Optimal trading ratios for pollution permit markets

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Abstract:

We demonstrate a novel method for improving the efficiency of pollution permit markets by optimizing the exchange of emissions through trade. Under full-information, it is optimal for emissions to exchange according to the ratio of marginal damages. Under asymmetric information, we derive necessary conditions for the marginal damage trading ratios to be optimal, illustrate that the marginal damage trading ratios are generally not optimal, and show how to improve efficiency using optimal trading ratios. We calculate the optimal trading ratios for a global carbon market. The gains from using optimal trading ratios rather than marginal damage trading ratios range from substantial to trivial, which suggests the need for careful consideration of asymmetric information when designing permit markets.

Keywords: Pollution markets | Asymmetric information | Trading ratios

Article:

1. Introduction

Incentive-based environmental regulations, such as permit markets or emissions taxes, have typically been designed to minimize the costs of achieving emissions targets.[2] Focusing on reducing abatement costs simplifies program implementation by eliminating the need to quantify damages from emissions of pollution. However, advances in air and water quality modeling now make it feasible to estimate damages precisely and thereby to incorporate them into program design. This suggests that regulators should turn from the narrow criterion of minimizing abatement costs to the more general criterion of efficiency that accounts for both abatement costs and damages (Muller and Mendelsohn, 2009).

We analyze a novel method for improving the efficiency of pollution permit markets by optimizing the way in which emissions are exchanged through trade. In our model there is asymmetric information between the regulator and the regulated sources of pollution (à la Weitzman, 1974), and the sources can be differentiated by the number of permits they are required to hold for each unit of emissions (à laMontgomery, 1972). When sources trade permits, these differentiated requirements govern the exchange of emissions, and hence are typically called trading ratios. Several recent studies have shown that selecting trading ratios equal to the ratio of expected marginal damages can substantially increase efficiency relative to the one-for-one trading found in many permit markets (Williams, 2002, Farrow et al., 2005, Muller and Mendelsohn, 2009, Henry et al., 2011 and Fowlie and Muller, 2013). Taking this as a point of departure, we ask if further efficiency improvements are possible. The rather surprising answer is yes. We derive necessary conditions for the marginal damage trading ratios to be optimal and characterize the optimal trading ratios. These results show that the optimal trading ratios generally depart from the marginal damage trading ratios.

The reason that marginal damage trading ratios may not be optimal is the presence of asymmetric information about the costs of reducing pollution between the sources and the regulator that designs the market. Indeed, in a first-best environment with full information, the marginal damage trading ratios are optimal. However, permit markets are generally employed to allow firms to respond flexibly to private information about their abatement costs. This information is typically not available to the regulator when the regulator designs the program (Weitzman, 1974). In such a second-best environment, the regulator must account for the damages from pollution as well as the uncertainty about abatement costs when selecting the optimal trading ratios.

To understand how this leads to a divergence between the optimal trading ratios and the marginal damage trading ratios, and to see the difference between our approach and other methods of accounting for asymmetric information, it is helpful to consider uniformly mixed pollution such as greenhouse gas (GHG) emissions. Here marginal damages are equal across sources, and the marginal damage trading ratios actually imply one-for-one trading. But one-for-one trading is generally not the most efficient structure. Due to the asymmetric information, the regulator cannot set the aggregate permit endowment (i.e. the “cap”) at the ex post optimal level. The cap is either too tight, in the case abatement costs are higher than expected, or is too loose, in the case abatement costs are lower than expected.

There are several ways to approach this problem. One might seek an answer in the mechanism design literature.[3] Here the sources would provide a report about their private information to the regulator in advance of the market. The regulator would design the market based on the information provided by the sources in such a way that the sources have the incentive to truthfully reveal their costs. In practice however, mechanisms of this type have not been used for permit market design, perhaps because they greatly increase the complexity of the market. Another approach is to allow the cap to change in response to market conditions.[4] For example, implementing a price ceiling allows the cap to expand when abatement costs are high, which improves efficiency. Several proposed permit markets have included provisions for a price ceiling. But the efficiency gains of a price ceiling may be mitigated by speculative attacks on it (Stocking, 2012 and Hasegawa and Salant, 2014), and adding a price ceiling to a permit market once again increases its complexity. These issues with the standard approaches suggest scope for an alternative way to improve efficiency that retains the simplicity of a basic permit market, at least from the point of view of the sources.

Our approach is to improve efficiency by adjusting the trading ratios away from marginal damages. This creates flexibility in total emissions even though the number of permits is fixed at the cap. For example, if a firm with a relatively low trading ratio sells a permit to a firm with a relatively high trading ratio, then the total emissions of pollution decrease. By selecting the trading ratios optimally, the regulator can, in effect, allow increased emissions when the costs are high and require decreased emissions when costs are low. Although this argument is most intuitive for uniformly mixed pollution, the general point applies to non-uniformly mixed pollution as well. In either case, efficiency can be improved by using optimal trading ratios rather than marginal damage trading ratios.

The importance of our analysis of optimal trading ratios is buttressed by three observations. First, regulators are incorporating trading ratios into a variety of existing and proposed permit markets. Despite this growing interest, optimal implementation of trading ratios has not been studied. Second, regulators are grappling with how to regulate non-uniformly mixed pollution. Permit markets with trading ratios are well suited for this task. Third, proposed markets to limit GHG emissions would swamp existing permit markets in size and scope. The massive scale of such programs implies that efficiency gains from using optimal trading ratios could be quite large in absolute terms, even if they are small in relative terms.

Given these observations, it is not sufficient to just delineate the optimal trading ratios, we must also investigate the practical importance of using optimal trading ratios rather than marginal damage trading ratios. We accomplish this through the numerical analysis of a multi-country carbon emission market. We show that the optimal trading ratios lead to efficiency improvements relative to marginal damage trading ratios. The magnitude of these improvements varies from significant to trivial, depending in particular on the regulators' uncertainty about abatement costs. This suggests that regulators should give careful consideration to the structure of asymmetric information when designing future permit markets.

Our analysis combines two prominent strands of the literature on incentive based regulations. The first is based on Montgomery (1972), who formally introduced the idea of trading ratios in permit markets. Montgomery recognized that, if damage from pollution differs across sources, then emissions licenses should not simply trade one-for-one. His proposed trading rules are consistent with marginal damage trading ratios.[5] The second is based on Weitzman (1974), who introduced the idea of informational asymmetries in permit markets. Because the parameters of permit markets must be set potentially years in advance, the regulator lacks information which will be available to market participants when they make abatement decisions. This asymmetric information has important implications for the choice of policy instruments and the resulting literature on "prices vs. quantities" is vast.[6] We are interested in a different question: What happens to Montgomery's trading ratios when we apply Weitzman's fundamental insight about asymmetric information? There has not been a systematic study of this issue.[7]

The authors who come closest to disentangling the relationship between trading ratios and asymmetric information are Fowlie and Muller (2013). In analyzing a model with quadratic abatement costs and linear damages, they observe that, under asymmetric information, the marginal damage trading ratios may not perform as well as simple one-for-one trading. This suggests, of course, that there may be a completely different set of trading ratios that dominate either benchmark. But they do not pursue this line of inquiry. To replicate their observation, we construct a simple numerical example in which one-for-one trading does indeed dominate the marginal damage trading ratios. We go on to calculate the optimal trading ratios and show that they perform better than either the marginal damage trading ratios or one-for-one trading.

Section 2 presents the model and derives the main results. We describe necessary conditions for the marginal damage trading ratios to be optimal and characterize the optimal trading ratios. We additionally analyze source-specific taxes and show that they generally depart from expected marginal damages. Section 3 analyzes a special case of the model in which the abatement costs and damages have the familiar linear-quadratic form. This additional structure enables us to determine necessary and sufficient conditions for the optimality of marginal damage trading ratios. In Section 4 we discuss information issues associated with implementing both marginal damage and optimal trading ratios. In Section 5, we use a simple two-source example of a linear-quadratic model to provide a numerical illustration of the main results. Section 6 presents a preliminary calculation of the gains from optimal trading ratios for a hypothetical global carbon trading market. Section 7 concludes.

