The Determinants of the Tax-Adjusted Real Interest Rate

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Abstract:
A reduced-form real interest rate equation, derived from an IS-LM-AS model, is estimated to examine the relationship between real interest rates, the federal debt, supply shocks, monetary policy and other variables. The federal debt coefficient is consistently positive and significantly related to the real interest rate in the levels form of the equation. When the equation is first-differenced to eliminate any problem of autocorrelation and intercept instability, the evidence shows that both the federal debt and the supply shock coefficients are positive and significant. The coefficient estimates of the first-difference equations are stable and robust to the estimated period.

Article:

1. Introduction

In this paper we test for the determinants of the tax-adjusted, short-term, real interest rate with special emphasis on the significance and stability of the coefficient estimates of the federal debt. In Section 2 an ex-ante, tax-adjusted, short-term, real interest rate equation that includes a measure of the federal debt, a liquidity effect, a supply shock, inflationary expectations and the dispersion of inflationary expectations is derived from an IS-LM-AS model. Given the empirical evidence presented in Section 3 that bond market participants are rational in their inflation forecasts, an ex-post, tax-adjusted, real interest rate equation is estimated in Section 4. The results show that the debt variable is positive and significantly related to the ex-post, tax-adjusted, short-term, real interest rate. Chow tests, reported in Section 5, reveal that the coefficient estimates are stable for the first-difference version of the model.

The results are not necessarily contrary to the rational expectations literature which has not found any positive and significant effect of federal deficits on real interest rates.[1] The evidence of a positive linkage between government debt and a real interest rate does not refute the rational expectations argument of no linkage between deficits and real interest rates since anticipated and un-anticipated changes in government expenditures and the budget deficit are not jointly tested. The results, however, do suggest that there is a positive relationship between the debt (relative to trend nominal GNP) and a tax-adjusted, real interest rate in the estimates of the log-level equation and between a change in the debt (relative to the change in trend nominal GNP) and the change in a tax-adjusted, real interest rate in the estimates of the first-difference equation.

2. The Model

An IS-LM-AS model is adapted from the Peek (1982), Wilcox (1983a, 1983b, 1983c), and Peek and Wilcox (1983, 1986) models. The IS and LM schedules are in inverse form and the *t* subscripts are suppressed. The model is:

$$\text{IS: } r = a_0 + a_1 D + a_2(M - P) - a_3 Q - a_4 SS + a_5 V_\Pi \,, \qquad (1)$$

$$\text{LM: } Q = b_0 + b_1(M - P) + b_2 i(1 - T) \,, \qquad (2)$$

$$\text{AS: } P = P^e + c_1 Q + c_2 SS \,, \qquad (3)$$

where all the coefficients are positive; Q , the output gap, is defined as the log of real GNP minus the log of potential real GNP; D is a federal debt variable; M is the log of M1; P is the log of the price level (the GNP deflator) so that $(M - P)$ are real balances; V_{Π} rep-resents the dispersion of inflationary expectations measured by the cross-sectional variance of the expected inflation rate in period *t;* SS is the external supply shock variable proxied as the log of the ratio of the GNP deflator for imports to the GNP price deflator; T is the marginal tax rate on interest income; and P^e is the log of the expected price level in period *t.* The ex-ante tax-adjusted real rate of interest in period *t,* r_t is related to the nominal interest rate, i_t by (4):

$$i_t(1 - T) = r_t + \Pi_t^e , \tag{4}$$

where i_t is the yield to maturity of a three-month Treasury bill and Π_t^e is a measure of inflationary expectations in period t based on the information set (Φ) in period $t - 1$.

The IS schedule depends on the real interest rate, real balances $(M - P)$, the output gap, the federal debt, a supply shock variable (SS) and the dispersion of inflationary expectations (V_{Π}). The Mundell (1963) and Tobin (1965) wealth effect, incorporated by $(M - P)$, suggests that an increase in the expected inflation rate causes a less than proportional change in nominal interest rates since people reduce their cash balance holdings and increase their savings, causing a decline in the real interest rate. The supply shock was included by Wilcox (1983a), who argued that a decrease in input supply and an increase in input prices lowers the marginal product of other inputs, thereby lowering investment demand and reducing the real interest rate.

