experimental section. However, in both sections, a majority of students failed to answer the questions, so that the results were tenuous at best.

Individual Learning Styles

The second area of inquiry dealt with how the student perceived her own "best" way of learning. A short-answer question was given at the beginning of the semester and a follow-up question was asked of both sections at the end of the semester. Four Likert-type questions were also given both sections at the end of the semester. These are summarized in Table 14 and Table 15.

The answers to the first question were quite varied, but they generally could be classified into one of six categories as indicated in Table 14. A large plurality for both sections thought that learning by means of the traditional lecture, class discussion, and use of the text was the best, most effective way. Again, the students in the comparison section were more definite in their choice of the combination of lecture, class discussion, and text (47% to 38%). But the comparison section also had the greater number of students who considered that activities and projects would be a better way to learn than the usual mix of homework and classwork. Also of note was the fact that several students,

Table 14

Learning Styles

1) How do you learn best--by following a teacher's lecture, reading the text, doing the homework problems, becoming involved in an activity of interest to you or by some other means? Explain.

Both Sections	Beginning of the semester	
	Experimental	Comparison
	Number of Respondents	
	29	32
	Percentages	
Homework, class discussion, test	13.8	12.5
Homework, reading, tests	17.2	9.4
Lecture, class discussion, test	37.9	46.9
Variety of methods	10.3	12.9
Individual work, tutoring	13.8	3.1
Activities and projects	6.9	15.6

2) Do you feel that taking tests is the only way an instructor can find out what you have learned? Should this course have had more tests, fewer tests, other activities as alternatives to tests . . . in order to maximize learning and arrive at a grade?

Both Sections — End of the semester

	Tests Only	Other	Can't Say	No Answer	Total
Experimental	5	18	1	6	30
Comparison	9	1	3	9	22

Table 15

Student Attitudes Toward the Best Way to Learn

Both Sections	End of the Semester					
	Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree	Total
1) The best way to learn and to retain what I learn over a long period of time is to follow a textbook two or three pages a day, work all the exercises and memorize the rules and the formulas.						
Experimental	9	9	4	6	2	30
Comparison	4	11	2	4	3	24
2) I learn best and retain what I learn over a long period of time if I study what the teacher and the textbook tell me to.						
Experimental	6	16	5	2	1	30
Comparison	4	8	5	5	2	24
3) I learn best and retain what I learn over a long period of time if I study what would be useful to me and in what I am interested.						
Experimental	13	13	3	1	0	30
Comparison	5	11	3	4	1	24
4) I learn best and learn more if I take initiative for my own learning.						
Experimental	7	11	10	1	1	30
Comparison	5	12	5	1	1	24

Table 15 (continued)

Summary

Scoring

Strongly Agree (SA)--1 pt.
Agree (A)--2 pts.
Undecided (U)--3 pts.
Disagree (D)--4 pts.
Strongly Disagree (SD)--5 pts.

Means

Question	Experimental	Comparison
1	2.17	2.29
2	2.20	2.71
3	1.73	2.38
4	2.27	2.21

Frequency Distributions--Percentages

Question	Experimental		Comparison	
	SA or A	D or SD	SA or A	D or SD
1	60.0	26.7	62.5	29.2
2	73.3	10.0	50.0	29.1
3	86.7	3.3	66.7	20.9
4	60.0	6.7	70.8	8.3

five in all, indicated their preference for an individualized approach to the learning of mathematics.

A fairly clear result emerged from the second question. The students who were exposed to alternative activities such as outside reading and investigations saw that an assessment of the quality of their work can be made in ways other than giving and taking tests, while those students in the comparison section, whose grades were determined by tests and homework (including a narrowly contrived outside project), were not aware of alternative ways of determining a grade. Some of the comments which the students made indicated a continued uncertainty about the alternatives to tests. Samples from four students are given:

> I personally feel that taking exams puts a person under strain; if one thing doesn't work, then [another attempt] is wrong, so most of the time I get tense when I can't start a problem off. But exams are necessary. (Experimental)
>
> I don't feel that test taking is the only way that an instructor can tell a student's progress. A lot of times a student cannot really put on a test what he or she has really learned. (Experimental)
>
> I think we should have had more quizzes in order to let you know how much we knew and then we could have spent a little more time on the math we didn't understand. (Comparison)
>
> I think that we should have more tests because then the student gets to see exactly what she knows or doesn't know. Also if a test is given once a month, then the student can take it upon herself to prepare. (Comparison)

The first two students quoted above (from the experimental class) grasped my suggestion early in the semester that tests were not a particularly valid way to assess what a student knew; clearly the students quoted above from the control section (and these quotations were very representative) had the impression that a test score somehow represents quite well the amount that one knows. I had discussed the issue of grading with the experimental class but not with the comparison section, and the responses and replies indicated that some of the students understood the questions I had raised.

It should first be noted that the data show considerable inconsistency and the responses on the latter two questions seem to contradict the point of view that the students expressed in the short-answer questions. The students in the experimental class very strongly agreed with statement 3 about learning best when studying what would be of interest to them.

When the process was stated in fairly theoretical terms, such as statements 3 and 4, the students found the idea of "taking initiative for their own learning" appealing to them. But the majority of students also felt that they learned best when studying the textbook and following the directions of the teacher. The students found it easy to agree with this statement involving a specific process. I would guess that if the specifics of what is meant by "useful to me and in what I am interested" were spelled out in detail, the students might not agree so overwhelmingly with statement 3.

It is interesting to note that the largest difference between the means of the two groups occurred with statement 3, so perhaps the theoretical idea of "student-directed learning" filtered through to the experimental group.

All of the evidence gathered--the answers to these questions and the remarks made in student notebooks and in conversation during the semester--indicated that a few students understood and developed some entuusiasm for student-directed learning, but most still favored the traditional approach to teaching as the "best" way to learn.

Comparisons of Experiences in Math 101 with Experiences in Math 102

A final set of questions given at the end of the semester asked the students to compare their experiences and the topics covered in the traditionally taught Math 101 with the experiences and topics covered in Math 102. Again the results were quite mixed. Some indicated that they worked harder in Math 101, since homework was not required in the experimental section of Math 102. Others indicated that they worked harder in Math 102 since the total requirements of doing investigations, learning specific material for success on topic tests, and doing reading reports and computer programs required more work than did Math 101. The majority of the students in both sections found the topics to be of more interest in Math 102, although several questioned how much the investigations really corresponded to solving the kinds of real-life problems they would be facing. As with the other short-answer questions, the answers tended to cluster in several overlapping categories. A summary of the results of these three questions is given in Table 16.

Those who answered "no" in Question 1 (Table 16) gave two different reasons. Some found the concepts presented in Math 102 more difficult than the fairly traditional topics (graphing, algebra, consumer mathematics) of Math 101. Others found the process of working the investigations and doing the other required work more confusing and frustrating. Most students did not answer the entire question, but answered only the first part.

Those who answered "yes" focused more on the issue of solving a real-life problem, although several simply reflected on learning more material. Here is a representative set of responses:

> Yes, I have learned more in this class than Math 101, mainly because I have had the chance to look over outside material and I also enjoyed learning about the mean, median, mode and standard deviation, mainly because this will help me in the future.
>
> Yes, I feel that I have learned a great deal more, due to the fact that the survey helped me a lot.
>
> I feel that I have learned more because I learned new ways of solving problems.
>
> I learned more, but it doesn't help in everyday life.

The second question, which was given to the experimental group, queried the students as to which class required the greater amount of work--Math 101 with the usual daily homework

Table 16

Comparisons of Experiences in Math 101

with Experiences in Math 102

1) Do you feel you learned more in this class [Math 102--Experimental section] than in Math 101 about how to use mathematics in solving a real-life problem? If so, why? If not, why not?

Experimental — End of the Semester

Yes	Same	No	No Answer	Total
12	8	2	8	30

2) Do you feel that you worked harder in this class than in Math 101? Why?

Experimental — End of the Semester

Yes	Same	No	No Answer	Total
10	3	9	8	30

3) Do you feel that the mathematics you have learned in this class--graphing, minimum-maximum problems, computers and writing your own programs, and statistics--will be useful in you in daily living and in later courses you will be taking? Explain. If not, what topics would be more useful?

Comparison — End of the Semester

Yes	Same	No	No Answer	Total
10	2	6	6	22

and periodic tests or Math 102 with no required daily homework, but instead investigations, topic tests, and other assignments. As with the other questions, the results were mixed. (See Table 16.)

Most students answered either "yes" or "no" without further explanation. Some of the comments were as follows:

> I feel that I worked harder in this class than in Math 101 because the material is harder, and in order to keep up, I studied harder and tried to do my best in class and on tests.
>
> Yes, and I feel that I have done extremely better in this math class due to my own personal interest.
>
> No, because I don't like the class doing different things because it is harder for me to understand.
>
> No, because I worked harder in 101 due to the fact that homework was due on certain days and I knew I had to work and do it to pass.

These few comments indicated that the experimental class was no different from other classes in the reasons students work; some work hard when given external pressure to do so and others work hard because of interest.

The third question, given to the comparison group, asked which class, Math 101 or Math 102, dealt with topics of greater usefulness. A slight plurality indicated that the topics in Math 102 seemed to be more useful, although most gave no specifics in support of their answers. One student expressed the feeling that the topics of statistics, computer programming, and graphing would be useful if she were majoring in mathematics, but since she was not a mathematics major, it followed that the topics would not be useful.

This observation would surprise mathematicians who tend to consider statistics and computer programming as applied. The complete results are given in Table 16 for statement 3.

Preference of Alternative Activities or Homework

One of the main reasons for attempting a curriculum based on a real-problem-solving strategy was to find a meaningful alternative to the traditional approach in order to generate interest, increase motivation, and alleviate boredom in the classroom. The final question asked of the experimental class was intended to assess how successfully that goal had been accomplished. As before, there was no clear consensus. The question and results are given in Table 17.

Most students gave only yes or no answers. Among the explanations were the following:

> No. I really would do homework.
>
> No, but [the activity] was worthwhile.
>
> In many cases I preferred the alternative activities to homework, but when something was very difficult, I wished that it had been gone over in class.

In summary it would be fair to say that, while many of the students liked some activity, they generally preferred the basic framework featuring the traditional approach. Any attempts to change the process of instruction must be undertaken cautiously. The instructor must discuss the reasons for changes in instructional methods and must convince the students of the value and importance of students developing

Table 17

Preference of Alternative Activities or Homework

Experimental Section — End of the Semester

Did you prefer the alternative activities of doing homework? Explain.

Yes	No	About the Same	No Answer	Total
11	9	2	8	30

their ability to take some responsibility for their own learning. The students need to come to understand the intentions of the teacher and to see the advantages of the activity-oriented approach. Despite the many student complaints about boring classes, changing the method of learning or the topics covered does not necessarily bring any sense of excitement into learning unless the students share a desire for new approaches to learning and teaching.

Conclusion

Looking at these various statistical results and student responses, we can see that there are some general yet definite trends. The students in the experimental class did slightly better in the acquisition of mathematical knowledge, their attitudes toward mathematics improved considerably (but not significantly) more, and they developed greater awareness of how to look at and analyze an open-ended, situational problem, than did their counterparts in the comparison section. But these gains were modest and the performances of the students in the experimental class on the problem-solving tasks during and at the end of the semester left room for considerable improvement.

The "USMES approach" is guided by several principles which are discussed in Chapters I and II. It should be noted again here that not all of them were followed completely as had been originally intended, particularly the following two:

> When the students find that certain facts and skills are necessary for continuing their investigation, they learn willingly and quickly in a more directed way to acquire these facts and skills. Consequently, the students should have available different resources that they may use as they recognize the need for them, but they should still be left with a wide scope to explore their own ideas and methods. (USMES, 1976b, p. 6)

> . . . [let the students] make their own mistakes and find their way. Offer assistance or point out sources of help for specific information (such as the "How to Cards") only when the children become frustrated in their approach to the problem. Conduct skill sessions as necessary. (USMES, 1976b, p. 9)

In order to alleviate some of the frustration and lack of progress the students were having, the instructor turned to using the textbook and giving lectures on topics which would be of use to the students in completing the investigations. This modification was made because the students did not apparently have the background and knowledge to carry out their investigations. Most students were able to formulate the problem for which they sought a solution, but they did not have any coherent idea of how to gather or analyze the data in a meaningful way (asking questions, making tests, computing means, finding frequency distributions, etc.). The fact that the instructor felt it necessary to make this change is itself a result to be noted here. It must be said, then, that the class procedure followed a <u>modified</u> USMES approach.

Despite the frustration and the confusion the students felt at the beginning of the investigations, after the projects were concluded and the students had reached firm conclusions, they generally expressed that they felt good about the experiences. However, with the exception of two groups, the quality of the data gathered and the reports written was not very high. It would probably have been helpful for the entire class to have worked through a short investigation with the instructor at the beginning of the semester in order to give the students a sense of one possible way to proceed. This would run the risk of violating one of the guidelines of the USMES approach, namely, letting the students figure out procedures for themselves, but it might also result in higher quality work on the part of the students.

The attempt to foster self-directed learning was not successful. There was no evidence to suggest that the students made any progress in this regard. Expecting students who came from the usual teacher-directed background in high school to become "self-directed learners" in one isolated freshman course was unrealistic. While some of the students indicated that theoretically they wanted to have a choice in topics and learning procedures, the majority continued to feel that the teacher should make the final determination of the topics and should decide what should be learned. The number of students responding to these questions was often less than half, so no conclusive finding can be stated. At best, it can be reported that the students in the experimental class were exposed to the possibility of their taking more initiative for their own learning.

Mathematical topics were taught by the lecture-discussion method with assigned homework in the traditional class throughout the semester, while a few of the topics used by all students in the experimental class were also taught by the lecture-discussion method with no required homework. Students in the experimental class were given a choice of which topics they wanted to work on, but they were required to take tests on four topics. The experimental group showed a greater increase from pretest to posttest scores when compared to the comparison group, but whether this was due to their being able to choose the "easiest topics" (for a particular student) or due to their being able to choose the "most interesting topic" (for a particular student) is not certain. If the latter is true, then doing better because of increased interest becomes a positive effect of the course.

The results of the problem-solving situational tests showed that the experimental group developed better problem-solving techniques in a "real-life, best-buy" situation--significant at the 10% but not at the 5% level. However, at the end of an entire semester during which real-problem solving had been emphasized, the problem-solving abilities demonstrated were not very sophisticated. This showed a need to try to find more meaningful, perhaps simpler kinds of investigations.

The experimental group, which had worked with questionnaires in connection with investigations, did better in evaluating a questionnaire on a test than the comparison section, which had only worked with questionnaires theoretically as part of a homework assignment.

Data from the Aiken Attitudinal Inventory showed that both groups made gains in attitudes toward mathematics and the gains in the experimental class were more (but not significantly more) than the gains in the comparison section. Student answers to other questions indicated that the students in the experimental class were fairly evenly divided concerning whether or not they preferred the use of investigations as a focus in a mathematics class. Several of the students strongly wanted to return to the usual approach; they felt threatened and confused by the USMES approach. In contrast, several others expressed positive feelings about the investigations and a sense of satisfaction in successfully reaching conclusions to their inquiries. Finally, a majority of students in the experimental class felt that they worked harder and learned more than they had in the previous semester of mathematics.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter consists of four sections: (1) a review of the problem, (2) responses to the guiding questions, (3) some comments concerning the methodology used to carry out the research and concerning the interpretation of the results, and (4) some observations and recommendations for the use of the USMES approach in the college liberal arts mathematics course. The argument is made that the results of the experiment discussed here support the proposition that the real-problem-solving approach can be used as part of a general education college mathematics course to give the students relevant, integrative, and enjoyable experiences in the use of mathematics to solve problems. However, certain modifications may need to be made depending on the instructor, goals of the course, and the students in the class.

Review of the Problem

Based upon a review of the literature, the 1971 report of the Committee on the Undergraduate Program in Mathematics, and the personal experiences of a number of instructors (including myself), the general education college mathematics course in the late 1970's could be described as mechanistic, skill oriented, and consisting of a collection of isolated topics. The content of the course had changed little during the 1970's and contained mainly mathematically oriented topics, a few applied topics, and some skill-building topics (review of arithmetic). The students came to the course with generally poor backgrounds, rather negative attitudes about mathematics, and previous experiences of being told what to learn by the teacher; they left with a somewhat better knowledge of isolated skills but "with very little conceptual basis of understanding of mathematics" (Fitzgerald, 1975, p. 41) and very little ability to apply the mathematics they learned to the solving of problems which involve decisions of importance to the students.

Curriculum planners and educators list several goals for the teaching of mathematics: develop informed, thinking citizens; provide the necessary background in skills and knowledge needed for making the informed decisions; and help the students develop problem-solving skills. The National Council of Teacher of Mathematics' Agenda for Action (1980), currently the most definitive statement on priorities in mathematics teaching (except for the mathematics major), recommended that mathematics courses should give students experience in posing questions, defining goals and strategies, trying out alternative approaches, reaching conclusions, and checking the solutions to see if the results do indeed solve the question or provide an optimal resolution of the problem. To complicate the task of the teacher, she or he is expected to teach the basic skills. In addition, recent research has lead to suggestions that teachers also become adept at teaching heuristics (Schoenfeld, 1980a; Suydam, 1980) and in providing classroom environments for practicing problem solving. Teachers of the general education course have been trained in a mathematics graduate program where specialization, not breadth, was desirable and the mathematics was taught in an abstract, deductive manner; these instructors are not prepared for alternative methods of teaching by such graduate school training.

The Unified Sciences and Mathematics for Elementary Schools program (USMES) has provided a curriculum, a rich source of teaching materials, and a guiding philosophy featuring real-problem solving and inquiry as a framework for the acquisition of mathematical and problem-solving skills. Although specifically intended for grades 1 through 8, it has the potential of being adapted to other situations, particularly the general education courses in which particular topics need not be taught for specific vocations, careers, or majors. A summary of the assumptions upon which USMES was developed was given by Davidman (1976):

1. the present and future conditions in our increasingly complex democracy make it necessary for elementary and junior high students to learn scientific methods of reasoning and problem solving;
2. such learning is best accomplished at the elementary and junior high school levels, when students confront local problems which are simultaneously real, manageable, relevant, and thus practical for the students in question;
3. in order to successfully cope with such real community problems, elementary students, like real

scientists and problem solvers, will need some resource tools and, unlike scientists, they will need carefully selected problems and well-organized learning environments if the goals of enhanced autonomous inquiry and more effective citizenship are to be ultimately realized;

4. the creation of this type of learning environment requires teachers who can easily move along the continuum from non-directive to directive teaching. (p. 8)

In order to develop a college mathematics course based on these assumptions, certain problems need to be addressed in addition to those involving the instructor. In particular, institutional requirements such as grading procedures, college catalog descriptions, and class schedules (three hours a week) must be met.

The real-problem-solving curriculum of USMES was tried out in one of two sections of a general education college mathematics course while the traditional selection of topics and the use of lecture, discussion, and homework characterized the other section. The goals of the instructor were to get the students interested in some aspects of mathematics, to motivate them to search out and learn mathematical skills and concepts in the context of solving problems, and to experience problem solving and gain confidence in the ability to solve real problems. The rationale of this approach came from considering how many people solve everyday problems outside of the classroom and applying a mode of self-learning to the classroom:

> . . . how do we, as individuals, learn something which we are not required to learn? I suggest that we, ourselves, gladly and energetically spend time and effort

> in study and research when we are . . . interested in solving a real problem of importance to us, a problem whose solution will lead to an improvement in the quality of our lives. When we have a repair problem in our house or car, and . . . we endeavor to find out how to solve the problem ourselves--we read manuals, learn about the fundamentals pertinent to proper functioning, consult knowledgeable friends, and experiment with various potential solutions. Thus we combine deductive and inductive reasoning in arriving at our conclusions. In short, when we have a problem of real interest to us to solve, then we are motivated to learn the fundamental underlying skills which we will need to know in order to solve the problem. (Treadway, 1981, pp. 3-4)

Response to the Guiding Questions

This study specifically addressed eight guiding questions given in Chapter I.

1. What happens in the classroom where the real-problem-solving approach is being used?

Chapter III gives a fairly detailed report of what happened in one classroom on one occasion. Both the instructor and the students had some "rough" times, particularly in the early stages. The instructor had to make out a course outline and a grading scheme which would satisfy the college requirements and follow the general description of the course in the college catalog. The students felt uncomfortable working in this new approach. Since they were accustomed to the teacher-directed approach of their high school mathematics, they were not always certain what they "were supposed to do" for a grade, they found it difficult to select and properly define the problem they wished to pursue, and they had some difficulty working with others in groups. The instructor found it necessary to deviate somewhat from the guiding principle of USMES and "teach" the entire class some underlying mathematical skills and strategies for collecting and analyzing data. Eventually, all groups completed the investigations satisfactorily with varying degrees of competence.

2. What changes in attitudes take place during a semester course?

An analysis of changes in attitude toward mathematics was made by giving both classes the Aiken Attitude Test at the beginning and at the end of the semester and by asking the students to write about previous experiences leading to a like or dislike of mathematics. Both classes showed improved attitudes toward mathematics. The increase in scores from the pretest to the posttest was slightly greater for the experimental class than for the comparison class. It was not clear how much, if any, of the special activities and procedures were responsible for these changes in attitudes.

A number of writers (Brevitt, 1975; Mitchell, 1974; Stein, 1973) have emphasized the role of a committed instructor. Brevitt claimed that the instructor, more than any particular method of teaching, contributes to attitude changes:

> Attitudinal change, I still believe, is a function of the instructor and of his [or her] rapport to the class, of his [or her] interest in his [or her] material, and of his [or her] abilities as a teacher. The significance

> of experimental data [the Aiken Attitudinal Scale] is in its replication. I don't believe an unsympathetic instructor, given all other variables constant, will duplicate the findings here. (1975, pp. 68-69)

The students in the experimental class met with me outside of class in small groups or individually. They knew I was willing to take extra time to help them work through the somewhat unusual experiences of the course, and they were aware that I had asked for their opinions about the grading procedure. They were also given choices on the topics they were to study. In both classes I tried to show enthusiasm for mathematics and to convey to the students its importance in making decisions. This involvement with the students as <u>collaborator</u>, inherent in the USMES approach, may well increase the interest and attitudes of students toward any subject.

3. What changes in the ability of students to take responsibility for their own learning takes place?

The results here were inconclusive partly because there were no specific attempts made to measure this attribute and partly because it would be necessary to observe the students in academic work in a succeeding semester. There was no observable change in their responses to the questionnaires, and at the end of the semester the students did not indicate any desire to take specific responsibility for their own learning. Indeed, in responding to a question about "taking responsibility for one's own learning" expressed in theoretical terms, a plurality of the students indicated support of the idea. However, when the students were asked to examine the specifics of how the course should be taught so that they would be given greater responsibility, the students indicated a preference for the teacher's choosing the topics and specifying the pages in the textbook to be read each class period. This was a clear contradiction.

