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Recent work by some researchers has involved decomposing the profession of teaching into core practices that can be discussed and accessed by novice teachers that can help them gain expertise in these practices. Core practices are central to the work of teaching, support student learning, and are fundamental to developing complex practice. Evidence is beginning to emerge on the benefits of taking a core practice approach to preservice education; however, little is known about how practicing teachers might improve their teaching through professional development taking a similar approach.

The primary purpose of this study is to understand teachers' learning in mathematics professional development focused on the core practice of leading mathematics discussions and their changes in classroom enactments of the practice. As a retrospective analysis of one cycle of a design experiment, the study investigated 13 K-5 teachers' learning and enactments of the core practice of leading mathematics discussions as they engaged in practice-focused professional learning tasks that were a part of 108 hours of professional development designed to become increasingly more aligned with classroom practice in terms of authenticity and complexity. Qualitative and quantitative analyses of video recordings of professional development sessions, classroom observation, field notes, interviews, and teacher reflections were conducted to understand the nature of professional learning tasks that supports teacher learning the core practice of leading mathematics discussions.

An analysis of teachers' participation in the professional development suggests that the professional learning tasks designed for teachers to value mathematics discussions provided opportunities for teachers to value and appreciate the practice of leading mathematics discussions, develop deeper understandings of the instructional moves and challenge their existing classroom practice. Results suggest that over time, teachers formed a community of practice marked by the shared practice of a framework for leading discussions learned in the mathematics professional development to make sense of mathematics teaching and learning and making their own practice public for collective reflection. Findings from the study indicate that teacher enactments of leading mathematics discussions in the classrooms were marked by an increased presence of probing and pressing moves focused on student mathematical thinking. Outcomes have implications for district leaders making decisions about professional development, teacher educators working with teachers to enhance their instructional practice in professional development settings, and researchers examining teacher learning and instructional change.
Key Words: Core Practices; Leading Mathematics Discussions; Professional Development; Elementary; Pedagogical Knowledge; CCSSM; Teacher Knowledge

ELEMENTARY TEACHERS' PARTICIPATION IN MATHEMATICS PROFESSIONAL LEARNING TASKS

by

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Approved by

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Committee Chair

For Ana - for being with me throughout this process, helping me grow and changing me for the better.

For Andrea, Kaili, and Daisy—for bringing so much joy to my life.

For mom and dad—for always believing in me.

I love you all!

APPROVAL PAGE

This dissertation, written by Wendy Deneen Rich, has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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CHAPTER I

INTRODUCTION

> These Common Core Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that these standards are not just promises to our children, but promises we intend to keep. (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010)

Calls for reform of the mathematical education of students in the United States are not new. Though the widely-adopted Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) represents the most recent wave in reform efforts of the past twenty years, emanating from a long tradition of standards-driven initiatives in mathematics education. Through a series of documents, the National Council of Teachers of Mathematics (NCTM) has called upon educators for decades to expand their notions of what school mathematics is and how it should be taught. NCTM (1980) first outlined a vision of improved mathematics education in *An Agenda for Action* by articulating stringent content standards, a push toward problem solving as the focus of school mathematics, and a call for additional research into the nature of problem solving and effective ways to develop good problem solvers. In 1989, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) continued the call for change through a reexamination of the content of school mathematics, the inclusion of standards for the processes of mathematics in addition to content, and expanded notions of the ways in which mathematics learning was evaluated. With its *Principles and Standards for School Mathematics* (NCTM, 2000), the Council elaborated their vision by proposing a set of principles to guide decisions affecting school mathematics, placing heightened emphasis on the processes of doing mathematics, and revising standards for the content that should be experienced by all students in all four grade bands; K-2, 3-5, 6-8, and 9-12. In response to a need for specificity beyond grade bands, *Curriculum Focal Points* (NCTM, 2006) highlighted areas of instructional emphasis at each grade level and served as an organizational structure for curriculum design and instruction at and across grade levels. In short, leaders in mathematics education have worked for over three decades to challenge deeply held notions of what mathematics is and how it should be taught.

During this time, NCTM was not alone in its call for change, with various reports and commissions echoing and informing NCTM's recommendations (Bransford, Brown, & Cocking, 2000; Mathematics Learning Study Committee, 2001; National Research Council, 2002). One significant contribution was the National Research Council's report, *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001). This report synthesized the research on mathematics teaching and learning and set forth recommendations addressing how teaching, curricula, and teacher education should change to improve mathematics learning. A key outcome of the report was an expanded definition of mathematical proficiency. This definition challenged the idea of mathematics as simply concepts and procedures and included five interwoven and interdependent strands of proficiency: computational understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

A consistent message throughout the mathematics education reform movement is a dual focus on deep understandings of mathematics beyond calculations and productive ways of engaging with mathematics that support those understandings. The CCSSM (2010) continues in this tradition by pairing its Standards for Mathematics Content with Standards for Mathematical Practice. Drawing upon NCTM's process standards (2000) and Kilpatrick et al.'s (2001) definition of mathematical proficiency, these standards describe the ways that mathematically proficient students engage with mathematics, including overarching habits of mind for productive mathematical thinkers, reasoning and explaining, modeling and using, and seeing structure and generalizing. These standards are new to many teachers in both their content (Porter, McMaken, Hwang, & Yang, 2011) and the instructional strategies needed to support students in meeting them (Krupa, 2011).

New education standards require a shift in the way teachers think about and practice their craft (Birman, Desimone, Porter, & Garet, 2000; Elmore, 2002; Guskey, 2002; Guskey & Yoon, 2009; Cohen & Hill, 2001). Models of mathematics teaching that rely heavily on direct instruction are insufficient to support students in engaging with mathematics in ways that meet these standards. Evidence suggests that mathematics instruction leading to the types of understandings and practices required by the CCSSM must be centered on students and students' mathematical thinking (Carpenter, Fennema, Peterson, Chaing, & Loef, 1989; Fernández, 2005; Han & Paine, 2010; Zhang & Cheng, 2011). In contrast to direct instruction, such a model of instruction is dialogic, i.e. teachers facilitate discussions of mathematics, encourage students to explain and justify, and engage students in problem solving and reasoning routinely (Munter, Stein, & Smith, 2013). This model of instruction is foreign to many teachers (Ball & Cohen, 1999; Stein, Smith, & Silver, 1999), takes time to learn (Berliner, 1994; Huberman, 1989; Richardson & Placier, 2001), is difficult to instantiate in classrooms (Lampert, 2001; McDonald, 1992), and requires an ability to adjust instruction in response to children's thinking (Jacobs & Empson, 2016). Meeting the goals of the reform movement has proven difficult, in part because of the challenges teachers experience in continuing their instruction on students' mathematical thinking. Professional development has been shown to be a significant factor in in assisting teachers in adopting new ways of teaching to meet changes in standards (Cohen & Hill, 2001).

Thus, the CCSSM renews the imperative of expanding what it means for students to do mathematics and what it means for teachers to teach mathematics. Though new standards represent a hope for a better mathematics education for all students, this hope will remain unrealized without a professional development agenda that directly addresses the need for expanded models of mathematics instruction (Sztajn, Marrongelle, & Smith, 2011). The research community has reached a consensus on the characteristics of effective professional development (Desimone, 2009; Elmore, 2002) and has identified various ways in which teachers learn (Ball & Forzani, 2009; Cochran-Smith & Lytle, 1999; Shulman, 1986; Sowder, 2007), yet mathematics teacher educators (MTEs) continue to struggle with helping teachers learn and implement reform teaching practices.

MTEs need a deeper understanding of the ways in which teachers learn new instructional practices to assist teachers in helping students meet the CCSSM.

Statement of Research Problem

Professional development must engage practicing teachers in examining and refining their knowledge of mathematics, beliefs about teaching and learning, and instructional practices in order to help teachers enact more rigorous standards. Yet professional development that leads to significant teacher learning and instructional change is challenging and complex (Stein et al., 1999). Though the research is clear that teachers require sufficient time and resources to expand their repertoire of instructional strategies (Elmore, 2002; Guskey, 2002; Guskey & Yoon, 2009; Krupa, 2011), many of the opportunities for teachers to learn do not meet criteria for effective professional development (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Elmore, 2002). Teachers expand their content knowledge and change their instructional practices by learning, collaborating, and supporting one another in communities (Borko, 2004; Darling-Hammond et al., 2009).

A reframing of professional development that connects teachers' learning in these settings with their classroom practice is needed (Garet, Porter, Desimone, Birman, & Yoon, 2001; Putnam & Borko, 2000, Webster-Wright, 2009). Stein et al. (1999) described challenges teachers and MTEs face as they undergo "transformative" changes for new practices and programs for mathematics instruction and claimed that MTEs will need to relearn their craft, just as teachers must relearn theirs. Until MTEs better understand how to design professional development for learning instructional practices centered on student mathematical thinking, the substantive changes that are the goal of reform are unlikely to occur.

Recently, scholars have worked to conceptualize professional learning opportunities that focus directly on *teaching* rather than teachers (Hiebert & Morris, 2012; Lampert, 2012). This reorientation requires a commitment by MTEs to create learning opportunities that focus not only mathematics content, but also instructional strategies. Some MTEs are carefully considering new ways of supporting novices of learning instructional practices (Boerst, Sleep, Ball, & Bass, 2011; Fernández, 2005; Ghousseini, 2009; Grossman, Compton, et al., 2009; Grossman & McDonald, 2008; Horn, 2010; Kazemi, Franke, & Lampert, 2009; Lampert, et al., 2013; McDonald, Kazemi, & Kavanaugh, 2013). Grossman, Compton, et al.'s (2009) seminal work calls for the decomposition of teaching into core practices. Core practices are central to the work of teaching, support student learning, and are fundamental to developing complex practice.

In addition, they describe three pedagogies of practice useful in relational professions: representations, decompositions, and approximations of practice. Representations of practice consist of different ways that instructional practice is represented and what those representations make visible to novices. Decompositions of practice involve breaking down practice into its parts for the purposes of teaching and learning. Approximations of practice comprise different ways that teachers can engage in practices that are more closely aligned to the practices in professional education settings.

Some MTEs are beginning to use these three pedagogies to explicitly design PLTs (Lampert et al., 2013; McDonald et al., 2013).

MPD that is connected to changes in instruction must be connected to classrooms. Though there has been some research on novices learning practice, little is known about how teachers learn about instructional practice in mathematics professional development (MPD). PLTs for novices learning core practices show promise but it is unclear how that research might inform PLTs for MPD.

Statement of Purpose

The purpose of this study is to investigate the ways that teachers learn one core practice in a professional development setting and the ways that learning is enacted in their practice. As part of a longer design-based research project, the research is driven by a set of conjectures built upon the literature on teacher learning and is guided by a set of design principles from the research base on effective professional development. From a situated perspective of learning, the study analyzes teacher participation in three types of PLTs in professional development and enactments of practice in classroom observations to examine the ways these PLTs supported teacher learning of the core practice of focus in the MPD.

Significance of the Study

This study investigates the changes in teachers' participation in a one year long MPD and describes the design of PLTs conjectured to support their learning. It aims to identify ways of supporting teacher learning to enact new instructional practices. The study seeks to contribute to the research base regarding teacher learning in professional development settings and the features of PLTs that support teachers in learning and enacting core practices. The study has the potential to inform MTE researchers by generalizing a theory of the processes by which teachers learn instructional practices in MPD and by developing a model of PLTs that assist teachers in that learning.

Research Context

The Core Math project was a multi-year professional development project with goals of supporting elementary grades teachers in implementing the CCSSM through a study of the mathematics learning trajectories underlying the standards (Daro, Mosher, & Corcoran, 2011) and instructional strategies centered on student reasoning. Funded by three awards from the ESEA Title II-A Improving Teacher Quality Grants program, Core Math partnered a local university in the Southeastern United States with three elementary schools from three different school districts. Year one of the project (Core Math I) involved an in-depth study of the CCSSM with two of the participating schools. In response to participants' request for further work on implementing the standards and project evaluation data from year one, the second year of the project (Core Math II) provided participating teachers, teacher leaders, and administrators from these two schools with 108 hours of professional development. The goals of the project were to create professional learning environments in which teachers and school leaders could safely question and improve their own knowledge of mathematics content and learn to enact instructional practices identified as effective in supporting children' mathematics learning.

To meet these goals, Core Math II was based on characteristics of effective professional development described in the literature (Darling-Hammond et al., 2009; Desimone, 2009; Elmore, 2002). Results from an analysis conducted by the project's research team indicated that the Core Math II professional development was successful at supporting teachers in adopting the core practice of focus (Wilson, Downs, & Duggan, 2014). Using a subset of the Instructional Quality Assessment (IQA) (Junker et al., 2004) to measure the academic rigor of instruction and classroom discourse of teachers' mathematics lessons, the research team assessed classroom instruction using pre and post video-recordings of mathematics lessons. Statistically significant ($0.00 \leq p \leq 0.09$) gains and moderate to strong effect sizes ($0.64 \leq d \leq 1.21$) in seven of the nine dimensions of the IQA reflected a shift from teacher-centered, traditional instruction to student-centered, reform-oriented instruction. Participants also completed the University of Michigan's Learning Mathematics for Teaching (LMT) (Hill & Ball, 2004) to measure gains in mathematics knowledge for teaching. The statistical significance ($p < 0.01$) and moderate effect size of the gains ($d = 0.595$) demonstrated that teachers' mathematical knowledge for teaching improved as a result of participation in the MPD. These findings, along with project evaluation reports of self-reported changes in practice and growth in MKT (Downs & Hargrove, 2014), suggest that Core Math II was effective at meeting its goals and serves as an appropriate context to investigate teacher learning of core practices in a professional development setting.

The dissertation study is a part of a design experiment (Cobb, 2000; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) investigating teacher learning of core practices in MPD. Prototypically, design experiments are organized according to three phases: a design phase, an ongoing analysis of the implementation, and a retrospective analysis (Cobb, 2000). This dissertation study is one cycle of the design experiment. Broadly, it investigates the ways that teachers learn to enact the core practice of leading mathematics discussions in MPD.

Organization of the Dissertation

In Chapter I, I have presented the introduction, statement of the research problem, research questions, purpose and significance, research context, and definition of terms. Chapter II reviews the research literature related to investigating teacher learning in MPD. After the review, I articulate an initial conceptual framework and refine my guiding question into three researchable questions. Chapter III outlines the methodology and procedures for data collection and analysis. In Chapter IV, I present the findings from the study. Chapter V contains a discussion and implications for research, policy, and mathematics teacher educators and also contains recommendations for future research.

CHAPTER II

LITERATURE REVIEW

This chapter provides a review of the research literature related to teacher learning with attention to changes in instructional practice in mathematics professional development (MPD). First, I review the literature on mathematics teacher learning in professional development and mathematics professional learning tasks that informed the design of the professional development and professional learning tasks for this study. Then, I identify the key points from the literature review and present a theoretical framework for the study. Finally, I conclude with a set of refined research questions.

Teacher Learning in Professional Development

Mathematics professional development designed to promote teacher learning should be built from the characteristics of effective professional development (Sztajn, 2011). Researchers in professional development have identified main features of effective professional development that represent a broad consensus in the field of academic research. Research has established effective professional development as: (a) intensive, ongoing, and connected to practice; (b) focused on student learning and addressing the teaching of specific curriculum content; (c) aligned with school improvement priorities and goals; and (d) structured to build strong working relationships among teachers (Darling-Hammond et al., 2009). Similarly, Desimone (2009) identified five critical features for effective professional development. She stated that effective professional development should include a strong content focus, active learning strategies, coherence, duration, and collective participation. Desimone argued that these components are substantiated through evidence in existing research to produce positive outcomes from professional development experiences.

Content focus refers to activities focused on specific content matter and how students learn that particular subject. Multiple studies have demonstrated that a focus on content in professional development is positively related to its effectiveness (Chapin, 1994; Hill & Ball, 2004; Lachance & Confrey, 2003). For example, in a study that looked at representations, connections, and misconceptions around mathematics content, Walker (2007) found that when mathematics content was at the forefront and placed within the experience of classroom practice, the professional development experiences became more beneficial. She also found that teachers were more able to design meaningful lessons and assignments. Walker suggested that by highlighting mathematics content teachers addressed their own misconceptions and improved their ability to use representations and make connections in their teaching. Similarly, Zaslavsky and Leikin's (2004) study noted that mathematics content-focused activities increased teachers' mathematical knowledge base and provided a space for professional growth.

Active learning refers to teachers taking an active role in their learning (Darling-Hammond et al., 2009; Desimone, 2009; Elmore, 2002; Garet, et al., 2001; Guskey, 2002; Guskey & Yoon, 2009). Some examples of active learning are: being observed by a mentor (Garet et al., 2001; Hargreaves & Fullan, 1992; Little, 1993; Loucks-Horsley, Stiles, & Hewson, 1996), participating in discussions (Ball, 1996; Elmore, 2002; Knapp, 1997; Talbert & McLaughlin, 1993), and working with other professionals (Goldsmith, Doerr, & Lewis, 2013; Lin, 2001; Tobin & Espinet, 1990). Tzur (2001) found support for active learning in professional development of mathematics teachers. He determined that through reflective activities, teachers increased their awareness of views that influence their teaching practices regarding how knowledge is constructed and how teachers can encourage and foster mathematics learning. Santagata and Angelici (2010) used video discussions to actively engage their participants in their own learning. They used a framework called the Lesson Analysis Framework to analyze the effects of teachers' decision making on the learning of their prospective teachers. They found that by interacting with the framework over time, participants thought deeply about student learning and instructional choices they made as teachers. Borko (2004) explored the use of video to foster active learning in professional development of mathematics teachers. They found that the detailed conversations around video-observed lessons greatly aided participant development as reflective practitioners and heightened their instructional practices. In another study, Kazemi and Franke (2004) found that opportunities to closely observe students can increase teacher belief in the mathematical competence of their students. Taken together, these studies support the notion that professional development should have participants take an active role in their learning.

Desimone (2009) described coherence as the degree the professional development aligns with district and school context as well as teacher knowledge and beliefs (Desimone, 2009; Elmore, 2002; Garet et al., 2001; Guskey, 2002; Guskey & Yoon, 2009; Heck, Banilower, Weiss, & Rosenberg, 2008). Garet and colleagues (2001) conducted a large-scale empirical comparison of effects of characteristics of professional development on teacher learning. They found that teachers who experienced coherent professional development that was connected to other professional experiences, aligned to their standards and assessments, and fostered professional communication were more likely to change their practice. The positive influence on change of teaching practice was greater than the effects of knowledge and skills.

Duration includes both the number of hours allotted and the time span across which the activities occur (Desimone, 2009; Elmore, 2002; Garet et al., 2001; Guskey, 2002). Farmer, Gerretston, and Lassak's (2003) study of mathematics teachers argued that the length of their 18-month duration model was directly related to their findings of teachers reflecting on their own practice after reflecting on authentic mathematics activities experienced during the professional development. They suggested that the success of their findings would not have occurred unless the teachers had time to grapple with the activities over an extended period. An additional study by Jacobs, Lamb, and Philipp (2010) offers further support for sustained time during professional development. In their study, they found additional gains were made in interpreting children's understanding when professional development continued through four or more years and included opportunities for leadership activities. This study provided strong evidence for the need for professional development to be sustained over many years.

Collective participation refers to professional development occurring with a group of colleagues from the same school or grade level to achieve similar learning goals (Desimone, 2009; Elmore, 2002; Garet et al., 2001; Guskey, 2002; Heck et al., 2008).

Collective participation offers several advantages to participants: (a) teachers who work together are more likely to have the opportunity to discuss concepts and problems that arise in professional development, (b) teachers who work together are likely to share common curricular materials, (c) teachers who share the same students can discuss needs across the school and grade levels, and (d) professional development may help sustain practice over time as attrition occurs within the school when a bulk of teachers participate in the same learning experience from the same school. Several studies support the notion of collection participation in professional development. First, Kazemi and Franke (2004) found that teachers from the same school developed their own community of practice and established some guiding principles about what it meant to teach at their school. Buczynski and Hansen (2010) found a direct correlation between the numbers of participants from a school site to the impact of the professional development on that site. They reported that the more teachers from a single school involved in a professional development cohort, the stronger the impact will be for that site.

Differential outcomes result from professional development, even when effective design characteristics are adhered to by professional development designers (Garet et al., 2010). Garet and colleagues discovered that even when all of the characteristics of effective professional development are present, the outcome can still not be predicted. A central claim of this study is that the under-specification of professional learning tasks (PLTs) in which teachers engage during professional development may contribute to these differing outcomes. The nature of PLTs is not represented as a critical component of effective professional development.

Professional Learning Tasks

Recent education reform efforts require teachers to foster mathematical classroom discussions to meet the needs of all of their student learners. Teachers are encouraged to move from a direct-instruction teaching model to a more student-centered teaching approach. Reform efforts have led to a new paradigm for professional development that discards the ineffective workshop model of the past and calls for more powerful learning opportunities for teachers (Stein et al., 1999). The dilemma with this new model for professional learning is the shift from working on *teachers* to working directly on *teaching* (Hiebert & Morris, 2012; Lampert, 2012). An essential part of transformative professional development is the inclusion of practice-based PLTs.

Ball and Cohen (1999) proposed a reconceptualization of professional development as the development of practice and practitioners. This shift would require the challenges, uncertainties, and complexities of teaching to be embedded in professional learning. They argued a need for the careful development of practice-based teacher education if instruction was to change as recommended by reformers. Teachers should learn in the context of their practice with support to develop the capacity to attend to, and learn about, students' knowledge, ideas, and intentions. To improve practice, they stated that tasks are essential for teachers to learn more about students' ideas and understanding. Centering professional development on practice requires tasks that depict the work of teaching, requiring professional developers to select, represent, or create opportunities for novice or experienced practitioners to learn. In the actual act of teaching, there is little time to reflect and puzzle over decisions made during the lesson.

In practice-based professional learning tasks, teachers are afforded opportunity to learn in manageable chunks and engage in more substantial professional discourse in communities of practice. Tasks built around artifacts such as student work, videotapes of classroom lessons, curriculum materials, and journals encourage inquiry and learning (Confrey, Castro-Filho, & Wilhelm, 2000; Jaberg, Lubinski, & Yazujian, 2002; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Ball and Cohen (1999) argued that professional discussions needed to be situated around tasks and artifacts of practice so a more useful language of practice could develop. They promoted centering the work of professional development in teachers' regular tasks of planning, selection, enactment, reflection, and assessment.

Recent work by some researchers has involved decomposing the profession of teaching into core practices that can be discussed and accessed by novice teachers that can help them gain expertise in these practices (Ball & Forzani, 2009; Grossman, Hammerness, & McDonald, 2009; Lampert, 2010; Lampert et al., 2013; McDonald et al., 2013). Core practices are central to the work of teaching, support student learning, and are fundamental to developing complex practice. Grossman, Hammerness, et al. (2009) identified preliminary criteria for core practices as practices that (a) occur with high frequency in teaching, (b) novices can enact in classrooms across different curricula or instructional approaches, (c) novices can actually begin to master, (d) allow novices to learn about students and about teaching, (e) preserve the integrity and complexity of teaching, and (f) are research-based and move the potential to improve student achievement (p. 277). Core practices can reduce the complexity of teaching so that they can be learned by highlighting certain aspects of practice over others (Jacobs & Spangler, in press).

Recent research efforts have encouraged grounding professional education in practice (Ball & Forzani, 2009; Franke, Kazemi, & Battey, 2007; Grossman, Hammerness, et al., 2009; Grossman & McDonald, 2008). Grossman, Compton, et al. (2009) investigated how people are prepared for professional practice in clergy, teaching, and clinical psychology. They identified three key characteristics of pedagogies of professional practice in education: representations, decomposition, and approximation of practice. In what follows, I organize what is known about PLTs by first discussing these three categories followed by a discussion of other characteristics of PLTs. While many PLTs have characteristics of each of these three categories, for the purpose of this review, I separate them using the following criteria: representations and decompositions of practice occur when teachers experience an aspect of teaching as observers or learners; approximations of practice occur when teachers engage in an aspect of teaching and receive feedback on their efforts.

Representations of Practice

Representations of practice refer to the different ways the work of practitioners is made visible to novices. One of the challenges for novices is to know what to look for and how to interpret what is observed (Grossman, 2011). Representations highlight some aspects of practice, and they mask other aspects. PLTs that use representations of practice often include a number of practice-based artifacts. For instance, the use of video cases has become a common way to make practice visible in professional development. Video allows for manageable sized chunks to be studied during a professional development setting (Le Fevre, 2004) and offers a window into teaching without the pressure of having to interact in a classroom situation (Sherin, 2004). Kinzer and Risko (1998) used multimedia cases in their pre-service classes and found that these cases led pre-service teachers to more increasingly refer to those cases when teaching in practicum classes in contrast to prior use of multimedia cases. The teachers also faced classroom challenges more successfully and reacted more flexibly to unexpected situations in class. Borko, Koellner, Jacobs, and Seago (2011) found that video cases in a professional development setting need to be guided and scaffolded. The clips should be purposefully selected to address specific program goals and embedded within activities that are carefully planned to scaffold teachers' progress toward those goals. They also stressed that clips need to be orchestrated carefully to promote productive discussion if they are to have an impact on teacher learning and student achievement.

