Uncovering the structure of a memorist's superior "basic" memory capacity

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Ericsson, K. A., Delaney, P. F., Weaver, G. A., & Mahadevan, S. (2004). Uncovering the structure of a memorist's superior "basic" memory capacity. *Cognitive Psychology, 49(3)*, 191-237.

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Abstract:
After extensive laboratory testing of the famous memorist Rajan, Thompson, Cowan, and Frieman (1993) proposed that he was innately endowed with a superior memory capacity for digits and letters and thus violated the hypothesis that exceptional memory fully reflects acquired "skilled memory." We successfully replicated the empirical phenomena that led them to their conclusions. From additional analyses and new experiments, we found support for an alternative hypothesis, namely that Rajan's superior memory for digits was mediated by learned encoding techniques that he acquired during nearly a thousand hours of practice memorizing the mathematical constant π. Our paper describes a general experimental approach for studying the structure of exceptional memory and how Rajan's unique structure is consistent with the general theoretical framework of long-term working memory (Ericsson & Kintsch, 1995).
Keywords: Memory skill; Exceptional performance; Skill acquisition; Mnemonics; Innate talent; Long-term working memory; Skilled memory; Expertise; Expert performance; Memorist

Article:

1. Introduction

Experimental psychologists generally point to Ebbinghaus' (1885/1964) pioneering studies as the birth of memory research as a laboratory science. Ebbinghaus sought general laws governing healthy adult memory, and his belief in the generality of those laws was so strong that he studied only a single participant: himself. Over several years of testing, he recorded his performance while memorizing over 2000 lists of nonsense syllables (Dukes, 1965). His faith in the value of his single-participant studies was apparently well founded, as 100 years after publication of his work, scientists met to discuss his impact and noted the validity and reproducibility of his original discoveries (Slamecka, 1985a, 1985b).

Many of Ebbinghaus' contemporaries gathered detailed introspective accounts of peoples' experiences rather than relying on controlled experimental studies. Because introspection was later rejected as a verifiable source of scientific evidence, the early case studies were largely dismissed (Ericsson & Crutcher, 1991; Humphrey, 1951). Investigators interested in performing case studies turned to other methods, including the use of in-depth interviews (cf. Allport, 1946; Skaggs, 1945). However, experimental psychologists have often questioned the scientific verifiability of the accounts derived from case studies. Ebbinghaus' (1885/1964) work suggests that the use of in-depth data from a single individual need not necessarily be open to criticisms about its empirical validity if the experimental approach is applied. In the natural sciences, it is commonly held that a single but potentially reproducible violation of the predictions of a theory would be sufficient to demonstrate that the theory is invalid. For example, it would be sufficient to find a *single individual* that could consistently predict the future (e.g., by picking the winning numbers of state lotteries) to counter a theory that such phenomena do not exist. Similarly, traditional theories of memory that incorporate the notion of a limited-capacity, short-term memory would be unable to account for even a single individual who can perform standardized memory tasks well outside the normal range, such as consistently accurate recall of 80 random digits presented in the a digit-span task (Ericsson, Chase, & Faloon, 1980). In fact, there are several influential contributions to psychology based on a single participant (Dukes, 1965), including the demonstration of unilateral color-blindness (Graham, Sperling, Hsia, & Coulson, 1961) and the memory performance of a chess master (Chase & Simon, 1973).

This paper reports a series of experiments with an individual with exceptional memory (Rajan) that meets the standards of traditional laboratory research, including reproducibility of performance across sessions, experiments, and laboratories. Rajan was previously tested by several independent investigators whose experiments were largely designed to understand the structure and generalizability of Rajan's superior memory performance on digits and letters (Baddeley, 1999; Biederman, Cooper, Fox, & Mahadevan, 1992; Thompson, Cowan, Frieman, & Mahadevan, 1991; Thompson, Cowan, & Frieman, 1993). While other memory experts' performance could be explained via Chase and Ericsson's skilled memory theory (1981, 1982; Ericsson, 1985, 1988; Ericsson & Chase, 1982), which proposed that memorists acquire various mechanisms to improve their memory through practice, Thompson et al. (1991, 1993) summarized the existing research on Rajan by concluding that skilled memory theory could not fully explain Rajan's memory performance.

In this paper, we use new experimental data to build on skilled memory theory by proposing an account of Rajan's memory within the framework of long-term working memory theory (Ericsson & Delaney, 1998, 1999; Ericsson & Kintsch, 1995). First, we will briefly sketch the skilled memory account of exceptional memory in terms of acquired memory skill, and then we will show how Thompson et al. (1991, 1993) interpreted inconsistent aspects of Rajan's performance to be evidence for a superior innate memory capacity for certain types of materials, such as numbers. Lastly, we will then outline a series of experimental studies that evaluate alternative accounts of these critical aspects of Rajan's superior memory.

1.1. Skilled memory theory and Rajan as a proposed exception

Skilled memory theory was developed by Chase and Ericsson (1981, 1982; Ericsson, 1985; Ericsson & Chase, 1982; Ericsson et al., 1980) to explain how college students after extensive practice were able to increase their performance on the digit-span task from around 7 digits to over 80 digits. With practice, the students went beyond merely rehearsing the digits and started to encode and store groups of 3–5 digits in long-term memory. The students acquired retrieval cues that allowed them to store and then later retrieve the digits groups in order from memory. Skilled memory theory proposed three principles that could explain exceptional memory performance in terms of acquired encoding skill without assuming exceptional basic capacity. First, to attain exceptional memory performance individuals need to rely on prior knowledge and patterns to encode the presented items and store the items as encoded groups in LTM (encoding principle). Second, encoded information needs to be associated with retrieval cues during study that can later trigger retrieval from LTM (retrieval structure principle). Finally, with additional practice individuals become more proficient in their encoding and can store the same amount of presented information in less time (speed-up principle).

More recent investigators have demonstrated dramatic improvements in memory performance after extended training that were consistent with the principles of skilled memory and its generalization into long-term working memory (Ericsson & Kintsch, 1995; Ericsson & Lehmann, 1996). Several studies even demonstrated impressive memory improvements in large samples of participants after extended practice either with instruction (Higbee, 1997; Kliegl, Smith, & Baltes, 1989; Kliegl, Smith, Heckhausen, & Baltes, 1987) or without instruction (Wenger & Payne, 1995). In addition, reviews of individuals with alleged exceptional memory (Ericsson, 1985, 1988) have shown that these individuals tended only to exhibit vastly superior memory performance for a particular type of material, such as digits. Furthermore, they were found to segment items into groups in the 3–5 item range, consistent with current estimates of the capacity of attention (Broadbent, 1975; Cowan, 2001) and frequently provided evidence for mnemonic associations while encoding groups of items of a particular type of material.

In a recent comprehensive review, Wilding and Valentine (1997) showed that most of the evidence on exceptional memory is consistent with Chase and Ericsson's (1981, 1982) proposal for skilled-memory—but they noted a few empirically supported exceptions. This paper will therefore focus on the best-supported exception of an individual with exceptional memory, namely Rajan Mahadevan, who was once the holder of a Guinness Book record and had committed over 30,000 decimals of the mathematical constant, π, to memory (Thompson et al., 1993).

Based on several years of memory testing, Thompson et al. (1991) showed that Rajan's exceptional memory for lists of digits was mediated by memory skills relying on retrieval structures and storage in LTM, consistent with skilled-memory theory. They also found, however, extensive evidence that Rajan segmented the lists into groups of 10–15 digits, which were much larger than the 3–5 digit groups reported by other memory experts. Thompson et al. (1991) proposed that Rajan was endowed with a superior basic capacity when he subsequently acquired memory skills to further improve his memory performance for digits, and that only these additional acquired skills were consistent with the principles of skilled-memory theory.

Their argument that Rajan was endowed with a superior basic memory capacity was supported by two types of additional evidence. First, the first available tests of Rajan's memory for digits suggested an exceptional memory (see Thompson et al., 1993, for a review). When Rajan first arrived in United States from India in 1980, his digit span was around 15 digits. This span is well outside the normal range of 4–10 digits and thus supports a qualitatively different memory. Rajan's superior memory for digits was neither mediated by the same encoding mechanisms as those acquired by college students attaining exceptional digit spans by practice nor those uncovered for other memory experts (Ericsson, 1988). He only occasionally reported using mnemonic associations (to dates and other meaningful numbers). They suggested that Rajan must be able to store individual digits by direct associations to a list of locations in memory (see Thompson et al., 1993).

Secondly, Rajan's exceptional memory was not limited to digits; he also had a memory span of 13 letters, which was well outside the normal range of memory span performance. However, consistent with skilled-memory theory and previously studied individuals with exceptional memory, Rajan's superior memory was limited to memory for certain types of materials, such as digits and letters. Several different investigators tested Rajan's spatial memory (Biederman et al., 1992), his visual recognition and recall (Baddeley, 1999) and his memory for word lists, stories and complex figures (Thompson et al., 1993) and found his memory performance to be within the normal range—in some cases even below the average—for college students. In an unpublished series of studies on Rajan's verbal short-term memory, Baddeley, Thompson and Mahadevan (see Baddeley, 1999; and Thompson et al., 1993, for summaries) found that Rajan's phonological loop or ability to rehearse verbal material was not exceptional and were thus unable to explain Rajan's memory performance. Thompson et al. (1993) were not able to develop a precise mechanism that could explain why Rajan's superior basic memory capacity was restricted to just numbers and letters. It is therefore possible that the differences in the structure of Rajan's superior memory could be explained by unique aspects of Rajan's experience and practice such as his early interest in memorizing numbers and his memorization of the many decimals of π.

1.2. Outline of studies

When Rajan consented to participating in memory studies at Florida State University, we designed a general plan to independently assess the structure of Rajan's exceptional memory performance for numbers and letters. We sought to examine the three characteristics of Rajan's ability that led Thompson et al. (1993) to conclude that Rajan was an exception to skilled memory theory. Specifically, our experiments explored the large size of Rajan's digit groups, his relatively rare report of mnemonic associations to prior knowledge, and his superior memory span for letters. More generally, we wanted to examine Thompson et al.'s (1993) argument that these three characteristics of Rajan's memory provided converging evidence for a superior innate basic memory capacity. In particular, we wished to explore alternative accounts of these characteristics that were based on acquired encoding techniques and domain-specific knowledge and experience.

Our initial experiments (Experiments 1 and 2) were designed to reproduce Rajan's superior memory for digits using methods akin to those used by Chase and Ericsson (1981, 1982). By using the same methodology we would find any differences between the self-study times of Rajan and the trained digit-span experts. In these experiments, we analyzed the patterns of times for presenting and recalling individual digits to gain more detailed information about the grouping structure and the mechanisms used to encode and retrieve sequences of digits. Based on these findings, we generated hypotheses about acquired encoding mechanisms that might mediate Rajan's performance and that would provide alternative accounts to Thompson et al.'s (1991, 1993) hypothesis of a superior basic memory capacity for digits. Next, we designed a series of experiments

(Experiments 3 and 4) to test our hypotheses by creating situations where his hypothesized encoding skills for list of digits would be impaired. It is, however, rarely possible to reduce the memory performance of individuals, such as Rajan, with extensive knowledge and experience of a type of material to a level approaching that of typical untrained college students (Ericsson & Polson, 1988a, 1988b). Only when the level of familiarity of the test material is equated for Rajan and typical college students would the acquired memory skill framework predict comparable memory performance.

In Experiment 5, we examined Rajan's superior memory span for letters to assess if the same mechanisms mediated both superior memory for digits and letters. We also examined Rajan's memory span for a relatively unfamiliar type of material, namely symbols on a keyboard, and compared his initial performance to that of typical college students. We monitored Rajan's memory performance for letters and symbols during several test sessions and collected verbal reports to assess any mediating encoding processes that might account for his superior memory performance and any improvements in performance with practice. As in our earlier experiments, we analyzed the encoding processes to determine whether acquired mechanisms specific to particular materials might account for Rajan's superior memory for letters and symbols, or whether a general basic memory capacity is implicated. We then designed Experiment 6 with conditions that were hypothesized to interfere with these encoding processes and thus reduce Rajan's memory span for the associated material.

2. Experiment 1: Self-paced memorization of digit lists

The primary goal of Experiment 1 was to reproduce Rajan's superior memory for digits using the methodology developed by Chase and Ericsson (1981, 1982; Ericsson, 1988), and to replicate two of the main pieces of evidence cited by Thompson et al. (1991, 1993) to support their claim that Rajan had a superior basic memory capacity for numbers: namely, that Rajan grouped as many as 15 digits together and that he rarely reported associations to prior knowledge of numbers. Rajan was asked to memorize lists of 15, 25, 50, and 75 digits. The digits in the lists were presented at a self-paced rate followed by recall and retrospective verbal reports on his thoughts during the trial.

2.1. Method

2.1.1. Stimuli

For each session, four new lists of random digits were generated. The lengths of the four lists were 15, 25, 50, and 75 digits, respectively. Throughout all of our experiments random numbers were generated by a computer program written in Think C (Version 6) using its library function for random number generation, where the generator was initialized using the system clock. The random seed was recorded so that the same sequence could be presented in the future as needed.

2.1.2. Apparatus and stimuli

A Power Macintosh computer using a 21 in. display with a 70 Hz refresh rate was used throughout all of our experiments with Rajan. Input was accomplished using a Macintosh keyboard and latencies were timed to $\pm$ 1 ms.

2.1.3. Procedure

Each of the 21 sessions consisted of four memory trials. The first memory trial always involved the shortest list (15 digits), and later memory trials used progressively longer lists, always ending with the longest list (75 digits).

Each list learning trial started with the presentation of a warning symbol in the center of the computer screen. The maximal rate of presentation was 5 digits/s, or 200 ms per presented digit.[1] Rajan was instructed to minimize the overall time taken to present all digits in the list without sacrificing accuracy of recall. Once the list had been presented, Rajan was instructed to recall the complete list and he was allowed to make any changes until he indicated that his recall was final. At this point, the experimenters requested any clarifications from Rajan, if necessary, to reconstruct a recalled list with the same length as the presented one. Rajan was also asked if he was unsure about any of any digits that he had reported. Finally, Rajan was asked to give a retrospective verbal report of his thoughts during the presentation and subsequent recall of the digits (Ericsson & Simon, 1993). After the verbal report, he was given feedback about the accuracy of his recall.

For half of the sessions, Rajan memorized the list silently (*silent* condition) and for the other half he was asked to verbalize his rehearsals and thoughts, following Ericsson and Simon's (1993) instructions for talk aloud (*aloud* condition). The order of silent and aloud sessions was counterbalanced.

All sessions were tape-recorded. Each session lasted about an hour, and they were held typically once or twice per week depending on Rajan's schedule. If more than two weeks elapsed between test sessions, then Rajan was given a separate warm-up session prior to resuming the regular experimental sessions. However, our data analysis was restricted to data from the regular experimental sessions.

2.2. Results

Rajan was able to reproduce his superior memory performance for digits with the self-paced presentation procedure. For the shorter lists, his overall recall rate was 98% and 97%, for 15- and 25-digit lists respectively, with perfect recall in over 50% of the lists. His accuracy of digit recall for the 50- and 75-digit lists was 90% and 91%, respectively, and he recalled these lists perfectly 60% and 50% of the time. One question of interest was whether Rajan would spend more time studying each digit on longer lists or not. We calculated the per-digit study time for perfectly recalled lists and found that the means were 0.54 s/digit for the 15-digit lists, 1.41 s/digit for the 25-digit list, 2.28 s/digit for the 50-digit lists, and 3.13 s/digit for the 75-digit lists. The average study times per digit for each perfectly recalled list were submitted to a one-way ANOVA, which showed that study time did depend on the list length, $F(3, 50) = 12.73$, $MSE = 1.06$ $p < .001$. Rajan took longer to study longer lists, as shown by the Bonferroni corrected post hoc *t*-tests which revealed that average study time for the 75-digit lists was significantly longer than for the 25-digit or 15-digit lists; and that the average study times for 50-digit lists were significantly longer than for the 15-digit lists. His rates of memorization were comparable to his previously observed rates of memorization for digits during memory span and for digits arranged in matrices (Thompson et al., 1991, 1993).

