Extraction capacity and the optimal order of extraction

By: Stephen P. Holland

Holland, Stephen P. (2003) "Extraction Capacity and the Optimal Order of Extraction," *Journal of Environmental Economics and Management* 45(3) May, p. 569-588.

Abstract:
The least-cost-first extraction rule for deposits with different extraction costs previously has been shown to be invalid in general equilibrium. This paper demonstrates that this rule also does not hold in partial equilibrium when extraction capacity is limited. Necessary and sufficient conditions for several surprising extraction orders are presented. If extraction from a high-cost resource is constrained, it may be optimal to begin extraction from a high-cost deposit (or backstop) strictly before extracting from a lower-cost deposit. If extraction from a low-cost resource is limited, it may be optimal to exhaust a high-cost deposit strictly before the low-cost deposit is exhausted or to abandon extraction temporarily from a high-cost deposit and then to exhaust it later. The analysis demonstrates how extraction constraints affect the order of extraction and shows that certain cost reversals are caused by limited extraction capacity rather than by the general equilibrium definition of extraction costs.
Keywords: Natural resource extraction; Capacity constraints; Order of extraction; Cost reversals; Backstop

Article:

1. Introduction

If there are multiple deposits of a natural resource, efficiency may depend on the order in which the deposits are extracted. Herfindahl [6] established that, with constant marginal extraction costs, deposits should be extracted in strict sequence from lowest to highest cost.[1] Kemp and Long [8] described two "folk theorems" from this least-cost-first intuition: (1) deposits should be extracted in strict sequence by order of cost and (2) all finite deposits should be exhausted before production begins from a high-cost backstop. They then demonstrate that these theorems are invalid in general equilibrium. With a similar model, Amigues et al. [1] showed that efficient production from a high-cost backstop can occur strictly before extraction begins from a lower-cost deposit. This demonstrates that weaker versions of Kemp and Long's folk theorems also are invalid in general equilibrium.[2] Chakravorty and Krulce [3] demonstrated that strict sequencing of extraction might not be efficient in partial equilibrium models with heterogeneous demand. However, the literature still implies both implicitly and explicitly that the original folk theorems identified by Kemp and Long hold for partial equilibrium models with homogeneous demand.[3]

This paper demonstrates that the folk theorems also do not hold in partial equilibrium with homogeneous demand when extraction capacity is limited.[4] Several efficient extraction orders are presented which are counter to the least-cost-first rule. In one, extraction from a high-cost deposit begins strictly before extraction from a lower-cost deposit. In another, a high-cost deposit is exhausted before a low-cost deposit. In a third, extraction from a high-cost deposit ceases after a period of initial extraction and later resumes.[5] Each extraction order contradicts Kemp and Long's first folk theorem. Their second folk theorem is contradicted by an extraction order where a high-cost deposit is not exhaustible but is utilized before lower-cost deposits are exhausted.[6]

While it is useful to know that folk theorems which do not hold in general equilibrium also do not hold in partial equilibrium, it is troubling that the appealing least-cost-first rule seems to have such limited applicability.[7] Herfindahl's result can be illustrated with a proof by contradiction. Consider an extraction path which has simultaneous extraction in two periods from two deposits with different costs. Present value costs could be lowered by exchanging one unit of extraction from the higher-cost deposit in the earlier period for one unit of extraction from the lower-cost deposit in the later period. Since consumption would be unaltered by the exchange, welfare would increase, and therefore simultaneous extraction cannot be efficient. Note that this argument is valid in both partial and general equilibrium if (a) the exchange is always feasible and (b) the timing of extraction affects welfare.[8] The exchange is feasible if extraction capacity is not limited. In partial equilibrium, the timing of extraction affects welfare since costs are discounted. In general equilibrium, welfare is determined by consumption of goods and leisure. Therefore, the timing of extraction may not affect welfare if extraction costs only change the remaining stock of the resource but do not affect consumption.[9] The Herfindahl result does not apply in this paper because extraction capacity is limited.

Limited extraction capacity affects the shadow values (scarcity costs) of both constrained and unconstrained deposits.[10] A binding flow constraint implies that a marginal increment to the constrained deposit cannot be consumed until the flow constraint no longer binds. Thus, the value of the marginal increment is less than if the marginal increment could be consumed immediately. Additionally, the flow constraint may increase the shadow values of unconstrained deposits since extraction from the unconstrained deposits is a substitute for the lack of extraction capacity in the constrained deposit. This paper shows that the optimal order of extraction depends on the opportunity cost of extraction—which depends on the scarcity costs (shadow values) as well as the extraction costs. Thus applications of the least-cost-first extraction rule must take into account the full opportunity costs of extraction.

This paper demonstrates that cost reversals can occur in partial equilibrium if extraction is limited. Extraction constraints tend to appear implicitly in general equilibrium models since labor supply is usually assumed to be limited at any point in time. Analyzing extraction constraints explicitly in a partial equilibrium framework allows one to conclude that the social indifference result found in Kemp and Long[11] and the deposit smoothing result found in Amigues et al.[12] are indeed general equilibrium phenomena. However, showing that the cost reversal identified in Amigues et al. also occurs in partial equilibrium demonstrates that the cost reversal is caused by the extraction constraint rather than by the definition of costs used in general equilibrium.

Although it may be obvious that a more expensive deposit might be exploited early if extraction from a cheaper deposit is constrained, Section 2 shows the opposite. In particular, it shows that a high-cost, constrained deposit should be exploited before a lower-cost, unconstrained deposit if the high-cost deposit is plentiful (or infinite) and the lower-cost deposit is scarce. This generalizes the cost reversal discovered by Amigues et al. by making the backstop exhaustible. Section 3 then shows that if a low-cost deposit is constrained, it may be optimal to exhaust a high-cost deposit strictly before a low-cost deposit is exhausted. This occurs if it is more valuable to use the unconstrained deposit as a substitute for the lack of extraction capacity than as a substitute for the lack of low-cost stock. In some cases it may be optimal to use the unconstrained deposit to substitute for both the limited extraction capacity and the limited stock. In particular, it may be optimal to extract initially from the high-cost deposit but then to abandon it and finally to exhaust it at a later time. Section 4 illustrates construction of numerical examples for the various cost reversals and Section 5 concludes.

2. A constrained, high-cost deposit

A commodity can be extracted from three deposits with constant marginal costs. For deposit $i \in \{1,2,3\}$, let c_i be the marginal extraction cost, let S_i be the size of the deposit, and let $q_i(t)$ be the amount extracted from deposit i at time t. Label the deposits such that $c_1 < c_2 < c_3$. At time t, consumption of the commodity, $Q(t) = \Sigma_i q_i(t)$, yields gross surplus $U(Q)$ where U is differentiable and concave, and $U' > 0$. Assume the choke price, $U'(0)$; is finite and greater than c_3 so that it is efficient to use all deposits and to exhaust all exhaustible deposits in finite time.[13] Costs and benefits are discounted at the common rate r. Assume extraction from the high-cost deposit is constrained at each time by the installed capacity $\bar{q}_3$.