The variance of the Π_t^e series computed by the University of Michigan's Survey Research Center (SRC), V_{Π}, has an ambiguous effect.[2] If disagreement over inflationary expectations can be perceived to represent uncertainty of the future flow of income from investment projects, then an increase in V_{Π} lowers the expected rate of return of investment projects for borrowers. An increase in V_{Π} also increases the uncertainty of the purchasing power of the return from savings, thereby requiring a higher real rate of return for savers. Thus, V_{Π} captures any change in the disagreement or uncertainty associated with the rising rate of inflation of the 1970s and the disinflation of the early 1980s. The evidence shows that the mean of Π_t^e more than doubled from 3.4% during 1961: i-1970:iv to 8.1% during 1975: i-1980:iv, while V_{Π} more than tripled.[3]

Two measures of fiscal stimulus were originally tested. The first measure was a government expenditure variable, defined as government purchases divided by trend nominal GNP.[4] These results are not reported because the government expenditure coefficient was insignificant. The second measure was a federal debt *(D)* variable, defined as the par value of the seasonally-adjusted net federal debt divided by trend nominal GNP.[5] The empirical results reported in Section 4 employ this measure of fiscal stimulus.

The Ricardian equivalence proposition predicts that the debt coefficient equals zero because rational economic agents increase their savings equal to the change in the debt. Evans (1985, 1987a, 1987b) tests the hypothesis that the government budget deficit affects real or nominal interest rates and finds no effect.[6] Barro (1981) argues that a permanent change in government expenditures has little or no effect on the real interest rate, while a temporary change in government expenditures induces an increase in goods prices relative to expected future prices and an increase in the real interest rate. A temporary increase in government purchases may be debt financed, so that the government can spread out the higher tax burden over a period of time. Thus, there could be a positive correlation between changes in the debt and changes in real interest rates that is consistent with the Ricardian equivalence proposition.

A loanable funds or portfolio balance approach argues that an increase in government spending or a decrease in taxes financed by bonds increases aggregate demand and real interest rates as long as taxpayers do not totally adjust to the increase in future tax liabilities. Thus, the debt coefficient must be greater than zero, though its significance would be an empirical question. If economic agents are forward-looking, then expected rather than current budget deficits would have a greater effect on current interest rates. Therefore, the debt coefficient may be small and insignificant if current budget deficits are a poor proxy variable for future budget deficits even though there may be a linkage between federal deficit spending and real interest rates.[7]

The LM equation is a standard representation where the opportunity cost variable is the after-tax nominal interest rate, $i(1 - T)$. The tax rate (T) is Seater's (1985, Table 1) annual measure of the average marginal federal personal income tax rate on adjusted gross income centered on the third quarter and interpolated for the other quarters. [8]

The model is closed by the AS equation where the gap between potential and actual real GNP *(Q)* is employed as the measure of excess demand. Increases in excess demand and the supply shock variable are predicted to have a positive effect on the price level.

The following reduced-form equation for the after-tax, short-term, real interest rate is derived from Equations (1)—(4):

$$r = \beta_0 + \beta_1 D + \beta_2 LIQ + \beta_3 SS + \beta_4 \Pi^e + \beta_5 V_\Pi , \qquad (5)$$

where *LIQ* is a liquidity measure employed by Carlson (1979), Wilcox (1983a, 1983b, 1983c) and Peek and Wilcox (1983, 1986) as a proxy variable for $(M - P^e)$. This measure is defined as the first difference of log M1 in period *t* minus the past three-year average of the first difference of log MI; so it represents the acceleration in nominal money supply growth.[9] While the sign of β_2 is ambiguous, β_2 will be negative if the liquidity effect dominates the Mundell-Tobin effect. The sign (positive or no effect at all) and significance of β_1 is open to debate between Ricardians and non-Ricardians according to the economic theory previously discussed.

