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**Using the area between two item response functions to index
differential item functioning: A generalized approach**

Penny, James Archie, Ph.D.

The University of North Carolina at Greensboro, 1994

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USING THE AREA BETWEEN TWO ITEM RESPONSE FUNCTIONS
TO INDEX DIFFERENTIAL ITEM FUNCTIONING:
A GENERALIZED APPROACH

by

James Archie Penny

A Dissertation Submitted to
the Faculty of the Graduate School at
The University of North Carolina at Greensboro
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The area between two item response functions was used to construct two indicators of differential item functioning. The first indicator was designed from the integral of the difference of the two response functions across six standard deviations of ability. The second index was designed from the integral of the square of the difference of the two response functions across the same range of ability. Both indices were developed using the three-parameter item response theory model such that they subsume the two-and one-parameter models as special cases; and both were designed to be sensitive to uniform and non-uniform differential item functioning.

The standard errors of the item response theory parameters were used to estimate the standard error of each integral. The ratio of the integral to the estimate of its standard error was shown to be normally distributed using Monte-Carlo data. Hence, both indices provide a multivariate assessment, a z test, of differences in the sets of item response theory parameters.

Three additional Monte-Carlo studies compared the performance of these two item response theory area indices with the Mantel-Haenszel χ^2 statistic and two other area indices. Each study uniformly varied only one of the three item response theory parameters allowing the effect of perturbations in each parameter to be studied in isolation. These studies demonstrated that extreme parameter values, especially $a < 0.5$, are often accompanied by large standard errors leading to questions about the

usefulness of indices of differential functioning that are based on item response theory parameters.

These studies also demonstrated that the Mantel-Haenszel χ^2 statistic is extremely sensitive to variations in the item response theory difficulty parameter, mildly sensitive to variations in the pseudo-guessing parameter, and completely insensitive to variations in the discrimination index. Hence, while providing an implicit multivariate test of differences in the sets of item response theory parameters using classical testing performance indices, the Mantel-Haenszel statistic implicitly weights parameter differences differentially. While the differential weighting of the difficulty and pseudo-guessing parameters may be appropriate given the differential effect changes in these parameters may have on subject responses, the complete insensitivity to changes in the discrimination parameter can result in items incorrectly not being identified as differentially functioning.

Lastly, the performance of the two item response theory area indices was compared to that of the Mantel-Haenszel χ^2 statistic using data from the Fall 1986 administration of the Graduate Management Admissions Test. Comparisons were made using groups of Black and White examinees and Female and Male examinees. As expected because of its differential weighting of parameter differences and the inclusion of estimates of errors of measurement in the area indices, the Mantel-Haenszel statistic flagged more items as differentially functioning.

APPROVAL PAGE

This dissertation has been approved by the following committee of the Faculty
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CHAPTER I

INTRODUCTION

Issues surrounding "test bias" and "unfairness in testing" are the source of some of the most virulent debates in social science research and educational policy. The reasons for the intensity of the debate go beyond scientific disputes about "nature versus nurture," and what has come to be called the "IQ controversy." Standardized tests and how they are used in practical situations affect people's lives in very real ways. The individual who does not score high enough on a given test may be denied admission to the college of her choice. The job applicant who does not score high enough on an employment test is denied the opportunity to demonstrate what she can do on the job. The individual who wants to enter a particular occupation may be barred from doing so if she does not pass the licensure test for that occupation. Tests have real consequences for individuals' life chances. It is therefore incumbent upon those who develop and use tests for placement in classes for the educable mentally retarded, for college admissions, for employment, for licensure, and for a host of other practical purposes to take every reasonable measure to insure that such tests are "fair," in some sense, to all segments of the population.

In the past quarter century, measurement professionals have devoted immense investigative energy to the study and, ostensibly, the elimination of sources of bias in tests. One has only to pick up any journal devoted to psychological or educational

measurement to realize the persuasiveness of the research. Virtually every issue of the leading measurement journals over the past 20 years contains at least one article devoted to some aspect of the subject of "test bias." While the research activity is incredibly wide-ranging, the bulk of the research can be categorized into three broad headings: research on the differential predictive validity of various tests for subgroups of the population; research on the construct and content validity of various tests; and research devoted to identifying individual items on tests that are "biased" against specific sub-populations. The focus of the present investigation involves the latter of these avenues of research.

A test item is said to be "biased" [or, to use the more current term, to exhibit "differential item functioning" (DIF)] against a given group if, when compared to others of comparable status on the trait being measure, the item is differentially more difficult for members of that group. A more complex form of DIF exists if the item is differentially discriminating between high and low scoring individuals across groups of comparable ability distributions. If equally able subjects of different groups exhibit differential responding to test items, then the question arises, "Is the test item in question measuring the same dimension or dimensions across demographic groups?" How can a test be considered "valid" for all examinees if it is tapping different dimensions for different groups of subjects?

This issue, it should be noted, goes beyond the mere confounding of, say, verbal and mathematical abilities on tests composed of "word problems." There, one would reasonably expect to be tapping two, or more, distinct dimensions; and the

resulting score would represent some composite of cognitive achievement along these dimensions. The threat of differential item functioning to test validity is a bit more insidious, and is not just a matter of multiple and confounded dimensions appearing on a single assessment. Rather, differential item functioning occurs regularly on tests that, by all of the usual criteria (including judgments of test content by experts) are measuring a single dimension across groups of interest. The differentially functioning items are virtually indistinguishable from items showing no DIF. Herein lies the threat to test validity; is such an instrument equally valid across the demographic groups?

Perhaps if the differentially functioning items can be identified, then those items can be removed and replaced with other items that function equally across demographic groups. The first difficulty here is with the identification of differential functioning. In the late 1960's and early 1970's test developers depended primarily, although not exclusively, upon the review of presumed experts to identify potentially biased items. Items that portrayed minorities in stereotypical situations, that contained specialized or difficult vocabulary, that used difficult or unfamiliar sentence structure, or that reflected situations that were unduly unfamiliar to some subgroups were removed from item banks. These methods, known as "judgmental review," while improving the quality of items (and consequently tests), at least from the perspective of simple fairness, did little to address the more subtle issue of test bias and differential item functioning. Detection of differential functioning, then, transcends simple visual analysis, requiring researchers to explore more objective

indices as a follow-up to judgmental review. The early numerical (i.e., objective) attempts to identify differential item functioning focused on the use of analysis of variance to assess differences across demographic groups via a group-by-item interaction term. The assumption was that a statistically significant interaction term indicated differential performance across demographic groups. Item performance, assessed typically in these studies as item score, item difficulty, and item discrimination, was a classical testing theory index and subject to the usual problem in classical test theory of sample dependency (the indices can vary from one sample to the next). However, the real problem with the use of analysis of variance (ANOVA) to identify differential item functioning is a matter of confounding; differential item functioning, assessed by ANOVA techniques, is confounded by genuine group differences. Hence, a statistically significant group-by-item interaction term could be indicative of differential functioning, or group differences, or both; but which of the three was at play could not be determined.

Attention eventually turned to the study of DIF via contingency tables based on the dichotomous item score crossed with grouping based on total score. This approach to the study of differential functioning is easily assessed and understood by both researchers and practitioners because of the underlying simplicity of contingency tables and the associated χ^2 statistic. The χ^2 , when interpreted with its associated degrees of freedom, not only provides an index of the amount of differential functioning but it also provides an index of statistical significance, the p-value. Refinements in χ^2 studies resulted in the adoption of a specialized χ^2 statistic with one

degree of freedom, known as the Mantel-Haenszel χ^2 , from biostatistics. Ensuing investigations have led to the adoption of the Mantel-Haenszel χ^2 statistic as the leading indicator of differential item functioning.

The item parameters of classical test theory, while easily explained and understood, suffer in that they are sample dependent. For instance, a test of algebraic skills given to a group of mathematicians will result in a particular set of indices for item difficulty; give the test to a group of auto mechanics, and an entirely different set of item difficulty indices will be found. Classical test theory assesses item performance in the context of the sample; as the sample changes, so does the assessment. In this last example, one might expect that the item difficulties would be on the easy side for the sample of mathematicians and on the difficult side for the auto mechanics.

Item response theory (IRT) attempts to assess items on a scale that is sample independent. That is, a set of item parameters would be invariant across samples. As one might expect, the mathematics leading to the IRT parameters is far more complex than in classical test theory. As well, IRT usually requires very large samples for stable parameter estimations. While a few hundred subjects are sufficient for most classical analyses, most IRT analyses require a few thousand subjects. The simpler IRT analyses may converge with as few as one thousand subjects, while the more complex analyses may require several thousand subjects for the same level of convergence. Given the mathematical sophistication and the sample size requirements

(and perhaps its relative youth), it comes as no surprise that IRT studies have not yet made their way to the classroom.

While the use of IRT parameters to study item performance is becoming the standard in the large testing arenas such as ETS, there are no accepted indices of differential functioning that are based on IRT parameters. There are several techniques being considered, all of which will be discussed in the forthcoming review of the literature, but none have achieved the status of the Mantel-Haenszel statistic. While many may attribute this to the relative ease with which the Mantel-Haenszel statistic can be computed, not to mention understood, the real problem with IRT indicators of differential item functioning (DIF) appears to be more of age: there simply has not been time yet to investigate the problem fully. Indeed, there is still argument over the merits and propriety of the assorted methods; in that light, arguments of index performance are secondary.

The need for an objective IRT indicator of DIF that is grounded in traditional inferential statistics is highly desirable and was the focus of this research. One such set of indicators are the IRT "area" indices. IRT analyses result in item parameters that can be used to define a graph for each item where subject ability is treated as the independent variable and the probability of a correct response to the item is treated as the dependent variable. This process is very similar to logistic regression; the resulting curve, called the Item Response Function (IRF), is "S-shaped" and numerically close to the normal ogive. If differential functioning exists for an item, then the IRT parameter estimates for each the demographic subgroups in question will

differ, resulting in different IRFs for different groups. If the IRFs of two groups of interest, say Blacks and Whites, are superimposed, then the area between the two curves can be considered as an index of differential functioning.

This research proposed two area-based indices of differential item functioning with associated tests of statistical significance. Some work has already been done in this field (Hambleton & Rogers, 1989; Raju, 1988; Raju, 1990) resulting in area indices of DIF that are dependent on the IRT model chosen, as well as the kind of DIF present (uniform and non-uniform). The indices derived for this research are general in that they treat the various IRT models as special cases of a single, more general model and they are applicable to both uniform and non-uniform differential item functioning.

Monte-Carlo methods constituted a significant portion of this research. These methods were used to investigate the distribution of the proposed indices and to investigate and compare the behavior of the proposed indices with other IRT-based indices and classically-based indices such as the Mantel-Haenszel χ^2 , for items of known differential functioning. Individual IRT parameters were varied uniformly across a range of values representing 99 percent of the values encountered in practice. Monte-Carlo "subjects" then took the simulated test and the assorted indices of DIF were compared. These procedures allowed an assessment of how the individual indices were performing under known circumstances.

The second phase of this research focused on data taken from the verbal sections of the 1986 administration of the Graduate Management Admissions Test

(GMAT). In this phase, the performances of the proposed indices were assessed by comparison to the Mantel-Haenszel χ^2 statistic; the Raju area indices, based on prior studies, were not used as they require adherence to special conditions that were considered impractical in this study. Comparisons were made between groups of Black and White male examinees and Female and Male white examinees. Large differences were expected between the Black and White examinees; lesser differences between the Female and Male examinees.

CHAPTER II

A HISTORY OF ITEM BIAS

Introduction

Certain expository conventions will be used throughout this research. Many measurement professionals now make a distinction between "item bias" and "differential item functioning," maintaining that differential item functioning (DIF) is actually a value-neutral, technical term that should be reserved for items that differ in certain statistical properties across groups, but may or may not be "biased." This distinction is acknowledged. However, for expository purposes, the two terms will be used interchangeably in this investigation. Following Holland and Thayer (1988), the term "focal group" will be used to refer to the group about which there is concern that bias may exist (i.e. blacks, Hispanics, women, etc.), and the term "reference group" will be used to refer to the group to which the focal group is being compared (e.g., whites or males). The term "studied item" will be used to refer to the item the bias of which is under investigation.

Efforts to identify item bias began in the late 1960's and early 1970's with the use of judgmental methods of item examination. These informal, subjective procedures for detecting DIF were followed by the use of correlational and ANOVA procedures to examine classical test theory indices such as item difficulty across groups. The latter methods gave way in the early 1980's to methods based on χ^2

techniques and item response theory (IRT). While the χ^2 techniques have converged on the Mantel-Haenszel approach (Holland and Thayer, 1988), methods based on IRT techniques are still being debated. Even more recently, Green, Crone, & Folk (1989) and Kelderman and Macready (1990) have suggested the use of log-linear models in DIF analysis, and have proposed tentative models for this purpose.

Judgmental Methods for Detecting DIF

Tittle (1982) has provided the most comprehensive review of judgmental methods in item bias studies. These methods involved the examination of individual items by persons presumed to be "expert" in identifying sources of bias in test items. Although there is undoubtedly a large, idiosyncratic element in such judgments, a set of "standards" for such evaluations has evolved over the years. These include (1) simple face validity, that is, is the item, on its face, believable to all examinees as valid measures of the construct in question; (2) fair representation of women and minorities (do reading comprehension items, for example, contain passages that can reasonably be presumed to have intrinsic interest to these groups) and (3) equal familiarity or experience with the nominal content of items. For example, the use of the word "regatta" in the stem of an SAT verbal (SAT-V) analogy item has been criticized by test critics as being biased against inner-city minority examinees.

In addition to reviewing items for bias and possibly offensive content, researchers at the Educational Testing Service have developed a rather elaborate "sensitivity" review process, one aspect of which is the identification of "caution" words and phrases to be avoided if possible (Hunter and Slaughter, 1980). Any SAT-

V item containing such words is flagged for special examination by a board of item reviewers. Typical words on the list include "backward," "barbarian," "birthrate," "class," "colonialism," "crime," "culturally disadvantaged," "developing nation," "gangs," "ignorant," "illegitimate," and "inferior."

Obviously, the passage of time, not to mention the literal context in which the item occurs, changes this list. The use of "backward" in the context of a vector problem on a physics test may be perfectly acceptable, yet quite offensive when used to describe a group of people. As well, the use of the word "crime" could be used quite innocuously in many sociological discussions, but might be offensive in the context of a discussion of race relations and civil rights.

In an attempt to bring some semblance of order and coherence to the burgeoning list of potentially offensive categories of words and topics, Hambleton (1980) offered the following "Item Bias Review Form" with which items could be rated:

1. Is the item free of offensive sexual, cultural, racial, and/or ethnic content?
2. Is the item free of sexual, cultural, racial, and/or ethnic stereotyping?
3. Is the item free of language that would be offensive to a segment of the examinee population?
4. Is the item free from descriptions that would be offensive to a segment of the examinee population?
5. Will the activities or situations described in the item be familiar to all examinees?
6. Will the words in the item have a common meaning to all examinees?
7. Is the item free of difficult vocabulary and/or sentence structure?
8. Will the item format be familiar to all examinees?

Major test publishers and other organizations that develop or sponsor tests (American Psychological Association, 1977; Macmillan, 1975; McGraw-Hill, 1968, 1974; Miller and Swift, 1980; National Center of Educational Media and Materials for the Handicapped, 1979) now have procedures and guidelines for the examination of items for possible offensiveness or bias. Such procedures are becoming a routine part of test development.

Despite judgmental attempts to remove obviously troubling language from test items, when statistical procedures to be described anon are used to detect and remove troublesome items, many of the remaining items are found to behave differentially across ethnic and gender groups. Moreover, it is often impossible to discern the reason for such differential performance. Consider, for example, the following item, discussed by Lord (1980, p. 218):

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly opposite in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all of the choices before deciding which is best.

- GEL: (A) glaze
 (B) debase
 (C) corrode
 (D) melt
 (E) infect

Clearly, this item contains no language which any group would find offensive, nor is it particularly worded to appeal to one group over another. Yet, on average, low scoring black examinees perform better on this item than low scoring white

examinees. For above-average examinees, just the reverse is true. Above-average white examinees outperform above-average black examinees.

O'Neil and McPeck (1993) present another example, a verbal analogy from the SAT, which contains stereotypical content of male interest, but which showed no DIF.

TORPEDO : WEAPON::

- (a) cannonball : gun
- (b) tractor : farm
- (c) submarine : fleet
- (d) fin : swimmer
- (e) *boat : vehicle*

The above examples are quite typical. Attempts to discover the sources of DIF identified by various statistical procedures via a content examination of the flagged items has proven difficult. The most comprehensive attempt to date at a substantive categorization of items that show gender and ethnic DIF was undertaken by O'Neil and McPeck (1993).

Given items that perform differentially and that defy identification by expert review, the process of test construction is far less rigorous than most professionals like. For example, many test developers have attempted to "balance the bias" (Jensen, 1980) in tests of reading comprehension by including content that reflects a variety of cultural experiences or by including content of (presumably) intrinsic interest to various focal groups. Thus, in an attempt to balance content of presumed interest to males (science, sports, etc.) many test developers now include passages of

presumed interest to females (the Arts, child-rearing practices, etc.). In this age of "political correctness," such transparent attempts to solve the problem of content or facial bias are, of course, fraught with danger. For example, the Princeton Review, an SAT commercial coaching company, advises its readers to eliminate immediately any distractor on a reading comprehension item that reflects unfavorably on African-Americans. Moreover, when in doubt, the examinee is advised to pick the alternative that is most favorable to African-Americans. A content examination by ETS staff of the reading comprehension items on the SAT confirmed the soundness of this advice (Bond, 1994).

Experimental Design and Statistical Approaches

One statistical approach to item bias involves the use of the analysis of variance to analyze the performance differences of particular groups of examinees via the examination of the group-by-item interaction term (Schmeiser, 1982). Performance differences by group are given typically by (1) score on the item, (2) the item discrimination index, (3) the count of omissions as an indicator of speededness, (4) KR_{20} , (5) item difficulty, and (6) the point biserial correlation. While the bulk of attention here is placed on experimental design, the real weakness of this approach, as will be shown, is with the use of ANOVA to study classically-based indices, which are population dependent, to identify item bias.

The use of ANOVA was the statistical method of choice in the early days of DIF analysis and was extremely popular (Dreger and Miller, 1960, 1968; Medley and Quirk, 1974; Shuey, 1966). The appeal of ANOVA as a technique for studying item

bias derives, of course, from the well-known advantages of rigorous experimental design (random assignment, control of experimental conditions, etc.) over more observational and correlational approaches. Moreover, the empirical results of ANOVA analyses reinforced the attractiveness of the procedure in that they tended to identify test items that favor white examinees over black, regardless of the index of performance used for the comparison.

While ANOVA is easily performed, understood, and explained, it suffers from one major drawback as an indicator of item bias -- it confounds differential item functioning with real group differences. This is explored in detail by Hunter (1975), Lord (1977), and Camilli and Shepard (1987). Camilli and Shepard show algebraically why this is so and confirm the algebraic development with both simulated and real data. They conclude from their analysis that "When there is a true difference in group achievement levels, the ANOVA interaction term is incapable of detecting bias that adds or subtracts from this true difference." As a consequence, ANOVA procedures for investigating DIF have all but disappeared in recent years.

Delta-Plot Method

The Delta-Plot method (Angoff, 1972; Angoff and Ford, 1973) is a graphical analysis of item difficulty values based upon Thurstone's (1925) classical Method of Absolute Scaling. In the Delta-Plot method, difficulty values are calculated for each group of interest. These values ("deltas") are normalized to a mean of 13 and a standard deviation of 4.

Each item, then, yields a pair of deltas, assuming two groups of interest. The deltas are plotted against each other on a bivariate graph where the deltas for one group are placed on the x-axis and the deltas for the other group are placed on the y-axis. If a given item is relatively more difficult for the x-axis group, it will be displaced toward the lower right-hand part of the scatter plot. If the item is relatively more difficult for the y-axis group, it will be displaced toward the upper left-hand part of the scatter plot. Typically, the points form an ellipse. If all items have similar relative difficulties for the two groups, the ellipse will be quite narrow. Conversely, the presence of substantial differences in relative difficulty among most items will result in a more circular ellipse.

In this approach, a distinction is made between the major axis of the ellipse and the 45 degree line. The distance between the 45 degree line and the major axis is considered an indicator of real, or legitimate, group differences. The perpendicular distance between a particular point and the major axis is considered an indicator of differential item functioning for the item associated with that point. Hence, while this method depends on the classical index of item difficulty, it does purport to distinguish between differential functioning and real group differences. In describing the rationale underlying this approach, Angoff (1982) notes:

When the samples differ in level of ability, the points will still fall in a long, narrow ellipse but will be displaced vertically or horizontally, depending on which group is the abler one . . . However, when the two groups differ in 'type', or when the items do not all have the same meaning for the two groups, the deltas will not fall in precisely the same rank order for the two groups, and the correlation represented by these points will be lower . . . The

items falling at some distance from the plot may be regarded as contributing to the item-by-group interaction. (p. 97)

It should be noted that, in addition to its use in ethnic, cultural, and gender item bias research, (Angoff & Ford, 1973; Angoff & Herring, 1976; Angoff & Modu, 1973; Angoff & Stern, 1973; Breland, Stocking, Pinchak, & Abrams, 1974; Coffman, 1961), the delta-plot method has been used in general score equating (Thurstone, 1925), and the standardization and equating of item difficulties (Thurstone, 1947). Additionally, delta-plots have been used to identify miskeyed items, items which have become obsolete, and items having different "psychological meaning" for different groups.

The simplicity, ease of calculation, cost, and relatively small sample sizes required for stable results have all contributed to the popularity of the delta-plot method of studying item bias. The two principal disadvantages of Angoff's method are (1) the delta values for the two groups being compared may not be equally reliable and (2) the confounding of differential item functioning with genuine group differences on the construct in question. The former problem can generally be solved by an appropriate scale transformation. Cardall and Coffman (1964) and Coffman (1961, 1963) suggest using an arcsine transformation of the p-values to control for different item variances (i.e., difficulties). Plake and Hoover (1979) obtained results which demonstrate that the arcsine transformation is indeed highly effective in equalizing item variances.

The second disadvantage (i.e., the confounding of DIF with true group differences) is not so easily resolved. As was shown by Cleary and Hilton (1968) and Angoff and Sharon (1974), the use of the within-groups items by subjects interaction as an error term is bound to yield statistically significant results. Even with small sampling of subjects and modest length test, the degrees of freedom will be large. This will render almost any effect statistically significant regardless of its practicality. For this and other reasons, the delta plot method is little used outside of ETS, and even there is being replaced by procedures to be discussed below.

Item Discrimination vs. Examinee Group

Green (1971) and Green and Draper (1972) present a procedure for the evaluation of item bias using classical item discrimination. Given two groups of examinees, the following procedure is used to identify items that perform differentially. First, for each group, the item point-biserials are dichotomized into groups of "high" and "low." Given this dichotomization of item discrimination within the focal and reference groups, items with "high" point-biserials for one group and "low" point-biserials for the other group are considered differentially functioning. Hunter (1975) has shown that, as with prior procedures, this method confounds item difficulty and group differences.

χ^2 Methods and the Mantel-Haenszel Technique

Scheuneman (1979) proposed a χ^2 procedure, based upon a total test score matching criterion, for the identification of DIF. She proposed that subjects from the two groups of interest should be matched by test score within each of three to five

intervals across the range of scores. In actuality, the number of matched groups can range up to $n+1$, where n is the number of items on the test.

For each item, a 2x2 contingency table is constructed at each of the K intervals. The data for a particular item are summarized in Table 1.

Table 1
Data for the j (th) Matched Set of Examinees

	Score on Studied Item		Total
	1	0	
Group 1	A_j	B_j	n_{1j}
Group 2	C_j	D_j	n_{2j}
Total	m_{1j}	m_{0j}	T_j

The test statistic for these K 2x2 contingency tables is expressed as the sum of the cell χ^2 s for the "A" and "C" cells of Table 1. This statistic was originally thought to be distributed, under the null hypothesis, as an approximate χ^2 with $K-1$ degrees of freedom. This is not the case as was discussed by Baker (1981) and Scheuneman (1981). They demonstrated that the expected value, under the null hypothesis, of Scheuneman's original test statistic also depends on upon the total number of incorrect responses in each 2x2 table. This dependency is corrected by the application of a minor multiplicative factor. The resulting test statistic is the "full information" Pearson χ^2 . Bishop, Fienberg, and Holland (1975) showed that the correct degrees of

freedom is the number of matched groups, given that the total number of examinees within each matched group is large.

While this technique does identify a differentially functioning item, it does not give an indication of the amount by which the item is functioning differently. This is a problem in that an impractically small difference in item functioning can be statistically significant, given enough examinees. Hence, large samples could lead to the identification of differentially functioning items when the actual between-group differences are trivially small.

The Mantel-Haenszel procedure (Mantel and Haenszel, 1959; Holland and Thayer, 1988), provides an index of the size of the departure of the data from the null hypothesis, as well as a test of statistical significance. This procedure is applied to matched groups where the data are in the form of $K (=n+1)$ 2×2 matched groups. If Table 1 is re-written using proportions instead of counts, where "p" indicates the proportion answering the item correctly, and "q" indicates the proportion of examinees answering the item incorrectly, then the Mantel-Haenszel procedure provides a χ^2 test against the following specific alternative hypothesis:

$$H_1 : \frac{p_{1j}}{q_{1j}} = \alpha \frac{p_{2j}}{q_{2j}} \quad \text{where } \alpha \neq 1$$

In this formulation, the null hypothesis of no DIF is rejected when the parameter " α " (the common odds ratio in the K 2×2 tables) is, statistically, significantly different from unity.

The Mantel-Haenszel χ^2

$$\chi_{MH}^2 = \frac{\left(\left| \sum_j (A_j) - \sum_j E(A_j) \right| - \frac{1}{2} \right)^2}{\sum_j \text{Var}(A_j)}$$

where

$$E(A_j) = \frac{n_{1j}m_{1j}}{T_j}$$

and

$$\text{Var}(A_j) = \frac{n_{1j}n_{2j}m_{1j}m_{0j}}{T_j^2(T_j - 1)}$$

is distributed approximately as χ^2 with one degree of freedom.

While this test statistic provides, as demonstrated by Birch (1964) and Cox (1970), the most powerful unbiased test of the null against the alternative, it also provides an indicator of the statistical significance of the differential functioning of the j_{th} item. As well, " α ", the common odds ratio, provides an indicator of the practical significance of the differential performance of two groups. The following equation gives an estimator of this common odds ratio:

$$\hat{\alpha}_{MH} = \frac{\sum_j \left(\frac{A_j D_j}{T_j} \right)}{\sum_j \left(\frac{B_j C_j}{T_j} \right)}$$

Holland and Thayer (1985) show that (with certain simplifying assumptions) the following transformation, which puts α_{MH} on a symmetrical scale where 0 is the null value, has the interpretation of being a measure of differential functioning in the scale of differences in item difficulty as measured by the ETS, or Angoff's, "delta scale."

$$\Delta_{MH} = -2.35 \ln(\hat{\alpha}_{MH})$$

Assuming Group 1 is the reference group and Group 2 is the focal group, α_{MH} is the average multiplicative factor by which the odds of a correct answer for a subject in the reference group exceeds the corresponding odds for a comparable member of the focal group. Values of α_{MH} which exceed unity indicate items on which the reference group performed better, on the average, than did comparable members of the focal group. The value of Δ_{MH} is the average amount more difficult that subjects in the reference group found the studied item than did comparable subjects in the focal group.

The Mantel-Haenszel χ^2 is a popular means of identifying differential item functioning. It does not confound differential functioning with real group differences because it matches examinees with identical total test score and hence, presumably, of

similar ability. It is easy to calculate, and is easily understood. It enjoys a favored status among psychometricians and is becoming increasingly popular with major test developers. Any new developments in this area must, therefore, be evaluated in light of the Mantel-Haenszel χ^2 statistic.

Logistic Regression Techniques

Swaminathan and Rogers (1990) extended a log-linear procedure presented by Mellenberg (1982) that predicts item responses from group membership, ability level, and the interaction between group membership and ability level. This interaction term is an indicator of DIF. Using simulation studies, they found that the logistic regression procedure was more powerful than the Mantel-Haenszel procedure for detecting non-uniform DIF, and that the two methods were equally powerful when detecting uniform DIF.

Promising studies continue in this area to investigate the power of the two procedures as the of ICC's (Item Characteristic Curve) for the focal and reference groups' intersection varies across the range of abilities. It is thought that the Mantel-Haenszel procedure may yet prove more powerful in identifying non-uniform DIF when the ICCs cross near one end of the ability range.

Methods Involving Item Response Theory

Birnbaum (1968), in Lord and Novick's (1968) classic text Statistical Theories of Mental Test Scores, laid the mathematical foundations for the use of the logistic function to describe a new approach to the measurement of cognitive abilities that focused on the relationship between the ability of an individual examinee and the

probability of a correct response to a test item. Earlier, Rasch (1960), starting from a somewhat different theoretical orientation, provided a similar mathematical formulation for open-ended measurements (i.e., where no guessing is involved). It is perhaps not an overstatement to say that these developments represent the most important advances in analytic measurement methodology since its classical development by Thorndike, Pearson, Thurstone, and others.

Before discussing the use of IRT in the detection of DIF, it will be helpful to review the essential features of the theory. Item response theory posits that an examinee's response to an item is a probabilistic function of the examinee's ability and the statistical properties of the item. Each item on a test is characterized by an item characteristic curve, or item response function (IRF), that specifies the probability, $P(\theta)$, of a person with ability " θ " correctly responding to the item. This function, as modeled by the following equation, follows the "logistic ogive" which is an "s-shaped" curve.

$$P(\theta) = c + \frac{1 - c}{1 + e^{(-1.7a(\theta - b))}}$$

The ability of the examinee is given by θ . For all intents and purposes, θ can be regarded as being normally distributed about 0 with a standard deviation of 1. In fact, a normal distribution is a popular Bayesian assumption for θ (Warm, 1978). The value, 1.7, appearing in the exponential is used to make the logistic ogive follow more closely the normal ogive.

The set $\{a, b, c\}$ are called the item parameters which, taken together, specify a family of logistic ogives. Any given set of parameters determines a unique logistic ogive. The item difficulty is given by "b" and is defined as the examinee ability at which the chance of knowing the correct response is 0.5. It should be noted that examinee ability (θ) and item difficulty (b) are placed on the same metric, lending heretofore unrealized insights into the interaction between item difficulty and examinee ability.

While the value of "b" can theoretically range from minus infinity to plus infinity, typical values range from -2.5 to +2.5. An item with a b-value of zero is considered to be of average difficulty. Larger "b" values indicate more difficult items, smaller "b" values indicate easier items.

Although there are no assumptions about the distribution of "b" because items are typically chosen for a specific purpose, the fact that the metric for item difficulty is identical to that of examinee ability lends some additional meaning to this index that is not available for the classical index of item difficulty. An item with a b-value of 2.5, say, can be answered correctly fifty percent of the time by examinees whose ability is 2.5, which corresponds, assuming an approximately normal ($\mu_\theta=0, \sigma=1$) distribution of ability, to examinees who are 2.5 standard deviations above the mean ability.

An item's ability to discriminate between persons of high and low ability is given by "a" and is defined as the slope of the logistic ogive where the probability of a correct response is 0.5, or where $\theta = b$. This parameter indicates the strength of

the relationship between the item response and examinee ability, and is a rough analogue to item discrimination indices (e.g., point biserial correlation) in classical test theory. While the value of "a" can, theoretically, range from negative to positive infinity, practical values typically range from about 0.5 to 2.0. Negative values usually indicate seriously flawed or miskeyed items.

The lower asymptote (or guessing parameter), "c", is the probability that examinees of very low ability will be able to respond correctly to the item. This parameter is also known as the "pseudo-guessing" parameter. While the value of "c" can theoretically range from zero to unity, most estimated values range from 0.00 to 0.40.

Ideal values of item difficulty and item discrimination are usually taken to be 1 and 0, respectively, although specific test uses will determine the ideal value of b. Commonly encountered (Warm, 1978) means and standard deviations of the a and b parameters are ($\mu_a \approx 0.95$, $\sigma_a \approx 0.28$) and ($\mu_b \approx 0.16$, $\sigma_b \approx 0.93$). The nominal value of the pseudo-guessing parameter (c) depends on the number of choices in the multiple-choice item. Theoretically, one would expect the guessing parameter to be 0.20 for a five choice item, but estimates based upon real test data are closer to 0.15. Lord (1980) and others have noted that the lower "c" estimates result from the inclusion of effective distractors that are likely to attract lower-ability examinees because they represent common misunderstanding or incorrect information.

The three-parameter model is considered to be the "full IRT model. Models with more parameters have been proposed (Lord, 1980), but these have been found to

be of limited value. Reduced models are preferred in some applications (e.g., non-multiple-choice tests). The two-parameter model, where the guessing parameter is set to zero, uses only the item difficulty and item discrimination parameters. The one-parameter model, also known as the Rasch model, uses only the item difficulty parameter.

In the Rasch model, Rasch (1960), the item discrimination parameter is set to one while the guessing parameter is set to zero. Many who espouse the Rasch model claim that it places ability on a ratio scale of measurement. This claim, although attractive, is arguable. Although it is true that the statement "John is twice as likely to get this item right as Henry," can be justified and in fact makes sense in the context of item response theory, it does not necessarily follow that summing the two examinees' probabilities of success on all of the items on a test results in scores that allow such statements as "Jane is one-half as able in quantitative reasoning as Barbara."

Of the three parameters, the item difficulty parameter (b) is the most easily estimated, as innumerable studies have shown. This is perhaps the source of the power and stability of the Rasch model. Item discrimination (a) is the next most easily estimated parameter. The guessing parameter (c) appears to be the most difficult to estimate primarily because of the relative paucity of subjects at the low end of the ability scale and because, as mentioned earlier, assumptions regarding random guessing may not be entirely appropriate.

Despite these estimation difficulties, the three-parameter model is still the model of choice for most applications involving standardized, multiple-choice tests. The reasons for this preference are easy to discern. First, the data predictably fits the three parameter model better. Second, examinees do guess on multiple-choice tests. Finally, to require all items to have the same or similar discrimination parameters would, it is felt, seriously restrict content coverage.

It should be emphasized that, with real data, the various IRT models may not fit particular items or particular examinees. This fact should be kept clearly in mind in the discussion of results that follows, and, in fact, in all applications of item response theory. This caution obviously does not apply to simulated subjects and items.

The Assumptions of Item Response Theory

There are four basic assumptions in item response theory. The first, fundamental to testing and not specific to IRT, is that if an examinee knows the correct answer, she will so respond. Absent such an assumption, there is little or no foundation for testing. This assumption ignores, and rightfully so, the existence of clerical errors where the examinee knows the correct response, but marks another. It should also be noted that the converse of this assumption is not true. That is, because of fortuitous guessing, it cannot be assumed that if examinees gets the correct answer, she knows the correct response.

The second assumption is that the item response function (IRF), or item characteristic curve (ICC), follows the normal ogive. Because of the computational

difficulties associated with the equation for the normal ogive, IRT uses a logistic approximation to the normal ogive. As a matter of historical interest, this assumption delayed Lord's development of IRT for 10 years because of the lack of computers with sufficient computing power and because of three difficulties with the normal ogive IRF. These were (a) the lower asymptote, (b) the upper asymptote, and (c) the middle part of the IRF where it rises most rapidly. One would think there is not much left about which to worry.

Lord (1965) published the results of a study using over 100,000 subjects in which he addressed these three difficulties. The problem with the lower asymptote was with its stability. Suppose a multiple choice item where $c=0.05$ based on an intended population were to be given to a group of extremely low-ability examinees. Would the IRF "rise" to $1/A$ where A is the number of responses for the item? Lord found no evidence of this phenomenon over his 100,000 subjects.

The problem with the upper asymptote of the IRF is described by Hoffman (1962) and is known as the Banesh-Hoffman Effect. The claim is that examinees with extremely high abilities would "know too much" and respond to the item outside its intended context. These examinees would get the item wrong and be penalized for "excessive" knowledge. Hence, the IRF should fall away from the upper asymptote for very high θ 's. Again, Lord found no evidence of this phenomenon over his 100,000 subjects.

The third problem concerned the assumed monotonicity of the IRF. It was not known at the time that the IRF would be monotonic, nor was it known that the shape

would be approximately that of the normal ogive. Lord's work demonstrated that the IRF was strictly monotonic, and that it did follow the normal ogive.

The third assumption of IRT is that of local independence. Hambleton, Swaminathan, and Rogers (1991) define local independence as follows:

Local independence means that when the abilities influencing test performance are held constant, examinees' responses to any pair of items are statistically independent. In other words, after taking examinees' abilities into account, no relationship exists between examinees' responses to test items. This set of abilities represent the *complete latent space*. (p. 10)

Formally, local independence is represented mathematically by

$$\begin{aligned} \text{Prob}(U_1, U_2, \dots, U_n | \theta) &= P(U_1 | \theta) P(U_2 | \theta) \dots P(U_i | \theta) \\ &= \prod_{i=1}^n P(U_i | \theta) \end{aligned}$$

where U_i represents a subject's response to item "i", θ is the subject's ability, and "n" is the number of items on the test. It follows from this definition that local independence is violated when, for example, the correct answer to one item depends upon the examinee having correctly answered another item on the test. The assumption of local independence may also be questionable in reading comprehension tests where sets of items are keyed to the same passages. Careful item writing should alleviate, if not remove entirely, this latter concern. Local independence is also violated when tests that are speeded. As Warm (1978) states "Not-reached items are not attempted . . . simply because of the presence of earlier items".

The fourth and last assumption of IRT is unidimensionality. Unidimensionality means that the items on the test measure only one ability, attribute, or area of knowledge. (A slightly less restrictive assumption is that the test measures a set of highly related attributes.) It should be noted that unidimensionality is a sufficient condition for local independence, but that local independence is not a sufficient condition for unidimensionality (Warm, 1978; Hambleton, et al., 1991). That is, when the assumption of unidimensionality is true, local independence is assured. But local independence may obtain even when the test in question is not unidimensional. "Local independence will be obtained whenever the complete latent space has been specified; that is, when all the ability dimensions influencing performance have been taken into account" (Hambleton, Swaminathan, & Rogers, 1991, p. 11).

Indicators of Unidimensionality

A universally accepted test of unidimensionality has not been developed. By far, the most widely used tests of unidimensionality involve a factor analysis of an item tetrachoric correlation matrix. It should be noted, however, that the tetrachoric correlation assumes that θ is normally distributed and that the guessing parameter is zero. While the distributions of most human cognitive abilities have been shown empirically to be approximately normal, we know for certain that examinees do, indeed, guess when given the opportunity.

Eight methods of testing for unidimensionality have been identified. Lord and Novick (1968) present what is called the eigenvalue test. Essentially, this is a scree plot of eigenvalues, or percent of common variance, against factor rank. A

unidimensional test is indicated by a sharply defined elbow in the scree plot where one factor dominates all the others .

McBride and Weiss (1974) extend the eigenvalue test by superimposing a plot based on random data. If the eigenvalue of only the first "real" factor is distinguishable from the random factors, then the test is said to be unidimensional. The implication here is that a single factor dominates the data, and the other factors are the result of noise in the data.

McBride and Weiss (1974) suggest the computation of the correlation of the item point-biserials with the item loadings on the first factor. If this correlation exceeds 0.80, then the test can be said to be unidimensional. They also suggest that unidimensionality can be assumed if the first factor loadings for all items are significant and of the same sign.

Rommell (1970) and Pine (1977) use a congruence test to examine the correspondence between the factor loadings of two groups of interest across the items on the test. As this Coefficient of Congruence, C_{AB} , approaches zero, the test is said to be unidimensional with respect to the grouping variable.

$$C_{AB} = \sqrt{\left(\sum_i (L_{iA} - L_{iB})^2\right)}$$

where

L_{iA} = loading of item i for the A group on the first factor,

L_{iB} = loading of item i for the B group on the first factor,

and

n = the number of items in the test.

Green, Lissitz, & Mulaik (1977) suggest a communality test for unidimensionality based on inter-item tetrachoric correlations and item communality. Warm (1978) applied this technique to the data published by McBride and Weiss (1974) and found it to be no better as an indicator of unidimensionality than the scree plots described in the first method listed, or the Eigenvalue Test. Additionally, Warm found that when the guessing parameter was non-zero, this coefficient of congruence gave no indication of unidimensionality one way or the other.

Bejar, Weiss, & Kingsbury (1977) suggested that the instrument under consideration be separated into two parts and the difficulty and discrimination indices for each part, as well as the whole, be calculated. If the parameter estimates, part vs. whole, correlate highly, then the instrument can be said to be unidimensional.

Bock and Lieberman (1970) suggested that, given the item parameters and assuming a normally distributed θ , calculated response patterns be compared to observed response patterns. A non-significant χ^2 , caveats on accepting the null hypothesis notwithstanding, supports unidimensionality.

Of these methods, the scree plot and the simple consideration of the percent variance accounted for by the individual factors (Lord and Novick's recommendation) enjoy, perhaps, the widest acceptance among practitioners as indicators of unidimensionality.

IRT-based Tests of Differential Item Functioning

Many researchers have attempted to devise indicators of differential functioning from the parameters of item response theory (Crovo & Philips, 1983; Curran, 1985; Hambleton & Rogers, 1989; Ironson, 1983; Kelderman, 1990; Kelderman & Macready, 1990; Lautenschlager & Park, 1988; Lord, 1977, 1980; Loyd, 1984; Mei & Harwell, 1993; Muthen & Lehman, 1985; Pine, 1977; Raju, 1988, 1990; Shealy & Stout, 1993; Thissen, Steinberg, & Wainer, 1988, 1993; Thissen & Wainer, 1985; Warm, 1978; Wilson-Burt, Mead, & Skaggs, 1986; and Zwick, 1990). They have met with limited success; no IRT DIF index has proven more powerful than the Mantel-Haenszel χ^2 statistic.

Lord (1980) has stated flatly that if an item has a different response function for one group than another, then the item is biased. He proposed tests of differential functioning based on tests of statistically significant differences between the estimated item parameters for the two groups of examinees (Lord, 1977, 1980). For the item difficulty parameter, this test is

$$z = \frac{b_1 - b_2}{\sigma_{(b_1 - b_2)}}$$

where

$$\sigma_{(b_1 - b_2)} = \left(\sigma_{b_1}^2 + \sigma_{b_2}^2 \right)^{\frac{1}{2}}$$

The z statistic is justified because of the very large sample sizes associated with IRT analyses. Alternatively, this statistic could be squared and treated as χ^2 with 1 degree of freedom. Similar tests can be performed for the item discrimination and guessing parameters, though tests of the guessing parameter are not presented in Lord's work.

Multiple Parameter Differences

Lord (1980) also proposed a simultaneous test, based upon the Mahalanobis distance (Mahalanobis, 1930, 1936) of the difference between the pairs of item difficulty and item discrimination parameters

$$D^2 = \mathbf{v}'\Sigma^{-1}\mathbf{v}$$

where \mathbf{v} is the vector of differences between the parameter estimates, $(a_1 - a_2, b_1 - b_2)$, and Σ is the corresponding covariance matrix. The Mahalanobis D^2 is distributed, under the null hypothesis of no parameter differences, as χ^2 (Pearson, 1900). This test is valid for one-, two-, and three-parameter models. The degrees of freedom is equal to the number of parameters being tested.

Warm (1978) gives a set of experientially developed rules of thumb which he has used to guide him in the identification of differentially functioning items. He offers these without apology for their lack of theoretical underpinning, asserting that

they are simply the best tools he has for the job. Warm identifies an item as "biased" if any one or more of the following conditions are met:

$$(a) \text{ABS}(a_1 - a_2) > 0.80$$

$$(b) \text{ABS}(b_1 - b_2) > 0.50$$

$$(c) \text{ABS}(c_1 - c_2) > 0.15$$

$$(d) \text{ABS}(a_1 - a_2) + \text{ABS}(b_1 - b_2) > 1.00.$$

These informal rules do not appear to have been adopted widely by the profession.

IRT Likelihood Ratio χ^2

Thissen, Steinberg, and Gerrard (1986) and Thissen and Wainer (1985) use likelihood ratio χ^2 s as described by Mislevy (1985) for testing the differences between two populations in the context of IRT. Muraki and Bock (1986) used this technique to demonstrate "item drift" which is essentially differential functioning over item administrations rather than examinee groups. A comparison of the Mantel-Haenszel χ^2 against the IRT likelihood ratio χ^2 by Holland and Thayer (1986) resulted in nearly identical DIF indicators.

It has been suggested by Thissen, Steinberg, and Wainer that these results are due to the relatively small differences in the parameters of the simulated data and the relatively small sample size. While this indeed may be the case, it appears that further investigation will be needed before the IRT likelihood ratio method replaces the Mantel-Haenszel technique. In the meantime, the likelihood ratio test, at best,

will remain a secondary technique to be applied to items already flagged by Mantel-Haenszel or some other method.

Area Techniques

The bulk of the current work on the detection of differential item functioning (DIF) is based on what is called the "IRT Area." If an item functions differentially for two groups of examinees, then two sets of item parameters can be estimated, one for each group. After an appropriate transformation of the θ scale, the item characteristic curve for each group is plotted and superimposed on the same graph. The area, expressed alternatively as a signed quantity by

$$\int (P_1(\theta) - P_2(\theta))d\theta$$

and an unsigned quantity by

$$\int |P_1(\theta) - P_2(\theta)|d\theta$$

between the two ICCs, given by P_1 and P_2 , is an index of differential functioning (Hambleton and Rogers, 1989; Ironson and Subkoviak, 1979; Shepard, Camilli, and Averill, 1981; Rudner, 1977; Rudner, Geston, and Knight, 1980a, 1980b).

The unsigned area (UA) is simply the absolute difference between the two ICCs integrated across some range of ability, usually minus to plus three or four standard deviations. The signed area (SA) is the difference between the two ICCs integrated over the same range.

Prior to the use of item response theory to investigate DIF, the plethora of ways in which items could function differentially for different groups was not fully appreciated. Item response theory made it clear that the extent to which an item favored one group over another can depend on where along the ability scale the comparison is made. This realization led to the distinction between *uniform* and *non-uniform* DIF. Uniform DIF is present when ICCs for the focal group and the reference group differ but do not cross. Although uniform DIF makes no statement about the relative difference at a given value of θ , it does require that one ICC is always "below" (i.e., shifted to the right of) the other. Non-uniform DIF is indicated by crossing ICCs. For items exhibiting non-uniform DIF, there is an ability (value of θ) where the two groups of examinees have equal probabilities of getting the item correct (i.e., where there is no DIF); above that intersection point, one group is favored by the item, and below the intersection point, the other group is favored.

Hambleton and Rogers (1989) examined the performance of both of the signed and unsigned area indices in light of the Mantel-Haenszel statistic. They found that the IRT area methods were slightly less consistent in the identification of differential item functioning, and suggest that the area methods be used only to flag an item as potentially biased. However, they do point out that the area methods will detect non-uniform differential functioning, while the Mantel-Haenszel method does not. This is an important finding, given that many items can be found which exhibit non-uniform differential functioning.

One important feature of the Mantel-Haenszel method is that it not only provides a measure of DIF which is interpretable in terms of classically-based indices of item difficulty, but its theoretical sampling distribution is known, so that tests of statistical significance are available. Hence, not only may the practitioner set an index level to be used as a practical indicator of DIF (e.g., the ETS log-odds ratio), but she also has available an indicator of statistical significance to direct evaluation of the DIF indicator in terms of sample size.

Raju (1988) has derived equations for the signed and unsigned area between two ICCs, but he does so separately for the one-, two-, and three-parameter models. Because of its close relevance to the work proposed here, Raju's work will be reviewed in some detail. It should be noted here that Raju uses $P(\theta)$ to denote the 2-parameter IRT model, and $F(\theta)$ to denote the 3-parameter model.

Let the signed and unsigned areas between two ICC's for groups 1 and 2 be defined as

$$SA = \int_{-\infty}^{+\infty} (F_1(\theta) - F_2(\theta))d\theta$$

and

$$UA = \int_{-\infty}^{+\infty} |F_1(\theta) - F_2(\theta)|d\theta$$

where

$$\begin{aligned} F_1(\theta) &= c_1 + (1 - c_1)P_1(\theta) \\ F_2(\theta) &= c_2 + (1 - c_2)P_2(\theta) \end{aligned}$$

and

$$\begin{aligned} P_1(\theta) &= \frac{1}{1 + e^{(-Da_1(\theta - b_1))}} \\ P_2(\theta) &= \frac{1}{1 + e^{(-Da_2(\theta - b_2))}} \end{aligned}$$

Raju considers two cases. Case I imposes the restriction that $c = c_1 = c_2$ for groups 1 and 2. Case II relaxes the above restriction, so that $c_1 \neq c_2$.

$$SA = (1 - c)(b_2 - b_1)$$

Under Case I, with the stipulation that $a_1 \neq a_2$, the signed and unsigned areas respectively are

$$UA = (1 - c) \left| \frac{2(a_2 - a_1)}{Da_1 a_2} \ln \left(1 + e^{\frac{(Da_1 a_2 (b_2 - b_1))}{(a_2 - a_1)}} \right) - (b_2 - b_1) \right|$$

For $a_1 = a_2$, the signed and unsigned areas simplify to

$$SA = (1 - c)(b_2 - b_1)$$

$$UA = (1 - c)|b_2 - b_1|$$

Substituting $c = 0$ in these two equations results in expressions for the signed and unsigned areas which are valid for both the Rasch (one-parameter) model and the two-parameter model where $a_1 = a_2$.

Under Case II (i.e., where $c_1 \neq c_2$), the signed areas is either plus infinity or minus infinity, and the unsigned area is plus infinity. Proofs may be found in Raju (1988). There are thus different indices of DIF depending not only on one's preference for the signed or unsigned model, but also on the number of parameters in the IRT model.

In later work, Raju (1990) developed statistical tests of significance for some of these area expressions. The ratio of the area to its standard error was posited by Raju to be distributed asymptotically as z .

IRT seems to be the wave of the future and it would seem highly desirable to developed all item analysis indices (including DIF) from an IRT perspective. A unified approach to a single, general IRT-based index of DIF is currently lacking, as in an accepted index of statistical significance for the difference between two item response curves. The next chapter delineates the problem a bit more fully, describes historical approaches to the problem, and examines Raju's approach in more detail. A procedure is then proposed and tested, and its statistical properties are examined using both simulated and real data.

CHAPTER III

STATEMENT OF THE PROBLEM

Researchers, test developers, test users, and others interested in test fairness in general and differential item functioning in particular must, in any given study or application, eventually derive a decision rule for deciding whether to flag a given item for further investigation. Prior to the mid 1970's, testing programs relied on either visual inspection or traditional inferential decision rules in connection with one of the classically-based DIF methods discussed earlier: the Angoff procedure, item x race interactions in the context of the ANOVA model, and χ^2 procedures. As indicated earlier, Holland and his colleagues at ETS have advocated use of the Mantel-Haenszel technique and, although the decision rule for flagging an item in this procedure is more-or-less arbitrary, the Mantel-Haenszel technique has in the past decade become the most popular classical-based procedure for identifying differential item functioning.

Despite the apparent adequacy of at least one of the classically-based methods (e.g., the Mantel-Haenszel), many measurement specialists have continued to seek viable IRT-based procedures for identifying DIF. The reason for this stems first from the fact that the majority of professional testing programs, state testing programs, and large district programs are rapidly moving toward tests that are constructed and scaled using the one- or three-parameter IRT model. Second, one can argue that the test

development process should be coherent in the sense that if it is IRT-based, then all relevant test and item statistics should be similarly IRT-based.

The search for an IRT-based index of DIF is problematical. First, decision rules for flagging differentially functioning items have not taken advantage of the long and rich history of developments in statistical inference. Rather, decision rules are often parochial, if not arbitrary, and sometimes based upon mere visual inspection of ICC's. Operationally, large testing programs cannot depend upon such subjective methods. Second, the most widely used programs for IRT analyses (e.g., LOGIST) have been plagued by large standard errors of parameter estimation (Thissen & Wainer, 1982; McLaughlin & Drasgow, 1987), although this problem has been lessened recently (Thissen, Steinberg, & Wainer, 1993). Third, the estimation procedures used by these programs to fit the three-parameter model with reasonable standard errors frequently require sample sizes larger than most computers can accommodate, or computing time beyond the resources of most researchers (Thissen & Wainer, 1982). Finally, IRT is still in its relative infancy, so that actual experience in judging whether two ICC's differ in important ways is meager.

The use of the area between two ICCs as an indicator of differential item functioning has been investigated by Hambleton and Rogers (1989) and Raju (1988, 1990). Hambleton and Rogers (1989) calculated the area between the ICCs over a defined range of abilities, and "scaled" those values using the Mantel-Haenszel χ^2 statistic. No attempt was made to establish an indicator of statistical or practical significance. Since both the signed area and the Mantel-Haenszel statistic are

insensitive to non-uniform differential functioning, a judicious choice of cut points for the signed area results in very similar performance by both indices. Given the relative ease with which the χ^2 statistic can be computed and interpreted, this research, while enlightening, did not lead to adoption of the IRT signed area as an indicator of DIF.

As mentioned in Chapter 2, Raju (1990), using asymptotic z's, developed indicators of DIF, with associated indices of statistical significance, based on the area between the item response curves of the focal and reference groups. He developed separate equations for each of the three basic IRT models and treated the signed area separately from the unsigned area. These separate mathematical models, combined with special cases for the IRT discrimination index, led Raju to eight different indices of DIF depending on the choice of signed or unsigned area, and choice of IRT model. The indices also require equal guessing parameters for the two comparison groups, although it has been shown repeatedly that the estimates of "c" for the focal and reference groups are rarely equal.

For the one-parameter model, Raju proposed an asymptotic z-test for both the signed area and the unsigned area. For the two-parameter model, he proposed asymptotic z-tests for the signed area, for the unsigned area when $a_1 = a_2$, and for the unsigned area when $a_1 \neq a_2$. For the three-parameter model, where the restriction $c_1 = c_2$ is imposed, he proposed an asymptotic z-test for the signed area (SA_{30}), for the unsigned area where $a_1 = a_2$, and for the unsigned area when $a_1 \neq a_2$ (UA_{32}).

Raju's work is a significant contribution to research on area-based indices of DIF, and it represents the first sustained attempt at flagging troublesome items via IRT-based decision rules that employ traditional inference procedures, rather than simple visual inspection. As noted above, however, in Raju's approach, the one-, two-, and three-parameter models are handled quite differently. Inasmuch as the one- and two-parameter models themselves are special cases of the three-parameter model, it would be more coherent to develop a general three-parameter DIF index that incorporates the one- and two-parameter models as special cases, rather than separate indices and statistics for each.

CHAPTER IV

METHODOLOGY

Considerations of parsimony suggest a single three-parameter DIF index that incorporates the one- and two-parameter models as special cases and that simultaneously estimates the area between two ICC's for both uniform and non-uniform DIF. Such an approach is the proximal kernel of this research. Two such indices were examined. Briefly, the first was a variation on the "unsigned" area which, while identical in magnitude to the unsigned area, carried an arbitrary sign. The absolute value function was not used in this index as it is not differentiable at zero. For two ICC's that cross, the area was computed in two stages. In the first stage, the area between the ICCs from the lower limit of theta (ability) up to the "critical" theta (where the ICCs meet) is computed. The second component of the unsigned area was the area between the ICCs starting from the critical theta and ending at the upper limit of theta. Since theta is assumed to be normally distributed, the lower limit of theta was selected to be -3, while the upper limit was defined as +3.

A second index, a variation of the first, was also explored. Rather than use the absolute value function or divide the integral into two parts, this index squares the integrand of the signed area index. Sometimes referred to as a "squared" area, this index is not the square of the area between the two ICCs; rather, it is the square of

the difference between the two ICCs integrated over a given range of ability, usually +3 and -3.

A test of statistical significance was developed for each indicator. A technique of error propagation borrowed from the physical sciences was used to calculate the uncertainty (standard error) of the index from the variance-covariance matrix of the IRT parameters. Raju (1990) referred to this technique as "the delta method." For the first index, based on the unsigned area (with arbitrary sign), the ratio of the index to its uncertainty is asymptotically distributed as z . For the second index, based on the "squared" area, the ratio of the index, which can never be negative, to its uncertainty is asymptotically distributed as the absolute value of z .

These two indicators of DIF were investigated by comparing them to the Mantel-Haenszel χ^2 , the appropriate Raju (1990) index, and to each other. First, Monte Carlo studies were used to examine the sensitivity of each indicator to changes in the individual IRT parameters. Second, these comparisons were extended to data from the 1986 administration of the Graduate Management Admissions Test (GMAT). The indices by Raju (1990) were not used with the data from the 1986 GMAT since all IRT analyses were performed using the three-parameter model, and none of Raju's work can be applied to data where the pseudo-guessing parameters are not equal.

Error Propagation

Errors of measurement, or uncertainties, are inherent in all measurements. The uncertainty in an estimated quantity is a function of the uncertainty in its individual components. Consider, for example, the estimation of the area of a

rectangle. The length and uncertainty of one side can be expressed as $x_{true} + \delta x$, while that of the second side can be expressed as $y_{true} + \delta y$. The area, and its corresponding uncertainty, can be computed as the product of length and width.

$$\begin{aligned}
 \text{Area} &= \text{length} * \text{width} \\
 &= (x_{true} + \delta x) * (y_{true} + \delta y) \\
 &= x_{true}y_{true} + x_{true}\delta y + y_{true}\delta x + \delta x\delta y
 \end{aligned}
 \tag{Eq. 4.1}$$

Viewed from this perspective, it is seen that the estimate of the area of the rectangle, when its sides are not perfectly measured, has four components, rather than the customary one. The first component, $x_{true}y_{true}$, is the traditional, true estimate of the area; the other three are uncertainties. The first of these, $x_{true}\delta y$, is the uncertainty in the estimate of the area based upon the measure of x_{true} and the uncertainty of y . The second, $y_{true}\delta x$, is the uncertainty in the estimate of the area based upon the measure of y_{true} and the uncertainty of x . The third, $\delta x\delta y$, is the uncertainty in the estimate of the area based upon the uncertainties in both x and y .

With most physical measurements, $\delta x\delta y$ is small and negligible in comparison to $x_{true}y_{true}$, $x_{true}\delta y$, and $y_{true}\delta x$. Hence, it is frequently ignored in further calculations. Consider now, the implications of the above reasoning to measurement and estimation problems in the social sciences. It is usually assumed that the errors, e_x , in the measurement or estimation of some quantity x are purely random and hence correlated with other quantities. It follows that all covariances involving error terms,

$$\begin{array}{c}
 r_{ye_x} S_y S_{e_x} \\
 r_{xe_y} S_y S_{e_x} \\
 r_{e_x e_y} S_{e_x} S_{e_y}
 \end{array}$$

are assumed to be zero. This is the usual assumption in classical test theory. In the context of IRT, it turns out that errors in estimating the a, b, and c parameters are correlated, sometimes highly so. For this reason, the covariance term, $\delta x \delta y$, was carried forward in the following calculations.

While the above treatment is sufficient for simple areas such as rectangles, it is far too cumbersome for more complex regions such as the space between two ICCs. Baird (1962) describes the method, again called the "delta method" in Raju (1990), by which the uncertainty in a computed quantity can be calculated from the uncertainties of its individual components. This method of error propagation was generalized for this research to incorporate the correlated measures of educational research. To wit, the area between two ICCs is a function not only of the two pairs of IRT parameters, but also of the variance-covariance structure resulting from parameter estimation. Note that this structure was viewed in this research as a result of parameter estimation, rather than any theoretical expectation of correlated parameters.

Consider a function of two variables,

$$z = f(x,y)$$

Eq. 4.2

The total differential of Eq. 4.2 is given by

$$dz = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy \quad \text{Eq. 4.3}$$

Treating this differential as a finite difference, we can write,

$$\delta z = \left(\frac{\partial f}{\partial x}\right)\delta x + \left(\frac{\partial f}{\partial y}\right)\delta y \quad \text{Eq. 4.4}$$

Evaluating Eq. 4.4 at a point, say x_0 and y_0 , and using the uncertainties in x and y for δx and δy , yields an estimate, δz , of the uncertainty in the estimate of z . This uncertainty, then, is used to calculate the standard deviation given that we have n values of z and that $x_0 = \mu_x$ and $y_0 = \mu_y$. Eq. 4.5 defines this calculation.

$$S_z = \left(\frac{\sum_{i=1}^n (\delta z_i)^2}{n} \right)^{\frac{1}{2}} \quad \text{Eq. 4.5}$$

Substituting Eq. 4.4 into Eq. 4.5, squaring, expanding the square within the summation, and simplifying with the following three equalities,

$$S_x^2 = \frac{\left(\sum_{i=1}^n (\delta x_i)^2 \right)}{n} \quad \text{Eq. 4.6}$$

$$S_y^2 = \frac{\left(\sum_{i=1}^n (\delta y_i)^2 \right)}{n} \quad \text{Eq. 4.7}$$

$$\sum_{i=1}^n (\delta x_i \delta y_i) = S_{xy}^2$$

Eq. 4.8

where Eq. 4.8 represents the covariance of x and y, yields Eq. 4.9, an expression for the variance of z given the values of x, y, and the elements of the xy variance-covariance matrix. This equation is easily generalized to more than two variables, and was generalized to six variables, the two sets of parameters from the three-parameter IRT model, in this research.

$$S_z^2 = \left(\frac{\partial z}{\partial x}\right)^2 S_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 S_y^2 + 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) S_{xy}$$

Eq. 4.9

The First Index of DIF

The signed area measure of DIF is based on the integral defined in Eq. 4.10.

$$DIF = \int_{-\infty}^{\infty} [P_1(\theta) - P_2(\theta)] d\theta$$

Eq. 4.10

The unsigned area measure of DIF is based on the integral defined in Eq. 4.11.

$$DIF = \int_{-\infty}^{\infty} |P_1(\theta) - P_2(\theta)| d\theta$$

Eq. 4.11

The work of Raju (1989, 1991) focuses on these integrals. However, given the assumption of a normally distributed θ , it is reasonable to consider the integral over a definite range of ability, say ± 3 , rather than $\pm \infty$. Such an approach avoids the difficulties of the unbounded integrals with which Raju's indices must deal (i.e.,

when the pseudo-guessing parameters differ between P_1 and P_2 , the area between the two ICCs may be infinite).

The present investigation used definite integrals without the explicit use of the absolute value function to eliminate the problem of unbounded expressions, as well as the difficulty of differentiating the absolute value function at zero. The following expression provides a single index of differential item functioning based on the area between the two ICC's that is sensitive to non-uniform DIF, and that subsumes the one-and two-parameter IRT models as special cases of the three-parameter model. It is the first of two indices developed as generalized solutions to the problem of using the area between two ICCs to index DIF.

$$Dif_1 = \int_{-\theta_0}^{\theta_c} [P_1(\theta) - P_2(\theta)] d\theta + \int_{\theta_c}^{+\theta_0} [P_2(\theta) - P_1(\theta)] d\theta$$

Eq. 4.12

where $\pm\theta_0$ represents the lower and upper limits of the ability range (± 3 , say), and θ_c is the value of the ability at which the ICCs cross. Note that the sign of Dif_1 is arbitrary, depending only on the choice of P_1 and P_2 .

Eq. 4.12 can be solved exactly.

$$Dif_1 = \left(\frac{1-c_1}{Da_1} \right) \ln \left(\frac{(1+e^{(-Da_1(\theta_c-b_1))})^2}{(1+e^{(Da_1(\theta_0+b_1))})(1+e^{(-Da_1(\theta_0-b_1))})} \right) - \left(\frac{1-c_2}{Da_2} \right) \ln \left(\frac{(1+e^{(-Da_2(\theta_c-b_2))})^2}{(1+e^{(Da_2(\theta_0+b_2))})(1+e^{(-Da_2(\theta_0-b_2))})} \right)$$

Eq. 4.13

Eq. 4.13 provides an exact expression for the area between two ICCs between $\pm\theta_0$, though with an arbitrary sign. The expression subsumes the one- and two-parameter IRT models as special cases of the three-parameter model. That is, using zero for c_1 and c_2 generates the expression for the two-parameter model; further substitution of unity for a_1 and a_2 gives the expression for the one-parameter, or Rasch, model.

Eq. 4.13 assumes non-uniform differential item functioning. That is, the inclusion of θ_c handles the case where the two ICCs cross within the range $\pm\theta_0$. If the ICCs do not cross (indicating uniform DIF), then θ_0 may be substituted for θ_c , resulting in the appropriate expression for uniform DIF. It should be noted that an exact expression for θ_c does not exist for the three-parameter model. The inclusion of the additive pseudo-guessing parameter prevents an exact solution. Hence, θ_c must be estimated using numerical techniques. Given the well-behaved nature of the logistic function, the binary search method proved quite efficient for the estimation of θ_c .

Sources of Error in Dif₁

Dif₁ is a function of six parameters, three for each of the two ICCs that bound the area in question. These parameters are a_1 , b_1 , c_1 , a_2 , b_2 , and c_2 . Given exact

values for the parameters, an exact value for the area can be computed. However, exact values of these parameters are never known. Instead, parameter estimates are computed using computer programs such as LOGIST by Wingersky, Barton, and Lord (1982) from examinees' response vectors. Using these estimates and associated uncertainties, Dif_1 and its uncertainty can be estimated.

Using Eq. 4.9, the uncertainty of Dif_1 can be expressed in terms of 6 partial derivatives, 6 variances, and 6 covariances.

$$\begin{aligned}
\sigma_{Dif_1}^2 = & \left(\frac{\partial Dif_1}{\partial a_1} \right)^2 \sigma_{a_1}^2 + \left(\frac{\partial Dif_1}{\partial b_1} \right)^2 \sigma_{b_1}^2 + \left(\frac{\partial Dif_1}{\partial c_1} \right)^2 \sigma_{c_1}^2 \\
& + \left(\frac{\partial Dif_1}{\partial a_2} \right)^2 \sigma_{a_2}^2 + \left(\frac{\partial Dif_1}{\partial b_2} \right)^2 \sigma_{b_2}^2 + \left(\frac{\partial Dif_1}{\partial c_2} \right)^2 \sigma_{c_2}^2 \\
& + 2 \left(\frac{\partial Dif_1}{\partial a_1} \right) \left(\frac{\partial Dif_1}{\partial b_1} \right) \sigma_{a_1 b_1} + 2 \left(\frac{\partial Dif_1}{\partial a_1} \right) \left(\frac{\partial Dif_1}{\partial c_1} \right) \sigma_{a_1 c_1} \\
& + 2 \left(\frac{\partial Dif_1}{\partial b_1} \right) \left(\frac{\partial Dif_1}{\partial c_1} \right) \sigma_{b_1 c_1} \\
& + 2 \left(\frac{\partial Dif_1}{\partial a_2} \right) \left(\frac{\partial Dif_1}{\partial b_2} \right) \sigma_{a_2 b_2} + 2 \left(\frac{\partial Dif_1}{\partial a_2} \right) \left(\frac{\partial Dif_1}{\partial c_2} \right) \sigma_{a_2 c_2} \\
& + 2 \left(\frac{\partial Dif_1}{\partial b_2} \right) \left(\frac{\partial Dif_1}{\partial c_2} \right) \sigma_{b_2 c_2}
\end{aligned}$$

Eq. 4.14

The ratio,

$$\frac{Dif_1}{\sigma_{Dif_1}},$$

defines the z-statistic, and provides a test statistic that can be used to determine if the area between the ICCs is statistically, significantly different from zero. Or, in terms of null and alternative hypotheses, to test $H_0: DIF = 0$ against the alternative hypothesis, $H_A: DIF \neq 0$. Henceforth in this research, the term, Dif_1 , will refer to the above ratio, not to the earlier definition of the area between the response functions.

The Second Index of DIF

While Dif_1 avoids the problems of different equations for different IRT models and the dependence on the distinction between the signed and unsigned areas to handle uniform and non-uniform DIF, it does introduce the not-so-minor complication of the estimation of θ_c , the ability at which the ICCs cross. The second index, Dif_2 , avoids the complication of estimating θ_c while providing a single index for all six conditions. That is, an index based on the three-parameter IRT model that subsumes the one- and two-parameter models that is also sensitive to non-uniform differential item functioning. It is obtained by computing the squared difference between the two ICCs and integrating between $\pm\theta_0$ as shown in the following equation.

$$Dif_2 = \int_{-\theta_0}^{\theta_0} (P_1(\theta) - P_2(\theta))^2 d\theta$$

Eq. 4.15

This research found no closed solution for Eq. 4.15. The "cross" term, $P_1(\theta)P_2(\theta)$, forces a numerical solution. Fortunately, the logistic function is very well behaved and a numerical solution was easily obtained using even the simplest techniques of numerical integration..

As with Dif_1 , Dif_2 is a function of the two sets of a, b, and c parameters and the two sets of uncertainties. The uncertainty of Dif_2 is given by the following equation, identical in form to that for Dif_1 .

Eq. 4.16

$$\begin{aligned} \sigma_{Dif_2}^2 = & \left(\frac{\partial Dif_2}{\partial a_1} \right)^2 \sigma_{a_1}^2 + \left(\frac{\partial Dif_2}{\partial b_1} \right)^2 \sigma_{b_1}^2 + \left(\frac{\partial Dif_2}{\partial c_1} \right)^2 \sigma_{c_1}^2 \\ & + \left(\frac{\partial Dif_2}{\partial a_2} \right)^2 \sigma_{a_2}^2 + \left(\frac{\partial Dif_2}{\partial b_2} \right)^2 \sigma_{b_2}^2 + \left(\frac{\partial Dif_2}{\partial c_2} \right)^2 \sigma_{c_2}^2 \\ & + 2 \left(\frac{\partial Dif_2}{\partial a_1} \right) \left(\frac{\partial Dif_2}{\partial b_1} \right) \sigma_{a_1 b_1} + 2 \left(\frac{\partial Dif_2}{\partial a_1} \right) \left(\frac{\partial Dif_2}{\partial c_1} \right) \sigma_{a_1 c_1} \\ & + 2 \left(\frac{\partial Dif_2}{\partial b_1} \right) \left(\frac{\partial Dif_2}{\partial c_1} \right) \sigma_{b_1 c_1} \\ & + 2 \left(\frac{\partial Dif_2}{\partial a_2} \right) \left(\frac{\partial Dif_2}{\partial b_2} \right) \sigma_{a_2 b_2} + 2 \left(\frac{\partial Dif_2}{\partial a_2} \right) \left(\frac{\partial Dif_2}{\partial c_2} \right) \sigma_{a_2 c_2} \\ & + 2 \left(\frac{\partial Dif_2}{\partial b_2} \right) \left(\frac{\partial Dif_2}{\partial c_2} \right) \sigma_{b_2 c_2} \end{aligned}$$

The partial derivatives of Eq. 4.16 include integrals with the $P_1(\theta)P_2(\theta)$ cross term, thus preventing a closed-form solution. Numerical techniques must be used to evaluate each of the twelve terms in Eq. 4.16. Again because of the well behaved nature of the logistic function, these integrals all converged rapidly in this research, even with simple numerical integration techniques.

The distribution of the ratio,

$$\frac{Dif_2}{\sigma_{Dif_2}},$$

is more difficult to determine than the earlier ratio involving Dif_1 , as it involves the integral of a squared difference, and will be examined in detail in the next chapter. Henceforth in this research, the term Dif_2 will refer to the above ratio, not the earlier definition of the integral of the squared difference of probabilities.

Study 1

To compare the sensitivity of Dif_1 , Dif_2 , Raju's (1990) UA_{32} , and the Mantel-Haenszel χ^2 statistic to fluctuations in the item discrimination parameter, 100 "items" were generated in which the difficulty and guessing parameters were held to the "ideal" values of 0 and 0.2 (for 5-choice multiple choice items) while the value of the discrimination parameter for the focal group, a_{focal} was allowed to range uniformly from 0.5 to 2.0. The value of the discrimination parameter for the reference group was held at 1.0, the "ideal" value.

A sample of 5000 examinee abilities was selected from a random normal distribution for the focal group; another sample of 5000 examinee abilities was selected from another random normal distribution. The mean of the abilities of the subjects in both groups was zero; the standard deviations were both unity. These two groups of Monte-Carlo subjects were used in all the Monte-Carlo studies in this research. While it has been demonstrated that IRT studies using LOGIST (Wingersky, Barton, & Lord, 1982) do not require such large sample sizes, this research involved large samples to remove any threat of estimation instability from the results.

Dif₁, Dif₂, and UA₃₂ were calculated directly from the data using the equations for the variance-covariance matrix (Hambleton, Swaminathan, & Rogers, 1990). The Mantel-Haenszel statistic was computed by "administering" the items of known IRT parameters to the two groups of Monte-Carlo subjects. The resulting response vectors were processed using the SAS (SAS is a registered trademark of SAS Institute Inc., of Cary, NC.) program from Harnisch (1991). In Study 1, as in Studies 2 and 3, the agreement between the four DIF indices was examined by plotting the indices as functions of a_{focal} .

Study 2

To compare the sensitivity of Dif₁, Dif₂, Raju's (1990) SA₃₀, and the Mantel-Haenszel χ^2 statistic to fluctuations in the item difficulty parameter, 100 "items" were generated in which the discrimination and guessing parameters were held to the ideal values of 1 and 0.2 (5-choice multiple choice items) while the value of the difficulty

parameter of the focal group, b_{focal} , is varied uniformly from -3 to +3. The difficulty parameter of the reference group was held at 0, the "ideal" value. Analytical procedures in Study 1 were repeated here.

Study 3

To compare the sensitivity of Dif_1 , Dif_2 , (Raju's work will not be considered in this study due to his requirement that $c_1 = c_2$.) and the Mantel-Haenszel χ^2 statistic to fluctuations in the item guessing parameter, 100 "items" were generated in which the difficulty and discrimination parameters are held to their "ideal" values of 0 and 1 while the value of the guessing parameter for the focal group, c_{focal} , was allowed to vary uniformly from 0.0 to 0.5. The guessing parameter for the reference group was held to 0.2, the ideal value for a five-option multiple choice question. As before, the analytical procedures of Study 1 were repeated here.

Study 4

To examine the behavior of the proposed indices with real data, an investigation of the indices using of the verbal reasoning section of the 1989 administration of the GMAT for male black and white examinees was undertaken. Males are chosen for this study because too few female blacks take this exam, and even fewer score high enough to meet the LOGIST entry requirement for analysis.

This study will include first an analysis of the items as a group for each race to examine the assumption of unidimensionality, followed by an examination of the individual items in which DIF was estimated using Dif_1 , Dif_2 and the Mantel-Haenszel statistic.

Principal axis factor analysis was used to examine the assumption of unidimensionality for both black and white examinees. Item loadings on the primary factor for black and white examinees were examined to determine whether the same general factor emerges for black and white examinees. Pearson correlations and scatter plots were used to assess the correspondence between the factor loadings.

The LOGIST program (Wingersky, Barton, and Lord, 1982) was used to estimate the item parameters for the black and white examinees. The groups of Black and White subjects were split into two halves each to allow "Black vs. Black" and "White vs. White" comparisons of the item parameters to examine the stability of the estimates within ethnic group. "Black vs. White" comparisons were made to establish between-group differences.

These comparisons were made by regressing the "a" parameters for one group onto the "a" parameters of the other group. This was repeated for the "b" and "c" parameters. Thus, the comparison of the three parameters across the three groups (Black-Black, White-White, and Black-White) resulted in nine separate regression analyses.

The differential functioning of the individual items between black and white examinees was indexed using Dif_1 , Dif_2 , and the Mantel-Haenszel χ^2 statistic. The three indices were listed by item in tabular form.

Study 5

This task involves the analysis of the items of the verbal reasoning section of the 1989 administration of the GMAT for white male and female examinees. This

first entailed an analysis of the items as a group for each gender to test the assumption of unidimensionality, followed by an examination of the individual items in which DIF was estimated by Dif_1 , Dif_2 and the Mantel-Haenszel statistic.

Principal axes factor analysis was used to examine the assumption of unidimensionality for both male and female examinees. Item loadings on the primary factor for male and female examinees were examined to test that the dimension for male examinees is equivalent to that for female examinees. As in Study 4, Pearson correlation and scatter plots were used to examine the correspondence between the factor loadings between male and female examinees.

The LOGIST program (Wingersky, Barton, and Lord, 1982) was used to estimate the item parameters for the male and female examinees. The two groups of male and female subjects were each split into halves to allow "Male vs. Male" and "Female vs. Female" comparisons of the item parameters to examine the stability of the estimates within gender group. "Male vs. Female" comparisons were made to establish between group differences.

These comparisons were made by the regression of the "a" parameters for one group onto the "a" parameters of the other group. This was repeated for the "b" and "c" parameters. Thus, the comparison of the three parameters across the three groups (Female-Female, Male-Male, and Female-Male) resulted in nine separate regression analyses.

The differential functioning between male and female examinees of the individual items was indexed using Dif_1 , Dif_2 , and the Mantel-Haenszel χ^2 statistic. The three indices were listed by item in tabular form.

Summary

Two new indicators of differential functioning were proposed. Each indicator has an associated test of statistical significance which was derived from the uncertainties of the IRT parameter estimates. These two new indicators were compared to the Mantel-Haenszel χ^2 statistic as a measure of effectiveness, the Mantel-Haenszel statistic being the only accepted indicator of differential functioning to date. Some comparisons to Raju's SA_{30} and UA_{32} were made, but these comparisons were limited due to Raju's requirement that $c_1 = c_2$. These indicators were compared using Monte-Carlo data where indicator sensitivity to changes in the individual IRT parameters was examined. They were also compared using data from the 1989 administration of the GMAT.

CHAPTER V

RESULTS OF THE STUDIES

Monte Carlo Study of the Distribution of Dif_1 Index

The Monte-Carlo simulation to examine the distribution of Dif_1 was based on two groups of 5000 subjects of identical ability taking a test of 200 items. The values of ability were generated by the SAS program listed in Appendix B; the values are listed in Appendix C. These abilities were used in all the Monte-Carlo studies conducted in this research. Maintaining this consistent group of abilities insures that the results of each study can be attributed to variations in item parameters, not to fluctuations in subjects' ability distributions.

The IRT parameters of the 200 items for the focal and reference groups were created by the SAS program listed in Appendix B; the parameter values are listed in Appendix D. Six independent streams of random normal deviates were used to create the two sets of parameters. For this study there was no relationship between any pair of parameters within or between the focal and reference groups; all covariances are zero. Admittedly, these simulation data are slightly different from data obtained in practice, where a weak-to-moderate covariance structure exists. The 9-by-9 correlation matrix of the IRT parameters for a sample of Black Males, White Males, and White Females who took the Fall 1986 GMAT is listed in Table 2. It is clear from this matrix that the within-group correlations are moderate at best, usually 0.5

		Black Males			White Males			White Females		
		a	b	c	a	b	c	a	b	c
Black Males	a	.82 (.00)	.36 (.00)	.49 (.00)	.34 (.00)	.02 (.84)	-.16 (.12)	.32 (.00)	.07 (.54)	-.15 (.18)
	b		.99 (.00)	.40 (.00)	.12 (.26)	.79 (.00)	.04 (.69)	-.01 (.95)	.81 (.00)	-.05 (.62)
	c			.80 (.00)	-.05 (.66)	.03 (.81)	.01 (.96)	-.06 (.62)	.03 (.76)	.07 (.55)
White Males	a				.70 (.00)	.29 (.01)	.09 (.43)	.82 (.00)	.25 (.02)	-.09 (.43)
	b					.95 (.00)	.18 (.10)	.12 (.25)	.97 (.00)	-.01 (.94)
	c						.57 (.00)	-.27 (.01)	.11 (.30)	.43 (.00)
White Females	a	Diagonal entries refer to the correlations of the parameters computed by splitting the demographic groups in half.						.82 (.00)	.14 (.19)	.00 (.98)
	b								.97 (.00)	.04 (.72)
	c									.57 (.00)

Table 2
Correlations of IRT parameters from GMAT data

or much lower. Between the groups, the only noteworthy correlations are those between corresponding parameters (i.e., The correlation of the "b" parameters between the Black and White Male groups was 0.79.) The only points at which the simulation data violate the trends we see in the correlation matrix from the GMAT data are the correlations between corresponding item parameters from different comparison groups. Here the simulation uses a zero correlation, where the GMAT data indicates a varying degree of association. For instance, while the correlation of the "b" parameters between the Black and White Male groups was 0.79, the correlation of the "c" parameters between the same groups was 0.005. Both of these are serious deviations from the ideal case where no differential functioning exists and where the parameters would be identical across comparison groups. Given the difficulty with which the correlation between corresponding indices between comparison groups can be anticipated and the "distance" that the GMAT data appears to be removed from the ideal situation of equal indices, this simulation was designed to model the "worst-case" situation where there is no association between any indices at all.

The population mean and standard deviation of subject ability, θ , was set at 0 and 1, respectively. Following Warm's (1978) recommendation based upon a theoretical and empirical examination of a large number of results, the mean and standard deviation of the discrimination index, "a", for both groups were set to 1 and 0.3, respectively; the mean and standard deviation of the pseudo-guessing parameter were set to 0.20 and 0.05, respectively.

Dif_1 was computed using the Turbo Pascal program listed in Appendix A; the user-defined procedures and functions are also listed in Appendix A. This code was later used in the study of the distribution of Dif_2 . Dif_1 , redefined as the ratio of the inter-IRF area to its standard error, was expected to follow a standard normal distribution; this simulation offered little technical statistical evidence of such a distribution. To wit, the value of the Kolmogorov Goodness-of-Fit (Conover, 1980) test statistic, T , is 0.125 ($T_0=0.0853$, $\alpha=.10$, $n=200$). However, there appears to be more to this story than can be represented in a goodness-of-fit index. Figure 1 illustrates the cumulative probability of the Dif_1 distribution in relation to that of the unit normal curve. While these data offer little statistical evidence to support the assumption of a normal distribution for Dif_1 the visual evidence was quite clear. Dif_1 follows a distribution that is approximately normal, though leptokurtic. Hence, Dif_1 can be interpreted as an approximate z statistic, though the value of α used in hypothesis testing will need to be amended to account for the conservative nature of the test due to its leptokurtic distribution.

Monte-Carlo Study of the Distribution of Dif_2

This simulation used the same subjects and items as used in the prior study; the simulated θ 's are listed in Appendix C; the item parameters are listed in Appendix D. As before, Dif_2 was computed with the Turbo Pascal program listed in Appendix A with user-defined procedures and functions also listed in Appendix A.

Dif_2 , as redefined to be the ratio of the integral of the "squared" inter-IRF area to its standard error, was expected to follow the distribution of the absolute value of

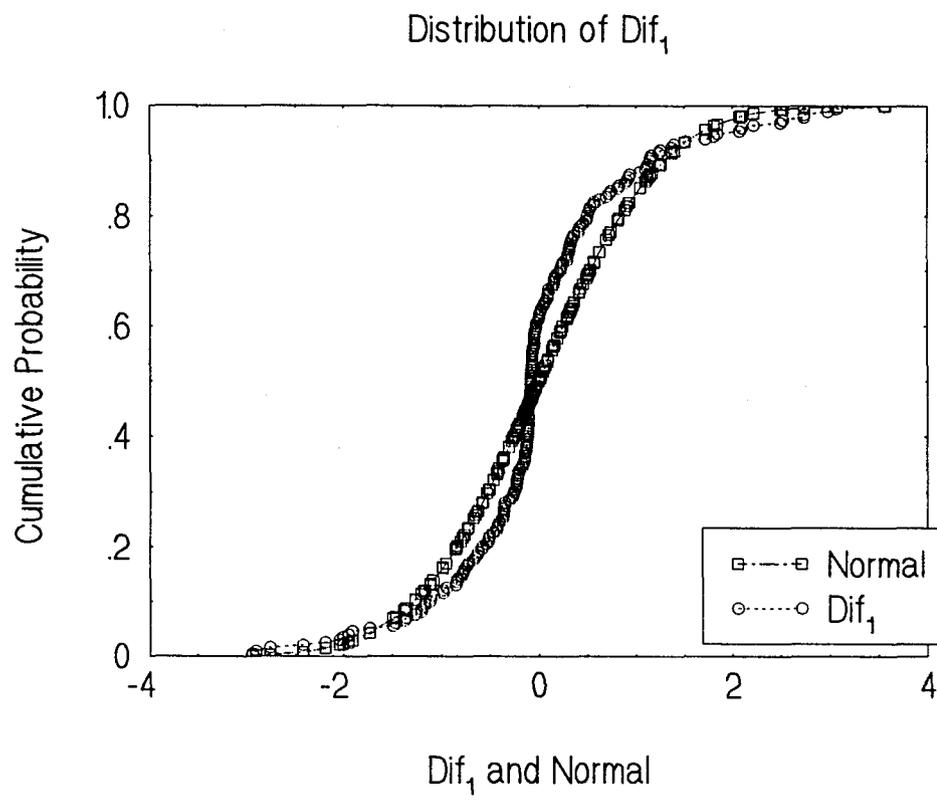


Figure 1

the normal curve. Raju (1990) posits a similar case in his discussion of the distribution of unsigned area indices. To assess this empirical distribution, half of the values of Dif_2 were selected at random and negated. If Dif_2 is distributed as the absolute value of z , then negating half of the values should form the "negative side" of the normal distribution. These data supported this contention. Figure 2 shows the cumulative probability curve of Dif_2 superimposed on that of the unit normal. The Kolmogorov Goodness-of-Fit test statistic (Conover, 1980), $T=0.0880$ ($T_0=0.0853$, $\alpha=0.10$, $n=200$) almost confirms that the distribution of Dif_2 is normal. In light of the graphical evidence presented in Figure 2 in conjunction with the goodness-of-fit index, it is clear that Dif_2 follows an approximately normal distribution, though without the leptokurtic nature seen in the distribution of Dif_1 .

Monte-Carlo Study of the Effect of Variations in the IRT "a" Parameter on Indices of DIF

To study the effect of variations of the "a" parameter on the values of Dif_1 , Dif_2 , UA_{32} , and the Mantel-Haenszel χ^2 , 100 items were generated. The IRT parameter values, (a, b, c), for the reference group were held at the ideal values (1.0, 0.0, 0.2, respectively), while, for the focal group, the dyad (b, c) was held at the ideal values of (0.0, 0.2) and the discrimination parameter, "a", was varied from 0.1 to 2.0 in increments of 0.019. Hence, the discrimination parameter for the first item was 1.0 in the reference group and 0.100 in the focal group. For the second item, it was 1.0 for the reference group and 0.119 for the focal group, and so on. This

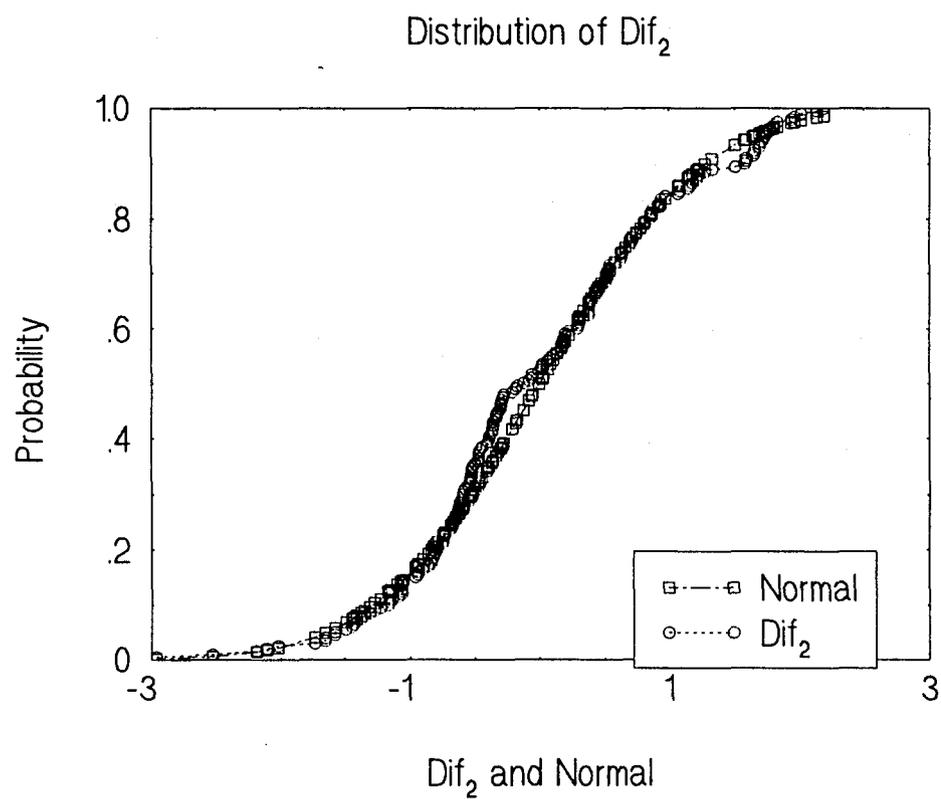


Figure 2

progression was continued for the entire set of 100 items. The 100th item thus had a discrimination index of 1.0 for the reference group and 2.0 for the focal group.

The same 5000 Monte-Carlo "subjects" mentioned earlier were used in this investigation, ensuring that the abilities of both the reference and focal subjects were equal. The only difference between the groups was the planned difference in the discrimination indices. Hence, any variations in the DIF indices can be attributed directly to differences in the a parameter.

Given that the only difference between the reference and focal groups is the value of the discrimination indices, the resulting DIF for all items will be non-uniform. Recall that the IRT discrimination index is defined as the slope of the item characteristic curve at the point where the probability of a correct response is 0.5. Different discrimination indices, paired with constant (and equal) difficulties and pseudo-guessing parameters across groups result in items that favor one group at abilities below the mean and the other group at abilities above the mean. While difficult to interpret, this type of differential functioning is commonly found in IRT studies of DIF.

The values of Dif_1 , Dif_2 , and UA_{32} were computed using the Turbo Pascal program listed in Appendix A. Procedures and functions called by this code are also listed in Appendix A. The Mantel-Haenszel statistic was computed using the SAS program by Harnisch (1991) listed in Appendix B. Data for the Mantel-Haenszel computation were generated by "administering" the test of 100 items to the 5000

Monte-Carlo subjects; this simulation was performed using the program listed in Appendix B.

Figure 3 displays the values of the four DIF indices as functions of the discrimination parameter. First, these results confirm a well known fact, namely, the Mantel-Haenszel statistic is not sensitive to non-uniform DIF. All Mantel-Haenszel χ^2 values appear to be random fluctuations above zero and none were statistically significant.

The Raju UA_{32} statistic increased slowly from $a_{focal}=0.1$ to $a_{focal}\approx 0.7$, then increased rapidly to an anomalous maximum at $a_{focal}\approx 0.9$, after which it fell sharply to 0 at $a_{focal}=1$. After that, it grew slowly until $a_{focal}\approx 1.4$ where it became uncomputable due to limitations in the Turbo Pascal, version 7.0, compiler to compute exponentials.

The Dif_1 statistic was negative for negative values of a_{focal} , reaching an absolute maximum at $a_{focal}\approx 0.45$. This appeared to be the result of a differential increase in the standard error of "a" as the item response curve "flattened," and is indicative of difficulties in estimating IRT parameters for items with poor discrimination. As the standard error of the area between two curves increases, the ratio of the area to its standard error grows smaller indicating that the apparent area is more the result of unstable parameter estimation than differential item functioning, per se. The Dif_1 statistic was positive for positive values of a_{focal} , growing consistently larger as a_{focal} grew.

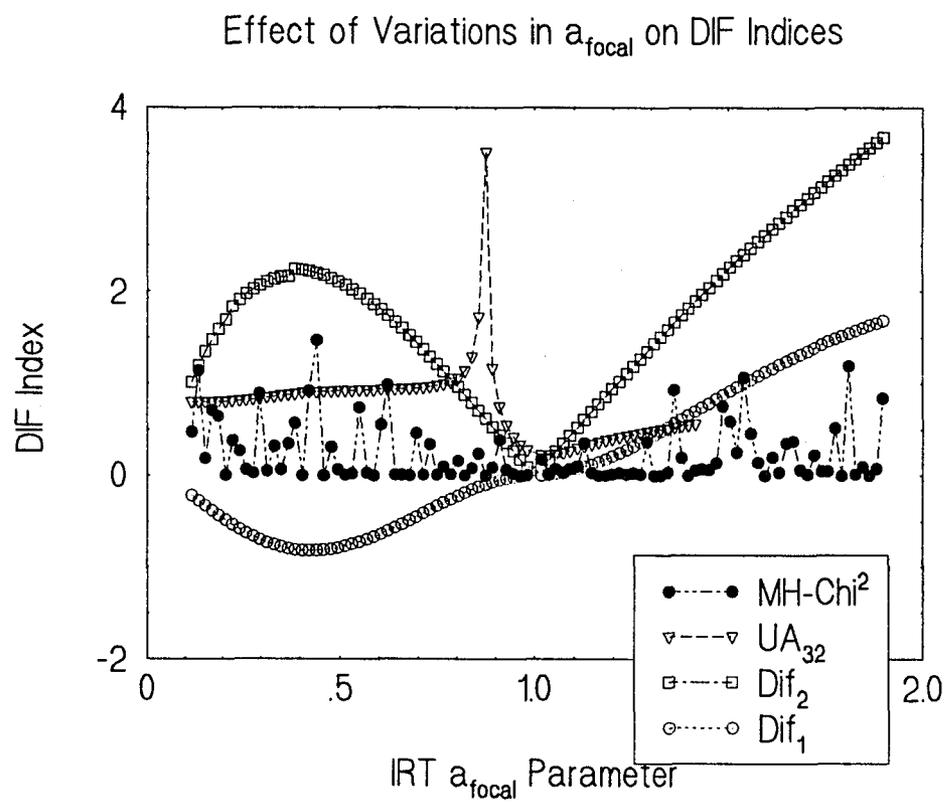


Figure 3

The Dif_2 statistic, based on the integral of the squared difference in the item characteristic curves, increases from a local minimum for $a_{focal} \approx 0.1$ to a local maximum at $a_{focal} \approx 0.5$. As in the case of Dif_1 , the parameter estimates of the "flatter" logistic functions are accompanied by relatively large standard errors. The Dif_2 statistic decreased to zero as a_{focal} went to 1.0. From there, it grew rapidly as a_{focal} grew.

As expected, the agreement, indexed by the Spearman correlation coefficient, between the Mantel-Haenszel statistic, which is insensitive to this type of differential functioning, and the area indices was zero; variations in the Mantel-Haenszel are due solely to statistical fluctuations, not to differences in item performance. The rank-order correlation between Dif_1 and Dif_2 exceeded 0.99; these two indices exhibit virtually identical performance. The correlations between the Raju statistic and Dif_1 and Dif_2 are 0.42 and 0.41. This reduced agreement is due primarily to our inability to compute UA_{32} for the larger values (steeper response functions) of "a." Inspection of the general trend of the UA_{32} graph indicates that, given computing resources with extended numerical precision, the agreement would be much higher, perhaps exceeding 0.90.

These results underscore two difficulties presented to DIF researchers when the underlying source of the differential functioning is a difference in the discrimination indices between the two comparison groups. First, the Mantel-Haenszel index, the DIF test statistic by which all others are judged, is completely insensitive to differences in discrimination. Second, the parameters of the "flatter"

response functions, representing poorly discriminating items, are accompanied by large standard errors resulting in reduced magnitude of the values of the area-based test statistics. Hence, the test developer attempting to construct an instrument of uniformly discriminating items while working within the IRT framework, is faced with the difficult task of simply identifying the items that perform differentially with respect to the discrimination index.

Monte-Carlo Study of the Effect of Variations in the IRT "b" Parameter on Indices of DIF

To study the effect of variations of the "b" parameter on the values of Dif_1 , Dif_2 , SA_{30} , and the Mantel-Haenszel χ^2 , 100 items were generated as in the prior study. The IRT parameter values, (a, b, c) for the reference group were held at the ideal values of (1.0, 0.0, 0.2, respectively), while, for the focal group, the dyad (a, c) was held at the ideal values of (1.0, 0.2) and the difficulty parameter, b, was varied from -3.0 to +3.0 in steps of 0.06. Hence, the value of the difficulty parameter for the first item was 0.0 for the reference group and -3.0 for the focal group. On the second item, it was 0.0 for the reference group and -2.94 for the focal group. This progression continued through the 100 items until item 100 was reached where the value of the difficulty parameter was 0.0 for the reference group and +3.0 for the focal group.

The values of the four DIF indices are presented as a function of item difficulty in Figure 4. It was clear from this plot that the Mantel-Haenszel χ^2 is extremely sensitive to differences in the IRT difficulty parameter. The scale chosen

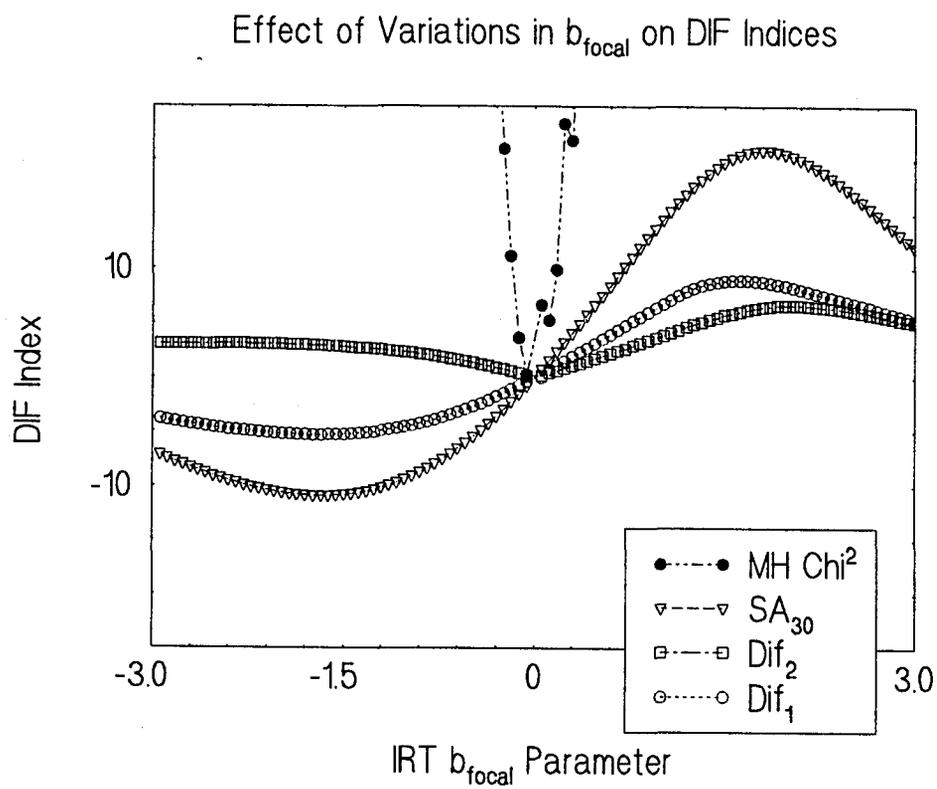


Figure 4

to illustrate the other indices, with their relatively smaller magnitudes, results in values of the Mantel-Haenszel statistic that are off-scale. Closer examination revealed values of this statistic in the thousands, compared to values between 40 and -20 for the other indices.

Raju's SA_{30} was well-behaved in this study. It ranged in value from a low of about -15 at $b_{focal} \approx -1.6$ to a high of about 20 at $b_{focal} \approx 1.6$. The value of SA_{30} is lower at the extreme values of difficulty even though the area between the item characteristic curves is higher because the standard error of the area estimate increases more rapidly than the estimate of the area for those extreme values of ability. This reducing phenomenon is seen again in the other two DIF indices; it was not seen in the Mantel-Haenszel statistic as it is not computed as a ratio like the other three.

Dif_2 was relatively small in magnitude and changed little for negative values of b_{focal} , falling to zero when b_{focal} went to zero. For positive values, it varied more, with a local maximum at about $b_{focal} \approx 2.0$; for larger values of b_{focal} , the magnitude decreased as the growth in the standard error of the estimate of the area exceeds the growth of the area.

The Dif_1 index was also well-behaved across the of b_{focal} ; it was also the largest in magnitude of the area indices. Again, the differential growth of the standard error of the area estimate results in a ratio that decreases rapidly as the value of b_{focal} approaches extreme values. All the DIF indices achieve statistical significance very quickly, usually within two or three steps, of $b_{focal} = 0$. This is indicative

of the differential effect changes in the difficulty parameter have on the probability of a correct response.

In general, the agreement between the three area-based indices and the Mantel-Haenszel (i.e., the extent to which the four indices identified the same items) was not expected to be high for items exhibiting non-uniform DIF since the Mantel-Haenszel is insensitive to such differences in item response functions. Comparisons are possible, however, among the four indices when DIF is simulated for the Rasch model, where the only differences in item response curves are differences in the location parameters. The correlations between Dif_1 , Dif_2 , Raju's SA_{30} , and the Mantel-Haenszel χ^2 are in Table 3.

Table 3

Correlation between Indices for Items that Simulate the Rasch Model

	Dif_1	Dif_2	SA_{30}	MH χ^2
Dif_1	1.00	0.90	0.99	0.39
Dif_2		1.00	0.91	0.59
SA_{30}			1.00	0.38
MH χ^2				1.00

First, note that the correlation between Dif_1 and SA_{30} is quite high. These indices identify virtually the same items as functioning differentially for the focal and reference groups. This result is to be expected inasmuch as the two indices differ only in the limits of the integral used to calculate the area. Raju's SA_{30} is an infinite

integral whereas Dif_1 is finite, stopping at ± 3 . For this reason, the numerator (the area between the response functions) will always be larger for the Raju index, the magnitude of that difference depending entirely on the difference in the two difficulty parameters. For two response functions widely separated, the Raju statistic will include area that lies beyond the ± 3 standard deviations of ability; area beyond those bounds is not included in Dif_1 (or in Dif_2)

To facilitate discussion of the other correlations, the behavior of the four indices as a function of differences in the difficulty parameter (x-axis) is depicted graphically in Figure 4. The y-axis in the figure is the z-statistic in the case of Dif_1 , Dif_2 , and SA_{30} , and is the χ^2 (df=1) statistic in the case of the Mantel-Haenszel. The first point to note is that the Mantel-Haenszel χ^2 quickly reaches statistical significance (viz., when

$$| b_{reference} - b_{focal} | \geq 0.18$$

and continues to increase with increasing differences in the population location (difficulty) parameters between the two groups. The critical value of the Mantel-Haenszel χ^2 is 6.6 for $\alpha=0.01$; the corresponding critical value of z is 2.8.

All three area-based indices also become statistically significant rather rapidly. Dif_1 becomes statistically significant when

$$| b_{reference} - b_{focal} | \geq 0.45 ;$$

Dif₂ becomes statistically significant when

$$| b_{reference} - b_{focal} | \geq 0.72 ;$$

and SA₃₀ becomes statistically significant when

$$| b_{reference} - b_{focal} | \geq 0.24 .$$

None of the IRT indices, however, are monotonically increasing or decreasing with increasing differences in the difficulty parameters. At first glance, this is surprising since the area between the response functions does grow monotonically with increasing differences in the difficulty parameters. The reason for this anomalous result is traceable directly to the standard errors of the parameter estimates. Recall that the ability distribution used for this simulation was normally distributed about zero with a standard deviation of one, and that the difficulty of the item in the reference group is zero. As the difficulty of the item for the focal group moves from zero, the area between the response function increases; so does the standard error of the area, but differentially faster. After about $b_{focal} = \pm 1.5 \theta$'s, the growth of the standard error of the area is such that it begins to dominate the area-based indices, turning them back down towards the x-axis. Why does the standard error grow like this as b_{focal} moves away from the $b_{reference}$? In doing so, it is also moving away from the center of the ability distribution. The farther away from zero b_{focal} is, the fewer subjects there are "under" it. This results in poorer IRT parameter estimations, and correspondingly higher estimates of standard error.

The very modest correlations between the area-based indices of DIF and the Mantel-Haenszel suggest that different items would be flagged by the area indices and the Mantel-Haenszel statistic. The modest correlations result from the non-monotonic behavior of the former, contrasted with the monotonic behavior of the latter. To be sure, the Mantel-Haenszel statistic is behaving "properly" here, while legitimate questions, mostly focused on the estimates of standard error, arise about IRT indices. Note, however, that within reasonable ranges of ability (i.e., $\pm 3 \theta$'s), the area indices for all differences in the location parameters remain highly statistically significant. We will return to a discussion of these points in the next chapter.

Monte-Carlo Study of the Effect of Variations in the IRT "c" Parameter on Indices of DIF

To study the effects of variations of the IRT "c", pseudo-guessing, parameter on the values of Dif_1 , Dif_2 , and the Mantel-Haenszel χ^2 , 100 items were generated as in the prior two studies. No Raju index is considered in this study due to the requirement that the values of the pseudo-guessing parameters of the focal and reference groups must be equal to use his three-parameter indices. The IRT parameter values, (a, b, c) for the reference group were held at the ideal values of (1.0, 0.0, 0.2), while, for the focal group, the dyad (a, b) was held at the ideal values of (1.0, 0.0) and the pseudo-guessing parameter, "c", was varied from 0.05 to 0.35 in steps of 0.003. Hence, the value of the pseudo-guessing parameter for the first item was 0.2 for the reference group and 0.05 for the focal group. On the second item, it was 0.2 for the reference group and 0.0053 for the focal group. This progression

continued through the 100 items until Item 100 was reached where the value of the pseudo-guessing parameter was 0.2 for the reference group and 0.35 for the focal group.

The values of the three DIF indices are presented as a function of the pseudo-guessing parameter in Figure 5. It is clear from this plot that the Mantel-Haenszel χ^2 statistic is quite sensitive to differences in the pseudo-guessing parameters, though not as sensitive as it was for variations in the difficulty parameters. Given a scale to support the other two indices with their relatively smaller magnitudes, the Mantel-Haenszel statistic moved off the plot at $c_{focal} \approx 0.14$ and $c_{focal} \approx 0.28$. A closer examination of the values shows that this statistic grows to nearly a value of 100, while the other indices fall between -1.5 and 1.5. This does not suggest that the Mantel-Haenszel is more sensitive to variations in the pseudo-guessing parameters than the other indices; rather, it indicates a simple disparity in magnitude.

Dif_1 and Dif_2 were both well-behaved in this study. Dif_2 declines smoothly and slowly from a value of about 2.0 at $c_{focal}=0.05$ to a value of zero at $c_{focal}=0.2$. From there, it rises, again smoothly and slowly, to about 2.0 at $c_{focal}=0.35$. This index is smaller in absolute magnitude than Dif_1 because it is computed from the square of the difference in two probabilities, and the square of a number is always smaller in magnitude than the number itself in the range $[0,1)$. It does not appear that the standard error of the estimate of the "squared" area grew disproportionately for the extreme values of c_{focal} as it did for the extreme values of a_{focal} and b_{focal} .

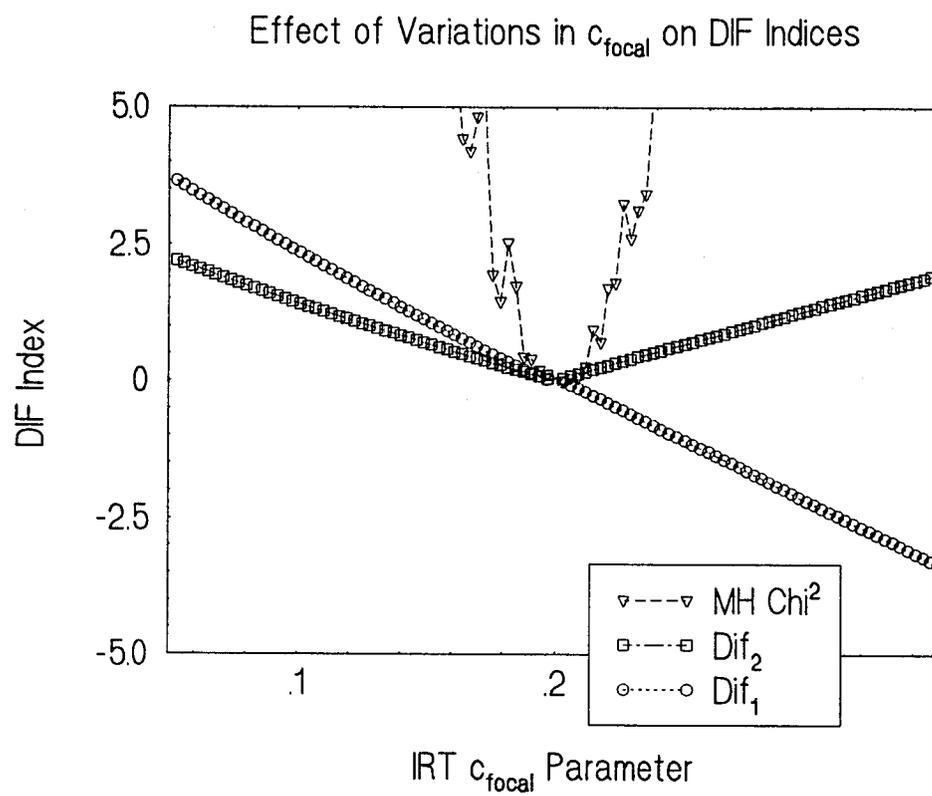


Figure 5

Dif₁ showed no anomalous behavior for any value of c_{focal} ; it declined gradually and smoothly from a maximum of over 10.0 at $c_{focal}=0.05$ to zero at $c_{focal}=0.2$. From there it continued a smooth and gradual decline to a minimum of less than -10.0 at $c_{focal}=0.35$. Again, the standard error of the estimate of the area did not grow disproportionately for the extreme values of c_{focal} as it did for the extreme values of a_{focal} and b_{focal} .

The correspondence between the DIF indices in this study was nothing short of phenomenal. The Spearman correlation between all three pairs exceeded 0.99. While Dif₂, being reduced in magnitude, did not achieve statistical significance over the range of values of c_{focal} investigated, it did vary directly with Dif₁ and the Mantel-Haenszel statistic. The latter achieved statistical significance very quickly as the difference in c_{focal} and $c_{reference}$ grew. Dif₁ became statistically significant at

$$| c_{focal} - c_{reference} | \geq 0.089 \quad ,$$

while the Mantel-Haenszel statistic grew to statistical significance at

$$| c_{focal} - c_{reference} | \geq 0.042 \quad .$$

For these comparisons, the critical values of Dif₁ and the Mantel-Haenszel χ^2 for $\alpha=0.01$ are 2.8 and 6.6, respectively.

The Performance of Dif₁, Dif₂, and the Mantel-Haenszel χ^2 on Data from the 1986 GMAT

Base-line Analysis and Model Fit. To assess the relative performance of Dif₁, Dif₂, and the Mantel-Haenszel χ^2 indices with real data, the three verbal sections of the Fall 1986 administration of the Graduate Management Admissions Test (GMAT), a total of 85 items, were analyzed. The test data were taken from data tapes made available for research purposes by the Graduate Management Admission Council. The tapes contain item-level data on over 200,000 candidates who took the GMAT in the 1986-87 academic year.

To insure trustworthy parameter estimation, 5000 black male, 5000 white male, and 5000 white female examinees were selected at random. Black females were not included since there were too few scoring sufficiently well on the test for stable estimation of parameters using LOGIST. Confirmatory factor analyses of the dichotomous response vectors for each demographic group were performed to examine the assumption of unidimensionality and factor equivalence. These data satisfy the IRT assumption of unidimensionality. The individual factor analyses resulted in a single dominant factor in every case. The percentage of variance accounted by the first factor in the white male, white female, and black male samples were 63, 62, and 62, respectively. Note that in the factor analysis, a small second factor also emerged, but it is relatively minor. Factor equivalence between male and female white examinees was also supported. The correlation between factor loadings on the first unrotated factor of the white male and white female examinees was 0.95.

Factor equivalence across race, however, is seriously questioned with this data set. The correlation between the first unrotated factor loadings for the white male data set and black male data set was a modest 0.50. The correlation between factor loadings on the first unrotated factors for black males and white females was even lower, 0.41. After rotation (varimax), the correlation of the factor loadings between the white male and white female examinees remained 0.95, while that between the black males and white males rose to 0.84. This increase, though a definite improvement, is not sufficient to abandon the question of factor equality. It does appear that the existence of the second factor, though small, is sufficient to suggest that a second dimension is being tapped with the black male examinees.