The proportion of correctly recalled lists for aloud sessions and silent sessions did not reliably differ for any of the list lengths. Therefore, the data from aloud and silent sessions were pooled in our analyses.

2.2.1. Assessment of number of digits in each group

Rajan's retrospective verbal reports indicated that he self-presented digits rapidly until he had seen as many as he was able to encode into a single group of digits. Then, he would rehearse the group of digits to commit it to memory. His retrospective reports virtually always contained information on how he had broken down a given list of digits into groups. For example, on one occasion he reported segmenting a 75-digit list into alternating groups of 9 digits and 6 digits, namely 9–6–9–6–9–6–9– 6–9–6. On error-free trials his reported group size ranged from 4 digits to 15 digits with a mean of 9.73 and a median of 10.0. The modal group size was 10 (42% of all groups), with the next most frequent sizes being 15 (15% of groups) and 5 (17% of all groups). No other group sizes were reported for more than 5% of the groups.

The validity of Rajan's self-reported sizes of digit groups was assessed by analyzing the latencies for key presses in his self-paced study of the digits. If Rajan's self-reports were accurate, the latencies for reading digits belonging to the same reported group (*within* times) would be expected to be reliably faster than the latencies that correspond to the start of a new group (*between* times). The data analysis was restricted to trials where Rajan recalled 75% or more of the digits and to digit groups within those lists that were error free.

For each list length, the latencies for digits within a group were substantially shorter than the between-group latencies, consistent with Rajan's report of the group sizes. Table 1 shows the means for between-group and within-group latencies by list length. Degrees of freedom for the paired t tests were adjusted to reflect no assumption of equality of variances. Some of the 15-digit lists were recalled as a single 15- digit group, and therefore did not contribute a value for between-group latencies. The average within-group latencies were in the

200–300 ms range, whereas the average between-group latencies for the longer lists are in the 10,000–20,000 ms range.

Table 1
Comparison of between and within study times for Experiments 1 and 2 (in s)

Experiment	List length (digits)	Study times				*df*	*t*
		Between		Within			
		M	*SD*	*M*	*SD*		
Experiment 1	15	6.70	5.45	0.29	0.17	9	3.74*
	25	16.00	14.73	0.31	0.21	29	5.83**
	50	19.53	25.27	0.31	0.69	79	6.81**
	75	24.42	22.29	0.36	1.40	139	12.77**
Experiment 2A	15	3.39	2.18	0.24	0.10	14	5.59**
	25	8.34	5.79	0.27	0.09	35	8.36**
	50	18.62	10.30	0.29	0.22	71	15.10**
	75	32.56	21.73	0.29	0.08	124	16.61**
Experiment 2B	15	3.50	1.52	0.17	0.10	6	5.79**
	25	6.79	3.43	0.19	0.07	15	7.69**
	50	9.90	7.99	0.40	1.57	47	8.13**
	75	21.50	17.09	0.27	0.82	61	9.78**

* $p < .01$.
** $p < .001$.

2.2.2. An analysis of reported mnemonic associations and patterns

Rajan’s verbal reports contained only a small number of reported patterns and associations—in accord with the findings reported by Thompson et al. (1993). He reported on the average fewer than one association per list of presented digits and the associations referred to 3 or 4 adjacent digits embedded in a larger group. For example, “895” was reported to be encoded as “1895 when Roentgen discovered Radiation” or “612” as “612 area code for Minnesota” or “7602” as “7602 as last part of phone number” and palindromes, such as “565” and “292,” were frequently commented on.

2.3. Discussion

Our first experiment reproduced Rajan’s superior memory performance under self-paced conditions. Rajan was found to rapidly present large groups of digits that were rehearsed and encoded in memory, with a preference for 5–15 digits per group. The principle differences between Rajan and previously studied memory experts (Ericsson, 1985, 1988) concerned how many digits were rapidly presented before taking a break and the duration of that study time. Whereas the trained memory experts (Chase & Ericsson, 1981, 1982) had longer study times after the presentation of each group of 3–4 digits, Rajan presented over 10 digits rapidly before stopping for a longer study time. On the other hand, the duration of each of Rajan’s study times were much longer (roughly between 5 and 25 s) than those observed for the trained subjects (roughly between 1 and 5 s, see Chase & Ericsson, 1981, 1982). Rajan also reported far fewer mnemonic associations than previously studied experts. In short, we were able to confirm the distinctive characteristics of Rajan’s memory for digits that led Thompson et al. (1991, 1993) to infer the mediation of basic memory ability.

Rajan’s segmentation of the digit list into groups was also far more variable than previously studied experts’ segmentation. This finding is particularly problematic as consistency across trials is necessary for any meaningful statistical aggregation of self-paced data on presentation and recall across sessions to allow identification of the underlying cognitive processes and representations.

3. Experiment 2: Self-paced memorization of digits using a standard group size and cued recall

Experiment 1 showed that Rajan’s segmentation of the list into digit groups differed markedly between lists of 15, 25, 50, and 75 digits. In fact, the segmentation even differed for lists of a given number of digits from one session to the next. After discussions about his grouping methods during the self-paced presentation, Rajan volunteered to use a consistent segmentation of the lists that would allow us to aggregate study times across sessions. Given that Rajan spent a long time rehearsing and studying each group of 10 digits (much longer than the trained memory experts did after each of 3–4 digit groups) we wanted to examine the factors that controlled the duration of these study times. Rajan's new segmentation scheme allowed us to assess the effects of the length of the lists by comparing study times for the first 10 digits in lists of 15, 25, 50, and 75 digits. If Rajan spent extra time consolidating groups of digits in expectation of a longer list than in expectation of a shorter list, then we would be able to infer strategic control of his encoding and rehearsal processes.

The consistent grouping of digits also allowed us to use cued-recall techniques to study Rajan's recall of digits in specified serial locations to test Thompson et al.'s (1993) paired-associate hypothesis for how Rajan could store information in LTM without relying on prior knowledge or associations between the digits. According to this hypothesis, Rajan memorized groups of 10–15 digits by forming a paired association between each spatial location and the corresponding digit (see the top panel of Fig. 6). If the paired-associate hypothesis were correct, we would expect a similar pattern of access speed to a digit regardless of where in a group it fell. If not, we hoped to propose an alternative hypothesis that could explain what encoding mechanisms might mediate Rajan's apparent superior basic capacity.

3.1. Method

3.1.1. Stimuli

For each session, lists of 15, 25, 50, and 75 random digits were generated using a computer program, as in Experiment 1.

3.1.2. Procedure

This experiment consisted of 27 sessions with four memory trials per session. List of 15, 25, 50, and 75 digits were presented according to the procedure outlined for Experiment 1, with two differences.

First, in this experiment Rajan applied a pre-determined segmentation of the digits in each list into groups. For the first 19 sessions, referred to as Experiment 2A, the 15-digit list was split into a 10- and a 5-digit group. The 25-digit list involved adding a group of 10; that is, 10–5–10. The 50-digit lists were segmented into five groups of 10 digits in the following manner: 10–10–10–10–10, and the 75-digit lists involved adding two additional 10-digit groups and a 5-digit group at the end: 10–10–10– 10–10–10–10–10–5.

Because we were interested in whether the results of Experiment 2A would prove consistent across different group sizes, during the final eight sessions (referred to as Experiment 2B), Rajan was given the opportunity to choose a new way of grouping the digits consistently across sessions. Rajan's groupings for the 15-digit and 25-digit lists were unchanged (10–5 and 10–5–10, respectively). However, he switched to groups of 15 digits broken down into subgroups of 10 and 5 digits for the longer lists—for the 50-digit lists, (10–5)–(10–5)–(10–5)–5, and for the 75-digit lists, (10– 5)–(10–5)–(10–5)–(10–5)–(10–5). In Session 2 of Experiment 2B, he used a grouping for the 75-digit list that deviated from the agreed-upon segmentation, so that trial has been excluded from all further analyses.

The second difference between Experiments 1 and 2 was introduced after Rajan had completed recall of all four memory lists in each experimental session. At that time, Rajan was given a cued-recall test on either the 50-digit or 75-digit list. First, Rajan was informed about which list would be tested (50 or 75) and was asked to recall the selected list one additional time. He received feedback and correction until he was able to recall the list perfectly in serial order. Then, during the cued recall test, Rajan was presented with a number on the screen that represented a given serial position within the list and that served as the signal that he should recall the digit in that location. For example, when Rajan saw the number 30 he had to type the 30th digit from the list on the keyboard as quickly as possible. All of the positions in the list were presented once in a randomized order and the reaction time between the presentation of the cue on the screen and the response was recorded.

3.2. Results

Rajan was able to reproduce his superior memory performance with the consistent segmentation of the lists into groups. His recall accuracy was 96% or higher for all four list lengths, and he recalled over half the presented lists perfectly for each of the four list lengths. For Experiment 2A, his speeds of memorizing digits for perfectly recalled lists were 0.44, 0.97, 1.80, and 3.36 s/digit for 15-, 25-, 50-, and 75-digit lists, respectively. For Experiment 2B, these same speeds were 0.41, 0.74, 1.75, and 2.80 s/digit. The rate of memorization and accuracy of recall matches those found in Experiment 1, as well as previously observed data for Rajan by other investigators.

3.2.1. Study times

The data analysis was restricted to correctly recalled digit groups for correctly recalled lists (at least 75%) in the same manner as in Experiment 1. To assess the validity of Rajan's reported encoding of groups of digits, the times between the self-paced presentation of digits *within* a given group were compared to the times *between* digits belonging to different digit groups (see Table 1). As in Experiment 1, for all four list lengths in both Experiment 2A and 2B, the between-group latencies were, on the average, around 50 times longer than the within-group latencies.

The self-paced presentation times for digits within a group were remarkably fast, averaging around 500 ms[2] between digits in 10-digit groups and around 300 ms[3] between digits in 5-digit groups. For each list length, one-way ANOVAs revealed no reliable effects associated with the serial position of the digits for 5-digit groups, all $Fs < 1$. For 10-digit groups, however, the pattern varied by list length. For the 75-digit lists, there was a significant effect of serial position, $F(8, 1044) = 8.61$, $MSE = 5.23$, $p < .001$. Bonferroni-corrected *t* tests revealed that the first digit in the group was read significantly more slowly than all others ($p < .05$ in each case), with an average difference of 48 ms. There was no serial position effect for the 50-digit lists, $F < 1$. For the 25-digit lists, there was again a serial position effect, $F(8, 135) = 4.84$, $MSE = 4.18$, $p < .001$. Bonferroni-corrected t tests revealed that the first digit was read significantly more slowly than digits 5–9 ($p < .05$ in each case). It was read approximately 81 ms slower than other digits, on average. Finally, for the 15-digit lists, the main effect of serial position approached but did not reach significance, $F(8, 117) = 1.74$, $MSE = 3.84$, $p < .10$. Although not significant, the pattern of times for 15-digit lists was consistent with the results from the other list lengths, with the first digit averaging 54 ms longer than the other digits in the group. Thus, the only serial position effects within a group, when we detected them, involved the first digit in the group being read slightly more slowly than later digits. These results are consistent with Rajan's verbal reports that he first simply presented the digits to himself and waited with time-consuming encoding and rehearsal until all digits in a group had been presented.

Two types of analyses were used to explore the extensive encoding and rehearsal of a digit group that occurred between the presentation of the last digit of one group and of the first digit of the next group. In the first analysis, we calculated the between-group latency for the first group of 10 digits for each of the four list lengths (i.e., 15, 25, 50, and 75 digits). For Experiment 2A, a one-way ANOVA revealed a significant effect of list length on between-group study time, $F(3, 65) = 14.92$, $p < .001$, $MSE = 53.43$. The group means showed a trend toward increasing study times with longer lists (see Fig. 1). Six Bonferroni-corrected post hoc tests showed that between-group study time following the first 10 digits for 75-digit lists was reliably longer than for all of the shorter lists. Similarly, less time was used to encode and rehearse the first 10 digits for the 15-digit lists than for the 50-digit lists. In short, when the list of digits was longer, Rajan spent more time encoding and rehearsing the first group of digits. This finding can only be explained by Rajan's deliberate anticipation of the challenges of recall with further memorization of additional digits. The results of Experiment 2B, also illustrated in Fig. 1, largely replicated the results of Experiment 2A, $F(3, 26) = 5.07$, $p < .01$, $MSE = 31.05$, with post hoc tests showing that less time was used to rehearse the first 10 digits for the 15-digit lists than for the 50- and 75-digit lists.

In a second type of analysis, we capitalized on the fact that Rajan had to memorize several consecutive 10-digit groups as parts of the longer lists with 50 and 75 digits. For each list length, the between-group study times for

10-digit groups as a function of their serial position within the list were analyzed with one-way ANOVAs. For the 50-digit lists in Experiment 2A the between-group study time for the first four[4] 10-digit groups are shown in Fig. 2 and were found to differ reliably as a function of serial position, $F(3, 68) = 10.62$, $p < .001$, $MSE = 75.44$. Six post hoc tests corrected for familywise error using the Bonferroni procedure revealed that the two middle groups were associated with longer *between* study times than the 1st and 4th 10-digit groups. Similarly, for the Experiment 2A 75-digit lists, *between* study times differed as a function of serial position $F(6, 110) = 4.94$, $p < .001$, $MSE = 392.03$. The means are illustrated in Fig. 2. Twenty-one Bonferroni-corrected post hoc tests showed that the 4th 10-digit group (in the middle) was associated with longer *between* times than the 1st and 2nd 10-digit groups and the 7th(final) 10-digit group. Rajan spent more processing time after the presentation of the middle 10-digit groups of both 50- and 75-digit lists.

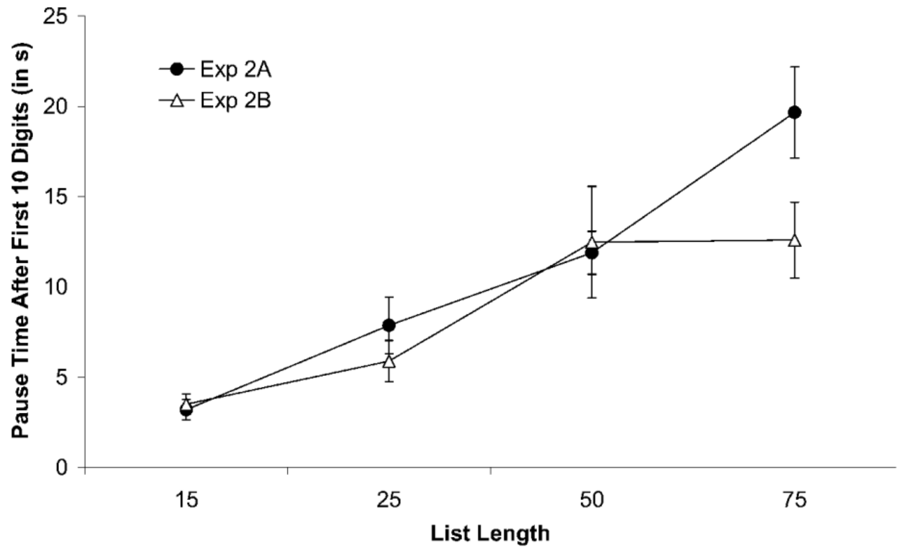

Fig. 1. The pause time following the presentation of the first 10 digits (*between* group times) in list of 15, 25, 50, and 75 digits.