The supply shock variable has an ambiguous effect because it reduces the marginal product of capital, lowers investment and the real interest rate while the higher input costs raise the price level and reduce the real money supply, thereby raising the real interest rate. Evidence reported by Peek and Wilcox has supported the hypothesis that the negative effect on investment and real interest rates dominates, as they have found β_3 to be negative and significant.[10] The inflationary expectation coefficient, β_4, is expected to be negative. The sign of the β_2, β_3, and β_5 coefficients are indeterminate according to the model. Equation (5) provides a framework for testing the significance of the federal debt and other macroeconomic variables in a real interest rate equation.

3. The Ex-Post Real Interest Rate

The dependent variable in Equation (5) is the ex-ante, tax-adjusted real interest rate. Although the ex-ante real rate is not observable, Huizinga and Mishkin (1984) note that information can be inferred about the relationship between the ex-ante real rate and the variables in the model by an ex-post real interest rate regression that will asymptotically yield the same coefficient estimate as the ex-ante real rate. The estimate of the coefficient vector according to Huizinga and Mishkin (1984, 702) "does not imply that X_t causes the ex-ante real interest rate only that X_t helps to predict it" where X_t is the vector of right-hand-side variables being employed to estimate r_{t+1}.

Mishkin (1981) and Huizinga and Mishkin (1984, 1986) have shown that if the forecast errors in Equation (6) are uncorrelated with the variables in the information set available in period $t - 1$,

$$E(\Pi_t - \Pi_t^e | \Phi_{t-1}) = 0 , \qquad (6)$$

Then an equation such as (5), where the dependent variable is the ex-post real rate, could be estimated by OLS to infer information about the relationship between macroeconomic variables and the ex-ante real interest rate.

We test for the necessary conditions of unbiasedness between the inflationary expectations variable and the measure of the inflation rate which requires that the residuals, e_t, are not correlated and that $\alpha_0 = 0$ and $\alpha_1 = 1$ as a joint hypothesis test. The following equation is estimated:

$$\Pi_t = \alpha_0 + \alpha_1 \Pi_t^e + e_t , \qquad (7)$$

where Π_t is the annualized quarterly rate of inflation of the GNP deflator and Π_t^e is the SRC expected rate of inflation for the next twelve months.

The SRC data are quarterly observations of the inflation rate for things people expect to buy for the next twelve months. Bryan and Gavin (1986) note the SRC household forecasts of inflation appear to be unbiased despite the well-known measurement problems of the series prior to 1966. They conclude (1986, 544) that SRC survey data of Π_t^e "appear to have statistical properties consistent with standard assumptions about the behavior of rational economic agents." The Π_t^e data and the annualized quarterly rate of change in the GNP deflator are not strictly comparable and may be subject to autocorrelation due to overlapping periods. The evidence to be presented, however, shows no problem with autocorrelation, no bias in the coefficient estimates, and no correlation between Φ_{t-1} and the forecast errors.

The biases from using the consumer price index to measure Π_t are well known (for example, Blinder 1980 and Fischer 1981). Because the CPI overstated the rate of inflation during the 1970s and early 1980s, Huizinga and Mishkin test alternative price indices. While the SRC data are compiled from a survey of expectations of the rate of consumer price changes, the consumer price index cannot be used to test for the rationality of the inflationary expectations survey due to its biases. Thus, the annualized quarterly rate of change of the GNP deflator is employed as Π_t, and the SRC data are employed as Π_t^e.

In the evidence reported in Table 1, we fail to reject the null joint hypothesis that $\alpha_0 = 0$ and $\alpha_1 = 1$ when Π_t is the annualized quarterly rate of inflation in period *t.* We do reject (evidence not reported) the same null joint hypothesis when Π_t is the actual annual rate of inflation over the next year (period *t* to *t* + 4). Thus, we reject a stronger form of unbiasedness, but we fail to reject a weaker form of unbiasedness where Π_t^e for the next year is an unbiased estimate of Π_t in the current period. Since our data are quarterly and i_t is a three-month Treasury bill rate, a weak form of unbiasedness is sufficient.[11]

To provide evidence of the sensitivity to the chosen sample period, Equation (7) is tested for three periods. The results for the joint hypothesis test that $\alpha_0 = 0$ and $\alpha_1 = 1$ are never rejected at the 5% level of significance (see Test 1, Table 1) for both the OLS and GLS results. As an additional test of autocorrelation, the errors from Equation (7) are regressed against a constant and the one to four period lagged errors. There is never a significant coefficient on any of the lagged error terms. The F-test that the constant and the lagged error coefficients are not significantly different from zero is never rejected. These test results are reported as Test 2 in Table 1.