2. Model

There are *n* regulated sources of pollution. The description of a source varies depending on the particular application of the model. For example, a source may correspond to a single facility, or it may correspond to a large group of firms within the same sector of a given country's economy. The abatement costs for source i are $C_i(e_i;\ \theta_i)$, where e_i is the emissions from source i and θ_i is a parameter that influences costs. Because abatement costs are in terms of emissions, we define marginal abatement costs as $MAC_i \equiv -\frac{\partial C_i}{\partial e_i}$. We assume costs are convex in emission reductions, so that $-\frac{\partial C_i}{\partial e_i} > 0$ and $C''_i \equiv \frac{\partial^2 C_i}{\partial e_i^2} > 0.$ The value of the cost parameter θ_i is known by source i when the abatement decision is made. In contrast, the regulator treats θ_i as a random variable. [8] Initially we assume that the regulator knows the distribution for each random variable. Later we discuss the extent to which this assumption can be relaxed. We use the expression "cost shocks" to refer to various realizations of these random variables. Let $E = (e_1, e_2, \cdots, e_n)$ denote the vector of emissions.

Emissions cause damages, which are specified by a convex damage function $D(E)$. The marginal damage from source i is $MD_i \equiv \frac{\partial D}{\partial e_i} > 0$. We say a damage function is *regular* if it can be written as

$D(E) = F\left(\sum \alpha_i e_i\right)$ equation(1)

for some convex function F and set of positive α_i's. Two familiar special cases of regular damage functions are uniformly mixed pollution, in which $\alpha_i = 1$ for every i, and constant marginal damage, in which F is linear.

The regulator uses a permit market to ameliorate the damages from pollution. We assume this permit market is competitive. Each source is given an endowment of permits w_i and $w = \Sigma\ w_i$ denotes the aggregate endowment. The sources face possibly different constraints on the number of permits they must surrender for each unit of emissions. These constraints are described by a source-specific variable r_i that is chosen by the regulator. In particular, if source i emits e_i units of pollution then they must surrender $r_i e_i$ permits. The ratio of r_i to r_j reflects the rate at which emissions of source i can be converted to emissions of source j through the trade of permits between the two sources. [9] If the ratio $\frac{r_i}{r_j}$ is the same for every i and j, then we have one-for-one trading of emissions. Following the literature, we refer to the r_i's as *trading ratios*.

The choice variables for the regulator are nominally the trading ratios and the permit endowments. However, because we assume the permit market is competitive and abatement costs are convex, the market equilibrium depends only on the aggregate endowment w and is independent of the distribution of the w_i. [10] Moreover, the permit market equilibrium is unchanged if the trading ratios and the aggregate endowment are all multiplied by the same constant. Without loss of generality, then, we can normalize w as convenient. In our theoretical analysis we normalize it to be equal to one.

Given a price p for permits, source i selects emissions to minimize the sum of abatement costs and expenditures in the permit market. Source i's problem is [11]

$$\min_{e_i} C_i(e_i; \theta_i) + p(r_i e_i - w_i).$$

The first-order condition for e_i is

$-\frac{\partial C_i}{\partial e_i} = r_i p,$ equation(2) which modifies the usual equality between marginal abatement cost and price to account for the trading ratio. An immediate consequence of this equation is that, if two sources have different trading ratios, then their marginal abatement costs will not be equal, i.e., the regulation is not cost-effective. Under our normalization of *w*, the permit market clearing equation is

$$\sum_i r_i e_i = 1. \quad \text{equation(3)}$$

The permit market equilibrium, conditioned on the regulator's choice of trading ratios, is summarized by Eqs. (2) and (3). This is a system of *n* + 1 equations and *n* + 1 unknowns (each of the e_i and *p*). We assume there is a unique solution as a function of the vector of trading ratios *R* and the vector of cost parameters Θ. Thus we have *ei*(*R*; Θ), *E*(*R*; Θ), and *p*(*R*; Θ).

The regulator selects values for the trading ratios to minimize the expected sum of abatement costs and damages. Thus the regulator's problem is to choose *R* to minimize

$$\mathcal{W} \equiv \mathbb{E}\left[\sum_i C_i(e_i(R;\Theta);\theta_i) + D(E(R;\Theta))\right].$$

Define *optimal trading ratios* as the trading ratios which minimize this objective. [12] The corresponding first-order condition for r_j is

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial e_i}{\partial r_j}\right] = 0 \ \text{ for } \ j = 1,2,\ldots n. \quad \text{equation(4)}$$

There is not a simple closed form solution to the first-order conditions, even in a standard case in which the abatement cost functions and the damage function are quadratic.[13]

Now consider the intuitive, but generally inferior, approach to selecting the trading ratios based on marginal damages. To motivate why one may want to use trading ratios of this type, suppose for the moment that there is no uncertainty about abatement costs. In this case, the efficient emissions solve

$$\min_E \sum C_i(e_i) + D(E).$$

The first-order condition for e_i is

$$-\frac{\partial C_i}{\partial e_i} = \frac{\partial D}{\partial e_i}.$$

Combining these with Eq. (2) gives

$$r_i p = \frac{\partial D}{\partial e_i}. \quad \text{equation(5)}$$

Thus, when there is no uncertainty, the trading ratios should be set according to marginal damages.

In our model, there is uncertainty about abatement costs. So the regulator cannot select trading ratios according to Eq. (5) because the e_j, and therefore marginal damages and p, are stochastic. But we can define marginal damage trading ratios by taking the expectation of Eq. (5). There are a couple of ways to proceed. Our preferred definition is constructed from the conditions [14]

$$\frac{r_j}{r_1} = \frac{\mathbb{E}\left[\frac{\partial D}{\partial e_j}\right]}{\mathbb{E}\left[\frac{\partial D}{\partial e_1}\right]} \text{ for every } j \neq 1. \qquad \text{equation(6)}$$

We use these conditions to pin down all of the trading ratios except r_1. This remaining trading ratio is determined by the solution to an optimization problem, as follows. Let the solution to Eq. (6) be denoted by $\tilde{r}_j(r_1)$ and let $\tilde{e}_i(r_1;\Theta) \equiv e_i(r1, \tilde{r}_2(r_1), \ldots, \tilde{r}_n(r_1); \Theta)$. The regulator's problem in this case is to find the value for $r_1$that minimizes total expected costs:

$$\min_{r_1} \mathbb{E}\left[\sum_i C_i(\tilde{e}_i(r_1;\Theta);\theta_i) + D\left(\tilde{E}(r_1;\Theta)\right)\right].$$

The first-order condition for r_1 is

$$\mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial \tilde{e}_i}{\partial r_1}\right] = 0. \qquad \text{equation(7)}$$

The first-order condition implies that the regulator sets marginal abatement costs equal to marginal damages on average where the average is weighted by the $\frac{\partial \tilde{e}_i}{\partial r_1}$'s.[15] Let $\tilde{r}_1$ be the solution to Eq. (7) and let $\tilde{r}_j \equiv \tilde{r}_j(\tilde{r}_1)$. We refer to the vector $\tilde{R} = (\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n)$ as the *marginal damage trading ratios*.

If the damage function is regular, then the conditions defining the marginal damage trading ratios simplify considerably. Combining Eq. (1) with Eq. (6) implies

$$\frac{r_j}{r_1} = \frac{\alpha_j}{\alpha_1}.$$

For example, if pollution is uniformly mixed, then the marginal damage trading ratios are all equal to a common value and hence imply one-for-one trading.

Our first main result is to show that the marginal damage trading ratios will generally not be optimal. This may seem a bit surprising, so let us first give intuition for why it is indeed true before turning to a more formal analysis.[16] Building on our discussion of this point in the Introduction, once again focus on the special case of uniformly mixed pollution. Here marginal damages are the same across sources, so one might expect that trading ratios should be equal across sources as well. To see why such one-for-one trading is, in fact, not generally optimal for uniformly mixed pollution, consider the market equilibrium condition (Eq. (3)). Evaluating this at the solution $e_i(R;\ \theta)$ gives

$$\sum_i r_i e_i(R;\Theta) = 1. \qquad \text{equation(8)}$$

Now suppose for the moment the market is indeed designed with one-for-one trading and let r be the common value for the trading ratios. It follows from Eq. (8) that the sum of emissions is equal to the constant $\frac{1}{r}$, which is the effective permit endowment, i.e., “the cap”. In general, however, when the trading ratios differ between sources, the sum of emissions will not be constant, and moreover, it will vary according to the realized values of Θ. This suggests that permit markets that do not use one-for-one trading have an interesting and under-appreciated feature. In these markets, sources in aggregate may emit more (or less) pollution depending on the actual values of the abatement cost functions, even though the aggregate permit endowment is fixed.