To determine the efficient extraction order of the deposits, consider the social planner's problem:

$$\max_{q_1(t),q_2(t),q_3(t)} \int_0^\infty e^{-rt}[U(Q(t)) - c_1q_1(t) - c_2q_2(t) - c_3q_3(t)]\,dt \quad (1)$$

subject to the constraints

$$\begin{aligned}&\int_0^\infty q_i(t)\,dt \leqslant S_i \quad \text{for } i = 1,2,3,\\ &q_3(t) \leqslant \bar{q}_3 \quad \forall t,\\ &q_i(t) \geqslant 0 \quad \forall t\ i = 1,2,3.\end{aligned} \quad (2)$$

Since the planner's objective function is strictly concave and the constraint set is convex, the maximization problem has a unique optimum. By the welfare theorems, this optimum can be supported as a Walrasian competitive equilibrium. Let $p(t) \equiv U'(Q^*(t))$; where $Q^*(t)$ is the efficient consumption in period t. Note that $p(t)$ is the equilibrium price path.[14] Let λ_i be the shadow value of the ith deposit and let $\mu(t)$ be the shadow value of the high-cost deposit's extraction constraint at time t. The Kuhn–Tucker first-order conditions for the Lagrangian of this problem can then be written[15]

$$\begin{array}{llll} q_i(t)\geqslant 0, & e^{-rt}[p(t)-c_i]-\lambda_i\leqslant 0 & \text{C.S.},\ i=1,2 & \forall t,\\ q_3(t)\geqslant 0, & e^{-rt}[p(t)-c_3]-\lambda_3-\mu(t)\leqslant 0 & \text{C.S.} & \forall t,\\ \lambda_i\geqslant 0, & S_i-\int_0^\infty q_i(t)\,dt\geqslant 0 & \text{C.S.},\ i=1,2,3, & \\ \mu(t)\geqslant 0, & \bar{q}_3-q_3(t)\geqslant 0 & \text{C.S.} & \forall t. \end{array} \quad (3)$$

Define the *augmented marginal cost* as the marginal extraction cost plus the scarcity cost at time t,[16] i.e., $AMC_i(t) \equiv c_i + \lambda_i e^{rt}$. The first two FOC's imply that whenever price is less than the augmented marginal cost of a deposit, there should be no extraction from that deposit. Furthermore, if there is positive extraction from a deposit, then either the price must equal the augmented marginal cost of that deposit, or extraction must be at capacity. Note that if the price equals the augmented marginal cost of a deposit, the net price grows at the rate of interest. This is the well-known result of Hotelling [7]. The third FOC requires that each deposit be exhausted unless its shadow value is zero, and the final equation requires that extraction be at capacity whenever the shadow value of the flow constraint is positive, i.e., whenever the price is greater than the augmented marginal cost.

Consider the sensitivity of the optimal extraction plan to the sizes of the flow constraint and the various deposits. If the flow constraint is sufficiently large, it may never affect extraction. Let $\dot{p} \equiv U'(\bar{q}_3)$; i.e., $\bar{p}$ is the marginal valuation of extraction at capacity. If $\bar{p} < c_3$, a firm would never extract from deposit 3 at capacity since the value of extracting the marginal unit would be less than the extraction cost. In this case, the order of extraction is exactly as described by Herfindahl, i.e., strict sequencing of extraction according to cost. However, if the extraction constraint is more limited, i.e., if $\bar{p} > c_3$. the optimal extraction order of the deposits may be affected.

If $\bar{p} > c_3$, first consider the extraction order in the special case where the high-cost deposit is infinite, e.g., the resource is renewable or a backstop.[17] In this case, the shadow value of the resource is zero, and the augmented marginal cost is constant, i.e., $AMC_3(t) = c_3$. The production rule for the backstop is the same in every period, namely: if $p(t) < c_3$, then there is no extraction; if $p(t) > c_3$, then extraction is at capacity; and if $p(t) = c_3$, then extraction is any amount up to capacity. The supply curve for this constrained backstop is illustrated in Fig. 1. A residual demand curve can be derived by subtracting the backstop supply from the marginal valuation curve. This residual demand curve shifts left for prices above c_3 and has a choke price, $\bar{p}$, since at this price the constrained backstop can satisfy demand. [18] The area of the shaded region in Fig. 1 is equal to the area under this residual demand curve.

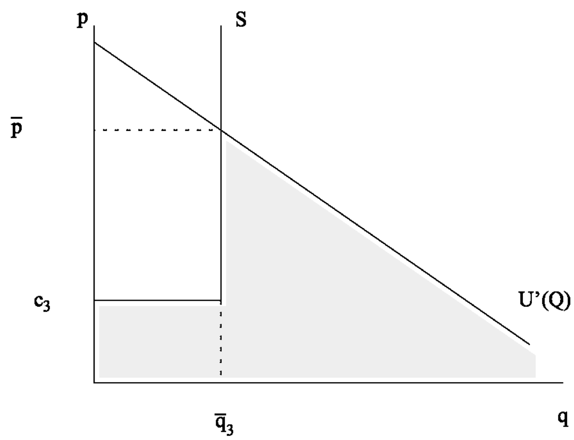

Fig. 1. Demand curve and the supply curve for the high-cost, constrained backstop. The shaded area is equal to the area under the residual demand curve.

Now consider the sub-problem of extracting the other two deposits, S_1 and S_2, based on the residual demand curve. Since extraction from these two deposits is not constrained, the Herfindahl result implies that extraction should be strictly sequential and that the low-cost deposit should be extracted first. Furthermore, the net price should grow at the rate of interest and should be continuous. Thus, the equilibrium price path is given by

$$p(t) = \begin{cases} c_1 + \lambda_1 e^{rt} & \text{for } t \in [0, T], \\ c_2 + \lambda_2 e^{rt} & \text{for } t \in [T, \bar{T}], \\ \bar{p} & \text{for } t \in [\bar{T}, \infty], \end{cases} \quad (4)$$

where T is when deposit 1 is exhausted and extraction begins from deposit 2, and $\bar{T}$ is when deposit 2 is exhausted.[19] This price path is illustrated in Fig. 2.[20]

Let t_3 be the time of initial extraction from deposit 3. The price at which production begins from the backstop, $p(t_3) = c_3$, is independent of the sizes of the other two deposits. In general, the price can reach $p(t_3)$ before or after deposit 1 is exhausted. Note in particular, that extraction begins from a high-cost, constrained backstop strictly before extraction begins from a lower-cost deposit, i.e., the cost reversal of Amigues et al. obtains, if and only if $p(T) > p(t_3)$. Since $p(T)$ depends on the size of deposit 2, this cost reversal occurs if deposit 2 is large. This situation is illustrated in Fig. 2.

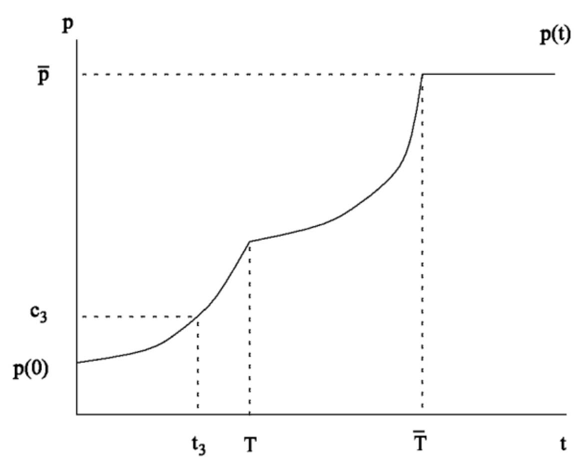

Fig. 2. Equilibrium price path with a capacity constrained backstop. Note that deposit 1 is extracted before time T, and deposit 2 is extracted from T to $\bar{T}$. Further note that in this figure, production from the constrained backstop begins at t_3 and the cost reversal obtains.

Under what conditions does this cost reversal obtain? To answer this question, define the critical size for deposit 2, $S_2^{max}(\infty)$, by[21]

$$S_2^{\max}(\infty) \equiv \int_0^x D(c_2 + (c_3 - c_2)e^{rt})\, dt - x\bar{q}_3, \qquad (5)$$

where x is defined by $c_2 + (c_3 - c_2)e^{rx} = \bar{p}$ and D is the demand curve.[22] Note that $S_2^{max}(\infty)$ is the amount of stock which would be extracted (i) if the price grew at the rate of interest net of c_2,[23] (ii) if the price started at c_3 and ended at $\bar{p}$, and (iii) if extraction from deposit 3 were at capacity. Further note that x is the time it would take for the price to grow from c_3 to $\bar{p}$ while growing at the rate of interest net of c_2. The proposition can now be stated which describes the conditions under which the cost reversal obtains if deposit 3 is inexhaustible.

Proposition 1. *Production begins from a high-cost, constrained backstop strictly before extraction begins from a lower-cost deposit if and only if* $U'(\bar{q}) > c_3$ *and* $S_2 < S_2^{max}(\infty)$.

