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Though there is a growing knowledge base emerging on Mathematics Professional Development (MPD), little is known about the practice of facilitating teachers learning in MPD. The purpose of this study was to explore the nature of facilitating practice-focused MPD for elementary grades teachers. This study of the practice of facilitation provides greater understanding of the purposes of instructional decisions made by facilitators leading professional learning tasks (PLTs) aimed to assist teachers in learning and enacting a core practice of mathematics instruction.

In the context of a MPD project that lead to significant improvements in teachers' instructional practices, this explanatory case study design examined the instructional decisions of two MPD leaders as they facilitated PLTs designed to support teacher learning about a core practice of mathematics instruction. Qualitative and quantitative analyses of observation, field notes, interviews, and planning guides from the Summer Institute were conducted to understand the nature of MPD facilitation that supports teacher learning of a core practice.

Findings indicated that although facilitators use similar instructional moves as teachers in mathematics classrooms, the practice of facilitating practice-focused PLTs is fundamentally different than classroom instruction. Facilitators' instructional moves when representing the core practice were most similar in their purpose. In PLTs that

decomposed the practice and provided opportunities for teachers to approximate the practice, facilitation differed from classroom instruction in its focus and purpose.

Results suggest facilitating MPD requires extensive MKT, as well as knowledge of teachers' context, in order to foster the relationships needed to support teachers' learning of enacting core instructional practices. The study's outcomes have implications for leaders making decisions about MPD and teacher educators preparing facilitators to lead MPDs.

FACILITATION OF MATHEMATICS PROFESSIONAL DEVELOPMENT: A CASE STUDY OF SUPPORTING TEACHERS' LEARNING OF THE CORE PRACTICE OF LEADING MATHEMATICAL DISCUSSIONS

by

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Approved by

Peter Holt Wilson
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This research study is dedicated in loving memory of those whom I have lost during my pursuit of this degree. My father Melvin Saunders who taught me the value of hard work. Also in memory of my brother Ronnie and sister Michelle whose constant love and support helped me fulfil my life's dreams and aspirations.

I miss you all and I know you are looking down on me with pride.

APPROVAL PAGE

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CHAPTER I

INTRODUCTION

Participation in professional development lies at the heart of almost every effort to reform mathematics education in the past 20 years in this country (Borko, 2004; Cochran-Smith & Fries, 2005; Darling-Hammond, 1996, 2000). In the United States, these learning experiences are not only encouraged but are often mandated by state laws and regulations requiring teachers to complete renewal credits for their professional licenses (de Vries, Jansen, & van de Grift, 2013). Teachers also participate in professional development for other reasons, including learning how to implement new curriculum, earning graduate credit, or simply addressing personal learning goals. Whether by choice or obligation, professional development is a necessary reality for most teachers.

This importance of professional development in enhancing instruction, student learning, and school improvement has led to extensive research by the teacher education community (Borko, Jacobs, & Koellner, 2010; Desimone, 2009; Little, 2002; Wei, Andree, & Darling-Hammond, 2009). In mathematics, extensive research suggests that mathematics professional development (MPD) may serve as a bridge connecting teachers' previous experiences and knowledge to new ways of thinking about teaching and learning mathematics (Ball, Lubienski, & Mewborn, 2001; Battey & Franke, 2008; Borko, 2004; Cavanagh & Prescott, 2010; Garet, Porter, Desimone, Birman, & Yoon, 2001). Researchers have shown that MPD has the potential to impact teachers' beliefs, practice, knowledge, and their students' learning (Avalos, 2011; Birman, Desimone, Porter, & Garet, 2000; Borko, 2004; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Guskey, 2009). Both in the field of mathematics teacher education and the policy arena, there is little debate about the necessity for quality MPD that leads to teacher learning and student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Research on professional development has highlighted characteristics that support teacher learning and instructional change (Birman et al., 2000; Elmore, 2002; Garet et al., 2001; Desimone, 2009; Heck, Banilower, Weiss, & Rosenberg, 2008; Webster-Wright, 2009). This research base underscores that "effective" professional development has both structural characteristics and core features. Structural characteristics include the types of tasks, the duration of the activities, and the ways that teachers participate (Birman et al., 2000; Desimone, 2009; Garet et al., 2001). These characteristics, coupled with core features, such as opportunities for active learning, activities that are content focused, and coherence with other school and district-based initiatives support changes in teachers' classroom instructional practices (Birman, Desimone, Porter, & Garet, 2000).

Among the first to suggest a "consensus view" of effective professional development, Elmore (2002) claimed that the research community had well-established characteristics of "effective" professional development. He posited that such professional development must be focused on supporting student learning, be anchored in adult learning theory, include opportunities for learning for both teachers and school leaders, and be sustained over time to ensure continuous learning opportunities.

Desimone (2009) consolidated and framed these characteristics as core components of professional development, urging researchers to adopt a common framework to advance research on professional development. These core components of her framework include content focus, coherence, duration, active learning, and collective participation. More recently, Darling-Hammond et al. (2009) added to these lists by suggesting that effective professional development must be intensive, ongoing, and connected to practice while building strong working relationships among teachers.

Taking these characteristics as a basis for effective MPD, a recent study conducted by Garet et al. (2010) suggests that, though these components may be associated with PD effectiveness as established by the literature, there is no causal link between them and teacher learning and increased student achievement. They examined the impact of a well-planned MPD for middle school mathematics teachers in 77 schools across 12 school districts using multiple facilitators, investigating the program's effects on teachers' content knowledge, implementation of instructional strategy, and student achievement. Garet et al. (2010) found no statistically significant impact on teacher content knowledge or student achievement. Although the study did find that the MPD affected some teachers' instructional practices, the results support Guskey's (2009) conclusion that the research base for professional development planned and implemented using the "consensus view" of effective PD is modest at best. Together, Garet et al.'s results and Guskey's (2009) claim suggest additional insights are needed to understand effective professional development.

Implicit in many of these characteristics from the consensus view is the role of facilitation in MPD. Facilitation is a key component of effective MPD but has been under-researched and examined. For example, facilitating active learning experiences requires skilled instructional moves to ensure that all members of the learning community are challenged, reflective, and collaborative. Fostering coherence with larger initiatives requires facilitation that draws upon broader resources and knowledge of the comprehensive needs of the learning community to design and implement learning activities connecting the goals of the larger initiative to the learning goals of the MPD. Though some researchers have investigated the characteristics of and preparation of facilitators (Borko, Koellner, & Jacobs, 2014; Linder, 2011; Sztajn, Hackenberg, White, & Allexsaht-Snider, 2007), very little is known about the nature of facilitation that supports effective MPD (Elliott et al., 2009; Goldsmith, Doerr, & Lewis, 2013). The central claim of this dissertation study is that the nature of facilitation is not fully understood and could be a possible explanation for differential outcomes of MPD that adheres to the consensus view.

Statement of Research Problem

Effective MPD may lead to changes in teachers' beliefs, knowledge, and instructional practices (Bennison & Goos, 2010; Carpenter et al., 1989; Franke, Fennema, Carpenter, & Ansell, 1992; Goos & Geiger, 2010; Guskey, 2002; Walshaw, 2010; Zehetmeier & Krainer, 2011). Additionally, extensive research regarding the characteristics of effective MPD including context, content, duration, learning modality, and connection to school and community expectations, suggests that there may be other factors that have a role in the effectiveness of PD including facilitation (Birman et al., 2000; Elmore, 2002; Garet et al., 2001; Desimone, 2009; Guskey, 2002, 2009; Heck et al., 2008; Webster-Wright, 2009). The practice of facilitation, though instrumental in shaping the outcomes of MPD, remains an under-researched and under-defined area. Since the role of facilitation has remained implicit in most of the research base, an investigation examining the practice of facilitation is warranted. This dissertation seeks to explicitly examine the practice of facilitation of MPD in supporting substantive teacher learning.

Purpose and Significance of Study

The study is guided by the question: *What is the nature of facilitation of MPD that supports teacher learning of a core instructional practice?* Its significance lies in its understanding of the practice of facilitation. The study aims to examine the instructional decisions made by facilitators of an MPD and coordination of these decisions in supporting teachers in learning the core practice of Leading Mathematical Discussions (LMD). The study has the potential to affect the preparation of MPD facilitators and aims to reveal the ways in which facilitation is similar to, and different from mathematics teaching. Such understandings may allow MPD designers to develop and include facilitation resources to increase teacher learning. This study seeks to contribute to the field's discussion about facilitation and the instructional decisions made by facilitators that support teacher learning of core practices.

Research Context

To understand the practice of facilitation, this case study (Merriam, 2002; Stake, 1995; Yin, 2011) investigated the facilitation of a yearlong practice-focused MPD with a goal of supporting teachers' learning to lead mathematics discussions (LMD). The MPD, Core Math II, was designed according to recommendations and characteristics of "effective" professional development (Floyd, in preparation; Rich, in preparation). Core Math II offered teachers intensive and on-going learning experiences directly connected to classroom practice. Additionally, the project provided opportunities for collaboration and active learning focused on specific mathematics and classroom mathematics teaching. Teachers' mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008) was measured three times during the MPD using the Learning Mathematics for Teaching (LMT) instrument (Hill, Schilling, & Ball, 2004). Teachers' enactment of LMD in their classrooms was measured three times using the Instructional Quality Assessment instrument (IQA) (Crosson et al., 2006). Results of these measures indicate that the Core Math II project effectively supported teachers in changing their instructional practices to be more student-centered with productive mathematics discussions by supporting teacher learning of mathematics knowledge of teaching (P. H. Wilson & Downs, 2014). Thus, the Core Math II project represents an "effective" MPD and an appropriate context to investigate facilitation. Such an investigation will elucidate the practice of facilitation that support teacher learning in MPD.

Definition of Terms

The following terms and definitions are used within the context of this study. The researcher developed all definitions not accompanied by citation.

Effective Professional Development—Professional development that leads to substantive changes in teachers' beliefs, knowledge or instructional practices and includes characteristics of the "consensus view" (Birman et al., 2000; Elmore, 2002; Garet et al., 2001; Desimone, 2009; Guskey, 2002, 2009; Heck et al., 2008; Webster-Wright, 2009).

Professional development facilitator—Professional who supports the transformative learning for teachers in a manner, which is aligned to the consensus view of effective professional development and yields the desired outcome for teacher change in beliefs, practices or knowledge. They use professional learning strategies to address the differing learning needs of teachers (Loukes-Horsley, Love, Stiles, Mundrey, & Hewson, 2003).

Mathematics Discussions—Talk or conversations in classrooms that center on mathematical concepts (Cobb, Yackel, & McClain, 2000; Rowland, 1999). Such conversations support learning of mathematics both directly and indirectly as classroom Instructional centered on mathematics support the discussion, dissection and understanding of mathematical concepts (Chapin, O'Connor, & Anderson, 2003).

Mathematical Knowledge of Teaching (MKT)—"mathematical knowledge needed to perform recurrent tasks of teaching mathematics to students" (Ball et al., 2008, p. 399). This domain of knowledge includes teacher explanation of the concepts and interpreting student responses as they relate to these concepts, modeling both procedural and thinking of processes and concepts, and adequately using curricular resources (Hill, Rowan, & Ball, 2005). MKT intertwines several types of teacher knowledge.

Core Practices in Teaching—Identifiable components fundamental to teaching that teachers enact to support learning Core practices consists of strategies, routines, and moves that can be unpacked and 'learned' by teachers. Core practices include both general and content specific practices. Examples of core practices include Leading mathematic discussions (LMD), modeling, and providing instructional explanations (http://corepracticeconsortium.com/about).

Organization of the Dissertation

In Chapter I, I have presented the introduction, statement of the research problem, research questions, purpose and significance, research context, and definition of terms. Chapter II will review the research literature related to facilitation of MPD and mathematics teaching. After the review, I articulate an initial conceptual framework and refine my guiding question into three research questions. Chapter III outlines the methodology and procedures for data collection and analysis. In chapter IV, I present the findings from the study. Chapter V includes a discussion including recommendations for future studies.

CHAPTER II

LITERATURE REVIEW

The purpose of this study was to explore the nature of facilitating practice-focused mathematics professional development (MPD) for elementary grades teachers. More specifically, it sought to examine the purposeful instructional decisions made by facilitators that supports teacher learning of a core practice. In this chapter, I first present a review of literature on the facilitation of MPD. Because there is such void in research on the nature of facilitation of MPD, I then review related research on classroom mathematics instruction following Kazemi, Elliott, et al. (2009) argument that using research on mathematics classroom practices for MPD facilitation is warranted because of the overlap in the ways that the field views student engagement with mathematical concepts and teachers' engagement with mathematics in MPD environment. Finally, I present a conceptual framework informed by the literature review that guided this study. I conclude by refining my original research question in relation to this framework.

Professional Development Facilitation

There has been extensive research in the past two decades on the characteristics of high quality MPD. The consensus is that high quality MPD:

1. Provides learning opportunities for teachers to deepen MKT (Borko et al., 2010; Desimone, 2009, Kazemi & Franke, 2004; Wei et all., 2009);

2. Uses skillfully selected artifacts to support teacher learning such as student work samples, lesson design, and video cases (Jacobs, Borko, & Koellner, 2009; Sherin, 2007; van Es & Sherin, 2010);
3. Provides learning experiences that are collaborative and situated in practice (Hawley & Valli, 2000; S. M. Wilson & Berne, 1999); and
4. Provides opportunities for teachers to see instructional practices modeled (Clark, Jacobs, Pittman, & Borko, 2005; Stein, Engle, Smith, & Hughes, 2008).

Yet the nature of facilitation in providing such learning experiences for teachers is rarely extensively or explicitly examined in scholarly research. When implicitly identified in literature, facilitation of MPD is seen as either promoting or constraining teachers' learning in MPD. For instance, Weiss and Pasley (2006) suggest that facilitation plays a pivotal role in supporting MPD effectiveness, claiming effective facilitation is necessary if MPD is to be coherent, focused, sustainable, scalable, active collaboration, and delivered over time. Further, they claimed that ineffective facilitation (e.g., missed opportunities to support teachers in deepening both content and pedagogical knowledge) can negatively impact teacher learning. Scholars studying mathematics teacher learning have called for research to better understand the ways that facilitation shapes teacher learning. In a recent synthesis of research on mathematics teacher learning, for example, Goldsmith and her colleagues (2013) concluded, "we need to know more about . . . the nature of facilitation and the importance of the facilitator's role and expertise in promoting teacher learning is gaining interest in the research community" (p. 20).

Characteristics of Facilitators

Several of the existing studies of facilitation identify particular characteristics of facilitators that teachers perceive as important in making MPD meaningful to them. Sztajn et al. (2007) examined the factors that supported the development of trust among the facilitators and teachers in MPD. In their study, 27 elementary grades mathematics teachers were asked questions about the goals and effectiveness of the MPD, the types of support or care they received from the facilitators of the MPD, and the factors that they felt contributed or inhibited the collaborative learning environment. Their findings concluded that there were three major factors that helped to build trust within the learning community: professionalism of the facilitators, the structure and organization of the PD, and establishment of relations between the facilitators and the teachers.

Professionalism of the facilitators was exhibited through flexibility, generosity with time, and respect for the experiences that the teachers brought to the MPD (Sztajn et al., 2007). The teachers stated that the facilitators were responsive through email or phone calls, which also contributed to building trust within the PD. The facilitators' abilities to relate the mathematics teaching and learning to real classroom contexts, and not just theory, supported the building of trusting relationships. Teachers felt that the facilitators sharing of practical, ready to use resources in the MPD also contributed to the building of trust. Finally, the relationship that was built between the university mathematics instructors and the school outside of the PD environment also contributed to trusting relationships among the teachers and facilitators.

Wanting to understand more about the qualities that engage teachers through motivation for optimal teacher learning, Linder (2011) conducted a study to identify these qualities in facilitators of elementary MPD. Linder explored teachers' perceived characteristics of "influential" facilitators with a phonological study of 20 elementary mathematics teachers receiving both traditional content-based 'workshop' type professional development and transformative professional development. Her findings revealed five descriptors or characteristics of influential MPD facilitators. The data also concluded that all five descriptors credibility, support, motivation, management and personality must be present in order for a facilitator to be considered influential in impacting changes in teachers' classroom practices.

Teachers in Linder's (2011) study perceived facilitators to be credible if they held knowledge about the content, had similar teaching experiences, interacted in a professional manner, and provide some empirical evidence of the effectiveness of the new instructional practices. They described support in MPD as efforts by the facilitator to ensure that their learning related to changes in their practice. Support also included the manner in which the facilitators interact with the teachers. Facilitators who listen, treat participants as equals, value what the participants bring to the table and establish personal connection with the participants are considered to be supportive. Teachers believed that this support should be provided before, during, and after the MPD. Another theme that emerged was motivation. When facilitators are passionate about the content and have a sincere desire to impact the lives of students and teachers, it comes across in the MPD and has an impact on how the teachers perceive the facilitators. Other characteristics teachers found influential were the ability to make in-the-moment decisions to best meet the needs of the teachers, those that challenge teachers to think deeper or differently about the mathematics support them in being considered more influential, and personality traits such as a sense of humor, pleasant demeanor and disposition, friendliness, energy level, calm, and confident.

Summary. Both Linder (2011) and Sztajn et al. (2007) examined the role of facilitators by exploring the factors or characteristics that make them "influential" and that build trust within MPD. Both studies reveal the significance of facilitators' credibility in supporting teacher learning. Credible facilitators are knowledgeable, professional, and have relatable experiences that assist them in fostering a community of learners working toward improving their students' mathematics learning.