The regulator, in turn, can use this feature to improve the performance of the permit market. Because of the uncertainty about abatement costs, the regulator does not know the efficient quantity of pollution. Loosely speaking, when aggregate marginal abatement costs are high, the efficient quantity of pollution is large. When the aggregate marginal abatement costs are low, the efficient quantity of pollution is small. The regulator can engender a similar relationship between emissions and abatement costs by optimally selecting the trading ratios.

Now return to the formal analysis of the general case of an arbitrary damage function. To show that the marginal damage trading ratios will generally not be optimal, we utilize the structure of the regulator's problem as well as the characteristics of the marginal damage trading ratios to evaluate the derivative of the regulator's objective function W at the marginal damage trading ratios. This gives us our first main result (all proofs are in the Appendix).

Proposition 1.

The derivative of the regulator's objective function W *with respect to* r_j*, evaluated at the marginal damage trading ratios* $\tilde{R}$*, is given by*

$$\left.\frac{\partial W}{\partial r_j}\right|_{\tilde{R}} = COV\left(p, e_j\right) + \sum_i COV\left(\tilde{r}_j \frac{\partial D}{\partial e_i} - \tilde{r}_i \frac{\partial D}{\partial e_j}, A^{-1}\frac{a_i p}{\tilde{r}_i C_j''}\right)$$

$$- \quad COV\left(A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}, e_j\right) + \mathbb{E}\left[\left(p - A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}\right)\right]\mathbb{E}\left[e_j\right]$$

where the covariances and the expectations are also evaluated at $\tilde{R}$*,* $a_i \equiv r_i^2/C_i''$ *, and* $A \equiv \Sigma_i a_i$.

We see that the derivative of W with respect to r_j, evaluated at the marginal damage trading ratios, can be written as the sum of $n + 2$ covariance terms plus an additional term which is the product of two expected values.

Proposition 1 has three main implications. First, setting the derivatives $\left.\frac{\partial W}{\partial r_j}\right|_{\tilde{R}}$ equal to zero provides necessary conditions for the optimality of marginal damage trading ratios. Conversely, it shows that if at least one of the derivatives is not equal to zero, then the marginal damage trading ratios are not optimal. We will analyze the properties of the derivatives through a variety of special cases and numerical examples. But at this point, it is important to stress that there is no reason, in general, that the overall sums of terms in the expressions for them should be equal to zero. In other words, the marginal damage trading ratios are generally not optimal.

Second, to a first-order approximation, Proposition 1 shows how to improve efficiency and gives the magnitude of the efficiency gains from this improvement. If $\left.\frac{\partial W}{\partial r_j}\right|_{\tilde{R}}$ is positive, then the objective function can be reduced by decreasing the trading ratio r_j below the marginal damage trading ratio $\tilde{r}_j$. If it is negative, then the objective function can be reduced by increasing r_j above the marginal damage trading ratio $\tilde{r}_j$. The magnitude of the derivative gives the efficiency gain in moving from a marginal damage trading ratio toward an optimal trading ratio. This magnitude depends on the marginal damages, the marginal damage trading ratios, and the uncertainty about price and emissions generated by the uncertainty about the abatement cost functions.

A final implication of Proposition 1 is that the optimal trading ratios lead to a loss of ex post cost effectiveness. Once again this is perhaps most clearly illustrated with the case of uniformly mixed pollution. If trading ratios are not one-for-one, then by Eq. (2), the marginal abatement costs are not equal. Aggregate abatement costs could be reduced by increasing abatement from a low-cost source and decreasing abatement from a high-cost source. The regulator tolerates this loss of ex post cost effectiveness to obtain the gain in ex ante efficiency from using the optimal trading ratios.

Next consider a special case in which damage functions are regular as defined in Eq. (1). For this special case, the derivative in Proposition 1 simplifies considerably.

Corollary 1.

Suppose that the damage function is regular. Then the derivative of the regulator's objective function W *with respect to* r_j*, evaluated at the marginal damage trading ratios* $\tilde{R}$*, is given by*

$$\frac{\partial W}{\partial r_j}\bigg|_{\tilde{R}} = COV\left(p, e_j\right)\Big|_{\tilde{R}}.$$

For regular damage functions, setting these derivatives equal to zero provides necessary conditions for the optimality of marginal damage trading ratios. The marginal damage trading ratios are not optimal if at least one $COV(p, e_j)$, evaluated at $\tilde{R}$, is not equal to zero. Although both the cases of uniformly mixed pollution and linear damages have received attention in the literature, Corollary 1 appears to be a novel insight.

To understand the key role that the covariance of emissions and prices plays in the determination of optimal trading ratios when damages are regular, it is helpful to consider a simple example with uniformly mixed pollution and two sources. In this example, Source 1's marginal abatement costs are known with certainty. Source 2's cost shock can either be high (H) or low (L) with equal probability. Fig. 1 shows the market with marginal damage trading ratios (one-for-one trading). There are marginal abatement costs for each source, two aggregate (or "market") marginal abatement costs corresponding to the high and low outcome, and marginal damages. [17] The first-best outcome occurs at the intersection of the appropriate market marginal abatement cost and marginal damages. Because of asymmetric information, the first-best outcome is not obtained. For example, in the event of the high cost shock, the total permit endowment is too small, marginal abatement costs exceed marginal damages, and there is a deadweight loss relative to the first-best outcome. This deadweight loss is indicated by the upper triangle in Fig. 1.

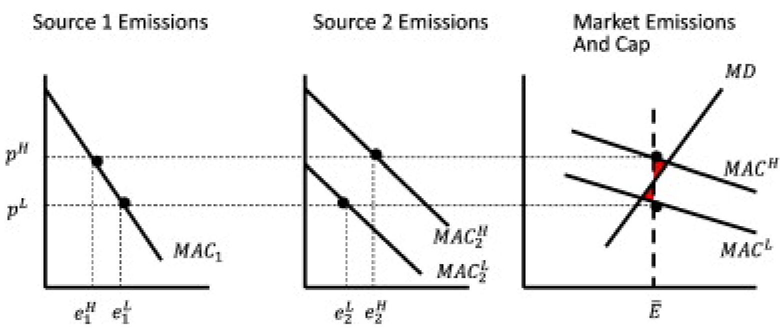

Fig. 1. Marginal damage trading ratios with uniformly mixed pollution.

The x-axis is in units of emissions (e.g., in tons) and the y-axis measures value per unit (e.g., dollar per ton).

Inspection of the relationship between the points in Fig. 1 reveals that $COV\left(p, e_j\right)\Big|_{\tilde{R}} \neq 0$ for either source. In fact, the covariance of emissions and prices is negative for Source 1, but positive for Source 2. Corollary 1 shows that efficiency can be improved by increasing Source 1's trading ratio, but decreasing Source 2's trading ratio. By giving a favorable trading ratio to the source whose emissions are large when the permit price is high, the regulator can, in essence, relax the aggregate emissions constraint in the event of high prices and hence improve efficiency.[18]

Having shown that the marginal damage trading ratios are generally not optimal, we now turn to characterizing the optimal trading ratios. This gives us our second main result.

Proposition 2.

The optimal trading ratios satisfy

$$r_j = \frac{\mathbb{E}\left[\frac{p}{C''_j}\frac{\partial D}{\partial e_j}\right] + COV\left(A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}, e_j\right) - COV\left(p, e_j\right)}{\mathbb{E}\left[\frac{p}{C''_j}A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right]}. \quad \text{equation(9)}$$

This result reinforces the distinction between optimal trading ratios and marginal damage trading ratios. In general, the ratio of one optimal trading ratio to another will not simply be equal to the ratio of expected marginal damages. Rather, the ratio will depend on more complicated expectations–that include price and the slope of marginal abatement costs in addition to marginal damages–as well as higher order moments of the regulator's uncertainty.[19] If there is no uncertainty, however, then the covariances in Proposition 2 are equal to zero and the other quantities are deterministic, so that Eq. (9) satisfies Eq. (6). As we would expect from our earlier discussion, the optimal trading ratios satisfy the conditions for the marginal damage trading ratios in this case.