When asked about who should choose the topics to be studied, a slight plurality (not all students expressed an opinion) of the students in the experimental class preferred that the teacher choose the topic, while a slight plurality of the students in the comparison section preferred that students choose what topics were covered and what assignments were required. Apparently students who have worked hard or struggled during a semester speculate that another method of teaching will be more successful and preferable for them.

4. What mathematical ideas and skills do the students learn?

Based on the results of the final examination, most students learned introductory skills in statistics, solving linear equations, graphing (points, lines, setting up linear equations when given points on a line), and probability. Some of the students, depending on individual preferences, learned about measurement (perimeter, area, and volume) and inequalities. It was difficult to assess those skills which were not measured by a classroom test, such as the ability to interpret data, determine a random sample or understand what a random sample is and why it is used, and to make inferences from the data gathered. The students were all exposed to these tasks; it was not clear whether they could use the skills again in a similar situation.

One of the expectations of the USMES program is that students, when faced with the need for certain skills to finish a problem, will study and learn those skills. There was no evidence that this took place. This process may work well in fairly routine problems such as fixing a malfunctioning object, but it probably won't work so well with the more ill-defined problems. As James Greeno (1980) pointed out:

> If a student has learned whatever knowledge is required for solving a problem, then the student can't get credit for problem solving, because the student is merely applying the knowledge that (s)he has been taught. Of course, if a student lacks the knowledge needed to solve a problem, the student won't be able to solve it. On this analysis students are bound to be poor problem solvers, because the only way they can be good problem-solvers is by doing things they haven't learned to do. (p. 12)

5. What are the unanticipated outcomes--both negative and positive--resulting from the use of the materials?

Three unanticipated outcomes and difficulties will be discussed: first, the problems of group interaction; second, the difficulty of introducing the challenges; and third, the the use of only one kind of problem-solving approach.

Group interaction. The strategy of having students work in groups of five to seven persons seemed to be a reasonable strategy at the beginning of the semester. If each individual had worked separately, the instructor would have had difficulty in monitoring the progress of each student. While there would be little time for the instructor to help each individual working independently in a class of 30, grouping the students would allow greater assistance by the instructor, sharing of tasks among the members of a group, and pooling of ideas, effort, and know-how.

However, in practice the outcome turned out somewhat differently. The students generally chose to be involved in a group based on a theme, not on the basis of compatibility among themselves. Since the students were trying to figure out what should be done, how to do it, and who should take responsibility for a particular part of the investigation, a need for leadership emerged, particularly in the larger groups (more than three persons). A leader was needed to coordinate activities and to make sure that tasks were completed, yet the members of the group needed to feel that they had some say in what their roles were and what went on. Some of the students did not always agree on leadership and were frequently concerned about other students in the group who "weren't doing their share of the work."

The instructor became involved in suggesting restructuring and other ways to work out the differences within the groups, in some cases splitting the larger groups. This involvement in the interpersonal relationships required the unexpected nonmathematical skill of consulting with individuals, concern for the students, and a commitment to the process of group investigation. Experiences in working out differences in a mature way and of cooperating with each other rather than competing were unexpected but valuable outcomes for the students.

Presenting the challenge. A second unexpected outcome was that the process of presenting the challenge to the students turned out to be quite a challenge for the instructor, as he tried to guide the students in finding a real problem of sufficient interest which used mathematical skills for its solution. The USMES Guide (1976b) suggested that the teacher choose the problem: "Introduce the challenge in a meaningful way that not only allows the [students] to relate it to their particular situation but also opens up various avenues of approach" (p. 9). The students then are expected to reformulate the challenge in their own terms. But this procedure puts the emphasis on the teacher's deciding rather than letting the students make that decision. Following the USMES approach seemed inconsistent with the goal of getting the students to consider the problems of interest to themselves.

The topic "Consumer Research," based on the ideas presented in the USMES teacher resource book Consumer Research (1976a) was originally introduced merely as a suggestion for the students to consider, inasmuch as the question "what is the best buy?" seemed to be one to which all the students could relate. Several groups followed through with this idea, but it was not clear how vital the students found this topic. Some authors have suggested that students at this age level aren't interested in "real-world" applications. The problems in which the students are truly interested--problems which occur in the dormitory (how to control noise, what are reasonable curfew regulations, how to improve the living conditions), boy or girl friends, or plans for Friday night--may not be amenable to quantitative solutions or involve the listed topics of the course, no matter how broad that list might be. It is a reasonable expectation that a course called Mathematics 102 deals with mathematics. Hence, a certain amount of artificiality in the choice of the problems to be investigated must be accepted.

One kind of approach. The third unexpected outcome was the fact that only one problem-solving process was used by the students, namely that of using statistical analysis including questionnaires and finding out the opinions of a sample of persons. The students did not use other methods to solve real problems, e.g., best-fit equations or minimum-maximum analysis. But how would the students know about these possible methods of solution unless they already had some experiences with these methods? This brings us again to the comments of Greeno above. A way must be found to incorporate a variety of problem-solving approaches, other than a statistical analysis.

6. What experiences, insights, frustrations and feelings of accomplishments does the teacher have in working with students in this kind of classroom situation?

Chapter III details the experiences of the instructor in trying to adapt to this "different" style of teaching. The personal conflicts between the accustomed teacher-directed teaching and the philosophically preferred student-directed teaching, particularly with regard to grading procedures and making out the course outline, are discussed there. My own uncertainty of when to give them specific instruction and direction plagued me throughout the course. The difficulties I had in changing teaching styles, from focusing on what I was doing to what was happening in class, paralleled the difficulties previously outlined by Malcolm Knowles. Knowles (1975) noted the difficulties an instructor wlll have, similar to my own, but he found that in time an instructor can be successful in developing self-directed learning in the classroom:

> When I started encouraging students to be self-directed learners, I initially assumed that they would change but I would perform the same. The only difference would be that they would take more responsibility for making use of my transmission, . . . I had a couple of early shocks as a result of this assumption. The first was when I discovered that my students didn't want to be self-directed learners; they wanted me to teach them. Then when I got them over that hurdle . . . they really got turned on about being self-directed learners and forced me to change my role. (p. 33)

In regard to this change of role, Davidman (1976) observed that USMES assumed that teachers in the program would be able (or would be trained through workshops) to "easily move along the continuum from non-directive to directive teaching" (1976, p. 8). It will probably be the rare mathematics teacher who can do this well, but a committed attempt during one semester will make it easier to change styles of teaching as is required during a succeeding semester.

A sense of accomplishment was derived from the enthusiasm expressed by approximately 10 of the students after they had completed their investigations.

7. Can a method of teaching based on a commitment to individualized instruction and the development of the autonomy of the learner be devised, which at the same time is effective in meeting the various goals of a good mathematics course?

The evidence provided by this course is not sufficient to answer this question. The experiences of the students in the experimental class fell short of providing a working example. Such a course needs to be improved and tried again for it contains the possibility of providing experiences designed to lead to self-directed learning and mathematics learning at the same time.

8. Can college students who have been through 12 years of traditional teaching change to a new system of learning? The class schedule of two or three 50-minute meetings a week is considerably different from the schedule in the elementary

schools; how will this affect instruction? Will materials appropriate for college students need to be substituted for USMES materials, and if so, how can this be done?

It has been observed that college students had difficulty adapting to the new system of learning and indeed most of them were not able to do so effectively. It is unrealistic to expect that an uninitiated student can be thrust into an unstructured learning situation and be expected to function productively right away.

The investigations were considered as semester-long projects and were treated differently from those in an elementary USMES classroom. Much of the work was done outside of regular class time. There was no disruption of the class or by the investigations due to the scheduling of the class.

The lack of appropriate nontext materials was a problem. The attempt to supply teacher-made "resource sheets" as substitute material was not particularly successful. The use of the "How To" cards and the "How To" booklets in the USMES program is a key to the student's being able to explore needed mathematical skills or problem-solving techniques as needed. The "resource sheets," which were handed out to the students in the experimental class and which were intended to serve the same function as the "How To" cards, turned out not to be very useful because they were dittoed (not attractive), difficult to read because of formal, mathematical style, and not requested often by the students. The "How To" booklets, written for junior high students, were designed to be of interest to students of that age level, with cartoons and attractive diagrams. I was reluctant to use these for fear of "insulting the intelligence" of the college students--although the subject matter was suitable. Some students did use a few of them, found them helpful, and did not express any feelings that they were too childish. There is a need, then, to create attractive, easy-to-read booklets for the college student--booklets which can be used in the same way as the "How To" booklet.

Comments about Research Methodology

In order to put the findings into proper perspective, three observations concerning research procedures and interpretation of test results are in order. First, the use of standardized tests and teacher-made tests presents methodological problems. As George Stalker (1978) pointed out in discussing his research on USMES classrooms:

> First, the scores of all students, whether or not they receive USMES training, are constantly changing as students grow. Thus, testing students "before and after" they receive USMES "treatment" is not, by and large, a useful method. Second, there are sufficiently many variables (such as teacher, school grades, "tracking," etc.) which have a dramatic effect on standardized test scores, that <u>also directly affect the administration of the USMES curriculum</u>, that even the grossest effects of basic skill instruction through USMES may be obscured. (p. 24)

Although in the comparison of the two methods in the investigation at hand, the instructor, the previous mathematics backgrounds, and the grade levels of the students were all roughly the same, nevertheless, the claims here must be modest. The combination of direct teaching skills and classroom investigations did not interfere with the acquisition of quantitative mathematics abilities; beyond this more cannot be claimed.

Second, the students in the experimental class were aware that they were part of a trial of a "new" teaching method. Frequently, such knowledge affects the attitudes of the participants and may alter the results in general. Campbell and Stanley (1963) called this factor "reactive arrangements" and they commented that:

> . . . a most prominent source of unrepresentativeness is the patent artificiality and the student's knowledge that he [or she] is participating in an experiment. . . . The playacting, outguessing, up-for-inspection, I'm-a-guinea-pig, or whatever attitudes so generated are unrepresentative of the school setting and seem to be qualifiers of the effect of [the treatment], seriously hampering generalization. (p. 20)

It is recognized, then, that if the students were aware that they were part of an experiment with unaccustomed attitudinal tests and questionnaires and a decidedly different approach in the classroom, this might make some difference in the replies on tests and comments in student journals.

However, the experimental class was neither "patently artificial" nor did the students know that they were subjects for a research project. Instead, they assumed that these questionnaires were merely the instructor's attempt to try to do something different, presumably to improve the course and to ask for "feedback." This approach in using the instructor to carry out the research minimizes the effects of the reactive arrangements, according to Campbell and Stanley:

> At present, there seem to be two main types of "experimentation" going on within the schools: (1) research "imposed" upon the school by an outsider, who has his own ax to grind and whose goal is not immediate action (change) by the school; and (2) the so-called "action" researcher, who tries to get teachers themselves to be "experimenters," using that word quite loosely. . . . An alternative model is for the ideas for the classroom research to originate with teachers and other school personnel, with designs to test these ideas worked out cooperatively with specialists in research methodology, and then for the bulk of the experimentation to be carried out by the idea-producers themselves. The appropriate statistical analyses could be done by the research methodologists. . . . Results should then be relevant and "correct." (p. 21)

In this investigation the idea originated and was carried out by the instructor; hence, reactive arrangements were minimized.

Third, as has been observed earlier, my role as teacher conflicted with my role as researcher. On the one hand, my dual role allowed for detailed descriptions of my personal change in attitudes as an instructor trying to adapt to a new method of teaching and it allowed for an analysis of the problems and difficulties which I faced and tried to resolve. On the other hand, the dual role also meant that since I would ultimately be determining the grades of the students, the student responses to the questionnaires might have been somewhat slanted; the students might have had the tendency to answer what they thought I would like to hear. Because of my desire not to put the students on the spot, I felt it necessary to keep the questionnaires anonymous in order to maximize the authenticity of the replies and thereby increase the validity of the results of the investigation. Hence, it was not possible to see how the attitudes of individuals changed and to get some insight as to the differing reactions of specific students in terms of academic background, expressed interest in a major, SAT and other test scores, and other factors.

Observations and Recommendations

The observations and recommendations given below are an attempt to generalize the results and experiences of the one-semester attempt to incorporate the real-problem-solving curriculum into the general education college mathematics course. Some of the features of the experimental approach can conceivably be used elsewhere and modifications can be made to adapt it to teaching situations with differing institutional regulations, student bodies, and instructors.

1. The USMES approach provides a framework and a list of guiding principles around which a general education mathematics course can be formed, but the implementation will need to be modified to meet the needs of the students and the instructor.

Most students in the general education course will need to be helped with coping with their first experiences of trying to solve a "real" problem. Knowles (1975) observed that one of the educational situations where teacher-directed learning would most likely be apropos is the encountering of a new "situation":

> No doubt there are learning situations in which we are indeed dependent (as when approaching an entirely new and strange area of inquiry), in which our readiness to learn is really determined by our level of maturation regarding the area of inquiry, in which we are rightly focusing on accumulating subject matter, and in which we are actually motivated by external pressures. . . . If self-directed learners recognize that there are occasions on which they will need to be taught, they will enter into those taught-learning situations in a searching, probing frame of mind and will exploit them as resources for learning without losing their self-directedness. (p. 21)

These observations lead directly to the next recommendation.

2. The use of a class investigation at the beginning of the semester, in which the teacher and the class go through the steps of posing the problem, working out a plan, gathering the data, evaluating the data mathematically, coming to a conclusion, writing up the conclusion, and checking the solution to see how accurate or reasonable it is, will form a good model for the students to follow. Later in the semester the students can undertake solving a problem of their own so they can benefit from the "learning by doing" and "student choice" features of the USMES approach.

While keeping in mind that the ultimate goal for the student (as a senior and beyond) is to become a self-directed learner as much as possible, we realize that a period of transition is necessary. Most students will need some sort of directions as to the process; there is no evidence that floundering without making some progress is a useful educational experience. Davidman (1976) observed that in one of the USMES classes he observed, the teacher used an approach which might "be viewed as a kind of apprenticeship program in real problem solving" (p. 248). Davidman continued:

> [The] extreme directive modeling of real problem solving, although shunned or overlooked by USMES/EDC, is not necessarily an educationally deficient alternative. To the contrary, I think that such directive modeling could be an important ingredient in an overall real problem solving curriculum provided that such teaching was informed by a teacher/change-agent training curriculum which explicitly discussed the strengths, weaknesses, limitations, and unknown dimensions, of the model. (p. 249)

There is always the danger of problem-solving modeling becoming just as teacher-directed as is the usual teaching of narrow, unrelated skills. However, modeling can serve as a first step, or beginning point, for the students; they can then work on investigations of real problems of their own during the latter half of the semester.

3. The teacher should not feel that he or she is obligated to intervene whenever a student indicates frustration in the problem-solving process. However, the students should be very clear about the expectations of the course and the requirements for a grade.

Frustration and temporarily being thwarted from finding a resolution to a problem are part of the real world and of problem solving in general. Davidman (1978) observed:

". . . because of teacher intervention or noninvolvement with the [USMES] program, the students in five of six USMES classes did not confront problems in a manner which allowed for the possibility of short-term frustration or failure" (p. 400). Frustration needs to be channeled eventually into productive learning experiences. Continued or excessive frustration and confusion will be a detriment to learning and increase negative attitudes toward mathematics.

4. A combination of exploration by the student and direction by the teacher will provide the best mixture of teaching/learning strategies. A variety of problem-solving experiences, not just "real" problem solving, will be enriching to the student. The teaching of heuristics for the closed types of problems should be explored as a stepping stone to the ability to solve the more open-ended problems.

There was not any evidence that problem-solving ability is developed simply by trying to solve a problem. Progress, practice, success, and insight must take place. This does not mean, however, that the ability to solve problems will develop in the absence of such experiences. Referring to higher level cognitive strategies of problem solving and decision making, Loman (n.d.) claimed that:

> . . . not only is the USMES approach directly related to such process skills, but it may be the only successful way of acquiring them. (This is not to say that a person with sufficient maturity and grasp of these processes may not further improve his or her capacity by explicit and formal consideration of individual problem-solving strategies.) (p. 1)

Recent research has suggested that direct teaching of heuristics may be useful in helping students with certain kinds of problem solving.

5. The students should be aware that there are many kinds of problems, and the problems can not all be solved with mathematics. Many problems do not have answers or the answer can only be expressed in terms of "being optimal" and some problems will involve value judgments, not mathematical results.

There are a wide variety of problems, as is noted in Chapter II, and any development of students' abilities to solve problems, whether those taken from a textbook, a puzzle or game, the writing of a computer program, or the proof of a mathematical theorem should be encouraged. Investigations on the relationships of vertices, edges, and faces of polyhedra can be fascinating to some students; what is required is that the problems help to motivate the learning of mathematical concepts which in turn are used in solving problems. Students may need to be asked to solve some problems which may not be directly of interest to them; nevertheless, we need to keep in mind the ultimate reason for their taking a course in mathematics, which should be to enable them to be able to solve problems which relate to their own lives.

Michael Scriven (1980) argued that there are a variety of problems, and students need to be aware that not all problems can be solved by technical means. His typology included three distinct kinds of increasing difficulty: (1) Within Paradigm problems such as those encountered in mathematics and physics in the classroom or closed real-life problems (what is the best buy? etc.); (2) New Paradigm problems which required a radically new approach or theory (puzzles, concept learning, some proofs); and (3) Third Kind problems which involved new and different cognitive and pedagogical difficulties. The distinction which some people make between real-problem solving and classroom-problem solving is misleading, Scriven argued, in that some real-life problems can be very routine while some problems which result from discussion of mathematical concepts can be very difficult. Scriven expressed optimism that progress can be made in helping students solve Within Paradigm problems, as indicated by the work of Polya, Wickelgren and others, but he had little optimism for being able to "teach" methods of problem solving of the "New Paradigm" problems; a "needed burst of brilliance" is required. Problem solving takes on a much different texture when dealing with Third Kind problems. Frequently there is not a clear solution and these problems require taking a stand and using moral or qualitative criteria. Evaluation must be an essential part of the solving of problems.

Any discussion of problem solving in the classroom should point out the many dimensions of problem solving and the many types of problems, and it should be realized that what is done in a mathematics classroom deals with only a small part of the possibilities.

6. The teacher needs to work hard at convincing the students of the importance of solving real problems and of student-directed learning.

The students obviously need to understand the rationale for the potentially strange approach and different emphasis as contrasted to other classes. The enforced use of inquiry learning can be just as authoritarian as the enforced use of drill and the textbook. The students should understand the rationale of new teaching methods and have some enthusiasm for participating in their use.

7. A variety of resource materials needs to be made available in lieu of the "How To" Booklets.

The teacher should review potential resources in the college library, in his or her own office, and in a mathematics laboratory, and should create suitable resource materials, so that when students need to look up mathematics skills or concepts which are not covered in the classroom, the possible sources of that information will be available to the student. This list of resources could include the "How To" booklets and should include materials of varying degrees of sophistication and reading abilities.

8. Students should be given a choice of working in groups or individually.

Group projects have many advantages, including the sharing of ideas and expertise and reducing the number of projects which the teacher must oversee. The social development of students in learning to work with others should not be overlooked. But, consistent with a democratic atmosphere in the classroom, students should not be forced to work in groups if they prefer to work individually.

9. The instructor should not expect to cover all possible topics and develop a variety of process skills in a one or two semester general education mathematics course.

The teacher needs to be given considerable latitude for the approach she or he uses but needs to do an honest and caring job of bringing to students a relevant, coherent selection of mathematics topics, which are "utilized, or tested, and thrown into fresh combinations" (Whitehead, 1956, p. 13).

The final recommendation follows from the findings and the previous recommendations:

10. The real-problem-solving approach, with modifications depending on the instructor, goals of the course, and the students, should be considered as one of several viable alternatives to the traditionally taught general education mathematics course.

Of the alternatives, such as the cultural, utilitarian, discovery, or historical approaches, the real-problem-solving approach will provide relevant, integrative, and enjoyable experiences in the use of mathematics and in problem solving. It can be used properly in the classroom if the instructor is convinced of its appropriateness and is committed to its guiding philosophy. Brevitt's statement is recalled here as being particularly apropos:

> If there is a particular selection of material which the instructor feels is within the grasp of students at this level of maturity and which he feels is important to their knowledge, understanding, appreciation, and growth, and if he feels strongly enough about such material that he is able to present it with enthusiasm and conviction, keeping in mind both the material and student are worthy of respect and concern, and if the instructor is willing to vary his approach from lecture to small-group to laboratory until he finds a style or combination of styles that serves his purpose best . . . then as an instructor he will find these students more than willing to respond favorably toward mathematics. (1975, pp. 56-57)

Conclusion

It is clear that the teaching of mathematics, if it is to be effective for all students and accomplish its goals, is a difficult task. No wonder so many teachers prefer to teach in a fairly routine manner covering content in an ordinary way. They want to be sure their students have learned the fundamental topics needed for further work in mathematics and in other courses, and they are comfortable in teaching in this way. But we cannot be satisfied. We would like to be sure that our students can also solve a variety of different kinds of problems--word problems, routine mathematical exercises, real-life problems, applications, and even "new paradigm" problems. Teachers are caught in the dilemma of needing to impart specific knowledge and at the same time wanting to let the students work on questions and problems of individual interest in order to enhance motivation. Furthermore, they are aware that, of all the disciplines, perhaps mathematics is the most <u>structured</u> and <u>sequential</u> in the way its content fits together. It is quite difficult to accomplish all of these goals.

To complicate matters, mathematics teachers are now being encouraged to focus on problem solving. Many of the instructors of mathematics courses learned to solve problems by simply lots of practice and by following the examples of their teachers. Few instructors have had much experience with teaching problem skills or in discussing the heuristics of George Polya. Expecting that students will learn to solve problems by seeing it done and by following specific algorithms which are taught as part of the content of the course leads to disappointment. This does not develop any ability to put knowledge together in new ways; it does not develop flexible and integrative thinking.

Mathematics teaching must focus on more than just teaching. One of the main features could be a major investigation, carried out by students individually or in groups, following the real-problem-solving guidelines of the USMES program. This would provide an open-ended problem and a variety of experiences needed for developing flexible problem-solving abilities.

A majority of researchers in problem solving are agreed that "extensive and accessible knowledge" (Simon, 1980, p. 82) must be present before problem solving can take place.