To gain some baseline information on the stability of the three IRT parameters for these data, six groups were created by splitting the original three groups of 5000 examinees into six groups of 2500 each. The comparison of parameters within groups provides a baseline for the approximate stability of the parameters and allows for an empirical test of the earlier assumption of zero covariances between the errors in estimating the parameters. The 9 x 9 correlation matrix of "a", "b", and "c" parameters between and within the two groups of white male, black male, and white female GMAT examinees is presented in Table 2. The diagonal entries are the correlations between the corresponding estimated parameters within the three groups, Black Male vs. Black Male, White Male vs. White Male, and White Female vs. White Female, and provide an index of the parameter estimates within the individual demographic groups.

The difficulty parameter was predictably the most accurately estimated of the three parameters. Within-group correlations of the "b" parameters for the 85 items ranged from 0.95 to 0.99. The within-group correlations for the "a" and "c" parameters range from .70 to .82 and .57 to .80, respectively. It is noteworthy that, in general, the cross correlations amongst the three parameters are uniformly low, with most hovering around zero. (The one exception to this general rule is the moderately high intercorrelation among the three parameters when estimated using the black male sample.) These results suggest that the assumption made earlier of a null covariance structure for the errors in the estimation of Dif_1 and Dif_2 is not unrealistic.

The relatively high correlations between the estimates of the "b" parameters when using different groups lends support to the "sample invariance" assumption of IRT for these data. But the relatively low correlations between the estimates of the "a" and "c" parameters when using different groups call into question the overall appropriateness of the IRT model for these data. That is, the assumption of sample invariance in estimating IRT parameters appears not to hold for the GMAT. To be sure, the low correlations stems, in part, from the well-known inability of LOGIST to estimate well the "a" and especially the "c" parameter. The high within-group correlations for the same parameter, however, suggest that the sample invariance assumption does hold for different random samples within the three populations.

The lack of factor equivalence for this test across race renders any substantive interpretation of differences in ICCs for black and white samples highly suspect. Inasmuch as the thrust of this investigation is the examination of the statistical

properties of the proposed indices of DIF, rather than substantive interpretation, per se, all difference in ICCs (that is, white male - black male, white male - white female, and black male - white female) were examined. In a substantive investigation, of course, only the within race differences by gender would be meaningful.

GMAT IRT Analysis. To provide a framework for the comparison of these measures of DIF using real data, the tracelines of the item response functions must be considered. These are given in Appendix E for the black and white male examinees; and in Appendix F for the male and female white examinees. To facilitate comparisons, the difficulty estimates of the focal group in the two studies have been transformed by an additive constant. This procedure adds the difference between the means of the difficulty estimates for the reference and focal groups back to the individual difficulty estimates of the focal group, removing any linear differences resulting from different starting points in the LOGIST parameter estimation procedures. Appendix G gives the values of the three parameters for the two groups of black and white males; Appendix H gives these values for the two groups of white males and females. It should be noted that the values in Appendices G and H are taken directly from LOGIST output; no transformation of difficulty values is made.

These data illustrated a technical problem with the Dif_1 index that was not anticipated; some of the IRF's, see Item 7-25 in Appendix E, cross twice, providing an example of a particularly complex form of DIF that has not been referenced in the literature. For ability starting at -3 and advancing to about -1.5, this item favors

black male examinees. This situation reverses for abilities between -1.5 and 1.5 where it favors the white males. Above 1.5, the item reverts back to favoring black males. In general, this item is poorly discriminating for the white males, the probability of a correct response rising essentially linearly from $\theta=-3$ to $\theta=+3$. For the black males it appears to be a fairly easy item to guess. The problem with the Dif_1 index is that it *assumes* that the IRF's cross no more than once, an assumption that is an integral component of the estimation of θ_c (critical theta). This research employed an algorithm that erroneously identified double-crossing IRF's as not crossing at all. The result was that the numerator represented the traditional signed-area, not the arbitrarily signed area, and was, hence, much lower in magnitude than it should have been.

The Turbo Pascal program to compute Dif_1 and Dif_2 for the black and white male examinees and white male and female examinees are given in Appendix A. The Mantel-Haenszel χ^2 statistic is computed using the program listed in Appendix B. Appendix G lists the values of Dif_1 , Dif_2 , and the Mantel-Haenszel χ^2 statistics for the black and white male examinees; Appendix H lists the corresponding information for the white male and female examinees.

The interpretation of the Mantel-Haenszel is usually made through a delta transformation, primarily used by ETS, which places the statistic on a scale with a mean of 13 and a standard deviation of 4. This research examined the Mantel-Haenszel statistic in its original form, a χ^2 with one degree of freedom. This approach was taken to facilitate comparisons with Dif_1 and Dif_2 , both hypothesized to

be distributed as a z statistic. For these comparisons, formal strict hypothesis tests were performed for each item. Since 85 statistical tests were performed for each index, a Bonferroni correction was made to insure an experiment-wise error rate of 0.05. To achieve this required an item-level alpha value of 0.000603; this was computed from $0.05 = 1.0 - (1 - \alpha)^{85}$, where 85 is the number of items being tested. Using this reduced value of α requires the following critical values to achieve statistical significance at the .05 alpha level: Dif_1 and Dif_2 at 3.4, and the Mantel-Haenszel χ^2 at 11.5.

The Mantel-Haenszel χ^2 flagged an astounding 65 of the 85 items as showing differential functioning between Black and White Males; it flagged 23 items between White Females and Males. Neither Dif_1 nor Dif_2 flagged any items as showing differential functioning between the gender groups. However, both Dif_1 and Dif_2 flagged 5 as exhibiting differential functioning between the racial groups. Of these items, 2 were flagged by both Dif_1 and Dif_2 . One item was flagged by Dif_2 only; it was missed by the Mantel-Haenszel χ^2 .

CHAPTER VI

DISCUSSION OF RESULTS

The present study was motivated by two considerations. First, as mentioned earlier, many have noted the desirability of coherent testing programs that are based upon the item response theory model in conception, development, and in all of the reported test statistics. Currently, most large-scale testing programs, while moving rapidly toward the IRT model for measures of cognitive ability, still use classically-based procedures for identifying DIF, primarily the Mantel-Haenszel statistic. Second, the investigation proposed alternatives to Raju's IRT-based indices, one of which, DIF_2 , is a more general area-based formulation of DIF that subsumes as special cases the eight indices Raju's developed for the one-, two-, and three-parameter models.

The complexity of the estimation procedures and the relatively large standard errors in estimating IRT parameters make it unlikely that the proposed index or similar ones advocated by Raju will replace the Mantel-Haenszel statistic in the immediate future. We will need to learn more about the distributional properties of area-based indices in actual applications. (The choice of the GMAT to investigate the indices with a real data set was unfortunate, since an improbably large number of items were identified as functioning differently for blacks and whites by the Mantel-Haenszel χ^2 statistic. This is examined more fully below.) On the basis of the simulation study, which will be discussed presently, the index Dif_2 appears to have

several advantages over the Mantel-Haenszel statistic and Raju's various indices. These are discussed in turn. We will then turn to a discussion of the results using real data, followed by an examination of some issues surrounding "non-uniform" versus "uniform" DIF. The study concludes with suggestions for further research.

Monte-Carlo Simulation

The results of the simulation studies are encouraging. As noted earlier, when the slope parameter alone was varied, that is, when non-uniform DIF, a form of differential functioning as difficult to detect as it is to understand, was introduced, only DIF₂ flagged some items as performing differently between the two comparison groups; the Mantel-Haenszel χ^2 statistic was uniformly non-significant, and Dif₁ and Raju's UA₃₂ never achieved statistical significance. Dif₂, while correlated 0.90 with Dif₁, achieved statistical significance for the steeper response functions. For items that were very poorly discriminating (indexed by response functions with low values of "a"), the standard errors of the parameter estimates were very high, resulting in ratios that were small.

When differences in the lower asymptote were introduced to simulate DIF, there was little to choose from among all four indices since none correlated less than 0.99. Given what we know about the imprecision with which the "c" parameter is estimated, the near-unity correlations between the indices when "c" is varied in the focal and reference group populations may come as a surprise. The answer to this seeming puzzle, however, is straightforward. In the simulation study, the "a" and "b" parameters were held constant, while varying only the "c" parameter. This has

the effect of restricting the possible values of the lower asymptote. The net result is that the resulting item response curves are very accurately estimated. Such is of course not the case with real data, where all three parameters vary simultaneously in complex ways, and where the "c" parameter, in particular, is very difficult to estimate accurately.

Before discussing the behavior of the four indices when the Rasch model holds (that is, when DIF occurs because of differences in the difficulty parameter), a couple of points should be made about the behavior of the Mantel-Haenszel statistic under this model. Holland and Thayer (1988) have shown that when the Rasch model holds, the Mantel-Haenszel χ^2 statistic is the uniformly most powerful statistic for testing the hypothesis that the Mantel-Haenszel odds-ratio equals one, against the alternative hypothesis that the Mantel-Haenszel odds-ratio is not equal to one.

However, Zwick (1990) has shown that when the studied item is included in the matching criterion (as recommended by Holland) and when the distribution of the two groups have different means (as is usually the case), then the Mantel-Haenszel statistic can erroneously produce a conclusion of DIF favoring either of the two groups under study. The actual extent to which the Mantel-Haenszel statistic is susceptible to Type II errors is still being investigated by ETS researchers. This problem notwithstanding, the Mantel-Haenszel statistic has a very good reputation as an index of DIF when the Rasch model holds. Thus, if any proposed index is to be a viable alternative to the Mantel-Haenszel, it should perform in a manner not too radically different from the Mantel-Haenszel.

The principal index proposed here, Dif_2 , and in fact the other area indices as well, meet this test. Although the correlation between Dif_2 and the Mantel-Haenszel statistic was only a modest 0.59 under the Rasch model, and even lower for the other two indices, the actual items identified as showing DIF were virtually identical. The reason for the modest linear correlation between the IRT-based indices of DIF and the Mantel-Haenszel statistic has to do with the behavior of the former indices at the extremes. That is, when simulated differences between the location parameters for the focal and reference group exceed about 1.5, the variability in the estimation of the parameters increased dramatically, compared to the increase in the actual area difference between the two ICCs. However, the erratic behavior of the area indices in extreme instances of DIF was never so great as to render the test statistic non-significant. In even the most extreme cases, the p-value for the area statistics was always less than .0001.

Comparing the Indices Using the GMAT Data Set

Actual tests contain a mixture of items that differ in complex ways with respect to the item parameters that describe them. The GMAT is no exception and, in fact, may be a better example of this fact than other tests because it contains complicated and unusual item types. Comparison of the area-based indices and the Mantel-Haenszel statistic using the GMAT data suggests that much more empirical work needs to be done before a definitive comparison can be made. With the GMAT data, the Mantel-Haenszel appears to be extraordinarily sensitive to differences in white and black ICCs. If the flagging criterion for DIF using the Mantel-Haenszel

Statistic is set at $\alpha=0.05$, then fully 60 of the 85 items on the verbal section of the GMAT would be flagged as functioning differentially for blacks or whites. One may well ask if such a result is reasonable. If 71 percent of the items in a test are problematical in the sense that they are functioning differently for two groups, in what sense can it be claimed that we have estimated the relative status of individuals on some unitary trait? The test developer seems to be in a Catch 22: She must either claim that, on the basis of 85 items, 60 of which are "flawed" in some sense; she can estimate the status of individuals with respect to some unitary ability; or she must claim that, on the basis of 25 "good" items, 60 other items developed to measure the same ability, are in some sense deficient. None of these options seems particularly attractive.

It should be noted that the sensitivity of the Mantel-Haenszel statistic is not merely a function of the large number of items on the GMAT (85) or the large sample sizes ($N = 5000$). Holland was aware of this sensitivity of Mantel-Haenszel χ^2 to small differences in two ICCs, and for this reason decided upon a "practical" criterion for DIF, namely, items are flagged for additional review if the Mantel-Haenszel odds-ratio is greater or equal to 1.10. Even using this criterion, however, 54 of the items were identified as functioning differentially for either blacks or whites.

The "Reality" of Non-Uniform DIF

Much has been made of the fact that the Delta Plot method, the Mantel-Haenszel statistic, and other classically-based indices of DIF are insensitive to non-

uniform DIF. When, for a given item, the population ICCs for the focal group and the reference group cross, members of one group with estimated thetas to the left of the point of intersection are favored, while members of the other group are favored to the right of the intersection point. Green (1991), despairing of discovering any reasonable cognitive, sociological, or psychological theory to explain such a phenomenon, has called it a "fluke" whenever it occurs. Thissen and Wainer's investigation of IRT standard errors and the present study would tend to suggest that such may well be the case. If the null hypothesis of no DIF is true, then Type I errors, when they occur, can result from poor estimation of any of the three parameters, either singly or in combination. Non-uniform DIF occurs when the slope parameters differ for the two groups. Since this parameter is not nearly as well estimated as the location parameter, it would seem that Type I errors involving non-uniform DIF are more likely than Type I errors involving differences in the difficulty parameter.

Whether non-uniform DIF is a fluke or a "real" phenomenon is a question that will ultimately have to be answered, if it is answerable at all, by psychologists and sociologists, not measurement and methodological specialists. If it turns out to be a methodological artifact of our inability to estimate the slope in the IRT model, then much time, intellectual energy, and needless post-hoc theorizing will have been wasted, but the effort is necessary given the importance of tests in the lives of people and the consequent necessity to ensure that they are fair and unbiased. If, on the other hand, it turns out that non-uniform DIF reflects genuine differences in the

populations of interest, then the general area-based index of DIF proposed here may be a viable alternative the Mantel-Haenszel and other classical indices.

A Word on the Mantel-Haenszel Statistic

The results of this study notwithstanding, the Mantel-Haenszel statistic has several desirable properties that should not be lightly dismissed. It is easily computed and understood; the χ^2 is well-known and familiar to virtually all competent researchers, and behaves "properly" in simulation studies where the extent of DIF is known. Furthermore, it is most sensitive to DIF that results from variations in the location parameter, and location is the most important item characteristic.

Its major shortcomings, as this and other studies have demonstrated, are that it is insensitive to non-uniform DIF and that it is overly sensitive to small differences in the location parameter and the lower asymptote. An additional problem with the Mantel-Haenszel statistic is that the odds-ratio, α_{MH} , may not be constant across score levels. The use of a single odds-ratio, then, is a convenient summary statistic that may conceal more than it reveals.

Extant Problems of Parameter Estimation

The inordinate and impractical sample sizes required to bring the standard errors of IRT parameters within reasonable bounds, first noted by Thissen and Wainer (1982) in connection with an earlier version of LOGIST, are still evident, though to a lesser degree, in the most recent version. This is particularly true when the parameters of the response function vary widely from the "ideal" values of

$$\begin{aligned} a &\approx 1.0, \\ b &\approx 0.0, \text{ and} \\ c &\approx 1/n \end{aligned}$$

where n is given by the number of choices in the multiple-choice item. For small values of "a", $a \leq 0.5$, the location (or difficulty) can be shifted well beyond the distribution of examinee ability. As well for these flatter response functions, the lower asymptote may not be approached until ability is several standard deviations below zero. Hence, poorly discriminating items are usually associated with large standard errors resulting in area indices of DIF that are characteristically low in magnitude. The steeper response functions, those where $a > 1.0$, do not show this phenomenon.

A similar phenomenon is seen when the difficulty parameter is beyond about 1.5 standard deviations above or below zero. These items are usually shifted from the mean examinee ability, resulting in fewer subjects under the inflection point of the response function and a corresponding decrease in the accuracy of the estimation of the difficulty parameter. Hence, while the area between the response functions may be growing rapidly, the standard error of the area, if one of the items is very easy or very hard, will grow even more rapidly, resulting in a depressed magnitude of the area-based index and a trend toward decreased statistical significance. Obviously, this trend does not indicate a lessening of differential functioning, but rather a lessening, due to estimation difficulties, of the power of the area-based index to detect the difference.

Cost of IRT Studies

The large sample sizes and the iterative procedures associated with IRT require relatively expensive computing resources compared to classical analyses. IRT area indices of DIF only add to this expense. However, it seems that much of the perspective of "expense" is historical artifact. The IRT analyses using LOGIST on a VAX computer at the University of North Carolina at Greensboro cost about fifty dollars, using local billing procedures, for an analysis of 85 items with 5000 subjects; this does not seem prohibitively expensive. The follow-up DIF studies were run within the Turbo Pascal Interactive Development Environment (IDE) on a 80486 33-megahertz DOS computer. The program to compute all three area DIF indices for 100 items and 5000 subjects rarely took more than five minutes to complete. Again, this did not seem like an inordinate burden. It is perhaps time to re-evaluate the costs of IRT analyses. Given the exponential growth in the power of relatively affordable computers, the benefits of IRT analyses over classical procedures may well be worth the increased expense.

Suggestions for Future Research

Weighting the Probability with the Distribution of Ability. Raju's area indices are computed with indefinite integrals; Dif_1 and Dif_2 developed herein are computed using definite integrals. There is reason to question the efficacy of an index that is computed on a range of abilities that extends beyond that of the examinees. To address this problem, the probabilities, $P_1(\theta)$ and $P_2(\theta)$, could be multiplied by density functions, $\phi_1(\theta)$ and $\phi_2(\theta)$, respectively, where $\Phi(\theta)$ is the normal distribution

function centered empirically on the respective mean abilities of the comparison groups. An alternative to the use of two density functions is the use of a single joint density function that is applied to the difference of $P_1(\theta)$ and $P_2(\theta)$ (e.g., $\phi_{12}(\theta)[P_1(\theta) - P_2(\theta)]$).

Differential Sensitivity of DIF Indices to Changes in Item Parameters. This research demonstrated the extreme sensitivity of the Mantel-Haenszel χ^2 statistic to changes in the IRT difficulty parameter, its moderate sensitivity changes in the pseudo-guessing parameter, and its complete insensitivity to changes in the discrimination parameter. The area indices also showed differential sensitivity to parameter differences. What would be desirable in a DIF index is a differential sensitivity of the *index* to parameter fluctuations that is directly proportional to the differential sensitivity of *total score* to parameter fluctuations. To understand the impact of differential sensitivity, a study of the effect of parameter changes, jointly with ability changes, on the probability of a correct response needs to be done. While a simple and straight-forward study, there is no evidence of such research in the literature. It should be noted that the "weighting" of a parameter's influence on the DIF index should correspond to the weighting of the parameter's influence on total score; this is one of the attractive features of the area indices, and is a theoretically based refinement an earlier definition of DIF (Lord, 1980) where DIF was indexed by a linear function of parameter differences known as the Mahalanobis distance.

Re-visit the Mahalanobis Distance. Given the relative success of the IRT area indices explored in this research, it seems time to re-visit the Mahalanobis multivariate distance statistic. While providing yet another multivariate index of the differences between the reference- and focal-group response functions, it would also provide insight into the effectiveness of weighting the influence of the parameters on the test statistic. On first glance, it would appear that the area between the response functions provides an implicit parameter weighting that is proportional to parameter effect on total test score. In comparison, the Mahalanobis distance, while still incorporating the standard errors of the parameter estimates, is the multivariate distance between the pair of parameter triplets; there is no weighting of one parameter over another in this formulation. Hence, the differences, regardless of origin, between the pairs of response function parameters would affect the test statistic equally, providing a direct assessment of DIF as defined by Lord (1980).

Better Estimation Procedures for Items that Perform Aberrantly. This is perhaps the most difficult problem to address. There will always be fundamental problems with estimating parameter values that require examinee abilities far from the mean. Obviously, the maximum likelihood estimators used in the current release of LOGIST are better than those used in the release studied by Thissen and Wainer (1982). However, this research did encounter problems with parameter values that were "far" from the ideal values in that the associated standard errors prevented the area indices from identifying differential functioning when it was painfully present. For instance, recall the Monte-Carlo study that examined the effect of differences in

the discrimination parameter on the DIF indices. For discrimination values below 0.5, the standard error of the area was so large that it "swamped" the index; the result being that no DIF for these items could be identified. However, the practical solution to this problem may not necessarily be the refinement of parameter estimators; rather, it could be the exclusion of such poorly discriminating items from the test.

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APPENDIX A
PASCAL PROGRAMS

```

Turbo Pascal 7.0 Program Used in Monte Carlo Study of Index
Distributions

program dist;

uses crt;

{$i incs-d12.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;

var
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  theta : array[1..5000] of real;

  theta_cross : real;

  dif_1 : real;
  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;

  dc2 : real;

  subjs : text;
  params : text;

  dist1 : text;
  dist2 : text;

  i : integer;
  j : integer;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading subject abilities');

  assign(subjs,'subjects.dat');
  reset(subjs);
  for i:= 1 to 5000 do readln(subjs,theta[i]);
  close(subjs);

  assign(params,'items.dat');
  reset(params);

  assign(dist1,'dist1.dat');
  rewrite(dist1);

  assign(dist2,'dist2.dat');
  rewrite(dist2);

  {
  calculate var-cov matrix for group 1
  }
  for i := 1 to 200 do
  begin
    clrscr;
    gotoxy(10,10);
    write('Item Number: ',i);

    readln(params,a1,b1,c1,a2,b2,c2);

    gotoxy(10,11);
    write('Computing dispersion for Group 1, Subj:');

    laa := 0; { ron, swami, and jane's mmss book. page
45}
    lab := 0;
    lac := 0;
    lbb := 0;
    lbc := 0;
    lcc := 0;
    for j := 1 to 5000 do
    begin
      gotoxy(50,11);

```

```

write(j);
p_temp := p(theta[j],a1,b1,c1);
q_temp := 1-p_temp;
Iaa := Iaa + sqrt(theta[j]-b1)
        *sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Iab := Iab + (theta[j]-b1)
        *sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Iac := Iac + (theta[j]-b1)
        *(p_temp-c1)
        *q_temp
        /p_temp;

Ibb := Ibb + sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Ibc := Ibc + (p_temp-c1)
        *q_temp
        /p_temp;

Icc := Icc + q_temp
        /p_temp;

end;

Iaa := sqrt(d)/sqrt(1-c1) * Iaa;
Iab := -sqrt(d)*a1/sqrt(1-c1) * Iab;
Iac := d/sqrt(1-c1) * Iac;
Ibb := sqrt(d)*sqrt(a1)/sqrt(1-c1) * Ibb;
Ibc := -D*a1/sqrt(1-c1) * Ibc;
Icc := 1/sqrt(1-c1) * Icc;

{
invert information matrix to get var-cov matrix
use formulae from Neter, Wasserman, and Kutner, page
202
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
        -Iab*(Iab*Icc-Ibc*Iac)
        + Iac*(Iab*Ibc-Ibb*Iac);

var_aa1 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab1 := -(Iab*Icc-Iac*Ibc) /det;
cov_ac1 := (Iab*Ibc-Iac*Ibb) /det;
var_bb1 := (Iaa*Icc-sqr(Iac))/det;
cov_bc1 := -(Iaa*Ibc-Iac*Iab) /det;
var_cc1 := (Iaa*Ibb-sqr(Iab))/det;

gotoxy(10,12);
write('Computing dispersion for Group 2, Subj:');

{
calculate var-cov matrix for group 2
}

```

```

Iaa := 0; { ron, swami, and jane's mmss book. page
45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to 5000 do
begin
gotoxy(50,12);
write(j);
p_temp := p(theta[j],a2,b2,c2);
q_temp := 1-p_temp;
Iaa := Iaa + sqrt(theta[j]-b2)
        *sqrt(p_temp-c2)
        *q_temp
        /p_temp;

Iab := Iab + (theta[j]-b2)
        *sqrt(p_temp-c2)
        *q_temp
        /p_temp;

Iac := Iac + (theta[j]-b2)
        *(p_temp-c2)
        *q_temp
        /p_temp;

Ibb := Ibb + sqrt(p_temp-c2)
        *q_temp
        /p_temp;

Ibc := Ibc + (p_temp-c2)
        *q_temp
        /p_temp;

Icc := Icc + q_temp
        /p_temp;

end; {loop over subject abilities}

Iaa := sqrt(d)/sqrt(1-c2) * Iaa;
Iab := -sqrt(d)*a2/sqrt(1-c2) * Iab;
Iac := d/sqrt(1-c2) * Iac;
Ibb := sqrt(d)*sqrt(a2)/sqrt(1-c2) * Ibb;
Ibc := -D*a2/sqrt(1-c2) * Ibc;
Icc := 1/sqrt(1-c2) * Icc;

{
invert information matrix to get var-cov matrix
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
        -Iab*(Iab*Icc-Ibc*Iac)
        + Iac*(Iab*Ibc-Ibb*Iac);

var_aa2 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab2 := -(Iab*Icc-Iac*Ibc) /det;
cov_ac2 := (Iab*Ibc-Iac*Ibb) /det;
var_bb2 := (Iaa*Icc-sqr(Iac))/det;
cov_bc2 := -(Iaa*Ibc-Iac*Iab) /det;
var_cc2 := (Iaa*Ibb-sqr(Iab))/det;

```

```

{
compute DIF 1 values
}

gotoxy(10,13);
write('Computing Theta_C');
theta_cross := theta_c(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

gotoxy(10,14);
write('Computing DIF 1');
dif_1 := dif1(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,15);
write('Computing DA1');
da1 := dd1_da1(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,16);
write('Computing DB1');
db1 := dd1_db1(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,17);
write('Computing DC1');
dc1 := dd1_dc1(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,18);
write('Computing DA2 for Item ',i);
da2 := dd1_da2(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,19);
write('Computing DB2');
db2 := dd1_db2(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

gotoxy(10,20);
write('Computing DC2');
dc2 := dd1_dc2(theta_0,
    theta_cross,
    a1,b1,c1,
    a2,b2,c2);

sd := sqrt(sqr(da1*var_aa1) +
    sqr(db1*var_bb1) +
    sqr(dc1*var_cc1) +
    sqr(da2*var_aa2) +
    sqr(db2*var_bb2) +
    sqr(dc2*var_cc2) +
    abs(2*da1*db1*cov_ab1) +
    abs(2*da1*dc1*cov_ac1) +
    abs(2*db1*dc1*cov_bc1) +
    abs(2*da2*db2*cov_ab2) +
    abs(2*da2*dc2*cov_ac2) +
    abs(2*db2*dc2*cov_bc2));

gotoxy(10,21);
write('Writing estimates to DIST1');
writeln(dist1,dif_1,' ,sd,' ,dif_1/sd);

{
compute DIF 2 values
}

gotoxy(40,13);
write('Computing DIF-2');
dif_2 := dif2(theta_0,a1,b1,c1,a2,b2,c2,precision);

gotoxy(40,14);
write('Computing DA1');
da1 := ddiff2_da1(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

gotoxy(40,15);
write('Computing DB1');
db1 := ddiff2_db1(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

gotoxy(40,16);
write('Computing DC1');
dc1 := ddiff2_dc1(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

gotoxy(40,17);
write('Computing DA2');
da2 := ddiff2_da2(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

gotoxy(40,18);
write('Computing DB2');
db2 := ddiff2_db2(theta_0,
    a1,b1,c1,
    a2,b2,c2,
    precision);

```

```

gotoxy(40,19);
write('Computing DC2');
dc2 := ddif2_dc2(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write('Writing estimates to DIST2');
writeln(dist2,dif_2,' ',sd,' ',dif_2/sd);

end; { loop over items}

close(params);
close(dist1);
close(dist2);

gotoxy(10,24);
write('Program ended normally. Press any key to exit');
repeat until(keypressed);

end.

```

Procedures and Functions used in Monte Carlo Study of Index Distributions

```
{
This function computes the probability of responding to an item
correctly.
}
```

```
function p(theta : real;
a : real;
b : real;
c : real) : real;
begin
p := c + (1 - c)/(1 + exp(-1.7*a*(theta - b)));
end;
```

```
{
This function computes 1-P.
}
```

```
function q(theta : real;
a : real;
b : real;
c : real) : real;
begin
```

```
q := 1 - p(theta,a,b,c);
end;
```

```
{
This function computes the theta at which the two ICCs cross
}
```

```
function theta_c(theta_0 : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real;
precision : real) : real;
```

```
var
hi_est : real;
lo_est : real;
middle : real;
```

```
begin
lo_est := -theta_0;
hi_est := theta_0;
repeat
middle := (hi_est + lo_est) / 2;
if (p(middle,a1,b1,c1) - p(middle,a2,b2,c2))
*
(p(lo_est,a1,b1,c1) - p(lo_est,a2,b2,c2)) < 0
then hi_est := middle;
else lo_est := middle;
until(abs(hi_est-lo_est)/((hi_est+lo_est)/2) < precision);
theta_c := middle;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
```

```
a1.
}
function dtc_dal (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real) : real;
```

```
const
d = 1.7;
begin
dte_dal := ((1-c1)*(-d)*(theta_c -
b1)*exp(-d*a1*(theta_c-b1))*
sqr(1 + exp(-d*a2*(theta_c - b2))))
/
(-(1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c - b2)))
+(1-c2)*(d*a2)*exp(-d*a2*(theta_c - b2))
*sqr(1 + exp(-d*a1*(theta_c-b1))))
;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
b1.
}
```

```
function dtc_db1 (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real) : real;
```

```
const
d = 1.7;
begin
dte_db1 := (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c-b2)))
/
((1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c-b2)))
-(1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
*sqr(1 + exp(-d*a1*(theta_c-b1))))
;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
c1.
}
```

```
function dtc_dc1 (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real) : real;
```

```
const
d = 1.7;
```

```

begin
  dtc_dc1 :=
    sqrt(1+exp(-d*a2*(theta_c-b2)))*(1+exp(-d*a1*(theta_c-b1)))
      *exp(-d*a1*(theta_c-b1))
    /
    ((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
      *sqrt(1+exp(-d*a1*(theta_c-b1)))
    -
    (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
      *sqrt(1+exp(-d*a2*(theta_c-b2))))
    ;
end;

{
This function computes the first partial derivative of theta_c wrt
a2.
}
function dtc_da2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_da2 := ((1-c2)*(-d)*(theta_c -
b2)*exp(-d*a2*(theta_c-b2))*
  sqrt(1 + exp(-d*a1*(theta_c - b1))))
  /
  (-((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqrt(1+exp(-d*a2*(theta_c - b2)))
  + (1-c1)*(d*a1)*exp(-d*a1*(theta_c - b1))
    *sqrt(1+exp(-d*a1*(theta_c-b1))))
  ;
end;

{
This function computes the first partial derivative of theta_c wrt
b2.
}
function dtc_db2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_db2 := (1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqrt(1+exp(-d*a1*(theta_c-b1)))
  /
  ((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqrt(1+exp(-d*a1*(theta_c-b1)))
  - (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
    *sqrt(1+exp(-d*a2*(theta_c-b2))))
  ;
end;

;

{
This function computes the first partial derivative of theta_c wrt
c2.
}
function dtc_dc2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_dc2 :=
    sqrt(1+exp(-d*a1*(theta_c-b1)))*(1+exp(-d*a2*(theta_c-b2)))
      *exp(-d*a2*(theta_c-b2))
    /
    ((1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
      *sqrt(1+exp(-d*a2*(theta_c-b2)))
    -
    (1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
      *sqrt(1+exp(-d*a1*(theta_c-b1))))
    ;
end;

{
This function computes the partial first derivative of DIF1 wrt
a1.
}
function dd1_da1 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_da1 := -(1-c1)/(d*a1)
    *ln(sqrt(1+exp(-d*a1*(theta_c-b1))))/
    ((1+exp(d*a1*(theta_0+b1)))*(1+exp(-d*a1*(theta_0-b1))))
    - ((1-c1)/a1)*2*(theta_c-b1
  + a1*dtc_da1(theta_c,a1,b1,c1,a2,b2,c2)/
    (1+exp(d*a1*(theta_c-b1)))
  +
  (theta_0+b1)/(1+exp(-d*a1*(theta_0+b1)))
  -
  (theta_0-b1)/(1+exp(d*a1*(theta_0-b1))))
  + 2*(1-c2)*dtc_da1(theta_c,a1,b1,c1,a2,b2,c2)
    /(1+exp(d*a2*(theta_c-b2)))
  ;
end;

```

```

end;

{
This function computes the partial first derivative of DIF1 wrt
b1.
}
function dd1_db1 (theta_0 : real;
                 theta_c : real;
                 a1 : real;
                 b1 : real;
                 c1 : real;
                 a2 : real;
                 b2 : real;
                 c2 : real) : real;

const
  d = 1.7;
begin
  dd1_db1 :=
(1-c1)*(-2*(d*c_db1(theta_c,a1,b1,c1,a2,b2,c2)-1)/
        (1+exp(d*a1*(theta_c-b1)))
        -
        1/(1+exp(-d*a1*(theta_0+b1)))
        -
        1/(1+exp(-d*a1*(theta_0-b1))))
+ (1-c2)*(2*d*c_db1(theta_c,a1,b1,c1,a2,b2,c2)/
        (1+exp(d*a2*(theta_c-b2))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
c1.
}
function dd1_dc1 (theta_0 : real;
                 theta_c : real;
                 a1 : real;
                 b1 : real;
                 c1 : real;
                 a2 : real;
                 b2 : real;
                 c2 : real) : real;

const
  d = 1.7;
begin
  dd1_dc1 := 2*d*c_dc1(theta_c,a1,b1,c1,a2,b2,c2)
            *(1/(1+exp(d*a1*(theta_c-b1)))
            +
            (1-c2)/(1+exp(d*a2*(theta_c-b2))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
a2.
}
function dd1_da2 (theta_0 : real;
                 theta_c : real;
                 a1 : real;
                 b1 : real;
                 c1 : real;
                 a2 : real;
                 b2 : real;
                 c2 : real) : real;

const
  d = 1.7;
begin
  dd1_da2 := (1-c2)/(d*a2)
            *ln(sqrt(1+exp(-d*a2*(theta_c-b2))))/
            ((1+exp(d*a2*(theta_0+b2)))*(1+exp(-d*a2*(theta_0-b2))))
            + ((1-c2)/a2)*(2*(theta_c-b2
            +a2*d*c_da2(theta_c,a1,b1,c1,a2,b2,c2)/
            (1+exp(d*a2*(theta_c-b2)))
            +
            (theta_0+b2)/(1+exp(-d*a2*(theta_0+b2)))
            -
            (theta_0-b2)/(1+exp(d*a2*(theta_0-b2))))
            -2*(1-c1)*d*c_da2(theta_c,a1,b1,c1,a2,b2,c2)
            /(1+exp(d*a1*(theta_c-b1)))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
b2.
}
function dd1_db2 (theta_0 : real;
                 theta_c : real;
                 a1 : real;
                 b1 : real;
                 c1 : real;
                 a2 : real;
                 b2 : real;
                 c2 : real) : real;

const
  d = 1.7;
begin
  dd1_db2 := (1-c2)*(
            2*(d*c_db2(theta_c,a1,b1,c1,a2,b2,c2)-1)/
            (1+exp(d*a2*(theta_c-b2)))
            +
            1/(1+exp(-d*a2*(theta_0+b2)))
            +
            1/(1+exp(-d*a2*(theta_0-b2))))
            - (1-c1)*(2*d*c_db2(theta_c,a1,b1,c1,a2,b2,c2)/
            (1+exp(d*a1*(theta_c-b1))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
c2.
}
function dd1_dc2 (theta_0 : real;
                 theta_c : real;
                 a1 : real;
                 b1 : real;
                 c1 : real;
                 a2 : real;
                 b2 : real;
                 c2 : real) : real;

```

```

a2 : real;
b2 : real;
c2 : real) : real;

const
  d = 1.7;
begin
  dd1_dc2 := -2*dtc_dc2(theta_c,a1,b1,c1,a2,b2,c2)
    *(1/(1+exp(d*a2*(theta_c-b2)))
    +
    (1-c1)/(1+exp(d*a1*(theta_c-b1))))
;
end;

{
This function computes DIF1.
}
function dif1 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dif1 := (1-c1)/(d*a1)*
    ln(sqr(1+exp(-d*a1*(theta_c-b1)))/
    ((1+exp(d*a1*(theta_0+b1)))*
    (1+exp(-d*a1*(theta_0-b1)))))
    -
    (1-c2)/(d*a2)*
    ln(sqr(1+exp(-d*a2*(theta_c-b2)))/
    ((1+exp(d*a2*(theta_0+b2)))*
    (1+exp(-d*a2*(theta_0-b2)))))
;
end;

{
DIF 2 function begin here
}

{
The following functions to compute partial first derivatives of P
wrt
(a,b,c) are based on equation 12-2 from Lord, 1980.
}

{
This function returns the partial first derivative of P wrt a.
}

function dp_da (theta : real;
  a : real;
  b : real;
  c : real) : real;

const
  d = 1.7;
begin
  dp_da := d*(theta-b)*q(theta,a,b,c)*(p(theta,a,b,c)-c)/(1-c);
end;

{
This function returns the partial first derivative of P wrt b.
}

function dp_db (theta : real;
  a : real;
  b : real;
  c : real) : real;

const
  d = 1.7;
begin
  dp_db := -d*a*q(theta,a,b,c)*(p(theta,a,b,c)-c)/(1-c);
end;

{
This function returns the partial first derivative of P wrt c.
}

function dp_dc (theta : real;
  a : real;
  b : real;
  c : real) : real;

const
  d = 1.7;
begin
  dp_dc := q(theta,a,b,c)/(1-c);
end;

{
This function returns the partial first derivative of DIF-2 wrt to
a1.
}

function ddif2_da1 (theta0 : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real;
  precision : real) : real;

var
  theta : real;
  theta_inc : real;
  int_sum : real;
  old_int_sum : real;
  ave : real;

  steps : integer;

```

```

i : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_da(theta,a1,b1,c1);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_da(theta,a1,b1,c1);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddiff2_da1 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
B1.
}

function ddiff2_db1 (theta0      : real;
                    a1          : real;
                    b1          : real;
                    c1          : real;
                    a2          : real;
                    b2          : real;
                    c2          : real;
                    precision: real) : real;
var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;

  i : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_dc(theta,a1,b1,c1);
  end;

  repeat
    for i := 1 to steps do begin;
      theta := -theta0+i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a1,b1,c1);
    end;
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a1,b1,c1);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddiff2_db1 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
C1.
}

function ddiff2_dc1 (theta0      : real;
                    a1          : real;
                    b1          : real;
                    c1          : real;
                    a2          : real;
                    b2          : real;
                    c2          : real;
                    precision: real) : real;
var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;

  i : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_dc(theta,a1,b1,c1);
  end;

  repeat
    for i := 1 to steps do begin;
      theta := -theta0+i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_dc(theta,a1,b1,c1);
    end;
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_dc(theta,a1,b1,c1);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddiff2_dc1 := ave;
end;

```

```

old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
  theta := -theta0 + i*theta_inc;
  int_sum := int_sum + theta_inc*
    2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
    dp_dc(theta,a1,b1,c1);
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

ddif2_dc1 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
A2.
}
function ddif2_da2 (theta0      : real;
  a1      : real;
  b1      : real;
  c1      : real;
  a2      : real;
  b2      : real;
  c2      : real;
  precision: real) : real;
var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;
  i          : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum - theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_da(theta,a2,b2,c2);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum - theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a2,b2,c2);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddif2_db2 := ave;
end;

dp_da(theta,a2,b2,c2);
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

ddif2_da2 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
B2.
}
function ddif2_db2 (theta0      : real;
  a1      : real;
  b1      : real;
  c1      : real;
  a2      : real;
  b2      : real;
  c2      : real;
  precision: real) : real;
var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;
  i          : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum - theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_db(theta,a2,b2,c2);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum - theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a2,b2,c2);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddif2_db2 := ave;
end;

```

```

{
This function returns the partial first derivative of DIF-2 wrt
C2.
}
function ddif2_dc2 (theta0      : real;
                   a1          : real;
                   b1          : real;
                   c1          : real;
                   a2          : real;
                   b2          : real;
                   c2          : real;
                   precision    : real) : real;
var
theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

steps     : integer;

i : integer;

begin
steps := 10;
theta_inc := 2*theta0/steps;

int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0+i*theta_inc;
int_sum := int_sum - theta_inc*
           2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
           dp_dc(theta,a2,b2,c2);
end;

repeat
old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0 + i*theta_inc;
int_sum := int_sum - theta_inc*
           2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
           dp_dc(theta,a2,b2,c2);
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

ddif2_dc2 := ave;
end;

{
This function returns DIF-2.
}
function dif2 (theta0      : real;
              a1          : real;
              b1          : real;
              c1          : real;
              a2          : real;
              b2          : real;
              c2          : real;
              precision    : real) : real;
var
theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

steps     : integer;

i : integer;

begin
steps := 10;
theta_inc := 2*theta0/steps;

int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0+i*theta_inc;
int_sum := int_sum + theta_inc*
           sqrt(p(theta,a1,b1,c1)-p(theta,a2,b2,c2));
end;

repeat
old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0 + i*theta_inc;
int_sum := int_sum + theta_inc*
           sqrt(p(theta,a1,b1,c1)-p(theta,a2,b2,c2));
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

dif2 := ave;
end;

```

```

Turbo Pascal program used to study the effects of
variations in  $a_{local}$  on the area-based indices of DIF

{
This program varies just a2 so that the effect of just differences
in a1 and a2 on Dif-1, Dif-2, and UA32 can be examined.
}

program vary_a;

uses crt;

{$i incs-abc.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;

  sd_a = 0.3;
  items = 100;

var
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  theta : array[1..5000] of real;

  theta_cross : real;

  dif_1 : real;

  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;
  dc2 : real;

  subjs : text;

  dif1_vary_a : text;
  dif2_vary_a : text;
  raju_vary_a : text;

  i : integer;
  j : integer;

  {vars for raju's UA32 computation}

  y : real;
  dhdb1 : real;
  dhdb2 : real;
  dhda1 : real;
  dhda2 : real;
  var_h : real;
  mu_h : real;
  z0 : real;
  mu_ua32 : real;
  var_ua32 : real;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading subject abilities');

  assign(subjs,'subjects.dat');
  reset(subjs);
  for i:= 1 to 5000 do readln(subjs,theta[i]);
  close(subjs);

  assign(dif1_vary_a,'dif1_va.dat');
  rewrite(dif1_vary_a);

  assign(dif2_vary_a,'dif2_va.dat');
  rewrite(dif2_vary_a);

  assign(raju_vary_a,'raju_va.dat');
  rewrite(raju_vary_a);

  a1 := 1;

  b1 := 0;
  b2 := 0.01;

```

```

c1 := 0.2;
c2 := 0.21;

{
compute var-cov matrix for group 1. note that this is
constant for all items.
}

gotoxy(10,11);
write('Computing dispersion for Group 1, Subj:');

Iaa := 0; { ron, swami, and jane's mmss book. page 45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to 5000 do
begin
gotoxy(50,11);
write(j);
p_temp := p(theta[j],a1,b1,c1);
q_temp := 1-p_temp;
Iaa := Iaa + sqrt(theta[j]-b1)
        *sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Iab := Iab + (theta[j]-b1)
        *sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Iac := Iac + (theta[j]-b1)
        *(p_temp-c1)
        *q_temp
        /p_temp;

Ibb := Ibb + sqrt(p_temp-c1)
        *q_temp
        /p_temp;

Ibc := Ibc + (p_temp-c1)
        *q_temp
        /p_temp;

Icc := Icc + q_temp
        /p_temp;
end;

Iaa := sqrt(d)/sqrt(1-c1) * Iaa;
Iab := -sqrt(d)*a1/sqrt(1-c1) * Iab;
Iac := d/sqrt(1-c1) * Iac;
Ibb := sqrt(d)*sqrt(a1)/sqrt(1-c1) * Ibb;
Ibc := -D*a1/sqrt(1-c1) * Ibc;
Icc := 1/sqrt(1-c1) * Icc;

{
invert information matrix to get var-cov matrix
use formulae from Neter, Wasserman, and Kutner, page 202
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
      -Iab*(Iab*Icc-Ibc*Iac)
      +Iac*(Iab*Ibc-Ibb*Iac);

var_aa1 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab1 := -(Iab*Icc-Iac*Ibc) /det;
cov_ac1 := (Iab*Ibc-Iac*Ibb) /det;
var_bb1 := (Iaa*Icc-sqr(Iac))/det;
cov_bc1 := -(Iaa*Ibc-Iac*Iab) /det;
var_cc1 := (Iaa*Ibb-sqr(Iab))/det;

{
begin item calculations
}
for i := 1 to items do
begin

clrscr;
gotoxy(10,10);
write('Item Number: ',i);

a2 := a1 - 3*sd_a + i*sd_a*6/items;

gotoxy(10,12);
write('Computing dispersion for Group 2, Subj:');

{
calculate var-cov matrix for group 2
}

Iaa := 0; { ron, swami, and jane's mmss book. page
45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to 5000 do
begin
gotoxy(50,12);
write(j);
p_temp := p(theta[j],a2,b2,c2);
q_temp := 1-p_temp;
Iaa := Iaa + sqrt(theta[j]-b2)
        *sqrt(p_temp-c2)
        *q_temp
        /p_temp;

Iab := Iab + (theta[j]-b2)
        *sqrt(p_temp-c2)
        *q_temp
        /p_temp;

Iac := Iac + (theta[j]-b2)
        *(p_temp-c2)
        *q_temp
        /p_temp;

Ibb := Ibb + sqrt(p_temp-c2)
        *q_temp
        /p_temp;
end;

```

```

Ibc := Ibc + (p_temp-c2)
      *q_temp
      /p_temp;

Icc := Icc + q_temp
      /p_temp;
end; {loop over subject abilities}

Iaa := sqrt(d)/sqrt(1-c2) * Iaa;
Iab := -sqrt(d)*a2/sqrt(1-c2) * Iab;
Iac := d/sqrt(1-c2) * Iac;
Ibb := sqrt(d)*sqrt(a2)/sqrt(1-c2) * Ibb;
Ibc := -D*a2/sqrt(1-c2) * Ibc;
Icc := 1/sqrt(1-c2) * Icc;

{
invert information matrix to get var-cov matrix
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
      -Iab*(Iab*Icc-Ibc*Iac)
      +Iac*(Iab*Ibc-Ibb*Iac);

var_aa2 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab2 := -(Iab*Icc-Ibc*Iac)/det;
cov_ac2 := (Iab*Ibc-Iac*Ibb)/det;
var_bb2 := (Iaa*Icc-sqr(Iac))/det;
cov_bc2 := -(Iaa*Ibc-Iac*Iab)/det;
var_cc2 := (Iaa*Ibb-sqr(Iab))/det;

{
compute DIF 1 values except for item 50. Here the IRFs
are
identical. At that point, the first derivatives of the
critical ability (where the trace lines cross) are undefined.
}

if i <> 50 then
begin

gotoxy(10,13);
write('Computing Theta_C');
theta_cross := theta_c(theta_0,
                    a1,b1,c1,
                    a2,b2,c2,
                    precision);

gotoxy(10,14);
write('Computing DIF 1');
dif_1 := dif1(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,15);
write('Computing DA1');
da1 := dd1_da1(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

gotoxy(10,16);
write('Computing DB1');
db1 := dd1_db1(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

gotoxy(10,17);
write('Computing DC1');
dc1 := dd1_dc1(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

gotoxy(10,18);
write('Computing DA2');
da2 := dd1_da2(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

gotoxy(10,19);
write('Computing DB2');
db2 := dd1_db2(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

gotoxy(10,20);
write('Computing DC2');
dc2 := dd1_dc2(theta_0,
               theta_cross,
               a1,b1,c1,
               a2,b2,c2);

sd := sqrt(sqr(da1*var_aa1) +
           sqr(db1*var_bb1) +
           sqr(dc1*var_cc1) +
           sqr(da2*var_aa2) +
           sqr(db2*var_bb2) +
           sqr(dc2*var_cc2) +
           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +
           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(10,21);
write('Writing estimates to DIF1_Va');
writeln(dif1_vary_a,a2,' ',dif_1,' ',sd,' ',dif_1/sd);
end; {compute dif-1 except for 50th item}

{
compute DIF 2 values except for the 50th item. the
identical
IRFs result in an area estimate that is too close to zero
for the estimation routine to handle. the problem is in

```

```

computing
the percent changes from one estimate to the next.
}

if i <> 50 then
begin

gotoxy(40,13);
write('Computing DIF-2');
dif_2 := dif2(theta_0,a1,b1,c1,a2,b2,c2,precision);

gotoxy(40,14);
write('Computing DA1');
da1 := ddiff2_da1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,15);
write('Computing DB1');
db1 := ddiff2_db1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,16);
write('Computing DC1');
dc1 := ddiff2_dc1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,17);
write('Computing DA2');
da2 := ddiff2_da2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,18);
write('Computing DB2');
db2 := ddiff2_db2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,19);
write('Computing DC2');
dc2 := ddiff2_dc2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write('Writing estimates to DIF2_Va');
writeln(dif2_vary_a,a2,' ',dif_2,' ',sd,' ',dif_2/sd);

end; {compute dif-2 except for 50th item }

{
compute UA32 from Raju 1990 except for item 50 where
Y is
undefined
}

if i <> 50 then
begin

gotoxy(40,21);
write('Computing UA32');

y := D*a1*a2*(b2-b1)/(a2-a1);
dhdb1 := 1-2*exp(y)/(1+exp(y));
dhdb2 := -dhdb1;
dhda1 :=
(2/sqrt(a2))*((a1*a2*(b2-b1)/(a2-a1)*exp(y)/(1+exp(y))
              -ln(1+exp(y))/d));
dhda2 := -sqrt(a1)/sqrt(a2)*dhda1;
var_h := sqrt(dhdb1)*var_bb1
         +sqrt(dhdb2)*var_bb2
         +sqrt(dhda1)*var_aa1
         +sqrt(dhda2)*var_aa2

         +2*abs(dhdb1*dhda1*cov_ab1)
         +2*abs(dhdb2*dhda2*cov_ab2);
mu_h :=
2*(a2-a1)/(d*a1*a2)*ln(1+exp(d*a1*a2*(b2-b1)/(a2-a1)))
        -(b2-b1);
z0 := -mu_h/sqrt(var_h);
if abs(z0) <= 9
then begin;
mu_ua32 := (1-c1)*(mu_h*(1-2*phi(z0,precision))
            +
            sqrt(2/pi)*sqrt(var_h)*exp(-sqr(z0)/2));
var_ua32 := sqrt(1-c1)*abs(var_h+sqrt(mu_h)
                          -(mu_h*(1-phi(z0,precision))
                          +sqrt(2/pi)*sqrt(var_h)*exp(-sqr(z0)/2)));
end
else begin
mu_ua32 := 0.0;
var_ua32 := 0.001;
end;
gotoxy(40,22);
write('Writing UA32 estimates to RAJU_VA');
writeln(raju_vary_a,a2,' ',mu_ua32,'

```

```
'mu_ua32/sqrt(var_ua32));  
    end; {UA32 computation except for item 50}  
  
end; { loop over items}  
  
close(dif1_vary_a);  
close(dif2_vary_a);  
close(raju_vary_a);  
  
gotoxy(10,24);  
write('Program ended normally. Press any key to exit');  
repeat until(keypressed);  
  
end.
```

User-defined procedures and functions used in the study
of the effects of variations of a_{focal} , b_{focal} , and c_{focal}
on the area-based indices of DIF

```
{
This function computes the probability of responding to an item
correctly.
}
```

```
function p(theta : real;
a : real;
b : real;
c : real) : real;
begin
p := c + (1 - c)/(1 + exp(-1.7*a*(theta - b)));
end;
```

```
{
This function computes 1-P.
}
```

```
function q(theta : real;
a : real;
b : real;
c : real) : real;
begin
q := 1 - p(theta,a,b,c);
end;
```

```
{
This function computes the theta at which the two ICCs cross
}
```

```
function theta_c(theta_0 : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real;
precision : real) : real;
```

```
var
hi_est : real;
lo_est : real;
middle : real;
```

```
begin
lo_est := -theta_0;
hi_est := theta_0;
repeat
middle := (hi_est + lo_est) / 2;
if (p(middle,a1,b1,c1) - p(middle,a2,b2,c2))
*
(p(lo_est,a1,b1,c1) - p(lo_est,a2,b2,c2)) < 0
then hi_est := middle
else lo_est := middle;
until(abs(hi_est-lo_est)/((hi_est+lo_est)/2) < precision);
theta_c := middle;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
```

```
a1.
}
function dtc_da1 (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real) : real;
const
d = 1.7;
begin
dtc_da1 := ((1-c1)*(-d)*(theta_c -
b1)*exp(-d*a1*(theta_c-b1))*
sqr(1 + exp(-d*a2*(theta_c - b2))))
/
(-(1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c - b2)))
+(1-c2)*(d*a2)*exp(-d*a2*(theta_c - b2))
*sqr(1 + exp(-d*a1*(theta_c-b1))))
;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
b1.
}
```

```
function dtc_db1 (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real) : real;
const
d = 1.7;
begin
dtc_db1 := (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c-b2)))
/
((1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
*sqr(1 + exp(-d*a2*(theta_c-b2)))
-(1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
*sqr(1 + exp(-d*a1*(theta_c-b1))))
;
end;
```

```
{
This function computes the first partial derivative of theta_c wrt
c1.
}
```

```
function dtc_dc1 (theta_c : real;
a1 : real;
b1 : real;
c1 : real;
a2 : real;
b2 : real;
c2 : real) : real;
```

```
const
d = 1.7;
```

```

begin
  dtc_dc1 :=
    sqr(1+exp(-d*a2*(theta_c-b2)))*(1+exp(-d*a1*(theta_c-b1)))
      *exp(-d*a1*(theta_c-b1))
      /
      ((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
        *sqr(1+exp(-d*a1*(theta_c-b1)))
        -
        (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
          *sqr(1+exp(-d*a2*(theta_c-b2))))
      ;
end;

{
This function computes the first partial derivative of theta_c wrt
a2.
}
function dtc_da2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_da2 := ((1-c2)*(-d)*(theta_c -
b2)*exp(-d*a2*(theta_c-b2))*
  sqr(1 + exp(-d*a1*(theta_c - b1))))
  /
  (-((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqr(1+exp(-d*a2*(theta_c - b2)))
    +
    (1-c1)*(d*a1)*exp(-d*a1*(theta_c - b1))
    *sqr(1+exp(-d*a1*(theta_c-b1))))
  ;
end;

{
This function computes the first partial derivative of theta_c wrt
b2.
}
function dtc_db2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_db2 := (1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqr(1+exp(-d*a1*(theta_c-b1)))
    /
    ((1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
    *sqr(1+exp(-d*a1*(theta_c-b1)))
    -
    (1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
    *sqr(1+exp(-d*a2*(theta_c-b2))))
  ;
end;

;

{
This function computes the first partial derivative of theta_c wrt
c2.
}
function dtc_dc2 (theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dtc_dc2 :=
    sqr(1+exp(-d*a1*(theta_c-b1)))*(1+exp(-d*a2*(theta_c-b2)))
      *exp(-d*a2*(theta_c-b2))
      /
      ((1-c1)*(d*a1)*exp(-d*a1*(theta_c-b1))
        *sqr(1+exp(-d*a2*(theta_c-b2)))
        -
        (1-c2)*(d*a2)*exp(-d*a2*(theta_c-b2))
          *sqr(1+exp(-d*a1*(theta_c-b1))))
      ;
end;

{
This function computes the partial first derivative of DIF1 wrt
a1.
}
function dd1_da1 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_da1 := -(1-c1)/(d*a1)
    *ln(sqr(1+exp(-d*a1*(theta_c-b1))))/
    ((1+exp(d*a1*(theta_0+b1)))*(1+exp(-d*a1*(theta_0-b1))))
    -
    ((1-c1)/a1)*(2*(theta_c-b1
    +a1*dtc_da1(theta_c,a1,b1,c1,a2,b2,c2)/
    (1+exp(d*a1*(theta_c-b1)))
    +
    (theta_0+b1)/(1+exp(-d*a1*(theta_0+b1)))
    -
    (theta_0-b1)/(1+exp(d*a1*(theta_0-b1))))
    +
    2*(1-c2)*dtc_da1(theta_c,a1,b1,c1,a2,b2,c2)
    /
    (1+exp(d*a2*(theta_c-b2)))
  ;
end;

```

```

end;

{
This function computes the partial first derivative of DIF1 wrt
b1.
}
function dd1_db1 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_db1 :=
(1-c1)*(-2*(d*c_db1(theta_c,a1,b1,c1,a2,b2,c2)-1)/
  (1+exp(d*a1*(theta_c-b1)))
  -
  1/(1+exp(-d*a1*(theta_0+b1)))
  -
  1/(1+exp(-d*a1*(theta_0-b1))))
+ (1-c2)*(2*d*c_db1(theta_c,a1,b1,c1,a2,b2,c2)/
  (1+exp(d*a2*(theta_c-b2))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
c1.
}
function dd1_dc1 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_dc1 := 2*d*c_dc1(theta_c,a1,b1,c1,a2,b2,c2)
  *(1/(1+exp(d*a1*(theta_c-b1)))
  +
  (1-c2)/(1+exp(d*a2*(theta_c-b2))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
a2.
}
function dd1_da2 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_da2 := (1-c2)/(d*a2)
  *ln(sqrt(1+exp(-d*a2*(theta_c-b2))))/
  ((1+exp(d*a2*(theta_0+b2)))*(1+exp(-d*a2*(theta_0-b2))))
  + ((1-c2)/a2)*(2*(theta_c-b2)
  +a2*d*c_da2(theta_c,a1,b1,c1,a2,b2,c2)/
  (1+exp(d*a2*(theta_c-b2)))
  +
  (theta_0+b2)/(1+exp(-d*a2*(theta_0+b2)))
  -
  (theta_0-b2)/(1+exp(d*a2*(theta_0-b2))))
  -2*(1-c1)*d*c_da2(theta_c,a1,b1,c1,a2,b2,c2)
  /(1+exp(d*a1*(theta_c-b1)))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
b2.
}
function dd1_db2 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

const
  d = 1.7;
begin
  dd1_db2 := (1-c2)*(
  2*(d*c_db2(theta_c,a1,b1,c1,a2,b2,c2)-1)/
  (1+exp(d*a2*(theta_c-b2)))
  +
  1/(1+exp(-d*a2*(theta_0+b2)))
  +
  1/(1+exp(-d*a2*(theta_0-b2))))
- (1-c1)*(2*d*c_db2(theta_c,a1,b1,c1,a2,b2,c2)/
  (1+exp(d*a1*(theta_c-b1))))
;
end;

{
This function computes the partial first derivative of DIF1 wrt
c2.
}
function dd1_dc2 (theta_0 : real;
  theta_c : real;
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real) : real;

```

```

a2      : real;
b2      : real;
c2      : real) : real;

const
  d = 1.7;
begin
  dd1_dc2 := -2*dtc_dc2(theta_c,a1,b1,c1,a2,b2,c2)
    *(1/(1+exp(d*a2*(theta_c-b2)))
    +
    (1-c1)/(1+exp(d*a1*(theta_c-b1))))
    ;
end;

{
This function computes DIF1.
}
function dif1 (theta_0 : real;
  theta_c : real;
  a1      : real;
  b1      : real;
  c1      : real;
  a2      : real;
  b2      : real;
  c2      : real) : real;

const
  d = 1.7;
begin
  dif1 := (1-c1)/(d*a1)*
    ln(sqr(1+exp(-d*a1*(theta_c-b1)))/
    ((1+exp(d*a1*(theta_0+b1)))*
    (1+exp(-d*a1*(theta_0-b1)))))
    -
    (1-c2)/(d*a2)*
    ln(sqr(1+exp(-d*a2*(theta_c-b2)))/
    ((1+exp(d*a2*(theta_0+b2)))*
    (1+exp(-d*a2*(theta_0-b2)))))
    ;
end;

{
DIF 2 function begin here
}

{
The following functions to compute partial first derivatives of P
wrt
(a,b,c) are based on equation 12-2 from Lord, 1980.
}

{
This function returns the partial first derivative of P wrt a.
}

function dp_da (theta : real;
  a      : real;
  b      : real;
  c      : real) : real;

const
  d = 1.7;
begin
  dp_da := d*(theta-b)*q(theta,a,b,c)*(p(theta,a,b,c)-c)/(1-c);
end;

{
This function returns the partial first derivative of P wrt b.
}

function dp_db (theta : real;
  a      : real;
  b      : real;
  c      : real) : real;

const
  d = 1.7;
begin
  dp_db := -d*a*q(theta,a,b,c)*(p(theta,a,b,c)-c)/(1-c);
end;

{
This function returns the partial first derivative of P wrt c.
}

function dp_dc (theta : real;
  a      : real;
  b      : real;
  c      : real) : real;

const
  d = 1.7;
begin
  dp_dc := q(theta,a,b,c)/(1-c);
end;

{
This function returns the partial first derivative of DIF-2 wrt to
a1.
}

function ddf2_da1 (theta0      : real;
  a1      : real;
  b1      : real;
  c1      : real;
  a2      : real;
  b2      : real;
  c2      : real;
  precision: real) : real;

var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;

```

```

i : integer;
begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_da(theta,a1,b1,c1);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_da(theta,a1,b1,c1);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddiff2_da1 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
B1.
}

```

```

function ddiff2_db1 (theta0      : real;
                    a1          : real;
                    b1          : real;
                    c1          : real;
                    a2          : real;
                    b2          : real;
                    c2          : real;
                    precision: real) : real;

```

```
var
```

```

theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

```

```
steps : integer;
```

```
i : integer;
```

```

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;

```

```

  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_db(theta,a1,b1,c1);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum + theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a1,b1,c1);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddiff2_db1 := ave;
end;

```

```

{
This function returns the partial first derivative of DIF-2 wrt
C1.
}

```

```

function ddiff2_dc1 (theta0      : real;
                    a1          : real;
                    b1          : real;
                    c1          : real;
                    a2          : real;
                    b2          : real;
                    c2          : real;
                    precision: real) : real;

```

```
var
```

```

theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

```

```
steps : integer;
```

```
i : integer;
```

```

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum + theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_dc(theta,a1,b1,c1);
  end;

  repeat

```

```

old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
  theta := -theta0 + i*theta_inc;
  int_sum := int_sum + theta_inc*
    2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
    dp_dc(theta,a1,b1,c1);
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

ddif2_dc1 := ave;
end;

{
This function returns the partial first derivative of DIF-2 wrt
A2.
}
function ddif2_da2 (theta0      : real;
  a1      : real;
  b1      : real;
  c1      : real;
  a2      : real;
  b2      : real;
  c2      : real;
  precision: real) : real;

var
  theta      : real;
  theta_inc  : real;
  int_sum    : real;
  old_int_sum : real;
  ave        : real;

  steps      : integer;

  i : integer;

begin
  steps := 10;
  theta_inc := 2*theta0/steps;

  int_sum := 0;
  for i := 1 to steps do begin;
    theta := -theta0+i*theta_inc;
    int_sum := int_sum - theta_inc*
      2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
      dp_da(theta,a2,b2,c2);
  end;

  repeat
    old_int_sum := int_sum;
    steps := steps*2;
    theta_inc := 2*theta0/steps;
    int_sum := 0;
    for i := 1 to steps do begin;
      theta := -theta0 + i*theta_inc;
      int_sum := int_sum - theta_inc*
        2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
        dp_db(theta,a2,b2,c2);
    end;
    ave := (old_int_sum+int_sum)/2;
    until((abs(old_int_sum-int_sum)/ave) < precision);

  ddif2_db2 := ave;
end;

```

```

{
This function returns the partial first derivative of DIF-2 wrt
C2.
}
function ddif2_dc2 (theta0      : real;
                   a1          : real;
                   b1          : real;
                   c1          : real;
                   a2          : real;
                   b2          : real;
                   c2          : real;
                   precision   : real) : real;
var
theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

steps     : integer;

i : integer;

begin
steps := 10;
theta_inc := 2*theta0/steps;

int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0+i*theta_inc;
int_sum := int_sum - theta_inc*
           2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
           dp_dc(theta,a2,b2,c2);
end;

repeat
old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0 + i*theta_inc;
int_sum := int_sum - theta_inc*
           2*(p(theta,a1,b1,c1)-p(theta,a2,b2,c2))*
           dp_dc(theta,a2,b2,c2);
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

ddif2_dc2 := ave;
end;

{
This function returns DIF-2.
}
function dif2 (theta0      : real;
              a1          : real;
              b1          : real;
              c1          : real;
              a2          : real;
              b2          : real;
              c2          : real;
              precision   : real) : real;
var
theta      : real;
theta_inc  : real;
int_sum    : real;
old_int_sum : real;
ave        : real;

steps     : integer;

i : integer;

begin
steps := 10;
theta_inc := 2*theta0/steps;

int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0+i*theta_inc;
int_sum := int_sum + theta_inc*
           sqrt(p(theta,a1,b1,c1)-p(theta,a2,b2,c2));
end;

repeat
old_int_sum := int_sum;
steps := steps*2;
theta_inc := 2*theta0/steps;
int_sum := 0;
for i := 1 to steps do begin;
theta := -theta0 + i*theta_inc;
int_sum := int_sum + theta_inc*
           sqrt(p(theta,a1,b1,c1)-p(theta,a2,b2,c2));
end;
ave := (old_int_sum+int_sum)/2;
until((abs(old_int_sum-int_sum)/ave) < precision);

dif2 := ave;
end;

{
this function, used by PHI, computes the unit normal for a
given
value of x.
}
function UN(x:real):real;
begin
UN := 1/sqrt(2*pi)*exp(-sqr(x)/2);
end; {function UN}

{
this function computes the area under the unit normal curve
from negative
infinity to Z0.
}
function phi(z0      : real;
            precision: real):real;
const
infinity = 4;

```

```
var
  i      : integer;
  steps  : integer;
  width  : real;
  sum    : real;
  old_sum : real;
begin
  steps := 100;
  width := (z0-(-infinity))/steps;
  sum := 0;
  for i := 1 to steps do
    sum := sum + UN(-infinity + i*width)*width;
  repeat
    steps := steps*2;
    width := (z0-(-infinity))/steps;
    old_sum := sum;
    sum := 0;
    for i := 1 to steps do
      sum := sum + UN(-infinity + i*width)*width;
      until ((abs(old_sum-sum)/sum) < precision);
    phi := sum;
  end; {end of Unit Normal function}
```

```

Turbo Pascal code used to compute the effects of variations of
bfocal on the area-based indices of DIF

{
This program varies just b2 so that the effect of just differences
in b1 and b2 on Dif-1 and Dif-2 can be examined.
}

program vary_b;

uses crt;

{$i incs-abc.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;

  sd_b = 1;
  items = 100;

var
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  theta : array[1..5000] of real;

  theta_cross : real;

  dif_1 : real;
  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;
  dc2 : real;

  subjs : text;

  dif1_vary_b : text;
  dif2_vary_b : text;
  raju_vary_b : text;

  i : integer;
  j : integer;

  mu_sa30 : real;
  var_sa30 : real;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading subject abilities');

  assign(subjs,'subjects.dat');
  reset(subjs);
  for i:= 1 to 5000 do readln(subjs,theta[i]);
  close(subjs);

  assign(dif1_vary_b,'dif1_vb.dat');
  rewrite(dif1_vary_b);

  assign(dif2_vary_b,'dif2_vb.dat');
  rewrite(dif2_vary_b);

  assign(raju_vary_b,'raju_vb.dat');
  rewrite(raju_vary_b);

  a1 := 1;
  a2 := 1;

  b1 := 0;

  c1 := 0.2;
  c2 := 0.2;

  {
  compute var-cov matrix for group 1. note that this is
  constant for all items.
  }

  gotoxy(10,11);

```

```

write('Computing dispersion for Group 1, Subj:');

Iaa := 0; { ron, swami, and jane's mmss book. page 45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to 5000 do
begin
  gotoxy(50,11);
  write(j);
  p_temp := p(theta[j],a1,b1,c1);
  q_temp := 1-p_temp;
  Iaa := Iaa + sqrt(theta[j]-b1)
    *sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  Iab := Iab + (theta[j]-b1)
    *sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  Iac := Iac + (theta[j]-b1)
    *(p_temp-c1)
    *q_temp
    /p_temp;

  Ibb := Ibb + sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  Ibc := Ibc + (p_temp-c1)
    *q_temp
    /p_temp;

  Icc := Icc + q_temp
    /p_temp;
end;

Iaa := sqrt(d)/sqrt(1-c1) * Iaa;
Iab := -sqrt(d)*a1/sqrt(1-c1) * Iab;
Iac := d/sqrt(1-c1) * Iac;
Ibb := sqrt(d)*sqrt(a1)/sqrt(1-c1) * Ibb;
Ibc := -D*a1/sqrt(1-c1) * Ibc;
Icc := 1/sqrt(1-c1) * Icc;

{
invert information matrix to get var-cov matrix
use formulae from Neter, Wasserman, and Kutner, page 202
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
  -Iab*(Iab*Icc-Ibc*Iac)
  + Iac*(Iab*Ibc-Ibb*Iac);

var_aa1 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab1 := -(Iab*Icc-Ibc*Iac)/det;
cov_ac1 := (Iab*Ibc-Iac*Ibb)/det;
var_bb1 := (Iaa*Icc-sqr(Iac))/det;

cov_bc1 := -(Iaa*Ibc-Iac*Iab) /det;
var_cc1 := (Iaa*Ibb-sqr(Iab))/det;

{
begin item calculations
}
for i := 1 to items do
begin
  clrscr;
  gotoxy(10,10);
  write('Item Number: ',i);

  b2 := b1 - 3*sd_b + i*sd_b*6/items;

  gotoxy(10,12);
  write('Computing dispersion for Group 2, Subj:');

  {
calculate var-cov matrix for group 2
}

Iaa := 0; { ron, swami, and jane's mmss book. page
45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to 5000 do
begin
  gotoxy(50,12);
  write(j);
  p_temp := p(theta[j],a2,b2,c2);
  q_temp := 1-p_temp;
  Iaa := Iaa + sqrt(theta[j]-b2)
    *sqrt(p_temp-c2)
    *q_temp
    /p_temp;

  Iab := Iab + (theta[j]-b2)
    *sqrt(p_temp-c2)
    *q_temp
    /p_temp;

  Iac := Iac + (theta[j]-b2)
    *(p_temp-c2)
    *q_temp
    /p_temp;

  Ibb := Ibb + sqrt(p_temp-c2)
    *q_temp
    /p_temp;

  Ibc := Ibc + (p_temp-c2)
    *q_temp
    /p_temp;

  Icc := Icc + q_temp
    /p_temp;
end; {loop over subject abilities}

```

```

Iaa := sqrt(d)/sqrt(1-c2) * Iaa;
Iab := -sqrt(d)*a2/sqrt(1-c2) * Iab;
Iac := d/sqrt(1-c2) * Iac;
Ibb := sqrt(d)*sqrt(a2)/sqrt(1-c2) * Ibb;
Ibc := -D*a2/sqrt(1-c2) * Ibc;
Icc := 1/sqrt(1-c2) * Icc;

{
invert information matrix to get var-cov matrix
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
      -Iab*(Iab*Icc-Ibc*Iac)
      + Iac*(Iab*Ibc-Ibb*Iac);

var_aa2 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab2 := -(Iab*Icc-Ibc*Iac)/det;
cov_ac2 := (Iab*Ibc-Iac*Ibb)/det;
var_bb2 := (Iaa*Icc-sqr(Iac))/det;
cov_bc2 := -(Iaa*Ibc-Iac*Iab)/det;
var_cc2 := (Iaa*Ibb-sqr(Iab))/det;

{
compute DIF 1 values except for item 50. Here the IRFs
are
identical. At that point, the first derivatives of the
critical ability (where the trace lines cross) are undefined.
}

if i <> 50 then
begin

gotoxy(10,13);
write('Computing Theta_C');
theta_cross := theta_c(theta_0,
                    a1,b1,c1,
                    a2,b2,c2,
                    precision);

gotoxy(10,14);
write('Computing DIF 1');
dif_1 := dif1(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,15);
write('Computing DA1');
da1 := dd1_da1(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,16);
write('Computing DB1');
db1 := dd1_db1(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,17);

write('Computing DC1');
dc1 := dd1_dc1(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,18);
write('Computing DA2');
da2 := dd1_da2(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,19);
write('Computing DB2');
db2 := dd1_db2(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

gotoxy(10,20);
write('Computing DC2');
dc2 := dd1_dc2(theta_0,
              theta_cross,
              a1,b1,c1,
              a2,b2,c2);

sd := sqrt(sqrt(da1*var_aa1) +
            sqrt(db1*var_bb1) +
            sqrt(dc1*var_cc1) +

            sqrt(da2*var_aa2) +
            sqrt(db2*var_bb2) +
            sqrt(dc2*var_cc2) +

            abs(2*da1*db1*cov_ab1) +
            abs(2*da1*dc1*cov_ac1) +
            abs(2*db1*dc1*cov_bc1) +

            abs(2*da2*db2*cov_ab2) +
            abs(2*da2*dc2*cov_ac2) +
            abs(2*db2*dc2*cov_bc2));

gotoxy(10,21);
write('Writing estimates to DIF1_Vb');
writeln(dif1_vary_b,b2,' ',dif_1,' ',sd,' ',dif_1/sd);
end; {compute dif-1 except for 50th item}

{
compute DIF 2 values except for the 50th item. the
identical
IRFs result in an area estimate that is too close to zero
for the estimation routine to handle. the problem is in
computing
the percent changes from one estimate to the next.
}

if i <> 50 then
begin

gotoxy(40,13);

```

```

write('Computing DIF-2');
dif_2 := dif2(theta_0,a1,b1,c1,a2,b2,c2,precision);

gotoxy(40,14);
write('Computing DA1');
da1 := ddiff2_da1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,15);
write('Computing DB1');
db1 := ddiff2_db1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,16);
write('Computing DC1');
dc1 := ddiff2_dc1(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,17);
write('Computing DA2');
da2 := ddiff2_da2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,18);
write('Computing DB2');
db2 := ddiff2_db2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

gotoxy(40,19);
write('Computing DC2');
dc2 := ddiff2_dc2(theta_0,
                 a1,b1,c1,
                 a2,b2,c2,
                 precision);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write('Writing estimates to DIF2_Vb');
writeln(dif2_vary_b,b2,' ',dif_2,' ',sd,' ',dif_2/sd);

end; {compute dif-2 except for 50th item }

{
compute raju's SA30 index
}
if i <> 50 then
begin
mu_sa30 := (1-c1)*(b2-b1);
var_sa30 := sqrt(1-c1)*(var_bb2 + var_bb1);
writeln(raju_vary_b,b2,' ',mu_sa30,'
',mu_sa30/sqrt(var_sa30));
end;

end; { loop over items}

close(dif1_vary_b);
close(dif2_vary_b);
close(raju_vary_b);

gotoxy(10,24);
write('Program ended normally. Press any key to exit');
repeat until(keypressed);

end.

```

```

Turbo Pascal code used to compute the effects of variations of
cfocal
    on the area-based indices of DIF
{
This program varies just C2 so that the effect of C2 on Dif-1
and
Dif-2 can be examined.
}

program vary_c;

uses crt;

{$i incs-abc.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;

  sd_c = 0.05;
  items = 100;

var
  a1 : real;
  b1 : real;
  c1 : real;
  a2 : real;
  b2 : real;
  c2 : real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  theta : array[1..5000] of real;

  theta_cross : real;

  dif_1 : real;
  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;
  dc2 : real;

  subjs : text;

  dif1_vary_c : text;
  dif2_vary_c : text;

  i : integer;
  j : integer;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading subject abilities');

  assign(subjs,'subjects.dat');
  reset(subjs);
  for i:= 1 to 5000 do readln(subjs,theta[i]);
  close(subjs);

  assign(dif1_vary_c,'dif1_vc.dat');
  rewrite(dif1_vary_c);

  assign(dif2_vary_c,'dif2_vc.dat');
  rewrite(dif2_vary_c);

  a1 := 1;
  a2 := 1;

  b1 := 0;
  b2 := 0;

  c1 := 0.2;

  {
  compute var-cov matrix for group 1. note that this is
  constant for all items.
  }

  gotoxy(10,11);
  write('Computing dispersion for Group 1, Subj:');

  laa := 0; { ron, swami, and jane's mmss book. page 45}
  lab := 0;
  lac := 0;
  lbb := 0;

```

```

lbc := 0;
lcc := 0;
for j := 1 to 5000 do
begin
  gotoxy(50,11);
  write(j);
  p_temp := p(theta[j],a1,b1,c1);
  q_temp := 1-p_temp;
  laa := laa + sqrt(theta[j]-b1)
    *sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  lab := lab + (theta[j]-b1)
    *sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  lac := lac + (theta[j]-b1)
    *(p_temp-c1)
    *q_temp
    /p_temp;

  lbb := lbb + sqrt(p_temp-c1)
    *q_temp
    /p_temp;

  lbc := lbc + (p_temp-c1)
    *q_temp
    /p_temp;

  lcc := lcc + q_temp
    /p_temp;
end;

laa := sqrt(d)/sqrt(1-c1) * laa;
lab := -sqrt(d)*a1/sqrt(1-c1) * lab;
lac := d/sqrt(1-c1) * lac;
lbb := sqrt(d)*sqrt(a1)/sqrt(1-c1) * lbb;
lbc := -D*a1/sqrt(1-c1) * lbc;
lcc := 1/sqrt(1-c1) * lcc;

{
invert information matrix to get var-cov matrix
use formulae from Neter, Wasserman, and Kutner, page 202
}

det := laa*(lbb*lcc-sqr(lbc))
  -lab*(lab*lcc-lbc*lab)
  +lac*(lab*lbc-lbb*lab);

var_aa1 := (lbb*lcc-sqr(lbc))/det;
cov_ab1 := -(lab*lcc-lbc*lab) /det;
cov_ac1 := (lab*lbc-lac*lbb) /det;
var_bb1 := (laa*lcc-sqr(lac))/det;
cov_bc1 := -(laa*lbc-lac*lab) /det;
var_cc1 := (laa*lbb-sqr(lab))/det;

{
begin item calculations
}

for i := 1 to items do
begin
  clrscr;
  gotoxy(10,10);
  write('Item Number: ',i);

  c2 := c1 - 3*sd_c + i*sd_c*6/items;

  gotoxy(10,12);
  write('Computing dispersion for Group 2, Subj:');

  {
  calculate var-cov matrix for group 2
  }

  laa := 0; { ron, swami, and jane's mmss book. page
45}
  lab := 0;
  lac := 0;
  lbb := 0;
  lbc := 0;
  lcc := 0;
  for j := 1 to 5000 do
  begin
    gotoxy(50,12);
    write(j);
    p_temp := p(theta[j],a2,b2,c2);
    q_temp := 1-p_temp;
    laa := laa + sqrt(theta[j]-b2)
      *sqrt(p_temp-c2)
      *q_temp
      /p_temp;

    lab := lab + (theta[j]-b2)
      *sqrt(p_temp-c2)
      *q_temp
      /p_temp;

    lac := lac + (theta[j]-b2)
      *(p_temp-c2)
      *q_temp
      /p_temp;

    lbb := lbb + sqrt(p_temp-c2)
      *q_temp
      /p_temp;

    lbc := lbc + (p_temp-c2)
      *q_temp
      /p_temp;

    lcc := lcc + q_temp
      /p_temp;
  end; {loop over subject abilities}

  laa := sqrt(d)/sqrt(1-c2) * laa;
  lab := -sqrt(d)*a2/sqrt(1-c2) * lab;
  lac := d/sqrt(1-c2) * lac;
  lbb := sqrt(d)*sqrt(a2)/sqrt(1-c2) * lbb;
  lbc := -D*a2/sqrt(1-c2) * lbc;

```



```

precision);

gotoxy(40,15);
write('Computing DB1');
db1 := ddiff2_db1(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

gotoxy(40,16);
write('Computing DC1');
dc1 := ddiff2_dc1(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

gotoxy(40,17);
write('Computing DA2');
da2 := ddiff2_da2(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

gotoxy(40,18);
write('Computing DB2');
db2 := ddiff2_db2(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

gotoxy(40,19);
write('Computing DC2');
dc2 := ddiff2_dc2(theta_0,
                a1,b1,c1,
                a2,b2,c2,
                precision);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write('Writing estimates to DIF2_VC');
writeln(dif2_vary_c,c2,' ',dif_2,' ',sd,' ',dif_2/sd);

end; {compute dif-2 except for 50th item }

end; { loop over items}

close(dif1_vary_c);
close(dif2_vary_c);

gotoxy(10,24);
write('Program ended normally. Press any key to exit');
repeat until(keypressed);

end.
```

```

{
This program computes Dif-1 and Dif-2 for the black and white
male
subjects.

In general, the "1" suffix indicates variables for the black
subjects,
while the "2" suffix indicates variables for the white subjects.
}

program bwm_dif;

uses crt;

{$i gmat.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;
  items = 85;
  black_subjects = 4988;
  white_subjects = 4999;

var
  a1 : array[1..85] of real;
  b1 : array[1..85] of real;
  c1 : array[1..85] of real;
  a2 : array[1..85] of real;
  b2 : array[1..85] of real;
  c2 : array[1..85] of real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  w_theta : array[1..white_subjects] of real;
  b_theta : array[1..black_subjects] of real;

  theta_cross : real;

  dif_1 : real;
  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;
  dc2 : real;

  subjs : text;
  parameters : text;

  dif1_bwm : text;
  dif2_bwm : text;

  i : integer;
  j : integer;

  bm_b_mean : real;
  wm_b_mean : real;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading black subject abilities');
  assign(subjs,'bm-theta.dat');
  reset(subjs);
  for i:= 1 to black_subjects do readln(subjs,b_theta[i]);
  close(subjs);

  gotoxy(10,10);
  write('Reading white subject abilities');
  assign(subjs,'wm-theta.dat');
  reset(subjs);
  for i:= 1 to white_subjects do readln(subjs,w_theta[i]);
  close(subjs);

  gotoxy(10,10);
  write('Reading Parameters for Black Subjects');
  assign(parameters,'bm-abc.dat');
  reset(parameters);
  for i:= 1 to items do readln(parameters,a1[i],b1[i],c1[i]);
  close(parameters);

  gotoxy(10,10);
  write('Reading Parameters for White Subjects');
  assign(parameters,'wm-abc.dat');
  reset(parameters);
  for i:= 1 to items do readln(parameters,a2[i],b2[i],c2[i]);
  close(parameters);

```

```

{
  compute means of the two sets of b's for translation. this
  puts the two sets of item difficulties on the same metric
}

gotoxy(10,10);
clreol;
write('Computing B-mean for Black subjects');
bm_b_mean := 0;
for i := 1 to items do bm_b_mean := bm_b_mean + b1[i];
bm_b_mean := bm_b_mean / items;

gotoxy(10,10);
clreol;
write('Computing B-mean for White subjects');
wm_b_mean := 0;
for i := 1 to items do wm_b_mean := wm_b_mean + b2[i];
wm_b_mean := wm_b_mean / items;

gotoxy(10,10);
clreol;
write('Adjusting item difficulties for Black subjects');
if wm_b_mean > bm_b_mean
then for i := 1 to items do b1[i] := b1[i] + (wm_b_mean -
bm_b_mean)
else for i := 1 to items do b1[i] := b1[i] - (wm_b_mean -
bm_b_mean);

assign(dif1_bwm,'dif1-bwm.dat');
rewrite(dif1_bwm);

assign(dif2_bwm,'dif2-bwm.dat');
rewrite(dif2_bwm);

{
begin item calculations
}
for i := 1 to items do
begin

  clrscr;
  gotoxy(10,10);
  write('Item Number: ',i);

  {
  compute var-cov matrix for black group.
  }

  gotoxy(10,11);
  write('Computing dispersion for Black group, Subj:');

  Iaa := 0; { ron, swami, and jane's mmss book. page
45}
  Iab := 0;
  Iac := 0;
  Ibb := 0;
  Ibc := 0;
  Icc := 0;
  for j := 1 to black_subjects do
begin
  gotoxy(55,11);
  write(j);
  p_temp := p(b_theta[j],a1[i],b1[i],c1[i]);
  q_temp := 1-p_temp;
  Iaa := Iaa + sqrt(b_theta[j]-b1[i])
    *sqrt(p_temp-c1[i])
    *q_temp
    /p_temp;

  Iab := Iab + (b_theta[j]-b1[i])
    *sqrt(p_temp-c1[i])
    *q_temp
    /p_temp;

  Iac := Iac + (b_theta[j]-b1[i])
    *(p_temp-c1[i])
    *q_temp
    /p_temp;

  Ibb := Ibb + sqrt(p_temp-c1[i])
    *q_temp
    /p_temp;

  Ibc := Ibc + (p_temp-c1[i])
    *q_temp
    /p_temp;

  Icc := Icc + q_temp
    /p_temp;
end; {loop over subjects}

  Iaa := sqrt(d)/sqrt(1-c1[i]) * Iaa;
  Iab := -sqrt(d)*a1[i]/sqrt(1-c1[i]) * Iab;
  Iac := d/sqrt(1-c1[i]) * Iac;
  Ibb := sqrt(d)*sqrt(a1[i])/sqrt(1-c1[i]) * Ibb;
  Ibc := -D*a1[i]/sqrt(1-c1[i]) * Ibc;
  Icc := 1/sqrt(1-c1[i]) * Icc;

  {
  invert information matrix to get var-cov matrix
  use formulae from Neter, Wasserman, and Kutner, page
202
  }

  det := Iaa*(Ibb*Icc-sqr(Ibc))
    -Iab*(Iab*Icc-Ibc*Iac)
    + Iac*(Iab*Ibc-Ibb*Iac);

  var_aa1 := (Ibb*Icc-sqr(Ibc))/det;
  cov_ab1 := -(Iab*Icc-Iac*Ibc) /det;
  cov_ac1 := (Iab*Ibc-Iac*Ibb) /det;
  var_bb1 := (Iaa*Icc-sqr(Iac))/det;
  cov_bc1 := -(Iaa*Ibc-Iac*Iab) /det;
  var_cc1 := (Iaa*Ibb-sqr(Iab))/det;

  gotoxy(10,12);
  write('Computing dispersion for White group, Subj:');
  {

```

```

calculate var-cov matrix for white group
}

45} Iaa := 0; { ron, swami, and jane's mms book. page
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to white_subjects do
begin
  gotoxy(55,12);
  write(j);
  p_temp := p(w_theta[j],a2[i],b2[i],c2[i]);
  q_temp := 1-p_temp;
  Iaa := Iaa + sqrt(w_theta[j]-b2[i])
    *sqrt(p_temp-c2[i])
    *q_temp
    /p_temp;

  Iab := Iab + (w_theta[j]-b2[i])
    *sqrt(p_temp-c2[i])
    *q_temp
    /p_temp;

  Iac := Iac + (w_theta[j]-b2[i])
    *(p_temp-c2[i])
    *q_temp
    /p_temp;

  Ibb := Ibb + sqrt(p_temp-c2[i])
    *q_temp
    /p_temp;

  Ibc := Ibc + (p_temp-c2[i])
    *q_temp
    /p_temp;

  Icc := Icc + q_temp
    /p_temp;
end; {loop over subject abilities}

Iaa := sqrt(d)/sqrt(1-c2[i]) * Iaa;
Iab := -sqrt(d)*a2[i]/sqrt(1-c2[i]) * Iab;
Iac := d/sqrt(1-c2[i]) * Iac;
Ibb := sqrt(d)*sqrt(a2[i])/sqrt(1-c2[i]) * Ibb;
Ibc := -D*a2[i]/sqrt(1-c2[i]) * Ibc;
Icc := 1/sqrt(1-c2[i]) * Icc;

{
invert information matrix to get var-cov matrix
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
  -Iab*(Iab*Icc-Ibc*Iac)
  +Iac*(Iab*Ibc-Ibb*Iac);

var_aa2 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab2 := -(Iab*Icc-Ibc*Iac)/det;
cov_ac2 := (Iab*Ibc-Iac*Ibb)/det;

var_bb2 := (Iaa*Icc-sqr(Iac))/det;
cov_bc2 := -(Iaa*Ibc-Iac*Iab)/det;
var_cc2 := (Iaa*Ibb-sqr(Iab))/det;

{
compute DIF 1 values
}

gotoxy(10,13);
write('Computing Theta_C');
theta_cross := theta_c(theta_0,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i],
  precision);

gotoxy(10,14);
write('Computing DIF 1');
dif_1 := dif1(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,15);
write('Computing DA1');
da1 := dd1_da1(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,16);
write('Computing DB1');
db1 := dd1_db1(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,17);
write('Computing DC1');
dc1 := dd1_dc1(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,18);
write('Computing DA2');
da2 := dd1_da2(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,19);
write('Computing DB2');
db2 := dd1_db2(theta_0,
  theta_cross,
  a1[i],b1[i],c1[i],
  a2[i],b2[i],c2[i]);

gotoxy(10,20);
write('Computing DC2');
dc2 := dd1_dc2(theta_0,
  theta_cross,

```

```

        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

sd := sqrt(sqrt(da1*var_aa1) +
sqrt(db1*var_bb1) +
sqrt(dc1*var_cc1) +

sqrt(da2*var_aa2) +
sqrt(db2*var_bb2) +
sqrt(dc2*var_cc2) +

abs(2*da1*db1*cov_ab1) +
abs(2*da1*dc1*cov_ac1) +
abs(2*db1*dc1*cov_bc1) +

abs(2*da2*db2*cov_ab2) +
abs(2*da2*dc2*cov_ac2) +
abs(2*db2*dc2*cov_bc2));

gotoxy(10,21);
write('Writing estimates to DIF1');
writeln(dif1_bwm,i,' ',dif_1,' ',sd,' ',dif_1/sd);

{
compute DIF 2 values
}

gotoxy(40,13);
write('Computing DIF-2');
dif_2 := dif2(theta_0,a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,14);
write('Computing DA1');
da1 := ddiff2_da1(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,15);
write('Computing DB1');
db1 := ddiff2_db1(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,16);
write('Computing DC1');
dc1 := ddiff2_dc1(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,17);
write('Computing DA2');
da2 := ddiff2_da2(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,18);
write('Computing DB2');
db2 := ddiff2_db2(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

gotoxy(40,19);
write('Computing DC2');
dc2 := ddiff2_dc2(theta_0,
a1[i],b1[i],c1[i],
a2[i],b2[i],c2[i],
precision);

sd := sqrt(sqrt(da1*var_aa1) +
sqrt(db1*var_bb1) +
sqrt(dc1*var_cc1) +

sqrt(da2*var_aa2) +
sqrt(db2*var_bb2) +
sqrt(dc2*var_cc2) +

abs(2*da1*db1*cov_ab1) +
abs(2*da1*dc1*cov_ac1) +
abs(2*db1*dc1*cov_bc1) +

abs(2*da2*db2*cov_ab2) +
abs(2*da2*dc2*cov_ac2) +
abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write('Writing estimates to DIF2');
writeln(dif2_bwm,i,' ',dif_2,' ',sd,' ',dif_2/sd);

end; { loop over items}

close(dif1_bwm);
close(dif2_bwm);

gotoxy(10,24);
write('Program ended normally. Press any key to exit');
repeat until(keypressed);

end.
```

```

{
This program computes Dif-1 and Dif-2 for the white male and
female
subjects.

In general, the "1" suffix indicates variables for the female
subjects,
while the "2" suffix indicates variables for the male subjects.
}

program wmf_dif;

uses crt;

{$i gmat.pas}

const
  theta_0 = 3.0;
  precision = 0.1;
  d = 1.7;
  items = 85;
  female_subjects = 5000;
  male_subjects = 4999;

var
  a1 : array[1..85] of real;
  b1 : array[1..85] of real;
  c1 : array[1..85] of real;
  a2 : array[1..85] of real;
  b2 : array[1..85] of real;
  c2 : array[1..85] of real;

  var_aa1 : real;
  cov_ab1 : real;
  cov_ac1 : real;
  var_bb1 : real;
  cov_bc1 : real;
  var_cc1 : real;

  var_aa2 : real;
  cov_ab2 : real;
  cov_ac2 : real;
  var_bb2 : real;
  cov_bc2 : real;
  var_cc2 : real;

  p_temp : real;
  q_temp : real;

  laa : real;
  lab : real;
  lac : real;
  lbb : real;
  lbc : real;
  lcc : real;

  det : real;

  m_theta : array[1..male_subjects] of real;
  f_theta : array[1..female_subjects] of real;

  theta_cross : real;

  dif_1 : real;
  dif_2 : real;
  sd : real;

  da1 : real;
  db1 : real;
  dc1 : real;
  da2 : real;
  db2 : real;
  dc2 : real;

  subjs : text;
  parameters : text;

  dif1_wmf : text;
  dif2_wmf : text;

  i : integer;
  j : integer;

  wm_b_mean : real;
  wf_b_mean : real;

begin
  textbackground(blue);
  textcolor(white);

  clrscr;

  gotoxy(10,10);
  write('Reading black subject abilities');
  assign(subjs,'wf-theta.dat');
  reset(subjs);
  for i:= 1 to female_subjects do readln(subjs,f_theta[i]);
  close(subjs);

  gotoxy(10,10);
  write('Reading male subject abilities');
  assign(subjs,'wm-theta.dat');
  reset(subjs);
  for i:= 1 to male_subjects do readln(subjs,m_theta[i]);
  close(subjs);

  gotoxy(10,10);
  write('Reading Parameters for Female Subjects');
  assign(parameters,'wf-abc.dat');
  reset(parameters);
  for i := 1 to items do readln(parameters,a1[i],b1[i],c1[i]);
  close(parameters);

  gotoxy(10,10);
  write('Reading Parameters for Male Subjects');
  assign(parameters,'wm-abc.dat');
  reset(parameters);
  for i := 1 to items do readln(parameters,a2[i],b2[i],c2[i]);
  close(parameters);

  {

```

```

compute means of the two sets of b's for translation. this
puts the two sets of item difficulties on the same metric
}

gotoxy(10,10);
clreol;
write('Computing B-mean for Female subjects');
wf_b_mean := 0;
for i := 1 to items do wf_b_mean := wf_b_mean + b1[i];
wf_b_mean := wf_b_mean / items;

gotoxy(10,10);
clreol;
write('Computing B-mean for Male subjects');
wm_b_mean := 0;
for i := 1 to items do wm_b_mean := wm_b_mean + b2[i];
wm_b_mean := wm_b_mean / items;

gotoxy(10,10);
clreol;
write('Adjusting item difficulties for Female subjects');
if wm_b_mean > wf_b_mean
then for i := 1 to items do b1[i] := b1[i] + (wm_b_mean -
wf_b_mean)
else for i := 1 to items do b1[i] := b1[i] - (wm_b_mean -
wf_b_mean);

assign(dif1_wmf,'dif1-wmf.dat');
rewrite(dif1_wmf);

assign(dif2_wmf,'dif2-wmf.dat');
rewrite(dif2_wmf);

{
begin item calculations
}
for i := 1 to items do
begin

  clrscr;
  gotoxy(10,10);
  write('Item Number: ',i);

  {
  compute var-cov matrix for female group.
  }

  gotoxy(10,11);
  write('Computing dispersion for Female group, Subj:');

  Iaa := 0; { ron, swami, and jane's mmss book. page
45}
  Iab := 0;
  Iac := 0;
  Ibb := 0;
  Ibc := 0;
  Icc := 0;
  for j := 1 to female_subjects do

begin
  gotoxy(55,11);
  write(j);
  p_temp := p(f_theta[j],a1[i],b1[i],c1[i]);
  q_temp := 1-p_temp;
  Iaa := Iaa + sqr(f_theta[j]-b1[i])
    *sqr(p_temp-c1[i])
    *q_temp
    /p_temp;

  Iab := Iab + (f_theta[j]-b1[i])
    *sqr(p_temp-c1[i])
    *q_temp
    /p_temp;

  Iac := Iac + (f_theta[j]-b1[i])
    *(p_temp-c1[i])
    *q_temp
    /p_temp;

  Ibb := Ibb + sqr(p_temp-c1[i])
    *q_temp
    /p_temp;

  Ibc := Ibc + (p_temp-c1[i])
    *q_temp
    /p_temp;

  Icc := Icc + q_temp
    /p_temp;
end; {loop over subjects}

  Iaa := sqr(d)/sqr(1-c1[i]) * Iaa;
  Iab := -sqr(d)*a1[i]/sqr(1-c1[i]) * Iab;
  Iac := d/sqr(1-c1[i]) * Iac;
  Ibb := sqr(d)*sqr(a1[i])/sqr(1-c1[i]) * Ibb;
  Ibc := -D*a1[i]/sqr(1-c1[i]) * Ibc;
  Icc := 1/sqr(1-c1[i]) * Icc;

  {
  invert information matrix to get var-cov matrix
  use formulae from Neter, Wasserman, and Kutner, page
202
  }

  det := Iaa*(Ibb*Icc-sqr(Ibc))
    -Iab*(Iab*Icc-Ibc*Iac)
    + Iac*(Iab*Ibc-Ibb*Iac);

  var_aa1 := (Ibb*Icc-sqr(Ibc))/det;
  cov_ab1 := -(Iab*Icc-Ibc*Iac) /det;
  cov_ac1 := (Iab*Ibc-Iac*Ibb) /det;
  var_bb1 := (Iaa*Icc-sqr(Iac))/det;
  cov_bc1 := -(Iaa*Ibc-Iac*Iab) /det;
  var_cc1 := (Iaa*Ibb-sqr(Iab))/det;

  gotoxy(10,12);
  write('Computing dispersion for Male group, Subj:');

  {

```

```

calculate var-cov matrix for male group
}

Iaa := 0; { ron, swami, and jane's mmss book. page
45}
Iab := 0;
Iac := 0;
Ibb := 0;
Ibc := 0;
Icc := 0;
for j := 1 to male_subjects do
begin
gotoxy(55,12);
write(j);
p_temp := p(m_theta[j],a2[i],b2[i],c2[i]);
q_temp := 1-p_temp;
Iaa := Iaa + sqrt(m_theta[j]-b2[i])
        *sqrt(p_temp-c2[i])
        *q_temp
        /p_temp;

Iab := Iab + (m_theta[j]-b2[i])
        *sqrt(p_temp-c2[i])
        *q_temp
        /p_temp;

Iac := Iac + (m_theta[j]-b2[i])
        *(p_temp-c2[i])
        *q_temp
        /p_temp;

Ibb := Ibb + sqrt(p_temp-c2[i])
        *q_temp
        /p_temp;

Ibc := Ibc + (p_temp-c2[i])
        *q_temp
        /p_temp;

Icc := Icc + q_temp
        /p_temp;
end; {loop over subject abilities}

Iaa := sqrt(d)/sqrt(1-c2[i]) * Iaa;
Iab := -sqrt(d)*a2[i]/sqrt(1-c2[i]) * Iab;
Iac := d/sqrt(1-c2[i]) * Iac;
Ibb := sqrt(d)*sqrt(a2[i])/sqrt(1-c2[i]) * Ibb;
Ibc := -D*a2[i]/sqrt(1-c2[i]) * Ibc;
Icc := 1/sqrt(1-c2[i]) * Icc;

{
invert information matrix to get var-cov matrix
}

det := Iaa*(Ibb*Icc-sqr(Ibc))
        -Iab*(Iab*Icc-Ibc*Iac)
        + Iac*(Iab*Ibc-Ibb*Iac);

var_aa2 := (Ibb*Icc-sqr(Ibc))/det;
cov_ab2 := -(Iab*Icc-Ibc*Iac) /det;
cov_ac2 := (Iab*Ibc-Iac*Ibb) /det;

var_bb2 := (Iaa*Icc-sqr(Iac))/det;
cov_bc2 := -(Iaa*Ibc-Iac*Iab) /det;
var_cc2 := (Iaa*Ibb-sqr(Iab))/det;

{
compute DIF 1 values
}

gotoxy(10,13);
write('Computing Theta_C');
theta_cross := theta_c(theta_0,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i],
        precision);

gotoxy(10,14);
write('Computing DIF 1');
dif_1 := dif1(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,15);
write('Computing DA1');
da1 := dd1_da1(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,16);
write('Computing DB1');
db1 := dd1_db1(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,17);
write('Computing DC1');
dc1 := dd1_dc1(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,18);
write('Computing DA2');
da2 := dd1_da2(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,19);
write('Computing DB2');
db2 := dd1_db2(theta_0,
        theta_cross,
        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

gotoxy(10,20);
write('Computing DC2');
dc2 := dd1_dc2(theta_0,
        theta_cross,

```

```

        a1[i],b1[i],c1[i],
        a2[i],b2[i],c2[i]);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(10,21);
write("Writing estimates to DIF1");
writeln(dif1_wmf,i,' ',dif_1,' ',sd,' ',dif_1/sd);

{
compute DIF 2 values
}

gotoxy(40,13);
write("Computing DIF-2");
dif_2 := dif2(theta_0,a1[i],b1[i],c1[i],
              a2[i],b2[i],c2[i],
              precision);

gotoxy(40,14);
write("Computing DA1");
da1 := ddiff2_da1(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

gotoxy(40,15);
write("Computing DB1");
db1 := ddiff2_db1(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

gotoxy(40,16);
write("Computing DC1");
dc1 := ddiff2_dc1(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

gotoxy(40,17);
write("Computing DA2");
da2 := ddiff2_da2(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

gotoxy(40,18);
write("Computing DB2");
db2 := ddiff2_db2(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

gotoxy(40,19);
write("Computing DC2");
dc2 := ddiff2_dc2(theta_0,
                 a1[i],b1[i],c1[i],
                 a2[i],b2[i],c2[i],
                 precision);

sd := sqrt(sqrt(da1*var_aa1) +
           sqrt(db1*var_bb1) +
           sqrt(dc1*var_cc1) +

           sqrt(da2*var_aa2) +
           sqrt(db2*var_bb2) +
           sqrt(dc2*var_cc2) +

           abs(2*da1*db1*cov_ab1) +
           abs(2*da1*dc1*cov_ac1) +
           abs(2*db1*dc1*cov_bc1) +

           abs(2*da2*db2*cov_ab2) +
           abs(2*da2*dc2*cov_ac2) +
           abs(2*db2*dc2*cov_bc2));

gotoxy(40,20);
write("Writing estimates to DIF2");
writeln(dif2_wmf,i,' ',dif_2,' ',sd,' ',dif_2/sd);

end; { loop over items}

close(dif1_wmf);
close(dif2_wmf);

gotoxy(10,24);
write("Program ended normally. Press any key to exit");
repeat until(keypressed);

end.
```

APPENDIX B
SAS PROGRAMS

(SAS is a registered trademark of SAS Institute, Inc., Cary, NC.)

```
/*
this program produces 5000 subjects.
*/
```

```
data _null_;
  filename subjs 'subjs1.dat';
  file subjs;
  seed1 = 789012345;
  theta = 0;
  retain seed1;
  do i = 1 to 5000 by 1;
    call rannor(seed1,theta);
    put theta;
  end;
run;
```

Program to Create 200 Items for Monte Carlo Study of Index Distributions

```
/*
This program generates two sets of items parameters for 200 items. One
set will be for the focal group; the other for the reference.
*/
```

```
data items;
  keep a1 a2 b1 b2 c1 c2;
  retain a1_seed b1_seed c1_seed
         a2_seed b2_seed c2_seed;
  a1_seed = 123456789;
  b1_seed = 234567891;
  c1_seed = 345678912;
  a2_seed = 456789123;
  b2_seed = 567891234;
  c2_seed = 678912345;
  a1 = 1;
  a2 = 2;
  b1 = 0;
  b2 = 0;
  c1 = 0.2;
  c2 = 0.2;
  do i = 1 to 200 by 1;
    call rannor(a1_seed,a1);
    call rannor(a2_seed,a2);
    call rannor(b1_seed,b1);
    call rannor(b2_seed,b2);
    call rannor(c1_seed,c1);
    call rannor(c2_seed,c2);
    output;
```

```
    end;
run;

/*
Transform parameters
*/

proc standard data=items mean=1 std=0.3 out=items;
  var a1 a2;
run;
proc standard data=items mean=0 std=1 out=items;
  var b1 b2;
run;
proc standard data=items mean=0.2 std=0.05 out=items;
  var c1 c2;
run;

/*
remove any items that are impossible
note: none were removed.
*/

data items;
  set items;
  if a1 gt 0;
  if a2 gt 0;
  if (b1 gt -3) and (b1 lt 3);
  if (b2 gt -3) and (b2 lt 3);
  if c1 ge 0;
  if c2 ge 0;
run;

/*
write parameters to data set for further use.
*/

data _null_;
  set items;
  filename items 'items.dat';
  file items;
  put a1 b1 c1 a2 b2 c2;
run;
```

Code to simulate test administration to 5000 Monte-Carlo
subjects to study the effect of variations in a_{focal} on
the Mantel-Haenszel χ^2 statistic

```
/*
this data step varies the A2 parameter uniformly over 6 standard deviations
while holding the other parameters constant and equal. a response vector is
created that will be processed using Harnisch's Mantel-Haentzel procedure.
*/
```

```
data _null_;
  length respv1 $ 200;
  length respv2 $ 200;
  filename subjects 'subjects.dat';
  filename group1 'group1-a.dat';
  filename group2 'group2-a.dat';
  infile subjects;
  items = 100;
  a1 = 1;
  b1 = 0;
  b2 = 0;
  c1 = 0.2;
  c2 = 0.2;
  seed = 543210987;
  noise = 0.5;
  do i = 1 to 5000 by 1;
    input theta;
    p1 = c1 + (1-c1)*exp(1.7*a1*(theta-b1))
        /
        (1+exp(1.7*a1*(theta-b1)));
    do j = 1 to items by 1;
      call ranuni(seed,noise);
      a2 = a1 - 3*0.3 + 6*0.3/items*j;
      p2 = c2 + (1-c2)*exp(1.7*a2*(theta-b2))
          /
          (1+exp(1.7*a2*(theta-b2)));
      if p1 ge noise then substr(respv1,j,1) = '1';
      else substr(respv1,j,1) = '0';
      if p2 ge noise then substr(respv2,j,1) = '1';
      else substr(respv2,j,1) = '0';
    end; /* loop over items */
    file group1 ls=100 lrecl=100;
    put respv1;
    file group2 ls=100 lrecl=100;
    put respv2;
  end; /* loop over subjects */
run;
```

SAS program to compute Mantel-Haenszel χ^2 statistic
from subject response vectors

/*

Harnisch, D.L. (1991) MHPROG SAS Macro Listing

Source: Harnisch, D.L. (1991). Techniques for Assessing Differential Item Performance on Achievement Tests. Proceedings of the Sixteenth Annual SAS Users Group International Conference. Cary, NC: SAS Institute Inc., 1503-1508.