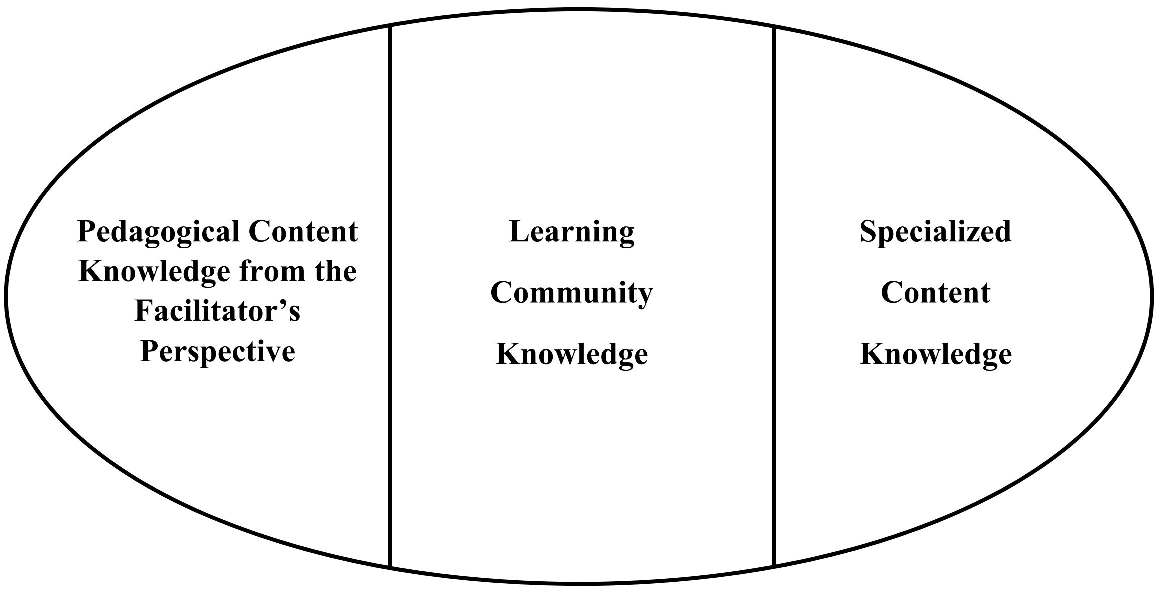
Learning to Facilitate MPD

Other existing studies of MPD facilitation address the ways in which mathematics teacher educators learn to facilitate MPD. Noting the absence of research on facilitation, Elliott et al. (2009) turned to literature on mathematics teaching and adapted frameworks from research on classroom instructional practices in their work investigating how facilitators learn to plan and lead mathematics discussions in MPD. They developed a framework for supporting facilitators that included establishing socio-mathematical norms (Yackel & Cobb, 1996) and a set of instructional practices for orchestrating mathematical discussions (Stein et al., 2008). The facilitators participated in a six-day seminar in which they practiced and learned how to establish socio-mathematical norms for discussing video cases to support teachers' mathematical reasoning. From collected data during the first two years a five-year professional development and research project, they found facilitators learned to (a) understand and effectively tackle teachers' varying mathematical understanding including misconceptions; (b) attend to these varying mathematical understandings in a manner that supports learning of all participants in the MPD; and (c) use mathematics problems that require teachers to focus on the big ideas rather than specific mathematical concepts. Additionally, they noted that the nature of the mathematics problem chosen had an impact on the type of knowledge the teachers would develop from engaging with the task. They recommended that facilitators should consider what knowledge they wanted teachers to develop and explicitly relay this with participating teachers. Their conclusions suggest that facilitators not only must develop deep mathematics knowledge for teaching (MKT) but also develop knowledge about how to facilitate the development of MKT for teachers.

Borko et al. (2014) provide a more recent and comprehensive look at how leaders can be supported in learning to facilitate MPD. They observed that there was an urgent need to prepare novice facilitators of MPD to meet the challenge of successfully facilitating in different settings using differing models of MPD in sustainable, scalable ways. Similar to Elliott et al. (2009), they looked at the types of supports needed by MPD facilitators and argued for characterizing the mathematical knowledge and skills required for effective facilitation. Their multi-year study examined three characteristics of facilitation: workshop culture, specialized content knowledge (SCK), and pedagogical content knowledge (PCK). Eight facilitators from six school districts participated in professional development on facilitating MPD, including a summer leadership academy, participation engaging in the MPD program as learners for one semester, and ongoing structured guidance by the research team. Using video recordings, artifacts, and interviews as data, they analyzed indicators of facilitators' SCK for the extent to which the facilitators were able to engage teachers in productive mathematical discussions and discussions regarding the MKT needed to support students in learning. They found that the novice facilitators rated high when looking at the how they supported teachers in generating and discussing varying ways of solving the learning task problem. An analysis of facilitators' use of PCK showed that they struggled to support teachers in deeply analyzing both student thinking and teacher instructional practices. These researchers concluded that some aspects of facilitation are more easily enacted than others. Establishing productive learning environments and using SCK to support teacher learning were less difficult for novice facilitator than using PCK to support teacher learning.

Borko and her colleagues (2014) used these findings to argue that the mathematics education community needs a way to conceptualize and discuss the types of mathematical knowledge needed for MPD facilitation. Extending the work of Ball et al. (2008) on MKT, they proposed a framework for Mathematics Knowledge for Professional Development (MKPD) (see Figure 1) comprised of three knowledge domains including SCK, PCK, and learning community (see Figure 1). They defined pedagogical content knowledge for facilitation as "the ability to engage teachers in the interpretation of students' mathematical ideas and purposeful analysis of instructional practices" (p. 165). Specialized content knowledge was defined as a

> deep understanding of the range of potential solution strategies and representations specific to a given mathematical context; the mathematical relationships within these sets of solution strategies and representations; the affordances and constraints of each strategy and representation. (p. 165)



Adapted from Borko, Koellner, & Jacobs (2014). Examining novice teacher leaders' facilitation of mathematics professional development. *Journal of Mathematical Behavior, 33,* 149–167.

Figure 1. MKPD Model.

Summary. These few existing studies of facilitation offer some characteristics of the nature of facilitation by illuminating some factors and characteristics that support teacher learning in MPD. Teachers from Linder (2011) and Sztajn et al.'s (2007) studies indicated that effective facilitators are credible, responsive, professional, knowledgeable about authentic classroom experience as well as content, and supportive. In outlining some of the challenges of learning to facilitate MPD, both Elliott et al. (2009) and Borko et al. (2014) underscored the importance of facilitators' MKT in supporting teacher learning. Though these studies represent initial attempts at understanding the nature of effective MPD facilitation, questions about the practice of facilitating remain. Following Elliot and colleagues (2009), I now examine research on classroom instructional practices.

Research on Classroom Instruction

Core Practices

In recent years, mathematics teacher educators have shifted looking at the knowledge that teachers have to specifying the instructional practices that teachers use to assist all students in learning mathematics (Ball & Forzani, 2009; McDonald, Kazemi, and Kavanagh, 2013)—instructional practices where teachers put their "knowledge to action" (Ball, & Forzani, 2009, Grossman, Hammerness, & McDonald, 2009; Lampert, 2009). Much of this work on characterizing the practices of teaching has been done in the arena of teacher education. For example, the work of the *Core Practice Consortium*, a collaboration effort from eight higher education institutions, focuses on addressing four problems of enactment: (a) there is a lack of common language to describe the work of teaching, (b) there is more to learn than time available in a teacher education program, (c) need to move from learning about teaching to learning how to teach, and (d) developing a more coherence within the teacher education community by establishing a common ground so that these entities can learn from one another (Core Practices Consortium, 2013, April). McDonald et al. (2013) argue that moving from research on teaching practices to actual enactment of these in the classroom requires that (a) there be a common language for practice, (b) identify common pedagogies, and (c) bridge the divide between preparatory coursework and practicum experiences (McDonald et al., 2013). Identifying these essential practices for successful teaching, educators are able to provide prospective teachers with learning opportunities that support their enactment in the classroom (Grossman et al., 2009; McDonald et al., 2013).

Identifying Core Practices of Mathematics Teaching

In their examination of three preparation programs for relational professions, Grossman et al. (2009) conclude that teacher education should move to decomposing the profession of teaching into core practices that are central to teaching and learnable by novices. Rather than suggest that there is one set of core practices or that they should be identified as competencies or techniques isolated from theory, their work focused on the development of a common understanding of what core practices and a common set of criteria for their specification. Their preliminary criteria for core practices are those practices that

- occur with high frequency in teaching;
- novices can enact in classrooms across different curricula or instructional approaches;
- novices can actually begin to master;
- allow novices to learn about students and about teaching;
- preserve the integrity and complexity of teaching; and
- are research-based and move the potential to improve student achievement. (p. 277)

In the mathematics education community, researchers have put forth a number sets of core practices. For example, the Teachingworks community (http://teachingworks.com/) from the University of Michigan list 21 high-leverage practices they suggest are the basic fundamentals of teaching. High-leverage practices are fundamental to skillful teaching and describe ways of engaging students in instructional tasks that promote learning. Some are focused on content while others include practices of ethics and equity. For example, high-leverage practices specific to content include leading whole class discussions, explaining content, posing questions about content, eliciting students' thinking, and monitoring student learning. Others are content-neutral, such as conducting a meeting with a parent or guardian or communicating about a student.

Another group, the Teacher Education by Design (TEDD) project (http://tedd.org/) also identifies a set of practices that are based on two guiding principles of *ambitious teaching* (Forzani, 2014; Kazemi, Franke, & Lampert, 2009; Lampert et al., 2013). First, ambitious teaching involves viewing students as competent individuals who are sense-makers. Second, ambitious teaching aims to provide equitable access to rigorous academic work for all students. The group works with six core practices for ambitious math instruction. These practices are:

- orienting students to each other's ideas and to the mathematical goal;
- eliciting and responding to student reasoning;
- setting and maintaining expectations for student participation;
- positioning students competently;
- teaching towards an instructional goal;

- assessing students' understanding; and
- using mathematical representations.

To address the need to support teachers in implementing the Common Core State Standards for Mathematics (CCSSM), the National Council of Teachers of Mathematics (2014) published *Principles to Actions*, where they specify eight mathematics teaching practices central to supporting students in learning mathematics with understanding. These eight practices represent a synthesis of research on mathematics teaching and represent yet another set of core practices of mathematics instruction and include facilitating meaningful mathematical discourse, posing purposeful questions, building procedural fluency from conceptual understanding, supporting productive struggle in learning mathematics, eliciting and using evidence of student thinking, establishing mathematics goals to focus learning, implementing tasks that promote reasoning and problem solving, and using and connecting mathematical representations.

Although these sets of practices vary in terms of the role of content, grain size, and definition, there is a consensus that leading mathematics discussions (LMD) is a core practice of mathematics teaching. Teacher-led mathematical discussions is one of three key features of classroom practices that support mathematical learning (Franke, Kazemi, & Battey, 2007). For the Core Practice Consortium, leading discussions involves "the teacher and all of the students work[ing] on specific content together using one another's ideas as resources . . . to build collective knowledge and capability in relation to specific instructional goals" (Grossman et al., 2014, webcast). Mathematics discussions help students to construct mathematical knowledge as well as support the deepening of mathematical understanding for all the students participating in these discussions (Franke et al., 2007; Franke et al., 2009; Hufferd-Ackles, Fuson, & Sherin, 2004). As students participate through sharing their mathematical thinking, teachers can elicit their reasoning, monitor their progress toward a learning goal, and make in-the-moment instructional decisions to support student learning. At the same time, all students benefit from these discussions by allowing others to make connections with the strategies to support their own mathematical thinking (Franke, Fennema, & Carpenter, 1997; Sfard & Kieran, 2001). In what follows, I briefly characterize and review research on the core practice of leading mathematics discussions.

Leading Mathematics Discussions

As a core practice of mathematics teaching, leading mathematics discussions (LMD) involves engaging students in mathematical discourse through questioning and other discourse moves. When leading discussions, teachers make a variety of contributions that engage students in collective mathematical reasoning through their questions, statements, and decisions about the direction and focus of the discussion (Chapin, O'Connor, & Anderson, 2009; Franke et al., 2007; Stein & Smith, 2011). In this section, I offer a broad overview of the research on discussions in mathematics classrooms to highlight the complexity of leading discussions that are instructionally productive.

Questioning. Questioning is a tool used by teachers to guide the direction for the mathematics discussions (Boaler & Brodie, 2004; Franke et al., 2009). Though some questions posed by teachers in discussions provide structure or manage non-mathematical aspects of students' contributions, high-level questions should deepen students' mathematical understandings by requiring them to reason and justify, synthesize, make inferences, or make connections among multiple concepts (Hiebert & Wearne, 1993). Another key function of posing questions when leading discussions is to elicit student thinking (Franke et al., 2009, Ghousseini, 2015). Eliciting questions encourage students to articulate their mathematical thinking aloud so that others can engage with, and respond to, their reasoning.

In the literature, there are numerous categorizations and terms for types of questions. For example, Franke et al. (2009) outlined three types of questioning techniques to support student learning: general, specific, and leading. General questions are those that are not connected to student responses or thinking but can generate a discussion around the mathematics content. These questions can be questions that invite students to participate or share their thinking such as, "Amy, would you like to share your strategy with the class?" Specific questions are those that are linked to students' responses and may be directed at one individual student or to several students. For example, probing questions are a type of specific questions that encourage students to elaborate on a contribution, clarify mathematical terminology or representations, or provide further explanation. Leading questions guide students toward a specific conclusion or explanation by including specific aspects of the problem upon which they want the students to focus.

Stein and Smith (2011) offer another categorization that extends an initial framework created by Boaler and Broadie (2004). They describe nine types of questions, including questions that explore mathematical relationships, probe student thinking, generate discussion, link and apply mathematical relationships, and extending mathematical thinking. For example, *generating questions* are questions that generate discussions that make mathematics available and understandable to other students. *Probing questions* are questions that ask students to explain their mathematical thinking in a manner that is clear and articulated precisely. Another question type is *linking and applying* questions where teachers go beyond asking students to share how they arrived at a solution to support students in making connections or recognizing relationships among differing strategies, solutions, or concepts. To advance students' mathematical understanding, teachers may ask *orienting and focusing questions* which helps students focus on key elements of the question which supports them in problem solving.

Establishing context questions, *exploring mathematical meaning questions*, *inserting technology*, *extending*, and *gathering information* questions types are also described as questions that support student learning.

Kazemi and Stipek (2001) describe yet another kind of question, those that press students to gain insight into the conceptual thinking of the students. Pressing questions require students to provide explanations that consist of mathematical arguments rather than procedural explanations for how they solved each problem. Teachers asking pressing questions encourage students to critically and thoughtfully analyze their solutions and strategies and those of their peers as all students are expected to participate in the solving of the problems and use mathematically based arguments and justifications.

As an integral part of the core practice of LMD, teachers pose questions to elicit or probe students about their mathematical thinking, press students to deeper levels of understanding for themselves as well as their peers, position students as competent mathematics thinkers, and support students in making generalizations about mathematical concepts (Fraivillig, Murphy, & Fuson, 1999; Lampert et al., 2013). Productive mathematical discussions require skillfully crafted questions to support student learning (Brodie, 2009; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Stein et al., 2008; Sherin, Jacobs, & Philipp, 2011)

Discourse moves. In addition to examining the questions mathematics teachers pose, researchers have also identified other discourse moves that teachers use when leading productive mathematical discussions, such as eliciting, orienting, and responding (Ghousseini, 2015). Chapin et al. (2009) describe five discourse moves that promote productive mathematical discussions in the classroom: revoicing, repeating, reasoning, adding on, and wait time. *Revoicing* moves clarify what a student has said, highlight ideas stated by a student, or ensure that all students hear what is said. *Repeating* moves restate a student's contribution to highlight or *mark* important statements made by students. Moves that promote *reasoning* allow students to engage with others' thinking and understanding and is often accomplished by asking students if they agree or disagree with a particular idea and why. *Adding on* moves elaborate or extend a particular contribution. Finally, allowing students to think is, according to Cazden and Beck (2003), a powerful instructional move that supports the deepening of the discussions in the classroom. Research on wait time suggests that increased wait time can actually increase student engagement in the mathematical discussion (Rowe, 1986).

Similarly, Ghousseini (2009) refers to these discourse moves as discourse routines. In addition to revoicing, she describes *orienting* as a move that directs students to another's idea, *pressing* as a move to encourage students to explain their reasoning, *connecting* moves that link students' ideas with the big idea of the lesson, and moves that *make the structure* of *the mathematical discussion visible* for all the participants is a necessary discourse routine for supporting student participation and learning.

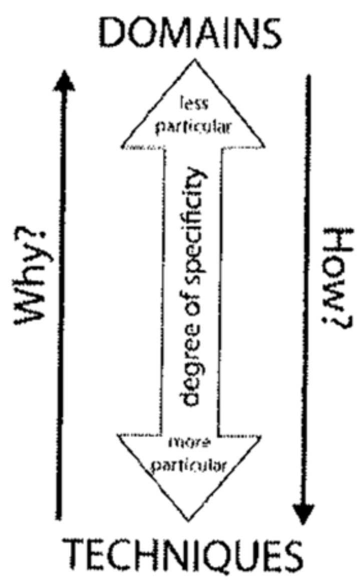
As a different grain size, Stein and Smith (2011) offered anticipating, monitoring, selecting, sequencing, and connecting as a way for teachers to structure preparing for and leading a mathematics discussion. To prepare, *anticipating* involved teachers making predictions about how students will think and solve mathematical problem, including students' approaches to problem solving, interpretations of the task, and strategies likely to be used. *Monitoring* involves attending to students' thinking while students engage in a learning task and includes interacting with students to deepen their understanding and preparing for the discussion. *Selecting* involves determining student solutions or approaches to share in the discussion, and *sequencing* refers to the order in which the solutions are to be shared. *Connecting* describes the actual mathematics discussion, where teachers use various questions, discourse moves, and selected students' ideas to relate the approaches and meet their lesson goal.