TABLE 1. *Equation (7)* $\Pi_t = \alpha_0 + \alpha_1 \Pi_t^e$

Period		α_0	α_1	R^2	*DW*	F-tests: Test 1 $r = 2$	Test 2 $r = 5$	Test 3 $r = 5$	Test 4 $r = 10$
1961:*i*–1975:*iv*	OLS	−0.20	1.12	0.63	1.95	1.45	0.19*	1.37*	1.73
		(0.37)	(9.98)			[3.15]	[2.57]	[2.39]	[2.03]
1961:*i*–1980:*iv*	OLS	0.42	0.95	0.66	1.72	0.82	0.61*	1.56*	1.20*
		(0.93)	(12.42)			[3.13]	[2.51]	[2.35]	[1.98]
1961:*i*–1985:*iv*	OLS	0.10	0.99	0.65	1.62	0.10	1.65*	1.62*	1.66*
		(0.24)	(13.57)			[3.11]	[2.49]	[2.33]	[1.95]
	GLS	0.36	0.94	0.54	2.05	0.40			
		(0.71)	(10.89)		$\rho = 0.20$	[3.11]			
					(2.03)				

NOTE: *T*-statistics are in parentheses.
r = the number of restrictions.
Critical values for the *F*-statistic at the 5% level of significance are in brackets.
*Indicates that no single coefficient estimate is statistically significant at the 5% level.

It is also necessary to show that the forecast errors, e_t in Equation (7) are not correlated with the information set (Φ) available in period *t* — 1. Thus, Equation (8) also is tested:

$$e_t = \gamma_i \Phi_{i,t-1} + u_t \qquad (8)$$

where $\Phi_{i,t-1}$ represents all of the one-period lagged values of the variables in the model which include *D*, *LIQ*, *SS*, V_{Π}, and *Q* (the output gap). An F-test that the coefficients of the one-period lag of the right-hand-side variables and the one- and two-period lags of the right-hand-side variables are jointly equal to zero when regressed against e_t fails to reject the null hypothesis (Tests 3 and 4). Thus, the forecast errors in Equation (6) are uncorrelated with the information set variables available in period t — 1, and Π_{t} can be substituted for Π_t^e in Equation (5) to compute an ex-post, tax-adjusted, short-term real interest rate. OLS estimates of Equation (5), where the dependent variable is an ex-post, tax-adjusted, short-term real rate, can be employed to infer information about the relationship between the real, ex-ante, short-term, tax-adjusted interest rate and macroeconomic variables.

4. Empirical Results

The dependent variable in Equation (5) is the ex-post tax-adjusted real interest rate which is the yield to maturity (not the discount rate) of the last month of the quarter three-month Treasury bill rate. Equation (5) includes a liquidity effect and deficit variable that were not tested by Huizinga and Mishkin (1986) in their real ex-post interest rate equation. The OLS estimates of Equation (5) are reported in Table 2 for the 1961:i, 1975:iv, 1961:i-1980:iv and 1961:i-1985:iv periods. The results show that the debt variable is positive and significant for a one-tailed test at the 5% level for the 1961:i-1975:i period and the 1% level for the other two periods. No other coefficient is consistently significant in the level equation.

The collinearity diagnostics developed by Belsley, Kuh, and Welsch (1980) reveal degrading levels of multicollinearity in the log-level version but not in the first-difference version of Equation (5).