This macro computes DIF indices based on the Mantel-Haenszel procedure. Users must specify the test questions using variable names of Q1-Qn where n represents the number of items on the test. These variables must be scored as 1 for correct and 0 for incorrect. The variable TOTAL must be created which represents the performance of the students on the criterion measure of interest. The SAS data set name which includes the binary coded test questions, total score, and the discriminating variable name (coded 1 for focal and 0 for reference) are used as arguments on the SAS macro MHPROG. For example, a SAS data set by the name of GR11MATH.DIF. containing 50 items binary coded and summed to create a variable TOTAL along with sex coded 1 for males and 0 for females would be written: MHPROG (GR11MATH.DIF,SEX,50);

*/

```
%MACRO MHPROG(DATA,COMVAR,NITEM);
```

```
PROC SUMMARY DATA = &DATA NWAY;
  CLASS TOTAL &COMVAR;
  VAR Q1 - Q&NITEM;
  OUTPUT OUT = MHSUM N = N1 - N&NITEM SUM = R1 - R&NITEM;
```

```
PROC SORT DATA=MHSUM; BY TOTAL ;
```

```
DATA MHSUMS ;
  SET MHSUM ;
  BY TOTAL;
  ARRAY NS{&NITEM} N1 - N&NITEM;
  ARRAY RS{&NITEM} R1 - R&NITEM;
  IF FIRST.TOTAL AND LAST.TOTAL THEN DO;
    OUTPUT ;
    DO I = 1 TO &NITEM;
      NS(I) = 0.0;
      RS(I) = 0;
    END;
    IF &COMVAR = 0 THEN &COMVAR = 1;ELSE &COMVAR = 0;
    OUTPUT;
  END;
ELSE OUTPUT;
```

```

PROC SORT ; BY TOTAL &COMVAR;

DATA MHBASE;
  SET MHSUMS (KEEP=N1-N&NITEM R1-R&NITEM);
  IF MOD(_N_,2)=1;

DATA MHFOCAL;
  SET MHSUMS (KEEP=N1-N&NITEM R1-R&NITEM);
  IF MOD(_N_,2)=0;

DATA DTOTS; SET MHSUMS; IF MOD(_N_,2)=0; KEEP TOTAL;

PROC IML;
  EPS = 0.0000001 ; /* 1.0E-7 */

  USE MHBASE;
  READ ALL INTO PREBS;
  NLEVEL=NROW(PREBS);
  RBS=PREBS(| 1:NLEVEL,&NITEM+1:&NITEM*2|);
  NBS=PREBS(| 1:NLEVEL,1:&NITEM|);
  FREE PREBS;

  USE MHFOCAL;
  READ ALL INTO PREFS;
  NLEVEL=NROW(PREFS);
  RFS=PREFS(| 1:NLEVEL,&NITEM+1:&NITEM*2|);
  NFS=PREFS(| 1:NLEVEL,1:&NITEM|);
  FREE PREFS;

  WBS = NBS - RBS;
  WFS = NFS - RFS;

/*
THERE ARE NOW FOUR MATRICIES WHICH CONSTITUTE THE TABLE CELLS. EACH
COLUMN OF THESE MATRICIES CORRESPONDS TO A TABLE CELL. THE COLUMNS
CORRESPOND TO THE DIFFERENT ITEMS.
*/

  ALPHAS = (RBS # WFS ) / ( RFS # WBS );

  MS = ( WBS # RFS ) / ( NFS + NBS );
  ALPHA_MS = (RBS # WFS) / (NFS + NBS);
  ALPHAMH = ALPHA_MS (| +, |) / MS(| +, |);

  CREATE SUMMARY1 FROM ALPHAS ;
  APPEND FROM ALPHAS ;

  FREE MS ALPHA_MS ALPHAS;

```

```

MUS = NBS # (RBS + RFS) / (NFS + NBS);
SIGMAS = ( NBS # NFS # ( RBS + RFS ) # ( WBS + WFS ) ) /
          ((NFS + NBS) # ( NFS + NBS ) # ( NFS + NBS -1 ));

TERM1 = RBS(| +, |);
TERM2 = MUS(| +, |);
TERM3 = SIGMAS(| +, |) <> (EPS # J(1, NCOL(SIGMAS), 1.0) );
TERM4 = ABS ( TERM1 - TERM2 ) - 0.5 ;

FREE WFS WBS TERM1 TERM2 ;

CHISQMH = ( TERM4 # TERM4 ) / TERM3 ;

FREE TERM3 TERM4 ;

PCHIMH = J(NROW(CHISQMH),NCOL(CHISQMH),1.0) - PROBCHI(CHISQMH , 1.0) ;
PFS = RFS / NFS ;
PBS = RBS / NBS ;
DS = PFS - PBS ;

*CREATING SAS DATA SETS FOR PLOTS;

USE DTOTS; READ ALL INTO DTOTSM;
NRTOT = NROW(DTOTSM);
DREFM = REPEAT(0,NRTOT,1);
DFOCM = REPEAT(1,NRTOT,1);
DIFFS = DS||DTOTSM;
REFPC = PBS||DTOTSM||DREFM;
FOCPC = PFS||DTOTSM||DFOCM;
ALLPC = REFPC//FOCPC;

*CREATE SDS.DDIFFS FROM DIFFS; *APPEND FROM DIFFS;
CREATE DALLPC FROM ALLPC; APPEND FROM ALLPC;

/*
removed SDS libname ref. -- jp
*/

CREATE SUMMARY2 FROM DS ;
APPEND FROM DS ;

CREATE SUMMARY3 FROM PFS ;
APPEND FROM PFS ;

CREATE SUMMARY4 FROM PBS ;
APPEND FROM PBS ;

FREE RBS RFS ;

EFS = NFS # PFS ;

```

```

EBS = NFS # PBS ;

FREE PFS PBS ;

PSUBF = ( EFS (| +, |) / NFS(| +, |) );
PHATF = ( EBS (| +, |) / NFS(| +, |) );
DSTD = PSUBF - PHATF ;

FREE NFS NBS ;

/* BOUND THE PROBABILITIES AWAY FROM THE ENDS */

PSUBF = (EPS # J(NROW(PSUBF),NCOL(PSUBF),1.0)) <> PSUBF ;
PSUBF = ((1.0 - EPS) # J(NROW(PSUBF),NCOL(PSUBF),1.0)) >< PSUBF ;
PHATF = (EPS # J(NROW(PHATF),NCOL(PHATF),1.0)) <> PHATF ;
PHATF = ((1.0 - EPS) # J(NROW(PHATF),NCOL(PHATF),1.0)) >< PHATF ;

CNAMS = { 'PSUBF ', 'PHATF' };
OUTPS = (PSUBF ' ') || (PHATF ' ');
CREATE PESTS FROM OUTPS (| COLNAME = CNAMS |);
APPEND FROM OUTPS;
FREE CNAMS OUTPS ;

DELTAPS = J(NROW(PSUBF), NCOL(PSUBF), 13.0) - 4.0 # PROBIT( PSUBF ) ;
DELTAPHS = J(NROW(PHATF), NCOL(PHATF), 13.0) - 4.0 # PROBIT( PHATF ) ;

* PRINT DELTAPS DELTAPS ;

ALPHAMH = (EPS # J(NROW(ALPHAMH),NCOL(ALPHAMH),1.0)) <> ALPHAMH ;

DELTAMH = -2.35 # LOG ( ALPHAMH ) ;
DELTASTD = -2.35 # LOG ((PHATF # (1 - PSUBF))/(PSUBF # (1 - PHATF)));

* PRINT DELTAMH DELTASTD ;

* PRINT ALPHAMH DELTAMH DSTD CHISQMH PCHIMH DELTASTD DELTAPS DELTAPHS;

* TRANSPOSING THE MATRIX TO YIELD ITEM BY INDEX TABLE;

TALPHAMH=ALPHAMH'; TDELTAMH=DELTAMH'; TDSTD=DSTD';
TCHISQMH=CHISQMH'; TPCHIMH=PCHIMH'; TDELSTD=DELTASTD';

*PRINT TALPHAMH TDELTAMH TDSTD TCHISQMH TPCHIMH TDELSTD;

OUTS = TALPHAMH || TDELTAMH || TDSTD || TCHISQMH || TPCHIMH
      || TDELSTD;
CNAME={'ALPHAMH','DELTAMH', 'DSTD', 'CHISQMH',
      'PCHIMH','DELSTD'};

CREATE RESULTS FROM OUTS (| COLNAME = CNAME |);

```

```

APPEND FROM OUTS;

DATA RESULTS;SET RESULTS; IF ALPHAMH=0 THEN DO ;
  DELTAMH=.; DSTD=.;CHISQMH=.;PCHIMH=.;DELSTD=.;END;
  IF DSTD>.10 THEN FLAG='M++';
  ELSE IF DSTD>.05 THEN FLAG='M+';
  ELSE IF DSTD<-.10 THEN FLAG='M--';
  ELSE IF DSTD<-.05 THEN FLAG='M-';
  ELSE FLAG='';

PROC PRINT DATA=RESULTS;
  TITLE2 "MANTEL-HAENSZEL STATISTICS: BY &COMVAR";
  TITLE3 'FLAG COLUMN INDICATES LEVEL OF DIF';

*PROC MEANS DATA=RESULTS;
*  VAR ALPHAMH DELTAMH DSTD DELSTD;
*  TITLE2 "DESCRIPTIVE STATISTICS OF MH-PARAMETERS: BY &COMVAR";

*PROC CORR DATA=RESULTS;
*  VAR ALPHAMH DELTAMH DSTD DELSTD;
*  TITLE2 "CORRELATIONS AMONG MH-PARAMETERS: BY &COMVAR";
*IF TTEST WANTED;

/*
PROC TTEST DATA=&DATA;
  CLASS &COMVAR;
  VAR TOTAL Q1-Q&NITEM;
  TITLE2 "TTEST OF PERFORMANCE ON ITEMS: BY &COMVAR";
*/

%MEND;

/*
the following code calls the preceeding macro.
*/

/*
Vary the A parameter
*/

filename group1_a 'group1-a.dat';
filename group2_a 'group2-a.dat';

/*
this is the reference group
*/

data group1_a;
  infile group1_a;
  array q[100] q1-q100;

```

```
input (q1-q100) (1.);
total = sum(of q1-q100);
group = 0;
run;

/*
this is the focal group.
*/

data group2_a;
infile group2_a;
array q[100] q1-q100;
input (q1-q100) (1.);
total = sum(of q1-q100);
group = 1;
run;

data vary_a;
set group1_a group2_a;
run;

%mhprog(vary_a,group,100);

/*
Vary the B parameter
*/

filename group1_b 'group1-b.dat';
filename group2_b 'group2-b.dat';

/*
this is the reference group
*/

data group1_b;
infile group1_b;
array q[100] q1-q100;
input (q1-q100) (1.);
total = sum(of q1-q100);
group = 0;
run;

/*
this is the focal group.
*/

data group2_b;
infile group2_b;
array q[100] q1-q100;
input (q1-q100) (1.);
```

```
total = sum(of q1-q100);
group = 1;
run;

data vary_b;
  set group1_b group2_b;
  run;

%mhprog(vary_b,group,100);

/*
Vary the C parameter
*/

filename group1_c 'group1-c.dat';
filename group2_c 'group2-c.dat';

/*
this is the reference group
*/

data group1_c;
  infile group1_c;
  array q[100] q1-q100;
  input (q1-q100) (1.);
  total = sum(of q1-q100);
  group = 0;
  run;

/*
this is the focal group.
*/

data group2_c;
  infile group2_c;
  array q[100] q1-q100;
  input (q1-q100) (1.);
  total = sum(of q1-q100);
  group = 1;
  run;

data vary_c;
  set group1_c group2_c;
  run;

%mhprog(vary_c,group,100);
```

Code to simulate test administration to 5000 Monte-Carlo
subjects to study the effect of variations in b_{focal} on
the Mantel-Haenszel χ^2 statistic

```
/*
this data step varies the B2 parameter uniformly over 6 standard deviations
while holding the other parameters constant and equal. a response vector is
created that will be processed using Harnisch's Mantel-Haentzel procedure.
*/
```

```
data _null_;
  length respv1 $ 200;
  length respv2 $ 200;

  filename subjects 'subjects.dat';
  filename group1 'group1-b.dat';
  filename group2 'group2-b.dat';

  infile subjects;
  items = 100;
  a1 = 1;
  a2 = 1;
  b1 = 0;
  c1 = 0.2;
  c2 = 0.2;
  seed = 543210987;
  noise = 0.5;

  do i = 1 to 5000 by 1;
    input theta;

    p1 = c1 + (1-c1)*exp(1.7*a1*(theta-b1))
        /
        (1+exp(1.7*a1*(theta-b1)));

    do j = 1 to items by 1;
      call ranuni(seed,noise);
      b2 = b1 - 3*1.0 + 6*1.0/items*j;
      p2 = c2 + (1-c2)*exp(1.7*a2*(theta-b2))
          /
          (1+exp(1.7*a2*(theta-b2)));
      if p1 ge noise then substr(respv1,j,1) = '1';
      else substr(respv1,j,1) = '0';
      if p2 ge noise then substr(respv2,j,1) = '1';
      else substr(respv2,j,1) = '0';

    end; /* loop over items */

  file group1 ls=100 lrecl=100;
  put respv1;
```

```

file group2 ls=100 lrecl=100;
put respv2;

end; /* loop over subjects */
run;

```

Code to simulate test administration to 5000 Monte-Carlo
subjects to study the effect of variations in c_{focal} on
the Mantel-Haenszel χ^2 statistic

```

/*
this data step varies the C2 parameter uniformly over 6 standard deviations
while holding the other parameters constant and equal. a response vector is
created that will be processed using Harnisch's Mantel-Haentzel procedure.
*/

data _null_;
length respv1 $ 200;
length respv2 $ 200;

filename subjects 'subjects.dat';
filename group1 'group1-c.dat';
filename group2 'group2-c.dat';

infile subjects;
items = 100;
a1 = 1;
a2 = 1;
b1 = 0;
b2 = 0;
c1 = 0.2;
seed = 543210987;
noise = 0.5;

do i = 1 to 5000 by 1;
input theta;

p1 = c1 + (1-c1)*exp(1.7*a1*(theta-b1))
/
(1+exp(1.7*a1*(theta-b1)));

do j = 1 to items by 1;
call ranuni(seed,noise);
c2 = c1 - 3*0.05 + 6*0.05/items*j;
p2 = c2 + (1-c2)*exp(1.7*a2*(theta-b2))
/
(1+exp(1.7*a2*(theta-b2)));
if p1 ge noise then substr(respv1,j,1) = '1';
else substr(respv1,j,1) = '0';

```

```
if p2 ge noise then substr(respv2,j,1) = '1';  
    else substr(respv2,j,1) = '0';  
  
end; /* loop over items */  
  
file group1 ls=100 lrecl=100;  
put respv1;  
  
file group2 ls=100 lrecl=100;  
put respv2;  
  
end; /* loop over subjects */  
run;
```

APPENDIX C
ABILITIES OF MONTE-CARLO SUBJECTS

-0.245103756	0.0594990637	-1.59866386	-2.35475141	-1.656724321	-0.011267772	-0.048615734
-1.464151601	1.6876372741	-0.037582955	1.146334505	0.2915715224	0.1805315162	0.6100777156
-0.838681944	1.7148875917	2.4566971113	0.0849612231	-1.302704663	-0.758566705	1.3291564671
0.831350507	-1.936179678	0.7161057011	-1.228606238	0.7405139528	0.3954602124	-0.049440555
1.366754965	0.2459673643	-1.513319888	2.8211918629	-1.215808458	0.1368220317	-0.28832377
-0.64528256	0.7199329539	0.372186633	-0.327542786	0.6062168092	-0.332746725	-1.071964952
-1.31920704	-0.28079621	-1.757585126	0.5789706094	0.2288940211	-1.289787147	0.2289767192
-1.87160576	-0.566843862	-0.659094709	1.9813992857	0.3155904706	-0.052546933	0.9304193295
0.2612790805	0.4391065041	-1.356167565	0.4947161417	2.1234946237	-1.102083726	1.0695711451
-0.674612196	-0.102089898	-1.458121689	0.7882071976	-0.828097337	0.7829049882	-0.414868328
1.0252025875	-0.3308229	-0.589680574	0.4423993866	-0.269032477	1.4493286938	-2.14394543
-0.93914156	-0.690954589	0.4095988467	-0.350097041	0.9094630046	-0.402976169	-0.616046692
0.4410898292	0.2848213877	0.5174029533	-0.195023578	-1.287272571	0.6910190291	-0.322169055
0.7500831701	0.285741721	1.0460498285	-0.422204708	0.8190496504	0.8039642397	-1.812955689
-1.54382796	1.1940688997	0.6070243177	-0.83503329	0.0820760656	-1.528874221	-0.262934885
0.037007709	1.1642467432	-0.243046449	2.0327822644	-0.715988524	-0.83784506	-0.529527486
-0.092097471	-0.346240641	-1.856525051	-0.62440291	-0.014576617	-0.218844484	-0.016455775
0.1545149109	1.8470746697	0.1791473596	0.7854693426	-1.762190869	-0.434980256	-1.243691127
-0.447030248	1.61993799	0.2620055093	-0.299144696	0.8877674907	-0.011850876	0.1186112026
2.2079111601	-1.237186406	-0.48874524	0.468113515	1.464704232	-0.555484102	-0.438178612
-2.020211901	-0.537406897	2.0952861759	-1.629261701	0.462594938	-0.555664288	-0.585268257
-1.0830135	1.8121610001	-1.935080507	0.4347209594	-1.177803829	1.0076887913	-1.042256947
0.1497461323	0.5170485056	0.3812157608	-1.344061581	-0.197508263	1.0510643055	-0.843490358
0.8321612633	1.1593187375	0.1171094862	-1.084330924	-0.973493147	0.9858396704	-0.690413127
-0.557637155	0.3455307457	0.12902261	-1.459182273	-1.044891049	0.8215635104	0.4731884936
1.8135645464	-0.874581256	-0.216103998	-0.775582099	0.4702056846	-0.073178341	-0.301172107
-0.57271085	-1.118656579	0.8165775749	-0.284276423	2.8962786094	1.0661697975	-0.857674415
-0.276988179	-0.658001544	-0.055802517	-0.315622476	-0.658732336	0.2054867324	0.8584242738
-0.701400495	0.9400905511	-0.141686361	1.5433259878	0.9554255506	0.9214019306	-1.170410033
0.3297237062	2.6854866207	0.2468399475	0.8507275052	0.3593806462	0.0668244033	-1.436029165
-0.078501858	0.5128875466	0.7051343407	1.3284259431	0.8093974199	-0.613868001	-0.041828492
0.6366472649	-0.671646069	-0.157498912	-2.024053793	0.3906019324	1.1232726936	0.9113687023
-0.411340091	-0.632202726	2.2472592001	1.3981612507	-0.80030649	0.7200253032	-0.322200185
0.2842167643	2.2401876706	0.6092358841	-0.936037176	0.0185363083	0.5663382913	1.3989335872
-0.311113714	-1.494217382	0.0193323468	-0.142075624	2.055232595	1.6099197562	-0.651475872
-1.080210621	0.9289838589	0.6084367209	1.2307680846	-0.979779482	-0.306798797	1.4620267926
-1.420507334	-0.598551812	-0.789321757	0.0994385416	1.165850783	-0.184196193	0.5602059543
-0.214178726	0.7023853001	0.0854089968	-0.180113754	-0.722444345	-0.242603638	0.7995191637
2.2999371802	-0.6389458	0.6629615762	0.7738175352	-1.177573462	2.3320287032	-0.259510414
-1.613907592	-1.219211152	-0.028491598	0.1966595503	0.4062744043	0.2654124538	-1.040139722
0.7161408169	-0.845009459	0.9929525405	0.572213072	0.52824724	-1.595302261	-1.046907974
-0.263586988	0.6783548424	1.5968916899	0.9019597487	-0.793714393	0.8074990204	1.6840868226
-1.926438367	-1.18076942	1.5265342494	0.4030396703	-0.748030411	-0.499311355	-0.24313164
0.5554994193	-1.864441087	-0.839692036	0.7116214421	-0.238640831	0.8098125021	1.000484966
-0.971149612	0.8957314872	-1.379471595	2.0169711267	-2.462780874	-0.332477682	-0.423145416
0.5815552283	-0.389287334	0.2016223173	0.4579620406	0.3456377208	0.114021532	0.6356252843
0.5039432816	0.9712763086	0.9587752057	0.0335672247	0.0908273345	-0.31261426	-0.787158146
1.5742600445	-0.035123355	0.7787624636	1.1327474512	-0.976630028	-1.190118752	-0.204867873
1.0091702289	-0.713884429	-0.735382481	1.1090417374	-0.356481639	0.6134981564	-0.062365108
-0.849935218	0.1371401212	-1.965122738	-0.511382284	-0.087388708	1.625009142	0.1990493997
1.2890464268	1.0768940275	0.6122844391	1.0124164458	0.3679723528	0.6967660496	-0.464957805
-1.432537374	-0.20842071	-2.625945962	-0.493276631	2.5341334767	-1.45514069	-0.512574555
0.0559253597	-0.042147349	1.1886885133	-0.853059501	1.185493965	-0.716891095	0.9737346707
0.5405089115	1.2994772256	-0.033250147	-0.455799445	-0.514596596	0.4852559393	-1.18079341
1.2743344912	-1.816213406	0.2734411171	0.4961869546	1.0188680993	-0.254808883	-1.254083484
-0.033306498	1.974894942	-0.570136457	-0.21731771	1.7183168696	-1.020177389	-0.674185314
1.5556157203	0.1322941992	0.5095603327	-0.543986015	0.872031548	-0.147321994	0.0972380243
1.6468119628	0.6370419888	-2.39262218	0.2659970487	-2.801877853	0.0823586561	-1.310999687
-0.344853205	2.4509157832	0.4957998928	-0.175673473	0.0918085045	0.1867325562	0.7549684143
-0.291519553	-1.068663497	0.4074618361	0.8230451922	1.3350201517	-0.426498332	1.0030678545
-0.473814178	0.8358794056	0.2333905293	-0.174329602	0.6707244557	0.1574671096	0.0922341644

-0.353500944	1.7771499879	1.8362476788	0.9301421149	-0.040898985	1.6915230398	0.2423006939
-1.781276079	-1.952789666	1.0738144096	0.6572256675	-1.642930549	1.4199351723	0.8993322209
-0.985963077	0.5674567416	1.3091178487	-1.677695105	0.8740999149	0.3523701126	0.5436032289
-0.358871942	-0.062750215	-1.5521215	-1.377236596	-0.017969128	0.1806449671	-0.674798722
-0.928393436	-0.06234918	-0.215226478	0.7776220015	0.6716574137	1.9234882509	1.0088267061
0.2025919469	0.5001478819	0.3936805693	-0.149325754	0.4842891747	-0.173419578	-1.063211892
-0.12360702	-1.262419111	-1.556436213	0.706033355	0.0761671759	-0.995065118	-1.182757357
-0.198754695	0.290132102	-0.031557623	0.2079559716	0.566237338	-1.052858586	0.6562745911
0.1598213592	-0.564917419	-0.381205296	-0.454419593	0.3245736827	-1.690902028	0.4684163014
2.1434043878	0.1168996301	0.6681212523	0.1876730418	-1.34969578	-0.372625199	-1.359265666
-0.977747902	-3.36368234	1.0950249297	-0.869909609	0.7882388348	-0.77267779	-0.961166734
-0.302427389	0.1365681738	-0.707020263	0.4643929092	1.2890894436	0.1112974358	-0.726500743
0.3220750894	0.3096621229	1.3291593762	-0.877378874	0.7111985689	0.4703350638	-1.400102888
0.4734500772	-0.73549594	-0.002258217	0.3175849307	-0.806587429	-0.44908097	-1.394431255
-1.067873752	0.9350729505	0.2060392217	0.6335367127	-0.522388046	0.8674765568	0.4019343529
-0.534973347	-1.839896295	-0.865438921	-0.520415205	1.0902359297	0.4710287784	-0.505678959
1.0284853581	-1.081167218	-1.428811672	0.3686363069	-0.52036515	-1.142815106	1.415295481
0.9996670913	0.7824055367	0.888188881	1.3517190654	-0.436565955	0.5274566489	0.4829140533
0.7114906817	0.3259209312	-0.829187143	0.4165295007	0.087228061	-0.856585421	-0.151644528
-0.152004206	-0.643999752	-0.188984863	-1.151272485	1.2490900685	-2.499922486	-1.762007539
-0.090199062	1.0027424574	-0.886736874	-0.104263748	-1.111267805	-1.146720368	-0.060917343
1.2636796866	-0.129261481	-1.685267041	2.0900325003	-0.373611352	-0.934989663	-0.043961177
0.6890966942	0.716216958	0.9396815646	-1.517685256	0.871305029	-1.158890515	0.7870619461
-0.450967136	-0.156265984	-0.987549391	-0.6452827024	1.1112305855	-0.105849333	0.6229565103
-1.890225325	0.6552840826	-2.643455151	-0.245514497	-1.274352765	-1.117121084	-0.221348605
-0.674280551	-0.367233425	0.1518374734	0.8623678835	1.0169370646	0.7343736353	-0.241931176
-0.101835526	1.4865949283	-0.240400848	0.698175354	-0.438935947	0.7902842092	-0.018456105
-0.478040662	0.0751338319	-0.367139869	-0.668021269	0.8425813691	1.4369761932	-0.683969192
0.3836408263	0.3797261382	0.9104988058	-0.468487053	-1.536452638	-1.070318926	-1.143191208
-0.870301635	-1.687453909	-0.863874551	-1.567709016	0.1466183311	0.4469434401	-0.250711193
-0.931095648	-0.035311921	-1.207436918	0.5743703454	0.4661076117	-1.145978192	-1.151620562
0.0509272778	0.0883433621	0.1881143813	-1.683543741	-0.900798043	-0.829323363	-0.606132361
3.2535164449	-2.337863759	-1.277135929	-1.28018325	1.6465932575	-0.90967014	-1.400577354
-0.781705878	0.9306506656	1.4467320949	2.0842956688	-0.376241476	-0.088318169	1.2044833844
-0.817588012	-0.934044156	-0.300773943	-0.152940414	0.2682222912	0.0485120279	-0.383724197
-1.540844837	-0.123466192	-0.581255175	0.1540241764	0.6839180448	1.8740558712	0.1012854732
0.1827465014	0.2390206343	-0.56939043	0.7605174501	-1.149413224	-0.554754162	1.0183982316
-1.415668281	-0.909923872	-1.462258198	0.0994611509	0.5596122418	0.1463684622	-0.318497939
-0.325493216	0.5727462546	0.1451074665	-0.196920225	1.9360683343	-0.269255663	0.9041100954
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-0.256979926	1.1927844122	0.9182314645	0.5765546151			
-0.958845524	0.7310259325	1.3761005571	1.2293767603			
1.7797452899	0.156809398	-1.30504274	0.5476280278			
-1.750451594	-0.556920415	-1.045844117	-2.037794553			
-1.573955113	-0.35655917	-0.533110366	-0.780465814			
-0.430222682	-1.200414707	0.7797408555	-1.994529141			
2.0416535563	0.014375525	-0.603538411	-0.31441796			
0.4815946499	-0.257455011	1.0169872045	1.0683928593			
0.0128595899	0.2864423788	0.615109876	0.2825208958			
0.7101603762	-1.149416845	-3.957636212	-0.66343806			
-1.155550558	-0.158727591	-1.440591256	-1.384620396			
2.1695510613	-0.829739457	0.4312408456	1.4755151101			
0.5686978245	0.0764110716	0.3734860947	1.3399120451			
-0.390546559	0.9116229513	0.6249413672	-0.485354054			
0.8465015544	1.513626087	-0.288344065	-0.48630578			
-2.382929913	-0.223852111	1.128403342	1.4000726216			
-0.438784582	1.0984385362	-0.416717572	2.4614315424			
1.5952174929	-0.923867505	-0.360827371	1.7645753801			
0.8973280686	-0.325080031	0.1542491895	-1.840476768			
-0.202504025	-1.122482026	-0.369715572	-0.001100513			
-0.361003794	1.7425471918	1.7443940087	-0.065873862			
-2.790379826	0.3699994788	0.0564221529	-1.395332832			
0.1745341957	1.3652403123	-1.697806819	-0.02802483			

-0.60178475
-0.95074428
1.9499954186
-0.733819501
1.6884104097
0.2071614763
0.6903950054
-0.463533261
-1.506412598
-2.396803011
0.2199943793

APPENDIX D

**IRT PARAMETER VALUES USED TO STUDY
THE DISTRIBUTION OF DIF_1 AND DIF_2 INDICES**

Group 1			Group 2		
A	B	C	A	B	C
1.03526	0.35374	0.20356	0.59730	0.98422	0.24870
0.98336	0.81627	0.09880	0.93589	0.30452	0.14085
0.96689	-0.39206	0.17587	0.54691	-0.00054	0.12479
1.25280	1.07481	0.19797	1.08731	-0.82544	0.17780
0.89784	-0.59125	0.22267	0.97017	-0.02262	0.13663
0.83129	-1.40125	0.29170	1.45191	1.47928	0.23557
1.28435	-0.86086	0.17979	1.18258	0.13773	0.17242
0.58195	-0.30337	0.21240	1.02806	0.98715	0.25313
1.40525	-1.29354	0.22158	1.75940	-1.53322	0.19745
1.28220	0.66892	0.18235	1.01806	0.35097	0.12853
0.65439	-1.69516	0.20449	0.47692	-0.56777	0.13163
0.57537	-0.02181	0.19174	1.22556	-1.49607	0.25585
1.13006	0.48220	0.24830	1.24321	0.76197	0.23435
0.50889	-1.28200	0.17961	0.70112	-0.97324	0.29110
1.28488	0.16590	0.19216	0.98752	-0.43377	0.12423
1.36452	-0.09718	0.23772	1.53543	-2.32731	0.22770
1.49607	-0.57731	0.32681	0.98296	-2.16007	0.20940
0.98672	0.12894	0.27441	0.91097	0.17240	0.17232
1.50484	-0.30986	0.18297	1.21900	-2.36164	0.14054
1.07991	1.00578	0.18215	0.72113	0.61593	0.11523
0.99722	0.58612	0.09749	1.05161	1.65483	0.12881
1.39462	-0.29905	0.22681	0.91998	-0.58041	0.17522
0.58880	-0.81573	0.14207	0.40937	-1.40549	0.26897
0.83213	-1.23879	0.14780	1.33579	0.49632	0.16493
0.92327	0.75074	0.22113	0.75846	0.45679	0.21785
1.61756	1.37266	0.26572	1.04924	-0.06155	0.18317
1.47425	0.09704	0.17341	0.89450	-0.52879	0.25802
1.33860	-1.14764	0.17856	0.98129	-1.65769	0.25424
1.22685	-1.21423	0.20762	1.13516	-0.17807	0.19438

1.05558	-0.64110	0.12879	0.96204	-0.26198	0.20575
0.92728	-0.55051	0.28809	1.15388	0.46160	0.21804
1.45035	-1.57487	0.14261	1.00815	-0.54140	0.18599
0.83400	0.29650	0.20606	1.10313	0.27186	0.23610
0.84895	1.47701	0.19114	1.26626	-1.10245	0.17589
1.03840	0.07216	0.12673	1.32775	-1.47667	0.22832
0.52885	1.18457	0.11530	1.46325	1.11012	0.21991
0.78739	-0.89027	0.31064	1.09525	-0.27848	0.20990
1.22427	-0.86536	0.15458	1.23839	-0.21472	0.22916
0.60089	2.96936	0.15447	1.66960	0.27323	0.18451
0.74640	-0.28741	0.15613	1.38511	-1.51957	0.31685
1.32003	0.14256	0.16322	0.80670	1.26339	0.21737
0.95154	1.62345	0.18473	0.95959	-0.67533	0.19646
1.32650	0.08797	0.20585	0.90897	0.22912	0.29528
1.04082	-0.63529	0.21446	0.98600	-0.54852	0.22582
0.82912	0.64541	0.21809	1.32034	-1.09722	0.16185
1.18867	-1.11018	0.11021	0.95304	0.13379	0.14076
0.82221	1.29858	0.24741	0.42144	0.20015	0.25098
1.24710	0.31855	0.27269	1.38695	0.95290	0.20103
0.89494	0.44460	0.21856	1.16602	0.50896	0.16077
0.39594	0.68026	0.16424	1.27891	-1.13626	0.20778
1.35472	0.31478	0.19039	0.63307	1.03702	0.15800
0.68105	0.87183	0.11706	0.71799	1.03352	0.32487
0.96123	1.06377	0.19262	1.11968	-0.23521	0.21341
1.20228	0.40963	0.14058	1.24765	1.41935	0.16459
1.09117	0.15999	0.17219	0.76075	-0.60125	0.20647
1.38849	-0.65597	0.26373	1.46569	0.14798	0.22565
0.79973	0.20148	0.22870	1.33351	-1.56703	0.14201
1.30410	-0.38777	0.24721	0.93422	-1.29783	0.29030
0.83561	-0.05064	0.19992	1.07218	0.44497	0.19374
1.16965	-0.42719	0.17701	0.71946	-1.64177	0.20284
1.18088	-0.05873	0.22707	0.66482	1.60033	0.24300
0.94708	-0.21302	0.25418	0.98120	1.28359	0.22226

0.77438	0.20053	0.17032	1.26385	0.40023	0.24003
0.75943	0.13497	0.22105	0.61866	1.27980	0.20982
1.15119	-1.46627	0.18930	1.13512	0.27494	0.21561
1.24938	0.70036	0.15093	0.84212	-0.17892	0.26183
0.60161	-0.00714	0.20609	1.00340	-1.20958	0.18028
0.85232	1.30505	0.13312	1.11638	-0.13029	0.19313
1.20483	1.17962	0.12686	0.69421	1.41691	0.24087
0.77374	1.66731	0.21824	1.02552	1.15507	0.20000
1.32126	-0.54387	0.20151	1.37950	1.50429	0.18604
0.76379	-0.50335	0.14086	0.97491	-0.19278	0.17051
1.14723	-1.26596	0.17383	1.06999	1.32741	0.19474
0.92460	-0.42006	0.17910	0.96932	0.73373	0.20375
1.66488	-0.35483	0.19304	0.67800	-1.96839	0.14941
0.60680	0.83291	0.23936	1.26640	0.71171	0.20095
0.83324	-1.57531	0.20977	0.56658	0.54958	0.19414
1.18681	-0.79976	0.22549	0.91283	0.30015	0.17249
1.21742	0.22492	0.26110	1.54505	-1.12592	0.23749
1.46911	-1.36913	0.19622	1.28420	-0.56192	0.10522
0.88971	1.26173	0.19594	0.83721	-0.59546	0.15361
1.22888	0.24359	0.15915	1.51253	0.85137	0.12510
0.36347	-0.03081	0.14129	0.93837	-0.13820	0.20004
1.24958	0.57903	0.22027	0.67879	-1.60560	0.18489
0.76568	0.93934	0.13701	0.64948	0.77433	0.14378
1.22430	-0.87835	0.17092	0.74676	-0.33966	0.21868
0.79711	-0.39976	0.17502	1.14782	-1.15159	0.19609
0.52058	0.11787	0.24928	1.03431	-1.11080	0.18948
0.85596	0.13837	0.12291	0.93270	-0.41875	0.22076
1.24774	-0.95036	0.16889	1.18718	-1.72747	0.22509
0.75219	-2.65295	0.25608	1.01555	-0.84931	0.22253
0.76880	-1.29672	0.20340	1.02623	-1.56027	0.08730
1.38576	0.51598	0.18415	0.76958	-0.08802	0.24857
1.32633	0.07642	0.25999	0.88253	-0.95534	0.19127
1.18278	1.20672	0.20874	0.71592	1.32311	0.15446

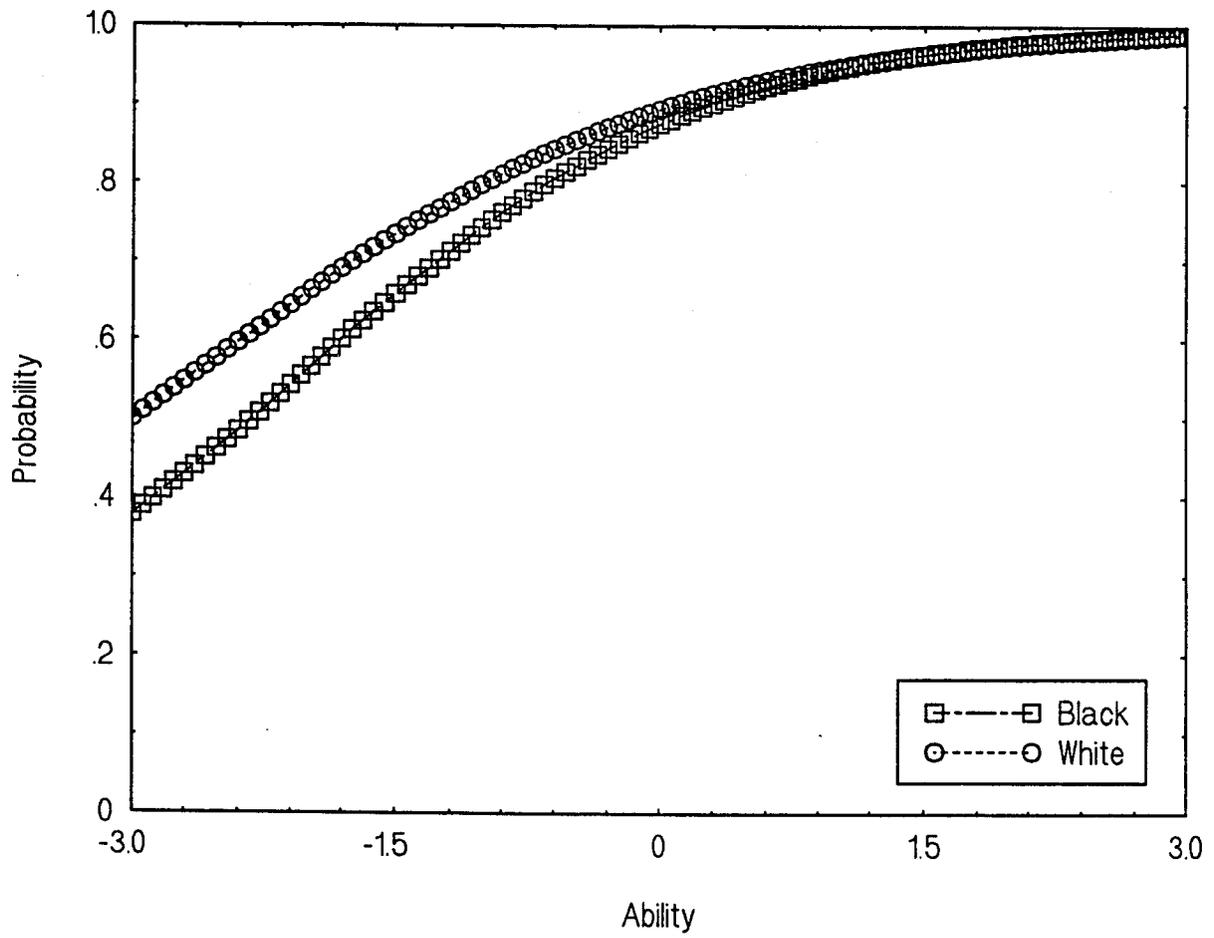
1.13368	-0.69435	0.19992	1.20220	-0.24744	0.19558
0.90818	-1.00641	0.14835	0.78753	-0.40492	0.15831
0.79891	2.39959	0.23721	0.94277	-0.31908	0.14571
1.52456	1.46609	0.25517	0.91620	0.70727	0.31523
0.97737	1.25207	0.30592	1.15562	0.26700	0.11098
1.23204	-1.11382	0.21205	0.45947	-0.81715	0.11737
1.20331	-0.61656	0.21105	1.29111	1.20707	0.24912
0.57798	-0.73088	0.21132	1.20449	0.16313	0.22463
1.56640	-1.44462	0.22227	1.04341	0.47462	0.13318
1.13191	0.51450	0.32240	0.81299	-0.05043	0.21921
0.46585	0.64694	0.27326	1.39967	-0.55990	0.09094
1.14464	0.57019	0.28383	0.50004	-1.19580	0.17564
0.56241	-0.15508	0.22079	0.79022	-0.16166	0.19451
0.96775	-0.28749	0.15939	1.49365	0.13141	0.12532
0.83199	0.30268	0.13039	1.13614	0.82303	0.15542
0.79509	-1.01719	0.24983	0.89502	2.11253	0.22763
1.07528	0.80802	0.23927	1.42553	-0.54885	0.17803
0.52552	0.67440	0.19962	0.79732	-0.84473	0.25641
0.68456	1.13266	0.22356	1.13163	-0.59166	0.17652
0.64650	-1.22219	0.23964	1.18247	-1.69486	0.24234
1.05301	0.74397	0.29024	0.86115	0.03011	0.23673
1.13800	1.02563	0.16849	0.90931	-0.54630	0.14615
0.99407	0.28204	0.22141	0.62631	-0.97119	0.21820
0.85951	-1.32550	0.21371	0.62987	-0.57174	0.26277
0.90802	-0.55573	0.21686	1.58098	1.75468	0.26582
0.97300	0.13753	0.15634	1.04720	-0.69742	0.12754
1.29192	0.34091	0.21605	1.21185	1.36045	0.11821
0.84657	-1.98048	0.17646	0.93933	-0.83352	0.14997
0.95622	0.61027	0.18231	0.49452	0.75396	0.17891
0.72309	-0.03447	0.19979	0.91402	1.09796	0.17890
1.05582	1.33158	0.28423	1.57121	0.48838	0.26709
1.20312	1.06002	0.19812	0.64236	0.66244	0.27001
1.40162	0.29022	0.17944	1.04687	0.75128	0.26116

0.79767	0.55550	0.21510	0.60245	1.34143	0.25420
0.69245	-1.59366	0.24846	1.57917	0.00226	0.24570
1.04125	1.21473	0.15159	1.04275	-2.14616	0.22566
0.79767	0.79802	0.19978	0.78273	-0.99568	0.15084
0.46197	1.17842	0.21609	1.23390	-0.44090	0.29246
0.82800	0.23052	0.19417	1.11100	0.12163	0.17815