Resnick and Ford (1981) pointed out that improving the likelihood of problem solving and invention requires three basic ingredients: prior knowledge, task environment, and strategy. They wrote:

> We can suggest instructional interventions that might improve the functioning of each component in the problem-solving situation. First, instruction can try to ensure the presence of well-structured knowledge and to maximize the links to related concepts and procedures. This means making sure mathematics instruction is mathematically honest, reveals the subject-matter structure, and provides the opportunity to practice new procedures and concepts in a wide variety of contexts. . . . From a purely intuitive standpoint, the more facts, procedures, and relations that characterize a person's knowledge structure, the more likely that the person is to invent or discover needed connections. (p. 234)

This implies that the teachers will continue to emphasize facts and procedures, skills and concepts, but that non-routine and interesting problems must be part of every course.

Developing and successfully implementing such a course as that considered in this study and recommended above will not be easy. It will take time and effort to develop a variety of activities, resources, and the ability to challenge students to solve real-life problems. But the rewards for the teacher and the student should be worth the effort: development of reasoning ability, ability to think for one's self, a better understanding of mathematics, and an ability to use mathematics in situations of concern to the student.

In conclusion, a quotation from Morris Kline (1975) is appropriate:

Teaching constructively . . . is by no means easy. But there is no royal road. To enjoy the view from a mountaintop one must get to the top. In mathematics there is no easy chairlift. The cable in young people's minds breaks down. But in the skillful employment of the process of discovery lies the true art of teaching. In this approach we arouse and develop the creative powers of the student and give him [or her] the delight of accomplishment. (p. 156)

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APPENDICES AND THEIR CONTENTS

APPENDIX A

THE MANY FACETS OF PROBLEM SOLVING

Problem solving, as viewed by psychologists, mathematicians, teachers and others, has a wide variety of characteristics and qualities. Claire Hill listed 11 different facets of problem solving in her annotated bibliography, Problem Solving: Learning and Teaching (1979).

1. Problem solving is using associations.
2. Problem solving is forming and testing hypotheses.
3. Problem solving is restructuring problems.
4. Problem solving is assimilation into schemas.
5. Problem solving can be simulated.
6. Problem solving can be encouraged by teaching styles.
7. Problem solving is modified by individual differences.
8. Problem solving aids learning.
9. Problem solving is a goal.
10. Problem solving is creative.
11. Problem solving is solving exercises.

APPENDIX B

USMES MATERIALS

UNIT	CHALLENGE
1. Advertising	Find the best way to advertise a product or event that you want to promote.
2. Bicycle Transportation	Find ways to make bicycle riding a safe and convenient way to travel.
3. Classroom Design	Change the classroom to make it a better place.
4. Classroom Management	Develop and maintain a well-run classroom.
5. Consumer Research	Determine which brand of product is the best buy for a specific purpose.
6. Describing People	Determine the best information to put in a description so that a person can be quickly and easily identified.
7. Designing for Human Proportions	Design or make changes in things that you use or wear so that they will be a good fit.
8. Design Lab Design	Improve or set up the Design Lab in your school or class for the benefit of those who use it.
9. Eating in School	Promote changes that will make eating in school more enjoyable.
10. Getting There	Overcome the difficulties in getting from one place to another.
11. Growing Plants	Grow plants for __________. Children determine the specific purpose, such as for gifts, for transplanting into a garden, for selling, etc.
12. Manufacturing	Find the best way to produce in quantity an item that is needed.
13. Mass Communications	Find a good way for us to tell many people about _________ (topic, problem).
14. Nature Trails	Develop an outdoor area to help others appreciate nature.
15. Orientation	Help yourselves and/or others to adapt to new situations.
16. Pedestrian Crossings	Recommend and try to have a change made that will improve the safety and convenience of a pedestrian crossing near the school.
17. Play Area Design	Promote changes which will improve the design or use of our school's play area.

UNIT	CHALLENGE
18. Protecting Property	Find a good way to protect your_______ (property in desks or lockers, bikes, tools, animals, Design Lab tools, etc.).
19. School Rules	Find ways of influencing rules and the decision-making process in the school.
20. School Supplies	Find effective ways to manage and/or conserve school supplies.
21. School Zoo	Collect and maintain a variety of animals in the classroom to help your class and others learn about them.
22. Soft Drink Design	Invent a new soft drink that will be popular and can be produced at a low cost.
23. Traffic Flow	Recommend and try to have a change accepted so that the flow of traffic will be improved at a nearby problem location.
24. Using Free Time	Find things to do during your ________ (recess time, lunch time) that would be ________ (educational, useful, fun).
25. Ways to Learn/ Teach	Find the best way to learn or teach someone else to do certain things.
26. Weather Predictions	Make your own weather predictions (for this afternoon, tomorrow, or a special occasion).

Taken from The USMES Study Guide, 1976, pp. 51-79.

List of Background Papers

As students working on real problem become involved in different types of investigations, teachers may need background information that is not readily accessible elsewhere. The Background Papers not only fulfill this need but also often include descriptions of activities and investigations that students might carry out.

Many papers provide information that is useful in many different units; others are focused on the problems in a specific unit. Each Teacher Resource Book contains a list of the papers pertinent to that unit. The papers are grouped in the categories shown. In some cases there is a great deal of overlap between categories. For example, some papers which deal with graphing also include probability and statistics; information in papers on design problems may extend into other categories. (USMES Study Guide, 1976, p. 83)

BIOLOGY

B 1 How to Love Frogs by Abraham Flexer
B 2 Raising Houseflies by Abraham Flexer
B 3 Identifying Organisms by Abraham Flexer
B 4 Hints for Growing Plants by Jay E. Anderson
B 5 How to Avoid Crop Failure and Disaster: Redundancy by Jay E. Anderson
B 6 How to Keep from Also Raising Aphids, Spider Mites, White Flies, Etc., When Growing Plants by Jay E. Anderson

DESIGN PROBLEMS

DP 1 Highway Intersections by Alan Holden
DP 2 Some Considerations on the Curvature of an Exit or Entrance Road by Earle Lomon
DP 3 Determining Taste Factors for Soft Drink Design (based on suggestions by Henry Pollak)
DP 4 Electromagnet Design by Earle Lomon
DP 6 What Can You Do with Tires and Ropes? by Playground Design Group--1972 Boston Summer Workshop
DP 7 Traffic Congestion by James Kneafsey
DP 8 Traffic Flow at Pedestrian Crossings by James Kneafsey
DP 9 Traffic Flow under Alternative Structural Conditions by James Kneafsey
DP 10 The Need for Traffic Signal Synchronization in Urban Areas by James Kneafsey
DP 11 The Impact of Parking Restrictions on Traffic Flow in Urban Areas during Peak Periods by James Kneafsey
DP 12 Traffic Flow at Rotaries by James Kneafsey
DP 13 People and Space by Gorman Gilbert
DP 14 Speed, Travel Time, Volume, and Density Relationships in Traffic Flow (based on suggestions by James Kneafsey)
DP 17 Bicycle Test Course by Frank O'Brien

ELECTRICITY

EC 1 Basic Electric Circuits (based on suggestions by Thacher Robinson)
EC 2 Trouble Shooting on Electric Circuits (based on suggestions by Thacher Robinson)

GEOMETRY AND SHAPES

G 1 Making Polyhedra by Alan Holden
G 2 Solids Made of Equilateral Triangles by Alan Holden
G 3 The Five Regular Solids by Alan Holden
G 4 Semi-Regular Solids by Alan Holden
G 5 "Fair" and "Regular" Polyhedra by Earle Lomon
G 6 Mass Production of Equilateral Triangles and Squares by Louise Buckner and Frank O'Brien

GRAPHING

GR 1 Notes on the Use of Histograms for Pedestrian Crossing Problems by Percy Pierre and Donald Coleman
GR 2 Notes on Data Handling by Percy Pierre
GR 3 Using Graphs to Understand Data by Earle Lomon
GR 4 Representing Several Sets of Data on One Graph by Betty Beck
GR 5 Plotting Weather Predictions Data on Three-Dimensional Pegboard Graphs (based on suggestions by Jack Borsting and Leland Webb)
GR 6 Using Scatter Graphs to Spot Trends by Earle Lomon
GR 7 Data Gathering and Generating Graphs at the Same Time (or Stack 'Em and Graph 'Em at One Fell Swoop!) by Edward Liddle

GROUP DYNAMICS

GD 2 A Voting Procedure Comparison That May Arise in USMES Activities by Earle Lomon

MEASUREMENT

M 1 Gulliver's Travels Activity by Abraham Flexer
M 2 Measuring Heights of Trees and Buildings by Earle Lomon
M 3 Determining the Best Instrument to Use for a Certain Measurement by USMES Staff
M 4 Measuring the Speed of Cars by Earle Lomon
M 5 Electric Trundle Wheel by Charles Donahoe
M 7 Weather Factors and Their Measurement by Ray Brady, Jr.

PROBABILITY AND STATISTICS

PS 1 Collecting Data in Sets or Samples by USMES Staff
PS 3 Weather Prediction by Bob Renad
PS 4 Design of Surveys and Samples by Susan J. Devlin and Anne E. Freeny
PS 5 Examining One and Two Sets of Data Part I: A General Strategy and One-Sample Methods by Lorraine Denby and James Landwehr
PS 6 Examining One and Two Sets of Data Part II: A Graphical Method for Comparing Two Samples by Lorraine Denby and James Landwehr
PS 7 Examining One and Two Sets of Data Part III: Assessing the Significance of the Differences between Two Samples by Lorraine Denby and James Landwehr

RATIOS, PROPORTIONS, AND SCALING

R 1 Graphic Comparison of Fractions by Merrill Goldberg
R 2 Geometric Comparison of Ratios by Earle Lomon
R 3 Making and Using a Scale Drawing by Earle Lomon

SIMULATION ACTIVITIES

SA 1 The Sit-Down Game by Merrill Goldberg
SA 2 Set Theory Activities: Rope Circles and Venn Diagrams by Merrill Goldberg
SA 3 Using Venn Diagrams to Find the Best Descriptions by USMES Staff
SA 4 Simulation/Modeling as a Tool in Assessing Various Solutions by Ludwig Braun and Betty Beck

Taken from The USMES Guide, pp. 83-85.

LIST OF "HOW TO" CARDS

The overall activity in USMES units is flexible in content and in discovery-learning experiences. In most instances the students learn through observing the results of their own and their classmates' experiments. However, in the course of their investigations the students will recognize the need for certain facts or skills. For example, they may need to find out how to use a stopwatch, how to make a scale drawing, or how to find the range of a set of measurements. In working with electric currents, they may want to find out why the wires get hot or why a light sometimes gets dim or goes out when another battery is added to the circuit. Students working in the Design Lab may want to know how to glue, nail, or cut wood, how to solder two wires, or how to cut wire.

Some children prefer to work things out for themselves and should be allowed to do so. However, a student may ask for help as he or she becomes stalled in carrying out a certain procedure. At this point the teacher may refer the student to the "How To" Cards as a source of information.

USMES "How To" Cards are designed to provide the assistance needed for acquisition of specific skills or knowledge. There is no definite sequence to the sets, and they should not be used as programmed learning material to be introduced one by one out of the context of the children's open investigation of a practical problem.

"How To" Cards are divided into two main groups: the regular "How To" Cards that cover math and science skills, such as graphing, measurement and electricity, and the Design Lab "How To" Cards that cover hands-on or construction skills usually encountered in the Design Lab. (The USMES Guide, 1976, p. 87)

BIOLOGY

B 1 How to Plan a Home for Your Small Animal

ELECTRICITY

EC 1 How to Make Simple Electric Circuits
EC 2 How to Check a Circuit by Tracing the Path of the Electricity
EC 3 How to Make Good Electrical Connections
EC 4 How to Find Out What Things to Use in an Electric Circuit
EC 5 How to Make a Battery Holder and Bulb Socket
EC 6 How to Make a Battery and Bulb Tester
EC 7 How to Find Out Why a Circuit Does Not Work
EC 8 How to Turn Things in Electric Circuits On and Off
EC 9 How to Find Out Why a Bulb Sometimes Gets Dim or Goes Out When Another Battery is Added to the Circuit
EC 10 How to Connect Several Things to One Source of Electricity
EC 11 How to Draw Simple Pictures of Electric Circuits

GEOMETRY

G 3 How to Construct a Circle Which is a Certain Distance Around

GRAPHING

GR 1 How to Make a Bar Graph Picture of Your Data
GR 2 How to Show the Differences in Many Measurements or Counts of the Same Thing by Making a Histogram
GR 3 How to Make a Line Graph Picture of Your Data
GR 4 How to Decide Whether to Make a Bar Graph Picture or a Line Graph Picture of Your Data
GR 5 How to Find Out If There is Any Relationship Between Two Things by Making a Scatter Graph
GR 6 How to Make Predictions by Using a Scatter Graph
GR 7 How to Show Several Sets of Data on One Graph

MEASUREMENT

M 1 How to Use a Stopwatch
M 2 How to Measure Distances
M 3 How to Measure Large Distances by Using a Trundle Wheel
M 6 How to Find the Speed of a Car
M 7 How to Find How Many Feet per Second is the Same Speed as 60 Miles per Hour
M 9 How to Make a Conversion Graph to Use in Changing Measurements from One Unit to Another Unit
M 10 How to Use a Conversion Graph to Change Any Measurement in One Unit to Another Unit
M 11 How to Measure the Amount of Water in the Air with a Psychrometer
M 12 How to Measure the Amount of Water in the Air with a Hygrometer
M 13 How to Measure the Amount of Rain with a Rain Gauge
M 14 How to Find Wind Speed by Watching What the Wind Does
M 15 How to Find Wind Direction with a Wind Vane
M 16 How to Measure the Pressure of the Air with a Mineral Oil Barometer

PROBABILITY AND STATISTICS

PS 2 How to Record Data by Tallying
PS 3 How to Describe Your Set of Data by Finding the Average
PS 4 How to Describe Your Set of Data by Using the Middle Piece (Median)
PS 5 How to Find the Median of a Set of Data from a Histogram

RATIOS, PROPORTIONS, AND SCALING

R 1 How to Compare Fractions or Ratios by Making a Triangle Diagram
R 2 How to Make a Drawing to Scale
R 3 How to Make Scale Drawings Bigger or Smaller

DESIGN LAB "HOW TO" CARDS

DL 1 How to Make Straight Cuts in Three-Layered Cardboard

DL 2 How to Make Curved Cuts in Three-Layered Cardboard

DL 3 How to Cut Slots in Three-Layered Cardboard

DL 4 How to Make Holes in Three-Layered Cardboard

DL 5 How to Glue Three-Layered Cardboard

DL 6 How to Make a Corner with Three-Layered Cardboard

DL 7 How to Cut Grooves in Three Layered Cardboard

DL 8 How to Hold Three-Layered Cardboard While You Work on It

DL 9 How to Make Straight Cuts in Wood

DL 10 How to Make Holes in Wood

DL 11 How to Nail Wood

DL 12 How to Make Curved Cuts in Wood

DL 13 How to Glue Wood

DL 14 How to Hold Wood While You Work on It

DL 15 How to Cut Grooves in Wood

DL 16 How to Make Wood Smooth

DL 17 How to Put Wood Screws in A Piece of Wood

DL 18 How to Put Screws in Hard-To-Get-At Holes

DL 19 How to Put Screws in Two Pieces of Hard or Soft Wood

DL 20 How to Hold Two Pieces of Sheet Metal Together

DL 21 How to Put Two Pieces of Wood on Metal Together with Nuts or Bolts

DL 22 How to Loosen or Tighten Nuts or Bolts

DL 23 How to Loosen Hard-To-Move Nuts and Bolts

DL 24 How to Cut Thin Pieces of Tin

DL 25 How to Cut Steel Rod

DL 26 How to Cut Glass

DL 27 How to Cut Wire

DL 28 How to Solder Two Pieces of Wire Together

DL 29 How to Use a Straight Edge when Cutting Wood or Three-Layered Cardboard

DL 30 How to Cut Straight Across a Piece of Wood

DL 31 How to Tell if Things Are Level

DL 32 How to Make a Sawhorse

Taken from The USMES Guide, 1976, pp. 88-91.

"HOW TO" BOOKLETS

In 1977 USMES published a series of "How To" Booklets as a supplement or alternative to the "How To" Cards. Each booklet consisted of sixteen pages. Two versions were made available--the Beginning "How To" Series and the Intermediate "How To" Series. The following explanation of the purpose and use of the booklets is taken from the inside cover of the USMES Intermediate "How To" booklets.

Purpose of "How To" Series

Materials that help students learn skills designing an opinion survey and choosing the appropriate measuring tools are not readily available for intermediate grade students. The USMES Intermediate grade "How To" series fills this gap. Its magazine-style format helps students acquire the skills and knowledge they need to do things like redesign their classroom, find the best buy in potato chips, or run a school store.

How to Use the "How To" Series

Wait for a need. When a student asks for help, refer him or her to the appropriate booklet. Having a student read a booklet before there is a need to do so will not only result in less effective learning but will defeat the USMES purpose of allowing students to decide what needs to be done.

When necessary, use the "How To" Series as a teaching aid. Most of the time students will be able to go through a booklet by themselves and learn the skills they need to learn. However, some material in some sets is difficult and somewhat abstract. When the booklet by itself is not doing the job, feel free to step in and help the students go through it.

Knowing how the contents of the booklet are organized may help in using the series effectively.

*The first page tells why or when a student may need the skill covered in the booklet, and includes a table of contents.

*Each booklet contains several examples of stories about students using the skill or process being taught. Each example emphasizes a different aspect of the skill or potential pitfall.

*When information in other booklets may help the student, the titles of the booklets are included in the text.

*The last pages of each booklet contain a summary of the points covered in the booklet.

THE "HOW TO" INTERMEDIATE SERIES

Measuring

- Use a Stopwatch
- Choose the Right Tool to Measure a Distance
- Use a Trundle Wheel
- Make a Scale Drawing
- Find the Speed of Things

Graphing

- Choose Which Graph to Make
- Make a Bar Graph
- Make a Histograph
- Make a Line Graph
- Use Graphs to Compare Two Sets of Data

Collecting Data
- Collect Good Data
- Round Off Data
- Record Data
- Do an Experiment
- Make an Opinion Survey
- Choose a Sample

Simplifying Data
- Tell What Your Data Show
- Find the Median
- Find the Mean
- Find the Mode
- Find Different Kinds of Ranges
- Use Key Numbers to Compare Two Sets of Data

APPENDIX C

KEY ELEMENTS IN A USMES INFORMATIONAL MEETING*

Introduction to USMES philosophy and units (30-60 minutes)

- slide/tape show
- videotape(s) of classroom activities
- question and answer session
- information brochures

Experiencing real problem solving by working on an USMES unit challenge at adult level (45-90 minutes)

- discussion of challenge
- tasks and priorities defined
- observation
- data collection
- construction work as needed
- data representation and analysis
- action on challenge
- group discussions on any aspect of process

Discussion of real problem solving and its place in the total school program (10-20 minutes)

Discussion of USMES resource materials (20-30 minutes)

- Design Lab (types, inventory, cost, scheduling, staffing, safety)
- "How To" Cards
- USMES Guide
- Teacher Resource Books
- Background Papers for teachers
- Design Lab Manual
- Curriculum Correlation Guide

Discussion of classroom strategy and teacher's role in USMES (5-10 minutes)

Discussion of skills and concepts in USMES units (5-10 minutes)

- Listing of skills/concepts covered in hands-on activity carried out during session.
- Review of skills charts in USMES Guide and Teacher Resource Books

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*From Preparing People for USMES: An Implementation Resource Book, 1975, p. 6.

Discussion of USMES evaluation and documentation program and data (5-10 minutes)

Discussion of USMES implementation program (5-10 minutes)

- Map showing extent of implementation
- Magazine articles and newspaper clippings

APPENDIX D

OBJECTIVES OF BENNETT COLLEGE*

1. To enable students to become liberally educated by acquiring a broad range of knowledge, understanding, and appreciation in the natural sciences and mathematics, the social sciences, the arts, and the humanities.
2. To insure adequate preparation in specialized fields to make possible the successful pursuit of a given career and/or graduate study.
3. To provide for the students those kinds of learning which will enhance and strengthen their critical and analytical abilities.
4. To provide a climate for the development of democratic values by emphasizing broad participation in discussion and decision-making, exercise of democratic freedoms, respect for the dignity and rights of all individuals, and concern for the common welfare.
5. To develop the desire to aid in the solution of social problems by a critical and objective analysis of current social issues on the local, national and international scene.
6. To provide opportunities for students to become meaningfully acquainted with the world of work.
7. To provide opportunities for growth in spiritual awareness and understanding which will enable the

student to formulate values and a philosophy of life in keeping with the principles of a church-related college.

8. To stimulate an appreciation for the aesthetic in everyday living.
9. To provide a core of experiences which contribute to the formation of values essential to the maintenance of sound and effective home and family life.
10. To provide an environment that encourages the discovery of new knowledge, and ways of using existing knowledge.

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*Bennett College Catalog (1974-1978), p. 11.

APPENDIX E

MATERIALS HANDED OUT OR USED IN THE EXPERIMENTAL AND COMPARISON SECTIONS OF MATH 102

APPENDIX E-1

MATHEMATICS 101
Modern Mathematics I

Fall Semester, 1978

Instructor: Ray Treadway

COURSE OUTLINE

Textbook: Mathematics, An Everyday Experience, C. D. Miller and V. E. Heeren. Scott, Foresman and Co., Glenview, Illinois, 1976.

Competencies

1. The student should show mastery of skills with integers, fractions, decimals, percents, negative numbers, graphing, and algebraic expressions.

2. The student should be able to analyze statements, in order to determine whether or not a given argument is logically sound.

3. The student should be able to work exercises and solve real-life problems involving number sequences, simple and compound interest, the geometry of shapes, and measurement.

4. The student should be able to solve simple linear equations and linear inequalities, to graph them, and be able to solve linear programming problems from applied situations.

5. Upon completion of the course, it is hoped that the student will have some insight into the nature and structure of mathematics, an increased ability to solve problems, an appreciation of the role of mathematics in our culture, and an increased ability to use logic in reasoning.

Implementation

The competencies will be gained through the use of (1) textbook reading and problem solving (homework), (2) classroom lecture and discussion, (3) computer-assisted instruction, (4) consultation with the instructor, and (5) tutoring.

UNITS

I. Review of fractions, decimals, signed numbers, percents and formulas.
   A. Pretest
   B. Analyzing results of pretest

II. Number Sequences
   A. Arithmetic Sequences
   B. Geometric Sequences
   C. Other sequences and applications
   D. Exponents and large numbers
   E. Prime Numbers

III. Consumer Mathematics
   A. Interest -- 1) Simple and 2) Compound
   B. Buying a car and/or house
   C. Life Insurance

IV. Ways of Thinking
   A. Sets
   B. Logic
   C. Applications: direct statement, converse, contrapositive

V. Geometry of Shapes
   A. Triangles
   B. Pythagorean Theorem
   C. Polygons
   D. Polyhedrons
   E. Similar Figures

VI. Geometry of Measures
   A. Metric System
   B. Areas
      1. Quadrilaterals
      2. Triangles
      3. Circles
   C. Volumes

VIII. Linear Programming*
   A. Coordinate system and graphing points
   B. Linear Equations and Their Graphs
   C. Linear Inequalities and Their Graphs
   D. Maximizing/Minimizing Linear Functions under restrictions

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*If time permits.