TABLE 2. *Log-Level and First Difference Estimates of Equation (5)*

	α	D	LIQ	SS	Π^e	V_{Π}	R^2/SE	DW
1961:i–1975:iv								
(2.1) Level	−0.89	13.94	1.47	−5.75	0.33	−0.04	0.44	2.05
	(0.26)	(2.26)*	(0.03)	(1.27)	(0.97)	(1.03)	1.67	
(2.2) FD		134.32	74.08	26.83	−0.72	0.10	0.17	2.52
		(2.28)*	(1.46)	(1.98)	(1.34)*	(1.41)	2.13	
(2.3) FD	0.43	190.92	69.38	28.92	−0.74	0.10	0.19	2.50
	(1.23)	(2.56)**	(1.37)	(2.12)#	(1.37)	(1.46)	2.12	
1961:i–1980:iv								
(2.4) Level	−9.81	21.97	20.05	1.18	0.64	−0.04	0.27	1.66
	(3.52)	(3.71)**	(0.60)	(0.35)	(1.97)*	(1.26)	1.92	
(2.5) FD		133.44	81.53	26.27	−0.26	0.05	0.19	2.38
		(2.26)	(2.63)	(2.17)#	(0.63)	(1.23)	2.27	
(2.6) FD	0.20	154.57	79.91	25.69	−0.25	0.05	0.19	2.38
	(0.66)	(2.29)*	(2.56)	(2.11)#	(0.60)	(1.20)	2.27	
1961:i–1985:iv								
(2.7) Level	−15.85	24.91	34.47	10.68	−0.29	0.03	0.16	2.11
	(3.12)	(2.64)**	(1.89)	(2.53)#	(0.85)	(0.76)	2.05	ρ = 0.52
(2.8) FD		92.50	39.74	28.29	−0.37	0.05	0.14	2.43
		(2.01)*	(2.47)	(2.45)#	(1.10)	(1.63)	2.28	
(2.9) FD	0.03	93.13	39.73	28.28	−0.37	0.05	0.14	2.43
	(0.13)	(2.01)*	(2.46)	(2.43)#	(1.10)	(1.62)	2.29	

NOTE:
D = the par value of the seasonally adjusted net federal debt divided by trend nominal GNP.
FD = first-difference results.
T-statistics are in parentheses.
*Significant at the 5% level for a one-tailed test.
**Significant at the 1% level for a one-tailed test.
#Significant at the 5% level for a two-tailed test.

The log-level results also require an autocorrelation correction for the 1961:i-1985:iv period. Furthermore, the constant term de-creases in size and becomes significant with the addition of data beyond 1975, which may indicate either a one-time shift or continuous drift in the intercept. As a result, Equation (5) is first-differenced and the results (with and without the intercept term) are reported in Table 2.[12] Honohan (1985) notes that there is a substantial errors-in-variables bias when the actual inflation rate is employed as a proxy for inflationary expectations in an equation that is first-differenced. Hence, β_4 is highly biased if the variance in the change in the inflation rate is large relative to the variance of the change in the expected rate of inflation.

These first-difference results also confirm the statistical significance of the debt coefficient for all three periods. The constant term, included to test for intercept drift, is insignificant. The supply-shock coefficient is consistently positive and significant, confirming the work of Wilcox, and Peek and Wilcox.[13] The *LIQ* coefficient is positive and significant for the 1961:i-1980:iv and 1961:i-1985:iv periods. The *LIQ* coefficient is hypothesized to be negative but a diminishing or positive liquidity effect has been documented by Mishkin (1981, 1982), Melvin (1982) and Mehra (1985). Thus, the federal debt, a liquidity effect, and a supply shock positively affect the ex-post, tax-adjusted real three-month Treasury bill rate.[14]

The Mishkin methodology of estimating an equation with the ex-post real interest rate as the dependent variable can only establish the correlation between the right-hand-side variables and the ex-ante real rate and not causation. Evidence of a positive debt coefficient cannot be interpreted to be evidence against the Ricardian equivalence proposition because temporary increases in government expenditures may raise the real rate of interest.

The effect of the budget deficit on the tax-adjusted, short-term real interest rate can be calculated using our equation. Consider the $79.9 billion increase in the nominal seasonally-adjusted net federal debt from 1985:iii to 1985:iv. The ratio of the nominal net federal debt to trend nominal GNP *(D)* increases from 0.3603 in the third quarter to 0.3736 in the fourth quarter of 1985. The change in the value of the ratio (0.0133) times the *D* coefficient for the 1961:i-1985:iv period (24.9) equals 0.33 or 33 basis points. There-fore, the 108 basis point decline of the tax-adjusted, ex-post, short-term real interest rate in 1985:iv would have been 33 basis points larger had it not been for the $79.9 billion increase in the nominal net federal debt.