Summary of leading mathematical discussions. This review examined different techniques and discourse moves that teachers use when leading productive mathematics discussions in classrooms. Though variations in terminology and grain size exist across researchers, there are commonalities across. For example, Chapin et al.'s (2009) revoicing and repeating moves are similar to Ghousseini's revoicing in that both share a rationale of marking significant contributions to discussions. Similarly, Franke et al. (2009) and Stein and Smith (2011) both include probing as a means of having students externalize their reasoning. Following Elliot et al. (2009), this study draws from these various discourse moves and questioning techniques to conceptualize the practice of facilitating MPD.

Conceptual Framework

Research on facilitating MPD is limited to identifying characteristics of facilitators and descriptions of the types of knowledge they use. However, research on teaching provides insight into the various instructional moves that one might make when orchestrating learning in discussions with learners. In this section, I draw from this literature and a recent conceptualization of core practices from the teacher education literature to build a framework for the practice of facilitating MPD.

Some researchers in teacher education conceptualize core practices as nested levels of varying grain sizes. For example, Boerst, Sleep, Ball, and Bass (2011) elaborate on Grossman et al.'s (2009) decomposition of teaching by combining the two ways that instructional practice has traditionally been treated in teacher education. One way is to decompose large domains of teaching practice, such as fostering a community of learners, establishing and maintaining norms, and planning for engaging instruction. One problem with a domain approach is that it is difficult for pre-service teachers to learn without significant scaffolding with smaller techniques. Thus, a second approach is to decompose as smaller techniques, such as how to use manipulatives, using the classroom technology, and procedural aspects of teaching. Yet with a technique approach, pre-service teachers leave programs of teacher education with a smaller set of skills that may or may not be usable in their classroom. As a potential solution to the problems of these two approaches, Boerst and his colleagues (2011) proposed a nested approach with intermediate subsets of a particular practice nested within a domain and techniques nested within the intermediate subsets (see Figure 2). For example, when working with pre-service teachers around a domain of practice such as LMD, one might begin with a technique such as eliciting student thinking followed by a more intermediate level of practice such as responding to student thinking.



Source: Boerst et al. (2011), p. 2855.

Figure 2. Continuum of Grain Size Practices.

Another approach for conceptualizing core practices was recently proposed by Janssen, Grossman, and Westbroek (2015). Borrowing from work with complex systems from other fields such as biology and organizational theory, they propose practice can be taken as a hierarchical modular system. Modular systems have a structure where subsystems with components can be identified. Within those subsystems, there may be smaller subsystems with components. Relationships across these levels of the system describe the effects of a subsystem on other subsystems and the system itself. Such systems have a property the relations within a particular subsystem are stronger than those across with other levels of the system.

Using Jansen et al.'s (2015) approach, one could conceptualize mathematics teaching as a hierarchical modular system comprised of two levels of subsystems (see Table 1). The first, the level of practice, is composed of core practices of teaching that act as subsystems of teaching, such as launching a task or closing a lesson. The second level, the level of decision, is comprised of instructional moves, such as posing a probing question or inviting a student to share. In teaching, instructional moves perform a particular function to achieve one or more specific goals. For example, a teacher might orient a particular student to another student's idea to connect a particular mathematical idea to another representation of that idea. Thus, an instructional move with its associated goal or goals represents an instructional decision. Because subsystems at lower levels perform the overall function of higher level systems, an instructional practice is a system of instructional decisions coordinated for a particular purpose. Thus, the level of instructional practice can be thought of as a decomposition of teaching, and the level of instructional decision as a decomposition of instructional practice.

Table 1

Facilitation as a Hierarchical Modular System

System		Teaching		Facilitation	
Level I Subsystems	Level of Practice	Core Practices	Purposes	Facilitation Practices	Purposes
Level II Subsystems	Level of Decision	Instructional Moves	Goals	Instructional Moves	Goals

Because of the similarities of the practice of facilitation with mathematics teaching, facilitation can be taken as analogous to teaching and thus conceptualized as a hierarchical modular system with two levels, the practice level and the decision level. Yet research on MPD facilitation has not addressed the practices of facilitation, the purposes of these practices, nor the decisions facilitators make when enacting these practices. Therefore, guided by a review of the literature on mathematics discussions and following the lead of other researchers, this conceptual framework takes seven instructional moves identified from research on teaching to begin to conceptualize the practices of facilitating MPD: marking, eliciting, explaining, pressing, instructing, orienting, and inviting moves.

Revised Research Questions

The guiding question for this study addresses the nature of MPD facilitation that supports teachers in learning a core practice. Informed by a review of research on mathematics professional development, teacher education, and mathematics teaching and using my conceptual framework for the practices of facilitation, I refined my overarching question into three research questions:

1. What instructional decisions do facilitators make when implementing practice-focused professional learning tasks during MPD? Specifically:
    a. What are the instructional moves that facilitators make when implementing practice-focused MPD?
    b. What are their goals for these moves?
    c. What is the relation between facilitators' instructional moves and goals?
2. Did facilitators' instructional decisions vary when implementing different practice-focused professional learning tasks? If so, in what ways do they vary?
3. When implementing practice-focused professional learning tasks, for what purposes do facilitators coordinate their instructional decisions?

CHAPTER III

METHODOLOGY

The purpose of this study was to explore the nature of facilitating practice-focused mathematics professional development (MPD) for elementary grades teachers. More specifically, this study seeks to examine the purposeful instructional decisions made by facilitators that supports teacher learning of a core practice. Because of the complex nature of the role of facilitation, a qualitative approach was warranted to provide the rich, holistic data, which helped to illuminate the practices of effective facilitation (Miles & Huberman, 1994). There are few existing studies related to examining the nature of facilitation of MPD, which makes this case revelatory (Yin, 2013) and makes the use of qualitative case study method appropriate for exploring this phenomenon. The explanatory case study on the nature of facilitation of MPD was used in an attempt to answer the following research questions:

1. What instructional decisions do facilitators make when implementing practice-focused professional learning tasks during MPD? Specifically:
    a. What are the instructional moves that facilitators make when implementing practice-focused MPD?
    b. What are their goals for these moves?
    c. What is the relation between facilitators' instructional moves and goals?

2. Did facilitators' instructional decisions vary when implementing different practice-focused professional learning tasks? If so, in what ways do they vary?
3. When implementing practice-focused professional learning tasks, for what purposes do facilitators coordinate their instructional decisions?

In this chapter, I provide an explanation for the research design, methods of collection, and analysis of data that were used for this study. First, the context of this single case study is described in detail. Next, I describe and justify the use of case study design as the best approach for answering the research questions for this study. Third, I describe the data sources, collection, and analysis procedures that were used in this study. Finally, I will provide a discussion of reliability and validity issues.

Theoretical Perspectives

The theoretical framework in which this research study is grounded is the situated learning theory (Lave, 1988; and Lave & Wenger, 1991), which posits that learning is contextual or situational as we are learning from our situations rather than through psychological mental processes of individuals (Anderson, Reder, & Simon, 1996; Cobb, 1994). Wenger, McDermott, and Snyder (2002) define communities of practice (CoP) as "groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis" (p. 4). For this study, learning was understood to involve participation in a (CoP). Teachers learn through their participation in the MPD community and this learning is reflected in changes in instructional practices in their classrooms. Through participation in the MPD, teachers have the opportunity to negotiate the meaning of LMD with other participants through active sharing, discussing, observing, reflecting and practicing.

MPD is conceptualized as boundary encounter between the teaching and mathematics teacher education communities (Wenger, 1998). Both teachers and facilitators serve as brokers and bring elements of their practice to MPD. Wenger (1998) contends that one complication in brokering is becoming full members of the community or being rejected because of lack of credibility: "Brokering therefore requires an ability to manage carefully the coexistence of membership …yielding enough distance to bring a different perspective but also enough legitimacy to be listened to" (Wenger, 1998, p. 110). Since the facilitators of the MPD are brokers in this boundary encounter, it is imperative that they bring credibility and extensive knowledge in order to be viewed as legitimate. Through participation in the MPD, the teachers and facilitators negotiate meanings of their understandings of the practice of LMD.

Context for the Study

This case study was part of a larger study of professional development in which the two facilitators of the MPD also served as researchers. The larger context of the study included professional development provided for two schools through a series of funded professional development projects. Two local school districts partnered with a local university for the project. The two participating schools were high-needs schools located in a rural county in the Southeastern United States. At the time of the study, Hillside Elementary School (all names are pseudonyms), a Title I school with 87% of the students classified as low-income, had only 30% of students considered proficient in mathematics, according to state summative assessments. The school did not meet federal requirements for No Child Left Behind in mathematics in 2011–2012. McDonald Elementary School, a Title I school in a non-high-needs district, was designated as a “school in need” by state requirements. Just over 90% of McDonald’s students were economically disadvantaged, and only 34% of students were considered proficient in mathematics, as determined by state assessments.

The first project, Core Math I, took place from the summer of 2011 through the summer of 2012. The emphasis of this first project was to support teachers in implementing the Common Core State Standards for Mathematics (CCSSM) using learning trajectories that are the foundation for the standards (Daro, Mosher, & Corcoran, 2011). Core Math I was funded by three awards from the ESEA Title II-A Improving Teacher Quality Grants program. The second project, Core Math II, originated from the identified needs of teachers to engage in sustained and focused learning opportunities to learn how to lead mathematics discussions in their classrooms. Specifically, the goal of the MPD within the context of the second project was to support teachers in learning the practice of leading mathematics discussions (LMD). This project took place from summer 2013 through summer 2014. There were 15 participants, all of whom also participated in the Core Math I project.

Core Math II Design

Core Math II was 108 hours of MPD during the 2013-2014 school year and was comprised of three phases each with the same goal of supporting teachers in learning about promoting productive discussions where the teacher and students orally contribute, actively listen, and consistently respond to and learn from one another's contributions. To effectively lead discussions, a teacher poses questions or tasks that allow students to share their thinking about specific mathematical concepts in order to surface ideas that will benefit other students.

The overall goal for the Core Math II MPD was to support teachers as they learned and implemented the core practice of LMD. The facilitators conceptualized the core practice of LMD by categorizing instructional moves, and goals for this these discussions. They defined an instructional move as an observable action taken by teachers, like probing or pressing student thinking. Goals were defined as the reason for making the instructional move.

The facilitators of Core Math II unpacked the practice of LMD into two broad stages- preparing and enacting. The preparation stage occurs when teachers first launch a high-demand task and allow students to explore the task individually or as a small group. During this time, teachers prepare for classroom discussion by anticipating, monitoring, selecting, and sequencing students' responses. The enacting stage is when students are sharing their responses and teachers support them in making connections with the differing strategies and solutions. Talk moves such as linking, re-voicing, pressing, and probing to achieve the mathematical goal of the lesson are used during both stages of LMD.

To provide the participating teachers with the support needed to learn the core practice of LMD, a multi-phase professional development model was designed. Phase 1 included traditional MPD experiences provided within a one week-long Summer

Institute, during which participants learned and implemented the core practice of LMD. During Phase 2, teachers worked with groups of students in an after-school setting as observers of the core practice of LMD. The sessions in Phase 2 were formal enough to support explicit learning of the core practice of LMD, but informal enough to allow for honest conversations among the participants about their challenges to their own classroom practices. During Phase 3, the facilitators provided in-the-moment support and feedback to teachers as they enacted the core practice of LMD in the authentic setting of their own classrooms.

During Phase 1, the context for this study, the leaders of the schools (administrators, lead teachers and coaches) also participated in whole group practice-based sessions, both with the teachers and in a smaller community of just leaders. The teachers and school leaders engaged in a sequence of professional learning tasks as learners of the core practice. Goals of the Summer Institute included making the core practice of LMD public, fostering value for the core practice, breaking-down or dissecting the core practice, and experimenting with the core practice. This goal was achieved through engagement with three distinct professional learning tasks (PLTs). Representing, Decomposing, and Approximating PLTs were designed based on the pedagogies of investigation work of Grossman and colleagues (Grossman et al., 2009).

Representing PLTs include the different ways the work of practitioners is made visible to novices; through stories, cases, videos, and/or artifacts from practice. One of the challenges is to know what to look for and how to interpret what is observed (Grossman, 2011). An important design aspect for representing practice is to determine what facets of the core practice of LMD is made visible to be seen and learned and what facets remain invisible.

Decomposing PLTs compose of breaking down complex practice into its parts for the purposes of teaching and learning. Decomposing practice allows novices to focus on an essential component of the work of teaching so it can be seen and enacted it more effectively; however, the ability to decompose practice depends on a common language and structure for describing practice, also called a “grammar of practice” (Grossman, 2011, p. 2839). Decomposing practice allows learners the opportunity to concentrate on enacting a set of moves or strategies of a complex practice because it has been explicitly decomposed to learners.

Approximating PLTs give teachers opportunities to practice the art of LMD on one another and to receive feedback on their progress. Approximations may consist of role-plays or types of simulation activities that allow opportunities for experimentation with new practices in easier conditions, often with instructors simplifying the demands of the work. Enacting PLTs provide teachers the opportunity to practice on students in authentic settings as they receive specific and targeted feedback.

Teachers and leaders participated in Representing PLTs, Decomposing PLTs, and Approximating PLTs during the week- long Summer Institute. During the Representing PLTs, teachers were able to experience the core practice of LMD as a learner by reflecting on the role that mathematical discussions have on their own mathematical thinking. Teachers were then provided opportunities to breakdown, examine, and discuss the core practice of LMD during the Decomposing PLTs. Finally, teachers were provided opportunities for teachers to experiment with the core practice of LMD during the Approximating PLTs.

Using a subset of the Instructional Quality Assessment (IQA) (Junker et al., 2005), the effectiveness of Core Math II was analyzed by the projects' research team. The results indicated that the Core Math II MPD was effective in supporting teachers in adapting the core practice of LMD. The research team assessed the enactment of the core practice of LMD in teachers' classroom instruction. Pre and post video recordings of teachers' mathematics instruction were assessed using the IQA assessment tool. Statistically significant ($0.00 \leq p \leq 0.09$) gains and moderate to strong effect sizes ($0.64 \leq d \leq 1.21$) in seven of the nine dimensions of the IQA indicated teachers learned about LMD and were able to implement the core practice in their classroom. Participants also completed the University of Michigan's Learning Mathematics for Teaching (LMT) (Hill & Ball, 2004) to measure gain in Mathematics Knowledge for Teaching (MKT). The statistical significance ($p < 0.01$) and the moderate effect size of the gains ($d = 0.595$) demonstrated that teachers' MKT improved as a result of their participation in the MPD (Floyd, forthcoming, Rich, Forthcoming). These findings empirically suggest that Core Math II effective in supporting teachers' learning of the core practice of LMD and thus serves as an appropriate context for examining the nature of facilitating problem-based MPD.

Research Design

Yin (2013) suggests that case studies are warranted when the researcher has little control over the events and when the focus of the study is a contemporary phenomenon with some real-life context (p. 1). He defines case study as comprising two parts with the first part addressing the scope of the design and the second part addressing the features. Scope according to Yin (2013) involves the empirical inquiry that investigates the phenomenon in a real world context where the boundaries between the context and the case are not clearly defined. The scope of this case study is the facilitation of an effective practice-based MPD. He goes on to describe the features of a case study as dealing with more variables than data points, using multiple data sources that can be triangulated, and using theoretical propositions to guide the data collection and analysis. For this particular study, I examined the instructional decisions made by the facilitators of the MPD. Although there are other variables that support teachers learning, only the facilitators' instructional decisions are examined.

Case Study Justification

According to Yin (2013), a case study design should be used when there is a need to explain how or why a particular phenomenon occurs and when the research questions require in-depth or deep descriptions of the phenomenon. This study used a single-case explanatory study design as it sought to develop a pertinent hypothesis regarding the nature of facilitation of practice-focused MPD for further inquiry (Yin 2013, p. 4).

This study investigated the nature of facilitation of practice-focused MPD and defined the case as the facilitation of the MPD. The practice of facilitation cannot be separated from the MPD in which it is situated; therefore, the context and topic of the study reflects one of the main features of case study. Additionally, the case study design was chosen because case studies are useful for understanding these types of complex situation. They allow for examination through intensive description. For this study, deeper investigation of the interactions between the facilitators and the teachers were achieved (Miles & Huberman, 1994; Stake, 2010; Yin, 2011, 2013).

According to Dede (2006), most research on the effectiveness of MPD relies on teacher self-reporting as the sole evidence to substantiate claims of effectiveness. The fact that the Core Math II MPD used for this study has additional empirical data to support its claims of effectiveness makes this particular context a "unique" case, which warrants a deeper examination of this phenomenon (Yin, 2013).

Definition of the Case

In order to study expert facilitation, this study used a single case study design. The instructional decisions of two facilitators, Marie and Deneen, served as the case of expert facilitation. In what follows, I provide descriptions of the experiences of the individual facilitators as well as a description of their collaborative efforts in facilitating MPD.