A particular way to make mathematics teaching visible is modeling instruction around mathematics tasks. Typically, these tasks are high-cognitive demand (Stein, Grover, & Henningsen, 1996). In their study of classrooms, Stein et al. (1996) found that the cognitive demands of mathematics tasks often decline during implementation. Teachers responded to these types of tasks by avoiding them, simplifying them, or softening their accountability. Therefore, experiencing the implementation and maintenance of demanding tasks is a key way of representing practice. Henningsen and Stein (1997) conducted a study to investigate factors that hindered or supported implementation of high cognitive demanding tasks. The most frequently occurring factors that contributed to the decline of the cognitive demand of tasks resulted when challenging aspects of the tasks were removed, focus shifted from understanding toward the completeness or correctness of answers, and too much or too little time was devoted to the task. Silver, Clark, Ghousseini, Charalambous, and Sealy (2007) offered a cycle of PLTs that used practice-based materials. The cycle always began with teachers solving a mathematics problem, then they read and discussed a narrative case. The cycle concluded with collaborative work on implications for their own classroom practice. The authors noted that the use of these practice-based materials helped interweave knowledge for mathematics, pedagogy, and student mathematical thinking.

Suzuka and colleagues' (2009) developed guiding principles for keeping a task focused on mathematical content knowledge (MKT) in professional development. Their first guiding principle was to engage teachers in the work of attending to one another's thinking by asking questions of colleagues to clarify their solutions, asking teachers to explain each other's thinking, and determining the confusing aspects of solutions. Their second principle was to ask teachers to explain their own thinking since explanations are important for the teaching and development of MKT.

In another study, Boston and Smith (2009) worked with 19 secondary mathematics teachers to focus on the selection and implementation of cognitively challenging mathematical tasks. They found that participating teachers improved their ability to select and implement challenging tasks and sustained this practice for the year following the project. The researchers attributed this improvement to the teachers' work with task-centric tools and frameworks and the ability to self-reflect during the professional development. Taken together, these studies provide support that teachers can learn about practice through it being represented for them and that this can be successfully attained through professional learning tasks with support during implementation.

Decompositions of Practice

Decomposition of practice is the breaking down of complex practice into its parts for the purposes of teaching and learning. Decomposition allows novices to focus on an essential component of a practice of teaching so it can be studied and enacted more effectively; however, the ability to decompose practice depends on a common language and structure for describing practice, also called a "grammar of practice" (Grossman, 2011, p. 2839). Decomposing practice allows learners the opportunity to concentrate on enacting a set of moves or strategies of a complex practice.

One popular way of decomposing practice is the use of interviews with students to focus on student thinking (Bransford et al., 2000; Darling-Hammond, 2008; Kilpatrick et al., 2001). For example, the initial purpose of the CGI project was to examine instruction of teachers who were provided with research on children's thinking (Carpenter et al., 1989). Teachers learned about problem types and strategies children use to solve problems through clinical interviews and conducting interviews with children solving addition and subtraction problems. The project teachers also had time to explore curricula materials and plan for the coming year. The researchers concluded that listening to and making sense of student strategies, CGI teachers were better able to adapt their instruction and provide learning activities for their students (Carpenter et al., 1989;

Fennema et al., 1996). In another study, Jacobs and Empson (2016) examined a skilled teacher's interactions with fraction story problems as she conducted interviews and classroom lessons with upper elementary students. They found support for their claim that conducting one-on-one interviews with students corresponds closely to classroom practice. They stated that engaging teachers in conducting interviews as part of professional development can provide opportunities to advance teachers' abilities to support and extend children's mathematical thinking in the classroom.

Another common approach for decomposing practice is the analysis of student work (e.g. Lampert, 2001; Kazemi & Franke, 2004). Teacher educators often have students analyze student work artifacts to help teachers learn to make sense of the complex work of teaching. Analyzing student work allows teachers opportunities to develop a common understanding of representations of good work, identification of common student misconceptions, and analysis of effective instructional strategies. The power of focusing student work in professional learning is evident in the success of elementary schools that consistently produce higher-than expected student achievement (Strahan, 2003). Kazemi and Franke (2004) conducted a study of one teacher workgroup as teachers analyzed student work in a professional development setting. They found that teachers had to first learn how to attend to student thinking. Initially, the teachers thought the student work was all they needed and did not realize that observing students and having conversations were also necessary to understanding student thinking. They noted a key outcome was the shift in how teachers used student work during the project. The teachers began to bring in anecdotal records or recounts of student thinking to accompany the work, which in turn impacted their classroom practice and what they noticed about student thinking. More recently, Goldsmith and Seago (2011) examined teachers' use of classroom artifacts in two professional development programs, one that used student work and the other that used video cases. Their findings show that the use of classroom artifacts provide opportunities for teachers to notice potential in student thinking and focus on the important mathematics as they delve deeper into student thinking. They also uncovered different purposes for different artifact usage: written artifacts help teachers follow or unpack thinking, while video artifacts are preferable for delving into details of student understanding.

Approximations of Practice

Approximations of practice refer to opportunities to engage in practices that are proximal to the practice of the profession. Approximations may consist of role-plays or types of simulation activities that allow opportunities for experimentation of new practices in easier conditions, often with instructors simplifying the demands of the work. Approximations provide occasions for specific and targeted feedback that occur in practice (Grossman, 2011).

Planning lessons and working with curricular materials is a common approach to approximating practice (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Doerr & English, 2006; Fernández, 2005; Remillard & Bryans, 2004). Lesson study is one method that develops teacher awareness and proficiency with the challenges of planning lessons (Fernandez & Yoshida, 2004). Corey, Peterson, Lewis, and Bukarau (2010) studied conversations between seven Japanese student teachers and three cooperating teachers from one school. These conversations typically occurred three times prior to the lesson being taught. The findings described three principles of high-quality lesson planning: an ideal lesson is guided by long and short term goals, created with clear connections to previous and future lessons, and requires anticipation of student thinking in relation to the goals. Santagata (2011) introduced a lesson analysis framework where teachers identified what students were supposed to learn in a lesson, analyzed to see if they did learn it or made progress toward learning it, and suggested alternative strategies for the lesson. The framework was found to help teachers attend to and reason about student learning. The authors also found that the video viewing of clips needs to be short with questions interspersed more frequently in order to assist teachers to effectively reflect on the lessons. These studies offer support for the importance of well-planned lessons in order to foster meaningful mathematical learning.

Rehearsals are becoming a popular approach for approximating practice. A rehearsal is defined as an interactive teaching experience that supports novices by providing opportunities to practice and reflect on important aspects of practice while receiving in-the-moment feedback (Lampert et al., 2013). Many researchers have focused on breaking down the practice of teaching mathematics into specific instances of importance. These researchers found that by using rehearsals, novice teachers were able to restructure how they viewed the teaching and learning of mathematics (Boerst et al., 2011; Ghoussieni, 2009; Hunter & Anthony, 2012; Horn, 2010; Lampert et al., 2013). These researchers concluded that novices improved in their ability to respond to the ways students' think about problems and use students' ideas to guide instruction toward the mathematical goals of the lesson, and that rehearsals created around specific decompositions of mathematics teaching allowed teacher educators to provide a safe environment for novices to practice enacting teaching that may be foreign to the ways in which they learned mathematics.

Another group of researchers worked with novice teachers to rehearse lessons with the goal of refining those lessons (Fernández, 2005; Han & Paine, 2010; Zhang & Cheng, 2011). These authors concluded that rehearsals focused on teaching allowed novice teachers to experience and practice teaching. They also concluded that even though their focus was on using rehearsals to refine a lesson, novices were able to extend their learning and generally apply it to other lessons. In each of these studies, rehearsals were found to be impactful for helping novices learn the art of teaching.

Other Characteristics of Professional Learning Tasks

A review of the literature indicates a number of key characteristics supporting teacher learning. In this section, I identify, elaborate, and offer examples of each of these key processes. Research suggests that teachers also learn through active engagement in processes of observation, discussion, and reflection (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley, Hewson, Love, & Stiles, 1998).

Observation is a key process in developing knowledge when teachers coordinate what they have learned through observation with what they already do (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley et al., 1998). Such coordination helps teachers make meaning of formal knowledge. For example, Cognitively Guided Instruction (CGI) used observation of student thinking as a way to develop teacher knowledge. In a CGI study, Carpenter and colleagues (1989) worked with 40 teachers studying children's thinking about addition and subtraction problems. The teachers learned about problem types and observed student interviews of solutions strategies. Twenty teachers were in a control group and 20 were in the treatment group. Teachers participating in CGI listened to students as they solved problems and encouraged children to solve problems significantly more than the control group. Researchers found that teachers who paid close attention to students' mathematical thinking were better equipped to support learning.

Similar findings from the Teaching to the Big Ideas (Schifter, Russell, & Bastable, 1998) project also supported the use of observing children's thinking as a way to increase knowledge for teaching. Teachers began their investigations into student thinking by analyzing other teachers' student work, observing and studying videotapes of clinical interviews, and studying written materials explaining student solutions. Schifter (1998) used cases of two teachers to illustrate how teachers called upon their learning from the Teaching to the Big Ideas project as they engaged students in their classroom in a study of fractions. For both classrooms, student thinking was central and the content from the professional development assisted the teachers in responding to students with greater mathematical fluency. Taken together, reports from both CGI and Teaching to the Big Idea provide examples of the power of observation as a key process of teacher learning. Using student thinking to learn about problem types in CGI or big ideas in the Teaching to the Big Idea project and then observing interviews of student solutions helped teachers better understand their experiences with their own students and led to their own learning.

Discussion is another key process for developing knowledge (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley et al., 1998). In a multi-year research study, Sowder and colleagues (1998) investigated the relation of middle grades teachers' understanding of mathematics to their teaching practices. The participants and researchers met weekly for the first year and monthly for the second year of the project. The three-hour meetings for both years focused heavily on prolonged discussions of the mathematics concept being studied. The participating teachers were observed in their classrooms over the two-year period of the project. The teachers' understanding of, and comfort with, content grew over the two-year period. Classroom discourse changed from a teacher-controlled environment to one where the teachers probed for student understanding and allowed students to share responses. The teachers attributed this change to the model they experienced during the professional development discussions they experienced.

In another study, Barnett (1998) used aspects of written cases to promote a deeper understanding of mathematics. She found that the discussions of transcripts of cases helped teachers extend their understanding of mathematics. She also found that the discussions of cases became a model of instruction for teachers so they learned about the practice of facilitating discussions through their participation in professional discussions. These reports provide evidence that teacher participation in fruitful discussions can lead to teacher learning; thereby supporting the notion that discussion is a key process to teacher learning.

Reflection is another key process for the development of knowledge (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley et al., 1998; Schön, 1995). Teachers learn as they enact and reflect on their practice. As teachers examine and reflect on the knowledge implicit in instruction, learning happens (Schön, 1995). This view of teacher learning acknowledges prior knowledge and learning over time. Through reflection and inquiry in practice, what teachers need to know and how to teach it becomes clear to the learner (Cochran-Smith & Lytle, 1999). Teachers construct knowledge in the midst of action, making choices, and creating rich learning opportunities for students. For example, in a study of an educational reform project oriented at helping economically disadvantaged middle school students understand mathematical ideas through engagement with challenging tasks, QUASAR researchers found that teachers could become successful maintaining high levels of cognitive demand when teachers were given opportunities to develop knowledge, skill, patience, and motivation for this type of instruction (Silver & Smith, 1996). Teachers were provided time to experience the tasks and reflect on the implementation of these tasks during the five-year professional development. The QUASAR researchers were convinced that reflection was so important to the development of knowledge for teachers that they developed paradigm cases from the project. Stein, Smith, Henningsen, and Silver (2000) used cases designed to help teachers understand tasks associated with cognitive demand that required teachers to critically reflect on their own practice. They claimed that cases were important tools that served as mediating devices between teachers' reflection on their own practice and their ability to interpret their own practice as instances of more general patterns of task enactment.

Kazemi and Franke (2004) studied the collective work of reflecting on participants' own students' mathematical thinking. They reported that participants developed new ways to work together around a particular focus, found ways to experiment within their own classrooms, and used the workgroup as a place to reflect on their experiment. Reflection helped the participants develop their own community of practice to study mathematical thinking. In another study, Borko and colleagues (2008) explored the use of video for the reflective professional development of mathematics teachers. They found that participants' reflective and detailed conversations around video-observed lessons aided development of reflective practitioners and heightened mathematics instructional practices of the participants. For example, in one instance a participant noted watching the leading nature of his own discussions on the video clips made him monitor his practice so as not to repeat it during future lessons. Taken together, these studies support the significant role reflection plays as teachers are developing knowledge for teaching. The practice of reflection in the professional learning setting can occur through written cases, video cases, or within and around the discussion artifacts of practice.

One example of such a PLT includes videotaping oneself as a reflective activity. Studies of videotape as a reflective tool have shown that participants gain independence, gain the personal and practical knowledge of teaching, and enhance their reflection (Armstrong & Priola, 2001). Sherin and van Es (2009) investigated mathematics teacher learning in a video-based professional development environment called video clubs. The researchers explored whether participation in the video clubs impacted the ability to notice and interpret significant features of classroom interactions. They found that when teachers reflected on the video clips during the meeting, they increased their capacity to notice and attend to student thinking, both on the clips and in their observed classroom lessons. The results support the use of video for professional learning experiences to promote reflective analysis.

Research by Seidel and colleagues (2005) reported on an experimental study in which they compared the experiences of teachers who watched video from their own classroom in a professional development environment with those of teachers who watched video from someone else's classroom. Teachers whose professional development was organized around their own videos found the experience to be more stimulating and reported that the program had greater potential for supporting their learning and for promoting change in their instructional practices. Another study by Borko et al. (2006) explored the use of classroom video as a tool for fostering productive discussions about teaching and learning. They found that their participants engaged in increasingly reflective and productive whole-group conversations around video from one another's classrooms. The teachers pointed to the watching and analysis of the video clips as the most valuable aspect of their participation of the professional development experience. Taken together, these studies support the use of teachers watching video clips of their own lessons as powerful professional learning opportunities.

Providing in-the-moment feedback to teachers during enactment of practice is another strategy that has proven successful. Rehearsal feedback consisting of "in the moment" feedback has been found to be beneficial to teachers (Lampert et al., 2013). In this study, Lampert and colleagues (2013) provided "in the moment" feedback to participating teachers during their rehearsal process. The researchers tracked the types of feedback practicing teachers received ranging from directive, evaluative, or scaffolding enactment. They found that facilitators provided directive feedback to teachers during rehearsals 60% of the time. This feedback included helping teachers with productive next moves during the rehearsal. About one-fourth of the time, feedback given to teachers was evaluative. The evaluative feedback mainly highlighted the effectiveness of moves made by the teacher. The facilitators scaffolded the rehearsal one-fifth of the time, including taking on the role of teacher or acting as a student to allow the teacher to see or experience specific moves or situations. The study found that feedback provided to teachers during rehearsals or enactments helped novice teachers know when and how to ask particular questions to elicit and make thinking public. Their research supports Grossman, Compton, and colleagues (2009) claim that approximations like rehearsals would allow facilitators to coach around specific strategies. Their findings support the powerful use of feedback during approximations of practice.

Theoretical Framework

The research is grounded in and influenced by a situated perspective on learning. From a situated perspective, learning is constituted through evolving practices and refined through a process of enactment and reflection (Cobb & Bowers, 1999; Greeno, 1997; Lave & Wenger, 1991). Learning is defined as participation and evidence of learning is change in practice (Wenger, 1998). Practice represents an individual's personalized enactment of the meaning they made through participation in the community of practice (Wenger, 1998). The situated learning perspective focuses on learning as a form of incrementally, but differentiated, participation in social practice

(Lave & Wenger, 1991) and considers the extent to which features of a social setting constrain or afford particular practices associated with learning, with enactment and reflection serving as mediators of learning or change.

Wenger (1998) notes two key ideas in the construction of meaning from a situated perspective. First, learning is bounded by context. Contexts for learning are formed by learners, other co-participants, and by the ideas, tools, and physical resources available. Contexts afford and constrain what learners do and come to know. A situated learning perspective places learning and knowing as transpiring within context, not within minds of individuals. Individuals begin to know and learn as they participate and engage with others in these contexts.

Second, learning is participation. Lave and Wenger (1991) discuss learning as movement along trajectories of participation within communities of practice. Their construct of *legitimate peripheral participation* describes the ways learners participate in communities and move from newcomers to more full participants in the community. New members come to participate in a more central manner within a community as they engage with the community's practice and develop knowledge, skill, and competence with respect to those practices. Learning occurs as participants engage more fully.

In summary, learning occurs through the development of practices and the ability to make meaning and transform through them. Learning is a matter of experience. It changes who we are by changing our abilities to participate, belong, and negotiate meaning within communities of practice (Wenger, 1998).

Communities of Practice

The community of practice in which one participates is central to knowing and learning in the situated perspective (Lave, 1991). A community of practice is a group of people who have a shared domain of interest and who learn how to do it better through regular interactions (Lave & Wenger, 1991; Wenger, 1998). According to Wenger (1998), there are two processes of making meaning in a community of practice: participation and reification. Participation is a process of creating meaning through direct and active involvement in a practice. Reification is a way of making an abstract representation easier to share within the community and can help clarify and explain meanings to a community's members and outsiders. Participation and reification are not mutually exclusive or opposing; they require and enable each other. Wenger (1998) describes this interaction as the duality of meaning. For example, Wenger explained how the reification of a constitution is just a form and is not equivalent to citizenry; however, it is empty without the participation of the citizens involved. The reified constitution must have the participation of citizens to bring it meaning and conversely, participation was necessary in order for the constitution to be reified in the first place.

A community itself is comprised of individuals, conceptual and physical tools, and cultural norms that guide practice and interactions among members. Members are enculturated in the community of practice as they develop competent practice within the culture (Brown, Collins, & Duguid, 1989). Wenger (1998) characterizes communities of practice as including four interconnected and mutually defining components: community, identity, meaning, and practice. The community component defines learning as belonging and refers to the connection or relation with others in the community itself. Identity characterizes learning as becoming where one develops with ways of knowing in practice and with understanding who we are and the potential we have (Lave & Wenger, 1991). Meaning defines learning as experience and affords participants a way of talking about their changing abilities and to experience the world as meaningful.

The fourth component, practice, characterizes learning as doing. Because this study focuses on teacher learning as change in practice, the following section elaborates three dimensions of practice as a property of a community. The three dimensions are mutual engagement, joint enterprise, and shared repertoire.

Practice. Practice exists in a community because participants are mutually engaged in actions and negotiate meanings with one another. Membership is defined by mutual engagement occurring around what members are there to do within that community. In order to be engaged, members must be included in what is important to the community. Relationships among members are formed, and the development of a shared practice depends on the mutual engagement where the members draw upon the contributions and knowledge of others.

Wenger (1998) states that it is joint enterprise that holds a community of practice together. Joint enterprise is defined as the result of collective negotiation by the participants of the community that creates relations of mutual accountability and becomes an integral part of the community. The enterprise is shared because it is negotiated within the community. Members’ responses are interconnected because they are engaged together in a common practice. Mutual accountability plays a central role in in defining circumstances where members attempt, neglect, or refuse to make sense of events and seek new meanings. Once perceptions and judgments become reified in a shared community of practice, members can negotiate the appropriateness of what they do.

A community of practice develops a shared repertoire over time that includes the creation of shared resources for negotiating meaning. The repertoire includes routines, words, tools, ways of doing things, stories, gestures, symbols, actions, or concepts produced or adopted by the community and has become a part of the communities' practice (Wenger, 1998). This repertoire increases in coherence because it belongs to the practice of a community pursuing an enterprise. The repertoire has "two characteristics that allow it to become a resource for the negotiation of meaning; 1) it reflects a history of mutual engagement and 2) it remains inherently ambiguous" (Wenger, 1998, p. 83). Actions and artifacts have recognizable histories of interpretation and can be used to create new meaning.

Boundary Encounters, Objects, and Practices

Communities of practice cannot be considered independent or isolated from the world. Learning through participation includes the individual in interaction with the world. Communities of practice are sources of boundaries and contexts for creating connections with other communities. Wenger (1998) characterizes the coming together of distinct communities of practice to negotiate new meaning or practices as a boundary encounter. In these encounters, boundary objects and brokers are needed to help members develop continuity between distinct communities of practice (Cobb & Smith, 2008).

Boundary encounters are events that can help negotiate meaning. Wenger (1998) describes several types of boundary encounters; one-on-one, immersion, and delegation. Encounters can occur as two members have a conversation (one-on-one), a visitor having impact on the community (immersion), or many members making meaning at the same time (delegation). Brokers participate in two or more groups and serve as a bridge enabling connections and meanings across communities (Wenger, 1998). Brokering provides connections by people who can introduce elements of one practice into another. It involves the complex work of translating, coordinating, and aligning perspectives. Brokering is participative, because the broker is a participating member in multiple communities and has the ability to negotiate and link practices. Brokers need enough distance to bring a different perspective to the community, but enough legitimacy to be listened to by the community.

Boundary objects coordinate the perspectives of different communities of practice (Cobb & Smith, 2008; Wenger, 1998). Boundary objects are reifications around which communities of practice organize their interconnections and negotiate meaning. Boundary objects have different meanings and purposes to different communities yet provide ways for different communities of practice to communicate (Cobb et al., 2003). A boundary object helps negotiate and stabilize various perspectives of the communities, generating shared meaning. When communities interact in a boundary encounter for an extended period of time and become mutually engaged, boundary practices can develop (Wenger, 1998).

Boundary practices integrate elements of practices from both communities to create a new practice, a form of collective brokering (Wenger, 1998). These boundary practices help connect communities and exchange knowledge. Mutual engagement, joint enterprise, and shared repertoire emerge as boundary practices in boundary encounters. For example, as teachers mutually engage with one another, as denoted by their interactions and relationships formed, the teachers are able to connect and exchange knowledge of what is being learned in the community. As teachers form a joint enterprise, they negotiate meaning and hold themselves accountable to that community. Finally, as teachers develop a shared repertoire, they refine their beliefs to be more closely aligned to that of the community. Each of these boundary practices emerge through boundary encounters within the community.

Sztajn, Wilson, Edgington, Myers, and Dick (2014) used ideas of brokering, boundary encounters, boundary objects, and boundary practices to conceptualize mathematics professional development and argued for design-based research methods to investigate teacher learning. They base their work on two premises: (a) the research community has knowledge about students' mathematics learning potentially useful to teachers, and (b) the teachers have knowledge about students' mathematical learning in context that is of critical importance to mathematics education researchers. In their conceptualization, teachers and researchers are brokers from their respective communities, both bringing learning goals to the boundary encounter. They argue that there is opportunity for the exchange of knowledge between these two communities if the professional development is structured around boundary objects – research-based knowledge that is recognizable and useful in both communities. Researchers design PLTs around these boundary objects to facilitate learning and the development of boundary practices with the goal of elements of the boundary practices being introduced into their respective communities practice.

This study examines teacher learning in MPD. I conceptualize the MPD as the boundary encounter. Boundary objects are frameworks of instructional practice as well as artifacts of practice, including student work, written and video case studies, articles, lesson plans, and mathematical problems. PLTs are designed to promote engagement and negotiate meaning of boundary objects. Overtime, new practices of examining and reflecting on teaching using frameworks can emerge as boundary practices. These boundary practices are characterized by mutual engagement, joint enterprise, and shared repertoire. Learning is outlined as the development of boundary practices in the MPD and change in instructional practice.

Summary and Refined Research Questions

A review of the literature indicated that professional development that is effective in promoting teacher learning and positive student outcomes include opportunities to uncover student thinking and closely work with colleagues in professional development settings that includes a strong content focus, active learning strategies, coherence, duration, and collective participation. Yet not all professional development with these characteristics yield consistent results in terms of teacher learning (Garet et al., 2010). A central claim of this study is the underspecified role of PLTs used in professional development. Representations, decompositions and approximations of practice that provide feedback and promote reflection are a promising way to conceptualize the design of PLTs.

Using a communities of practice framework, in particular the conceptualization of mathematics professional development as a boundary encounter, the study seeks to understand the role of PLTs in assisting elementary grades mathematics teachers in learning to implement the core practice of leading mathematics discussions within their classroom. Drawing upon my review of the literature and this theoretical framework, I conclude this chapter by refining my overarching question by specifying three research questions:

1. What opportunities to learn the core practice of leading mathematics discussions do the PLTs afford?
2. In what ways did teachers learn the practice of leading mathematics discussions in the PLTs?
3. To what extent did teachers' enactments of the practice of leading mathematics discussions in their classrooms change as they participated in the professional development?

CHAPTER III

METHODOLOGY

In this chapter, I begin with an overview of design-based research, offer a justification for its use in investigating my research questions, and articulate my design principles and initial learning conjectures. I next describe the context, participants, and professional development that served as context for the research and briefly report on the completed design and ongoing analysis phase of the greater project. After describing data sources and methods of analysis, I conclude with a discussion of reliability, validity, researcher subjectivity, and potential ethical dilemmas.

Design-based Research

Design-based research is a systematic study of instructional strategies and tools that can help "create and extend knowledge about developing, enacting, and sustaining innovative learning environments" (DBR Collective, 2003). According to the Design-Based Research Collective (2003), there are five proposed characteristics of design-based research methods:

1. The central goals of designing learning environments and developing theories are intertwined.
2. Development and research take place through continuous cycles of design, enactment, analysis, and design.
3. Research on designs must lead to sharable theories that help communicate relevant implications to practitioners and other educational designers.
4. Research must account for how designs function in authentic settings.
5. The development of such accounts relies on methods that can document and connect processes of enactment to outcomes of interest. (p. 5)

As an emerging approach to investigating processes of learning, design-based research draws on multiple learning theories to build an understanding of learning, cognition, and development (Barab & Squire, 2004). For learning and cognition, context matters (Barab & Squire, 2004; A. Brown, 1992). Barab and Squire (2004) emphasized that design-based research is a process where learning cannot be separated from the environment in which it occurs. Contexts can be systematically engineered in ways that allow researchers to generate claims about learning. Cobb et al. (2003) characterize design-based research as “engineering”:

> Prototypically, design experiments entail both “engineering” particular forms of learning and systematically studying those forms of learning within the context defines by the means of supporting them. This designed context is subject to test and revision, and the successive iterations that result play a role similar to that of systematic variation in experiment. (p. 9)

Design studies emerged from multiple traditions of research on learning, including Constructivist, Pragmatism, and Socio-cultural Learning Theories (Confrey, 2006). Although design experiments have traditionally been used in classrooms or laboratories to investigate student learning, researchers are beginning to use the methodology to study teacher learning (Fishman & Davis, 2006). Sztajn, Wilson, Edgington, Myers, and Dick (2013) argue that the use of design experiments in professional development will provide better understanding of teacher learning and lead to improvements in mathematics professional development as an area of practice and research.