TABLE 3. *Log-Level Estimate of Debt Coefficient*

	Nominal Debt	Real Debt	Nominal Debt/ Nominal GNP	Nominal Debt/ Potential Nominal GNP
1961:*i*–1975:*iv*	−0.12 (2.71)**	−0.02 (0.72)	11.5 (1.95)*	13.9 (2.29)*
1962:*i*–1976:*iv*	−0.09 (1.96)*	−0.00 (0.01)	11.6 (1.78)*	13.1 (2.05)*
1963:*i*–1977:*iv*	−0.07 (1.70)*	0.01 (0.31)	19.3 (2.69)**	19.6 (2.88)**
1964:*i*–1978:*iv*	−0.06 (1.46)	0.01 (0.23)	19.2 (2.17)*	18.2 (2.35)*
1965:*i*–1979:*iv*	−0.07 (1.38)	0.01 (0.48)	22.4 (1.71)*	21.8 (2.05)*
1966:*i*–1980:*iv*	−0.11 (1.63)	0.01 (0.23)	41.9 (2.05)*	37.8 (2.42)**
1967:*i*–1981:*iv*	−0.13 (2.05)*	0.00 (0.09)	53.5 (1.74)*	56.4 (2.29)*
1968:*i*–1982:*iv*	−0.09 (1.97)*	−0.00 (0.13)	36.1 (1.09)	50.6 (1.65)
1969:*i*–1983:*iv*	−0.06 (1.51)	0.00 (0.11)	63.0 (2.39)*	91.2 (3.20)**
1970:*i*–1984:*iv*	−0.03 (0.76)	0.03 (1.01)	72.1 (3.65)**	93.9 (4.29)**
1971:*i*–1985:*iv*	0.00 (0.12)	0.03 (1.13)	48.1 (3.20)**	57.3 (3.47)**

NOTE: *T*-statistics are in parentheses.
*Significant at 5% level for a one-tailed test.
**Significant at the 1% level for a one-tailed test.

5. Stability

In order to test for the stability of the coefficient estimates, Chow (1960) tests are computed for the log-level and the first-difference versions of Equation (5) for every breakpoint between 1969:i and 1977:iv. By this method no prior information as to the likely breakpoint is assumed. For Equation (5), the critical value of $F_{6,88}$ at the 5% level is 2.22. The F-statistic for the Chow test for the log-level version of the equation is above this critical value for every breakpoint during this nine-year period. Thus, the evidence from the Chow test for the levels version of the model indicates that there has been at least one shift in the coefficient estimate over this period.[15] The critical value of the F-statistic at the 5% level for the Chow test for the first-difference estimate of Equation (5) is $F_{5,90} = 2.33$. The results reveal that the null hypothesis of coefficient stability is never rejected for any breakpoint over the nine-year test period when the real interest rate equation is first-differenced.

A rolling regression technique is employed where Equation (5) is estimated over fifteen-year periods beginning with 1961:i-1975:iv for both the log-level and first-difference versions. Equation (5) is reestimated by adding four successive quarters (1976) and dropping the first four quarters (1961) from the sample period. Eleven fifteen-year regressions are estimated. The debt coefficients and their *t*-statistics are recorded in Table 3 for the log-level equation and in Table 4 for the first-difference equation. The estimates of the debt coefficient using the rolling regression technique are also provided for three other measures of the debt: the nominal seasonally-adjusted net federal debt (Debt), the real debt, and the debt divided by nominal GNP.