Deneen

At the time of the study, Deneen served as the Director for Elementary Education for Oaktown City Schools. She has worked in this capacity for two years. She was the facilitator and co-designer of the Core Math II Project. Deneen began her path as a mathematics professional developer within the first three years of her teaching career. During that time, Deneen was a fifth-grade teacher and was provided the opportunity to participate in a statewide project called Teach Stat, a three-year intensive professional development experience. She was given feedback on her classroom implementation of the strategies and content taught during the MPD. She was then chosen to be one of the teachers who facilitated workshops and MPD across the state to other teachers. She then went on to support mathematics curriculum with the Department of Public Instruction for the state and became the lead teacher for mathematics in Maple County. Deneen earned her Masters of Education in Curriculum and instruction (2003), earned an Elementary Mathematics Specialist License (2011), and was currently pursuing a PhD in Mathematics Education.

Deneen was a successful classroom teacher who enjoyed teaching and was conscientious about the learning experiences that she gave her students. She was chosen to be Maple County Teacher of the Year. Deneen's position as lead teacher allowed her the opportunity to observe classroom instruction throughout the district. This experience solidified her belief in the need for MPD, as she assumed that all teachers were doing the same thing in their classrooms that she was doing in her classroom.

Marie

Marie served as Lead Teacher for Mathematics and Science for Maple County Schools. She started her career as a fourth grade teacher and during her second year of teaching, Deneen, who was working as her Lead Teacher, recommended Marie the same statewide project called Teach Stat program in which she had previously attended. Following this experience, she participated in a three-year project called Team 2 Teaching Excellence in Mathematics, which was designed to identify and groom young leaders in the mathematics education community. Marie became involved in curriculum development with the Department of Public Instruction for the state. Because of the implementation of CCSSM standards, MPD was necessary to support teachers across the state in the implementation of these new standards. In her fifth year of teaching, Marie was given the opportunity to travel throughout the state supporting teachers in learning the new state-adopted standards. Marie earned her Masters of Education in Curriculum and instruction (2003), earned Elementary Mathematics Specialist License (2011) and is currently pursuing a PhD in Mathematics Education as she wants to learn more about MPD and why some teachers go back and implement what they learn while others do not.

Marie and Deneen's Collaboration

Marie and Deneen have over 15 years of experience in co-facilitation of mathematics professional development for a variety of school and district communities throughout the state. Together, they have facilitated MPD on assessment, mathematics instructional strategies, implementation of CCSSM, and use of hands-on materials for conceptual understanding of mathematics. They have also worked extensively with the state department to write standardized assessment questions, and to review and revise the mathematics curriculum. At the university level, both have served as evaluators for the Elementary Mathematics Add-on Licensure as well as teaching online classes for this program.

Data and Analysis

In this section, I describe in detail the data sources that were used for this study to address my research questions. Following this description, I provide a detailed description about the data analysis process utilized to reduce the data and draw conclusions (Miles & Huberman, 1994).

Data Sources

One of the major principles of case study design is the use of multiple sources of evidence, including evidence of documentation, archival records, interviews, direct observation, participant observation, and physical artifacts (Yin, 2013). For this study, primary data sources consisted of observations of Representing, Decomposing, and Approximating PLTs from the Core Math II Summer Institute and field notes. Secondary data sources included interviews, and artifacts that supported the triangulation process such as recordings of planning sessions, planning documents, and agendas. See Table 2 for the data sources used for each research question.

Observations. The primary sources of data for this case study were video recordings of Deneen and Marie's facilitation of practice-focused PLTs with accompanying field notes from the Summer Institute. These observations were necessary to examine the instructional moves of expert facilitation of MPD, infer and member-check goals for these moves, in order to identify and describe the instructional decisions made to support teacher learning. These videos were transcribed.

Interviews. Supplementary data sources included interviews conducted with Deneen and Marie. Three interviews occurred during the implementation of the MPD. All interviews were audio-recorded with the researcher taking notes. Three interview protocols were developed in consultation with another researcher in preparation for this study. These interviews occurred while Marie and Deneen implemented the MPD. The interviews were semi-structured and included prepared questions but allowed the interviewer to pose follow up questions.

Table 2

Data Crosswalk

Research Questions	**Data Source #1** ***Videos and field notes of episodes from MPD***	**Data Source #2** ***Facilitator interviews***	**Data Source #3 Secondary data sources:** ***Artifacts***
What is the nature of the instructional decisions that facilitators make when implementing practice-focused PLTs? Specifically:			
a) What are the instructional moves that facilitators make when implementing practice-focused PLTs?	**X**	**X**	**X**
b) What are their goals for these moves?	**X**	**X**	**X**
c) What is the relation between facilitators' instructional moves and goals?	**X**	**X**	**X**
Did facilitators' instructional decisions vary when implementing different practice-focused professional learning tasks? If so, in what ways do they vary?	**X**	**X**	
When implementing practice-focused PLTs, for what purposes do facilitators coordinate their instructional decisions?	**X**	**X**	

The first protocol had ten questions and served as the introduction interview. The interview was designed to learn about the Marie and Deneen about their professional background and experiences with facilitating MPD, both as individuals and as a team (see Appendix A). The second protocol (Appendix B) posed 17 questions and served to gather insights into the Marie and Deneen's reflection processes following Summer Institute. The questions were specific to the MPD session and were developed based on the field notes of the researcher. The third protocol consisted of 44 questions divided into six subsections. These included: background information, prior experience as a learner in professional development, beliefs about teaching and learning, beliefs about facilitation, roles as facilitators, and the overall project (see Appendix C). The purpose of this interview was to gather information and insights from Marie and Deneen about the nature of facilitation. Audio recordings of all interviews were transcribed.

Artifacts. Artifacts from the planning and implementation of the MPD were used also used as a secondary data source for this study. These include researcher field notes, PD planning guides, PD presentation slides, handouts given to teachers, and agendas.

Data Analysis

To answer my research questions, I followed Yin's (2013) recommendations of explanation-building approach to analyzing my data. The analysis for this case study occurred in three sequential phases.

Phase 1. Two problem cycles (Ostrich-Giraffe problem and Horse problem; see Appendix E) from the Summer Institute were videotaped and transcribed into a Microsoft word document. The transcriptions were then uploaded into Atlas 7.0 software where they were reviewed for individual instructional moves. The first step in the analysis was to isolate the facilitators' instructional moves during the MPD. Each instructional move was initially labeled as either a question or a statement. Once the questions were labeled and grouped, further analysis was done to determine the nature of each question. It was observed that some questions invited teachers to participate in the discussion. Some questions elicited the thinking of teachers, while others pressed teachers to think deeper about the mathematical content. These questions were also examined to determine if they were focused on mathematical content or classroom mathematics teaching. There were some questions that were dismissed as not being relevant if they did not contribute to the discussion (e.g., "did everyone get a snack?").

Next, the statements were analyzed using the conceptual framework to identify the facilitators' instructional moves as explaining, marking, re-voicing, instructing, clarifying, inviting, formalizing, or performing some "other" instructional move. These were also initially categorized as content or classroom mathematics teaching focused. Each statement was coded. These codes were combined to more complex groups of instructional moves. For example, all revoicing about mathematics, revoicing about teaching, and revoicing about students were combined with marking to create a more complex group called "marking." Additionally, clarifying and formalizing were combined to create a more complex group called "explaining."

Ultimately, seven instructional moves were identified from the analysis and these were aligned to the instructional moves identified in educational literature on teacher practices. These moves include: inviting, orienting, pressing, marking, eliciting, instructing, and explaining. Transcriptions of observed MPD and field notes were coded using these initial codes (see Table 3). This first set of coding was organized in Atlas.ti as family: move (see Table 3 for start codes and definitions).

Table 3

Codes and Definitions for Data Analysis

Code	Description
Marking	An instructional move was coded marking when the facilitator highlighted key aspects of the practice, either by restating an important idea that a teacher had contributed to the discussion or through sharing the goals for their instructional practice.
Eliciting	An instructional move was coded eliciting when the facilitator prompted teachers to share their thinking with the group.
Explaining	An instructional move was coded explaining when the facilitators made ideas emerging in the MPD explicit for the teachers. These moves either clarified a mathematical or classroom mathematics teaching concept for teachers or explicitly explained a concept.
Pressing	An instructional move was coded pressing when the facilitators asked a series of questions that built on previous questions to encourage them to think deeper about the content.
Instructing	An instructional move was coded instructing when the facilitator was providing information to the teachers to support them in engaging in the PLT.
Orienting	An instructional move was coded orienting when the facilitator prompted teachers to share their thinking in response to the thinking of others. One type of orienting move was used when the facilitators wanted the teachers to orient to a particular strategy.
Inviting	An instructional move was coded inviting when the facilitators encouraged teachers to share their strategy or to participate in the discussion.

Following this initial round of coding, the instructional moves were then coded as Representing, Decomposing, or Approximating based on the PLT type during which they occurred. Marie and Deneen identified these PLTs on the transcription of the MPD. The third set of codes assigned was organized in Atlas.ti as family: PTLtype. They were also coded as domain specific if the move was about mathematics and domain-neutral if the move was not about mathematics. No code was assigned in Atlas.ti for this code. Query reports were run and each move was coded manually. Next, each instructional move was coded for a fourth time for inferred goals using an open coding approach. These goals were then verified by Marie and Deneen through a member checking interview. Marie and Deneen read through the transcripts and provided their insights on their goals for each instructional move. Any discrepancies were discussed until a consensus was established. These results yielded seven goals including positioning, fostering a community, establishing norms, scaffolding, pressing, deepening MKT, and relating (see Table 4 for codes of goals and description). The fourth type of code assigned was organized in Atlas.ti as family: goals.

Table 4

Codes for Goals of Instructional Moves

Code	Description
Positioning	When the facilitator uses a talk move that positions all teachers as competent. This can be done by revoicing something that a teacher says in an effort to bolster the speaker's confidence or signal to other teachers that there is value in what is being said.
Fostering a community	When the facilitator uses talk moves with the explicit goal of establishing a community of learners in which all members feel valued and safe.

Table 4

Cont.

Code	Description
Establishing norms	When the facilitator uses a talk move that supports the establishment of socio-mathematical norms including: valuing and learning from mistakes.
Scaffolding	When facilitators provide support to help teachers complete tasks that they could not complete alone. This includes supporting teachers in learning to work together, steering the conversation towards to particular mathematics direction, or summarizing a lengthy discussion by drawing attention to particular ideas.
Probing	When the facilitator asks a series of questions with the goal of getting information from teachers about meaning or justification.
Deepening MKT	When the facilitators use a talk move to support teacher learning of some aspect MKT.
Relating	When the facilitators use talk moves to help teachers relate to the learning experiences of students.

Finally, Atlas.ti was used to analyze each coded instructional move paired with its goal to identify a new set of codes of instructional decision. These super coded were organized in Atlas.ti as super code: decisions. For example, Deneen had an exchange with one of the participants, Ann, regarding her thinking when choosing the order in which she would ask the students to share their solutions. This instructional move was coded as eliciting. I conjectured that her goal for this move was positioning Ann as a competent teacher. Upon consultation with Deneen, it was ultimately coded as probing because Deneen wanted Ann to explain or justify why she would choose the particular order in which to allow students to share their solutions. Because there is no definite right or wrong way to do this, the Deneen used the series of questions to probe Ann regarding the intentionality of her choices. This instructional decision was coded as *eliciting to probe, domain specific, and fostering learning.* Again query reports were run for each instructional decision. These were coded by hand as either fostering learning or fostering community, and also analyzed to determine in what ways these differences occurred as each decision was coded as either fostering learning or fostering community.

Phase 2. Once the instructional decisions were identified, Atlas.ti was used to query each instructional decision by PLT type. This allowed me to compare the decisions across the PLTs. Using the number of occurrences for each decision in each PLT type, I compared decisions across PLTs using SPSS and Fisher Exact Tests to determine if there were statistically significant differences across the PLTs.

Phase 3. Analysis during this phase consisted of a comparative, holistic analysis by PLT type. Each PLT type was analyzed to determine the overall purpose of the decisions in each PLT. For this analysis, the entire transcripts were considered to support the understanding of each purpose.

Validity and Reliability

According to Merriam (1998), internal validity is the extent to which the results from a qualitative study match the reality (p. 201). Additionally, interval validity relies on the trustworthiness of the participants and the researcher to accurately, honestly, and clearly interpret the data (Creswell, 2013). The three strategies that were employed to ensure validity of this dissertation were persistent observation, member checking, and triangulation.

Prolonged engagement or persistent observation according to Creswell (2013) allows the researcher to build trust with participants as well as learning about the context in a meaningful way to safeguard against distortions that participants or the researcher may introduce. The Core Math II project took place over the course of nine months. As the researcher, I was able to witness first hand all of the face-to-face MPD as well as some of the planning and debriefing sessions of the facilitators. This prolonged engagement with context and the case allowed me to look at the MPD in its entirety to determine which segments were relevant to the focus of my study. This prolonged engagement allowed me to closely observe the interactions of the facilitators and the teachers thus checking and confirming my observations along the way.

Member checking according to Maxwell (2013) means respondent validation. This requires that the researcher get feedback from the participants in the study. For this study, member checking consisted of allowing Marie and Deneen, who serve as the case for this study, the opportunity to review the episodes from the MPD to check for accuracy in the researchers' assessment of the interactions between the facilitators and the teachers. Marie and Deneen checked the accuracy of the inferred goals assigned to the discursive moves. Member checking served as a safeguard against misinterpretations regarding the meanings of Marie and Deneen's instructional moves. Additionally, Marie and Deneen were provided the transcripts for review ensuring accuracy and transparency.

Evidence and claims were triangulated from the multiple data sources. According to Creswell (2013), "Triangulation is the process of corroborating evidence from different individuals, types of data or data collection in descriptions and themes in qualitative research" (p. 252). For this study, many of the secondary sources such as the interviews, observations and artifacts were used to triangulate the findings.

Reliability for this case study refers to the "stability of responses to multiple coders of data set" (Creswell, 2013, p. 253). For this study, I conferred with a fellow researcher to externally check my interpretations of the data and coding definitions to ensure that these interpretations and analyses were reliable.

Summary

In this chapter, I presented the design and methodology used for this case study on the instructional decisions made by facilitators of MPD and coordination of these decisions in supporting teachers in learning the core practice of Leading Mathematical Discussions (LMD). I presented an explanation for why case study was the most appropriate method to answer my research questions. Additionally, I provided details regarding sources of data used, the process used to analyze the data, and measures that were taken to reduce the risk of threats to validity and reliability of the study.

CHAPTER IV

FINDINGS

This chapter offers an analysis of the data and presents the findings of this study. The purpose of this study was to explore the nature of facilitating practice-focused mathematics professional development (MPD) for elementary grades teachers. More specifically, this study seeks to examine the purposeful instructional decisions made by facilitators that supports teacher learning of a core practice. Using qualitative and quantitative methods, I examined data from the Summer Institute portion of a yearlong practice-focused MPD program, including observations of all practice-focused PLTs in the summer institute, facilitator interviews, and field notes. Transcribed video-recordings of the PLTs and facilitator interviews, along with my field notes, were analyzed using the conceptual framework developed in Chapter 2. The analysis provided detailed descriptions of the ways facilitators supported teacher learning when implementing practice-focused PLTs. The research questions that guided the analysis asked:

1. What is the nature of the instructional decisions that facilitators make when implementing practice-focused PLTs? Specifically:
   a) What are the instructional moves that facilitators make when implementing practice-focused PLTs?
   b) What are their goals for these moves?
   c) What is the relation between facilitators' instructional moves and goals?

2. Did facilitators' instructional decisions vary when implementing different practice-focused professional learning tasks? If so, in what ways do they vary?
3. When implementing practice-focused PLTs, for what purposes do facilitators coordinate their instructional decisions?

The findings that follow are organized by these three questions and detail emergent themes from the analysis. Examples are included where appropriate and were selected for clarity and brevity.

Instructional Decisions When Implementing Practice-Focused PLTs

Facilitators' Instructional Moves

Seven instructional moves emerged from the analysis of the implementation of the three PLT types in the Summer Institute. As depicted in Table 5, Marking about LMD (69 times or 28.5%) and about mathematics (52 times or 21.5%) were the most used moves as this instructional move represents half of all the moves made by the facilitators. Facilitators' elicitations of teachers' thinking (39 times or 16.1%) and explaining (33 times or 13.6%) were also prevalent. Less common were invitations for teachers to participate in the discussion, share their solutions or strategies with others, and share their thinking (6 times or 2.5%). The analysis did not identify any additional instructional moves that facilitators used beyond those represented in the literature on mathematics classroom instruction. In what follows, I briefly describe and illustrate each identified instructional move.

Table 5

Frequencies of Instructional Moves When Facilitating Practice-Focused PLTs

Instructional Move	Mathematical Focus	Classroom Mathematics Teaching Focus	Total
Marking	52 (21.5%)	69 (28.5%)	121 (50%)
Eliciting	20 (8.3%)	19 (7.9%)	39(16.1%)
Explaining	11 (4.5%)	22 (9.1%)	33 (13.6%)
Pressing	14 (5.8%)	6(2.5%)	20 (8.3%)
Instructing			14 (5.8 %)
Orienting			9 (3.7%)
Inviting			6 (2.5%)
Total			242 (100%)

Moves to mark. Moves to mark important ideas were made by the facilitators to direct teachers' learning toward a specific learning target, and these moves represented exactly half of the facilitators' statements and questions across the three types of PLTs. These also included facilitators revoicing teachers' contributions to discussions. The analysis revealed that these moves focused on the core practice of LMD, teachers' mathematical thinking, and students as mathematical thinkers and learners.