Computer-Assisted Lab

Programs written by Bennett Mathematics faculty will be used to cover the following topics at the computer terminals. The student will progress at a pace suitable for the student, although a minimum number of topics is required.

A) Fractions B) Decimals C) Negative Numbers
D) Evaluating Expressions E) Properties of real numbers
F) Linear Equations G) Linear Inequalities

[Unit VII was not covered in Math 101 due to lack of time. This unit was covered in the control section Math 102-02 in the Spring.]

APPENDIX E-2

Mathematics 102-02
Modern Mathematics

Spring Semester 1979

Instructor: Ray Treadway

COURSE OUTLINE

Textbook: Mathematics, An Everyday Experience, C. D. Miller and V. E. Heeren. Scott, Foresman and Co., Glenview, Illinois, 1976.

Competencies

1. The student should show mastery of arithmetic skills, using formulas and solving linear equations, graphing simple sentences in one and two variables, and using basic statistics.

2. The student should be able to work exercises involving linear inequalities (solving and graphing), mean, standard deviation and frequency distributions of a set of data, interpretation of data from a normal distribution, area and volume of geometric figures, and computer programs in BASIC.

3. Upon completion of the course, it is hoped that the student will have an increased ability to solve real-life and word problems.

Implementation

The competencies will be gained through the use of (1) textbook reading, (2) homework, (3) classroom lecture and discussion, (4) computer-assisted instruction, (5) consultation with instructor, (6) feedback from quizzes and homework, and (7) tutoring.

UNITS

I. BASIC GRAPHS
   A. Coordinate system in two variables; locating points
   B. Graphing straight lines
   C. Graphing parabolas, ellipses, circles, and hyperbolas
   D. Functions

II. More GRAPHS
  A. Linear Inequalities
  B. Linear Programming

III. PROBABILITY
  A. Basic Definitions
  B. Multiple Part Experiments
  C. Complementary Problems
  D. Counting Methods
  E. Multiplying and Adding Probabilities

IV. Statistics
  A. Tables and Graphs
  B. Averages (Mean, Median, and Mode)
  C. Variability
  D. The Normal Curve
  E. Sampling
  F. Misuses of Statistics

V. Applications of Probability and Statistics
  Units will be chosen from the following depending on time
  A. Survey--A project
  B. Curve Fitting
  C. Probability in Genetics, Health, Games and Puzzles
  D. Further use of normal curve--Binomial Distribution

VI. Computers
  A. History
  B. Computer Arithmetic
  C. Parts of a Computer
  D. Simple Program Statements in BASIC

The Computer-Assisted Instruction Laboratory

I. Linear Equations
II. Linear Inequalities
III. Measurement
IV. Computer Program
V. Functions
VI. Factoring*
VII. Quadratics**

* and ** denote extra credit

Grading

I. The Class grade
   A. Homework (120 points)
   B. Test 1 (On Units 1, 2 and 3) (100 points)
   C. Test 2 (On Units 4, 5 and 6) (100 points)
   D. Project (statistics and normal curve) (100 points)
   1. Semester average = (sum of scores of A, B, C, and D)/4
   2. Final Exam
   Class grade: If the semester average is greater than the final exam, then class grade = .67 (sem. ave) + .33 (final exam)
   If the semester average is less than the final exam, then class grade = .6 (sem. ave.) + .4 (final exam)

II. Lab grade
   1. Semester lab ave. = (sum of points on required topics/total possible) x 100
      Required topics: linear equations, inequalities, measurement, functions, working computer program
   2. Written final exam on computer topics
   Lab grade = 2/3 (semester lab ave.) + 1/2 (final exam in lab)

III. Overall grade: (2/3) Class grade + (1/3) lab grade
   90 or above, A; 80-89, B; 70-79, C; 60-69, D; Below 60 F

Attendance

Attendance in class and laboratory is required. More than three unexcused absences will result in the student being dropped from the course, either with a DP or a DF (after the first nine weeks, a DF is automatic). Excuses are to be negotiated between the student and the instructor, but a written excuse from the proper authority is helpful. Excuses for absences on days of tests must be approved before the time of the class (contact the instructor ahead of time); in case of an emergency, a written note is necessary.

APPENDIX E-3

CAI MATHEMATICS

1978-1979

GRADING

The units required for each course during the first semester are listed below:

<u>Math 001</u>
FRACTIONS
DECIMALS
NEGATIVES
REAL NUMBERS
ALGEBRAIC EXPRESSIONS

<u>Math 101</u>
FRACTIONS
DECIMALS
NEGATIVES
REAL NUMBERS
ALGEBRAIC EXPRESSIONS
LINEAR EQUATIONS

<u>Math 102</u>
INEQUALITIES
MEASUREMENT
FUNCTIONS
FACTORING
QUADRATICS

<u>Math 103</u>
FRACTIONS
DECIMALS
NEGATIVES
REAL NUMBERS
ALGEBRAIC EXPRESSIONS
LINEAR EQUATIONS
INEQUALITIES

<u>Math 104</u>
INEQUALITIES
MEASUREMENT
FUNCTIONS
FACTORING
QUADRATIC EQUATIONS
POLYNOMIALS

<u>Math 105</u>
ALGEBRAIC EXPRESSIONS
LINEAR EQUATIONS
INEQUALITIES
MEASUREMENT
FUNCTIONS
FACTORING

Grading Procedures

1. The minimum number of required units must be completed. A grade of zero will be assigned for lessons not completed.

2. The grade will be determined by

$$\frac{\text{number of correct responses}}{\text{total number of possible responses}} \times 100$$

3. If a student completes work beyond the required units, her score can be increased as follows:

   For each additional topic completed the score will be increased by the number of points indicated

90-100	5 points
80-89	4 points
70-79	3 points
60-69	2 points

4. If a student completes all the required units, she may do some additional drill and retake tests on which she made grades less than 70.

APPENDIX E-4

Preliminary Tests

MATH 102 TOPIC TESTS Name_________________

PRETEST Show work for Partial Credit. Give the answer clearly.

I) Algebraic Expressions

A) Evaluate $4x + 2x^2/3 + 3$ when x is 6

Answer____________________

B) Simplify (Combine Like Terms)

$4x^2 - 3x + 1 + 8x^2 + 2y + 5x - 4$

Answer____________________

C) If x and y stand for numbers which are related by

$y = 3-4\ (x+5) - x^2$, find the value of y which makes the equation true when x is -1.

Answer____________________

II) Linear Equations and Inequalities

A) Solve for x:

$5x - 3 = 2x + 1$

B) Solve for x

$\frac{2}{3}x = \frac{4}{5}$

Answer____________________

C) What values of x (where x can be any real number) satisfies $-3x + 4 \geqslant - 8$

Answer____________________

III) Use the quadratic formula to solve the equation $3x^2 + 8x - 3 = 0$ for x.

The quadratic formula states that if x satisfies $ax^2 + bx + c = 0$, then

either $x = \frac{-b + \sqrt{b^2 - 4sc}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

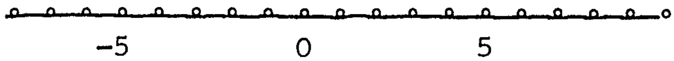
Answer____________ or __________________

Math 102 Topic Tests (Pretest) p. 2 Name____________

IV) Graphing

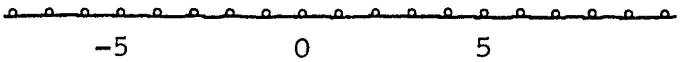
A) Graph the set of integers satisfying

$$-5 \leqslant x < 3$$

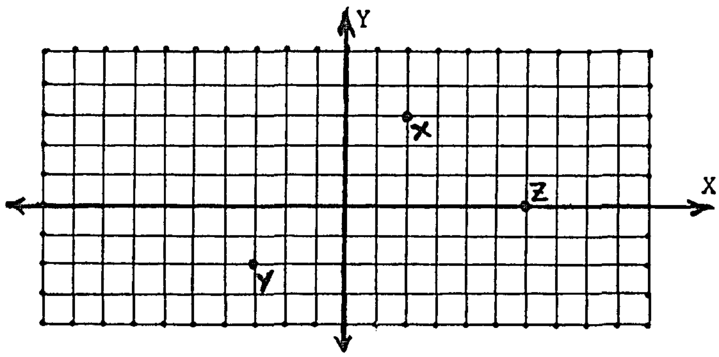


B) Graph the set of real numbers satisfying

$$x \geqslant -2$$



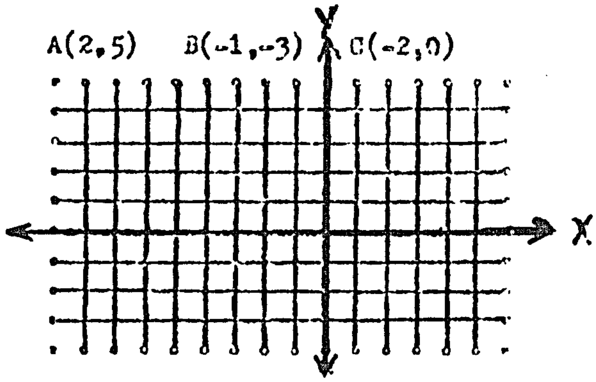
C) Use the coordinate system below to find the coordinates of the labeled points.



X(,) Y(,) Z(,)

D) Graph the following points by marking a heavy dot in the appropriate place and labeling with the corresponding capital letter.

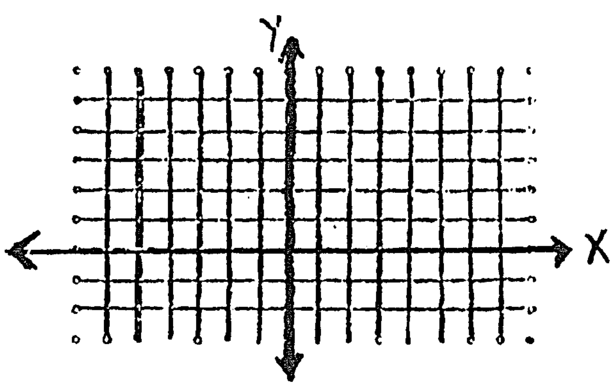
A(2,5) B(-1,-3) C(-2,0)



E) Graph the set of all points which satisfy the equation y = x + 2. Start by filling in the table below.

x	-2	0	2
y			

Draw the graph below.



V. Statistics

A) Find the mean, median, and mode of 7, 2, 4, 3, 7, 8, 2, 7

Mean____________________

Median____________________

Mode____________________

B) Fill in a frequency distribution table for the scores

7, 9, 9, 8, 4, 6, 6, 7, 4, 3, 6, 7, 6, 4, 5, 6, 2

	tally	frequency
2 or 3		
4 or 5		
6 or 7		
8 or 9		

APPENDIX E-5

QUESTIONNAIRE
Spring 1979

In an attempt to improve our mathematics courses at Bennett, and in particular Mathematics 102, members of the mathematics faculty would like your honest evaluation of your present feelings toward mathematics and your honest opinion toward the course material and method of teaching. Furthermore, in a survey that I am doing, I would also appreciate your reactions to some statements about school and how you approach solving your problems. You will not be graded on your answers; in fact, you are asked to put your answer sheets in a (sealed) envelope without putting your name on your sheets. Your answers will be anonymous.

I-Please respond to each of these statements circling one of the five choices.

SA Strongly agree A Agree
U undecided or no opinion
D Disagree SD Strongly disagree

1.	I am always under a terrible strain in math class.	SA	A	U	D	SD
2.	I do not like mathematics and it scares me to have to take it.	SA	A	U	D	SD
3.	Mathematics is very interesting to me, and I enjoy arithmetic and mathematics courses.	SA	A	U	D	SD
4.	Mathematics is fascinating and fun.	SA	A	U	D	SD
5.	Mathematics makes me feel secure and, at the same time, it is stimulating.	SA	A	U	D	SD
6.	My mind goes blank and I am unable to think clearly when working mathematics.	SA	A	U	D	SD
7.	I feel a sense of insecurity when attempting mathematics.	SA	A	U	D	SD
8.	Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	SA	A	U	D	SD
9.	The feeling that I have toward mathematics is a good feeling.	SA	A	U	D	SD

10.	Mathematics makes me feel as if I were lost in a jungle of numbers and I can't find my way out.	SA	A	U	D	SD
11.	Mathematics is something which I enjoy a great deal.	SA	A	U	D	SD
12.	When I hear the word mathematics, I have a feeling of dislike.	SA	A	U	D	SD
13.	I approach mathematics with a feeling of hesitation, resulting from a fear of not being able to do math.	SA	A	U	D	SD
14.	I really like mathematics.	SA	A	U	D	SD
15.	Mathematics is a course in school which I have always enjoyed studying.	SA	A	U	D	SD
16.	It makes me nervous to even think about having to do a mathematics problem.	SA	A	U	D	SD
17.	I have never liked mathematics and it is my most dreaded subject.	SA	A	U	D	SD
18.	I am happier in a mathematics class than in any other class.	SA	A	U	D	SD
19.	I feel at ease in mathematics and I like it very much.	SA	A	U	D	SD
20.	I feel a definite positive reaction to mathematics; it is enjoyable.	SA	A	U	D	SD

Write a few sentences on the back about how you feel about your previous mathematics courses, in particular, Math 101 or a similar "general education" course, in response to each of these questions.

21. Do you feel that the instructor ONLY should choose the topics in mathematics for the student to learn or should the students have some say in what topics are studied? Explain.

22. How do you learn best?--by following a teacher's lecture, reading the text, doing the homework problems, becoming involved in an activity or interest to you, or by some other means? Explain.

23. Do you feel frustrated and lost in learning if the instructor does not indicate what pages in the textbook you should read and which questions should be answered?

24. What previous experiences in math classes have led to your like or dislike of mathematics?

APPENDIX E-6

Information Sheet--Background and Interests of Students

Name___________________ Campus Box Number_____________

Course: Math_________ Section____________

Previous Math Courses: High School

at Bennett (or other college)

What is your major or special interest________________________

If you could choose any topic in mathematics which you might find interesting and/or useful, what would the topic be? Why?

What sort of uses for mathematics do you expect to have in the near future or in your major field?

Lab ID Number (four digit number)_______ Lab Day and Time_____

How far did you get in the CAI lab last semester (what topic are you working on now)?

Grading	Problem Solving		Questionnaire		Homework #1	#2	#3	#4
Absences	#1	#2	#3	#4	#5	#6		

Topic Tests Scores
#1 #2 #3 #4 #5 #6

Test scores #1 #2 #3 Final

S/U scores grade

Mid-sem ave grade

APPENDIX E-7

COURSE OUTLINE

MATHEMATICS 102-01
Modern Mathematics
Spring Semester, 1979

Instructor: Mr. Ray T. Treadway

Textbook (reference): Mathematics, An Everyday Experience. C. D. Miller and V. E. Heeren. Scott, Foresman and Co., Glenview, Illinois, 1976.

Special Note

This course is being taught in an experimental manner in connection with a research project. Statements about implementation and content may be modified on the basis of the experiences of the research.

Objectives

1. The student should be able to confront a problem from a real-life situation, to make observations and gather data, to analyze the data using mathematical and other skills, and to arrive at a workable and reasonable solution to the problem.

2. The student should be able to perform arithmetic operations by hand and calculator; solve linear equations; solve linear inequalities; find perimeters, areas, and volumes; and evaluate functions (computer-assisted instruction competencies).

3. The student should be familiar with the basic concepts of mathematics such as number, equations and inequalities, functions, graphing, deductive and inductive thinking, and statistics. (The specific skills acquired in these areas will be determined by the interests of the students and the requirements for solving a chosen problem.)

4. The student should learn the basic principles of computer programming and be able to write a simple program in the BASIC language.

5. The student should be able to take responsibility for her own learning, for keeping track of progress she is making, and for self-assessment of the quality of her learning.

6. Upon completion of the course, it is hoped that the student will be able to pose a question in connection with solving a problem of relevance to her, to find some simple mathematical models, and to manipulate the model mathematically to find a solution, using observation, reasoning, and factual knowledge.

7. Upon completion of the course, it is hoped that the student will have a feeling that mathematics can be worthwhile and relevant and that she will have increased confidence in her ability to think and to use mathematics.

Implementation

1. The students will be presented several real and practical problems which (it is hoped) will apply to some aspect of their daily lives and which will be of some direct interest to the students. Each student will select a problem to pursue and will work in groups with others also interested in the same problem.

2. The students are to decide how to proceed in solving the problem, to make observations, gather data, draw graphs, analyze the data, form and test one or more hypotheses, and take some final action or form conclusions.

3. When the students find that certain facts, skills, and concepts are necessary to pursue their investigation, they will consult references, including portions of the text book (reading and working problems). The teacher may, when appropriate, spend part or all of a class period discussing a needed topic, thereby serving as a resource person.

4. The teacher will stimulate activities in the class as a coordinator and collaborator, assisting groups or individuals as they need help.

5. The students will keep a record of the work they are doing, produce graphs, models, and reports, and choose mathematical topics they wish to study and be checked on. By keeping a record of the work a student is doing, it will help the student to see the development of skills.

6. Work on the computer terminals (CAI) and study of mathematical references--reading explanations, drill, practice, and testing on specific topics--will allow the student to learn at an individual pace.

UNITS

I. Problem Solving Challenges (to be covered by students depending on interest).

   A. Using Free Time
   B. Consumer Research
   C. More Effective Ways to Learn/Teach
   D. Ways to Serve Large Numbers of People
   E. Maximum Profit, Minimum Cost
   F. Other

II. Mathematical Concepts (to be considered by students depending on requirements for solving problems and interest)

   A. Fractions/Percentages/Decimals
   B. Consumer Math (money/finance
   C. Measuring (length, area, volume, time, weight)
   D. Estimating, Approximating, Rounding Off
   E. Organizing Data (tallying, ordering)
   F. Statistical Analysis (mean, median, mode, ranges, probability, interpretation of graphs, normal curve, correlation)
   G. Opinion Surveys/Sampling Techniques
   H. Functions/Graphing
   I. Ratios (rates/slopes)
   J. Maximum and Minimum values
   K. Equations and Inequalities
   L. Other

III. Computer Topics

   A. Parts of Computer
   B. Components of a Program
   C. Keywords of BASIC

IV. Computer-Assisted Instruction

   A. Solving Linear Equations*
   B. Linear Inequalities*
   C. Measurement*
   D. Functions*

*Required.

E. Writing a Computer Program*
F. Factoring
G. Quadratics

References

How To Booklets, Educational Developmental Center/USMES, Newton, Mass., 1976.

APPENDIX E-8

MATHEMATICS 101-01

Possible Grading Procedures

Evaluation and Grading:

Since this course is designed to help a student learn to solve problems (using mathematics) so as to better function in daily life, rather than learning specific mathematical facts and skills, it follows that the process of evaluation could (or should) be drastically different than the usual process of homework and tests.

The following argument can be made. Tests are sometimes artificial and not consistent with the objectives of a real-life problem solving approach. Tests put pressure on students to memorize for the moment (forgetting soon afterward), create anxiety, and force students to compete against one another. An alternative approach is to set guidelines for A, B, C, and D work, against which the students can judge their own progress.

Grading alternative I: Homework, Two tests, Topic tests, Project, and Final. Each of these will be scored numerically 1-100 and a weighted average computed. A student's grade is reduced to a numerical value which determines the grade.

Grading alternative II: Mutual assessment of self-collected evidence. (I) The student will keep a written document, i.e., journal or notebook, in which she will write up the work she has done, the attempts, the results, and the impressions of that work. This written document will include any drill work done by the student in the process of learning a particular topic or skill. Continual evaluation by the teacher and self-evaluation by the student will take place throughout the semester. Furthermore, the student will choose topics of interest to her which she will study, using a textbook, teacher lecture, or tutorial practice, and in which she will demonstrate competency by taking a short test (which can be repeated up to three times).

Format for Grading

Activities	Evidence
I--Problem Solving Skills	
A) Analyze what the problem is, make a clear statement of the various aspects of the problem, suggest approaches, set priorities	Written statement

B) Collect data and information (Surveys, experiments)	List of findings graphs
C) Analyze data and interpret data	graphs computations charts written statement
D) Use materials	description of sources, findings, work done
E) Come to conclusions and give supporting evidence	written statement
II--Learn specific mathematical topics and skills (with approval of instructor)	pass "attainment" tests --3 tries allowed
III--Self-evaluation Write a summary (at least 3 pages) of impressions of and attitudes toward the problem-solving approach. Indicate "what I have learned."	written statement
IV. Final Examination	

Due Monday: Write a page (or so) on "how I think the grade ought to be determined." Include the kind of evaluation--do you want to take tests? etc.--what is fair? If you feel that alternative II is reasonable, then indicate how an A, B, C, and D should be determined. How should the final exam be counted? Should a numerical grade be determined? If so, how?

Computer-assisted instruction: 1/3 of overall grade.

The grading procedure for the Lab has been determined by the Mathematics Department and a numerical grade will be determined as follows:

Required Topics: Linear Equations, Linear Inequalities, Measurement, Functions, Computer Program (Extra credit may be earned by working beyond these topics): Grade (Points on the tests--total/No. of possible points) x 100

WRITTEN FINAL EXAM

LAB SCORE = 1/3 (final) + (2/3) average of test scores

APPENDIX E-9

Math 102-01 Grading Procedure

Rationale: Since this experimental course is designed to help a student learn to solve problems (using mathematics) so as to better function in daily life rather than to learn specific mathematical facts and skills, it follows that the process of evaluation should put an emphasis on the activities of problem solving rather than on tests which emphasize memorization.

Format for grading

The student will keep a written document, i.e., a journal or notebook, in which she will write up the work she has done, the attempts, the results, and the impressions of that work. This written document will include any drill work done by the student in the process of learning a particular topic or skill. The notebook will be turned in once every two weeks so that the instructor will be aware of the progress the student is making and can inform the student of her status. Continual evaluation by the instructor and self-evaluation by the student will take place throughout the semester. Furthermore, the student will choose topics of interest to her or of importance to her, which she will study using a textbook, teacher lecture, or other source, and of which she will demonstrate mastery by taking a short test (which can be repeated up to three times). Finally, each student will prepare a report on outside reading on a topic dealing with mathematics or a project requiring several written pages.