TABLE 4. *First-Difference Estimate of Debt Coefficient*

	Nominal Debt	Real Debt	Nominal Debt/ Nominal GNP	Nominal Debt/ Potential Nominal GNP
1961:*i*–1975:*iv*	−0.07 (0.72)	0.09 (2.63)**	81.7 (1.55)	134.3 (2.28)*
1962:*i*–1976:*iv*	−0.04 (0.48)	0.10 (2.80)**	91.9 (1.72)*	134.5 (2.30)*
1963:*i*–1977:*iv*	0.02 (0.30)	0.10 (3.21)**	88.1 (1.65)	133.7 (2.32)*
1964:*i*–1978:*iv*	0.02 (0.29)	0.11 (3.33)**	105.2 (1.88)*	133.7 (2.18)*
1965:*i*–1979:*iv*	0.03 (0.39)	0.11 (3.34)**	116.6 (2.06)*	146.6 (2.39)*
1966:*i*–1980:*iv*	−0.07 (0.80)	0.07 (1.83)*	49.0 (0.71)	110.9 (1.57)
1967:*i*–1981:*iv*	−0.10 (1.56)	0.04 (1.05)	24.8 (0.34)	114.1 (1.50)
1968:*i*–1982:*iv*	−0.13 (2.53)**	0.02 (0.43)	−11.4 (0.17)	91.1 (1.17)
1969:*i*–1983:*iv*	−0.10 (2.21)*	0.01 (0.29)	16.1 (0.25)	108.7 (1.52)
1970:*i*–1984:*iv*	−0.08 (1.93)*	0.01 (0.39)	46.8 (0.75)	158.1 (2.28)*
1971:*i*–1985:*iv*	−0.04 (1.33)	0.01 (0.50)	45.2 (0.79)	116.2 (1.81)*

NOTE: *T*-statistics are in parentheses.
*Significant at the 5% level for a one-tailed test.
**Significant at the 1% level for a one-tailed test.

The log-level results show mixed evidence. The nominal and real debt coefficients are never positive and significant. The nominal debt divided by nominal GNP and the nominal debt divided by trend nominal GNP coefficients are positive and significant in ten of the eleven rolling regressions. Therefore, the evidence of a positive relationship between a measure of the federal debt and the tax-adjusted, short-term, real interest rate is robust when the nominal debt is divided by either nominal GNP or nominal potential GNP.

The first-difference results show that the nominal debt coefficients are never positive and significant. The coefficients for the real debt and nominal debt divided by nominal GNP are positive and statistically significant primarily in the fifteen-year periods through 1965:i-79:iv. The coefficient of the nominal debt divided by trend nominal GNP is positive and significant in seven of the eleven periods. If zero-one dummy variables are employed for the credit controls of 1980:ii and 1980:iii, then the nominal debt divided by potential GNP coefficients are positive and significant (not reported) in the other four periods. The log level and the first-difference results suggest that the positive relationship between a federal debt measure and a tax-adjusted, ex-post, short-term real interest rate is robust to the estimated sample period as long as the nominal debt divided by trend nominal GNP measure is employed.

6. Conclusion

The empirical methodology of estimating an ex-post, real interest rate in place of an ex-ante, real interest rate which was developed by Mishkin and was employed by Huizinga and Mishkin requires a careful interpretation of the results. A reduced-form equation for the real interest rate equation has been derived from an IS-LM-AS macroeconomic model and estimated in order to examine the relationship between real interest rates and macroeconomic variables such as fiscal and monetary policy variables. Empirical evidence is presented indicating a positive relationship between a measure of the debt *(D),* defined as the ratio of the nominal par value of the seasonally-adjusted net federal debt to trend (potential) nominal GNP, and the ex-post, tax-adjusted real interest rate on three-month Treasury bills. The debt coefficient maintains its positive sign and statistical significance in almost all of the time period estimated for both log level and first-difference estimates of the equation. In addition, the coefficient estimates are stable according to Chow tests when the equation is first-differenced. Thus, the results provide evidence of a positive relationship between the federal debt and a three-month ex-post, tax-adjusted real interest rate.

Notes:

[1] See Evans (1985, 1987a, 1987b) and Plosser (1982, 1987).

[2] See Hoffman and Schlagenhauf (1985), Makin (1983), and Wilcox (1983c).

[3]Engle (1983) has argued that the higher rate of inflation in the 1970s partially was expected so that the increase in inflation uncertainty was only slightly greater than in the 1960s. Therefore, the variance of the inflation rate obtained from survey data will overstate the increase in inflation uncertainty.