Marking about LMD. Instances of facilitators marking ideas related to LMD occurred 69 times, more than any other move across the PLTs examined. Some of these moves highlighted key aspects of the practice, either by restating an important idea that a teacher had contributed to the discussion or through sharing the goals for their instructional decisions during Representing PLTs when reflecting. Others focused on instructional decisions to prepare for a mathematics discussion. For example, as Deneen reflected on her facilitation of the Ostrich and Giraffe problem during a Decomposing PLT, she discussed how she anticipated the teachers would engage with the problem and some of the decisions she made while teachers explored the problem to prepare for the discussion:

> I was anticipating all of these different kinds of strategies happening, and then we had a lot of people that were just using guess and check and once you started talking, you reinforced each other's guess and check [approach] and it became more about how to do guess and check . . . So some of the questions for participants that were guessing all over the place or having difficulty keeping track or guessing a pattern . . . Can you find a systematic approach to organizing all of this data? . . . Have you tested the eyes and legs to see that your solution works?

In this reflection, Deneen marked two strategies that she took to prepare for the mathematics discussion, anticipating teachers' approaches and posing pressing questions to focus teachers on the mathematics of the problem.

Other moves in this category marked teachers' perceptions of the challenges of implementing LMD or marked the complexity of the practice. For example, one teacher shared her reflection regarding posing non-leading question to students. She found this to be very challenging. Deneen responded to this teacher by first acknowledging that posing non-leading questions is "a hard thing to struggle with." After recounting her own struggles when leading a discussion of the Horse problem, Deneen stated,

> It is very hard not to try to step in and lead, but the main thing is we want to make sure that at the very end, people do know the right answer, that they're still not left with that thinking I'm unresolved, yeah, they've shared all of that, and my whole group got +20, but I still think that my way of +10 here is right.

Marking teachers' mathematical thinking. Facilitators used 52 marking moves to highlight important aspects of teachers' mathematical thinking across the practice-focused PLTs. These moves brought forward a particular aspect of a teacher's explanation of how he or she was thinking about a mathematics problem or some aspect of the mathematical concept during a Representing PLT. For instance, after one teacher named Sarah recounted her initial approach to the Ostrich and Giraffe problem, Deneen marked an important shift in Sarah's thinking:

> Sarah, you were saying when you started, you were just going—you would add some to this column, and add some to this column, and add some to this column or that column . . . until you got the right number of legs and the right number of animals. You didn't do that. How did you start the first guess again?

This move marked Sarah's shift from an unsystematic guess and check approach to a more sophisticated strategy.

Moves in this category also marked teachers' mathematical contributions to discussions to introduce and/or reinforce more precise language. To illustrate, the following example shows how Marie introduced more formal language into the discussion in response to Quinn's understanding of Sarah's approach.

> **Quinn:** It's more of an organized version of guess and check – she's not necessarily guessing and checking, she's solving problems to get to a solution. To me, that's not guess and check . . . I mean it was more she had a problem and she worked it out until she got where she needed to be.
>
> **Marie:** She was decomposing that number at the different ways you could decompose to find the right combination.

Marking about students as mathematical thinkers and learners. A third use of marking moves related to students' mathematical thinking and the ways in which they learn. In seven instances, facilitators marked and highlighted some aspect of student mathematical thinking. For example, when discussing the Ostrich and Giraffe problem, Deneen remarked,

> That's the kind of thinking we want to see our kids doing, those kinds of questions—but why wouldn't you just take off a 2 legged ostrich? That's what the kids would go to immediately and thinking about that, that's what makes sense.

Later in the Summer Institute, she came back to the discussion:

> So for a kid going through that process—if the kid is using guess and check, when we talked yesterday, a lot of times kids don't really use guess and check accurately. They use guess, and guess again, and guess again, guess again, and they don't make adjustments based on their first guess.

In these instances, marking moves focused on making sense of what students do mathematically.

Other statements focused on students as learners. During a discussion where teachers were talking about their reservations about LMD, for example, Deneen marked their concerns as important and acknowledged the complexity of the practice:

> Do we have students who don't have confidence? If we have students that have this lack of confidence, how do we help to also create their confidence but also keep that risk taking environment that we're talking about? . . . And then that one you brought up yesterday, perseverance. Really getting the perseverance so we can get at the mathematics.

Here, Deneen restated some of the teachers' concerns about aspects beyond mathematics ("confidence" and "perseverance") that affect students as learners.

Moves to elicit. The facilitators made 39 moves (16.1%) to elicit teachers' thinking. Twenty of them focused on the mathematical ideas contained within teachers' strategies for solving mathematics problems. For example, Deneen asked the group about what they were thinking about a strategy shared by one of the teachers.

> How many legs there are? There's a lot going on mentally that she's having to try to keep up with here because that's why I said it's really hard to see from this. It looks like she magically figured out there's 8 and 7. But there's a lot that went on when she was actually doing her work there. Okay, thank you Sarah. Anybody, what would you call this strategy? She said she didn't know if that was mathematical or not. What kind of strategy would you call that?

In another, Deneen prompted a teacher to share a guess and check strategy and how her thinking had changed from the previous day as a results of engaging with others' ideas, stating, "I want Heather to share her guess and check and the logical reasoning, and I want you to talk about your strategy for today and how it may have been different from the way you approached it yesterday." Moves such as these brought the details of teachers' mathematical thinking to the group for consideration.

The remaining 19 elicitations prompted teachers to share their thinking about classroom mathematical teaching. For example, Deneen used a series of eliciting moves when Ann was sharing her group's sequencing of student work for discussion of the Horse problem:

> **Deneen:** So, she's going to actually start with an incorrect solution which is interesting . . . So she's going to start with this and then what are you going to do

> with this answer? Are you going to let them know that it's wrong or just kind of get it out there?
>
> **Ann:** No. I just, [was] going to show it to them.
>
> **Deneen:** She's going to get it out there to get it out as a solution and let the kids see the thought process. Then where are you going to go?
>
> **Ann:** Then I'm going to go with the one with the $30 . . .
>
> **Deneen:** Ann would go here. Why?
>
> **Ann:** Just because it's a different answer.
>
> **Deneen:** Okay, she wants to get out a different answer.
>
> **Ann:** And again, I didn't say whether or not it was right or wrong.
>
> **Deneen:** Okay. So she's just going to get out, now she's going to have a student share a different answer just to get the kids thinking, 'whoa, now there's a different way to solve it.' Where would you go next?

In this interaction, Deneen elicited Ann's reasoning for her selection of students' solutions to share in discussion. Similar to those focused on mathematical thinking, these instructional moves brought the details of the teachers' thinking to the group for their consideration.

Moves to explain. Facilitators made 33 moves (13.6%) to make emerging ideas in the MPD explicit for the teachers. These moves either clarified a mathematical or classroom mathematics teaching concepts for teachers or explicitly explained a concept. For example, Deneen clarified the mathematical strategy that a group of teachers used when solving the Ostrich and Giraffe problem. After teachers shared their approach with the whole group, she stated,

> Yes. That's a great strategy to get there. But, yea, grouping, multiplication. I love that she was thinking about those 2 legged animals and those 4 legged animals, if I just write out my 44 in sets of twos. I can keep track of 2 legs I can make some of them twos and I can group some and make them fours. So she's thinking about that strategy, but once again, be careful at keeping track. That's the thing that makes this a difficult problem.

Her comments on the strategy explained how the teachers grouped and used repeated addition to solve the problem.

Deneen and Marie also used explaining moves to clarify key ideas about the core practices of focus for the PLT. For instance, after teachers analyzed written work to plan a hypothetical classroom discussion in small groups, each group shared their plan with the whole group. The teachers disagreed over which work samples they would share, and one teacher posed the question about whether Deneen would select a particular solution. In response, Deneen made a move to clarify the process of selecting:

> It depends on your learning target and where your class is. It's kind of the exact same thing where you all were saying about this one—is it going to confuse most of your kids? If most of my kids already understand it and they're not making this mistake, and it's an issue for one child, I wouldn't bring that . . . I don't think you have to select all of them. Think about which ones are going to—think in mind two things. What's my learning target? What do I want the class to get out of this?

Her statements about selecting further explained some of the considerations teachers make when preparing for a mathematics discussion based on students' ideas.

Though these moves were used to explain both mathematical and thinking about classroom mathematics teaching, the facilitators used them more frequently to clarify and explain the core practice, rather than mathematics. Of the 33 moves in this category, 22 of them focused on the practice of LMD, with the remaining 11 focusing on mathematical ideas.

Moves to press. Facilitators pressed teachers' mathematical or thinking about classroom mathematics teaching 20 times across the PLTs. Fourteen of these moves focused on challenging teachers to think deeply about the mathematics they were using. For example, Deneen stated,

> So more concrete with the pictures. This is a little more abstract. Anything else? To the yellow group out there, tell me a little bit about the two, the difference in the two. You started telling me, Heather. Talk to me more about the table.

When making this statement, Deneen is pressing teachers regarding the use of guess and check strategy. Although teachers used pictures to represent the legs of the animals, they did not see that they were still essentially the same strategy. Deneen pressed them to think deeper about this mathematical strategy.

Additionally, there were six pressing moves that challenged teachers' thinking about the practice of LMD. In the following instance, Deneen presses teachers regarding the types of questions they would ask students given student responses examples:

> So, let's say student B did this as you're walking around. Student B solved it this way, they've written on their paper 2 giraffes, 2 x 4 is 8. 20 ostriches, 20 x 2 is 40. What are you going to say? This time I want you to think about what's wrong with it and you can look at it. What questions would you ask?

This line of pressing questions helps teachers to think deeply about their questions to help move students' thinking forward.

Moves to instruct. Instructional moves were coded as instructing when the facilitator was providing information to the teachers to support them in engaging in the PLT. Facilitators used 14 times which represents 5.8% of the total moves across the PLTs. There were no distinct differences in the type of instructive moves the facilitators used between instructions about mathematics problem solving and instruction about PLT. Deneen provided basic instructions to teachers regarding the expectation for engagement in the activity:

> I want you to do individually to start with. Give you a chance to reflect and think through it yourself and then I'm going to ask you after a few minutes go by, I'm going to let you talk and share with a partner . . . We'll be taking one paper per partner. SO this table should give me three sheets of paper, three sheets of paper, two sheets of paper.

The instructions provided were clear and explicit, providing little room for confusion regarding the facilitator's expectations.

Moves to orient. There was a total of 9 instructional moves to orient teachers to each other's thinking. This category represents 3.7 % of the total moves across the PLTs. There were no distinctions between moves used by facilitators regarding mathematics and classroom mathematics teaching. Facilitator instructional moves were coded as moves to orient when the facilitator prompted teachers to share their thinking in response to the thinking of others. One type of orienting move was used when the facilitators wanted the teachers to orient to a particular strategy. Deneen invited Ann to share her strategy with the whole group:

> **Deneen:** So we've got a couple of other strategies I want to highlight. Ann, would you mind sharing yours?
>
> **Ann:** I started out first by figuring out how many animals I would need, which is 15. And you wanted 44 legs, so I wrote down 2 until I got to 44 and then I started circling groups of 4 and then I got over here I went back and counted to see how many animals I had and then I knew I wasn't going to have enough so I had to back off and change it to make these ostriches and I came up with 6 ostriches and 8 giraffes.
>
> **Deneen:** Questions for Ann? What would you name her strategy?

Using this move allowed the entire group of teachers to consider the thinking of Ann's strategy at a deep enough level that they are able to identify or name the strategy she employed.

Orienting moves were also used by the facilitators to explain the mathematical thinking of other teachers or share how their strategies or thinking were similar to or different from the others. Following the orienting to Ann's strategy, Deneen then asked teachers to think about how the strategy they used differed or was similar:

> Did anybody have a different one that they wanted to bring out and think mine is completely different, we haven't really talked about that yet? What was important about this problem to know?

When she used this move, teachers were provided the opportunity to think deeply about the differences in the strategy that they utilized and the one that Ann shared; with this move, she supported them in thinking deeply about the mathematics.

Moves to invite. The facilitators invited teachers to participate in the conversation six times which represents 2.5% of the total moves across the PLTs. Instructional moves were coded as inviting when the facilitators encouraged teachers to share their thinking.

Most of the invitations by the facilitators were directed to the entire group of teachers and were almost exclusively asking them to share their strategy, such as “Who feels like they can explain it to the group?” or “Did anybody have a different one that they wanted to bring out and think, ‘mine is completely different—haven’t really talked about that yet?’” These inviting moves supported the sharing of ideas during the PLT. In a few instances, the facilitators called on specific teachers to invite them to share their thinking with the group. These inviting moves supported the mathematics discussions in the MPD. Without these invitations, there may not have been opportunities provided for discussions about the mathematics or classroom mathematics teaching.

Facilitators’ Goals for Instructional Moves

Goals for fostering teacher learning. The analysis of the data revealed seven goals for the instructional moves used by the facilitators during the PLTs. Four of the seven goals for the facilitators’ instructional moves supported teacher learning. These four goals include: scaffolding, probing, deepening MKT, and relating. Both domain-specific and domain-neutral instructional moves were driven by these four goals.

Scaffolding to foster learning. There were 81 instances when the goal of the facilitator was scaffolding to support teacher learning. This was done when facilitators provided support to help teachers to complete problems that they could not complete alone. Facilitators chose to scaffold to support teachers in collaborating together to complete a particular problem. They also used scaffolding to support teachers in reaching the desired learning problem by steering the conversation towards a particular mathematics or classroom mathematics teaching direction. To ensure that the particular ideas were emphasized, facilitators also summarizing a lengthy discussion by drawing attention to these specific ideas.

Probing to foster learning. There were 50 instances of facilitators probing teachers regarding their thinking. This occurred when the facilitator would ask a series of questions with the goal of getting information from teachers about the meaning of a mathematical concept or justification of their thinking or strategy. Once the teacher responded, the facilitator would continue to ask more questions of that particular teacher or of the group as a whole in order to support the teachers in making their thinking visible to the entire group as well as to themselves.

Deepening MKT. There were 38 instances of facilitators using instructional moves with the goal of deepening the MKT of the teachers. Moves used by the facilitators supported teachers' learning of mathematics, the teaching of mathematics, how students think about mathematics, common mistakes made by students, as well as strategies for solving mathematical problems. Facilitators used moves to deepen teachers' understanding of the teaching and learning of mathematics.

Relating to foster learning. There were 19 instances when the facilitators' goal for their instructing move was to support teachers in relating to the learning experiences of students. When teachers are able to relate to their students' learning experiences, they are better able to support student learning by providing learning opportunities in which students have a greater opportunity for success. This goal supports teachers in both the teaching and the learning of mathematics.

The four goals—scaffolding, probing, deepening MKT, and relating—are all aims that facilitators have when using instructional moves to support teacher learning. These were the goals for instructional moves for a total of 188 instances out of 242 which represents 78% of the total number of instructional moves.

Goals for fostering community. The three remaining goals for the facilitators' instructional moves supported the fostering of community among the participants and facilitators of the MPD. These three goals include: positioning, establishing norms, and fostering community. As with the goals for fostering learning, both domain-specific and domain-neutral instructional moves were driven by these three goals. There were a total of 53 instances in which the facilitators' goal was to foster community.

Positioning to foster community. There were 26 instances in which the facilitators used an instructional move that positioned all teachers as competent. This can be done by marking or re-voicing something that a teacher says in an effort to bolster the speaker's confidence or signal to other teachers that there is value in what is being said. This is also accomplished when the facilitator validates the participants' responses with non-verbal affirmations.

Establishing norm to foster community. There were 21 instances when the facilitators used instructional moves that supported the establishment of both socio-mathematical and professional norms in the MPD. The socio-mathematical norms included how teachers were expected to participate in collaborative problem solving as well as expectations for critiquing or providing judgments regarding the viability of another person's solution or procedure. Additionally, the Deneen and Marie established professional norms about how the teachers would not only relate to each other as learners but also how they would discuss students.

Fostering a Community: There were seven instances when the facilitators used instructional moves to establish a community of learners in which all members of the learning community were challenged, reflective, and collaborative. The goal for these instructional moves was to ensure that all members of the community had learning experiences in which they felt valued and competent professionals contributed valuable knowledge with the group. Additionally, the goal was for the teachers to feel safe sharing their mathematical thinking which ultimately enhanced the learning of the entire community.

Though one of the explicit goals for the instructional moves the facilitators used was to promote a community of learners, the other two goals also supported the overall purpose of fostering a community of learners. Positioning teachers as competent mathematicians and establishing norms (both socio-mathematical and professional) also helped to foster a community in which all members had a shared meaning of what the core practice of LMD entail.

Facilitators' Instructional Decisions

As Table 6 depicts, results from the analysis relating facilitators' instructional moves with specific goals identified a total of 49 possible instructional decisions made when implementing practice-focused PLTs. Each cell represents the number of instructional moves associated with a particular goal occurring across the PLTs. Overall, there were four broad types of decisions: decisions that used content-neutral moves to

Table 6

Instructional Decisions

	Fostering Learning					Fostering Community				
	Scaffolding	Probing	MKT	Relating	*Sub-Total*	Positioning	Norms	Community	*Sub-Total*	**Total**
Content-Neutral										
Instructing	6	3	0	1	*10*	3	1	0	4	**14**
Orienting	1	4	0	0	*5*	2	0	2	4	**9**
Inviting	1	2	1	0	*4*	1	1	0	2	**6**
Sub-Total	*8*	*9*	*1*	*1*	*19*	*5*	*2*	*2*	*10*	**29**
Content-Specific										
Marking	52*	10	23*	13*	**98**	9*	9*	5	23	**121**
Eliciting	6	19*	4	3	**32**	2	5	0	7	**39**
Explaining	13*	3	4	2	**22**	8*	3	0	11	**33**
Pressing	2	9	6	0	**17**	1	2	0	3	**20**
Sub-Total	*73*	*41*	*37*	*18*	*169*	*20*	*19*	*5*	*44*	*213*
Total	**81**	**50**	**38**	**19**	**188**	**26**	**21**	**7**	**53**	**242**

*Indicates key instructional decisions.

foster teacher learning, decisions that used content-neutral moves to foster community, decisions that used content-specific moves to foster community, and decisions that used content-specific moves to foster learning.