Proof. The cost reversal obtains if and only if $p(T) > p(t_3)$ where T and $p(t)$ are as defined in Eq. (4). If $\bar{p} < c_3$, then the flow constraint never binds, extraction is by order of costs, and $p(T) < p(t_3)$. If $\bar{p} = c_3$, then deposit 2 must be exhausted when the price equals c_3, so $p(T) < p(t_3)$.

Consider $\bar{p} > c_3$. Recall that when extraction from deposit 2 is positive, the price must grow at the rate of interest net of c_2, and deposit 2 should be exhausted when the price reaches $\bar{p}$. Recall that $p(t_3)$ does not depend on S_2. If $S_2 = S_2^{max}(\infty)$, the definition of $S_2^{max}(\infty)$ implies that $p(T) = p(t_3)$: This follows since if $p(T)$ were greater than $p(t_3)$, there would be excess supply from deposit 2, i.e., deposit 2 would not be exhausted when the price reached $\bar{p}$, and if $p(T)$ were less than $p(t_3)$, there would be excess demand from deposit 2. Thus if $S_2 = S_2^{max}(\infty)$, extraction from the backstop and deposit 2 begin simultaneously. If $S_2 > S_2^{max}(\infty)$, when does extraction begin from deposit 2? If $p(T)$ were equal to $p(t_3)$, total extraction from deposit 2 would be precisely $S_2^{max}(\infty)$ since extraction from deposit 2 ceases when $p(t) \geq \bar{p}$. But this implies that there is excess supply from deposit 2. Similarly, if $p(T)$ were greater than $p(t_3)$, extraction from deposit 2 would be less than $S_2^{max}(\infty)$, and there would again be excess supply. Therefore, $p(T)$ must be less than $p(t_3)$ if $S_2 > S_2^{max}(\infty)$. If $S_2 < S_2^{max}(\infty)$, a symmetric argument shows that $p(T)$ must be greater than $p(t_3)$.

Proposition 1 demonstrates that extraction from a high-cost backstop may begin strictly before extraction from a lower-cost deposit, if extraction from the backstop is sufficiently limited and the lower-cost deposit is sufficiently scarce. This result is the partial equilibrium equivalent of the cost reversal discovered by Amigues et al. in a general equilibrium framework.[24] The extraction order does violate the simple least-cost-first extraction rule, especially if we note that the marginal extraction cost of the backstop would be infinite for extracting more than $\bar{q}_3$ units. Yet despite this high marginal extraction cost, extraction occurs from the backstop strictly before any extraction from a deposit with lower marginal extraction cost. This occurs because the scarcity cost of the low-cost deposit is enhanced as a substitute for the limited extraction capacity, whereas the value of the high-cost deposit is decreased by its lack of extraction capacity.[25] Thus, the proper intuition to extend the Herfindahl result is that the order of extraction is governed by the sum of the extraction and the scarcity costs, not by marginal extraction costs alone.

Although Proposition 1 does violate the least-cost-first extraction rule, the economic intuition behind the proposition is clear. Since the backstop would produce whenever $p(t) > c_3$, strictly sequential extraction would require that all other deposits be exhausted when the price reaches c_3. If extraction is only from deposit 3, the price cannot be less than $\bar{p}$. This requires that the price path would need to jump from c_3 to $\bar{p}$. If $\bar{p} > c_3$ this implies a discontinuous price jump which cannot occur in equilibrium. Thus intertemporal arbitrage requires that some of deposits 1 and 2 be available for extraction while there is production from the backstop. If deposit 2 is small, the proposition shows that both deposits should be used while there is production from the backstop, i.e., production should begin from the backstop strictly before extraction from deposit 2.

Does the cost reversal disappear when the high-cost deposit is exhaustible?[26] Analysis of the model where S_3 is finite indicates that the cost reversal can still obtain. Consider how extraction changes as the size of the high-cost deposit varies. The extraction rule from this deposit is now: if $p(t) < AMC_3(t)$, then no extraction; if $p(t) > AMC_3(t)$, extraction is at capacity; and if $p(t) = AMC_3(t)$, extraction is any amount up to capacity. Because AMC_3 is increasing, eventually AMC_3 becomes greater than $\bar{p}$. If $AMC_3(t) > \bar{p}$, extraction cannot be at capacity since demand is less than capacity. In this case, the Herfindahl argument implies that deposits 1 and 2 should be exhausted first and that the price should be growing at the rate of interest net of c_3 when it reaches the choke price.

If S_3 were very small or the flow constraint were large, extraction from the high-cost deposit would always be less than capacity and optimal extraction would be determined by the Herfindahl rule. If the deposit were more plentiful or the flow constraint smaller, then the extraction constraint might bind. Under what conditions might the flow constraint bind? To answer this question, define the critical size of deposit 3, S_3^{min}, by[27]

$$S_3^{\min} \equiv \int_0^{\infty} D(c_3 + (\bar{p} - c_3)e^{rt})\, dt. \tag{6}$$

Note that S_3^{min} is the amount of stock which would be extracted if the price were to start at $\bar{p}$ and grow at the rate of interest net of c_3. Further, note that S_3^{min} is independent of the parameters of deposits 1 and 2.

Lemma 1. *Optimal extraction from the high-cost deposit is at capacity at some time if and only if* $U'(\bar{q}_3) \geq c_3$ *and* $S_3 \geq S_3^{min}$.

Proof. If $\bar{p} < c_3$, then capacity never binds. If $\bar{p} = c_3$, then capacity will bind only if the scarcity cost of deposit 3 is zero, i.e., the deposit is not exhaustible. Consider $\bar{p} > c_3$. The definition of t_3 implies that extraction is at capacity at some time if and only if $p(t_3) \leq \bar{p}$. Recall that deposit 3 should be exhausted when the price reaches the choke price and that the price should grow at the rate of interest net of c_3 when extraction is below capacity. If $S_3 = S_3^{min}$, then the definition of S_3^{min} implies that $p(t_3) = \bar{p}$ since for $p(t_3) > \bar{p}$ there would be excess supply from deposit 3, and for $p(t_3) < \bar{p}$ there would be excess demand from deposit 3. Thus $p(t_3) = \bar{p}$ if $S_3 = S_3^{min}$. If $S_3 < S_3^{min}$, $p(t_3)$ must be greater than $\bar{p}$ because otherwise there would be excess demand from deposit 3. Similarly if $S_3 > S_3^{min}$, then $p(t_3) \leq \bar{p}$.

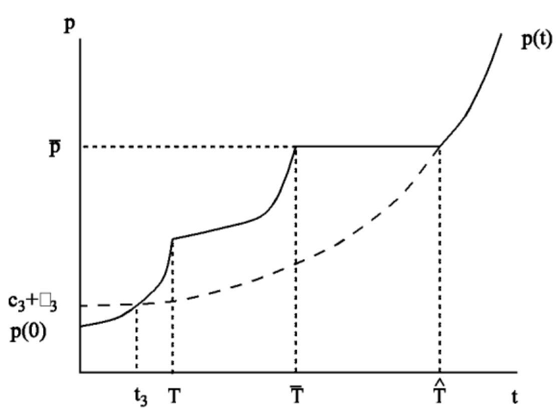

Fig. 3. Equilibrium price path for three exhaustible deposits where the cost reversal obtains. The dashed line is the augmented marginal cost of the high-cost, constrained deposit. Note that between t_3 and $\hat{T}$, the price is above the augmented marginal cost of the high-cost deposit and extraction is at capacity from this deposit.

If extraction from deposit 3 is at capacity for some period of time, the Herfindahl argument implies that deposits 1 and 2 should be extracted strictly sequentially based on the (time-dependent) residual demand curve. In this case, the equilibrium price path has four parts, is continuous, and is given by[28]

$$p(t) = \begin{cases} c_1 + \lambda_1 e^{rt} & \text{for } t \in [0, T], \\ c_2 + \lambda_2 e^{rt} & \text{for } t \in [T, \bar{T}], \\ \bar{p} & \text{for } t \in [\bar{T}, \hat{T}], \\ c_3 + \lambda_3 e^{rt} & \text{for } t \in [\hat{T}, \infty], \end{cases} \tag{7}$$

where $\hat{T}$ is the time when extraction from deposit 3 is last at capacity. This price path is illustrated by the solid line in Fig. 3.