2.1. Optimal source-specific taxes

Due to asymmetric information, the optimal trading ratios depend on more than just expected marginal damages. This raises the question as to whether pricing mechanisms, such as pollution taxes, should be set equal to marginal damages or whether they too should be adjusted under asymmetric information.[20]

Suppose t_i is the tax per unit of emissions for source i, and T is the vector of source-specific taxes. As is well-known, the source will equate its marginal abatement costs and the tax, so the first-order condition fore_i in the source's cost minimization problem is

$$-\frac{\partial C_i}{\partial e_i} = t_i. \quad \text{equation(10)}$$

Let the solution to this equation be $ei(T;\ \Theta)$. The regulator selects the source-specific taxes to minimize the expected sum of abatement costs and damages. Thus the regulator's problem is to choose T to minimize

$$W \equiv \mathbb{E}\left[\sum_i C_i(e_i(T;\Theta);\theta_i) + D(E(T;\Theta))\right].$$

The first-order condition of the regulator's objective with respect to t_j is [21]

$$\frac{\partial W}{\partial t_j} = \mathbb{E}\left[\sum_i \left(\frac{\partial C_i}{\partial e_i} + \frac{\partial D}{\partial e_i}\right)\frac{\partial e_i}{\partial t_j}\right] = \mathbb{E}\left[\left(-t_j + \frac{\partial D}{\partial e_j}\right)\left(\frac{-1}{C''_j}\right)\right] = 0. \quad \text{equation(11)}$$

Solving for t_j implies that

$$t_j = \frac{\mathbb{E}\left[\frac{\partial D}{\partial e_j}/C''_j\right]}{\mathbb{E}\left[1/C''_j\right]} = \mathbb{E}\left[\frac{\partial D}{\partial e_j}\right] + \frac{COV\left(\frac{\partial D}{\partial e_j}, 1/C''_j\right)}{\mathbb{E}\left[1/C''_j\right]}. \quad \text{equation(12)}$$

Since in general there is no reason the covariance term in Eq. (12) should equal zero, it is generally not the case that optimal source-specific taxes should equal expected marginal damages. The optimal source-specific taxes should be adjusted by a factor that depends on the second derivative of the abatement cost function.[22]

The optimal source-specific taxes in Eq. (12) are related to the theory of optimal taxation first studied by Ramsey (1927). In optimal Ramsey taxation, larger taxes are applied to more inelastic goods. Note that $1/C''$ is related to the abatement cost elasticity. [23] Thus if marginal damages are high when the abatement cost elasticity is high, then the second term in Eq. (12) is positive and the optimal source-specific tax exceeds expected marginal damages (i.e., is larger in the inelastic good). This intuition is illustrated graphically in Additional Appendix D.

3. A linear-quadratic example

Additional insight into the structure of the optimal trading ratios, the marginal damage trading ratios, and the differences between them can be gleaned from an example with specific functional forms. In this example, the abatement cost function

$$C_i(e_i;\theta_i) = \frac{\lambda_i}{2}\left(\frac{\theta_i}{\lambda_i} - e_i\right)^2 \quad \text{equation(13)}$$

is quadratic and the marginal abatement cost function

$$-\frac{\partial C_i}{\partial e_i} = \theta_i - \lambda_i e_i \quad \text{equation(14)}$$

is linear. We interpret θi as the intercept and λi as the slope of the marginal abatement cost function. It is convenient to collect the λi into a diagonal matrix Λ. We assume that the random variables θi are independent, and we let the expected values and variances be denoted by $\bar{\theta}_i$ and σi^2, respectively.

The damage function is quadratic as well. We have

$$D(E) = W^t E + \frac{1}{2} E^t \mathbf{V} E, \quad \text{equation(15)}$$

where W is a vector with entries ωi and $\mathbf{V}$ is a symmetric matrix with entries vij. Marginal damages are given by

$$\frac{\partial D}{\partial e_i} = \omega_i + \sum_k v_{ik} e_k.$$

Some special cases are worth noting. First, if $\mathbf{V}$ is the zero matrix, then damages are linear and marginal damages are constant. Second, if $\omega i = \omega$ for every i and $vij = v$ for every i and j, then pollution is uniformly mixed.

A distinct advantage of the linear-quadratic example is that we can obtain simple closed-form expressions for p and ej. These, in turn, enable us to give an explicit expression for $COV(p, ej)$. We state this as the first of several results for the linear-quadratic example (The proofs are in Additional Appendix A.).

Result 1.

In the linear-quadratic example,

$$COV\left(p, e_j\right) = \frac{A^{-1} r_j}{\lambda_j}\left(\frac{\sigma_j^2}{\lambda_j} - A^{-1}\sum_i a_i \frac{\sigma_i^2}{\lambda_i}\right). \quad \text{equation(16)}$$

It follows that, if the damage function is regular, then the marginal damage trading ratios are optimal if and only if $\frac{\sigma_j^2}{\lambda_j}$ is the same for every source j.

The additional structure of the quadratic example enables us to give necessary and sufficient conditions for the optimality of the marginal damage trading ratios. As expected from our discussion of Proposition 1, marginal damage trading ratios are optimal only under fairly restrictive conditions. Result 1 shows that, for regular damages, the optimality of the marginal damages trading ratios depends on whether or not the abatement cost functions exhibit a specific type of homogeneity. If the ratio of the variance of the cost parameter σj^2 to the slope of the marginal abatement cost function λj is the same across all sources, then the marginal damage trading ratios are optimal. If, however, the ratios of the variance to the slope vary across sources, then marginal damage trading ratios are not optimal. This includes the case in which the distributions for the θi's are exactly the same across sources, but the λi's are different.

Building on Result 1, we can quantify the efficiency gains from moving from marginal damage trading ratios toward the optimal trading ratios. The slope of the regulator's objective at the marginal damage trading ratios gives a first-order approximation of these efficiency gains. For regular damage functions, this first-order approximation for a small change in r_j is given by Eq. (16). Thus the relative gain from a small change in r_j is larger if σ_j^2/λ_j is further from the weighted average of the σ_i^2/λ_i's. If we adjust all the r_j's from the marginal damage trading ratios toward the optimal trading ratios, the first-order approximation of the gain will be larger if the σ_j^2/λ_j's are further from their weighted average, intuitively, if the dispersion of the σ_j^2/λ_j's is larger. A special case of regular damage functions illustrates this intuition most clearly.

Result 2.

In the linear-quadratic example, suppose that pollution is uniformly mixed and that $\lambda_i = 1$ *for every i. To a first-order approximation, the efficiency advantage of taking a step of unit length from the marginal damage trading ratios toward the optimal trading ratios is given by*

$$\Delta = \frac{1}{\bar{r}\sqrt{n}}\sqrt{\left(\frac{1}{n}\right)\sum_{i}^{n}\left(\sigma_j^2 - \frac{1}{n}\sum_{i}\sigma_i^2\right)^2}.$$

The square root term in Δ corresponds to the standard deviation of the list of numbers σ_1^2, σ_2^2, $\cdots$, σ_n^2. As expected, an increase in this standard deviation leads to an increase in Δ. The effect of an increase in n is not as straightforward because it depends on the assumption we make about what happens to the distribution of the σ_i. For example, suppose we make the rather obvious assumption that the mean and variance of this distribution are constant with respect to n. This yields the unambiguous result that Δ is decreasing in n. But this assumption also implies that the regulator's uncertainty about *aggregate*abatement costs is decreasing in n. [24] In other words, the magnitude of the regulator's asymmetric information problem becomes less severe as n increases. An alternative assumption is that the distribution of the σ_i changes in such a way that the magnitude of the regulator's problem remains constant with respect to n. Here it is easy to construct examples in which Δ is increasing in n, even with our assumption that the uncertainty is uncorrelated across sources. [25]

In summary, for regular damage functions, heterogeneity of abatement costs (through differences in the ratio of variance of cost uncertainty to slope of marginal abatement cost) leads to a wedge between the optimal trading ratios and the marginal damage trading ratios. The greater the degree of this heterogeneity, the greater the efficiency advantage of the optimal trading ratios.

Next consider arbitrary damage functions. To study these, we simplify the linear-quadratic case by eliminating the abatement cost heterogeneity that was critical in our discussion of regular damage functions. Accordingly, we have

$$\sigma_i^2 = \sigma \text{ and } \lambda_i = \lambda \text{ for every } i. \quad \text{equation(17)}$$

Under this assumption, we can characterize the regulator's objective as follows.

Result 3.