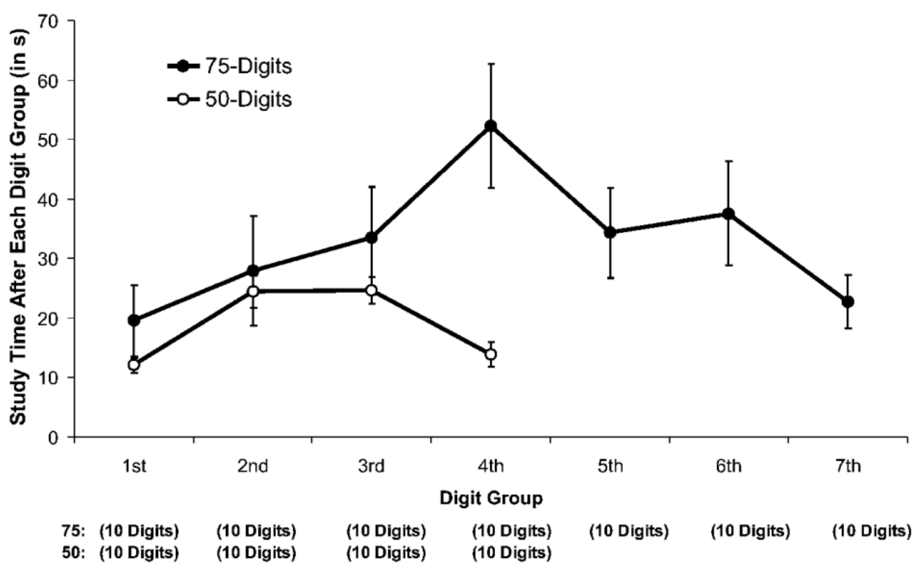

Fig. 2. ***Between*** **study times as a function of serial position of the digit group for lists of 50 and 75 digits in Experiment 2A.**

This pattern is consistent with his verbal reports that the 1st , 2nd, and 3rd 10-digit groups were cumulated together into a super-group for 50-digit list, and thus the *between* times following the 2nd and 3rd 10-digit group reflected additional time rehearsing and relating the first three 10-digit groups to each other. Similarly, the 1st, 2nd, 3rd and 4th 10-digit groups were cumulated together into a super-group for 75-digit lists. The strategy of generating encodings between the current digit group and earlier presented digit groups to build incrementally a super-group has been observed for most other memory experts who are capable of memorizing long lists of numbers (Ericsson, 1985, 1988).

According to his retrospective reports during Experiment 2B, Rajan generated similar super-groups by combining 10-digit and 5-digit groups into groups of 15 digits which he combined into a super-group of 30 digits. For the 50-digit lists, an AN-OVA revealed that between-study times differed for different digit groups, $F(5, 37) = 4.85$, $p < .005$, MSE = 46.35, with the means shown in Fig. 3. We conducted 15 Bonferroni-corrected post hoc tests that confirmed that Rajan spent more time after he had completed his first super-group that consisted of the first 30 digits ((10–5)(10–5)) than at other digit group boundaries at 25, 40, or 45 digits. No other differences between means were statistically reliable. For the 75-digit lists, between-group differences in study times were reliable, $F(8, 53) = 9.24$, $p < .001$, MSE = 140.34, with the means shown in Fig. 3. Thirty-six Bonferroni-corrected post hoc tests showed that Rajan spent reliably more time at his reported super-groups, namely after the first 30 digits (after 4th group with 5 digits) and after 60 digits (after 7th group with 5 digits) than after any of the other between-group boundaries at 10, 15, 25, 40, 55, and 70 digits. These results are consistent with Rajan's reports of encoding several digit groups into super-groups for 75-digit lists, because he spent more time after the presentation of the shorter 5-digit groups (M = 32.79 s) than after the longer 10-digit groups (M = 12.79 s), $t(29) = 4.98$, $p < .001$.

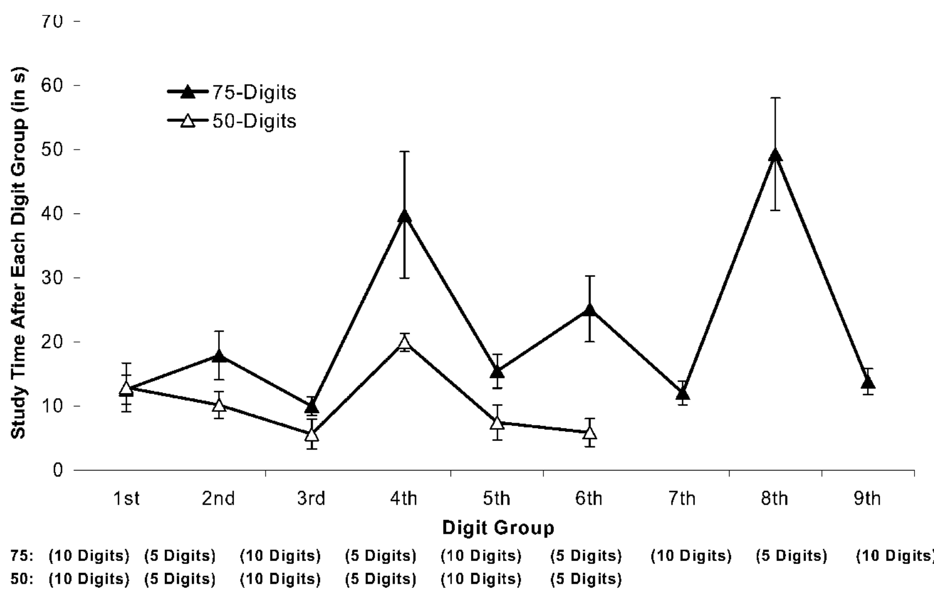

Fig. 3. ***Between*** **study times as a function of serial position of the digit group for lists of 50 and 75 digits in Experiment 2B.**

3.2.2. Retrospective reports of mnemonic associations and patterns

An analysis of retrospective reports from the last six sessions (Sessions 22–27) was restricted to perfectly recalled sequences of digits. Rajan reported mnemonic codes for an average of 24.3% of the recalled digits. Most mnemonic codes referred to relations between two digits (46%), such as "49" = 7**2 and "52" = "1952 Wechsler's book," between three digits (26%), such as "007" = "James Bond" and "953" = "1953—Watson & Crick," and between four digits (23%), such as "1965" = "Waugh & Norman paper" and "2025" = 45**2. Only one sequence referred to relations of more than four digits, namely "30609" = "every digit has curved lines."

Mnemonic associations and patterns between individual digits were almost exclusively reported for digits that were presented in immediate succession in the lists. The only exception to that rule was that Rajan sometimes reported relating the first digit of several 10-digit groups to each other. Furthermore, mnemonic patterns referred to consecutive digits within the same digit group with one exception. Rajan reported that "1866" referred to Bechterov's paper on the reflex, where "186" were the last three digits in a group of 10 digits and "6" was the first digit in the final 5-digit group for a list of 15 digits. A more detailed analysis suggested that Rajan' reports of mnemonic associations to 3-digit groups might not be uniformly distributed across serial positions with the 10-digit groups. Out of 17 mnemonic reports, 6 groups referred to the first three digits, 7 groups referred the second group of three digits (the 4th, 5th, and 6th digits) and 2 groups referred to digits in the last four digits (the 7th, 8th, 9th, and 10th digits). In two 10-digit groups, that is 2899975794 and 0144197419, the 3- digit pattern (in bold digits) violated a grouping pattern where 10-digit groups were broken down into two 3-digit groups followed by a 4-digit group. Similarly, 9 of the 15 4-digit groups with reported mnemonic associations were either embedded in 5-digit groups or were the last four digits (the 7th, 8th, 9th and 10th digits). In the remaining exceptions the 4-digit groups violated the hypothesized internal grouping pattern for 10-digit groups. For example, in 195-757-9902 Rajan reported thinking of his birth year “1957.” These observations on Rajan’s encoding of digits are consistent with a grouping of digits into 3-digit and 4-digit groups that is quite flexible to allow for discovered patterns and meaningful associations.

3.2.3. Cued recall of individual digits by their presented serial position

Rajan responded correctly on 90.6% of all cued-recall trials for the 50-digit lists and 80.6% of all trials for the 75-digit lists. When we exclude a single aberrant session testing of a 75-digit list, where Rajan gave up giving deliberate responses and thus did not meet our criterion of 75% accuracy for inclusion in the data analysis, his accuracy for 75-digit lists rose to a more respectable 87.5%. The data analysis was restricted to correct responses with times less than 3 SD above the mean.

For each list length (50 and 75), we conducted a separate ANOVA of Rajan’s time to access the digit from memory and type it in on the keyboard (henceforth, retrieval time). Each ANOVA included two independent variables, namely the digit group that contained the cued serial position (i.e., the first group in the list, the second group, etc.) and the serial position within that digit group. There was no reliable interaction of reaction times between digit groups and position within digit group for Experiment 2A for either list length: for the 50-digit lists in Experiment 2A, $F(36,398) = 1.02$, the 75-digit lists in Experiment 2A, $F(54, 523) = 1.30$, both $p > .05$. There was a significant main effect of the serial order of the digit group for both lists—for the 50-digit lists in Experiment 2A, $F(4, 398) = 6.70$, $p < .001$, $MSE = 3.78$; for the 75-digit lists in Experiment 2A, $F(4, 523) = 12.61$, $p < .001$, $MSE = 4.34$. There was also a significant main effect of position within the group—for the 50-digit lists, $F(9, 398) = 4.20$, $p < .001$, $MSE = 3.78$, and for the 75-digit lists, $F(9, 523) = 10.49$, $p < .001$, $MSE = 4.34$.[5]

The mean retrieval times for the serially ordered digit groups are plotted in Fig. 4. Ten Bonferroni-corrected post hoc tests showed that the retrieval times for the 1st group in 50-digit lists were faster than any other group except the last 5th group, and that retrieval of the 5th group was faster than the middle (3rd) group. This pattern is consistent with the earlier evidence for Rajan’s encoding of a super-group that combine the initial three 10-digit groups and an assumption that the retrieval of the last (5th) digit group is benefited by recency (Ericsson & Kintsch, 1995). A similar pattern was observed for the 75-digit lists, for which we conducted 28 Bonferroni-corrected pairwise comparisons ($\alpha = .0018$). As with the 50-digit lists, there was an advantage for the 1st 10-digit group and the last (8th) 5-digit group,[6] which were both accessed faster than all other 10-digit groups except the 2nd 10-digit group. The 6th 10-digit group took significantly longer to access than the 1st, 2nd, and 8th groups, and approached significance for several other groups (for the 3rd group, $p < .002$; 4th group, $p < .002$; 5th group, $p < .006$; and 7th group, $p < .02$). In addition, the 2nd 10-digit group was accessed reliably faster than the 7th 10-digit group. The pattern for retrieval times is largely consistent with the structure inferred from the study times and the retrospective reports. There is a trend toward longer longer retrieval times for the last two digit groups (3rd and 4th 10-digit groups) within the first super group consisting of the 1st, 2nd, 3rd, and 4th 10-digit groups. The longer retrieval time for the 6th 10-digit group would be consistent with formation of a second supergroup consisting of the 5th and 6th 10-digit groups.

Mean retrieval times, as a function of position within each digit-group for Experiment 2A, are shown as Fig. 5. Post hoc analyses for the 75-digit lists, showed that the 1st digit was retrieved faster than the digits in the 4th through 9th positions. Most of the other digits were retrieved reliably faster than the digits in the 7th and 8th positions. The post hoc analyses revealed a similar pattern for the 50-digit lists, where the 8th digit was retrieved slower than the 1st, 2nd, 6th, 9th, and 10th digits. In addition, the 10th digit was retrieved faster than the 5th and 8th digits. There were no other statistically reliable differences.

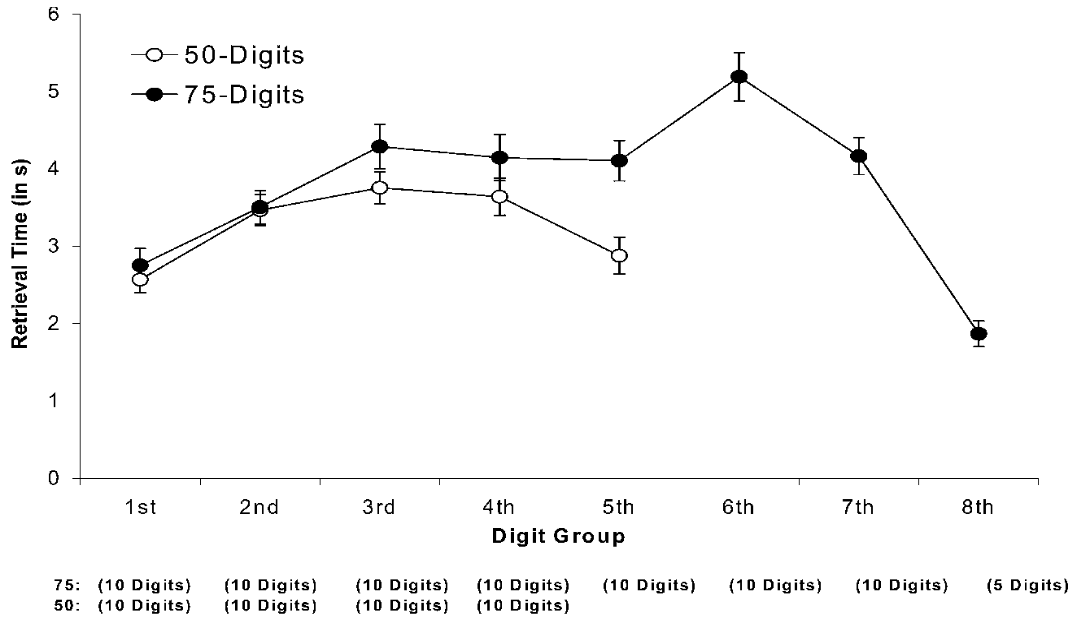

Fig. 4. The mean retrieval time for digits in a given digit group as a function of the groups' serial position with the lists of 50, and 75 digits for Experiment 2A.