At what price does extraction begin from the high-cost deposit if $S_3 > S_3^{min}$? The production rule for deposit 3 states that extraction should be at capacity when the price exceeds the augmented marginal cost. Since only S_3^{min} units of stock can be extracted when the price equals the augmented marginal cost, the remainder of the stock must be extracted at capacity when $p(t)$ exceeds $AMC_3(t)$. Note that $y \equiv (S_3 — S_3^{min})/\bar{q}_3$ is the length of time extraction from deposit 3 should be at capacity. Since $AMC_3(t_3 + y) = \bar{p}$ and AMC_3 must grow at the rate of interest net of c_3, it follows that $AMC_3(t_3) = c_3 + (\bar{p} - c_3)e^{-ry}$.[29] Note in particular, that $AMC_3(t_3)$ is independent of the parameters of deposits 1 and 2. Further note that the price at which extraction begins from the high-cost deposit is $AMC3(t_3)$ unless $p(0)$ is already greater than $AMC3(t_3)$, i.e., $p(t_3) = \max\{p(0), AMC_3(t3)\}$.

As above, the cost reversal obtains if deposit 2 is relatively scarce. To determine the conditions for the cost reversal, first define the critical size of deposit 2, $S_2^{max}(S_3)$; as[30]

$$S_2^{\max}(S_3) \equiv \int_0^x D(c_2 + (\bar{p} - c_2)e^{r(t-x)})\, dt - x\bar{q}_3, \tag{8}$$

where x is determined from

$$c_2 + (\bar{p} - c_2)e^{-rx} = AMC_3(t_3) \tag{9}$$

Eq. (8) defines $S_2^{max}(S_3)$ as the amount of stock which would be extracted if (i) the price grew at the rate of interest net of c_2 to $\bar{p}$, (ii) the initial price were $AMC_3(t_2)$ (implied by Eq. (9)), and (iii) extraction from deposit 3 were at capacity. Note that S_2^{max} is a function of S_3 since $AMC_3(t_3)$ depends on S_3. A proposition which establishes necessary and sufficient conditions under which the cost reversal obtains can now be stated.

Proposition 2. *Extraction begins from a high-cost, constrained deposit strictly before extraction begins from a lower-cost deposit if and only if $\bar{p} \geq c_3$, $S_3 \geq S_3^{min}$, and $S_2 < S_2^{max}(S_3)$.*

Proof. The cost reversal obtains if and only if $p(t_3) < p(T)$: If the first two conditions do not hold, then extraction is never at capacity and Herfindahl implies that $p(t_3) > p(T)$.

If the first two conditions hold, then Lemma 1 implies that extraction is at capacity for some time and the price path is as in Eq. (7). Recall that $AMC_3(t_3)$ does not depend on the parameters of deposits 1 and 2 and that deposit 2 should be exhausted when the price reaches $\bar{p}$. If $S_2 = S_2^{max}(S_3)$; then deposit 3 will be exhausted if $p(t_3) = AMC_3(t_3)$. The definition of $S_2^{max}(S_3)$ then implies that $p(\mathrm{T}) = p(t_3)$. This follows since if $p(T)$ were greater (less) than $p(t_3)$, there would be excess supply (demand) from deposit 2. Now consider $S_2 > S_2^{max}(S_3)$. If $p(T)$ were greater than or equal to $p(t_3)$; there would be excess supply from deposit 2. Therefore $p(T) < p(t_3)$. If $S_2 < S_2^{max}(S3)$, there would be excess demand from deposit 2 if $p(T) \leq AMC_3(t_3)$: Therefore, $p(T) > p(t_3)$.[31]

Proposition 1 demonstrated that extraction from a high-cost backstop may begin strictly before extraction from a lower-cost deposit. Proposition 2 then extends this result to the case of a finite deposit which is sufficiently large. The logic remains the same: (i) the scarcity cost of the high-cost deposit is decreased by its abundance and by its limited extraction capacity, but (ii) the scarcity cost of the lower-cost deposit is increased by its scarcity and by the limited extraction capacity of the other deposit, therefore (iii) the opportunity cost of extraction from deposit 3 is initially lower than the opportunity cost of extraction from deposit 2 and extraction begins from deposit 2 first.

As above, the economic intuition for Proposition 2 follows from the continuity of the price path. If $\bar{p} \geq c_3$ and $S_3 \geq S_3^{min}$, extraction from deposit 3 should initially be at capacity. Thus, strictly sequential extraction would again require a discontinuous price jump which cannot occur in equilibrium. If deposit 2 is small, both deposits are utilized while extraction from deposit 3 is positive, and the cost reversal obtains.

3. A constrained, low-cost deposit

Section 2 demonstrates that limited extraction capacity may affect the extraction order in surprising ways if a high-cost deposit is constrained. This section demonstrates how limited extraction capacity affects the extraction order if a low-cost deposit is constrained.

To illustrate the effects of a flow constraint on a low-cost deposit, consider the optimal extraction order from two deposits where extraction from the low-cost deposit is constrained. The above model can be modified to explore this case by adding the constraints $S_3 = 0$ and $q_1(t) \leq \bar{q}_1 \forall t$ where $\bar{q}_1$ is the flow constraint on the low-cost deposit. The Kuhn–Tucker conditions for the social planner's problem are now:

$$\begin{array}{llll} q_1(t) \geqslant 0, & e^{-rt}[p(t) - c_1] - \lambda_1 - \mu_1(t) \leqslant 0 & \text{C.S.} & \forall t, \\ q_2(t) \geqslant 0, & e^{-rt}[p(t) - c_2] - \lambda_2 \leqslant 0 & \text{C.S.} & \forall t, \\ \lambda_i \geqslant 0, & S_i - \int_0^\infty q_i(t)\,dt \geqslant 0 & \text{C.S.}, & i = 1,2, \\ \mu_1(t) \geqslant 0, & \bar{q}_1 - q_1(t) \geqslant 0 & \text{C.S.} & \forall t, \end{array} \tag{10}$$

where μ_1(t) is the shadow value of the extraction constraint for the low-cost deposit. As above, the Kuhn–Tucker conditions imply that whenever the price is above the augmented marginal cost, extraction is at capacity, and whenever price equals the augmented marginal cost, i.e., the net price grows at the rate of interest, extraction satisfies demand.

To illustrate the effect of the flow constraint on the extraction plan, first define $\bar{p}_1 \equiv U'(\bar{q}_1)$ as the marginal utility at capacity from deposit 1. Clearly, if $\bar{p}_1 < c_1$, then the constraint never binds and the Herfindahl solution holds. If $\bar{p}_1 \geq c_1$, the extraction constraint may still not bind. Recall that both deposits should be exhausted when the price reaches the choke price. Thus, if both deposits are small, the initial price is greater than the capacity price, i.e., $p(0) = c_1 + \lambda_1 > \bar{p}_1$. In this case, the extraction constraint never binds, and the Herfindahl solution obtains. If the deposits were larger, the initial price would be lower. In fact if the deposits were large enough such that $c_1 + \lambda_1 < \bar{p}_1$, then the extraction constraint would bind initially.[32] Note that the price cannot be below the capacity price with extraction only from deposit 1. Thus if $c_2 + \lambda_2 > \bar{p}_1$, initial extraction is only from deposit 1 and the price path is

$$p(t) = \begin{cases} \bar{p}_1 & \text{for } t \in [0, \bar{T}], \\ c_1 + \lambda_1 e^{rt} & \text{for } t \in [\bar{T}, \hat{T}], \\ c_2 + \lambda_2 e^{rt} & \text{for } t \in [\hat{T}, \infty], \end{cases} \tag{11}$$

where $\bar{T}$ is the instant before the extraction constraint no longer binds and $\hat{T}$ is when deposit 1 is exhausted. Until time $\bar{T}$, extraction is only from deposit 1 and is at capacity. Between time $\bar{T}$ and $\hat{T}$ extraction is only from deposit 1 and is below capacity. After $\hat{T}$, extraction is only from deposit 2. This price path, illustrated in Fig. 4, is constant, then grows at the rate of interest net of c_1, and later grows net of c_2.