Activities	Evidence	Evaluation (by instructor and student)
I-Problem Solving Skills		
1. Analyze what the problem is, make a clear statement of the various aspects of the problem, suggest approaches, set priorities	Written statement	Satisfactory or unsatisfactory (rework)
2. Collect data and information	List of findings	Satisfactory or unsatisfactory (rework)
3. Analyze data and interpret data	Graphs Computations Charts	Satisfactory or unsatisfactory (rework)

4. Use reference materials	Written statement (description of sources, findings, work done)	Satisfactory or unsatisfactory (redo)
5. Write up conclusion with supporting evidence	Written statement	Satisfactory or unsatisfactory (redo)
II-Learn specific mathematical topics and skills (with approval of instructor)	Pass attainment tests--3 tries allowed	Success/Not successful
III-Report on outside reading or completion of (math textbook project)	Written evidence	Level 2: Good work Level 1: Adequate work Level 0: Not adequate

IV. Self-evaluation
Write a summary (at least 3 pages) of impressions of and attitudes toward the problem-solving approach. Indicate "what I have learned." — Completed/Not completed

Semester - Class grades

D Level: Complete all 5 activities in solving a real-life problem satisfactorily. Pass 3 topic tests. Turn in self-evaluation. Write 1 report at level 2 quality.

C Level--in addition
Complete three activities in a second problem-solving activity satisfactorily. Pass an additional topic test (4 in all)

B Level--in addition
Complete all 5 activities in a second problem-solving activity satisfactorily. Pass an additional topic test (5 in all)

A Level--in addition
Write a second report at level 2 quality.
Pass an additional topic test (6 in all).

Letter grades will be determined by the above requirements. If one activity less than the required number is completed, a minus (-) will be attached to the grade. If one activity more than the required number is complete, a plus (+) will be attached to the grade.

In counting activities, each topic test passed is worth one activity; a report of level 1 quality is worth one, while a report of level 2 quality is worth two.

Class Grade consists of:	Semester grade	75%
	Final Exam	25%

In order to assign a numeric value to the semester grade, use the following:

F depends on number of activities

D- 62	D 65	D+ 68	C- 72	C 75	C+ 78	B- 82	B 85
B+ 88	A- 92	A 95	A+ 98				

Important Notice: In view of the experimental nature of the course and the grading system, adjustments may be made if it becomes necessary to do so as the semester progresses. Any adjustment will be discussed in class and clearly announced.

LAB GRADES:

Required Topics: Linear Equations, Inequalities, Measurement, Functions, Computer Programming

If a student has not completed a required unit and taken the test, a grade of zero will be assigned for that unit and averaged with the other test scores.

> Extra Credit: If a student completes work beyond the required unit, her score will be increased as follows: For each additional topic completed the score will be increased by the number of points indicated:
>
> | | |
> |---|---|
> | 90-100 5 points | 80-89 4 points |
> | 70-79 3 points | 60-69 2 points |

If a student completes all the required units, she may do some additional drill and retake the tests on which she made grades of less than 70 in order to improve her score.

Semester Lab Score = (Total points made)/(Total required) x 100

WRITTEN FINAL EXAM

Lab score = 1/3 Final exam score + 2/3 semester lab score

COMBINED CLASS AND LAB SCORE = 1/3 lab score + 2/3 class grade

COMBINED CLASS AND LAB GRADE: A 90 and above, B 80-89; C 70-79; D 60-69.

APPENDIX E-10

Mathematics 102-01

Suggested Questions for Planning an Investigation

1. How do you decide which product to buy and which brand to buy? Do you consider cost?...usefulness?...availability?...advertising?...cost per unit of weight?...cost per amount of use?

2. How can we determine whether one product brand is better than another?

3. What does the price of a brand tell you about its quality? How can you prove to someone else that the price does or does not show how good the brand is?

4. How can you compare the prices of different brands of the same product? Why should you calculate the price per length, per ounce, etc.?

5. What are the manufacturer's claims about the product? How true are they?

6. What are the most important properties to test?

7. How can you find out which product brand people like the best?

8. What tests could you make on your product? What will your tests and test procedures be? What supplies will you need to conduct the tests?

9. How can you keep a record of your test procedures and results?

10. How can you be sure that your test will give valid results, i.e., that the data pertains to the question being asked? How can you be sure that the sample item or people involved in your test aren't special in some way and are truly representative of all the items or people being tested?

11. How can you best make a picture or diagram of your test results?

12. How can you keep a record of your test procedures?

13. What things affect your testing? Why should you make sure that only one thing changes at a time?

14. How can you make sure that your test is the same for each brand?

15. Why might it be useful to do a test more than once? If you do the same test several times, what do the results tell you about accuracy? your test? your product?

16. Why might it be useful to test more than one sample of each product brand?

17. Who might be interested in your findings?

18. How can you display your data so that it is easily understood by other people?

19. If you are testing two different qualities of your product, is there a relationship between the two? Can you find a formula (an equation with two or more unknowns) which shows what the relationship is?

Most of these questions are adapted from Consumer Research (Teacher Resource Book), USMES/Educational Development Center, 1976, pp. 39-40.

APPENDIX E-11

Math 102-01

SUGGESTED TOPICS

(These topics will be covered in class if there is sufficient interest.) No one is required to master any one of these topics, but each student must pass from three to six of these topics during the semester. This list of topics serves as a suggestion of the possibilities; if you are interested in other ideas, please have a conference with the instructor.

A) Summarizing Data (Central Measures: mean, median, mode; Variability: range and standard deviation; Grouping: frequency distribution and histogram; Graphing: line graph)
References: Hand out Sheet #2; Text: Chpt. 8, Sections 1-2
Saunders Modular Math Services: Statistics Parts of pp. 1-50, Tables & Graphs, pp. 1-17; Any statistics text

B) Normal Curve and Hypothesis Testing (Normal Curve, Percentages, Testing hypotheses, Binomial problems)
References: Text: Chpt. 8, Section 4; Handout # 3;
Saunders Modular Math Series, Statistics, pp. 50-63.

C) Graphs of Points and Linear Equations (Graph points, graph a linear equation, given two points find the equation of the line through them)
References: Text: Chpt. 6, Sections 1 & 2, Handout # 4.

D) Graphing Parabolas, Ellipses, Circles, Hyperbolas
References: Text: Chapter 6: Sections 3-5.

E) Graphing linear inequalities/linear programming
References: Text: Chapter 11, Sections 7-8
Saunders Modular Series: Algebra 2, pp. 41-48; Linear Programming.

F) Comparing Two Sets of Data (Scattergrams, Correlations, q-q graphs, prediction equations)
References: Text: Chpt. 9, Section 1, Handout # 5.
Saunders Modular Math Series 26-31, USMES How To Booklets: Use Graphs to Compare Two Sets of Data and Use Key Numbers to Compare Two Sets of Data

G) Probabilities--The Mathematics of Chance
References: Text: Chapter 7, Sections 1-3, 7
Saunders Modular Series, Probability; pp. 1-39; Jacobs, Mathematics, a Human Endeavor (Chapter 8).

H) Permutation and Combinations, Fundamental Principle of Counting
References: Text: Ch. 7, Sections 4-6/Saunders Modular Math Series Probability, pp. 39-60.

I) Numeration and Counting Systems (for Education majors) (Numeration systems of other civilizations, other bases)
References: Text: Chapter 1, Sect. 1-4, also pp. 407-9. Saunders Modular Math Series Numeration.

J) Computers (Computer Arithmetic, Parts of a computer, Programming)
Text: Chapter 10; Saunders Modular Math Series Computers.

K) Sets and Logic (For Education majors)
Text: Introduction Saunders Modular Math Series: Sets and Logic
Many other books

L) Other topics of interest to you

APPENDIX E-12

Math 102-01 Assignment Sheet and Guidelines to outside reading report

Feb. 19: Statement about project: analyze what the problem is, make a clear statement of the various aspects of the problem, suggest what questions will be asked and tests made, how the sample will be determined, and how the data will be analyzed. Approximately one page.

Feb. 22--Questionnaires prepared--start gathering data

Feb. 26--Take first topic test

Mar. 2--Turn in title of book and chapter heading of the reading material or mathematics textbook material. (See guidelines, suggestions below.)

March 16: Take second (or third) topic test.

March 19: Turn in list of findings on the data and information (surveys, experiments) which you or your group has gathered.

March 26: Conclusions on your project due: include a written statement of the sources and findings from reference materials, and a conclusion of what your experiment has shown you together with a summary of the data. (Start second project or equivalent)

April 6: Report on outside reading due.

Guidelines for outside reading.

Read a chapter (ten to twenty pages) from a book or periodical on a topic in mathematics of interest to you. The following are on reserve in the library and you should look through these to get an indication of the kinds of articles which are available:

L. Osen, Women of Mathematics, Cambridge, MA: MIT Press, 1974.
E. Bell, Men of Mathematics, New York: Simon and Schuster, 1937.
J. Newman, World of Mathematics, New York: Simon and Schuster, 1956.
Vol. 1: Part II Historical and Biographical
Vol. 2: Part VI Mathematics and Social Science
Vol. 4: Part XXI Mathematical Theory of Art

Vol. 4: Part XXIII: Mathematics in Literature
Part XXIV: Mathematics and Music
Part XXV: Mathematics as a Culture Clue
Part XXVI: Amusements, Puzzles, Fancies

H. Rademacher, Enjoyment of Math, Princeton: Princeton Press, 1957.

T. Dantzig, Number: the Language of Science, New York: MacMillan, 1954.

J. Tanur, Statistics: A Guide to the Unknown, San Francisco: Holden Day, 1972 (for nonmathematicians--how statistics are used: Units include: Man in his Biological World-Staying Well, Getting Sick and Dying, Men and Animals); Man in his Political World; Man in his Social World--Communicating with others, Man at Work, Man at School and Play, Counting Man and his Goods, Forecasting Population and the Economy, Measuring Segregation and Inequality; Man in his Physical World)

Other: Consult with Instructor

The paper should be the equivalent of 2 or 3 typewritten pages. About half of your paper should summarize what you have read. The other part should discuss what you feel is important, if anything, about the article you read. Any quoting of sentences from the article should be in quotation marks or indented with footnotes. The main purpose of the paper is to show that you have read the material with understanding and you have reflected on what you have read.

Extended Mathematical Problems: May be substituted for outside reading.

See D. Campbell: The Whole Craft of Number, Boston: Prindle, Weber & Schmidt, 1976.
S. Stein: Mathematics: the Man-Made Universe, San Francisco, Freeman, 1976.
Belk, Beisher and Crowe, Excursions into Mathematics, New York: Worth, 1969.

APPENDIX E-13

MATH 102 CHECKLIST OF COMPLETED ACTIVITIES

NAME______________________ To be used by student

Assignment Date Completed

TOPIC TESTS

First________________	Fourth________________
Second________________	Fifth________________
Third________________	Sixth________________

Project	First	Second
1. Written statement of problem	(Feb. 19)	(Apr. 9)
2. List of findings: data from surveys	(Mar. 19)	
3. Analysis of Data	(Mar. 26)	(Apr. 23)
4. Written statement of use of references	(Mar. 26)	(Apr. 23)
5. Written conclusion: what were the findings?	(Mar. 26)	(Apr. 23)
Report of outside readings		
Good work		
Satisfactory	(Apr. 6)	(Apr. 20)
Evaluation & Summary	(Apr. 27)	

Deadlines are given in parentheses below each activity. Work in the left column is required for a D. Work in both columns is required for an A.

APPENDIX E-14

MATHEMATICS 102-01

Student Notebooks

The student is to keep a written document (notebook) of self-collected evidence of the progress and work she is doing.

The notebook should be a record of the work you are doing day by day, including the efforts to work problems and to answer questions. It can include impressions of your experiences. It should summarize material you have read, outline activities undertaken, contain exercises you have worked in doing homework, and describe progress made in doing your project.

Topic Tests

Show the work done in doing exercises from the text.
Indicate plans for taking and passing topic tests, e.g., what material you are studying.
Include topic tests attempted and those already passed.
Record time spent in tutoring and in conferences with the instructor on the topics.

Activities

Indicate what you have done day by day to accomplish the activities.
Summarize what you have read, giving the source.
Include preliminary drafts of analyzing the problem, collect- of data, interpreting data, using reference materials, and forming conclusions.
Evidence: Include the specific evidence to indicate that you have completed the activity as outlined on the grading sheet. These artifacts include written statements, graphs, tabulated data, computations, charts, and summaries of use of reference materials.
Include impressions of your experiences.
Outline any problems and difficulties you have had and what you have done to overcome them.

Project

Indicate and summarize the reading you have done and progress made. Give sources.
If you are working on a report, indicate what the topic is and give an outline as soon as you have developed one.

If you are working on a mathematical problem such as a computer program, research into teaching arithmetic, use of mathematics in business, etc., indicate progress you are making.
Evidence: Written report (see guidelines on separate paper). Indicate conferences with instructor or with outside persons who are giving advice.

Other
What topic in CAI are you working on, what progress are you having, and what difficulties are you having?
Which lessons have you completed?

Important Note: The above guidelines are only suggestions. It may not be appropriate to include all of the suggested tasks, while other kinds of statements or evidence may be desirable.

APPENDIX E-15

RESOURCE SHEETS

RESOURCE SHEET #1

Choosing a Sample and Collecting Data

1. Before you begin a statistical experiment, you must decide carefully what question you want answered and you must identify what group of people or things about which you are asking the question or questions.

2. You should decide whether to conduct your experiment on all possible items or on just part of it. In most cases asking questions about all possible items (all the batteries in the state or all the students at Bennett College) is too involved, expensive, and time-consuming, so questions are asked about a sample of the items, a relatively few items chosen so as to be representative of the entire population.

3. The sample must be as unbiased as possible; otherwise your sample might not give you the right results.
   a. Decide what groups or group you are interested in.
   b. Decide whether you want to compare two or more groups.
   c. Decide how big the sample should be by looking at the table below. (The table reflects statistical theory, particularly that involving the normal curve--see Miller and Heeren, *Mathematics an Everyday Experience*, pp. 328-341)

Table I

Size of Group	Size of Sample if not comparing groups	Size of Sample if comparing groups
50	23	
100	29	31
200	34	45
300	36	53
400	40	58
500	40	62
600	40	65
1000	40	71
Over 1000	40	80

(From *How To Choose a Sample*, USMES/Education Development Center Publication)

   d. Pick the sample from the population by random sampling, stratified sampling (see any statistics book), or by some means in which every item in the population has an equal chance of being chosen to be in the sample. Then collect the data.

4. The questions should be clearly thought out, unambiguous, with no more than five choices for an opinion survey. A trial run, perhaps among your classmates, to ensure clarity, is very useful. Confusing questions should be clarified.
   If you are doing an experiment on products, such as testing durability, time, temperature, speed, scores on a test, percentage of yes answers to a yes/no question, etc., here are some suggestions.
   a) Make a trial or test measurement before beginning, so you can see what kind of problems will come up.
   b) Keep everything the same except the one property you are testing.
   c) Repeat your measurements several times, or have several people do the measuring, trying to keep the brand name hidden so that no bias can occur.

5. Record your data, rounding off as necessary.

The above is adapted from How To Booklets, Collecting Data, USMES/Educational Development Center; from Marshall Gordon and Norman Schaumberger, A First Course in Statistics (New York: MacMillian, 1978), Chapter 8; and from Norma Gilbert, Statistics (Philadelphia: Saunders, 1976), Chp. 8.

RESOURCE SHEET #2

CLASSIFYING DATA AND SIMPLIFYING DATA

TOPIC A

Once you have obtained a large number of scores or numerical values from your survey or experiment, you need some way of analyzing or classifying the scores. It is difficult to organize or summarize a set of scores such as

0,1,10,3,8,4,4,5,7,7,3,7,5,6,7,3,4,5,7,3,4,4,10,7,1

because there are so many that the mind cannot interpret them all at once. How would you summarize the results, compare with a similar set of scores, or make a prediction based on the score? One frequently used method is to find one number which in some sense represents an <u>average</u> or <u>central tendency</u> of the number. A second method is to group the data into a few groups (or intervals), between five and fifteen groups, and then count the number of scores in each group. From this you form a <u>frequency distribution</u>.

A) Measures of Central Tendency (Averages)

1. <u>Median</u>: The median is the middle number or measurement in a set of data. Half the numbers are above the median and half are below the median. Arrange all the scores in order from lowest to highest. Count how many items of data you have and divide the number of items by 2. If your answer is a whole number plus one half, change it to the next highest number and count that many scores starting with the smallest. If your first answer is a whole number, count that many scores starting with the smallest and then add the score you stop on to the next highest score and divide by 2.

Medians are used when one or two of the scores are unusually high or low and may be due to a mistake or strange circumstances. They are also used to indicate the "middle" number. The median is not altered by an extreme score (which may be an advantage), but it does not take into consideration a legitimate extreme score (which may be a disadvantage).

2. <u>Mode</u>: The mode is the score which occurs most often. If you are counting the number of times choices for things or opinions occur (qualitative data), the mode is the thing that is chosen most often.
   To find the mode count the number of times each score occurs. This can be done easily if you arrange the scores in order.

The mode can be used when your data are categories (such as freshman, sophomore, etc.) and number counts. For one set of data, the mode is used to find the most frequent count, to find the most favored choice, and to find the score which occurs most often. The mode is not affected by extreme values. However, it is of limited use because it does not take into account the value of any other scores.

3. Mean. The mean is the number obtained by adding all the scores and dividing by the number of scores in the set of data. It represents the number you get when the total of your data is split into equal pieces.

The mean can be used at all times, but it is particularly useful when you want to take in account all scores, even if some are extreme, because they all are meaningful or are likely to occur again. The mean is most useful in making predictions about what will happen if the experiment is repeated. The mean is also used as a typical number if you only know the total amount (sum of scores) and the number of scores.

Example: Scores listed above: Arrange in order
0,1,1,3,3,3,3,4,4,4,4,4,5,5,5,6,7,7,7,7,7,7,8,10,10
Median is 5 (12 numbers above, 12 numbers below), Mode: 7;
Mean = 125/25=5.

B) FREQUENCY DISTRIBUTION: The example will explain the process.
The frequency is the number of scores in each class; here only six numbers need to be considered instead of 25.

Class Frequency	-.5 to 1.5 3	1.5 to 3.5 4	3.5 to 5.5 8
	5.5 to 7.5 7	7.5 to 9.5 1	9.5 to 11.5 2

Resource Sheet #3-1

Frequently knowledge of the mean, mode, and median of a collection of data is insufficient, by itself, to allow adequate analysis of the meaning of the numbers. In addition to knowing the measures of central tendency we also need to know something about how much the scores vary from each other or how much the scores are dispersed. In some situations, for example, drilling a hole of diameter 5 cm, a mean of 5 may result from holes of 4.5, 5, and 5.5 and also from holes of 4.9, 5.0, and 5.1. The two sets of data have means of 5 but the second set is preferable since it displays less variation.

Consider the scores on three versions of a ten question quiz given to five students each. Which is the "best" quiz, i.e., which is neither too hard nor too easy and tells the instructor how well students are doing on the material:

Quiz 1	0,5,5,5,10	Mean 5	Median 5	Mode 5
Quiz 2	4,5,5,5,6	Mean 5	Median 5	Mode 5
Quiz 3	1,3,5,7,9	Mean 5	Median 5	No mode

The mean and median are the same and do not give enough information to distinguish about the quizzes.

The <u>range</u> is determined by largest number - smallest number
Quiz 1 10 Quiz 2 2 Quiz 3 8

This measure of dispersion tells how far apart the largest and smallest numbers are, but it does not involve any of the middle numbers.

We need an "average distance of the scores from the mean" as a better indicator of dispersion. However, if we subtract each score from the mean, add the differences, and divide the number of scores, we will always get zero:
Example quiz 3

$$((1-5) + (3-5) + (5-5) + (7-5) + (9-5))/5 =$$

$$(-4 + (-2) + 0 + 2 + 4)/5 = 0/5 = 0$$

The negatives cancel out the positives. Hence to make each difference positive, we square the differences before adding; then to compensate at the end of the computations, we take the square root.

Algorithm for finding the STANDARD DEVIATION of a set of data:

1. Determine the mean. (Col. 1) Add. Divide sum by number of scores = m
2. Subtract the mean from each score. (Col. 2) x-m
3. Square each value in Col. 2, record in Col. 3. $(x-m)^2$
4. Add the values in Column 3.
5. Divide the result by the number of scores.
6. Take the square root = s

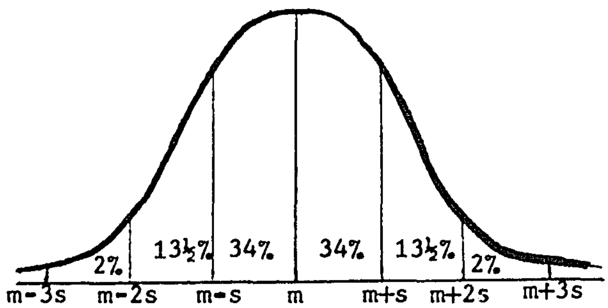
	X	X-m	$(x-m)^2$
	1	-4	16
	3	-2	4
	5	0	0
	7	2	4
	9	4	16
Sum	25		40

$\frac{25}{5} = 5$ $\qquad$ $\frac{40}{5} = 8$

(s) $\sqrt{8} = 2.83$

The average spread--the standard deviation--is 2.83.

Standard deviations are very useful in computing percentages, proportions or total numbers for large amounts of data taken from natural circumstances--test scores (sometimes), measurements of heights, and weights and lengths, and many other measurements. The frequency distribution of these large number of scores takes the form of a NORMAL CURVE with the peak at the mean scores. It can be shown 34% of the scores will be between the mean and the mean plus 1 standard deviation. See the chart for other values.



Resource Sheet #3-2

Notes on the Normal Curve

If 1) a large number of scores are obtained (by testing, weighing, measuring, etc.) from items (animals, plants, people, naturally occurring phenomenon, etc.) taken 2) from the general population, and a line graph is drawn, the graph (scores on horizontal axis, frequency on vertical axle) will almost always look like a normal curve (see page 330 of your text). The items to be measured must be chosen at random from the general population.

In a normal distribution the mean, median, and the mode all have the same value--which is associated with the axis of symmetry running vertically through the highest point of the normal curve.

34% of the scores will occur between the mean and the mean + 1 standard deviation
13.5% of the scores will occur between the (mean + 1 st. dev) and (mean + 2 st. deviations)
2% of the scores will occur between the (mean + 2 st. deviations) and (mean + 3 st. deviations)

A more accurate determination of the percentage of scores between the mean and the mean + z standard deviations, where z is a decimal fraction, is given by the table on p. 330. This percentage of scores is also the probability that an item chosen at random will fall between the mean and the mean + z standard deviations.

Example If the mean is 50 and the standard deviation 10, what is the probability of a score falling between 50 and 63? Use the formula

$$\text{score } X = \text{mean} + z\ (\text{st. dev})$$
$$63 = 50 + z\ (10)$$

Solving the linear equation for z, we get $z = 1.3$. From the tables, z corresponds to 40.3%.