Content-neutral moves to foster learning were used made a total of 19 times, which represented 7.9% of the total number of instructional decisions made across the PLTs. These decisions were made when the focus was not on mathematics, but supporting teachers in learning about teaching. The most prevalent type of decision which was content-neutral yet supported teacher learning was an instructing move to scaffold. For example, during the Horse problem Deneen instructed teachers on how they were to complete the problem. The directions given were not content-specific but explicitly instructed teachers on how they were to solve the problems and discuss later. They were also instructed not to help or share until the appropriate time in order to allow all the participants to engage in the learning problem.

> We're going to give you a math problem that is for the leadership team we have and what we want you to do is we want you to solve this on your own first. While you're eating that last few bites of pizza or finishing up dessert, we want you to solve this and then you're going to talk about it with your group members. What we're going to ask is those teachers that stay here that have already solved it, you're just going to hush up, not say anything, observe, kind of watch the process as it happens. Marie? and I will facilitate your question.

Although these instructional decisions supported teachers in learning, I determined that they are not key instructional decisions as they only comprise 8% of the instructional decisions made by the facilitators. Additionally, because they are content-neutral, they are decisions that are not specific to the facilitation of MPD. Content-neutral moves to foster community represented 4.1% of the total number of instructional decisions made across the PLTs. These decisions were made by the facilitators in an effort to foster community among the participants but did not explicitly support learning of the mathematical content. The most prevalent type of decision was instructing with the goal of positioning the teachers as competent. The facilitators made sure to include praise or affirmation for the teachers to bolster confidence. For example, during both the problem cycles, the facilitators complimented the teachers' responses with such statements as, "Nice," "Excellent explanation," and "That's an excellent approach."

As with the content-neutral instructional strategies for foster learning, these strategies with a goal to foster community are not considered key instructional decisions for facilitation because they are instructional moves that would be employed in any classroom but with a different purpose. Additionally, they are not content specific and therefore could be used for the facilitation of any professional development.

Of the 49 possible instructional decisions, the facilitators used 43 of them during their facilitation of the MPD. Of these 43, eight types most utilized by the facilitators represent the key instructional decisions for facilitating practice-focused MPD. Domain-specific instructional moves used to foster community were made 44 times representing 18% of all instructional decisions. These decisions were the second most prevalent category and supported teachers in identifying themselves as part of a community of practitioners who happened to also be learners of LMD. Three of the most frequently occurring types of decision within this category are considered to be instrumental in implementing the PLTs.

The most frequently occurring decisions made when facilitating the PLTs were those that used content-specific moves to foster learning, representing 70% of the total number of decisions. Six of the most commonly occurring types of instructional decisions within this category are also considered to be instrumental in implementing the PLTs.

Key instructional decisions. In what follows, I elaborate these eight key decisions for facilitating practice-focused PLTs. These key decisions represent 60% (145) of all instructional decisions used by the facilitators when implementing the PLTs.

Marking to scaffold. The most prevalent of these decisions involved marking to scaffold teachers' learning of mathematics, a category that represented 21% of all the decisions and 36% of the key decisions. These decisions occurred when the facilitator marked about the mathematics or LMD with the specific goal of scaffolding to support teachers in completing the problem or providing support for future completion of problems. For example, Deneen responds to Ashely who shares how on the previous day while doing a similar problem, she used the guess and check strategy but did something different on this day by using a picture. She explained that using the picture helped her to solve the problem more efficiently. Deneen marked that the strategy of using the picture actually represented a more sophisticated strategy then the one previously used by Beth. Deneen stated,

> This is the beauty of this picture that I pointed out yesterday briefly. This is nice because it is here for us to see. Once you understand this picture, it leads you to a higher strategy that you could actually do logically. She was able to think about it without the pictures because she understands the structure of the picture and that's the beauty picture that I really want us to, one of the things that I wanted to come back and touch on from yesterday is that we really need to make sure that we give a variety of strategies, a variety of structures to think about. Not only does a

> picture model your thinking, but sometimes it helps understand the structure of the mathematics so that I can move to a more efficient strategy where I don't need the picture. So, yes, pictures are very important and they're not just for kindergarten through second grade.

Following Beth's sharing of her strategy with the group, the facilitators wanted to ensure that the entire group were able to understand that using pictures can support problem solving in a manner that can be more sophisticated than some other strategies. Employing marking moves to highlight the use of pictures supported all the teachers in possibly trying the strategy going forward, by building their awareness that the use of pictures can sometimes represent deeper conceptual understanding of the mathematics.

Marking to deepen MKT. Facilitators marked about mathematics or teaching with the specific goal of supporting teachers deepening of their MKT. They made this instructional decision 23 times across the two PLTs. This represents 9.5% of all instructional decisions and 16% of the key instructional decisions. This instructional decision was used to help deepen both Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) of teachers. For example, Deneen shared with teachers the importance of ensuring that they have a clear understanding of the learning target/goal that they want their students to achieve. Additionally, she marked about the importance of knowing the level of number sense and basic skills needed to solve using different strategies. She stated,

> What is the math that I'm trying to get? The reason that I gave you the problem. What is it that I wanted the class to know? What is it that I really want to do to move the kids forward? So, like in the horse problem, really from that one to me the mathematics are how are they moving forward with these numbers? What's their number sense and the approach of dealing with numbers? You know, how

> do they deal with when they have to subtract and they're going to get a negative number? So those kinds of things. And you could even have a, it could be an algebra thing. Do they understand how to put like terms together? Because depending on the grade level on target. That's the thing I want to make sure that I highlight.

The instructional decision was made to support teachers in deepening their PCK by highlighting knowledge regarding students and common mistakes, such as not being able to combine like terms, or subtraction with a negative solution. Additionally, this marking supported teachers' understanding that knowing the curriculum and learning targets for the particular grade level dictates the type of strategy that teachers will emphasize and support through explicit instruction.

Eliciting to probe. Facilitators elicited or prompted teachers about mathematics or classroom mathematics teaching with the goal of probing teachers to get information or justification of the mathematics and mathematical strategies. They made this instructional decision 19 times across the PLTs. This represents 8% of all instructional decisions and 13% of the key instructional decisions. For example, Deneen had an exchange with one of the participants, Ann, regarding her thinking when choosing the order in which she would ask the students to share their solutions.

> **Deneen:** Okay. So then let me ask Ann first because I just heard Ann and Beth and Quinn talking. Tell me your order. Whatcha thinking?
>
> **Ann:** I would start with the broke even.
>
> **Deneen:** Okay. You would start with this one?
>
> **Ann:** Yea.
>
> **Deneen:** And why?

Ann: I just think it would be good because they're by themselves just to start with zero. I mean it was nothing.

Deneen: Okay. So she's going to actually start with an incorrect solution which is interesting. She's going to share this. Especially if there are several kids in the class that have this thinking. So you still, you might have several kids in the class that have this as their answer and they're unresolved at what's going on. So she's going to start with this and then what are you going to do with this answer? Are you going to let them know that it's wrong or just kind of get it out there?

Ann: No. I just, going to show it to them.

Deneen: She's going to get it out there to get it out as a solution and let the kids see the thought process. Then where are you going to go?

Ann: Then I'm going to go with the one with the $30.

Carol: We would've went first on that.

Deneen: Hang on. Let's get another one in a minute. But Ann would go here why?

Ann: Just because it's a different answer.

In the exchange between the facilitator and the teacher, the facilitator wanted the teacher to explain or justify why they would choose the particular order in which to allow students to share their solutions. Because there is no definite right or wrong way to do this, the facilitator used the series of questions to probe Ann regarding the intentionality of her choices.

Marking to relate. Facilitators marked about mathematics or teaching with the goal of helping teachers relate to the learning experiences of students. They made this instructional decision 13 times across the PLTs. This represents 5% of all the instructional decision and 9% of the key instructional decisions. One way that the facilitators marked is by revoicing what one of the teachers has said or by elaborating on a statement made by the teachers. For example, Nicole brought up the fact that students are not very flexible in their thinking as they often use the same strategies to solve problems. She mentioned that although they had done similar problems previous to the Ostrich-Giraffe problem, but they still basically used the same guess and check strategy to solve. The facilitators wanted to elaborate on this statement so Marie commented,

> And I want to go back. I want Sally to say something. But Nicole brought up such a good point in this group we were talking because she said I've done this problem before with cows and chickens, y'all have all seen that problem, I've done this before with cows and chickens and Deneen has shown me this same strategy with the cows and chickens. Mellissa, did you use Deneen's, this strategy? No. I mean and not saying that this is the best strategy, this is one of many strategies to say, but it's the same way I think what we do with our kids a lot of times is we well, I showed them that strategy. Why did they not use them? I taught them that lesson, why are they not using it? And we revert back to what we're comfortable with. Even so we were having that discussion earlier at lunch about how to move kids, you know through those learning objectives, move kids to those higher strategies. What will that take in order to, it doesn't take 1 exposure. It may not take 10 exposures, you know, how do we help kids? And I thought that was a good point.

In this marking statement made by Marie, she helps teachers relate to the thinking of students by first marking statements made by Nicole and her personal experiences solving the problems. She then went on to elaborate about how difficult it is to move students to more sophisticated strategies. She highlighted the fact that even though Deneen shared a more sophisticated strategy, the teachers still reverted back to an old strategy. Having teachers relate to students' thinking will support them in understanding what is necessary to move students.

Explaining to scaffold. Facilitators explained concepts related to mathematics or instructional practices with the goal of scaffolding the concepts to support teachers in completing problems. They made this instructional decision a total of 13 times across the PLTs. This represents 5% of all the instructional decision and 9% of the key instructional decisions. One example in which the facilitators decided to use an explanation to support teachers in being successful was during the Ostrich Giraffe problem; Deneen explained that although making a table is considered a different strategy from guess and check, the strategies are very connected. She stated,

> Maybe it's a little of both but technically a different strategy; [it] is called make a table or make an organized table. So you actually can use that as a separate strategy. It's guess and check really if you're just guessing and then checking and then using that to revise so that they wouldn't have to use an entire table. So that's really a, it's kind of a different strategy but they are connected, very connected.

Deneen's explanation regarding the connection between guess and check and the strategies of making a table support teachers in their understanding that, because the strategies are grounded in mathematical thinking, they are connected and can both be used to solve the problem efficiently.

Marking to position. This key instructional decision occurred eight times. This represented 3% of the total or 5% of the key instructional decisions. For example, during the Ostrich-Giraffe problem, Deneen wanted to make sure to mark as important and correct a response by Kelly. Kelly stated,

> They understand multiplication and groups of which is completely utterly foundational for anyone who doesn't live in third grade land and they understand that they are going to have groups of animals. Deneen responds by stating, "Okay, good. I want to make sure everybody heard Kelly that was very good and her analysis what does the student understand about the problem and what does the student know?"

This statement made by Deneen not only positioned Kelly as knowledgeable, it also marked the important conceptual understanding that the students would need to know in order to be successful using a particular strategy to solve the problem.

Marking to establish norms. The facilitators used this key instructional decision eight times which represents 3% of the total instructional decisions and 5% of the key instructional decisions. The facilitators marked mathematical discussions as important with the goal of establishing socio or professional norms for the MPD. In this example, Deneen wants to establish the importance of using errors as a springboard for learning. She explicitly identified the goal of learning from not only our own mistakes but also from the mistakes of others. She stated,

> Where is the error in the thinking so we can all learn from it and I love that having that risk taking environment in the classroom that's happening so we can put up an incorrect response and say, we're learning from this? And hopefully the person that's doing this is hopefully realizing that oh, this is a great learning opportunity, I'm glad that Katherine shared this strategy because there's several of us that not only had that this time but many of us would keep making a mistake in the future if we don't resolve it. So, thank you for sharing yours and being a risk taker.

Deneen made this instructional decision to emphasize for teachers the understanding that not only is it important for them to learn from their mistakes, but that as teachers it is important to establish socio-mathematical norms which support students' learning from their mistakes. Her use of the word "risk taker" supported the establishment of this norm for the fostering of a community of learners.

Explaining to position. This domain-neutral key instructional decision was made eight times which represents 3% of the total instructional decisions made and 5% of the key instructional decisions. The facilitators explained some key idea about classroom mathematics teaching or content for the purpose of positioning the teachers as competent mathematicians or professionals. For example, Deneen wanted to position Sarah as a competent mathematician by highlighting the fact that using pictures does not always mean that the strategy is less sophisticated than other strategies. Deneen stated,

> No. We want to be real careful too that we don't disassociate everything picture wise. Because Sarah started with a picture, a lot of us were using pictures but not necessarily just because that's the way that our brain works. We also want to think about what's important about this picture.

In this statement, Deneen emphasized that the use of pictures can be a viable start to solving the mathematical problem. She explained that it's not just the use of the picture alone but rather the important elements of Sarah's picture that made it useful for solving the problem.

Summary of facilitators' instructional decisions. When implementing practice-focused PLTs, the analysis showed that facilitators used eight types of instructional moves as summarized in Table 7. Four of these moves were content-specific marked, elicited, explained, and pressed with a focus on mathematics or the practice of LMD. Three other content-neutral instructional moves instructed, oriented, and invited participants. These moves had two broad types of goals. One set of goals related to fostering community, specifically by positioning teachers as competent mathematicians, establishing and maintaining productive norms, and fostering community. The other set related to fostering teacher learning by scaffolding, probing teachers for justifications, and deepening teachers' MKT.

Table 7

Eight Key Instructional Decisions

Instructional Decision	Description
Marking to Scaffold	Facilitator marked about the mathematics or LMD with the specific goal of scaffolding to support teachers in completing the problem or providing support for future completion of problems.
Marking to Deepen MKT	This instructional decision was used to help deepen both subject matter knowledge and Pedagogical Content Knowledge (PCK) of teachers.
Eliciting to Probe	Facilitators elicited or prompted teachers about mathematics or instructional practices with the goal of probing teachers to get information or justification of the mathematics and mathematical strategies.
Marking to Relate	Facilitators marked about mathematics or teaching with the goal of helping teachers relate to the learning experiences of students.
Explaining to Scaffold	Facilitators explained concepts related to mathematics or instructional practices with the goal of scaffolding the concepts to support teachers in completing problems
Marking for Norms	Facilitators marked mathematical discussions as important with the goal of establishing socio or professional norms for the MPD
Marking to Position	Facilitators re-voiced or highlighted contributions made by teachers in order to position the teacher as knowledgeable about mathematics or teaching.
Explaining to Position	The facilitators explained some key ideas about classroom mathematics teaching or content for the purpose of positioning the teachers as competent mathematicians or professionals.

An examination of the relation between the specific moves and goals revealed four broad categories of instructional decisions. Whereas two of these categories of decisions were not mathematical in nature, two categories focused on mathematics learning or mathematics teaching. Within these categories, eight key instructional decisions for facilitating practice-focused PLTs emerged. Three of these decision types used content-specific instructional moves with the goal of fostering community, and five used content-specific instructional moves with the goal of fostering learning. Thus, the facilitation of practice-focused PLTs within this study entailed instructional decisions that predominantly foster learning with moves focused on mathematics or instructional practice as well as those that use similar moves to foster community.

Instructional Decisions in Practice-focused PLTs

In this section, I present results related to my second research question addressing whether there was variance across the three types of practice-focused PLTs. Quantitative analyses were used to answer the first part of the research question. Qualitative analysis was necessary to answer the second part of the question. Table 8 shows the frequency of each key instructional decision for each of the PLTs.

Table 8

Key Instructional Decisions Across Practice-focused PLTs

Instructional Decision	Representing	Decomposing	Approximating	Total
Marking to Scaffold	6	25	18	49
Marking to Deepen MKT	9	8	6	23
Eliciting to Probe	6	3	10	19
Marking to Relate	4	6	3	13
Explaining to Scaffold	1	2	10	13
Marking for Norms	2	4	3	9

Table 8

Cont.

Instructional Decision	Representing	Decomposing	Approximating	Total
Marking to Position	5	2	2	9
Explaining to Position	4	2	2	8
Total	37	52	54	143

Instructional Decisions across PLTs

Results from a Fisher's exact test indicated that there was an association between the instructional decisions the facilitators made and the type of practice-focused PLT they were facilitating, suggesting that the facilitation practices used for at least one of the PLT types differed from the others. Post-hoc pairwise Fisher's Exact Tests with Bonferroni corrections indicated that overall, there is no statistically significant difference between the frequency of instructional decisions when facilitating Decomposing and Approximating PLTs. However, the frequency of instructional decisions made when facilitating Representing PLTs significantly differed from frequencies when facilitating Decomposing and Approximating PLTs. See Table 9.

These results were supported by a qualitative comparison of the instructional decisions made across the PLT types. The focus of four key instructional decisions in Representing PLTs was different for the focus in the Decomposing and Approximating PLTs. In the Representing PLTs, instructional decisions were largely made to support teachers in learning mathematics. In contrast, the focus of instructional decisions made during the Decomposing and Approximating PLTs was more varied, with some addressing student mathematical thinking while others focused on LMD. In what follows, I describe and provide examples of these differences for each of the four key decisions.