For the case shown in Fig. 4, there is no initial extraction from the high-cost deposit since $p(0) = \bar{p}_1 < c_2 + \lambda_2$. However, this inequality need not hold. If the high-cost deposit is sufficiently large, it may be that $c_2 + \lambda_2 < \bar{p}_1$. This initial configuration of augmented marginal costs is illustrated in Fig. 5. Since the augmented marginal cost of deposit 1 is below the capacity price, extraction from this deposit should be at capacity. However, this implies positive residual demand below $\bar{p}_1$. Since the augmented marginal cost of deposit 2 is also below the capacity price, extraction from the high-cost deposit fills the residual demand. Thus in Fig. 5, the initial price is $c_2 + \lambda_2$, and initial extraction from the high-cost deposit is positive, i.e., $q_2(0) = Q(0) - \bar{q}_1 > 0$.

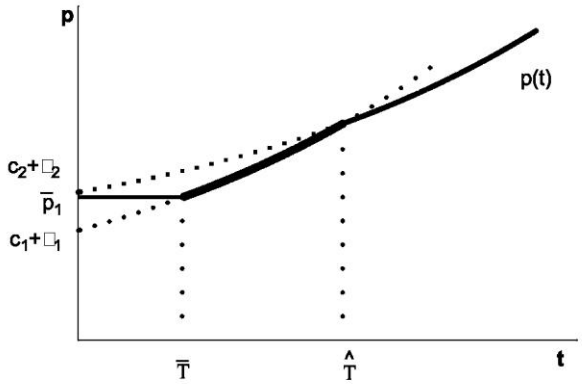

Fig. 4. Price path if extraction from a low-cost deposit is constrained. Note that before $\bar{T}$ the price is above the augmented marginal cost of deposit 1 so extraction is at capacity in this range.

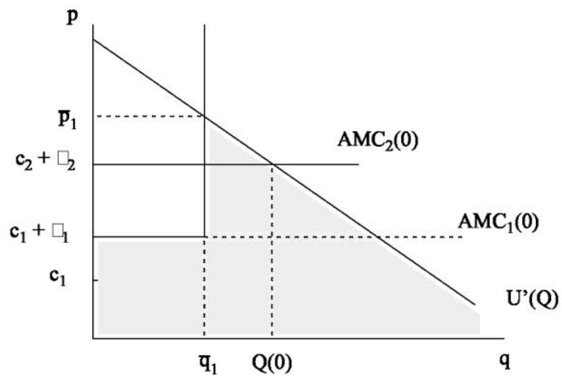

Fig. 5. Demand curve and initial augmented marginal cost curves where initial extraction from the high-cost deposit is positive. The area under the residual demand curve is shaded and the high-cost deposit here fills the residual demand.

Under what conditions will initial extraction be from the high-cost deposit? To answer this question, the critical deposit size, $S_2^{min}(S_1)$; is defined in Appendix A. S_2^{min}(S1) is the deposit size for which a firm extracting from deposit 2 would be indifferent between extracting at $t = 0$ at a price $\bar{p}_1$ and extracting after deposit 1 is exhausted. The following lemma can now be stated:

Lemma 2. *Initial extraction is from both deposits if and only if* $\bar{p}_1 > c_2$ *and* $S_2 > S_2^{min}(S_1)$.

Proof. See Appendix A.

Lemma 2 shows that initial extraction should be from the high-cost deposit if the low-cost deposit is sufficiently constrained and the high-cost deposit is sufficiently large. This shows that the unconstrained deposit might sometimes be best used as a substitute for the lack of extraction capacity rather than saving it until the low-cost deposit is exhausted. However, since augmented marginal costs are increasing, it may be that $AMC_1(t) < \bar{p}_1 < AMC_2(t)$ at some time $t > 0$.[33] At this time, extraction would be only from deposit 1 and would be at capacity. Note that if extraction was initially from both deposits, extraction from the higher-cost deposit 2 must have ceased at some point. The price path for this case would be

$$p(t) = \begin{cases} c_2 + \lambda_2 e^{rt}, & \text{for } t \in [0, T], \\ \bar{p}_1, & \text{for } t \in [T, \bar{T}], \\ c_1 + \lambda_1 e^{rt}, & \text{for } t \in [\bar{T}, \hat{T}], \\ c_2 + \lambda_2 e^{rt}, & \text{for } t \in [\hat{T}, \infty], \end{cases} \tag{12}$$

where T is when extraction from deposit 2 temporarily ceases. This price path is illustrated in Fig. 6. Before time T, the price grows at the rate of interest net of c_2, the low-cost deposit produces at capacity and the high-cost deposit fills the residual demand. After T, the augmented marginal cost of the high-cost deposit exceeds the capacity price, extraction from the high-cost deposit temporarily ceases, and extraction is from deposit 1 at capacity. After time $\bar{T}$, $AMC_1(t)$ exceeds the capacity price, extraction from deposit 1 is below capacity, and the price rises at the rate of interest net of c_1.[34]

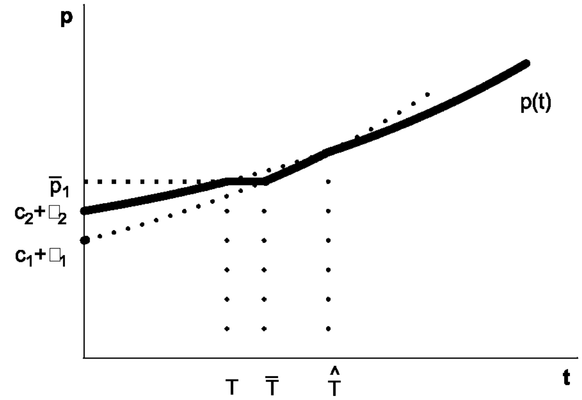

Fig. 6. Price path for the case where the high-cost deposit is abandoned. Note that before T, extraction from deposit 1 is at capacity and deposit 2 fills the residual demand. At time T, the high-cost deposit 2 is abandoned and conserved until $\hat{T}$.

Under what conditions would extraction from the high-cost deposit be initially positive but later zero? To answer this question, the critical deposit size, $S_2^{max}(S_1)$ is defined in the appendix. $S_2^{max}(S_1)$ is the smallest high-cost deposit size for which the price grows at the rate of interest net of c_2 for all t. The following lemma can now be stated:

Lemma 3. *If extraction from a low-cost deposit is constrained, a high-cost deposit should be extracted initially and then abandoned at a price less than* $U'(0)$ *if and only if* $\bar{p}_1 > c_2$, $S_2^{min}(S_1) < S_2$, *and* $S_2 < S_2^{max}(S_1)$.

Proof. See appendix.

Lemma 3 indicates that a high-cost deposit may be abandoned after a period of initial extraction if the deposit is not too abundant. Although the lemma provides conditions under which the deposit should be abandoned, it does not indicate whether or not the deposit should be exhausted when it is abandoned. In fact, it may be better to reserve some of the high-cost deposit for use after the low-cost deposit is exhausted if the low-cost deposit is scarce. Alternatively, it may be optimal to exhaust the high-cost deposit. Note that if the deposit is exhausted before it is abandoned, then the high-cost deposit would be exhausted strictly before the low-cost deposit. To determine conditions under which the high-cost deposit is exhausted strictly before the low-cost deposit, the critical deposit size, $\tilde{S}_1(S_2)$; is defined in Appendix A. $\tilde{S}_1(S_2)$ is the size of deposit 1 for which a firm extracting from deposit 2 would be indifferent between initial extraction and extraction when deposit 1 is exhausted at the choke price. The following proposition can now be stated:

Proposition 3. *If extraction from a low-cost deposit is constrained, a high-cost deposit should be exhausted before the low-cost deposit if and only if* $\bar{p}_1 > c_2$, $S_2^{min}(S_1) < S_2$, $S_2 < S_2^{max}(S1)$; *and* $S_1 \geq \tilde{S}_1(S_2)$

Proof. See appendix.