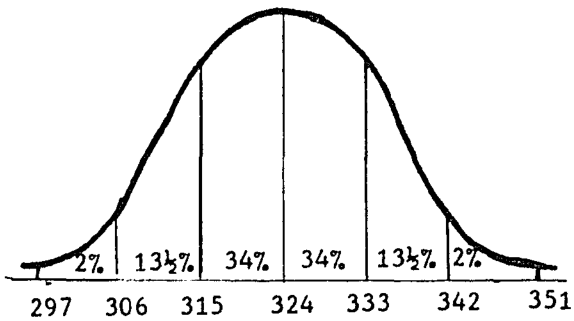
Note that the normal curve is symmetric, so that if z were negative, we can use the absolute value of z to look up probabilities in the tables.

BINOMIAL DISTRIBUTION Suppose an experiment is performed by repeating the same process (called a trial) n times, and on each trial, there are only two possible outcomes, called success or failure. The score for the experiment is the number of successes out of the n trials. These scores will "fall on a normal curve." If we know that p is the probability of a success on a trial, then

$$\text{mean} = np$$
$$\text{st. dev} = \sqrt{np(1-p)}$$

Example A manufacturer of ping pong balls, after a quality control survey, announced that 75% of its balls were guaranteed to be good. The balls were shipped in boxes of 432 each. (The experiment then is to take each of the 432 balls and check to see if they are good or not. Then the number of good balls is determined, and this is the score X. Here the outcomes are good or bad, n = 432 and p = .75). Now suppose 1000 boxes (of 432 balls each) are shipped to the famous Jordan Sears store, and an employee counts the number of good balls in each box, records the 1000 scores, and draws a line graph. The graph will be a normal curve with mean = np= (432) (.75) = 324 and standard deviation = $\sqrt{np(1-p)}$ = $\sqrt{432 X .75 X .25} = \sqrt{81} = 9$.

34% of the boxes will have between 324 and 333 good balls, 13% of the boxes will have between 333 and 342 good balls, etc.



Resource Sheet #3-3

Using a Sample to Estimate Results for a Population

Suppose we want to find the mean and standard deviation of a large population (in statistics, this means the entire set of scores for any collection of items being tested, not just people). Frequently, the population is so large that we can not take and measure all the members of the population. Hence we are forced to take and choose, at random, a relatively small number of items in the population. We call this collection a sample. Mathematicians have determined that in order for the sample to be a useful indication about the population, at least thirty, and preferably more items must be chosen.

A population distribution has a unique mean and standard deviation: M(P) and S(P). If we take a random sample from this population, the sample would determine a particular sample mean and a sample standard deviation: M(S) and S(S). The sample mean would differ from the population mean, and the sample st. dev. would differ from the population st. dev. If we took another sample and got another sample mean and sample st. dev., these too would differ. How can we know how accurate the estimates from the sample are to the true population mean and standard deviation? This is where the normal curve and some formulas help us.

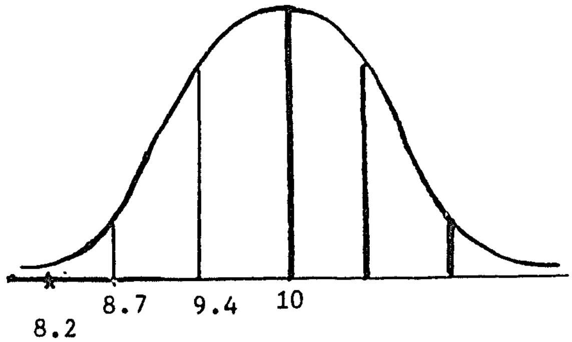
If we take all possible samples of a given number, n, of items, and compute the means for each of the samples, we get a collection of scores (i.e., means) which form a new set of scores. These means can be analyzed as any other set of scores--we can get the mean of the means M(M), the standard deviation, and find a frequency distribution and histogram. THE HISTOGRAM OF THE MEANS OF ALL SAMPLES OF SIZE n FORMS A NORMAL CURVE, if the original data forms a normal curve.

This set of means of samples is called the distribution of samples.

1) If the original distribution is normal with mean M(P) and standard deviation S(P), then the sampling distribution of the means is normal with mean M(M) = M(P) and standard deviation $S(M) = S(P)/\sqrt{n}$

EXAMPLE A weight-reducing program states that in 2 weeks time an average person will lose on the average 10 pounds if the program is followed exactly. Thirty people who followed the program exactly have lost on the average 8.2 pounds, with a standard deviation of 3.5 pounds. Can we claim the sample findings are reasonable to accept, or are the sample findings different enough from the population to cast doubt on the claim?

Claim that M(P) = 10. We will allow the results to occur 95% of the time by chance; but if the sample results fall into the tail end regions of a normal curve ($2\frac{1}{2}$% on each side), then we will be suspicious of the claim that M(P) = 10.



Sampling Distribution
of Means

n = 30

$$S(M) = \frac{S(P)}{\sqrt{30}} \quad \frac{S(\text{sample})}{\sqrt{30}} = \frac{3.5}{\sqrt{30}} = .64$$

$$X = 10.0 + z(0.64)$$

$$8.2 = 10.0 + z(.64)$$

$$z = \frac{-1.8}{.64} = -2.81$$

which corresponds (see tables) to 49.7% .

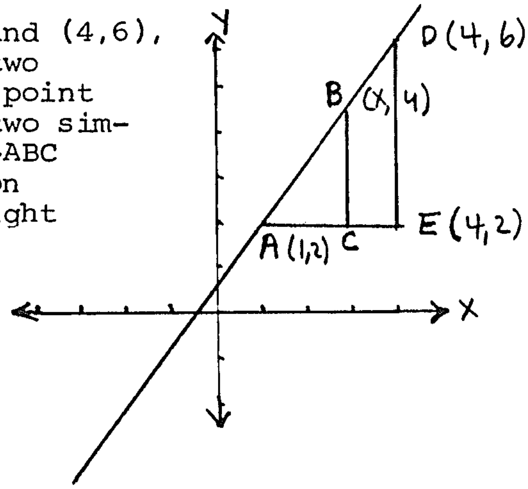
Reject the claim that the mean is 10.

Resource Sheet #4

Graphs of First Degree Equations
Equations of Lines

I. // If two points of a line are given, how to find the equation of all points (x,y) on the line.

Example Given two points (1,2) and (4,6), you can draw a line through the two points. If you let (x,y) be ANY point on the line, as in the drawing, two similar right triangles are formed--ABC and ADE--since they share a common angle at A, and they both have right angles. Then the ratios of corresponding sides are equal.



$$\frac{\text{vertical of small}}{\text{horizontal of small}} = \frac{\text{vertical of large}}{\text{horizontal of large}}$$

Hence $\frac{BC}{CA} = \frac{DE}{EA}$ Since BC is y-2, CA is x-1 DE is 6-2 and EA is 4-1

we have $\frac{y-2}{x-1} = \frac{6-2}{4-1}$ or $\frac{y-2}{x-1} = \frac{4}{3}$. This ratio

is called the slope of the line: it is the ratio of the vertical change divided by the horizontal change:

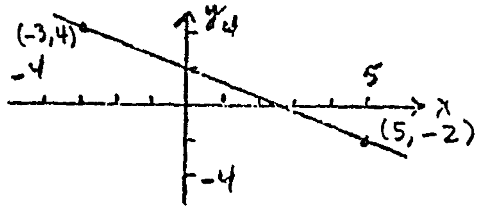
$$\frac{\text{change in y}}{\text{change in x}}$$

The equation is usually written after multiplying both sides by x-1:

$$y-2 = \frac{4}{3}(x-1)$$

General formula If a line goes through two given points (x_1, y_1) and (y_1, y_2) and if (x,y) is any point on the line, then the equation of the line is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$



Example: The equation of the line through (-3,4) and (5,-2) is

$$y-4 = \frac{-2-4}{5-(-3)}(x-(-3))$$

which simplifies to $y-4 = \frac{-6}{8}(x+3)$ or $y-4 = \frac{3}{-4}x - \frac{9}{4}$

or finally $y = -\frac{3}{4}x + \frac{7}{4}$.

Note that the two points (-3,4) and (5,-2) satisfy the equation.

II // If an equation in x and y of first degree (no squares, cubes, etc.) is given, then the set of all points (x,y) forms a straight line through any two points satisfying the equation.

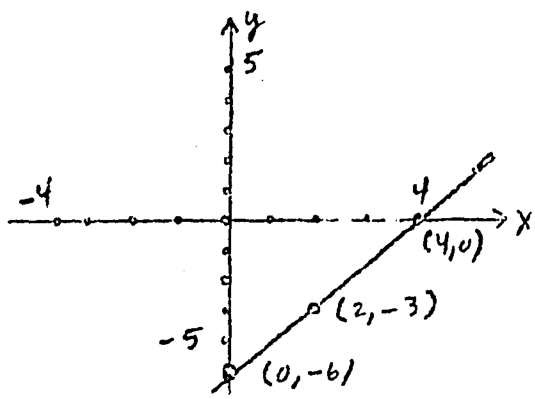
Example: $6x-4y=24$ is a first degree equation.

To find any points satisfying the equation, choose any number for x or for y, substitute, and solve for the other letter. This gives a pair (x,y).

If x is 0, the equation becomes $-4y = 24$, so $y = -6$ (divide both sides by coeff of y)
If y is 0, the equation becomes $6x = 24$, so $x = 4$ (divide both sides by coeff of x).
If x is 2, the equation becomes

$$\begin{aligned} 6(2)-4y &= 24 \\ 12-4y &= 24 \\ -4y &= 12 \\ y &= -3 \end{aligned}$$

The three points line on a straight line: (0,-6), (4,0) (2,-3)



Resource Sheet #5-1

SCATTER PLOTS

In mathematics courses you are usually given the equation of some relationship and asked to graph the formula. For example, you are told that if

$$y = 5x + 2,$$

then graph the set of all points (x,y) which satisfy the equation. (The graph will be a straight line.)
Usually, you are not told where the equation comes from in real situations. In the unit on LINEAR PROGRAMMING, we saw that one source of formulas was the rules or regulations of business operations or physical relationships--e.g., the price of a ticket is reduced $20 for each additional person.

Another source of formulas is the gathering data and seeing if there is a relationship or an approximate relationship which can be put into a formula. We seek a formula which will be accurate "within three units 95% of the time," etc. This involves statistical considerations, particularly the normal curve, scatterplots, regression, correlation coefficients, etc.

Consider the following: (From David Moore: Statistics, Concepts and Controversies, p. 155,6)

A homeowner is interested in how the demand for heating in cold weather affects the amount of natural gas his home consumes. Demand for heating is often measured in "degree days." (To find the number of degree days for a certain day, record the high and low temperature on that day and find the temperature midway between the high and low. If this temperature is less than 65°F, there is one degree day for every degree below 65°F). The homeowner recorded his natural gas consumption in cubic feet per day, and also the average number of degree days per day, for nine consecutive months. Here are the data.

Degree days per day	15.6	28.8	37.8	36.4	35.5
Cu feet gas per day	5.2	6.1	8.7	8.5	8.8
Degree days per day	18.6	15.3	7.9	0	
Cu feet gas per day	4.9	4.5	2.5	1.1	

1) Use graph paper and use only the upper right quadrant--positive values only. Mark a scale for degree days on the horizontal axes and cu feet on the vertical axes. Make sure your markings are uniform and consistent with the number of squares on the graph paper.

2) Plot the 9 pairs of points in the table above.

3. From the scatterplot, give a rough estimate of gas consumption on a day with 20 degree days.

4) The points will not lie on a line, but they will be "linearly" oriented. Draw a line through the points lightly in such a way that your line comes as close to all the points as possible (i.e., minimize the vertical distance from the point to the line).

5) Recall that if you know two points (x_1, y_1) and (x_2, y_2) on a line, then you can write a formula for the line by the relationship.

6) Use your formula to check your answer in 3.

Assignment: Read Chp. 9, Section 1.

RESOURCE SHEET #5-2

Finding the Best-Fit Linear Relationship Between Data of Two Variables

OR

USING A COMPUTER PROGRAM SO WE WILL HAVE TIME TO WATCH TELEVISION

We sometimes seek to find an "exact" relationship between two variables so we can make predictions--if we know something about one variable, we can use an equation to find out a corresponding value of another related variable. "When data are processed, the occasion arises for fitting lines, curves, or surfaces to represent relations between two or more variables. The astronomer predicts the path of a satellite; the businessman projects a trend; the biologist relates the size of a growing population to time; the educator compares students' college grades with their high school grades; the meteorologist forecasts the path of storms; and the doctor compares the rate of relief from pain with the size of the analgesic dose. All such tasks involve the fitting of curves. For some problems the work is easy and can be done by hand; for others the calculations are horrendous and would not even be considered without modern high-speed computers. Nevertheless, the central ideas behind work in curve-fitting are relatively simple."[1]

On a simpler level, we may want to find out if there is any relationship between

the cost of a wooden pencil and
the number of strokes it takes to wear a sharpened lead down to wood

Step 1: Plot on a graph a dot for each pair (cost, number of strokes).

Step 2: If the dots are approximately in a linear grouping, then try to draw a line through the dots until the line fits the points well. ("An old and simple method of fitting a straight line is to stretch a black thread across a graph, and rearrange its position until one's eye is satisfied that the thread fits the points well. Then the coordinates are read at convenient points at the left and right edges of the figure."[2])

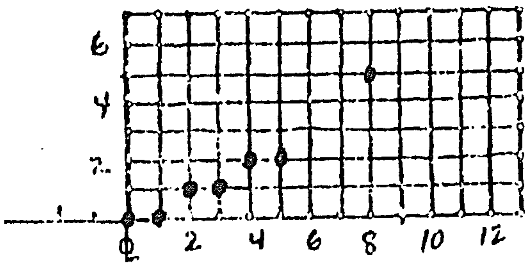
[1]Mosteller, F., Rourke, R., Thomas, G. Probability with Statistical Applications (Reading, MA: Addison-Wesley Publishing Co., 1970, 2nd ed.), p. 382.
[2]Ibid., p. 385.

Step 3: Find the coordinates of two points on the line and use these coordinates to find the equation of line--see sheet 4.

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1) \text{ and change to the form } y=ax+b$$

Step 4: Use this equation or the graph to predict. Take a value for a first coordinate, the x-value, and substitute into the equation. Then evaluate for y.

Example: Use the black-thread method to find an equation for a line fitting the points (0,0), (1,0), (2,1), (3,1) (4,2), (5,2) (8,5).



Class discussion: for each person. Find the points (0,?) (6,?) on the line you have drawn. Then find the slope a in your equation above. How much difference is there among members of the class?

RESOURCE SHEET #5-3

We need a simpler, more direct, more accurate, and quicker way to determine the equation of the line which best fits all of the observed points. Using the equation in the form $y = ax + b$, we can, knowing a value of x, find a predicted value of y.

Example: Given Observed values:

x	y
10	10
15	40
35	40

Which is the best fit line?

$y_c = 30$

$y_c = 6x-50$

$y_c = .86x+12.8$

We will take the best-fit line as the line $y_c=ax+b$ for which the sum of all $(\text{y-value on line} - \text{observed value of } y)^2$ is a minimum.

We understand in summing these squares up that we take the values of y_c and y for each point (x,y) in the observations.

The difference for each point between the given score Y and the predicted Y_c score is the <u>error of estimation</u>. The <u>regression</u> line is the line for which the sum of squares of the errors of estimation is minimized.

<u>Solution of Example</u>

$y_c = 30$

y	y_c	$y-y_c$	$(y-y_c)^2$
30	30	-20	400
40	30	10	100
40	30	10	100
			600

$y_c = 6x-50$

y	y_c	$y-y_c$	$(y-y_c)^2$

$Y_C = .86x + 12.8$

y	Y_C	$y-y_C$	$(y-y_C)^2$
10	21.4	-11.4	130.0
40	25.7	14.3	204.5
40	42.9	2.9	8.4
			342.9

There is a formula, proved by the use of calculus, which will give the slope and the equation of the best-fit line. If you like formulas, to find the best fit equation, use

$$y = ax+b, \text{ where}$$

$$a = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} \quad \text{and } b = \bar{Y} - a\bar{X},$$

where n is the number of points and $\sum$ means sum all terms.

If you don't like formulas, then fill in a table like this one.

	XY	Y	X	X^2
SUM	A	B	C	D
$(SUM)^2$			E	
SUM/n		F	G	

Step 1: Fill in table.

Step 2: Sum all four columns, obtaining values for A,B,C,D.

Step 3: Square the sum of the x-values and obtain E.

Step 4: Divide the sums of Y and X by n, to obtain means F,G.

Step 5: Substitute and compute:

$$a= \frac{nA-BC}{nD-E} \qquad b=F-aG$$

Answer: Y = aX+b

OR WRITE A COMPUTER PROGRAM TO FOLLOW THE STEPS ABOVE.

RESOURCE SHEET #6

SUMMARY OF BASIC STATEMENTS

(from Robert Lynch and John Rice, Computers: Their Impact and Use, Basic Languages)

+ addition - subtraction *multiplication / division

∧ or ↑ exponent

$y = \frac{3(5 + x^2)}{5-3}$ is written on one line in BASIC (3*(5+X∧2))/ (5-3)

Symbolic Names: One or two characters; the first one is a letter, the second one, if present, is a digit. Examples A X B4 G2

Constant: One to 8 digits, with or without sign and with or without a decimal point. May also include the letter E followed by a plus or minus sign and an integer (when using exponential notation). The maximum size of the integer after the E is less than 38. Examples: 37, -37, -34.56, 1234.45E+9 -23.45E-13

Let of Assignment Statement: The Keyword LET followed by a variable name, followed by an = sign and ending with an expression. Example Let A = Bl + 4.5

Label A number which precedes each statement in a BASIC program. The label is the order in which the statements are executed. Examples:

```
3 LET A = 5
99 LET B = A+5.4
10 PRINT A
```

Expression An expression is a collection of variables and constants with arithmetic operations indicated. The expression is correct or legal if the result of performing the operations is a single value. A-B*C+5/(6-G2)

Message A message is a list of string characters in quote marks. Any BASIC characters may be used in the message except quote marks: "The ANSWER IS"

Print The word print followed by a list of items separated by commas. The items in the list may be variables, numbers, expressions, or messages. Example: 22 PRINT X, 17.42, 1.2*Y, "THE ANSWER IS"

Data The keyword DATA is followed by a list of constants separated by commas. The constants are placed on the bottom of the data stack (program) in the order in which they appear. Examples DATA: 17,18,999,-66.1

Read The keyword READ is followed by a list of variable names separated by commas. Values are taken off the top of the data stack and assigned in order to the variables in the READ list. A number taken off the data stack is not replaced. Examples 10 READ A, B, C

Input The keyword INPUT is followed by a list of variable names separated by commas. When the computer reaches the word INPUT, it stops and displays ?. The operator then types the numbers, separated by commas, as corresponding to the variables in the input statement. Example 10 INPUT X,Y The operator would type in two numbers: the first for X, the second for Y.

End The keyword END is placed as the last step of any program. It tells the control section of the computer to cease processing this program.

Relational Expression Takes the form: (expression) (relational operator) (expression)
The operators can be =, <, >, < >, >=, <=. These are equals, less than, greater than, less than or greater than (i.e., not equal to), greater than or equal to, less than or equal to.

If-then The IF-THEN statement is of the general form
IF (relational expression) THEN (label)
If the relational expression is true then a transfer is made to the statement whose label is given. That is to say that the normal sequence of execution of the statements is broken and the next statement to be executed is the one whose label is specified. Examples 50 IF K-1 = 0 THEN 200

Go to The statement uses the keyword GOTO followed by the label of some other statement. When this statement is encountered in execution the next statement executed is that whose label is given. Examples GOTO 999

Functions ABS (variable) absolute value SQR (variable) square root SIN, COS, TAN, ATN used in trig EXP and LOG used for logarithms INT integer part, SGN sign, RND random number

APPENDIX E-16

TOPIC TESTS

Name________________________________

Topic Test A   Central Tendency, Grouping, Variability

Show work for partial credit. Put answer in blank at the right.

1) Find the mean, median, and mode of 75,53,42,81,42,49,53,80

Show work below.

A) Mean__________

B) Median________

C) Mode__________

2) Find the range and the standard deviation of 4,5,7,7,11,14.
Show work for the standard deviation in the chart below.

x	x-mean	$(x\text{-mean})^2$
4		
5		
7		
7		
7		
11		
14		
______	______	______

Use the square root table:

number	square root
8	2.83
9	3.00
10	3.16
11	3.32
12	3.56
13	3.61
14	3.74
15	3.87
16	4.00

A) Range______________

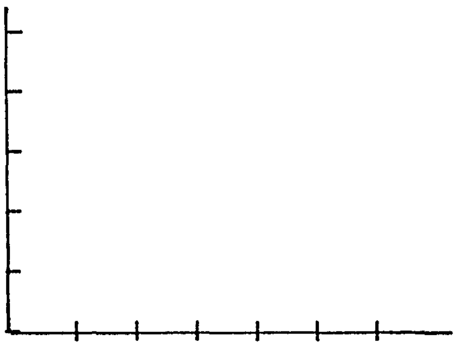
B) Standard deviation________________

3) Thirty people are asked to rate how much they enjoyed a movie on a scale of 1 (very much), 2 (a lot), 3 (some), 4 (little), 5 (not at all). The following scores were reported:

3,4,3,2,5,3,4,2,5,3,4,1,2,5,3,4,3,3,3,3,4,2,1,5,1,5,4,2,2,4

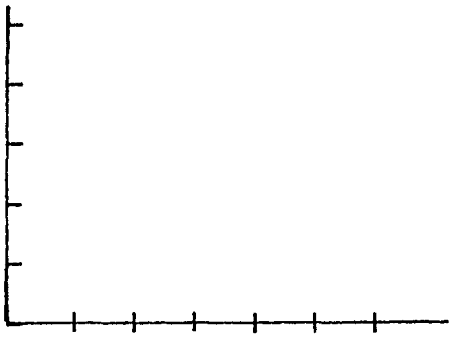
Make a frquuency distribution, histogram, and line graph.

	tally	frequency
1		
2		
3		
4		
5		



histogram

mark the scales



line graph

TOPIC B SAMPLE TEST Normal Curve

1) On a test the average grade was 75 with a standard deviation of 7. If the scores are distributed approximately in a normal curve, find the percentages of the scores in each of the following intervals:

   a) 75-82 b) 61-75 c) less than 68

   d) greater than 96

2) Use tables on p. 332 of your text to find the score on the test in problem #1 above which you would expect 20% of the students to score.
(Hint: Use the formula $\bar{X}$ = mean + z (st. dev.), where you want to find the score X for which 20% of the students would score higher, which is the same as finding the score X for which 30% of the students would score between the mean and X, and the other 50% of the students score below the mean). Choose z to be the number in the z col. (p. 332) closest to P = 30%.