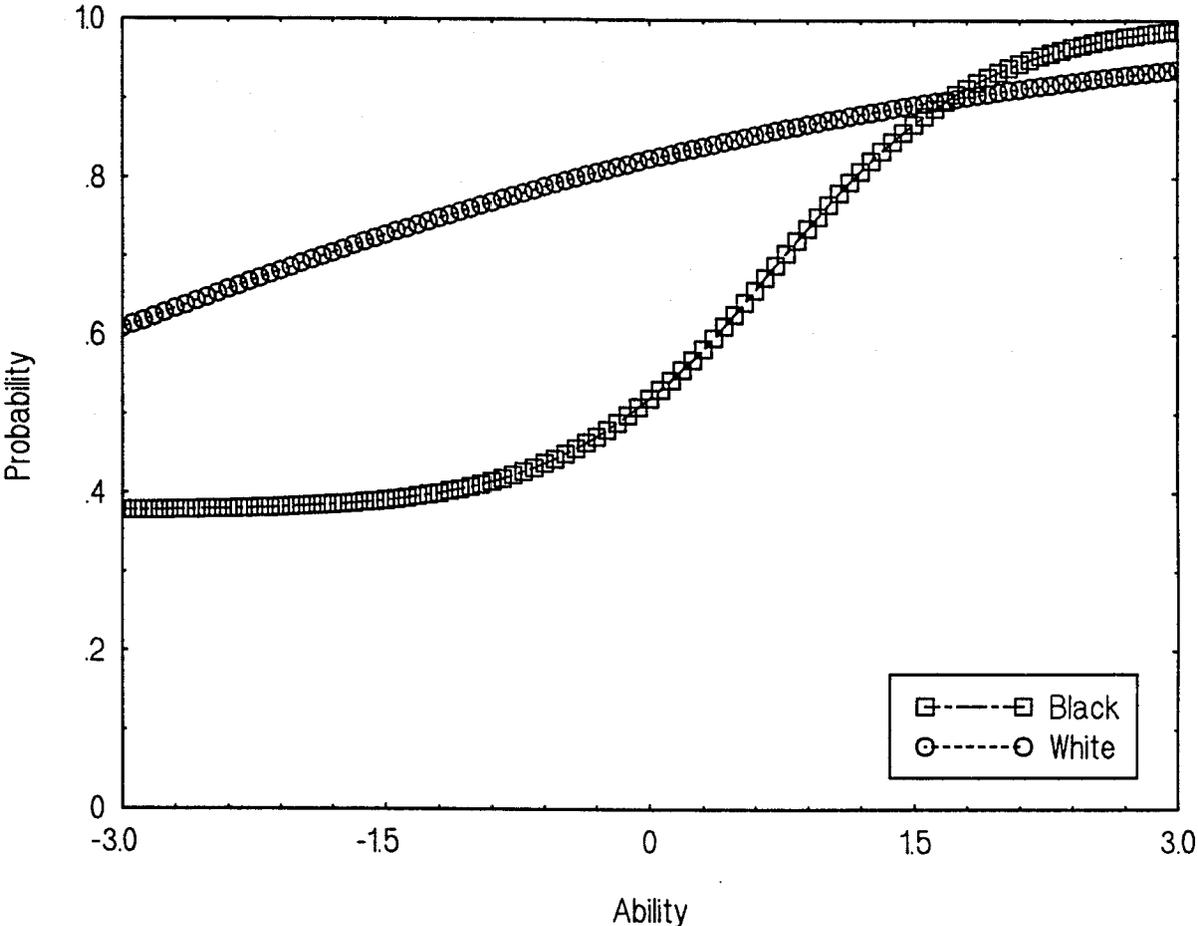
APPENDIX E

TRACELINES FOR BLACK AND WHITE MALE EXAMINEES

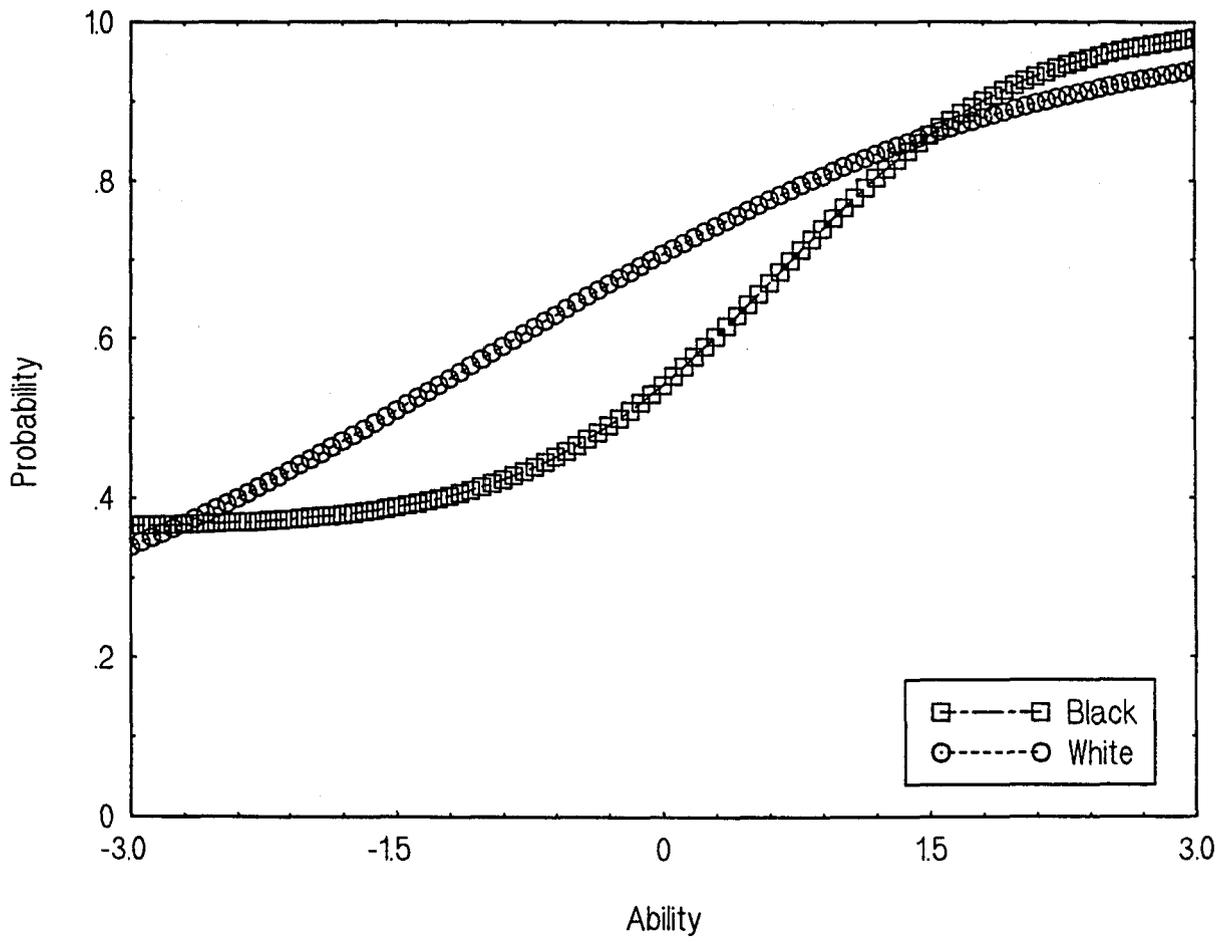
Black vs. White: Item 2-1



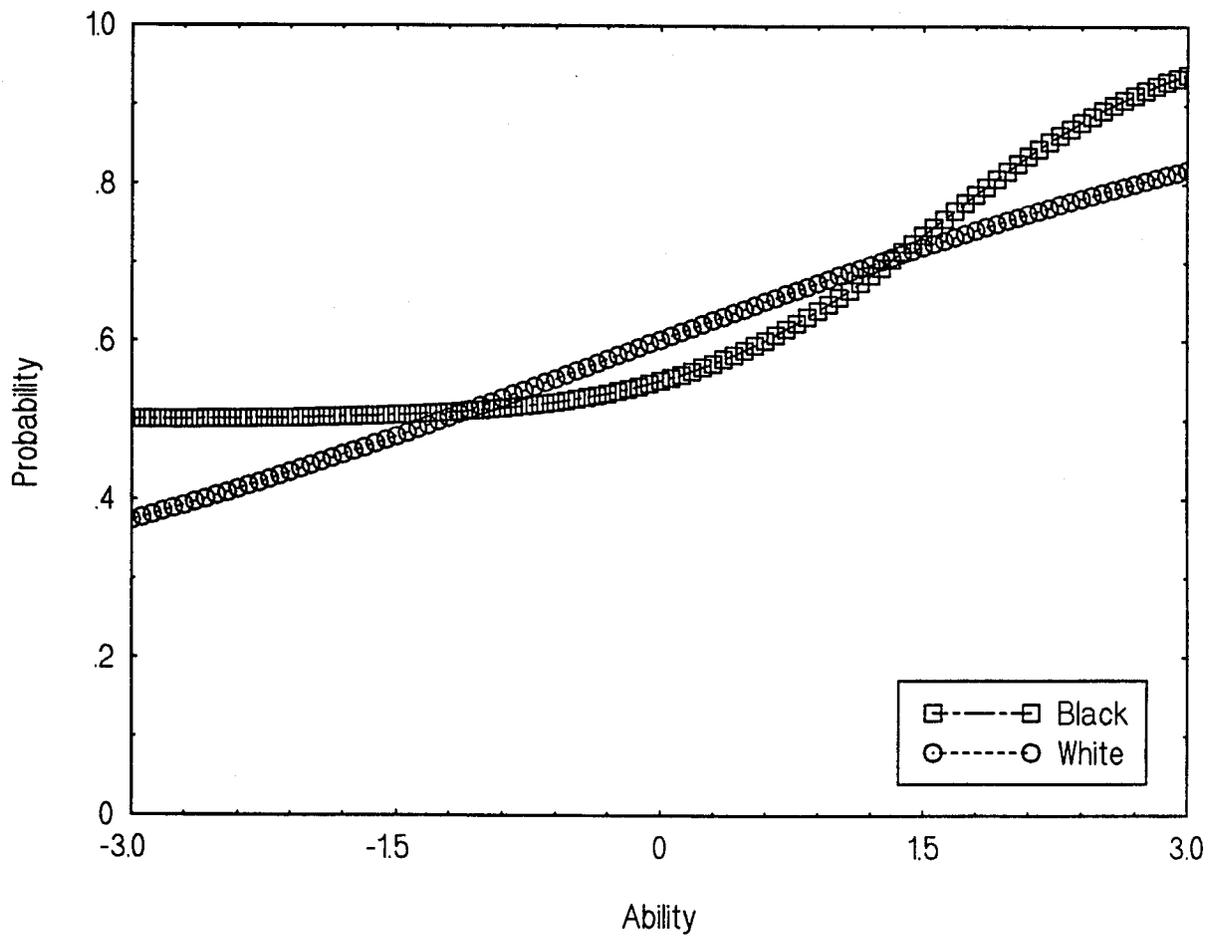
Black vs. White: Item 2-2



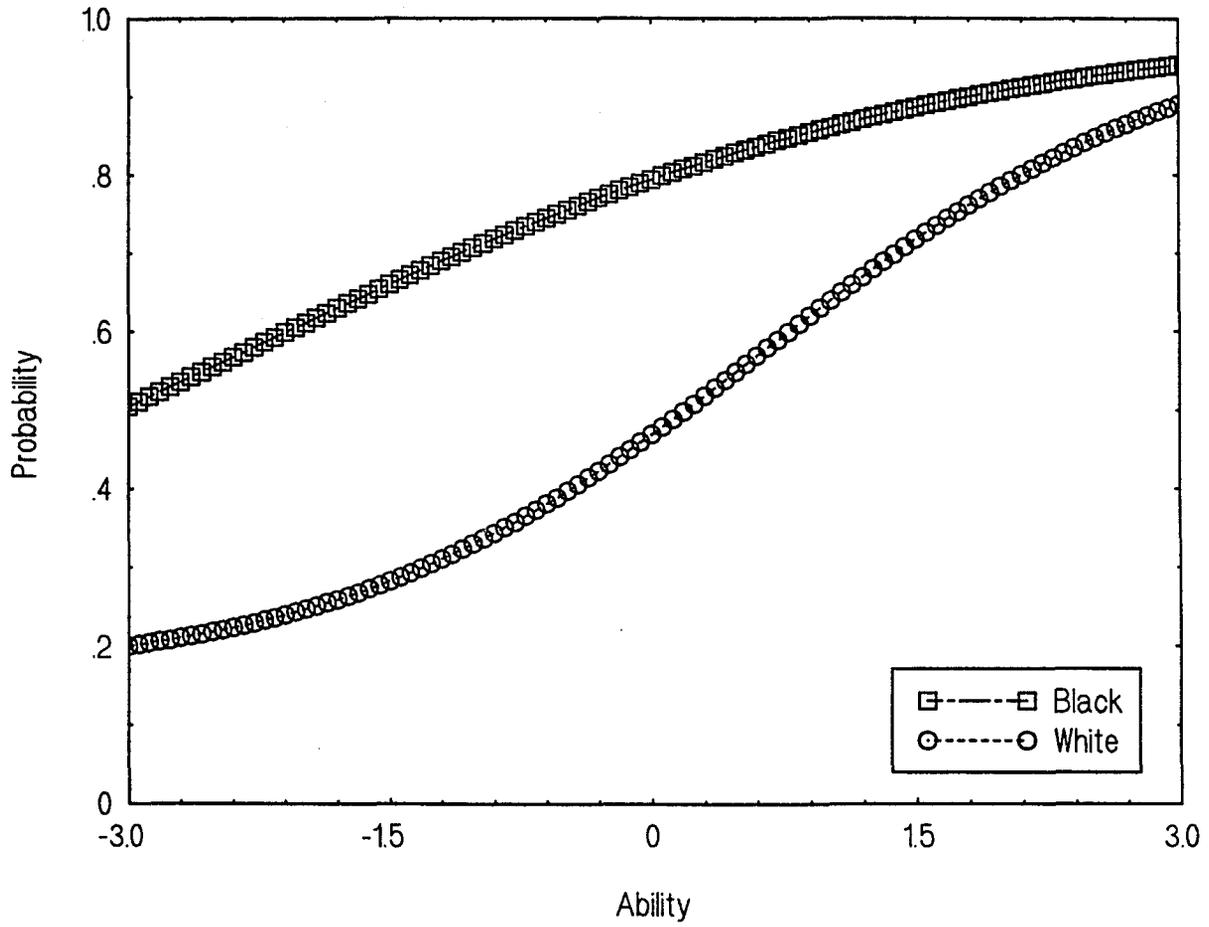
Black vs. White: Item 2-3



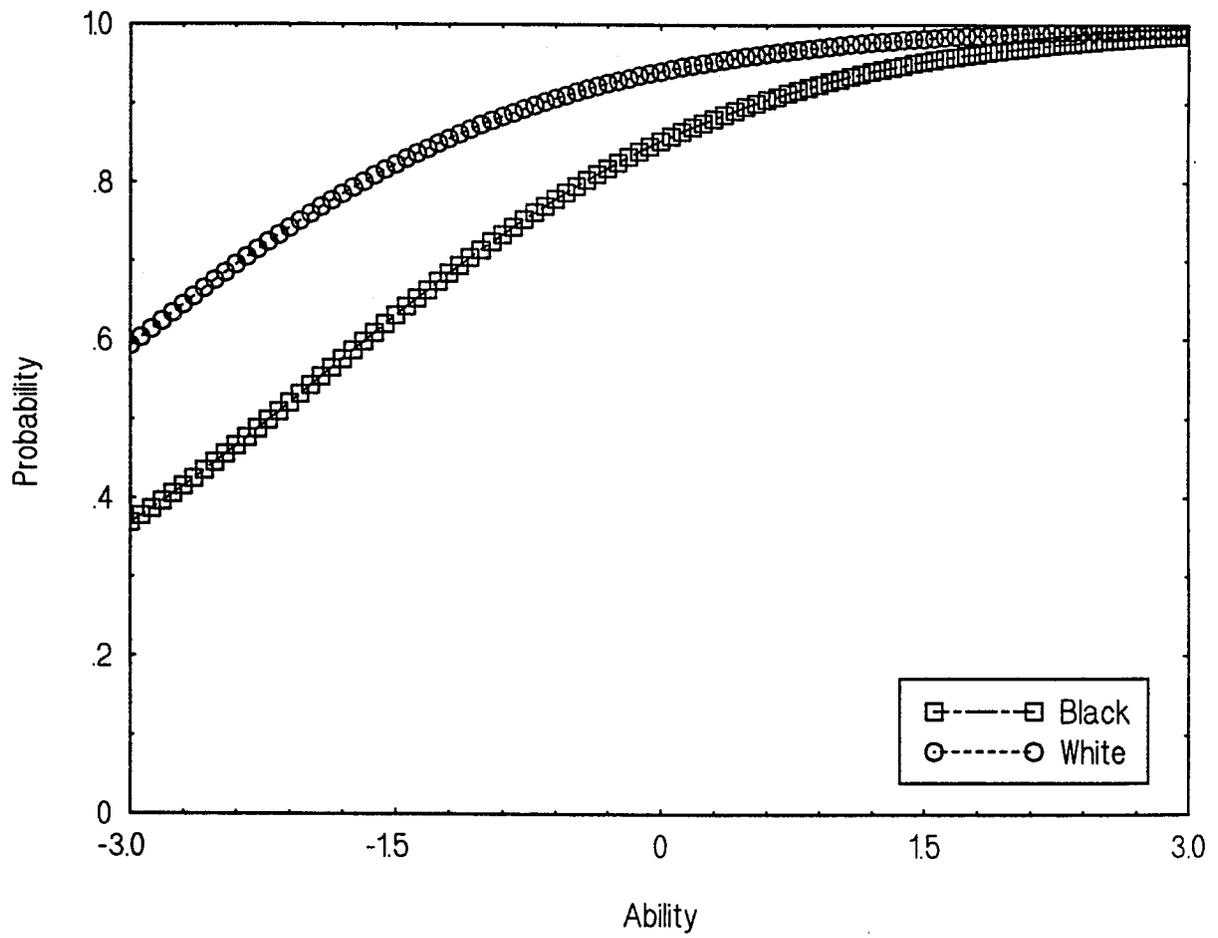
Black vs. White: Item 2-4



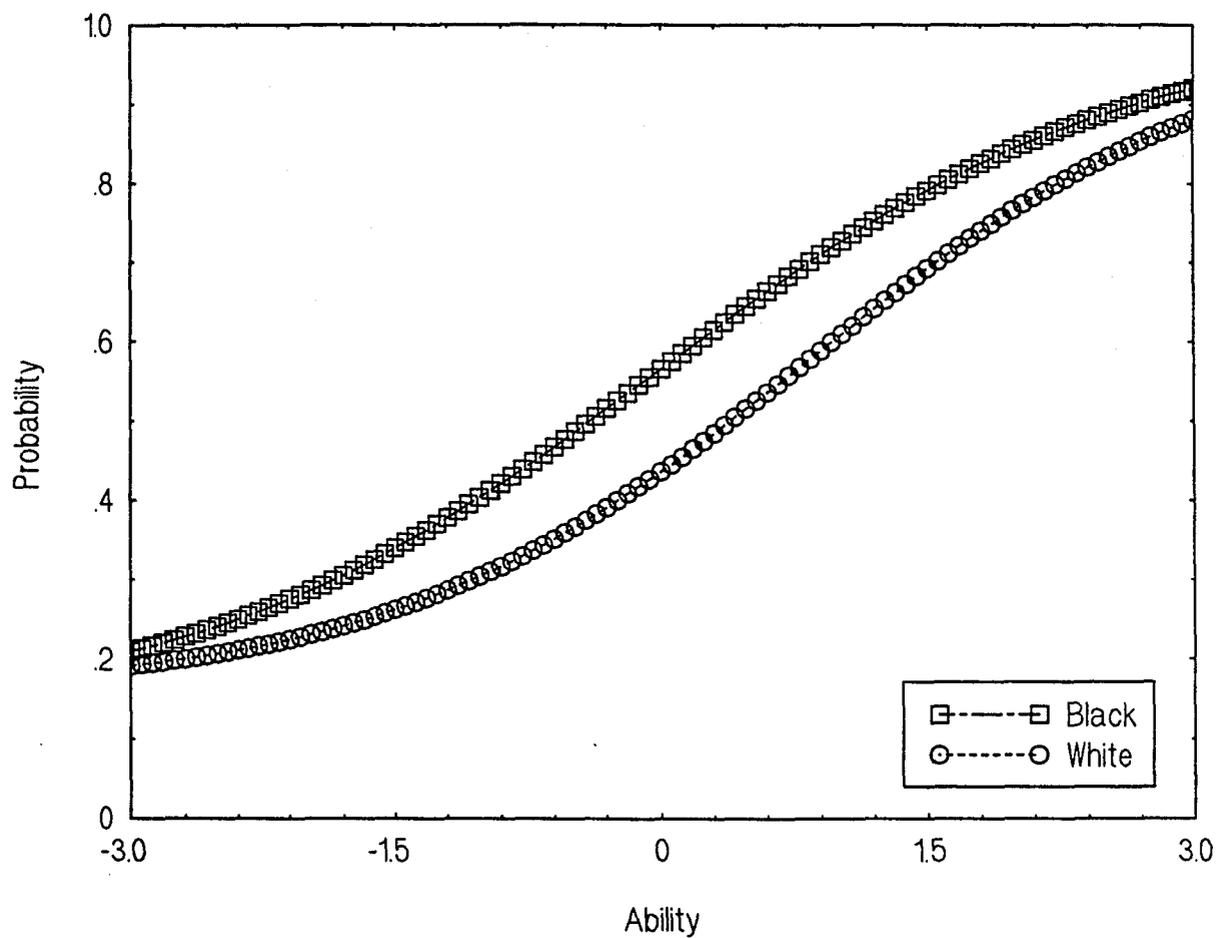
Black vs. White: Item 2-5



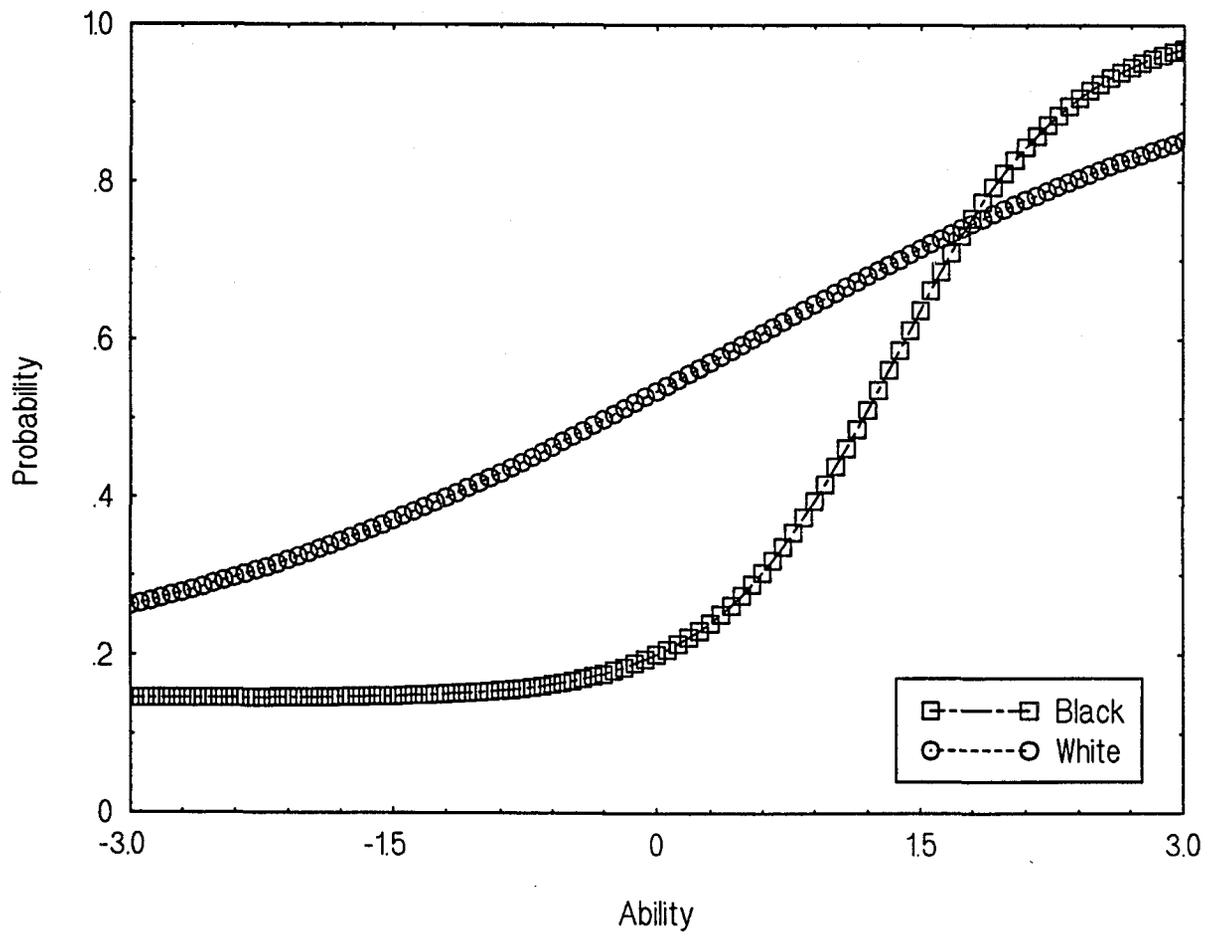
Black vs. White: Item 2-6



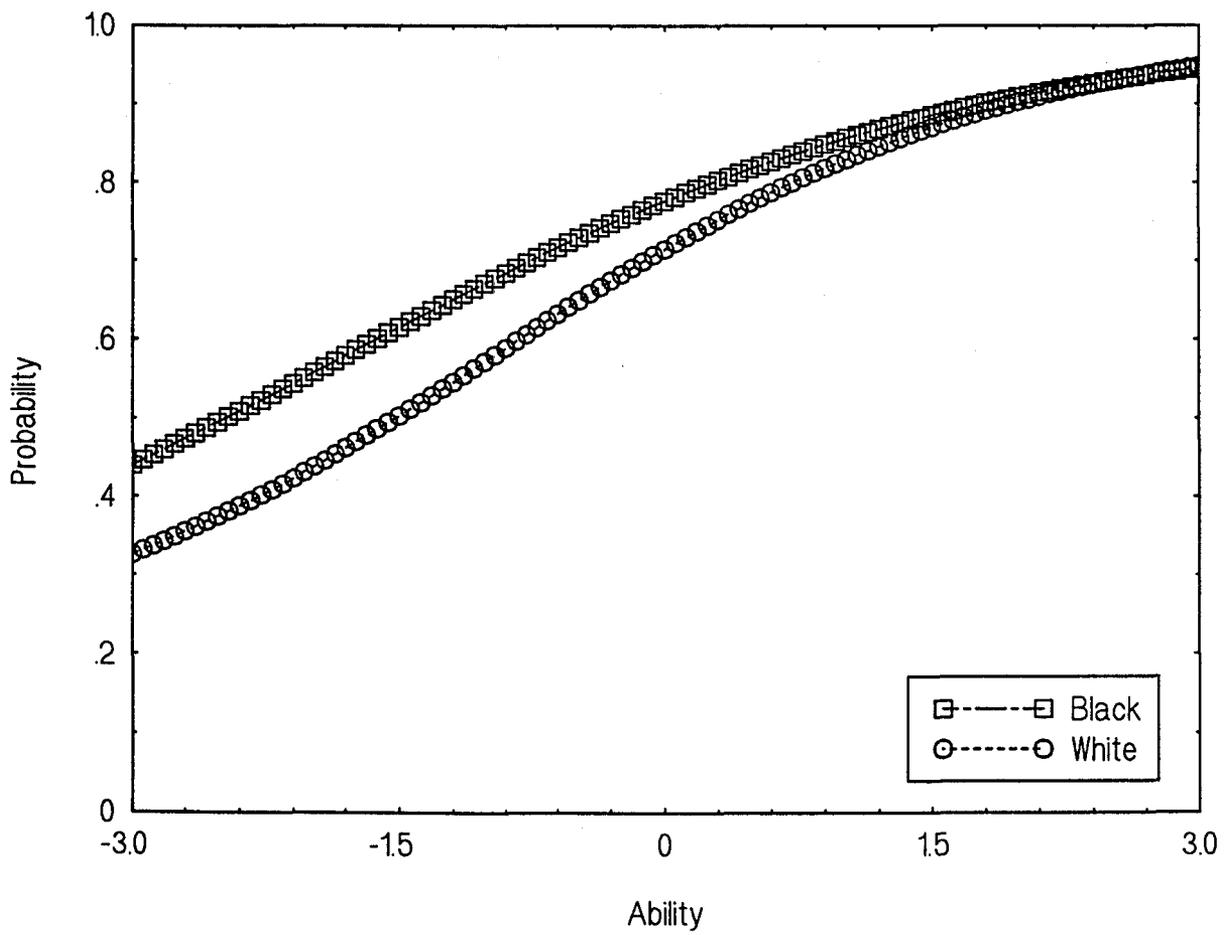
Black vs. White: Item 2-7



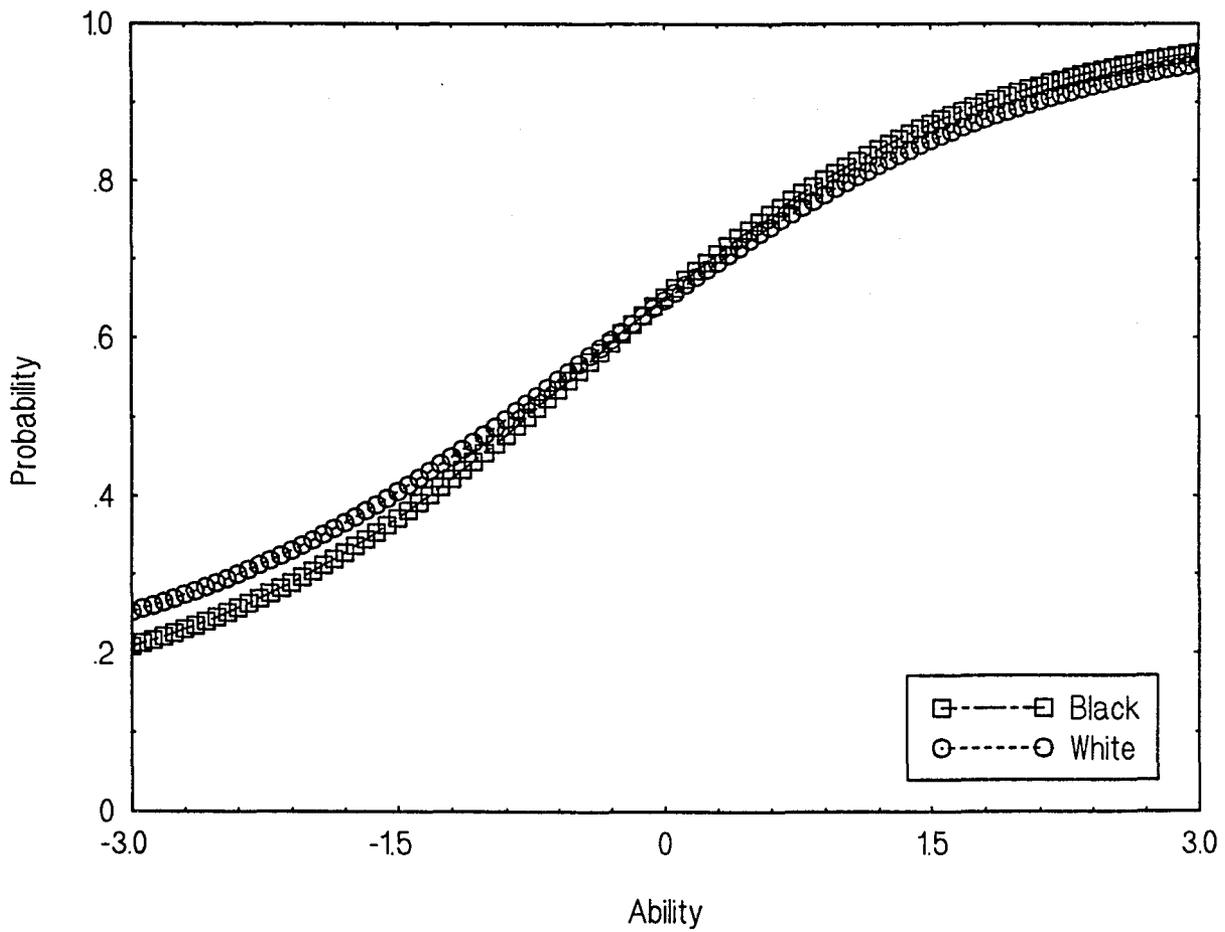
Black vs. White: Item 2-8



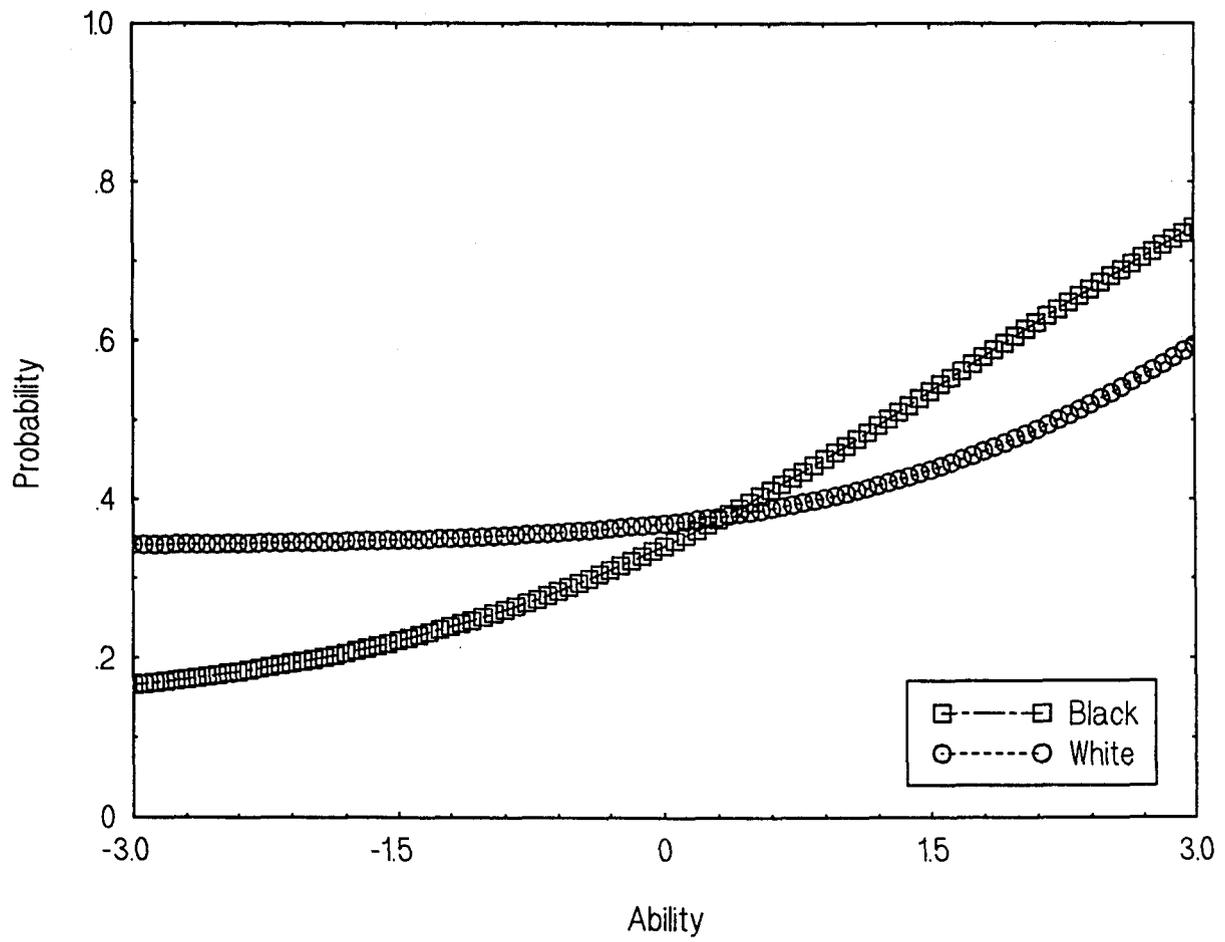
Black vs. White: Item 2-9



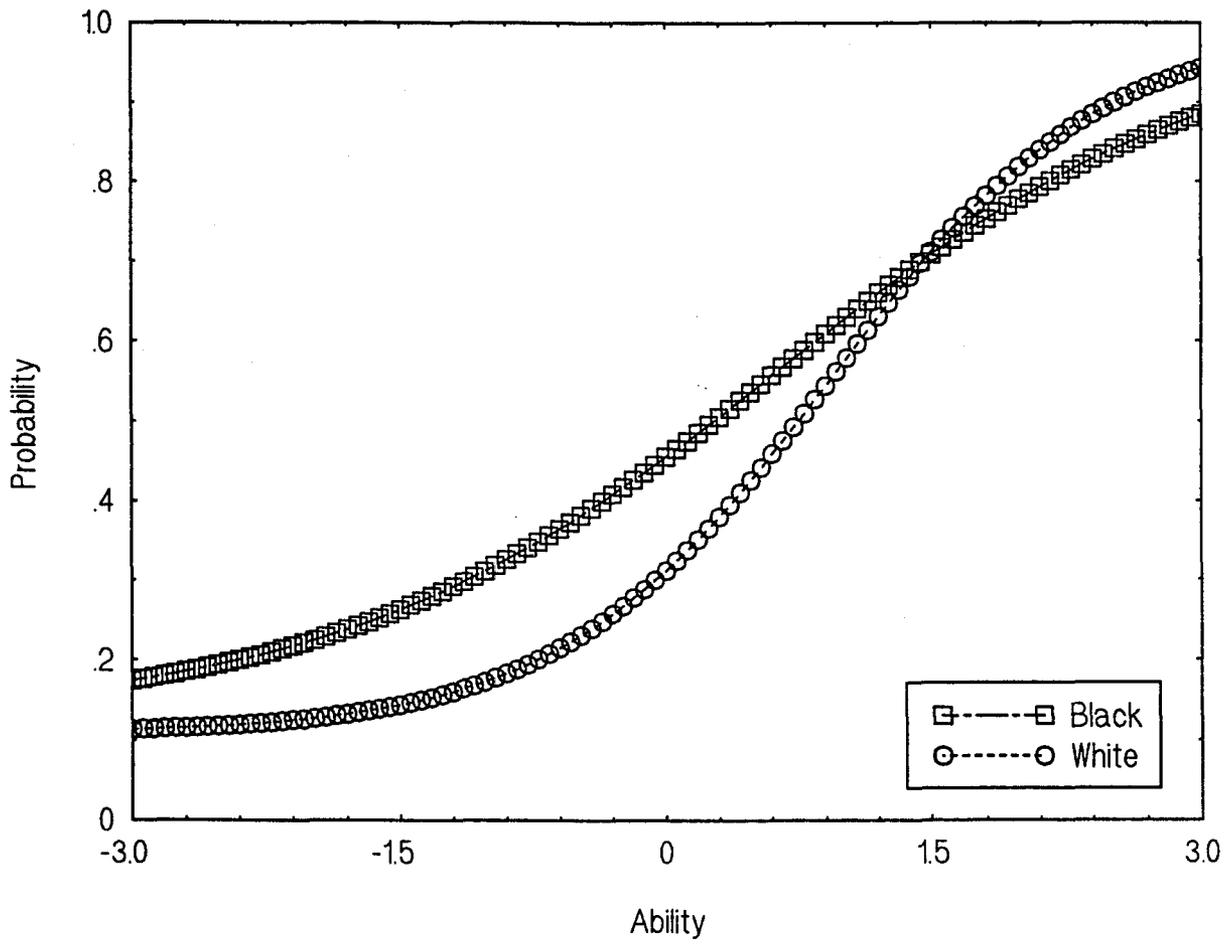
Black vs. White: Item 2-10



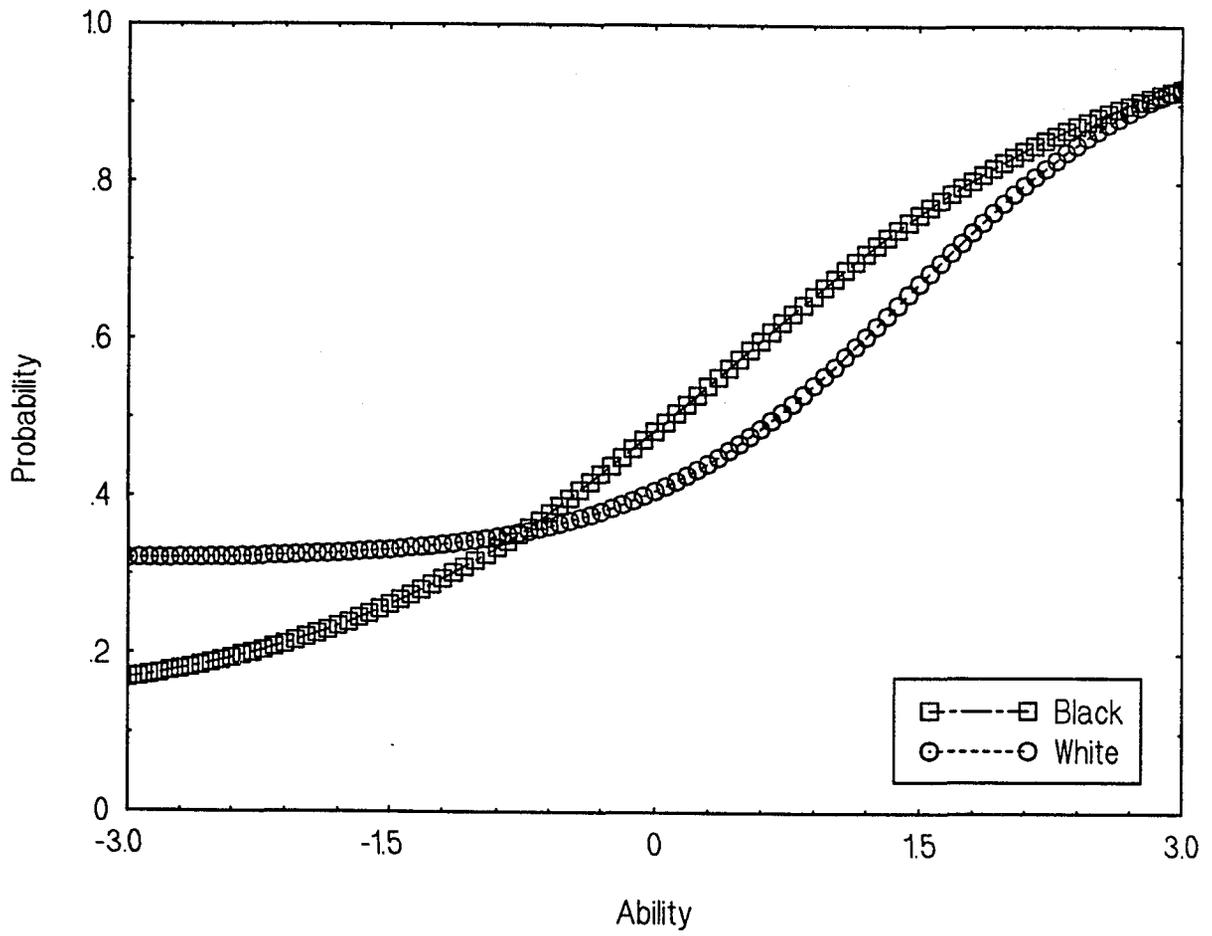
Black vs. White: Item 2-11



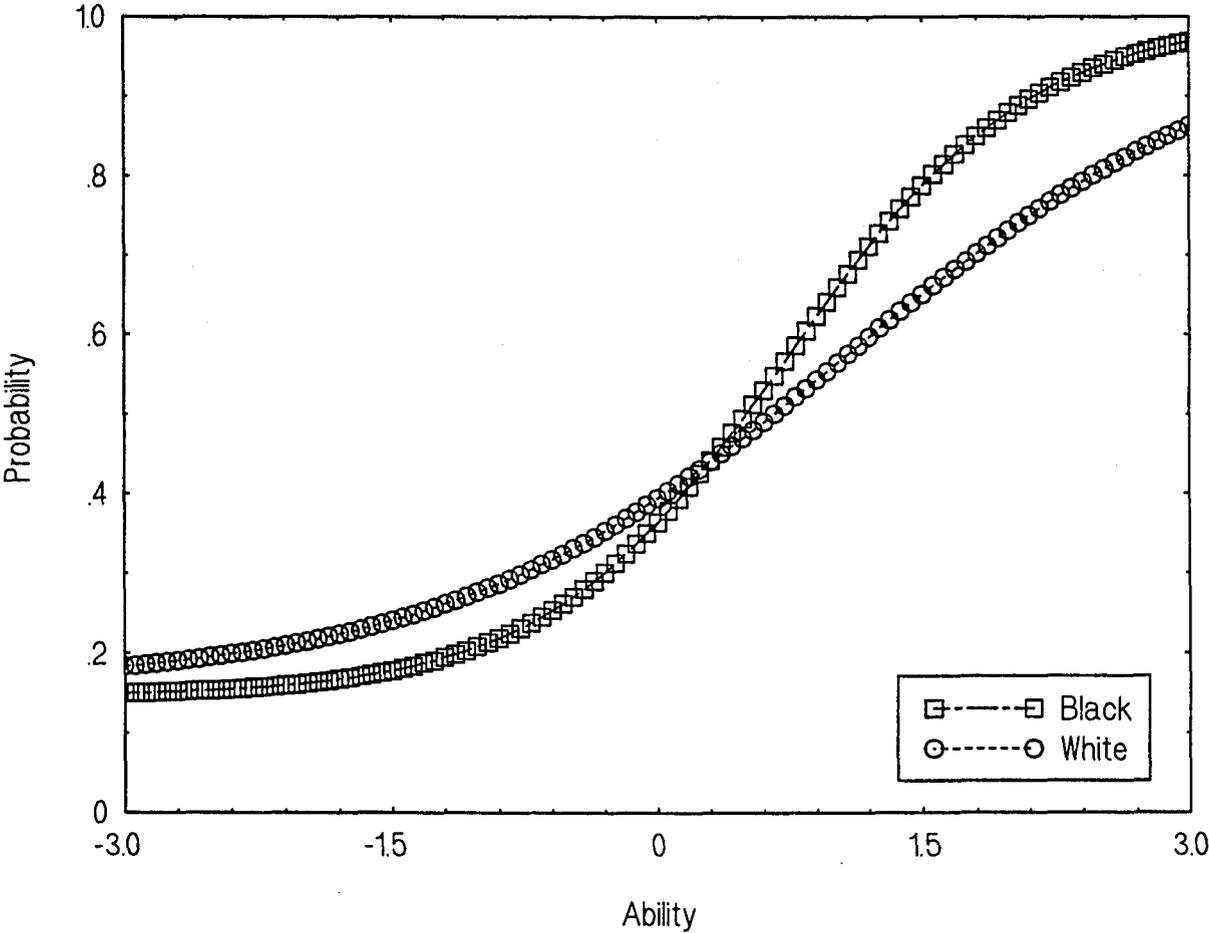
Black vs. White: Item 2-12



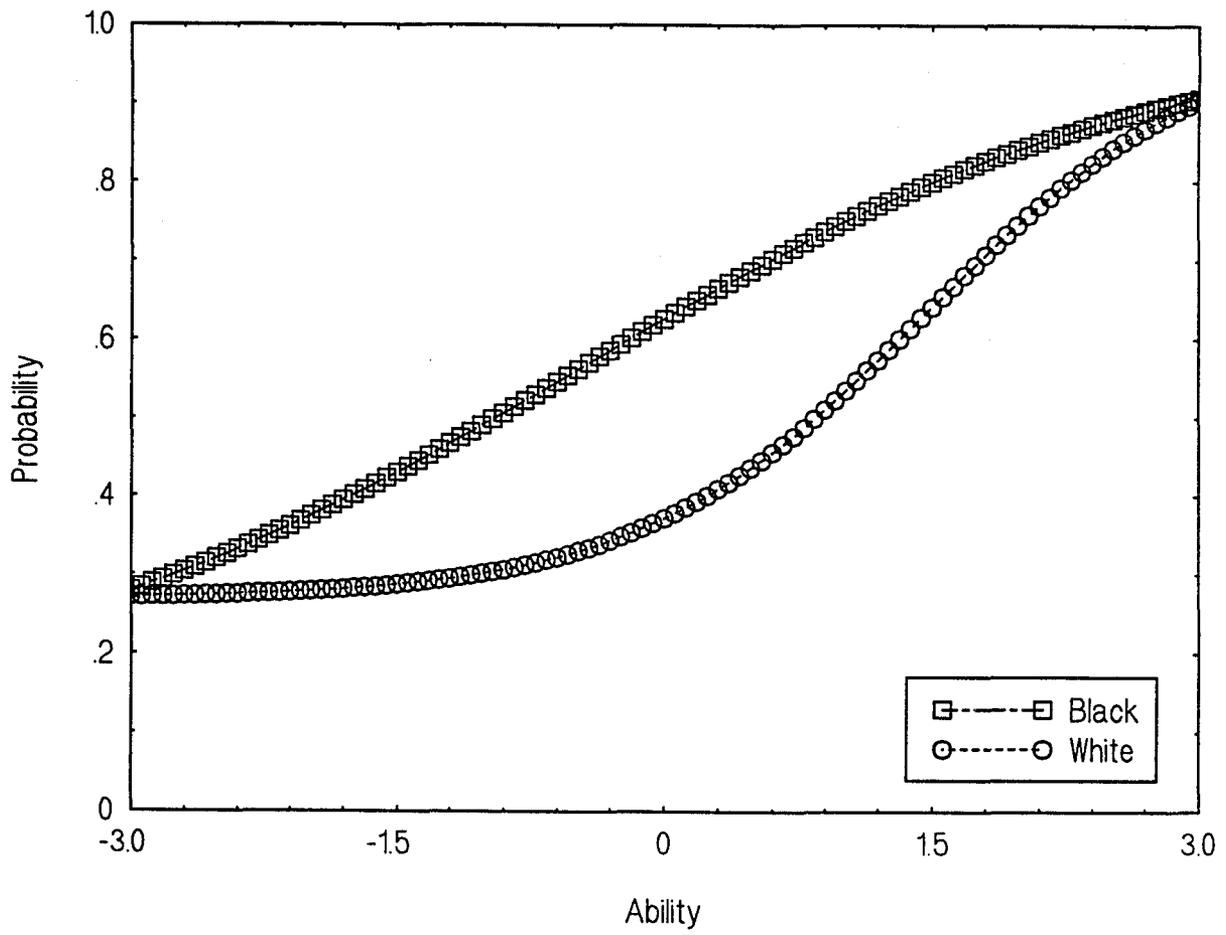
Black vs. White: Item 2-13



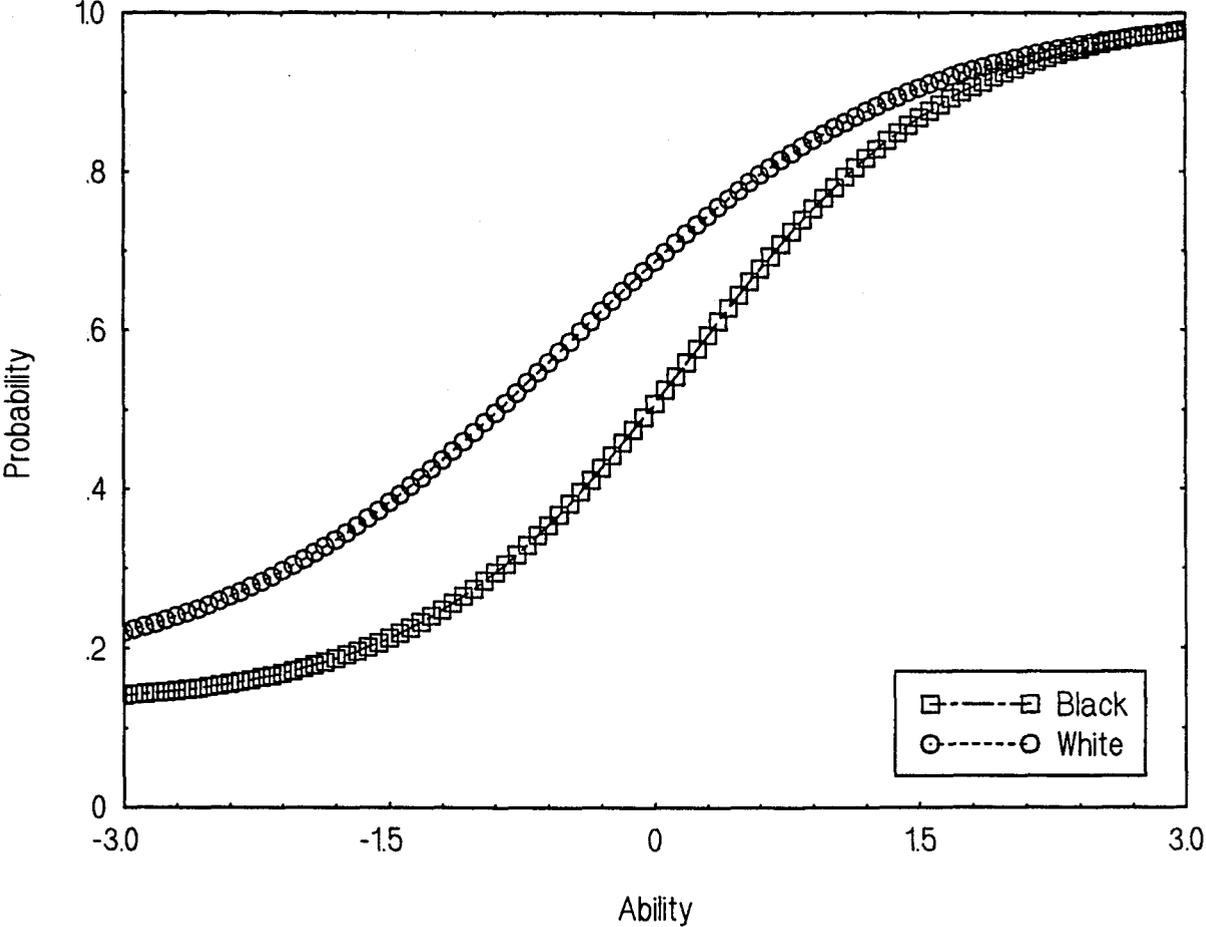
Black vs. White: Item 2-14



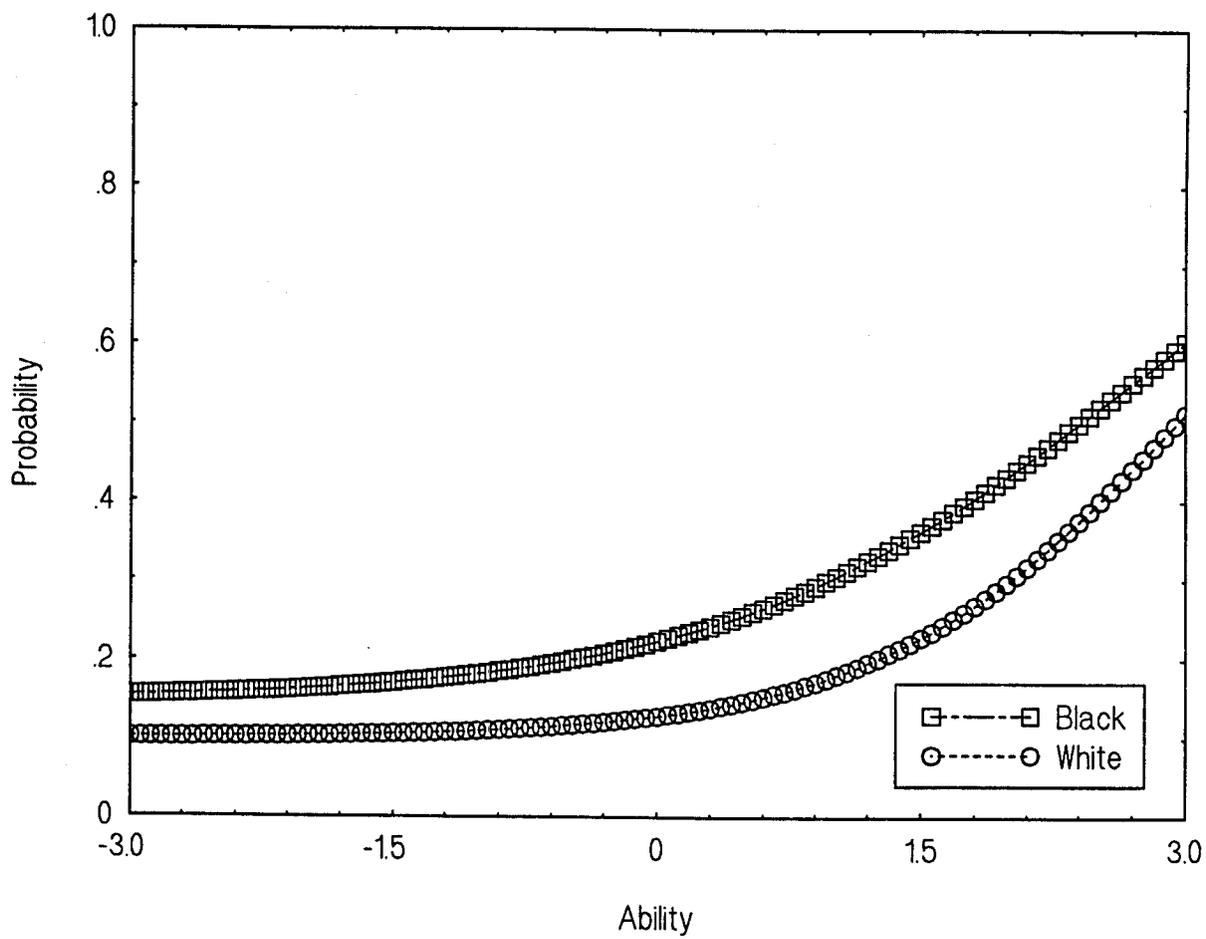
Black vs. White: Item 2-15



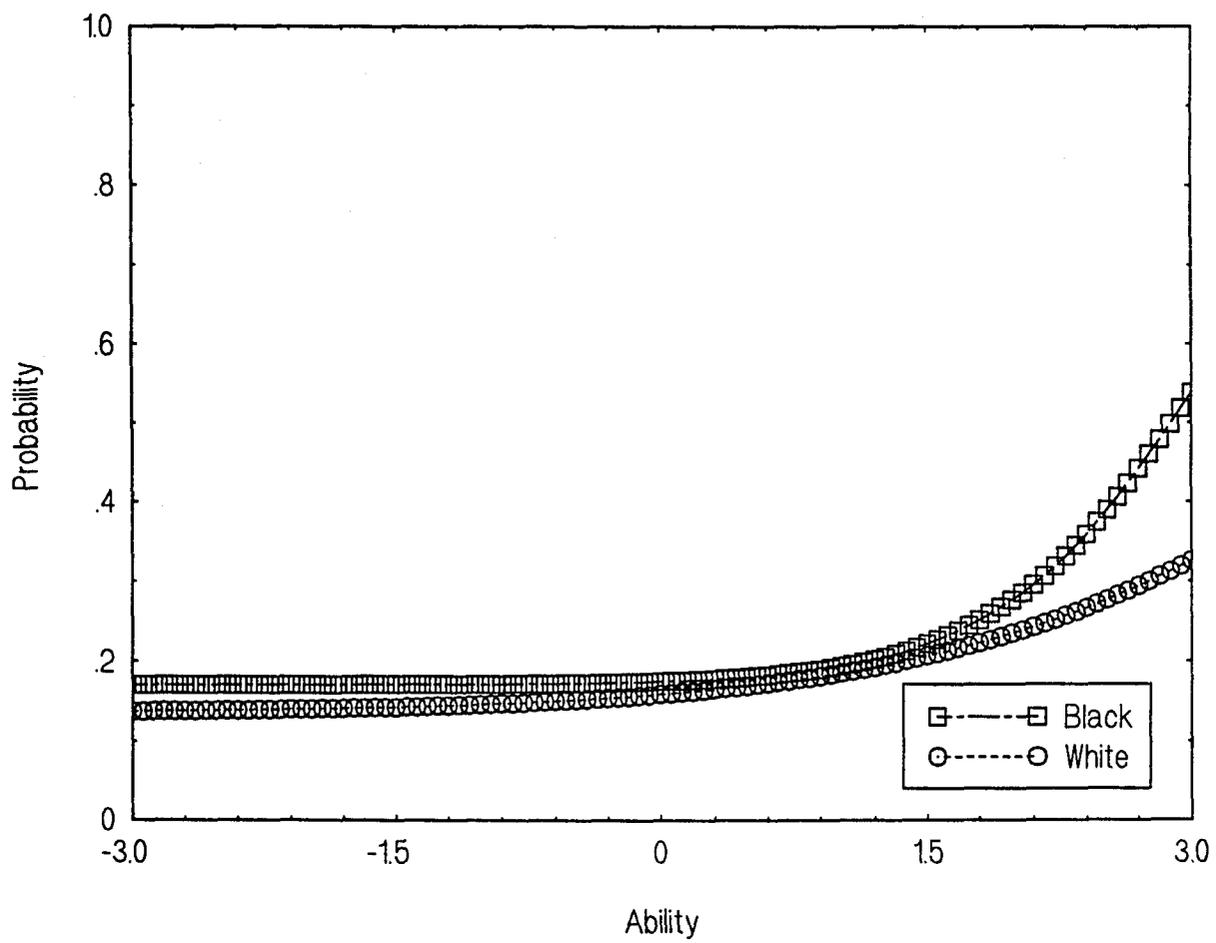
Black vs. White: Item 2-16



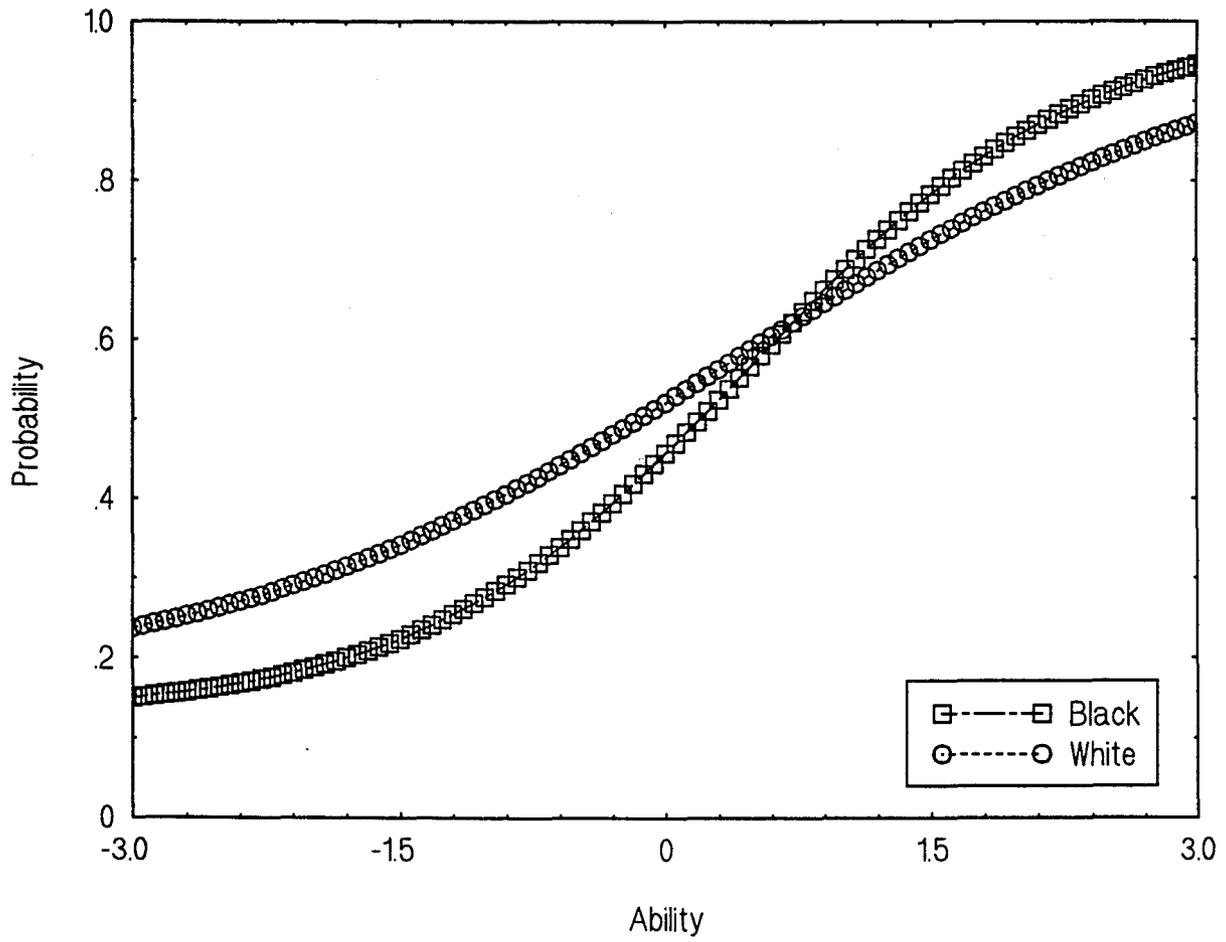
Black vs. White: Item 2-17



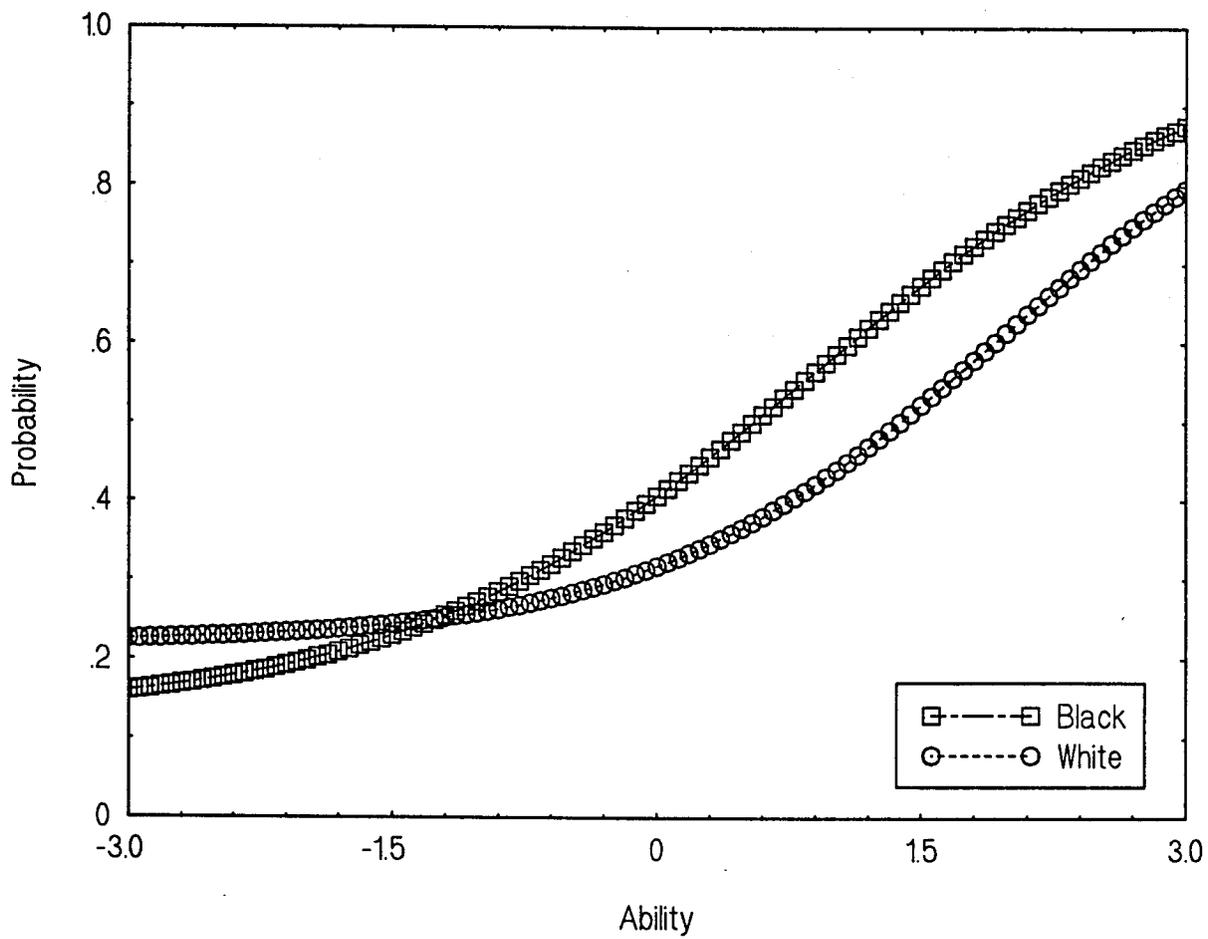
Black vs. White: Item 2-18



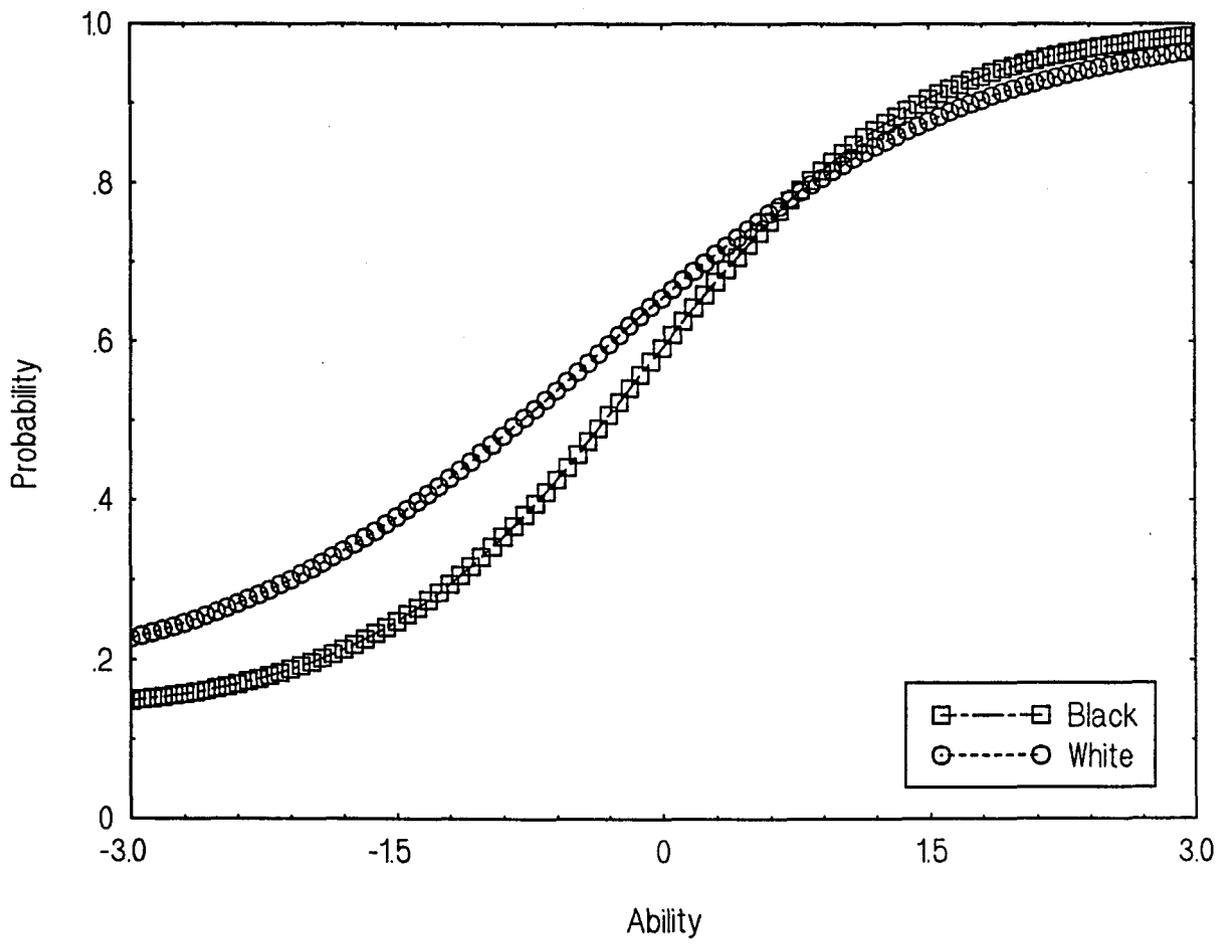
Black vs. White: Item 2-19



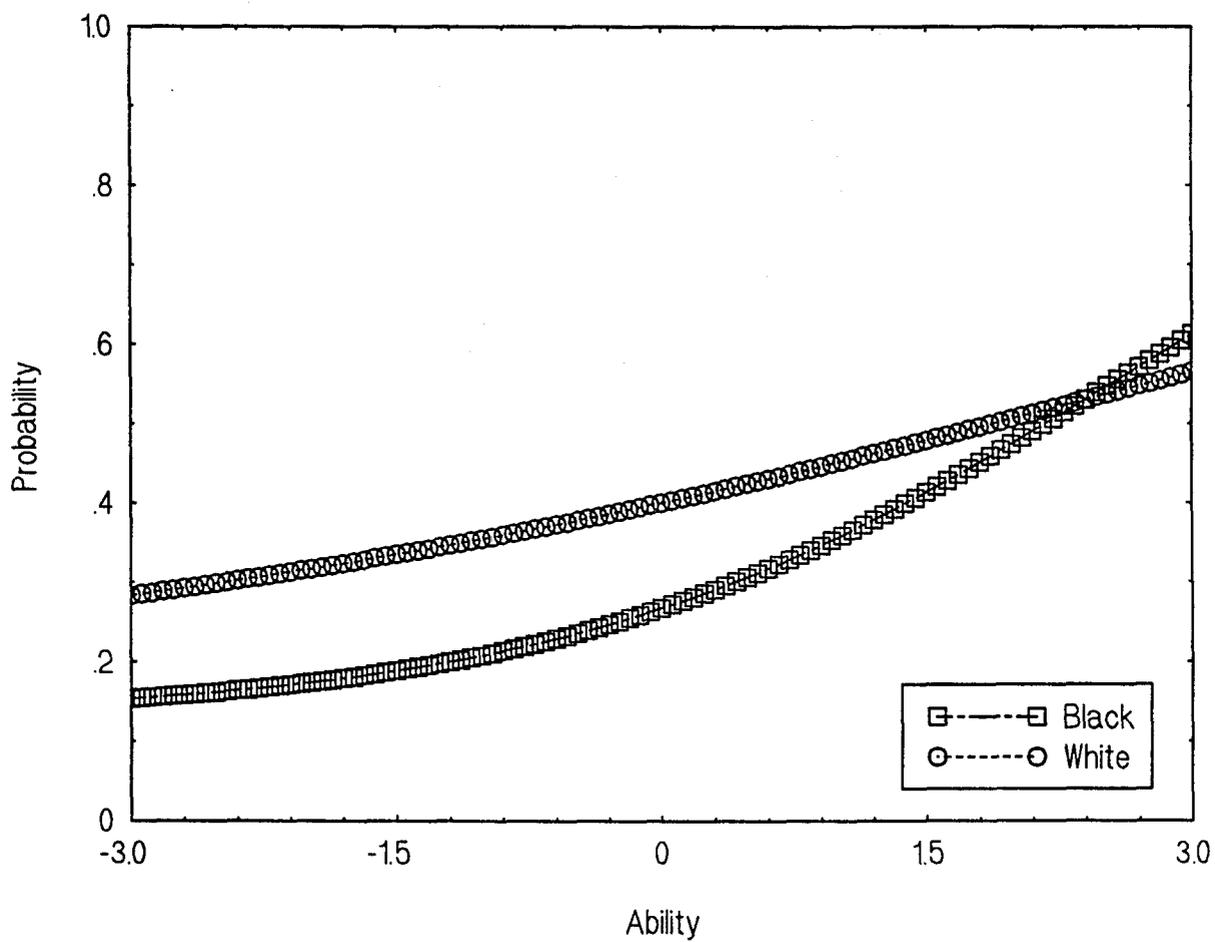
Black vs. White: Item 2-20



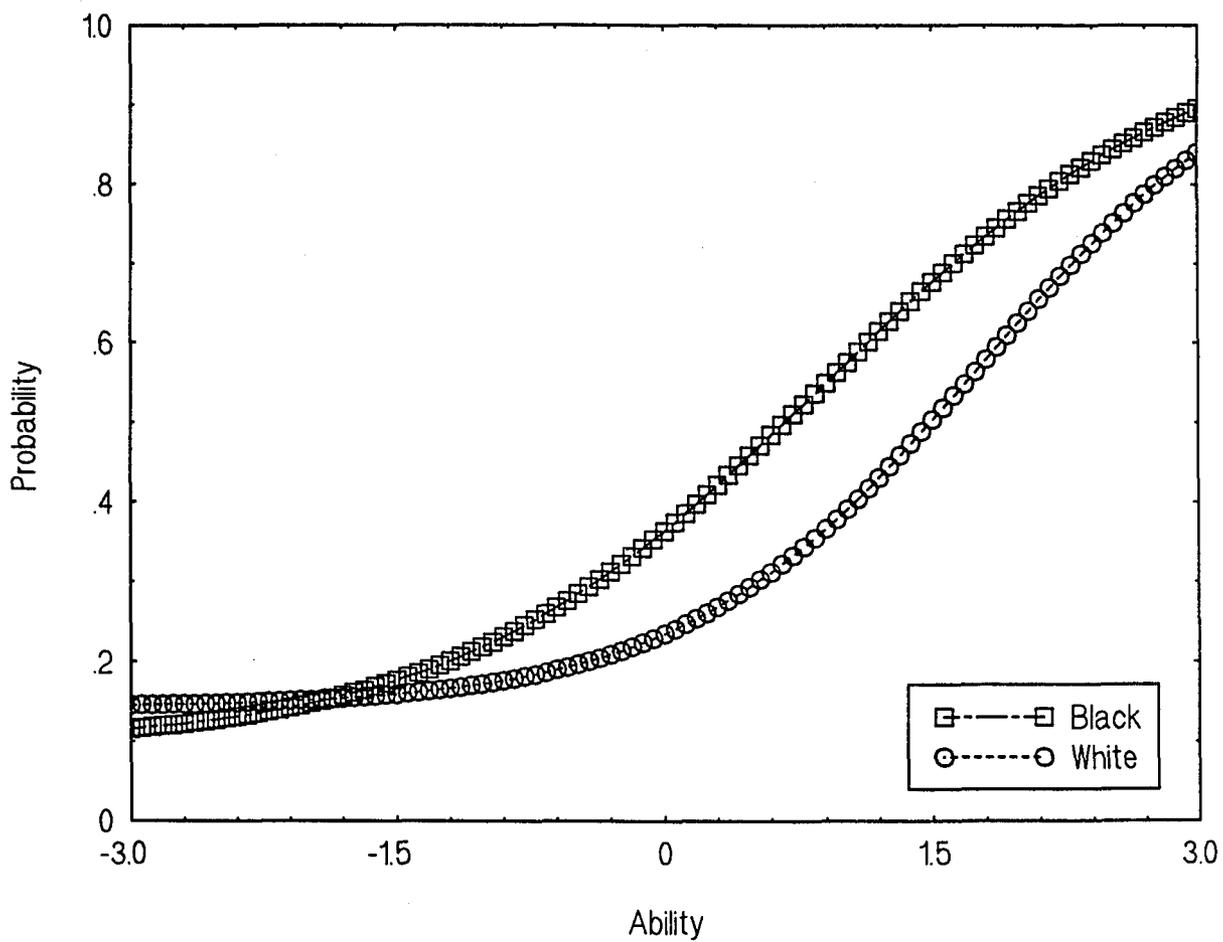
Black vs. White: Item 2-21



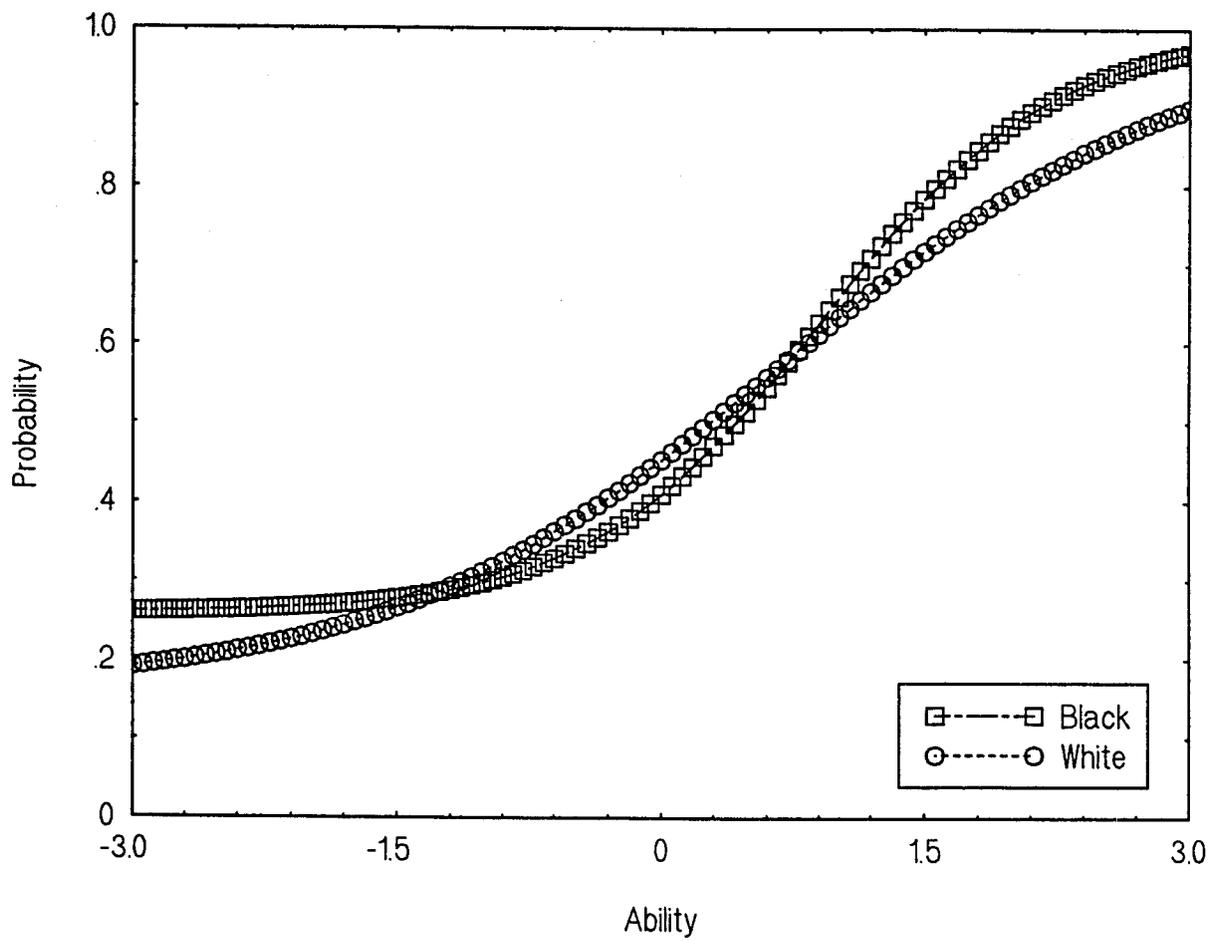
Black vs. White: Item 2-22



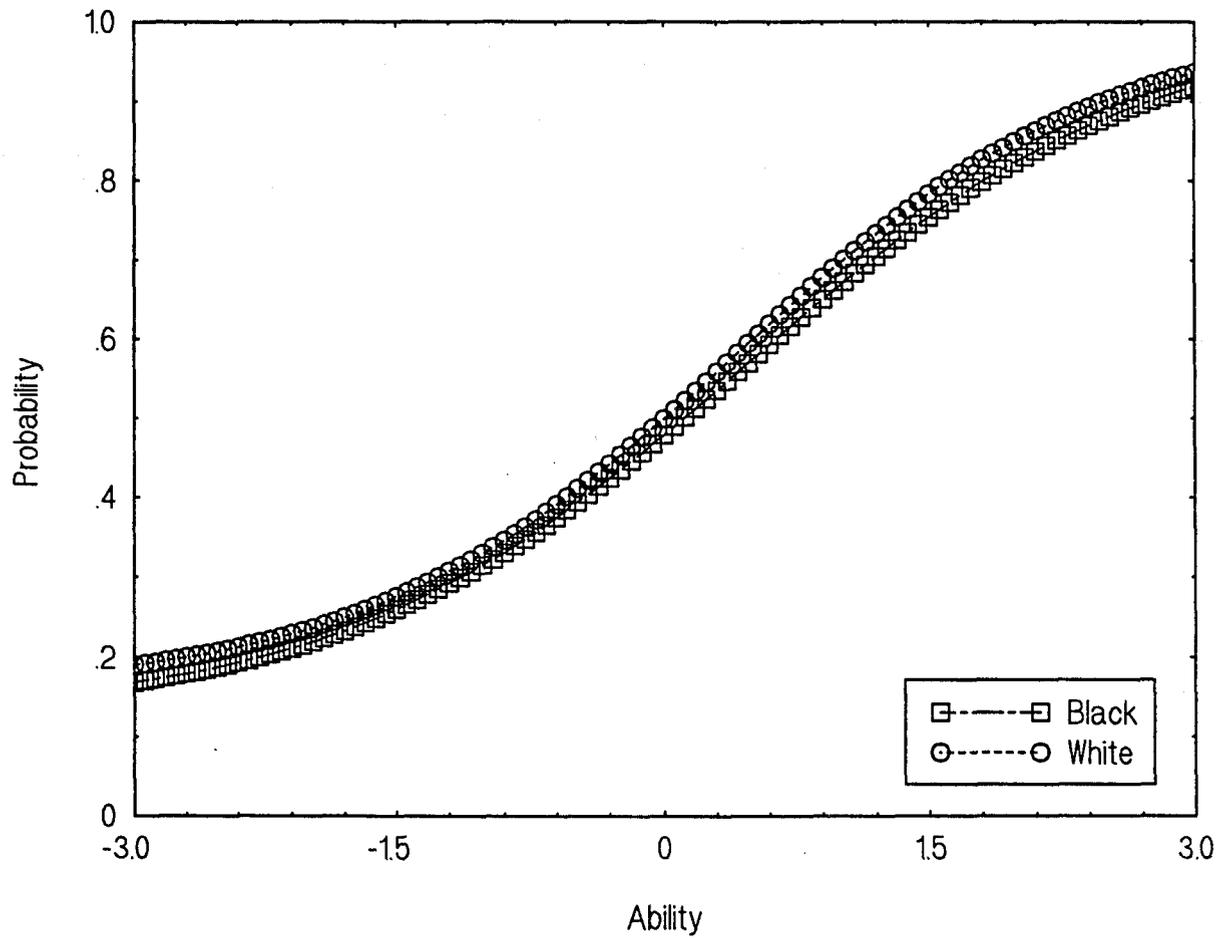
Black vs. White: Item 2-23



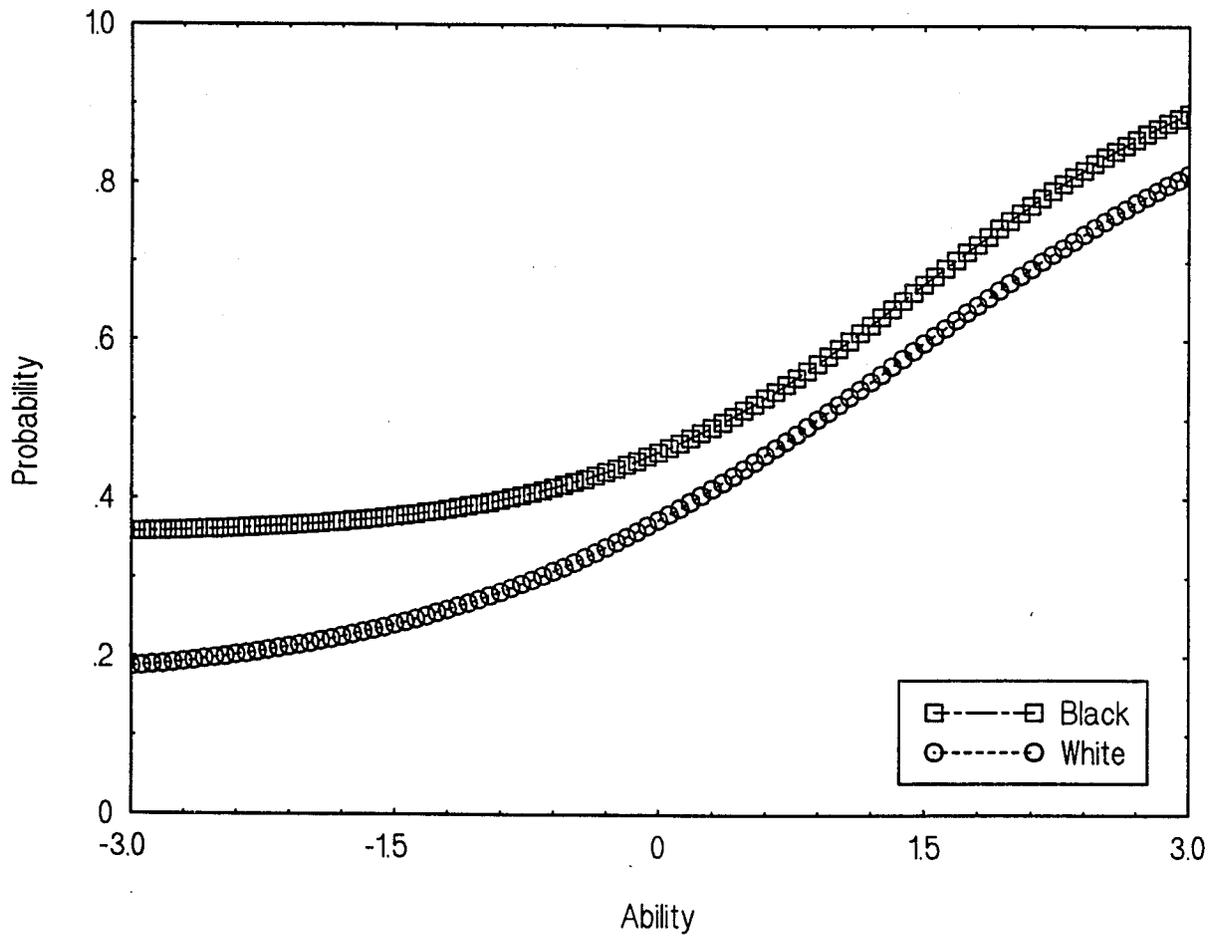
Black vs. White: Item 2-24



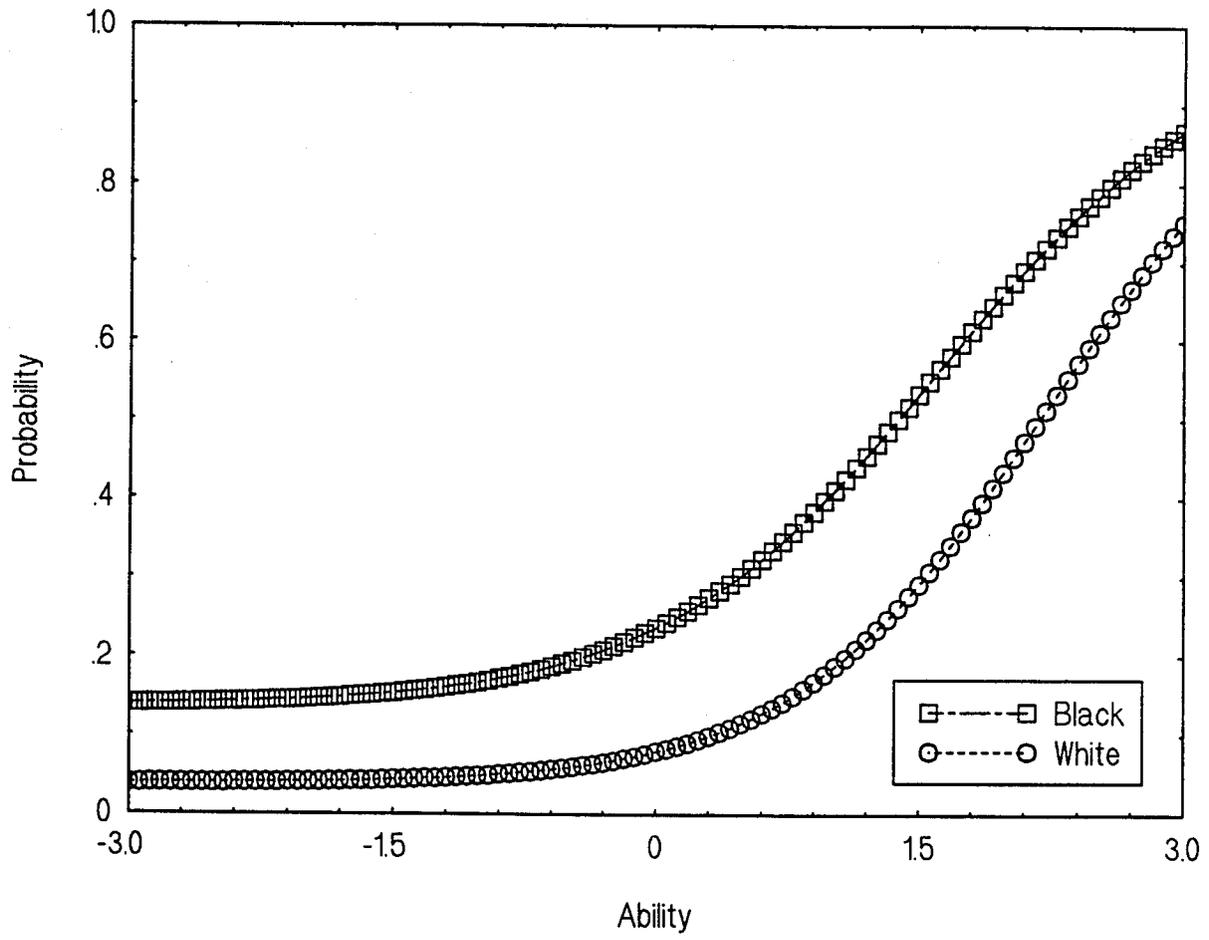
Black vs. White: Item 2-25



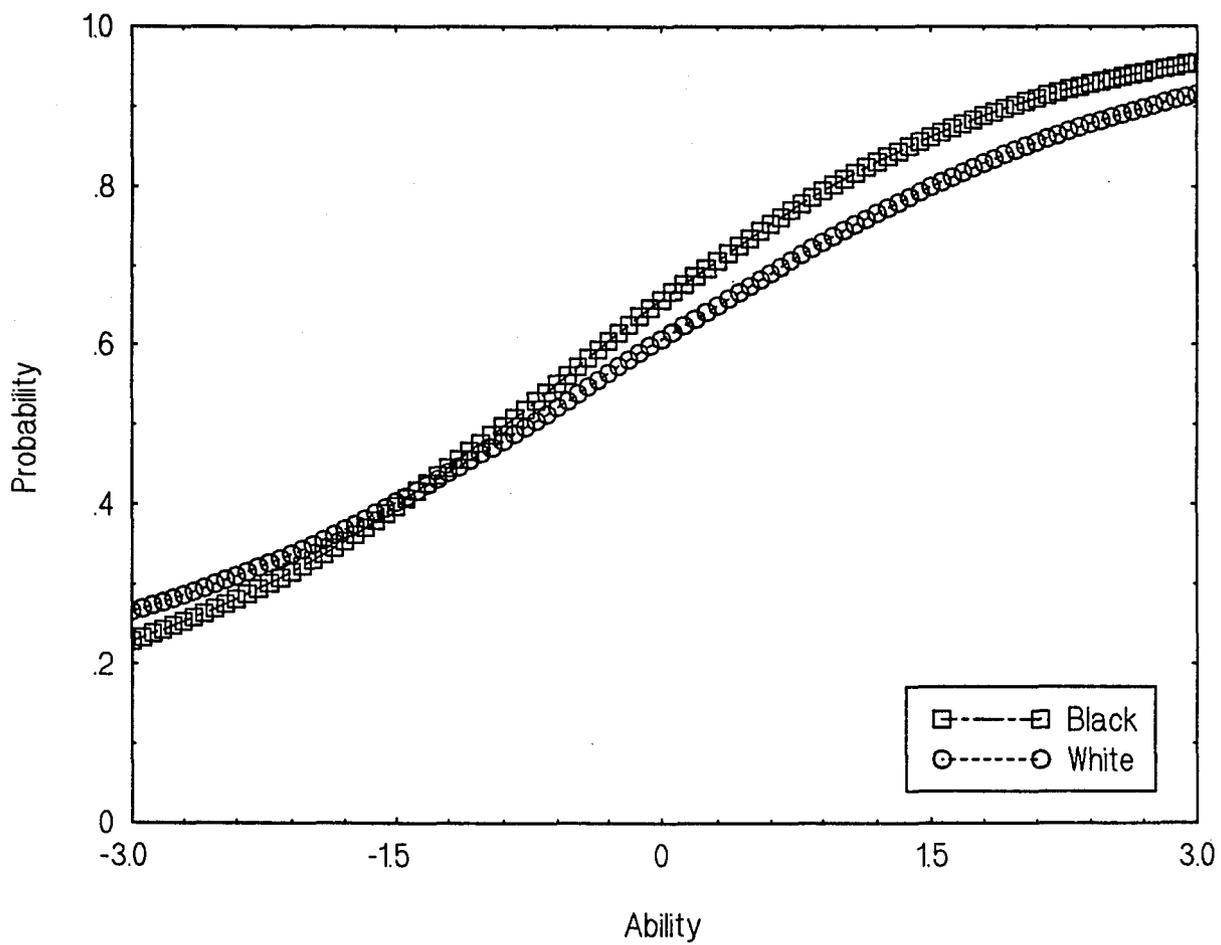
Black vs. White: Item 6-1



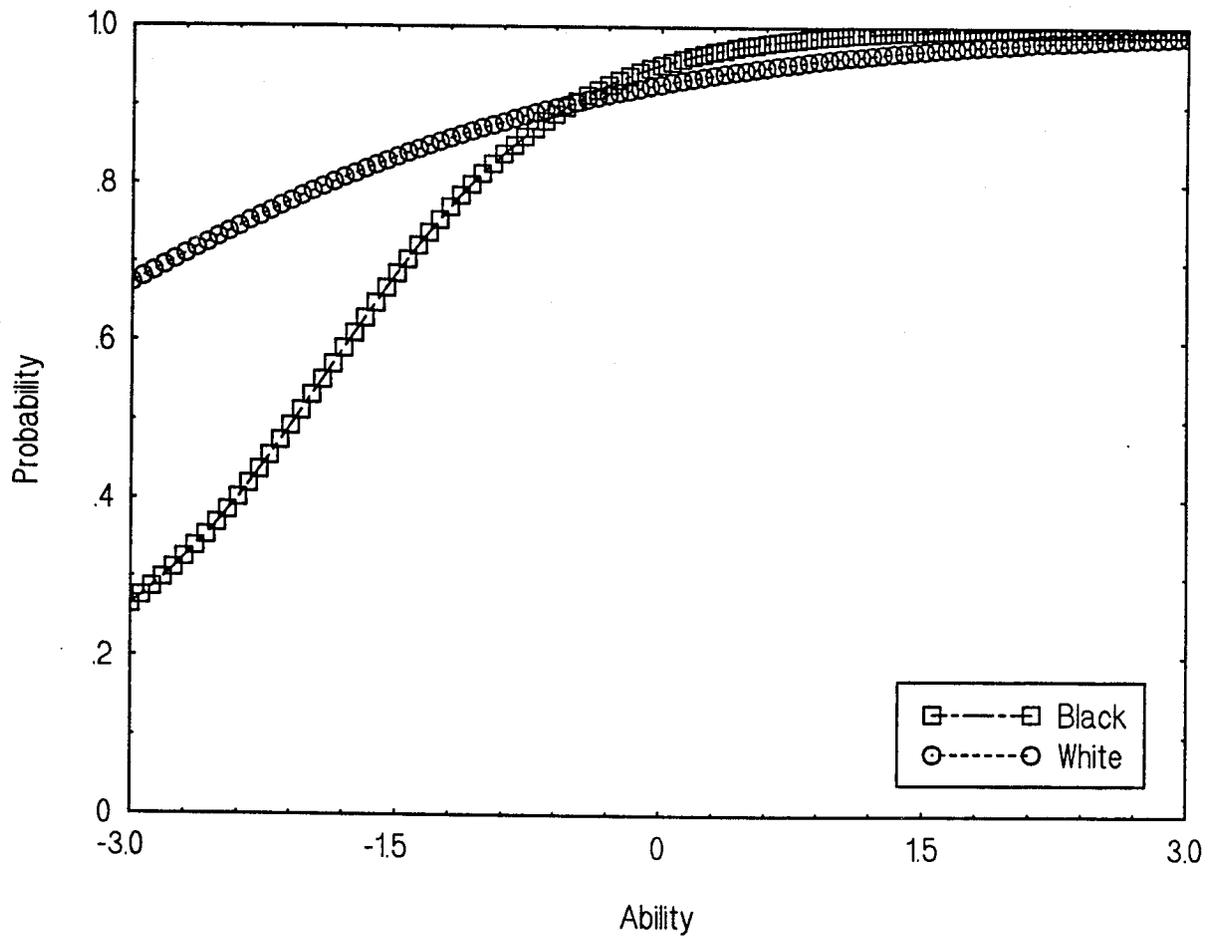
Black vs. White: Item 6-2



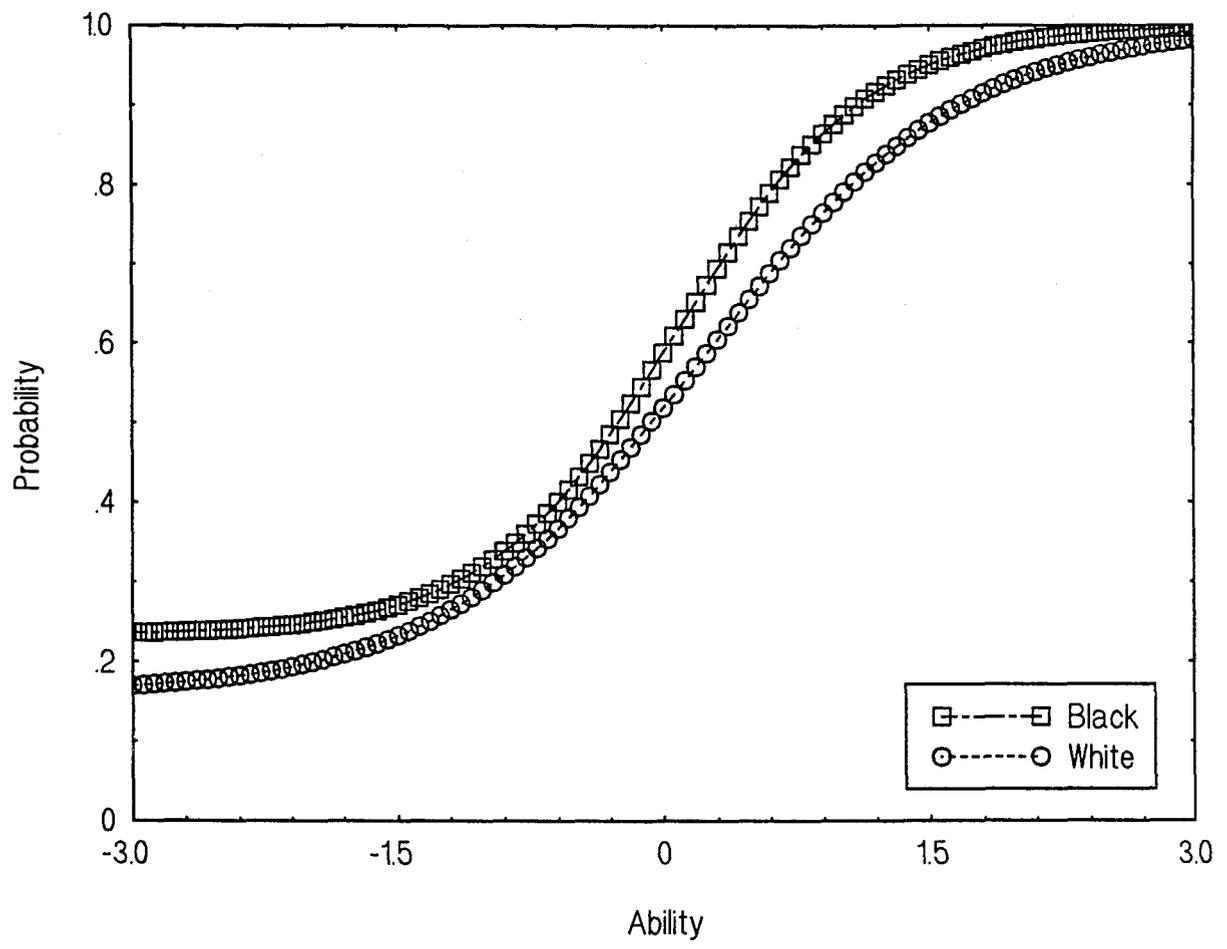
Black vs. White: Item 6-3



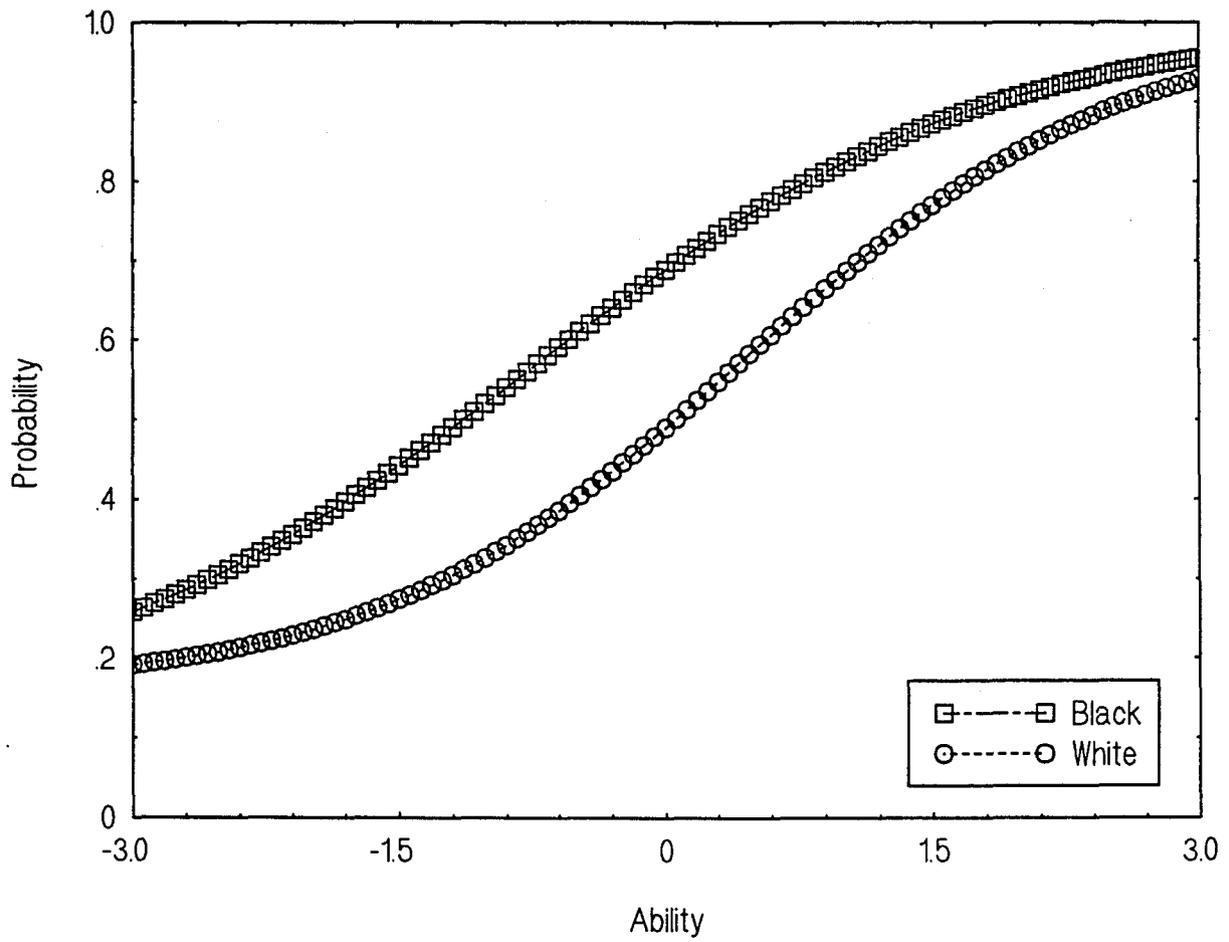
Black vs. White: Item 6-4



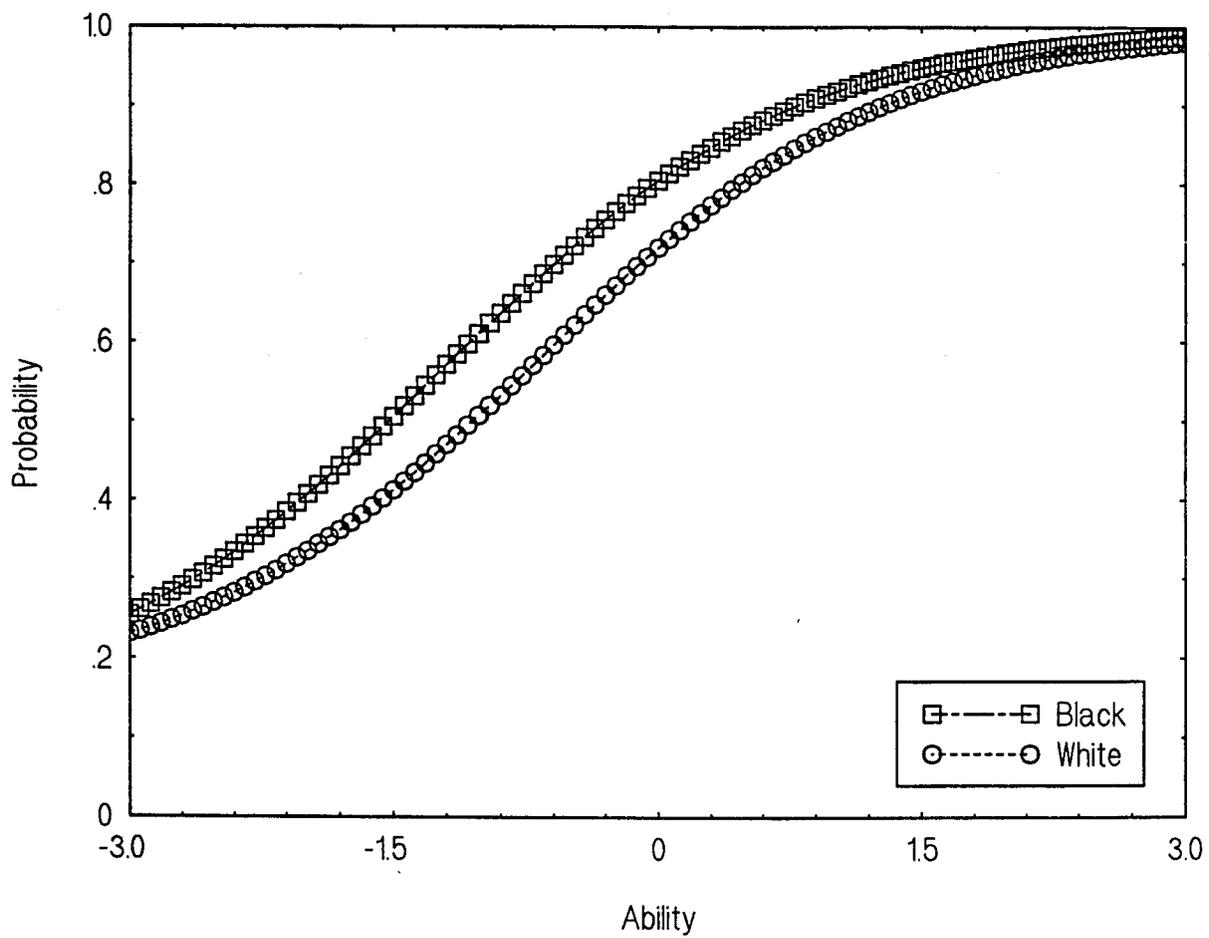
Black vs. White: Item 6-5



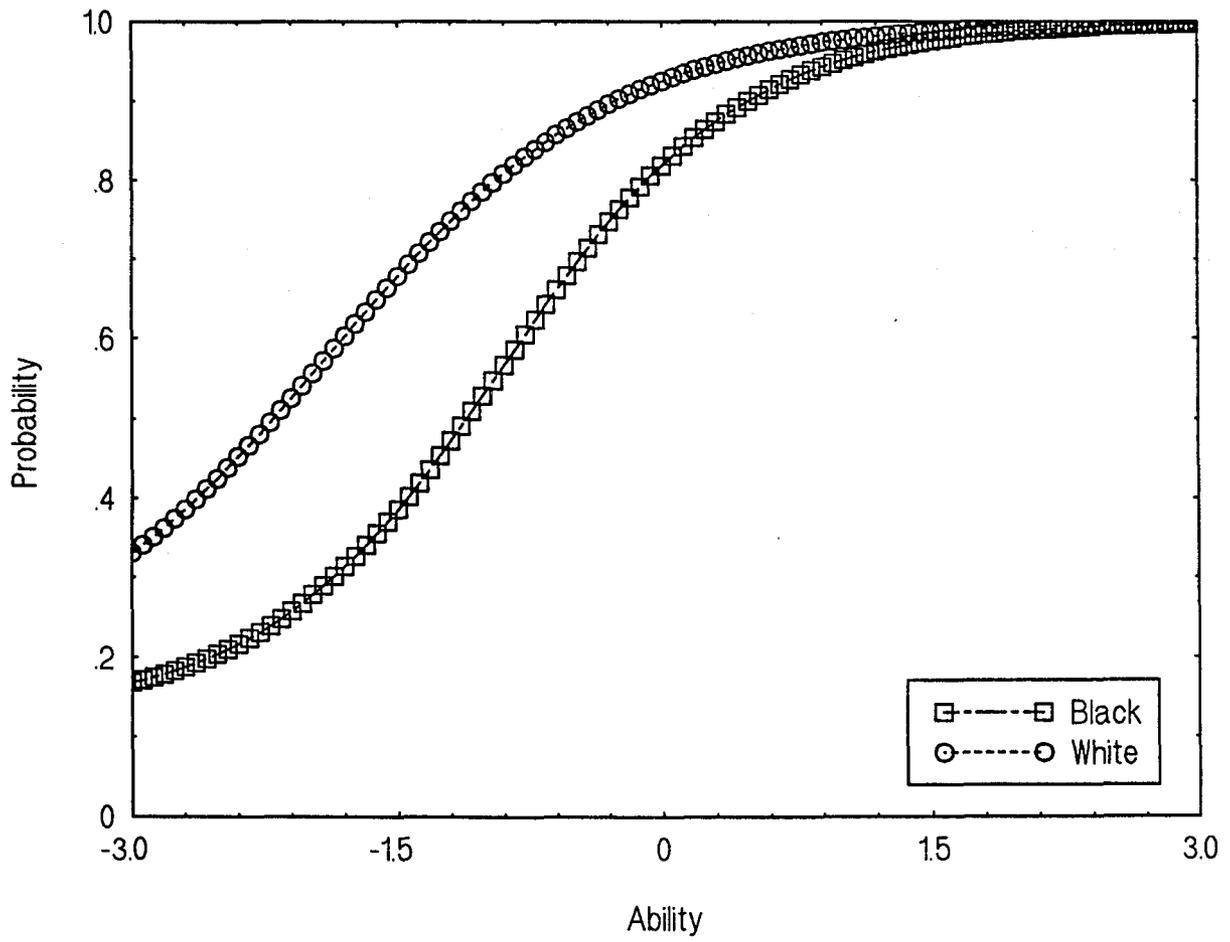
Black vs. White: Item 6-6



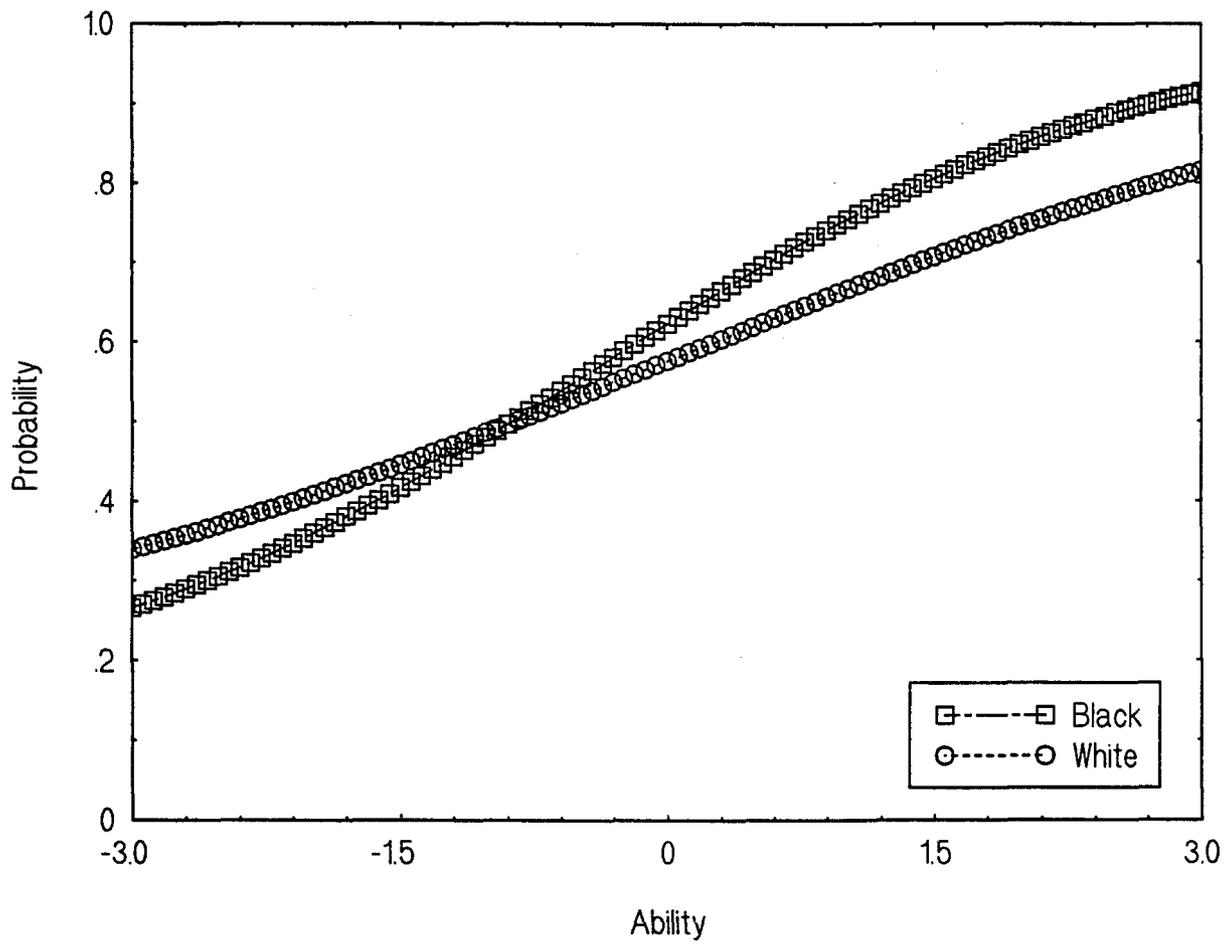
Black vs. White: Item 6-7



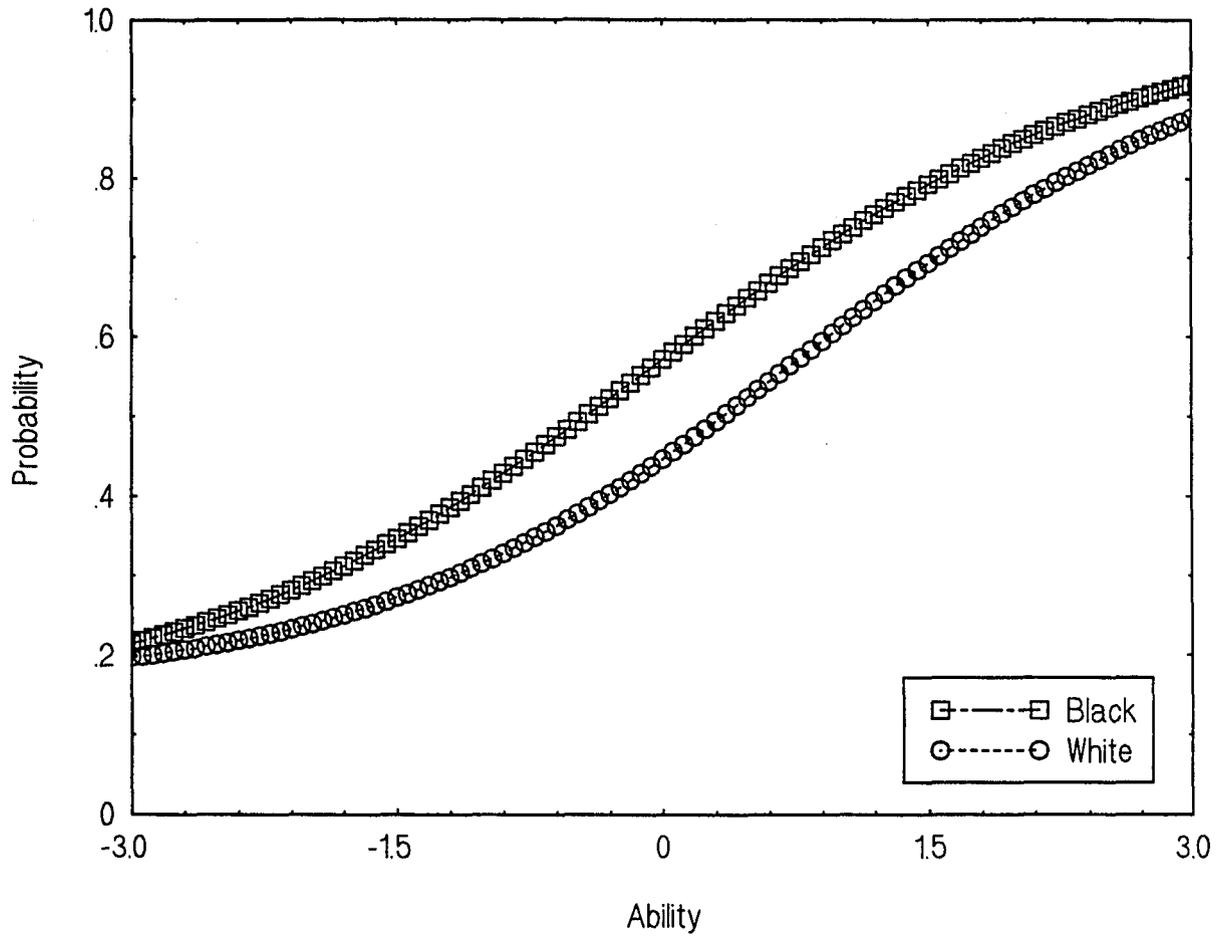
Black vs. White: Item 6-8



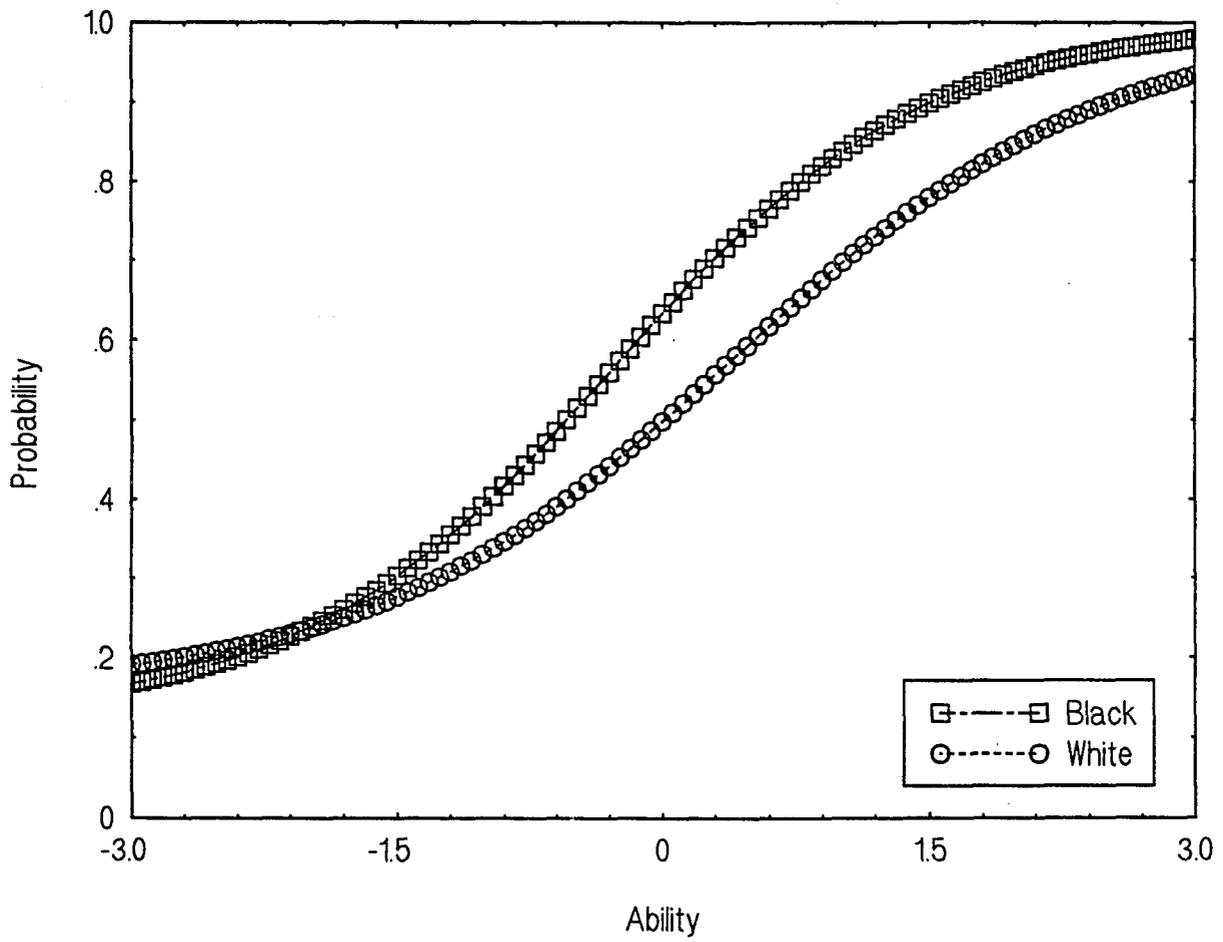
Black vs. White: Item 6-9



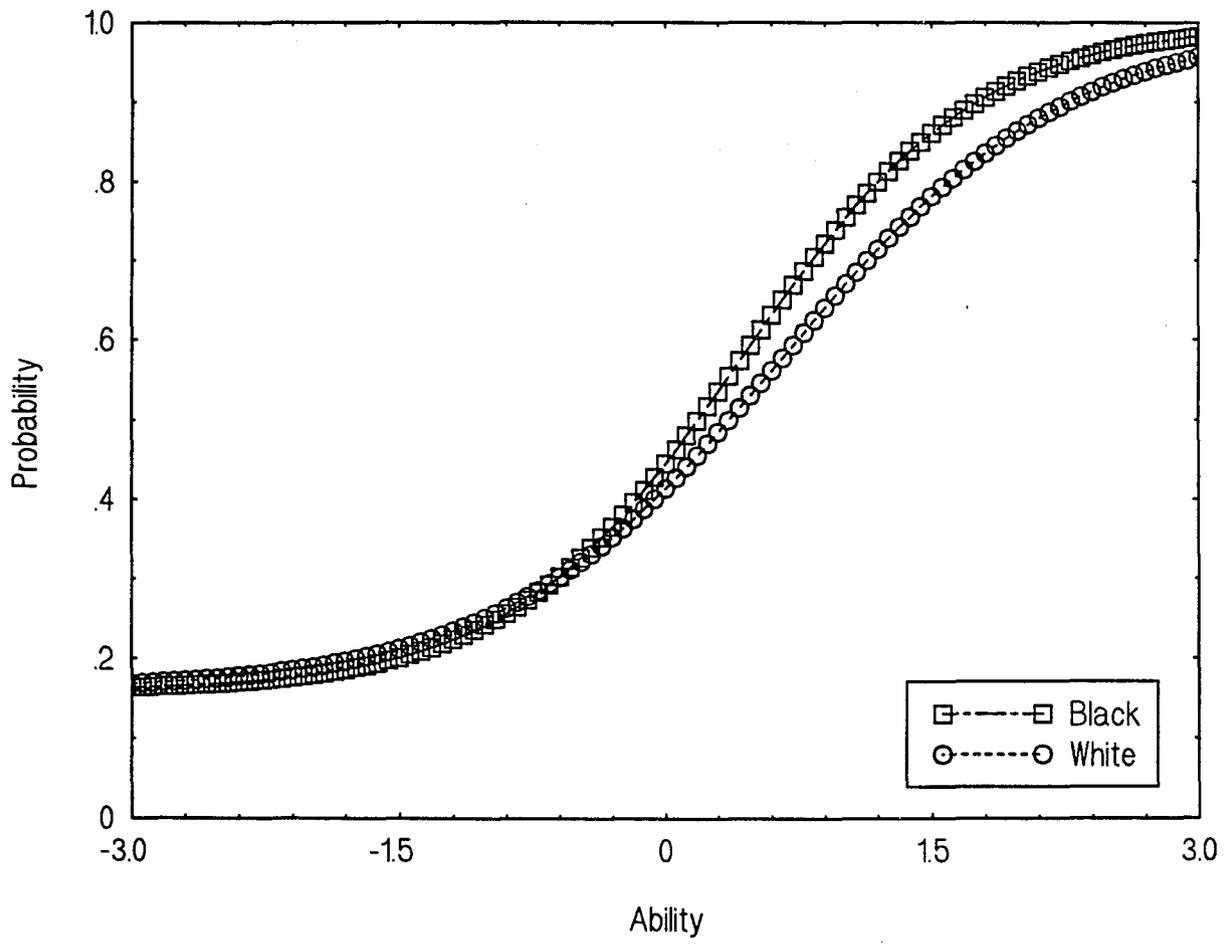
Black vs. White: Item 6-10



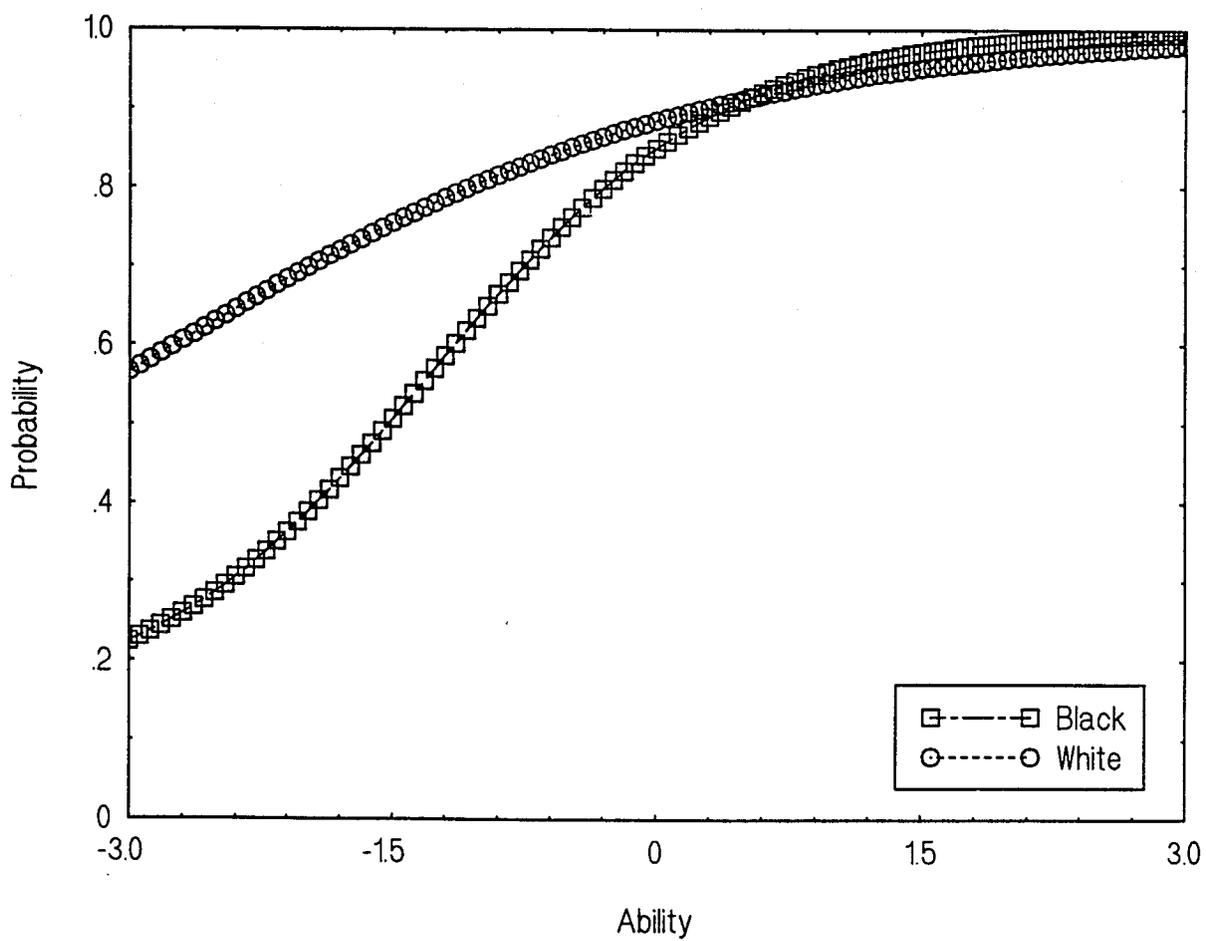
Black vs. White: Item 6-11



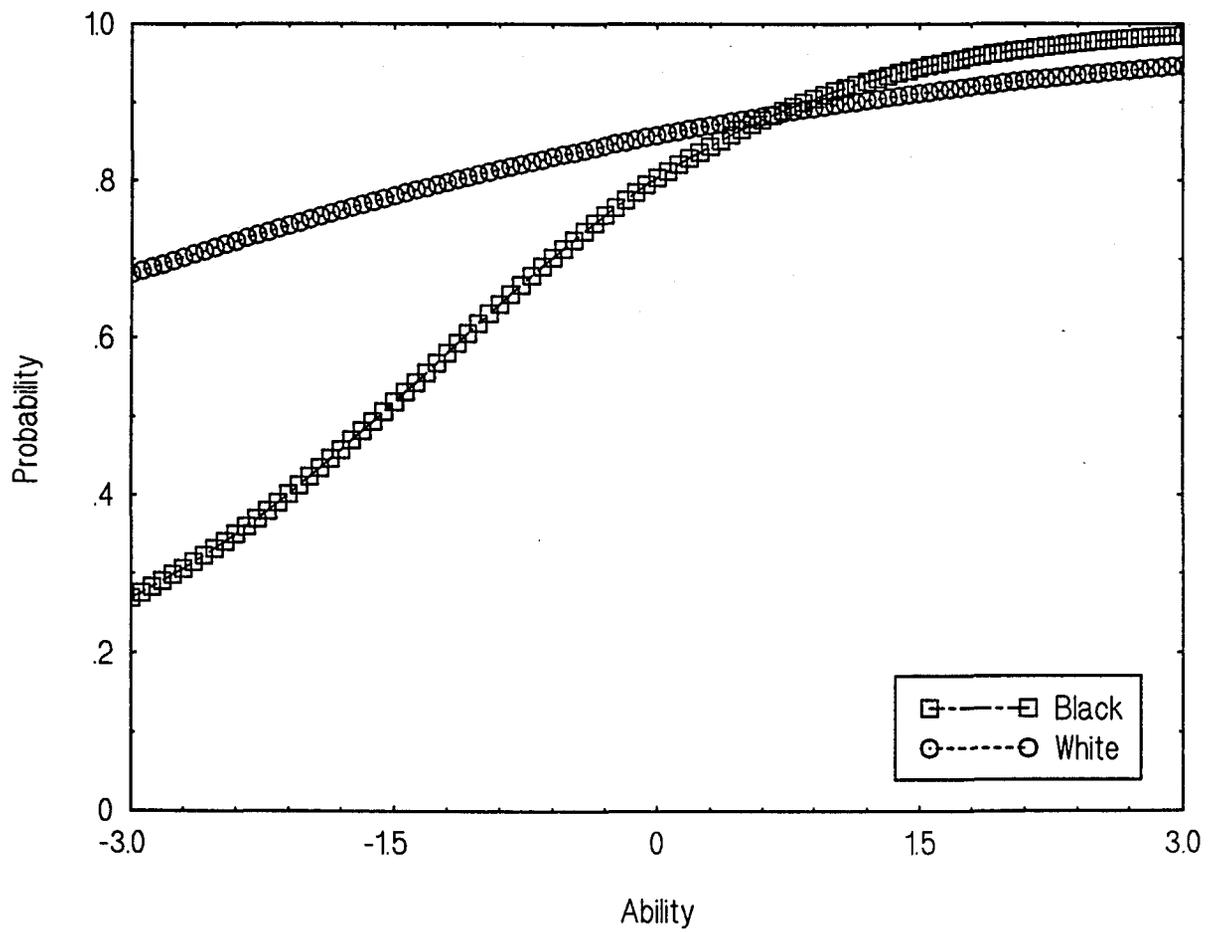
Black vs. White: Item 6-12



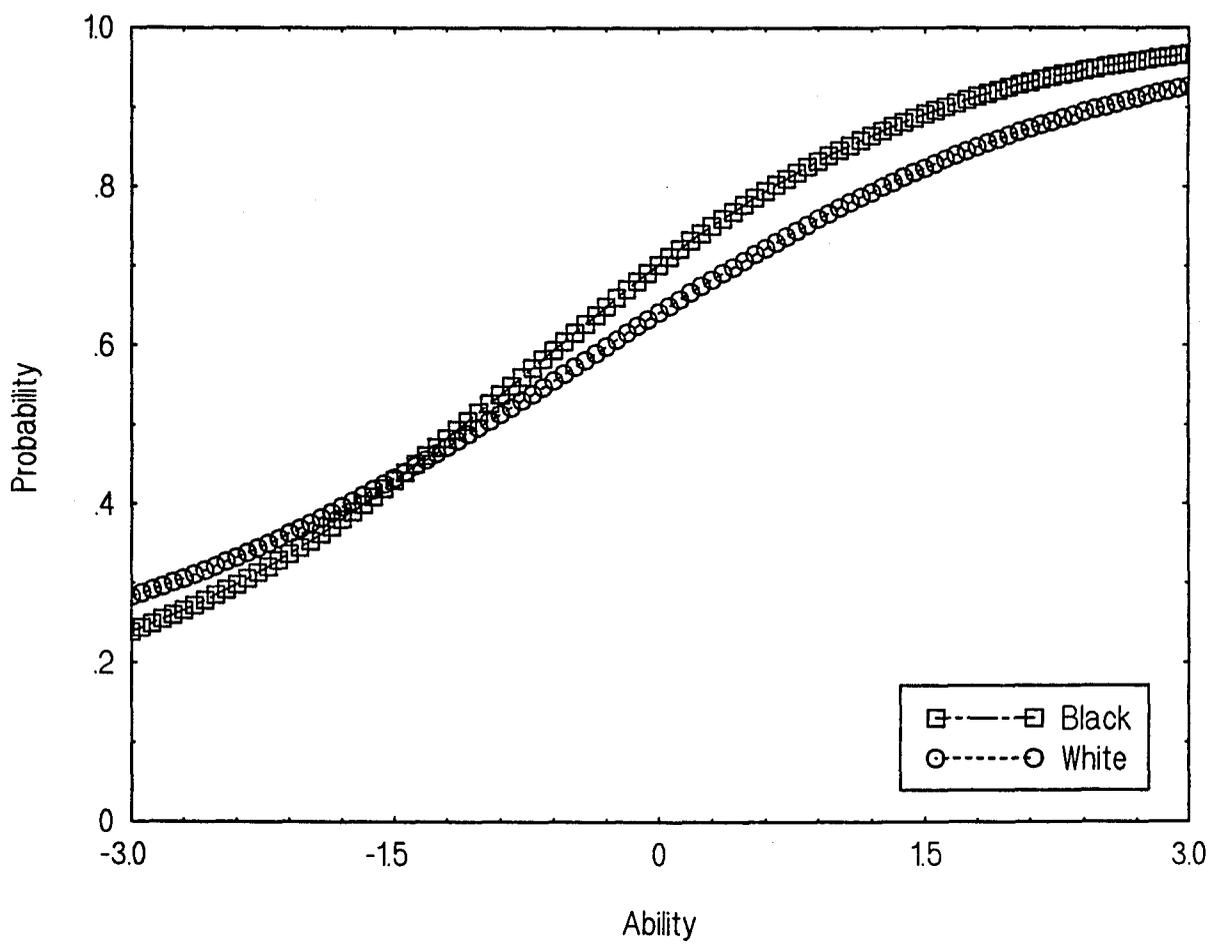
Black vs. White: Item 6-13



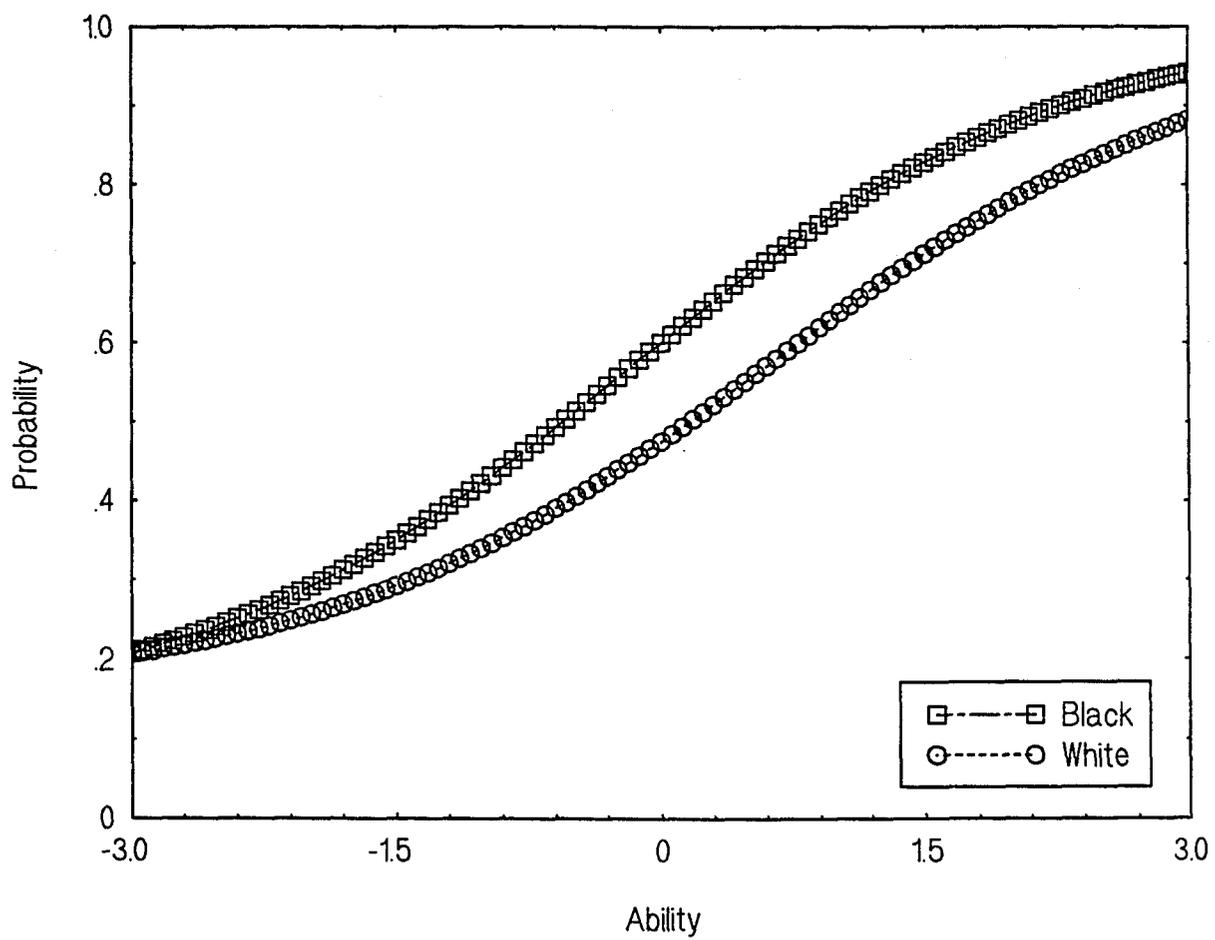
Black vs. White: Item 6-14



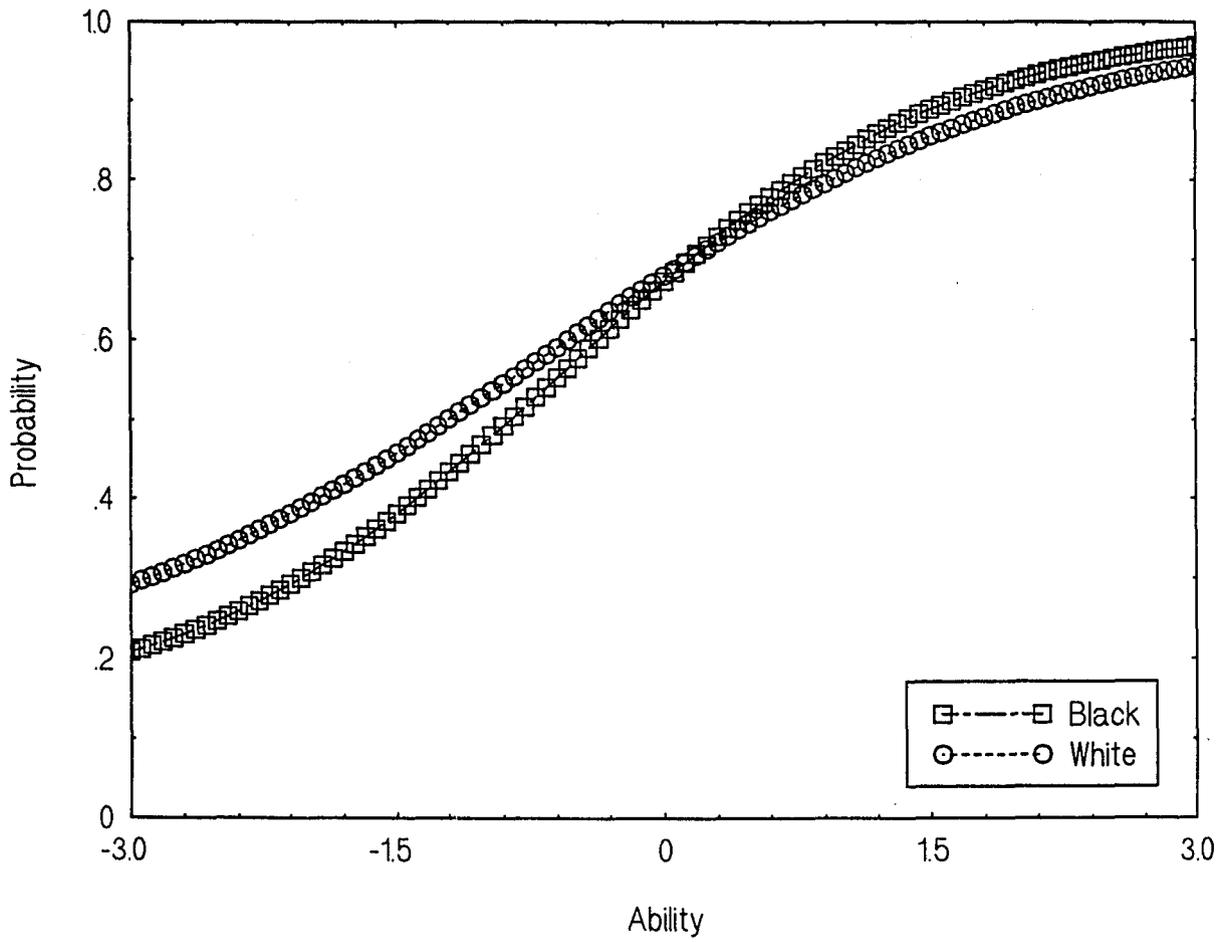
Black vs. White: Item 6-15



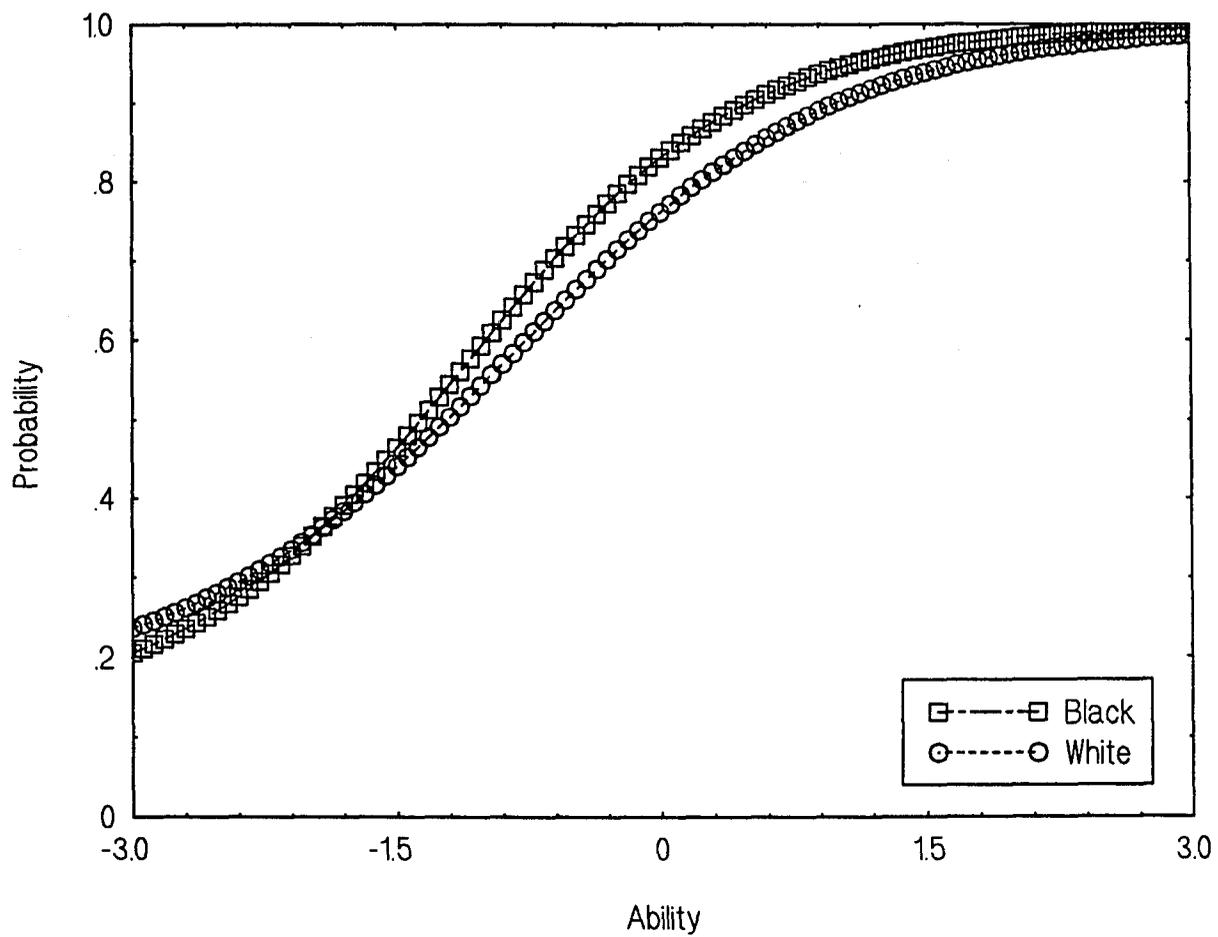
Black vs. White: Item 6-16



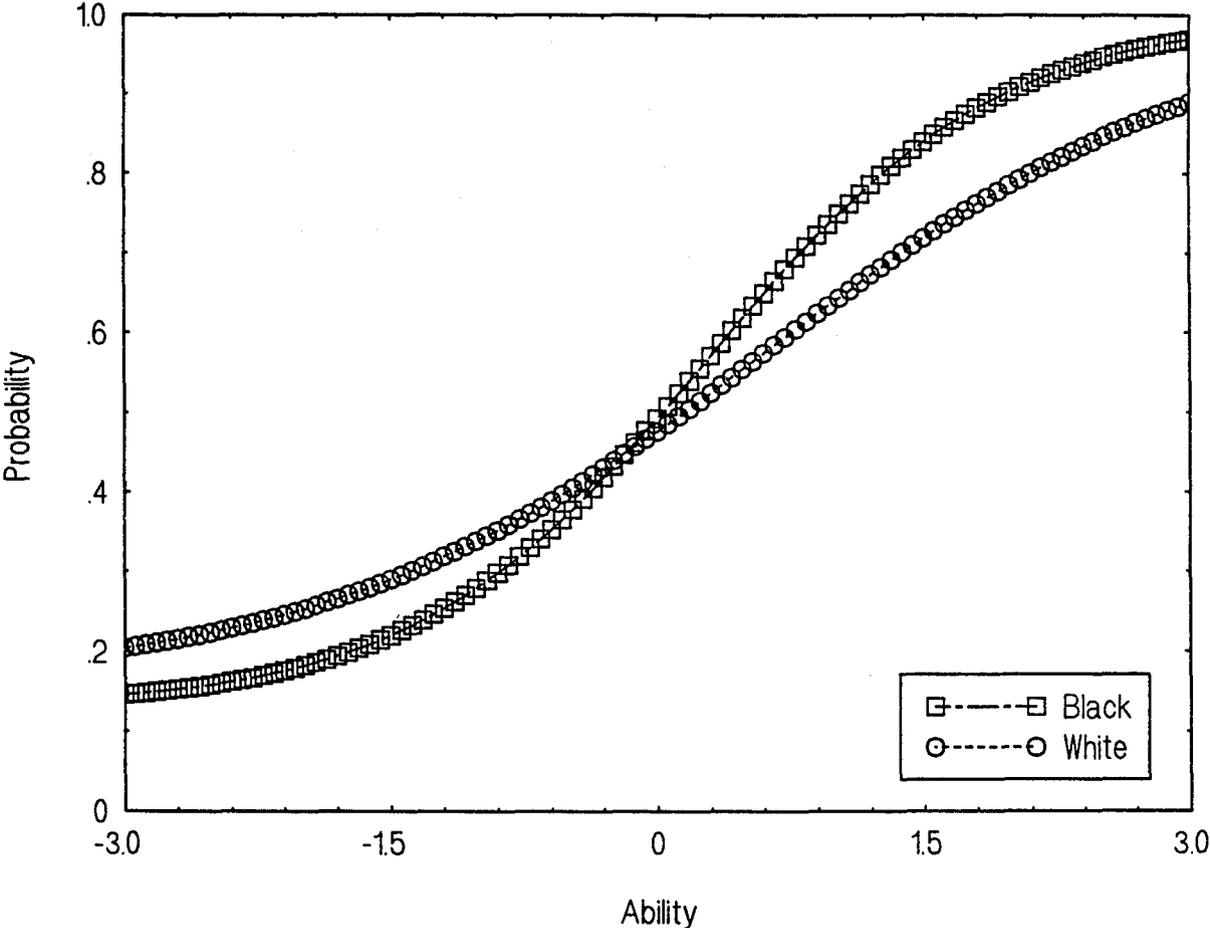
Black vs. White: Item 6-17



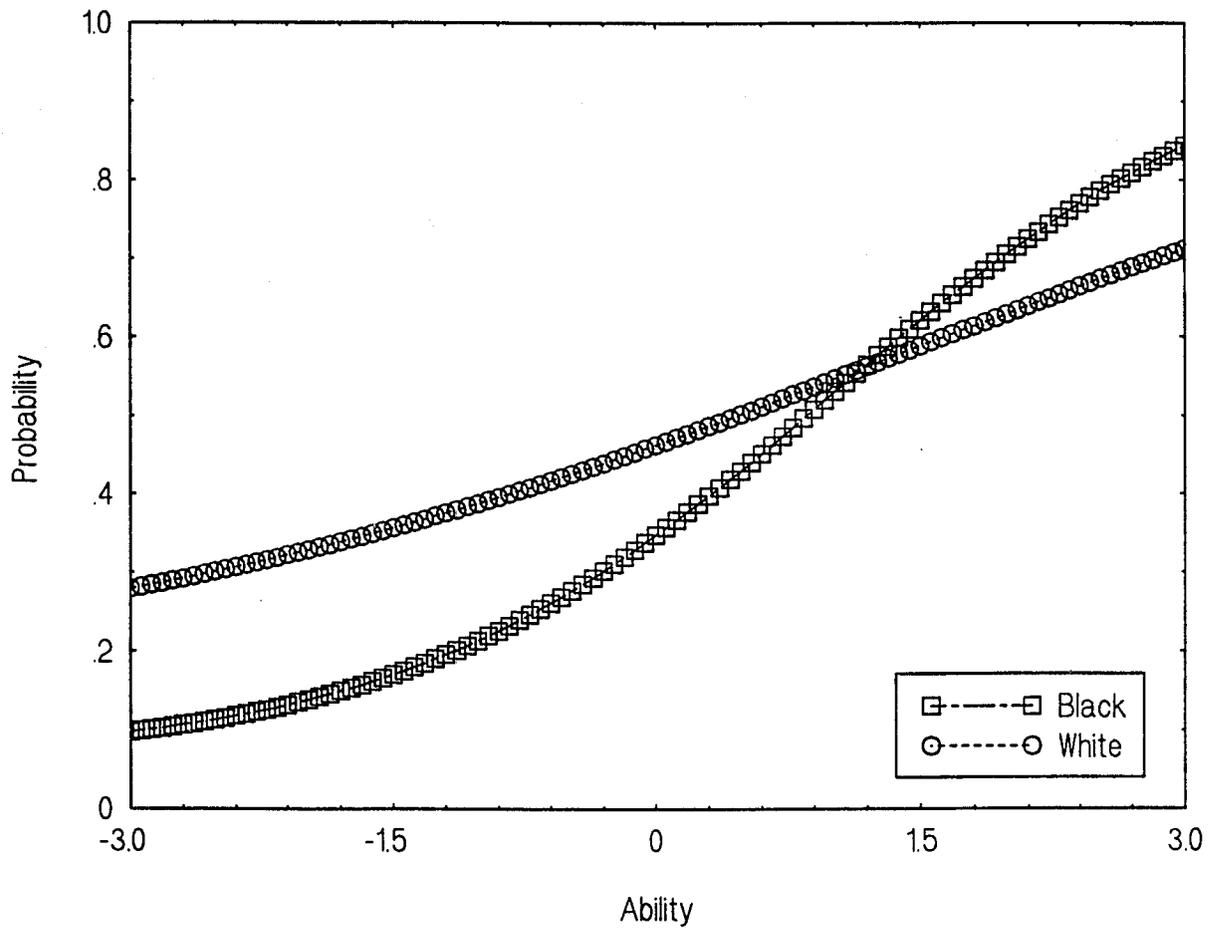
Black vs. White: Item 6-18



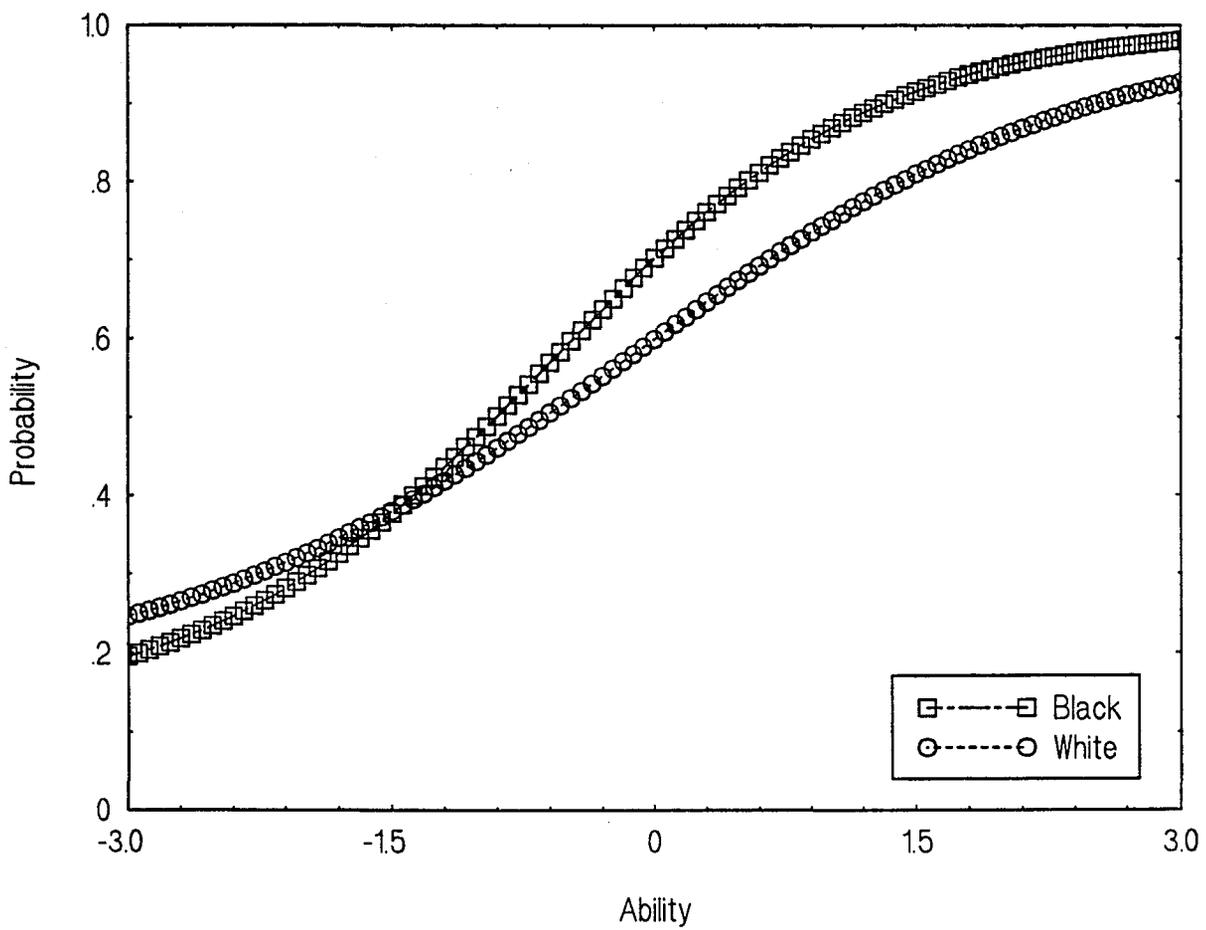
Black vs. White: Item 6-19



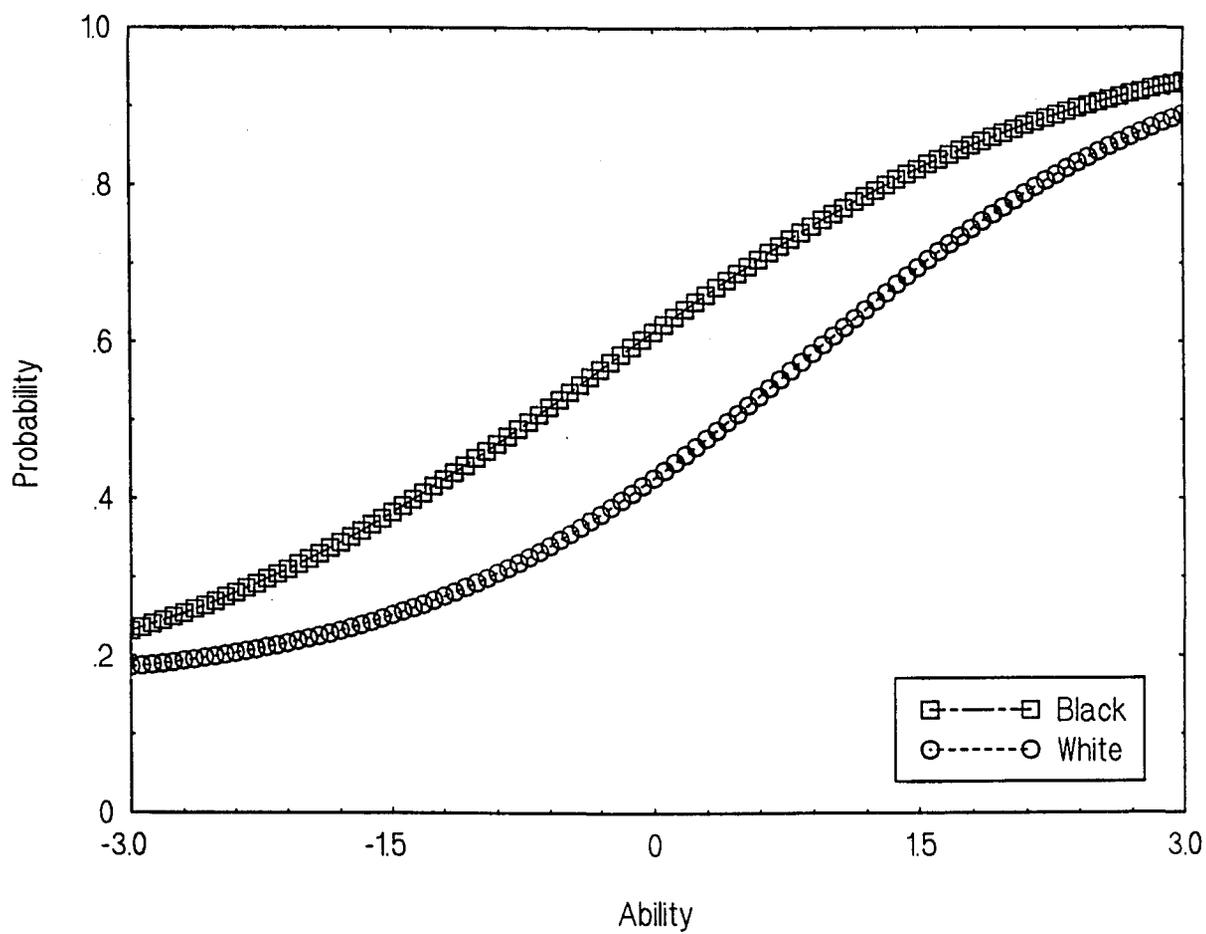
Black vs. White: Item 6-20



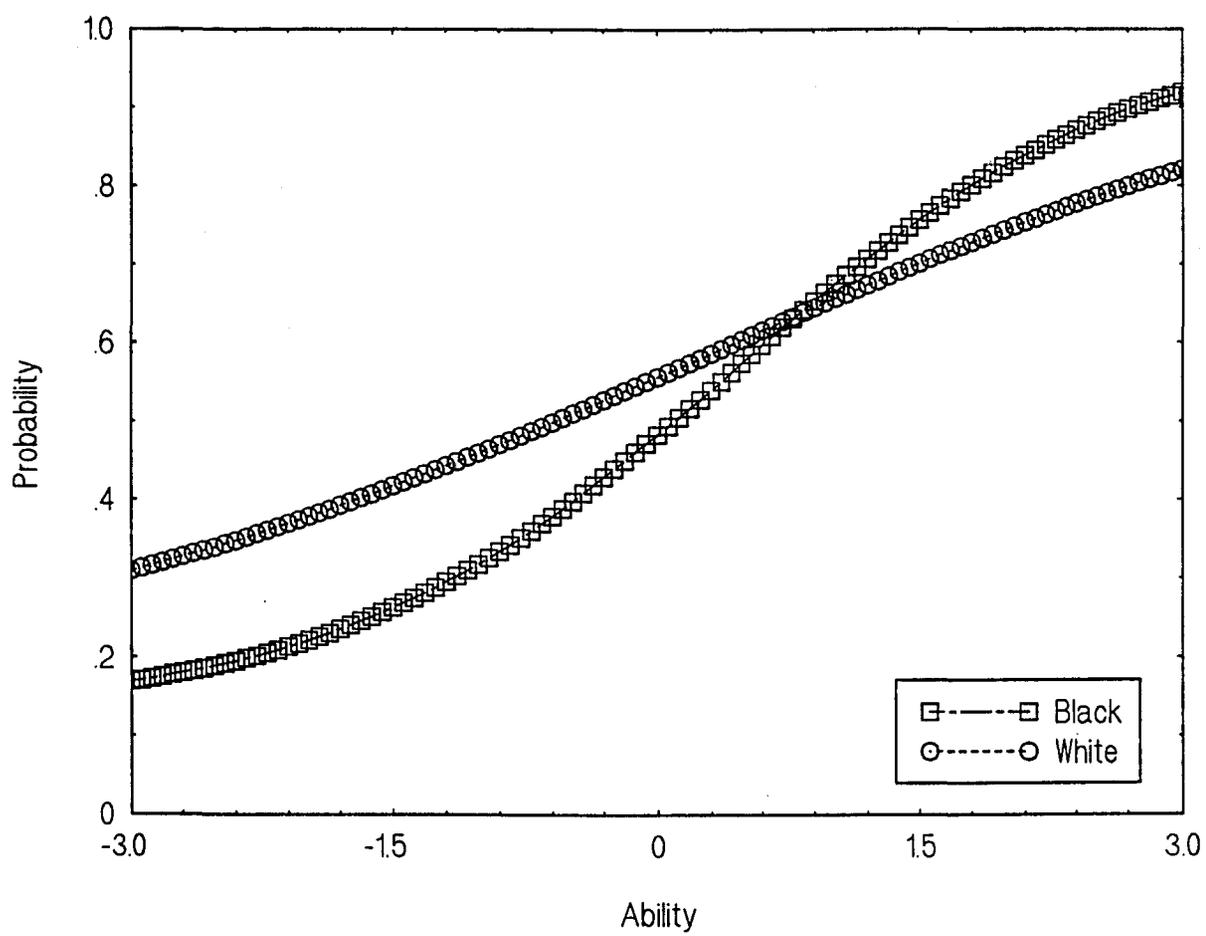
Black vs. White: Item 6-21



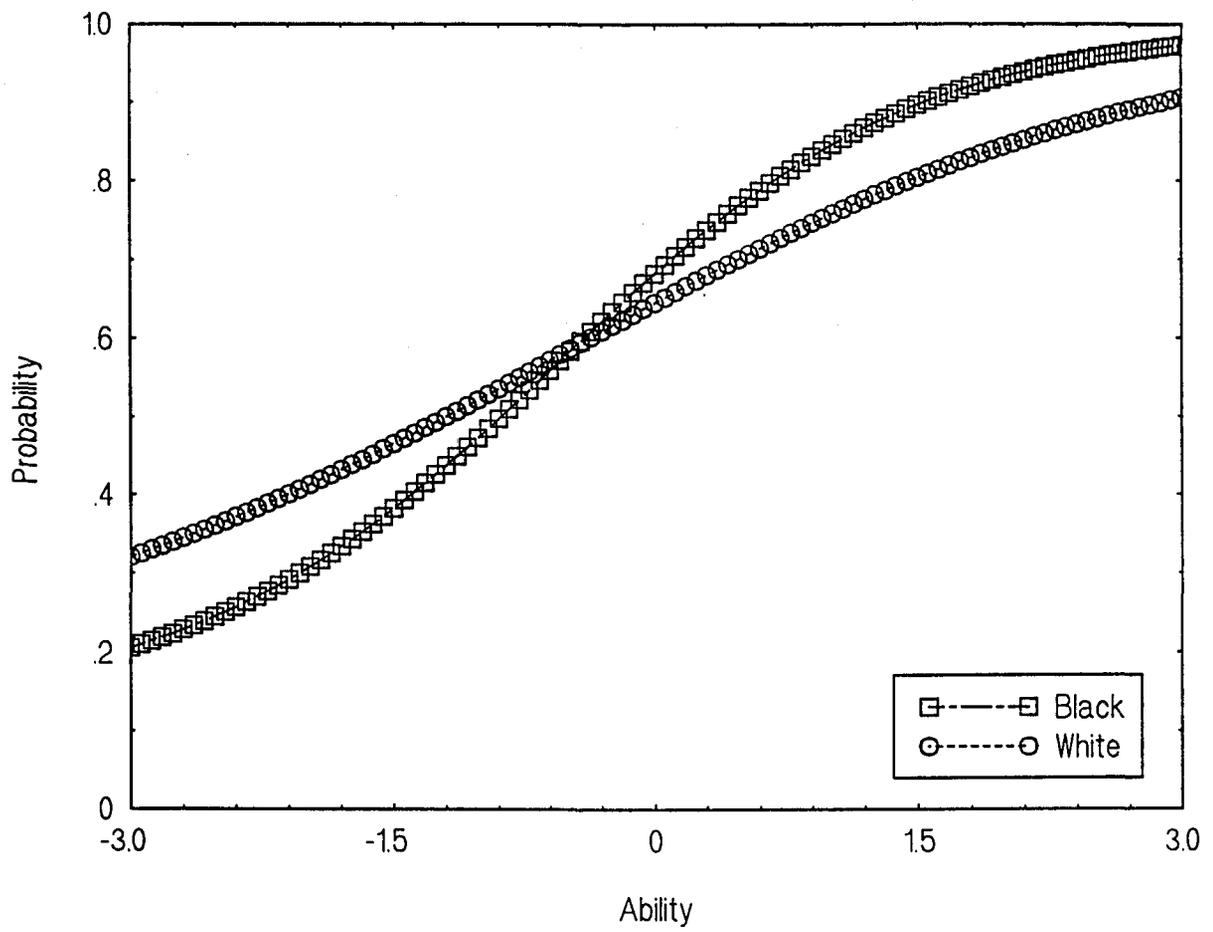
Black vs. White: Item 6-22



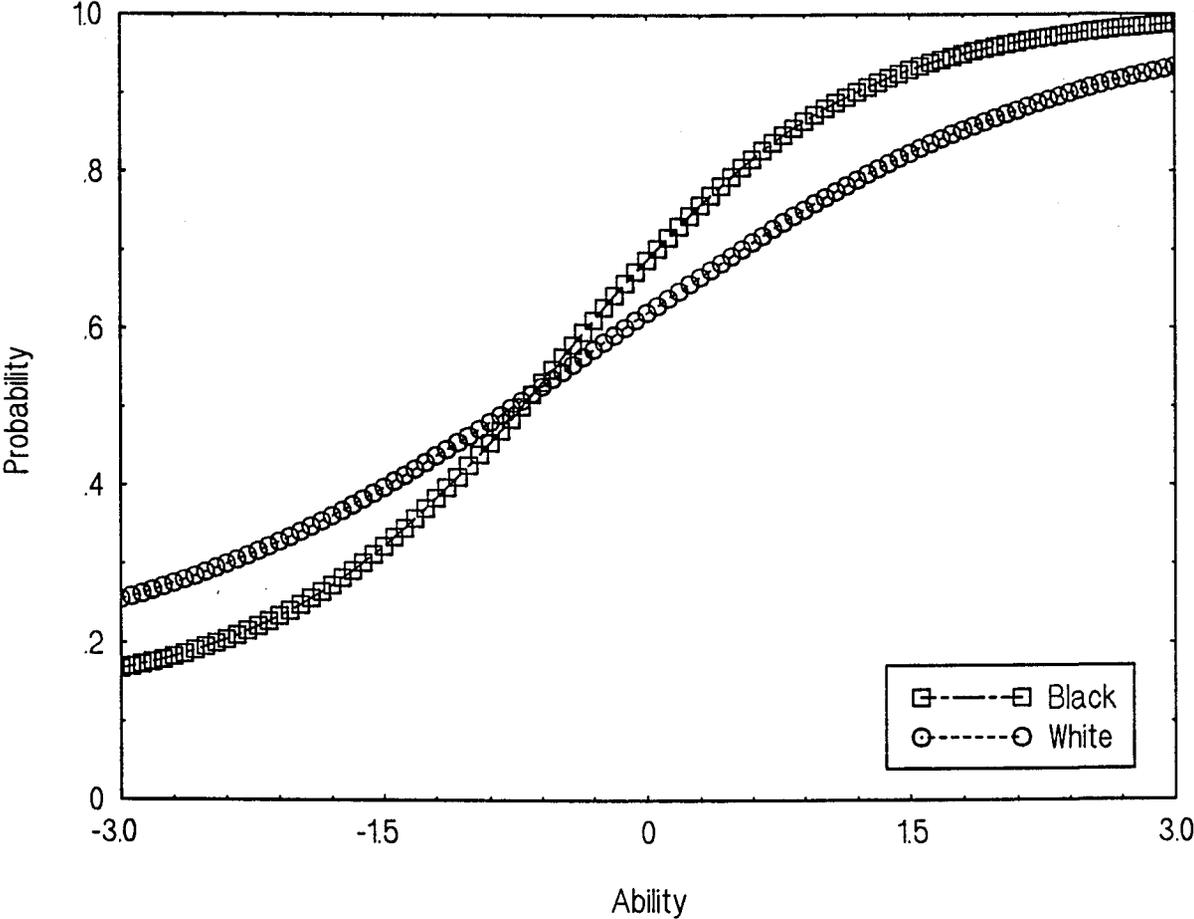
Black vs. White: Item 6-23



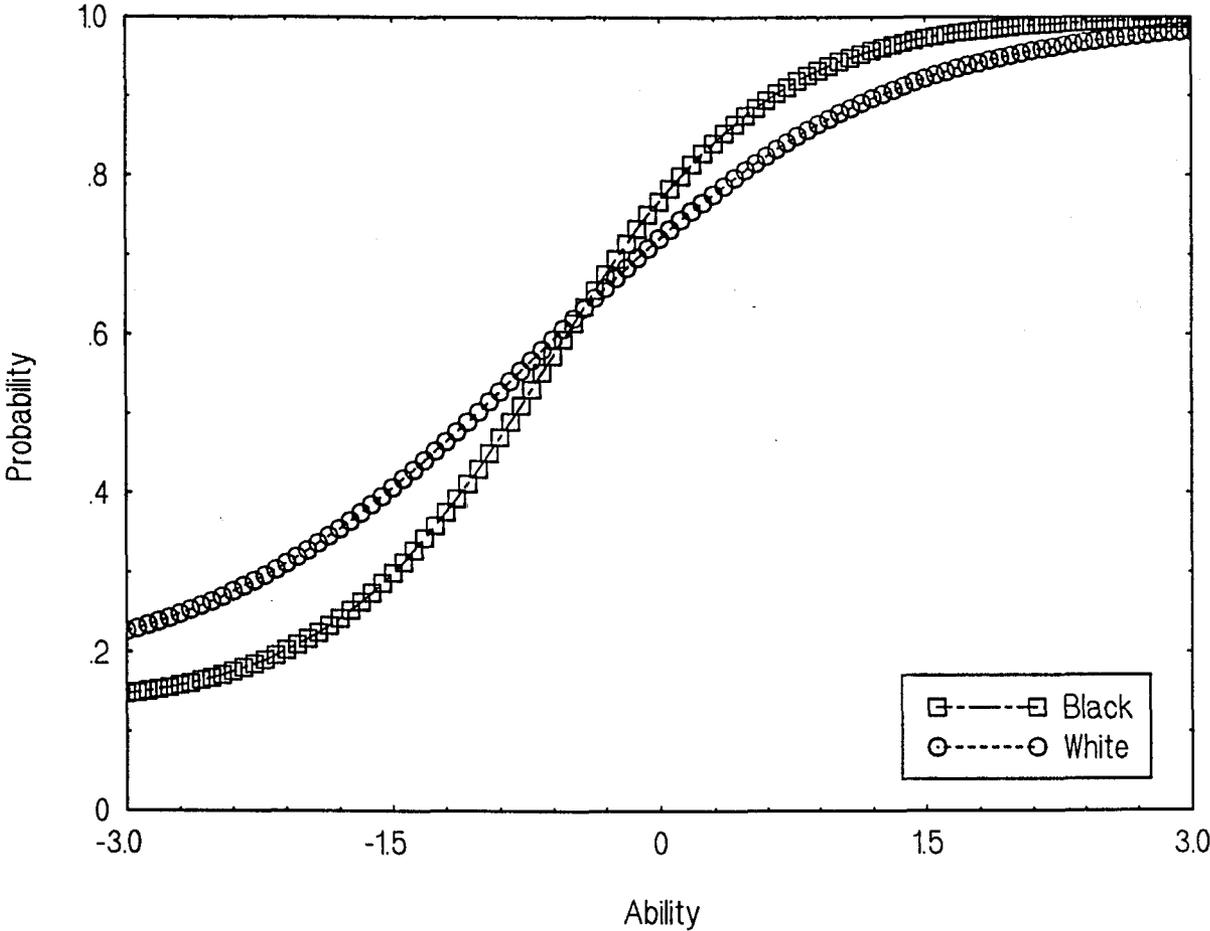
Black vs. White: Item 6-24



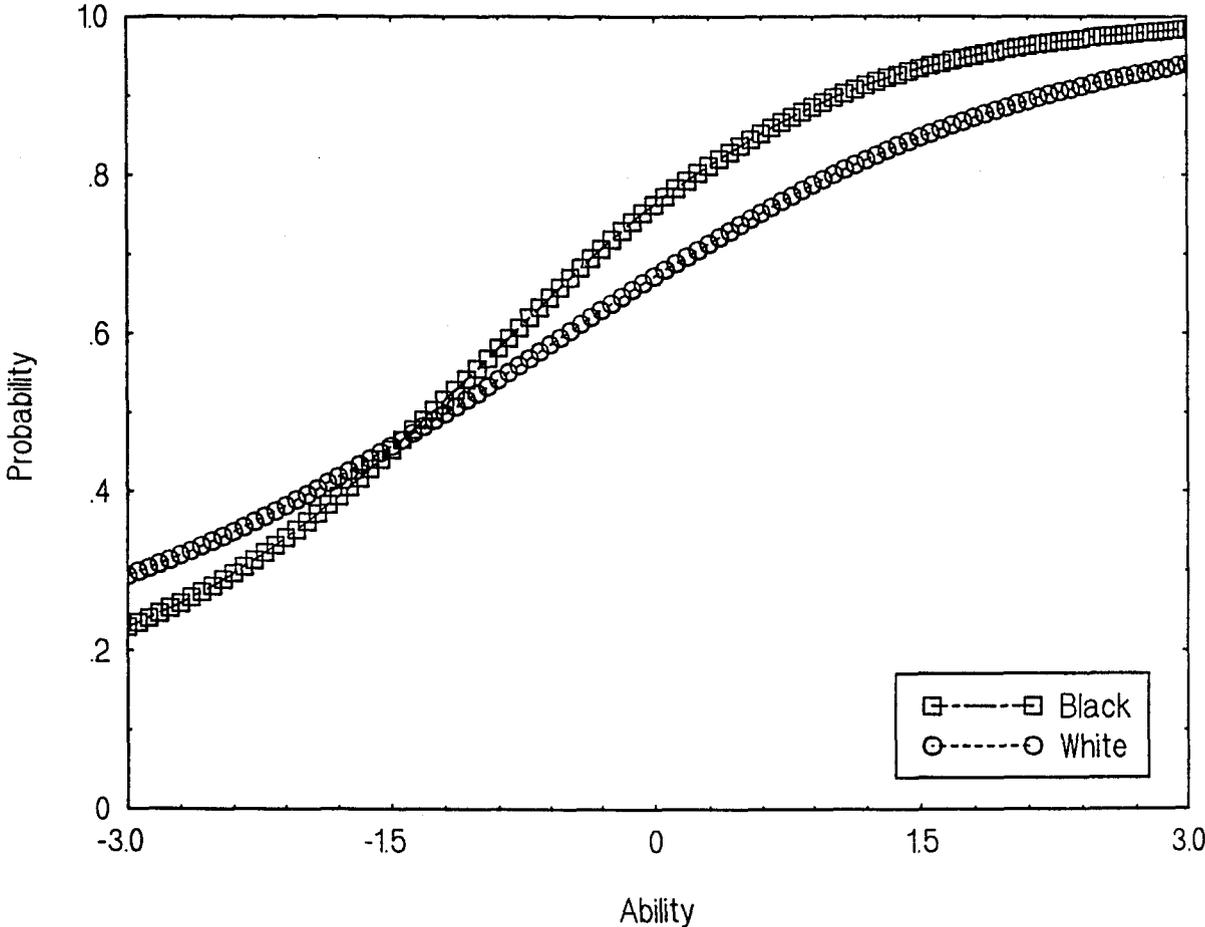
Black vs. White: Item 6-25



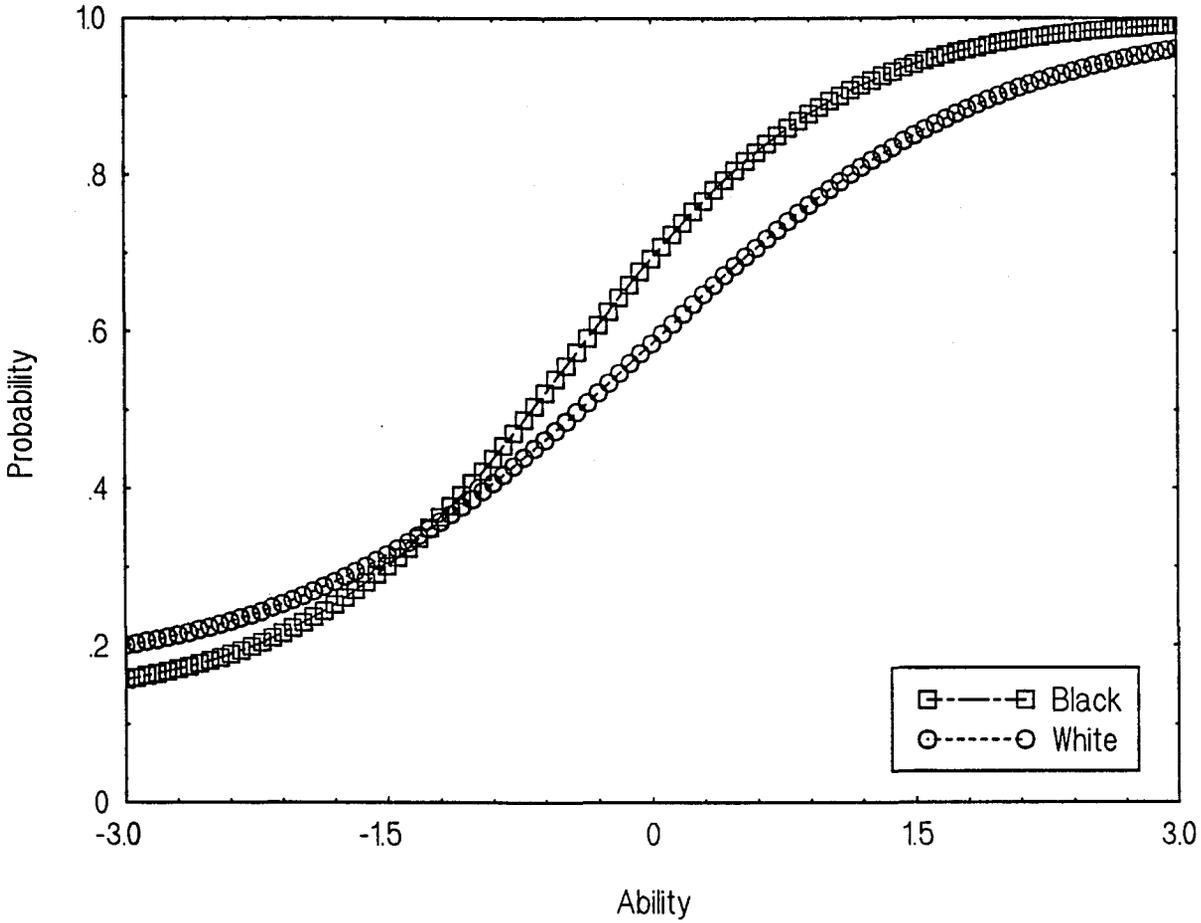
Black vs. White: Item 6-26



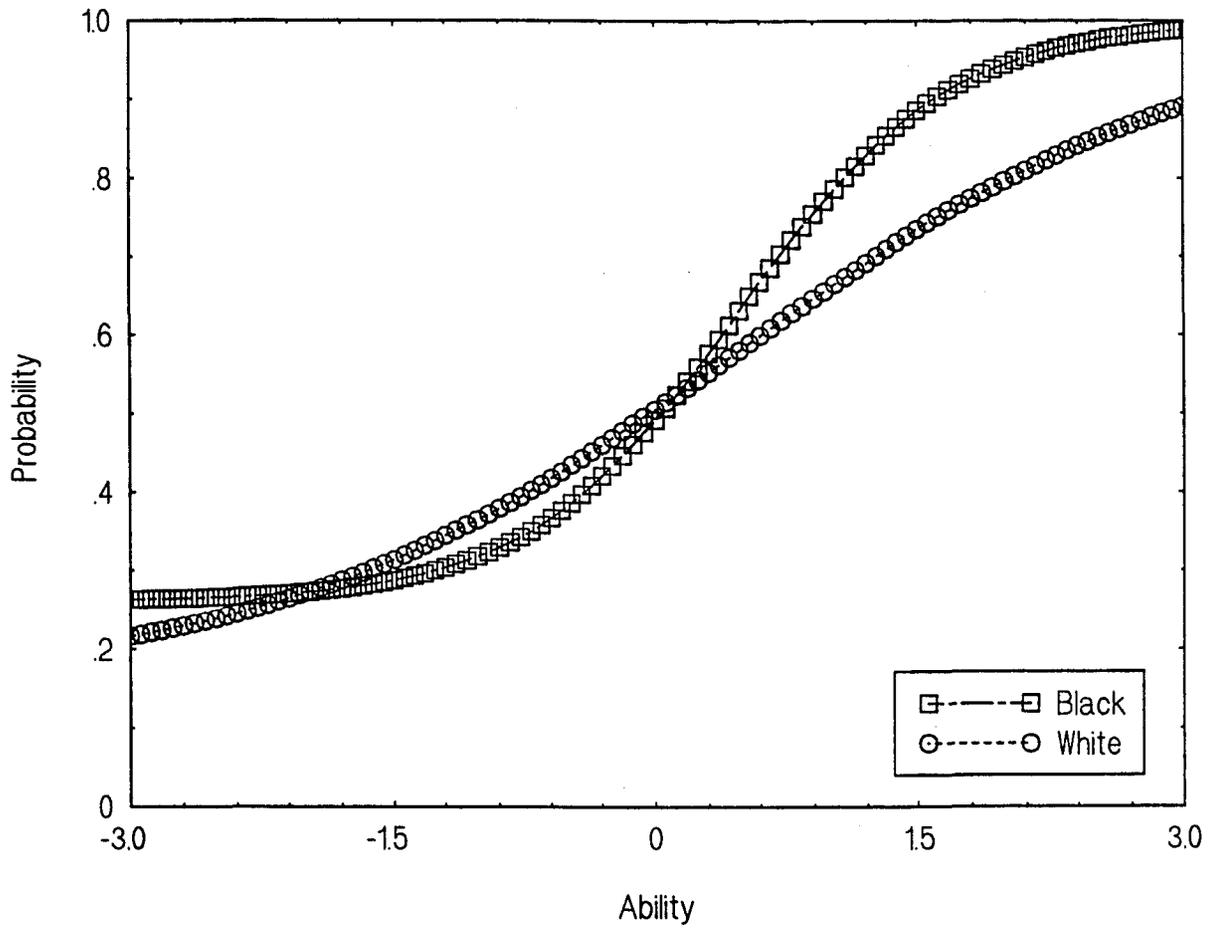
Black vs. White: Item 6-27



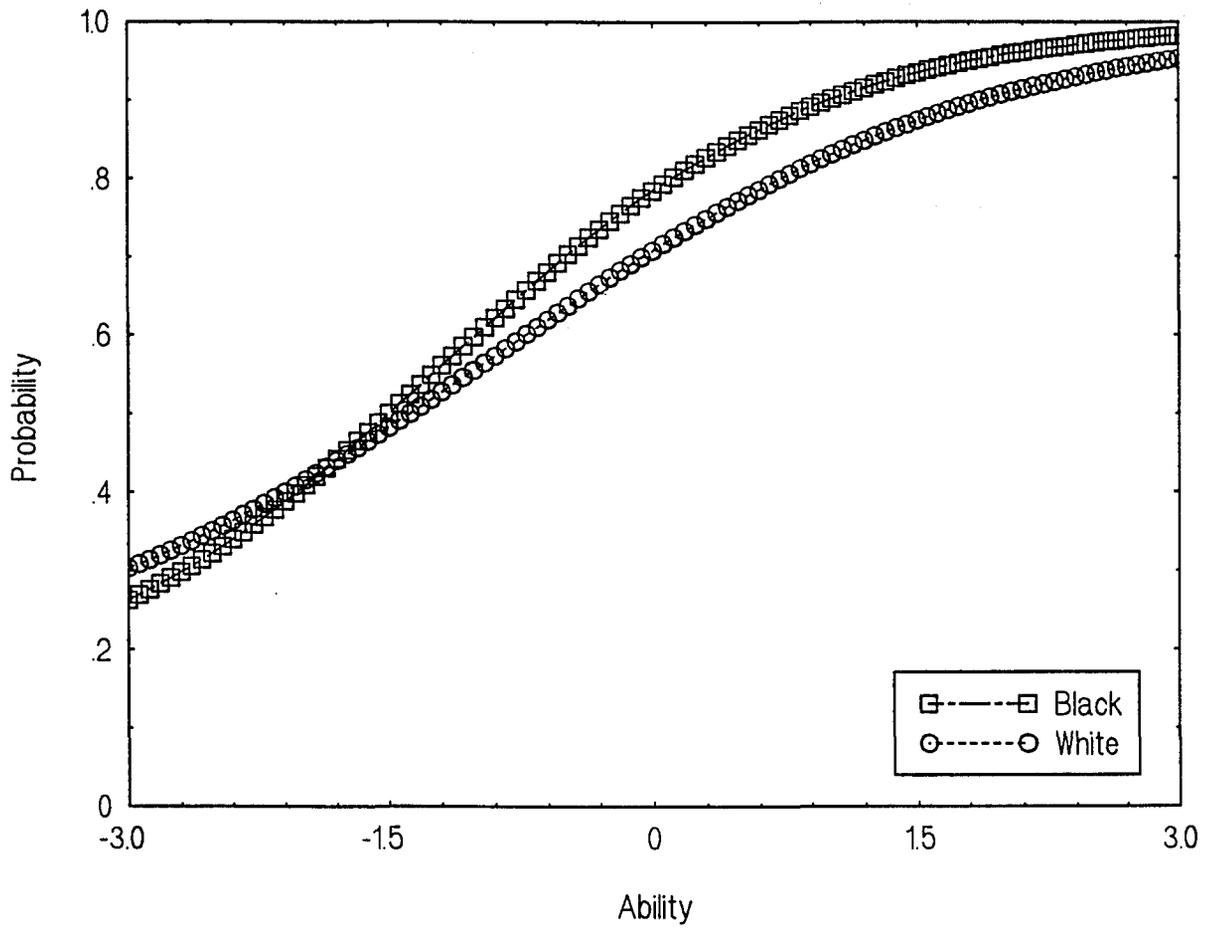
Black vs. White: Item 6-28



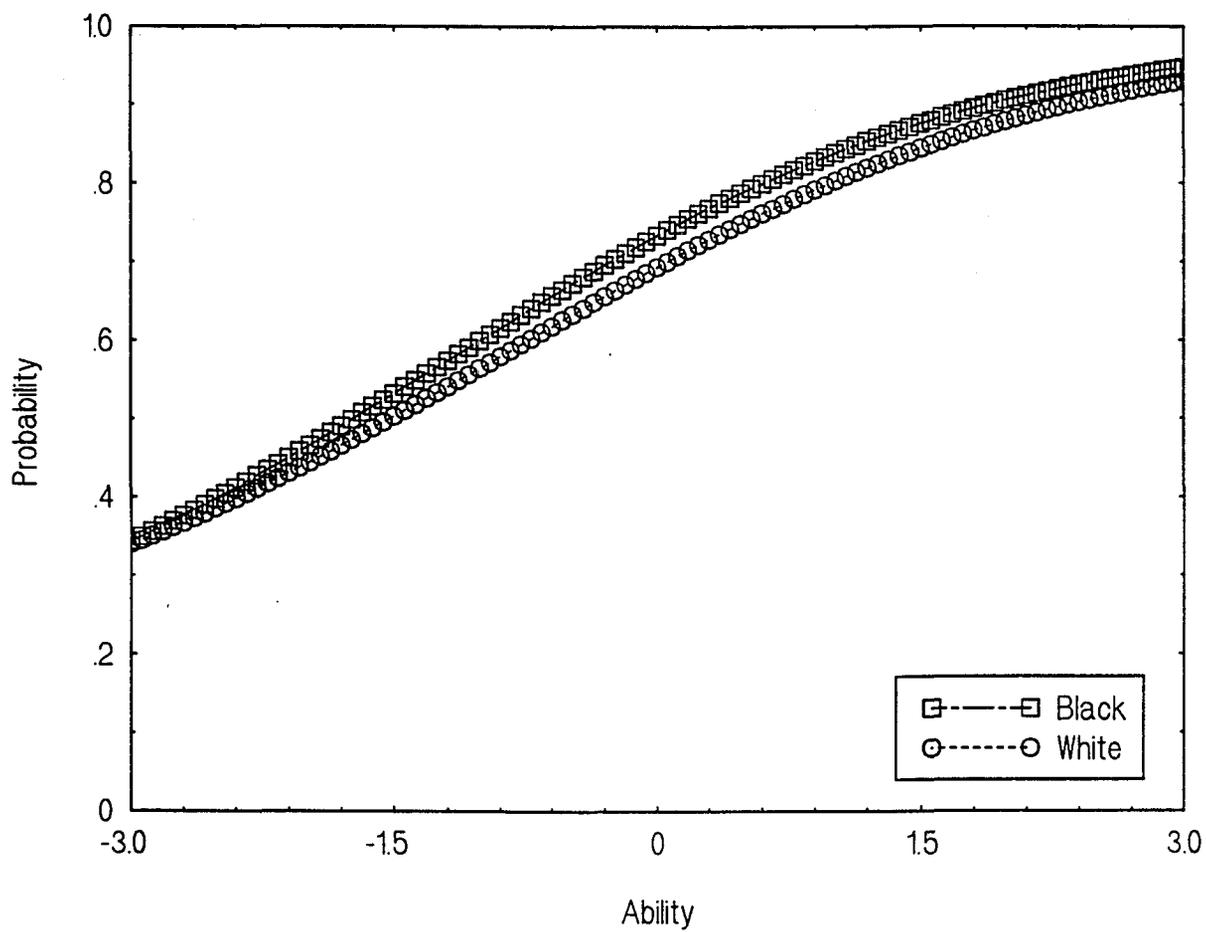
Black vs. White: Item 6-29



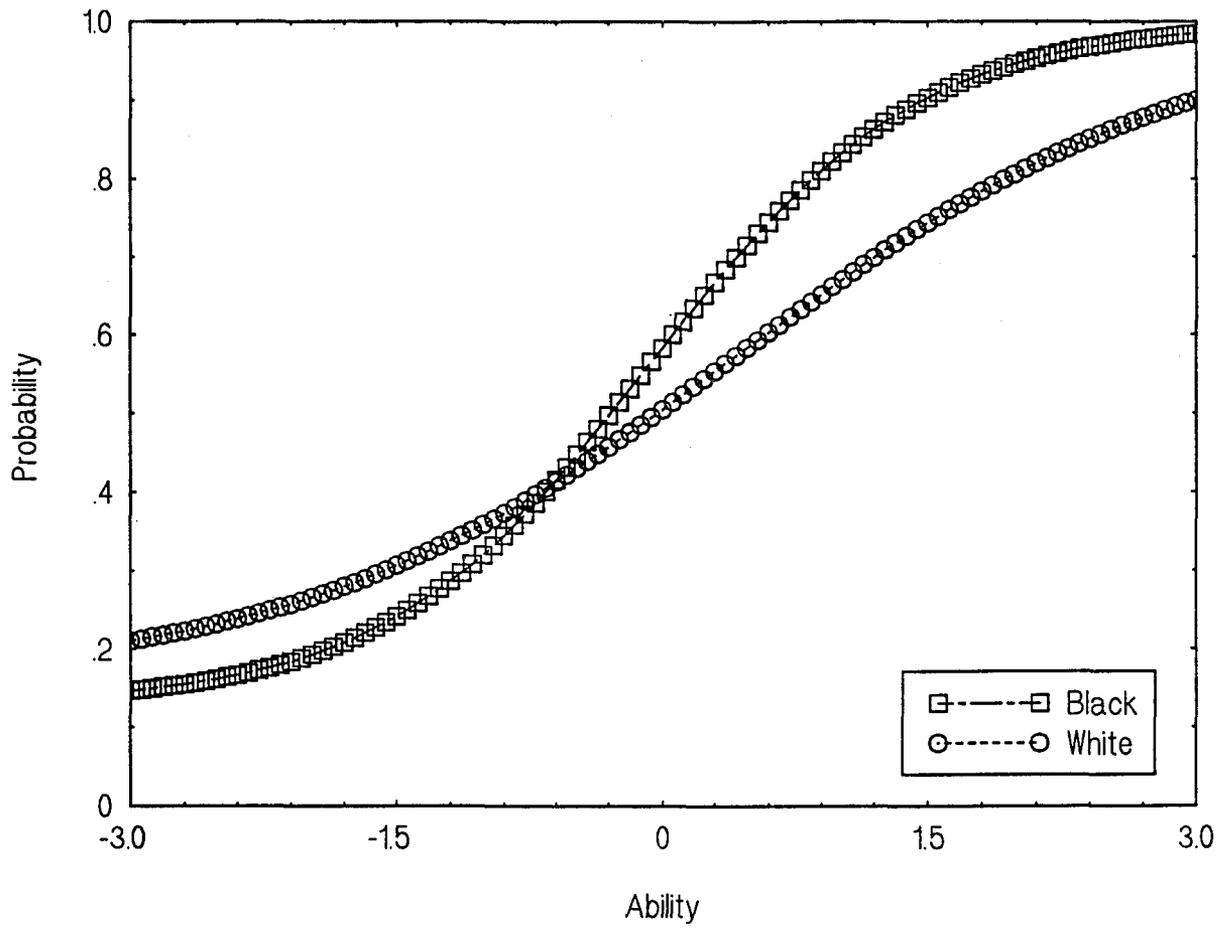
Black vs. White: Item 6-30



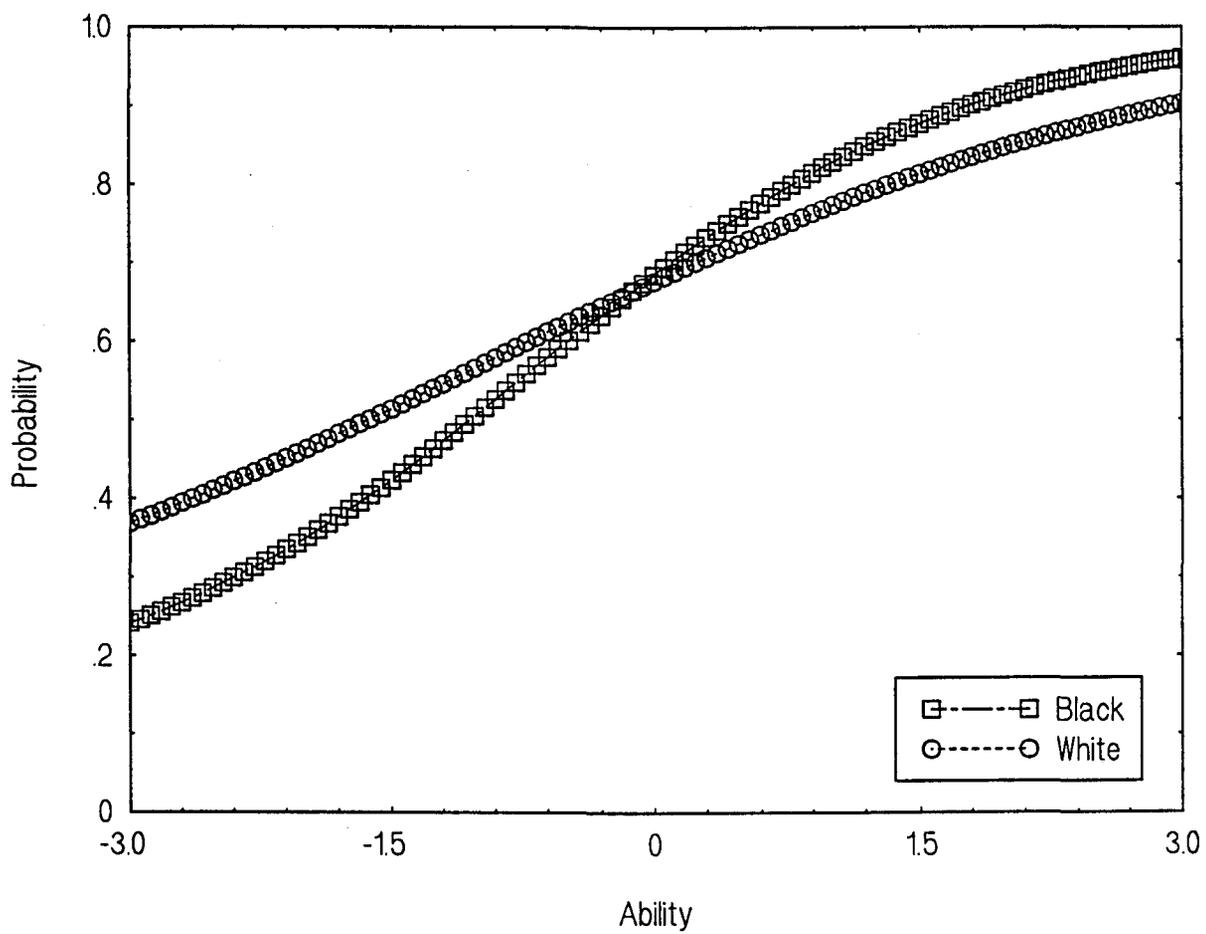
Black vs. White: Item 6-31



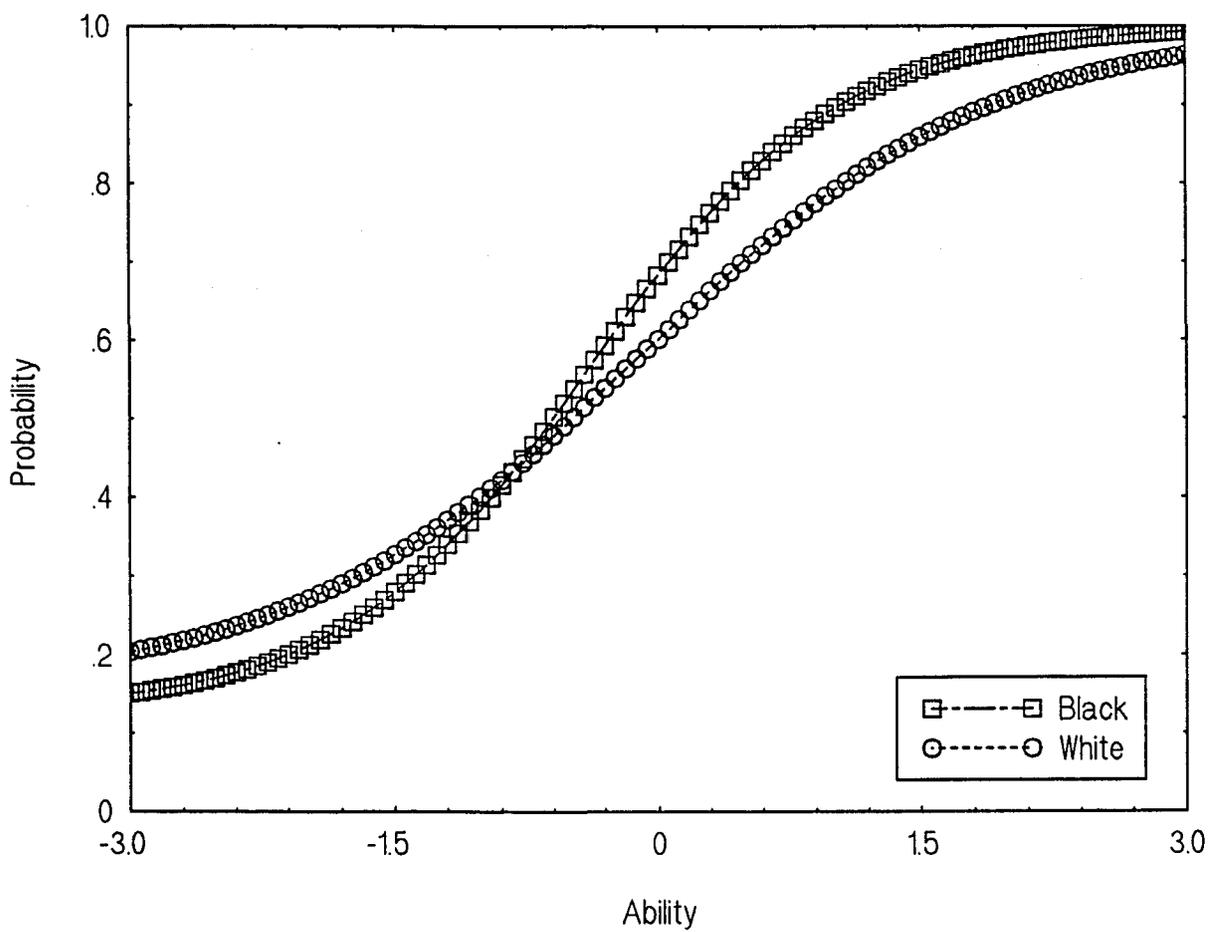
Black vs. White: Item 6-32



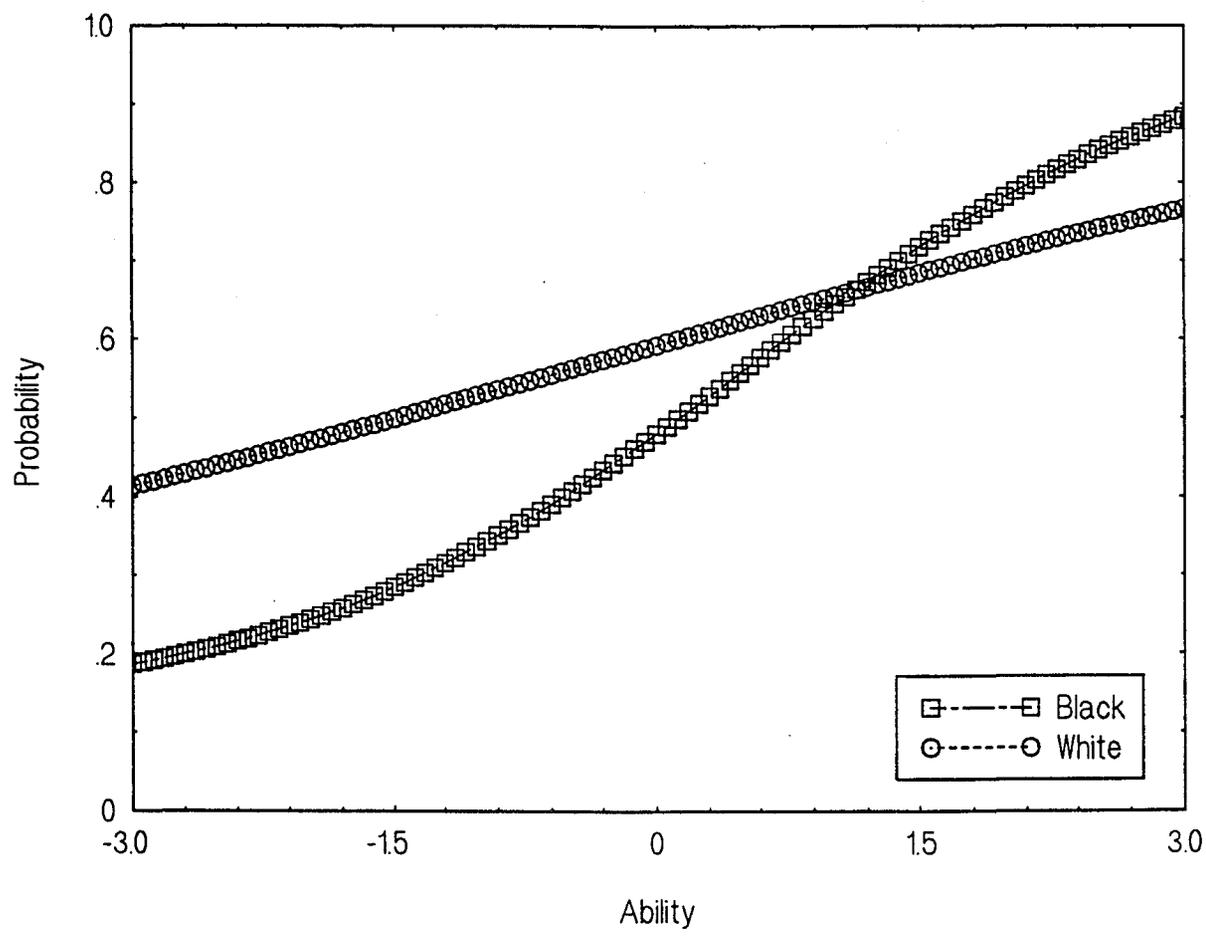
Black vs. White: Item 6-33



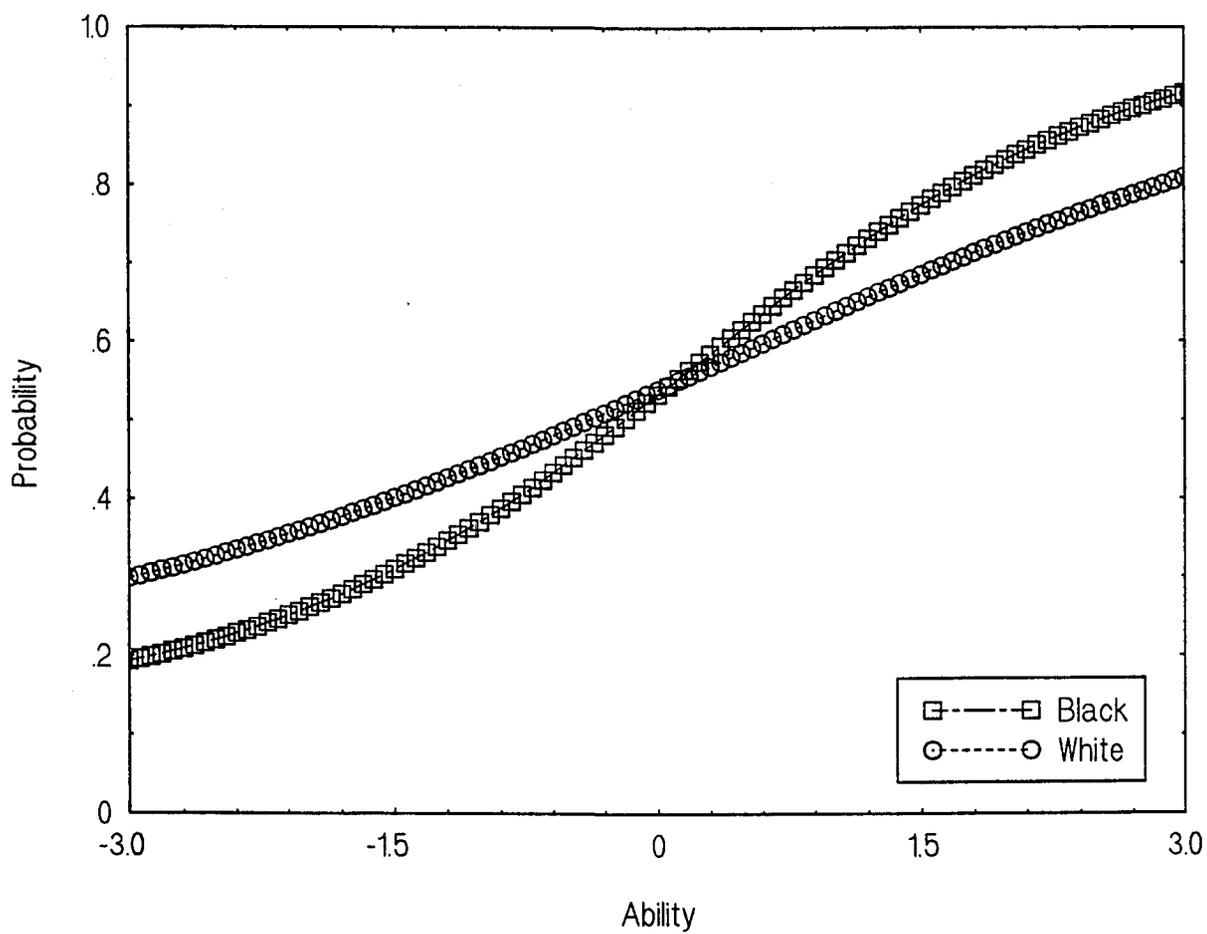
Black vs. White: Item 6-34



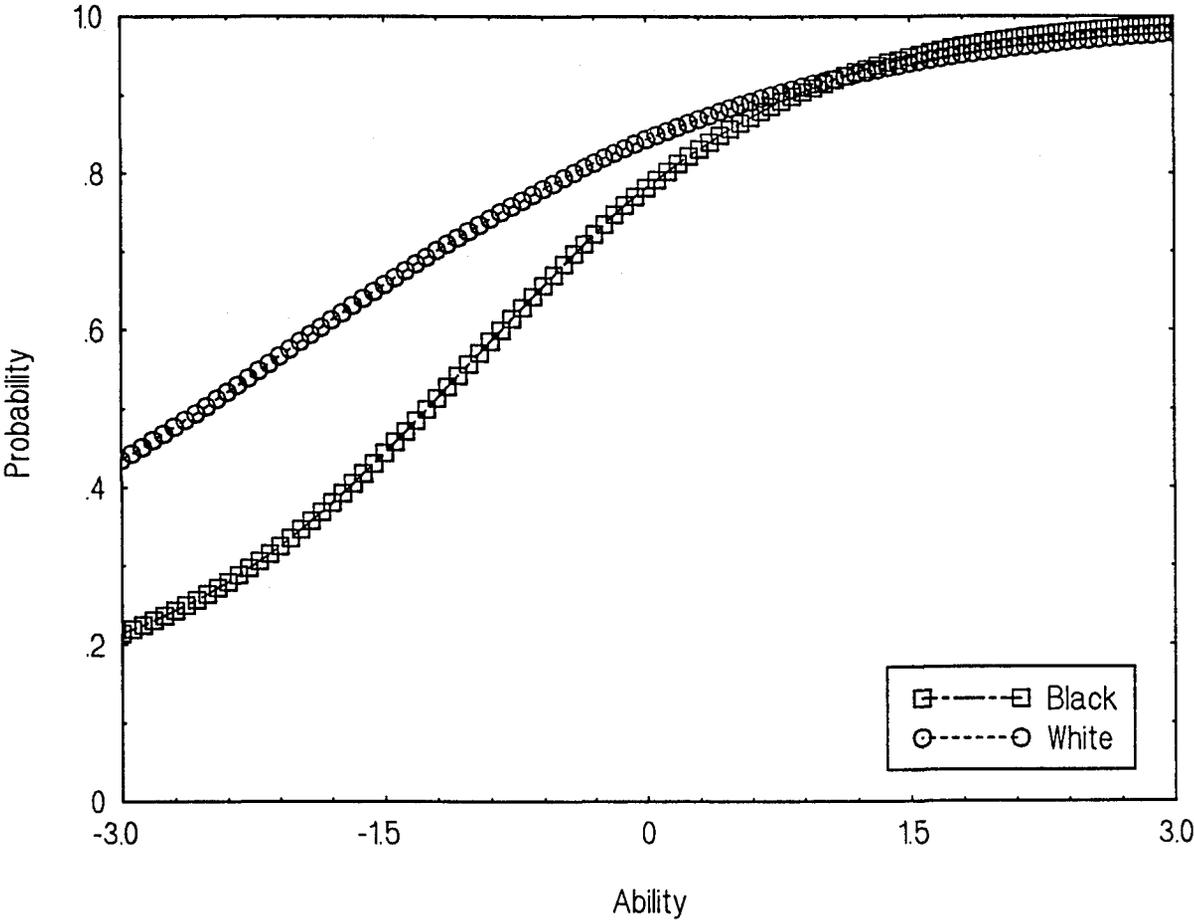
Black vs. White: Item 6-35



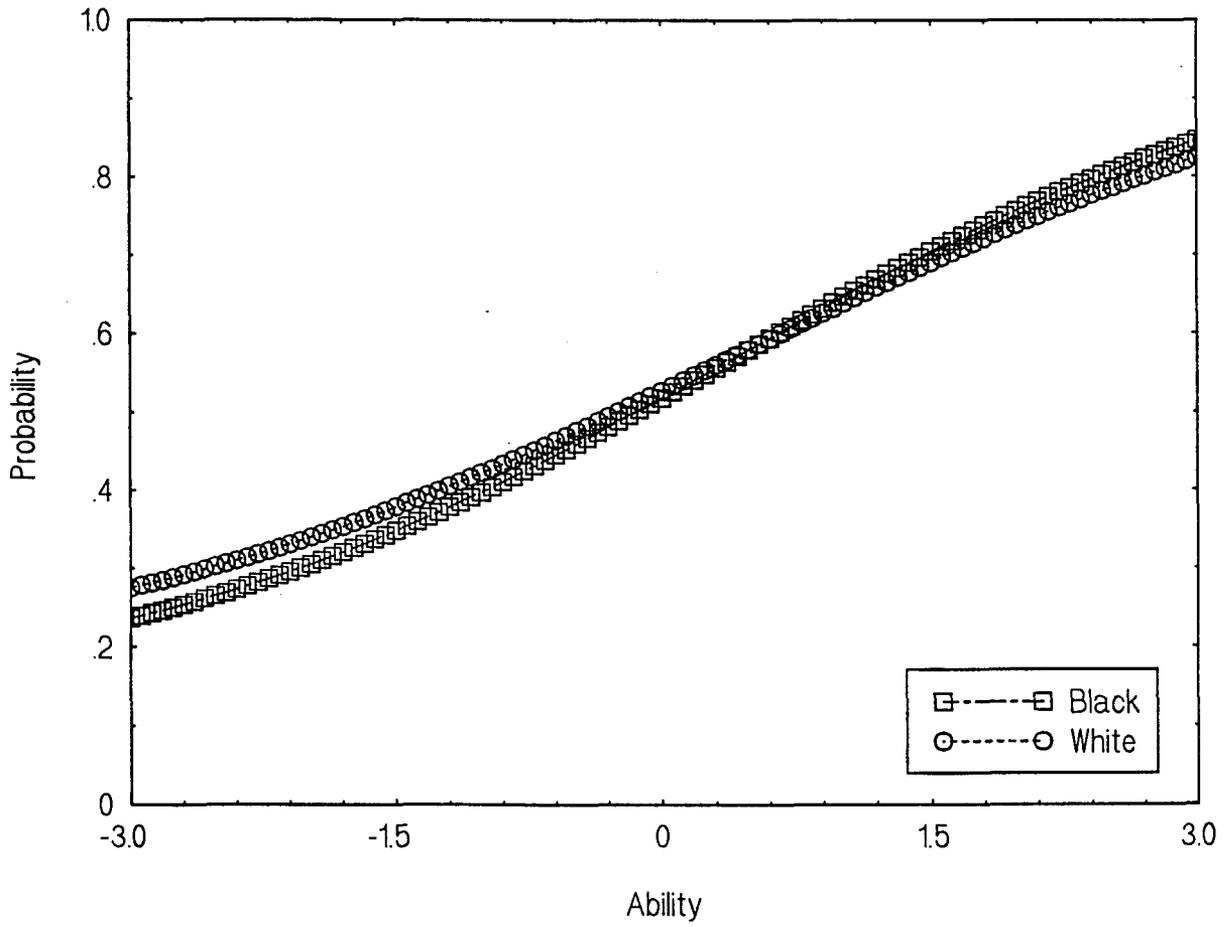
Black vs. White: Item 7-1



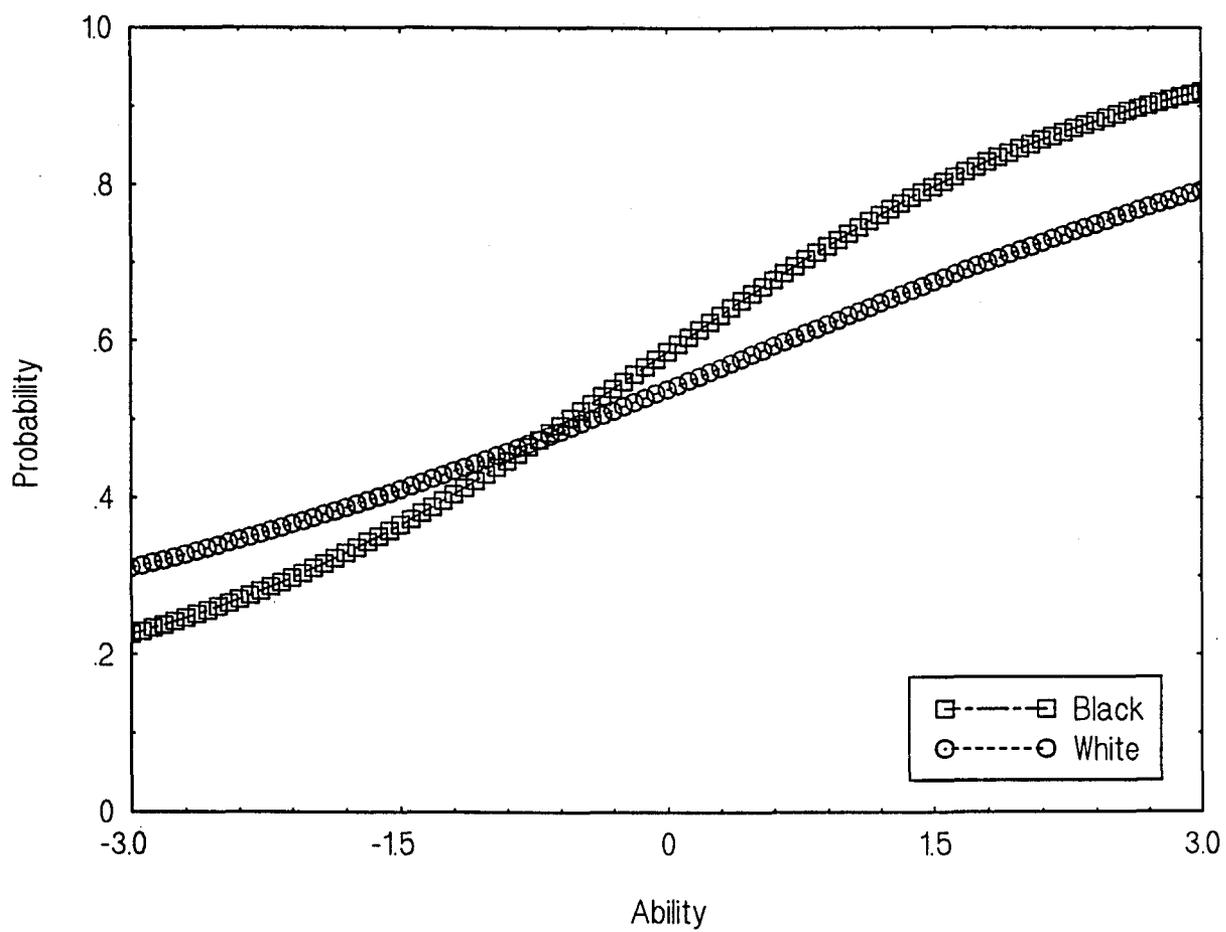
Black vs. White: Item 7-2



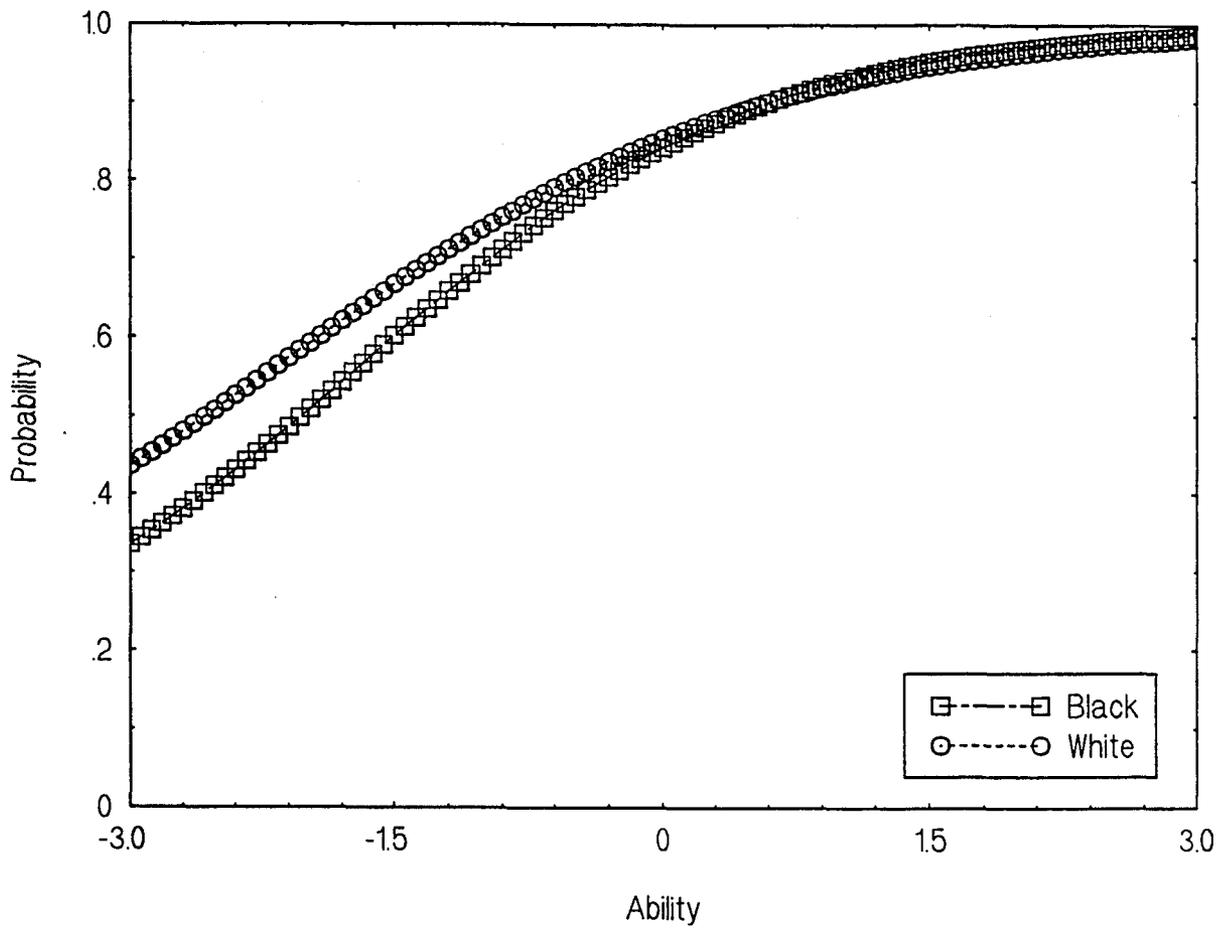
Black vs. White: Item 7-3



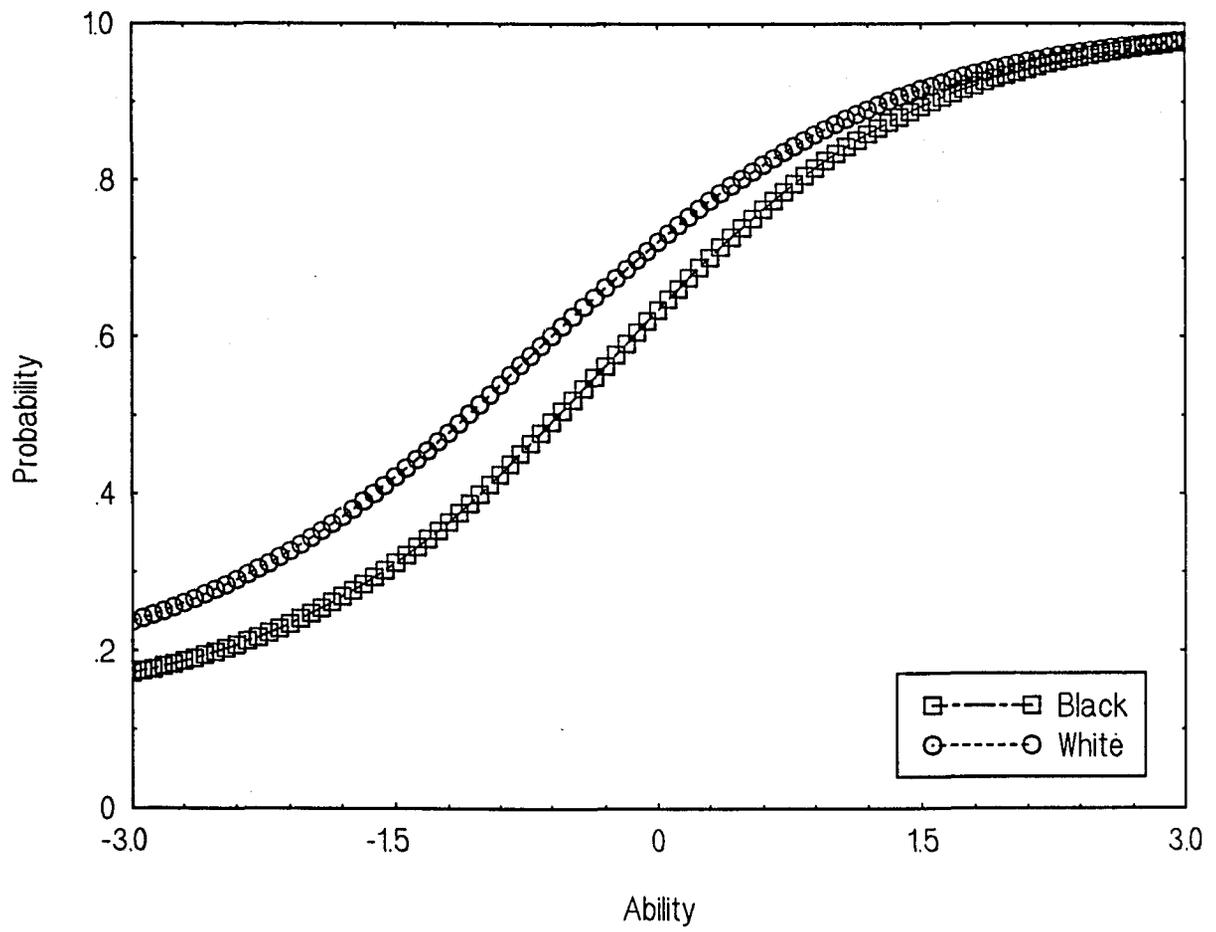
Black vs. White: Item 7-4



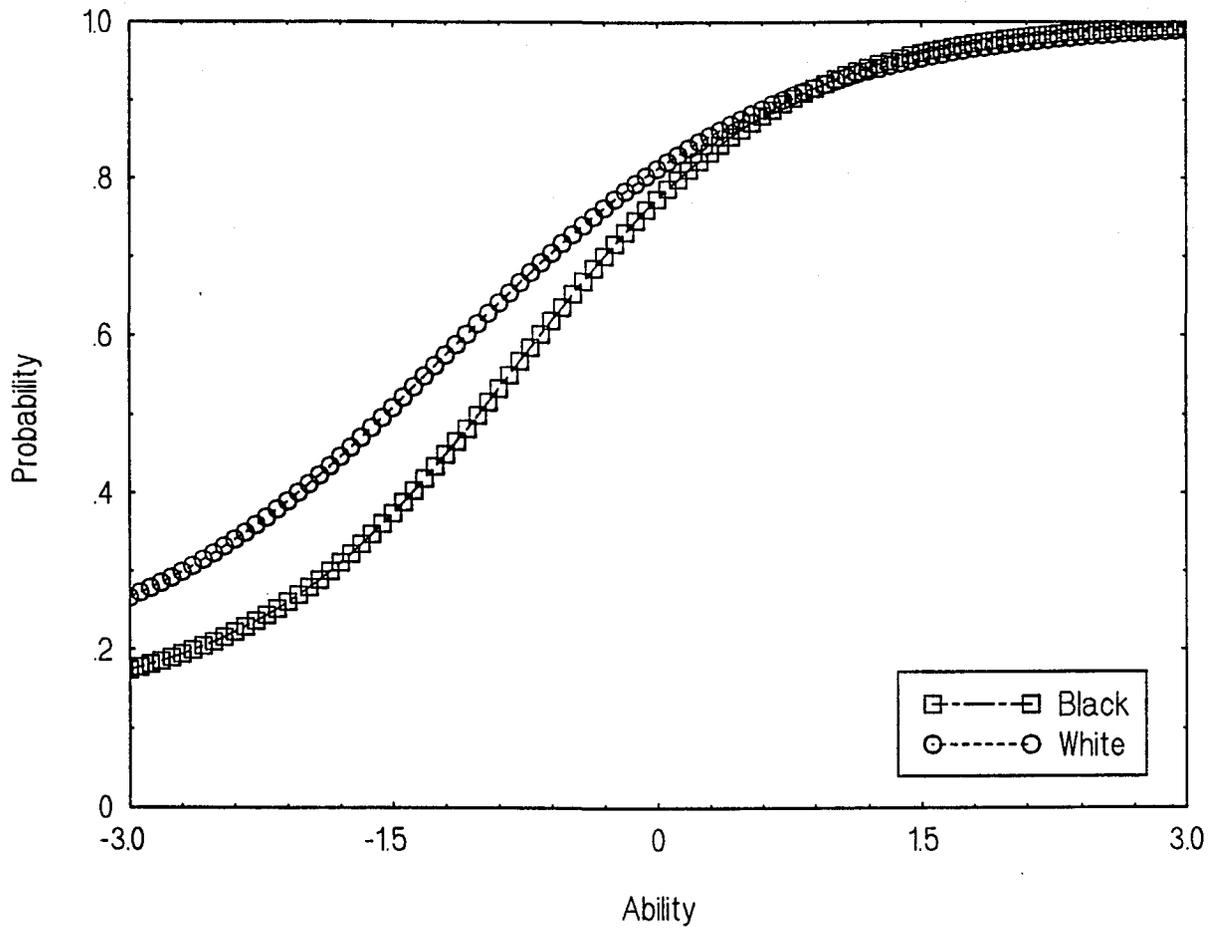
Black vs. White: Item 7-5



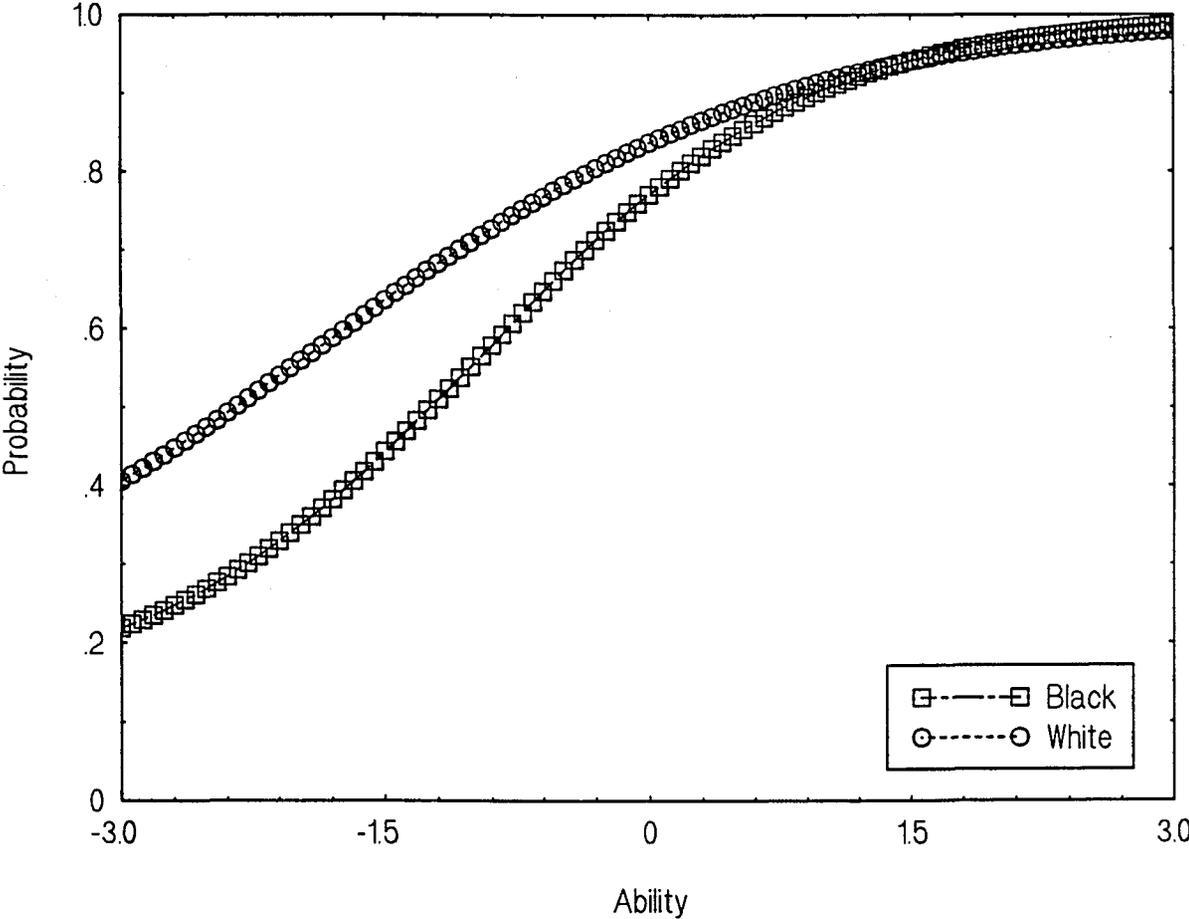
Black vs. White: Item 7-6



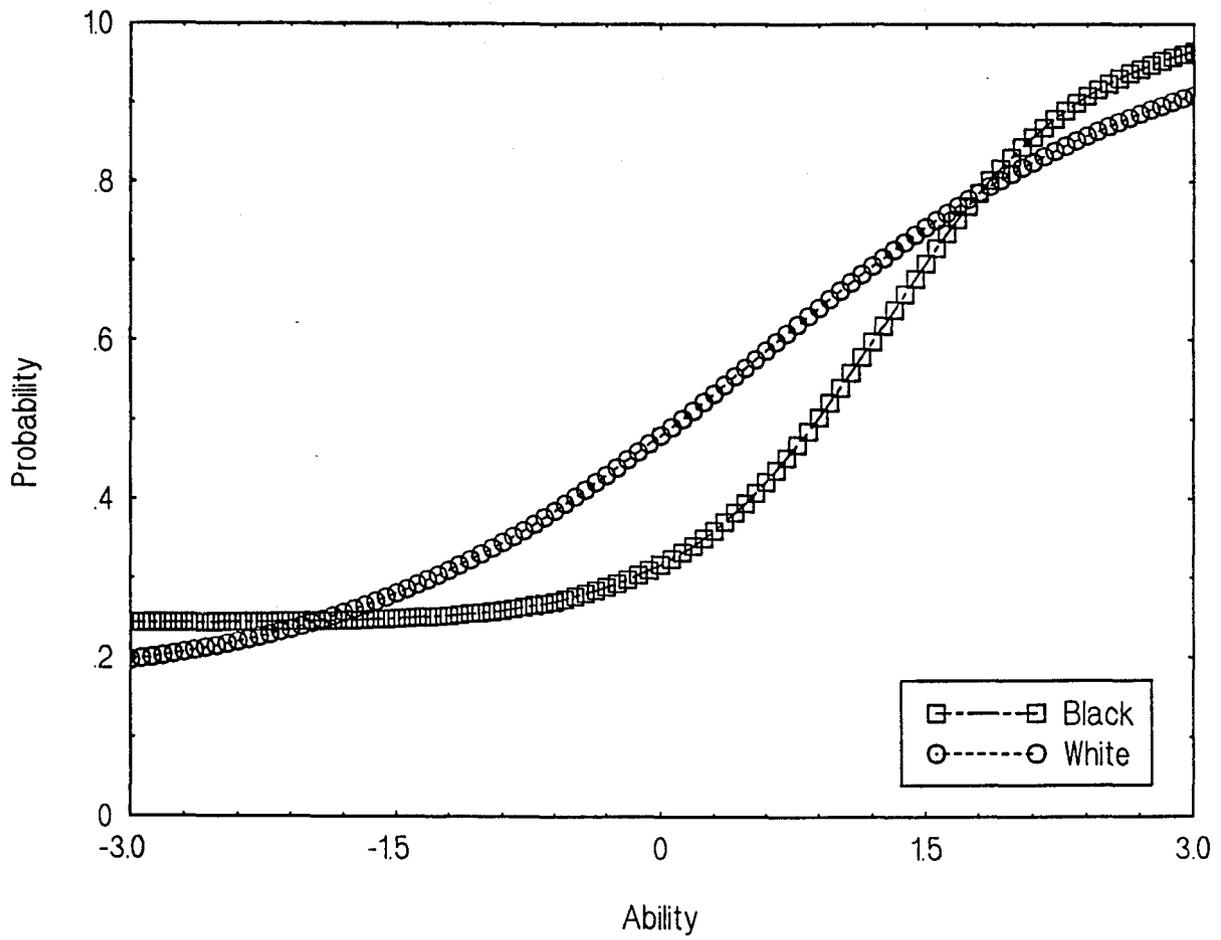
Black vs. White: Item 7-7



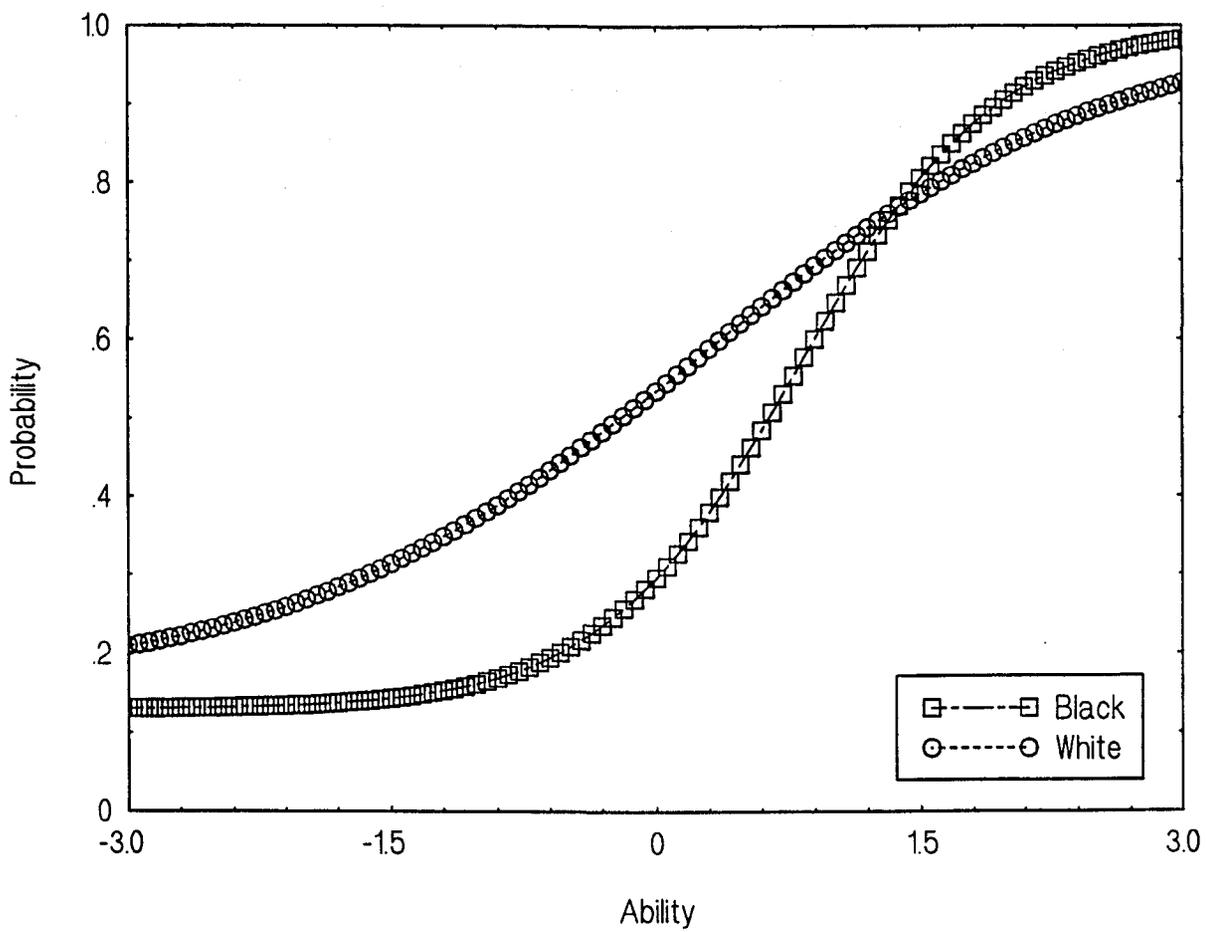
Black vs. White: Item 7-8



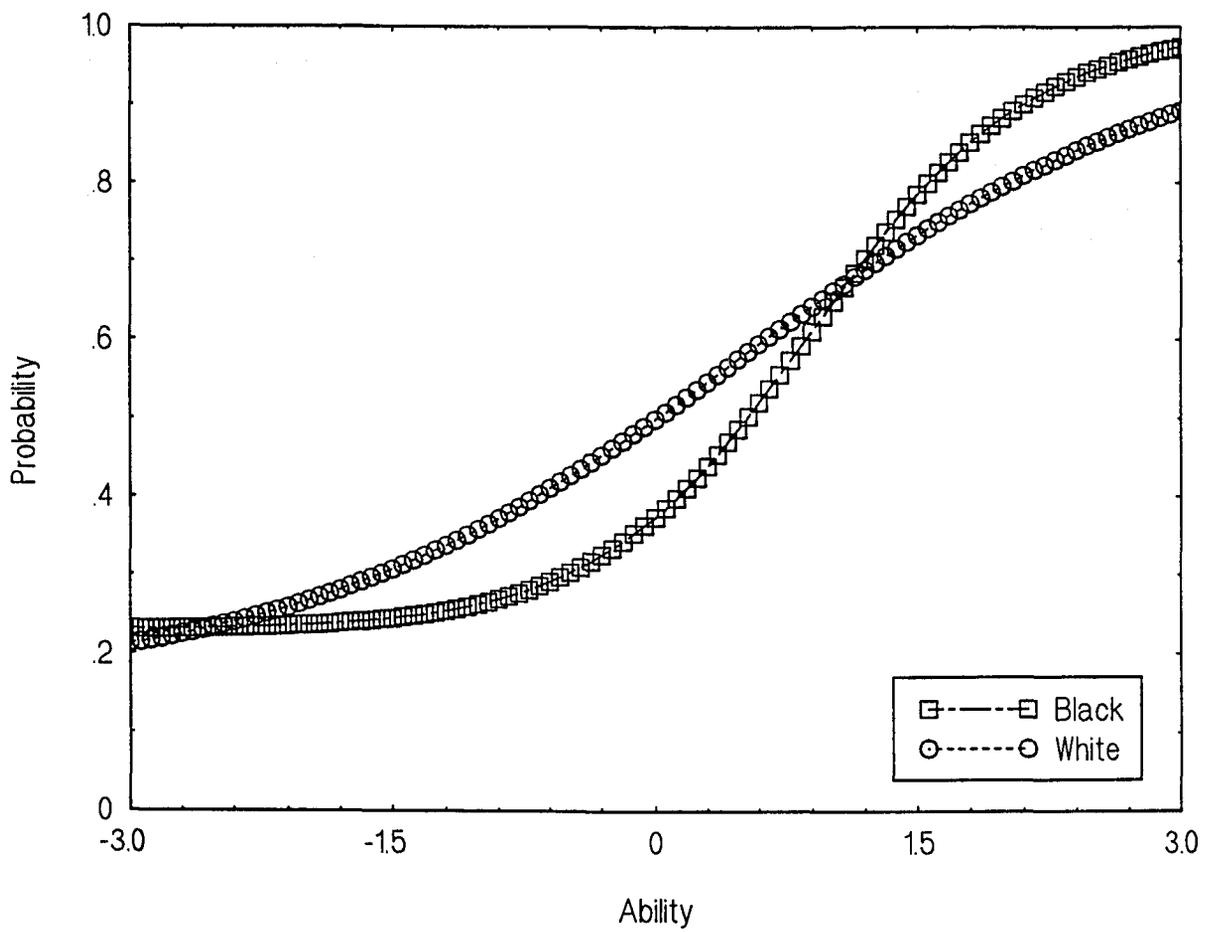
Black vs. White: Item 7-9



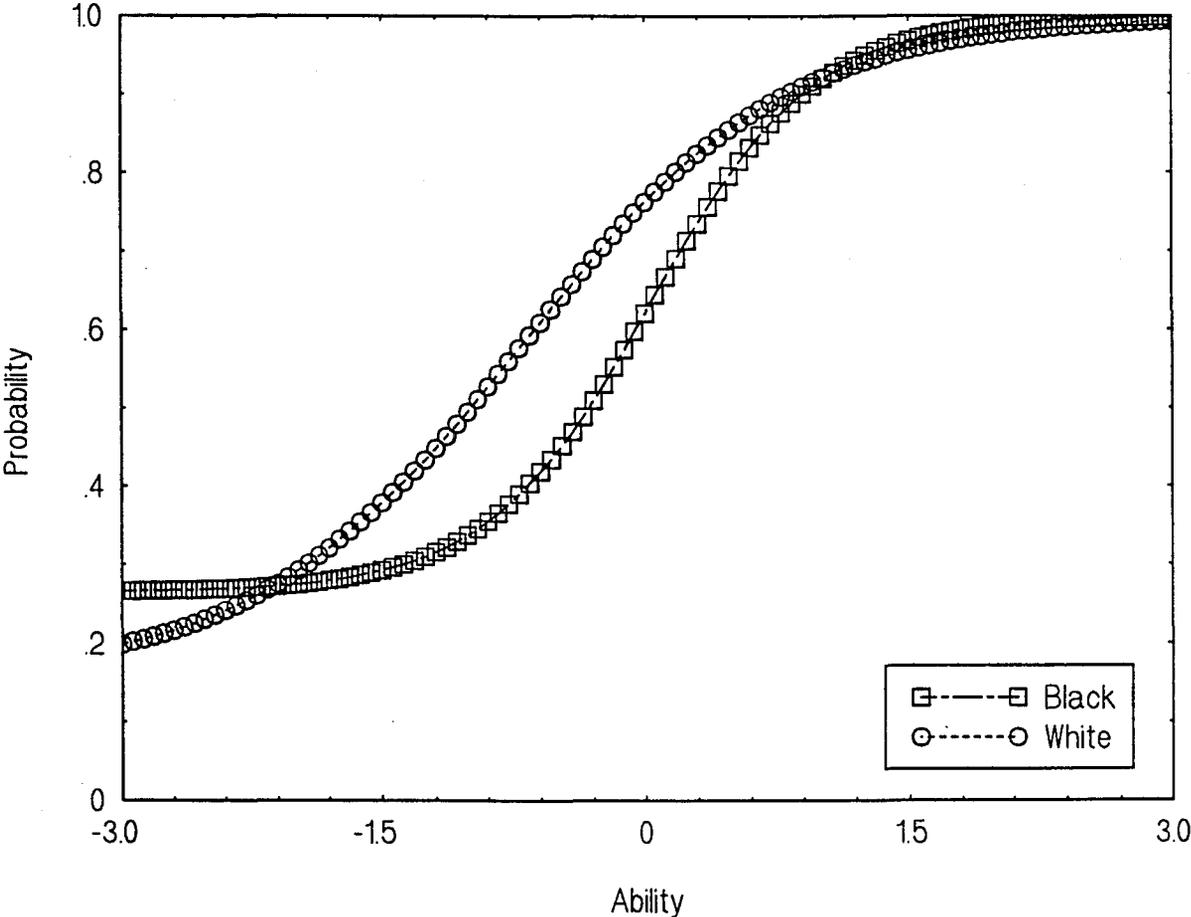
Black vs. White: Item 7-10



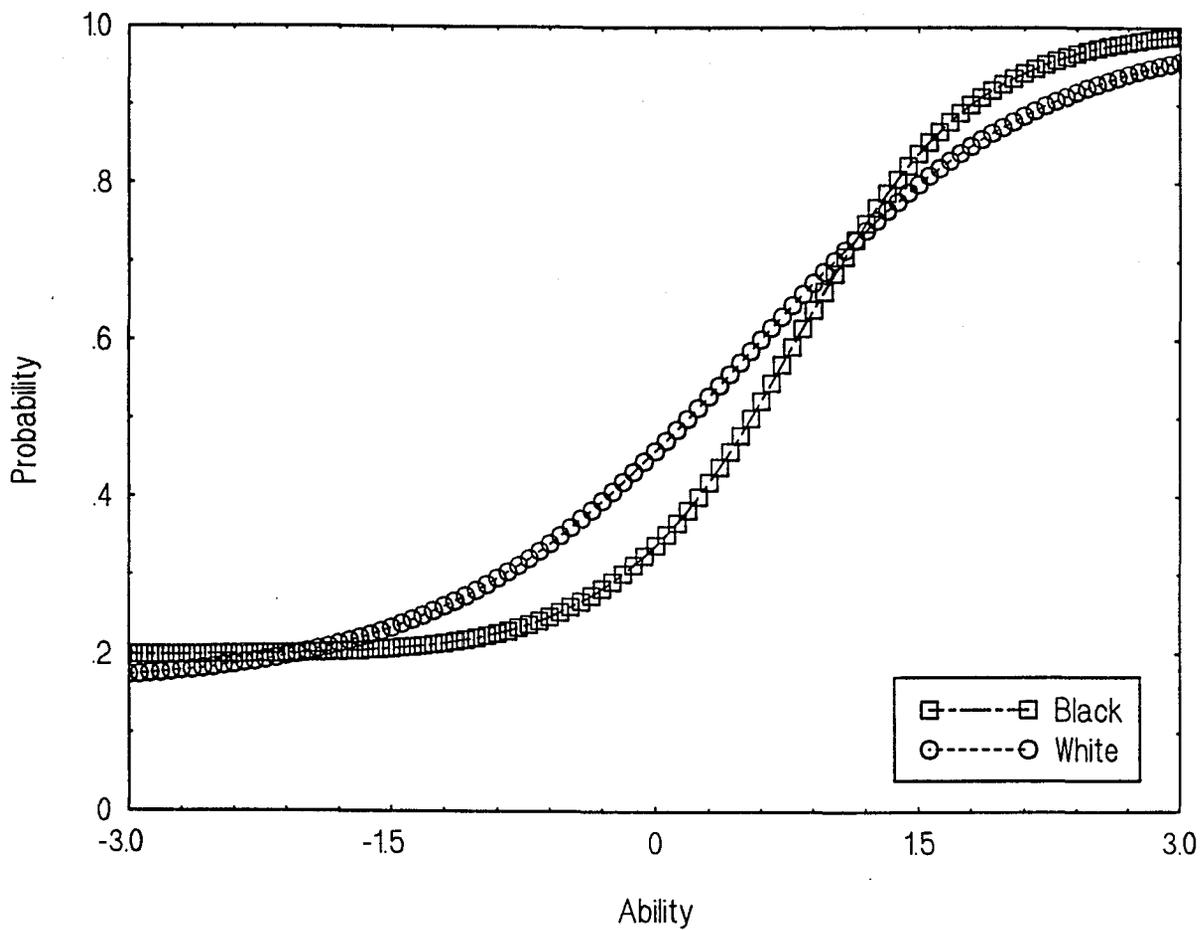
Black vs. White: Item 7-11



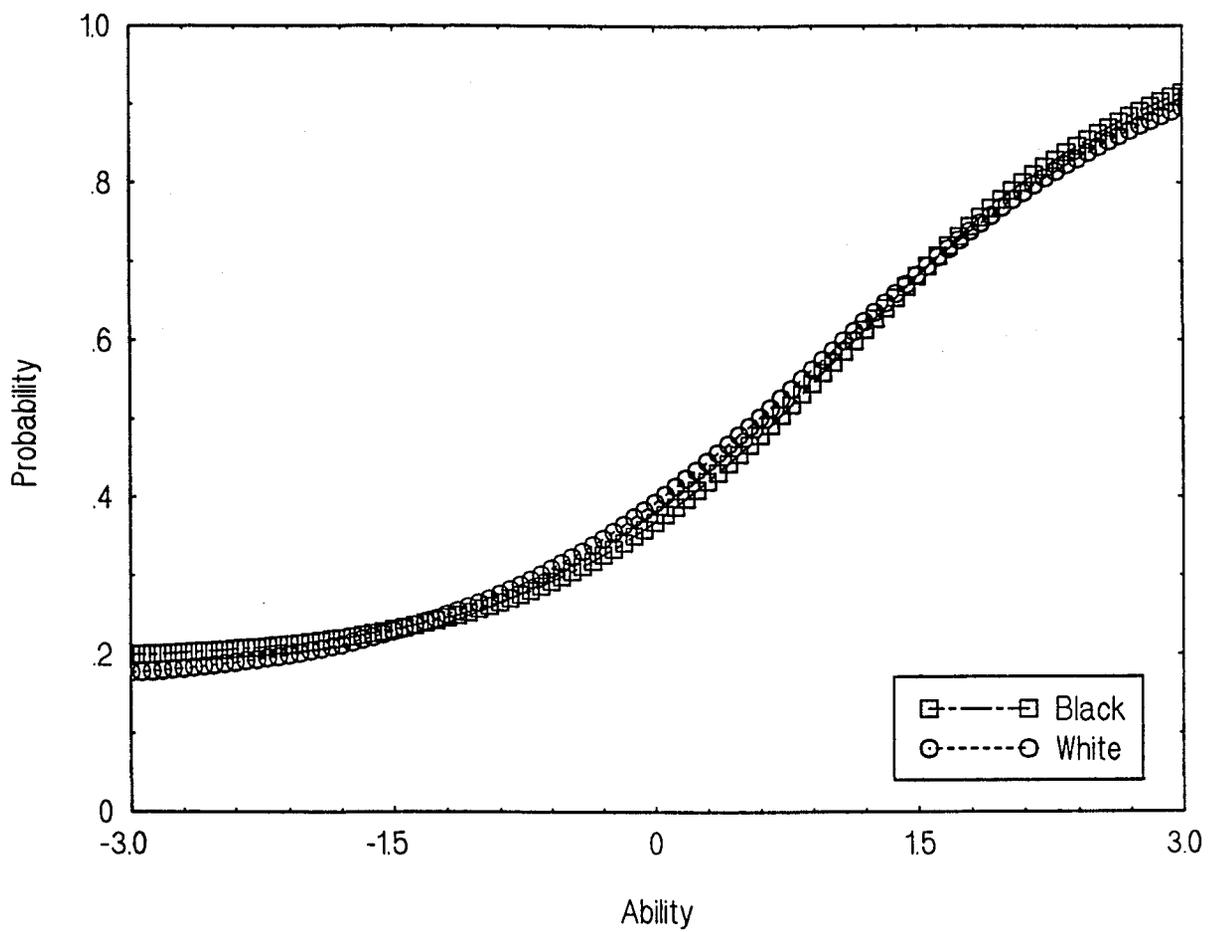
Black vs. White: Item 7-12



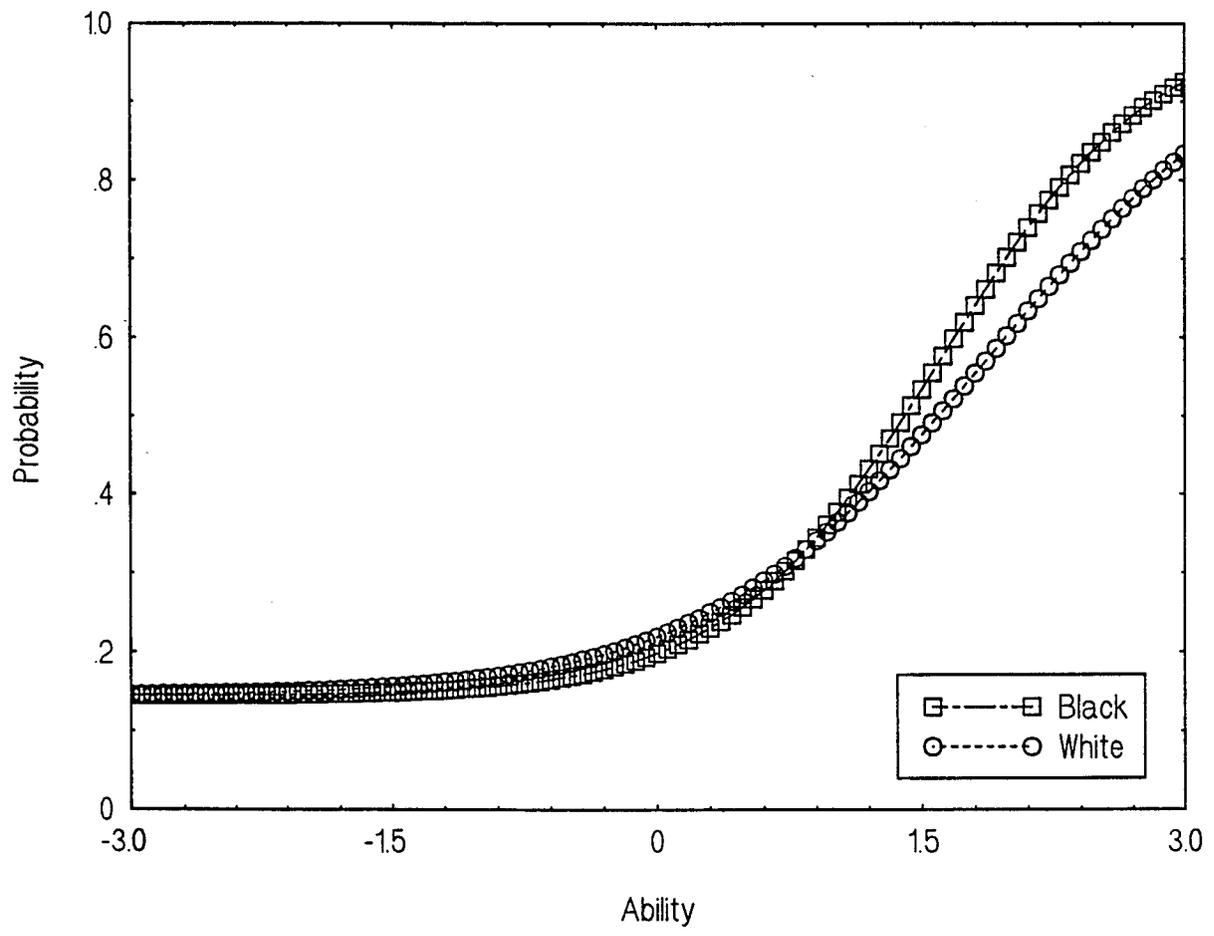
Black vs. White: Item 7-13



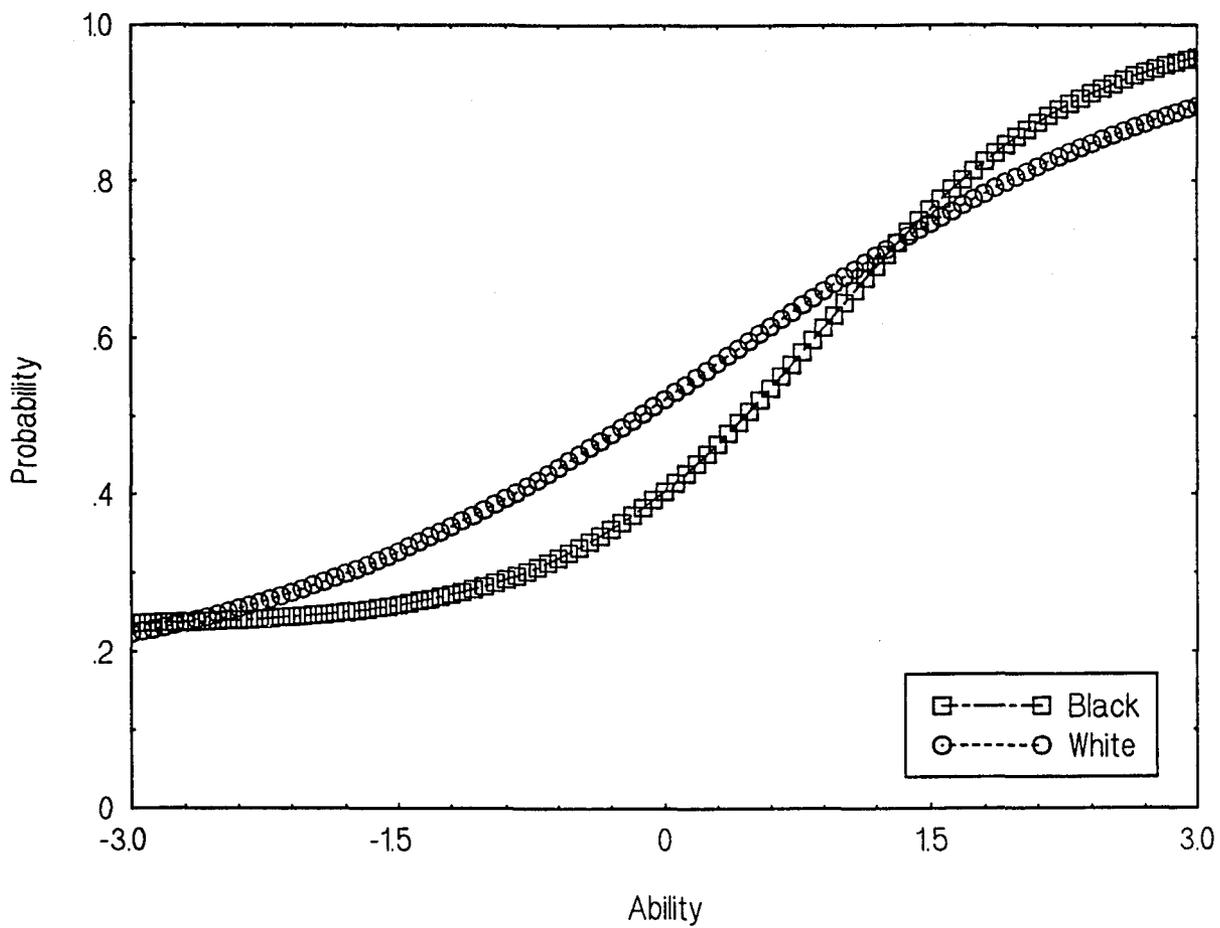
Black vs. White: Item 7-14