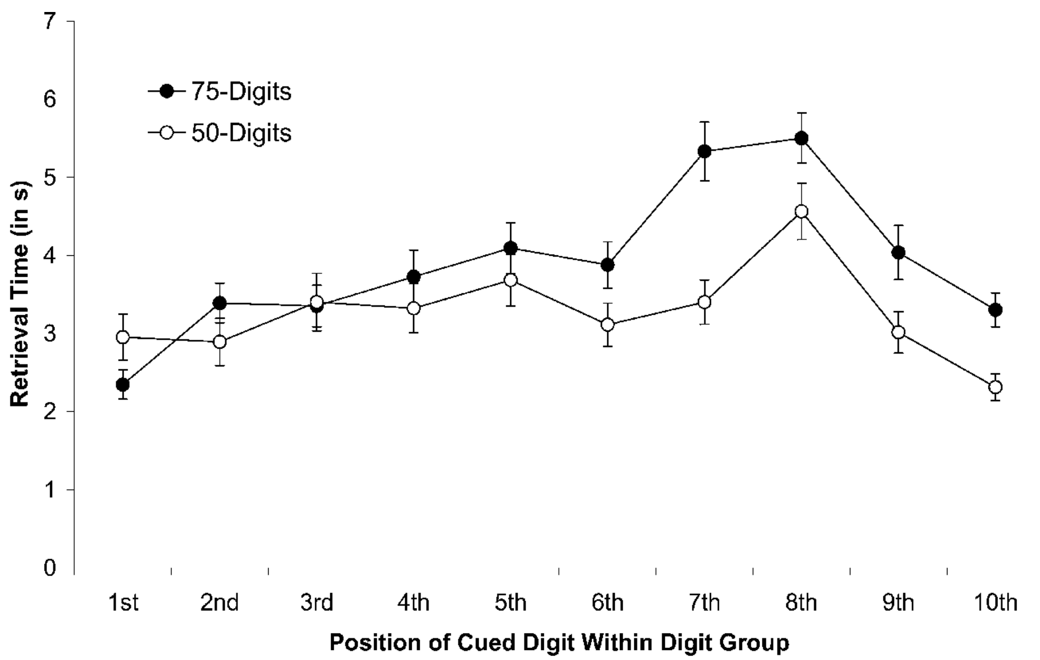

Fig. 5. The mean retrieval time for digits as a function of their serial position within the digit group for lists of 50, and 75 digits for Experiment 2A.

The results were similar for Experiment 2B, but the interpretation of the results are complicated by the complete confounding between serial position of digit groups and the number of digits within each digit group. Hence, these results were not reported.

Table 2
Probability and frequencies of errors in Experiments 1 and 2 as a function of their position within digit groups of 6 or more digits

Rehearsal	Error position						Total
	1	2	3	$L-2$	$L-1$	L	
Error probability	.04	.08	.14	.24	.27	.23	—
Number of errors	4	7	12	21	24	20	88
Single errors	1	1	4	4	5	6	21

In the bottom row, frequencies of errors in digit groups with only a single error per digit group.

3.2.4. Error analyses

The error analysis was based on the 192 lists from both Experiment 1 and 2, but our analysis excluded seven trials where Rajan's proportion of errors exceeded 25%. Out of the remaining 185 digit lists, only 50 lists contained a least a single error. In these lists the errors were clustered into 79 events involving erroneous digit-subsequences containing one or more digits that were incorrectly recalled or omitted, yielding a total of 191 different classifiable digit errors. In all but one instance, the errors so identified occurred *exclusively* within a particular digit group—while recall was correct for the digits just prior to and just subsequent to the boundaries of the digit group. In summary, recall errors did not span adjacent digit groups (with a single exception), which implies that these groups were encoded and recalled as unitary segments within the total list.

About half of the error events involved single digit errors (a single item omitted, inserted, or substituted) while the remaining error events involved multiple errors in the digit group, including 10 instances where an entire digit group was omitted. Of particular interest was the location of errors in digit groups with a length (L) of six or greater, where 55 of the 78 within-group error events (some involving more than one digit) occurred. Because digit groups varied in size, particularly in Experiment 1, individual digit errors (a total of 88) were tabulated for positions 1, 2, and 3, and for positions L, L-1, and L-2 as an approximation of a serial position function even for digit groups of differing sizes and the frequencies are shown in Table 2. The probability of error is remarkably low for the first couple of positions in a digit group and increases toward the end. This finding is consistent with the earlier finding of the priority of the beginning of a digit group during cued retrieval, where Rajan accesses the beginning of the list and finds the item using a search process.

3.3. Discussion

Rajan was able to reproduce his exceptional memory performance even under the constrained grouping conditions—if anything his accuracy and speed, especially for the longer 50- and 75-digit lists, was somewhat higher than in Experiment 1.

Most of our findings replicated and extended the earlier findings by Thompson et al. (1993) that led them to distinguish Rajan's exceptional basic memory capacity for encoding 10–15 digit groups from the complex acquired skill that he used to encode and recall longer lists of digits in terms of sequences of such digit groups. Our analyses of Rajan's self-paced study times for encoding and rehearsing digit groups (between-group encoding latencies) showed that these times were influenced by the context for memorizing a given 10-digit group. The between-group encoding latencies were longer when Rajan anticipated encountering additional 10-digit groups (that is, with longer lists). The study times were also shown to depend on the serial position of a given 10-digit group within a longer list and corresponded well to Rajan's reported encoding of super-groups, where several 10-digit groups are related to previously encoded digit groups. More generally, these findings are consistent with the skilled memory theory and the previous pattern of encoding times observed for trained memory experts (Ericsson, 1985, 1988; Ericsson & Polson, 1988a, 1988b; Staszewski, 1988). The distinctive aspect of Rajan's memory performance is his rapid rate of presenting digits within his 10-digit and 15-digit groups. With practice, however, trained memory experts have been able to reduce the duration of the between-group study times from 2 to 5 s to durations approaching 1 s (Chase & Ericsson, 1981, 1982; Staszewski, 1988).

After extended practice, the differences between within-group and between-group times average less than 0.5 s (Staszewski, 1988), suggesting the development of skills for rapid encoding and smooth integration of digit groups in memory.

Our analysis of the cued-recall latencies showed that retrieval times were influenced by main effects of two factors, namely, the location of the designated digit group and the particular serial position within the digit group. The estimated retrieval times for accessing the digit group were consistent with the organization of Rajan's reported super-groups. Given that Thompson et al. (1991, 1993) had already found evidence that Rajan relied on retrieval structures and super groups, these findings are essentially consistent with their proposal. One minor difference was that Rajan's encoding of 15-digit groups was often the result of a relatively rapid association of a 10-digit and 5-digit group, consistent with our observations in Experiment 1, when Rajan was completely free to control the rate of presentation of digits.

More importantly, Experiment 2 also revealed details about the internal structure of the 10-digit groups that were inconsistent with Thompson et al.'s (1991, 1993) hypothesis of an underlying basic "raw" capacity to store digits. They hypothesized that Rajan stored digits by associating each digit with its corresponding spatial location, resulting in 10–15 "slots" within each digit group (illustrated in the top panel of Fig. 6). Their original hypothesis would predict uniform retrieval times for digits within a digit group, but our analysis found systematic differences as a function of serial position within the 10-digit group. Retrieval of digits in the 7th and 8th position was reliably slower than retrieval from the earlier part of the 10-digit groups for the 75-digit lists, with a similar pattern for 50-digit lists. Furthermore, our error analysis showed clear serial position effects. The initial digits of a digit group contained fewer errors, suggesting a different and more reliable encoding of the beginning of the list of digits in the encoded digit group. Finally, our analysis of the digit sequences that had verbally reported associations suggested that Rajan would group immediately adjacent digits together into 2-digit, 3-digit and 4-digit groups. However, within a group of 10 digits, Rajan was flexible and based his grouping in part on his discovery of memorable patterns.

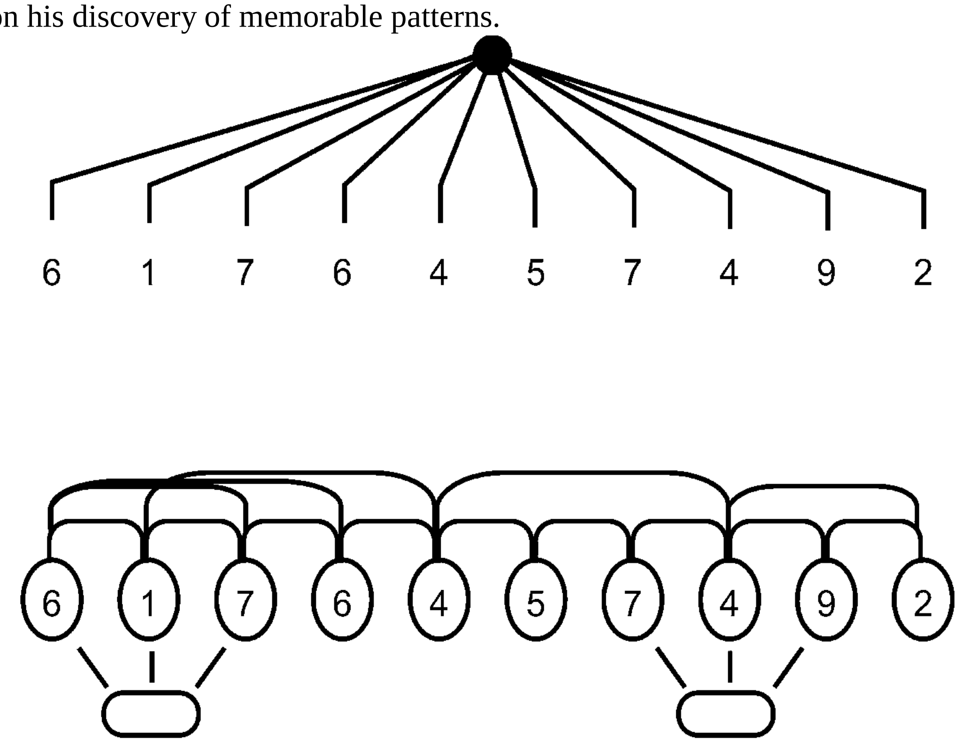

Fig. 6. Schematic illustration of Thompson et al.'s (1993) paired-associate model for direct associations between digits and serial location with "chunk" (upper panel) and an illustration of our hypothesis of list-like structure with associations between adjacent and more distant digits within the same list (lower panel).

Our new findings can be better accounted for by an alternative hypothesis that does not assume that Rajan was endowed with a superior basic memory capacity of 10–15 "slots." In contrast to the model with slots for individual digits the alternative hypothesis proposes that the digits comprising each 10-digit group are primarily encoded by associations between items of the list, such as digits, groups of digits, and encoded patterns, leading to a list-like structure with overlapping encodings (illustrated in the lower panel of Fig. 6). Direct access to digits would be limited to the beginning and perhaps the end of the digit group. Retrieval of interior digits requires mediated access that leads to slower access of digits toward the end of the interior of the list. More generally, our hypothesis proposes a possible connection between Rajan's method of encoding digits in the self-paced task and his previously acquired methods for memorizing the 30,000 decimals of π—which incidentally are presented in tables organized by 10-digit groups. Studies of Rajan's retrieval of digits from π by Thompson et al. (1993) showed a pattern consistent with the one observed in Experiment 2. Within seconds, Rajan could access the location of a unique 5-digit sub-string among the first 5000 digits of π, if the digit string consisted of the first five digits of one of the 10-digit groups of π. In contrast, when the sub-string formed the interior part of a 10-digit group then Rajan took several hundred times longer because he had to serially search thousands of digits to find the string. Our subsequent experiments attempted to provide evidence for this account by varying the types of lists that Rajan was asked to learn.

4. Experiment 3: Effects of constrained digit lists

If Rajan had a superior basic memory capacity, he should show excellent generalization to other types of lists of digits. Consequently, if we could design lists that would reliably decrease Rajan's performance, it would strengthen alternative hypotheses that rely on acquired encoding skills. In particular, we hypothesized that distinctive associations between digits, groups of digits and patterns were formed within the long groups, and that these associations typically mediate Rajan's superior memory performance for digits. In Experiment 3, we therefore tried to reduce the distinctiveness of these associations, making it harder for Rajan to distinctively store the lists of digits in LTM. We presented him with sequences of constrained digits, such as 388383338838. If Rajan's performance matched his performance for random digits, it would support the view that Rajan had a superior basic memory capacity for digits. If not, it would support our alternative hypothesis of mediating distinctive associations between elements of lists of digits.

4.1. Method

4.1.1. Stimuli

Four lists of 50 random digits were generated with a computer program before each test session. Of these, two were *regular* lists that were randomly generated involving the complete set of digits ranging from 0 to 9. The other two lists were *constrained* and generated by randomly sampling only from two different digits. During pilot testing Rajan proposed that his difficulties with the constrained lists might be attributable to the perceptual similarity of the two digits. To be able to address this possibility, half of the constrained lists were generated from two digits that Rajan judged to be very different perceptually, namely 1s and 3s, and the other half with digits perceived by Rajan to be perceptually very similar, namely 3s and 8s. The order of presentation of the regular lists and two types of constrained lists was counterbalanced across the eight sessions.

4.1.2. Procedure

The general procedure for the eight sessions was the same as that used in the silent control condition of Experiment 1, with the following exceptions. First, Rajan was informed prior to the presentation of a list what type of list he would be encountering (i.e., regular digits, 3s and 8s or 1s and 3s). Second, the task was not self-paced; digits appeared at a rate of 1 digit/s—the typical rate for presentation in the memory span. Rajan was instructed to recall accurately as many digits as possible as long as he specified the exact serial position for each recalled digit. Finally, following the completion of the four trials there was a post-session recall phase in which Rajan was asked to recall as many digits as possible from each of the lists in an attempt to assess the involvement of LTM in Rajan's memory for the lists.

4.2. Results

For the regular digits, Rajan recalled 85.6% of the presented digits correctly. He rarely guessed and only around 2% of his recalled digits were incorrect. For 37.5% of the trials, he was completely correct on all 50 digits. He recalled 43.9% of the constrained digits and only 7.5% of his recalled digits were inaccurate. He never recalled all of the 50 digits correctly for a constrained list.

Although Rajan reported that he experienced that memorization of 3s and 8s was more difficult than 1s and 3s, our analyses of his memory performance did not provide any objective support for these experiences. Consequently, the memory performance for the two sets of constrained digits was aggregated in the subsequent data analyses.

A two-factor ANOVA with test session and condition (regular vs. constrained) showed that the number of correct items recalled was not influenced by a reliable interaction, $F < 1$, but both main effects were reliable. There was a significant main effect of experimental condition, $F(1, 16) = 54.28$, $p < .001$, $MSE = 68.53$, with the regular lists ($M = 42.6$ items) being recalled better than the constrained lists ($M = 21.0$ items). There was also a significant effect of session, $F(7, 16) = 3.17$, $p < .05$, $MSE = 68.53$. Post hoc tests using 28 Bonferroni corrected t tests found that this difference reflected an advantage for the final session ($M = 42.5$ items) as compared to the first session ($M = 19.3$ items).[7]

The above analysis provides an inflated estimate for Rajan's immediate memory (memory span) for these lists. Rajan reported that he frequently memorized the constrained lists as two separate shorter lists. He even would look away from the computer screen during the presentation of some of the middle digits of the constrained 50-digit lists and use the time to rehearse earlier items. To get a better estimate of Rajan's actual memory span, the same two-factor ANOVA was conducted for the number of digits that Rajan was able to recall prior to any error, beginning from the first presented digit on a trial (referred to as "forward recall" in the following discussion). There was no reliable interaction for these data, $F(7, 28) = 1.74$, $p > .05$, $MSE = 75.94$, but there was a significant main effect of experimental condition, $F(1, 16) = 50.41$, $p < .001$, $MSE = 75.94$, reflecting higher forward recall in the regular condition ($M = 35.4$) than in the constrained condition ($M = 13.5$). There was also a significant effect of session, $F(7, 16) = 2.67$, $p < .05$, $MSE = 75.94$. The mean forward recalls for Sessions 1–8 were, respectively: 10.0, 22.8, 19.8, 28.8, 24.3, 31.5, 31.0, and 27.5 digits. Post hoc analysis of the effect of session using all 28 possible Bonferroni corrected t-tests revealed no differences significant at the .05 level.[8]

Because Rajan was always presented with 50-digit lists in this experiment, we could not calculate "memory span" in the traditional sense. The median (50^{th} percentile) of the correct forward recall across trials provided our best estimate of memory span—40 for regular lists, and only 16 for constrained lists.

4.2.1. Analysis of errors in immediate recall

The mean number of errors per trial was 0.25 for the regular condition and 0.94 for the constrained condition (the medians were 0 and 1, respectively).