Phases of Design-based Research

Cobb (2000) describes three phases of design studies: (a) the design of the intervention, (b) an ongoing analysis of its implementation, and (c) a retrospective analysis after the intervention. During the design phase, the researcher formulates the potential instructional goals and materials guided by a set of design principles and learning conjectures. The researcher considers assumptions about the types of norms necessary for the intervention to support the learning that takes place. Assumptions are made about the cognitive starting points that occur at this phase of the experiment (Gravemeijer & van Eerde, 2009).

The second phase is marked by the implementation of the intervention. Through an ongoing analysis of the implementation, revisions or alterations are made based on evidence of learning in relation to the conjectures. This phase involves revisions of the initial conjectures, the possible refutation of them, and/or the creation of new conjectures to test. Evidence of learning in relation to these conjectures may be documented in audio records of meetings or conjecture logs (Cobb et al., 2003) with the goal of systematically linking variations in learning outcomes to the designed intervention.

During the third phase, a retrospective analysis is conducted to answer original research questions. A thorough retrospective analysis requires multiple data sources, including video tapes, conjecture logs, and artifacts of learning. Although ongoing data analysis takes place throughout the entire experiment, the retrospective analysis follows the conclusion of the intervention. This analysis often employs qualitative methods to discern patterns in the data, using these patterns to generate conjectures about the data, testing the conjectures on the entire data set, and analyzing the data again (Gravemeijer & van Eerde, 2009).

Replicability is not the goal with this methodology; instead, “petite generalizations” (Stake, 1995) and trustworthiness of the data are valued (Cobb, 2000). The treatment of classroom activities and events as prototypes allows researchers to understand the ways in which contextual factors influenced learning from the design. Inferences resulting from analysis must be reasonable and well documented in order for resulting claims to be trustworthy. Claims and conjectures must be open to a continual refining and refuting process.

Justification of Design-based Research

Design-based research is the appropriate methodology for this dissertation study because of its commitment to understanding relationships among theory, artifacts, and practice. Further, the study sought to understand the mechanisms by which teachers learned to lead mathematics discussions, and questions of process in education research are best answered by design experiments (National Research Council, 2002). The methodology was an ideal way to understand learning that results from professional learning tasks (PLTs) that assist teachers with the implementation of core practices into their classrooms. This study represents a retrospective analysis of the first cycle of a design study.

Design Principles

A central component of design-based research is the articulation of, and adherence to, a set of principles that guide the design of the intervention. These design principles (Collins, Joseph, & Bielaczyc, 2004) are central to the study because they specify aspects critical for learning represented by the designed intervention. After progressive refinements of the design in relation to these principles (Collins et al., 2004), the intervention and theories produced are the “designed” products of the study.

Guided by my review of the literature on teacher learning, mathematics professional development (MPD), and PLTs, I specified four design principles that informed the design of PLTs for learning the core practice of leading mathematics discussions (LMD). First, *PLTs for learning core practices should represent, decompose, or approximate/enact the practice and embed opportunities for reflection.* This principle is derived from research on teacher learning (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley et al., 1998). PLTs that address how teachers learn by providing a space for modeling of new strategies and opportunities to practice and reflect on them are associated with instructional change (Garet et al., 2001). Further, this principle draws heavily on the research of Grossman and McDonald (2008) on pedagogies of practice and providing opportunities for teachers to engage in the kinds of thinking, reasoning, and communicating used in ambitious mathematics instruction (Lampert et al., 2013). It incorporates the importance of reflection and feedback as essential components of teacher development (Stein et al., 1999; Horn, 2010; Lampert et al., 2013).

Second, *PLTs for learning core practices should include mathematical goals that develop conceptual understandings and attend to reasoning.* This principle builds from existing literature suggesting the importance a mathematically worthwhile and relevant purpose for PLTs and the need to be explicit about these purposes in the professional development (Ball, 1993; Ball, Hill, & Bass, 2005; Boerst et al., 2011; Sleep, 2012; Suzuka et al., 2009) and is informed by Suzuka and colleagues' (2009) recommendations for keeping PLTs focused on Mathematical Knowledge for Teaching (MKT). Their first guiding principles engaged teachers in the work of attending to one another's thinking by asking questions of colleagues to clarify their solutions, asking teachers to explain each other's thinking, and determining the confusing aspects of solutions. Their second guiding principle prompted teachers to explain their own thinking since explanations are important for teaching and developing MKT. These authors expressed how teachers needed opportunities to practice talking mathematics (Suzuka et al., 2009). Further, this principle highlights the interactive nature of core practices; these practices are contingent on students' mathematical thinking, thus learning to enact a practice requires explicit opportunities to link instructional moves with student reasoning (Franke et al., 2007; Franke et al., 2009; Lampert, 2010; McDonald et al., 2013).

Third, *PLTs for learning core practices should attend to teachers' prior knowledge of mathematics, instruction, and students*. This principle draws on (a) research indicating a learner's prior knowledge influences what they learn (Bransford et al., 2000) and (b) Suzuka and colleagues' (2009) guiding principles for keeping PLTs focused on MKT during their enactment. It draws on research focused on supporting teachers in learning how to use knowledge in action (Ball & Forzani, 2009; Grossman, Hammerness, et al., 2009; Lampert, 2010) and making the work of practitioners the center of professional study (Ball & Cohen, 1999; Campbell, 2014). Further, it highlights that aspects of core practices are already a part of teachers' existing practice and that learning to enact core practices is likely about reorganizing and repurposing teachers' instructional moves rather that the introduction of new moves.

Lastly, *PLTs for learning core practices should be based upon artifacts of mathematical thinking that vary in authenticity*. This principle is based on recommendations for PLTs to utilize artifacts such as student work, videotapes of classroom lessons, curriculum materials, and journals that encourage inquiry and learning (Confrey et al., 2000; Jaberg et al., 2002; Jacobs et al., 2007). Ball and Cohen (1999) argued that professional discussions needed to be situated around tasks and artifacts of practice. They promoted centering the work of professional development in teachers' regular tasks of planning, selection, enactment, reflection, and assessment.

Initial Learning Conjectures

Learning conjectures relative to the design are another central component of design-based research (Cobb et al., 2003; Confrey & LaChance, 2000). Conjectures are inferences based on incomplete or inconclusive evidence from the literature and represent the researcher's hypothesis for how the intervention will support or constrain learning. These conjectures evolve constantly as the research progresses through the ongoing analysis and serve as a basis for the retrospective analysis. Evidence supporting or challenging the conjectures is documented in the conjecture log (Cobb et al., 2003), linking their revisions to data.

For this study, I articulated three initial learning conjectures related to the ways the designed PLTs would support teacher learning of the core practice of leading mathematics discussions (LMD). These conjectures were informed by my review of the research literature, consultation with the Core Math II team, as well as my own personal practice and experiences in mathematics professional development. These initial conjectures about teacher learning are:

- Conjecture One: *PLTs that highlight the role of leading mathematics discussions, that elicit and use students' mathematical thinking, and that use authentic artifacts of mathematics learning support teachers in learning to lead mathematics discussions over time.*
- Conjecture Two: *Participation in PLTs that represent, decompose, approximate the core practice of leading discussions and reflect on the role of leading mathematics discussions will lead to the development of a professional learning community focused on improving instructional practice over time.*
- Conjecture Three: *Teachers' enactments of the core practice of leading mathematics discussions will increasingly incorporate instructional moves discussed in the MPD over time.*

Conjecture One is informed by Grossman, Compton, and colleagues' (2008) notions of using representations, decompositions, and approximations of practice to assist novices in learning to enact ambitious mathematical teaching practices. It also draws from research suggesting feedback as a critical factor in teacher learning (Horn, 2010; Lampert et al., 2013), Philipp and colleagues' (2007) argument that teacher beliefs and practices change when provided opportunities to reflect upon their own students' mathematical thinking or on other aspects of their practice, and research demonstrating that learning is the result of reflection and inquiry in practice (Cochran-Smith & Lytle, 1999).

Conjectures Two and Three are informed by a situated perspective on learning. Members are enculturated in the community of practice as they develop competent practice within the culture (J. S. Brown et al., 1989). A community of practice forms that includes four interconnected and mutually defining components: community, identity, meaning, and practice (Lave & Wenger, 1991; Wenger, 1998). Participants learn to do practices better through regular interactions in a community of practice (Wenger, 1998) thereby impacting researcher and teacher practices (Sztajn et al., 2013).

Study Design

Context

Funded by three awards from a Southeastern state's ESEA Title II-A Improving Teacher Quality Grants program, the Core Math projects partnered a local university and two schools within two different school districts. In the first project (Core Math I), the team's focus was on enhancing teachers' mathematics content knowledge through understanding mathematics learning trajectories and the study of instructional practices that allowed teachers to focus on their students' mathematical thinking. Thirty K-5 teachers from two high-needs elementary schools participated in 120 hours of professional development. At the conclusion of the project, participating teachers had made moderate gains in their mathematics knowledge for teaching. On the Learning Mathematics for Teaching Instrument (Hill & Ball, 2004), one of two measures for mathematics knowledge for teaching, participant gains were statistically significant (p =

0.006) with a moderate effect size (*d* = 0.525). On the Mathematical Sophistication Instrument (MSI) participant gains were statistically significant ($p < 0.001$) with a strong effect size (*d* = 0.804). Participating teachers requested additional support with enacting the instructional practices studies in Core Math I in their classrooms and indicated a desire for future professional development.

The second project, Core Math II, responded to these request and evaluation data in two ways. First, the project provided participating teachers with 108 hours of professional development on LMD. Second, it provided a subset of teacher leaders and school administrators with professional development on supporting, spreading, and sustaining pedagogical change. Core Math II served as context for this study.

The overall goal of Core Math II was to enhance teacher content and pedagogical knowledge to support implementation of the CCSSM. To meet this goal, the project designed professional development to meet the following three measurable objectives:

1. Participating teachers and school leaders will increase their mathematics knowledge for teaching, particularly their *specialized content knowledge* and their *knowledge of content and students*.
2. Participating teachers and school leaders will demonstrate reform-oriented instructional practices and mathematics knowledge for teaching, particularly their *knowledge of content and teaching*.
3. Participating teacher leaders and administrators will provide leadership for supporting and sustaining instructional change.

The two participating schools were high-needs schools located in a rural county in the Southeastern United States. At the time of the study, Hillside Elementary School (all names are pseudonyms), a Title I school with 87% of the students classified as low-income, had only 30% of students considered proficient in mathematics. The school did not meet federal requirements for No Child Left Behind in mathematics in 2012–2013. McDonald Elementary School, a Title I school in a non-high-needs district, was designated as a school in need by state requirements. 90.5% of McDonald’s students were economically disadvantaged, and only 34% of students were considered proficient in mathematics by state assessments.

Two members of the Core Math team served dual roles for this project; one served as a district lead mathematics teacher and the co-designer and facilitator of the professional development; the other served as the director of elementary education and the co-designer and facilitator of the professional development. The schools profited from the additional time these researchers spent in both schools, as well as the intensive professional development opportunities afforded through the project. Additionally, the school districts benefitted from the partnership established and sustained with higher education faculty.

The Core Practice of Leading Mathematics Discussions

Recent work by some researchers has involved decomposing the profession of teaching into core practices that can be discussed and accessed by novice teachers that can help them gain expertise in these practices (Ball & Forzani, 2009; Grossman, Hammerness, et al., 2009; Lampert, 2010; Lampert et al., 2013; McDonald et al., 2013).

Core practices are central to the work of teaching, support student learning, and are fundamental to developing complex practice. Core practices can reduce the complexity of teaching by highlighting certain aspects of practice over others (Jacobs & Spangler, in press).

In the mathematics education community, researchers have suggested lists of core practices. For instance, TeachingWorks (2013) from the University of Michigan list 21 high-leverage practices they recommend as basic fundamentals of teaching. High-leverage practices describe ways of engaging students in instructional tasks that promote learning. For example, high-leverage practices specific to content include leading whole class discussions, posing questions about content, eliciting students' thinking, and monitoring student learning. Some high-leverage practices are content-neutral, such as conducting a meeting with a parent or guardian or communicating about a student.

Another group, the Teacher Education by Design (TEDD) project (University of Washington, 2014) also identifies practices that are based on two guiding principles of *ambitious teaching* (Forzani, 2014; Kazemi et al., 2009; Lampert et al., 2013). Ambitious teaching is described as teaching that views students as competent individuals who are sense-makers and provides equitable access to rigorous academic work for all students. The group works with six core practices for ambitious math instruction. These practices are: (a) orienting students to each other's ideas and to the mathematical goal, (b) eliciting and responding to student reasoning, (c) setting and maintaining expectations for student participation, (d) positioning students competently, (e) teaching towards an instructional goal, and (f) assessing students' understanding and use of mathematical representations.

The NCTM (2014) published *Principles to Actions*, where they identify eight mathematics teaching practices essential to supporting students in learning mathematics. These eight practices represent a synthesis of research on mathematics teaching and provide another list of core practices of mathematics instruction. These practices include facilitating meaningful mathematical discourse, posing purposeful questions, building procedural fluency from conceptual understanding, supporting productive struggle in learning mathematics, eliciting and using evidence of student thinking, establishing mathematics goals to focus learning, implementing tasks that promote reasoning and problem solving, and using and connecting mathematical representations.

Although these sets of practices vary, there is a consensus that leading mathematics discussions (LMD) is a core practice of mathematics teaching. Mathematics discussions help students construct mathematical knowledge and support the deepening of mathematical understanding for all the students participating in these discussions (Franke et al., 2007; Franke et al., 2009; Hufferd-Ackles, Fuson, & Sherin, 2004). Teachers elicit student reasoning, monitor student progress toward a learning goal, and make in-the-moment instructional decisions to support student learning as students share their thinking during the discussion. Students benefit from collective participation in these discussions through opportunities to make connections with the strategies to support their own mathematical thinking (Franke, Fennema, & Carpenter, 1997; Sfard & Kieran, 2001). In what follows, I share the conception for the core practice of leading mathematics discussions for this dissertation.

Framework for Leading Mathematics Discussions

Leading mathematics discussions (LMD) involves engaging students in mathematical discourse using questioning and other discourse moves. When leading discussions, teachers employ moves to engage students in collective mathematical reasoning through their questions, statements, and decisions on the direction and focus of the discussion (Chapin, O'Connor, & Anderson, 2009; Franke et al., 2007; Smith & Stein, 2011). In this section, I offer my conception of the core practice of leading discussions that informed the design for this study.

The MPD was designed with the vision of the core practice of LMD as the teacher and students all working on specific content and using one another's ideas to build knowledge of a specific mathematical goal of the lesson. The purpose for LMD was defined as meeting the mathematical goal of the lesson by having students share and refine their mathematical thinking collectively. The framework was built around the following research (see Table 1).

Table 1

Conceptual Framework for the Core Practice of LMD

<table>
<tr><th>Lesson Model</th><th>Organizational Framework</th><th>Instructional Moves</th></tr>
<tr><td>Planning</td><td>Anticipating</td><td></td></tr>
<tr><td>Launch</td><td></td><td rowspan="3">Inviting
Probing
Orienting/Focusing
Pressing
Connecting/Linking</td></tr>
<tr><td>Explore</td><td>Monitoring
Selecting
Sequencing</td></tr>
<tr><td>Discuss</td><td>Connecting</td></tr>
</table>

Smith and Stein's (2011) structure that prepares teachers for leading a mathematics discussion was used as an integral part of the framework shared with teachers. This framework offered anticipating, monitoring, selecting, sequencing, and connecting as a way for teachers to productively orchestrate mathematics discussions. To prepare for the discussions, *anticipating* allows teachers to make predictions about how students may reason through and solve the mathematical problem, including students' approaches to problem solving, interpretations of the task, and strategies likely to be used. *Monitoring* involves attending to students' thinking while students engage in a learning task and includes interacting with students to deepen their understanding and preparing for the discussion. *Selecting* involves determining student solutions or approaches to share in the discussion, and *sequencing* refers to the order in which the solutions are to be shared. *Connecting* describes the actual mathematics discussion, where teachers use various questions, discourse moves, and selected students' ideas to relate the approaches and meet their lesson goal. The purpose of this model is to help make teaching with high-level tasks more manageable for teachers by keeping the richness of a task as they facilitate the sharing of ideas related to the mathematical task.

As a fundamental part of the core practice of LMD, teachers pose questions or probe students to elicit understanding about their mathematical thinking, press students to deeper levels of understanding, position students as competent mathematics thinkers, and support students in making generalizations about mathematical concepts (Fraivillig, Murphy, & Fuson, 1999; Lampert et al., 2013). Productive mathematical discussions require skillfully crafted questions to support student learning (Brodie, 2010; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Stein, Engle, & Smith, 2008; Sherin, Jacobs, & Philipp, 2011). This conceptual framework was employed to assist participating teachers to be able to lead productive mathematics discussions within their own classroom practice.

The discourse moves in this framework were predominantly based on the work of Ghousseini (2009) and Smith and Stein (2011). Ghousseini's work refers to discourse moves as discourse routines. She describes *orienting* as a move that directs students to another's idea, *pressing* as a move to encourage students to explain their reasoning, *connecting* moves that link students' ideas with the big idea of the lesson, and moves that make the structure of the mathematics discussion visible for all the participants is a necessary discourse routine for supporting student participation and learning. Pressing and orienting were used in this conceptual framework for the following purposes; pressing helps push student thinking forward or deepen understanding, and orienting helps students focus on key aspects of the task and advance mathematical understanding.

Smith and Stein's (2011) categorization that extends an initial framework created by Boaler and Brodie (2004) describe nine types of questions, including questions that explore mathematical relationships, probe student thinking, generate discussion, link and apply mathematical relationships, and extending mathematical thinking. For example, *probing questions* are questions that ask students to explain their mathematical thinking in a manner that is clear and articulated precisely. Another question type is *linking* questions where teachers go beyond asking students to share how they arrived at a solution to support students in making connections or recognizing relationships among differing strategies, solutions, or concepts. To advance students' mathematical understanding, teachers may ask *orienting and focusing questions* which helps students focus on key elements of the question which supports them in problem solving.

This framework assists teachers in making intentional instructional moves when LMD in order to meaningfully engage their students in productive discussions. It also helps teachers learn the instructional moves necessary for productive discussions and the goals for making these moves. The MPD is designed to help teachers learn to enact the moves so they will positively impact student discussions.

Professional Development Outline

The Core Math II professional development took the literature's recommendations on effective professional development as a basis for its design (Sztajn et al., 2011). The two student researchers from the Core Math team co-designed and co-facilitated all of the professional development sessions. Full details of the intervention have been previously reported (Floyd, 2014; Rich, 2014) and are included in the following sections. In this section, I briefly provide an overview of the professional development and PLT design.

To provide the participating teachers with the support needed and remain consistent to the elements of effective professional development, a multi-phase professional development model was designed. Phase 1 of the model was a traditional summer institute where teachers came together for 30 hours to learn about the core practice of LMD. Teachers engaged in sequences of PLTs designed according to the aforementioned principles. During Phase 2, teachers worked with a group of students in an afterschool setting. This phase consisted of 30 hours of MPD (18 face-to-face hours and 12 hours of classroom-based activities). Phase 3 was a combination of classroom visits where researchers provided feedback to teachers as they enacted the core practice in the authentic settings of their classrooms. Teachers met monthly to discuss their learning from the classroom observations. Phase 3 was comprised of 20 contact hours (10 hours face to face and at least 10 hours of in-class support). Phase 4 of the professional development represented a culmination of teachers' learning from the project. In a 28-hour summer symposium, teachers from the Hillside and McDonald shared their experiences and led PLTs with a new group of elementary grades teachers. Additional details for the professional development phases are available in Appendices A-D. Sample PLTs from each phase are included in Appendix A with characteristics explained in Appendix B.

Participants

Participants for the study were elementary grades classroom teachers who were a part of the Core Math projects and volunteered for the accompanying research. From the thirty teachers who participated in the Core Math I project during the 2010-2011 school year, fifteen were invited to participate in Core Math II. In consultation with their administrators, 6 teachers at Hillside Elementary from Grades 2 through 5 were given first priority, with remaining spots offered to first grade and kindergarten teachers. At this school, administrators selected teachers on the basis of their ability to share their learning with others on their grade level that did not attend. Six teachers from McDonald Elementary were asked to volunteer.

In all, 13 teachers were a part of Core Math II, seven from Hillside and six from McDonald. These 13 teachers scored the highest on instruments from the Core Math I project measuring mathematics knowledge for teaching. Administrators and lead teachers also participated in the professional development. All thirteen of these teachers volunteered to be participants in the research component of the project and received a stipend for participation. The purposeful selection of professional development participants, and thus research participants, ensured the greatest likelihood of teacher learning and implementing the core practice of LMD in their classrooms and provided a setting where this phenomenon could be studied (Miles & Huberman, 1994).

Professional Learning Task Design

The PLTs were designed based on the recent work by Grossman, Compton, and colleagues (2009) on the learning of professional practice: representations, decompositions, and approximations of practice. In this study, the professional practice learned in each of the PLTs was centered on the core practice of LMD. Representing PLTs were activities that illustrated one or more facet of the practice of leading discussions and allowed participants ways of participating in it. Decomposing PLTs were activities in which the facets of the practice of LMD were parsed into components that were named and explicated. Approximating or Enacting PLTs were activities in which the teachers engaged in experiences of LMD in increasingly authentic environments in order to reduce some of the complexity of the facets learned. For the purpose of this study, the PLTs were bound by the primary learning goal of the PLT. For example, representing is the teacher learning goal of the Representing PLTs. Decomposing is the teacher learning goal of the Decomposing PLTs and approximating or enacting is the goal of the Approximating or Enacting PLTs. Even though there are components of each of these integrated within each PLT, this study refers to the different ways of engaging teachers in learning practice. The categorization of the tasks shares a similar learning goal.

Data Collection and Analysis

In this section, I first provide a description of data sources that were used to investigate my research questions. I then outline and describe how the theoretical framework that informed analysis of these data. I conclude this chapter by outlining reliability, validity, and researcher subjectivity.

Data Sources

Data for this study consisted of video recordings of PLT sequences from the first three phases of the professional development, 34 classroom lesson observations during Phase 3 with related field notes, artifacts from the PLT sequences and from teachers' classrooms, and the conjecture log. A crosswalk relating my research question to data sources is included in Table 2.

Video recordings of the PLT sequences and the classroom observations serve as primary data sources for the study. Roschelle (2000) warns against video-biases and recommends using a pilot study to improve videographic techniques. In Core Math I, the pilot study, video data was analyzed for evaluation purposes and provided such a learning opportunity. This analysis yielded several lessons that informed collection of data for the

Table 2

Crosswalk with Research Questions

Research Questions	Data Source #1	Data Source #2	Data Source #3	Data Source #4
What opportunities to learn the core practice of LMD do the PLTs afford?	Video recordings of the PLT sequences during the MPD to see how tasks helped teachers come to understand this core practice.	34 Classroom Observations and video-taped lessons during Phase 3 where teachers received feedback and in the moment support during enactment.	Conjecture Log to indicate the characteristics of the PLTs and denote the significant moments that support the adoption of this core practice.	Collection of Artifacts (lesson plans, student work, and assessments) from each of the PLTs and from the teachers' classrooms to provide evidence for adoption of the core practice during enactment.
In what ways did teachers learn the practice of LMD in the PLTs?	Video recordings of the PLT sequences during the MPD to track participation using the theoretical framework.	34 Classroom Observations and video-taped lessons during Phase 3 where teachers received feedback and in the moment support during enactment.	Conjecture Log to indicate the frameworks (boundary objects) in the MPD and how these frameworks supported teacher participation.	Collection of Artifacts to show how participation influenced shared repertoire, mutual engagement, and joint enterprise.
To what extent did teachers' enactments of the practice of LMD in their classrooms change as they participated in the MPD?	Video recordings of the PLT sequences to track the presence of academic rigor and accountable talk in the classroom, using the IQA.			

study, including the effect of the camera on the participants' behavior and the placement of cameras to capture key aspects of teachers' participation.

Video recordings of PLT sequences representative of the design were selected for this dissertation study. Four PLT sequences were selected for analysis, two from Phase 1, and one each from Phase 2 and Phase 3. Video data was processed using Powell, Francisco, and Maher's (2003) model for videotape analysis. This model begins with attentively viewing and describing the video data and culminates with the identification and transcription of "critical moments." For this study, I define critical moments to be episodes of teachers' participation: (a) related to the core practice of LMD; (b) aspects of the PLT sequences related to the design principles; (c) shared repertoire, mutual engagement, and joint enterprise constructs; and (d) aspects of the PLT sequences related to the conjectures. These critical moments served as the analytic unit for the study.

After carefully reviewing the videotaped task sequences, I identified significant moments or critical events (Powell et al., 2003). Video recorded lessons of the three PLT sequences were viewed to identify critical moments. According to Powell and colleagues (2003), a method for video analysis follows seven nonlinear, interactive phases. The phases are as follows: viewing the video attentively, describing the video data, identifying critical events, transcribing, coding, constructing a storyline, and composing a narrative. This model was adapted by the researcher to analyze the video recorded lessons and identify critical moments in which an opportunity existed to draw upon the ways teachers participate during the task sequences.