Table 9

Fisher's Exact Tests with Monte Carlo Simulations (*n*=10000 samples)

	T	*df*	*p*
Instructional Decisions x PLT Types	27.283	14	0.009*
Post-Hoc Pairwise Comparisons			
Decisions x Rep/Decomp	14.167	7	0.036*
Decisions x Rep/App	14.625	7	0.033*
Decisions x Dec/App	11.816	7	0.091

Note. $^{*}p < 0.05$; $^{**}p < 0.0475$ for Bonferroni Correction.

Marking to deepen MKT. Decisions during Representing PLTs focused mainly on the subject matter knowledge domain of MKT. For example, during the Representing PLT using the Ostrich-Giraffe problem, Deneen remarked,

> Right. You really need to know the number of animals. The total number of animals is what's really critical to helping you think about a strategy that works and this problem doesn't come right out and really tell you how many animals there were. You had to think about that, and if you weren't careful, you would have kind of glossed over that and gone right by it. So nice job with knowing exactly what the important information is in this problem. I'd like to share a different solution.

This decision supported teachers in deepening their own understanding of the mathematics of the problem.

During the Decomposing and Approximating PLTs, decisions that marked ideas to deepen teachers' MKT focused more on deepening of PCK domain of MKT. For example, during the Decomposing PLT relating to the Horse problem, an exchange occurred among four teachers (Quinn, Ann, Beth, and Nicole) regarding the guess and check strategy. They described how the strategies look different because some solutions were more organized than others. Deneen marked the interchange by stating,

> Right. Good. So is that guess and check or is that a different strategy entirely? Maybe it's a little of both. But technically, a different cross off strategy is called make a table or make an organized table. So you actually can use that as a separate strategy. It's guess and check really if you're just guessing and then checking and then using that to revise so that they would have to use an entire table. So that's really, it's kind of a different strategy but they are connected, very connected. You can see how all of these strategies are connected actually and so that's why we wanted to see a variety.

Her decision here supported teachers in making connections between the varying strategies and in learning that the connections are grounded in mathematical thinking.

Similarly, during Approximating PLTs, these decisions focused on enhancing teachers' PCK. As teachers examined student work samples during one such PLT, Deneen marked one small group's observation about one of the strategies, saying, "you're grouping your like terms here, which is where algebra is really going." Her decision supported teachers in understanding the connection between students' strategies in the elementary grades and the mathematics that will later develop as students' progress through the grades.

Marking to scaffold. Decisions to mark ideas with the goal of scaffolding teachers' learning also varied in focus across the three types of PLTs. In the

Representing PLTs, all six of these decisions were focused on mathematics and assisting teachers in engaging with the problems. For example, while teachers explored the Ostrich-Giraffe problem, Deneen made the decision to restate what teachers had recounted to her while she monitored: “So you know there’s going to be eight ostriches because this has to be a total of 15 animals.” This decision led to teachers attending to the constraint of the total number of animals in the problem.

In contrast, the foci of marking to scaffold decisions in the Decomposing and Approximating PLTs were not restricted to mathematics. During Decomposing PLTs, these decisions addressed understanding the practice of LMD and focused on supporting teachers in understanding how to enact the practice in their classrooms. For example, Deneen marked about the practice of anticipating students’ responses as a way to set up a mathematics discussion:

> So, anticipating. In the anticipating phase you’re supposed to think through the possible solutions, brainstorm a path to your learning target and create possible questions, okay. So, I wanted to share a little bit with you about my anticipations, and what I was anticipating when coming through, and the solutions I was anticipating and then how that would help me to set up my launch and then my monitoring . . . And I will tell you that what I thought and anticipated would happen did not happen that way. So that happens a lot of times so on the fly, I was on the fly quite a bit because my anticipating was a little off from where I thought it would be.

Marking for scaffolding decisions during the Approximating PLTs focused on student learning. For example, Deneen marked an idea a teacher made about how to move forward when many of the students are making common mistakes. She stated,

> And that's a different way to kind of direct the student . . . "Now, let's try to use a different model," or "let's use a different way to get you to think about it." And that might be a really—that might be a move if you have a lot of kids doing something like this is to say, "people are losing sight of the 17 and can't keep the 48," and that might be a group decision that you can kind of all go back to the problem, and whatever you sit down and do something manipulative wise to help them get to that point. So really it would depend on where the child is and where the class was to see what your next moves were.

Unlike the focus of these types of decisions in Representing PLTs, marking to scaffold decisions in Decomposing and Approximating PLTs attended to the practices of teaching and supporting students' learning.

Eliciting to probe. Decisions to elicit teachers' ideas to probe their current understandings also varied in focus across the PLT types. In Representing PLTs, all six instances again had a mathematical focus and related to a mathematical strategy for solving the Ostrich-Giraffe or Horse problems. When discussing these problems, these decisions elicited a teacher's thinking about her strategy with statements such as, "talk to me more about the table," and "tell us how you did it." During Decomposing and Approximating PLTs however, these decisions focused more on students and student thinking than on their own strategies and mathematical thinking. For example, Deneen made a decision to try and understand how teachers would continue to engage a student who arrived at an incorrect solution:

> Something that I read in here popped out in my mind about 80 minus 50 = 30. So I'm just curious, which step, if you have 1 person in your classroom as you're walking around and you see a child who does 80 - 50 = 30, he made $30, and they stop working. Which step do you address that at?

Explaining to scaffold. There was also a difference in the nature of decisions to explain ideas across the PLT types. For Representing PLTs, explanations focused on mathematics, for example ensuring that the teachers remembered that, when guessing and checking to solve the Ostrich-Giraffe problem, they could not forget the total number of 15 legs. In contrast, this type of decision during Decomposing and Approximating PLTs addressed the practice of LMD and its relation to students' mathematical thinking. For example, during a discussion during the Ostrich and Giraffe problem, Deneen responded to a question from a teacher regarding when and how to select and sequence a particular student solution:

> I would, depending. If I had several kids in my class that had that thinking and I still had people that were unresolved and they were so passionate about their response, yes I would show it and then we'd come back and I'd do the exact same thing I did with you guys, can you tell me why this doesn't work now? What was the thinking so we now understand that? But only, if I only have 1 child basically that did this, I would probably try to work with that child.

In summary, key instructional decisions differed across the PLTs in both frequency and focus. Whereas decisions when facilitating Representing PLTs focused on mathematics content, decisions made when facilitating both Decomposing and Approximating PLT addressed issues of instruction and student thinking.

Identifying Practices for Facilitating PLTs

In this section, I answer the same third research question regarding the purposes toward which facilitators coordinated their instructional decisions across the PLT types. My analysis indicated that instructional decisions were coordinated for multiple purposes that varied by the type of PLT.

Facilitating Representing PLTs

In Representing PLTs, the facilitators coordinated their instructional decisions for the purposes of engaging teachers in learning through participating in mathematics discussions and by modeling to make the practice public and available for collective discussion. Through instructional decisions with goals of supporting teachers' mathematics learning and fostering a community suited for supporting mathematics learning, they provided opportunities for teachers to experience learning mathematics in discussions and foster value for mathematical discussions as a part of learning. The following vignette illustrates the ways the facilitators coordinated their key instructional decisions.

Vignette of a Representing PLT. On day two of the week-long Summer Institute, the facilitators began with a brief discussion about their work from the previous day. After discussing the teachers' feedback in their daily debriefing and planning session for the next day, the facilitators decided that they would provide more independent thinking time and less support for solving the mathematics problems that day. They shared with teachers the nature of the feedback from the previous day and how they would alter the learning experience for the teachers on this day. Marie invited one particular teacher to share with the group why she felt that more independent thinking time was needed. That teacher expressed her desire to have more thinking time and related her experience the previous day with that of her students. She shared how she needed time to think and draw pictures to explore the problem prior to digging into the mathematics.

After eliciting the types of questions that might support students in solving the mathematical problem from the day before, Deneen marked several ideas for questions that the teachers offered. Next, she posed the Horse problem to the group. They instructed the teachers to work independently and then to work at their tables as groups. They emphasized throughout that the table must come up with a consensus. As they circulated, they noticed that several teachers at the same table had differing solutions. Teachers began to ask for assistance, but the facilitators did not engage them and continued to circulate. Once all the tables came to consensus, the facilitators advanced their goal of fostering community by marking various teachers' mathematical contributions to a discussion to position teachers as competent. They began with decisions to highlight several teachers' strategies, deliberately not verifying whether solutions were correct or incorrect, with goals of making teachers feel safe and validating their thinking. Several teachers then commented that they were certain that their solutions were correct even if they differed from others at their tables. Next, the facilitators stated that each table must come up with a true consensus so they allowed them additional time to discuss at their tables. Two particular teachers who were working together were unable to come to a consensus because they were both holding on to their own solutions. At this point, the facilitators decided to allow groups to share their solutions with the whole group.

The first group of three teachers shared their solution and determined that there was a $20 profit. They modeled the problem using strips of paper to represent the money and each person represented a buyer or seller. They visually showed each transaction.

The facilitators pressed this group regarding why they felt they needed manipulatives to make the problem less confusing. This prompted a mathematical discussion with other teachers in the whole group posing questions of the presenting group, as well as commenting on their strategy. With decisions to probe, press, and explain, the facilitators sought to deepen the teachers' MKT, clarifying the strategy and pressing teachers to use larger amounts of money to confirm their conjectures.

Next, they invited a second group to share their solution and strategy. After this group shared that they worked backwards from the $80, the facilitators elicited the teachers' reasoning to ensure that they understood the strategy that was shared by this group. After one teacher shared, Deneen oriented another teacher to the idea. As the discussion progresses, Marie positioned another teacher as competent by inviting her to share more details about the working backwards strategy.

As illustrated by this vignette, the facilitators' decisions worked together to allow teachers to engage in learning mathematics through discussions. In this way, the practice of leading discussions was made visible in the MPD and was a collective experience upon which the group could reflect. Although their facilitation of the Representing PLT was visibly identical to classroom mathematics teaching, the purpose of their practice was to not only support teachers in learning mathematics but to also represent the practice of learning the core practice of LMD for the teachers and create opportunities so that teachers can foster value of the core practice in the learning process.

Facilitating Decomposing and Approximating PLTs

In the Decomposing and Approximating PLTs, the facilitators' instructional decisions worked together to make explicit the components of the practice of LMD and underscore how various instructional moves relied on students' mathematical thinking. Their main goal for these PLTs was to have teachers about LMD. The facilitators' decisions made during the Decomposing and Approximating PLTs supported the purposed of breaking down the core practice into smaller grain sized strategies. These decisions also provided opportunities for teachers to experiment with the core practice grounded in students' mathematical thinking as illustrated by the following vignette from later on the second day of the Summer Institute.

Vignette of a Decomposing and Approximating PLT. After engaging teachers in the Horse Problem during the Representing PLT, the facilitators reminded teachers of the importance of anticipating when preparing to lead a discussion. When one teacher stated that she did not feel very strong in mathematics and was concerned that she could not anticipate incorrect responses, Marie marked her concern and invited others to share their own. Deneen then elicited teachers' thinking about how a student might get a solution of plus 80 to the Horse problem. After several teachers offered their ideas, Deneen then explained the challenges students have with keeping track of buys and sells.

In the Approximating PLT that followed, teachers prepared to lead a hypothetical discussion by selecting and sequencing various samples of students' written work. After explaining that there was no correct or incorrect way to select and sequence and instructing teachers on the process for the activity, the facilitators monitored as teachers discussed the work samples and developed a plan for the discussion. Next in a whole-group discussion, the facilitators invited groups to share different sequences, pressed them for their reasoning, elicited their understandings of what the students might know, and explained what evidence in the work samples suggested about students' mathematical thinking. Throughout, their decisions worked to foster a safe learning environment teachers shared their discomfort with some of the approaches. When a particularly sophisticated approach taken by a student came up, the facilitators quickly related their confusion to their facilitation of the Horse problem during the Representing PLT, validated their fears, but explained how preparing to lead discussions by anticipating, selecting and sequencing, and eliciting students' thinking can help one be prepared.

As groups shared their understandings of the work samples and the student's thinking, the facilitators elicited their understandings of the practice of LMD with questions such as, "What should you do when several students have the wrong solution?" and "When and how do you let students know that their solution is incorrect?" As teachers continued to share their order for the discussion and their conjectures about the mathematical thinking represented in the work samples, the facilitators marked teachers' contributions about teaching to the discussion and elicited their understandings of how students' thinking related to the practice.

Following the Approximating PLT, teachers read an article on leading discussions. After reading, the facilitators invited several teachers to react to the article. First, several teachers discussed their challenges with anticipating incorrect solutions or different strategies. Marie marked these challenges to position teachers as safe and a part of a community learning together. She reminded them that they have deep knowledge about the curriculum and what students should know at each grade level and explained that they already anticipate, and that focusing on the practice of preparing for and leading discussions together could help them improve. Later in the discussion, another teacher posed a question about students finishing before other students. After inviting other teachers to respond, both Deneen and Marie offered thoughts on ways to support students in remaining engaged. Throughout the discussion of the article, the facilitators marked important contributions made by teachers relating to leading discussions and relating these ideas to the experiences that teachers had when engaging in the Horse problem.

This vignette shows how facilitators' instructional decisions during Decomposing and Approximating PLTs worked together for the purposes of linking the practice of LMD to students' mathematical thinking and relating their experiences as a learner in discussions to their students and their own teaching. Through making its components explicit and supporting teachers in engaging with the practice as a teacher, the purpose of their practice was to support teachers in understanding how to lead discussions built upon students' mathematical thinking, engaging in the practice, and reflecting on their own practice in relation to their learning in the MPD. Unlike facilitating Representing PLTs, facilitating Decomposing and Approximating PLTs did not resemble classroom mathematics teaching. Instructional decisions oriented teachers to students' mathematical thinking, explicitly focused on ways of building a mathematics discussion from students' work.

CHAPTER V

DISCUSSION

This study explored the nature of facilitating practice-focused mathematics professional development (MPD) for elementary grade teachers in order to identify and describe the instructional decisions by facilitators to support teacher learning. Through an examination of expert facilitation, this case study documented facilitators' instructional decisions across three types of PLTs, and how these decisions were purposefully coordinated for different purposes. The final chapter of this dissertation is organized into three sections. First, I answer the three specific research questions that guided the study and integrate the findings to describe the nature of instructional decisions for facilitating mathematics professional development. Next, I situate the results of this study within the existing literature on professional development facilitation and research on mathematics teaching. Finally, I conclude with implications for research, policy, and mathematics teacher educators.

Facilitating Mathematics Professional Development

Although there is extensive research on MPD in general, there are very few studies that seek to examine the practices of facilitation that support teacher learning. This research was guided by three specific research questions that investigated the instructional decisions made by expert facilitators, the variation of these decisions across different types of PLTs, and the purposes of these decisions. In this section, I summarize key findings from the analysis to answer each research question. I then synthesize these results to characterize two practices that support teacher learning when facilitating practice-focused MPD.

The Nature of Instructional Decisions

The facilitators of these practice-focused PLTs on LMD utilized seven distinguishable instructional moves. These moves were marking, eliciting, explaining, pressing, instructing, orienting, and inviting. These instructional moves are not unique to facilitation as they are similar to the moves identified in research on mathematics teaching. Four of the moves—marking, eliciting, explaining, and pressing—were specific to either the mathematics content or the core practice of LMD. The other three moves – inviting, instructing, and orienting – were not content or practice-specific.

Specific goals were identified for each of the instructional moves made by the facilitators during the MPD. The seven goals include: scaffolding, probing, deepening MKT, relating, positioning, establishing norms, and fostering community. Four of the goals were content-specific to supporting teacher learning. The other three goals were specific to fostering community among the learners.

The goals for which a particular instructional move was made by the facilitators constituted and instructional decision. In this study, there were a total of 49 possible instructional decisions that the facilitators could have made. They actually made a total of 43 of these possible decisions. Of these 43 possible instructional decisions, there were eight key instructional decisions which supported teacher learning of LMD.

In this study, the nature of the instructional decisions that facilitators made when implementing practice-focused PLTs on LMD supported teacher learning or fostered a learning community. These instructional decisions included moves similar to those used by classroom teachers aligned with goals some of which were content or classroom mathematics teaching focused. There are eight key instructional decisions were pervasive, and these decisions represented 60% (145) of all decisions made when facilitating the PLTs. These key decisions were: marking to scaffold, marking to establish norms, marking to deepen MKT, eliciting to probe, marking to relate, explaining to scaffold, explaining to position, and marking to position.

Variation Across Professional Learning Tasks

In this study, the instructional decisions differed in both frequency and focus by PLT type, with the differences occurring between the Representing PLT and both Decomposing and Approximating PLT. There was no significant difference between the Decomposing and Approximating PLT. In addition to the differences in frequency, the focus of the instructional decisions also varied with the focus during the Representing PLT on mathematics and the focus of Decomposing and Approximating PLT on LMD grounded in student mathematical thinking. To answer this research question, quantitative data were analyzed using Fisher's Exact Test with Boniferroni correction. Qualitative analysis was also used to answer the second part of this question.

Characterizing Purposes of Instructional Decisions

Results from this study indicate the practice of facilitating Representing PLTs may look very similar to classroom mathematics instruction in many ways. In this case study, teachers were given the opportunity to deepen their MKT in mathematics discussions. Facilitators used these learning experiences to support teachers' learning about the core practice of LMD by relating their learning experiences to the experiences of their students. Research suggests that when teachers recognize the value of the discussions to their own learning and understanding of the mathematics, they are more likely to implement the core practice in their classrooms (Boerst et al., 2011). Unlike mathematics instruction, which has the sole purpose of deepening content knowledge, the instructional decisions made by the facilitators in the Representing PLT in this study had the additional purposes of making the core practice visible and valuable.