Extraction from the unconstrained, high-cost deposit can substitute either for the lack of extraction capacity or for the lack of low-cost stock. If the unconstrained deposit substitutes for the lack of extraction capacity, then the unconstrained deposit should be used when extraction capacity binds. If it substitutes for the lack of low-cost stock, then it should be used after the low-cost deposit is exhausted. Lemma 2 describes conditions under which the unconstrained deposit is used as a substitute for the lack of extraction capacity. The lemma demonstrates that initial extraction should be from both deposits if (i) the extraction constraint binds sufficiently ($\bar{p}_1 > c_2$) and (ii) the high-cost deposit is sufficiently plentiful ($S_2 > S_2^{min}(S_1)$) such that its scarcity cost is low and it is a good substitute for the lack of capacity. Lemma 3 shows that extraction from deposit 2 should be abandoned at a price less than the choke price if the deposit is sufficiently scarce that its shadow value grows above $\bar{p}_1$ and it is no longer a good substitute for the lack of extraction capacity. However, Lemma 3 does not indicate whether deposit 2 should be exhausted or not when it is abandoned. In general, deposit 2 need not be exhausted when it is abandoned, but Proposition 3 shows that it should be exhausted if deposit 1 is sufficiently large that it is not optimal to save some of deposit 2 for extraction after deposit 1 is exhausted. If deposit 2 is exhausted before deposit 1, the unconstrained deposit is used solely as a substitute for the lack of extraction capacity.

Proposition 3 and Lemma 3 imply that deposit 2 may not be exhausted when it is abandoned. In this case, extraction would be initially from both deposits, then from only deposit 1, and finally from only deposit 2. Here the unconstrained deposit would substitute first for the extraction constraint and later for the stock constraint. A numerical example in the next section illustrates that this case can obtain for certain parameters.

The economic intuition for these cases is again explained by the arbitrage conditions on the price path. Since extraction from deposit 1 is constrained, the price cannot be below $\bar{p}_1$ when extraction is solely from deposit 1. If deposit 2 is large, initial extraction at price $\bar{p}_1$ might yield greater present value profits for deposit 2 than waiting until deposit 1 is exhausted. This arbitrage from deposit 2 will cause simultaneous extraction. Deposit 2 is then abandoned before exhaustion if the present value profits from waiting becoming greater than the present value profits from extracting.

4. Numerical examples

Proposition 3 and Lemma 3 imply that deposit 2 may not be exhausted when it is abandoned. The following numerical example illustrates that this case can obtain for certain parameters and also illustrates how numerical examples can be constructed for the other cost reversals. The example outlines a simple numerical procedure which can be used to solve more realistic parameterizations of the model.

Consider the parameters $U'(Q) = 10 — Q$, $r = 0.\ 1$, $\bar{q}_1 = 4$, $c_1 = 1$, $c_2 = 3$, $S_1 = 26.84$ and $S_2 = 11.34$. Note that this implies $\bar{p}_1 = 6$ and $U'(0) = 10$. If there were no capacity constraints, the solution to the model would be easy to compute since deposits would be extracted strictly sequentially. For this simple demand parameterization, analytic methods would be sufficient. However, for more realistic demand parameterizations, numerical approximation methods would likely be required. A numerical method could simply guess shadow values (λ's) for the two deposits and then calculate demand based on the shadow values. If demand were greater (less) than supply, the shadow values would be raised (lowered) until there was no longer excess demand or supply from either deposit.

With limited extraction capacity, one could describe the extraction order by calculating values for $S_2^{min}(S_1)$, $S_2^{max}(S_1)$, and $\tilde{S}_1(S_2)$ and comparing the values with the actual deposit sizes. However, these calculations would be complicated. A numerical method to solve the model could simply guess shadow values for the two deposits and exploit the basic relationship between shadow values and excess demand. Note however, that calculation of the price path and demand must take into account any extraction constraints.[35] In this example, if the shadow values $\lambda_1 = 3$ and $\lambda_2 = 2$ were guessed, the price path would be as in Eq. (12) and Fig. 6 where $e^{rT} = 1.5$, $e^{r\bar{T}} = 1.667$, $e^{r\hat{T}} = 2$, and $e^{\mathrm{r}T\mathrm{c}} = 3.5$. Note that this is the equilibrium price path with $p(0) = 5$, $p(T) = 6$, $p(\bar{T}) = 6$, $p(\hat{T}) = 7$ and $p(T^c) = 10$ since $\bar{T}\bar{q}_1 + \int_{\bar{T}}^{\hat{T}} D(p(t))\,dt = 26.84$ and $\int_0^T D(p(t))\,dt — T\bar{q}_1 + \int_{\hat{T}}^{Tc} D(p(t))\,dt = 11.34$.

Similarly, an example can be constructed in which the high-cost deposit should be exhausted strictly before the low-cost deposit. Consider the same demand and cost parameters with deposit sizes $S_1 = 40.626$ and $S_2 = 2.164$. Note that if the shadow values $\lambda_1 = 2.5$ and $\lambda_2 = 2$ were guessed, then the price path would be as in Eq. (12) and

Fig. 6 except that the choke price would be reached before $\hat{T}$. This implies that $e^{rT} = 1:5$; $e^{r\bar{T}} = 2$; and $e^{rTc} = 3:6$: Note that this is the equilibrium price path since $\bar{T}\bar{q}_1 + \int_{\bar{T}}^{Tc} D(p(t))\, dt = 40.626$ and $\int_0^T D(p(t))\, dt — T\bar{q}_1 = 2:164$:

5. Conclusion

It is known that the least-cost-first extraction rule does not hold in general equilibrium. This paper shows that the least-cost-first rule also must be modified in partial equilibrium if extraction capacity is limited. Limited extraction capacity affects the shadow values of constrained and unconstrained deposits. A marginal increment to a constrained deposit is less valuable since it cannot be used immediately. On the other hand, a marginal increment to an unconstrained deposit is more valuable since it can be used as a substitute for the lack of extraction capacity. By changing these opportunity costs, extraction constraints can affect the order of extraction.

Several extraction orders are presented which violate the least-cost-first extraction rule. If a high-cost deposit is constrained, it may be optimal to extract from a high-cost deposit (or backstop) strictly before extraction begins from a low-cost deposit if the high-cost deposit is sufficiently plentiful and the low-cost deposit is sufficiently scarce. This order results because the opportunity cost of extraction from the high-cost deposit is decreased by the constraint, but the opportunity cost of the low-cost deposit is increased by the constraint.

If a low-cost deposit is constrained, it may be optimal to exhaust a high-cost deposit strictly before a low-cost deposit if the low-cost deposit is sufficiently plentiful. In this case, the unconstrained, high-cost deposit is used solely as a substitute for the lack of extraction capacity. However, if the low-cost deposit is sufficiently plentiful, it may be optimal to save some of it for extraction after the low-cost deposit is exhausted. In this case, the high-cost deposit would be used initially, then abandoned and later extracted until exhausted. Here the high-cost deposit first substitutes for lack of the extraction capacity then later substitutes for the lack of low-cost stock. These cases illustrate the effect of limited capacity on the order of extraction.

The results of this paper emphasize the importance of scarcity costs in applications of the least-cost-first extraction rule. Moreover, the cases clarify that certain cost reversals are caused by limited extraction capacity and not by the general equilibrium definition of extraction costs as suggested by the previous literature. Although the cases violate the least-cost-first extraction rule, they are readily explained using the arbitrage conditions on the competitive equilibrium price path.