Video recordings of the teachers' classroom instruction and 34 lesson observations with field notes where I provided in-class support as teachers enacted the lesson was the primary data source for this study. During each phase of the professional development, teachers recorded a lesson and submitted it for review by the project team. These recorded observations provided a way to see the phenomenon of the core practice of LMD in the actual setting of the classroom (Creswell, 2013). The first lesson was collected at the end of Phase 1 in early September, the second at the end of Phase 2, in early January, and the last at the end of Phase 3 in May.

As part of the ongoing analysis, these recorded observations were rated using the Instructional Quality Assessment (IQA) (Junker et al., 2004). The IQA measures the overall academic rigor of teachers' lessons and quality of classroom discourse. For this study, the overall composite scores for academic rigor and accountable talk were utilized along with three specific measures; AR3 denoting the presence of students' mathematical thinking in discussions, ARQ specifying the presence of probing and pressing moves, and AT2 denoting the presence of inviting and orienting moves. Academically rigorous questions support students' mathematical thinking and reasoning and promote learning. Probing questions ask students to clarify their thinking or enables students to elaborate their own thinking for their benefit or for the class. Rigorous questions also point to underlying mathematical relationships or make links between mathematical ideas. Questions can also enable other class members to contribute and comment on ideas during the discussion.

Two secondary sources of data assisted in answering the research questions. First, artifacts from the three PLT sequences and from teachers' classrooms was used as supportive evidence. Teachers' written work on PLTs, lesson plans, student work, and assessments allowed me to view the core practice used on these artifacts. Second, the project's conjecture log provided additional evidence and direction for the analysis. Cobb et al. (2003) underscore the importance of documenting decisions made about the adjustments to the intervention relative to evidence of learning during the implementation. I used the conjecture log as data for evidence of each conjecture and to document the decisions made to address changes and next steps for the intervention. Since both the artifacts and conjecture log are text-based, no data processing was required.

Analysis of Data

To analyze my first research question, critical moments from the MPD and teachers' classrooms, accompanying artifacts, and the project's conjecture log was analyzed using the pedagogical framework. To shape this framework into an analytic tool, I began with the work of Grossman, Compton, and colleagues (2009) that described the three pedagogies of practice: representations, decompositions, and approximations of practice. In order to determine what opportunities the PLTs afforded teachers to learn the core practice of LMD, I followed a two-pronged analysis: one focused on the transcripts of the PLT discussions during the MPD and another focused on the classroom observations and conjecture log of field notes. I used artifacts to capture evidence or provide support as needed. I followed a process that relied on both a priori codes and emergent ones (Strauss & Corbin, 1998). The a priori codes were based on each of the components listed in my framework for LMD. Using these codes, I took an analytic pass on the data to identify episodes that attended to one or more of the components of the framework. For example, I coded elements of the five practices as orchestrating mathematics discussions and instances of using discourse moves as eliciting reasoning. I followed in my coding the sequence in which the pedagogies of practice were designed and compared them across the three phases of the MPD.

I continued with open coding to identify themes that emerged. For example, a theme that emerged was the roles that teachers play while LMD. During the first phase of the MPD, teachers took on the role as a learner as they participated in the sequences of the PLTs. During the second phase, the teachers took on the role of observer with little to no responsibility for student outcomes. In the third phase, the role shifted to that of teacher with full responsibility for student outcomes. Some additional themes that emerged were the differing ways the teachers viewed artifacts during the phases of the MPD, what they attended to as the focus of their learning across the MPD, how teachers structured a discussion, and how teachers thought about or valued LMD. I grouped the themes into categories and matched categories across data sets.

For my second research question, critical moments from the MPD and teachers' classrooms, accompanying artifacts, and the project's conjecture log was analyzed using the communities of practice theoretical framework. To shape this framework into an analytic tool, I began with Raynes and Jacobs's (2001) operationalization of the shared repertoire, mutual engagement, and joint enterprise constructs to analyze teacher participation in a community of practice. These researchers defined each as follows: a shared repertoire includes artifacts, stories, tools, discourses and historical events that are common to the community members, mutual engagement includes the relationships built and maintained as members engage in ongoing activities, and joint enterprise involves community members negotiating meaning and holding one another accountable in a set of related practices. Taken in relation to the focus of this research, I define these constructs and provide examples of each from the data in Table 3.

Table 3

Definitions and Examples of Mutual Engagement, Joint Enterprise, and Shared Repertoire Constructs

A priori codes	Description	Examples
	Mutual Engagement	
Interactions	How teachers are interacting with one another as they participate in PLTs centered on the core practice of LMD.	Participants building on and valuing one another's ideas. Example Interaction: Quinn: So they would make 2 groups of 4 and that would be their 2 giraffes. Is what you're saying that they'd make 20 groups of 2? Valerie: That's the way I would start and then you would only make 17 animals. Quinn: Okay. I was just trying to visualize it in my head but that makes sense now, okay.
Relationships	What the interactions mean about the relationships teachers are forming as they participate in PLTs.	Ex: Teachers respectfully sharing different moves they would make from one another during the discussion. Ex: Victoria staying after a session to ask for extra help.

Table 3

Cont.

A priori codes	Description	Examples
	Joint Enterprise	
Negotiation	Teachers state or allude to negotiations made as they learn and/or enact the core practice of LMD. These negotiations inform how they are making sense of one another's thinking and of the boundary practices.	Ex: Teachers trying to make sense of one another's strategies by questioning for clarification Ex: Teachers brainstorming ways to meet school requirements of posting learning targets without giving away important mathematical concepts to be learned in the lesson.
Accountability	The teachers state or allude to an implicit value of the core practice and demonstrate accountability for its use.	Teacher notices that she is not uses instructional moves and becomes angry at herself. Heather: No kids come up and lead the discussion because I'm still like taking the whole thing. What am I doing?
	Shared Repertoire	
Shared Beliefs	Teachers state or allude to beliefs about mathematics learning and teaching.	Ex: Teacher voiced how she grappled in past spending time letting students struggle through a misconception, but realizing how important it is now. She stated that previously she would have simply told them how to do it.

Using these descriptions, I iteratively developed a codebook for qualitative data analysis (Decuir-Gunby, Marshall, & McCulloch, 2011). Drawing from Decuir-Gunby and colleagues work, I initially used a theory-based coding system, using my analytic tool to develop codes a priori from the existing theory. Then, I moved to a more data-driven coding system to include codes that emerge from the raw data. After the codes were stabilized, I verified them with an independent mathematics education researcher to establish reliability.

After the initial coding process, I examined the reduced data corpus using constant comparative techniques (Glaser, 1992) to collapse codes, discern patterns, and search for discrepancies. This procedure allowed me to track changes in teacher learning by examining shifts in their practices across the professional development.

To analyze the last research question that asked to what extent the teachers' classroom practice of LMD changed, I used the IQA instrument and followed up with classroom examples to support the scores from this instrument. This instrument measures the extent to which teachers consistently press students to provide explanations, explain their reasoning, and make connections between ideas, strategies, or representations. Two fellow researchers and I independently rated the moves teachers made on the IQA rubrics and then we verified the ratings and established reliability. Then, Friedman Tests were run for the academic rigor and accountable talk summation scores along with three specific measures; AR3 denoting the presence of students' mathematical thinking in discussions, ARQ specifying the presence of probing and pressing moves, and AT2 denoting the presence of inviting and orienting moves.

Reliability and Validity

During analysis, I used many of Creswell's (2013) recommendations for validation. First, I clarified my researcher biases upfront. I continued to reflect upon my biases as a researcher, and with this awareness I attempted to monitor and control these biases. One such bias is a firm belief in the power of professional development as a vehicle for instructional change. In addition, my connections with the school district, the schools, and participants could also be a source of bias. Further, I am a strong advocate of the core practice of LMD and attending to student reasoning and believe that mathematics instruction should shift to include this model.

To address these biases, I utilized “negative case sampling” in an attempt to purposively search for examples that disconfirm my conjectures (Johnson, 1997). Another critical strategy I used is prolonged engagement and persistent observation. I already had a trusted relationship with the participants, but I intentionally worked to continue to deepen and nurture that trust during this study.

Triangulation of inferences based on data was a key factor for validation. The research design included multiple data sources to corroborate the evidence for my findings. I used video recordings of PLT sequences, classroom observations, artifacts, and my conjecture log to validate emerging themes about the ways teachers learn the core practice of LMD. I provided the opportunity for participants to review my analyses and interpretations in order to give opportunities for member checking (Merriam, 1998). This allowed the participants a chance to judge the accuracy and credibility of my accounts and hopefully avoid researcher bias. Additionally, to avoid bias, I tried to attend to how close I am to the data by collaborating with fellow researchers that supported the role of out-of-district observer to help with reality checks and validate my interpretations throughout the study.

In order for a study to be reliable, the research must be dependable and consistent. Replicability is not the goal with design-based research; it is more important to have reasonable inferences in order for the data to be trustworthy (Cobb, 2000). The goal of design-based research is to problematize the design and implementation and to advance theory (Barab & Squire, 2004). For this study, details of data collection and analysis are disclosed. The Design-Based Research Collective (2003) suggestions for enhancing reliability were followed for the on-going analysis and retrospective analysis. These suggestions include using triangulated data collection methods, repetition of analysis across cycles of enactment, and use of standardized measures or instruments.

Subjectivity and Potential Ethical Issues

My role as Director of Elementary Education provided an opportunity to bridge school and district-level administration with the practice of teaching. My position also afforded me unlimited access to the teachers in the school. I began my work in this district as mathematics lead teacher and have worked with the majority of participants in this manner given me credibility in mathematics and the art of teaching. My promotion to Director of Elementary Education two years ago has increased my position of power and privilege. The greatest potential threat my district position presented for my research occurred during the data collection and reporting phases. To ensure validity, teachers need to feel comfortable being completely honest on their interviews and know that their participation and actions will not be used in an evaluative manner. To address those threats, I relied on my professional relationships with the teachers, full disclosure of the purposes of my study, and collaboration with other researchers to make sure I was aware of the ways my professional position affects this research. I have also assured the teachers that the information they provide is for purposes of my professional growth and will not be used in anyway against them. My intention was to reduce the teachers' view of my presence as evaluative and to help them view it as more as seeking understanding.

Definition of Terms

The following terms and definitions are used within the context of this study. The researcher developed all definitions not accompanied by citation.

Ambitious Mathematics Instruction—Teachers teach in response to what students do as they engage in high-quality, authentic problem solving tasks. They adjust content and methods to what they observe in student performance.

Approximating Practice Tasks—A type of PLT where teachers enact what they learned from studying a particular instructional practice (Grossman, Compton, et al., 2009), such as analyzing student work, anticipating students' approaches to a mathematics task, and rehearsals.

Core Practices in Teaching—Identifiable components fundamental to teaching that teachers enact to support learning Core practices consists of strategies, routines, and moves that can be unpacked and 'learned' by teachers. Core practices include both general and content specific practices. Examples of core practices include leading mathematics discussions (LMD), modeling, and providing instructional explanations (Core Practice Consortium, 2016).

Decomposing Practice Tasks—A type of PLT where instructional practices are decomposed into manageable components to be studied (Grossman, Compton, et al., 2009), such as viewing a video of classroom practice with a focus on questioning or learning how to launch a student-centered lesson.

Effective Professional Development - Professional development that leads to substantive changes in teachers' beliefs, knowledge or instructional practices and includes characteristics of the “consensus view” (Birman et al., 2000; Elmore, 2002; Garet et al., 2001; Desimone, 2009; Guskey, 2002; Guskey & Yoon, 2009; Heck et al., 2008; Webster-Wright, 2009).

Leading Mathematics Discussions—The teacher and all of the students work on a particular mathematics goal using one another’s ideas as resources. The purpose is to build collective knowledge and capability of specific learning goals to allow students to practice listening, speaking, and interpreting. The teacher and a wide range of students contribute to the discussion, listen actively, and respond to and learn from one another (adapted from Grossman et al., 2014).

Representing Practice Tasks—A type of PLT where instructional practices are made visible so teachers may study it (Grossman, Compton, et al., 2009), such as video recordings of classroom teaching and modeling of particular strategies.

CHAPTER IV

RESULTS

The primary purpose of this study was to better understand the ways in which teachers learned the core practice of leading mathematics discussions (LMD) in professional development. Professional learning tasks (PLTs) were designed according to a set of design principles outlined in Chapter 3 to facilitate teachers' learning of this core practice. Three specific questions guided the research: (a) What opportunities to learn the core practice of LMD do the PLTs afford?; (b) In what ways did teachers learn the practice of LMD in the Practice-Focused PLTs?; and (c) To what extent did teachers' enactments of the practice of LMD in their classrooms change as they participated in the professional development?

In this chapter, I first detail findings on the opportunities to learn the core practice of LMD afforded by the PLTs across the three different phases of the mathematics professional development (MPD). Next, I present findings on the ways in which teachers' enactments of the core practice of LMD changed throughout the MPD. I then conclude with findings on the ways teachers learned in the MPD based on an analysis of changes in their practice as evidenced in their discourse throughout the MPD. Findings are illustrated with examples selected to clearly and concisely represent teachers' discourse throughout the MPD. All names in this report are pseudonyms, and the term "teachers" in this chapter is used to refer to the collective group of teachers participating in the study.

PLTs for Learning the Core Practice of LMD in Professional Development

Four principles guided the design of the PLTs for the MPD. First, PLTs represent, decompose, or approximate/enact the core practice of LMD and embed opportunities for reflection. Second, PLTs have both pedagogical and mathematical goals and aim to support teachers in learning the core practice as well as develop conceptual understandings of mathematics through attention to reasoning. Third, PLTs attend to teachers' prior knowledge of mathematics, instruction, and students. Last, PLTs are based upon artifacts of mathematical thinking that vary in authenticity. In the following sections, I present results showing the opportunities for learning the core practice of LMD that were afforded by the PLTs.

Representation PLTs

PLTs representing the core practice of Leading Mathematics Discussions (LMD) fostered a value for the practice by providing opportunities for teachers to recognize the central role of mathematical thinking in instruction. In the initial phase of the MPD, teachers came to value the core practice of LMD by experiencing the role of mathematical thinking in teaching and their own learning. In later phases, these PLTs provided opportunities for teachers to consider how LMD might support student learning. In the following section, I will elaborate these results.

Phase 1. Representing PLTs provided opportunities for teachers to value the role of mathematical thinking in teaching through experience as a learner. In Phase 1, these PLTs allowed participants to: (a) engage with others' thinking through comparison and questioning and (b) refine their mathematical reasoning collectively. The analysis indicated that experiencing LMD as a learner fostered a value for LMD.

Engaging with others' thinking through comparison and questioning. In Representing PLTs, teachers engaged with others' explanations to further the mathematical understanding of the entire group. As teachers compared the reasoning behind their solutions and questioned one another to understand the solutions, they deepened their understanding of the mathematics under consideration and came to appreciate the central role of mathematical thinking in their own learning. In turn, the valuing of mathematical thinking led teachers to value the instructional moves that elicited and responded to it. For instance, during the first representing PLT, teachers adopted a more sophisticated approach for solving the Ostrich and Giraffe Problem: *A zoo has several ostriches and several giraffes. They have 30 eyes and 44 legs. How many ostriches and how many giraffes are in the zoo?* When solving the problem, all of the teachers used a guess and check approach without a sophisticated way to select their initial guess. In the mathematics discussion, Beth explained her thinking to the group and clarified her reasoning for selecting the numbers to solve the problem:

Beth: I started somewhere in the middle because I knew that if I started in the middle, and once I got my number, I could adjust easier instead of starting with like 1 and 14. So when I first started, I guessed 7 ostriches and 8 giraffes and I got a total of 46 legs. And since I was close to what my goal of 44 legs I knew that I had to take, that there had to be less giraffes. So, since I was pretty close, I just switched the numbers for 8 ostriches and 7 giraffes and I got a total of 44 legs.

Fac1: I think I have a couple questions for the group. I want to know comparison wise, tell me about this strategy in relation to Sarah's strategy. How are they alike?

Linda: They're both guess and check.

Fac1: They're both guess and check, okay. How are they different?

Heather: Theirs [the group with the table] had an aspect of guess and check as well, I think, right? But with a system, yeah. And just the way she put ostriches and giraffes separately and they kind of figured out what combinations she could, kind of thing. That's what they kind of did a little bit. And then you took, you had to write down, you know, the amount of legs whereas Sarah had drawn it.

Fac1: So it's a little different way. So let me ask Sarah then, because Sarah, you were saying when you started you were just going and you would add some to this column and add some to this column, and add some to this column or that column, and add some more to this column and you were going until you got the right number of legs and the right number of animals. [Beth] You didn't do that. How did you start the first guess again?

Beth: Umm, I just started in the middle where I could get, you know, as close to the center. If it had been 16 animals, yea if I knew I had to have 16 animals, I would've started with 8 and 8 just so that I could start somewhere in the middle and then adjust from there. Like do I need to go more giraffes or less giraffes instead of starting at a broad 14 and 1, I'd rather get somewhere in the middle.

The discussion assisted the group in refining the guess and check approach and illustrated the importance for explanations of mathematical thinking in order to move the group toward understanding. Beth's explanation provided an opportunity for others to understand her mathematical thinking and refine their approaches. Heather and Linda engaged with Beth's mathematical thinking, naming and comparing nuances in different strategies. The group revisited Beth's thinking to understand how to efficiently use guess and check by considering the initial guess and using that guess to make appropriate adjustments.

In other instances, teachers questioned one another to solidify their understanding of the problem and to unpack the mathematical ideas with other members of the group. For example, Valerie asked, “Why didn’t you just take off another ostrich?” when she was confused about how Beth’s adjustment helped her determine how to account for the number of legs. Before Beth could respond, Danielle interjected and explained, “Because you’ve gotta have 15 animals.” Here, the teachers were recognizing the role others’ mathematical thinking played in their own learning as they engaged with each other’s thinking by questioning one another.

In a second Representing PLT, teachers experienced the core practice of LMD as a learner when solving the Buying the Horse Problem: *A man buys a horse for $50. He sells it for $60. He then buys it back for $70. Then he sells it one last time for $80. How much money, if any, did he make or lose on his trades?* As teachers grappled with understanding the problem, they relied heavily on one another’s reasoning to reach consensus of a solution. The following exchange occurred during the mathematics discussion after teachers explored the horse problem and demonstrates another opportunity for the teachers to attend to one another’s’ thinking. Valerie explained her incorrect reasoning:

> So, I started trying to keep track of how he was moving. So I went from 50 to 60 and went, okay, well that’s a plus 10. Then I went from 60 to 70, but then that was confusing to me too because that should have been a plus 10 but really it wasn’t, it was a minus 10. So I had to re-vamp my plan. So I went from 50 to 60, here that’s a plus 10, then I went from 60 to 70, going back to zero. Then I went from 70 to 80, which was 10. But I just looked at this spot kind of as a place holder saying, alright that was overall product instead of saying, keeping track of this 10 [pointing to her paper]. Because to me this 10 was gone.

Kara interjected that she was having the same issue by stating, “That’s exactly what was happening to me!” Nicole added, “The thing is that I don’t think we ever fully convinced people. We have a few of us that did it differently but got the same answer the first time. And the other two have the same answer.” After that statement, there was still so much confusion about whether the answer was the man made $10 or made $20, the teachers in the group decided to try a new strategy. For approximately 15 minutes, teachers collectively acted out the problem by exchanging money they made out of paper. Throughout the process, they were determined to help one another understand their thinking and reach a consensus on a solution.

Quinn: Okay. So that’s plus 10.

Valerie: Now you decide you want.

Quinn: To buy it for 70.

Valerie: To buy it for 70. So now you needed an additional $10 because you give me all of that.

Quinn: Right.

Valerie: And then 10 additional dollars so now you’re at 0.

Quinn: Now I’m at 0.

Beth: No you’re not because you’ve got $10 profit right here and you buy it for 70 so you’re spending $60 that you don’t have. Does that make sense?

Quinn: No.

Beth spent several more minutes helping the teachers understand the transaction until the solution finally makes sense to the group:

Fac1: Did you all agree?

Quinn: Yes.

Valerie: Yes.

Fac1: What are you all agreeing on?

Quinn: Beth's.

Brenda: 20. Making 20.

Quinn: Or I guess any of them.

Fac1: Right.

Quinn: Except mine.

Fac1: You two were there already.

Valerie: Yes but I had 10.

Fac1: You couldn't agree it makes sense?

Quinn: I've come to agree it makes sense.

Valerie: I agree that that makes sense but I don't know how mine doesn't make sense.

Even though all teachers agreed that the transaction made sense and agreed on the solution, Valerie was left not understanding why her reasoning did not work. Through representing the core practice of LMD, teachers had an opportunity to recognize the way one another's mathematical thinking supported their own learning as they collectively refined their reasoning of this problem.

Refining reasoning collectively. Representing PLTs also provided opportunities for teachers to value the role of mathematical thinking in teaching through refining mathematical reasoning collectively in discussions. For example, Valerie's residual question about her own approach to the Buying a Horse Problem was taken as the problem for the whole group to discuss. The group worked on thinking through Valerie's problem by connecting strategies and adding onto the thinking of others in the pursuit of determining why this solution was incorrect. After the facilitator elicited several teachers' thinking in discussion, Carol noted, "We all got the same answer for the first [every strategy used resulted in a solution of made $10 for the first step]. It's after he buys it back is where all of us are different." Through Carol's observation and comparison of all presented strategies, the group was able to determine that each strategy used similar thinking for the first step, and that different interpretations of the next step was resulting in different solutions.

The group was finally able to understand the reasoning when one facilitator asked Erin to share her current reasoning with the group. She stated,

> When, after the first pair of transactions where the horse was bought for 50 and sold for 60 and there was the extra $10 there. Yea, right there. Then he bought the horse for 70 and what was happening I think when you do it in a string, you think well the difference between, umm, you get 60. After he has $10 he sells it for 70, he's out $60 at that point total for all of the transactions so far. And you think the difference between 70 and 60 is 10, so that's where that 10 came back in, it was a confusing little part. He was actually out $60 at that point.

Danielle responded to Erin's reasoning, suggesting that they think about the problem as, "order of operations" since the string of operations was causing confusion. Through the mathematics discussion, the teachers were able to draw upon each other's mathematical thinking and collectively refine their reasoning.

An analysis of teachers' reflections of the Representing PLTs in Phase 1 provided further evidence of opportunities for teachers to find value in other's mathematical thinking and the ways the core practice of LMD elicited and used it to support their learning. Several teachers commented on connecting strategies in discussion, stating for example, "I have not had students look at the similarities between strategies. This was quite the revelation to me." Another teacher commented, "The way pairs were picked to share similar strategies was very beneficial." Several teachers appreciated independent time to solve the problem and then discussion time with others to clarify understanding, with one stating that, "getting time to work it out on my own, discuss with a partner, and then share with the whole group helped me to see different ways to solve the problem." Another teacher noted, "Trying to solve on our own first made the discussion much better and gave us more to discuss."

Phase 2. Whereas Representing PLTs in Phase 1 allowed teachers to experience the core practice of LMD as learners, these PLTs in Phase 2 were designed for teachers to observe the practice of facilitating student learning. Teachers were to observe one of the facilitators lead a discussion of a mathematics problem with students in an after-school program, focus on the instructional moves, and make note of the facilitator's questions. During the facilitation of the problem however, the teachers abandoned their role of observer and interacted with the students. While many expressed the importance of eliciting students' thinking, explanations, and sense-making, the presence of students during the PLTs led teachers to focus on students' mathematical thinking opposed to the core practice.

During the first Representing PLT in Phase 2 for example, as soon as the facilitator had launched the problem with students, some of the teachers physically moved onto the carpeted area to assist students before they had an opportunity to think about the problem at all. Other teachers began posing questions to the students that were near them and led them to specific approaches. The students were not allowed appropriate time to think about the problem, and the presence of multiple adults working with students on the problem rendered the representation of LMD unrealistic and inauthentic, and few teachers focused on the facilitator's instructional moves.

In Phase 2, the teachers transferred their value of learning from others' mathematical thinking during discussions from Phase 1 to students' mathematical thinking. While the facilitator represented LMD with the students, teachers attended to how the students solved the problems and noted specific examples of their mathematical thinking. Yet, the goal of the PLT was for teachers to focus on the ways that LMD elicited and used students' mathematical thinking.

Summary. Across the MPD, Representing PLTs fostered a value of the core practice of LMD by providing opportunities to recognize the central role of mathematical thinking in teaching and learning. Embedded opportunities for reflection allowed teachers to recognize the ways understanding others' mathematical thinking supported their own learning (Principle 1). Experiencing LMD as a learner allowed teachers to relate their own mathematical learning to the instructional moves of the core practice and recognize its role in their learning (Principle 2). These PLTs allowed teachers to draw upon their prior knowledge about teaching to appreciate the practice as well as express concerns about enacting LMD in their classrooms (Principle 3). PLTs based on their own mathematical thinking and their colleagues' thinking provided the chance for teachers to value understanding others' thinking as a part of the learning process, yet the presence of students shifted teachers' foci from the role of LMD in eliciting and using mathematical thinking to the students themselves (Principle 4).

Decomposing PLTs

PLTs decomposing the core practice of LMD provided opportunities for teachers to make sense of the practice and reflect on the ways it fosters learning. These PLTs allowed teachers to make sense of the instructional decisions and related goals for each decision. In contexts of experiencing the practice, teachers were able to reflect on the ways the practice supported their own learning. In contexts of observing the practice however, the opportunity to observe the practice was moderated by teachers' foci on students' mathematical thinking. Similar to Representing PLTs, Decomposing PLTs also presented opportunities for teachers to value the practice of LMD.