3) Any distribution of data for which there are only two possible events (heads/tails; good/bad; success/failure) is called a binomial distribution. If the experiment consists of repeating the same activity n times and the number of successes is counted, and if it is known that the probability of a success on each trial is p, then

   a) the mean number of successes out of n attempts if the experiment is repeated many times is

$$\text{mean} = \frac{p}{n}$$

   b) the standard deviation is $\sqrt{np(1-p)}$

A) What is the mean number of fives to be expected in 180 tosses of a single die? (for a fair die, $p = \frac{1}{6}$)

B) What is the standard deviation?

C) Using the normal curve, within what limits can we expect the number of fives to fall approximately 95% of the time?

D) Use the normal curve to predict the probability, rolling a fair die, of getting between 40 and 45 fives when you roll a die 180 times.

4) Testing indicates that the lifetimes of a new shipment of 1000 BurnBrite lightbulbs are approximately normally distributed with a mean of 100 hours and a standard deviation of 10. How many of the lightbulbs will last between 83 and 117 hours? (Use tables on p. 332)

These problems will be discussed April 20. Look them over.

Notebooks will be due during the week of April 23 through 27. Please hand in the homework you have been doing and any reports (book reports, projects, etc.) which you have not previously handed in and which I have <u>not</u> looked at already. Your notebook should also include a listing of the work you have done, material covered in the CAI lab, and any impressions of work you have done.

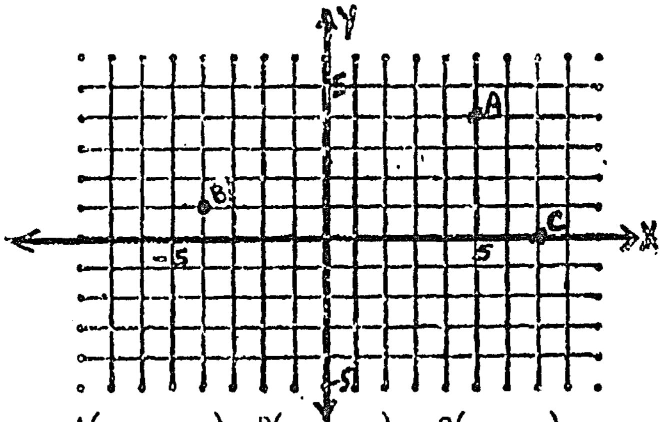
Include in your notebook a sheet which describes your computer program, what it does, how it is run, and what the name of the program is so that I may call for it and test it.

Name______________________

TOPIC GRAPHS, LINES, & LINEAR EQUATIONS

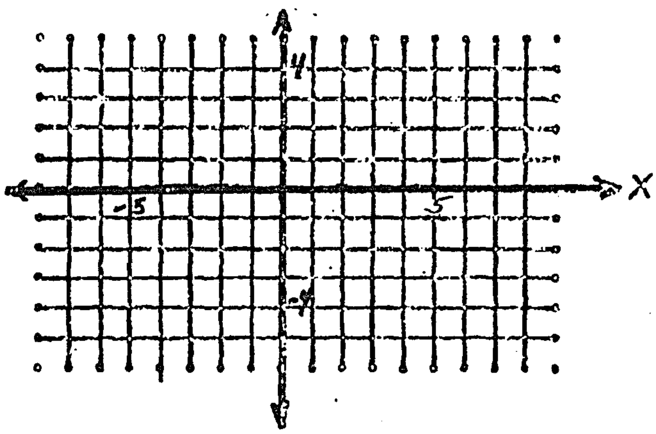
Show all work.

1) Use the coordinate system below to find the coordinates of the labeled points.



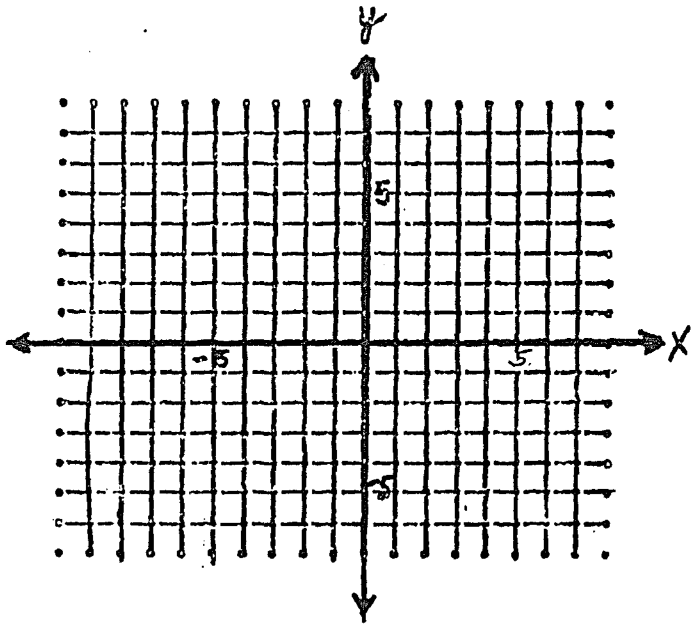
A(,) B(,) C(,)

2) Graph the following points by making a heavy dot in the appropriate place and labeling with the corresponding capital letter. X(3,-5) Y(-4,-3) Z(0,4)



3) Graph the set of all points which satisfy the equation $y = -x + 2$. Start by filling in the chart:

x	-2	0	2
y			



4) Find the equation satisfied by all points on the line through (1,-2) and (4,7).

Show your work.

Answer______________________

MATH 102 Homework and Suggested Topic Test E

INEQUALITIES AND LINEAR PROGRAMMING

Name________________

Show all work and answers on this sheet.
Part I) Inequalities in one variable

1) How many integers satisfy $-4 \leq x < 5$?

2) How many real numbers satisfy $-4 \leq x < 5$?

3) Identify the graph of $-4 \leq x \leq 5$ for x a real number
(LINE, SEGMENT, RAY, OR HALFLINE)

4) Identify the graph of $x \geq 6$ as in #3

5) Identify the graph of $x < 6$ or $x > 2$ as in #3

6) Solve for x: $4x + 2 < 10$

____

7) Solve for x
$-2 \leq 3x + 4 \leq 13$

____

8) Solve for x
$-5x > 15$

____

PART II) Inequalities in two variables

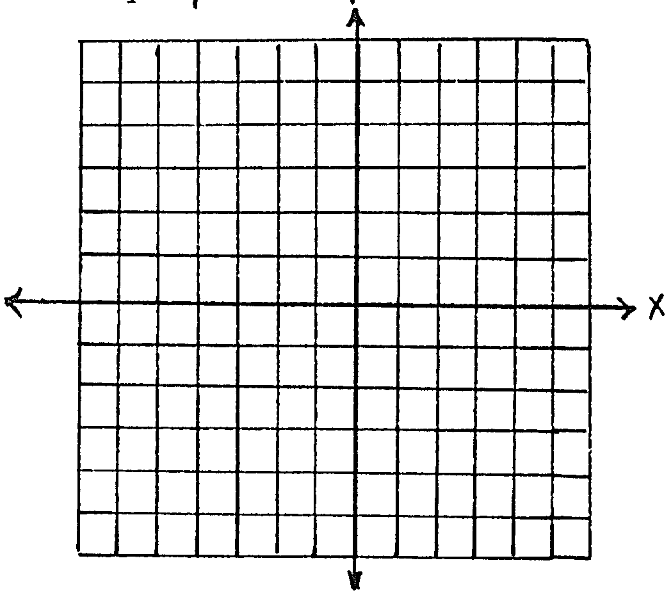
1) Does (4,5) satisfy $3x-2y < 7$ (YES/NO) __________

2) Shade in the region on the graph below

$2x + y \leq 4$

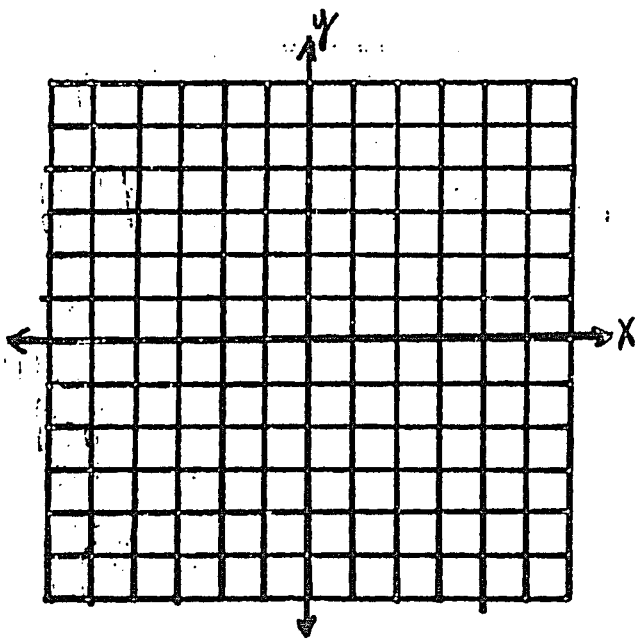
Plot points for boundary line here:

x	0	
y		0



3) Shade in the region on the graph below:

$2x + y \leq 4$ and $x \geq -4$



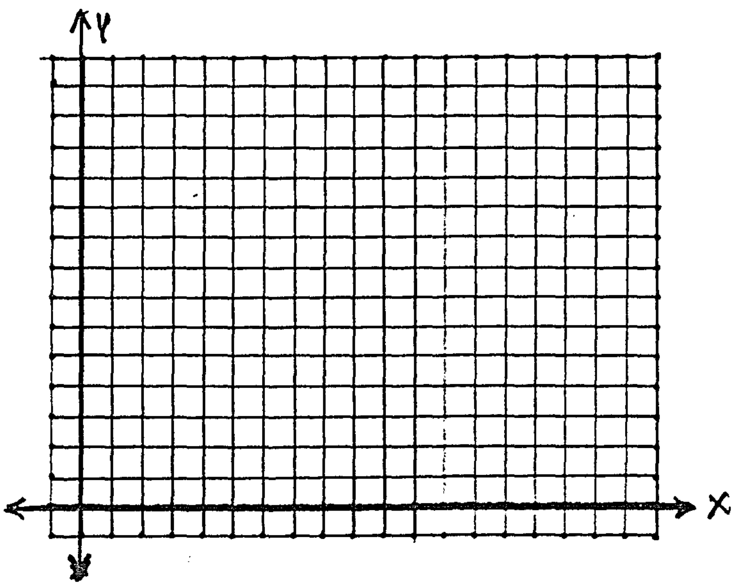
4) Find the maximum value of the expression $7x-3y$ subject to the constraints:

a) $x \geq 1$; b) $y \leq 6$; c) $x \leq 5$; d) $x \leq y$; e) $y \geq 0$

(A) Fill in the chart

Intersection of lines	In region? Yes/No	If yes, value of $7x-3y$
a,b		

(B) Draw the graph below



Name____________________

TOPIC G PROBABILITY

Show work.

1. A box contains three green balls, two red balls, and one yellow ball. If one ball is drawn at random, what is the probability of drawing a red ball?

Answer____________________

2. Suppose a ball is drawn from the box containing three green, two red, and one yellow ball and then another ball is drawn without replacing the first.

   A) Draw a tree diagram showing all possible outcomes. Use G for green, R for red, and Y for yellow.

   B) Find the probability of drawing two red balls. Show your work.

   Answer____________________

   C) Find the probability of drawing a green and a yellow ball.

   Answer____________________

3. In a group of 50 students, 20 are women, 12 are freshmen, and 8 are both freshmen and women. What is the probability that one student, chosen at random, will be either a woman or a freshman?

Answer____________________

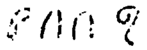
4. In a survey of 100 students, 13 were psychology majors, 15 were biology majors, 35 were education majors, 10 were English majors, 7 were political science majors, and 20 were business majors. If one person is chosen from the group at random, what is the experimental probability that she is an education major?

Answer____________________

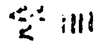
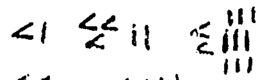
TOPIC I: Numeration and Number Bases

Homework and Sample Topic Test

1) What numbers are represented (in the ordinary decimal system) by

(a)  (b) 

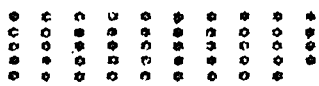
(c) 

(d)  (e) 

(f)  (g) 

(h) LIX (i) CCXXIX (j) XLDCCVII

2) Represent the number of elements in the set below in the indicated numeration systems.



Egyptian__________ Roman__________

Zambian__________ Base 10__________

Babylonian__________ Base Two__________

Base Five__________

3) Write 3013 in Egyptian__________ Roman__________

Base Five ________ Hint $5^4 = 625$ and $5^5 = 3125$

4) Convert to Base ten numerals

32_{five} __________

3214_{five} __________

5) Convert to Base five numerals

32_{ten} __________

254_{ten} __________

6) Add: a) 𓍢𓎆 |||||

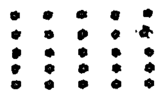
𓎆𓎆𓎆 ||||||||

b) 2034five

+3142five

7) Let the following symbols represent the following quantities

A 1   B 2   C 3   D 4   E 16   F 64



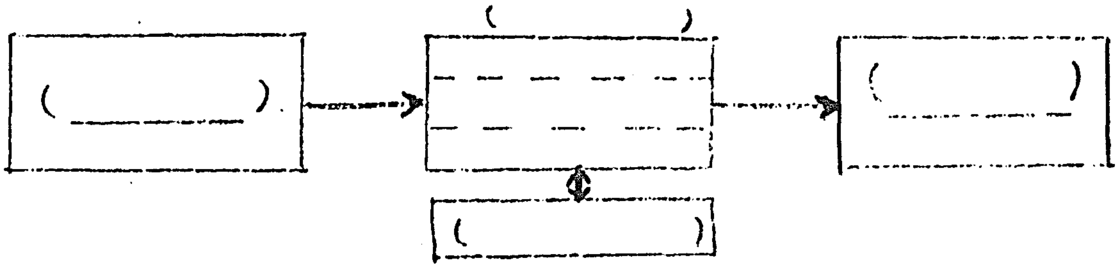
a) Using these symbols, represent the number of elements in the set to the right in an additive-repetitive system with base 4.

b) Using these symbols, represent the number of elements in the set to the right in a place-value, base 4 system.

TOPIC J

Some Suggested Sample Questions for the Test on Computers

1. Fill in the following chart which shows the parts and interrelationships between the parts of a computer



2. List four devices or methods of putting data into a computer.
3. Limited data and temporary programs, stored inside the computer, are stored on____________________.
4. List four devices on which data and programs are stored outside the computer.
5. List four kinds of output units.
6. Explain why data is stored and transmitted in binary form.
7. Write the decimal number 79 in binary form.
8. Write the binary number 10110011 in decimal form.
9. Write 5.943E+07 in standard form.
10. Write 6.7E-06 in standard form.
11. Write -79200000 in scientific form (computer).
12. Write .00000792 in scientific form (computer).
13. Fill in blanks: When you type the following commands or program instructions, what part (or parts) of the computer (from your diagram in problem 1) are activated or used?

    A. GET-STEEV Transfers program from_____ to_____
    B. RUN __________
    C. LIST __________
    D. IF _____, THEN_____ __________
    E. LET X = 5 __________
    F. ..... X + 2*Y __________
    G. INPUT X (transfers data from _______ to _______)
    H. PRINT Y (transfers data from _______ to _______)
    I. SAVE--MAT (transfers program from _______ to _______)

14. Write the algebraic expression in computer symbols

$$z = \frac{2+x^2}{3y - 5}$$

15. Step through the following program, completing the lists of values for S, X, and F at line 10 and 30, as many times as the program passes through these lines. Find the printout.

```
 5  LET S=0
10  READ X,F
20  IF X=0 THEN 50
30  LET S=S+X*F
40  GOTO 10
50  PRINT "SUM IS", S
60  DATA 1,7,2,4,3,8,0,2,4,7
70  END
```

	S	X	F
line 10			
line 30			
line 10			
line 30			
.			
.			
.			
etc.			
.			
.			
.			
line 50			

TOPIC K LOGIC AND SETS

Homework and Sample Topic Test

1) Let p: John rode a bicycle q: Mary skipped rope

Write in words:

A) p and q_ _

B) q $\longrightarrow$ (not p) _

C) (not q) or p _

2) If she runs with the ball, then she will not score two points.

A) Converse _

B) CONTRAPOSITIVE _

C) Inverse _

3) Indicate whether each statement below is true or false. Use "Ice cream has sugar" as true and "Donuts are sour" as false.

A) Ice Cream has sugar and donuts are not sour._________

B) If Ice Cream has sugar, then donuts are sour. ________

C) Ice cream has sugar OR donuts are sour. _ _ _ _ _ _ _

D) If donuts are not sour, then ice cream does not have sugar.

4) A solemn promise was made: "If you loan me \$5.00 I will come see you on Monday." What conclusion can you draw? (Write Yes, No, or Can't Tell)

A) You loaned me \$5.00. Did I visit you on Monday?________

B) I came to see you on Monday. Did you loan me \$5.00?____

C) I did not come to see you on Monday. Did you loan me \$5.00?__________

D) You did not loan me \$5.00. Did I come to see you on Monday?____________

5) Negate the following: A) All students in this class will get an A. ____________________

B) Some students in the class will get an A.__________

C) Either she will get an A or get a B.____________

D) She will take Math and she will take English.________

6) Let the universal set be U = set of letters of the alphabet

$A = \{\text{first ten letters of the alphabet}\}$

$B = \{g, h, j, k, m, p, u, v\}$

$C = \{u, v, w, x, y, z\}$

i) List the elements in A______________________

ii) Write a verbal description of C________________

iii) List the elements of $B \cup C$____________________

iv) List the elements of $B \cap A$__________

of $B' \cap A$__________

v) Which are true? $c \in A$______, $m \notin B$________

$A \subset C'$ ________

7) Income tax statement: "If you are married and filing a joint return and if the total income is under $13,125 and allowable deductions total more than $2100 or your income is between $13,125 and $17,500 and deductions exceed $2800, then you should itemize." Should the following couple itemize?--married, joint filing, income $17,000, deductions $2900? Show work__________

Math 102 Measurement Topic L1

Name______________________

Show work. Nearest hundredth. Give units.

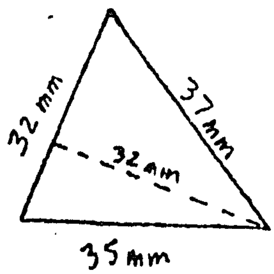
1) A right triangle is also isosceles with the length of the arms equal to 10 cm. What is the length of the hypotenuse? (Use a calculator)

____

2) The area of rectangle whose length is 15cm is 180 sq cm. What is the width of the rectangle?

____

3) Find the perimeter and the area of the triangle below.



____

____

4) Find the circumference and area of a circle whose radius is 12 inches.

____

____

5) A cone has a circular base with radius 10 cm. Its height is 24 cm. Find the volume.

____

6) The base of a prism is a square with side of length 6 feet. Its height is 10 feet. Find the volume.

____

7) Find the volume of a sphere with radius equal to 12 inches.

____

8) A pentagon has regular sides of 5 inches each. The apothem is 4 inches. Find the perimeter and area.

____

Homework and Suggested Topic Test Questions

Topic L2 Consumer and Business Mathematics

1) Mary N. Happy, who sells cosmetics, makes $50 a week plus 15% of her sales.

   A) If she sells $800 worth of cosmetics in a week, how much will she make in that week?

   B) Find 15% of 180.

2) A) A calculator, whose original price is $75, is being sold at 30% discount. What is the sale price of the calculator?

   B) What percent of 200 is 160?

3) A) 18% of what number is 160?

   B) If the price of a coat is raised from $75 to $90, what is the percent of increase?

4) A) Use the tables of compound interest in the text to find the total amount (including interest) in a savings account after 3 years at 6% per year, compounded quarterly, if $500 is the original amount in the account.

   B) Use the tables of compound interest in the text to find the amount you must put in a savings account now (present value) at 18% per year, compounded monthly, if you are to have $200 in 2 years.

5) How long would you need to leave $200 in a simple interest savings account at 8% per year in order to get an interest of $30?

6) You buy some furniture costing $1400. The store charges 10% addon interest. Find the total interest if you finance the furniture for 30 months.

APPENDIX E-17

Computer Programming

Math 102 Assignment for Writing

Each student will need to write a program of her own on the HP2000 in the BASIC language.

Requirements: The program should be original.
It should consist of at least ten steps.
It should involve at least one IF THEN and one GOTO statement.
It should involve at least one READ or INPUT statement and a PRINT statement.
It should involve at least three data entries.

Producing the program consists of four steps:

1) Experimenting with a program on the computer and consulting with the instructor.

2) Entering the program in final form into main storage. Checking to see that the program is entered correctly is done by typing LIST and then by typing RUN. Be sure your answers obtained are correct.

3) Once the program is entered in its correct, final form, it should be named and saved on magnetic tape. Please use your initials to name your program: type
NAME-(three letter name).
SAVE

4) Present a written copy of your program and the name you have given to the program to the instructor.

Examples of possible programs:

1) Evaluate a complicated formula.
2) Keep track of payments and deposits in a check book.
3) Solve a linear programming problem by testing many different ordered pairs.
4) Keep track of an inventory of three or four items in a store.
5) Take a set of numbers and arrange them in ascending order.
6) Find the mean of a set of grouped data--do other statistics.
7) Other

A successfully run program, approved by the instructor, satisfying the requirements above, will count 100 points, and will be equivalent to a test score on your lab grade.

APPENDIX E-18

Suggestions for Writing Your Conclusion

Each student should write her own report, indicating the results of the survey or project as she interprets them. The report should be concise but it should cover the following items.

1. What are you trying to find out? Why is the question important to you?

2. How was your sample chosen--with a table of random numbers, by handing out questionnaires to people you met, or by some other means?

3. What is the summary of your data? Use tables and charts (see below). Here you present means and modes, percentages, histograms, standard deviations, cost comparisons, graphs, etc., depending on which of these are appropriate.

4. What outside resources and references did you use?

5. What is your conclusion? How do the data support your conclusion? If you did not use a random sample, how much can you generalize your results to the entire "population" of people under discussion, i.e., how do you know that you didn't ask persons with just one point of view which would bias your results?

Example of a table:

The question "how much time, on the average, do you spend doing homework outside shhool?" This was asked of high school students in four categories: boys in a co-ed school, girls in a co-ed school, boys in an all-boys school and girls in an all-girls school.