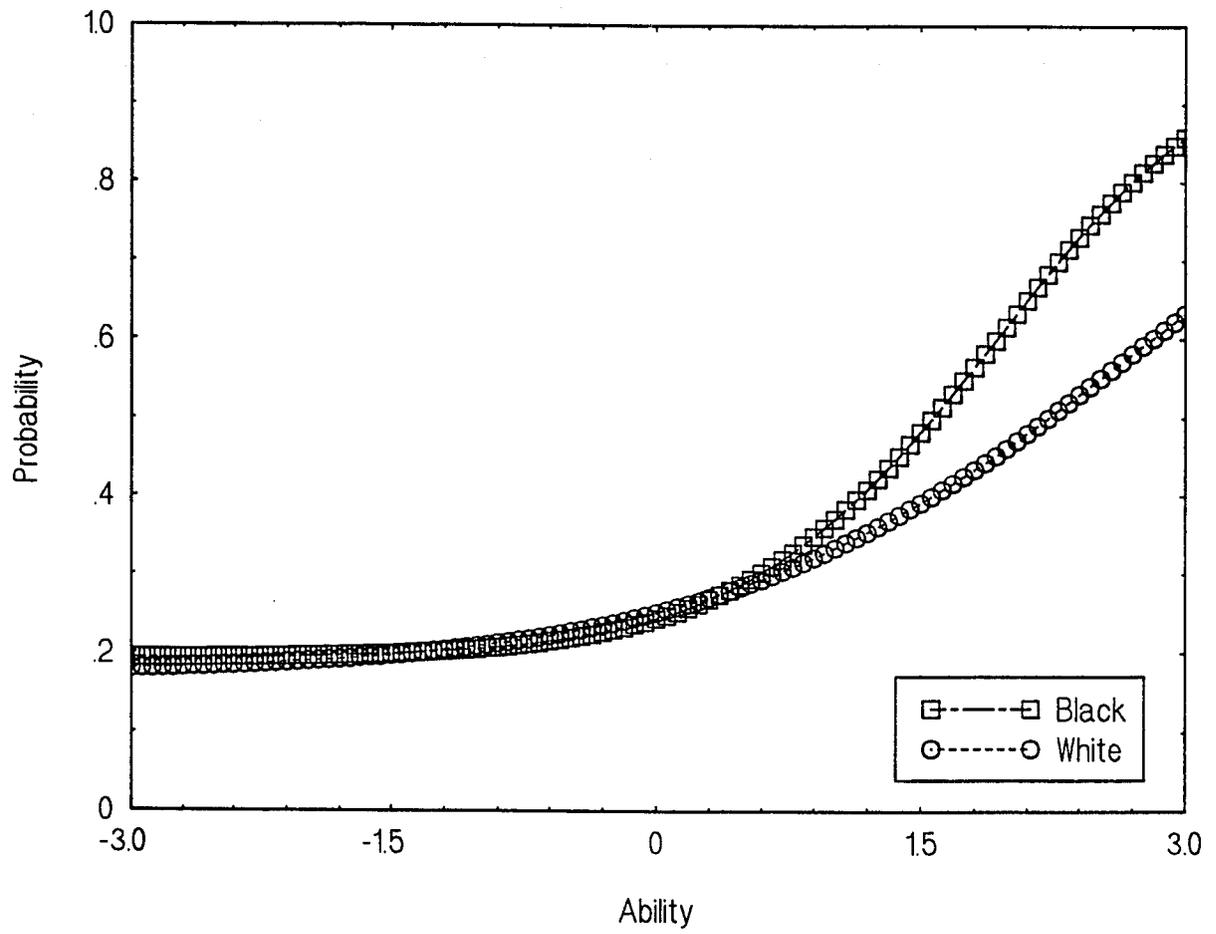
Black vs. White: Item 7-15



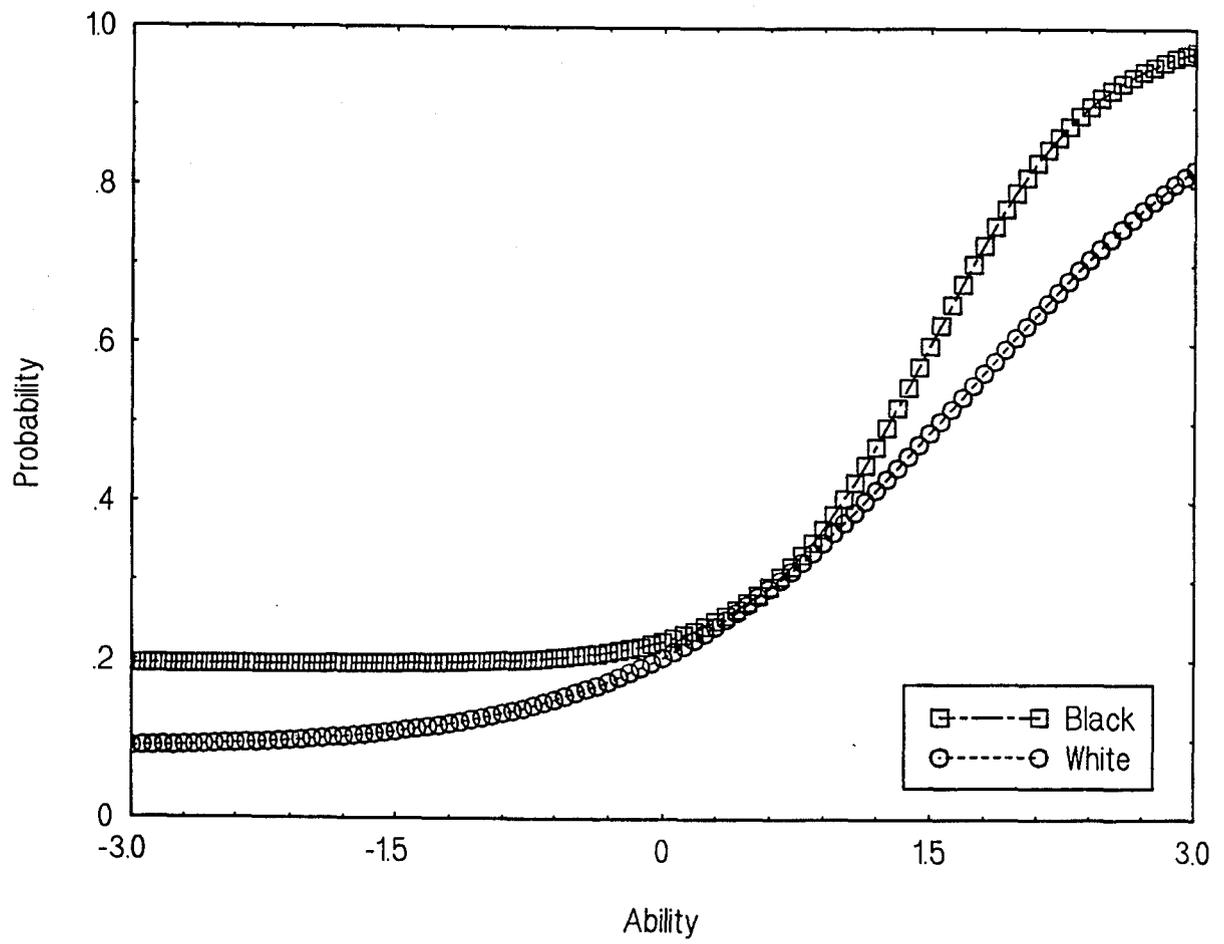
Black vs. White: Item 7-16



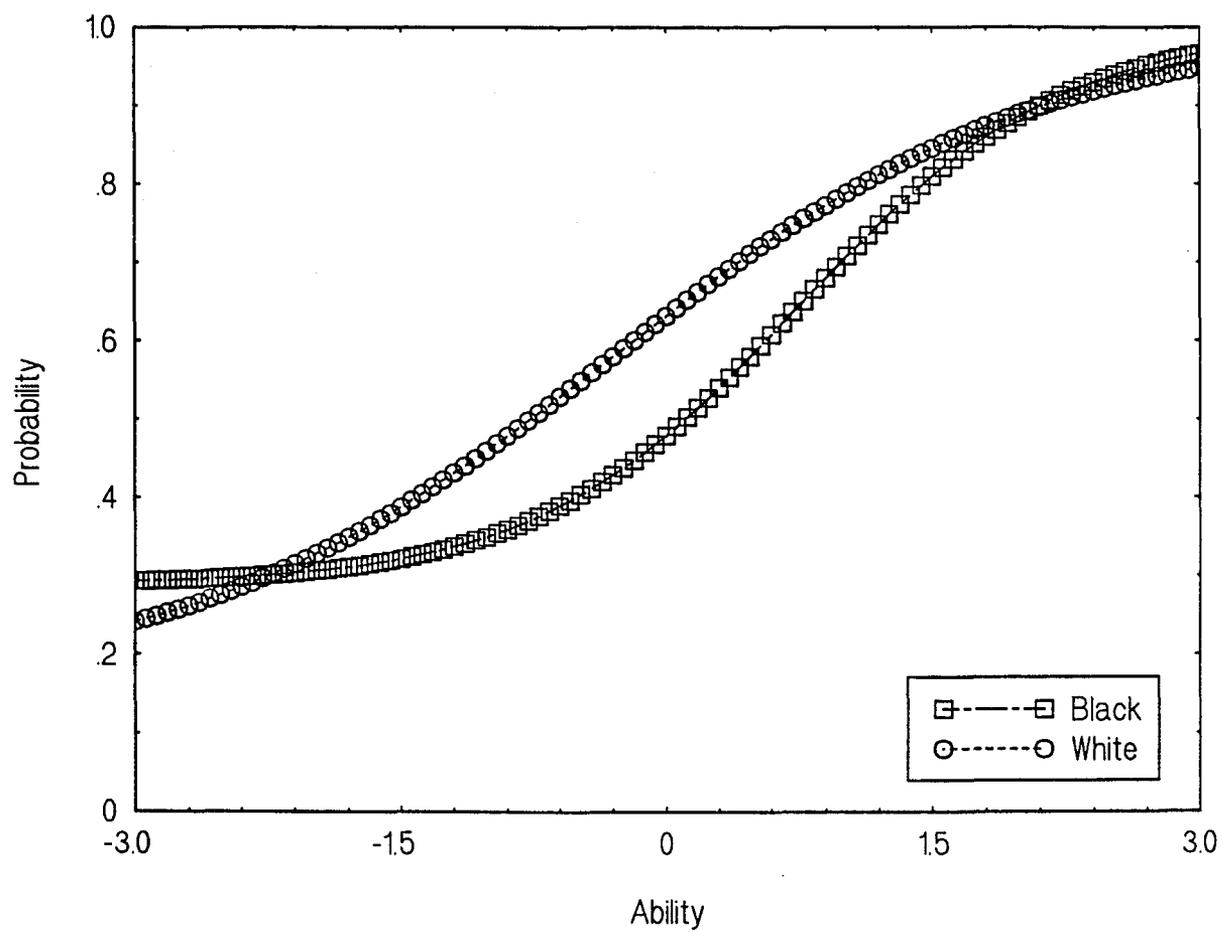
Black vs. White: Item 7-17



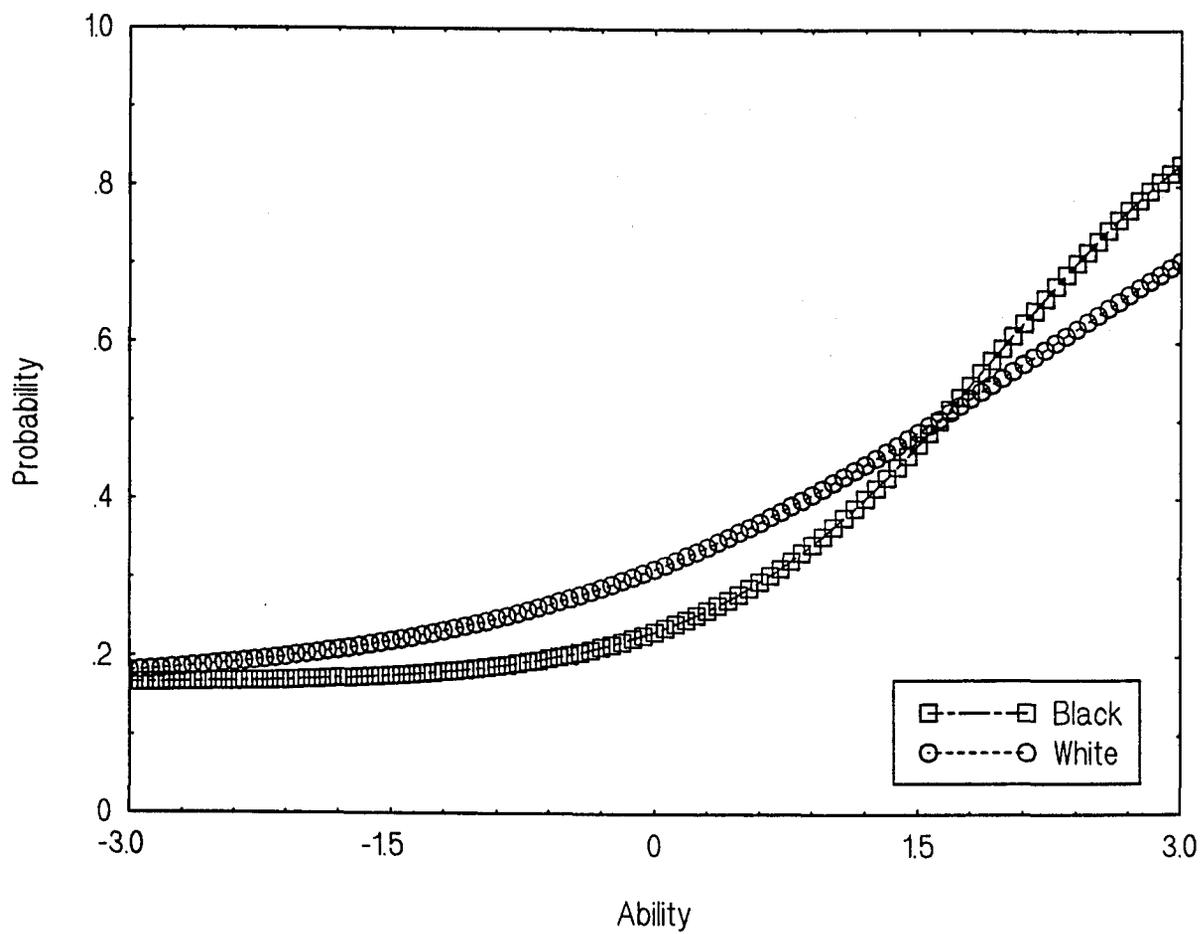
Black vs. White: Item 7-18



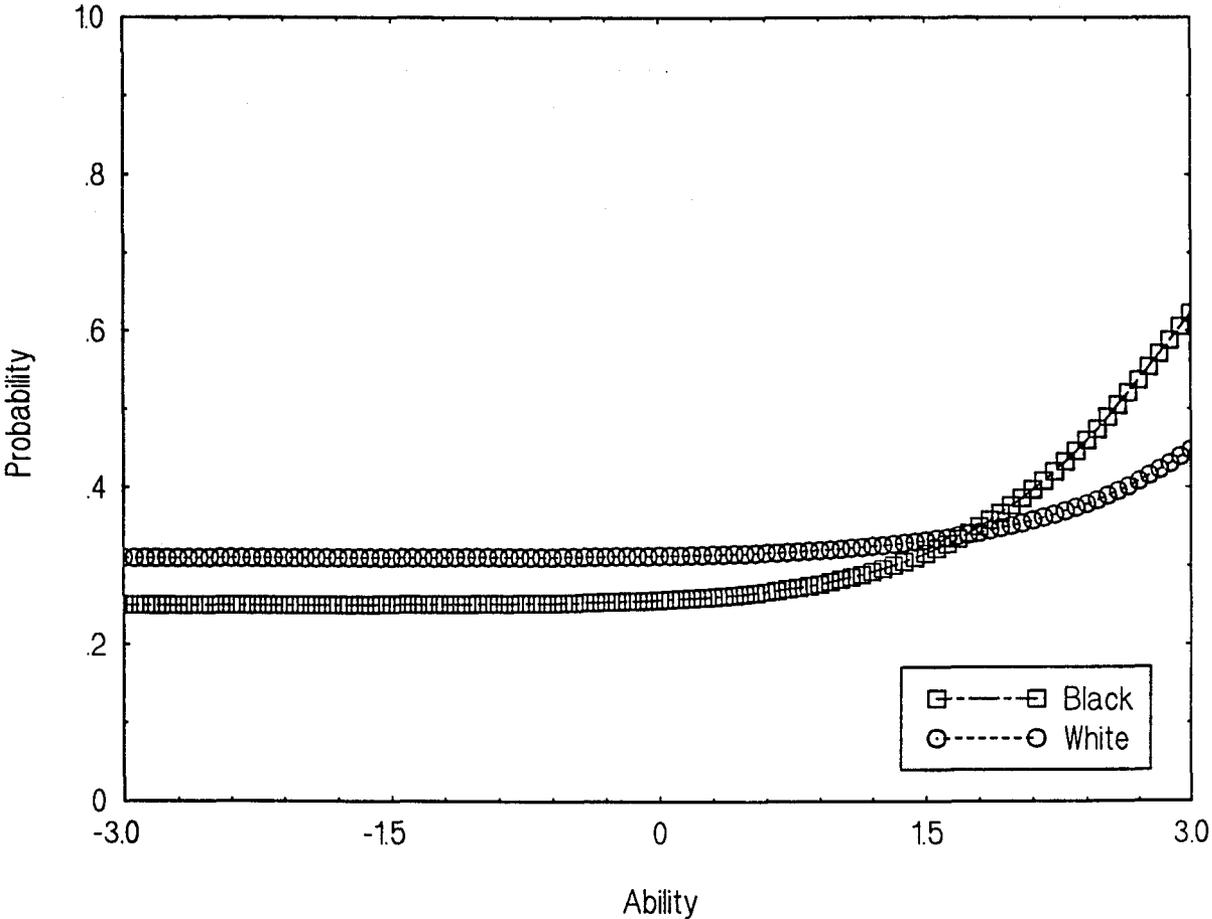
Black vs. White: Item 7-19



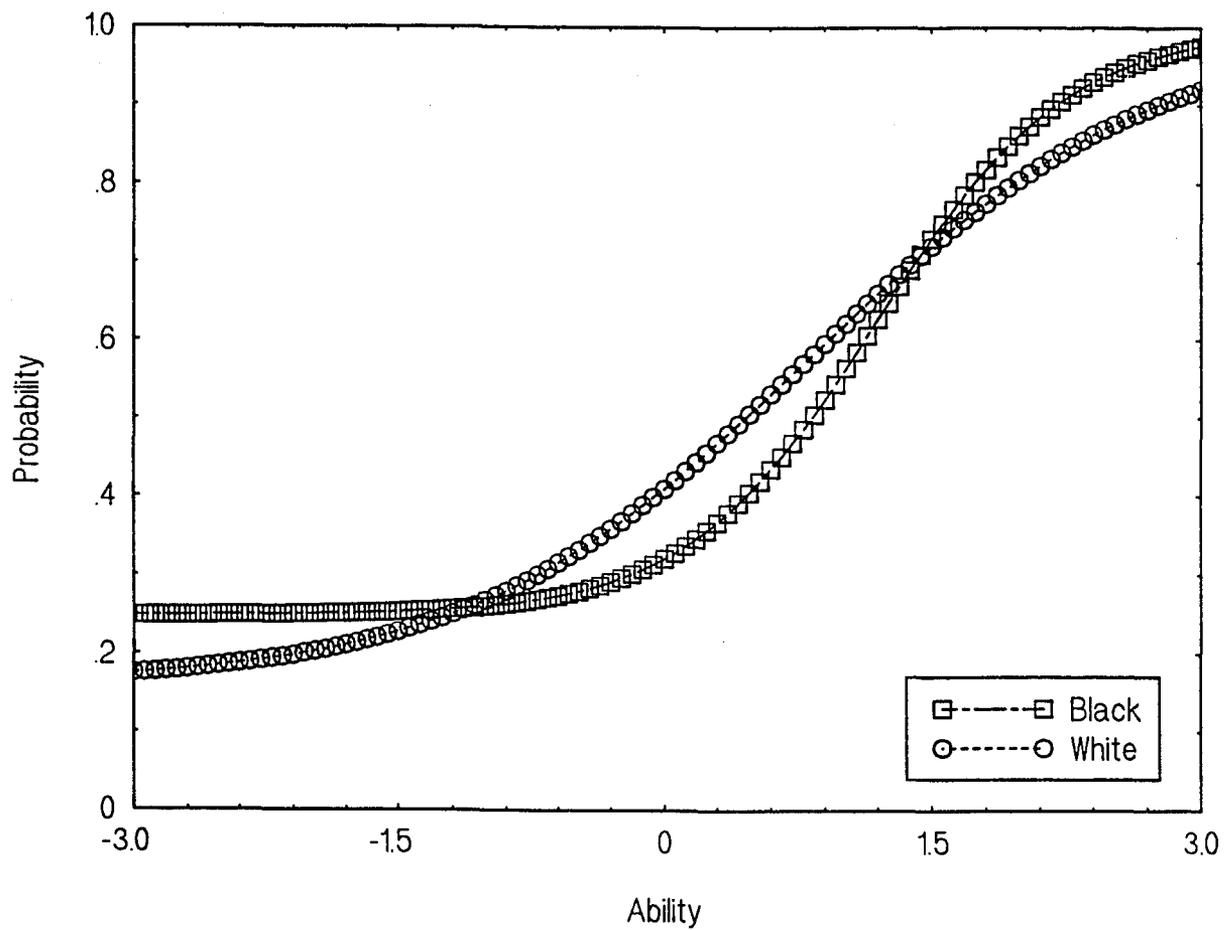
Black vs. White: Item 7-20



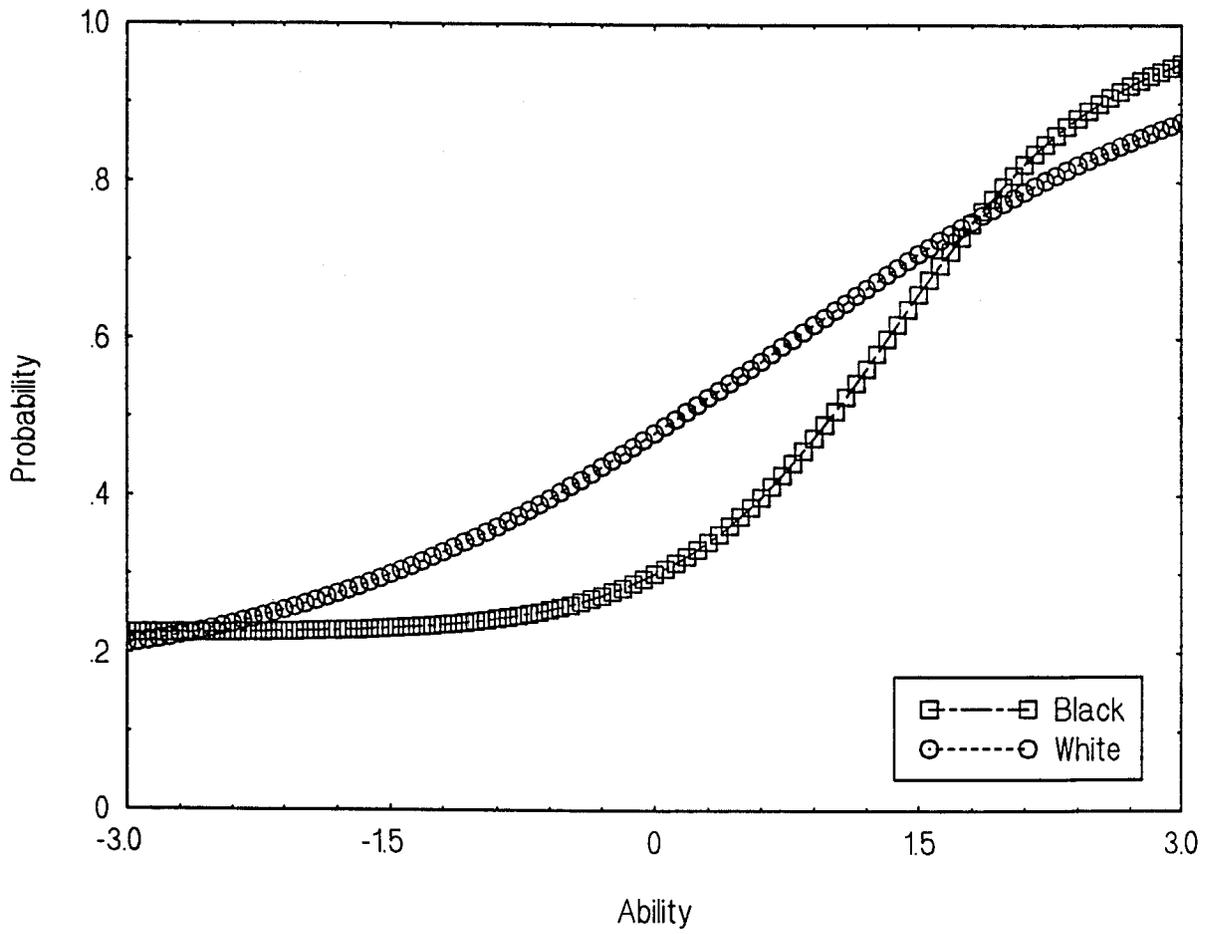
Black vs. White: Item 7-21



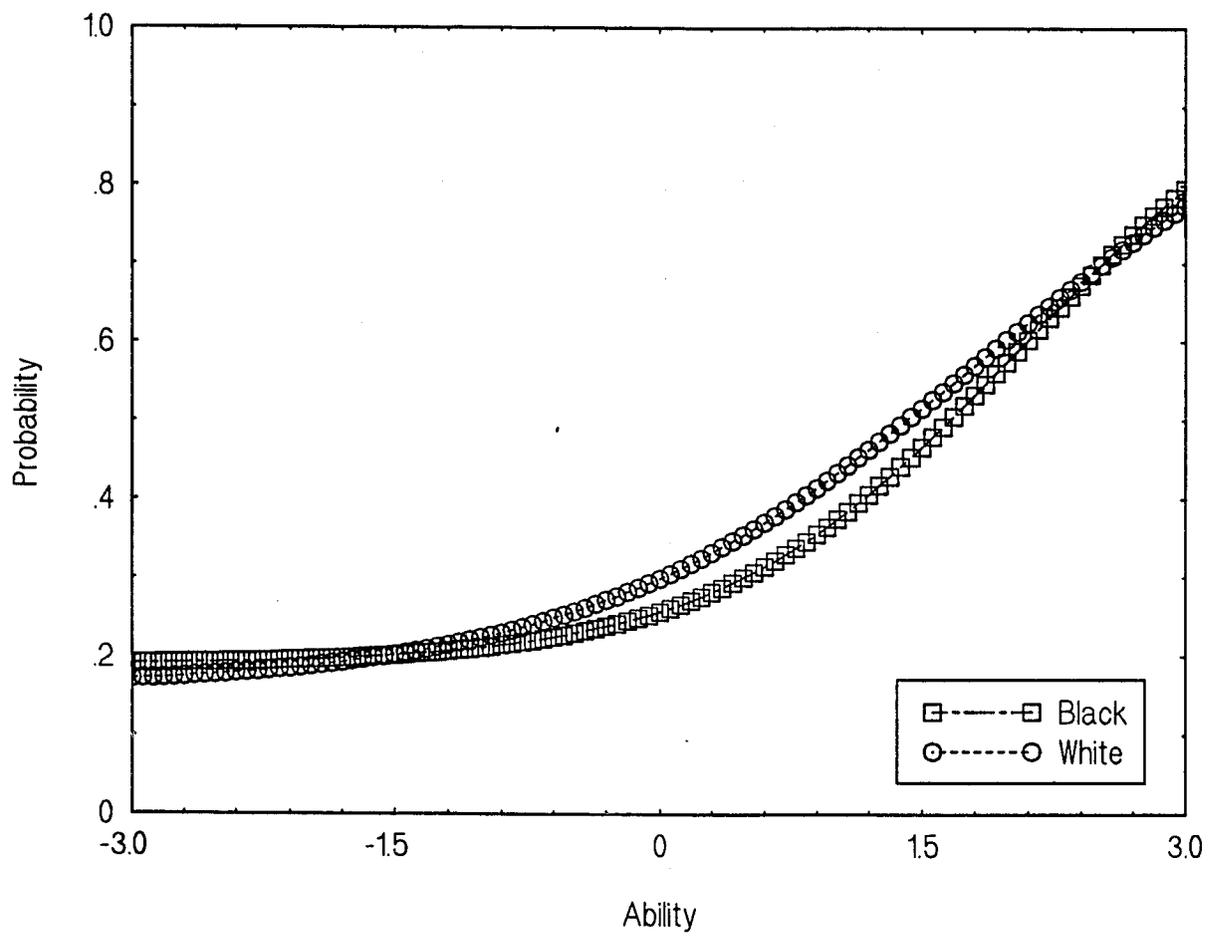
Black vs. White: Item 7-22



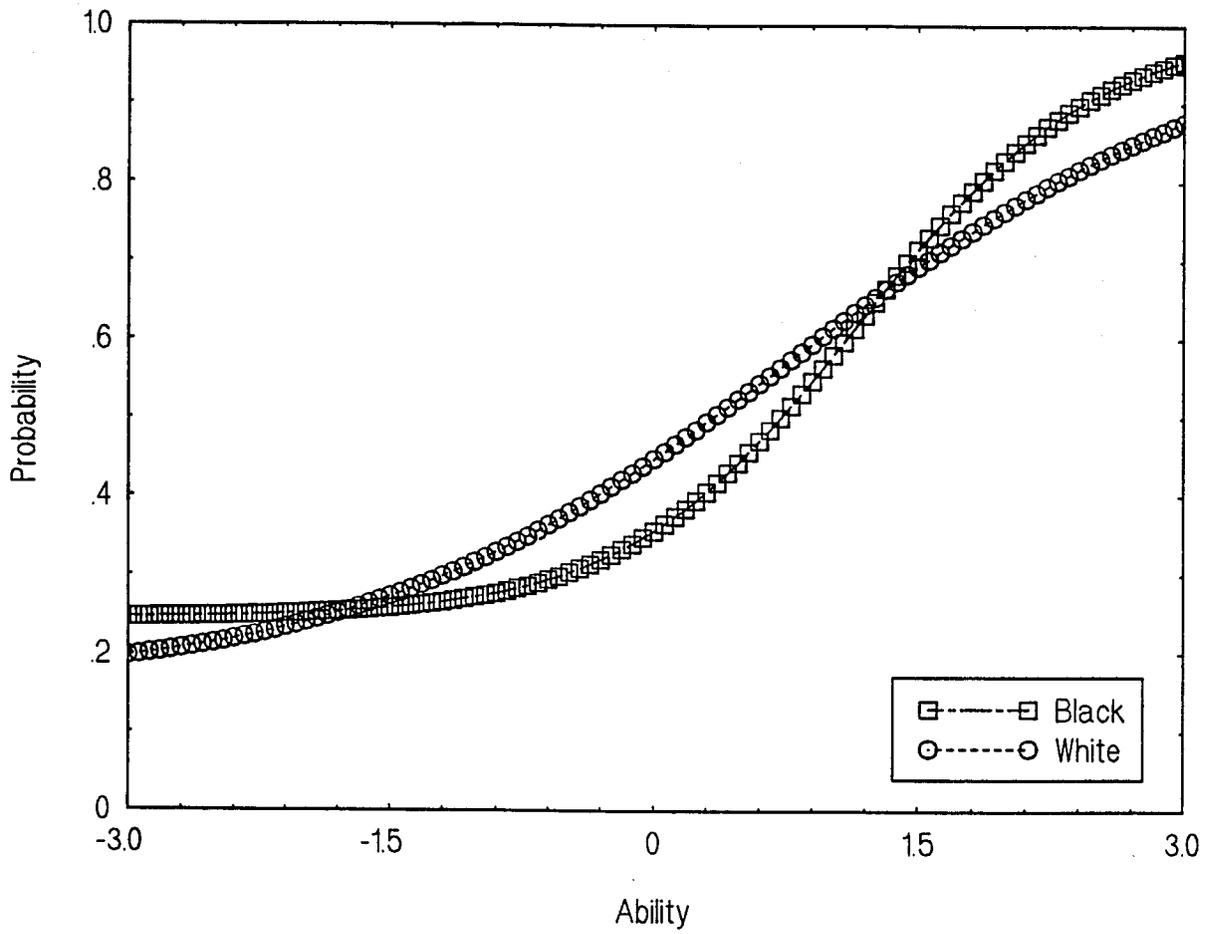
Black vs. White: Item 7-23



Black vs. White: Item 7-24



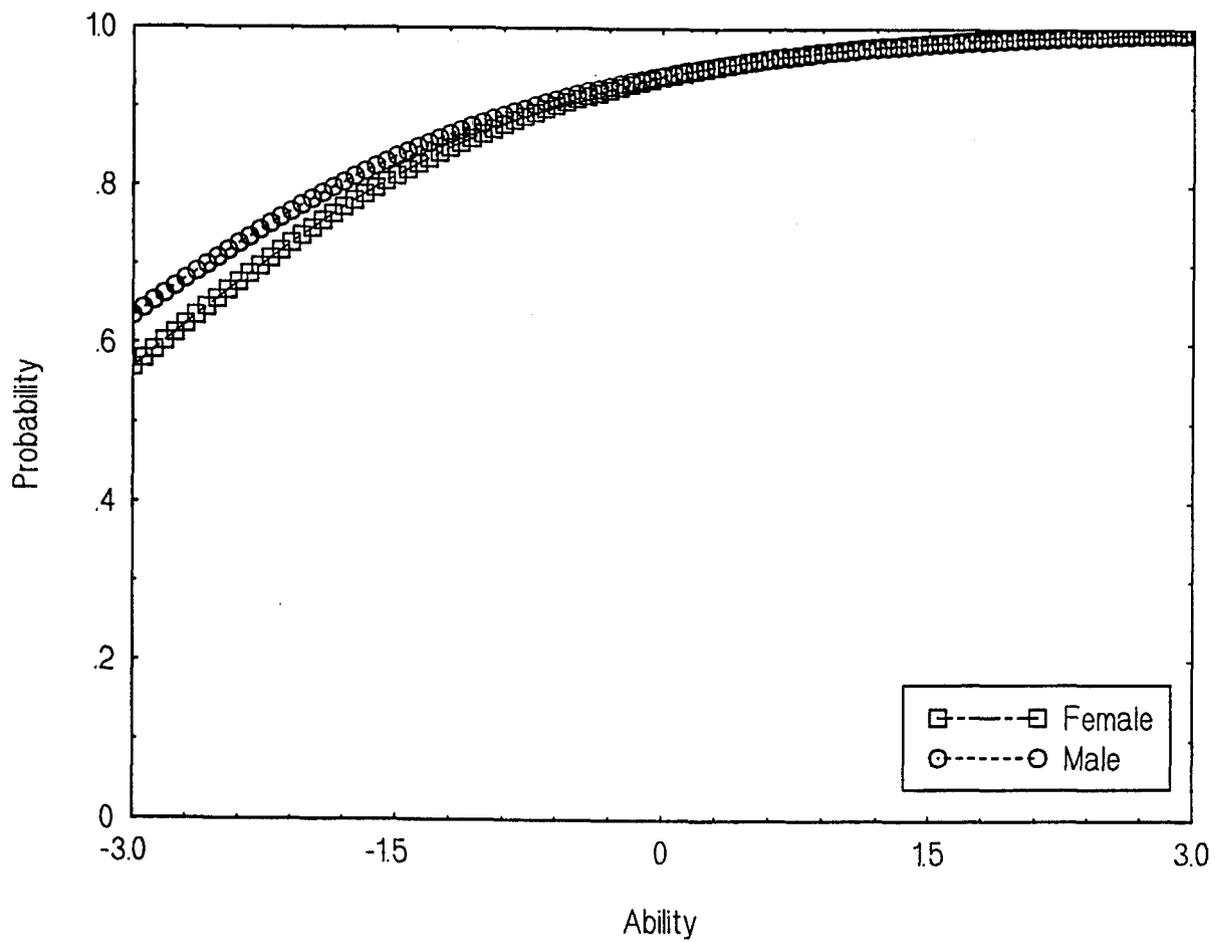
Black vs. White: Item 7-25



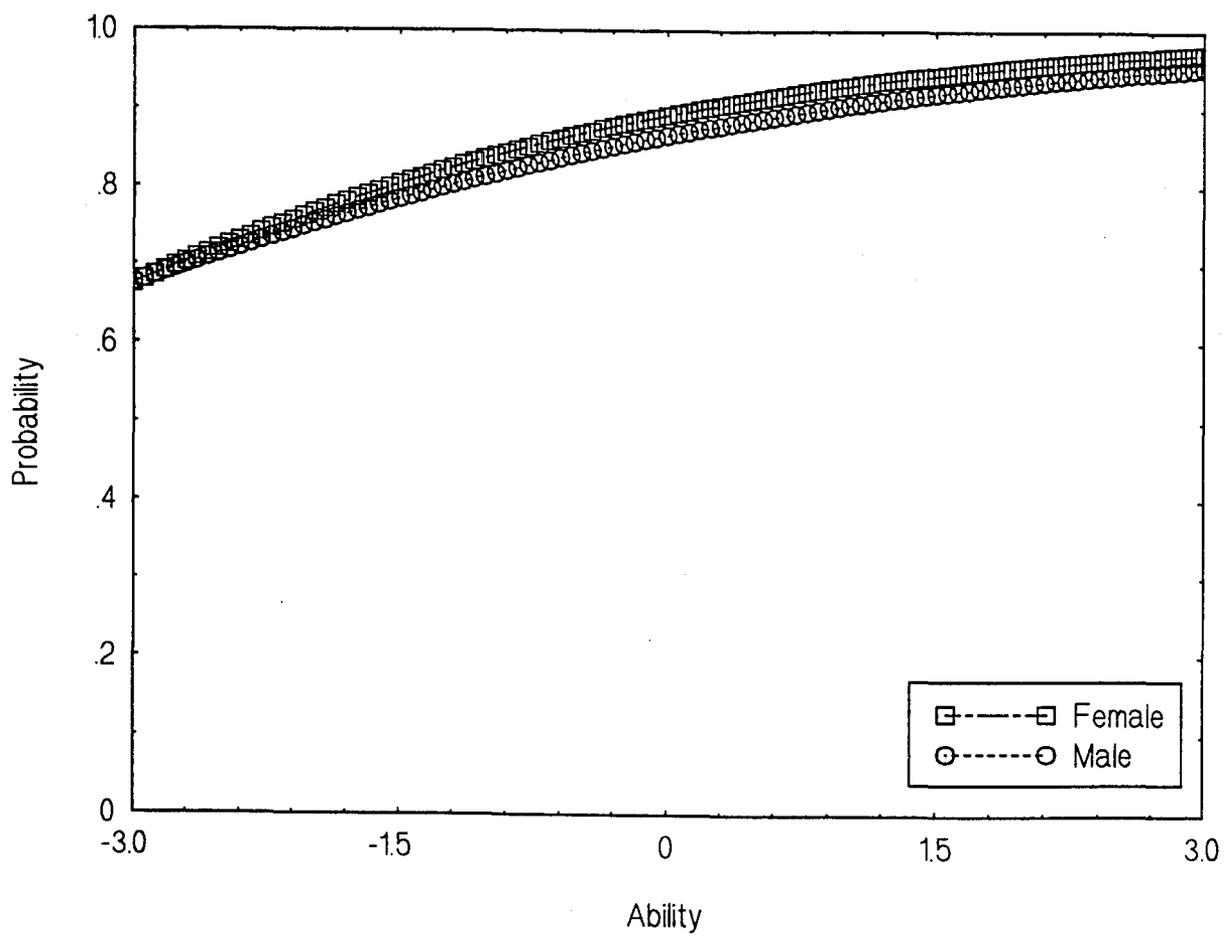
APPENDIX F

TRACELINES FOR MALE AND FEMALE WHITE EXAMINEES

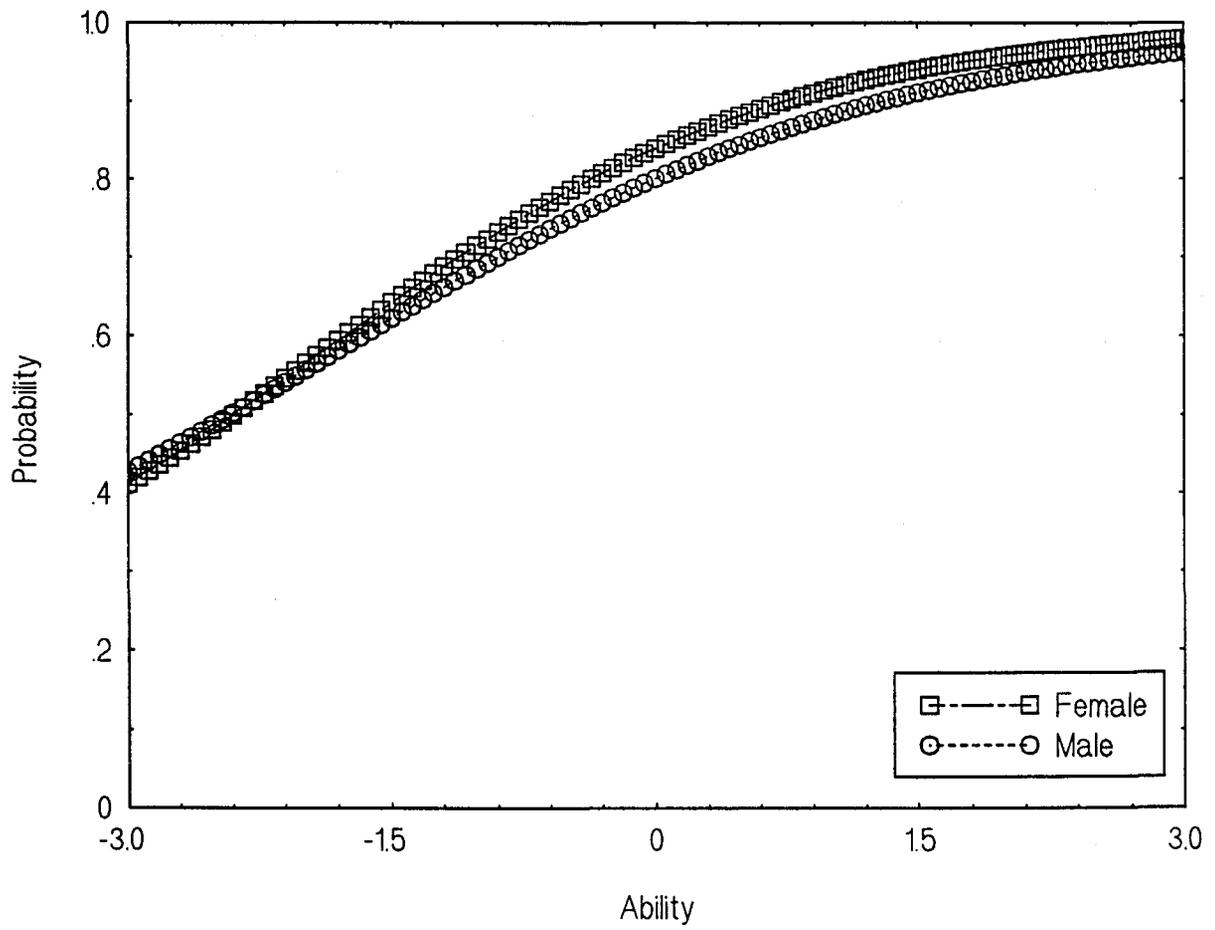
Male vs. Female: Item 2-1



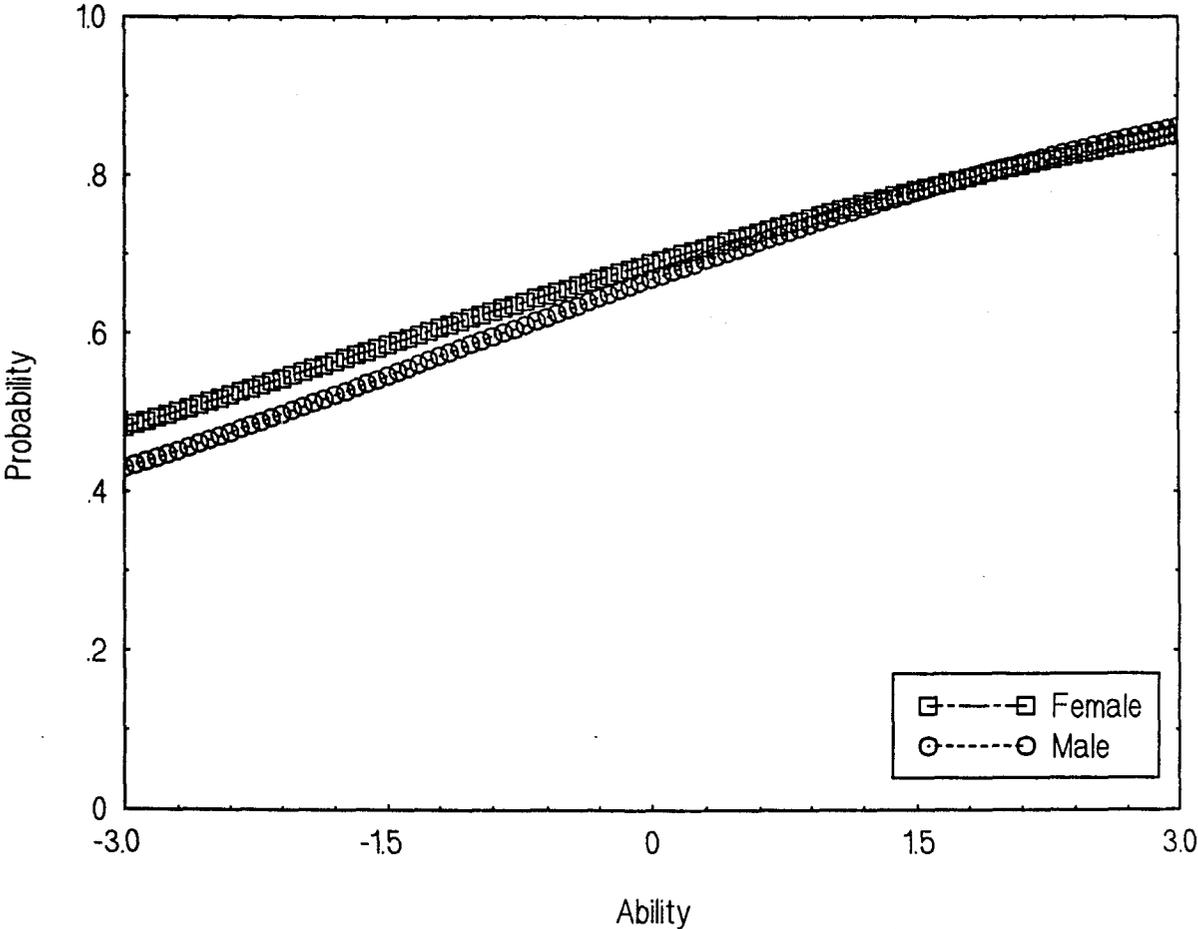
Male vs. Female: Item 2-2



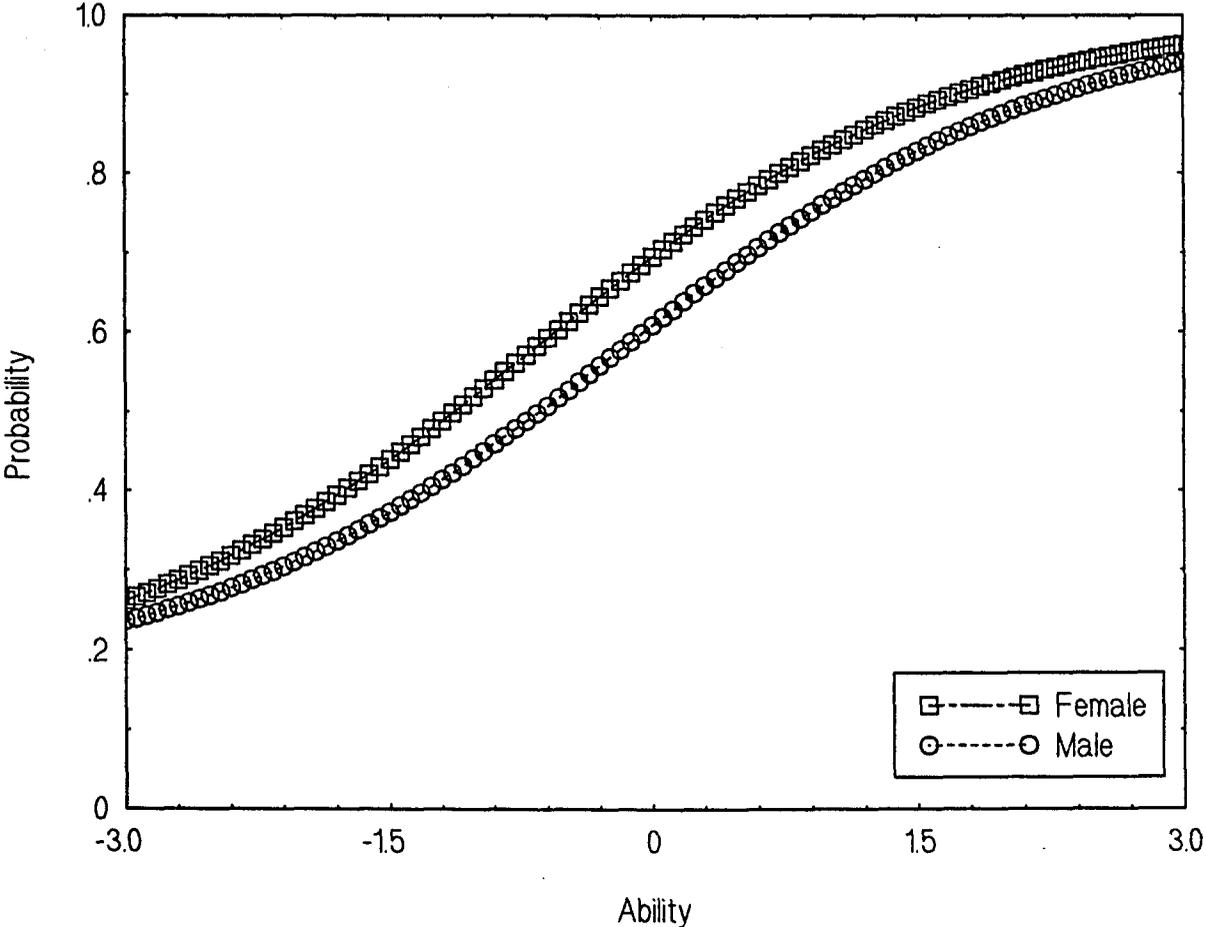
Male vs. Female: Item 2-3



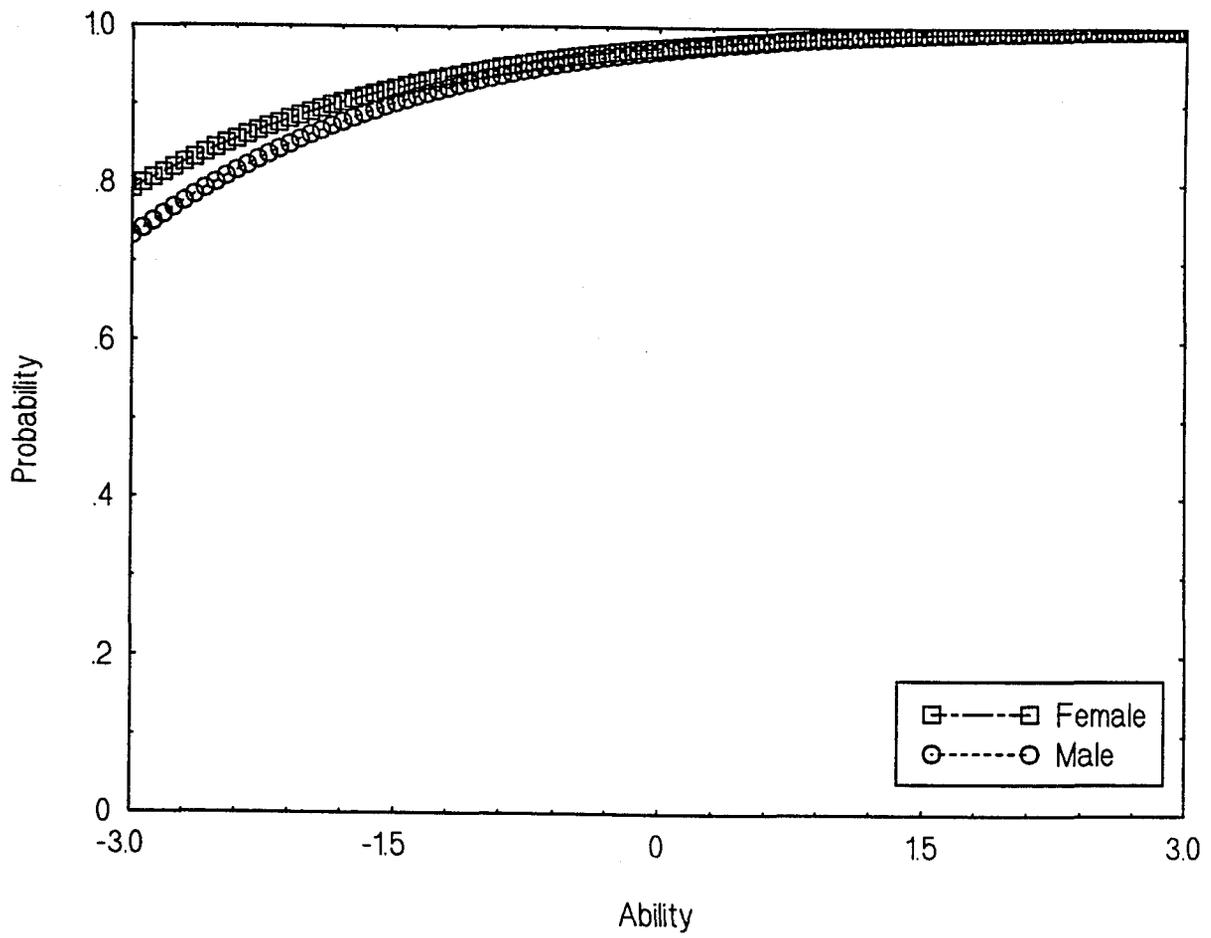
Male vs. Female: Item 2-4



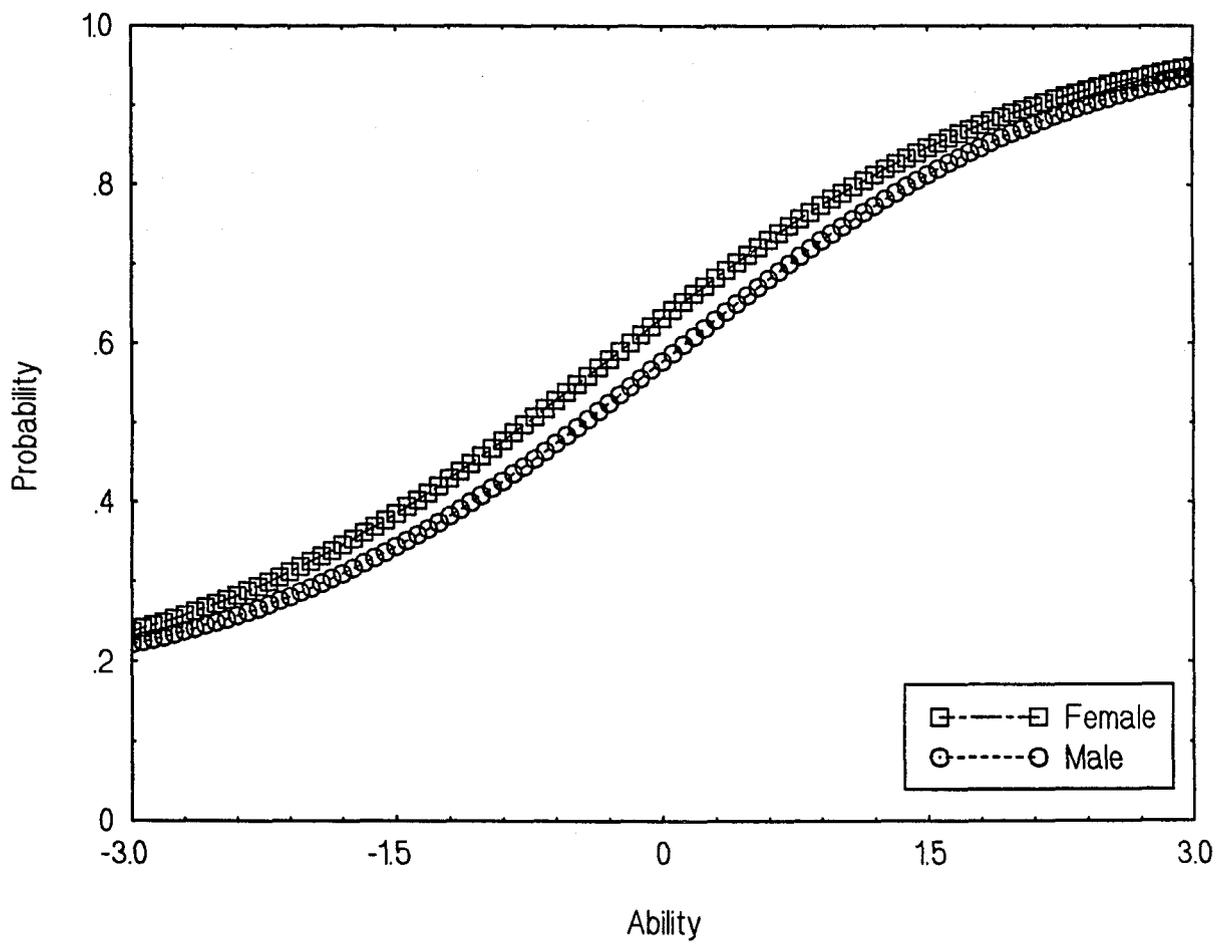
Male vs. Female: Item 2-5



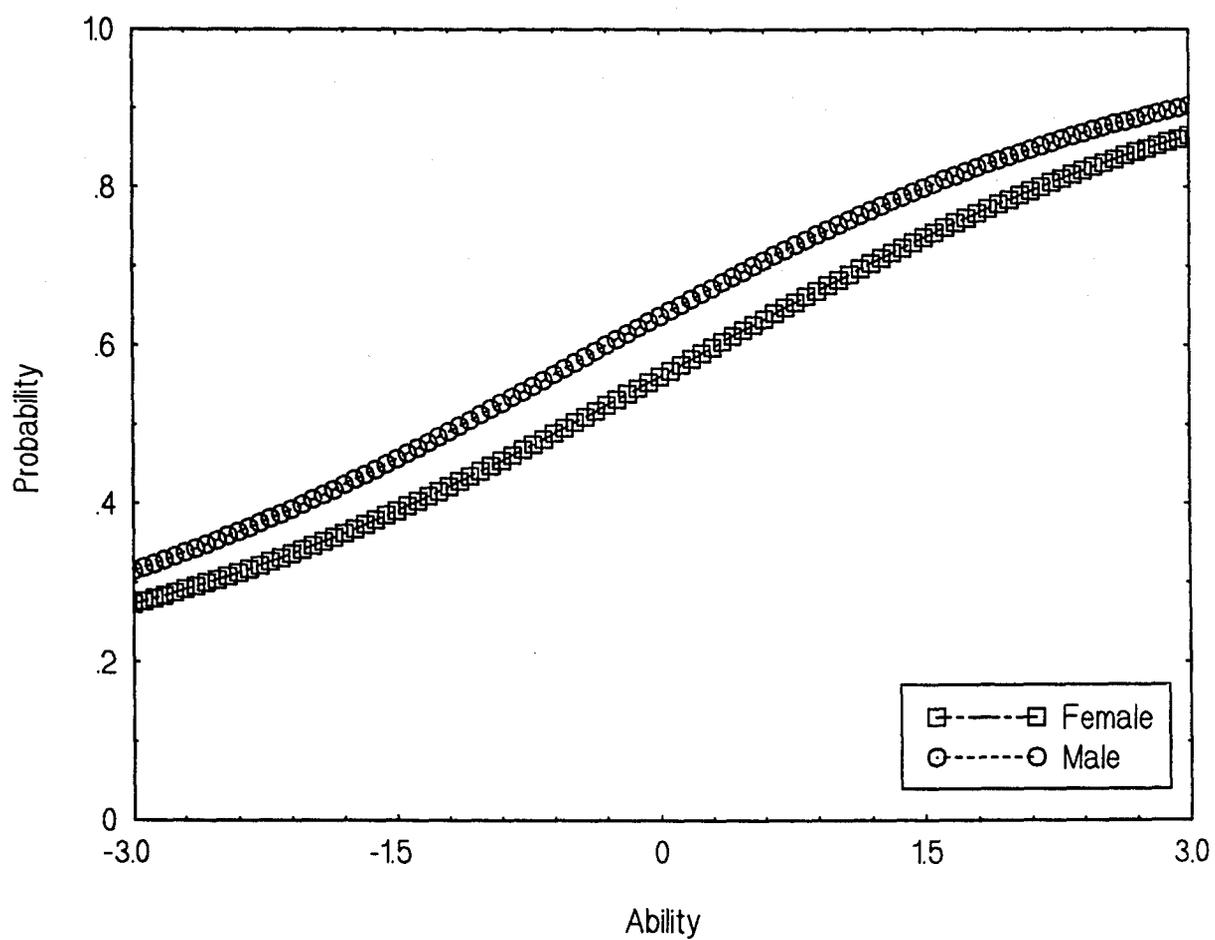
Male vs. Female: Item 2-6



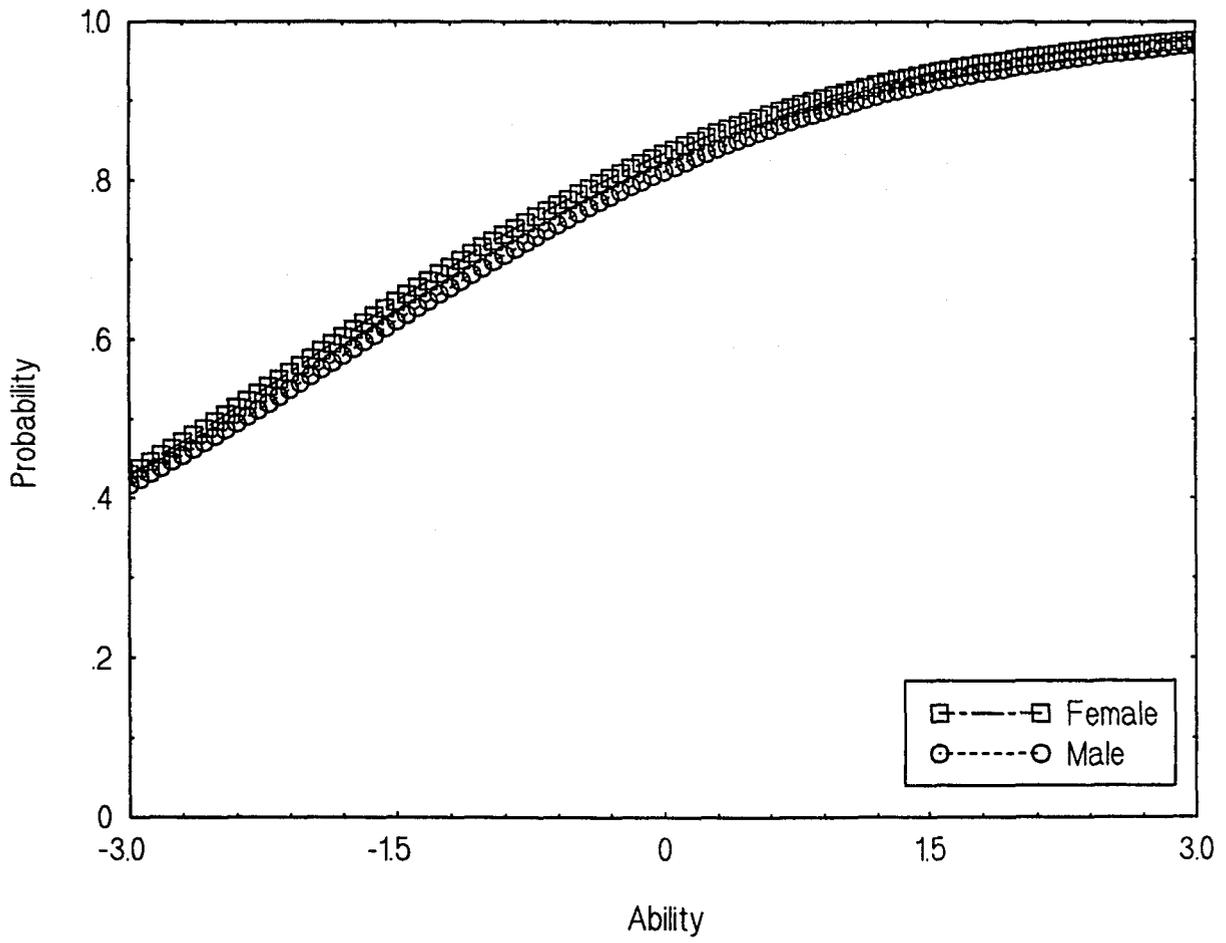
Male vs. Female: Item 2-7



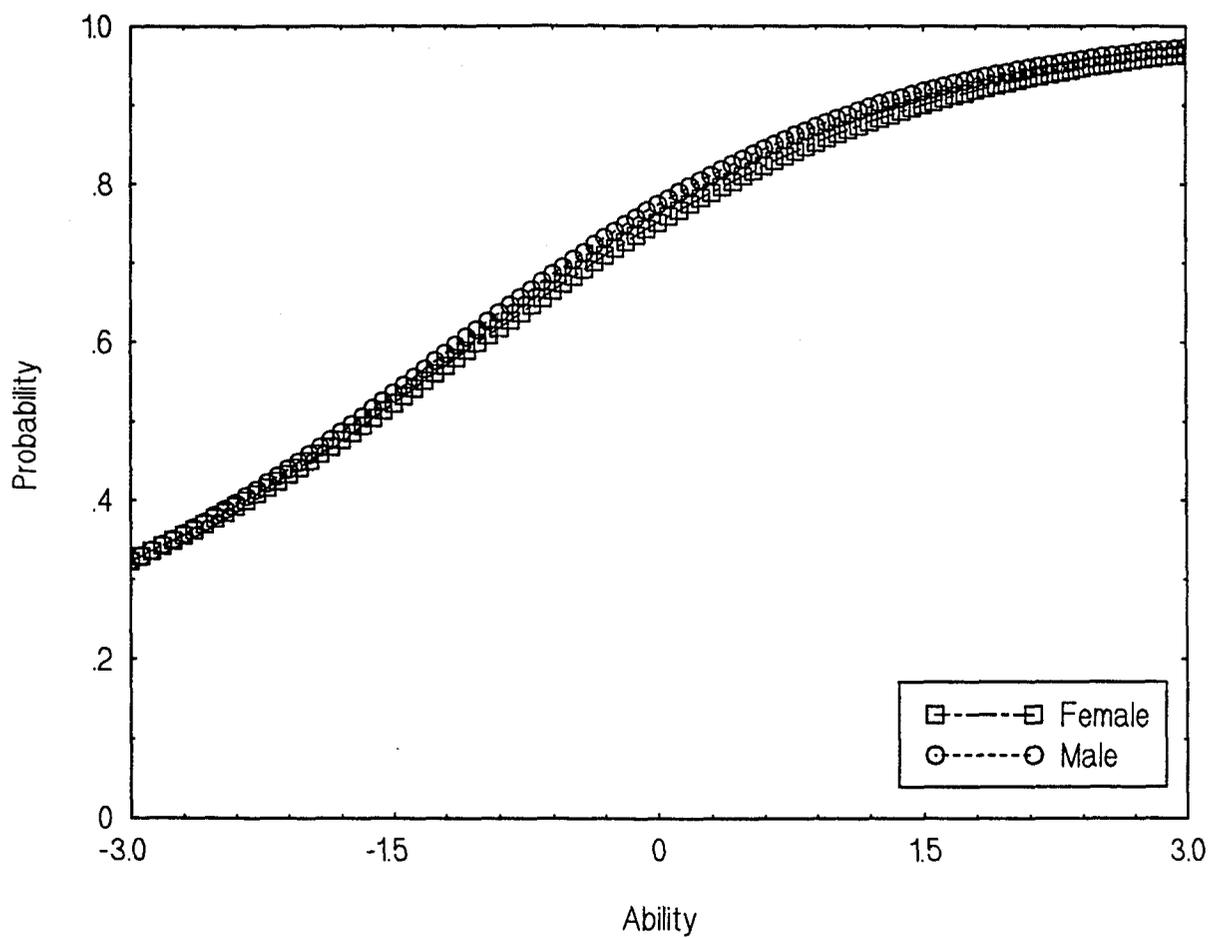
Male vs. Female: Item 2-8



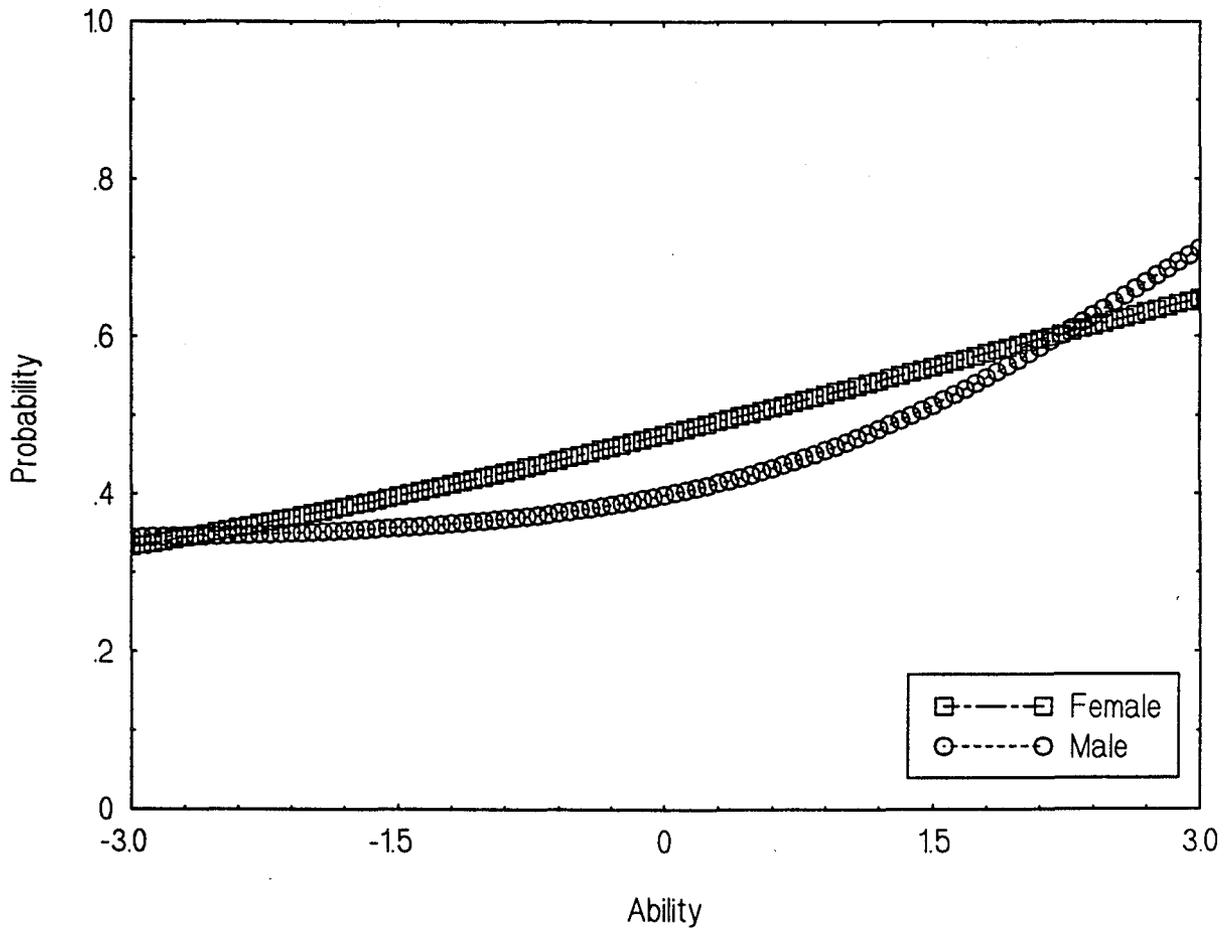
Male vs. Female: Item 2-9



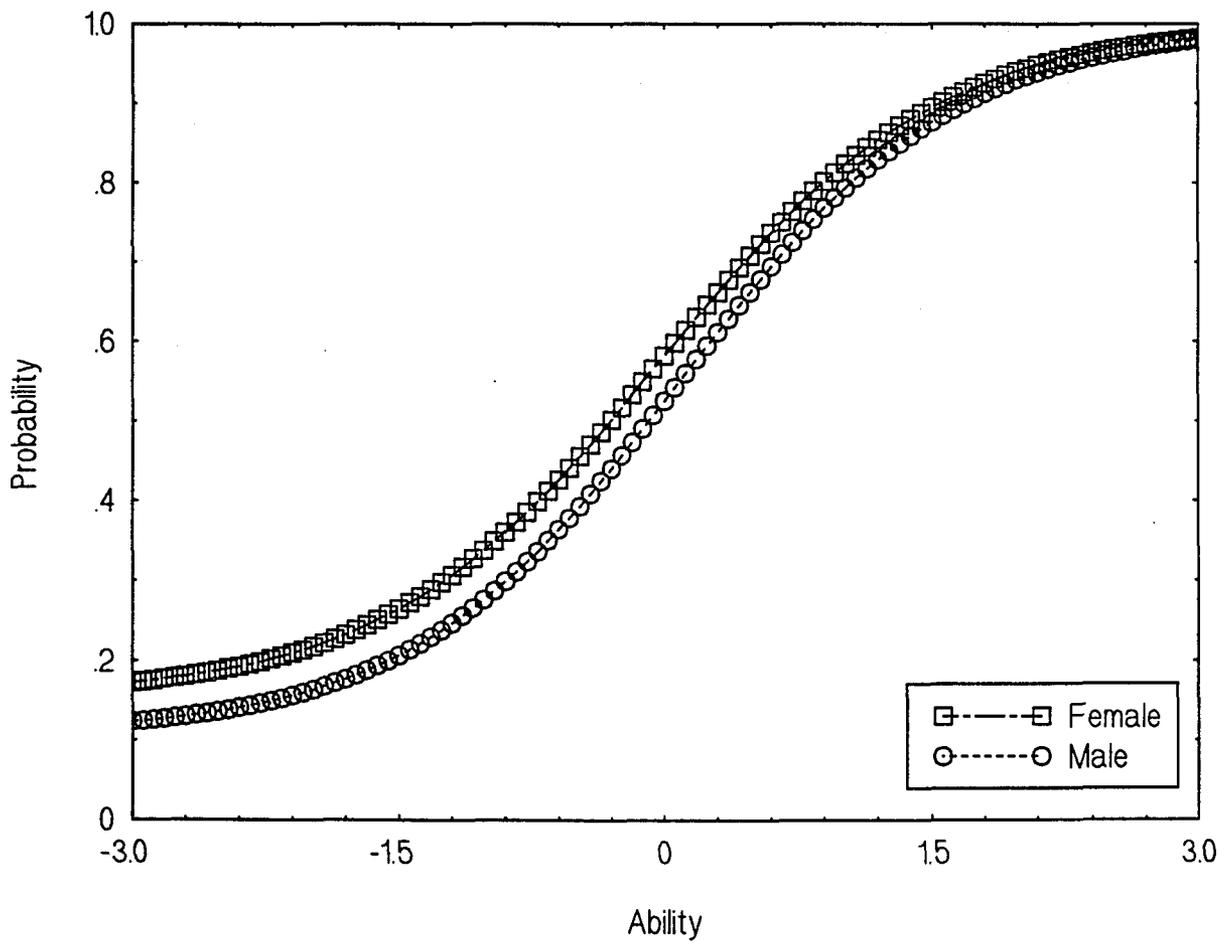
Male vs. Female: Item 2-10



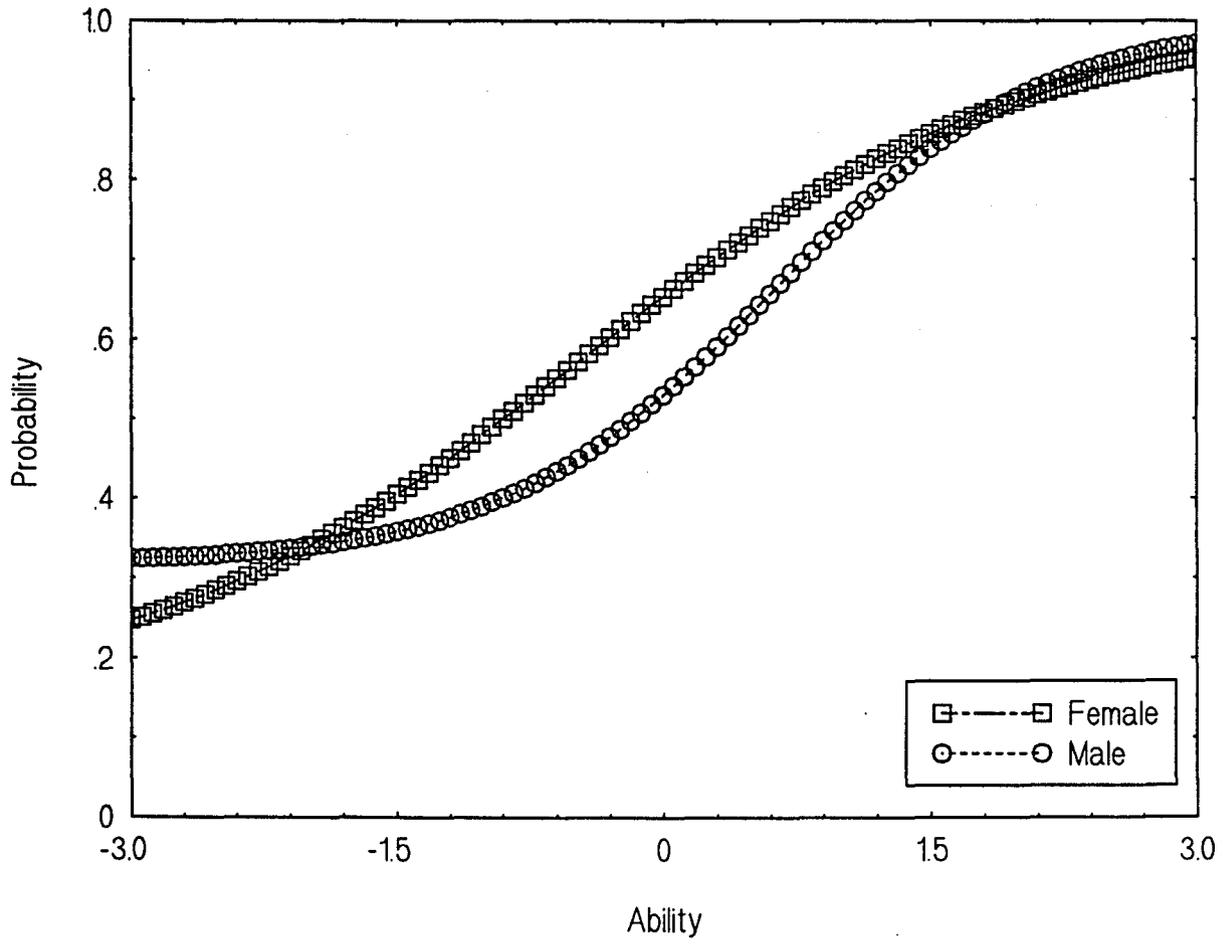
Male vs. Female: Item 2-11



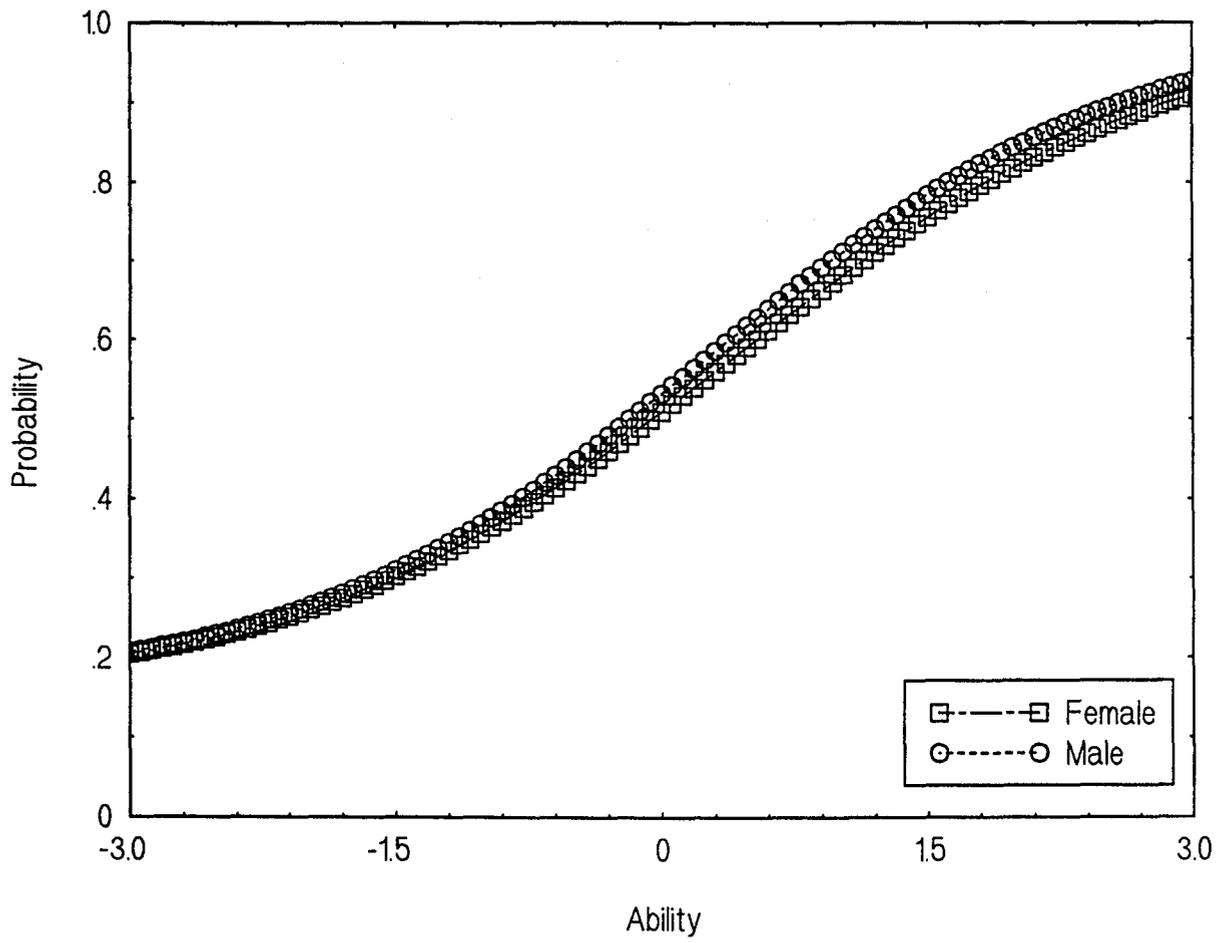
Male vs. Female: Item 2-12



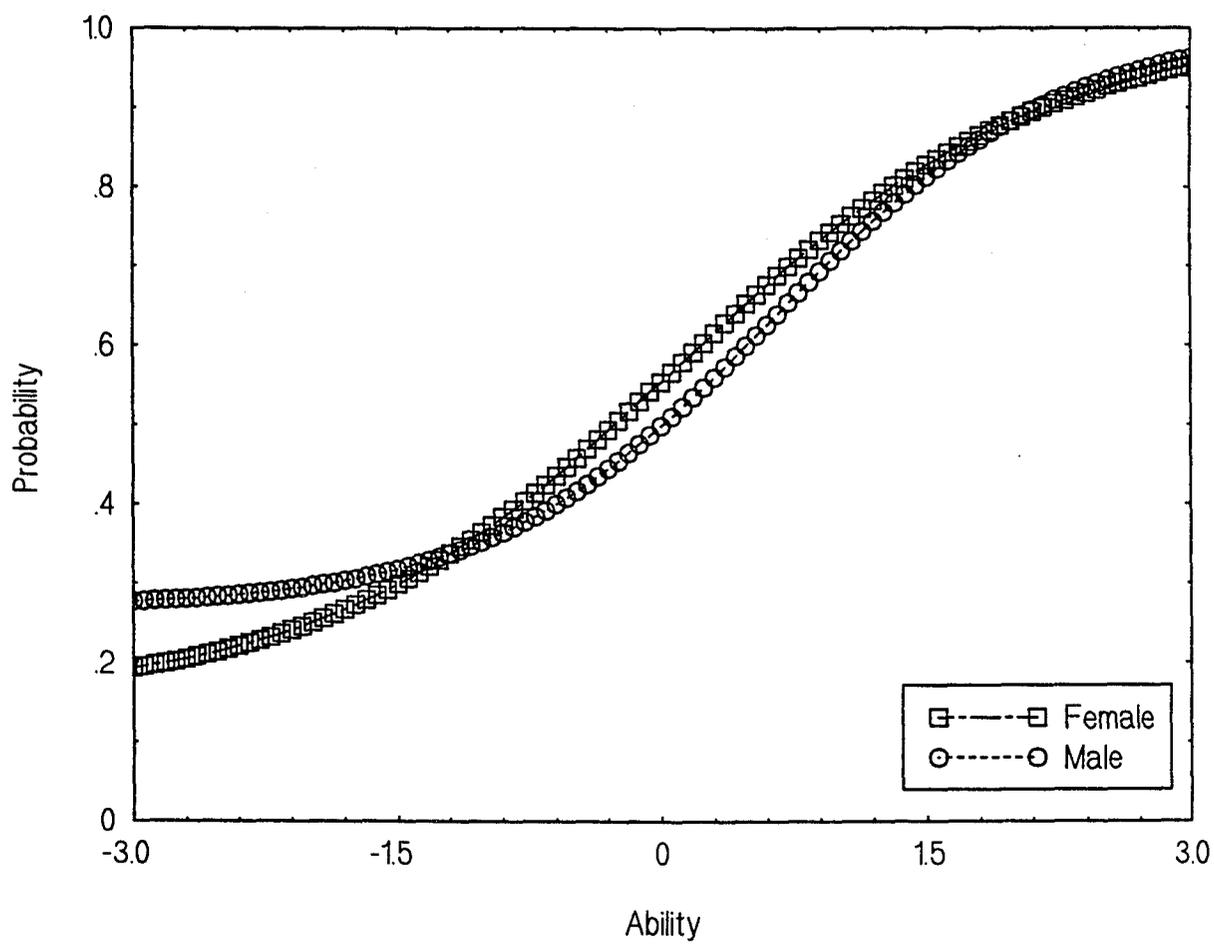
Male vs. Female: Item 2-13



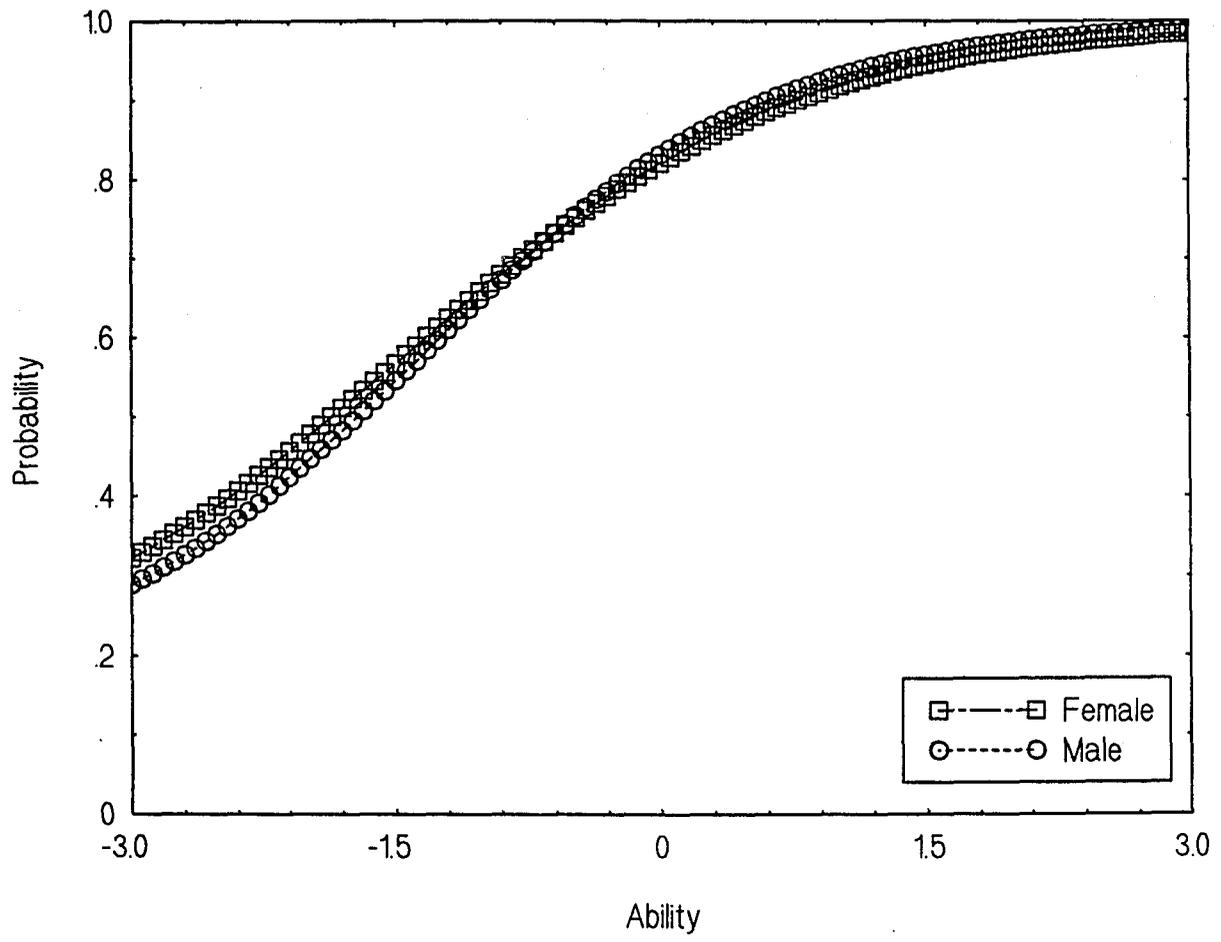
Male vs. Female: Item 2-14



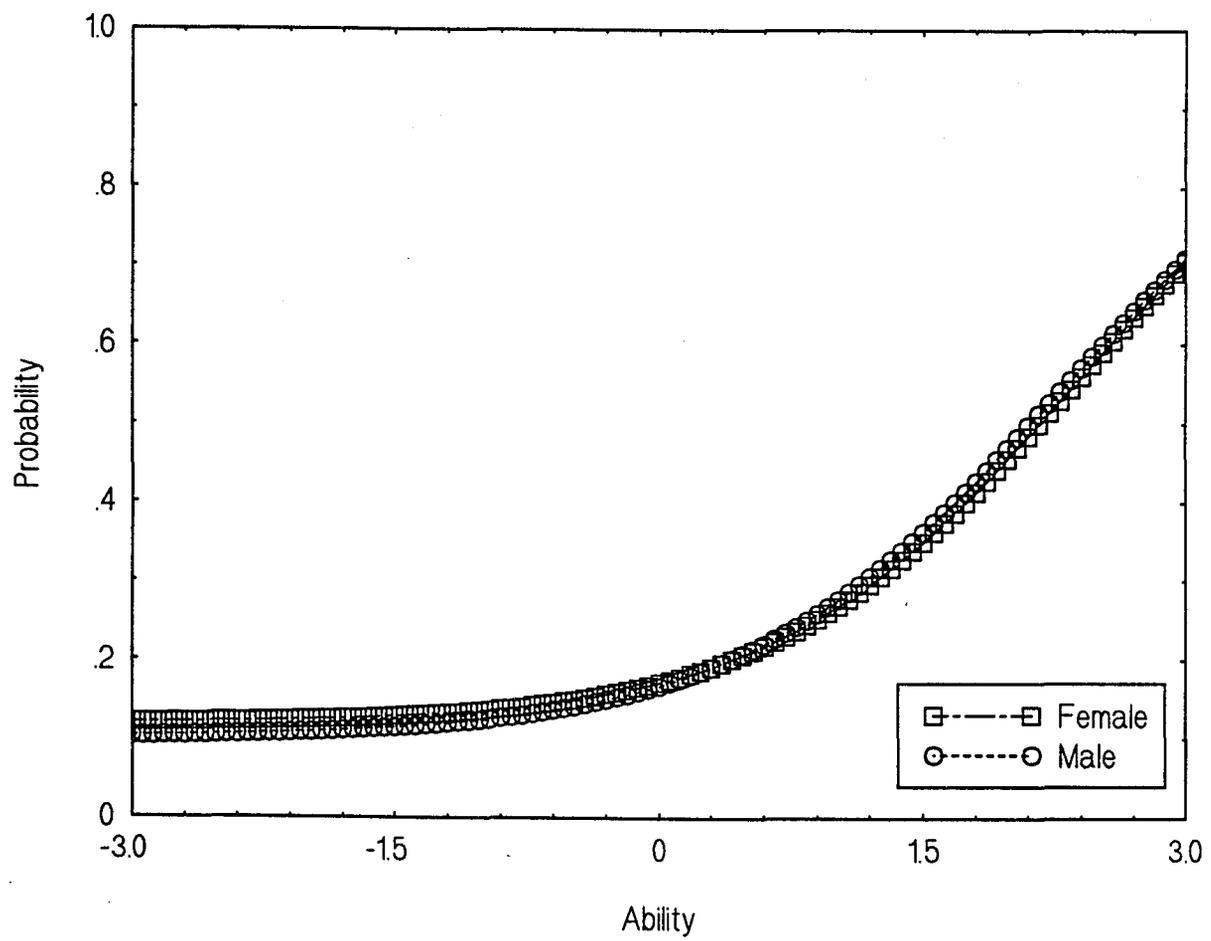
Male vs. Female: Item 2-15



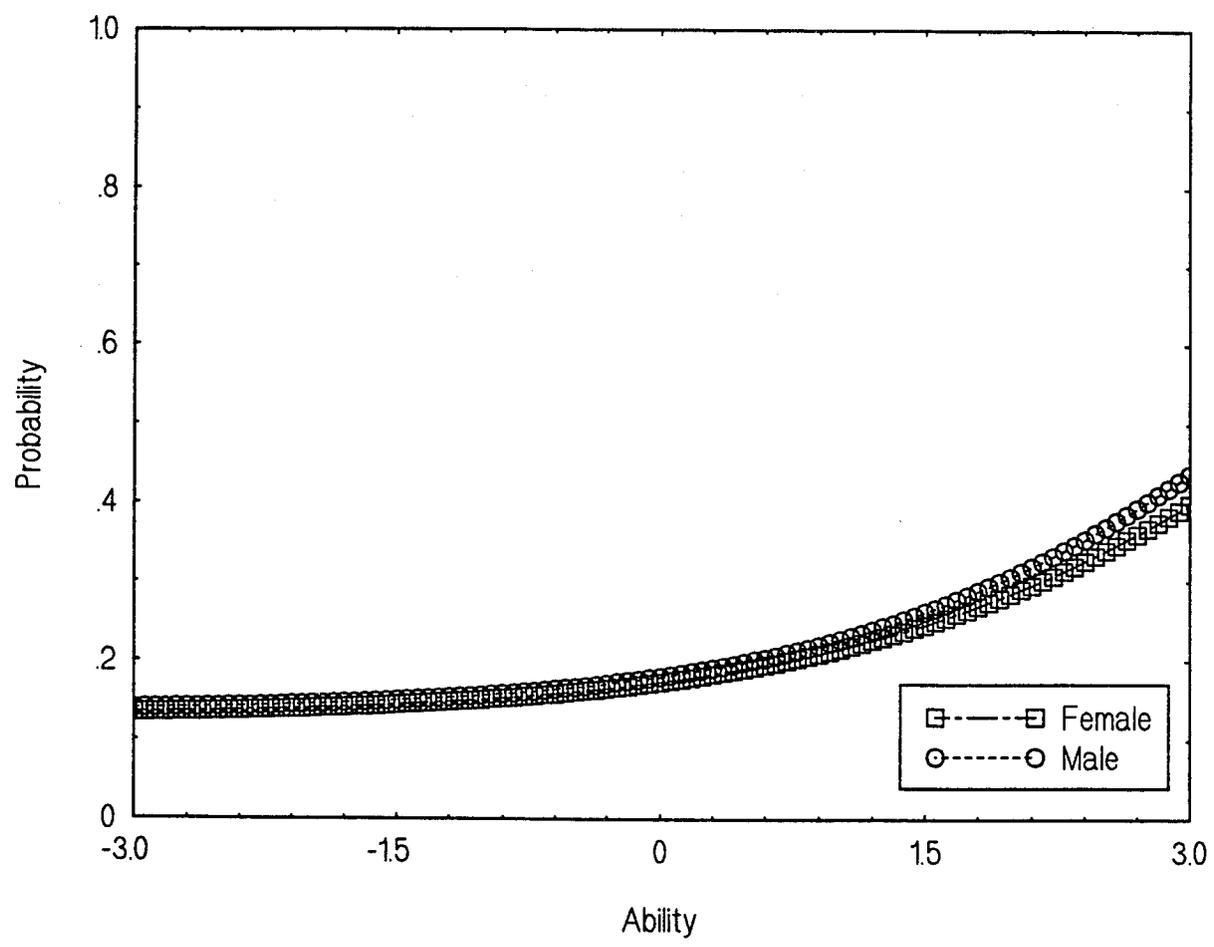
Male vs. Female: Item 2-16



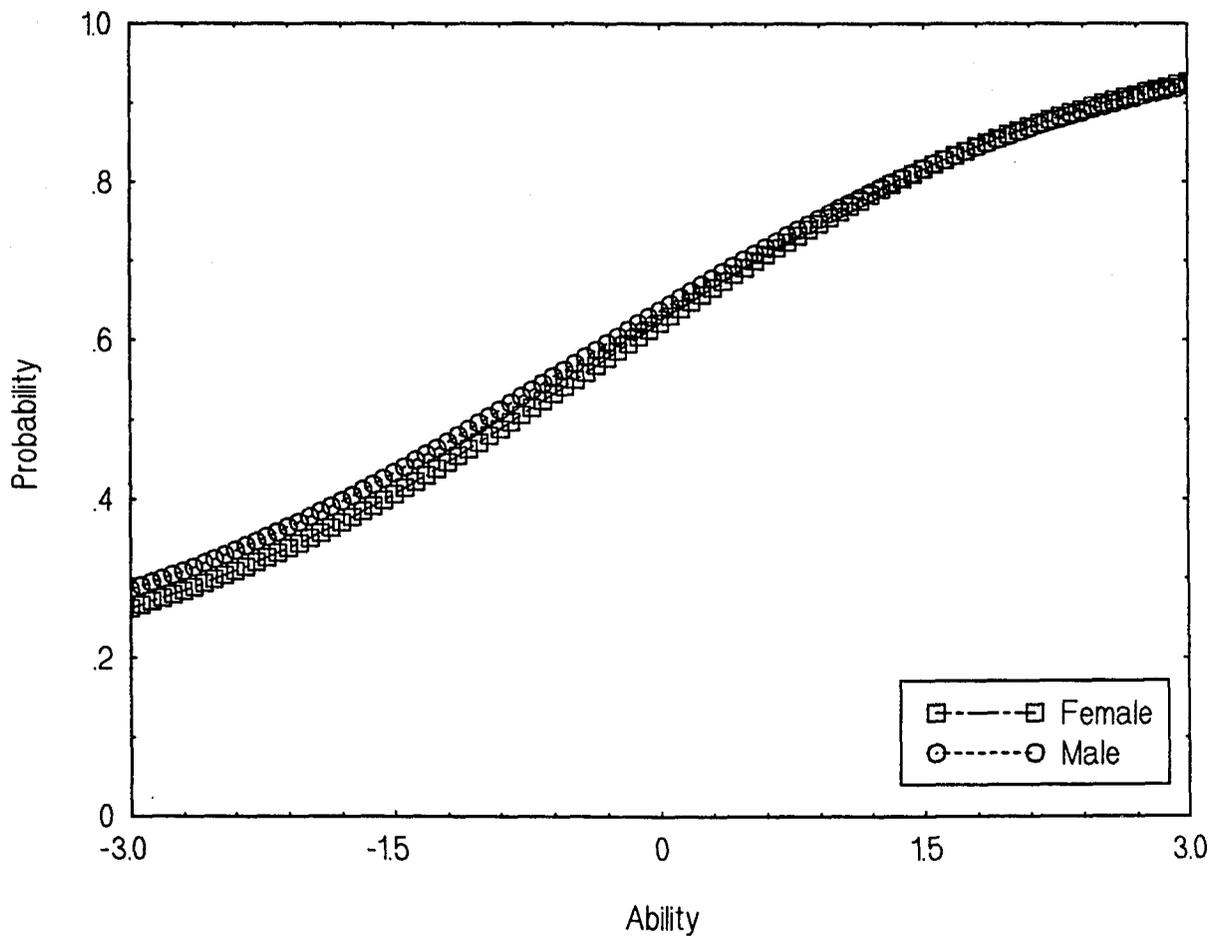
Male vs. Female: Item 2-17



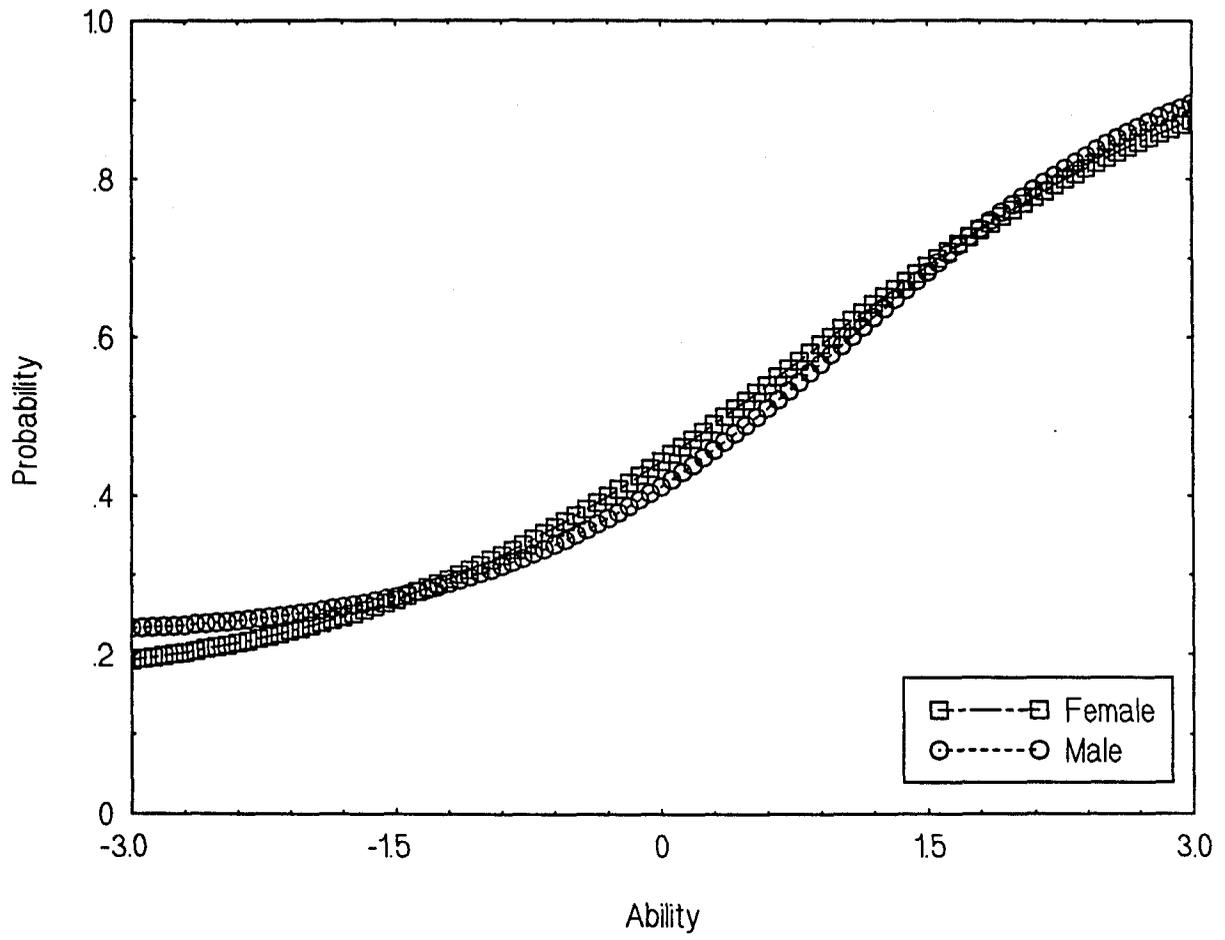
Male vs. Female: Item 2-18



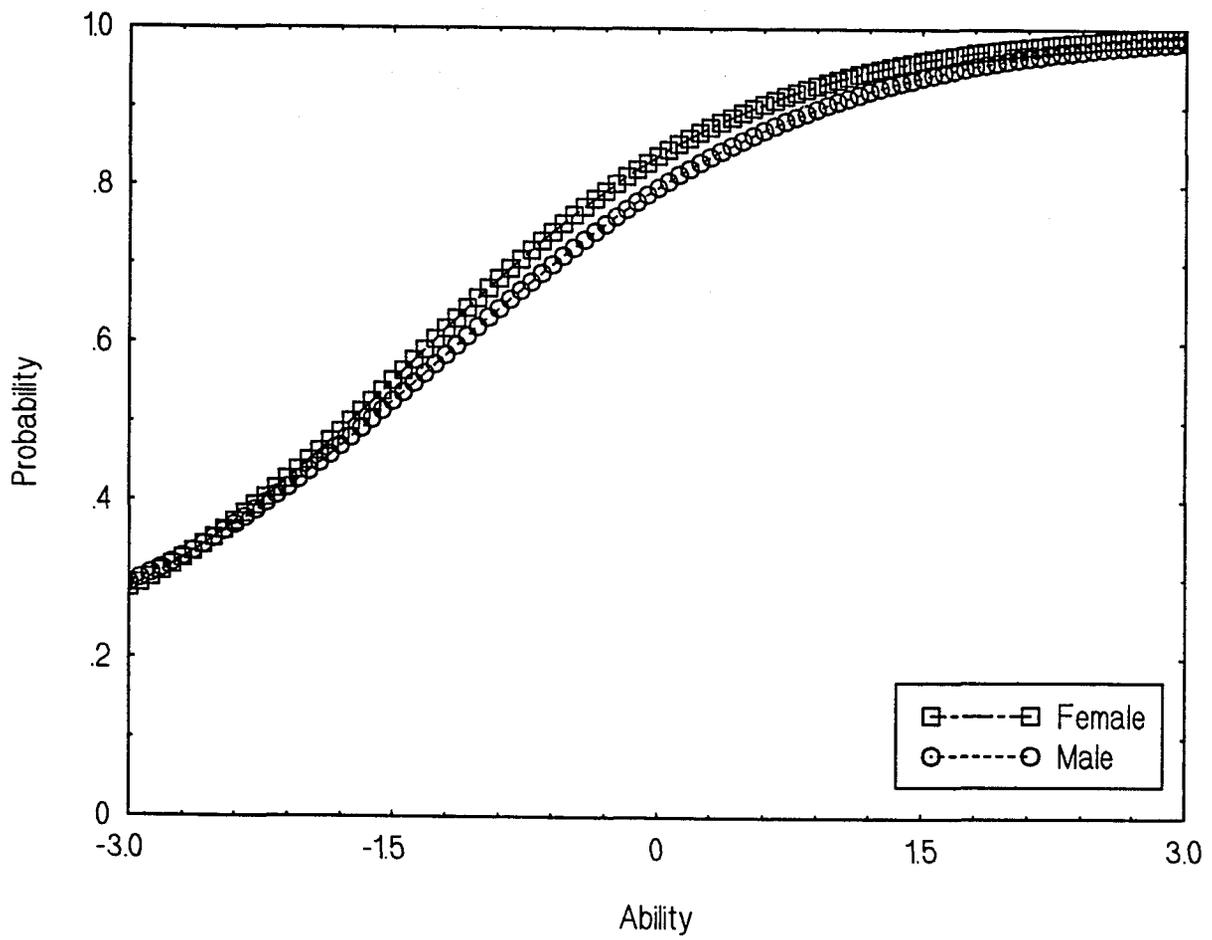
Male vs. Female: Item 2-19



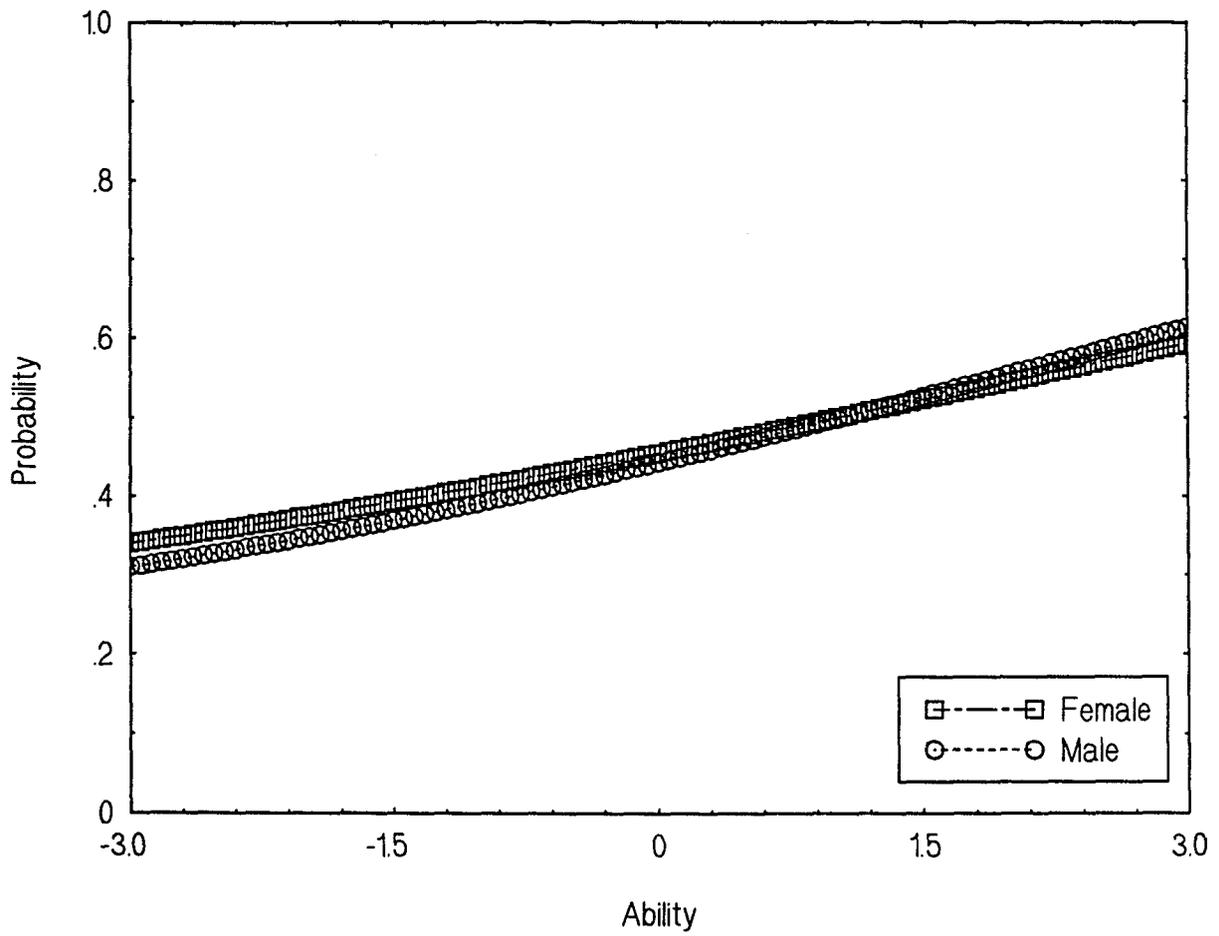
Male vs. Female: Item 2-20



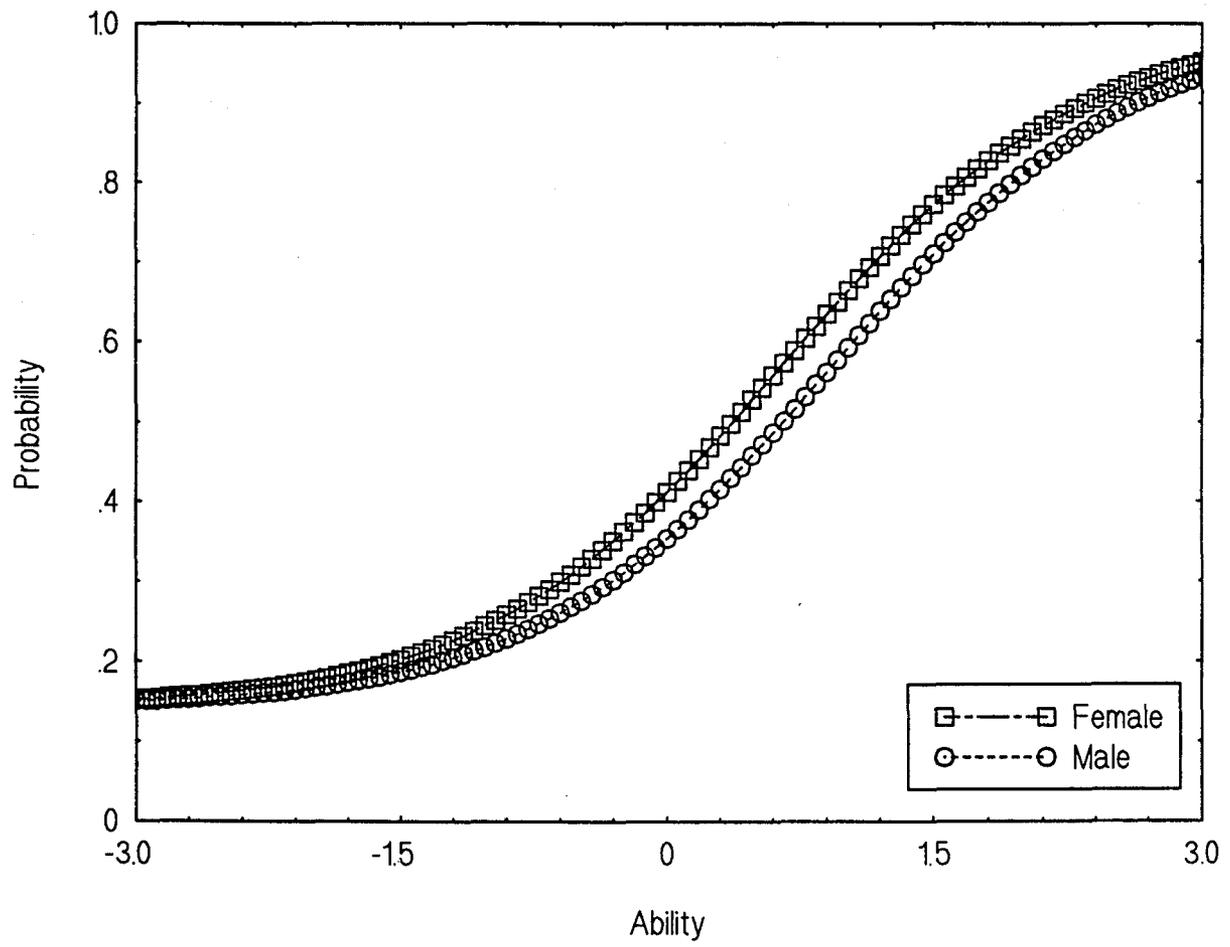
Male vs. Female: Item 2-21



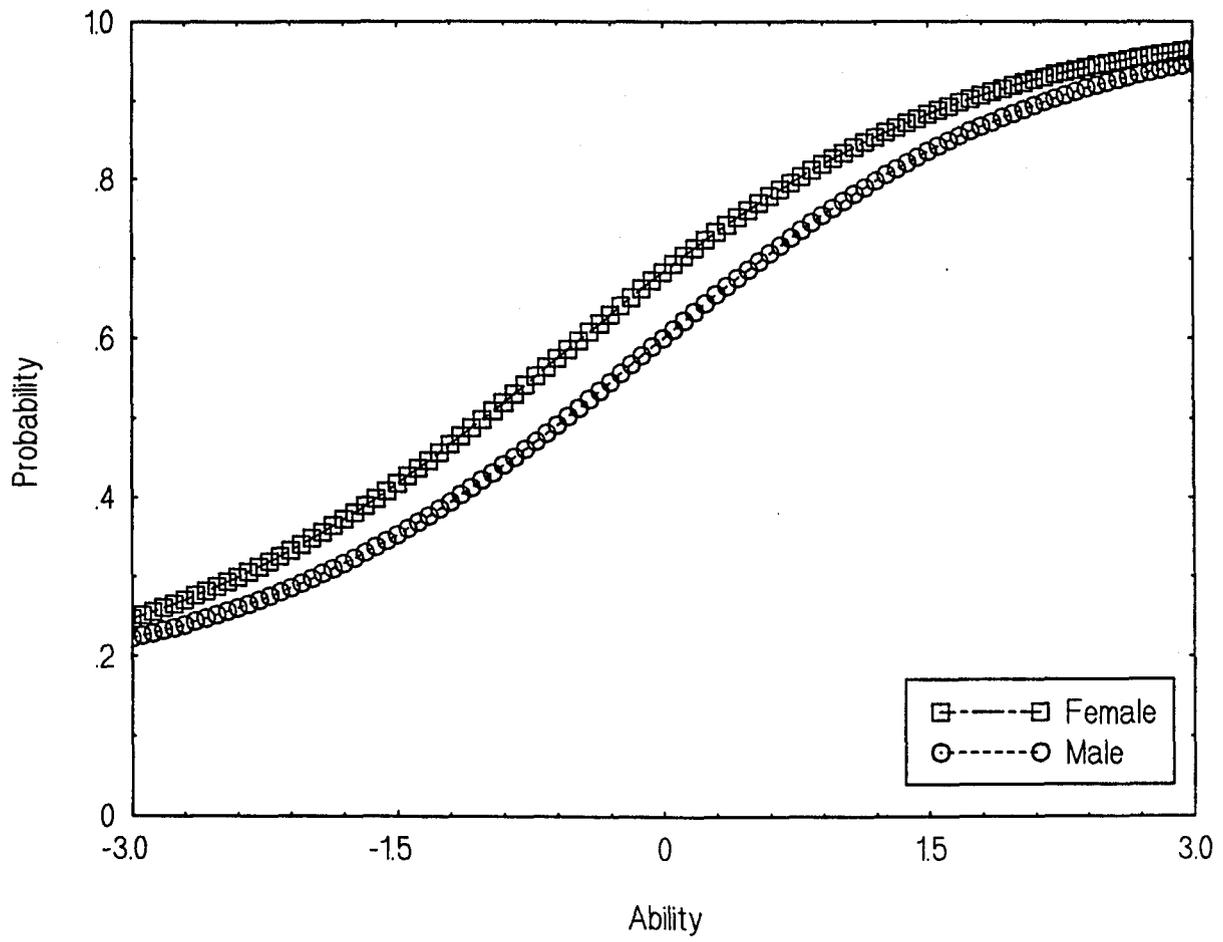
Male vs. Female: Item 2-22



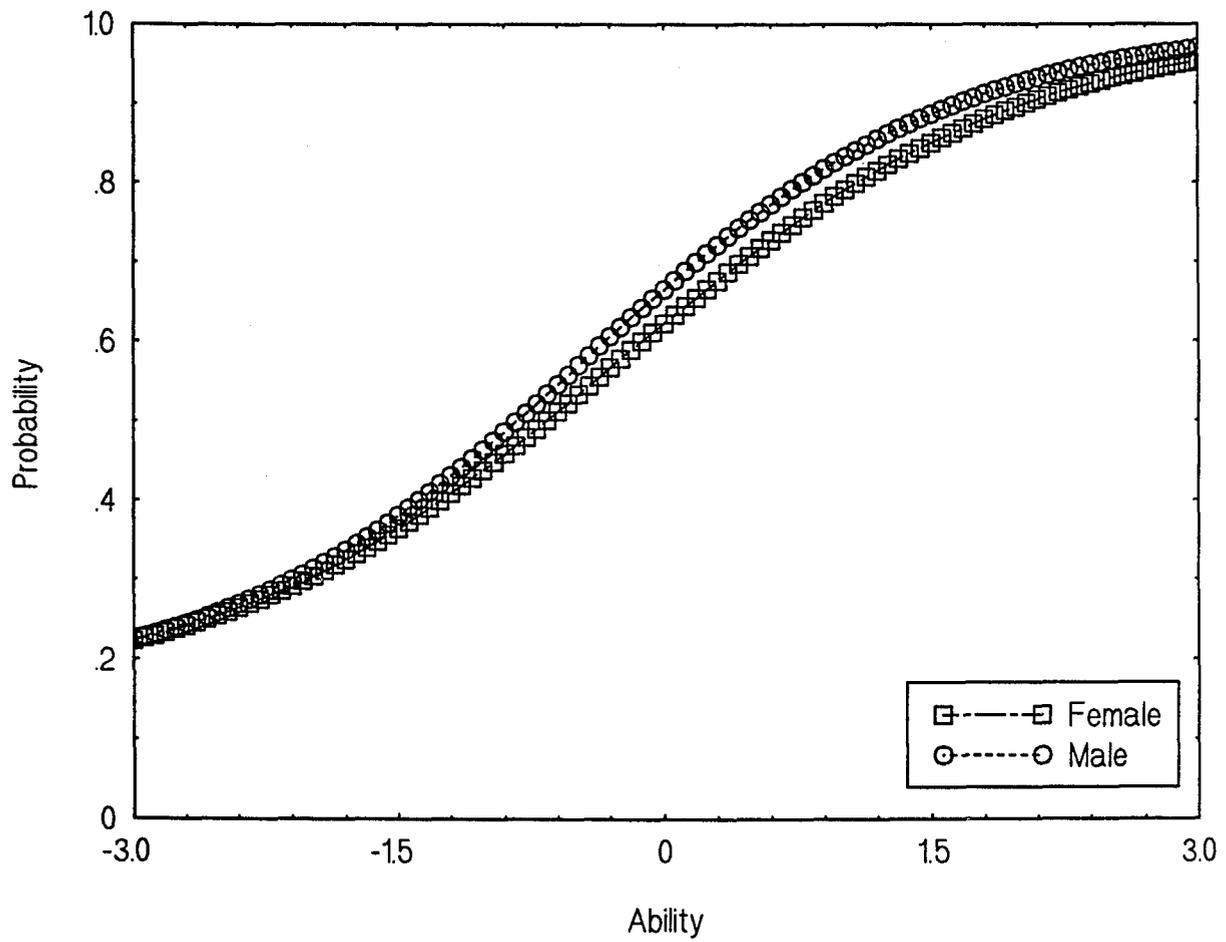
Male vs. Female: Item 2-23



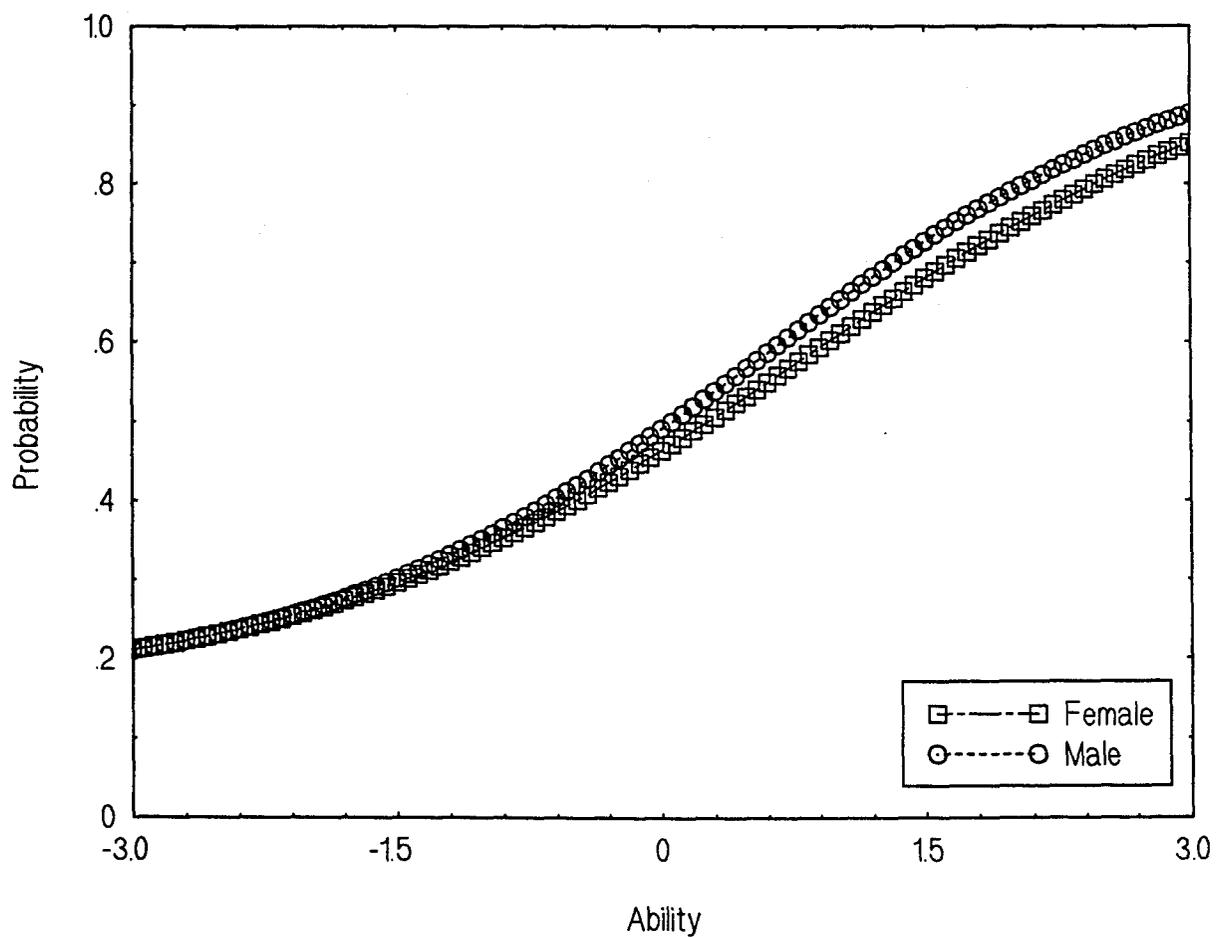
Male vs. Female: Item 2-24



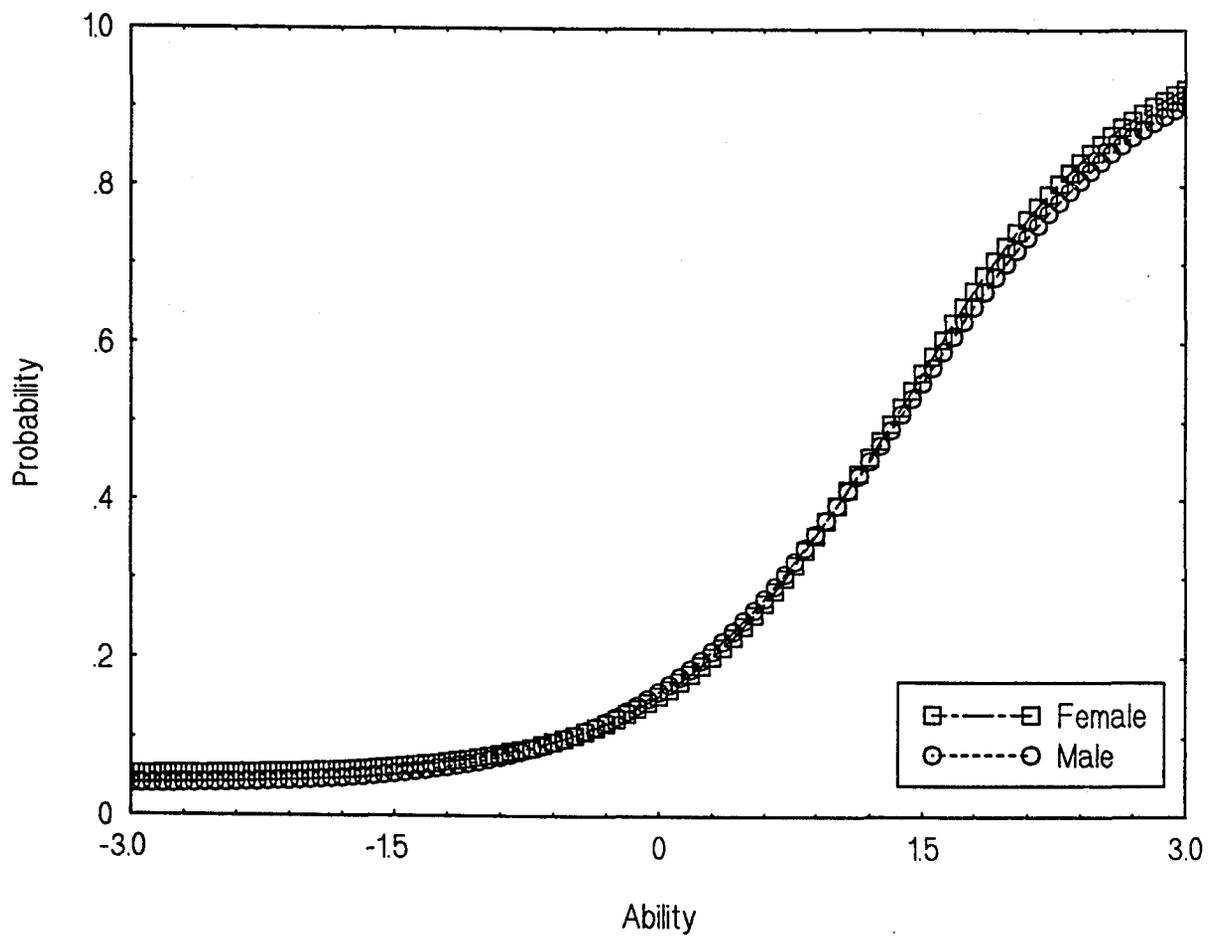
Male vs. Female: Item 2-25



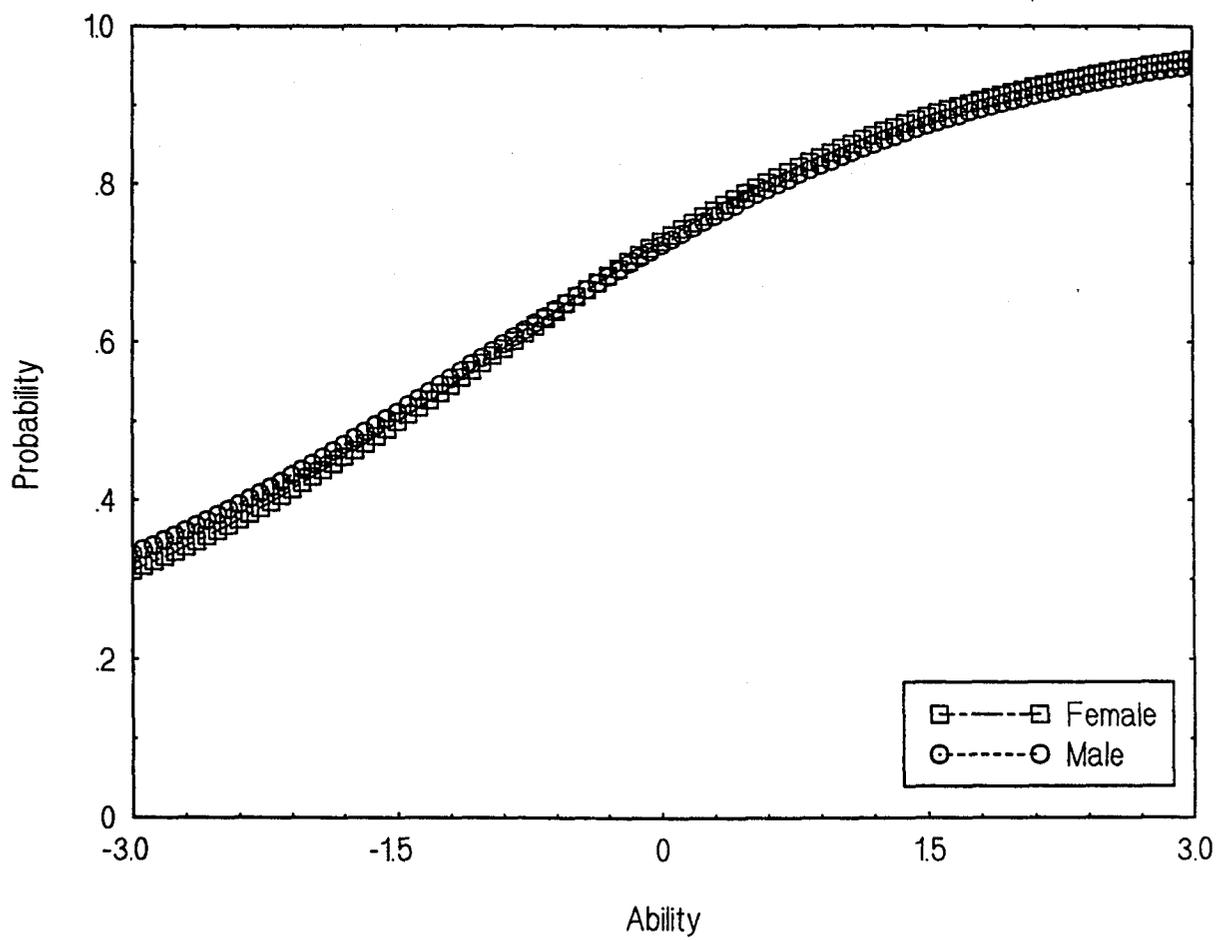
Male vs. Female: Item 6-1



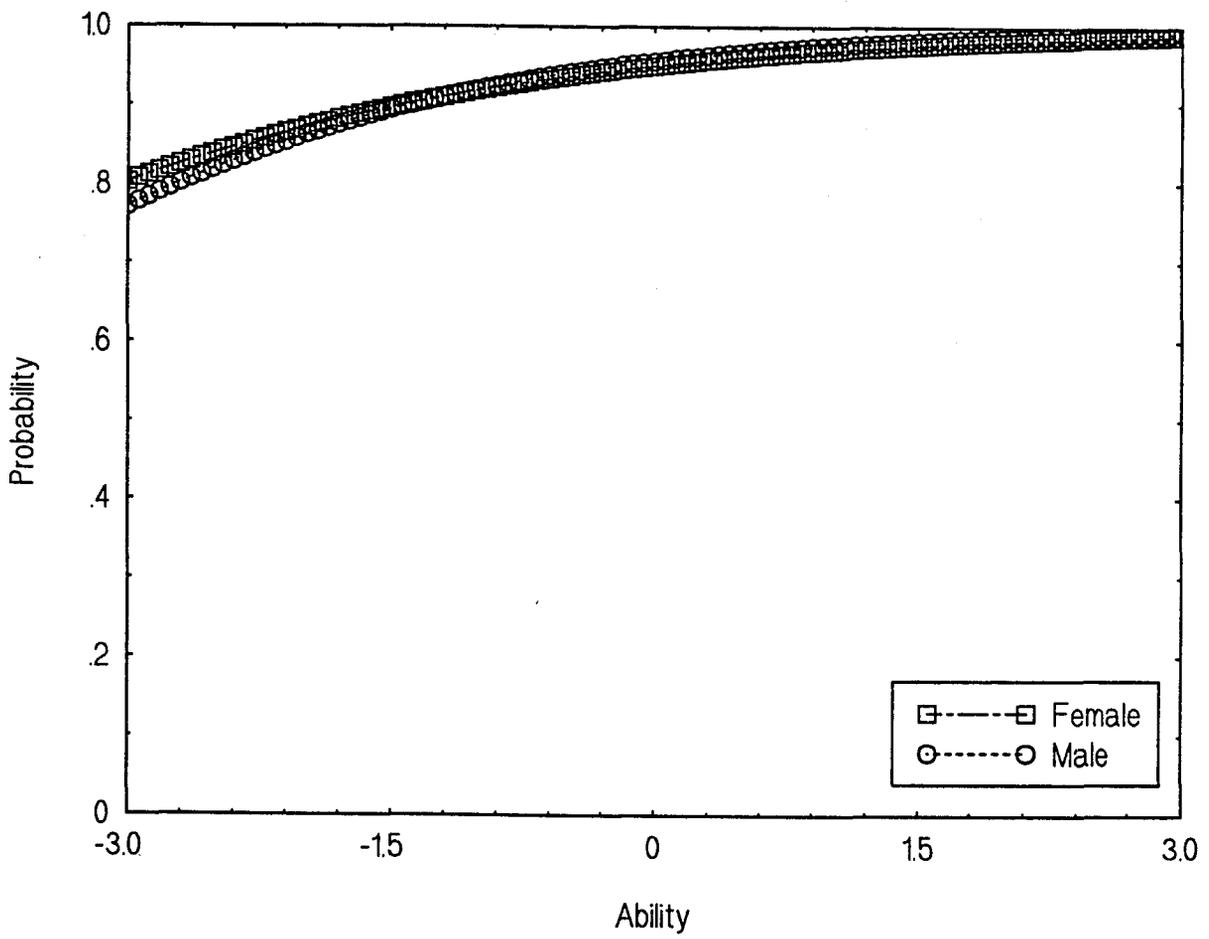
Male vs. Female: Item 6-2



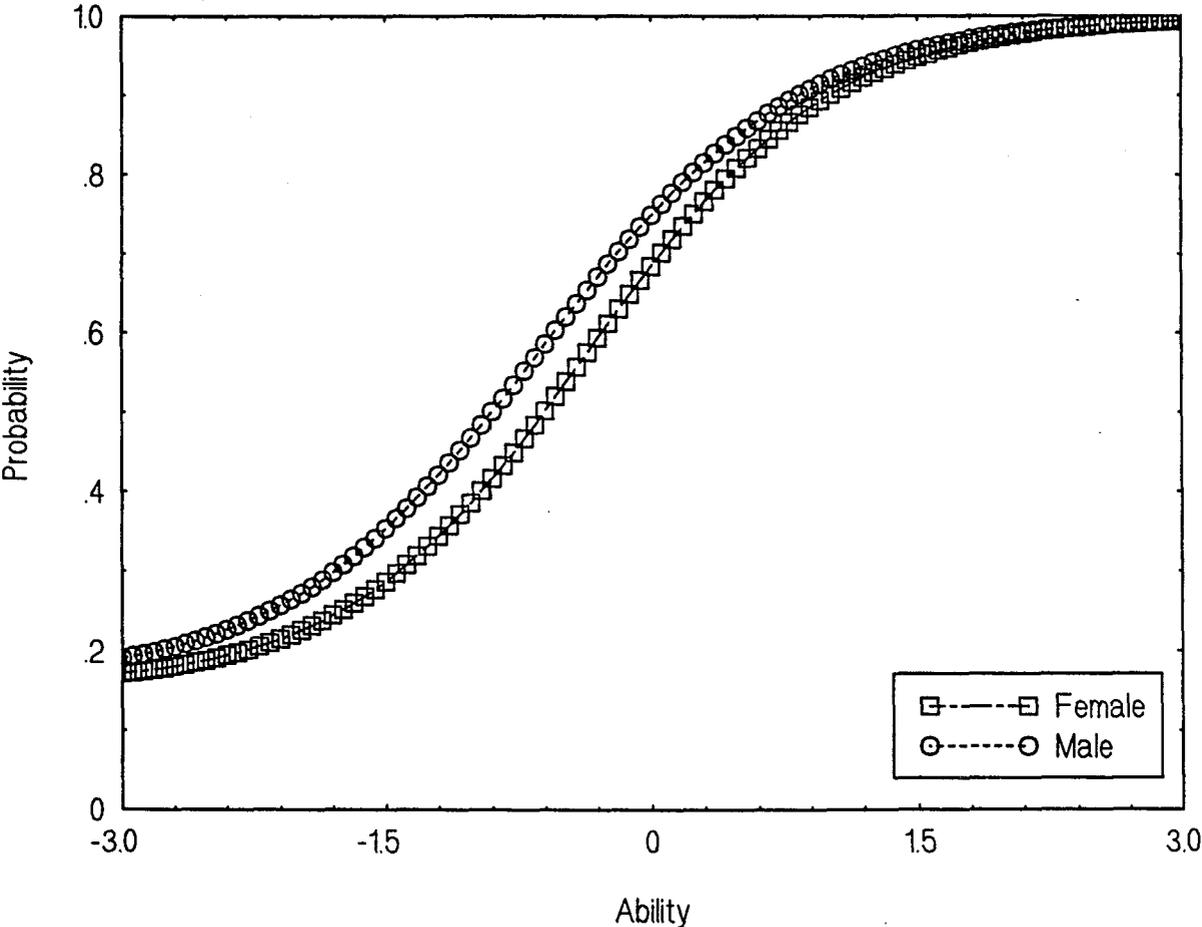
Male vs. Female: Item 6-3



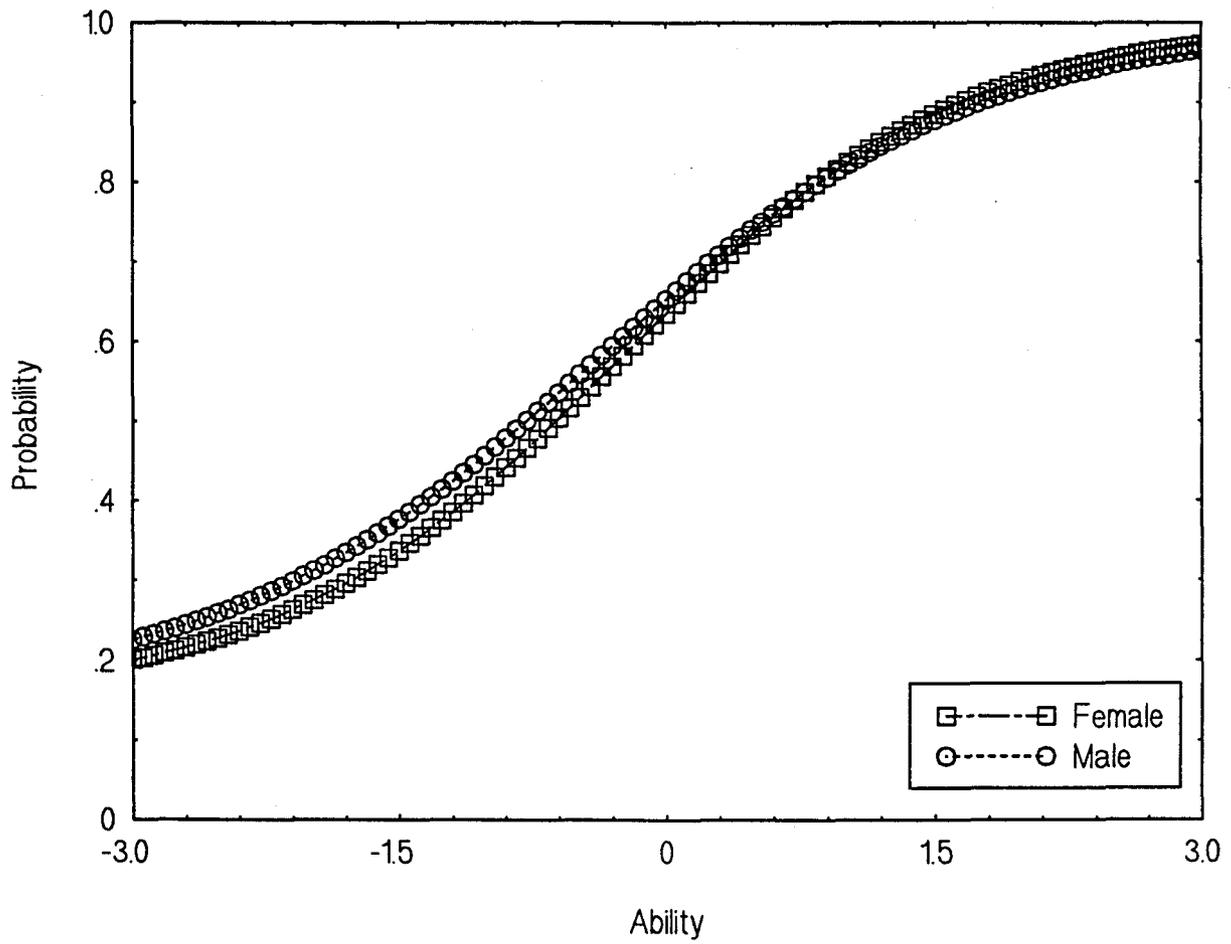
Male vs. Female: Item 6-4

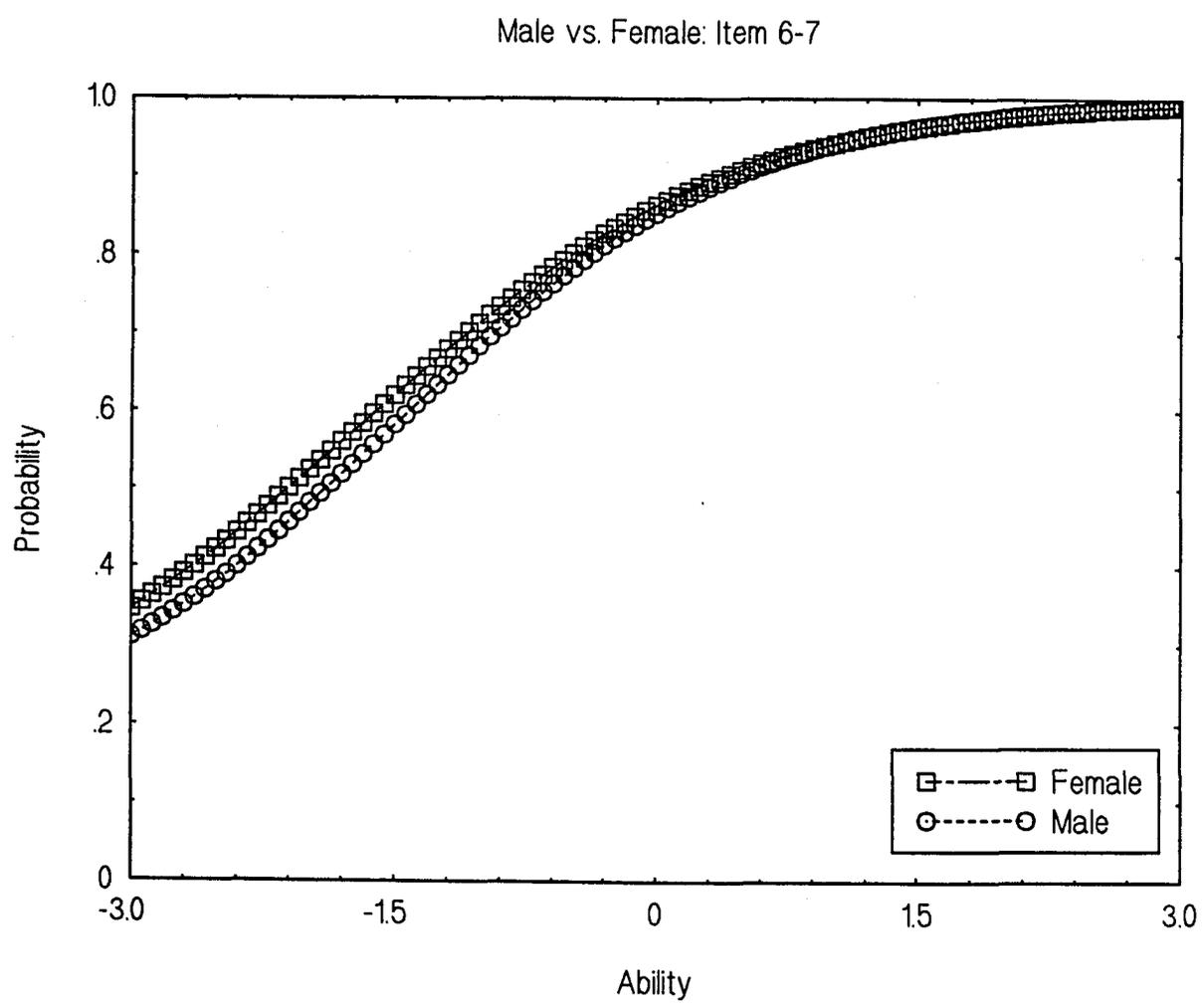


Male vs. Female: Item 6-5

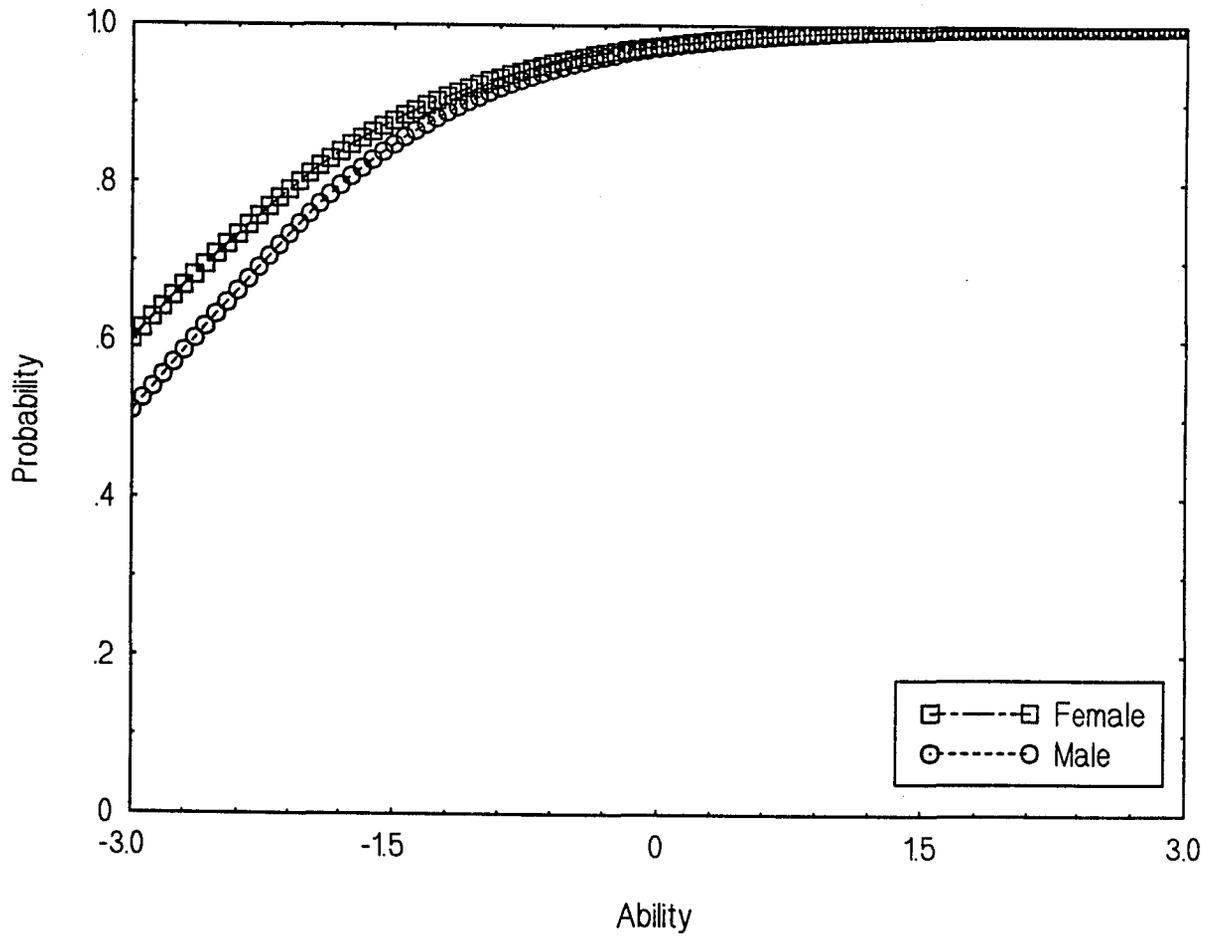


Male vs. Female: Item 6-6

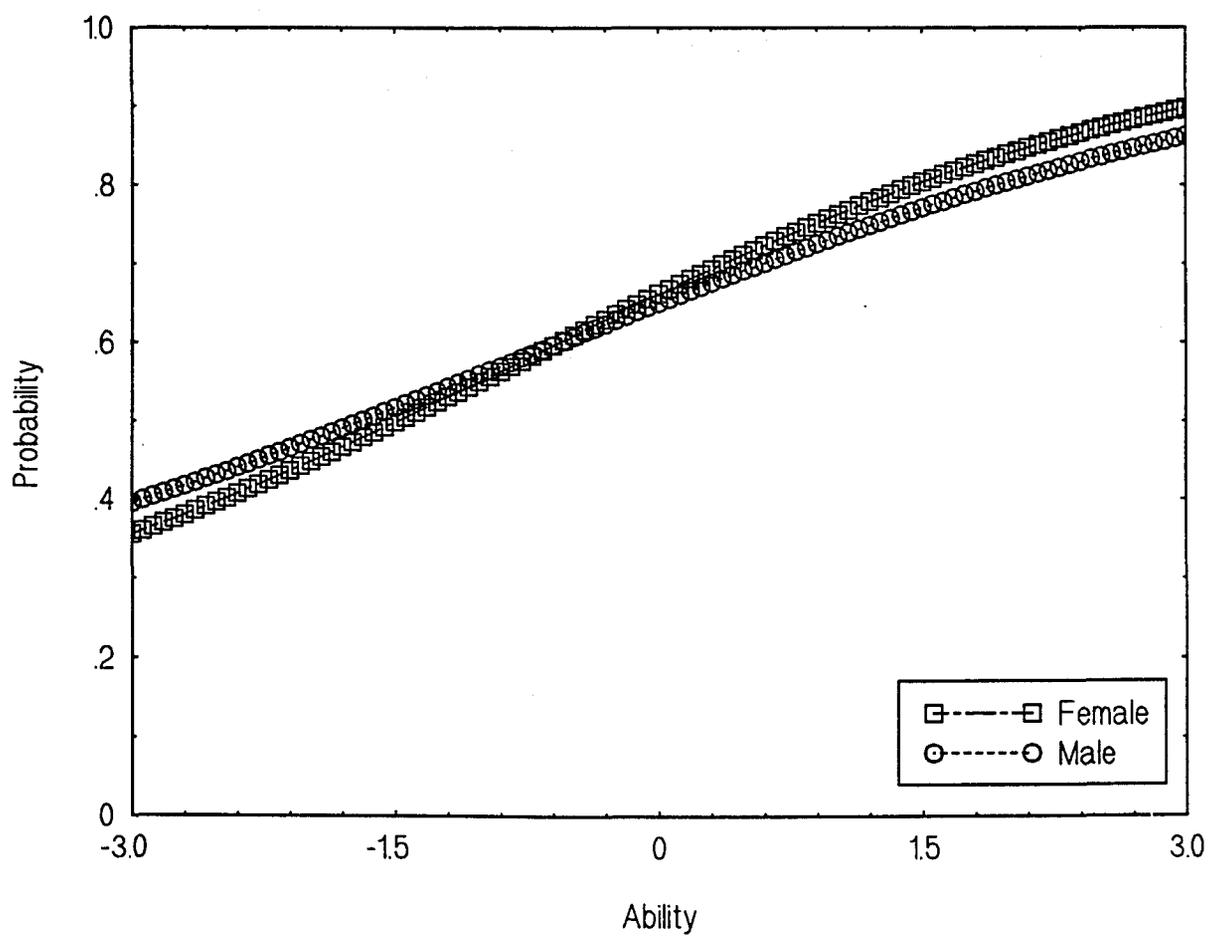




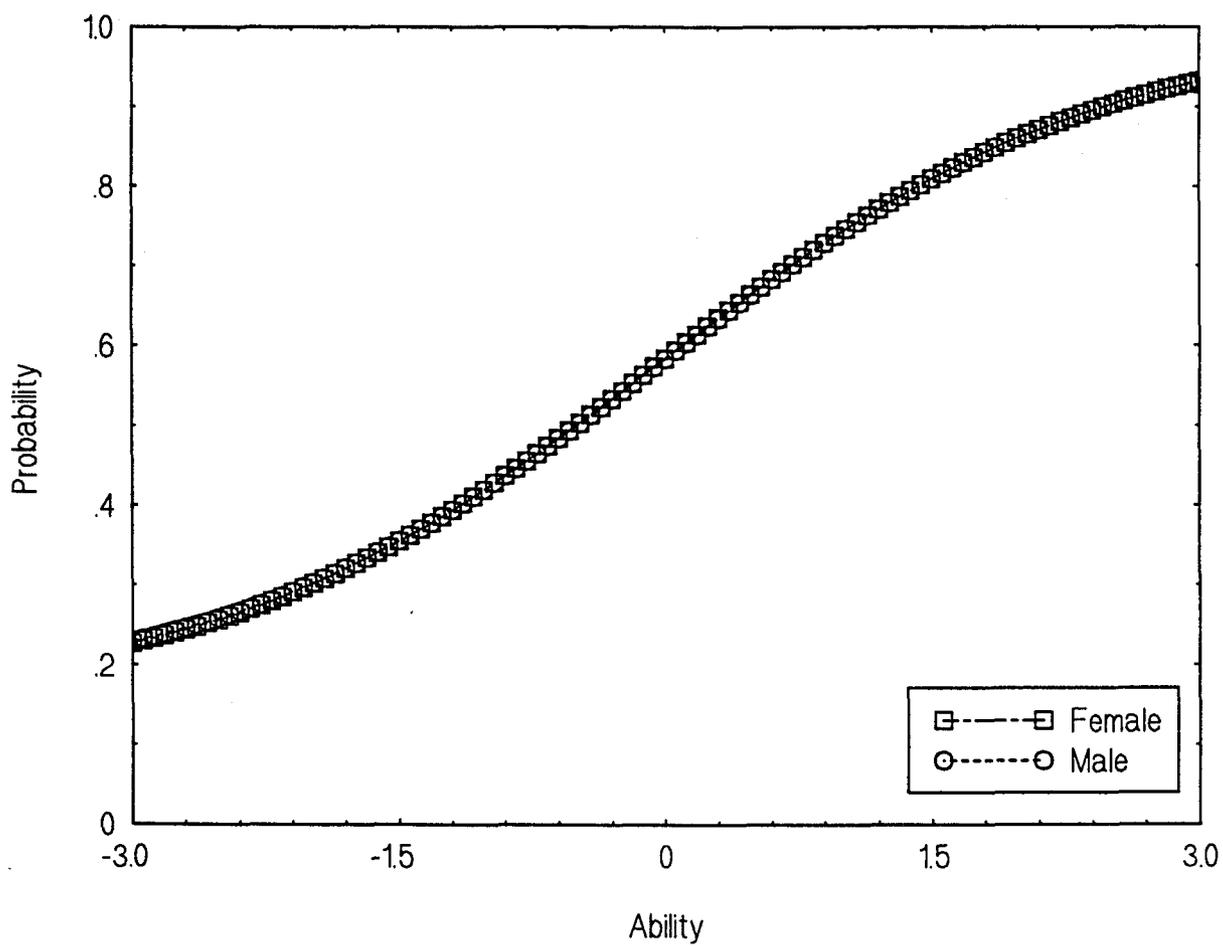
Male vs. Female: Item 6-8



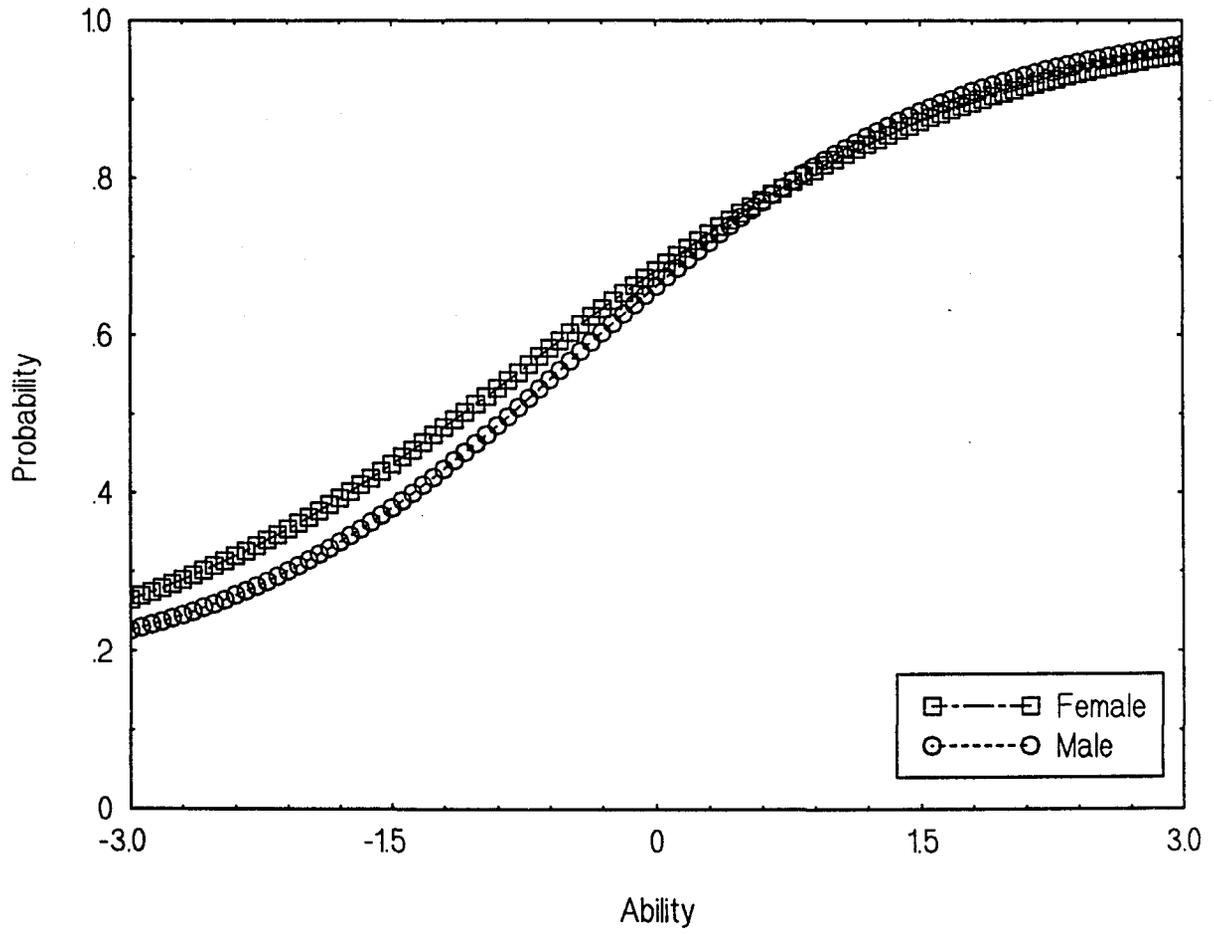
Male vs. Female: Item 6-9



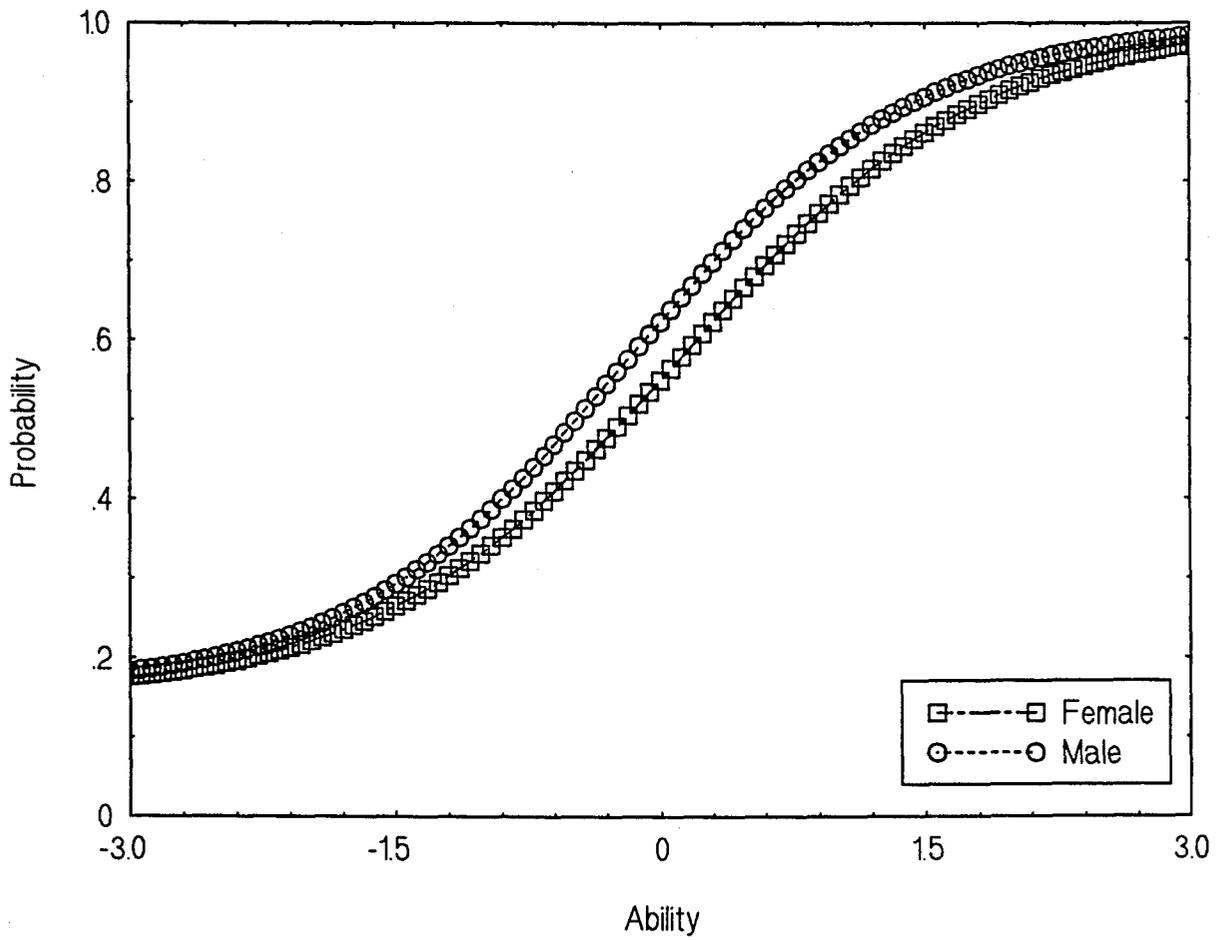
Male vs. Female: Item 6-10



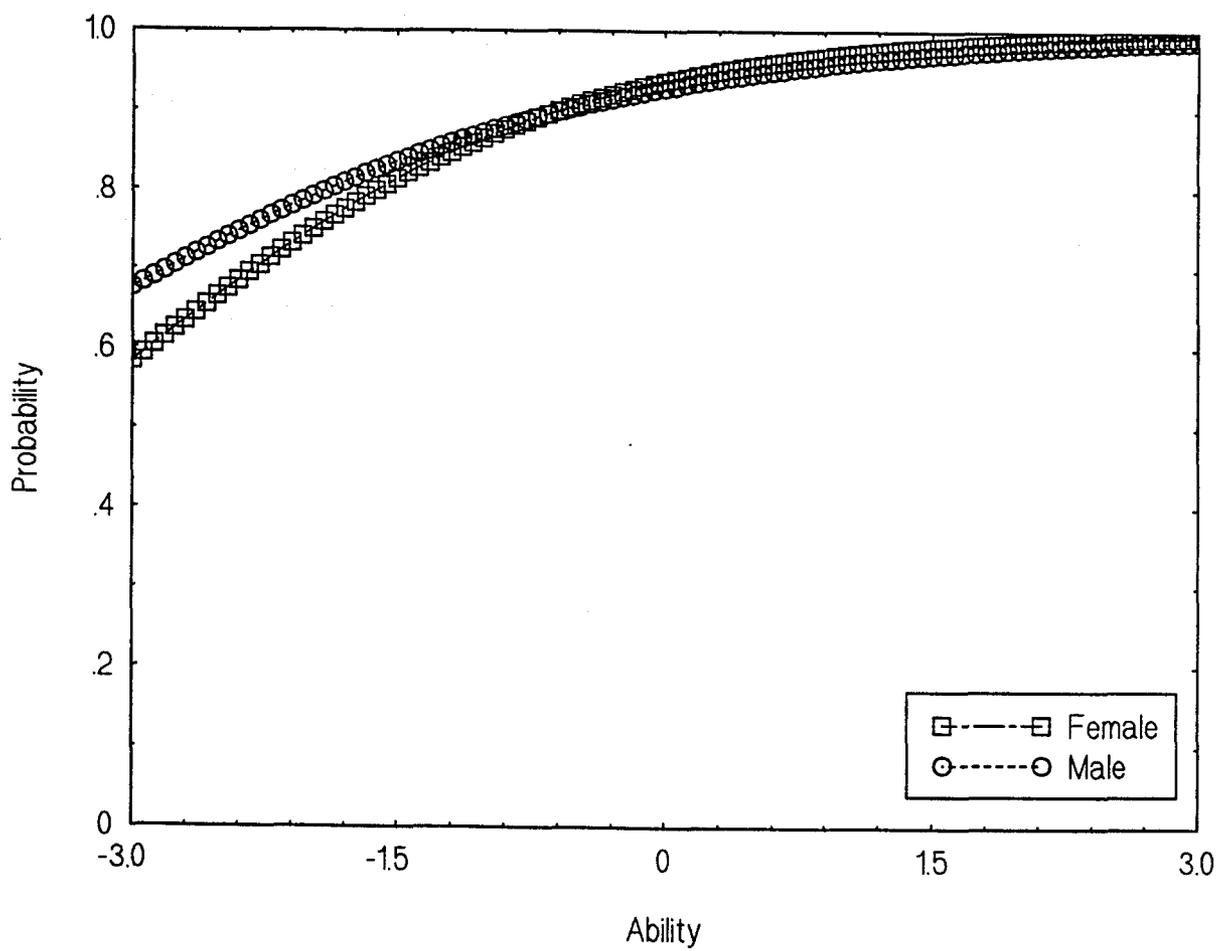
Male vs. Female: Item 6-11



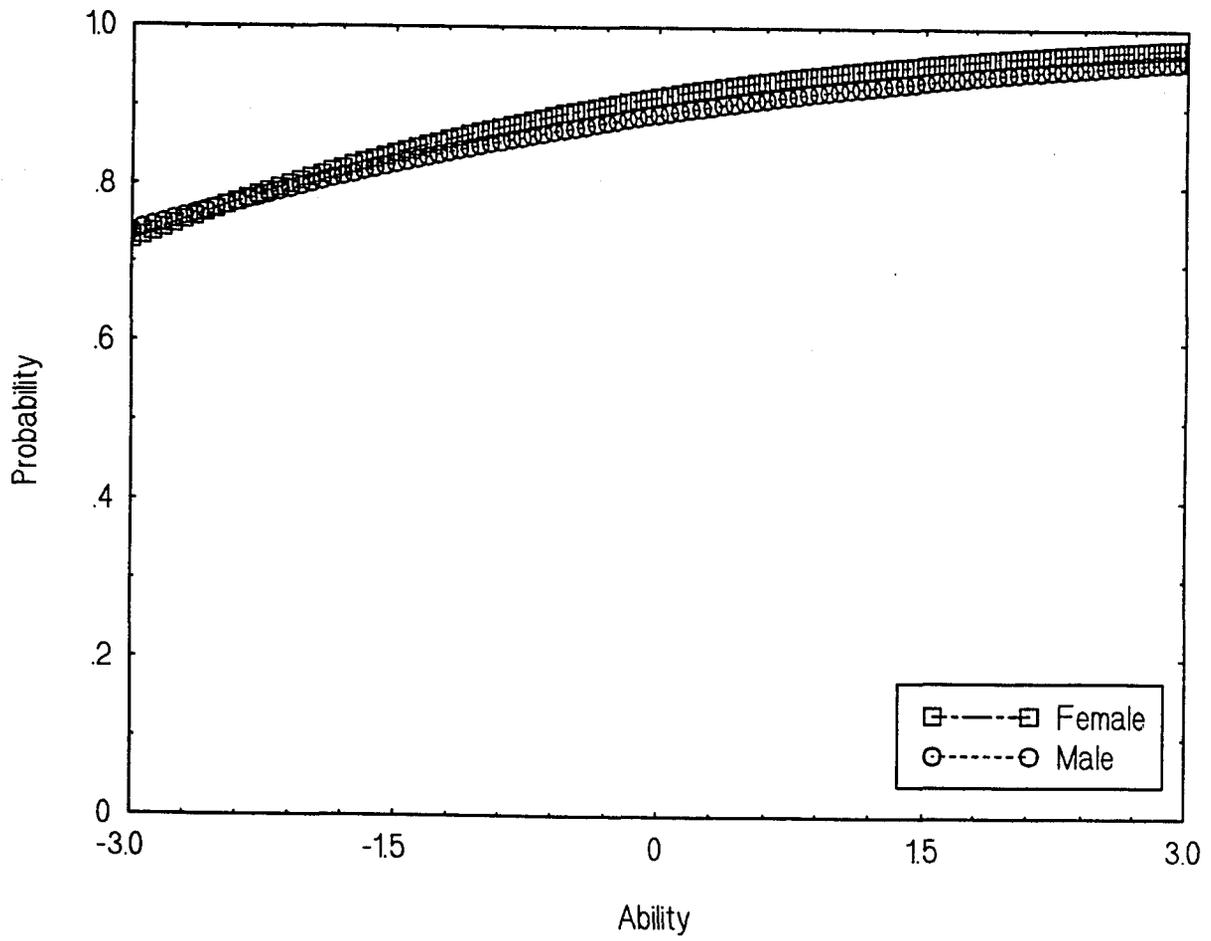
Male vs. Female: Item 6-12



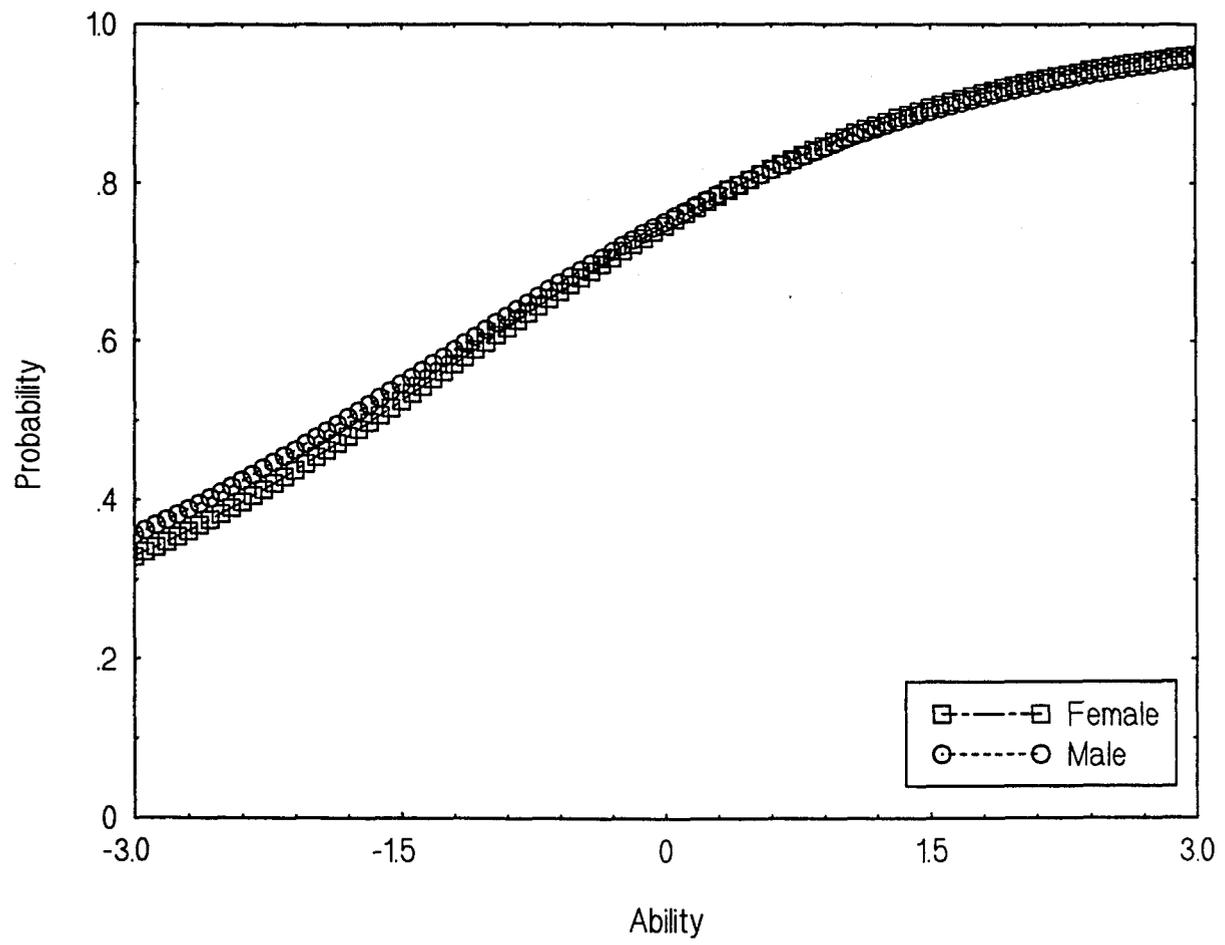
Male vs. Female: Item 6-13



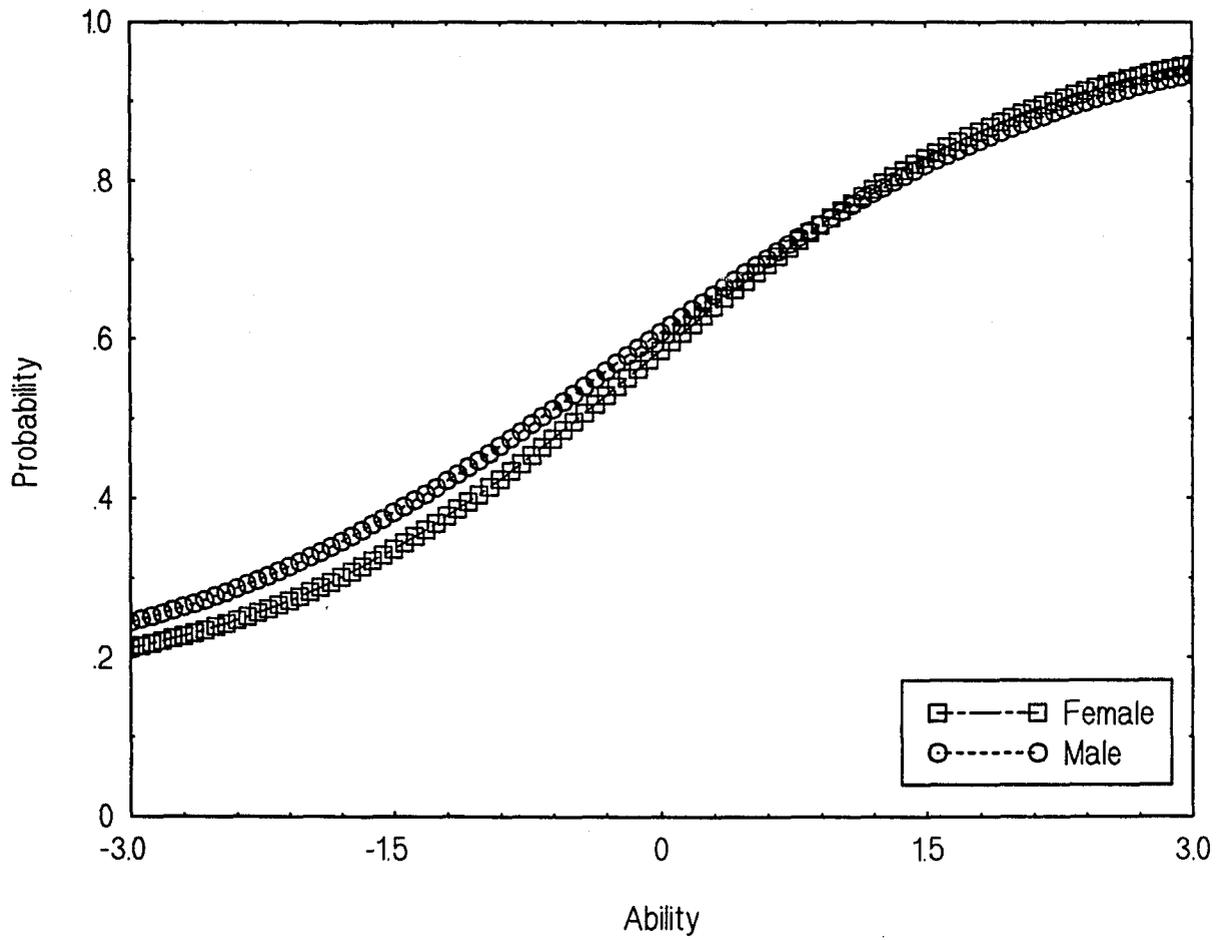
Male vs. Female: Item 6-14



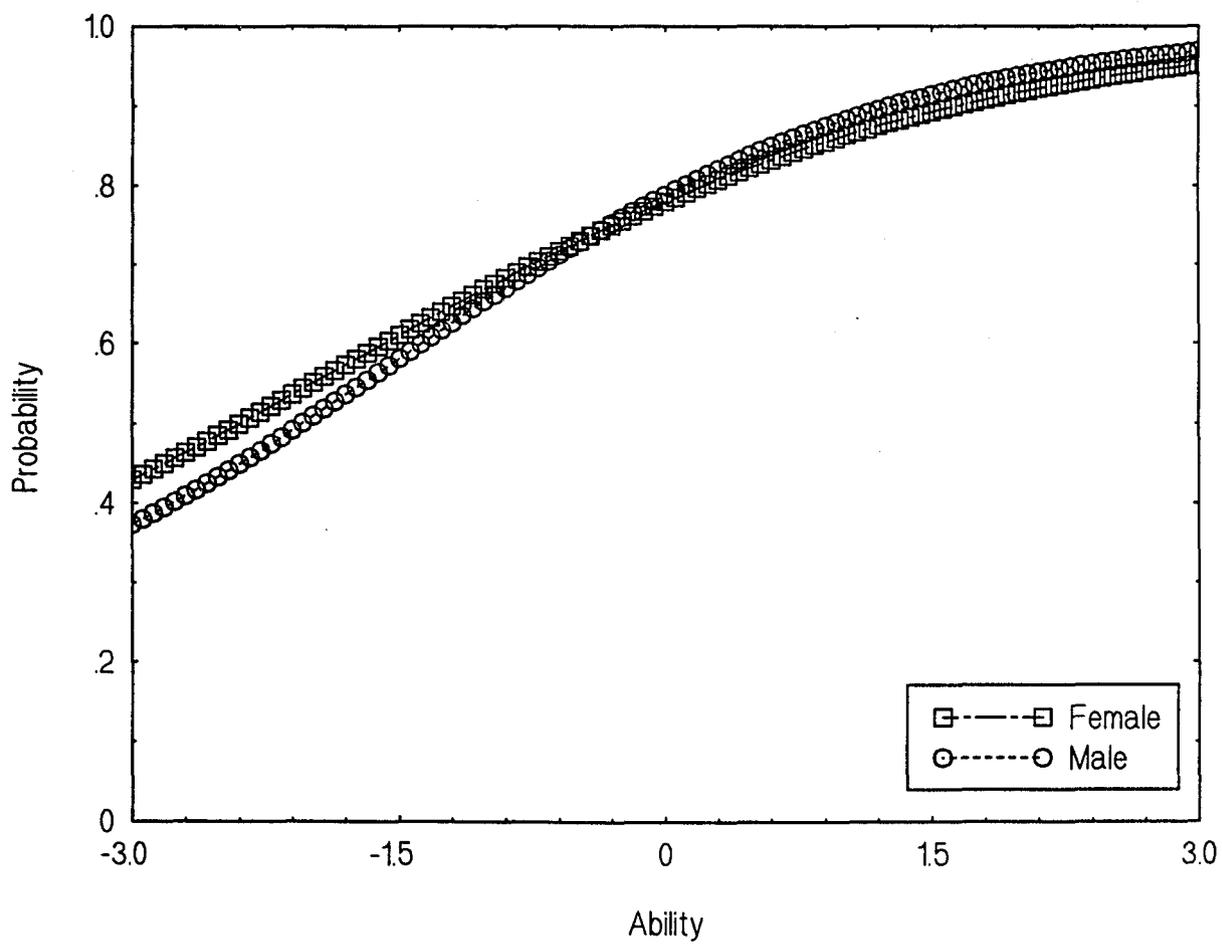
Male vs. Female: Item 6-15



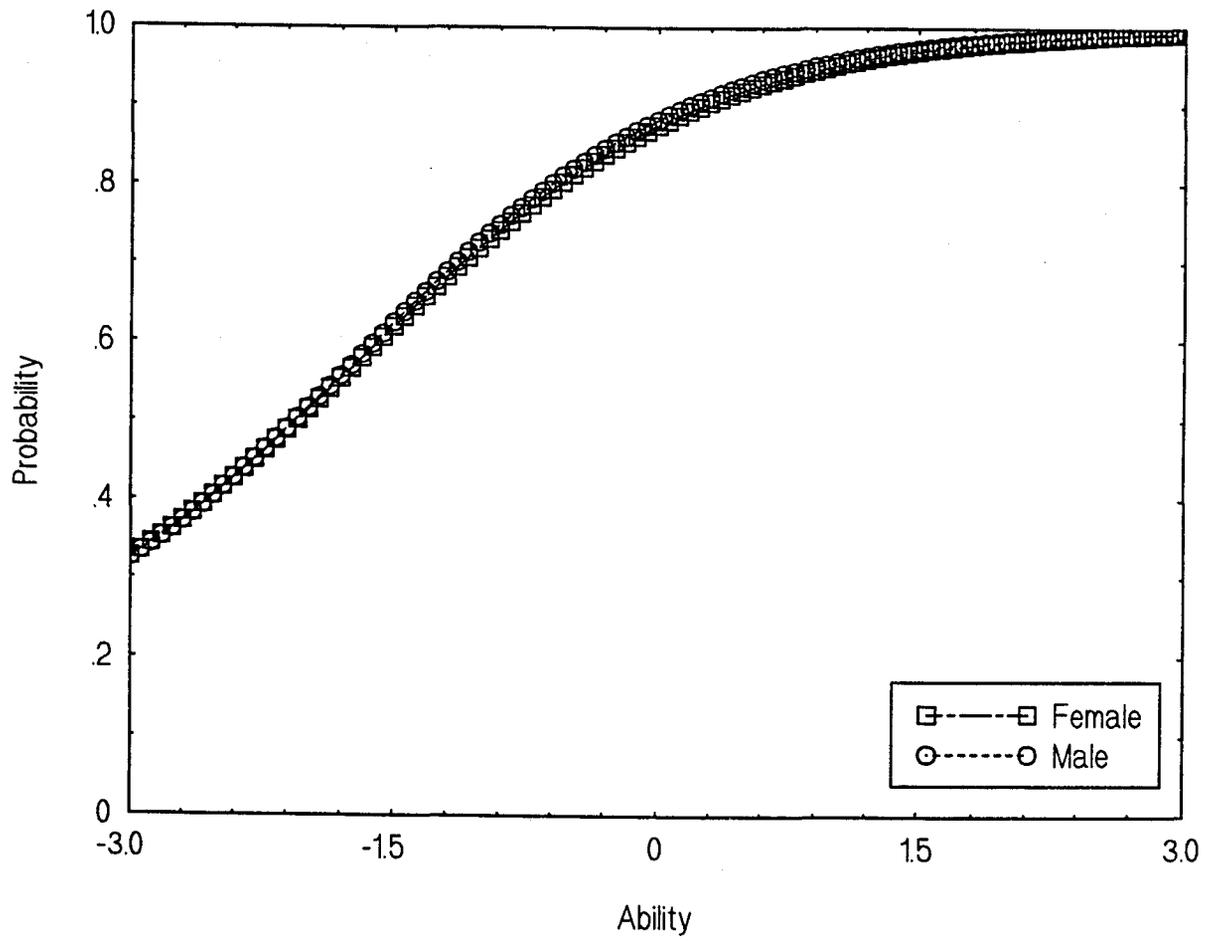
Male vs. Female: Item 6-16



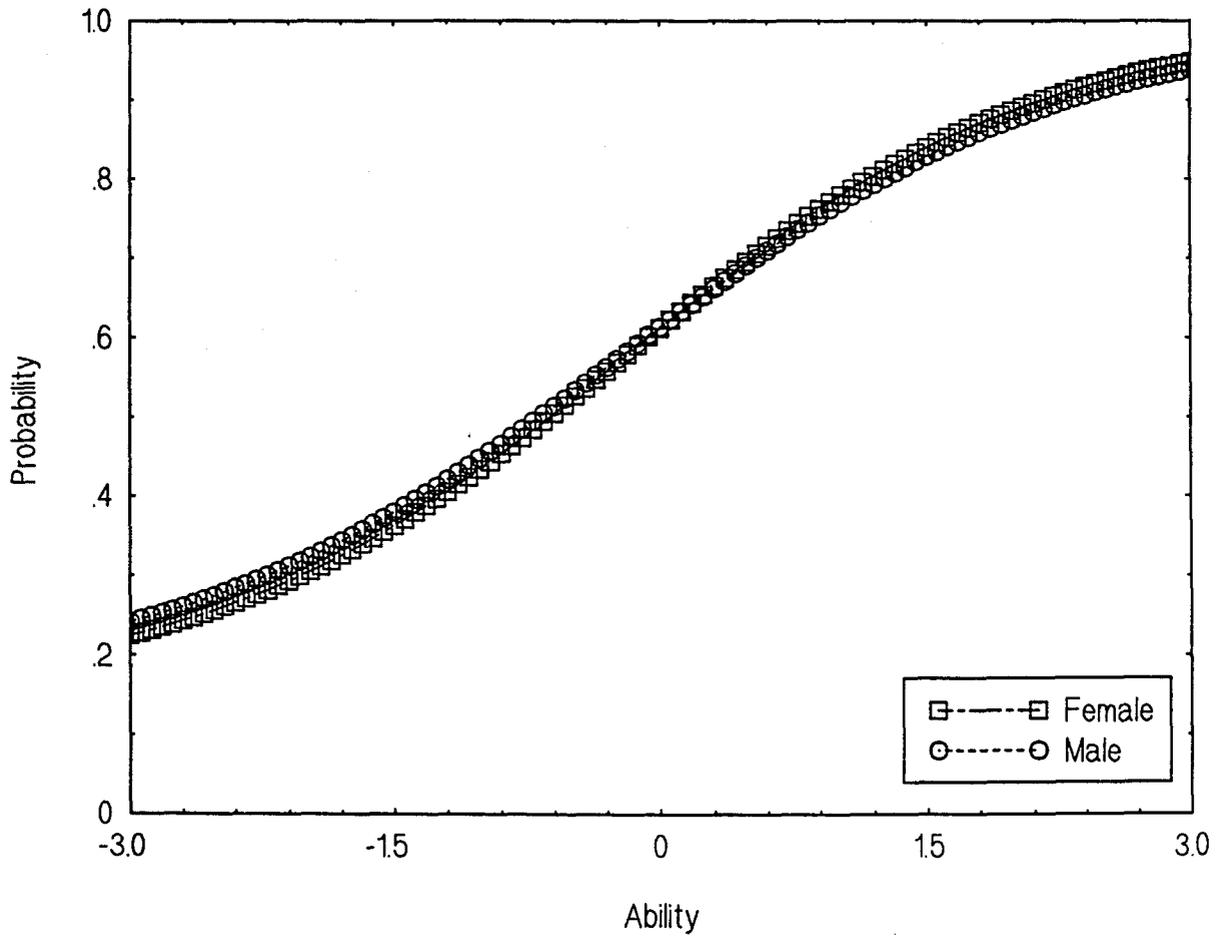
Male vs. Female: Item 6-17



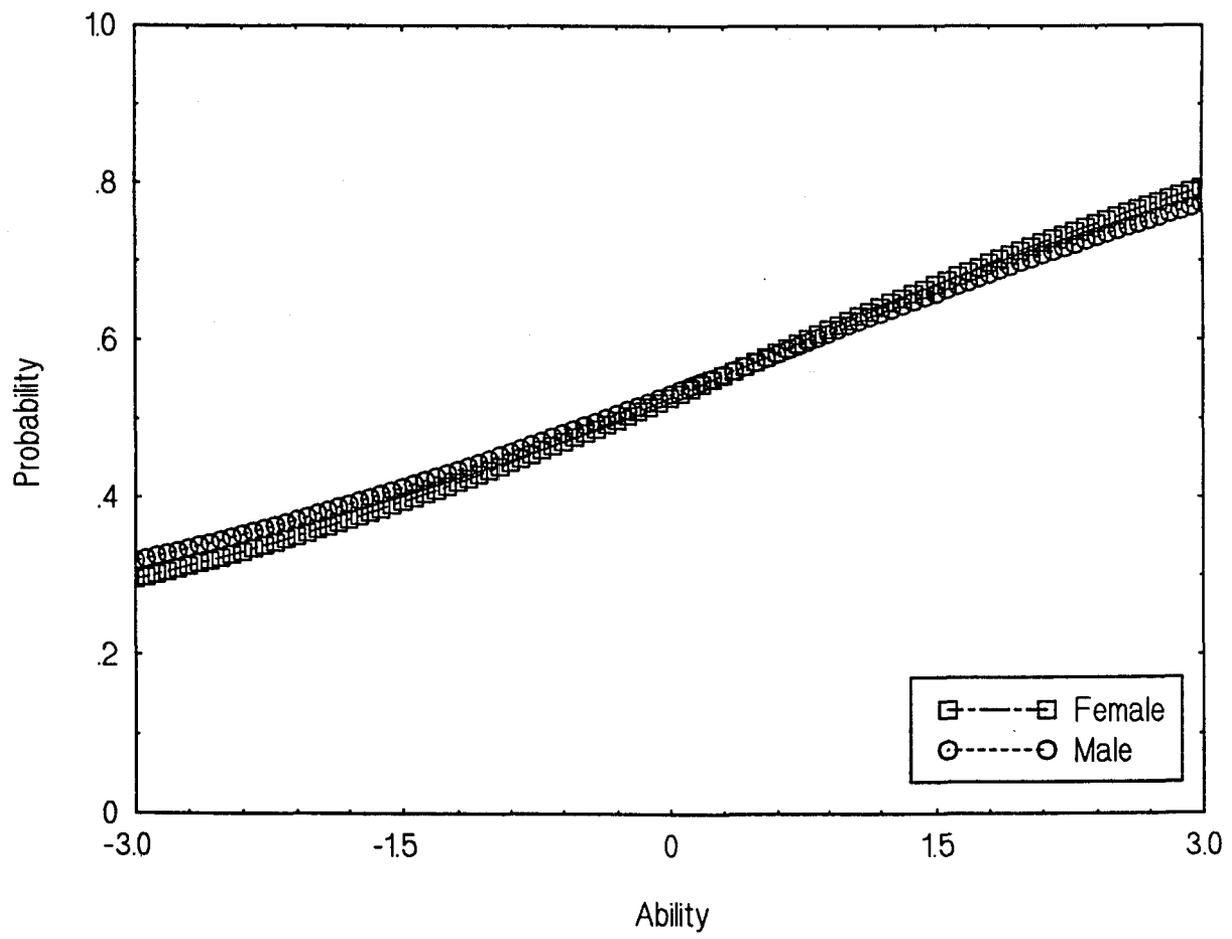
Male vs. Female: Item 6-18



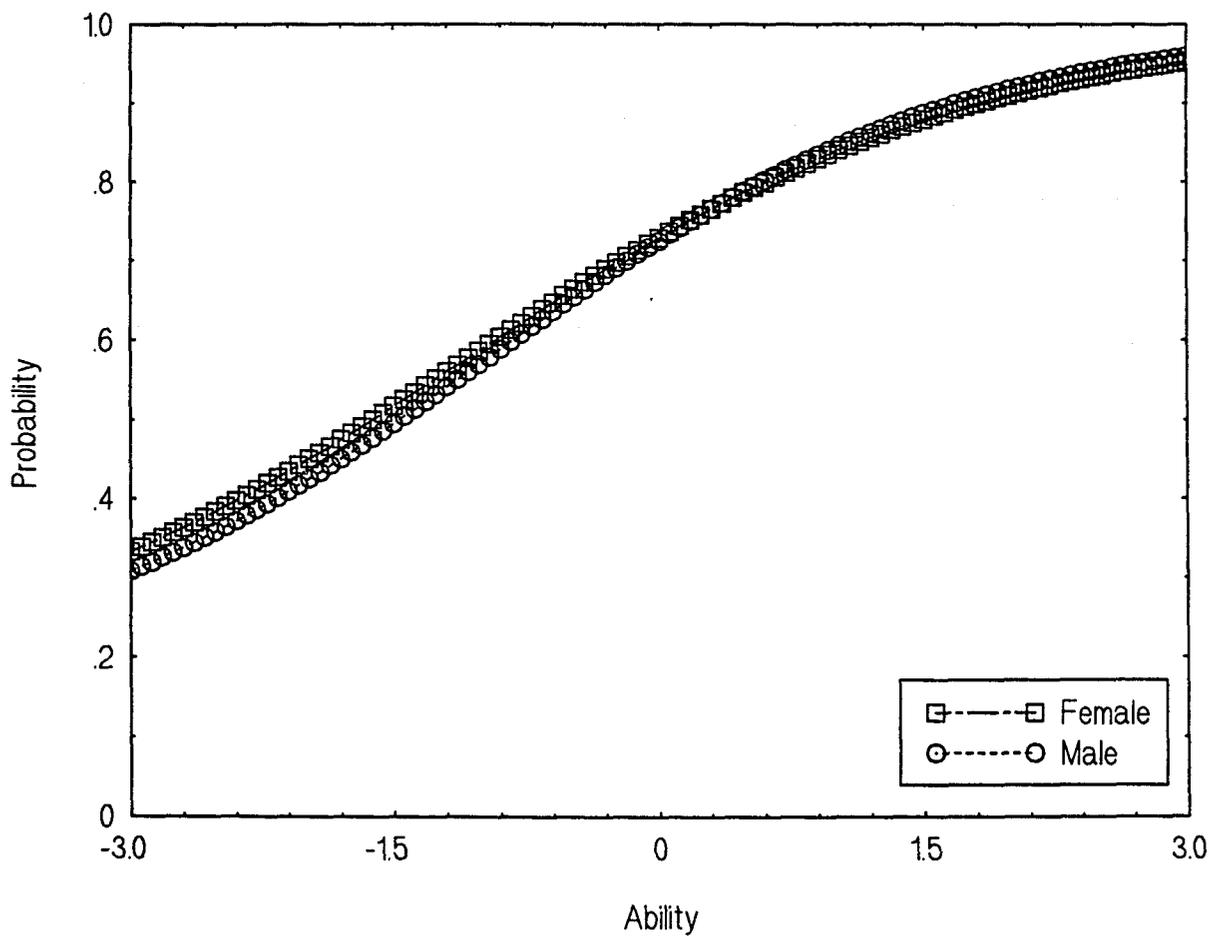
Male vs. Female: Item 6-19



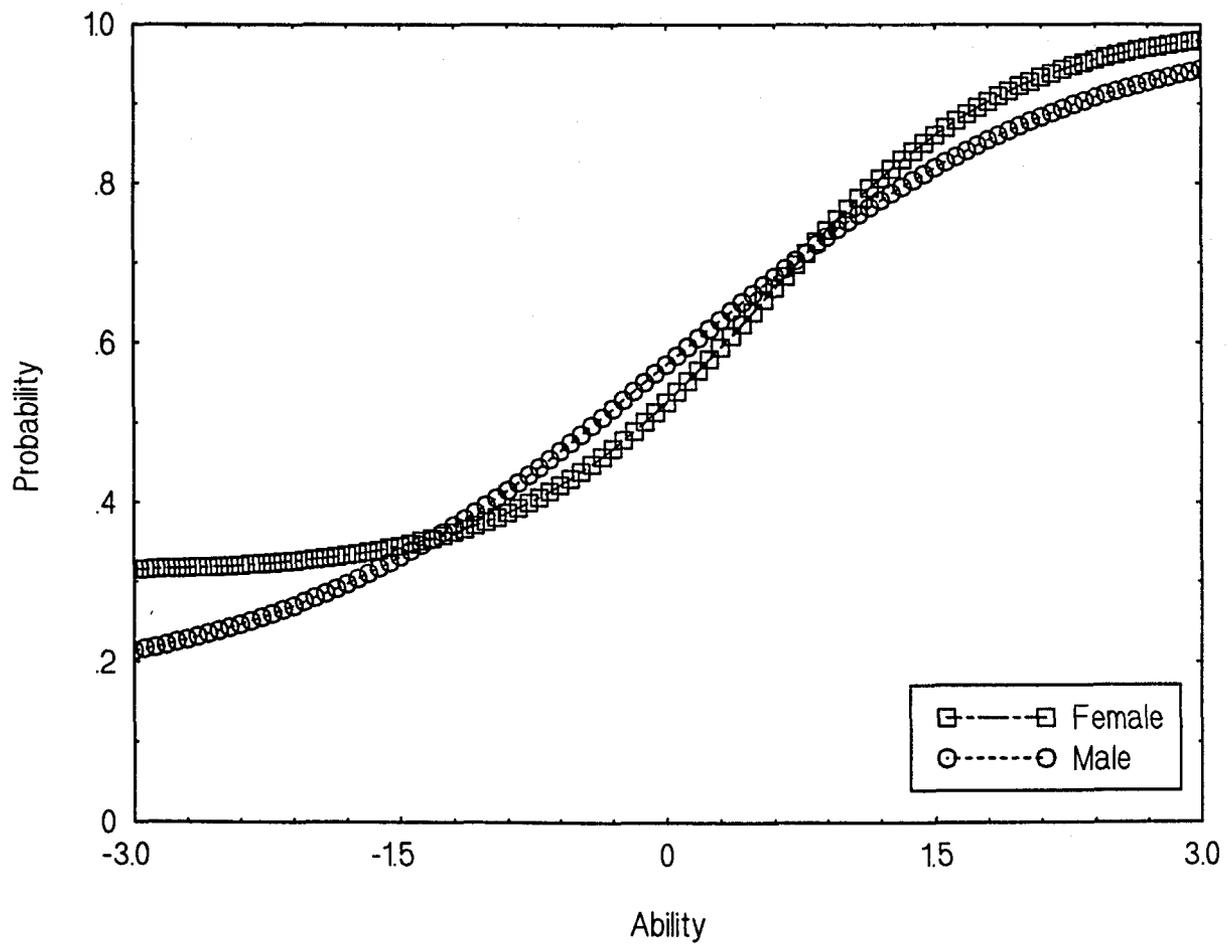
Male vs. Female: Item 6-20



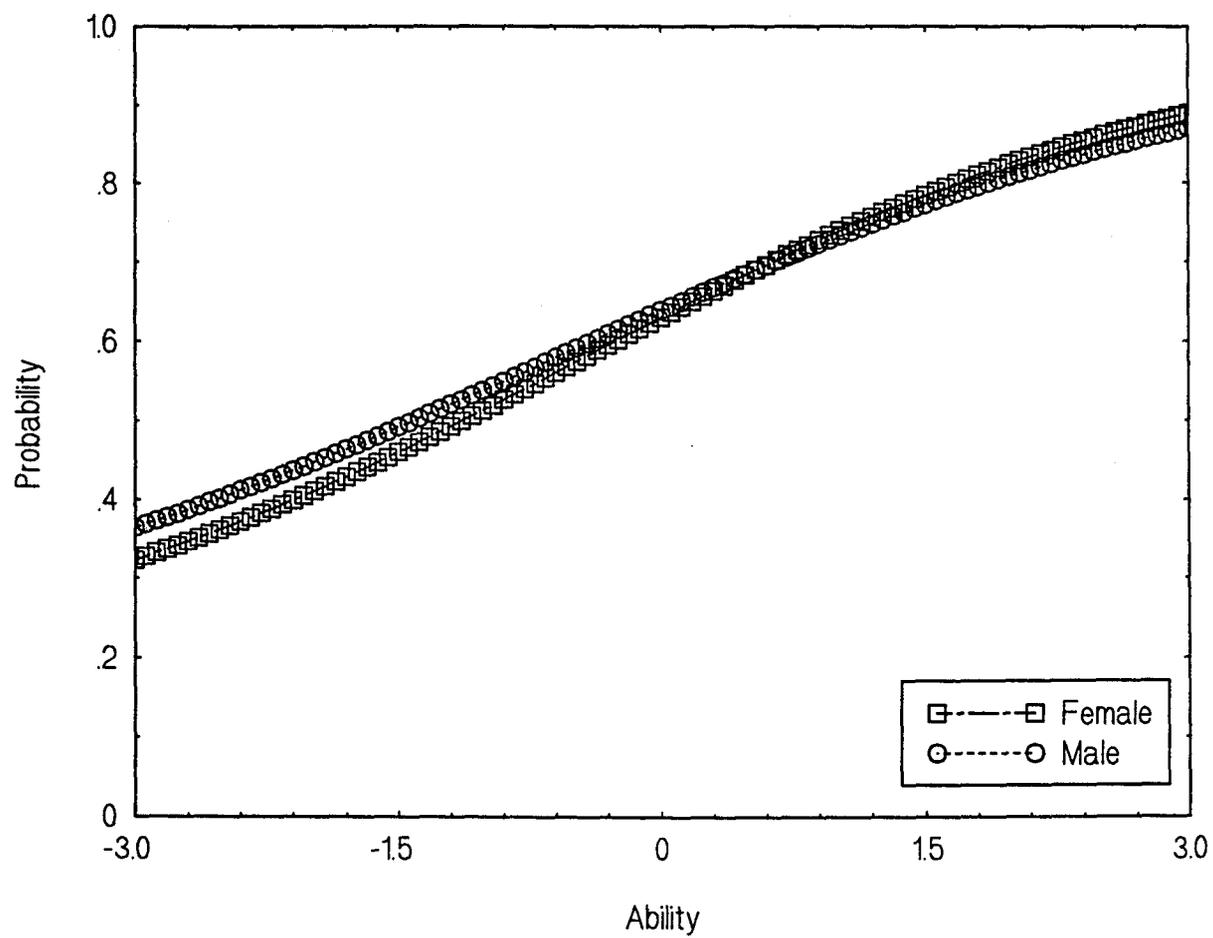
Male vs. Female: Item 6-21



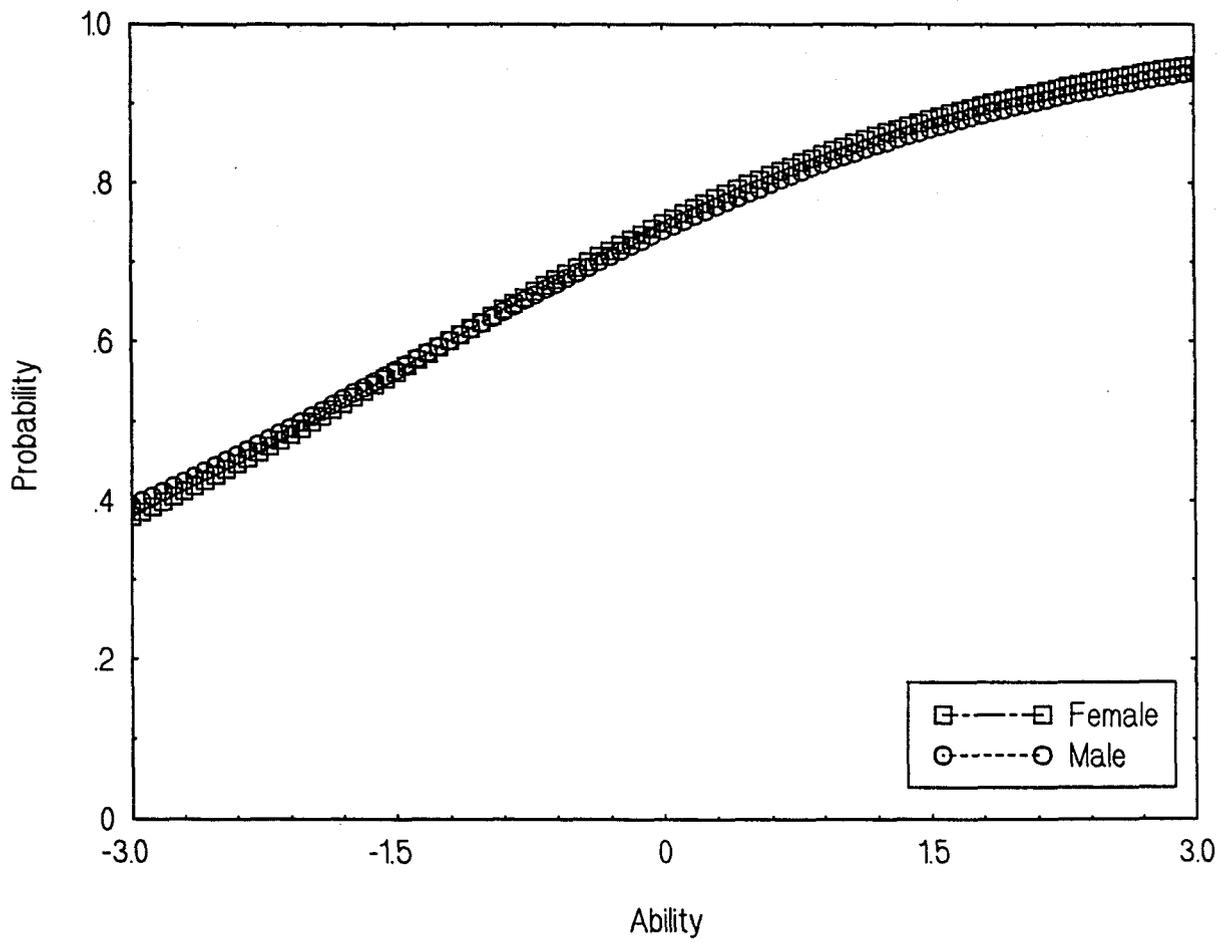
Male vs. Female: Item 6-22



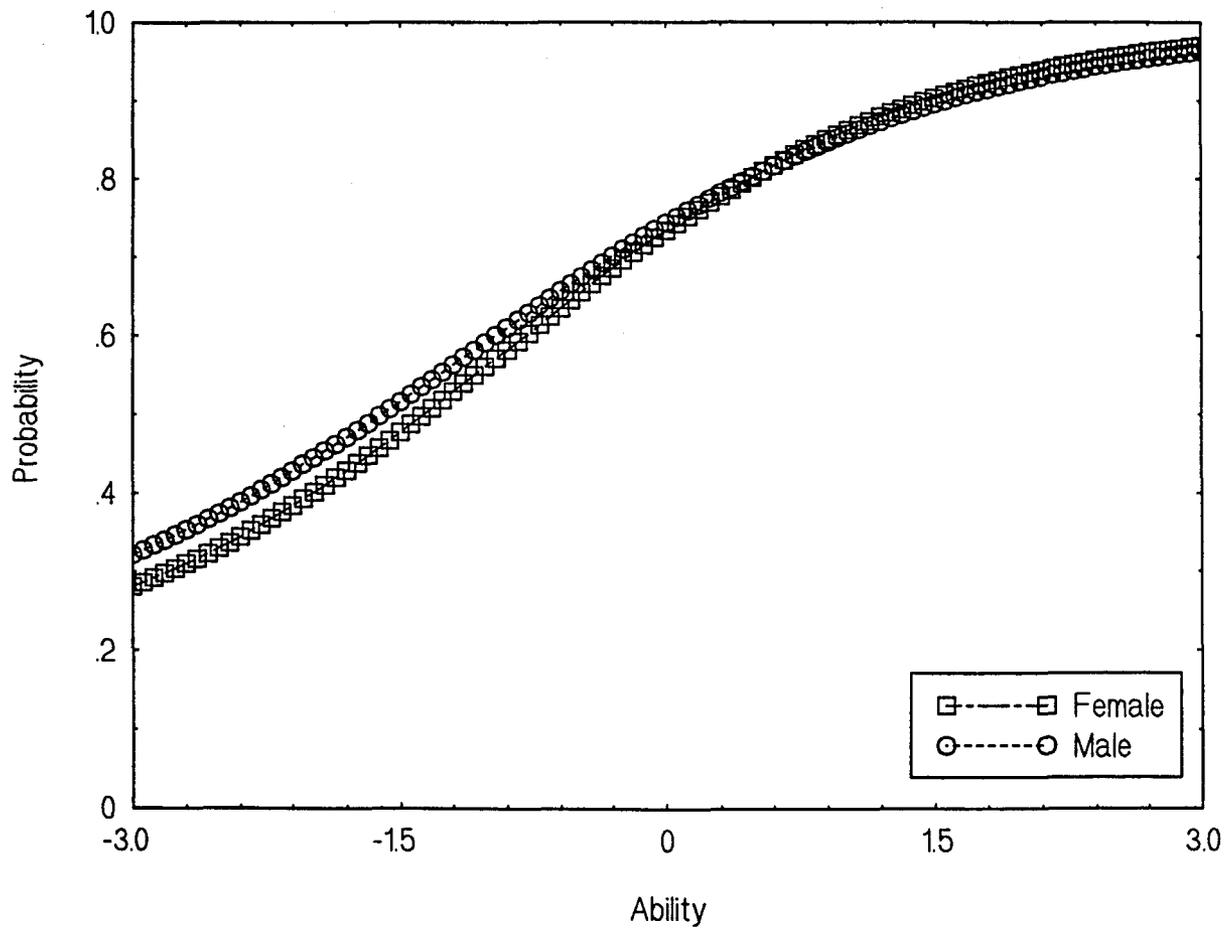
Male vs. Female: Item 6-23



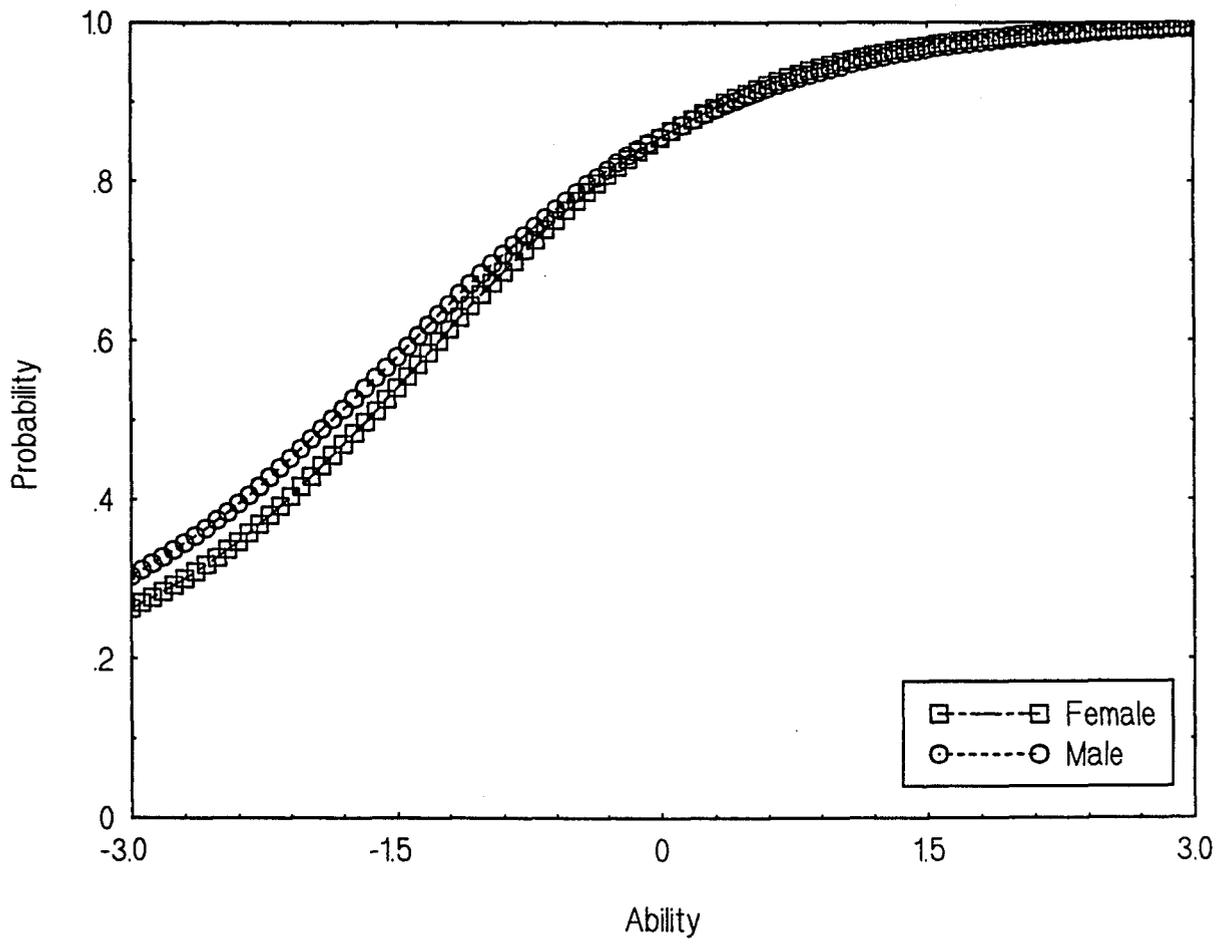
Male vs. Female: Item 6-24



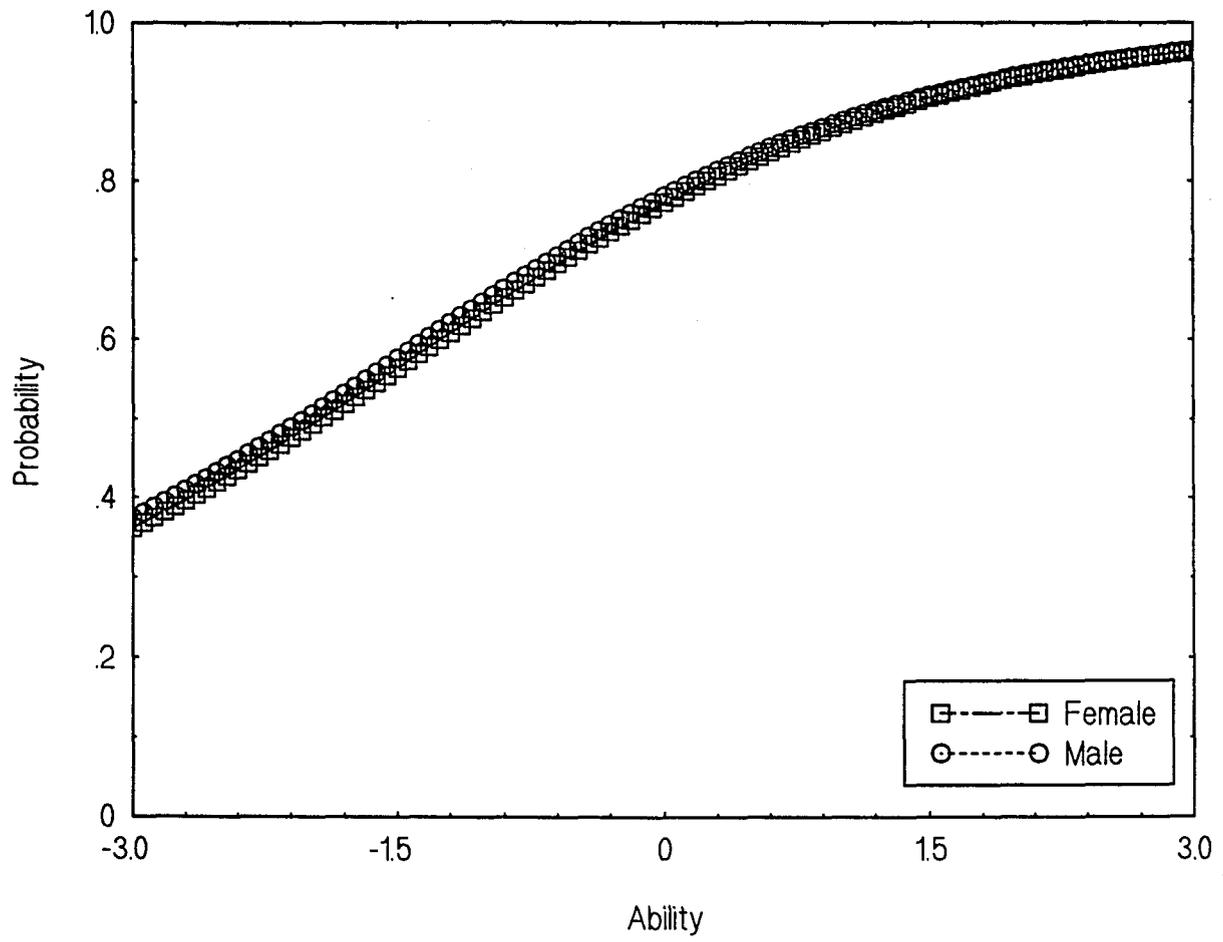
Male vs. Female: Item 6-25



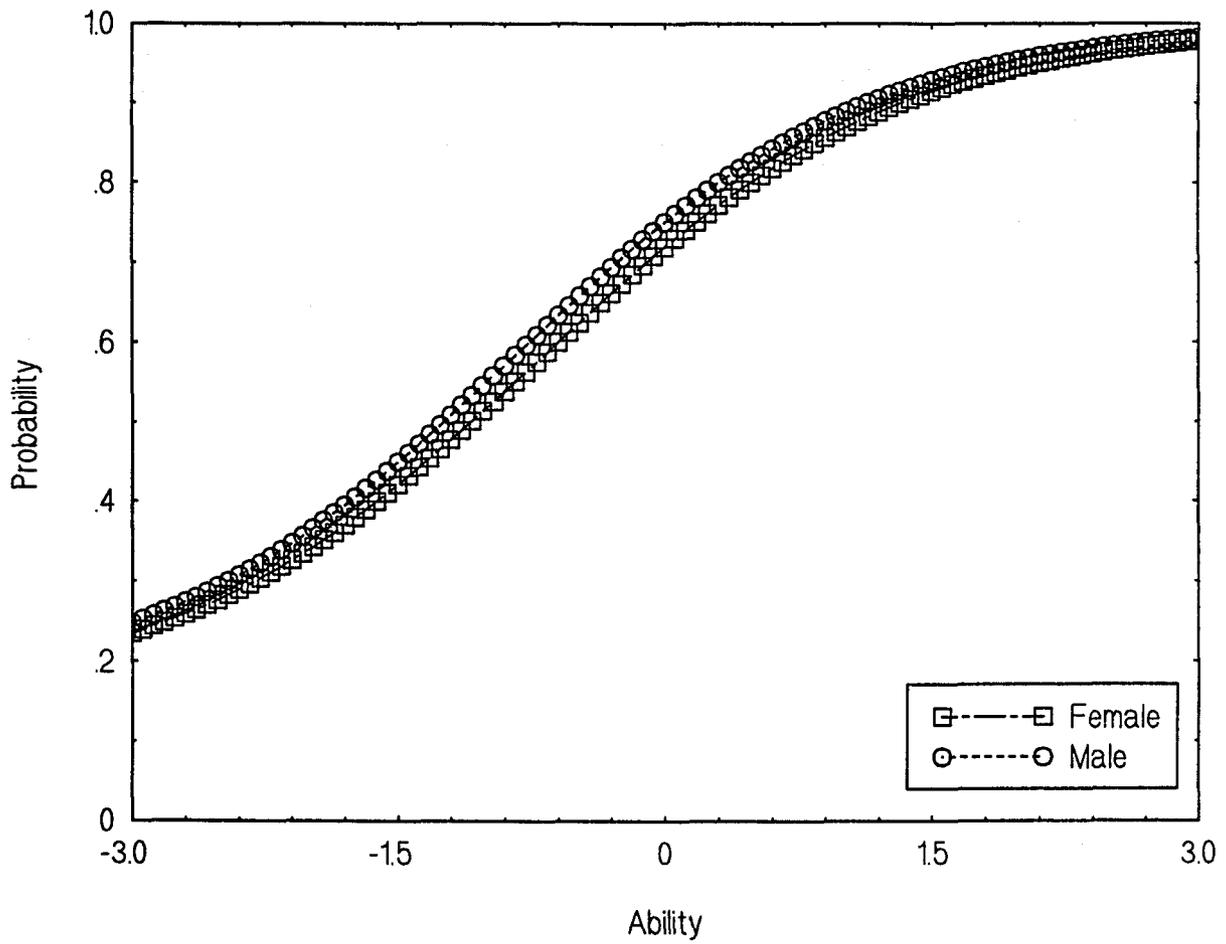
Male vs. Female: Item 6-26



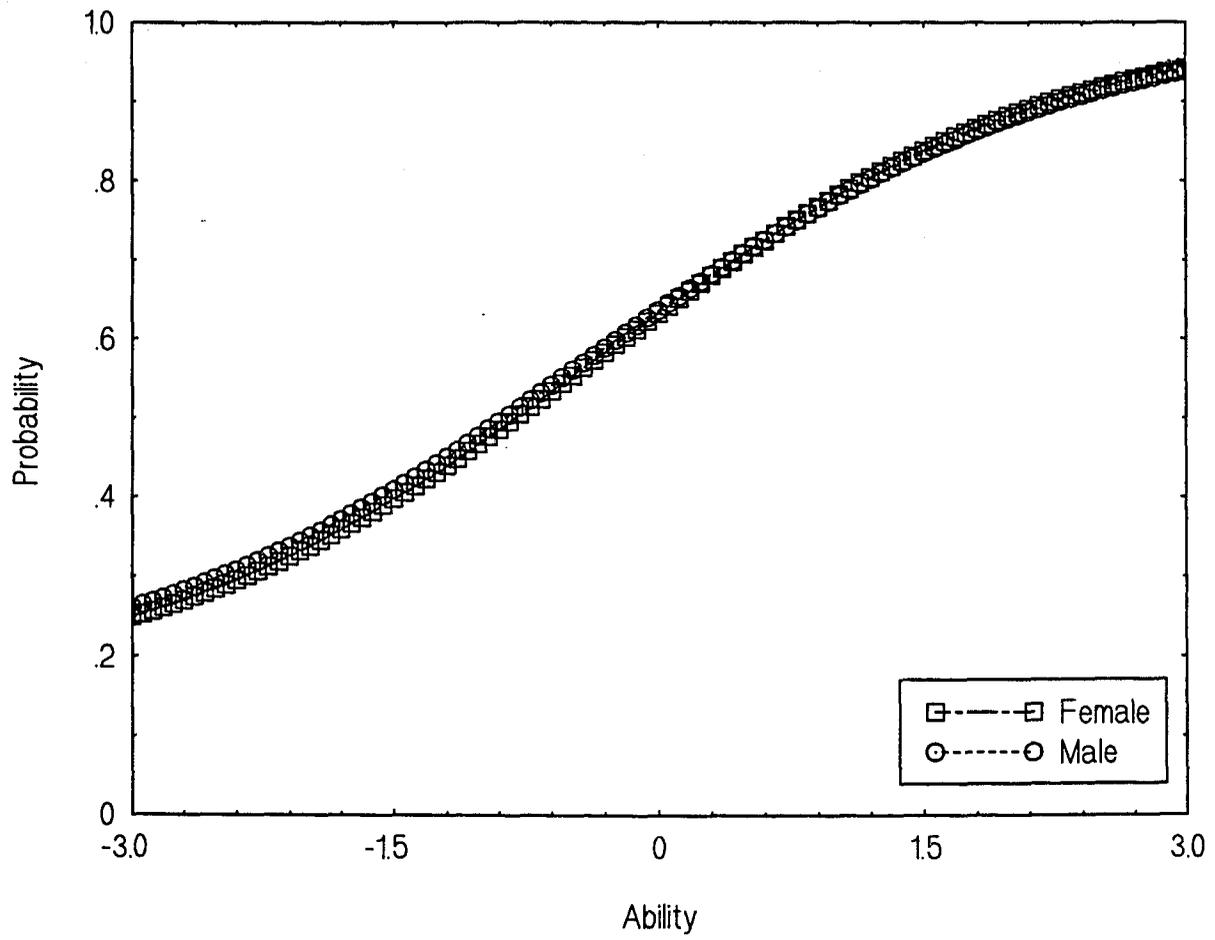
Male vs. Female: Item 6-27



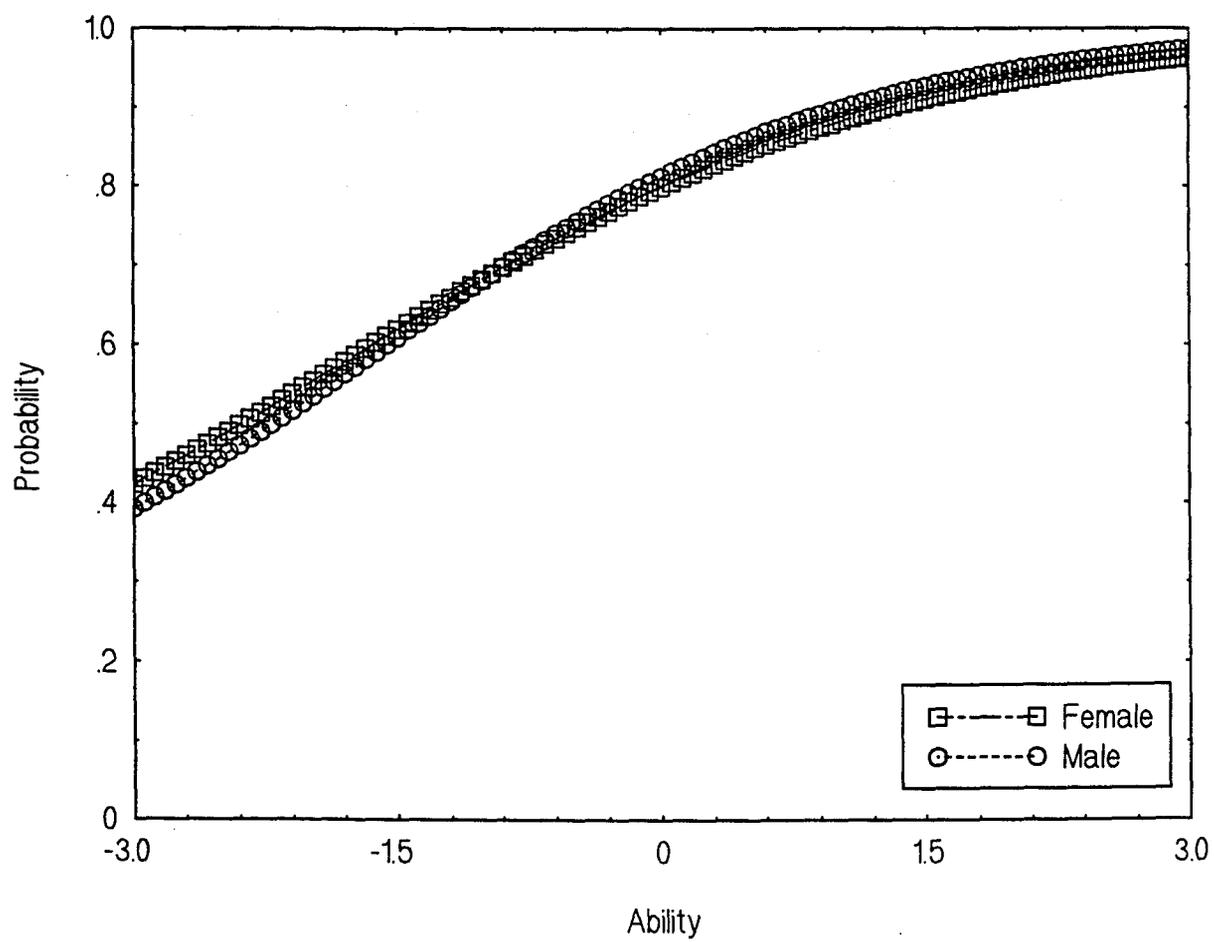
Male vs. Female: Item 6-28



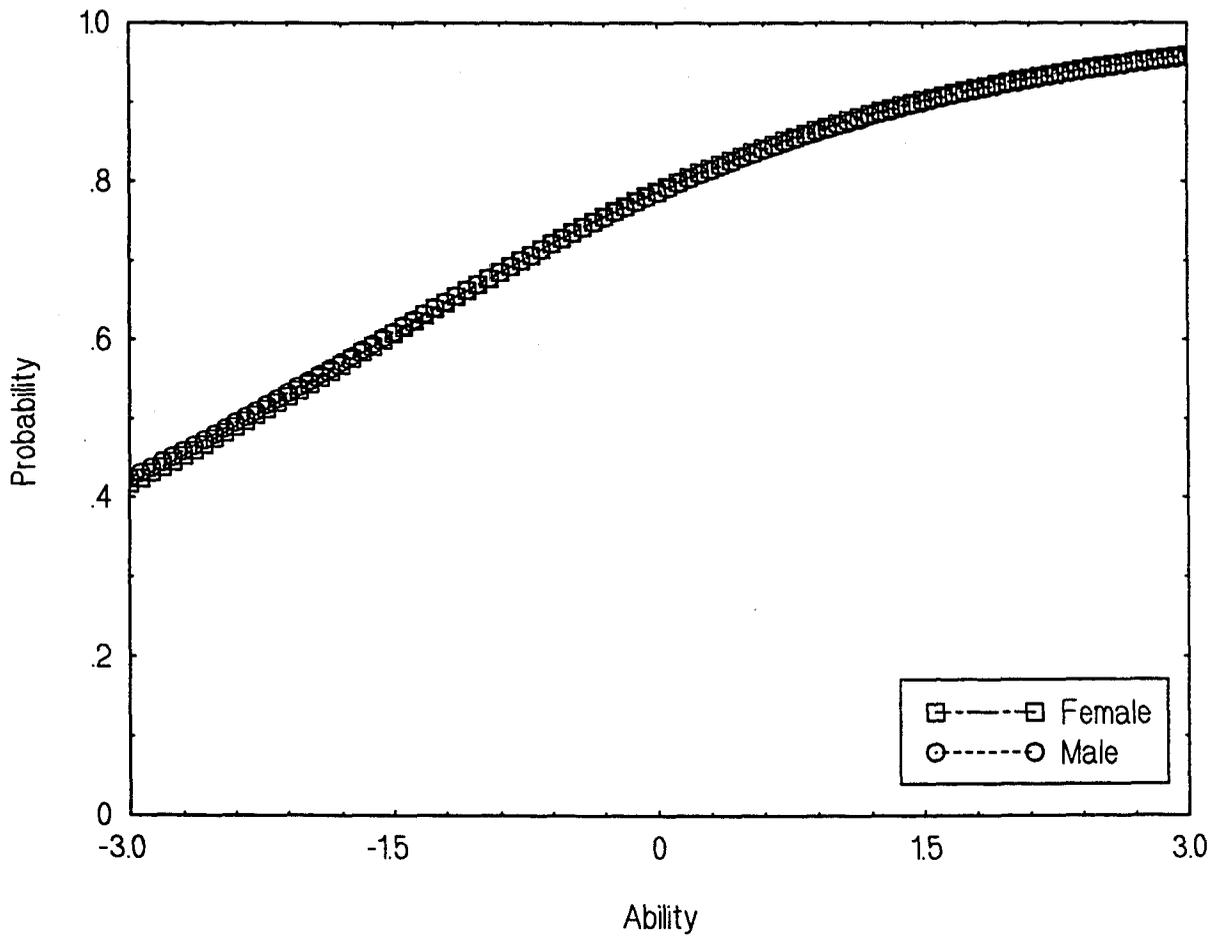
Male vs. Female: Item 6-29



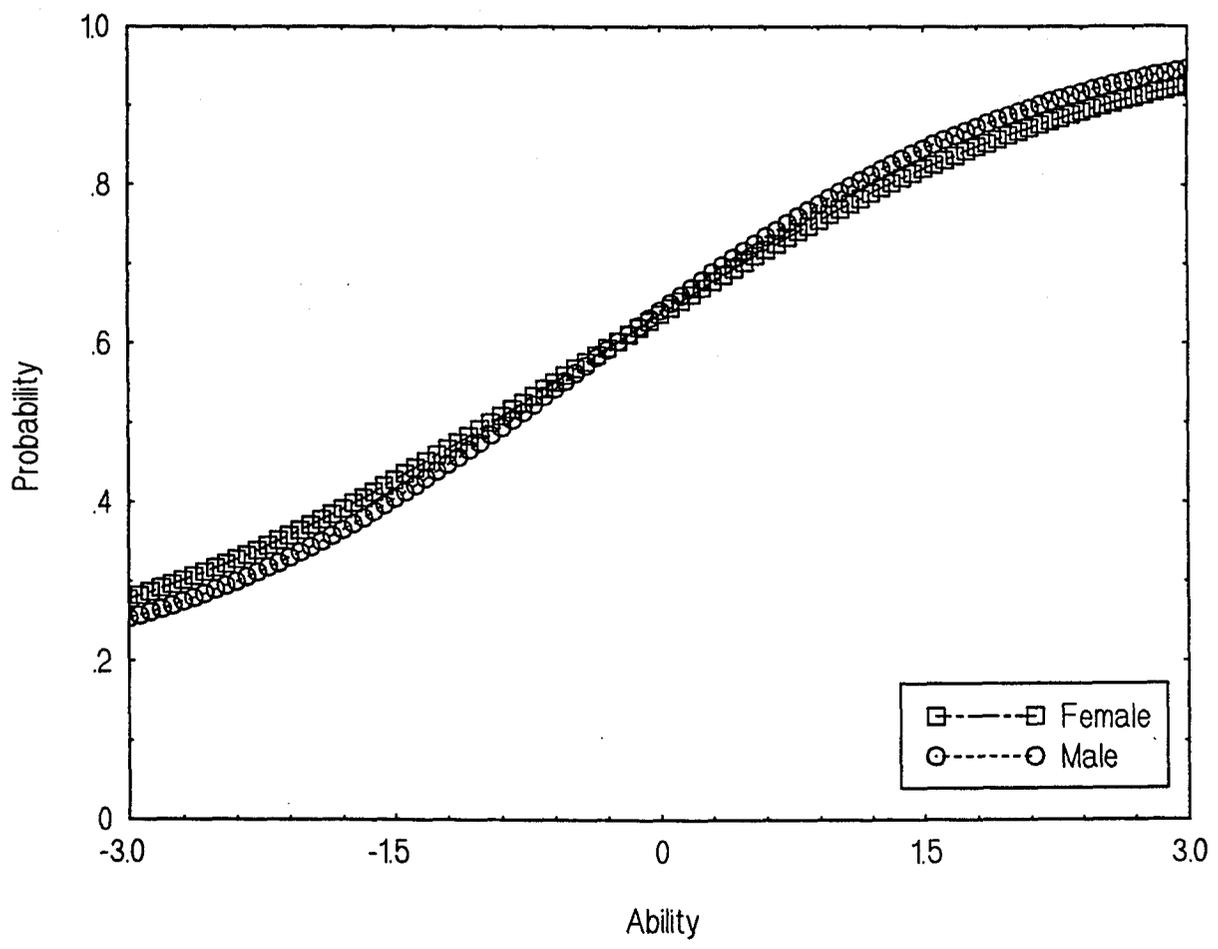
Male vs. Female: Item 6-30



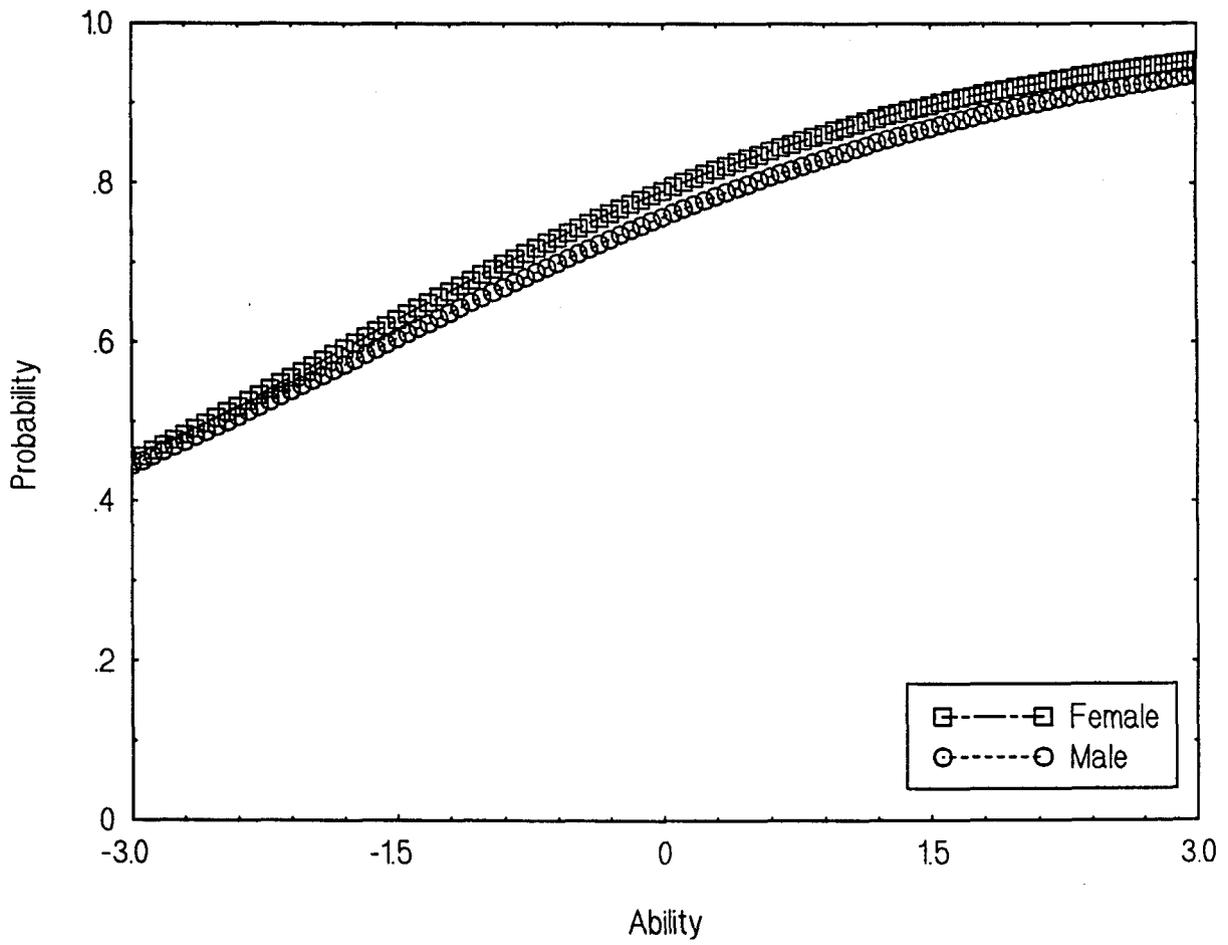
Male vs. Female: Item 6-31



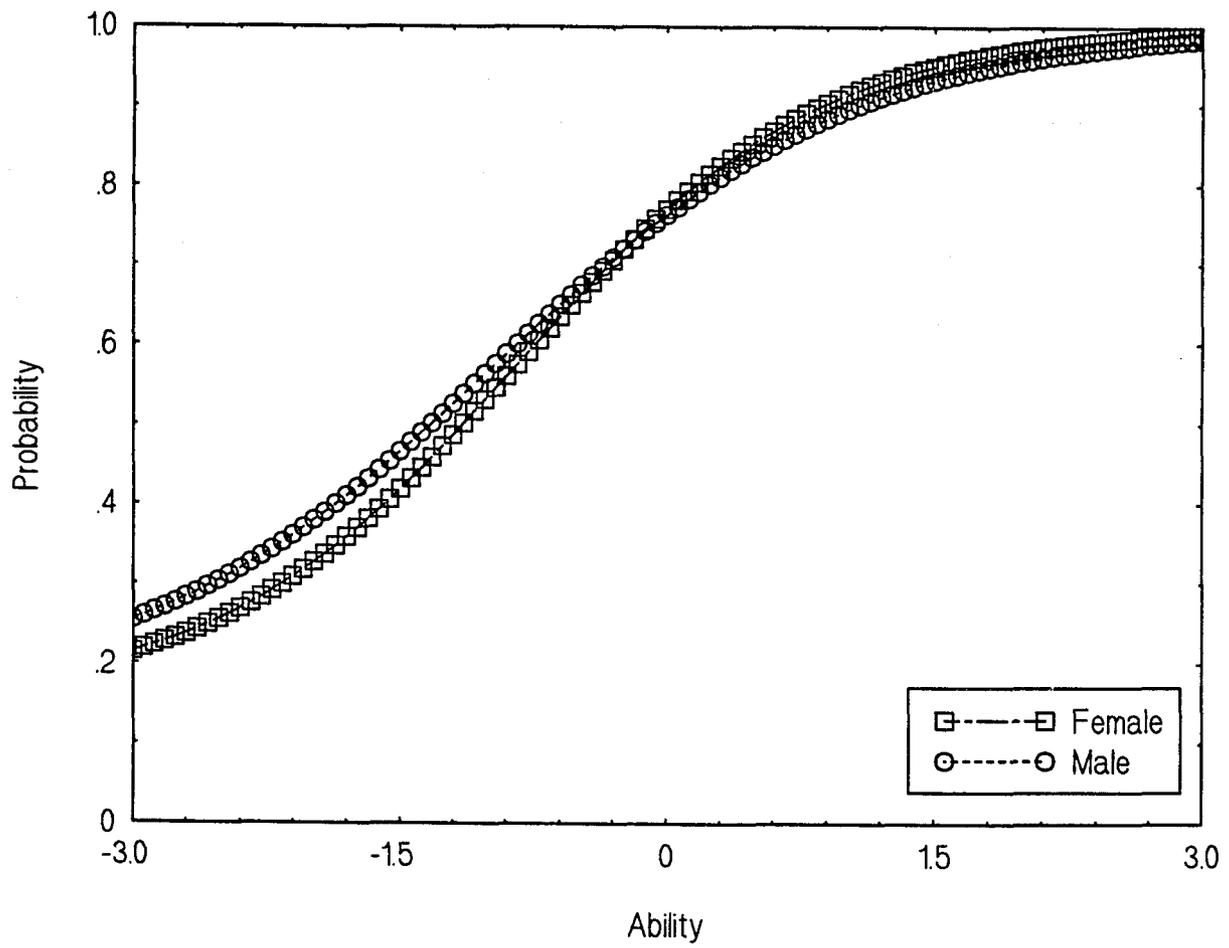
Male vs. Female: Item 6-32



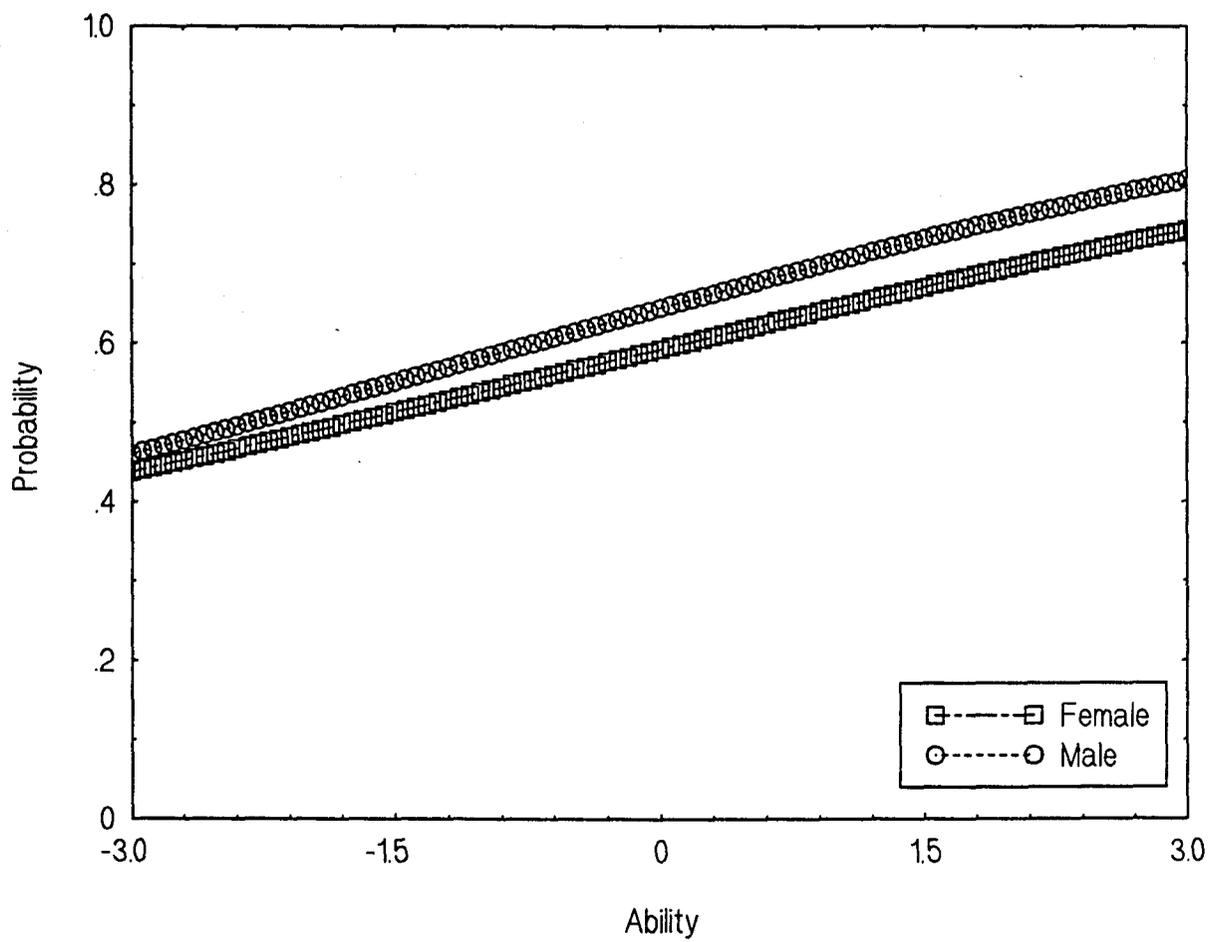
Male vs. Female: Item 6-33



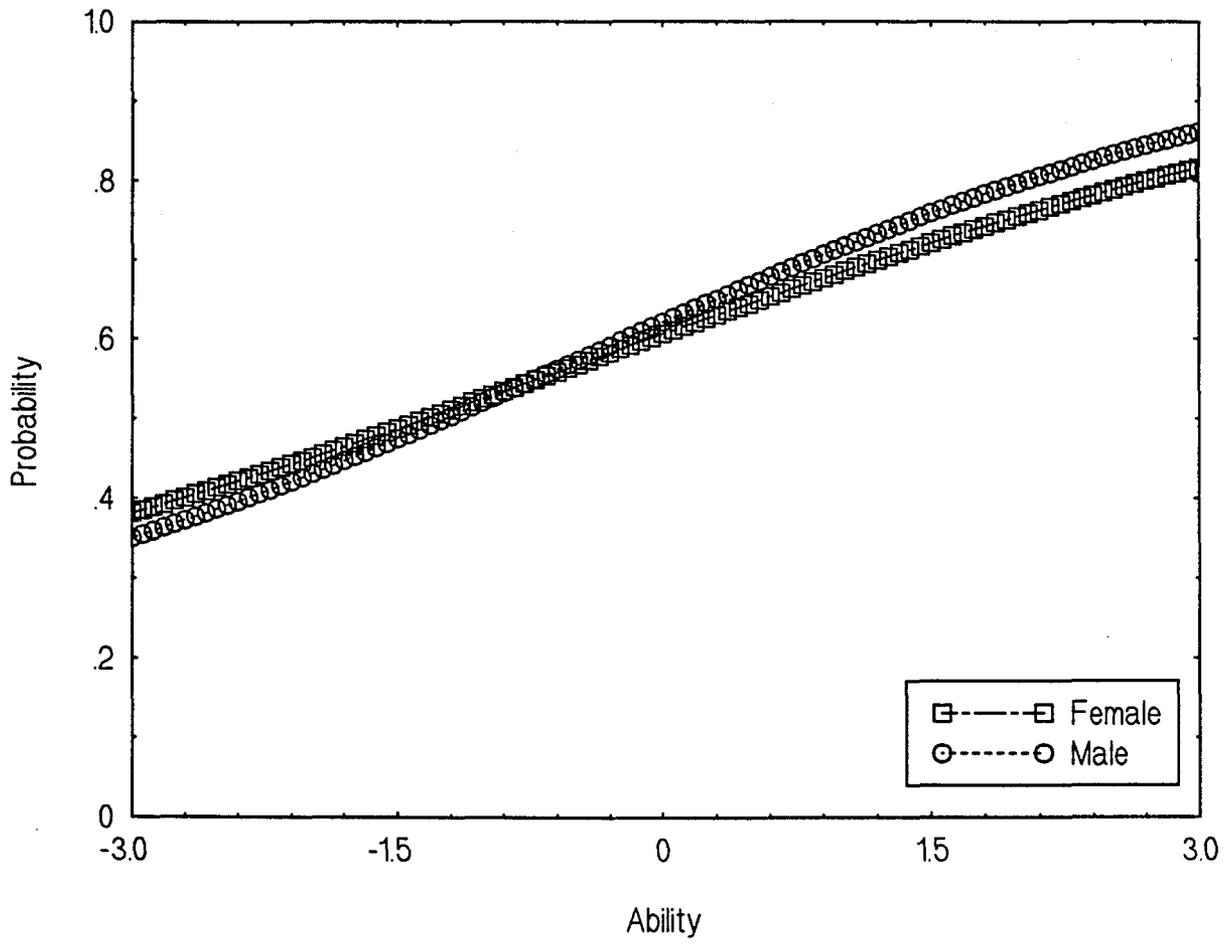
Male vs. Female: Item 6-34



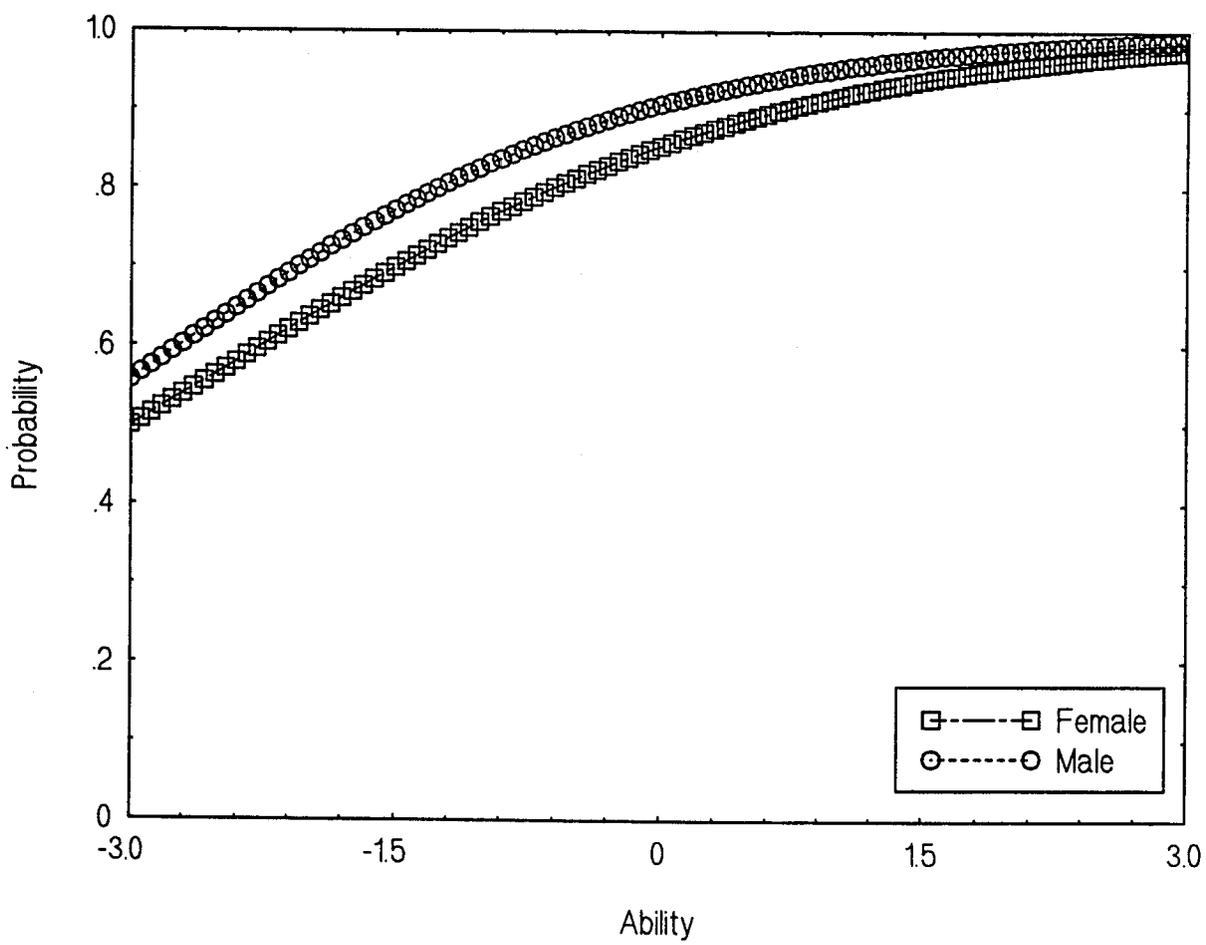
Male vs. Female: Item 6-35



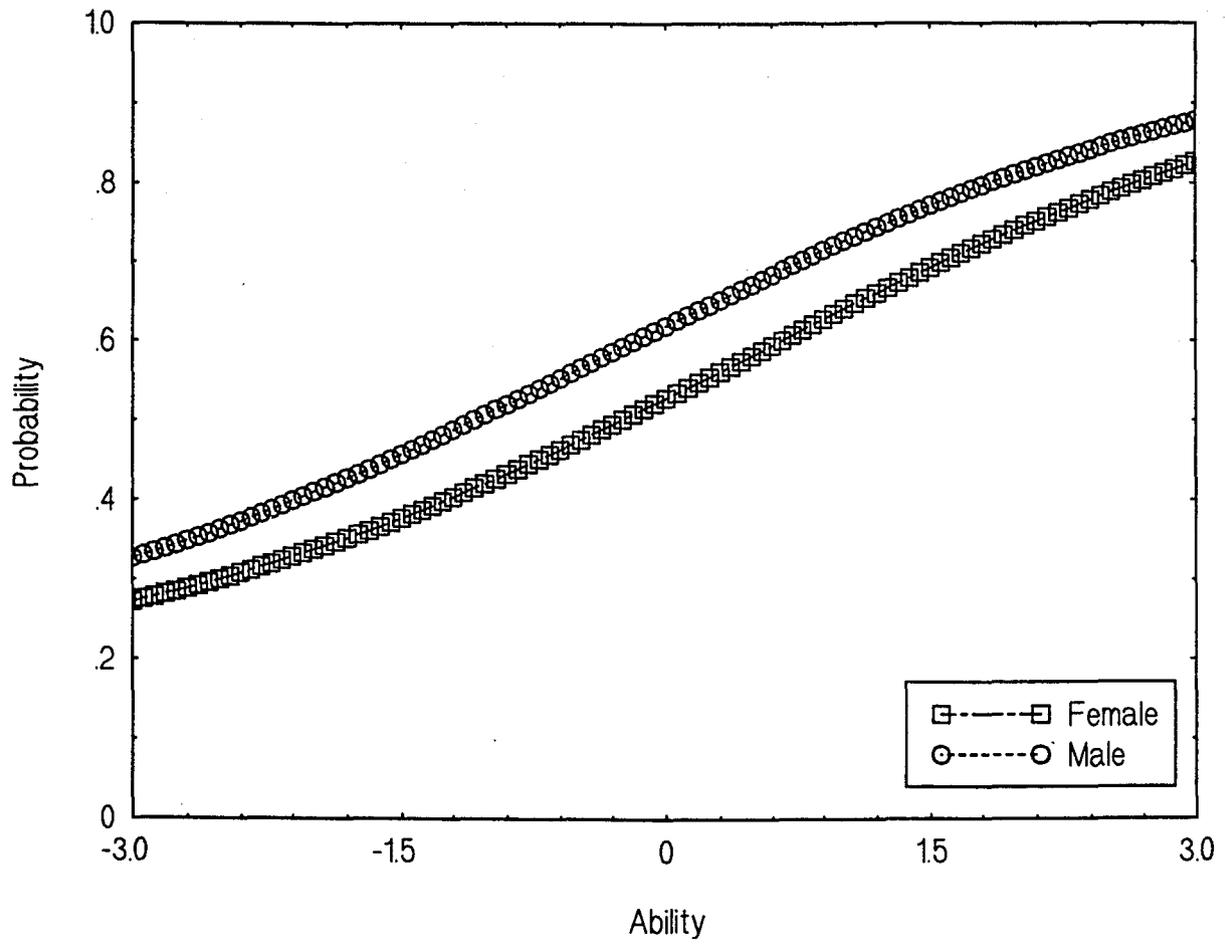
Male vs. Female: Item 7-1



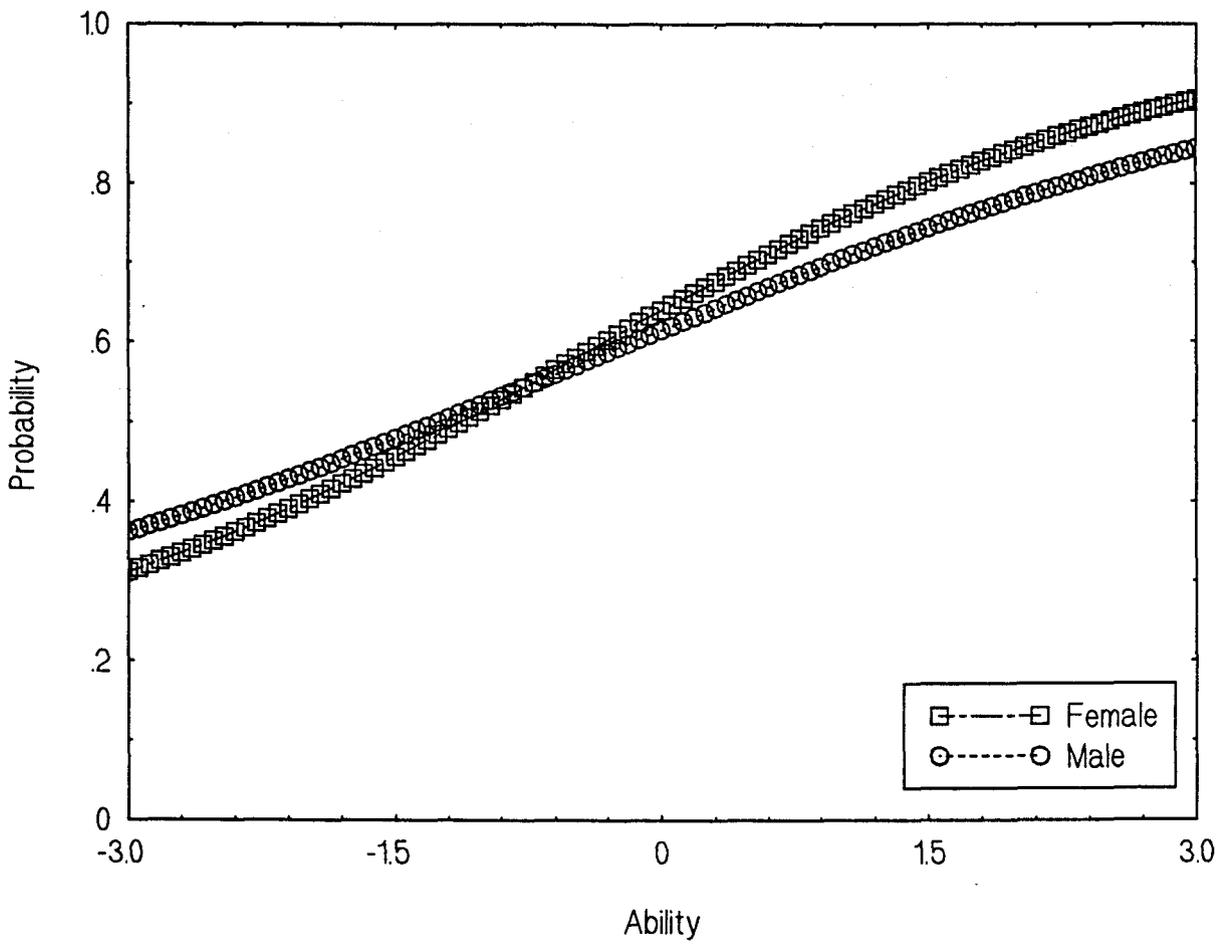
Male vs. Female: Item 7-2



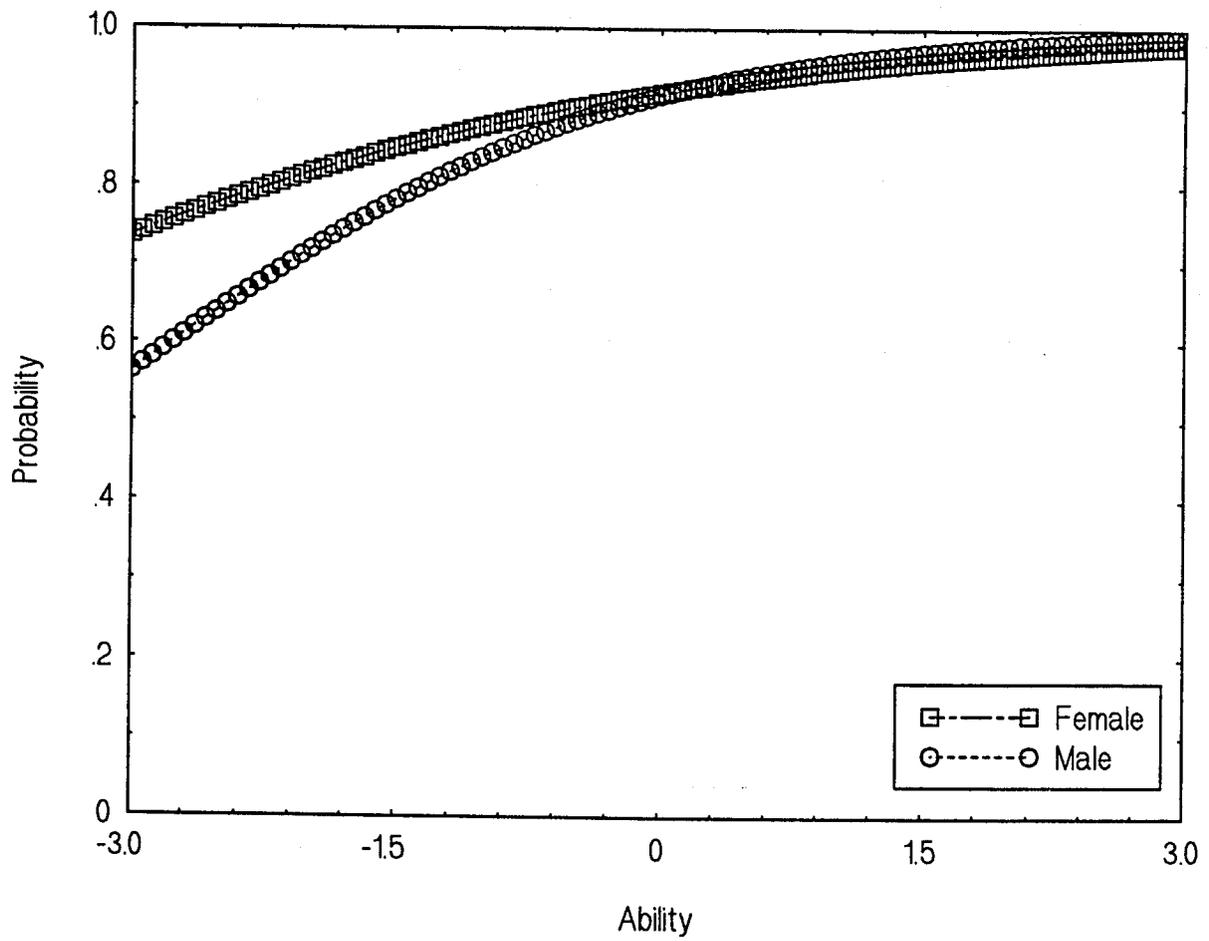
Male vs. Female: Item 7-3



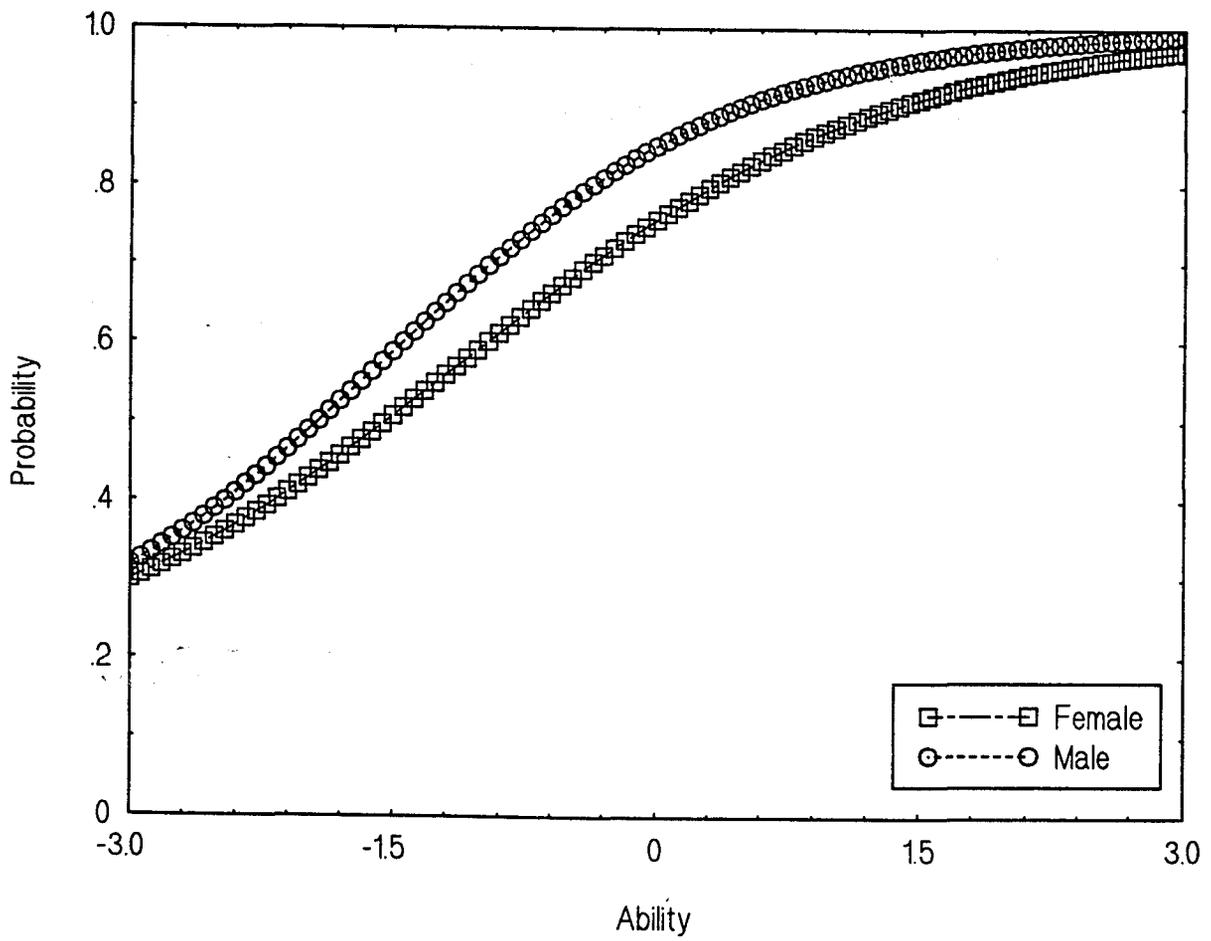