Developing understanding. Decomposing PLTs provided opportunities to develop an understanding of the core practice of LMD. Decomposing PLTs gave teachers opportunities to understand the instructional moves and related goals comprising the practice. These PLTs served as a context for teachers to understand how these instructional decisions work together to engage students in considering one another's ideas to advance the mathematical goal of the lesson. In the following section, I will elaborate these results.

Understanding instructional decisions. Decomposing PLTs provided teachers with opportunities to understand the instructional decisions that comprise LMD. These PLTs highlighted the moves and goals of the facilitators from the Representing PLTs. In Phase 1, teachers were able to examine the facilitators' decisions in relation to their experiences as a learner. In Phase 2, teachers were able to examine the decisions made in relation to student learning.

Phase 1. Focusing on the facilitators' goals and instructional decisions when teaching mathematics problems during Representing PLTs provided a context for teachers to better understand the practice and reflect on it in relation to their learning. To illustrate, one of the Decomposing PLTs in Phase 1 focused on one facilitator's use of the practice during the Ostrich and Giraffe Problem. During the task, the facilitator shared her various instructional moves and what she hoped to accomplish with them. Teachers asked questions to determine the rationale behind the decisions made and to determine which decisions were planned prior to the lesson.

One instructional decision she made was to have teachers first work individually before pairs were permitted to share and discuss their solutions with one another. She explained that the move was primarily aimed at generating more responses for the whole class discussion. Additionally, she wanted everyone to have time to think through the problem in order to have a place to begin the problem with their partner. The discussion around this decision was a significant moment for the teachers as they came to understand more about LMD. In their reflections of this task made at the end of the day, twelve of the sixteen teachers mentioned this discussion as an important moment in their learning about the practice. One teacher reflected, “During the ostrich and giraffe task, I like that we were first given the chance to begin the task on our own. I find that when I immediately dive into problems with a partner or in a group, I find myself thinking like them, when that may not be a strategy I would typically use. I feel as though I could do this more in my classroom to get a much wider variety of ways to solve the problem.” Another teacher also noted, “I did enjoy how we had to solve it by ourselves first and then we were allowed to discuss it in pairs. If we had been told to do it in pairs first, I don't know that I would have solved it well at first. When taking my strategy into pairs, it was easy to share it and listen to another’s ideas.”

An analysis of teachers’ reflections at the conclusion of the PLT provided additional evidence that teachers had opportunities to understand the instructional moves and related goals that comprise LMD. Multiple teachers commented how detailing why individual moves were made helped them understand the practice. For example, one teacher commented, “When our instructors decomposed the task it showed that their moves were not random, but were planned out. Once taught I was able to put myself in their shoes and begin to use those instructional moves in my classroom.” Another stated,

> During Phase 1, it was very helpful to me when you took the time to decompose the instructional moves that you made. It allowed me to experience the task as a student and "see" what you were thinking from the teachers’ point of view. This helped me make connections in my head as a teacher so that I would be able to

> implement the same moves in my own classroom while still feeling what the student would feel.

Across their reflections, teachers noted how dissecting the practice of LMD during the PLTs helped them understand the practice better.

Phase 2. In Phase 2, one of the Representing PLTs involved teachers observing a facilitator leading a mathematics discussion with a small group of students after school. During the Decomposing PLT, she detailed her instructional decisions to make the goals for her moves visible to the teachers. The goal of the PLT was to help teachers understand the practice and to relate it to their own classroom practice. As she shared her reasons for various questions posed to students, teachers had a chance to consider how questions were adjusted based on the students' responses. She explained her goal of uncovering the students' thinking and then press for more productive thinking about the problem, not to lead the student to the answer. When reflecting at the end of the session, one participant stated, "I was glad to hear your reasoning. It helped me remember to keep my own questions focused on helping students reach the mathematical goal." Another stated, "Questioning students without leading them can be very challenging. Being able to observe you do it, and understand what you were thinking, as well as ask clarifying questions, helped me when it came to questioning students. Having the opportunity to immediately try this made the experience concrete for me." As in Phase 1, Decomposing PLTs provided contexts for teachers to develop an understanding of LMD.

Though the goal of the Decomposing PLTs was to focus on the practice of LMD, representing the practice with actual students shifted some teachers' focus away from the facilitator's instructional moves to the students' thinking. One teacher noted on her end of the phase reflection, "Many of the students did not do what I initially thought they would and it was nice to see student thinking so that I could anticipate better when I got back to my own classroom." A teacher phrased this realization well by stating that students are "able to build on their own mathematical thinking and they can see what other students are doing and be able to build on what they're doing."

Valuing the practice. As with Representing PLTs, Decomposing PLTs provided opportunities to see the value in LMD. Teachers had a chance to reflect on their levels of comfort with LMD and identify areas where they wanted to grow. Teachers demonstrated that they found the practice useful and worthwhile for teaching in discussions where they tried to coordinate their developing understandings of the instructional decisions with their own practice. For example, one teacher reflected at the conclusion of the MPD how the Decomposing PLTs affected her learning, "I think decomposing moves was the most important aspect of the MPD. It started to break down the steps of teaching a task, not just into a list of things to do, but to the nitty-gritty details so that teachers could understand the thought process that should occur in their own minds as it happens." Another offered,

> I'm generally willing to just jump in and try something but having access to the rationale for the instructional moves helped me visualize how implementing a high demand task would look in my classroom and what I needed to do on my end to make it successful. Without knowing this I would have just presented the task and tried to help students work on it, but I would not have been able to take it to the next level and have students connect ideas and build their own understanding. Honestly I don't know that my teaching practices would have changed much if

> you hadn't decomposed your teacher moves—for me that was where the real learning and connecting of ideas at a pedagogical level happened for me.

By providing opportunities for teachers to understand the practice and consider the ways it supports learning, Decomposing PLTs were contexts for teachers to make sense of and value the practice.

Summary. Across the MPD, Decomposing PLTs provided opportunities for teachers to develop their understanding of the core practice of LMD and fostered a value for the practice. The tasks served as contexts for teachers to relate the learning to their own teaching through embedded opportunities for reflecting on the instructional moves, related goals, and ultimate purpose of LMD in relation to their own learning (Principles 1 & 3). Though experiencing the practice as a learner provided contexts for teachers to develop their understanding, observing the practice with authentic students led to a shift in focus from practice to students' thinking thus moderating the opportunities afforded by the Decomposing PLTs (Principle 4).

Approximating and Enacting PLTs

PLTs approximating or enacting the core practice of LMD provided opportunities for teachers to challenge their existing practice, continue to develop their understanding of it, and continue to foster value of LMD. Whereas Approximating PLTs in Phase 1 involved engaging in the practice with on one another, the PLTs in Phase 2 involved students in an afterschool setting. Enactment PLTs occurred in Phase 3 with the teachers' actual students in their own classrooms.

Challenging understanding. Teachers participated in multiple Approximating PLTs that provided opportunities to explore the practice that they previously experienced in Representing PLTs and focused on in Decomposing PLTs. The Approximating and Enacting PLTs moved to more authentic environments with increasingly individualized support as the MPD progressed. In Phase 1, Approximation PLTs provided opportunities to explore how to lead discussions with one another in a safe environment that did not involve students and there were no stakes. In Phase 2, the PLTs explored the practice with students in after school settings and were thus low-stakes. In Phase 3, the PLTs were constructed around enactments of the practice in their own classrooms where teachers were responsible for the outcome of the practice.

Phase 1. Approximating PLTs provided opportunities for teachers to explore LMD by simulating contexts to use the practice in which they were not responsible for student learning. These opportunities challenged teachers' existing practice of LMD as they reflected on how they typically led discussions when teaching. For instance, in a PLT following the Ostrich and Giraffe Problem, teachers worked with a set of written responses to the problem and discussed how they would lead a discussion based on the mathematical thinking represented in the responses. In discussing their reasoning for using a particular piece of work with an incorrect solution with the whole group, they disagreed on what instructional moves they would make. They recounted the disagreement to the whole group:

Sarah: I was saying, you know, go back and look at the problem and how many animals you know the question talked about.

Nicole: I said I would probably start with, I would just ask why did you start with 20 and 2? And I would expect, based on what I see with your multiplication, there would be some relationship to the 48 and how they broke it up and they saw the multiplication part. At which point, I would follow it up with ‘does your solution match the question or the facts within the question?’ And then, they’d say, ‘yeah there’s 48 legs’ and I would say ‘okay, what else does the problem tell you? There is 17 animals’ and I would kind of stop there, and before I would leave them, I would ask them ‘where are you going to go next and why?’

Fac1: Okay. And why did you like that approach better for you?

Nicole: It makes them do the thinking and realize it, as opposed to just telling them ‘oh, great thinking with the multiplication but, you know, you have 22 animals and you only need 17.’ To say ‘re-read the problem,’ my kids typically will re-read it and say ‘yep, I’m right’ because they read it and they know they’re right until I ask them to point out parts. I try not to lead them too much.

Fac1: Okay, so without being leading Nicole was, their group was really grappling with how we give feedback that’s going to be helpful. And most helpful feedback to the child is going to help them get to the solution without me having to pinpoint here’s where you went wrong in your thinking, now fix this. You don’t have 17 animals. If I point-blank ask them how many animals are you supposed to have? Nicole says, I want them to do that thinking.

Nicole disagreed with Sarah’s move, noting it was too leading and would take away student thinking. The public disagreement allowed teachers to consider issues of leading students or trying to “fix” incorrect solutions with moves that do the thinking for them. In similar exchanges in this whole group discussion, teachers’ existing practice was challenged as they coordinated what they were learning in the MPD with the ways they lead discussions in the classroom.

Another challenge to teachers' existing practice resulted from the anonymous student work samples. In discussions, teachers repeatedly made comments such as, "None of the kids did it like I did," "That one is too confusing for me to explain," and "I wouldn't select that one because it would confuse my entire class." As the teachers looked at the anonymous student work samples they were quick to judge the strategies and quick to label students as a high, low, or gifted kid just by looking the strategy on the paper. As the selecting and sequencing PLT continued, teachers began to appreciate the more unusual strategies as they talked through them and learned to analyze them. They realized that they were too often quick to dismiss strategies and did not often ask the students to explain these strategies. Although the students were not present to ask further questions, through the discussion of the strategies and trying to predict what the students may have been thinking, the teachers came to challenge their existing beliefs on analyzing student work and using strategies that they may not immediately understand in the class discussion. They seemed to gain comfort in trusting students to explain by questioning their reasoning.

Phase 2. Approximating PLTs in Phase 2 provided additional opportunities to challenge practice by providing experiences to lead mathematics discussions with students in an afterschool setting. Though their understanding of the practice had developed through Phase 1 of the MPD, many of the instructional decisions they made when leading discussions with students conflicted with the ultimate purpose of PLTs as a context for teachers to understand how these instructional decisions work together to engage students in considering one another's ideas to advance the mathematical goal of the lesson. Through reflection and feedback from peers and facilitators, teachers had opportunities to recognize both the difficulty of coordinating instructional decisions in real time to achieve the purpose of the practice as well as the disparity between their developing understandings of practice in the MPD and their existing practice.

These PLTs were designed to be low stakes contexts where teachers could focus on their instructional decisions. However, the presence of students moderated a focus on practice by shifting attention from making moves to advance their mathematical goal to students' success on the problem. During Phase 2, approximation shifted to include working with small groups of students. Teachers participated in several approximation PLTs that provided opportunities to explore the focus practices they were experiencing in the representation and decomposition PLTs. Working with actual students afforded participants opportunities to try out questioning techniques and receive feedback from other participants and the facilitator on what they were trying. This task was particularly valued by the participants.

In an approximation task that followed the decomposition of a student interview, teachers were partnered with a small group of students with one teacher assigned to lead a mathematics discussion and another teacher assigned to be in the role of observer. The observers were asked not to interact with the teachers or students but only to record questions and student thinking. The teachers leading the discussions were asked to question student thinking on the same problems the teachers just observed from the interview. The teachers had their list of questions that they generated prior to the interview and the questions that they wrote down during the interview to assist them with questioning the students; however, several teachers expressed as the facilitator monitored the task that this was difficult. Quinn beckoned the facilitator immediately after beginning to work with her group to ask, “What should I ask next? This is really hard!”

As the task progressed the teachers began to feel more comfortable and began to pay attention to the students work and ask specific questions based on what the students were doing. For example, specific questions being asked from various teachers, were: “Where did you get the 50 from; show me how you counted that 4; how can you represent 20 more; is there a way you can draw the rods that are left over; what would the equation look like; can you think of all the different ways the ants could march?”

There were still many instances of leading questions or comments by many participants. For example, several participants were heard saying statements like: “Add 4 to find four more; use a number line to help you see that; put the larger number in your head and then count on; this is subtracting, try that.” Some of these comments came from the observers that were assigned the role of only recording questions asked to elicit student thinking for their partner teacher.

As teachers debriefed when the students left, many teachers commented how difficult it was to ask questions without leading the students to an answer. It was apparent that the teachers were not asking their students questions to understand the thinking behind their strategies. The Approximation PLTS in Phase 2 allowed teachers the opportunity to notice how difficult questioning is when LMD so that thinking is not taken away from the students. This appeared to challenge the teachers in their current practice of leading discussions as they began to question if they were too leading with their students also.

Phase 3. PLTs structured around teachers' enactments of LMD in their classrooms challenged their practice by providing opportunities to reflect and receive feedback on their instructional decisions, identify sites of improvement, and request additional support. When viewing recordings with the facilitators, teachers analyzed their instructional decisions and reflected on how those decisions work together to engage students in considering one another's ideas to advance the mathematical goal of the lesson. For example, Katherine realized that most of her students were not participating in the discussion when viewing the recording with the facilitator. As they discussed, she admitted that she had difficulties in getting students to participate in discussion.

Fac1: Did you try putting a couple strategies up and asking the kids to talk about, so tell me how this kid did it and how this kid did it and how are they alike? How are they different?

Katherine: I didn't do that. And that's something that I need to do. I definitely need to do that.

Fac1: I think that might generate some discussion too.

Katherine: That is probably my weakest area. What to do with, you know, what to ask them and how to get them talking.

Fac1: And at any point in time, like in a discuss [Discuss portion of the lesson], did you have them model it with the frogs or even with themselves? I wonder if even having them come up and say?

Katherine: Yea. I think yea, if I'd had somebody come up and bring the logs and bring them and say now show me what you did with this.

Fac1: Yeah, show me what you did with this and then let's see. Everybody talk about that. What do these yellow frogs represent and these green frogs? These are my frogs that are on the log and I have them talk about it so everybody sees that.

Katherine: The discussion part is the hardest part for me as far as trying to get them, trying to get it to be more student centered. I mean that's just a whole flip for me and it's something that I definitely see the value of.

In this discussion, Katherine's reflection on her teaching challenged a part of her practice. She identified a goal of increasing student participation and received feedback on instructional moves to help achieve her goals.

Another teacher's practice was challenged as she debriefed her lesson. Heather noticed that though she was using instructional moves learned in the MPD to lead a discussion, she was not allowing students to refine their mathematical ideas together. Below are a series of comments she made while viewing her classes discussion:

> He wants to share. I'm like, why don't I let him talk?
>
> I'm like okay, at this part, I wanted them to figure out what they did so one child shared another child's strategy, and I never allowed them to do any because I shared the other strategy.
>
> I wanted him to build off what the answer they got and then deal with thousands, and he kept saying I want the 14. So, I'm telling them this is where I want you to go! Really?
>
> Still like no kids come up and lead the discussion because I'm still like taking the whole time. What am I doing?
>
> And that's where I like, I need help with that because I'm like that's how I am, I'm like okay I have this goal, this goal, this goal. We're going to do this person for this, this person for this but they're not going to go exactly where I want. You know?

In the lesson debrief, Heather reflected on the misalignment between the goals of her moves while enacting and what she wanted them to be. She came to realize that because she had a mathematical goal in mind, she focused on steering the discussion to where she wanted. Her recognition of needing to improve and request for assistance suggests that the PLT challenged her current practice.

At the end of the MPD, teachers reflected over the different PLTs throughout the year. Almost all of them reported that the Enacting PLTs were the most beneficial in learning to lead mathematics discussions. In the words of Danielle, the PLTs "challenged the way that I teach, the way that I think about teaching – in a positive way." Through opportunities for reflection and feedback, Enacting PLTs challenged teachers' practice

Developing understanding. Similar to Decomposing PLTs, Approximating and Enacting PLTs provided opportunities for teachers to develop their understanding of the practice of LMD through analyzing instructional moves and related goals and how they worked together to meet the mathematical goal of the lesson by having students share and refine their mathematical thinking collectively. In Phase 1, Approximating PLTs provided opportunities for teachers to understand LMD by grappling with instructional moves and goals they needed to make as they practiced in a "no stakes" environment. In Phase 2, the PLTs provided opportunities for teachers to further develop their understanding of the practice through working with students in an after school, collectively and individually reflecting, and receiving feedback in a low stakes environment. In Phase 3, Enacting PLTs provided opportunities for teachers to further develop their understanding of the practice through guided reflection and individualized, specific feedback on their enactment in their classroom.

Phase 1. [Posing questions about moves and goals]. In Phase 1, Approximating PLTs allowed teachers to make sense of how one's instructional moves when leading discussions shape its quality. Teachers had a chance to pose questions of one another and facilitators as they made sense of the practice. For example, when discussing how they would select and use students' ideas based on written work in small groups, a particular work sample led to varied opinions. When sharing out, Quinn asked the facilitator where she would select student work for the discussion, providing an opportunity for the facilitator to discuss possible outcomes if it was selected:

Fac1: Would I pick that one for the class [discussion]?

Quinn: Right. How would you do it?

Fac1: Okay. Here's the way I would do it. I would decide on my class, if I have one child in the entire classroom that has done this solution and I don't think that using that solution is going to move my class forward, I wouldn't share this one.

Quinn: Okay.

Fac1: So this is probably one that I would not select.

Quinn: Okay.

Fac1: Even though I see the benefit for it. It depends on your learning target and where your class is. It's kind of the exact same thing where y'all were saying about this one is - it's going to confuse most of your kids. If most of my kids already understand it and they're not making this issue and it's an issue with one child, I wouldn't bring that up. This [pointing to the student work sample in question], as I'm walking around, I might pull that child and say okay this child needs some more work, I want to come back and I want to work with this child. I

> think you don't have to select all of them. I really would think, I press you to think . . . What's my learning target? What do I want the class to get out of this?

In their small groups, teachers struggled with the idea of selecting all responses for discussion in order to not leave a student out. The PLT provided them an opportunity to question possible moves they could take based on their goals and pose questions about what others would do.

[Experiment with moves] Simulating the practice in a no-stakes environment also enabled teachers to experience the outcomes of various instructional decisions to develop their understanding of the practice. In another PLT during Phase 1, teachers were able to simulate leading a discussion with a small group of teachers based on the Buying a Horse Problem. The teachers leading the discussion expressed their difficulty in making eliciting moves but not achieving their goal for the move:

> Fac1: Was it hard to figure out what kinds of questions you needed to ask and why? So let's break that down a little bit. Anyone want to share their thinking?
>
> Nicole: Initially asking questions wasn't that hard, but then they weren't going where I needed them to go, or they kept reinforcing the wrong answer. And I was like, can you show it another way? No, but this is what I did. Okay. And it became a point where it was how do I ask, what do I say next? Because I've tried to ask questions I can think of that would work and ones that I thought worked this morning with our group and they had almost a different way of doing it than anybody in our group really did. It looked different. So it was neither a way that we had up there and I had not anticipated.
>
> Kara: I'll say it's really hard that she kept, in my brain I know it's right and then Sarah was like, Kara's smiling, you're not right and it was hard for me because I didn't know what the questions to ask. She was asking, you know, the questions that you would think of, and I just

wanted to be like check that number again. Like, I wanted to give a hint more than like a guiding question. It was really hard to just sit and watch.

Carol: I think a lot of the way we question, the problem is, is how do I say this, our question to get the answer we want to hear, or is it the answer that we should be hearing, because I think that's the problem for me. It's how can I get them to really say that 20 is the answer. Oooh was I supposed to give that answer? But, I mean…that's hard.

Through experimenting in the PLT, teachers were able to appreciate the complexity of making instructional moves in response to students, consider the importance of wait time when eliciting, and develop their understanding of the practice.

Phase 2. Teachers also had opportunities to develop their understanding in Approximating PLTs by considering how their goals for particular instructional moves when LMD advanced the mathematical goal of the discussion. Prior to interviewing the group of students, teachers were asked to prepare questions to elicit students' thinking on three mathematics problems. Many generated generic questions to invite students to explore the problem rather than specific questions to elicit thinking, such as "Is there another way you could solve that?," "what do you know that can help you?," and "what do you already know?," or "can you show me how you got that?" Some questions asked students to explain how they got their answer or how they drew their representation. A few teachers created more specific questions but they were unable to articulate how those moves would advance the mathematical goal of the discussion. For example, Carol suggested the question, "Why are you putting that number in that specific spot?" The facilitator asked when one might make that move when leading a discussion, to which Carol replied, "After the student placed all of the numbers." Another teacher, Erin, added that the move would be productive after a student placed the first digit, “to see if they know why or if they are just randomly doing it.” Valerie then added, “I think it also depends on what they do. If they put the 0 in the hundredths place that would be a good time to ask that question.” Through the conversation, the facilitators helped teachers to focus on their goals for particular moves in relation to the mathematical goal of the discussion.

Opportunities to develop understandings of LMD were moderated when leading the discussion with actual students, however. Teachers’ focus shifted from the instructional decisions made when leading the discussion to the students’ strategies. Rather than focusing on instructional moves to help students understand the important mathematical ideas, most of the teachers focused exclusively on the students. Though asked to record instructional moves during the discussion, few teachers observing recorded any of the moves. Instead, they were completely distracted by the students, with some leaving their role of observing to help students solve the problem. In debriefing after the students left, Quinn, who was assigned to observe and record her colleague’s moves, stated, “The little one listed up to 10, Brenda and I just sat back watching him, but after he did that he started drawing pictures. He used skinnies and bits to add. And so I leaned over the table and asked why did you do that?” Danielle, also an observer, reported, “I wanted to ask questions - there was this one boy who just randomly grabbed cards…I said what did you do differently here because you were closer to 20 but I couldn't get anywhere. I finally said were you picking your cards for a reason or were you just picking them. He said I was just picking them. So I don't know if there was more behind it—I couldn't get anything out of him." The presence of actual students during this PLT constrained the opportunities to develop understandings of the practice afforded by the others.

Phase 3. Teachers had opportunities to develop their understanding of the practice of LMD during Enacting PLTs through reflecting and receiving feedback on their enactments from others. In addition, knowing their own students allowed them to customize their instructional moves to meet the needs of their students. In one PLT in Phase 3, teachers collectively viewed selected recordings of their class discussions. After viewing the recordings, the group discussed what they observed, posed questions, and requested feedback. After viewing Valerie's recording, Valerie asked the facilitator to comment on her discussion. The facilitator responded:

> You had the kids interacting at 3 different levels with each other's strategies. The first kid's strategy you had another kid explain, so you have one kid interacting. The second one, I love the move you made with the second strategy when you asked them all, is this, can you do this? You know is this a strategy you can use? So you had all of the kids interacting with that one kid's idea. So that was perfect because you've got them all engaged and thinking about that one strategy. But what's really nice about it is you had them all interacting around a pivotal, critical, mathematical concept. Now that's where I want to make sure, why I want to make sure and press you about the learning target a little more is because I want to make sure that's not haphazard.

Valerie reflected and then commented, "Okay, so here's what I need help thinking through. I don't understand learning targets. I know my standards inside and outside. But I have a lego missing that I can't quite stick them together. I know what I want my kids to do. I think that I do fine learning targets but in comparison to the other stuff that I'm doing that is an area of weakness of mine."

The facilitators were able to keep drilling in on learning targets with Valerie to help her narrow down her focus for the discussions so the students would end with more understanding of the big idea of the lesson. Valerie realized, “But considering I’m sometimes vague on my learning target there’s a possibility that they are sometimes vague on their learning targets.”

In their enactments, knowing their own students allowed some teachers to make instructional moves in response to students’ thinking. For example, Quinn made a move to have students explore the task more when leading her discussion. After viewing her recording, she summarized the lesson and another teacher, Beth, posed a question:

> It was a launch-explore-discuss when I started it and then as we, it took them forever to make ½ and so once I found one that made ½ we stopped. And I showed them [by having a he class discuss this child’s solution as evidenced on the recording] and then they explored some more. And then when I found another one, we brought it up, we explored some more. So it was more of a launch-explore-launch-explore-launch-explore.

In the discussion that followed, Quinn noted that often when she monitored students while exploring, she needed to bring her students back together to re-launch the problem or have a mini-discussion and then let the students explore the problem further. Her knowledge of her students assisted her in making instructional decisions that led to engaging students in considering one another’s ideas to advance the mathematical goal of the lesson. Working with her own students allowed her to deepen her understanding of the practice.

Across discussions of the recordings, teachers were able to reflect on their own practice in relation to these enactments. At the end of the MPD meeting, Erin reflected, “I think it was nice to be able to talk through the little things that go wrong in each part and we were able to say, hey you know what I do spend too long on the launch or I do spend too much time with one group during the explore. And we were able to make it better, wherever you were." Collectively viewing and discussing their classroom enactments allowed the teachers to deepen their own understanding of the practice by reflecting on their practice in light of others.

Fostering value. As with Representing and Decomposing PLTs, Approximating and Enacting PLTs provided opportunities for teachers to express their desire and commitment to improve their practice, demonstrating their value of LMD in supporting learning. As these PLTs became increasingly authentic across the MPD phases, teachers were able to take ownership of their learning and responsibility for the outcomes of their enactments.