[4]Trend nominal GNP is calculated as the GNP deflator *(P)* times potential real GNP (*yp*). See Gordon (1984, Table B-2) for the *yp* data. The government debt (Debt) measure is the nominal par value of the seasonally adjusted net federal debt published by the Federal Reserve Bank of St. Louis. Thus, the debt measure estimated in Equation (5) is $D_t = DEBT_t/P_t*yp_t$.

[5] The value of this debt/trend GNP measure declined from 0.43 in 1961:i to 0.22 in 1974:iv, fluctuated between 0.23 and 0.27 from 1975:i to 1983:iv and then rose to 0.37 by the end of 1985, thereby capturing the increase in deficit spending under the Reagan administration.

[6] Plosser (1982) has shown there is no statistically significant positive effect on nominal interest rates from an increase in deficit spending, though he does confirm a significant negative effect from an increase in government purchases on asset prices.

[7] Blanchard (1985) and Feldstein (1986) argue that expected deficits are a determinant of current long-term interest rates.

[8] The data for 1981-1985 (the most recent data available) are computed by the author using Seater's methodology. Peek and Wilcox (1986) present evidence that households and not corporations or tax-exempt institutions are the effective marginal investor of Treasury bills.

[9] The reduced form coefficients are related to the structural parameters by the following equations: $\beta_0 = Z(a_0 + VWb_0)$, $= Za_1$, $\beta_1 = Z(a_2 + VWb_1)$, $\beta_3 = —Z(VWc_1c_2, + a_2c_2 + a_4)$, $\beta_4 = (ZVWb_2)$, and $\beta_5 = Za_5$, where $W = (1/1 — b_1c_1) > 0$, $V = —(a_3 + a_2c_1) < 0$ and $Z = (1/(1 — VWb_2)) > O$.

[10] Peek and Wilcox (1983, 1115) eliminate exchange-rate changes from the import deflator while our measure of SS does not.

[11] A diligent referee helped to clarify this issue and argued for a strong form of unbiasedness if a one-year Treasury bill rate is employed. The evidence of a positive and significant debt coefficient is still robust if a one-year Treasury bill rate replaces the three-month rate. Another referee suggested using the consumer price index (minus housing and shelter). This series would lead to a rejection of the joint null hypothesis in Equation (7) and would result in insignificant debt coefficients in Equation (5), if the series is used to compute the dependent variable.

[12] The Durbin-Watson statistics indicate that the equations have been overdifferenced. Plosser and Schwert (1978) note that overdifferencing does not create biased or inconsistent estimates and Plosser, Schwert, and White (1982) indicate that differencing can be a diagnostic check for model specification.

[13] A referee suggested that the 1980:ii—I980:iii credit control period should be dummied out with zero-one dummy variables for 1980:ii (D802) and 1980:iii (D803). The results do not change the log-level or first-difference estimates reported in Table 2, though the D802 coefficient is consistently negative and significant.

[14] Plosser (1982, 1987) and Evans (1985, 1987a, I987b) find no evidence of a positive correlation between budget deficits and interest rates. Evans (1987a, 42) employs -the change in the real market value of the privately held gross federal debt- which was deflated by the CPI and then divided by Gordon's measure of potential real GNP. Evans (1987b) employs the change in the real par value of the government debt divided by trend real GNP, and he deflates the nominal government debt by either the GNP price deflator or the CPI in his study of the effect of the unanticipated component of the deficit on nominal interest rates for the U.S. and five other countries. Plosser (1987, 366) notes that there is -more of a tendency for debt shocks to be associated with higher nominal interest rates but the coefficient estimates remain insignificant by the usual criteria- for the second half of the sample: 1977-1985.

[15] Makin does not reject the null hypothesis of structural stability for 1959-1981 with a break point at the end of 1969 for a nominal three-month Treasury bill rate. Peek and Wikox (1983) do not reject the null hypothesis of coefficient stability for a Chow test conducted on their tax-adjusted and non-tax-adjusted nominal interest rate equation for semi-annual data from 1952:vi-1979:xii with a break at the mid-point of their data (between

1965:xii-1966:vi). These studies tested for a break at the mid-point of the data and did not have sufficient degrees of freedom to test for a break in the 1970s.

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