Appendix A

Definition of $S_2^{min}(S_1)$. Define the critical size of deposit 2, $S_2^{min}(S_1)$; by

$$S_2^{min}(S_1) = \int_y^\infty D\,(c_2 + (\bar{p}_1 — c_2)e^{rt})\, dt, \tag{A:1}$$

where x, y and p_y are defined from

$$S_1 = x\bar{q}_1 + \int_x^y D(c_1 + (\bar{p}_1 - c_1)e^{r(t-x)})\, dt \tag{A.2}$$

and

$$p_y = c_2 + (\bar{p}_1 - c_2)e^{ry}, \tag{A.3}$$

$$p_y = c_1 + (\bar{p}_1 - c_1)e^{r(y-x)}. \tag{A.4}$$

Eq. (A. 1) defines $S_2^{min}(S_1)$ as the amount of stock which would be extracted if extraction started at p_y (defined in Eq. (A.3)) and grew at the rate of interest net of c_2. Eq. (A.2) shows that x is the length of time extraction from deposit 1 would be at capacity given that it must be exhausted when the price rises to p_y (defined in Eq. (A.4)) from an initial price of $\bar{p}_1$. Eq. (A.3) then sets both the final extraction price from deposit 1 and the initial extraction price from deposit 2 to p_y. Furthermore, note that $AMC2(0) = \bar{p}_1$ as can be seen by setting $t = 0$ in Eq.

(A.1). This implies that a firm would be indifferent between extraction from deposit 2 at time zero at a price $\bar{p}_1$ or after time y.

Proof of Lemma 2. If $\bar{p}_1 < c_2$, then either the initial price would be less than c_2, or the capacity constraint would never bind. If $\bar{p}_1 = c_2$, then it is cheaper to fill initial demand with extraction solely from deposit 1.

Consider $\bar{p}_1 > c_2$. If $S_2 = S_2^{min}(S_1)$; then the definition of $S_2^{min}(S_1)$ implies that the equilibrium price path is as given in Eq. (11) where $\bar{T} = x$ and $\hat{T} = y$ and there is no initial extraction from deposit 2. If $S_2 < S_2^{min}(S_1)$, then if extraction from deposit 2 started at p_y, there would be excess demand. Thus, extraction must start at a higher price and there is no initial extraction from deposit 2. If $S_2 < S_2^{min}(S_1)$, then extraction must start at a price lower than p_y. Thus $p(y) = AMC_2(y) < p_y$. Recall from the definition of $S_2^{min}(S_1)$ that the initial augmented marginal cost was equal to $\bar{p}_1$, so a firm would have been indifferent between extraction at time 0 at a price $\bar{p}_1$ and extraction at time y at a price p_y. But now the firm would strictly prefer extraction at time 0 at a price $\bar{p}_1$. Therefore, $p(0) = AMC_2(0)$ must now be less than $\bar{p}_1$ so initial extraction from deposit 2 is, positive.

Definition of $S_2^{max}(S_1)$. Define the critical size of deposit 2, $S_2^{max}(S_1)$, by

$$S_2^{\max}(S_1) = \int_0^\infty D(c_2 + (\bar{p}_1 - c_2)e^{r(t-z)})\,dt - z\bar{q}_1, \tag{A.5}$$

where z is defined from $S_1 = z\bar{q}_1$. Note that z is the length of time extraction from deposit 1 could be at capacity. Eq. (A.5) shows that $S_2^{max}(S_1)$ is the amount of stock which would be extracted if the price grew at the rate of interest net of c_2 and was equal to $\bar{p}_1$ at time z.

Proof of Lemma 3. If the first two conditions do not hold, initial extraction from deposit 2 is not positive.

If the first two conditions do hold, then $p(0) < \bar{p}_1$. If $S_2 = S_2^{max}(S_1)$, the definition of S_2^{max}(S1) implies that the price always grows at the rate of interest net of c_2, deposit 2 is never abandoned, deposit 1 is extracted at capacity until exhaustion at time z, and $p(z) = \bar{p}_1$. If $S_2 > S_2^{max}(S_1)$, then $p(z) < \bar{p}_1$ otherwise there would be excess supply from deposit 2. Since deposit 1 would be exhausted at z, deposit 2 is never abandoned. If $S_2 < S_2^{max}(S_1)$, then $p(z) > \bar{p}_1$ otherwise there would be excess demand from deposit 2. But then at some time before z, it must be that the price is greater than $\bar{p}_1$ but deposit 1 is not exhausted. Since extraction is not at capacity, the Herfindahl argument implies that extraction must be only from deposits 1 and 2 must have been abandoned when the price reached $\bar{p}_1$.

Definition of $\tilde{S}_1(S_2)$. Define the critical size of deposit 1, $\tilde{S}_1(S_2)$, by

$$\tilde{S}_1(S_2) = l\bar{q}_1 + \int_l^m D(c_1 + (\bar{p}_1 - c_1)e^{r(t-l)})\,dt, \tag{A.6}$$

where l and m are defined from

$$U'(0) = c_1 + (\bar{p}_1 - c_1)e^{r(m-l)}, \tag{A.7}$$

$$U'(0) = c_2 + (\bar{p}_1 - c_2)e^{r(m-k)}, \tag{A.8}$$

where k is defined by

$$S_2 = \int_0^k D(c_2 + (\bar{p}_1 - c_2)e^{r(t-k)})\,dt - k\bar{q}_1. \tag{A.9}$$

Note that $\tilde{S}_1(S_2)$ is the amount of stock which would be extracted if extraction were at capacity for l units of time and then filled demand as the price rose at the rate of interest net of c_1 from $\bar{p}_1$ to the choke price (from Eq. (A.7)). Eq. (A.9) defines k as the length of time it would take to exhaust deposit 2 if the price rose to $\bar{p}_1$ at the rate of interest net of c_2. Eq. (A. 8) requires that the shadow value of deposit 2 be such that if the price were to grow from $\bar{p}_1$ at the rate of interest net of c_2, it would reach the choke price after m — k units of time.[36] Note that this implies that extraction from deposit 2 would yield identical profits at time 0 at price $p(0)$ and at time m at price $U'(0)$.

Proof of Proposition 3. If the first three conditions do not hold, extraction is not initially from deposit 2 and the deposit is not abandoned.

If the first three conditions hold, then initial extraction is from deposit 2 which is then later abandoned. If $S_1 = \tilde{S}_1(S_2)$, then the definition of $\tilde{S}_1(S_2)$ implies that supply equals demand if the price grows at the rate of interest net of c_2 until time k, then is constant until time l, and finally grows at the rate of interest net of c_1 to the choke price at time m. Thus deposit 2 is exhausted before deposit 1. If $S_1 > \tilde{S}_1(S_2)$, then $p(m)$ must be less than $U'(0)$, so deposit 2 is again exhausted before deposit 1. If $S_1 < \tilde{S}_1(S_2)$, then deposit 1 must be exhausted before m, so the price must reach the choke price before m. Recall that from the definition of $\tilde{S}_1(S_2)$ extraction from deposit 2 had the same present value profits at time 0 and time m. Thus, if the price is slightly higher before m, it would be optimal to extract from deposit 2 at this time. Therefore, deposit 2 is not exhausted before deposit 1.

Notes:

1Solow and Wan [14] also derive this result for a model with productive capital. Weitzman [15] extends the model to include arbitrary extraction costs and Gaudet et al. [5] extend the result to spatially distributed users.
2 Lewis [10] argued that if the resource can be converted to productive capital earning a positive rate of return, then strict sequencing of extraction is restored in general equilibrium.
3 Lewis conjectures that "in partial equilibrium models of resource production, it seems to be almost transparent that it is efficient to exploit low cost deposits first". Amigues et al. claim that their result "completely reverses the ordering
which would appear optimal from partial equilibrium analysis." 4 See Krautkraemer [9] for a discussion of the importance of extraction capacity in the analysis of natural resource
extraction. See also [2,4,11,12] for a discussion of investment in extraction capacity.
5 Gaudet et al. find that deposits may be optimally abandoned in spatial models with setup costs. Mason [13] shows that firms may abandon non-trivial deposits with switching costs and uncertainty.
6 Kemp and Long are not precise about conditions under which their folk theorems might hold. The models of Kemp and Long and Amigues et al. both limit extraction capacity through the constraint on labor supply. Thus, it is unclear whether their results are driven by limited extraction capacity or by their general equilibrium definition of costs.
7 Kemp and Long recommend that the theorems "be separately verified for each model of production and for each notion of cost constancy."
8 With non-constant marginal extraction costs, simultaneous extraction can occur as augmented marginal extraction costs are equated across deposits. See also Amigues et al. for an interesting example where simultaneous extraction occurs to smooth the transition from extraction of one deposit to extraction of another while the marginal utility of leisure grows at the rate of interest.
9 In Kemp and Long's model, consumption is determined by the optimal mix of leisure and consumption. In their third proposition, available hours are "sufficiently large" such that leisure is positive in every period. Since leisure and consumption are perfect substitutes, increasing current consumption (and decreasing current leisure) while decreasing future consumption (and increasing future leisure) leaves utility unchanged. Their second proposition demonstrates that this exchange changes the extraction path of the resource, but does not affect welfare. See also footnote 11 for an example where the order of extraction does not affect welfare.
10 If extraction capacity is chosen endogenously, similar cost reversals may obtain depending on the costs of installing capacity. Note that an exogenous extraction constraint implies that it is prohibitively expensive to install additional capacity.
11 Kemp and Long showed that if "cost is constant in terms of the resource itself," then under certain conditions the extraction order is "a matter of social indifference." The intuition behind this surprising result is illustrated by a model where utility is derived solely from consumption of "oil." Suppose there are two deposits of oil with extraction costs of, respectively, a half barrel of oil and a quarter barrel of oil per barrel extracted. In this