4.2.2. Post-session recall

Ninety nine percent of the digits that were recalled after each of the trials during the session were also correctly recalled during the post-session recall.

4.3. Discussion

Our hypothesis that Rajan's superior memory for regular digits is mediated, at least in part, by an ability to form distinctive associations between digits and groups of digits within the same list was unequivocally supported. His performance on constrained lists of digits was 60% lower than his performance for regular lists. In fact, his median forward recall for the constrained lists, namely 16 digits, is approaching but still outside the range of the average memory span for binary numbers for college students, which is 7.9 digits (Slak, 1974). However, Rajan's immediate memory for constrained digit lists does not appear to provide a "pure" estimate of his superior basic memory capacity for numbers. The fact that Rajan was able to recall both regular and constrained sequences nearly perfectly during the post-session recall of all presented lists shows the likely involvement of storage in LTM. Unfortunately, the current data do not prove that the originally presented digits were immediately stored in LTM, because the subsequent act of recalling the list may influence post-session recall (McDaniel & Masson, 1985). Finally, Rajan's memory performance for constrained digits would still be mediated by patterns and distinctive associations to some degree, because our hypothesis merely predicted greater interference in memory for similar patterns and associations between constrained digits compared to more variable and distinctive patterns and associations for regular digits. In Experiment 4, we attempted to minimize the role of previously acquired patterns and associations by studying Rajan's transient memory capacity in a task where retrievable storage in LTM should be saturated.

5. Experiment 4: Running memory span for regular and constrained lists

In Experiment 3, we found evidence that Rajan's memory performance relied on LTM and on the presence of patterns and distinctive associations in digit lists. To assess Thompson et al.'s (1993) hypothesis that Rajan had a superior basic memory capacity we would need to estimate his basic memory performance under experimental conditions where opportunities for storage in LTM with patterns and distinctive associations were minimized. Experiment 4 was designed to determine the lower limits of Rajan's memory capacity for both regular and constrained lists of digits. We attempted to assess Rajan's reliable transient memory capacity using the procedure for running memory span, where a long list of digits (well above his memory span) is presented sequentially until the presentation is unexpectedly stopped. At this point the participant is asked to recall as many digits as possible from the end of the list. The number of perfectly recalled digits–without omissions or errors–estimates the running span of digits that can be kept accessible during the presentation.

The design of a running memory span task for a memory expert is much more complex than for normal participants because the memory expert has access to a variety of encoding skills that might be useful in enhancing his performance. Rajan's memory span for regular digits approached 50 digits and if presentation were stopped before the 50th digit then his performance would likely be perfect and reflect his normal encoding with storage in LTM. Hence, to assess his running span the lists of digits would have to be substantially longer than 50-digits to reliably estimate his immediate memory capacity. On the other hand, if Rajan knew that the presentation would never be stopped until the 75–90th digit he could ignore the first 25–30 digits and only then start to memorize the digits in his normal manner. To address this problem with strategies our procedure involved presenting digits such that presentation would stop unpredictably anywhere after 10–150 digits.

If we were able to prevent Rajan from relying on LTM, we would expect to find regular and constrained lists would result in similar memory performance. However, if Rajan were able to find methods to continue to use his exceptional encoding skills, then he would recall regular lists better than constrained lists. We predicted that Rajan's new methods would correspond to different thought processes that would be reflected in his retrospective verbal reports after each trial.

5.1. Method

5.1.1. Stimuli

For each test session, a computer generated three lists of regular digits and three lists of constrained digits where the lengths of the lists were randomly determined and ranged between 10 and 150 digits. The constrained lists were generated in the same manner as in Experiment 3, with the exception that the two digits defining the constrained set were also randomly determined prior to the generation of each constrained list.

5.1.2. Procedure

In each of the 16 test sessions, Rajan was presented with six lists of digits in a manner consistent with Experiment 3 (and using the same 1 digit/s presentation rate). The type of list (regular or constrained) alternated within a session, and which type of list was presented first in the session was counterbalanced. When Rajan was told before each trial which type of list he would be presented, he was also told for the constrained lists which two digits had been used to generate them. When the presentation of digits on the computer unexpectedly (for Rajan) stopped, Rajan was instructed to recall as many consecutive digits as possible from the end of the list. Rajan was allowed to recall the digits in any order as long as he specified the sequential position of each recalled digit. Correct recall was measured as the number of consecutive error-free digits reported immediately prior to the final digit.

5.2. Results

For each trial the running memory performance was defined by the number of consecutive correctly recalled digits beginning from the last item in the list working backwards to the first encountered error or omission in the list. Digit span is usually computed as the length of list that a person can get right 50% of the time. In order to analyze these data with an ANOVA the list length was grouped into four categories based on Rajan's digit span as estimated in Experiments 1 and 2: (1) *Short* lists with a length between 10 and 45 digits and thus within Rajan's digit span for regular digits; (2) *Medium* lists with 46–80 digits and judged to be at or above Rajan's digit span; (3) *Long* lists with 81–115 digits and considered well above his span; and (4) *Extremely long* lists with 116–150 digits and considered far outside Rajan's digit span. Because list length was allowed to vary randomly, the test sessions were grouped into four 4- session blocks to increase the chances that each length category would have at least one observation per session block. Preliminary data analyses showed that the pattern of results reported below do not substantially change with smaller or larger blocking of sessions. Given the aggregation across sessions there was no meaningful method for assigning serial position to the data in the analysis and therefore the Judd and Kenny (1981) test for auto-correlated residuals was not conducted.

A three-factor ANOVA with Experimental Condition (regular and constrained digits), List Length (short, medium, long and extremely long lists of digits) and Blocked Sessions showed that none of the interactions achieved significance. The main effect of list length was significant, $F(3, 65) = 6.78$, $p < .01$, $MSE = 24.11$. Post hoc tests on the effects of list length using all six possible Bonferroni-corrected t tests showed that Rajan recalled more items on the medium length lists ($M = 12.79$) than on the extremely long lists ($M = 6.85$). The difference between short lists ($M = 12.83$) and extremely long lists approached significance, $t(48) = 1.97$, $p < .10$. The long lists averaged 7.74 items. There was also a main effect of session block, $F(3, 65) = 8.52$, $p < .001$, $MSE = 24.11$. Post hoc tests using all six possible Bonferroni-corrected t tests showed that Rajan recalled more items in the third session block ($M = 14.96$) than in the other three session blocks (Session Block 1, $M = 6.98$; Session Block 2, $M = 8.69$; and Session Block 4, $M = 9.12$). Finally and most importantly, there was a main effect of experimental condition, $F(1, 65) = 15.35$, $p < .001$, $MSE = 24.11$, with Rajan performing better on regular lists ($M = 12.06$) than on constrained lists ($M = 7.72$).

The best estimate for Rajan's immediate memory capacity in the running memory-span task is provided by the average number of digits that he recalled perfectly starting with the last digit going backwards on at least 50% of the trials. These estimates were 11.1 for regular and 7.98 for constrained digits, respectively. However, Rajan's verbal reports suggested that this performance on the running memory task did not provide a pure estimate of his immediate memory that was independent of memory in LTM.

5.2. 1. Retrospective verbal reports

Rajan reported that he tried to encode the presented digits in much the same way as he memorized the lists in Experiments 1–3. An analysis of the pattern of recall of digits was conducted to assess the validity of those reports. If Rajan consistently segmented the presented list into groups of 10 starting from the first digit, then we would expect his recall of digits to reflect 10-digit groups, namely 1st–10th, 11th– 20th, 21st–30th, and so on. For example, for a list of length 127 digits, we would expect him to recall the last seven items plus one (111th–120th digit) or more groups of ten, such as 101st–110th digits. Consistent with our prediction, for the 29 cases where Rajan's backward span exceeded 11 digits but for which he did not recall the whole list correctly, 24 of 29 cases involved recalling a group of 10 digits plus the remaining few items from the end of the list. In other words, on 83% of the trials where Rajan scored 12 or higher, the list length modulo 10 and his recall score modulo 10 were equal. Furthermore, consistent with the method of segmenting the lists into 10-digit lists Rajan's memory performance was quite variable from trial to trial. For example, if he had just started to encode a new list of 10 digits when the presentation ended then he might only recall the last couple of digits. Therefore we estimated his *reliable* working memory capacity during this memory experiment as the number of digits that he could recall on *at least 90% of the trials* (Broadbent, 1975)—1 0th percentile of his correct backward recall across trials. Consistent with a large variability across trials his reliable memory capacity for regular digits was quite limited (less than 3 digits) and even more so for constrained digits (around 1 digit).

5.3. Discussion

Our experiment was only partially successful in estimating Rajan's transient basic memory capacity for digits. The conditions of running memory span testing reduced Rajan's memory performance and his ability to draw on retrievable storage in LTM. Rajan's backward span averaged 11 digits for regular digits, which was substantially lower than his forward span in Experiment 3, namely 40 digits. Rajan's running span for the constrained digits is reliably lower than for regular digits and averages around 8 digits. This differential in performance indicates that Rajan relied on similar encoding methods as in Experiment 3 for the tests of running memory span. Furthermore, an analysis of his memory performance revealed a large variability across trials with a reliable (90% or more of the trials) memory capacity of less than four digits, which is much lower than his average performance for regular and constrained digits. Rajan's retrospective reports provided insight into the causes of the variability in performance across trials by revealing his reliance on complex encoding strategies for both lists with regular and constrained digits. These findings show that Rajan's average running memory span does not accurately measure his "basic" memory capacity, but it also reflects his use of previously acquired encoding strategies.

Our inability to inhibit Rajan's use of encoding strategies in Experiment 4 makes it impossible for us to estimate accurately Rajan's "basic" memory capacity for digits. On the other hand, Rajan's average running memory span performance appeared to exceed that of untrained college students (Pollack, Johnson, & Knaff, 1959) and appeared to be clearly above the upper bound of the normal range of their running memory spans (around 6 digits). However, Pollack et al. (1959) also showed that when untrained college students were given substantial practice on running digit span for regular and constrained lists of digits, their spans increased by around 100%. Given that the conditions of our study differed in several ways from Pollack et al.'s (1959) study, their performance cannot be directly compared to that of Rajan. Pollack et al. (1959) did, however, describe the encoding methods relying on encoding and storage in LTM that their trained subjects used for their improved performance. The similarity of their encoding methods and those of Rajan is compelling.

In sum, Experiment 4 revealed the difficulty of assessing cleanly the basic memory capacity of a memory expert, such as Rajan, when that expert has extensive knowledge and acquired encoding methods for that material. In fact, both Rajan and students who had received extensive training with running digit span reported using encoding methods in which incoming digits were deliberately grouped and encoded in LTM. Hence, memory performance at test reflects directly accessible digits in short-term working memory as well as retrievable information about encoded digits that can be recalled from LTM (cf. LTWM as proposed by Ericsson & Kintsch, 1995). Even under the conditions of running memory span that clearly limit Rajan's reliance on LTM his memory performance was found to be mediated by encoding in LTM. These findings do not rule out the possibility that Rajan's basic memory capacity is outside the normal range, but they allow us to propose alternative accounts of Rajan's superior memory performance for digits in terms of acquired memory skill.

6. Experiment 5: Memory for symbols and letters

During the replication and experimental analysis of Rajan's exceptional memory for digits, we developed a LTWM model for his 10-digit groups based on material-specific encodings without recourse to a superior generalizable basic memory capacity. However, this model cannot account for Rajan's exceptionally high memory span for another tye of material, namely letters. Thompson et al. (1993) argued that Rajan's initial memory span for digits (around 15 digits) recorded in 1980 prior to their extensive memory testing at Kansas State University was mediated by the same basic memory capacity as his superior memory span for letters (around 13 letters). In this experiment, we attempted to replicate Rajan's memory span for letters and to assess any mediating memory and encoding mechanisms. We also assessed the generalizability of Rajan's basic memory capacity by testing Rajan's memory span for other types of symbols, such as *, +, and @. Finally, in order to get baseline estimates of typical memory spans for symbols and letters, we compared Rajan's memory span against a control sample of undergraduate students' memory spans.

If Rajan had a superior basic memory capacity, one would expect this superiority to hold for other types of materials that are similar to digits and letters. Furthermore, we would expect his superior memory span for letters to be mediated by similar types of mechanisms as his superior memory for digits. In contrast, skilled memory theory would predict that Rajan should rely on encodings that were specific to the associations between letters and letter groups. Since Rajan never engaged in extensive memorization of random letter strings, as he had done with digits during the memorization of n, skilled memory theory would predict that Rajan would group letters and retrieve mnemonic associations with methods similar to those observed for other individuals with superior memory performance (Ericsson, 1985).

One further issue was that naming letters during recall is a highly entrenched skill whereas naming the symbols would be slower. We therefore measured how quickly both Rajan and control participants could read lists of letters and symbols aloud.

6.1. Method

6.1.1. Participants

In addition to Rajan, 10 undergraduate students enrolled in a general psychology course participated for course credit. The students were tested individually.

6.1.2. Apparatus and stimuli

For Experiment 5C only, we presented stimuli on a PC-compatible computer running Windows 98 using PowerPoint.

6.1.3. Stimuli

Lists of letters and symbols were randomly generated using a computer program. Following Conrad (1964), the letters were drawn from the following consonants: B, C, F, M, N, P, S, T, V, and X. In collaboration with Rajan, we selected a set of symbols available on the standard keyboard that minimized perceptual confusions and that Rajan could easily name: !, @, #,^,*, (, +,], \, and ?. The undergraduate participants were also instructed to come up with names for each symbol that they could remember and that were quick to say.

6.1.4. Procedure: Rajan (Experiments 5A and 5B)

In each of 16 sessions, Rajan completed memory span trials for letters and symbols. Memory trials with symbols alternated with trials with letters and the order of presentation was counter-balanced across sessions. During each of Rajan's first 8 sessions (and during the control group's sessions), there were twenty memory-span trials. As Rajan's memory performance increased during this experiment the time to complete the trials increased and during the last eight sessions the number of memory-span trials was reduced to 14.

The general procedure for memory span testing was the same as the one used in Experiment 3 with a presentation rate of 1 item/s, but modified to implement a span procedure with varying list lengths similar to that used to assess digit span. In the typical memory-span procedure, participants are presented with a list length near their span and then the length is adjusted upwards or downwards depending on the accuracy of reproduction. In this experiment, correctly recalling a list of letters resulted in the next list of letters being one item longer, while incorrectly recalling the list resulted in the next list of letters being one item shorter. The same was true for symbols (we assessed memory span for symbols and letters separately). In order to find Rajan's original span we started the first test session by presenting Rajan with lists of four items of each type. On the subsequent test sessions, Rajan was presented the list length determined by his performance on the last trial of the preceding session for that type of material. To prepare Rajan prior to each trial, the experimenters informed him how many items and the type of items that would be presented. For example, if Rajan had correctly recalled a 15-item sequence of symbols at the end of the prior experimental session, his first sequence of symbols in the next session would number 16.

Given that Rajan's familiarity with letters differed from his experience with the symbols, Rajan's ability to encode and process the two types of items was measured during the course of the experiment. Before and after the memory-span testing portion of each session Rajan was instructed to name, as fast as possible, items of each type of material without making errors. For the first four sessions, Rajan provided names for three lists of 30 letters and three lists of 30 symbols, alternating between list types both before and after the memory-span testing. Rajan controlled the rate of presentation of items by pressing the space bar on the computer, as in the self-paced presentation of digit lists in Experiment 1.

