A Cubic Estimate of the Term Structure of Interest Rates for a Money Demand Function

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Abstract:
Heller and Khan (1979) estimate the term structure of interest rates by a quadratic equation in order to employ the estimated parameters of the term structure in a demand-for-money function. An important improvement of their estimates can be made by specifying a cubic rather than a quadratic equation in order to estimate "humped" term structures. Furthermore, the ordinal numbering of the maturities must be changed to the actual length of the maturity of each security to avoid estimation error.

Article:
Heller and Khan (1979) incorporate the term structure of interest rates of United States government securities in a demand-for-money function for the United States for the 1960 / iii-1976 / iv period. Their work is an important advance in the demand-for-money literature for a number of theoretical and empirical reasons. They estimate the term structure of interest for each point in time by a quadratic equation and then include the time-series of the estimated parameters of the quadratic equation in the demand-for-money function. The purpose of this paper is to note that their results can be significantly improved by an estimate of a non-logged cubic equation which explicitly considers the time to maturity of each government security.

From a theoretical standpoint, Heller and Khan [(1979), p. 110] note that " [i] f the holding of money is viewed as part of a general portfolio decision process, then clearly the rates of return on all alternative financial assets are relevant."[1] Such an approach would avoid the dilemma of whether a short- or long-term interest rate is the appropriate opportunity cost of holding money and would improve upon studies which have included two interest rates.[2] Milton Friedman (1977) concludes that an increase in the demand for real money balances would result if short-term interest rates rose while long-term rates declined even though the average level of interest rates is held constant. While this theoretical conclusion has not been directly confirmed by empirical work, it does under-score the necessity of considering the spectrum of interest rates in the demand-for-money function. Finally, the expected rate of inflation is also incorporated into the demand-for-money function because inflation expectations are incorporated into long-term interest rates according to term structure theory.

Heller and Khan (1979) use a quadratic function to estimate the term structure for 1960/i-1976 /iv:

$$\log R_i = \alpha + \beta\tau_i + \gamma\tau_i^2, \; i = 1, \ldots, n, \qquad (1)$$

where "R_i (i = 1,…n) is the rate of interest on the financial asset with the *i*-th maturity period," (p. 112) and τ is the maturity period. The first and second derivatives are $(d \log R_i/d\tau_i) = \beta + 2\gamma\tau_i$ and $(d^2 \log R_i/d\tau_i^2) = 2\gamma$, respectively. If γ is negative, then a maximum results and the term structure is an ascending curve where $\beta = -2\gamma$ is positive. If γ is positive, then a minimum results and the term structure is a descending curve where β is negative. Because a quadratic function has only one turning point, there will be points in time when a quadratic function will produce a poor fit. Malkiel [(1966), pp. 13] , who investigated yields on U.S. government securities for 1953 through 1966, has noted that humped term structure curves have occurred for many of the months in 1957, 1959, 1960, and 1966. Similar humped term structure curves have appeared during 1970, 1974,

and 1978. Therefore, a cubic function which can handle the typical humped term structure curve should provide a significantly better fit.

The cubic function used in this study is

$$R_i = \alpha + \beta\tau_i + \gamma\tau_i^2 + \delta\tau_i^3 . \qquad (2)$$

However, in our case $i \neq 1\ldots$ n, but i is equal to the length of the maturity of the Treasury obligation. For the 1960 /*i*-1976 /*iv* period seven Treasury bills or bonds were utilized with maturities of 3, 6, and 9 through 12 months and 3, 5, 10, and 20 years. Therefore i = 0.25, 0.50, 0.875, 3, 5, 10, and 20, and not the ordinal number of 1, 2, ..., 7 as in the case of Heller and Khan.

The first and second derivatives are $(dR_i/ d\tau_i) = \beta + 2\gamma\tau_i + 3\delta\tau_i^2$ and $(d^2R_i / d\tau_i) = 2\gamma + 6\delta\tau_i$, respectively. For the typical humped term structure curve to exist, a maximum must first occur so that $2\gamma + 6\delta\tau_i < 0$; later a local minimum will occur so that $2\gamma + 6\delta\tau_i > 0$.