Results from this study suggest the practice of facilitating Decomposing and Approximating PLTs differs from mathematics instructional practices in both focus and purpose. Within this study, the majority of the instructional decisions made during the PLTs focused teachers on students' mathematics thinking by probing their understandings and scaffolding their learning of the core practice. Unlike classroom mathematics teaching, the focus of these PLTs was not mathematics content but rather how students think about the math and strategies to support their learning. When leading these PLTs, facilitators centered discussions on the core practice and its relation to students' mathematical thinking.

In summary, results of this study suggest that in many ways, instructional moves of facilitation are similar to mathematics teaching. While modeling the core practice of LMD, facilitators engaged teachers in mathematics to provide opportunities to experience the core practice as a learner. However, a distinguishing difference between classroom mathematics teaching and MPD facilitation are the purposes toward which the decisions are made. In addition to engaging teachers in mathematics, facilitators worked to make the core practice clear and observable. In doing so, they provided common experiences for teachers that serve as a basis for discussion of the practice and create opportunities to reflect on the value of core practices in learning. Other facilitation practices were distinct from research on classroom mathematics teaching. Although moment-to-moment moves were similar, facilitators' goals for these moves, and the ways the moves were coordinated, differed. These practices of facilitation aimed to provide opportunities for teachers to learn about the core practice by closely reflecting, examining, and experimenting.

Discussion

With little research addressing MPD facilitation, researchers have looked to classroom mathematics teaching to inform research about facilitation. Although such an approach proved valuable, this study identified ways in which the practices of facilitating are distinct. Findings from this study indicate facilitating utilize extensive MKT to support teachers in reflecting on, examining, and experimenting with instructional practice. Similar to Elliott et al.'s (2009) study of facilitating MPD, the facilitators in this study understood and addressed teachers' various mathematical understandings to their learning. They used mathematics problems and PLTs to focus teachers on big mathematical ideas rather than specific concepts. Throughout this study, the facilitators made in-the-moment instructional decisions to support teachers' learning by attending to teachers' mathematical thinking and misconceptions.

Additionally, findings from this study provide additional evidence for the critical role of facilitators' MKT outlined by Borko and her colleagues (2014). In their examination of the types of supports needed by MPD facilitators, they argued that the mathematics education community needs a way to discuss the types of mathematical knowledge needed for MPD facilitation. In their proposed framework (see Figure 1), they specify that facilitators must be able to anticipate possible solutions strategies that teachers will offer when planning for practice-focused MPD. Likewise, facilitators need "the ability to engage teachers in the interpretation of students' mathematical ideas and purposeful analysis of instructional practices" (p. 165).

Facilitators drew upon their extensive knowledge of mathematics, classroom mathematics teaching. Eighty-eight percent or 213 out of 242 of their instructional decisions were either about mathematics or classroom mathematics teaching (see Table 5 and Table 6). Facilitators' use of MKT is evidenced by their strategic selection or construction of learning tasks that considered the learning goals for teachers as wells as providing opportunities for teachers to 'stumble' while solving which deepens the teachers' mathematical understanding. For example, the facilitators chose the Ostrich-Giraffe problem and the Horse problem because they lend themselves to varying or alternative strategies to solve. The facilitators launched both problems in such a manner that they were able to help teachers understand not only their own misconceptions but possible student misconceptions. Using their MKT, they were able to support teachers in deepening their own MKT. Using their extensive MKT, the facilitators used their extensive MKT to plan MPD that was both engaging and supportive of teachers' learning. When providing opportunities for teachers to ask questions of each other to clarify the others' solutions, the facilitators had to make in-the moment decisions to ensure that the questions were clear, concise, and targeted to support the learning of all. Finally, facilitators' use of MKT is evidenced by the explicit explanations provided about the connection of the learning task and the teachers' classroom mathematics teaching ensuring that all teachers have access to the content in meaningful ways. For example, Deneen states,

> But Melissa brought up such a good point in this group we were talking because she said I've done this problem before with cows and chickens, y'all have all seen that problem, I've done this before with cows and chickens . . . but it's the same way I think what we do with our kids a lot of times is we well, I showed them that strategy. Why did they not use them? I taught them that lesson, why are they not using it? And we revert back to what we're comfortable with . . . It may not take 10 exposures, you know, how do we help kids?

Results also suggest that facilitators also drew on their knowledge about the teachers and the contexts of teachers' work to make decisions about supporting teacher learning. Similar to Linder's (2011) findings, the facilitators' credibility was crucial and was established and maintained with their knowledge of the content, professionalism, and experience. Confidence in a facilitator is important, because teachers must feel secure to take risks, share their practice, and invest effort in changing their practice. Linder (2011) also found that teachers valued facilitators who were able to answer their questions about content or help them find the answer for themselves, as well as those who shared the latest and newest educational practices.

The facilitators in this study had knowledge and experiences working with these 15 teachers prior to the Core Math II. Deneen and Marie had led previous MPD including Core Math I project. Additionally, because of their roles in school districts, the facilitators had extensive knowledge about the curriculum, the school climates and cultures, as well as administrative expectations and demands established for the teacher participants. This professional relationship allowed them to build interpersonal relationships within the MPD. Both professional and interpersonal relationships with the teachers helped the teachers build confidence that their contributions to the mathematical discussions were welcomed and appreciated.

The facilitators also share similar teaching experiences as the teachers since both Deneen and Marie are former elementary mathematics teachers. Using their personal experiences, they created a learning community with the teachers and were viewed as credible sources of information about improving instructional practice. This is evidenced in the free exchange of ideas, questions, and engagement in the mathematical discussions by all the participants. The importance of teachers collaborating in an honest, reflective manner is well documented in research as being pivotal to the success of professional development (Burbank & Kauchak, 2003; Johnson, Fargo, & Butler Kahle, 2010; Butler, Lauscher, Jarvis-Selinger, & Beckingham, 2004; Cawelti, 2003; Fullan, 2004; Fullan & Stiegelbauer, 1991; Robbins & Alvy, 2009; Yang & Liu, 2004). This type of exchange can only occur when the facilitators are viewed as being credible by the teachers and thus trustworthy (Linder, 2011). Trust occurs when there is mutual respect among the members of the learning community (Wenger et al., 2002). Through their long standing relationships with the teachers, understandings of their school and district contexts, and their experiences as elementary mathematics teachers, Deneen and Marie nurtured a community of teachers of which they were a part. Facilitation required extensive use of MKT and experiences with teaching mathematics are two factors that distinguish facilitating from mathematics teaching.

Implications and Further Research

There is much to learn about the practices facilitators use in MPD to support teacher learning. Results from this study add to the small research base on facilitation by describing the ways in which facilitation is similar to, and different from, mathematics teaching. Conceptualizing the core practices of teaching as a hierarchical modular system, Janssen, Grossman, and Westbroek (2015). was analytically useful to show that though the instructional moves made when facilitating MPD are identical to those made when teaching mathematics, some of the goals for these moves are quite different. Moreover, the cumulative result of these instructional decisions resulted in facilitation practices with purposes that move beyond deepening content knowledge to include relating content, students' mathematical thinking, and instructional practice.

For policy, school, and district leaders making decisions about MPD, the results of this study indicate careful attention should be given, not just to a particular PD program, but also the professional who will facilitate it. Though quality professional development can lead to improvements in instructional practice and student learning (Darling-Hammond et al., 2009), allocating resources and providing high-quality, effective professional development opportunities for teachers remains problematic for education leaders. Though common solutions include commercially available programs, instituting mandated professional learning communities in schools, or having teachers develop and lead MPD, this study suggests the importance of the knowledge, experiences, and relational skills of those facilitating these solutions. Deneen and Marie drew upon their MKT, knowledge of teachers' contexts, and their relationships to facilitate teachers' learning in this study. The practice of facilitation, which was shown to be distinct from the practice of mathematics teaching, contributed to participating teachers' improvements in their own MKT and their classroom instruction (Floyd, forthcoming; Rich, forthcoming).

The findings of this study also have implications for mathematics teacher educators working to prepare MPD facilitators. Practices for facilitating MPD include, but are not limited to, instructional practices of mathematics teaching. As such, identifying accomplished teachers is only a first step in preparing MPD facilitators. Mathematics teacher educators should work with those preparing to lead MPD to deepen their MKT, reflect upon their own experiences with teaching, and develop skills for both becoming familiar with the contexts of teachers and building and sustaining the kinds of professional relationships that support teachers in instructional change.

Finally, results of this study have a number of implications for mathematics teacher education researchers. The limited research on facilitation has used research on mathematics teaching to inform studies of facilitation (Elliott et al., 2009). While this approach was useful for this study, its outcomes suggest there are additional facets of the practice of facilitation that need examination. In addition, future research should examine facilitation of different types of PLTs, in various settings in which MPD takes place (Rich, forthcoming), across different PD programs (Borko, 2004), and across time (Floyd, forthcoming). For example, Deneen and Marie used PLTs that engaged teachers in mathematics problems as learners to represent practice. Additionally, researchers may examine further the nuanced differences in the instructional decisions between Decomposing and Approximating PLTs. Researchers might also investigate PLTs that use classroom videos to represent practice, for example, to elaborate or differentiate characteristics of facilitation practices identified by this study. Others might examine the ways facilitators represent practice with case studies (Stein, Smith, Henningson, & Silver, 1999). Another approach to learn more about the practice of facilitation would be comparative studies of facilitating established models of MPD designed to support teachers' learning of mathematics, instructional practices, and students' mathematical thinking, such as Borko et al.'s (2014) Problem-Solving Cycle or van Es and Sherin's (2008) Mathematics Video Clubs.

Summary

The purpose of this study was to examine the practices of facilitating effective MPD. Informed by the limited research on MPD facilitation and research on mathematics teaching, this case study investigated the instructional decisions made during three types of practice-focused PLTs. Several ways in which practices of facilitation are similar to, and different from, classroom mathematics teaching were identified, suggesting different goals for the instructional moves made and broader overarching purposes for the practices. These findings contribute to the research about facilitation beginning to emerge, provide additional direction for education leaders making decisions about MPD, and offer direction for mathematics teacher educators in preparing mathematics teachers to become professional development facilitators.

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APPENDIX A

INTRODUCTION INTERVIEW PROTOCOL FOR DENEEN AND MARIE

1. Please introduce yourselves for me.
2. Tell me about your experience just in education, teaching experience, and what brought you to core math 2.
3. What was your role in your school and with the Core Math II project?
4. What are your previous experiences with mathematics professional development as a teacher?
5. What aspects of these experiences would you say contributed most to your learning as a teacher?
6. Do you believe that Mathematics Professional Development make a difference for teachers? If so in what ways? If not, why not?
7. What is your philosophy on teacher learning? How do teachers learn? Is there a difference in the learning of pre-service and in-service teachers?
8. When you are designing, planning and facilitating a professional development, how do you ensure that you have the components that you believe are necessary for teachers to learn?
9. I know you cannot always prioritize, but if you had to prioritize one that you think has a greater impact on teacher learning?
10. So what is your ultimate goal with Core Math II? What do you want teachers to know when they leave at the end of the Core Math II project?

APPENDIX B

DEBRIEFING/PLANNING SESSION INTERVIEW (AUGUST 4)

1. I noticed that you have the norms as part of the presentation for each day of the PD. Why is this important to you?
2. I noticed that you collected comment cards each day of the PD. What information was hoped to be gained by doing this? How is this information used for planning?
3. What is your evidence that the teachers are learning? Is it their language, their practice, or the way that they participate in the PD?
4. What will be your evidences of implementation?
5. Why did you go back and review the orchestrating classroom discussions if this is something that they learned in Core Math I?
6. Describe the differences in the content in Core Math II from Core Math I?
7. Describe the three phases for Core Math II? How will each phase support teacher learning?
8. During the PD, you did a task called ‘Crossing the River.’ What was the learning goal of this task?
9. I noticed that one of the participants (Tyler) did not collaborate well during that task. Share your strategies for supporting her in feeling more comfortable being collaborative with the rest of the group?
10. Can you identify a couple of key learning moment when you realized that teachers or administrators were learning?

11. I noticed that some of the participants, especially one administrator felt uncomfortable watching one of the videos with the children thinking through a problem?
12. Why did you choose to address her in this manner?
13. Do you think she now has a better understanding of the importance of a classroom closing discussion?
14. What is the role of differentiation in MPD? How do you plan for this?
15. What does your reflection and planning sessions look like?
16. What type of decisions do you make during these sessions?
17. When do you know that you need to make adjustments to the PD that you've planned?

APPENDIX C

PROTOCOL FOR DENEEN AND MARIE INTERVIEW QUESTIONS (MAY 17, 2014)

Background information

1. Please introduce yourselves again for me.
2. Tell me why you decided to become a mathematics professional developer
3. Tell me what you know to be true about teachers? MPD? Students?

Prior experiences as a learner in PD?

1. Describe some of your experiences with professional development in mathematics.
2. What tasks were you asked to do?
3. Describe the structure of the experience.
4. Describe the environment in which the PD occurred.
5. Was there a time when you did not get a lot of a professional development experience?
6. Describe this experience.
7. Why was it not rewarding?
8. Did the facilitator do anything to hinder this experience? What?

Beliefs about teaching and learning

1. What makes a good mathematics teacher?
2. How do you measure student learning?

3. What does it mean when the assessment data does not match the instructional practices? What could be some reasons for this?
4. What is the teachers' role in the classroom in supporting relationship building among students?

Beliefs about facilitation

1. How would you define a mathematics professional development facilitator?
2. Does the facilitator of MPD have a responsibility to motivate their participants? If so how?
3. What does it mean to be credible as a MPD facilitator?
4. Do you consider yourself a credible MPD facilitator? If so why?
5. Can credibility be loss or gained? If so, how does this occur?
6. What does professionalism in MPD mean? What does this look like?
7. What is the role does personality?
8. What is the role of humor in effective MPD? Give examples of when using humor contributed to the learning experience of the participants?
9. How does one learn to become an effective MPD facilitator?

Role as MPD facilitator and Examples

1. What makes MPD different from PD in other content areas?
2. What knowledge do you believe is essential for MPD facilitators to possess in order to be effective?
3. Give an example from your experience in which this knowledge was pivotal in the learning experience of the participants.

4. What does it mean to support participants in MPD? What does this look like? Give examples of how this has been done?
5. What are some of the management/logistical components of MPD that you feel impact the effectiveness of the MPD?

Core Math II

1. Describe your participants
2. Describe your relationship with your participants
3. What was your role in building the working relationship with your participants? What were some of the challenges?
4. What was the impact of these participants being chosen have on the relationship?
5. How has the relationship dynamics contributed to the learning experiences of the participants? Give examples?
6. Looking back over the last two years of the Core Math experiences, is there anything that you would have done differently, more or less to enhance the relationship with the teachers in this experience.
7. What are the power dynamics of the relationship that you have with your teachers in this experience?
8. What is the role of discussion in your Core Math PD? Was this by design? Why?

9. Looking back over the last two years, is there anything that you would have done differently, more or less to increase the level of deep discussions about the learning of the participants?
10. Were there norms that you set during the Core Math II? Were the norms social? Professional? If so, what were they? Why did you choose these? How did you set them?
11. How would you describe the receptiveness of these norms by the participants?
12. Describe for me how you and the participants 'do' mathematics within the PD?
13. How do you handle mathematical errors made by teachers while doing math in the MPD? Why?
14. Were there times when participants did not adhere to the established norms? How did you handle this?
15. What do you what your teachers to learn from Core Math II? If they only walked away knowing one thing, what would that be?

APPENDIX D

INSTRUCTIONAL DECISION CODES

Decisions that are domain-specific and foster learning	Marking to scaffold Eliciting to scaffold Explaining to scaffold Pressing to scaffold Marking to Probe Eliciting to probe Explaining to Probe Pressing to Probe	Marking to deepen MKT Eliciting to deepen MKT Explaining to deepen MKT Pressing to deepen MKT Marking to relate Eliciting to relate	Explaining to relate Pressing to relate
Decisions that are domain-specific to foster community	Marking to position Eliciting to position Explaining to position Pressing to position	Marking to establish norms Eliciting to establish norms Explaining to establish norms Pressing to establish norms	Marking to foster community Eliciting to foster community Explaining to foster community Pressing to foster community

Decisions that are domain-neutral that foster learning	Instructing to scaffold Orienting to scaffold Inviting to scaffold Instructing to probe Orienting to probe Inviting to probe	Instructing to relate Orienting to relate Inviting to relate
Decisions that are domain-neutral that foster community	Instructing to position Orienting to position Inviting to position Instructing to establish norms Orienting to establish norms	Inviting to establish norms Instructing to foster community Orienting to establish community Inviting to establish community

APPENDIX E

OSTRICH-GIRAFFE AND HORSE PROBLEMS

Ostrich-Giraffe Problem

A zoo has several ostriches and several giraffes. They have 30 eyes and a total of 44 legs. How many ostriches and how many giraffes are in the zoo?

Horse Problem

A man buys a horse for $50.00. So remember he buys the horse for $50.00. He turns around and he sells it for $60.00. Then he thinks nah, nah I really need that horse back so he goes back and he buys it but the man says, oh $70.00. So he buys the horse back for $70.00. And then he thinks about it and says you know what I really didn't need that horse after all. So he sells it for $80.00 his last time. So I need you to think through what happened with all of the transactions. Did the man end up making money, losing money, or breaking even?