After the first few test sessions, Rajan's retrospective reports showed that he did not memorize the symbols as symbols but he re-coded each symbol to a corresponding digit and then memorized the list as a sequence of digits. Consequently, the reading task was extended with one additional reading condition, and to preserve the procedures of the ongoing experiment, the additional testing was conducted after the completion of the complete experimental session (including the reading after the memory testing). Starting with session five, Rajan was presented with three additional lists of 30 symbols that he was instructed to encode into the appropriate associated digits and to name that digit. Subsequently, Rajan started to report encoding letters as digits, as well. Hence, starting with the seventh session an additional reading task was presented at the very end of the session (after the added reading task for recoded symbols). This task involved the presentation of three lists of 30 letters that were to be read as digits. The procedure was revised at the start of the second half of the 16 test sessions. From that session on three trials each of all four types of reading (symbols, letters, symbols as digits, and letters as digits) were performed both before and after each experimental session. The order of presentation was counterbalanced with all lists of a particular type being read in a single block.

Because of the procedural changes in the experiment, performance was analyzed separately for the two halves of the experiment. Sessions 1–8 will be referred to as Experiment 5A and sessions 9–16 as Experiment 5B.

6.1.5. Procedure: Control group (Experiment 5C)

The procedures for the control group of undergraduates and Rajan were identical except that the undergraduates completed only 1 session. This session was in all respects identical to Rajan's first session in Experiment 5A, including using the same lists in the same order. Like Rajan, the control participants completed the reading task before and after the session.

6.2. Results

6.2. 1. Rajan's memory span over time: Experiments 5A and 5B

An individual's memory span is typically estimated by two different types of events when memory testing is extended over many lists and sessions (Ericsson et al., 1980). When Rajan was able to correctly recall a list of length (N) but failed at the subsequent trial with a longer list (N + 1) his memory span was estimated as N + 0.5. Likewise, when he failed on a list of length M but succeeded the subsequent trial with a list length of M — 1 his span was estimated as M — 0.5. (This procedure also corresponds to the standard "staircase" procedure used to estimate psychophysical thresholds.)

Owing to changes in the procedure between the first eight sessions (Experiment 5A) and final eight sessions (Experiment 5B), we analyzed the span scores separately for each experiment using a 2 Type of Material (letters vs. symbols) x 8 Session AN-OVA. For Experiment 5A there were reliable effects of test session, $F(7, 59) = 116.72$, $p < .001$, MSE = 1. 11, and of type of material, $F(1, 59) = 525.16$, $p < .001$, MSE = 1. 11, but these effects were qualified by a significant interaction, $F(7,47) = 74.45$, $p < .001$, MSE = 1.11.[9] The interaction between test session and type of material (letters and symbols) is illustrated in the left panel of Fig. 7. The post hoc analysis focused on the eight symbol vs. letter comparisons for each of the eight sessions and the 28 possible session vs. session comparisons within each of the two experimental conditions and thus resulted in a total of 64 Bonferroni-corrected t tests. For symbols, Rajan improved his span from 5 items to 26 items over the eight sessions of Experiment 5A. The post hoc comparisons for symbols showed that his performance in Session 8 was better than in any other session; in Session 7 was better than in Sessions 1–5; in Session 6 was better than in Sessions 1–4; in Sessions 4 and 5 was better than in Sessions 1–3; in Session 3 was better than in

Sessions 1 and 2; and in Session 2 was better than in Session 1. For letters, Rajan's span remained in the range of 10–13 items, though the post hoc comparisons showed he performed worse in Session 1 than in Sessions 3, 4, 5, 6, and 8; and that he performed better in his best session (Session 4) than in his worst two sessions (Sessions 1 and 7). From Session 4 and onward, Rajan's performance on the symbols was significantly better than in his performance on the letters.

For Experiment 5B, the same 2 Type of Material (letters vs. symbols) x 8 Session ANOVA showed reliable effects of session, $F(7, 30) = 32.39$, $p < .001$, $MSE = 1.07$, and of type of material, $F(1, 30) = 245.46$, $p < .001$, $MSE = 1.07$, but these were qualified by a significant interaction, $F(7,30) = 7.48$, $p < .001$, $MSE = 1.07$.[10] Rajan's estimated spans are shown as a function of session in the right panel of Fig. 7. The same 64 post hoc Bonferroni corrected t tests were run for Experiment 5B as were run for 5A. Rajan's average memory span for symbols was higher than that for letters for all sessions, but only the difference for Sessions 9 and 13 reached the level of significance. For letters, his performance in Session 16 was higher than during Session 15, and his performance in Session 9 was lower than his performance in Sessions 12, 13, 15, and 16. None of the comparisons between sessions for symbols reached significance. Hence, the post hoc comparisons are consistent with a greater improvement across sessions for letters than symbols.

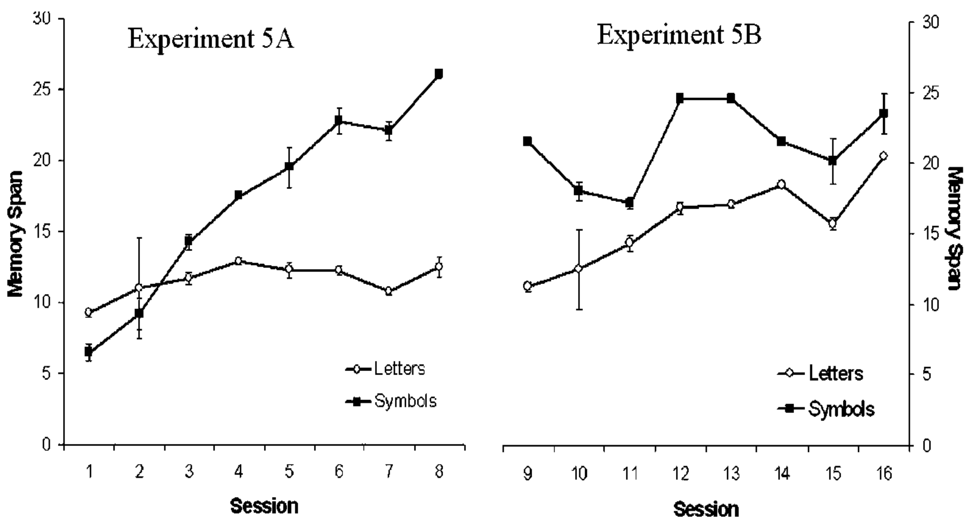

Fig. 7. Average digit span for two types of material (symbols and letters) as a function of session in Experiment 5A (left panel) and in Experiment 5B (right panel).

6.2.2. Retrospective verbal reports

Rajan's retrospective reports from memory span testing with letters and symbols contained very different types of information from his reports from memorizing digits.

For the letters Rajan tried several other methods. His initial strategy, reported for Sessions 1–8 (Experiment 5A), involved trying to create words from letter sequences. During the first session he reported encoding the list of 9 letters "TVPCXMNNC" as a group of three "TVP" with "TV" and "VP" (famous mnemonist) and a group of six "CXMNNC" as "CoX MaN for NarCotics." For example, in session 3, he encoded the sequence "MSXTBTSVPCV" as "MSX," a popular license plate prefix in India (Madras State X); TBT, a sequence that was the same forwards and backwards; and finally, "SVPCV," Hunt and Love's (1972) Subject *VP's* Curriculum Vitae, remembering that the S came before VP instead of after. Starting with session 4, he began deliberately using group sizes of 3 letters to facilitate recall, and followed by groups of 2 letters. Each of these groups would be encoded using a common word (e.g., TV became television, etc.). An analysis of all correctly recalled letter sequences during Sessions 3, 4, and 5, showed a clear pattern. The letter strings were segmented into small groups, where 96% of the groups ranged from 2 to 4 letters. The only exceptions occurred when he rehearsed the last 5–6 letters. For most of the letters (76% of the correctly recalled letters) he reported mnemonic associations and patterns.

Table 3
Rajan's conversion table mapping letters into digits

Letter	Digit
N	1
B	2
C	3
X	4
S	5
V	6
F	7
P	8
T	9
M	0

Starting with session 6, he reported trying a new strategy to recode letters into digits, but he found that he was not fast enough to be able to recode all of the letters into digits. He therefore recoded as many letters into digits as possible in the beginning of the list, and then rehearsed the remaining few as letters. As he became more skilled at recoding letters into digits, he switched to an all-digits strategy. Table 3 shows the mapping Rajan used to convert letters to digits. He eventually used a strategy where he encoded as many letters as digits as possible and then remembered the remaining part of the sequence as letters. During the last six sessions (Sessions 11– 16), the correctly recalled sequences consisted, virtually without exception, of a long group of 9, 10 or 15 recoded digits followed by 2–6 letters. On average, 80% of the letters were encoded as digits and 34% of these recoded digits were given mnemonic associations. For the remaining 20% of the letters, Rajan reported making mnemonic associations for 26%.

For symbols, Rajan's memory span was estimated at around seven items during Session 1. Rajan reported that his disappointing performance motivated him to think about ways for him to improve his performance for the symbols—our agreement with him was that he would not engage in practice outside the test sessions. From Session 2 to the end of the Experiment 5B he re-coded symbols into digits using a conversion table similar to the one shown in Table 4, which was his final mapping between symbols and digits. An analysis of correctly recalled symbol sequences from the last six sessions (Sessions 11–16) revealed a clear pattern. All symbols were re-coded as digits (100%) and for 31% of these digits Rajan reported forming mnemonic associations. He segmented the first 15 symbols into a group, occasionally by merging groups of 10 and 5 or 9 and 6. Following this group Rajan formed another group of between 1 and 9 symbols recoded as digits.

In a few sessions, Rajan was asked to recall the list as it had been presented as symbols and also as digits (using his internal recoding). He was substantially faster to recall the lists as digits ($M = 5.34$ s, $SD = 2.41$) than as symbols ($M = 36.03$ s, $SD = 13.00$), paired $t(10) = 7.50$, $p < .001$, supporting the hypothesis that he maintained the list in memory as recoded digits.

6.2.3. Reading times

Averaging over Experiments 5A and 5B, we found that the before-session reading was highly accurate. Rajan read 99.9% of the symbols correctly and 98.3% of the letters correctly. When asked to recode presented items into digits, he converted and read 99.9% of the symbols and 99.6% of the letters in a manner consistent with his conversion tables. For the after-session reading times, he read 98.3% of the symbols and 98.5% of the letters correctly. When converting to digits, he converted 98.8% of symbols as digits correctly and 97.8% of letters were converted correctly into digits in the time allotted.

Table 4
Names of symbols and Rajan's conversion table mapping old symbols to digits

Symbol set	Symbol	Name	Digit
Old symbols	!	Exclamation	1
	(	Paren	2
	[	Bracket	3
	+	Plus	4
	#	Number	5
	^	hat	6
	?	Question	7
	\	Slash	8
	*	Star	9
	@	At	0
New symbols	<	Lesser	—
	~	Approx	—
	:	Colon	—
	-	Dash	—
	%	Percent	—
	“	Quote	—
	=	Equal	—
	$	Dollar	—
	∞	Infinity	—

Our main interest was whether reading times constrained memorization performance. If Rajan could not recognize the items quickly enough, or translate them to digits, then his reported strategy would be impossible. Considering recognition time alone, the mean time to read lists of letters over both experiments was 11.87 s (SD = 1.57s) while the mean time to read lists of symbols was 22.51 s (SD = 4.23 s). This gives reading time estimates of 2.5letters/s and 1.3 symbols/s. It seemed plausible that the reading rate for symbols might have constrained his performance, but session-by-session estimates of mean reading time for each list type accounted for no additional variance in the memorization task in either Experiment 5A or 5B (all Fs < 1).

In Experiment 5B, we also collected reading time estimates of time to convert symbols or letters to digits. The mean time to read letters as digits was 25.87 s (SD = 4.87) while that for symbols as digits was 17.19 s (SD = 2.20s). This gives respectable translation rates of 1.16 letters/s and 1.75 symbols/s. These recoding speeds makes Rajan's reports of concurrent recoding plausible with a 1 digit/s presentation rate, and would leave an average of about 140 ms/letter or 430 ms/symbol for additional processing once the item was translated into a digit. Rajan claimed that he did not have enough time to recode entire lists of letters into digits. To test Rajan's claim, we compared Rajan's speed to translate and read letters as digits and symbols as digits by examining the proportion of each that took over 1 s to emit. For letters, 13% of Rajan's latencies during a test session took over 1 s to verbalize the corresponding digit—a significantly higher proportion than for verbalizing the digits corresponding to symbols, where only 1% of the latencies exceeded 1 s for their production, $t(9) = 7.38$, $p < .001$.

We also recorded ordinary students' reading times for lists of 30 items in order to see how Rajan's reading rate differed from the average. For participants in Experiment 5C, the reading times for symbols averaged 26.42 s per list, and for letters 20.71 s per list. The comparable Session 1 reading times for Rajan were 19.65 s per list for symbols and 10.37 s per list for letters. Rajan was reliably faster for symbols than control participants, $t(75) = 3.86$, $p < .001$, as well as for letters, $t(75) = 4.74$, $p < .001$. Thus, Rajan was significantly faster than the control participants at reading both types of lists. The students were 99.8% accurate reading lists of letters and 99.4% accurate reading lists of symbols.

6.2.4. Rajan's memory span compared to control participants (Experiment 5C)

Experiment 5C sought to compare Rajan's initial memory span for letters and symbols to a control group of undergraduate students. For each participant, we took the first three estimates of span for letters and for symbols to get a stable estimate of initial span. We then conducted a 2 Type of Material (letters vs. symbols) x 2 Group (Rajan vs. controls) ANOVA, with Type of Material as a repeated measures factor. There was a significant main effect of type of material, $F(1, 31) = 40.57$, $MSE = .81$, $p < .001$, with letters being recalled better ($M = 7.97$) than symbols ($M = 5.52$). There was also a significant main effect of group, $F(1, 31) = 24.30$, $MSE = .77$, $p < .001$, with Rajan recalling more items ($M = 7.67$) than the control group ($M = 5.82$). However, there was also a significant Type of Material x Group interaction, $F(1, 31) = 10.01$, $MSE = .81$, $p < .001$. We followed up the interaction with two t tests comparing Rajan's memory span to the control group's memory span for each type of material. For letters, Rajan recalled significantly more items ($M = 9.50$) than the control group did ($M = 6.43$), $t(31) = 6.33$, $p < .001$. However, for symbols, Rajan did not recall significantly more items ($M = 5.83$) than the control group did ($M = 5.20$), $t(31) = 1.08$. The results suggest that Rajan's superior span was initially limited to letters.

6.3. Discussion

Our initial estimate of Rajan's memory span for symbols was around six symbols, which did not reliably differ from ordinary college students' memory span for symbols. We therefore rejected the hypothesis that Rajan has a generalized superior memory capacity for symbols. In contrast, Rajan's superior memory span for letters was substantially higher and his average span after a few test sessions agreed well with previous estimates reported by Thompson et al. (1993). An analysis of Rajan's retrospective reports from trials with lists of letters revealed a different pattern of thoughts than he reported for digit lists in Experiments 1–4. For lists of letters, Rajan reported segmenting the sequence into groups of 2, 3 or 4 letters and then deliberately generating a mnemonic association for the group. Rajan's encoding methods for letters differed completely from his methods with numbers, yet agreed remarkably well with those methods reported by previously trained digit experts and other exceptional memorists (Ericsson, 1985, 1988). Most importantly, the data from Experiment 5A showed that Rajan's superior memory span for letters was not mediated by the same mechanisms as his superior memory for digits and did not reflect some uniform superior basic memory capacity. Furthermore, his encoding mechanisms for letters or for digits did not permit immediate transfer to other types of symbols.