In the linear-quadratic example, suppose that Eq. (17) *holds. Then we have*

$$W = \sum C_i(\mathbb{E}[e_i]; \bar{\theta}_i) + D(\mathbb{E}[E]) + \frac{\sigma^2}{2\lambda^2}\left(\lambda + \sum_i v_{ii} - \frac{R^t \mathbf{V} R}{R^t \mathbf{I} R}\right), \quad \text{equation(18)}$$

where **I** *is the identity matrix*.

As we might expect, the quadratic functions yield a mean-variance structure for the regulator's objective.[26] We can also give a necessary and sufficient condition for the optimality of the marginal damage trading ratios.

Result 4.

In the linear-quadratic example, suppose Eq. (17) *holds,* $W = 0$*, and* **V** *is invertible. Then the marginal damage trading ratios are optimal if and only if* $\mathrm{E}[\Theta]$ *is an eigenvector of* **V**.

Result 4 is similar in structure to Result 1 in that it provides a fairly restrictive necessary and sufficient condition for marginal damages trading ratios to be optimal. In this case, with heterogeneity on the damages side, the condition is defined with respect to the vector of expected values, rather than being a condition on the variances. If it holds, then the marginal damage trading ratios, the expected emissions, and the expected cost parameters all lie on the same ray from the origin. This ray is also an eigenvector of **V**. It turns out that this eigenvector maximizes the quadratic form in Eq. (18), which effectively eliminates concerns about uncertainty. Hence the optimal trading ratios, which in general differ from the marginal damage trading ratio on account of such uncertainty, offer no improvement relative to the marginal damage trading ratios in this case.[27]

Taken as a whole, the results for the quadratic example reinforce and enhance our findings from the general model. Under the necessary and sufficient conditions, the marginal damage trading ratios are optimal, but this will generally not occur. The efficiency gains from using the optimal trading ratios depend in a complicated manner on distributions of both the expected value and variances of the random variables in the cost functions as well as the interaction of these distributions with the properties of the damage function.

4. Information and implementation issues

Up to now we have assumed that the regulator knows the entire distribution for each of the random variables θi. In this section we consider the extent to which this can be relaxed for both the marginal damage trading ratios and the optimal trading ratios. We also consider how the regulator can obtain the needed information.

First consider the optimal trading ratios. The regulator needs enough information to evaluate the expectation of the products of the partial derivative terms in the first-order conditions (Eq. (4)). These terms will generally be at least linear in the θi's so that the product will be at least quadratic. This implies that the regulator would need to know at least the expected values and variances of the distributions for the θi's. In the linear-quadratic example, this information is actually all that the regulator needs to know. This can easily be seen from Result 3, but it is also true under more general conditions than those used in Result 3 because, in the linear-quadratic model, each of the partial derivative terms in Eq. (4) is precisely linear in the θi's.

Next consider the marginal damage trading ratios. Here the regulator needs enough information to evaluate the expectation of the products of the partial derivative terms in the first-order conditions (Eq. (7)) as well as the marginal damage trading ratio constraint set (Eq. (6)). This suggests that the regulator will generally need the same kind of information that is required to implement the optimal trading ratios. But, if damages are regular, then the regulator may need less information. Under regular damages, the relative values of the marginal damage trading ratios are determined directly by the αi's. Thus it is tempting to conclude that the regulator need not have any information about the distributions for the random variables θi. This is not correct, however, as the regulator must also determine the optimal value for r_1, which requires enough information to evaluate the expected market price. [28] So, for example, if the abatement costs are quadratic and damages are regular, then the regulator needs to know the expected values of the random variables but not the variances.

How can the regulator obtain the expected values and variances of the distributions for the θi's? One obvious procedure would be to conduct an econometric analysis of the relationship between emissions and abatement costs for each source. This procedure provides a direct estimate of the desired parameters, but it has significant data requirements. In addition, it is inherently backward looking, whereas the usual interpretation of the regulator's uncertainty is with respect to abatement costs in the future. Alternatively, one could assume that the parameters are proportional to some easily observable characteristic of the sources. For example, in their analysis of Nitrogen pollution from waste water treatment plants, Yates et al. (2013) assume that the expected value and variance are proportional to the plant size. [29] The cost of this approach is that it may introduce inaccuracies in the determination of the expected value and variances, which would in turn reduce the efficiency gains from implementing the optimal trading ratios. Yet another procedure would be to take a subjective Bayesian approach and directly assess the regulator's beliefs about abatement cost uncertainty. Although this procedure is in many ways the most theoretically appealing, making policy choices in this manner may raise significant practical problems.

It may also be possible to determine a crude but direct estimate of optimal trading ratios in particular markets. For example, consider the European Union's Emission Trading Scheme (EU-ETS) for carbon pollution. This permit market has been operating under the marginal damage trading ratios (one-for-one trading) for about seven years. Therefore, one should be able to calculate the covariances in Corollary 1 based on existing market data. Trading ratios adjusted from unity according to the estimated covariances may give an efficiency improvement relative to one-for-one trading. But again some caution is warranted, as these trading ratios are based on past variation in abatement costs, not future variation.

There is also the issue of the size of the market and how many trading ratios a regulator may feasibly determine. For example, the EU-ETS contains over 12,000 sources. It may be impractical to determine trading ratios for all of them. In this case, it may be useful to divide the sources into groups (for example by country and sector of the economy) and then determine a trading ratio for each group. In Additional Appendix F, we give an example in which we determine trading ratios for approximately 50 sources, so it should be feasible to determine at least this number of trading ratios in actual practice. In the EU-ETS, this would allow for two sectors in each member state.

Turning now to implementation issues, optimal trading ratios may conflict with the requirements of some environmental regulations in ways that marginal damage trading ratios do not. Consider water pollution regulation under the Clean Water Act. Suppose there is an estuary for which a total maximum daily load has been specified and there are a number of waste water treatment plants that discharge emissions upstream of the estuary. The Clean Water Act allows permit trade between the plants, provided that the water quality at the estuary remains constant. This can be insured by setting trading ratios equal to the transfer coefficients that describe the percentage of emissions from a given plant that reach the estuary. If we further assume that damages are regular, then these trading ratios are in fact marginal damage trading ratios. The optimal trading ratios will lead to lower expected total costs, but they may not be consistent with the regulatory requirements because permit trade may lead to a change in the water quality at the estuary. Similar issues may arise for air pollution due to National Ambient Air Quality Standards.

Another implementation issue arises for both optimal trading ratios and marginal damage trading ratios. One-for-one trading has an appealing uniformity–each source is treated the same by the regulator. To implement optimal trading ratios, or to implement marginal damage trading ratios when pollution is not uniformly mixed, the regulator must select source specific regulation. This may create an opportunity for sources to lobby for a more favorable trading ratio. As is well known from the literature on rent-seeking, such lobbying activities often decrease welfare.

5. Numerical calculations

In this section we use a simple example of the linear-quadratic model to illustrate Corollary 1 with numerical calculations.[30] There are two sources, and the slope of marginal abatement costs are equal across sources. Source 1's marginal abatement costs are known with certainty but Source 2's cost shock can either be high (H) or low (L) with equal probability. Damages are linear and differ across the two sources. Source 1 has low marginal damages ($MD_1 = 10$) and

Source 2 has high marginal damages (MD_2 = 12). This example is also consistent with the model employed by Fowlie and Muller (2013).

Table 1 illustrates the results for the marginal damage trading ratios, one-for-one trading, and the optimal trading ratios. From Eq. (6), the marginal damage trading ratios satisfy $r_2 = 12/10 * r_1$. Panel A of Table 1shows that, under marginal damage trading ratios, the value for r_1 is 0.92, so that r_2 = 1.10. Thus the low damage source (Source 1) pays a relatively low effective price for its emissions and the high damage source (Source 2) pays a relatively high effective price for its emissions. The marginal damage trading ratios hold damages constant across the two cost shocks, but allow aggregate emissions to vary.

Table 1. Numerical example: linear damages with MD_1 = 10, MD_2 = 12 [a].