In Phase 1 PLTs for example, teachers noted in their reflections that experiencing the practice as a learner during the Ostrich and Giraffe Problem motivated them to want to learn to select the right ideas to share in discussions. In their reflections on PLTs in Phase 2, teachers expressed that experimenting with students helped them appreciate the complexity of the practice and realize how they needed to improve. In Phase 3, the reflections from the Enacting PLTs acknowledged the ways they had grown, the parts of the practice they enacted well, and the areas in which they wanted to improve. Throughout the MPD, Approximating and Enacting PLTs were opportunities to come to value the practice of LMD as a means of supporting learning.

Summary. Across the MPD, Approximating PLTs were contexts where teachers could come to value the practice of LMD, develop their understandings of it, and have their existing practice challenged (See Table 4). The opportunities for reflection embedded within them allowed teachers to value the importance of using students' mathematical thinking as a resource for learning (Principle 1). Increasingly authentic settings allowed teachers to develop an understanding of LMD in relation to their mathematical goals for students (Principle 2). As teachers experienced these PLTs, they came to value LMD as a legitimate instructional practice as they negotiated and reconciled their existing practice with what they were learning in the MPD. (Principle 3). These PLTs provided an opportunity for teachers to challenge their understanding of LMD. Enacting LMD with the focus on their own students' mathematical thinking resulted in increased ownership of the practice and enactments that met the needs of their students (Principle 4).

Opportunities to Learn Afforded by Professional Learning Tasks

As summarized in Table 4, my analysis reveals three key opportunities for teachers to learn to lead mathematics discussions afforded by the PLTs. All three types of PLTs created instances where teachers could see the role of mathematical thinking and mathematics discussions in learning and thus value the practice of leading discussions. Decomposing and Approximating PLTs also allowed opportunities for teachers develop and understanding of the techniques of leading discussions. Approximating PLTs had an additional characteristic of challenging teachers' understandings based on their existing teaching practices.

Table 4

Summarization of PLT Findings

	PLTs Representing LMD	PLTs Decomposing LMD	PLTs Approximating and Enacting LMD
Opportunities to learn the Practice of LMD	Fostered a value of the practice of LMD	Fostered a value of the practice of LMD Developed understandings of the practice of LMD	Fostered a value of the practice of LMD Developed understandings of the practice of LMD Challenged understandings of the practice of LMD

Teacher Learning Findings

The initial learning conjecture that guided this study was that as the Representing, Decomposing, and Approximating and Enacting PLTs for learning the core practice of LMD become more closely aligned with the work of teaching, participants' enactments would become more aligned with the purpose of the practice shared in the MPD. Findings from my analysis indicate that teachers learned by becoming a boundary community with the facilitators with two distinct practices of a) using the framework of LMD to negotiate meaning of mathematics teaching and learning, and b) by making their own practice public.

Learning by Becoming a Community of Practice (CoP)

As teachers negotiated meanings of the practice of LMD throughout the MPD, the boundary encounter became a CoP with increased mutuality, aligned enterprise, and shared beliefs among its members. In this way, teachers learned as they participated in the ongoing practices of the community. These boundary practices enabled them to adopt new elements of the practice of LMD as well as make new meanings. In this section, I describe three aspects of teachers' participation within the community throughout each phase of the MPD; their interactions and relationships, their accountability to one another as they negotiated meaning, and the emergence of shared beliefs about mathematics teaching and learning.

Phase 1. Wenger (1998) defined mutual engagement as the phenomenon of people interacting and forming relationships in collective activity. Interactions during the initial phase of the MPD were mainly initiated and sustained by the facilitators or were a facilitator's response to a participant. The few interactions that occurred from participant to participant were mostly to clarify understanding of one another's strategies on the mathematical problems experienced during the PLTs. Their interactions, such as affirming one another's thinking or expressing interest in one another's classroom practice, suggested that they were beginning to develop trusting relationships and value one another's input.

On Day 1 of the summer institute, teachers worked together to make sense of the Ostrich and Giraffe Problem as a whole group. As example of their patterns of interaction early in the MPD, the following excerpt illustrates the ways teachers typically engaged with one another to build trust.

> Valerie: In order to scaffold them [students] through their thinking after already probing them about how many animals do we have, how many do you have? Where is your mistake? So that they don't abandon the

complexity of their answer with the multiplication to give them cubes, the poppy cubes, and have them build small arrays so that they're able to say, "Alright, I can only have 17 animals. Each chunk of cubes would represent 1 animal. How could I manipulate my arrays thereby manipulating my multiplication to be able to make animals that represent the 17?"

Quinn: So, they would make two groups of 4 and that would be their two giraffes. Is what you're saying that they'd make 20 groups of 2? Which would be their . . .

Valerie: That's the way I would start and then I . . .

Quinn: Right.

Valerie: And then you would only make 17 animals.

Quinn: Okay. I was just trying to visualize it in my head but that makes sense now, okay.

During this interaction, Quinn questioned Valerie in order to clarify the strategy she wanted to use with her students, and Valerie trusted the group enough to share her classroom practice. The group did not evaluate her idea and remained neutral as ideas were presented by the others. These ways of engaging with one another in Phase 1 enabled them to collectively recognize and specify aspects of the core practice of focus during the PLTs.

As the teachers negotiated meaning of LMD during the summer institute, they came to appreciate the ways that the facilitators' instructional moves supported them as learners, such as "think time" move to ensure they understood the mathematical problems posed. In turn, their appreciation led to realizations that students might benefit from these moves as well. For example, during an Approximating PLT, teachers provided feedback on sample student solutions to the horse problem when monitoring. One solution that was incorrect prompted a debate about how much information to provide students when monitoring:

Sarah: I was saying, you know, go back and look at the problem and how many animals, you know, the question talked about.

Nicole: I know. I said I would probably start with – I would just ask why did you start with 20 and 2? And, I would expect based on what I see with your multiplication, and there would be some relationship to the 48 and how they broke it up and they saw the multiplication part. At which point, I would follow it up with, "does your solution match the question or the facts within the question?" And then they'd say, "yeah, there's 48 legs," and I would say "okay, what else does the problem tell you? There is 17 animals," and I would kind of stop there, and before I would leave them, I would ask them, "well, where are you going to go next and why?"

Fac1: Okay. And why did you like that approach?

Nicole: It makes them do the thinking and realize it as opposed to just telling them, "oh, great thinking with the multiplication but, you know, you have 22 animals and you only need 17."

Later in the discussion, other participants reflected on their difficulties providing feedback while monitoring. Carol stated, "Where I was stuck is I assume that they're going to get that when I ask that question, but what do I do when they don't?" Beth stated how much she struggled with leading students, "I don't come out and necessarily say, 'Yes, you're right,' but I feel like I ask questions that lead to know that they are correct and the other student is not. So I think that's something that I struggle with." These statements, indicative of how the teachers negotiated meaning during Phase 1, suggest that teachers' primary focus was on making sense of the instructional moves and understanding the launch-explore-discussion lesson structure. Their focus on relating what they were learning in the MPD to their own practice demonstrates a sense of accountability to themselves and their own learning. This phase allowed participants the space to experience learning in discussions and to engage with in a low stakes environment.

As they participated in the PLTs, teachers also began to build a collection of stories, tools, and beliefs that were shared and that evolved throughout the year. They came to the MPD as members of a larger teaching community of practice, with beliefs about mathematics, teaching, and learning, such as a belief that mathematics is abstract, that good mathematicians arrive at solutions quickly, and that learning should not be confusing. In part as a result of these beliefs, some deferred to others that solved the problems quickly, whether they were correct or not. Many did not value concrete or pictorial representations, often viewing such representations as unsophisticated and non-mathematical with comments. When discussing solutions to the Ostrich and Giraffe Problem for example, Nicole explained, “I started out with something apparently pre-kindergarten with my little giraffes with their legs and the little ostriches.” Quinn responded, ″She teaches 5th grade. So, it kind of shows you where her brain goes. As a kindergarten teacher, how a kindergartener would see this is “I’m going to draw this.” And she went more for the numbers aspect because she’s not used to having to draw the picture when she teaches because her 5th graders already have that.” Interactions such as this suggested that the belief that some strategies were more mathematically valid than others was initially shared among the teachers in the boundary encounter.

Another shared belief was the idea that teachers should not use certain strategies in discussions because they would confuse students. When analyzing student work in preparation for a discussion in an early Approximating PLT for example, several teachers made comments like, "I think that would be very hard for many students to access," as they made selections of ideas to share. In the following small group interaction from this PLT, Quinn, Beth, Nicole, and one of the facilitators discussed whether a particular student solution should be selected:

Fac1: Would you go to the red [student solution written in red], I guess is my question?

Quinn: My brain says no.

Fac1: Okay.

Quinn: But . . .

Beth: That one is a little confusing to me.

Quinn: See that's what I was thinking. That's confusing to me. It made sense in her table, when she showed me her table and how it worked, I was like, oh yeah, that makes sense. But I guess that seems too abstract for me. To think about negatives really boggles my mind. I wasn't so good with negatives anyway. So to think about a kid coming up with that and then having to, if I've never taught negatives before, to think that my students would go, I don't understand, what's a negative? Huh? Wait, there's no negatives in numbers. You know and I have, I don't want to go into that whole other ballgame of negatives.

Nicole: Well fourth graders have a hard time with negatives.

By the end of Phase 1, discussions began to include contributions from teachers suggesting the importance of mathematical thinking and the processes involved in arriving at a solution emerged. For example, after experiencing the Buying a Horse

Problem and working in small groups to determine which solution was correct, Erin stated how her group realized the importance of understanding each other's thinking in order to determine which answer made sense. She said, "We said wait a minute, that's different than what we did. And so that was where we could say, well tell me again what you did, tell me slowly, let me think that through, let me compare that to what I did and see which one makes sense mathematically." Valerie later summed a group discussion on how important it is to uncover mathematical thinking by stating, "Ask questions to everybody all the time, no matter what." Statements like these were uncontested by other teachers in whole group discussions and were present across PLTs and in teachers' individual reflections, suggesting that some of the shared beliefs many of the teachers initially held about mathematics, teaching, and learning were aligning to those embodied by the MPD.

This phase allowed participants the space to experience learning through discussions and to engage with the core practice in a low stakes environment. As designed, teachers came to appreciate the need to understand other's mathematical thinking as they engaged with one another during the PLTs. For many, this collective realization began to transfer from themselves to their own students and highlighted the ways in which they might lead mathematics discussions to better understand the processes students use to arrive at their solutions as opposed to a singular focus on the solution.

Phase 2. Whereas interactions in the summer institute were primarily initiated by the facilitator, teachers began to initiate and participate in conversations during the second phase of the MPD, with far more teacher-to-teacher interactions than the initial phase. The relationships marked by trust for, and interest in, one another formed during the summer institute enabled the group to begin to consider and build upon one other's ideas as they shared stories of their classrooms, reflected upon their teaching during the PLTs, and collectively analyzed their own practice of leading discussions with a small group of students after school.

In the following example, teachers were generating questions that they planned to pose to elicit the students' thinking when solving problem: *Sam has the numbers 2, 7, 5, 0, 6, and 8. How should he arrange the numbers in the spaces (_ _ _ + _ _ _) to get the largest sum?* Carol shared a question that she would ask to understand a student's placement of a number in a certain spot on the following mathematics problem:

Carol: Why are you, just why are you putting that number in that specific spot?

Fac1: Why are you putting that number in that spot? So let me ask you, when do you ask that question?

Carol: After they've placed the specific numbers wherever they want to put them.

Fac1: Okay so you would wait until after they…

Carol: I would.

Fac1: Placed all of them. Would anybody do it a different way?

Carol: Once they've . . . ask them for each number?

Erin: Well, just the first digit to see if they know why or if they're just randomly doing it.

Valerie: I think it also depends on what they do. If they put the 0 in the hundredths place that would be a good time to ask that question.

Carol: [pause] Yeah, that's true.

Though unclear if Carol had considered why she might ask that question, Erin responded by proposing a time she might pose her question and offered a reason by explaining what information would be gleamed from the response. Valerie followed up with another way to think about asking Carol's question. Carol paused to consider their ideas and then agreed that both would work. In this example, Erin and Valerie built upon Carol's original response with less support from the facilitator than Phase 1. The interaction illustrates a deeper trust that was developing in the community as teachers were engaging with one another's ideas about teaching.

As the teachers negotiated meaning of LMD in Phase 2, they became increasingly accountable to themselves as teachers. Though teachers came to appreciate the role of mathematical thinking and value process over product for themselves and students in Phase 1, their realizations did not transfer to their work with small groups of students in the Approximating PLTs. As a result, teachers' focus on the instructional moves and portion of the lesson from Phase 1 shifted to the goals for particular moves and the overall purpose of LMD.

In the summer institute, teachers recognized that it was difficult to allow others to struggle through mathematical problems as they worked with their colleagues and reflected on teaching, often commenting that their questions led to telling. During a Representing PLT in Phase 2, teachers were asked to observe one of the facilitators lead a discussion with a group of students. Yet as the facilitator worked with the students, many of the teachers began questioning the students from their seats or even working with students on the carpet. During an Approximating PLT, pairs of teachers were to practice leading a small group discussion with students, where one teacher practiced and the other observed. Though asked not to interject questions or interfere with the lesson, most were unable to stay in the role of observer and assisted the practicing teacher by asking questions. When debriefing, Danielle explained:

> I wanted to ask questions - there was this one boy who just randomly grabbed cards . . . I said what did you do differently here because you were closer to 20, but I couldn't get anywhere. I finally said were you picking your cards for a reason or were you just picking them. He said I was just picking them. So I don't know if there was more behind it—I couldn't get anything out of him.

In the same debrief, Carol reflected on the difficulty of probing moves stating, "I'm trying to pull, pull, pull—and I wish I had more of that from them and not me." These examples illustrate a shift in accountability from themselves as learners to themselves as teachers.

During Phase 2, teachers continued to use their shared beliefs about mathematics, teaching, and learning to engage with one another. Teachers expressed that mathematics is abstract and that students should not be confused while learning. For example, Valerie recounted a student in her class using manipulatives to solve a problem stating, "I had a little guy that was doing complex thinking until the manipulatives were given. I wanted to rip those away." Another teacher was upset when a student used a counting strategy but then moved to concrete manipulatives to add. She said, "The little one listed up to 10.

Ann and I just sat back watching him . . . After he did that he started drawing pictures. He used skinnies and bits to add!"

To these shared meanings, teachers came to believe that that correct answers do not always equate to an understanding of the concept during Phase 2 that became evident in one of the Approximating PLTs. In the PLT, teachers led discussions with a small group of students using the Close to 100 problem, where students drew cards and then made two-digit numbers to try to get as close to 100 as possible. One teacher stated during the debrief, "One thing I noticed was the child who had the lowest score was the lowest thinker of the two I was observing [lowest score is the winner in this problem]. It brought home the point, just because you win the game…he just happened to get good cards every time." Another teacher affirmed this observation about that student by stating, "I asked him to tell me what you're doing to get you close every time, and he couldn't explain anything. It was just like it was random. I'm close, but I don't know why." This realization suggested an increased value of inviting and probing moves, and their concern with leading discussions without lowering the cognitive demand for the students persisted.

Phase 3. By final phase of the MPD, teachers initiated most interactions and drove the discussion. As teachers worked to enact LMD in their classrooms, they discussed with one another areas where they felt they needed the most support and feedback. Through sharing their successes and struggles when leading discussions, it was evident that the teachers had developed trusting relationships and truly formed a community of practice.

The following illustrates the kind of mutual engagement that emerged in the MPD. In one of the final meetings, Kara asked Heather how she records and keeps track of all of the different strategies that she shared her students were using as they solved mathematical problems in class.

Heather: I have all intentions of writing notes. It's on my wall and there is nothing up on it. I have this math work wall and I saw it at a Grad School conference and I was like I'm doing that. And you put all of the student work on paper clips and they each have a name and they can like flip through like 3 really good student works in the past. Not all of it, but the good ones and they can flip through. Oh, that was a strategy. My intention, the work hasn't gotten up there yet. But that's one way. I don't personally like record anything paper chart wise up there because we're constantly doing the problems.

Valerie: Do you have any type of interactive notebook that your kids use for anything?

Heather: I mean they have a math journal, yeah.

Valerie: We did something that you might really like from what you said.

Heather: Yea, mmm hmmm.

Valerie: We did these little flippy things that they're just fourths of paper that I had my kids kind of glue one on top of another but then they were able to record the strategies that they liked that we did in class and kind of so you know at the top it's the problem and there are like 4 different strategies that you could, that they could kind of flip through. And then you could have them record how they're the same. Like along the side.

Heather: Yea. We could definitely use that I think.

Erin: I used to do, I used to record if it was something that the kids all thought was a really cool strategy that I felt like it applied to a lot of things we were doing, not something obscure or something that wasn't very efficient, then we would name it after that kid.

Kara: Oh my gosh! I was just about to say that.

Valerie: Yes, do that.

Erin: Yes, so it might be Julie's strategy or it might be . . . You write Julie's strategy on the board and kind of put it up there. So we were worrying about multiplication, division, whatever every day. Different kids would be like well I'm going to try Julie's strategy today and you know so and it gave that kid they were the really, the expert on it.

Heather: Right.

Erin: Yes, so if you got stuck and you were trying to use, yea then Julie could come and help you with it.

Kara: We did that too and it's really like, it was a strategy that I would have told them anyways you know. It was just one that he kind of, one of my students just brought up before I mentioned it and so we named it after him and if you ask everybody knows what like, remember that day exactly what it was.

This episode illustrates two shifts in the ways the teachers were engaging with one another from Phase 2. First, the entire interaction was teacher initiated and needed no direction from the facilitator. Second, teachers brainstormed collectively and offered suggestions to help Heather with her concern. In the exchange, it is evident that the teachers had developed a deep trust as they sought assistance and provided support to one another.

As the teachers negotiated meaning of the instructional moves and purpose of LMD in this final phase of the MPD, they became responsible to students in practice, which fostered change in their practice. For example, Heather reflected how routinely having mathematics discussions gave her and her students a structure that helped her empower her students, stating, "I would have never given my kids this much power. Not that they have power, but as much trust as I do have for them to. I know that if I go over there that's what they're going to be working on. They're not going to be playing. Whereas in the past classes I would never have done that. But it's that structure, they're used to seeing it on the very first day and they knew it was going to be part of their day every day." Several teachers had adapted the launch-explore-discuss structure for setting up and leading discussions based on their enactments, reflecting a deep understanding of the practice. For example, Quinn discussed how she re-launched for exploration after a short discussion with her students based on their needs:

> Their job today is to make half with a geoboard. So they were supposed to come up with all of these ways to make half and figure it out. And so it turned, it was a launch-explore-discuss when I started it, and then as we—it took them forever to make half and so once I found one that made half we stopped. And I showed them and then they explored some more. And then when I found another one, we brought it up, we explored some more. So it was more of a launch-explore-launch-explore-launch-explore.

Another teacher, Beth, sought validation for adapting the framework during her lesson, "So, is it okay or is it best practice I guess if you kind of stop and have that discussion like in between sometimes?" These examples illustrate the ways that teachers were negotiating meaning of LMD, no longer just in relation to the framework shared in the MPD, but based on their experiences in the classroom. By adapting to the needs of their students when leading discussions, the community was generating new meaning for the practice.

By the end of the MPD, there was little evidence of the beliefs about mathematics, teaching, and learning that the teachers brought with them to the MPD, indicating that the CoP had evolved and created a new shared repertoire for engaging with one another around the practice of LMD. In addition to the new belief from Phase 2, a new belief that teachers should not lead students to the answer emerged. As the teachers viewed and reflected on videos of their classroom discussions during Phase 3, most realized that they were far too leading and expressed that they wanted additional coaching and support in for the remainder of the MPD. For example, Sarah commented, "They probably had it because I had showed them . . . They could've figured it out." She later followed by stating:

> I think too I say too much. Like I don't give enough wait time for them to, for them to, like I guess I can see their wheels spinning and when I see them start to struggle instead of letting them work through it, I rescue.

Another teacher, Heather, was outraged with herself as she noticed how leading her moves were and how much of the discussion she took away from her students: "Still, like no kids come up and lead the discussion because I'm still taking the whole thing. What am I doing?" Toward the end of her video recording, she reflected, "My weakness is my discussion by far . . . what do I do, I start leading? I start leading to get them to this learning target." Carol also noticed she was taking away opportunities for student thinking, stating, "Instead of me telling them, I really have to listen to how I phrase things or what I'm [saying], because I know I can be very leading." As Beth viewed the video recording of her lesson, she noticed a pattern of when the instances of when leading occurred. She observed, "If one person has a really good idea, I want to like jump in, and I need to just, I need to stop." Kelly struggled not providing enough wait time because she felt the need to keep the discussion moving in order to maintain engagement in the problem as she stated, “I know I don’t want to ask too quickly; however, I want to keep them motivated.” These representative comments highlighted reflect that teachers had refined their beliefs about teaching and learning away from a teacher-led model of discussions. The new experiences of teaching generated by enacting the practice allowed them to create the shared belief that students do not learn conceptually by being led but require time to use what they know to solve problems.

Summary. Teachers learned to enact the core practice of LMD by becoming a community of practice. They grew to mutually engage and negotiate meaning about LMD in the MPD. Increased mutuality led to changes in the nature and substance of their interactions and led to the formation of trusting relationships with one another. Interactions shifted from being initiated and led by the facilitators to being driven by teachers with one another. Relationships changed from valuing and supporting one another to trusted colleagues who would offer critical feedback about their teaching.

As they negotiated meaning of the practice of LMD over time, teachers’ engagement increasingly aligned toward a joint enterprise of using the framework shared in the MPD to support their students in learning mathematics conceptually. Their initial focus on the instructional moves and segments of a lesson shifted over time to making sense of how to orchestrate a mathematics discussion with students in increasingly complex environments. They came to share freely about their enactments and adapted the practice during their instruction in their classrooms based on their students. Whereas teachers were initially accountable to themselves and each other as learners, they became increasingly responsible for one another's learning about LMD and ultimately for their students' learning.

The emergent community was supported by a shared repertoire of beliefs about mathematics, teaching, and learning as they engaged with one another in the MPD. Initially, many of their beliefs were grounded in their own experiences as a teacher and learner. Over time, they developed shared beliefs about student engagement and their role in supporting student learning.

Learning by Developing Boundary Practices

A defining characteristic of a community is practice—a common, shared way of engaging in a joint enterprise. As a community emerged from the initial boundary encounter, teachers formed two distinct boundary practices that enabled their learning of the core practice. First, the community developed a practice of using the LMD framework to negotiate meaning of mathematics teaching and learning. Supporting this was another practice of publicly sharing one's instructional decisions, successes, and challenges of leading discussions. In this section, I describe shifts in teachers' uses of the framework for LMD and their engagement with one another's practice of leading discussions throughout each phase of the MPD to illustrate how these two boundary practices enabled teachers' learning in the emerging community.

Negotiating meaning with the framework. Over time, teachers came to use the framework for LMD to negotiate meanings of the core practice. The framework helped teachers identify instructional moves used when leading discussions as well as reflecting on how such moves might support students' learning. As the PLTs became more authentic and increasingly complex, teachers came to understand the purpose of leading mathematics discussions in new ways.

Phase 1. Initially, teachers used the framework to make sense of the instructional moves of the practice. During Phase 1, they identified developed shared meanings for the various moves and how one might organize them with the framework, such as probing and pressing moves and launching and monitoring. As a boundary object, they used the framework to recognize and negotiate their existing understandings of leading discussions as they engaged with it in the PLTs. As described in the findings section for PLTs for Core Practices, the Representing PLTs provided common examples of the core practice and its value in learning. In the Decomposing and Approximating PLTs, teachers used the framework to identify, reflect upon, and experiment with the components of the practice.

Nicole: Last year when I was doing these tasks . . . I haven't anticipated as much at all, in some cases hardly. I've experienced firsthand what it's like trying to come up with the right questions and especially with selecting and sequencing portion. Trying to do that on the fly without having thought about it that was one of my 'ah-ha moments' when I read this. I was like, oh you have to order those before, I never thought about the order I would put them in beforehand. I could do that first . . . kind of taking some of those strategies that I would use, that I would and just to come up with what and kind of have an idea in my head of which one I would do in which order and then being able to mold it to my class as I go. But when it says rather than have the mathematical discussions consist of separate presentations, which is what mine turned into, and different ways to solve the problem the goal is to have student presentations build on each other and develop that powerful mathematical idea. Mine sometimes turned the glorified show and tell.

Valerie: This would be a time to go in your planning stage and go okay this is my skill, this is how my kids are going to progress through this thought pattern, let's see if I can find a student that falls here, a student that falls here, a student that falls here . . .

Quinn: Well I think that would be a way to anticipate. That makes perfect sense how you could anticipate their response.

Carol: That's the difference from last year to this year. Things have now started to make sense.

In this discussion, Nicole used the framework to relate her existing meanings of anticipating, selecting and sequencing student responses to what she experienced as a learner as she read the article that explained these components of the framework. By having experienced how the horse problem discussion was led, she was able to contrast that to how she formerly led discussions in with her classroom to better understand these components of the framework from the article. Across Phase 1, teachers began to develop a practice of using the framework to make sense of what the practice of LMD was in relation to their own understanding.

Phase 2. In Phase 2, teachers began to use the framework to not only understand what the practice of LMD was but also to understand the goals for instructional moves and the ultimate purposes of mathematics discussions. This shift in how teachers engaged with the framework—from using it to identify and relate the techniques of leading discussions to their own teaching to reflecting on the reasoning behind the techniques—suggests an evolution of the boundary practice as a way of engaging in the community meaningfully. In the PLTs, the framework structured the ways that teachers engaged in discussions about the mechanics of LMD as well as the underlying purpose of the practice For example, teachers began to see that probing and pressing moves could be made to advance the mathematical goal of a lesson. Rather than simply posing questions to elicit students' thinking, these instructional moves could work towards their ultimate learning goal. As a result, their questions moved from generic to mathematically specific, such as, "Is 2 rows of 6 the same as 6 rows of 2?" or "Explain how you got the number 38, you know so that they could say whether they counted by ones or if they actually counted by tens and ones." Throughout Phase 2, teachers used the framework to generate new meanings of why one might make particular instructional moves or lead mathematics discussions in their teaching.