Heller and Khan also elected to log the dependent variable of the term structure equation. The results of estimating equation (2) with R_i and log R_i as the dependent variable show that the coefficient of determination (R^2) for the non-logged version of the equation is higher than the logged version over 80 percent of the time for the 1960 /iii-1976 / iv period. These results can only be suggestive because the dependent variables are not identical. There-fore as an empirical matter, researchers should compare the results of including the parameters of a log and a non-log estimate of the term structure in the demand-for-money function.[3]

TABLE 1. ***Estimate of the Term Structure of Interest Rates*** **1974/*i*–1975/*iv***

	α	β	γ	δ	R^2_C	R^2_Q	$\bar{R}^2_C$	$\bar{R}^2_Q$
1974/*i*	7.6179*	−0.2635+	0.0239	−0.0005	0.9555	0.9084	0.9110	0.8626
1974/*ii*	8.4544*	−0.1007	−0.0011	0.0003	0.7472	0.7366	0.4944	0.5999
1974/*iii*	8.5702*	0.4315	−0.0197	0.0009	0.4841	0.3766	−0.0318	0.0644
1974/*iv*	7.3793*	0.1492	−0.0249	0.0010	0.9004	0.6540	0.0008	0.4810
1975/*i*	5.7369*	0.5442*	−0.0618*	0.0020*	0.9967	0.8563	0.9434	0.7845
1975/*ii*	5.3320*	0.9513*	−0.1119*	0.0036+	0.9779	0.7615	0.9558	0.6423
1975/*iii*	6.3933*	0.7942+	−0.1020+	0.0034+	0.9443	0.5662	0.8886	0.3393
1975/*iv*	5.6734*	0.7612*	−0.0827+	0.0026+	0.9801	0.8459	0.9602	0.7684

*Significant for a two-tailed test at the 1 percent level.
+Significant for a two-tailed test at the 5 percent level.

Table 1 presents the coefficients of estimating equation (2) for 1974 and 1975 which includes a period of both a humped and an ascending term structure.[4] The results show that all of the coefficients of the cubic equation are significant at the 5 percent level for 1975. The superiority of the cubic equation results is confirmed by a comparison of the R^2's for the quadratic (R^2_Q) and cubic (R^2_C) equations. While Heller and Khan [(1979), p. 121] note that their estimate of equation (1) "is able to capture well over 90 percent of the variation in the seven interest-rate series in each period," their equation is misspecified because i = 1,… n. Our results of estimating a non-logged term structure equation where τ is specified as the length to maturity reveals that $R^2_Q > 0.90$ for 22 of the 66 quarters, while $R^2_C > 0.90$ for 51 of the 66 quarters. Furthermore, a computation of $\bar{R}^2_C$, for the cubic equation is usually greater than $\bar{R}^2_Q$ for the quadratic equation. Similar conclusions are also reached when the time period is expanded to 1947 / 1978 /iv.

Heller and Khan make an important contribution to the demand-for-money literature with the inclusion of the term structure of interest rates into the function. However, this paper shows that the term structure of interest rates should be estimated by a non-logged cubic equation rather than a logged quadratic function where the actual length of the maturity of the government securities is explicitly entered into the equation.

Notes:

[1] Laidler [(1969), p. 91] , Feige and Pearce [(1977), p. 45] , and Friedman (1977) made the argument on theoretical grounds prior to Heller and Khan.

[2] Coldfeld (1976), Hamburger (1977), and Klein (1974 and 1977) have included two or more interest rate terms.

[3] Klein (1974) elects to estimate a partial log demand-for-money function where all variables are logged except for his two or three interest rate variables. He provides a theoretical justification for his decision by noting that (p. 939) " [m] odern portfolio theory does not suggest that interest rates or interest differentials should enter in a logarithmic form . . . . The commonly used logarithmic functional form implies a proportionately greater effect for every percentage point change in interest the lower the rate of interest, and an undefined demand for money at a zero rate of interest. But as long as there are some increasing marginal costs associated with holding cash balances, we should expect the demand for money curve to cut the axis and a finite determinant money demand at negative interest rates."

[4] The definition of the interest rate observations are given by Heller and Khan in their appendix. However our data sources included:

(a) *Business Statistics,* 1977 edition, appendix, pp. 255-56;

(b) Standard & Poor's Statistics' *Banking and Finance,* 7/ 78;

(c) Standard & Poor's Statistical Service, Current *Statistics,* / 79;

(d) Federal Reserve Board's Banking and *Monetary Statistics,* 1941-1970 and Annual *Statistical Digest,* various issues; and

(e) Moody's *Municipal and Government Manual,* pp. a6-7.

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