	MAC_1	Price	MAC_2	e_1	e_2	$e_1 + e_2$	Damages
Panel A: Marginal damage trading ratios.							
$r_1 = 0.92$; $r_2 = 12/10 r_1 = 1.10$; DWL = 7.38							
Low cost	7.54	8.21	9.05	12.46	5.95	18.41	196.0
High cost	12.46	13.57	14.95	7.54	10.05	17.59	196.0
Panel B: One-for-one trading.							
$r_1 = 1$; $r_2 = 1$; DWL = 7.25							
Low cost	8.5	8.5	8.5	11.5	6.5	18	193
High cost	13.5	13.5	13.5	6.5	11.5	18	203
Panel C: Optimal trading ratios.							
$r_1 = 0.96$; $r_2 = 1.04$; DWL = 7.06							
Low cost	8.09	8.39	8.76	11.91	6.24	18.14	193.9
High cost	13.08	13.56	14.16	6.92	10.84	17.77	199.3

[a]The example is parameterized by $MAC_1 = 20 - e_1$; $MAC_2L = 15 - e_1$; $MAC_2H = 25 - e_1$; where high and low costs occur with equal probability. Total permits are normalized to 18, which would be the optimal emissions with 1:1 trading.

Interestingly, one-for-one trading actually performs better than marginal damage trading, even though pollution is not uniformly mixed in this example. As shown in Panel B of Table 1, under one-for-one trading, the damages are not held constant across the cost shocks, but the aggregate emissions are held constant. This leads to a lower deadweight loss than marginal damage trading ratios, which verifies Fowlie and Muller's observation that such an outcome is possible in their model.

The optimal trading ratios have a lower deadweight loss than either of the other schemes. Panel C of Table 1 shows calculations for the optimal trading ratios. At the marginal damage trading ratios, we have$COV(p, e_1) < 0$. It follows from Corollary 1 that the optimal trading ratio for source 1 is greater than the marginal damage trading ratio (0.96 vs. 0.93). On the other hand $COV(p, e_2) > 0$, so the optimal trading ratio for source 2 is lower than the marginal damage trading ratio (1.04 vs 1.10). Under the optimal trading ratios, neither the aggregate emissions nor the damages are constant across the cost shocks. This flexibility improves efficiency.

6. Application: carbon trading

We have established that the optimal trading ratios will generally be different from the marginal damage trading ratios, even for uniformly mixed pollution. We now investigate potential policy implications of this observation by considering a permit trading application with uniformly mixed pollution. In Additional Appendix F, we give an additional application with non-uniformly mixed pollution.

Consider a stylized global carbon trading market. Ackerman and Bueno (2011) determine simple two-parameter functions that characterize the cost of reducing carbon emissions for various geographic regions of the world.[31] To apply these to our model, we interpret our sources as regions and write Ackerman and Bueno's functions in terms of emissions rather than emission reductions. This gives

$$-\frac{\partial C_i}{\partial e_i} = \frac{a_i(b_i(1+\theta_i)-e_i)}{e_i},$$

where a_i and b_i are constants determined by Ackerman and Bueno. We interpret $b_i(1+\theta_i)$ as the stochastic business-as-usual (BAU) emissions. For simplicity we model the random variable θ_i with a three point symmetric distribution with zero expectation so that θ_i takes on the values $\{-k_i, 0, k_i\}$ with probabilities $\{\rho_i, 1-2\rho_i, \rho_i\}$. For example, ρ_i is the probability that BAU emissions increase by k_i percent over their expected value. We also assume that the θ_i are independent across regions. To complete the model we specify the marginal damage function as

$$\frac{\partial D}{\partial e_i} = \beta\sum(e_i-b_i)+s,$$

where β (the slope of marginal damage) comes from Newell and Pizer (2003) and s (the social cost of carbon at expected BAU) comes from IWGSSC (2010).

For simplicity, we focus on the industrial sectors of the four regions with the largest emissions: China, Europe, South/South East Asia, and the U.S. The results are given in Table 2 and Table 3. For a given set of parameters ρ_i and k_i, we calculate the total expected costs (expected sum of abatement costs and damages) under the optimal trading ratios and the marginal damage trading ratios.

Table 2. Carbon trading with optimal trading ratios: symmetric scenarios[a,b].

	$\rho_i = 50\ \%$				$k_i = 0.2$	
	$k_i = 0.5$	$k_i = 0.33$	$k_i = 0.2$	$k_i = 0.1$	$\rho_i = 25\ \%$	$\rho_i = 10\ \%$

Optimal trading ratios						
China	0.877	0.946	0.981	0.995	0.990	0.996
Europe	1.082	1.037	1.013	1.003	1.007	1.003
S/SE Asia	1.133	1.055	1.019	1.005	1.010	1.004
U.S.A.	1.063	1.030	1.011	1.003	1.006	1.002
St. dev. price						
Marginal damage TR	38.534	25.694	15.418	7.709	10.902	6.895
Optimal TR	37.649	25.463	15.371	7.704	10.886	6.891
Expected price						
Marginal damage TR	74.022	74.022	74.022	74.022	74.022	74.022
Optimal TR	73.838	73.997	74.023	74.023	74.024	74.023
Total cost						
First best	305.699	305.699	305.699	305.699	305.699	305.699
Marginal damage TR	321.374	312.554	308.148	306.309	306.921	306.188
Optimal TR	320.665	312.424	308.132	306.308	306.917	306.187
Deadweight loss						
Marginal damage TR	15.674	6.855	2.449	0.610	1.222	0.488
Optimal TR	14.966	6.725	2.432	0.609	1.218	0.488
Percent reduction	4.5%	1.9%	0.7%	0.16%	0.33%	0.13%

[a]The permit endowment is set so that the marginal damage trading ratios are 1. This represents approximately a 50% reduction from BAU emissions. [b]Costs and deadweight loss (DWL) in billions of dollars. Prices in 2007 dollars per ton carbon.

Table 3. Carbon trading with optimal trading ratios: asymmetric scenarios[a,b,c].

	$k_{\text{China}} = 0.5$	$k_{\text{China}} = 0.33$	$k_{\text{China}} = 0.2$	$k_{\text{China}} = 0.1$
Optimal trading ratios				
China	0.798	0.903	0.965	0.991
Europe	1.146	1.071	1.026	1.007
S/SE Asia	1.154	1.073	1.027	1.007
U.S.A.	1.152	1.072	1.027	1.007
St. dev. price				
Marginal damage TR	30.426	20.285	12.171	6.085
Optimal TR	27.113	19.312	11.966	6.060
Expected price				
Marginal damage TR	74.023	74.022	74.022	74.022
Optimal TR	73.635	73.968	74.031	74.028
Total cost				
First best	305.699	305.699	305.699	305.699
Marginal damage TR	315.270	309.934	307.220	306.079
Optimal TR	313.194	309.516	307.166	306.076
Deadweight loss				
Marginal damage TR	9.571	4.235	1.521	0.380
Optimal TR	7.495	3.817	1.467	0.377
Percent reduction	21.7%	9.9%	3.6%	0.9%

[a]For each column, the tail probabilities are ρ_{China} = 50 % for China and ρ_{ROW} = 0 % for the other three regions. [b]The permit endowment is set so that the marginal damage trading ratios are 1. This represents approximately a 50% reduction from BAU emissions. [c] Costs and deadweight loss (DWL) in billions of dollars. Prices in 2007 dollars per ton carbon.

In Table 2, we consider symmetric abatement cost shocks (the tail probabilities ρ_i and the percentage change in BAU emissions k_i are the same across regions). Because China has the largest BAU emissions, shocks to Chinese abatement costs drive the carbon price. Hence Chinese emissions covary positively with price under marginal damages trading ratios, and Corollary 1 implies that efficiency can be improved by lowering China's trading ratio. Indeed, China's optimal trading ratios are below one in each scenario, whereas the trading ratios for the other regions exceed one in each scenario. For the largest uncertainty (k_i = 0.5 and ρ_i = 50 %), optimal trading ratios reduce the deadweight loss by \$0.5 billion or about 5%. For lower levels of uncertainty, the gains from optimal trading ratios are more modest.

Up to now, we have focused exclusively on ex ante expected total costs. This is consistent with Weitzman (1974), but we can also analyze differences in ex post outcomes. For a given realization of the Θ vector, ex post total costs may be higher or lower with optimal trading ratios relative to marginal damage trading ratios. Consider the parameters corresponding to the first column of Table 2. For approximately 75% of the realizations, the ex post total costs with optimal trading ratios are lower than the ex post total costs with marginal damage trading ratios. Given that ex post total costs are lower, the average decrease is \$1.40 billion. Given that ex post total costs are higher, the average increase is \$1.38 billion. The results for the other columns of Table 2 are similar.