example, the order of extraction from the two deposits is irrelevant since extraction costs affect only the stock of oil remaining, and hence there is no benefit from delaying costs.
12 The deposit smoothing result is explained in footnote 8.
13 The existence of a choke price is not essential to the issues addressed here.
14 After all the deposits are exhausted, the equilibrium price is no longer unique. For ease of exposition, let $p(t)$ continue to grow at the rate of interest after all deposits are exhausted.
15 Since the state variable does not appear in the objective function, the co-state variables (shadow values) are constant. Therefore, Lagrangian techniques are sufficient for the optimization.
16 The scarcity cost of a deposit, λ_i, depends not only on that deposit, but on the other deposits as well as the capacity constraint.
17 The backstop technology is usually assumed to have sufficient capacity such that the marginal production cost determines the choke price for extraction of exhaustible resources. Here the installation of capacity is sufficiently costly that it is the production capacity of the backstop which determines the choke price. In many situations, this may be a very reasonable model of a backstop.
18 The residual demand shifts left at c_3 since for prices above c3 the backstop produces at capacity and for lower prices produces nothing.
19 The shadow values λ_1 and λ_2 can be found by solving the system of equations such that demand equals supply and the price path is continuous. See also Section 4.
20 As in the Herfindahl analysis, both shadow values are decreasing functions of the stock sizes of the two exhaustible deposits. To see this, note that if one stock increased, but the shadow prices—and hence price path—remained unchanged or increased, there would be excess supply from that deposit. Therefore, the equilibrium price path must be lower, i.e., the shadow values must decrease.
21 If $c_3 > \bar{p}$, define $S_2^{max}(\infty) \equiv 0$. The label $S_2^{max}(\infty)$ indicates that this critical size for deposit 2 depends on the fact that deposit 3 is here assumed to be infinite. The critical size for deposit 2 if deposit 3 is finite will be derived below.
22 Note that x can be substituted into Eq. (5), so $S_2^{max}(\infty)$ depends solely on exogenous parameters.
23 By this statement I mean that the growth rate of $p(t) — c_2$ is r. This is clearly true since $p(t) = c_2 + (c_3 — c_2)e^{rt}$.
24The extraction order here is the same as the extraction order in Amigues et al. since the model here is a special case of their model where utility is quasi-linear.
25 To see this dependence on capacity, note that as extraction becomes more limited, $\bar{p}$ increases and the residual demand at every price also increases. The comparative static results from the Herfindahl model then indicate that scarcity costs increase on both finite deposits since the residual demand for these deposits has increased.
26 Amigues et al. claim that “the presence of [a higher-cost inexhaustible substitute] is crucial to the issue raised in this paper.”
27 If $\bar{p} \leq c_3$, let $S_3^{min} \equiv \infty$.
28 Similar price paths which are flat when extraction is at capacity are derived by Campbell [2], Lewis [1 1], Lozada [12], and Farzin [4].
29 In other words, since the current value shadow value at time $t_3 + y$ must equal $\bar{p}$ - c_3, the current value shadow value at time t_3 must equal $(\bar{p} — c_3)e^{-ry}$.
30 If $S_3 < S_3^{min}$, let $S_2^{max}(S_3) \equiv 0$: Note that $S_2^{max}(S_3)$ is a function solely of exogenous parameters and can be written as such by substituting x from Eq. (9) into Eq. (8).
31 If deposits 1 and 2 were very small, $p(0)$ may be greater than $AMC_3(t_3)$. In this case, extraction begins immediately from deposit 3 and $p(t_3) < p(T)$.
32 The conditions under which this statement holds can be easily derived.
33 It may be that deposit 2 is so large relative to deposit 1 that this condition never holds. In this case, $p(t) = AMC_2(t)$ for all t and deposit 1 is extracted at capacity until it is exhausted.
34 As illustrated, the price path then grows at the rate of interest net of c_2 after $\hat{T}$. Whether or not extraction resumes from deposit 2 is addressed below. The choke price—at which all deposits should be exhausted—is not illustrated here, but generally could be above or below $p(\hat{T})$.

35 The general price path for multiple deposits with capacity constraints indexed by i can be written $p(t) = \min_i\{\max\{c_i + \lambda_1 e^{rt}, U'(\bar{q}_i)\}\}$. This formulation takes into account the trial shadow values and the extraction constraints.

36 In other words, the current value shadow value of deposit 2 is $\bar{p}_1 - c_2$ at time k and thus is $(\bar{p}_1 - c_2)e^{r(m-k)}$ at time m when deposit 1 is exhausted at the choke price.

References

[1] J.-P. Amigues, P. Favard, G. Gaudet, M. Moreaux, On the optimal order of natural resource use when the capacity of the inexhaustible substitute is limited, J. Econom. Theory 80 (1998) 153–170.

[2] H. Campbell, The effect of capital intensity on the optimal rate of extraction of a mineral deposit, Canad. J. Econom. 13 (2) (1980) 349–356.

[3] U. Chakravorty, D.L. Krulce, Heterogeneous demand and order of resource extraction, Econometrica 62 (1994) 1445–1452.

[4] Y.H. Farzin, The effect of the discount rate on depletion of exhaustible resources, J. Political Economy 92 (5) (1984) 841–851.

[5] G. Gaudet, M. Moreaux, S. Salant, Intertemporal depletion of resource sites by spatially distributed users, Amer. Econom. Rev. 90 (2001) 1149–1159.

[6] O.C. Herfindahl, Depletion and economic theory, in: Mason Gaffney (Ed.), Extractive Resources and Taxation: Proceedings, University of Wisconsin Press, Madison, 1967.

[7] H. Hotelling, The economics of exhaustible resources, J. Political Economy 39 (1931) 137–175.

[8] M.C. Kemp, N.V. Long, On two folk theorems concerning the extraction of exhaustible resources, Econometrica 48 (1980) 663–673.

[9] J. Krautkraemer, Nonrenewable resource scarcity, J. Econom. Literature 36 (1998) 2065–2107.

[10] T. Lewis, Sufficient conditions for extracting least cost resource first, Econometrica 50 (1982) 1081–1083.

[11] T. Lewis, A note on mining with investment in capital [The Effect of Capital Intensity on the Optimal Rate of Extraction of a Mineral Deposit], Canad. J. Econom. 18 (3) (1985) 665–667.

[12] G. Lozada, The conservationist's dilemma, Internat. Econom. Rev. 34 (3) (1993) 647–662.

[13] C. Mason, Nonrenewable resources with switching costs, J. Environ. Econom. Management 42 (2001) 65–81.

[14] R.M. Solow, F.Y. Wan, Extraction costs in the theory of exhaustible resources, Bell J. Econom. 7 (1976) 359–370.

[15] M.L. Weitzman, The optimal development of resource pools, J. Econom. Theory 12 (1976) 351–364.