Male vs. Female: Item 7-4



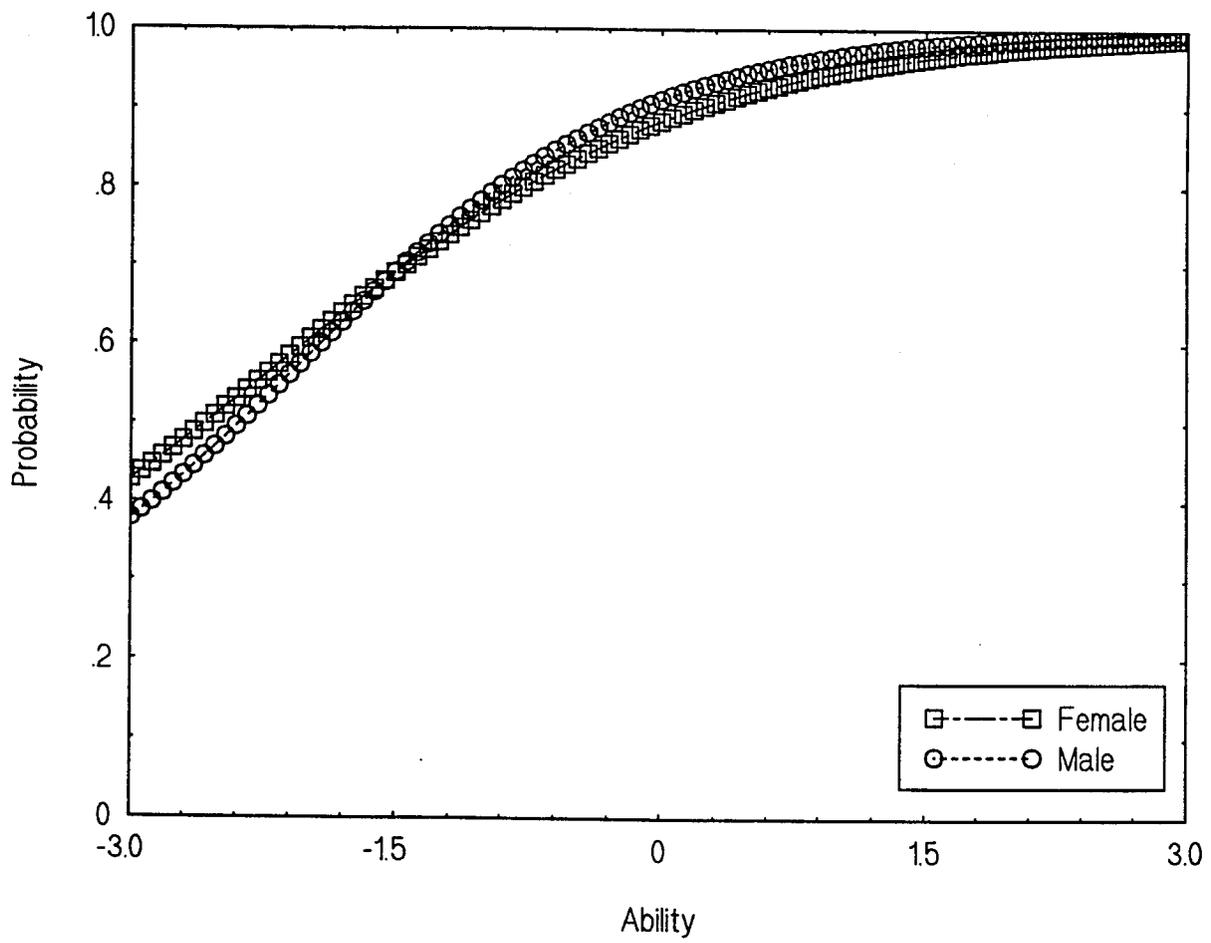
Male vs. Female: Item 7-5



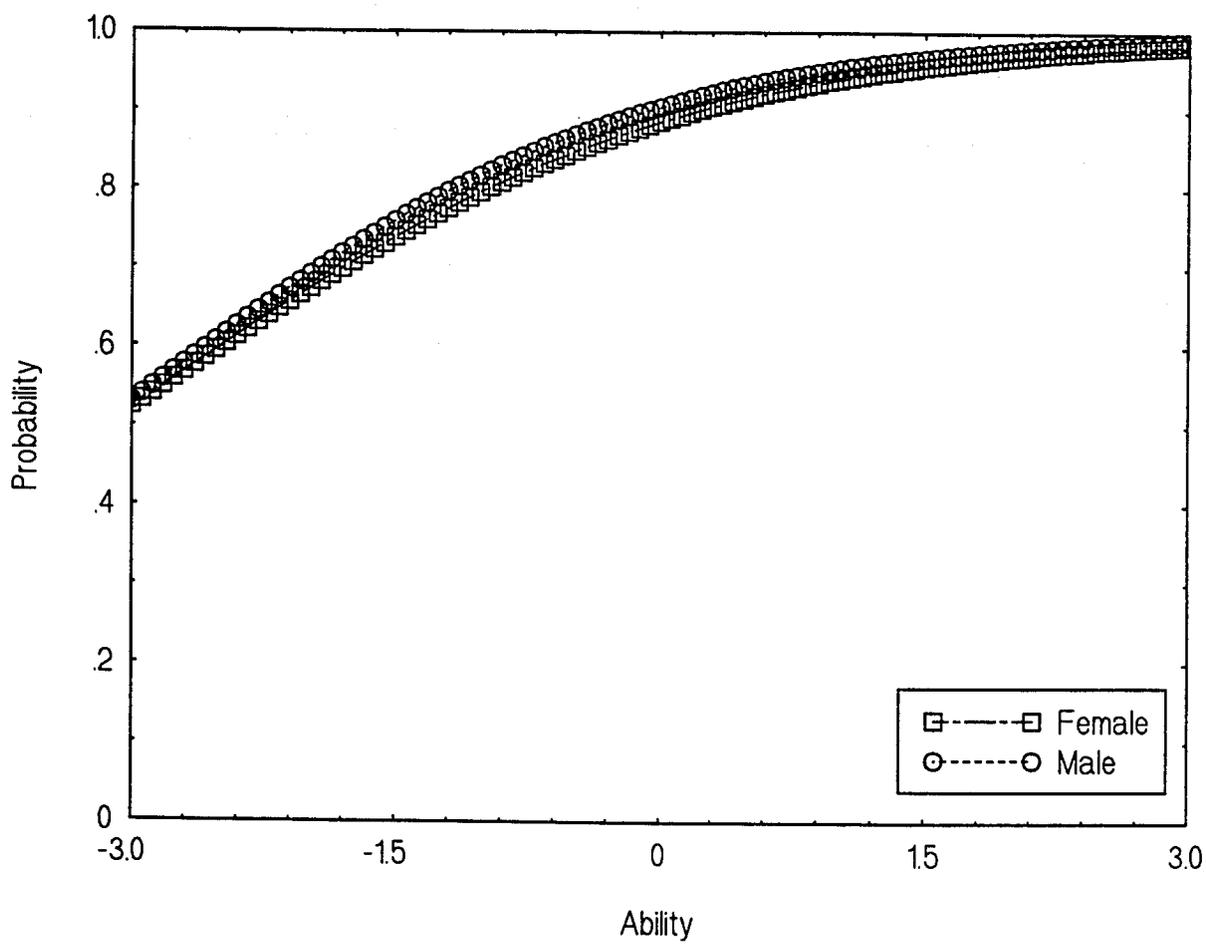
Male vs. Female: Item 7-6



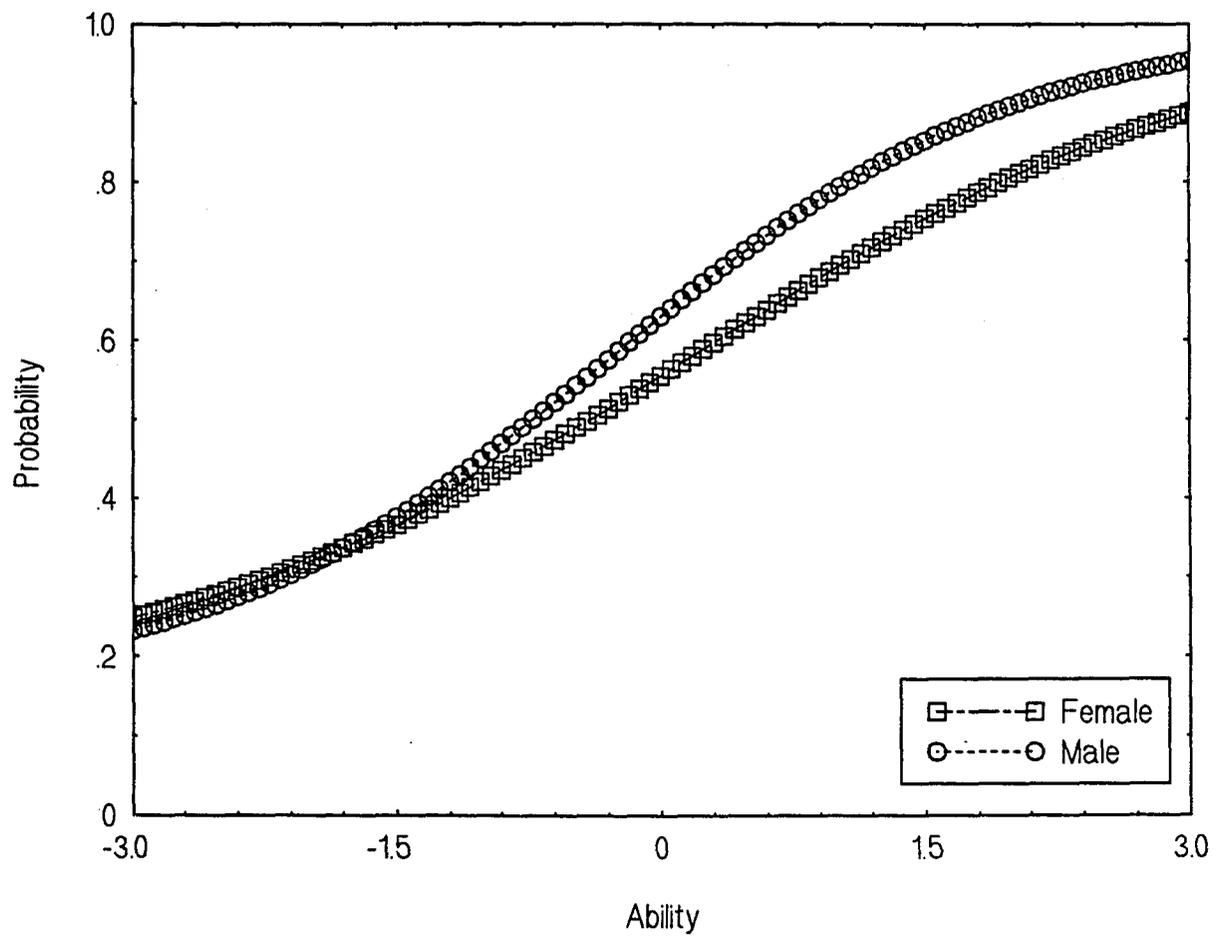
Male vs. Female: Item 7-7



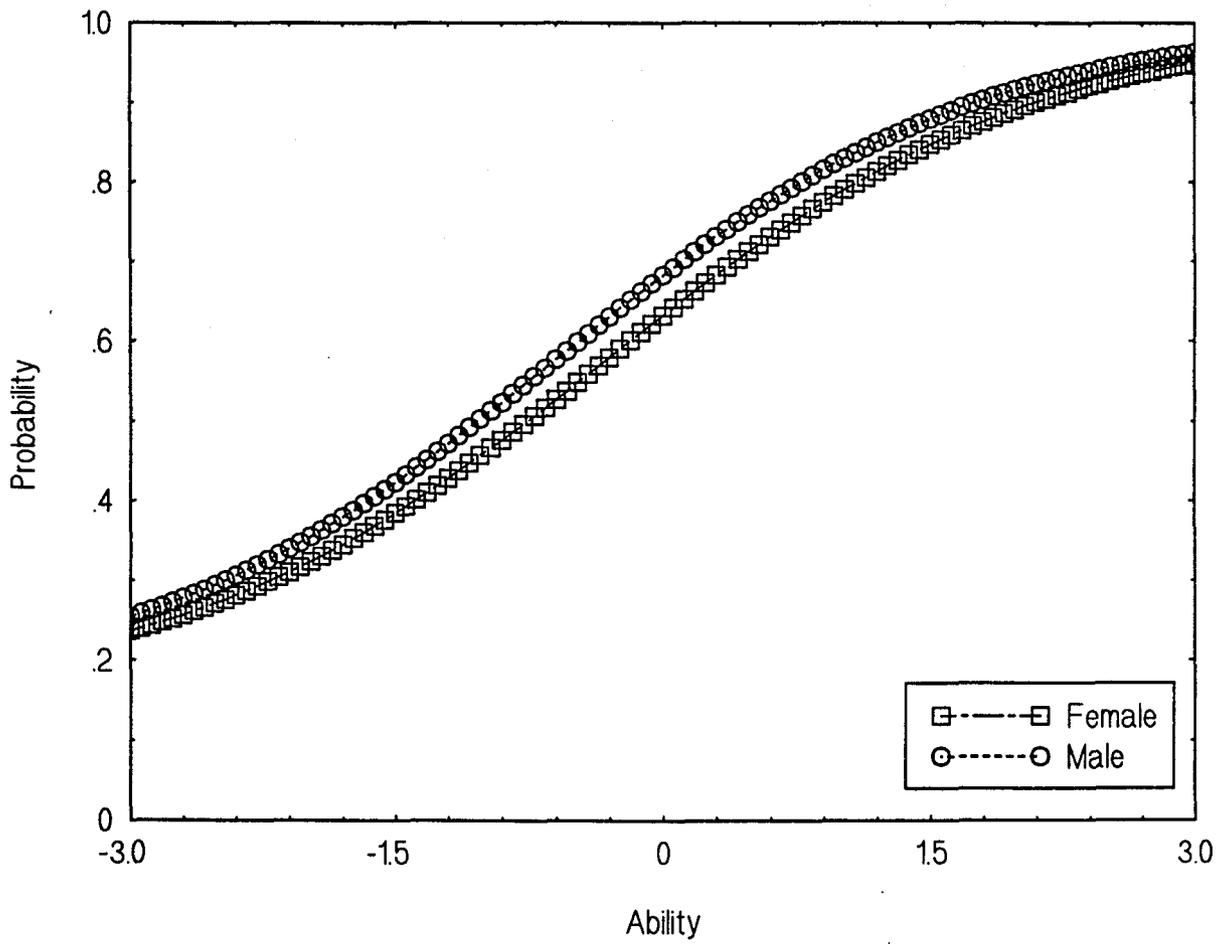
Male vs. Female: Item 7-8



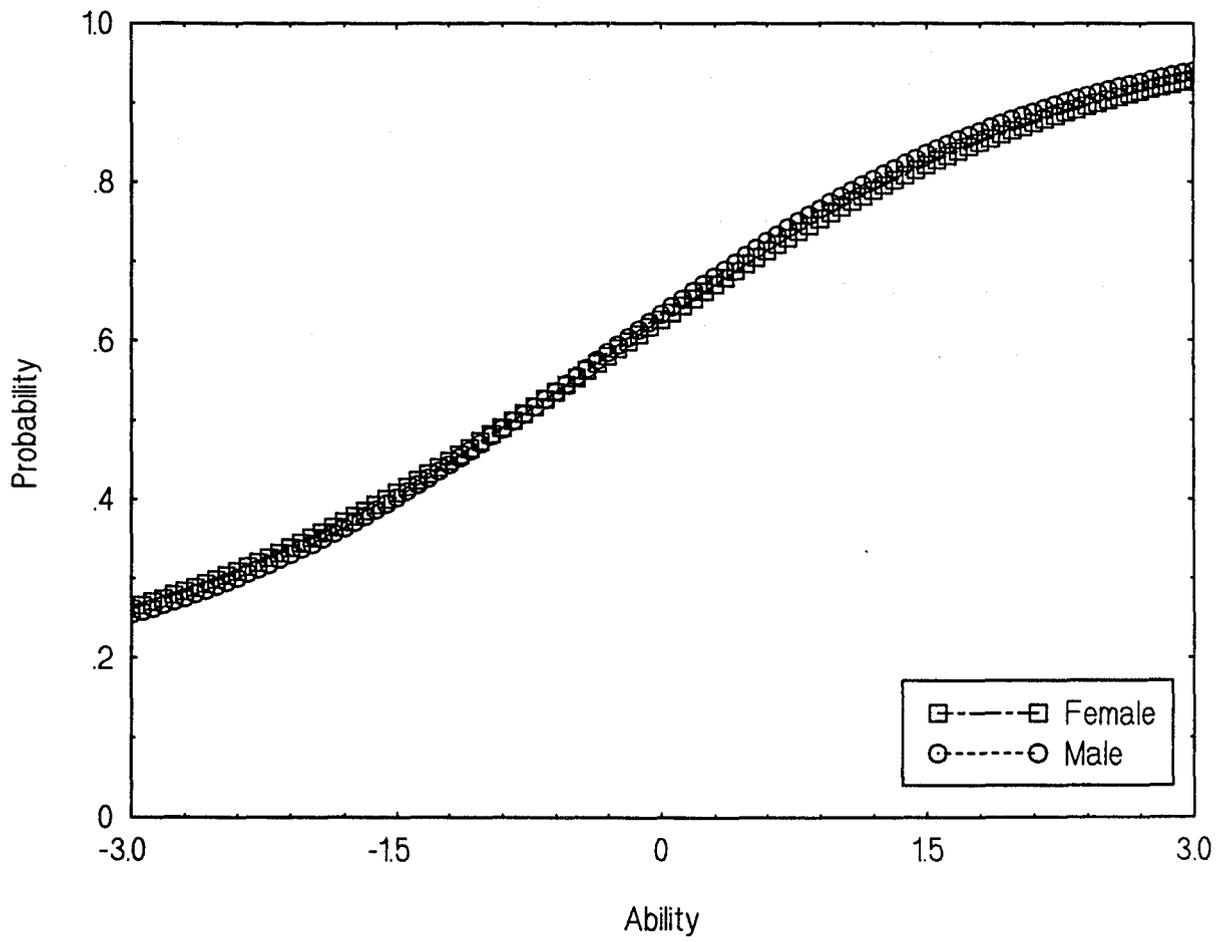
Male vs. Female: Item 7-9



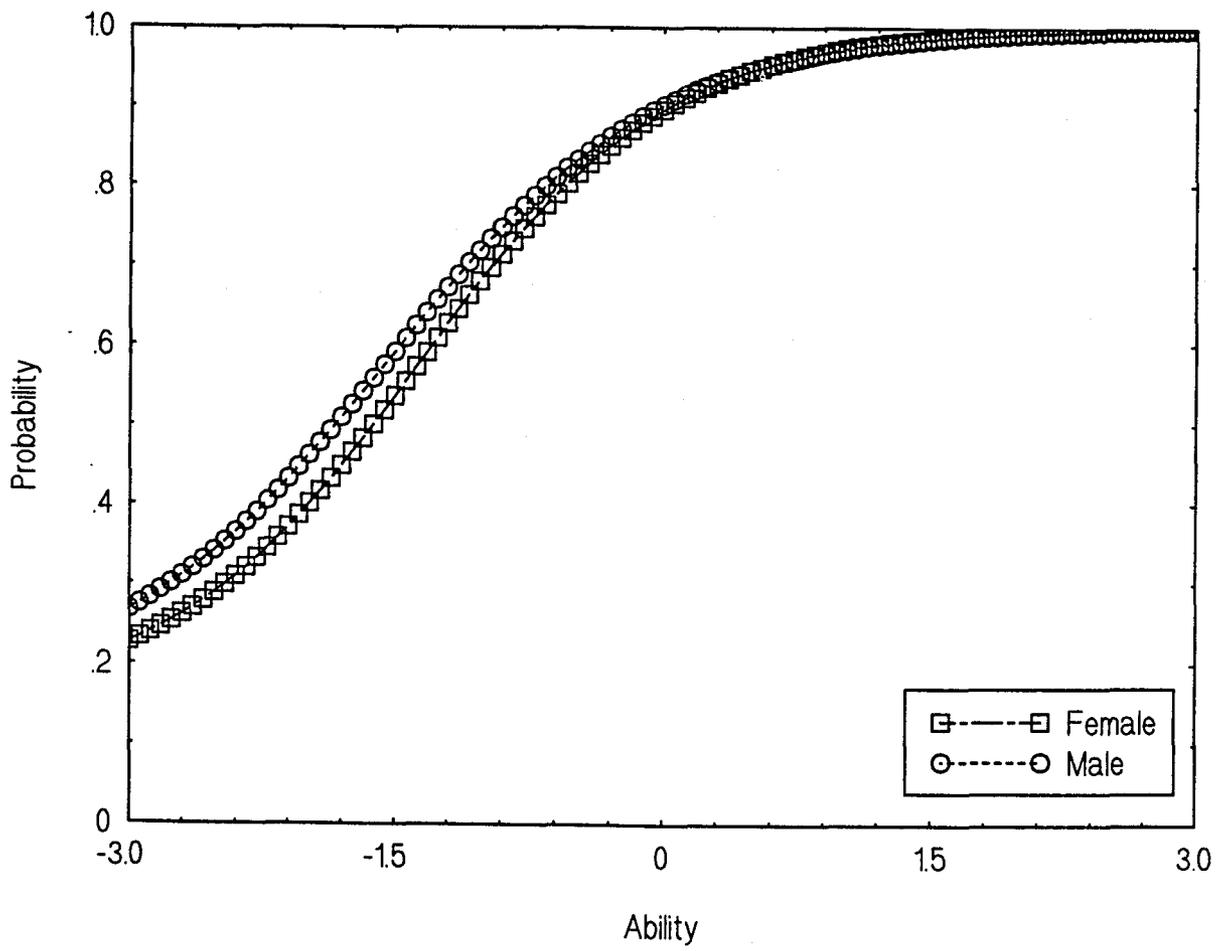
Male vs. Female: Item 7-10



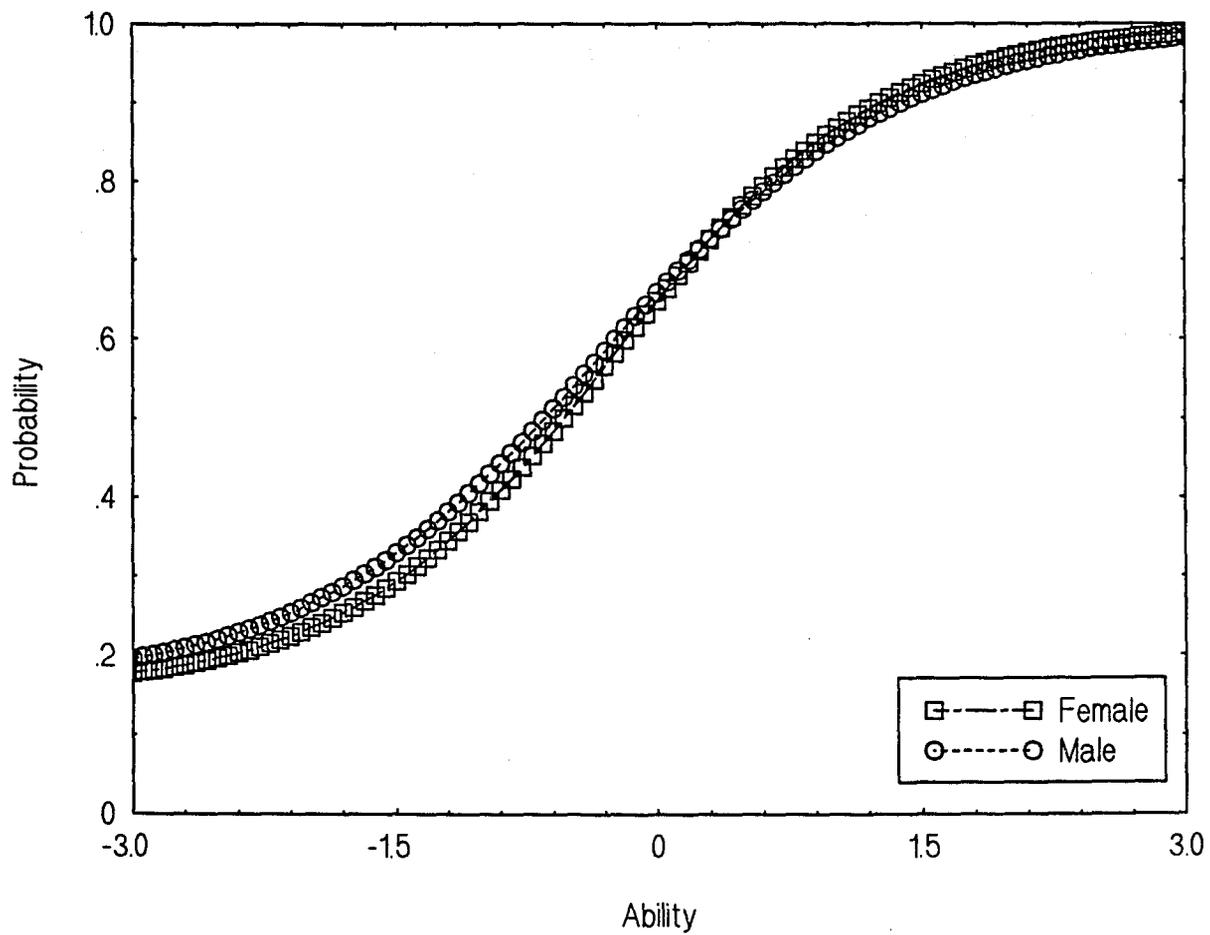
Male vs. Female: Item 7-11



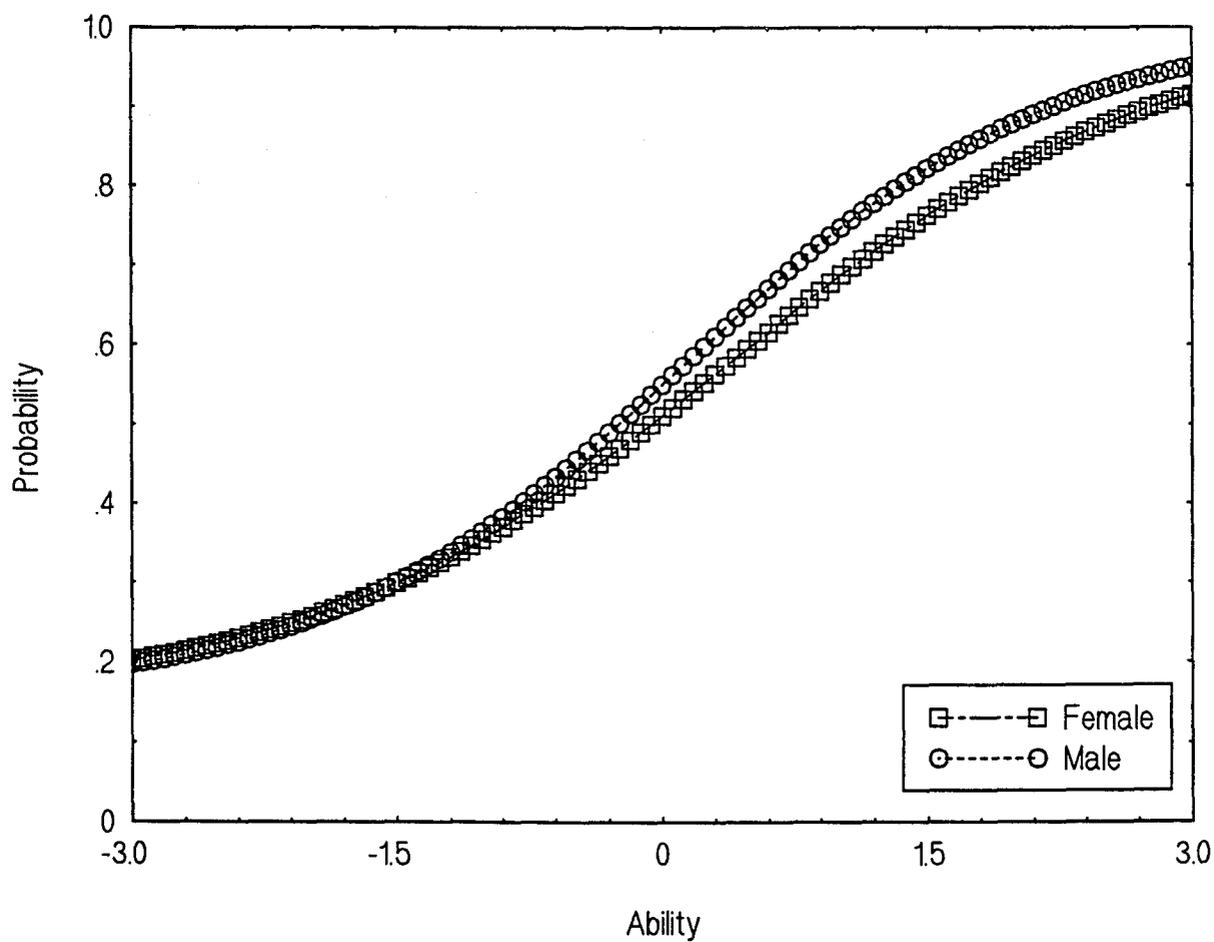
Male vs. Female: Item 7-12



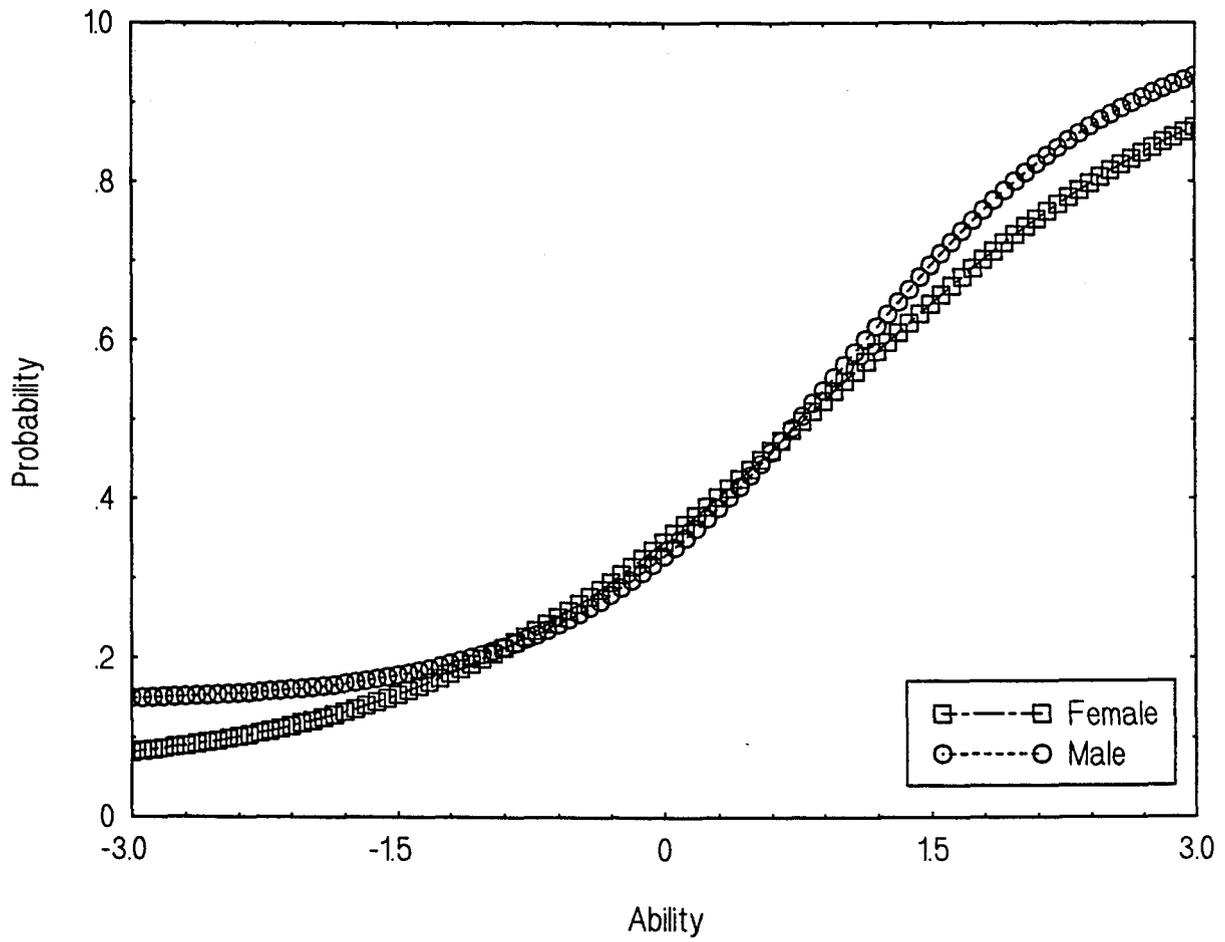
Male vs. Female: Item 7-13



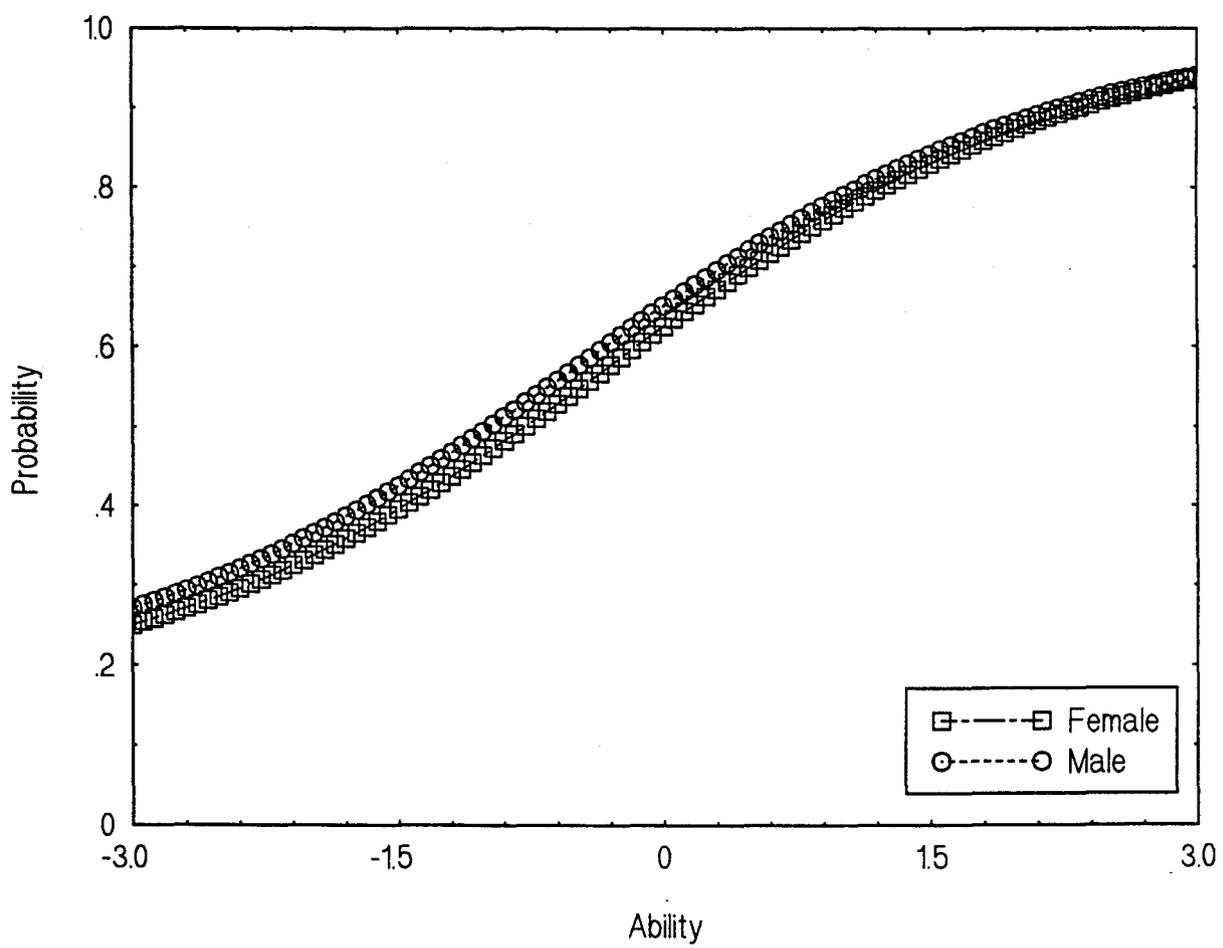
Male vs. Female: Item 7-14



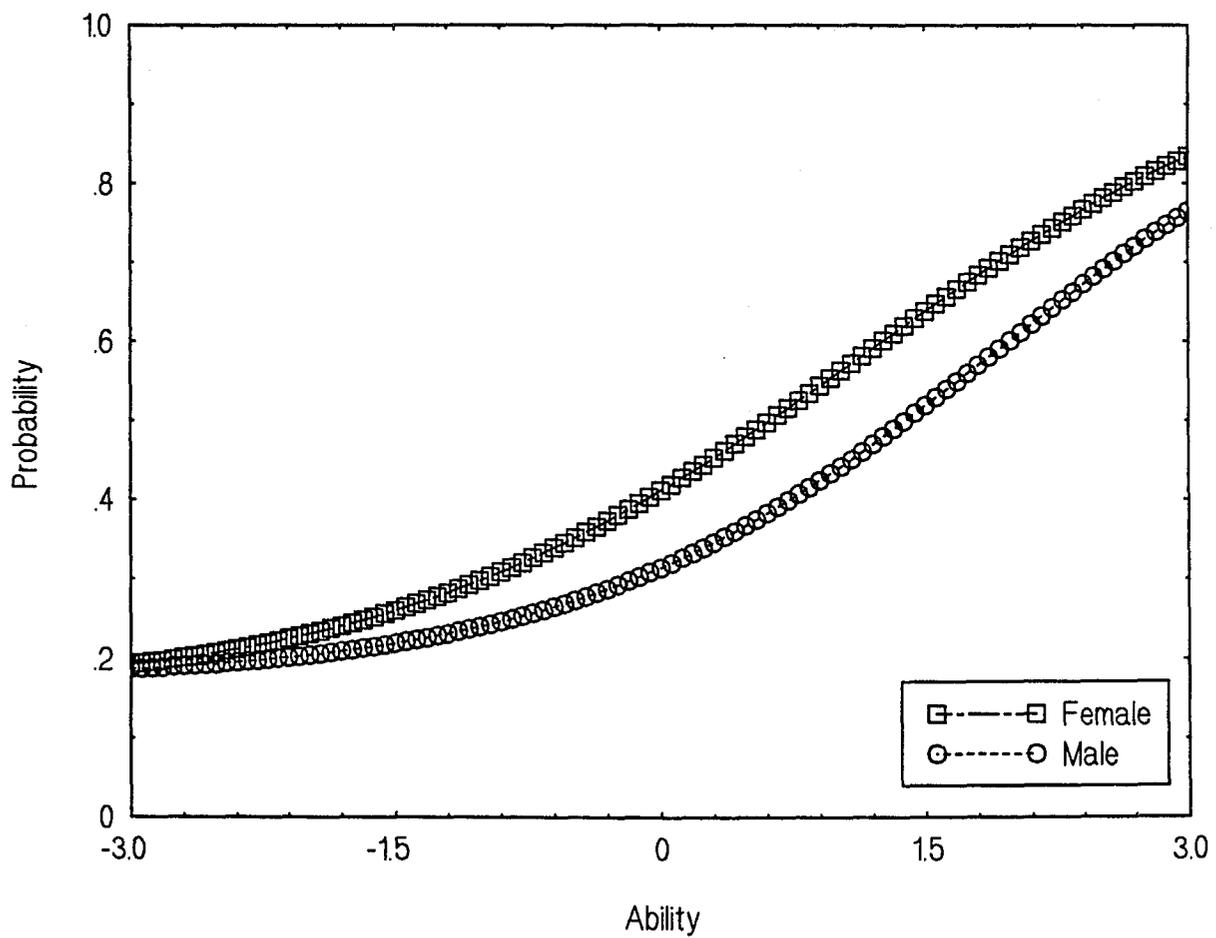
Male vs. Female: Item 7-15



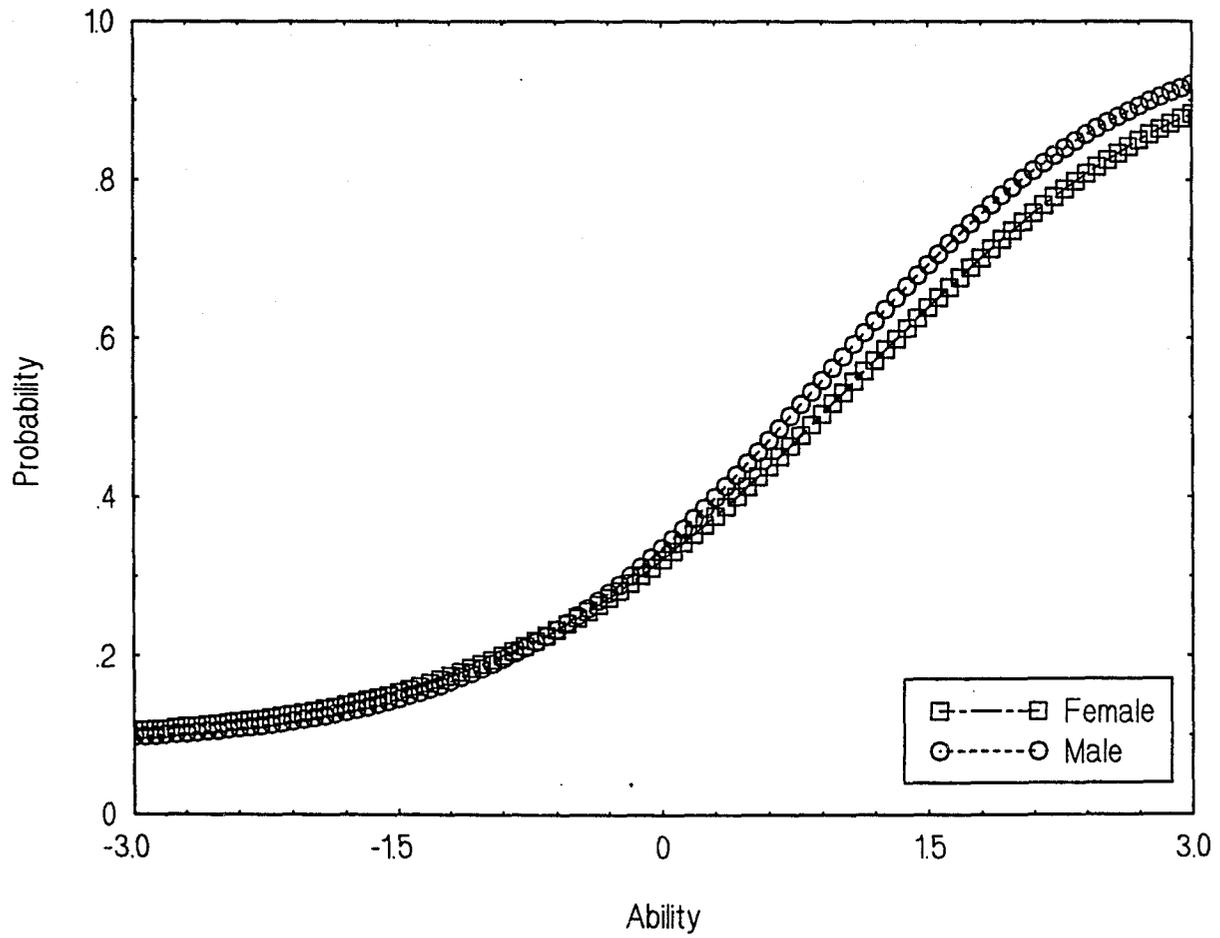
Male vs. Female: Item 7-16



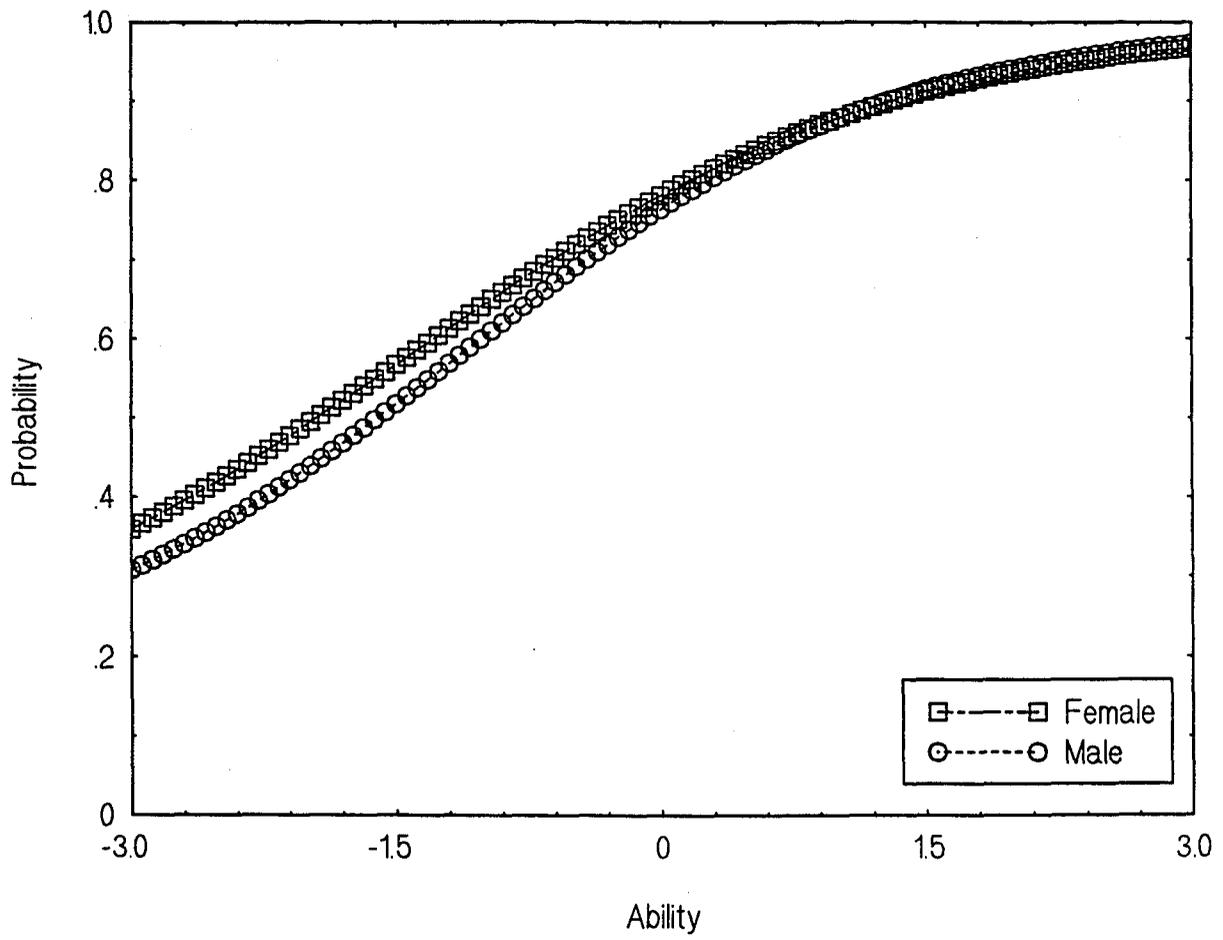
Male vs. Female: Item 7-17



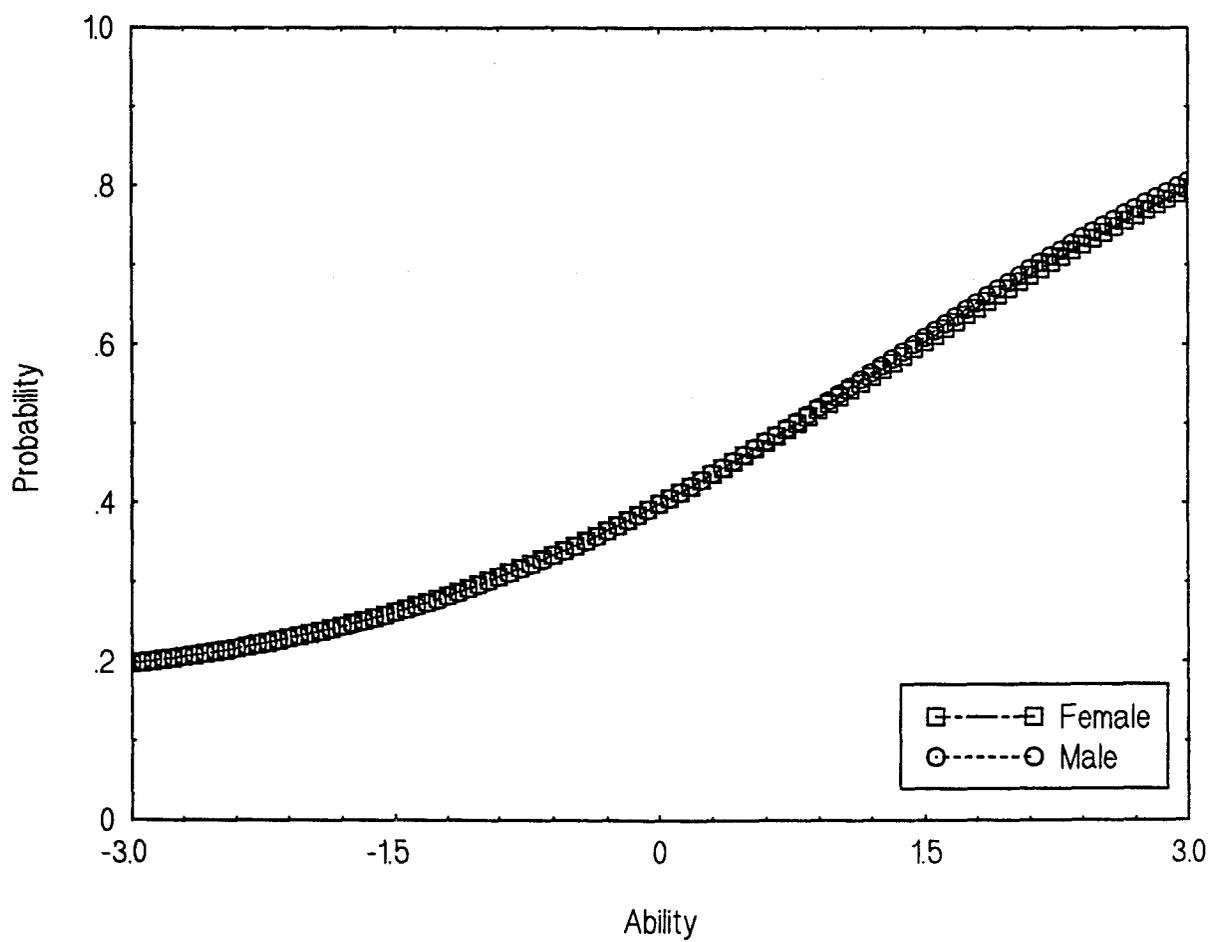
Male vs. Female: Item 7-18



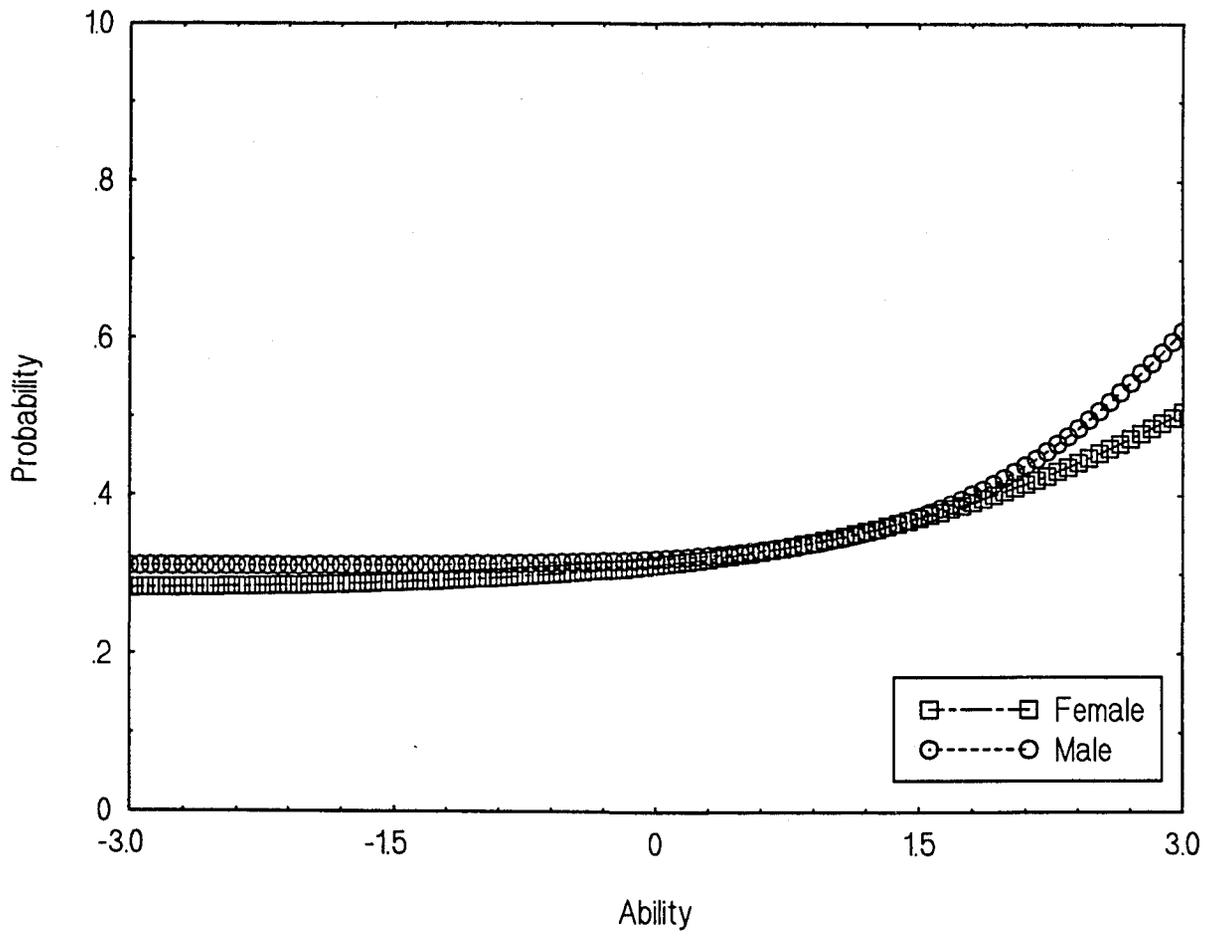
Male vs. Female: Item 7-19



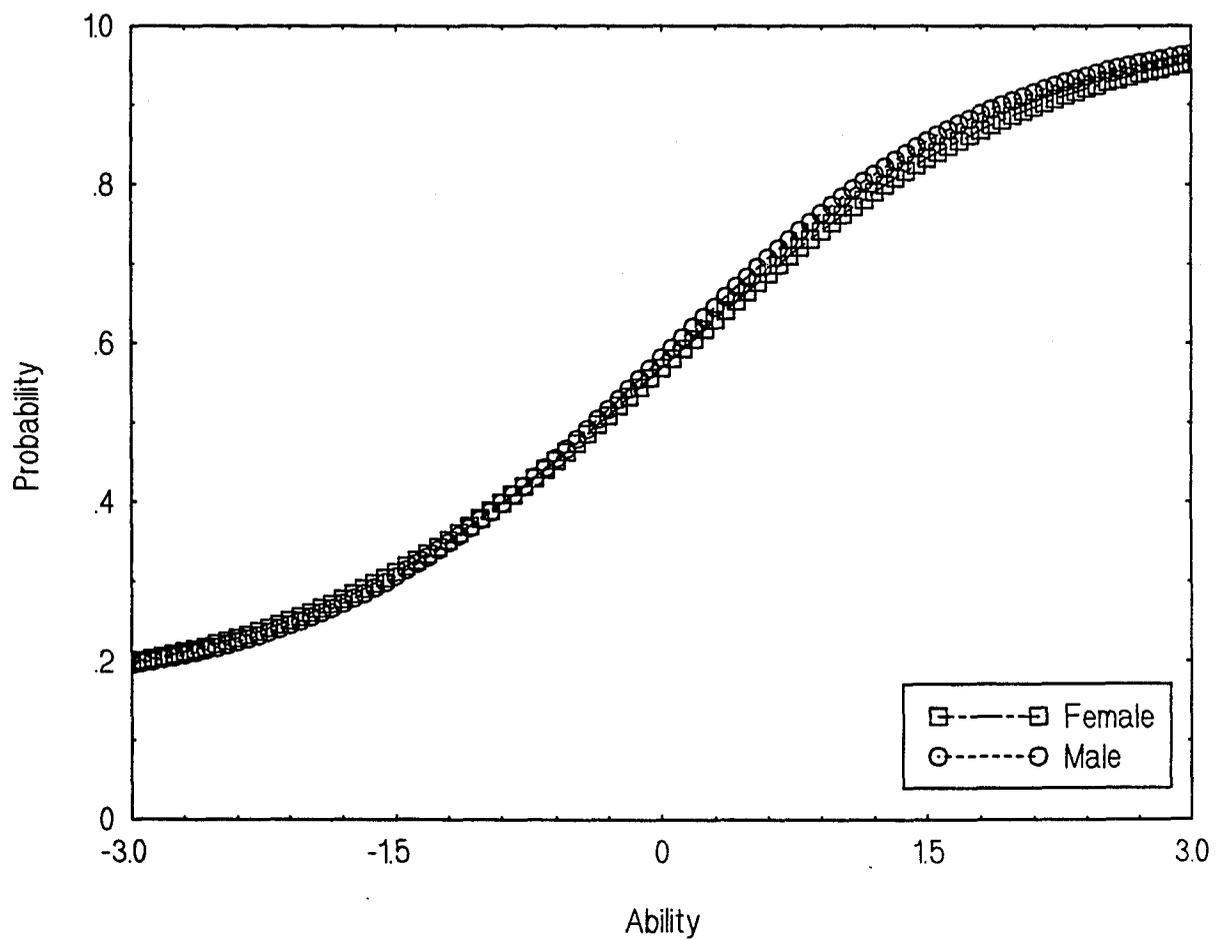
Male vs. Female: Item 7-20



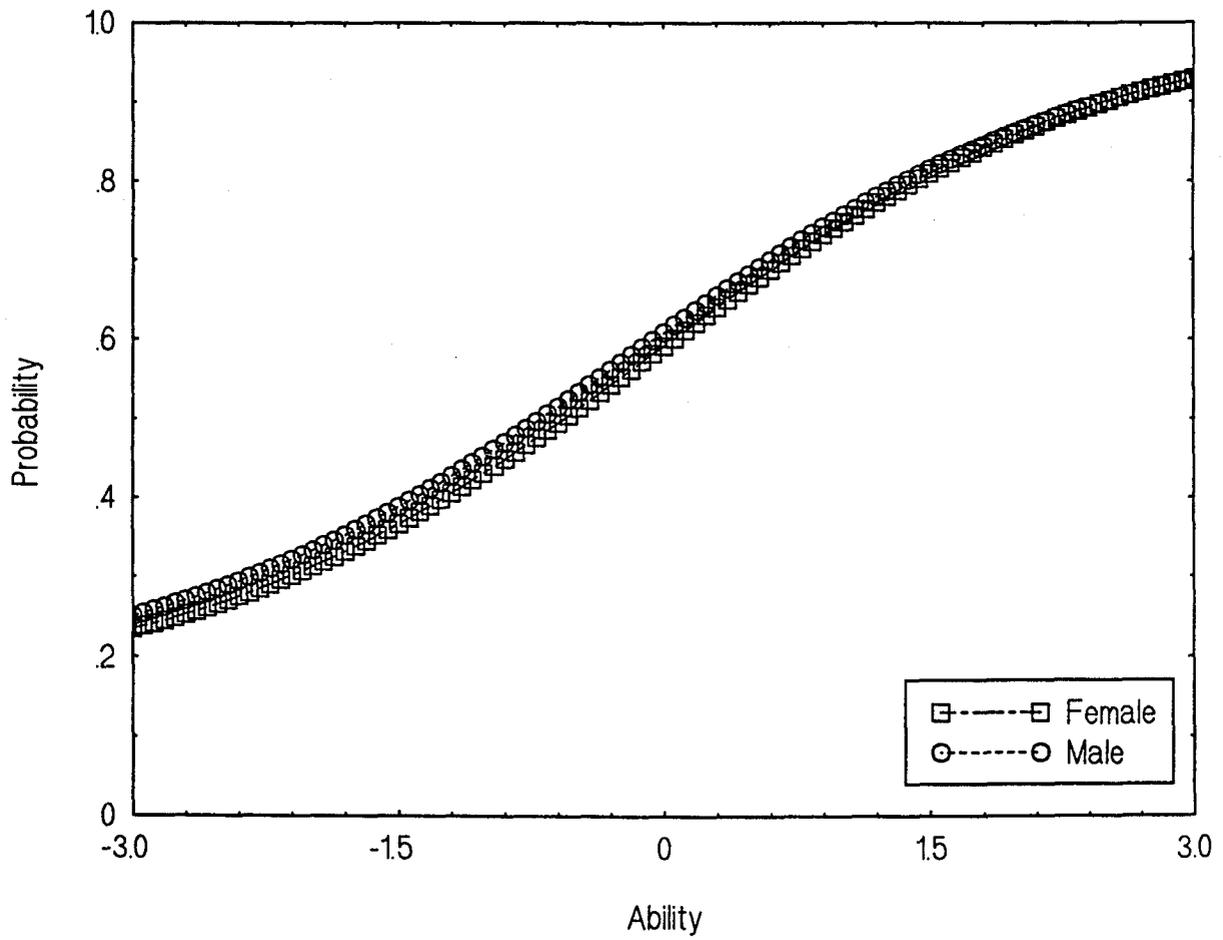
Male vs. Female: Item 7-21



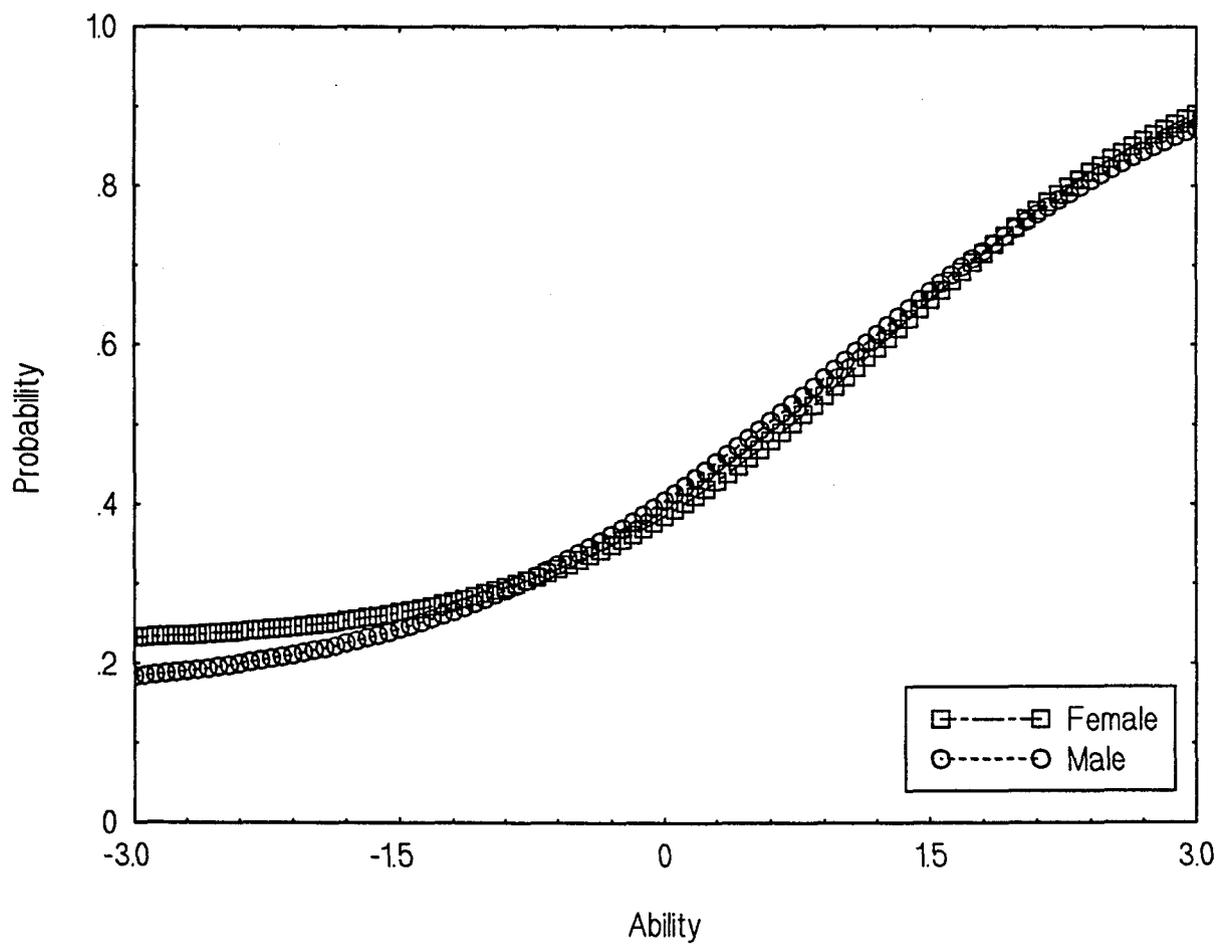
Male vs. Female: Item 7-22



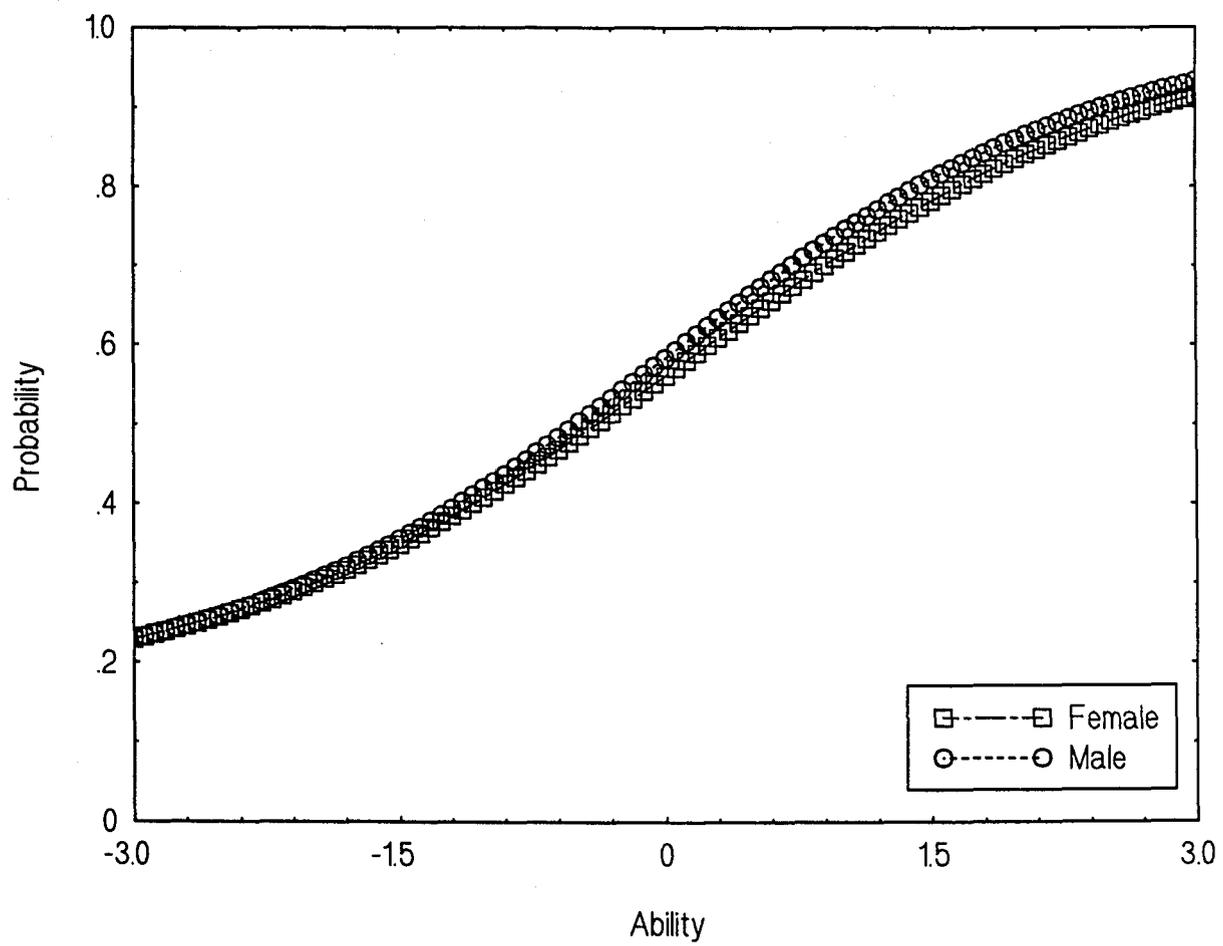
Male vs. Female: Item 7-23



Male vs. Female: Item 7-24



Male vs. Female: Item 7-25



APPENDIX G
CORRESPONDENCE OF DIF INDICES USING
BLACK AND WHITE MALE COMPARISON GROUPS

Item	Dif ₁ (3.4)	Dif ₂ (3.4)	MH χ^2 (11.5)
2-1	-0.64964	0.519413	4.872
2-2	-0.31043	0.328223	479.135
2-3	-1.10115	0.847162	128.166
2-4	0.058167	0.253559	8.159
2-5	0.266046	0.304524	685.924
2-6	-0.69606	0.661489	127.305
2-7	-0.76355	0.826071	97.613
2-8	-2.04791	1.569118	651.043
2-9	-0.25722	0.230893	26.861
2-10	-1.52403	1.019072	0.017
2-11	-0.05917	1.053892	0.146
2-12	1.12465	3.163341	43.324
2-13	-2.39132	2.338798	29.592
2-14	-2.53864	2.121813	0.215
2-15	0.125326	0.797485	351.365
2-16	-4.19126	2.54255	203.098
2-17	0.309277	0.969846	100.226
2-18	0.119592	0.462981	0.339
2-19	-1.55826	1.228778	15.795
2-20	-0.98043	1.498228	74.664
2-21	-3.03054	1.852684	13.768
2-22	-0.17878	0.303256	103.467
2-23	-1.46969	2.449283	144.371
2-24	0.445543	1.996878	1.393
2-25	-2.35845	2.021726	7.224

6-1	0.580154	1.774853	36.952
6-2	1.673464	3.177691	266.602
6-3	-0.89609	0.675712	22.299
6-4	-0.32464	0.44894	8.038
6-5	0.122568	3.395087	36.799
6-6	-0.54047	0.521808	206.371
6-7	-1.55794	0.912099	51.449
6-8	-3.18247	2.172177	147.88
6-9	-0.24636	0.292999	32.053
6-10	-0.70596	0.750217	87.256
6-11	-1.8867	1.286216	113.208
6-12	-5.08458	3.461485	3.148
6-13	-0.59008	0.580567	15.834
6-14	-0.15492	0.226021	13.965
6-15	-0.8559	0.627967	32.143
6-16	-0.79946	0.711832	115.082
6-17	-1.27374	0.863482	0.059
6-18	-2.22935	1.256712	40.256
6-19	-1.96651	1.496441	16.22
6-20	-0.60599	0.631229	42.053
6-21	-1.13713	0.825043	86.963
6-22	-0.4785	0.591308	184.909
6-23	-0.68711	0.631767	9.8
6-24	-0.75131	0.611824	27.099
6-25	-1.44447	1.023374	55.619
6-26	-3.09247	1.812167	19.932
6-27	-0.88154	0.650122	82.09

6-28	-2.5238	1.547636	80.927
6-29	0.002603	1.284336	14.851
6-30	-0.87372	0.603504	55.421
6-31	-0.4469	0.36074	14.103
6-32	-1.6881	1.274427	96.157
6-33	-0.54721	0.464123	4.232
6-34	-2.75012	1.676795	62.913
6-35	-0.25671	0.283418	30.973
7-1	-0.51965	0.514353	3.773
7-2	-1.1147	0.840992	34.779
7-3	-0.43409	0.523111	1.262
7-4	-0.30044	0.352646	37.185
7-5	-0.78221	0.579669	2.563
7-6	-2.98447	1.795085	59.828
7-7	-2.8716	1.673463	15.72
7-8	-1.27401	0.906784	43.509
7-9	0.859231	2.601182	215.206
7-10	-3.78601	2.616706	301.33
7-11	-2.19533	1.76851	59.945
7-12	-4.86836	3.341092	87.778
7-13	1.156864	4.084359	92.176
7-14	-2.31893	2.607933	19.304
7-15	-3.62111	3.904668	20.216
7-16	-1.9322	1.553176	84.829
7-17	0.347081	1.448056	1.765
7-18	2.349248	4.178345	2.859
7-19	-2.19753	1.660793	189.09

7-20	-1.31423	1.789342	104.384
7-21	-0.25442	0.825863	61.076
7-22	1.628449	3.520946	95.36
7-23	-2.36988	2.035747	285.542
7-24	0.848236	2.196327	100.227
7-25	0.482019	1.984289	72.316

APPENDIX H
CORRESPONDENCE OF DIF INDICES USING
FEMALE AND MALE WHITE COMPARISON GROUPS

Item	Dif ₁ (3.4)	Dif ₂ (3.4)	MH χ^2 (11.5)
2-1	-0.02116	0.045424	0.3805
2-2	-0.00009	0.018038	14.884
2-3	-0.00054	0.084513	20.4591
2-4	0.00675	0.029194	5.1737
2-5	0.378952	0.306525	79.4318
2-6	0.012687	0.025036	3.2784
2-7	0.232176	0.219868	30.8996
2-8	-0.13643	0.189241	59.8351
2-9	0.04805	0.04118	9.967
2-10	-0.05941	0.060069	3.2491
2-11	0.000238	0.098148	38.9461
2-12	0.881675	0.489424	37.4822
2-13	0.004731	0.592271	96.0802
2-14	-0.08884	0.157657	3.5105
2-15	-0.01824	1.092795	15.1682
2-16	0.026516	0.09427	0.0567
2-17	0.013197	0.4189	0.0058
2-18	-0.01335	0.08145	0.5924
2-19	-0.01831	0.06365	1.2592
2-20	-0.00235	0.56385	6.4202
2-21	0.000443	0.140449	23.8451
2-22	0.001626	0.024125	0.9855
2-23	0.712977	0.759731	45.4509
2-24	0.411745	0.322824	89.3192
2-25	-0.2158	0.208677	9.6133

6-1	0.014584	0.234757	6.0103
6-2	-0.00182	0.454688	0.4161
6-3	-0.00633	0.066299	0.52
6-4	0.001229	0.014256	0.6048
6-5	-0.90046	0.486865	29.5433
6-6	-0.0766	0.158994	1.1471
6-7	0.065727	0.087008	5.9577
6-8	0.073361	0.087389	5.296
6-9	-0.01149	0.158706	1.519
6-10	-1.70e-08	0.013201	0.1904
6-11	0.093235	0.171384	7.1747
6-12	-0.73834	0.540385	32.8373
6-13	-0.0188	0.05587	0.5264
