Book Reviews: Polynomial and Matrix Computations — Volume 1: Fundamental Algorithms. By *Dario Bini* and *Victor Pan*. Birkhauser, 1994. xvi+415 pages. ISBN 0-8176-3786-9. $64.50.

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S. R. Tate. Review of Polynomial and Matrix Computations; Volume 1: Fundamental Algorithms by Dario Bini and Victor Pan. Appeared in SIGACT News, Vol. 26, No. 2, June 1995.

Article:

The past few decades have produced a wealth of interesting and useful work in the area of algorithms for algebraic, symbolic, and numerical computing. Unfortunately, there has been a huge void in the area of books summarizing and bringing together the core results of algebraic computation, with the two main exceptions being Borodin and Munro's excellent book *The Computational Complexity of Algebraic and Numeric Problems* (published in 1975), and selected chapters of the oft-cited algorithms book by Aho, Hoperoft, and Ullman (published in 1974). Both of these books are now over two decades old and cover fairly small sets of selected topics.

To fill this void we are now greeted with the release of a book covering the basic, foundational material of the algebraic algorithms field, written by authors who are leading researchers in the field and are responsible for many of the current best algorithms. *Polynomial and Matrix Computations* covers the topics in the title, using both sequential and parallel models of computation, and with most attention given to operations on dense structured matrices (Toeplitz, Hankel, Vandermonde, etc.), and their relations to polynomial problems. Also included is a chapter on algorithms under a bit-cost model of computation, which might not be guessed from the title of the book, but is an interesting and welcome addition. Clearly, a single book cannot be everything to everyone, and one goal of this review is to identify what audiences this book successfully addresses, and those which it doesn't.

For researchers in the field of algebraic algorithms, this is a "must-have" book, both as a reference and as a review of basic material. While reading the book I constantly found myself re-discovering results that I had forgotten, and even discovering some new results. The coverage is quite complete, and includes some material that I was quite pleasantly surprised to find; for example, the inclusion of Schwartz's results on randomized polynomial identity testing, the treatment of general algebraic Newton iteration, the inclusion of material on the computation over arbitrary rings (including those that do not support FFT), and the chapter on bit-cost algorithms are all very welcome topics. The inclusion of parallel algorithms is not surprising but still very nice, as the only previous treatment of parallel algebraic algorithms has been as topics in general parallel algorithms books (such as the books by JáJá and Leighton, and the book edited by Reif).

Topics in the book are very thoroughly cross-referenced both to other topics within the book and to outside sources. While this is extremely nice for someone researching a specific topic, the inclusion of *so* many cross-references is actually quite distracting when trying to read completely through the book. For a specific example, in one 9 line paragraph of explanatory material the authors have 11 references to other parts of the book, and 8 references to outside sources.

For reasons of presentation, I don't feel the book serves well for teaching. The book simply doesn't read easily enough to capture most students' attention. Furthermore, while many of the results are quite beautiful, the authors seem more interested in simply conveying the mathematical facts rather than taking time to point out the beauty of the results. For example, the authors mention that there can be no finite rational algorithm for exactly computing the roots of an arbitrary polynomial (this follows from the fact that even for degree 2 polynomials the roots may be irrational), and a few pages later give the well-known method for computing sums of powers of the roots of a polynomial. While these are fascinating results, the juxtaposition of them is even more fascinating, and yet no time is taken to point out how interesting this is. Furthermore, this might have been a good place to put a discussion of more powerful models of computation that allow taking arbitrary roots, and that while all zeros can be computed for polynomials of degree up to 4, such an algorithm is not possible for degree 5 or more (this is the Galois/Abel result on the insolvability of degree 5 polynomials). While such an omission doesn't matter much to a researcher familiar with the area, if the book is to be used as a text for students it's important to take time to simply marvel at some beautiful results.

As for the target of the book with respect to teaching, I leave it in the authors' own words (from the Preface):

> The authors' teaching experience at the Graduate Center of the City University of New York and at the University of Pisa suggests that the book may serve as a text for advanced graduate students in mathematics and computer science who have some knowledge of algorithm design and wish to enter the exciting area of algebraic and numerical computing.

While I don't disagree with that assessment of the book's potential use, what bothers me is that the background required for the *substance* of the book is not as extensive as suggested in that sentence. A student with rudimentary knowledge of abstract algebra and an undergraduate-level knowledge of algorithm analysis should be prepared for the material of the book (certainly the first two chapters: 227 pages). However, the *presentation* of the book's material is perhaps not appropriate for students without a high degree of mathematical sophistication. In fairness to the authors, it doesn't appear that they intended to write a book to the less advanced audience, but due to the lack of books for this purpose I feel that a warning is in order for people looking for a book to fill this niche.

Technically, the book is outstanding. Many such technical books are released with large numbers of errors, both mathematical and typographical. Bini and Pan's book suffers from very few such problems. There is an occasional minor typographical slip (writing n + 1 as opposed to n, for example), but overall the book seems to have been proof-read and edited well. I think more care should have been given to the basic algebraic definitions and properties, as they did for the linear algebra material at the beginning of Chapter 2 — for example, the authors don't give a good general definition of the characteristic of a ring, but rather simply define the characteristic with respect to the rings that are of interest in the book (the "number rings" — Z_m, Z, R, and C). While most readers should be familiar with the notion of the characteristic, a marginal amount of extra explanation could have defined the characteristic in full generality, and then used that definition to give the characteristic of rings that are interesting in this book. The terminology is a little confusing in a few isolated parts of the book — for example, in the material on computations over arbitrary rings, Bini and Pan use the term "generator" in a non-standard way, which is momentarily confusing.

In conclusion, for researchers in the field of algebraic computing, I highly recommend this book as an essential addition to your bookshelf. For people interested in learning the field, I would recommend starting off mastering the basic material in some of the earlier books mentioned in the first paragraph of this review. With that amount of basic background, I think strong students should be able to handle (and enjoy) the material in Bini and Pan's book. What I cannot recommend is using this book as a first introduction to the entire area of algebraic algorithms — the book simply doesn't have enough patience for such readers. What's really unfortunate is that there *are no* books that are current and serve this purpose!