	Coed boys		Boys single		Coed girls		Girls single	
Response	N	%	N	%	N	%	N	%
None or almost none	10	4.1	15	3.3	14	8.5	11	3.0
Less than ½ hr.	11	4.6	10	2.2	9	5.5	19	5.2
About ½ hr.	31	12.9	16	3.5	15	9.1	21	5.8
About 1 hr.	53	22.0	75	16.6	22	13.3	58	15.9
About 1½ hr.	43	17.8	86	19.0	33	20.0	55	15.1
About 2 hrs	55	22.8	167	36.9	49	29.7	145	39.8
3 hours or more	38	15.8	84	18.5	23	13.9	55	15.1
Totals	241		453		165		364	
Mean # of hrs	1.49		1.76		1.50		1.68	

(This example was taken from David R. Cook and N. Kenneth LaFleur, A Guide to Educational Research, second ed. (Boston: Allyn and Bacon, 1975), p. 65.

The conclusion made in this experiment was that significant differences were found with co-ed students, both boys and girls, reporting that they spent less time than students in the single-sex schools, in studying.

APPENDIX E-19

FINAL EXAMINATIONS

Both Classes

FINAL EXAM - COMPARISON SECTION

Name______________________________

Math 102-02

This test has two parts. The first will be done without notes, text, or calculator. When you have finished, bring your first two pages to the desk or raise your hand and receive the second part of the test. There are 110 points possible, so you will have choice or a chance for extra credit.

1 (12 points) True/False Circle T or F depending on whether the statement is true or is false.

T F A) (-2,3) satisfies the equation $2y = -4x + 2$.

T F B) (4,3) satisfies $2x - 3y \geqslant 7$

T F C) In linear programming, given a region R of points satisfying some constraints, the maximum of a profit formula $P = 3x + 4y$ will occur at one of the corner points of the region R.

T F D) Every vertical line crosses the graph of a function exactly once.

T F E) The graph of $5x = 7$ is a horizontal line.

T F F) The graph of $y = 2x^2 + 1$ is a parabola.

T F G) The graph of $3x^2 = 5 - 2y^2$ is a hyperbola.

T F H) The set of all points 3 units from a fixed point is a circle.

T F I) In a computer program PRINT X transfers data from main storage to output.

T F J) In a computer program SAVE transfers a program from auxiliary storage to main storage.

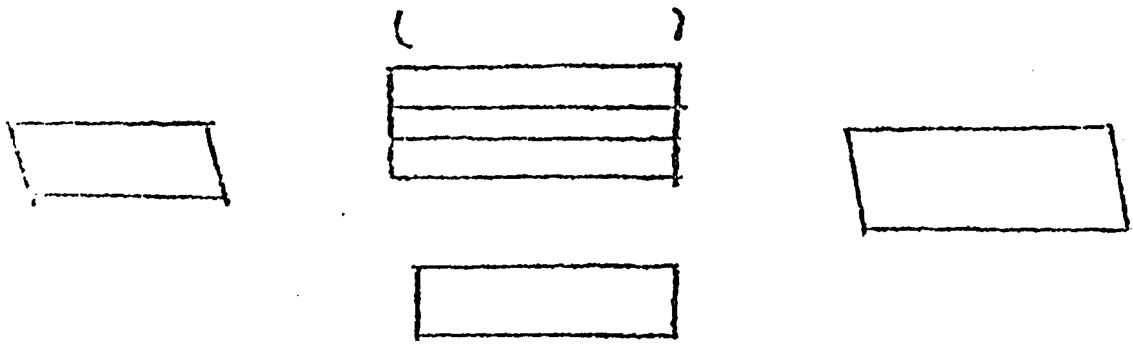
T F K) In a computer command, LIST transfers a program from main storage to output.

T F L) On some computers, a card reader is used as an input device.

FINAL EXAM - COMPARISON

2) Fill in the following chart which shows the parts and interrelationships between the parts of a computer.

(4 points)



3) (6 points)

A) Write 7-34E+10 in standard form

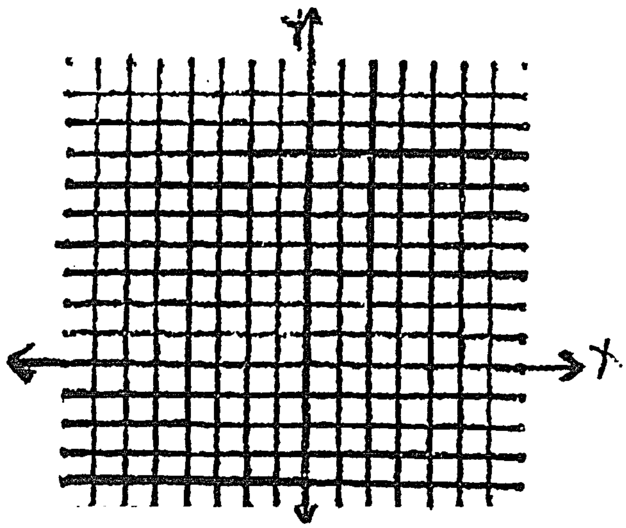
B) Change the binary number 10011001 to base ten form.

__________ __________

4) (8 points) Consider the equation $y = 2x^2-3$. Complete each ordered pair below so that it satisfies the equation. Then graph the equation.

(0,   )   (1,   )   (-2,   )   (2,   )   (3,   )

Show work for the above ordered pairs.



FINAL EXAM

EXPERIMENTAL AND COMPARISON

Math 102 Final Exam ____________________

TOPIC A STATISTICS

Part II: Put answers where they can easily be found at the right.

1) Find the mean, mode, and median of 24, 26, 23, 29, 23, 29, 20, 23, 28 (Show work)

(20 points) Mean____________ Mode__________

Median__________

2) Fill in the chart below to find the standard deviation of the numbers 8,8,10,14,16,16. Also find the range.

Score	Score-mean	$(\text{score-mean})^2$
8		
8		
10		
14		
16		
16		
	Sum	
	Sum/total	

Square root table

Number	Square root
9	3.00
10	3.16
11	3.32
12	3.46
13	3.61
14	3.74
15	3.87
16	4.00

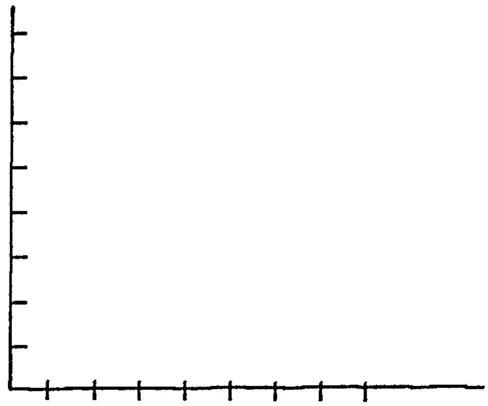
Range________________

Standard dev.

____

3) Draw a histogram for the following frequency distribution. Label your two axes.

Score	frequency
20	4
21	6
22	7
23	5
24	3



FINAL EXAM

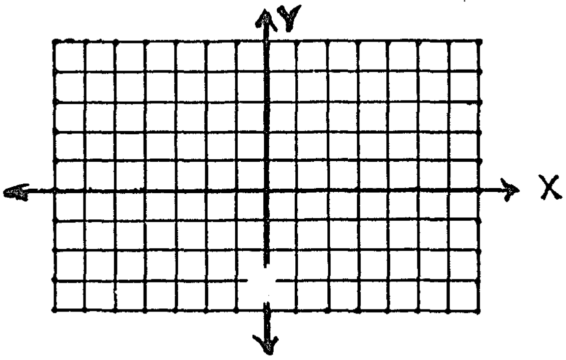
EXPERIMENTAL AND COMPARISON

TOPIC C - GRAPHING

Part III (10 points)

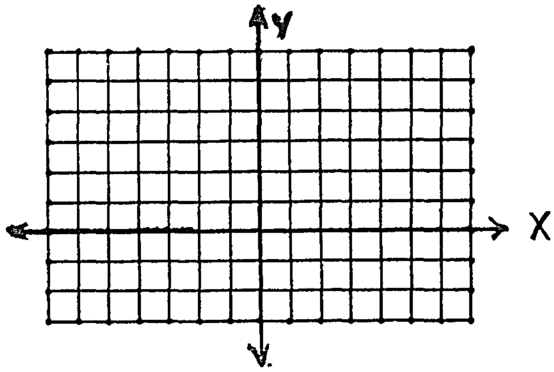
1) Graph the points by placing a dot and the capital letter in the appropriate place.

X (6,2)   Y(-3,2)   Z(0,-4)



2) Graph the set of all points which satisfy the equation 2x+3y = 12

x	0	
y		0



3) Find the equation of the line which goes through (4,-3) and (-2,5).

FINAL EXAM - COMPARISON

Math 102

TOPIC: NORMAL CURVE

OPEN BOOK You may use your text, notebook and calculator.

PART IV (15 points) 1) On a test the average grade was 75 with standard deviation of 7.

A) Draw the normal curve on the axes below and put in the scores and percentages in the appropriate places.

B) If the scores on this test are distributed approximately in a normal curve, find the percentages of the scores in each of the following intervals:

68-75__________ B) 82-96____________ C) Below 68____________

2) A manufacturing firm makes aluminum cans for Eoca Aloc and its quality control research shows that 90% of the cans made are satisfactory. The cans are sent in boxes of 400 each. (This is a binomial distribution with a probability of success equal to .9.) 1000 boxes (of 400 cans each) are sent, and the number of cans which are satisfactory in each box are counted. The 1000 scores are distributed approximately in a normal curve.

A) The number of satisfactory cans out of the 400 are counted. What is the mean? (show your computations)

____

B) What is the standard deviation?

____

C) What percentage of the scores would you expect to get more than 366 satisfactory cans in a batch of 400?

____

3) If scores on a scientific experiment are normally distributed with a mean of 100 and a standard deviation of 20, what percent of the scores will be between 100 and 118? Use tables in text.

____

FINAL EXAM - COMPARISON

Name________________________

102-02 p. 4

Part V 15 points (Open book)

1. Write the algebraic expression $s = 3y + x^3/5 + 2.5$ in the symbols of a computer program.

2. Step through the following program, completing what is printed out whenever line 50 is reached, as many times as the program passes through line 50.

```
 10  READ X,Y
 20  IF X=0 THEN 100
 30  LET Z = 4*X + Y
 40  IF Y<0 THEN 50
 45  LET Z = X∧2 - Y
 50  PRINT "X AND Y ARE"X,Y,"AND Z IS"Z
 60  GO TO 10
 80  DATA  4,7,4,-7,-2,3,0,0,2,-4
100  END
```

PART VI (15 POINTS) (Open book)

1) Graph the following linear inequality:

$y \geq 2x - 3$

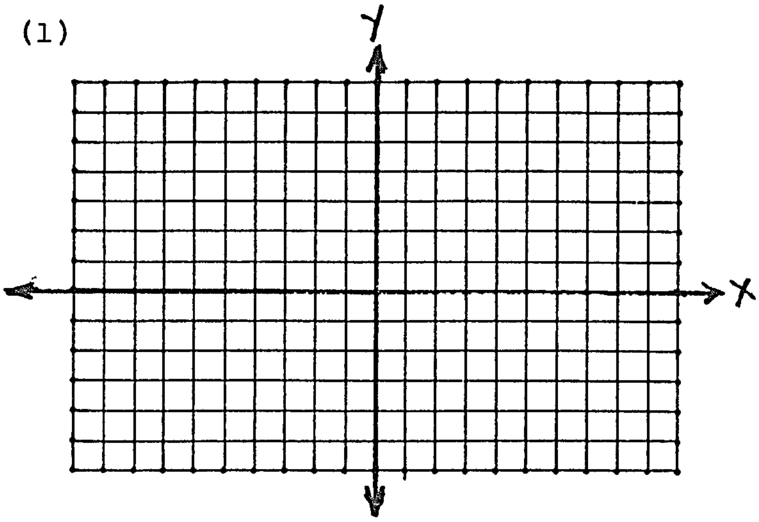
Show work for finding two points on the line $y = 2x-3$ first. Then draw the line, shade the region.

2) Graph the following: shade in the points satisfying the constraints.

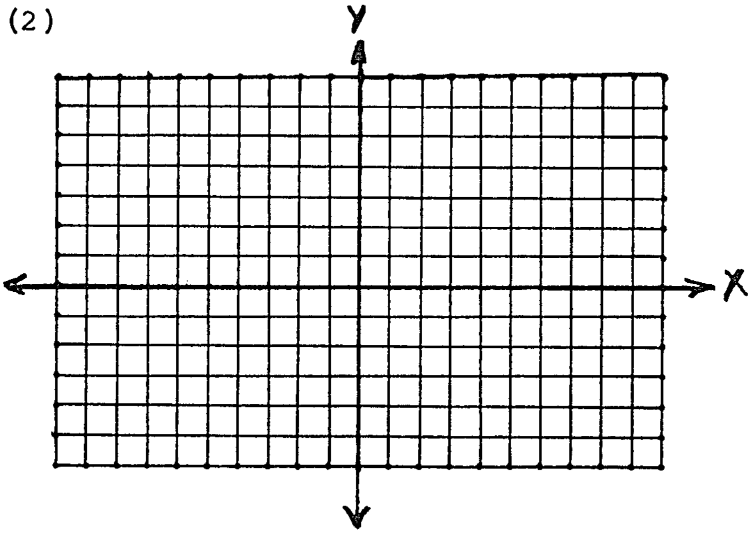
$y \leq 2x-3$
$x \leq 2$

(see following page for graphs)

(1)



(2)



FINAL EXAM

COMPARISON AND EXPERIMENTAL

General Questions

1) Suppose in a store you wish to find out which is the best buy: a dozen small eggs, a dozen medium eggs, a dozen large eggs, or a dozen extra large eggs. You have as much time as you need to investigate. How would you go about deciding what the best buy is?

FINAL EXAM

COMPARISON AND EXPERIMENTAL

2) A questionnaire is given to 50 people at random with the following results. A score of

1 is assigned to a choice of strongly agree,
2 is assigned to a choice of agree
3 is assigned to a choice of no opinion
4 is assigned to a choice of disagree
and 5 is assigned to a choice of strongly disagree.

The frequency distributions of the two questions are given:

Question 1: Boys do better in math than girls

	Score	frequency
Strongly agree	1	15
Agree	2	17
no opinion	3	3
disagree	4	10
strongly disagree	5	5

Question 2: Boys do better in English than girls.

	Score	frequency
Strongly agree	1	6
Agree	2	10
no opinion	3	10
disagree	4	15
strongly disagree	5	9

Find the mean for each question.

#1 _________ #2____________

How would you analyze the results of the survey as to the opinions of those questioned?

FINAL EXAM

EXPERIMENTAL SECTION

TOPIC TESTS: Choose four topics. Put answers on separate sheets of paper. Put your name on each sheet.

Topic A - Statistics (Experimental students were given the same questions as Part II of the Control section test)

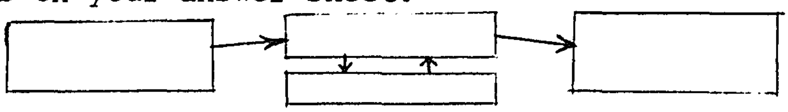
Topic B - Normal Curve (Experimental students were given the same questions as Part IV of the Control section test)

Topic C - Graphing (Experimental students were given the same questions as Part III of the Control section test)

COMPUTER: J

1) Write the word true or false:
   A) In a computer program PRINT X transfers data from main storage to output.
   B) After running a computer program, SAVE transfers a program from auxiliary storage to main storage.
   C) The computer command LIST transfers a program from main storage to output.
   d) On some computers a card reader is used as an input device.

2) Fill in the following chart which shows the parts and interrelationships between the parts of a computer. Do this on your answer sheet.



3) A) Write 7.34E+10 in standard form.

   B) Change the binary number 10011001 to base ten form.

   C) Change 75 to base two (binary) form.

4) Write the algebraic expression $x = 3y + x^3 / 5 + 2.5$ in the symbols of a computer program.

5) Step through the following program, completing what is printed out whenever line 50 is reached, as many times as the program passes through line 50.

```
 10  READ X,Y
 20  IF X=0 THEN 100
 30  LET Z = 4*X+Y
 40  IF Y < 0 THEN 50
 45  LET Z = X↑2 - Y
 50  PRINT "X AND Y ARE"X,Y, "Z IS" Z
 80  DATA  4,7,4,-7,-2,3,0,0,2,-4
100  END
```

PROBABILITY G

1) A bag contains 3 red, 4 yellow, and 2 blue balls.

   A) What is the probability of drawing 1 red ball?

   B) A ball is drawn, removed, and a second ball is drawn. What is the probability of drawing two red balls? Draw a tree and label the branches.

   C) A ball is drawn, removed, and second ball is drawn. What is the probability of drawing a yellow and a blue?

2) A toy set has 22 red pieces, 15 small pieces, and 8 small red pieces. There are 50 pieces in all. What is the probability of choosing a red OR a small piece if you choose a piece at random.

INEQUALITIES E

1) How many integers satisfy $-3 \leq x < 4$?
2) How many real numbers satisfy $-3 \leq x < 4$?
3) Identify the graph of $x \geq 3$ for x a real number? LINE, SEGMENT, RAY, HALFLINE)
4) Identify $-3 \leq x \leq 6$ as in question 3.
5) Identify the graph of $x < 2$ or $x > -5$.
6) Solve for x: $2x - 3 < 4$
7) Solve for x: $7x + 1 \geq -3x - 3$
8) Solve for x: $-3 \leq 2x + 1 \leq 5$
9) Solve for x: $-2x > 4$

FINAL EXAM

EXPERIMENTAL SECTION

TOPIC L1 LINEAR EQUATIONS

Find the solution for each of these equations. Write your answer in the corresponding blanks. For equations involving integral and fractional coefficients, your answer should be an integer or common fraction, written in REDUCED IMPROPER FORM. For equations involving decimal coefficients, your answer should be written as a decimal, to the nearest tenth. Show your work.

1) $-6x = 24$ ________ 2) $\frac{4}{5}x = 20$ ________

3) $7x - 14x = 7 - 17$ ________ 4) $\frac{4}{9}x - \frac{5}{18}x = \frac{1}{2} - \frac{7}{18}$ ________

5) $6x + 5 = 9x - 7$ ________ 6) $7x + \frac{3}{4} = 4x - \frac{7}{4}$ ________

7) $4(x+8) = 5(x+7)$ ________ 8) $7(x-14) = -63$ ________

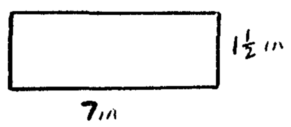
FINAL EXAM

EXPERIMENTAL SECTION

Topic L2 MEASUREMENT

Show work. Round answers to the nearest hundredth. Always include unit.

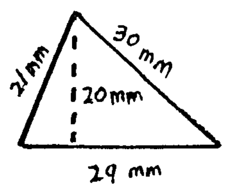
1) Find the perimeter and area of the rectangle below.



Perimeter____________

Area________________

2) Find the perimeter and area of the triangle below.



Perimeter____________

Area________________

3) Find the circumference and area of a circle whose radius is 5 cm. Use π = 3.24.

Area________________

Circumference__________

4) The area of a circle is 154 inches. Find the radius.
Use $\pi = \frac{22}{7}$

Radius_______________

5) A regular pentagon has sides of length 2.5 meters and apothem 4 meters. Find the total area of the pentagon.

____

6) The length of a box is 2 inches, its width is 1.5 inches, and its height is 4 inches. Find the volume.

____

7) The base of a square pyramid has an edge of 7 inches. The height of the pyramid is 15 inches. Find the volume.

____

8) The base of a triangular prism is a right triangle with arms (sides) 3 inches and 4 inches, and hypotenuse of 5 inches. The height of the prism is 8 inches. Find the volume.

____

9) Find the volume of a sphere whose radius is 6 cm.

____

APPENDIX F

Group on Crime
Preliminary Response

Part 1 Feb. 9, 1979

I feel that there are some changes in the questionnaire which are pretty important in order to make the results more meaningful. First, there are some misspelled words (fell instead of feel) which should be corrected. Second, you need a sentence or two of introduction explaining the purpose of the questionnaire and telling the respondents how to answer.

In question #1 you need to complete the sentence: "more prevalent in today's society than what? Put strongly agree and agree in order. Do you want the students to circle the correct answer; if so, give directions.

In order to have a more meaningful questionnaire, I suggest that questions 6,7,9,10 allow five possible answers: strongly agree, agree, undecided, disagree, and strongly disagree. Why not include these with #1 at the beginning, and give just one set of instructions?

Question #2: Do you expect the student to answer the question about feelings toward source of crime in her own words? If so, you should give more space.

In question #8, you have not left enough space for an answer. Perhaps it should be the last question, and you can direct students to answer on the reverse side.

Please note the importance in questions 1,6,7,9,10 of having five different answers with a scale of 1,2,3,4,&5. Yes/No questions don't tell you very much in complicated issues such as this. The respondent needs more latitude in her choices. Then your group can take the scores for each question separately and find the mean, median, mode, standard deviations, and histograms.

How do you expect to choose your sample of students? It must be random--each person in the total group must have an equal chance of being chosen.

How do you expect to get the questionnaires filled out?

APPENDIX F

Comments to Juvenile Crime Group
Final Response

Part 2 April 20, 1979

This is a thorough report. The questionnaire shows a lot of thought and the random sampling method is appropriate. Your presentation of the data could have been improved if all the data, frequency distribution, a table of percentages, and the mean for each of the questions were to have been put on a single page. For example:

Question	1 Strongly agree	2 Agree	3 Undecided	4 Disagree	5 Strongly disagree	Mean
1	23 54.8%	15 35.7%	3 7.1%	1 2.4%	0 0%	1.57
2	6 14.3%	3 7.1%	2 4.8%	36 85.7%	0 0%	3.85

Etc.

This chart, along with question in a compact form, all on one page, or perhaps two pages, would have made analyzing the data much easier. The work in computing the data is done correctly. The histograms are nicely done and give a good visual display. Your data from question 8 is categorical data and does not have any meaning when computing a mean. Simply find the mode.

The outside reading and reproduction of tables from the two experimental reports shows a good deal of work and attention to detail. The data show how statistics is used in surveys in political science. But there is no summary of the results, no statement of what the data show and I can not see any relationship to the questions on the questionnaire. I thought the purpose of the outside reading was to try to find data indicating the "real" answers to the questions on the questionnaire, so as to compare the opinions of the students at Bennett with what is "actually true." I don't believe that this was done at all.

The reports from the students in the group are interesting, thoughtful, and rather brief. I expected to see a more detailed analysis of the data from the questionnaire--to take the frequency distributions, means, and modes, and analyze what each result means in terms of the questions.

While a lot of work went into preparing the questionnaires, passing them out, and tallying the data, more work in saying what the data implied would have made this report more useful.

To make this a really outstanding report, data concerning your questions, rather than projects involving helping juvenile offenders, should have been part of the report.