6-14	-0.00041	0.014186	10.9051
6-15	-0.00945	0.043711	0.0765
6-16	-0.07229	0.16156	2.6301
6-17	0.022871	0.096847	0.0405
6-18	-0.00023	0.025892	0.2431
6-19	-0.01096	0.097167	0.0008
6-20	-0.00354	0.065228	0.2222
6-21	0.011165	0.05141	0.8608
6-22	-0.00506	1.51409	1.3431
6-23	-0.01179	0.065897	0.515
6-24	-0.00198	0.068917	1.122
6-25	-0.0372	0.095521	0.693
6-26	-0.06405	0.113944	0.193
6-27	-0.01269	0.022463	0.56

6-28	-0.18795	0.127459	6.247
6-29	-0.00388	0.035899	0
6-30	0.008415	0.094913	0.613
6-31	-0.00025	0.01289	0.194
6-32	0.022612	0.192656	0.067
6-33	0.033364	0.045985	14.274
6-34	-0.10002	0.200121	0.165
6-35	-0.00012	0.035361	32.051
7-1	0.008465	0.10299	2.143
7-2	-0.11537	0.087549	56.185
7-3	-0.12834	0.206071	102.713
7-4	-0.03028	0.238463	4.688
7-5	0.027489	0.060131	4.139
7-6	-0.41349	0.279763	105.162
7-7	0.004403	0.226619	7.747
7-8	-0.03677	0.030968	8.811
7-9	0.076551	0.41245	46.481
7-10	-0.22334	0.188654	20.484
7-11	0.004507	0.11008	0.281
7-12	-0.18844	0.180467	1.33
7-13	-0.09888	0.208663	0.179
7-14	0.045707	0.375784	13.726
7-15	0.00194	0.827033	0.024
7-16	-0.06831	0.090974	4.786
7-17	0.422071	0.790255	90.654
7-18	0.055272	0.677572	4.156
7-19	0.054623	0.099943	4.197

7-20	0.000317	0.054064	0.106
7-21	-0.06155	0.241549	2.337
7-22	0.013001	0.183673	2.585
7-23	-0.02928	0.067398	4.095
7-24	0.015148	0.741596	2.663
7-25	-0.06632	0.122088	6.729

APPENDIX I
IRT PARAMETERS OF BLACK AND WHITE MALE
EXAMINEES FROM THE 10-86 GMAT

Item	Black Males			White Males		
	a	b	c	a	b	c
2-1	0.53052	-1.98475	0.12978	0.45587	-3.33676	0.15961
2-2	0.99724	0.71732	0.37682	0.23404	-4.17799	0.15961
2-3	0.85744	0.64137	0.36226	0.379	-1.81481	0.15961
2-4	0.81895	1.58369	0.5	0.23198	-1.10348	0.15961
2-5	0.28471	-2.4265	0.12978	0.47902	-0.1717	0.15961
2-6	0.50213	-1.86032	0.12978	0.49493	-3.90713	0.15961
2-7	0.45053	-0.00286	0.12978	0.49525	0.01404	0.15961
2-8	1.17114	1.3462	0.14543	0.34553	-0.46831	0.15961
2-9	0.32432	-1.92778	0.12978	0.4059	-1.80399	0.15961
2-10	0.53248	-0.44661	0.12978	0.47439	-1.23933	0.15961
2-11	0.39398	1.70058	0.12978	0.50651	2.7022	0.33947
2-12	0.47126	0.64496	0.12978	0.76704	0.09828	0.10798
2-13	0.52654	0.42424	0.12978	0.77424	0.60739	0.31804
2-14	0.85471	0.73852	0.14612	0.5068	0.26813	0.15961
2-15	0.36273	-0.45627	0.12978	0.72633	0.63567	0.2688
2-16	0.77795	0.1977	0.12903	0.60477	-1.34277	0.15961
2-17	0.4857	2.82298	0.14717	0.64174	2.3161	0.10015
2-18	0.98093	3.13556	0.17035	0.44904	3.81415	0.13463
2-19	0.63099	0.47518	0.12978	0.39381	-0.41193	0.15961
2-20	0.50144	0.89934	0.12978	0.58693	1.12034	0.21951
2-21	0.77427	-0.09567	0.12978	0.55125	-1.2217	0.15961
2-22	0.37185	2.6518	0.12978	0.16567	2.41233	0.15961
2-23	0.56887	0.8933	0.09531	0.70432	0.93578	0.14256
2-24	0.88485	0.90759	0.25829	0.51354	-0.11745	0.15961
2-25	0.52464	0.45205	0.12978	0.56749	-0.43024	0.15961

6-1	0.62898	1.52479	0.35226	0.4559	0.5563	0.15961
6-2	0.73574	1.63424	0.13665	0.81854	1.40504	0.0381
6-3	0.48666	-0.5206	0.12978	0.40474	-1.02554	0.15961
6-4	0.88963	-1.87791	0.12978	0.37412	-4.54816	0.15961
6-5	1.10653	0.08166	0.23356	0.80905	-0.62074	0.15961
6-6	0.45858	-0.75468	0.12978	0.55339	-0.37349	0.15961
6-7	0.59455	-1.2293	0.12978	0.60291	-1.51206	0.15961
6-8	0.86726	-0.9063	0.12978	0.72481	-2.72487	0.15961
6-9	0.3838	-0.41017	0.12978	0.25248	-0.79179	0.15961
6-10	0.44379	-0.03858	0.12978	0.47476	-0.02058	0.15961
6-11	0.67157	-0.27836	0.12978	0.56181	-0.42141	0.15961
6-12	0.89442	0.43601	0.15882	0.73608	-0.16623	0.15961
6-13	0.72122	-1.27981	0.12978	0.37399	-3.7307	0.15961
6-14	0.56733	-1.27866	0.12978	0.21628	-5.16926	0.15961
6-15	0.50926	-0.75333	0.12978	0.40344	-1.26597	0.15961
6-16	0.48695	-0.19587	0.12978	0.4557	-0.17789	0.15961
6-17	0.55459	-0.54083	0.12978	0.42499	-1.50892	0.15961
6-18	0.74016	-1.12585	0.12978	0.63352	-1.69447	0.15961
6-19	0.71733	0.27539	0.12978	0.4688	-0.20252	0.15961
6-20	0.47909	1.02691	0.06408	0.24053	0.56687	0.15961
6-21	0.61922	-0.62059	0.12978	0.44405	-0.96017	0.15961
6-22	0.43905	-0.29826	0.12978	0.52136	0.03376	0.15961
6-23	0.52228	0.43229	0.12978	0.27809	-0.60183	0.15961
6-24	0.56803	-0.57238	0.12978	0.34392	-1.37248	0.15961
6-25	0.72273	-0.47543	0.12978	0.44251	-1.09469	0.15961
6-26	0.95393	-0.62361	0.12978	0.61992	-1.50012	0.15961
6-27	0.59439	-0.97524	0.12978	0.41427	-1.47822	0.15961

6-28	0.78976	-0.44999	0.12978	0.59046	-0.85844	0.15961
6-29	0.9776	0.47601	0.26044	0.44548	-0.36011	0.15961
6-30	0.55253	-1.1796	0.12978	0.4331	-1.69091	0.15961
6-31	0.38076	-1.2643	0.12978	0.36329	-1.73281	0.15961
6-32	0.78285	-0.06166	0.12978	0.46661	-0.37539	0.15961
6-33	0.48737	-0.68203	0.12978	0.30842	-1.72586	0.15961
6-34	0.83446	-0.39372	0.12978	0.58737	-0.94038	0.15961
6-35	0.44426	0.52607	0.12978	0.17624	-1.03604	0.15961
7-1	0.46955	0.18576	0.12978	0.27936	-0.41968	0.15961
7-2	0.65328	-0.98449	0.12978	0.43083	-2.84974	0.15961
7-3	0.34516	0.3661	0.12978	0.31127	-0.35778	0.15961
7-4	0.42784	-0.13382	0.12978	0.25839	-0.38139	0.15961
7-5	0.52565	-1.68876	0.12978	0.4454	-2.89459	0.15961
7-6	0.64846	-0.29158	0.12978	0.58685	-1.53791	0.15961
7-7	0.77867	-0.78876	0.12978	0.62382	-2.01353	0.15961
7-8	0.62836	-0.96119	0.12978	0.45211	-2.6883	0.15961
7-9	1.03201	1.26762	0.2429	0.51077	-0.27824	0.15961
7-10	1.05478	0.80585	0.12909	0.50429	-0.58507	0.15961
7-11	0.9509	0.90922	0.22926	0.45453	-0.32906	0.15961
7-12	1.24731	0.02893	0.26465	0.77469	-1.54241	0.15961