Phase 3. Near the end of the MPD, the boundary practice of using the framework enabled not only learning within the community but also enactments in their classrooms. The common language and shared understandings about the reasons for leading discussions supported teachers in leading discussions in their classrooms and generating new meanings of mathematics teaching and learning in their teaching. Whereas earlier they used the framework in prescriptive ways, teachers came to use it generatively as they made new meanings of leading discussions in their classrooms with their students.

For example, Quinn discussed how she re-launched a lesson after noticing many of the same misconceptions of fractions in one of her lessons. Because her purpose for the coming discussion was for students to meet her learning goal, she decided to collectively address the issue because their exploration would not be productive. Her decision to come together, clarify, and re-launch the problem suggests that her use of the framework in the MPD to learn about the practice supported her in enacting it in her teaching. In the discussion around this episode, her description of the lesson as "launch-explore-launch-explore-discuss" indicated that her learning about the practice became generative as she enacted it in ways that were in response to her students.

Negotiating meaning by making practice public. Similar to the development of the boundary practice of using the framework, teachers developed a practice of discussing their own teaching in the MPD as a way to relate their learning to their practice in classrooms. This practice of making one's own teaching public for discussion complemented the shifts in use of the framework from identifying instructional moves of the core practice to using it to enact and generate new meanings in their teaching.

Phase 1. As teachers experienced the core practice as learners in the PLTs, they began to relate these experiences by sharing about their own teaching. They described how they lead discussions within their own classrooms as they negotiated meanings of instructional moves from the framework. In sharing their histories as teachers and existing beliefs about mathematics teaching and learning through stories of their classroom practice, they mutually engaged in negotiating meaning of the framework in the MPD.

Some teachers' contributions to whole group discussions in Phase 1 reflected contrasts between what they were learning in the MPD and how they actually lead discussions in their own classrooms. For instance, Beth commented on questions she posed of students, saying "I feel like I ask questions that lead to know that they are correct and the other student is not. So I think that's something that I struggle with." Others identified similarities to their teaching, such as Brenda's description of her strategy for posing questions in her classroom. She explained, "I taught my kindergarten class at the beginning of the year, I would start telling them that when I ask you questions about what you're doing, it's not because you're right or wrong. I want to understand your thinking and we want everyone else to understand your thinking too." Some teachers even shared their personal difficulties with leading discussions. Nicole described the challenge of selecting student solutions for a classroom discussion in her teaching:

> Last year when I was doing these tasks quite a bit, I look at this and I think, I've been through like trying at the moment. I haven't anticipated as much at all, in some cases hardly just like the general, oh yeah, that's what they'll say and then move on. And I've experienced firsthand what it's like trying to come up with the right questions and especially with selecting and sequencing portion. Trying to do that on the fly without having thought about it that was one of my ah-ha moments when I read this. I was like oh you have to order those before, even if I've anticipated I never thought about the order I would put them in beforehand.

As teachers shared their own practice, they were able to relate the meanings of LMD in their teaching community with those represented by the framework in the MPD. The emerging boundary practice of using the framework to negotiate meaning of LMD within the MPD and the practice of sharing about one's own teaching enabled teachers to relate their learning to their practice in their own classrooms.

Phase 2. PLTs in Phase 2 created opportunities for teachers to publicly reflect on their own teaching practice, not just in recounting anecdotes from their classrooms, but by engaging in the practice with students and collective reflection. In conversations during the MPD sessions, teachers shared stories of their experiences in leading discussions since the summer institute and also focused on the instructional moves they made during the Approximating PLTs. As they collectively reflected on their practice, their focus shifted from the techniques of LMD to their goals for various moves and ultimately what they hoped to gain by having mathematics discussions in their classrooms.

Most discussions of their own practice centered around the difficulty of moves that elicit student thinking. During one of the Approximating PLTs for example, Quinn almost panicked when she did not know what to ask as she experimented with leading a discussion with a student. As a facilitator walked past, she reached out and stated that she couldn't think of a single question to ask. Later in the debrief, Quinn admitted her difficulty with thinking of questions to pose to uncover student thinking. She expressed how her mind went blank and she felt unsure of her questioning skills to provoke student reasoning. Other teachers sympathized with Quinn and shared their own difficulties in trying to elicit students' thinking. Beth added her own experience, commenting, "I asked him twice, but I still don't understand what he was thinking!" As a boundary community emerged from the MPD, shifts towards more trusting relationships coincided with a refined boundary practice of sharing one's own practice of LMD with the community that included the reasoning behind instructional moves.

Phase 3. PLTs in Phase 3 provided opportunities for teachers to further discuss their own teaching publicly. These tasks involved recording their classroom teaching and individually discussing their practice with the facilitators, receiving coaching and in-the-moment feedback during actual classroom lessons, and sharing their successes and struggles with one another in the afterschool meetings. During this phase, teachers began to identify aspects of their own practice of LMD where they felt competent, as well as where they wanted to grow. As the community's practice of making their own teaching available for collective learning evolved, teachers not only focused on their own teaching, but offered feedback to one another.

In a number of discussions during Phase 3 PLTs, teachers noted where they were satisfied with their practice and identified areas for improvement. For example, Katherine reflected that helping students make sense of and build on other students' ideas was an area where she wanted to grow. She stated, "That is probably my weakest area. What to ask them and how to get them talking." When sharing her episodes from her lesson, Carol commented,

> I can do the launch in a matter of minutes and have them do it like I was talking to you the other day, and I was just really comfortable with it . . . (My students are) good as far as you know trying to understand what they have to do and then that's the hard part . . . I'm trying to pull, pull, pull and I wish I had more of that from them and not me.

As they shared how they led discussions with students with one another, some teachers invited feedback and suggestions for improving their practice of leading discussions. For instance, Danielle commented, "My discussion is still, I feel my weakest point. I try to have a really clear goal when I go in of what I want to talk about, but that's not always what I get from the student work." The group validated her struggle and collectively discussed strategies for making instructional moves aligned with her learning goal. In her lesson debrief, Heather noted something similar, stating, "that's where I need help, because I have this goal… but they're not going to go exactly where I want. You know?" By developing a practice of discussing their teaching with the community, teachers identified aspects of their practice that they wanted to improve and sought feedback from others to continue to refine their practice.

Summary. As a boundary community emerged, teachers developed a boundary practice of making their teaching available for collective discussion. In Phase 1, this practice assisted teachers in relating their experiences in the MPD to their own classrooms and assisted in negotiating meaning of the instructional moves represented by the framework. In Phase 2, discussing their teaching with one another assisted them in moving beyond the techniques of discussions to reflect on their own goals for particular instructional moves and their overall purpose of mathematics discussions. By the end of the MPD, teachers noted problematic aspects of their practice, sought support and guidance for improvement, and provided feedback to one another as they shared their teaching with one another.

Teacher Enactment Changes during the Professional Development

Results from the analyses of lesson ratings using the IQA indicate that the overall academic rigor of teachers' lessons and quality of classroom discourse improved significantly as teachers participated in the professional development focused on the core practice of LMD (see Appendix E for all scores). As displayed in Table 5, changes in the Academic Rigor composite score (ARC) indicate improvements in the degree to which the observed lessons engaged students in active mathematical reasoning and developing understanding (Boston, 2012). A Friedman Test to detect differences in means among the ARC scores for Time 1 (M=11.540, SD=2.436), for Time 2 (M=13.000, SD=3.055), and for Time 3 (M=14.850, SD=2.267) was significant χ^2 (2, N = 13) = 9.702, p = 0.008, indicating a difference of at least one mean score at the three time points. The increase in means suggests an increase in the presence of students' mathematical thinking in classroom discussions.

Table 5

Friedman Tests for Selected Dimensions of the IQA

	Time 1			Time 2			Time 3				
	N	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*	$\chi(2)$	*p*
ARC	13	11.540	2.436	13	13.000	3.055	13	14.850	2.267	9.702	.008**
ATC	13	9.540	2.259	13	10.620	3.453	13	12.690	3.591	6.125	.047**
AR3	13	2.000	1.000	13	2.770	0.927	13	2.770	0.725	7.517	0.023**
ARQ	13	2.000	0.577	13	2.770	0.725	13	2.770	0.725	10.138	0.006**
AT2	13	2.080	0.760	13	2.000	0.816	13	2.310	0.855	3.100	0.212

Note: ** denotes significance at the 0.10 level.

A similar analysis of the Accountable Talk composite score (ATC) also shows that the discourse in teachers' classrooms became more accountable to the learning community and knowledge and rigor of mathematics (Boston, 2012). A Friedman Test to detect differences in means among the ATC scores for time 1 (M = 9.540, SD = 2.259), for time 2 (M = 10.620, SD = 3.453), and for time 3 (M = 12.690, SD = 3.591) was significant χ^2 (2, N = 13) = 9.702, p = 0.008), indicating a difference of at least one mean score at the three time points. The increase in means suggests an increase in the quality of classroom discourse in the observed lessons. To further understand changes in teachers' enactments of LMD, I detail changes in two dimensions of the Academic Rigor (AR) rubrics and one dimension of the Accountable Talk (AT) rubrics most closely measuring in the practice of LMD in the next sections.

Presence of Students' Mathematical Thinking in Discussions

To enact the practice of LMD, classroom discussions must involve students' mathematical thinking. With the exception of two lessons at Time 1, all lessons across the three time periods indicated that students' mathematical thinking was made public for discussion. Across time, the quality of teachers' uses of student thinking improved, with the majority of students making their thinking public for discussion. A Friedman Test of AR3 scores, along with an examination of the means at Time 1–3, indicated that there were marked increases in the use of students' mathematical thinking in classroom discussions (χ^2 (2) = 7.517, p = 0.023; M=2.000, SD=1.000; M = 2.700, SD = 0.927; M = 2.700, SD = 0.725). Follow-up pairwise comparisons using a Wilcoxon test and controlling for the Type I errors across theses comparisons at the .10 level using a Bonferroni correction procedure summarized in Table 6 indicated a significant increase in AR3 scores after Phase 1 (p = 0.007) and across the MPD (p = 0.003). In this section, I characterize the ways the presence of students' mathematical thinking occurred over time during lesson observations at the end of each phase of the MPD.

Table 6

Pairwise Comparisons of the Presence of Students' Mathematical Thinking

	Differences				
	M	*SD*	*S*	*p*	corrected *p*
Time 1 to Time 2	-0.154	0.689	11.000	0.047	0.007**
Time 2 to Time 3	0.000	0.707	0.000	0.500	0.167
Time 1 to Time 3	0.769	0.927	14.000	0.008	0.003**

Note: *N*=13, ** denotes significance at the 0.10 level

After Phase 1. Two lessons were scored 0 after the summer institute. In these lessons, there was no discussion of students' mathematical ideas. In each, the lesson ended after students explored a problem with no opportunities for students to discuss as a class. Seven lessons received a score of 2. In these lessons, teachers provided opportunities for students to share their work or explanations of their strategies by leading a discussion, but the discussion did not include evidence of students providing justifications or explanations. For example, in one lesson, three students presented their solutions to the class by showing their work and reading off what they did to solve the problem. None of the students explained or discussed their solutions.

The remaining four lessons scored a 3, indicating that students shared their ideas in the whole class discussion and offered incomplete explanations or justifications. In one of these lessons for example, a group shared their solution and explained anytime you add a double number, the answer is always even. A student went to the board and provided examples of problems her group used to prove the statement; however, the students were unable to provide a justification for why this worked as they shared their examples. They were able to find evidence that it worked and make generalizable statements for adding two even numbers to get an even number, but their explanations were incomplete as to why this worked for all even numbers.

After Phase 2. In observations after Phase 2, one lesson received a score of 1 because the mathematics task the teacher selected did not have the potential to engage students in rigorous thinking. In the discussion, all student responses were brief answers to convergent questions, such as “How many blue M&M’s do you have?” The AR3 ratings of four teachers’ lessons remained a 2 in this observation, with no evidence of the presence of students explaining or justifying, similar to the example above. Three teachers’ lessons remained at a 3, with two additional teachers’ lessons improving from a 2. In these lessons, teachers’ practice of LMD provided opportunities for students to explain their thinking. In addition, some of these level 3 lessons included evidence of students making incomplete connections across the ideas shared. For example, one class was able to compare sets and determine which set had more or less than the others to order them from the largest number to the smallest; however, the ordering of the numbers caused confusion when there were equal sets to be compared. Many students continued to use strategies for when the sets were not equal and could not represent how the sets were equal.

Three teachers’ lessons were rated as 4, one of which improved from a score of 0 from the first observation. In these lessons, students explained their strategies for solving problems, provided explanations, and made connections to the mathematical ideas of the lesson. For example, one class arrived at multiple answers when adding decimal numbers. Students compared their answers and used a variety or representations to determine how to properly add decimals. The students connected their representations to conceptually understand the size of 0.085 and 0.3 to make sense of their sum.

After Phase 3. Five teachers' lessons scored a 2 on their final observation, with two of them improving from the previous observation. In these lessons, students shared and described their strategies with no explanations or connections. Six teachers' lessons were rated a 3, with four having previously scored 0's or 2's. In these lessons, students shared strategies and explained their reasoning, many without prompting from the teacher. The remaining two lessons scored a 4. In both, students consistently provided explanations for why their strategies worked and made connections among the strategies and the mathematical concept of the lesson.

Summary. Across the MPD, teachers' lessons increasingly provided opportunities for students to make their mathematical thinking public for discussion. By the end of the MPD, eight of the thirteen lessons rated 3 or 4 on the AR3 rubrics, indicating that many students shared their work, explained their reasoning, and attempted to connect their thinking to the underlying mathematical ideas of the lesson. While the remaining five lessons were rated as 2, there was still evidence that students shared their mathematical thinking during a discussion.

Presence of Probing and Pressing Moves

ARQ was used to measure the presence of probing and pressing moves in the observed lessons. With the exception of two lessons at Time1, all lessons across the three time periods indicated there was at least one attempt to ask academically relevant questions. Across time, the quality of teacher questioning improved as more academically relevant questions were asked during the lesson.

A Friedman Test of ARQ scores, along with an examination of the means at Time 1 – 3, indicated that there were marked increases in the use of probing and pressing moves (χ^2 (2) = 10.138, p = 0.006; M = 2.000, SD = 0.577; M = 2.770, SD = 0.927, M = 2.770, SD = 0.725. Follow-up pairwise comparisons using a Wilcoxon test and controlling for the Type I errors across theses comparisons at the .10 level using a Bonferroni correction procedure summarized in Table 7 indicated a significant increase in ARQ scores after Phase 1 (p = 0.003) and across the MPD (p = 0.006). In this section, I characterize the ways the presence of probing and pressing questions occurred over time during lesson observations at the end of each phase of the MPD.

Table 7

Pairwise Comparisons of the Presence of Probing and Pressing Moves

	Differences				
	M	*SD*	*S*	*p*	Bonferroni corrected *p*
Time 1 to Time 2	0.769	0.832	14.000	0.008	0.003**
Time 2 to Time 3	0.000	0.707	0.000	0.500	0.167
Time 1 to Time 3	0.769	0.927	14.000	0.017	0.006**

Note. *N*=13, ** denotes significance at the 0.10 level

After Phase 1. Two lessons were scored 1 after the summer Institute. In these lessons, there was limited evidence of probing or pressing moves by the teacher. The students provided brief explanations and the teachers did not probe the students for further clarification or press them for justifications. For example, in one lesson a student had been given a number card with the number 10 and a picture of ten moons listed in two rows of five. The student was asked what number he had and if he could build it on the ten frame. The child said ten and then placed ten counters on the ten frame. The teacher asked how many do you have there and the child counted the ten moons on the card by one and said, "ten!". Then the teacher said is that ten on the ten frame there and the child said yes. The teacher walked away without probing for understanding to know if the child understood the quantity of ten or if the child merely matched the picture on the card he was given.

Nine teachers received a score of 2 indicating that overall the teachers were probing and pressing students to explain their thinking after participating in Phase 1 of the MPD. For example, in one lesson a third-grade class was discussing the problem given to them to prove if the sum of two double numbers is always even. The first group agreed and proved by showing they tested all numbers 1+1 through 20+20 and they were satisfied that the answer was yes. The teacher pressed them by asking if they tried any large numbers to be sure it always worked. They did not, but another student eagerly jumped in to try 100+100 and the students all yelled out, “it’s even!” Then the teacher pressed them to try numbers that don’t end in 0 or 1. They tested 36+36 and then tried two odd numbers that were doubles, like 21+21. This classroom discussion illustrated how many teachers were beginning to probe and press students to explain their thinking, but they had not pressed for the academically relevant mathematics of the problems; for example, the *WHY* is the sum always even when two doubles are added.

The remaining two lessons scored a level 3 as they had both asked at least two probing questions that explored mathematical relationships important to their lessons. For example, a fourth-grade class was solving a set of multiplicative comparison problems for the first time. As the teacher monitored the students as they were exploring the problems, she noticed that the students had no difficulty understanding the concept that these were all multiplication problems and why multiplication was needed; however, she realized that their methods for multiplication were not sophisticated. During the classroom discussion, she pressed the students to look at the methods for multiplication used to help students understand multiplication. She pressed the students to look at four strategies and determine why they work. Strategy one was a model of 12 boxes drawn with 6 dots in each box. Strategy two was repeated addition using 12 + 12 + 12 + 12 + 12 + 12. Strategy three was 12 x 6 decomposed as 10 x 6 +2 x 6. The last strategy was 6 x 6 + 6 x 6. Each of the strategies was discussed and students were pressed to explain the mathematical relationships as to why these strategies worked.

After Phase 2. In observations after Phase 2, only five lessons remained at a level 2 indicating that overall the teachers were asking their students to explain their reasoning. For example, six teachers' lessons were rated a level 3 indicating that these teachers were following up with probing moves in the discussion that prompted students to explain their thinking. In each of these lessons at least two probing questions were asked that explored mathematical relationships important to their lessons.

Two lessons scored a level 4 as these teachers consistently asked academically relevant questions that provided opportunities for students to explain their reasoning and to describe the important mathematical ideas in the lesson. For example, in one second grade classroom, the teacher had her students solve the horse problem. The teacher consistently asked the students why and to explain their own reasoning as well as the reasoning of the others sharing their thinking. She pressed them to continually look at the problem to determine why a number was added or taken away to keep them in the context of buying and selling. The teacher was strategic in having two groups share their strategies and then asked the class to discover connections between the two solutions. One solution computed in order of the story and the other group combined the sums and the differences to determine the solution. This lesson was a great example of a level 4 response because the teacher probed students as they were working in their small groups, she pressed the class to understand each separate solution as it was presented, and then she pressed the class to look for connections between the two strategies and why they both gave the same answer. She kept her questioning on the relevant mathematical relationships students needed in order to understand this problem and to deepen their content knowledge.

After Phase 3. Five lessons scored at a level 2 at the end of the MPD. Three of the remaining teachers dropped one level and three improved one level. Two of the three teachers that dropped a level both missed most of Phase 3 as they were on maternity leave for the majority of the classroom coaching and support portion of the MPD.

Summary. Across the MPD, teachers' lessons provided opportunities for students to clarify and justify their thinking through teacher utilization of probing and pressing moves. By the end of the MPD, the teachers consistently asked students to explain their reasoning as they were LMD in their classrooms. Attending to the important mathematical ideas in the lesson as they were probing students proved to be more difficult to implement into practice. Eight of the 13 teachers were able to do this by the end of the MPD at a level 3 or 4; however, only two were consistent throughout their entire lesson at keeping the questioning related to the important mathematical ideas of the lesson.

Presence of Inviting and Orienting Moves

Talking with others is fundamental to learning. Accountable Talk is talk that responds to and further develops what other members of the group have said and it sharpens student thinking by reinforcing the ability to use and create knowledge (Boston, 2012). In order to enact the practice of LMD, classroom discussions must provide opportunities for students to engage with one another. These opportunities present themselves as teachers use inviting and orienting moves during the discussion. Across time, the quality of inviting and orienting moves did not improve. A Friedman Test of AT2 scores, along with an examination of the means at Time 1-3, indicated that there were no significant increases in the use of inviting and orienting moves $\chi^2 (2) = 3.100$, p = 0.212; M = 2.080, SD = 0.760; M = 2.000, SD = 0.816, M = 2.310, SD = 0.855. Follow-up pairwise comparisons using a Wilcoxon test and controlling for the Type I errors across theses comparisons at the .10 level using a Bonferroni correction procedure summarized in Table 8 indicated a significant increase in AT2 scores after Phase 1 (p = 0.167) and across the MPD (p = 0.094). In this section, I characterize the ways the presence of inviting and orienting moves occurred over time during lesson observations at the end of each phase of the MPD with specific examples of teachers who represent those who made noteworthy improvements in using the moves, some improvements, and no improvements, and one who used the moves less over time.

Table 8

Pairwise Comparisons of the Presence of Inviting and Orienting Moves

	Differences				
	M	*SD*	*S*	*p*	corrected *p*
Time 1 to Time 2	-0.154	0.689	-1.000	0.500	0.167
Time 2 to Time 3	0.000	0.707	5.000	0.156	0.052
Time 1 to Time 3	0.769	0.927	3.000	0.281	0.094

Note. *N*=13, ** denotes significance at the 0.10 level

Marked improvements. Though not statistically significant as a group, 5 teachers made improvements in their use of inviting and orienting moves (see Appendix E). Danielle, for example, came to use these moves when leading mathematics discussions as the MPD progressed. After Phase 1, Danielle's lesson scored a 0 because she did not invite students to contribute to the discussion at all. After Phase 2, Danielle scored a level 3 as she was consistent asking students to explain one another's thinking. She stopped students during parts of their solution sharing and invited other students to explain what that student said or did or why that worked. She also supported students in connecting ideas to build coherence in the discussion as she showed two strategies and asked the class to now discuss how these two strategies got the same answer but used completely different numbers.

She continued to score a level 3 in her final lesson observed as there were two clear instances where she used inviting and orienting moves to have students to relate to one another's ideas. In one, she asked a student to add on to another student's thinking. In another, she identified a connection between two different solutions, oriented the students to the connection, and asked them to describe how the ideas were related.

Some improvements. Three teachers showed some improvements in using inviting and orienting moves on their observations over time. Quinn, for example, improved from a level 1 to a level 2 on the observation at Time 3 by making one strong effort to connect the speakers' contributions to each other. During her first two observations, Quinn did not invite other students into the discussion. The discussion was primarily led by her although she did call a couple students to share their solutions; however, she did not invite other students to engage with those students or their solutions, nor did she support students in connecting ideas or building coherence in the discussion. By her final observation, Quinn did make some improvements by supporting students to connect ideas during the discussion when she asked students to compare two different fraction representations. This questioning supported a link between the two ideas and built some coherence in the discussion. Unfortunately, there was only one attempt during the entire lesson to link ideas or re-voice student contributions.

No improvements. Eight teachers out of the 13 made no improvements on using inviting and orienting moves during their classroom discussions. One teacher remained at a level 1 throughout the observations as she never made an effort to link or re-voice student contributions. The majority of her kindergarten students answered questions chorally or one number/word responses without any discussion of their thinking. Five teachers remained a level 2 throughout the classroom observations. While these teachers did re-voice student responses or have students repeat responses, they did not attempt to show how the responses were related to one another. Two teachers remained at a level 3 throughout all observations. Both of these teachers demonstrated at least two instances of connecting student ideas to one another and showing how the ideas were related. Neither of these two teachers consistently connected throughout the lessons. There were missed opportunities for linking that could have deepened knowledge for the entire class.

Decline. One teacher, Sarah, actually went backwards on using inviting and orienting moves in her classroom. Sarah scored a level 3 at Time1, a level 2 at Time2, and a level 1 at Time3. Sarah expressed difficulty with LMD with kindergarten students. Her previous instruction was more direct and leading because she thought they would not be able to do a lot of the thinking on their own. She asked the facilitators to observe her first lesson and debrief with her afterwards. Facilitator coaching could have prompted some of the linking that occurred during this initial observation. As the students were working, the teacher questioned the facilitators on some of the solutions. Part of the discussion that occurred with the teacher was brought out during the discussion with the students. On subsequent lessons, the teacher had no support and there was some re-voicing and prompting for agreement by the students; however, there were no instances of connections between student contributions.

Summary. While many teachers' lessons did include explicit linking moves where teachers asked students to add-on to another student or asked whether they agreed or disagreed with statements made by other students, the majority of the teachers did not show how the ideas were connected or lead the students into a deeper conversation or press for further mathematical understanding for the class. In several instances, students were building upon each other's ideas without explicitly identifying how the ideas were linked for a coherent discussion. Many of the teachers that were connecting student ideas and did score a level 3 by the end of the MPD never reached a consistent level using these teacher-linking moves. Their classroom discussions had at least two attempts to connect ideas for a coherent discussion; however, the entire discussion was still fragmented into students sharing isolated strategies or solutions without a rich discussion of their thinking to build knowledge for the group. The teachers expressed having difficulty bringing the discussion back to the important mathematical ideas for the lesson. This may have been a deterrent with the teachers appropriately using inviting and orienting moves in the classroom. The teachers were able to invite students into the conversation, but then the discussion was often left hanging without explicit connections made between the ideas or to the important mathematical concept of the lesson.

Summary of Teachers' Enactments of the Practice

Teachers' enactments of LMD in their classrooms improved as they participated in the MPD. The results showed an increased presence of student mathematical thinking in discussions and an increased use of probing and pressing moves as teachers practiced LMD with their own students in their own classrooms. However, using inviting and orienting moves that were connected to the important mathematical idea of the lesson proved to be more difficult for the teachers.