In Table 3, we consider asymmetric cost shocks. Here China's abatement costs are uncertain and the other regions' abatement costs are known. The gains from using optimal trading ratios are more dramatic than in the symmetric case of Table 2. With a high level of uncertainty about China's abatement costs (k_{China} = 0.5), optimal trading ratios reduce the deadweight loss by about 22% or around \$2 billion per year. Turning to ex post costs, the asymmetric cost shock example only has two possible realizations of the Θ vector, as costs are either high or low in China. Ex post costs are lower under optimal trading ratios for both of these realizations, for all values of k_i.

7. Conclusion

We analyze a model of asymmetric information between a regulator and sources of pollution and show that optimal policies are generally not based simply on expected marginal damages. In the context of pollution permit markets, we find that optimal trading ratios generally depart from marginal damage trading ratios. The regulator can improve efficiency by adjusting the marginal damage trading ratios in a manner determined in part by the covariance of the permit price and a source's emissions. In simple cases, such as uniformly mixed pollution or linear damages, if a source's emissions covary positively with the market price of permits, then the regulator should give the source a relatively favorable trading ratio. Intuitively, this favorable trading ratio allows additional emissions‒despite a fixed cap‒in precisely the case when the cap is set too tight from an ex post perspective. In the context of an emissions tax, our results imply that the regulator can improve ex ante efficiency by setting source-specific taxes according to a Ramsey-like rule which adjusts the expected marginal damages to account for the covariance of marginal damages with the slope of the marginal abatement costs.

Our theoretical analysis shows that it is possible for a regulator to improve the efficiency of pollution permit markets by using optimal trading ratios. However, whether the regulator should implement optimal trading ratios depends crucially on whether the benefits of optimal trading ratios are sufficient to offset any additional regulatory costs which might arise from their use. To estimate the magnitude of possible benefits, we compare optimal trading ratios to marginal damage trading ratios in a global carbon trading market. The results show that the benefits vary from significant to trivial depending primarily on the characteristics of the regulator's uncertainty about abatement costs.

As for additional regulatory costs, our discussion of information and implementation issues reveals that for non-regular damages optimal trading ratios may not be much more burdensome than marginal damage trading ratios. With respect to information, both require the regulator to estimate marginal damages by analyzing models of emission transport through the relevant physical space in conjunction with models mapping emissions into harm to humans and ecosystems. And both require estimating the parameters for the random variables in the abatement cost functions. With respect to implementation, both require moving away from the intuitively appealing and easy to explain cost-effectiveness criterion. And both give the regulator discretion to give differential regulatory requirements to the various sources of pollution, thereby potentially opening the door for the sources to lobby or litigate for a more favorable treatment.

Our results suggest that a reevaluation of the standard treatment of the regulator's uncertainty in environmental regulation is in order. Guided by Weitzman (1974), the focus has traditionally been on the expected values of the random variables that describe this uncertainty. In his model, the regulator need only use expected values to set policy; there is no need to use any higher-order moments. In contrast, we find that higher-order moments play a central role for determining optimal trading ratios, emission taxes, and even marginal damage trading ratios.

More generally, the type of regulations we consider illustrate how environmental markets may evolve into a middle ground between traditional command and control regulation and what one might call mechanism design regulation. Command and control regulation essentially ignores the fact that sources have private information about abatement costs. Mechanism design regulation induces sources to reveal their private information, and uses the revealed information to tailor the regulation for each source. In contrast to these two extremes, to implement optimal trading ratios the regulator uses the distribution of each source's private information to tailor the market design and improve efficiency.

Appendix A.

Proof of Proposition 1

From Eqs. (2) and (3), the equilibrium for the optimal trading ratios is defined by the $n-1$ equations $\frac{\partial C_i}{\partial e_i}/r_i = \frac{\partial C_j}{\partial e_j}/r_j$ for each $i \neq j$ and by the equation $\Sigma\, iriei = 1$. Differentiating the $n-1$ equations with respect to*rj* gives

$$\frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_j} = \frac{-\frac{\partial C_j}{\partial e_j}}{r_j^2} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} = \frac{p}{r_j} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} \quad \text{for each} \quad i \neq j, \quad \text{equation(19)}$$

where the first equation follows from differentiating and the second equation follows from the definition of Eq. (2). Differentiating $\Sigma\, iriei = 1$ with respect to *rj* implies that

$$\sum_i r_i \frac{\partial e_i}{\partial r_j} + e_j = 0 \quad \text{equation(20)}$$

which implies that

$$\begin{aligned} -e_j = \sum_i r_i \frac{\partial e_i}{\partial r_j} &= \sum_{i \neq j} \frac{r_i^2}{C_i''}\frac{p}{r_j} + \sum_i \frac{r_i^2}{C_i''}\frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} \\ &= \frac{p}{r_j}\sum_i \frac{r_i^2}{C_i''} - \frac{pr_j}{C_j''} + \frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j}\sum_i \frac{r_i^2}{C_i''}, \end{aligned}$$

where the first equality follows from rearranging Eq. (20), the second equality follows from substituting in Eq. (19), and the third equality follows from algebra. Solving this equation implies that

$$\frac{C_j''}{r_j}\frac{\partial e_j}{\partial r_j} = \left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_j p}{C_j''} - e_j\right) - \frac{p}{r_j} \quad \text{equation(21)}$$

which implies from Eq. (19) that

$$\frac{C_i''}{r_i}\frac{\partial e_i}{\partial r_j} = \left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_j p}{C_j''} - e_j\right) \quad \text{for each} \quad i \neq j. \quad \text{equation(22)}$$

Substituting Eq. (2) into the derivative of the regulator's objective with respect to *rj* as shown in Eq. (4) gives

$$\frac{\partial W}{\partial r_j} = \mathbb{E}\left[\sum_i -pr_i\frac{\partial e_i}{\partial r_j} + \sum_i \frac{\partial D}{\partial e_i}\frac{\partial e_i}{\partial r_j}\right] = \mathbb{E}\left[pe_j + \sum_i \frac{\partial D}{\partial e_i}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1}\left(\frac{r_j p}{C_j''} - e_j\right)\frac{r_i}{C_i''} - \frac{\partial D}{\partial e_j}\frac{p}{C_j''}\right] = \mathbb{E}\left[pe_j + A^{-1}\left[\frac{p}{C_j''}\left\{\sum_i \frac{r_j r_i}{C_i''}\frac{\partial D}{\partial e_i} - \sum_i \frac{r_i^2}{C_i''}\frac{\partial D}{\partial e_j}\right\} - e_j \sum_i \frac{r_i}{C_i''}\frac{\partial D}{\partial e_i}\right]\right]$$
$$= \mathbb{E}\left[A^{-1}\sum_i\left[\frac{pr_i}{C_j''C_i''}\left(r_j\frac{\partial D}{\partial e_i} - r_i\frac{\partial D}{\partial e_j}\right)\right] + \left(p - A^{-1}\sum_i \frac{r_i}{C_i''}\frac{\partial D}{\partial e_i}\right)e_j\right] = \mathbf{E}\left[\sum_i A^{-1}\left[\frac{a_i p}{r_i C_j''}\left(r_j\frac{\partial D}{\partial e_i} - r_i\frac{\partial D}{\partial e_j}\right)\right] + \left(p - A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right)e_j\right]$$

equation(23)

where the second equality follows from substituting Eqs. (20), (21) and (22), and the rest follow from algebra and the definition of *ai* and *A*.

Now Eq. (6) implies that, at the marginal damage trading ratios, we have $\tilde{r}_j \mathbb{E}\left[\frac{\partial D}{\partial e_i}\right] = \tilde{r}_i \mathbb{E}\left[\frac{\partial D}{\partial e_j}\right]$. Using this in conjunction with the well known result that COV(XY)=E[XY]−E[X]E[Y] implies that Eq. (23) can be written as

$$\left.\frac{\partial W}{\partial r_j}\right|_{\tilde{R}} = \sum_i COV\left(\tilde{r}_j\frac{\partial D}{\partial e_i} - \tilde{r}_i\frac{\partial D}{\partial e_j}, A^{-1}\frac{a_i p}{\tilde{r}_i C_j''}\right) + \mathbb{E}\left[\left(p - A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}\right)e_j\right].$$

Applying the covariance formula to the expected value term on the right gives us the equation in the proposition. ■

Before proving Corollary 1, we first prove a Lemma about the marginal damage trading ratios that holds provided damages are regular.

Lemma 1.