7-13	1.14978	0.79657	0.19763	0.68802	-0.32414	0.15961
7-14	0.66981	1.12272	0.1918	0.57251	0.14937	0.15961
7-15	0.98899	1.60692	0.14447	0.73787	1.0318	0.14395
7-16	0.80638	0.90872	0.23198	0.43773	-0.46225	0.15961
7-17	0.83369	1.91068	0.19207	0.49134	1.89532	0.17332
7-18	1.28722	1.49835	0.19454	0.65354	0.88255	0.08568
7-19	0.78895	0.74898	0.28822	0.48638	-1.13791	0.15961

7-20	0.74477	1.93342	0.16481	0.41686	1.30259	0.15961
7-21	0.91608	3.00799	0.24999	0.79562	3.19387	0.30992
7-22	1.10885	1.19109	0.24843	0.61139	-0.00816	0.15961
7-23	0.96446	1.35617	0.22448	0.44123	-0.18683	0.15961
7-24	0.68477	2.06433	0.18873	0.51437	1.01879	0.15961
7-25	0.88704	1.16745	0.24442	0.47411	-0.02178	0.15961

APPENDIX J
IRT PARAMETERS OF FEMALE AND MALE WHITE
EXAMINEES FROM THE 10-86 GMAT

Item	Female			Male		
	a	b	c	a	b	c
2-1	0.50795	-3.12694	0.1548	0.45587	-3.33676	0.15961
2-2	0.28457	-4.14263	0.1548	0.23404	-4.17799	0.15961
2-3	0.44728	-2.07731	0.1548	0.379	-1.81481	0.15961
2-4	0.19749	-1.79303	0.1548	0.23198	-1.10348	0.15961
2-5	0.49194	-0.85764	0.1548	0.47902	-0.1717	0.15961
2-6	0.45308	-4.62314	0.1548	0.49493	-3.90713	0.15961
2-7	0.48779	-0.48623	0.1548	0.49525	0.01404	0.15961
2-8	0.34148	-0.033	0.1548	0.34553	-0.46831	0.15961
2-9	0.41935	-2.14749	0.1548	0.4059	-1.80399	0.15961
2-10	0.44524	-1.32722	0.1548	0.47439	-1.23933	0.15961
2-11	0.16327	1.60637	0.1548	0.50651	2.7022	0.33947
2-12	0.75569	-0.18917	0.1548	0.76704	0.09828	0.10798
2-13	0.48381	-0.61588	0.1548	0.77424	0.60739	0.31804
2-14	0.48033	0.23152	0.1548	0.5068	0.26813	0.15961
2-15	0.57695	-0.05693	0.1548	0.72633	0.63567	0.2688
2-16	0.53202	-1.6214	0.1548	0.60477	-1.34277	0.15961
2-17	0.67633	2.22721	0.11795	0.64174	2.3161	0.10015
2-18	0.41856	3.95325	0.12868	0.44904	3.81415	0.13463
2-19	0.41908	-0.46815	0.1548	0.39381	-0.41193	0.15961
2-20	0.47116	0.64801	0.1548	0.58693	1.12034	0.21951
2-21	0.61203	-1.55109	0.1548	0.55125	-1.2217	0.15961
2-22	0.13219	2.49458	0.1548	0.16567	2.41233	0.15961
2-23	0.70472	0.47848	0.14161	0.70432	0.93578	0.14256
2-24	0.51371	-0.76127	0.1548	0.51354	-0.11745	0.15961
2-25	0.51974	-0.41277	0.1548	0.56749	-0.43024	0.15961

6-1	0.41293	0.61857	0.1548	0.4559	0.5563	0.15961
6-2	0.89672	1.22614	0.05145	0.81854	1.40504	0.0381
6-3	0.43626	-1.16678	0.1548	0.40474	-1.02554	0.15961
6-4	0.30923	-5.42927	0.1548	0.37412	-4.54816	0.15961
6-5	0.86546	-0.52338	0.1548	0.80905	-0.62074	0.15961
6-6	0.61565	-0.43141	0.1548	0.55339	-0.37349	0.15961
6-7	0.5633	-1.88477	0.1548	0.60291	-1.51206	0.15961
6-8	0.65677	-3.26619	0.1548	0.72481	-2.72487	0.15961
6-9	0.30938	-0.94547	0.1548	0.25248	-0.79179	0.15961
6-10	0.47215	-0.21059	0.1548	0.47476	-0.02058	0.15961
6-11	0.47359	-0.8079	0.1548	0.56181	-0.42141	0.15961
6-12	0.6923	-0.0564	0.1548	0.73608	-0.16623	0.15961
6-13	0.48579	-3.20521	0.1548	0.37399	-3.7307	0.15961
6-14	0.2773	-4.75975	0.1548	0.21628	-5.16926	0.15961
6-15	0.43116	-1.33065	0.1548	0.40344	-1.26597	0.15961
6-16	0.52289	-0.21232	0.1548	0.4557	-0.17789	0.15961
6-17	0.34803	-1.93523	0.1548	0.42499	-1.50892	0.15961
6-18	0.61386	-1.85804	0.1548	0.63352	-1.69447	0.15961
6-19	0.50357	-0.36445	0.1548	0.4688	-0.20252	0.15961
6-20	0.26882	0.35692	0.1548	0.24053	0.56687	0.15961
6-21	0.40779	-1.25651	0.1548	0.44405	-0.96017	0.15961
6-22	0.8492	0.37588	0.31115	0.52136	0.03376	0.15961
6-23	0.32249	-0.63045	0.1548	0.27809	-0.60183	0.15961
6-24	0.36764	-1.54739	0.1548	0.34392	-1.37248	0.15961
6-25	0.49516	-1.09696	0.1548	0.44251	-1.09469	0.15961
6-26	0.68667	-1.52131	0.1548	0.61992	-1.50012	0.15961
6-27	0.41762	-1.57926	0.1548	0.41427	-1.47822	0.15961

6-28	0.57956	-0.88032	0.1548	0.59046	-0.85844	0.15961
6-29	0.46027	-0.50756	0.1548	0.44548	-0.36011	0.15961
6-30	0.37949	-1.98154	0.1548	0.4331	-1.69091	0.15961
6-31	0.37038	-1.89838	0.1548	0.36329	-1.73281	0.15961
6-32	0.40274	-0.58653	0.1548	0.46661	-0.37539	0.15961
6-33	0.34038	-2.09096	0.1548	0.30842	-1.72586	0.15961
6-34	0.69778	-1.00231	0.1548	0.58737	-0.94038	0.15961
6-35	0.14879	-0.45003	0.1548	0.17624	-1.03604	0.15961
7-1	0.22457	-0.53474	0.1548	0.27936	-0.41968	0.15961
7-2	0.38169	-2.58857	0.1548	0.43083	-2.84974	0.15961
7-3	0.31188	0.27746	0.1548	0.31127	-0.35778	0.15961
7-4	0.34893	-0.67274	0.1548	0.25839	-0.38139	0.15961
7-5	0.28428	-4.80906	0.1548	0.4454	-2.89459	0.15961
7-6	0.48801	-1.26389	0.1548	0.58685	-1.53791	0.15961
7-7	0.50506	-2.3093	0.1548	0.62382	-2.01353	0.15961
7-8	0.41532	-2.80851	0.1548	0.45211	-2.6883	0.15961
7-9	0.38896	-0.01322	0.1548	0.51077	-0.27824	0.15961
7-10	0.49081	-0.48448	0.1548	0.50429	-0.58507	0.15961
7-11	0.42485	-0.49138	0.1548	0.45453	-0.32906	0.15961
7-12	0.84986	-1.53502	0.1548	0.77469	-1.54241	0.15961
7-13	0.77417	-0.42945	0.1548	0.68802	-0.32414	0.15961
7-14	0.49389	0.21098	0.1548	0.57251	0.14937	0.15961
7-15	0.51595	0.74289	0.05212	0.73787	1.0318	0.14395
7-16	0.45024	-0.46799	0.1548	0.43773	-0.46225	0.15961
7-17	0.43984	0.9412	0.1548	0.49134	1.89532	0.17332
7-18	0.58708	0.90769	0.08946	0.65354	0.88255	0.08568
7-19	0.42994	-1.60755	0.1548	0.48638	-1.13791	0.15961

7-20	0.4025	1.14788	0.1548	0.41686	1.30259	0.15961
7-21	0.44604	3.85403	0.27885	0.79562	3.19387	0.30992
7-22	0.5653	-0.12986	0.1548	0.61139	-0.00816	0.15961
7-23	0.4573	-0.26115	0.1548	0.44123	-0.18683	0.15961
7-24	0.61668	1.10896	0.22394	0.51437	1.01879	0.15961
7-25	0.44453	-0.06814	0.1548	0.47411	-0.02178	0.15961