Experiments 5A and 5B offered an unexpected opportunity to observe how an exceptional memory performance for a new type of material (symbols) can be attained and how a superior memory performance for one type of material (letters) can be increased into the exceptional range by further training. Rajan was able to develop new strategies for encoding symbols and letters that led to memory spans of over 20 items for both types of materials. Rajan reported recoding each symbol into a corresponding unique digit and then remembering the resulting list of digits. He similarly recoded letters as digits. Our data on his maximal recoding rates showed that he had the ability to recode symbols into digits even at the 1 symbol/s rate. In contrast, our undergraduate controls in Experiment 5C could barely even read the symbols at a rate of 1 symbol/s, suggesting that Rajan's faster reading speed may leave him with more time to engage in encoding processes than most people. At the time of recall, Rajan would re-convert the digits into the associated symbol or letter, as evidenced by his 10-fold faster speed to recall the presented string as digits compared to as symbols.

7. Experiment 6: Interfering with Rajan's superior memory span for symbols

Rajan's superior memory span for symbols depended on learning a unique mapping between each symbol and a digit. It would be easy to interfere with this mapping by expanding the set of symbols by adding a new set of ten symbols and then generating random lists from the expanded set of twenty different symbols. We reasoned that the most direct extension of Rajan's current method to deal with this new situation would be to develop a recoding table for the new symbols. Rajan would then memorize the lists sampled from the 20 symbols as recoded digits along with a list of additional binary tags that informed Rajan whether the recoded digit was from the old or the new set. By this approach, Rajan would have to memorize the new lists by memorizing two lists: one list of recoded numbers and another binary list telling him whether the recoded digit came from the old or new set. However, the binary information list would be structurally similar to the lists that Rajan struggled with in Experiments 3 and 4. Consequently, we needed to assess how well

Rajan could remember such lists by themselves and thus Rajan was presented with random lists generated from two items, one old symbol and one digit, such as 444^4^44^4^^.

Another simple test of Rajan's ability to transfer his skill would be to generate lists from a set of 20 symbols that he was already familiar with, namely a mixture of digits and the old symbols, such as "23@!2?0+." To remember these mixed strings using his recoding method, Rajan would have to remember which were re-coded digits and which corresponded to actual presented digits.

7.1. Method

7.1.1. Stimuli

For the memory-span portion of this experiment, we generated four different types of random lists. (1) *Old-symbols* lists were generated using the ten symbols in Experiment 5. (2) The 10 old symbols were augmented with a new set of ten symbols, called the new symbol set, namely {%, 1, ~, —, =, $, <, ", :, &} . *Mixed symbols* lists were generated by randomly selecting from a set of five symbols (randomly drawn from the new set) and five symbols (randomly drawn from the old set). (3) *Symbol/digit mixed* lists were generated from a set of 10 items, where five digits were randomly picked and five symbols were randomly drawn from the old symbol set. (4) *Constrained* lists were generated from sets consisting of a single digit and a single old symbol. For Sessions 1–15, the digit and symbol were chosen by randomly selecting one of the following pairs for each trial: {1,!},{2,@},{3,#},{4,^},{5,*},{6, (},{7,1/2},{8,+},{9,\}, and {0,?}. During Session 16, a random digit was selected and paired with a random symbol for each trial.

7.1.2. Procedure

The experiment consisted of 16 sessions. The general procedure was identical to that in Experiment 5, except for a few minor differences. There were 12 memory-span trials per session, consisting of three trials each of the four types of lists, namely old symbols, mixed symbols, symbol/digit mix, and constrained. The first list of each type presented in the entire experiment was 12 items long. For the first test session successful recall of given length list led to increases in list length of four items and incorrect recall led to decrease in list length by four items. In all subsequent sessions, the increments and decrements of list length were two items.

As in Experiment 5, Rajan read lists of different types both before and after the memory-span portion of the experiment. Additional lists for the reading portion of the experiment were generated using the same procedure as the lists used for memorization. The two reading phases were identical to those from the final session of Experiment 5, except that three lists each of the old symbols, the new symbols, the symbol/digit mix and the mixed symbols were presented.

7.2. Results

Following the analysis in Experiment 5, we estimated Rajan's memory span for each type of material. When Rajan correctly recalled N items but missed N + 2 items, he received a span score of N + 1. When Rajan missed N items but correctly recalled N — 2 items, he received a span score of N — 1. Given that each session only contained three trials for each type of material, it was necessary to aggregate over sessions to form two session blocks (of four sessions per block) to get sufficient estimates of memory span for each cell of the design. Consequently, the Judd and Kenny (1981) test for auto-correlated residuals could not be applied meaningfully. A 2 Session Block x 4 Type of Material ANOVA on span score was conducted. There was no significant main effect of session block, $F(1, 90) = 1.32$, $MSE = 7.69$, but there was a reliable main effect of type of material, $F(3, 90) = 232.79$, $p < .001$, $MSE = 7.69$, which was qualified by a significant 2-way interaction, $F(3, 90) = 12.21$, $p < .001$, $MSE = 7.69$. The memory span as function of type of material and blocked session is shown in Fig. 8. A post hoc analysis with 28 Bonferroni-corrected t tests showed a superior performance with old symbols lists as well as constrained lists during both sessions compared to performance with mixed symbols or symbol/digit mixed lists in either block of sessions. The difference in performance between old symbols and constrained lists interacted with session block and was only reliable during the first session block. No other differences achieved significance.

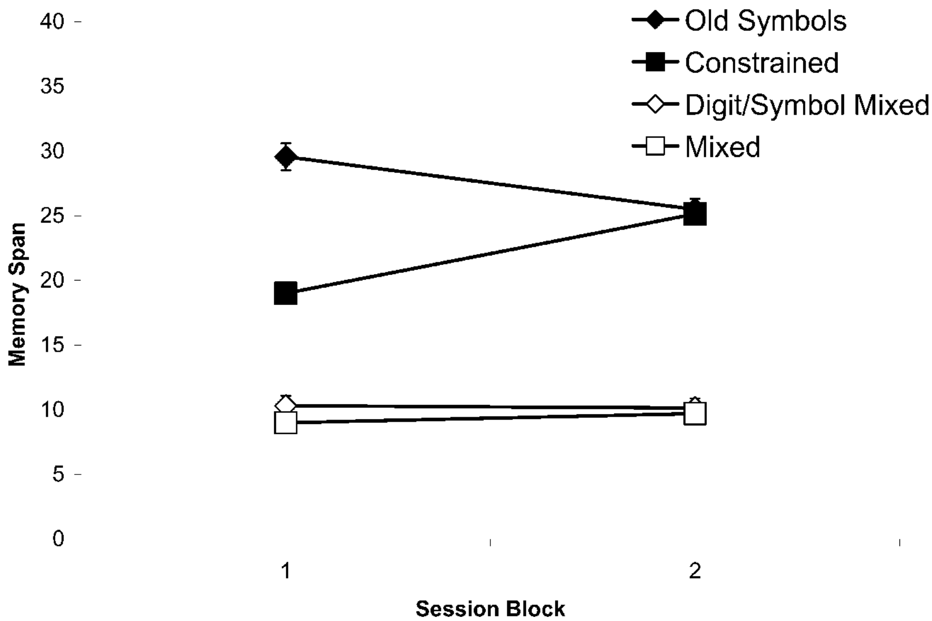

Fig. 8. Average digit span for four types of material (old symbols, constrained symbols, mixed symbols, and symbol/digit mixed lists) as a function of two blocks of sessions for Experiment 6.

7.2.1. Retrospective verbal reports

For old symbols lists, Rajan reported using encoding methods similar to those from Experiment 5 and recoded them as digits. During Sessions 11–16, all correctly recalled sequences were completely recoded into digits (100%) and for 30% of the re-coded digits mnemonic associations were reported. Rajan generally reported segmenting the sequences into either an initial group of 15 items (75% of lists) or 8 groups of 3 digits (17%). For the constrained lists, Rajan coded the symbol as a digit and used his standard methods. For example in Session 13, Rajan encoded *******5*55*5*5**5****5*55 by recoding * → 9 and forming groups of 3 items: 999-999-959-559-595-995-999-959-55, where 959 was coded as a date when his brother was born, 995 and 999 as a recent dates, 1995 and 1999. He would also notice and encode repetitions of the same 3-digit group. However, in the 5 cases when the digit code of the symbol matched the presented digit (for example, the 1 and ! case), Rajan reported being very frustrated and could not apply his normal methods. During Sessions 11–16 all the symbols in the correctly recalled sequences were re-coded into digits (100%) and the sequences memorized as list of digits, where mnemonic associations were reported for 27% of the digits. The sequences were segmented into an initial super-group of 8 groups of 3 digits (60%) or 5 groups of 3 digits (30%). The only exception consisted of a report of super-group consisting of 11 and 4 items, where by chance the initial 11 items were identical (00000000000–??00).

For the other types of list (mixed symbols and symbol/digit mixed lists), Rajan reported considerable difficulties because his method of encoding all non-digit items in the list into digits made it impossible to recall what the original item was. For example, if Rajan remembered 3 in the symbol/digit mixed lists, it could either be the symbol # or the number 3 that was originally presented. For the mixed symbols lists, Rajan would recode the old symbols and try to remember the new symbols as symbols. During Sessions 11–16 the correctly recalled sequences contained 35% symbols as recoded digits but no reports of mnemonic associations. The initial group of items ranged markedly from 3 to 10 items with no predominant pattern.

For the symbol/digit mixed lists, Rajan reported trying several unsuccessful strategies, but eventually he seemed to remember them without much recoding. During Sessions 11–16, successfully recalled lists had a somewhat larger number of presented digits (59%, where chance-level performance equals 50%) and only 16% of the old symbols were reported as having been re-coded into digits. The grouping of the sequences was quite variable from trial to trial and the groups ranged from 3 to 10 items.

7.2.2. Reading times

Following the procedure of Experiment 5, we collected reading times for the different types of materials. Rajan's error rates for all the four types of materials were below 2%. The mean reading times for the 30-item lists were 20.0 s for mixed lists, 22.4 s for new symbols, 18.2 s for old symbols, and 17.6 s for symbol/digit mixed lists.

7.3. Discussion

Rajan's memory span performance for the old symbols lists essentially remained at the same high level as in Experiment 5. However, when we interfered with the application of his recoding strategy by mixing old and new symbols together or by mixing old symbols with digits, his memory span remained at around 9 items. Thus, even after 16 sessions of testing, his memory span for these two types of materials was near the range of the standard 7 ± 2 items that untrained participants can achieve for many types of materials (Miller, 1956). Although we have only measured college students' memory span for the old symbols in Experiment 5C, other investigators (Myer & O'Connell, 1972; Williams & Fish, 1965) have estimated memory span for lists of similar types of symbols for untrained students and found them to be within the 7 ± 2 item range. These reference data suggest that Rajan's memory span may well have been above average, but his memory performance for these two types of materials did not appear to be outside the range of performance of typical subjects, as had been the case for both digits and letters.

The most unexpected result of the earlier Experiment 5 was that Rajan's memory span for constrained lists was around 12 items in Session 1—roughly his span for the constrained lists in Experiment 3. In contrast to the stability of his performance in Experiment 3, Rajan's memory span gradually increased in Experiment 6 to almost double, about 24 items. The improvement in Experiment 6 was associated with a very different encoding strategy than the one used in Experiment 3. In the current experiment, Rajan grouped the lists into sets of 3 digits and frequently generated mnemonic associations and encoded symbols as patterns. The latter encoding method is very similar to Rajan's method for encoding letters (Experiment 5) as well as the methods described by most other memory experts (Ericsson, 1985, 1988).

8. General discussion

In the Introduction, we reviewed the evidence on individuals with exceptional memory performance and found that it is consistent with theoretical proposals for acquired memory skills (Chase & Ericsson, 1981, 1982; Ericsson, 1985; Ericsson & Chase, 1982; and subsequent developments of these ideas by Ericsson & Kintsch, 1995 and Ericsson, Patel, & Kintsch, 2000) with a small number of exceptions (Wilding & Valentine, 1997). Although Thompson et al. (1991, 1993) claimed that there are some aspects of Rajan's basic memory capacity that cannot be explained as an acquired skill, they found that most evidence for Rajan's exceptional memory for digits were the results of extended practice and acquired memory skill. In fact, Thompson et al.'s (1993) review showed that Rajan does not have a generalized superior memory capacity (cf. photographic memory) and his superior performance is limited to digits and letters. Furthermore, Baddeley, Thompson and Mahadevan in an unpublished study see Baddeley, 1999; Thompson et al., 1993, for summaries found that the most plausible theoretical hypotheses involving general mechanisms, such as a superior phonological loop or an enhanced ability to rehearse verbal material, were not supported and thus unable to explain Rajan's superior memory performance for digits and letters. In our Experiment 5, we examined the possibility that Rajan might have superior memory capacity for abstract familiar symbols and examined his memory span for familiar symbols other than digits and letters. Rajan's initial memory span for such symbols was indistinguishable from college students' memory span for symbols. However, after a few weeks, Rajan acquired specific memory skills resulting in a superior memory span for that type of material. Consequently, Thompson et al.'s (1991, 1993) argument for the innate exceptionality of Rajan's memory is based on his observed performance with two types materials, namely digits and letters, that they claim cannot be explained by acquired memory skills.

8.1. The evidence for Rajan's superior basic memory capacity for digits and letters

Thompson et al.'s (1991, 1993) proposal that Rajan must be endowed with innately superior memory was based on two types of independent evidence of a common superior basic memory capacity. First and most importantly, they found that long "chunks" of 10–15 digits mediated Rajan's superior memory for digits and was evident in the first tests of Rajan's memory. These long "chunks," unlike those of earlier memory experts, did not appear to require mnemonic associations to be formed between the various digits, thus providing the foundation for Rajan's naturally superior memory. Rajan's later gains in memory performance with practice appeared to build on this superior memory capacity with the addition of acquired memory skills like those proposed by Chase and Ericsson's (1981, 1982) skilled memory theory. Thompson et al.'s second type of evidence was that Rajan displayed a superior memory span for letters (10–15 letters), which they interpreted as implying a superior basic memory capacity that generalized to different types of materials.

8.1.1. Rajan's superior memory for digits

Using self-paced memorization, we found that Rajan encoded lists of digits with "chunks" exceeding four digits, with a preferred length of 10 or 15 digits. Each list's "chunks" were encoded together into a retrieval structure akin to those used by other memory experts (Ericsson, 1985, 1988). Rajan reported a low frequency of patterns and mnemonic associations compared to other memory experts. Thus, our initial results confirmed the earlier reported findings concerning Rajan's exceptional memory for numbers (Baddeley, 1999; Biederman et al., 1992; Thompson et al., 1991, 1993) his use of long chunks, few mnemonic encodings, and retrieval structures.

Although our initial experiments supported several aspects of Thompson et al.'s (1991, 1993) findings, some of our new evidence was inconsistent with their hypothesized mechanisms. In particular, our evidence on systematic differences in retrieval speed for individual digits was problematic for their proposal of a superior basic memory capacity. They hypothesized that Rajan's memory within a "chunk" could be modeled using a slot model in which access to individual digits should be direct and immediate once the chunk was retrieved (see Richman, Staszewski, & Simon, 1995; for a similar theoretical proposal for another superior memory expert and a critique of that proposal in Ericsson & Kintsch, 2000). Our evidence on retrieval times and errors supported a more complex internal structure for these long "chunks." We proposed that "chunks" of digits are list-like structures with associations between the digits within the same digit group, where smaller numbers of immediately adjacent digits are frequently grouped together in sub-groups with or without reportable mnemonic associations. These associations can be viewed as a generalization of the structure of the encodings of 3-, 4-, and 5-digit groups observed for trained memory-span experts (Chase & Ericsson, 1981, 1982), as proposed in Ericsson and Kintsch's (1995, 2000) theoretical framework of long-term working memory. The list-structure account of Rajan's encodings of digit groups can explain why the end-points of the list (especially the beginning of the lists) are accessed rapidly. In contrast, access to interior digits is slower due to additional retrieval processing. The same hypothesis can also explain why errors were more prevalent for the last half of longer chunks.