Suppose that damages are regular. For the marginal damage trading ratios, the regulator selects $\tilde{R}$*such that*

$$\mathbb{E}\left[p - A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right] = 0$$

where $a_i \equiv r_i^2/C_i''$ *and* $A \equiv \Sigma_i a_i$

Proof of Lemma 1

From Eqs. (2) and (3), the equilibrium for marginal damage trading ratios is defined by the $n-1$ equations $\frac{\partial C_i}{\partial e_i}/r_i = \frac{\partial C_1}{\partial e_1}/r_1$ for each $i \neq 1$ and by the equation $\Sigma_i r_i \tilde{e}_i = 1$. Differentiating the $n-1$ equations with respect to r_1 gives

$$\frac{-\frac{\partial C_i}{\partial e_i}}{r_i^2}\frac{r_i}{r_1} + \frac{C_i''}{r_i}\frac{\partial \tilde{e}_i}{\partial r_1} = \frac{-\frac{\partial C_1}{\partial e_1}}{r_1^2} + \frac{C_1''}{r_1}\frac{\partial \tilde{e}_1}{\partial r_1}.$$

(To derive this equation, we have used the fact that Eqs. (6) and (1) imply that $\partial r_i/\partial r_1 = r_i/r_1$). Using Eq.(2) we have

$$\frac{C_i''}{r_i}\frac{\partial\tilde{e}_i}{\partial r_1} = \frac{C_1''}{r_1}\frac{\partial\tilde{e}_1}{\partial r_1}$$ equation(24)

for each $i \neq 1$. Differentiating $\Sigma_i r_i \tilde{e}_i = 1$ implies that

equation(25)

$$\sum_i r_i \frac{\partial\tilde{e}_i}{\partial r_1} + \sum_i \frac{r_i}{r_1}\tilde{e}_i = 0$$ which implies that

$$\frac{-1}{r_1} = \sum_i r_i \frac{\partial\tilde{e}_i}{\partial r_1} = \sum_i r_i \frac{C_1'' r_i}{r_1 C_i''}\frac{\partial\tilde{e}_1}{\partial r_1} = \frac{\partial\tilde{e}_1}{\partial r_1}\frac{C_1''}{r_1}\sum_i \frac{r_i^2}{C_i''},$$ equation(26)

where the first equality comes from $\sum_i r_i \tilde{e}_1 = 1$ and rearranging Eq. (25), the second equality follows from substituting in Eq. (24), and the third equality follows from algebra. Solving this equation shows that

$$\frac{\partial\tilde{e}_1}{\partial r_1} = \frac{-1}{C_1''}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1} = \frac{-a_1}{r_1^2}A^{-1}$$

which implies

$$\frac{\partial\tilde{e}_i}{\partial r_1} = \frac{r_i}{r_1}\frac{-1}{C_i''}\left[\sum_i \frac{r_i^2}{C_i''}\right]^{-1} = \frac{-a_i}{r_1 r_i}A^{-1}$$ equation(27)

from Eq. (24).

Substituting Eq. (2) into Eq. (7), the first-order condition for r_1, gives

equation(28)

$$\begin{aligned} 0 &= \mathbb{E}\left[\sum_i -p r_i \frac{\partial\tilde{e}_i}{\partial r_1} + \sum_i \frac{\partial D}{\partial e_i}\frac{\partial\tilde{e}_i}{\partial r_1}\right] \\ &= \mathbb{E}\left[\frac{p}{r_1} - \sum_i \frac{\partial D}{\partial e_i}\frac{a_i}{r_i r_1}A^{-1}\right] \end{aligned}$$

where the second equality follows from substituting in Eqs. (25) and (27). Multiplying through by r_1, and noting that $e_i\left(\tilde{R},\Theta\right) = \tilde{e}_i(\tilde{r}_1,\Theta)$ gives the desired result. ■

Proof of Corollary 1

From Lemma 1, it follows that the formula in Proposition 1 can be written as

$$\left.\frac{\partial W}{\partial r_j}\right|_{\tilde{R}} = COV\left(p,e_j\right) - COV\left(A^{-1}\sum_i \frac{a_i}{\tilde{r}_i}\frac{\partial D}{\partial e_i}, e_j\right) + \sum_i COV\left(\tilde{r}_j\frac{\partial D}{\partial e_i} - \tilde{r}_i\frac{\partial D}{\partial e_j}, A^{-1}\frac{a_i p}{\tilde{r}_i C_j''}\right).$$

Because the damage function is regular, we have

$$\frac{\tilde{r}_j}{\tilde{r}_1} = \frac{\alpha_j}{\alpha_1}.$$

It follows from Eq. (3) that $\sum \alpha_i e_i = \sum \frac{\alpha_1}{\tilde{r}_1} \tilde{r}_i e_i = \frac{\alpha_1}{\tilde{r}_1}$ is a constant. Next consider the marginal damage function

$$\frac{\partial D}{\partial e_i} = F'\left(\sum \alpha_i e_i\right)\alpha_i = F'\left(\frac{\alpha_1}{\tilde{r}_1}\right)\alpha_i.$$

This is non-stochastic, and so Eq. (6) implies that $\frac{\partial D}{\partial e_i} / \tilde{r}_i = \frac{\partial D}{\partial e_1} / \tilde{r}_1$ for every *i*. Substituting these expressions into the partial derivative above gives

$$\begin{aligned} \left.\frac{\partial \mathcal{W}}{\partial r_j}\right|_{\tilde{R}} &= COV\left(p, e_j\right) - COV\left(A^{-1} \sum_i \frac{a_i}{\tilde{r}_1} \frac{\partial D}{\partial e_1}, e_j\right) + \sum_i COV\left(0, A^{-1} \frac{a_i p}{\tilde{r} C_j'}\right) \\ &= COV\left(p, e_j\right) - COV\left(\frac{\frac{\partial D}{\partial e_1}}{\tilde{r}_1} A^{-1} \sum_i a_i, e_j\right) = COV\left(p, e_j\right) \end{aligned}$$

where the third equality follows since $\tilde{r}_1$ and $\frac{\partial D}{\partial e_1}$ are non-stochastic. ■

Proof of Proposition 2

Start with derivative of regulator's objective function. Expanding Eq. (23) gives

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[A^{-1} \frac{p r_j}{C''_j} \sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i} - A^{-1} \frac{p}{C''_j} \frac{\partial D}{\partial e_j} \sum a_i + p e_j - A^{-1} e_j \sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i}\right] = 0.$$

Simplifying and then collecting terms gives

$$\frac{\partial \mathcal{W}}{\partial r_j} = \mathbb{E}\left[A^{-1}\left(\frac{p r_j}{C''_j} - e_j\right) \sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i} - \frac{p}{C''_j} \frac{\partial D}{\partial e_j} + p e_j\right] = 0. \quad \text{equation(29)}$$

Now take the weighted sum of the first-order conditions:

$$\sum r_i \frac{\partial \mathcal{W}}{\partial r_i} = \mathbb{E}\left[A^{-1}\left(p \sum_i \frac{r_i^2}{C_i''} - \sum_i r_i e_i\right)\left(\sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i}\right) - p \sum_i \frac{r_i}{C_i''} \frac{\partial D}{\partial e_i} + p \sum r_i e_i\right] = 0.$$

Using Eq. (3) and the definition of *ai* it follows that

$$\mathbb{E}\left[A^{-1}\left(p \sum_i a_i - 1\right)\left(\sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i}\right) - p \sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i} + p\right] = \mathbb{E}\left[\left(p - A^{-1} - p\right)\left(\sum_i \frac{a_i}{r_i} \frac{\partial D}{\partial e_i}\right) + p\right] = 0.$$

It follows that

$$\mathbb{E}[p] = \mathbb{E}\left[A^{-1}\sum_i a_i \frac{1}{r_i}\frac{\partial D}{\partial e_i}\right].$$ equation(30)

Returning to Eq. (29), we solve this equation for *rj*. This gives

$$r_j = \frac{\mathbb{E}\left[\frac{p}{C_j''}\frac{\partial D}{\partial e_j}\right] - \mathbb{E}\left[\left(p - A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right)e_j\right]}{\mathbb{E}\left[\frac{p}{C_j''}A^{-1}\sum_i \frac{a_i}{r_i}\frac{\partial D}{\partial e_i}\right]}.$$

Applying the formula COV(XY)=E[XY]−E[X]E[Y] and Eq. (30) to the second term in the numerator gives the desired result.

Appendix B. Supplementary online material

Available at http://dx.doi.org10.1016/j.jpubeco.2015.03.005

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