CHAPTER V

DISCUSSION

This study explored the ways in which teachers learned and enacted the core practice of leading mathematics discussions (LMD) in professional development. Through an examination of teachers' participation in practice-focused PLTs during a yearlong MPD and their enactments of the practice in their classrooms, the study documented teachers' learning throughout the different stages of the MPD. The final chapter of this dissertation is organized into three sections. First, I answer the three specific research questions that guided the study and integrate the findings to describe the ways teachers in professional development learned to enact an instructional practice. Next, I situate the results of this study within the existing research on professional learning tasks and mathematics teacher learning. Finally, I conclude with implications for research, policy, and mathematics teacher educators and recommendations for future research.

Teacher Learning of Instructional Practice

As a part of a larger design experiment, this study investigated the ways teachers learned an instructional practice in MPD through an examination of their participation in professional learning tasks focused on the core practice of LMD and their ongoing enactments of the practice in their teaching. Three research questions helped me study how teachers learned this instructional practice. The first research question addressed the design of the task. The second addressed how teachers participated in those tasks. And the third addressed what enactment of the practice looked like in participating teachers' classrooms.

Practice-focused Professional Learning Tasks

Teachers learned the core practice of LMD as they engaged in practice-focused PLTs. In this study, design principles for the PLTs contributed to the overall understanding of this core practice. The PLTs provided teachers with opportunities to foster value of the core practice, develop understanding of the core practice, and challenge existing belief of the core practice of LMD. The design principles and opportunities afforded by the PLTs both were key parts that contributed to the teachers' learning and enactment of LMD.

Four principles guided the design of the PLTs for the MPD. First, PLTs represent, decompose, or approximate/enact the core practice of LMD and embed opportunities for reflection. Second, PLTs have both pedagogical and mathematical goals and aim to support teachers in learning the core practice as well as develop conceptual understandings of mathematics through attention to reasoning. Third, PLTs attend to teachers' prior knowledge of mathematics, instruction, and students. Last, PLTs are based upon artifacts of mathematical thinking that vary in authenticity. This study suggests that the design principles for these PLTs supported learning of the core practice of LMD through increasingly authentic settings. Increasingly authentic settings allowed teachers to develop an understanding of LMD in relation to their mathematical goals for students. Teachers came to value LMD as a legitimate instructional practice as they negotiated and reconciled their existing practice with what they were learning in the MPD. Enacting LMD with ones' own students resulted in increased ownership of the practice and enactments that met the needs of their students.

Across the MPD, Representing PLTs fostered a value of the core practice of LMD by providing opportunities to recognize the central role of mathematical thinking in teaching and learning. Embedded opportunities for reflection allowed teachers to recognize the ways understanding others' mathematical thinking supported their own learning. Decomposing PLTs provided opportunities for teachers to develop their understanding of the core practice of LMD and fostered a value for the practice. The tasks served as contexts for teachers to relate the learning to their own teaching through embedded opportunities for reflecting on the instructional moves, related goals, and ultimate purpose of LMD in relation to their own learning. Approximating PLTs were contexts where teachers could come to value the practice of LMD, develop their understandings of it, and have their existing practice challenged. The opportunities for reflection embedded within them allowed teachers to recognize the importance of using students' mathematical thinking as a resource for learning.

Learning from Participating in Professional Learning Tasks

Teachers learned the practice of LMD with boundary practices that emerged as they became a community of practice. Over time, the teachers formed a community characterized by increasingly trustful relationships, aligned enterprise, and shared beliefs among its members. As this community emerged, they first developed a boundary practice of using the framework for LMD, and then by de-privatizing their teaching and making it available for public reflection. Both of these practices enabled the teachers' learning and enactment of LMD.

In this study, teachers learned to enact the core practice of LMD by becoming a community of practice through the use of practice-focused PLTs. Teacher interactions shifted from being initiated and led by the facilitators to being driven by teachers with one another. By the end of MPD, the teachers provided constructive feedback to one another instead of just being supportive colleagues of one another. As they negotiated meaning of the practice of LMD over time, their initial focus on the instructional moves and segments of a lesson shifted to making sense of how to orchestrate a mathematics discussion with students in increasingly complex environments. They became increasingly responsible for one another's learning about LMD and ultimately responsible for their own students' learning. As they engaged with one another in the PLTs, they developed shared beliefs about student engagement and their role in supporting student learning.

In this study, teachers formed the boundary practice of using the framework for LMD. This framework guided their participation during the practice-focused PLTs. Findings suggest teachers used the framework as a way of engaging in the community in meaningful ways. They evolved from using it to identify and relate techniques of LMD to their own teaching to reflecting on the reasoning behind the techniques. By the end of the MPD, the common language and shared understandings about the reasons for leading discussions supported teachers in leading discussions in their classrooms and generating new meanings of mathematics teaching and learning in their teaching. The framework helped teachers identify instructional moves used when leading discussions as well as reflecting on how such moves might support students' learning. As the PLTs became more authentic and increasingly complex, teachers came to understand the purpose of leading mathematics discussions in new ways.

Teachers also formed the boundary practice of making their teaching public and available for reflection and increased learning of the entire community. As teachers made their teaching available for collective discussion in this study, they noted problematic aspects of their practice, sought support and guidance for improvement, and provided feedback to one another as they shared their teaching with one another. The boundary practice of de-privatizing their teaching supported teachers to learn the practice of LMD in this study.

Leading Discussions in Classrooms

In this study, teachers had overall improvements in their enactment of the core practice of LMD with their students. Their enactments had an increased presence of student mathematical thinking (SMT) and an increased use of pressing and probing moves as teachers practiced LMD with their own students in their own classrooms. Some teachers had a strong presence of inviting and orienting moves; however, the use of the inviting and orienting moves overtime did not improve.

Results of this study suggest that participation in practice-focused PLTs promoted SMT. Across the MPD, teachers' lessons increasingly provided opportunities for students to make their mathematical thinking public for discussion. By the end of the MPD, eight of the thirteen lessons rated 3 or 4 on the AR3 rubrics, indicating that many students shared their work, explained their reasoning, and attempted to connect their thinking to the underlying mathematical ideas of the lesson. There was evidence that all teachers asked students to share their mathematical thinking during a discussion.

This study also suggests participation in practice-focused PLTs promoted increased use of pressing and probing moves by participating teachers. Across the MPD, teachers' lessons provided opportunities for students to clarify and justify their thinking through teacher utilization of probing and pressing moves. By the end of the MPD, the teachers consistently asked students to explain their reasoning as they were LMD in their classrooms. However, attending to the important mathematical ideas in the lesson as teachers were probing students proved to be more difficult to implement into practice.

The study suggests that participation in practice-focused PLTs did not improve the use of inviting and orienting moves by teachers. While many teachers' lessons did include explicit linking moves where teachers asked students to add-on to another student or asked whether they agreed or disagreed with statements made by other students, the majority of the teachers did not show how the ideas were connected or led the students into a deeper conversation or pressed for further mathematical understanding for the class. The teachers expressed having difficulty bringing the discussion back to the important mathematical ideas for the lesson. This may have been a deterrent with the teachers appropriately using inviting and orienting moves in the classroom. The teachers were able to invite students into the conversation, but then the discussion was often left hanging without explicit connections made between the ideas or to the important mathematical concept of the lesson.

Together, these findings suggest that practice-focused PLTs can support teachers in learning and enacting new instructional practice. Four principles guided the design of the PLTs for the MPD. First, PLTs represent, decompose, or approximate/enact the core practice of LMD and embed opportunities for reflection. Second, PLTs have both pedagogical and mathematical goals and aim to support teachers in learning the core practice as well as develop conceptual understandings of mathematics through attention to reasoning. Third, PLTs attend to teachers' prior knowledge of mathematics, instruction, and students. Last, PLTs are based upon artifacts of mathematical thinking that vary in authenticity. In what follows, I situate my findings in the literature.

Discussion

Designing Professional Learning Tasks

Design Principle 1. Similar to Grossman, Compton, and colleagues (2009), results of this study indicate that teachers benefit from MPD that is designed on the pedagogies of practice, just as novice teachers in teacher education programs do. This study adds to the findings of Ghousseini and Herbst's (2014) study that targeted aspects focused on during the three pedagogies of practice when working with novice teachers using Hammerness and colleagues' (2005) Framework for Learning to Teach. Ghousseini and Herbst (2014) found that opportunities inside representations of practice were limited to developing visions of the practice and content knowledge. Decompositions afforded opportunities to work on the vision, the repertoire of tools for that practice, and dispositions for teaching. They did not allow for development of content knowledge. Approximations allowed teachers opportunities to work on all four aspects. My findings add to this work and expand it to accommodate non-novice learners. For experienced teachers, representations of practice helped them value the core practice of LMD. Decompositions of practice allowed for opportunities to value and develop understanding of the core practice. Approximations and Enactments of Practice allowed for development of value for the practice, deepening understanding of the practice, and it also challenged existing beliefs of the core practice. Challenging beliefs was critical to shift teachers to implementing this practice in their actual classrooms. Results of this study suggest MPD should be designed considering the pedagogies of practice to allow non-novice learners opportunities to come to understand core practices that are new to them.

Design Principles 2 & 3. The PLTs were designed using reflection as a tool to build on and respect prior knowledge as well as to support attention to reasoning and thinking during the core practice of LMD. Similar to Stein et al. (2000) teachers in this study critically reflected on their own practice. For Stein and colleagues, the cases were important tools that served as mediating devices between teachers' reflection on their own practice and their ability to interpret their own practice as instances of more general patterns of task enactment. In this study, the teacher's own participation and video analysis became the way they de-privatized their own practice to make it public for reflection and feedback. The PLTs required reflection to what they were learning and how they attended to reasoning as classroom teachers. Situating their reflection in their practice allowed teachers to value aspects of their own practice and critically reflect on improvements for LMD.

Similarly, Borko and colleagues (2008) explored the use of video for the reflective professional development of mathematics teachers. My findings support that participants' reflective and detailed conversations around video-observed lessons aided development of reflective practitioners and heightened the instructional practice of the participants. In this study, many teachers commented on the reflective nature of their own discussions and promoted a desire to correct this practice so they attend to student reasoning. Results of this study suggest that PLTs should attend to teachers' prior knowledge and aim to support teachers in attention to reasoning.

Design Principle 4. PLTs in this study moved from less to more complex settings over the course of the MPD. In contrast to Ghousseini's (2009) findings that found rehearsals helped the participants shift focus to student thinking. Enactment of practice in increasingly authentic settings does not always allow an opportunity to learn the intended skill. Results of this study indicate that teachers did not benefit from the early introduction of students in Phase 2. The teachers in this study became too fixated on the strategies that the students were using during a PLT that they failed to take up the researchers' goal about the practice they were to be observing as it was being represented. This was not the case in Phase 1 when the teachers were the actual learners or they were studying the core practice using anonymous students. However, in Phase 2 when teachers were using students from the school to study the core practice, the moves and goals to be studied took a back seat to the students. This study suggests a need to be cautious when introducing students into the MPD too soon in order for teachers to focus on the intended core practice being learned.

I recommend people develop PLTs for MPD using the four design principles used in this study. I would not recommend changes to the first three design principles. However, the fourth design principle could be altered to be *PLTs are based upon artifacts of mathematical thinking that vary in authenticity and are introduced in increasingly complex settings, using known students as the most complex setting.*

Practicing Teachers Learning Core Practice

Research suggests that teachers learn through active engagement in processes of observation, discussion, enactment of practice, and reflection (Garet et al., 2001; Lieberman, 1996; Loucks-Horsley et al., 1998). In this section, I situate my findings of how the teachers in this project came to learn the core practice of LMD in the literature suggesting how teachers learn. At the end of this section, I discuss my learning conjectures and propose any necessary revisions.

Teachers in this study watched their own video recorded lessons and analyzed them with facilitators. Similar to Seidel and colleagues (2005), this study showed that teacher observation and reflection on their own teaching supported their learning and promoted change in the core practice within their own classrooms. This study also supports similar findings from Borko et al. (2006) that showed teachers benefitted from engagement in increasingly reflective and collective conversations around videos of one another's classrooms. In these studies, and my current study, teachers pointed to the watching and analysis of their video clips as the most valuable aspect of the MPD experience. My study adds further support to the use of teachers watching video recordings of their own classroom lessons as powerful learning opportunities.

Discussion has been shown to be important to teacher learning, but collective discussion of one's own practice is critical. Similar to Sowder and colleagues (1998), discussions of the practice being learned were prominent over the course of this study. In their study, teachers attributed the change in their observations showing an increase in probing student thinking and allowing students to share responses to their participation in discussions of these practices in the MPD. Similarly, Barnett (1998) found that the discussion of written cases became a model of instruction for participating teachers in his study. My study adds to these findings by highlighting the significance of the formation of the boundary practice of making one's own teaching public (Ghousseini & Sleep, 2011) and available for reflection to increase learning of the entire community. As teachers made their practice public for collective discussion in this study, they had deeper discussions where they are able to give and receive critical feedback on their practice. The formation of the community of practice (CoP) allowed teachers to be comfortable giving and receiving feedback from one another.

The use of a framework for LMD led to a shared understanding and common language of the core practice as teachers enacted this practice into their own classrooms. Similar to findings from Sztajn and colleagues (2014), a framework was beneficial to help teachers initially understand the practice being learned. During the LTBI project, teachers used the shared repertoire of tools to negotiate meaning as they deepened their understanding of the framework. In contrast, however, my study offered a framework for the practice of leading discussions. Findings suggest that there may be similarities between teachers' learning of frameworks of children's thinking and those for instructional practice.

The initial learning conjectures that guided this study were;

- Conjecture One: *PLTs that highlight the role of leading mathematics discussions that elicit and use students' mathematical thinking and that use authentic artifacts of mathematics learning support teachers in learning to lead mathematics discussions over time.*
- Conjecture Two: *Participation in PLTs that represent, decompose, approximate the core practice of leading discussions and reflect on the role of leading mathematics discussions will lead to the development of a professional learning community focused on improving instructional practice over time.*
- Conjecture Three: *Teachers' enactments of the core practice of leading mathematics discussions will increasingly incorporate instructional moves discussed in the MPD over time.*

My study provided evidence in support of the first learning conjecture and the additional understandings of the relationship between PLTs and teacher learning. This learning conjecture can be revised to include the promotion of de-privatizing teacher practice as well as the role of varying authenticity of records of student thinking. There was solid evidence for learning conjecture two. PLTs that represent, decompose, and approximate the core practice of LMD are beneficial for supporting practicing teachers in learning to lead discussions in their own classroom practice.

After completion of the retrospective analysis of the first cycle of this design study, there was partial evidence in support of the third conjecture as it occurred with most, but not all, instructional moves for LMD. I recommend amending the third learning conjecture to address strengthening teacher inviting and orienting moves while learning the practices in MPD. Teachers in this study expressed difficulty knowing what mathematical ideas to follow or highlight during the discussion in part because their learning goals were underspecified. I propose the revision of the third conjecture to state: *With an emphasis on articulation of clear mathematical purposes for classroom tasks, teachers' enactments of the core practice of leading mathematics discussions will increasingly incorporate instructional moves discussed in the MPD over time.*

Outcomes of MPD

Research has established effective professional development as: (a) intensive, ongoing, and connected to practice; (b) focused on student learning and addressing the teaching of specific curriculum content; (c) aligned with school improvement priorities and goals; and (d) structured to build strong working relationships among teachers (Darling-Hammond et al., 2009). Similarly, Desimone (2009) identified five critical features for effective professional development. She stated that effective professional development should include a strong content focus, active learning strategies, coherence, duration, and collective participation. In this section, I situate my findings for the outcomes of this MPD in the literature of effective MPD.

Similar to Tzur (2001), this study found support for active learning in professional development. In both studies, teachers participated in reflective activities that increased awareness of their views that influence how they foster mathematics learning of their students. While the active learning strategies occurred in many forms in this MPD, all of the forms were focused on student learning. Similar to Kazemi and Franke's (2004) findings that closely observing students can increase teacher belief in the mathematical competence of their students, this study's focus on student mathematical thinking increased their value for the core practice of LMD which led to increased use of the practice in their own classrooms.

Active learning refers to teachers taking an active role in their learning (Darling-Hammond, 2008; Desimone, 2009; Elmore, 2002; Garet et al., 2001; Guskey, 2002; Guskey & Yoon, 2009). Santagata and Angelici (2010) promoted the use of video discussions to actively engage their participants in their own learning. Sherin and van Es (2009) also used video excerpts from teachers' classrooms to promote active learning. My study used video in a similar way that supported the notion that engagement is key to teacher learning.

MPD should be aligned to school improvement priorities and goals so that it is coherent. Similar to Garet and colleagues (2001), this study also had a positive influence on change of teacher practice as evidenced by the lesson observations over the course of the MPD. Both studies had MPD that was connected to professional experiences, aligned to the standards, supported by administration, and fostered professional communication. My findings provide additional evidence that coherence is a critical factor for a positive outcome and an effective MPD.

Research has shown that MPD needs to be structured to build strong relationships with teachers from the same school in order for collective participation to occur. A community of practice is a group of people who have a shared domain of interest and who learn how to do it better through regular interactions (Lave & Wenger, 1991; Wenger, 1998). In the Learning Trajectory Based Instruction project (LTBI), Sztajn et al. (2013), found that teachers developed a sense of belonging as they worked together and learned from one another. Buczynski and Hansen (2010) reported the more teachers from a single school were involved in the MPD cohort, the stronger the impact will be for that school. Kazemi and Franke (2004) found that teachers developed their own CoP and collectively defined what it meant to teach mathematics at their school. Similar to the findings from Kazemi and Franke (2004), the teachers in this study became a strong CoP that collectively defined what it meant to LMD at their respective schools. Results from this study add to the findings on effective MPD by demonstrating how all of the elements of effective MPD can occur within one design and get positive outcome data.

Implications and Further Research

The goal of this research was to investigate teacher learning of the core practice of LMD. The results indicate that practice-focused PLTs designed to be increasingly connected to classroom practice can offer teachers' opportunities to better understand students' mathematical thinking and lead discussions that are built on the reasoning of their students. This section highlights characteristics of the PLTs that assisted teachers in enacting the core practice of LMD, as well as recommendations of additional factors for consideration to extend the potential of practice-focused PLTs.

The findings from this study has implications for practitioners who design and implement MPD on the core practice of LMD. Representing, Decomposing, and Approximating and Enactment PLTs proved to be productive spaces for teachers to learn to enact core practices for LMD. It is important to remember to attend to what teachers know and design or select tasks that build upon their prior knowledge. PLT designers and MPD facilitators should consider explicit ways of connecting teachers' current practice with what they are learning. Though such PLTs should be authentic for teachers and closely related to the practice of teaching, careful attention to the degree of authenticity of student artifacts may prevent teachers from shifting focus on the core practice to student thinking.

For researchers, findings from the study add to the current literature by providing an initial glimpse of how the work around core practices might be adapted for non-novice learners. MPD should be designed with the explicit assumption that teachers already enact elements of core practices and provide opportunities to problematize and refine their existing meanings of them. Further research is needed to identity principles of practice-focused PLTs and the role they play in learning for experienced teachers. While the field is currently exploring sets of Instructional Activities (Lampert et al., 2013; McDonald et al., 2013) as a "container" to learn to enact the core practices, this study showed that another approach is viable for practicing teachers. Mathematics teacher education researchers would benefit from studies exploring the similarities and differences in the ways that teachers learn and enact new practices in less and more authentic settings.

With respect to teacher learning, the study's revised learning conjectures require empirical investigation. This study provided additional understandings of the relationship between PLTs and teacher learning. *Teachers' participation in sequences of PLTs that represent, decompose, and approximate/enact practice and provide opportunities to de-privatize teacher practice, as well as vary the authenticity of records of student thinking over time, promotes the formation of boundary practices aligned with the core practice of LMD.* The revision of learning conjecture one includes the promotion of de-privatizing teacher practice so it becomes public for critical reflection. Challenging teacher beliefs of how they previously LMD were critical in shifting teachers' implementation of this practice in their actual classrooms. Future research needs to be conducted that determines how these pedagogies of practice occur as PLTs for non-novice learners. Do they need to occur sequentially or are all three embedded in into each PLT? Research also needs to investigate the appropriate grain size for MPD for practicing teachers verses teacher candidates.

Another implication from this study involves policy. With the addition of the mathematical practices to standards, policy-makers have an opportunity to further promote student reasoning by emphasizing ways of engaging in mathematics in addition to content learning. Significant time and resources are required for teachers to enhance their instruction if the integrity of practice-oriented standards is to be maintained. Without substantial commitment, changes in standards will likely not yield results that are intended by policy. A significant investment in developing capacity is required for increased expectations (Elmore, 2002). Without resources, we may not be able to “keep the promise for these standards” for children.

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APPENDIX A

PLT DESCRIPTION DATA MAP

Activity	Goal	Rationale
Ostrich & Giraffe Problem: "Exploring Launch-Explore-Discuss" Structure	Teachers experience the components of a task (launch- explore-discuss) and reflect on how strategies used build mathematical understanding.	Leaders use the problem to assess what strategies participants use and to refer back to it in future sessions to break down components.
Buying a Horse Problem: The Five Practices for Orchestrating Productive Mathematics Discussions	Teachers review understanding of the 5 practices for orchestrating mathematical tasks. They breakdown teacher decisions and discuss how choices open or close different opportunities for student understanding.	To emphasize the importance of high-demand tasks. Revisit the five practices for classroom discourse and help teachers become more familiar and comfortable with them.
Student Interview Task: Working with Students II Task 1: Largest Sum (Addition) Task 2: Base 10 Bags (Subtraction) Task 3: Marching Ants (Multiplication Arrays)	Teachers observe a live student interview. The questions are broken down in between tasks to discuss the rationale for making specific discourse moves. Then teachers have an opportunity to work with a student as they solve the same three tasks to try out the discourse moves.	To help teachers understand how questions help clarify thinking and uncover mathematical understanding. To provide opportunities to practice the art of mathematical questioning and use of discourse moves observed.
After the Fact Coaching Task: One-on-one debriefing of video clips.	To allow teachers time to reflect on decisions made during their lesson and to give them specific, actionable feedback.	Teachers need time to reflect on their practice and receive feedback that helps them refine their practice.
In-the-Moment Coaching Task: Observing teachers teaching lessons.	To provide "in the moment" feedback and support to teachers in an authentic situation.	Professional development tasks completed in authentic learning environments increase the likelihood that teachers will implement these types of tasks within their own classrooms.

APPENDIX B

PLT ANALYSIS CHART

PLT Sequence 1: Ostrich & Giraffe Problem

	Characteristics	Task Type
Experiencing the Task – Modeling of lesson Launch-Explore-Discuss Structure	Experiencing Task Launch Experiencing Explore: Participants solved task, individually first then discussed as partners/groups Experiencing Discussion	Representing Practice
Breaking Down the Experience	Anticipation Selecting & Sequencing of Solutions Connections between solutions Highlighting discourse moves by facilitator and purpose of moves.	Decomposing Practice
Analysis of Student Work	Attending to the Mathematics – Moving to sophisticated strategies (How do you help students as you are monitoring?)	Approximating Practice

PLT Sequence 2: Buying a Horse Problem

	Characteristics	Task Type
Experiencing the Task – Modeling of Lesson	Experiencing Task Launch Experiencing Explore: Participants solved task, individually first then discussed as partners/groups Experiencing Discussion	Representing Practice
Breaking Down the Experience	Anticipating student responses and misconceptions. Highlighting discourse moves by facilitator and purpose of moves.	Decomposing Practice

	Characteristics	Task Type
Analysis of Student Work	Selecting and Sequencing student work samples of the horse problem to be shared in a classroom.	Approximating Practice

PLT Sequence 3: Student Interview Task

	Characteristics	Task Type
Experiencing the Task – Modeling Lesson	Working out the Task Participants write possible questions to ask students on these three tasks Modeling of Discourse Moves (Facilitator interviewed student in front of the group to model pathways afforded by certain types of questions asked.)	Representing Practice
Breaking Down the Experience	Group discusses questions asks and how and why they differed from the questions planned. Highlighting discourse moves and purpose for the moves.	Decomposing Practice
Model with Students	Participants practice questioning a student on the same tasks (One student was assigned to two participants so one questions as the other records.)	Approximating Practice

PLT Sequence 4: After the Fact Coaching

	Characteristics	Task Type
Enact the Lesson	Teachers taught and recorded the lesson with their classroom of students.	Enacting Practice
Coaching Feedback	Teachers sat with one or both project facilitators while watching their lesson and pausing to discuss and reflect on decisions made. Teachers received feedback on the components of their lesson (LED) and the discourse moves made during their lesson.	Embedded Reflection and Feedback

APPENDIX C

INDIVIDUAL TEACHER IQA SCORES

	AR3			ARQ			AT2		
Teacher	Time 1	Time 2	Time 3	Time 1	Time 2	Time 3	Time 1	Time 2	Time 3
Victoria	2	2	2	2	3	2	2	2	2
Katherine	2	2	3	1	3	4	2	1	3
Brenda	0	3	2	1	3	3	-1	1	1
Beth	3	4	4	3	4	4	3	3	3
Heather	2	3	3	2	3	3	3	3	3
Quinn	2	2	3	2	2	3	1	1	2
Valerie	2	2	2	2	2	2	2	1	2
Danielle	0	4	3	2	4	3	-1	3	3
Sarah	2	1	2	2	2	2	3	2	1
Nicole	3	4	4	3	3	3	3	3	4
Carol	3	3	3	2	2	3	2	2	2
Kelly	2	3	2	2	3	2	2	2	2
Kara	3	3	3	2	2	2	2	2	2