According to our list-structure account, encoding multiple "chunks" in memory depends on the distinctiveness of the sequential associations between digits and— groups of digits within the "chunks." Experiments 3 and 4 tested our hypothesis that Rajan's memory depended on rich and varied associative connections between digits and groups of digits in the list. When Rajan was presented with both regular random sequences of digits and constrained sequences (random combinations of two digits), his performance was much less exceptional for constrained sequences. Laboratory studies have demonstrated that college students show the opposite pattern: their memory performance on constrained lists of digits is higher than normal (Slak, 1974).

Our experiments were designed to minimize the chance that his reduced performance resulted from lack of effort. In both Experiments 3 and 4, we showed that Rajan maintained sufficient motivation to maintain his superior memory for regular digits. Our argument that Rajan's low performance in Experiments 3 and 4 was due to an ineffective encoding method was strengthened by observations in Experiment 6, where he acquired vastly superior performance for constrained lists. It is of great theoretical significance that when Rajan attained an exceptional memory for the constrained sequences in Experiment 6 he changed his encoding methods from those used in Experiments 1–4. In Experiment 6, he relied on a highly consistent segmentation of the lists into groups of 3 digits with more extensive reports of patterns and mnemonic associations. This is the pattern that

Chase and Ericsson (1981, 1982) originally observed for their trained subjects and that other investigators have found to be critical for improved performance in regular adults (Ericsson, 1985, 1988; Higbee, 1997; Kliegl et al., 1987, 1989).

Finally, in Experiment 5 Rajan increased his memory span for symbols by recoding the symbols into digits during their presentation, which allowed him to memorize them as digits. At the time of recall, he translated the digits back into their corresponding symbols. It is interesting to note that Rajan's grouping of these recoded digits reflected his normal grouping of random digits with groups ranging between 9 and 15 digits. A similar pattern of grouping was observed for the old-symbols lists in Experiment 6, although on several occasions he would group the recoded digits as 3-digit groups—consistent with his encoding of constrained digits. In sum, Rajan's reliance on encoding long groups of digits, involving 10 or more digits, was found to be highly restricted to a particular type of encoding, namely those applicable to random digits (from 0 to 9). This high degree of specialization is consistent with the mediation of prior knowledge and acquired patterns and encoding methods.

8.1.2. Rajan's superior memory span for letters and other symbols

Thompson et al.'s (1991, 1993) second claim was that Rajan's superior memory span for letters reflected the same underlying capacity mediating long "chunks" of digits. In the first five sessions of Experiment 5, we replicated Rajan's superior span—around 12 letters. Rajan's retrospective reports and the structure of his recall of letters indicated that he used a very different grouping structure for letters than he did for the long lists of regular digits. Rajan reported splitting off a few groups of 2–4 letters that he would encode as patterns or by mnemonic associations. The last part of the letter sequence was rehearsed as a group of 4–6 letters. Hence, Rajan's encoding processes for letters matched that observed for digit-span experts in the recall of digits (Chase & Ericsson, 1981; Ericsson et al., 1980). Furthermore, when Rajan decided to improve his span for letters in Experiment 5B he did not use the same encoding methods mediating his exceptional memory for regular digits. Instead, Rajan adapted the method that he discovered for improving his memory span for symbols in Experiment 5A. Rajan recoded the letters into digits and grouped the lists into groups of 3 recoded digits, which were encoded and stored in memory. With this encoding method, Rajan was able to increase his memory span for letters to over 20 letters (an increase of about 70% over his initial letter span). Perhaps the most compelling evidence that Rajan recoded the letters into digits in Experiment 5B comes from unexpected tests during which Rajan was asked to recall the presented sequence either as letters or as recoded digits. After his successful recall of a list, Rajan was able to recall the associated list in terms of the recoded digits almost 10 times faster than he was able to reproduce the list of presented letters.

In sum, Rajan's initial exceptional span for letters was mediated by segmentation of items in groups of 2–4 digits and frequent reliance on mnemonic encoding method—consistent with the principles of skilled memory (Chase & Ericsson, 1981, 1982; Ericsson, 1985). The evidence relating to Rajan's memory span for letters did not support the mediation of a similar mechanism observed for random digits nor any other claims for a superior basic memory capacity.

8.2. Rajan's ability to encode digits as long "chunks": Anecdotal evidence on its development

Our research could only find support for one exceptional aspect of Rajan's memory that differed from trained memory-span experts and thus was inconsistent with a skilled memory account (Chase & Ericsson, 1982; Ericsson, 1988). Rajan's exceptional mechanism is specific to a certain material, i.e., digits, and concerns his encoding of 10–15 digits as a single "chunk" or list. Thompson et al. (1991, 1993) reviewed evidence against the possibility that the mechanism reflected acquired skill by presenting evidence for its existence in 1980—well before the start of the extended memory testing of Rajan. Their own studies show that Rajan possessed this mechanism in 1986–1990. They inferred that Rajan must have had that mechanism when he was first formally tested in Minnesota in 1980 and displayed a span of 15 digits. They even suggested that Rajan had a similar superior memory ability as a child based on an anecdote in which an 6-year-old Rajan memorized the license plates of cars for party guests. These findings led Thompson et al. (1991, 1993) to infer that Rajan must possess an innately superior memory capacity.

Our interviews with Rajan confirmed his recollection of the anecdote with the license plates and the original testing of his span in Minnesota. Rajan's father was also able to confirm his memorization of license plates. However, our interview led us to raise some questions about the evidentiary value of this event, even under the assumption that the account is accurate. Given that all the party guests were family friends, one cannot know how many license plates Rajan had memorized prior to the party. Neither do we know how long Rajan spent looking at the plates during the party prior to the public recall of them. As part of our more extended interview, Rajan was asked to recall as many episodes as possible from his development (critical incident method of Flanagan, 1954) where he could recall both the number of actual digits memorized and where the associated time for presentation could also be recalled. The first recalled incident concerned an event when Rajan was administered an IQ test at age 14. On the digit-span subtest, Rajan's span was 8–9 digits both forwards and backwards. During another incident as a first year engineering student, Rajan was asked to memorize strings of 15 digits by his roommates and took between 45 and 60 s-in the memory span procedure only 15 s would be available for presentation of the digits. He claimed that this episode was especially vivid, as he had originally said that he might need 120 s, but then one of his roommates exclaimed that even he could memorize the digits with that much time available. Our new evidence is more consistent with a gradual development of Rajan's memory performance. However, the most prudent approach is to disregard the anecdotal evidence altogether and focus on the first public evidence on Rajan's superior memory collected under controlled and standardized conditions.

8.3. The development of Rajan's ability to encode digits as long "chunks" Were there any verifiable public achievements concerning memorization of digits that might suggest a gradual development of this ability that could explain Rajan's superior memory span performance during the first memory tests by Fox and his colleagues at the University of Minnesota in 1980? Rajan told us that he always liked to memorize numbers, such as phone numbers, dates and cricket scores and statistics, but in college he got interested in memorizing the decimals of π. He memorized the first 100 digits in an unsystematic manner prior to 1979. He was then offered a trip to the US if he could memorize the first 10,000 decimals. Between January and August of 1979, he reported spending roughly 2 h per day memorizing the first 10,000 decimals, which took an estimated total of 200–400h. Hence, prior to the original test in Minnesota in 1980, Rajan had spent a lot of time memorizing decimals of π, but he had not practiced with the memory span test. From January to June of 1981, Rajan spent 3 h/day memorizing an additional 28,000 decimals, which took an estimated total of 400–500h. Consequently, at the time Thompson et al. (1991, 1993) started testing Rajan in 1987, he had spent at least around 1000h on memorizing and refreshing his memory of the first 30,000 to 40,000 decimals of π.

How might the extended period of memorizing decimals of π have led to the mechanism that Rajan used for memorizing presented lists of digits? The tables of π are presented 100 digits per line that are blocked into groups of 10 digits. Rajan reported memorizing the decimals in groups of 10 digits—a claim substantiated by Thompson et al.'s (1993) cued-recall studies testing Rajan's memory for π. The primary challenge with memorizing decimals of π is to encode each group of 10 digits uniquely to avoid confusion with all the previously memorized 10-digit groups of decimals. Rajan's method of uniquely encoding the beginning of each 10-digit group and then associating the other digits in the group was a very adaptive strategy for learning new 10-digit groups of decimals of π. The same encoding methods can be applied to memorizing other types of numbers. Based on our interviews we estimated that Rajan had, in addition to the 5000 groups of 10 digits for π, memorized 300 groups of cricket scores, around 100 phone numbers, 150 dates, and numerous other numbers and number combinations.

Our proposed account is that Rajan acquired encoding methods adapted to memorization of the numerous 10-digit groups of decimals of π. When he was given standardized memory tests for digits in 1980, he applied the same methods to memorize digits. These methods were suitable only for memorizing rich and varied lists of regular digits and when used to memorize lists of constrained digits then performance was dramatically reduced. When Rajan discovered the methods of fixed grouping and the use of mnemonic associations in

Experiments 5 and 6, he then adopted methods that match those used by other memory experts (Ericsson, 1985, 1988).

In sum, our interviews and experimental studies reveal evidence for an alternative account of Rajan's memory for digits and letters that is not based on innately superior memory capacity. The structure and development of his superior memory performance is consistent with many of the characteristics of the skilled memory theory (Chase & Ericsson, 1981, 1982; Ericsson, 1985) and fully consistent with the subsequent extension into LTWM (Ericsson & Delaney, 1999; Ericsson & Kintsch, 1995; Ericsson et al., 2000; Staszewski, 1990). At the same time, we agree with Thompson et al. (1991, 1993) that Rajan's memory for numbers differs from memory for numbers in other memory experts (Ericsson, 1988). Unlike the many individuals who deliberately acquired memory skills specific to particular memory tasks as adults, Rajan was interested in committing numbers to memory for most of his life. When Rajan memorized list of digits, he did not report spontaneously accessing specific associations to number. At the same time, he was quite able to generate mnemonic associations to related numbers when he deliberately tried to do so. With his extensive experience in committing numbers to long-term memory, it may no longer be useful to explicitly access mnemonic associations. New information is represented in terms of previously acquired information with meaningful associations in the manner that everyone adds new factual information to their existing knowledge base. This type of meaningful encoding of information was shown by Ericsson and Kintsch (1995; Ericsson et al., 2000) to provide an account in terms of LTWM for the superior memory of experts in their domain of expertise.

8.4. Theories of superior memory and case studies of exceptional individuals

Our paper exemplifies a general approach for the study of exceptional individuals like Rajan. Our approach was an adaptation of the expert-performance approach proposed by Ericsson and Smith (1991; Ericsson & Charness, 1994; Ericsson et al., 1993; Ericsson & Lehmann, 1996) for studying consistently superior performance of experts for representative tasks that capture the essence of expertise in a domain. Evidence should consist of reproducible performances that are clearly superior to everyday adults' performance in standardized testing situations. Ideally, the superiority should be so great that the exceptional individuals' performance on a single trial would be an outlier in the distribution of the performance of the general population. In our experiments, for example, Rajan's exceptional memory performance for digits, symbols, and letters exceeded average college students' performance by as much as 10–20 SD. When standardized tasks are employed, it is often possible for many different investigators to replicate the same or similar performance in different laboratories (Baddeley, 1999; Biederman et al., 1992; Thompson et al., 1991, 1993). Our disagreements with Thompson and his colleagues never concerned the reproducibility of their experimental findings, only their interpretations of these findings.

We sought experimental conditions under which Rajan's exceptional memory performance could be repeatedly reproduced, particularly for the most theoretically relevant phenomena from earlier studies (e.g., his 10–15 digit groups and his superior memory span for letters). Then we collected detailed data on the mediating cognitive processes to generate alternative hypotheses about the mediating mechanisms and/or capacities. These hypotheses were later examined with additional experimental tests and analysis of other converging evidence on the mediating processes. Because the effect sizes in our experiments were often very large, in most cases we could identify mediating mechanisms with unusual precision. For example, the effect size of the difference in the between-group and within-group study times in Experiment 2A was 2.8 SD for the 75-digit lists and 3.1 SD for the 50-digit lists and the difference between the number of old symbols recalled and the number of items recalled in the mixed condition in Experiment 6 was 1.9 SD. We hypothesized that the process of memorizing thousands of decimals of π for a few hundred or even a few thousand hours might change the cognitive representations of numbers and thus improve an individual's memory performance for digits. All of our research findings on Rajan's memory is consistent with the emerging effects of such an extended focused practice. Recent research employing brain imaging techniques (Ericsson, 2003; Maguire, Valentine, Wilding, & Kapur, 2003) has also suggested that exceptional memory primarily reflects acquired skill. The brains of individuals with exceptional memory have not been found to differ reliably from the brains of control participants. Even more interestingly, Maguire et al. (2003) found that differences in brain activation during exceptional memory performance could be explained by the exceptional individuals' unique strategies and encoding techniques.

In sum, the research on truly exceptional memory suggests that it is mediated by a small set of possible mechanisms (Ericsson, 1985, 1988, 1998, 2003; Maguire et al., 2003; Wilding & Valentine, 1997). We believe that future research with individuals who can display consistently exceptional performance will provide some of the most challenging evidence for theories of human cognition and the best evidence on the actual limits of human performance.

Notes:
[1] Rajan could view each consecutive digit of the list by pressing a key on the keyboard. Regardless of how rapidly the key was pressed, there was a minimum presentation time of 200 ms. For 100 ms, the digit was presented and for 100 ms the computer screen was blank to ensure perceptible separation of consecutively presented digits.

[2] This value consists of 200 ms for the actual presentation of the digit on the screen and a latency of around 300 ms until the subsequent keystroke.

[3] This value consists of 200 ms for the actual presentation of the digit on the screen and a latency of around 100 ms until the subsequent keystroke.

[4]In 50-digit lists, the last keystroke to present the 50th digit provides no information on the subsequent processing of the last 10-digit group.

[5] To assess whether the single-subject data met the assumptions for ANOVA Judd and Kenny's (1981) χ^2 test for autocorrelation of the residuals was calculated and no significant violations of the independence assumption were detected for 50-digit lists, $\chi^2(5) = 1.43$, $p > .05$ and the 75-digit lists $\chi^2(5) = 8.76$, $p > .05$ (Ericsson & Polson, 1988a).

[6] The advantage for the 5-group remained even when the analysis was restricted to only the first five digits of each group.

[7] The Judd–Kenny x2 test revealed no evidence of autocorrelation among the residuals, $\chi(5) = 10.05$, $p > .05$.

[8] The Judd–Kenny x^2 test revealed no evidence of autocorrelation among the residuals, $\chi(5) = 8.96$, $p > .05$.

[9] For Experiment 5A, the total model accounted for 96.6% of the variance. There was no evidence for serial dependence among residuals, $\chi(5)=10.36$, $p > .05$.

[10] The model accounted for 94.3% of the variance in estimates of memory span and was no evidence for serial dependence among residuals, $\chi^2(5) = 4.38$, $p > .05$.

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