INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

University Microfilms International
300 North Zeeb Road
Ann Arbor, Michigan 48106 USA
St. John's Road, Tyler's Green
High Wycombe, Bucks, England HP10 8HR

7824892

KOSTAKI, STAVROULA ERIKETTA
THE MATHEMATICS PROGRAM OF THE GREEK HIGH SCHOOL IN TERMS OF INTERNATIONALLY ACCEPTABLE PATTERNS AND PRACTICES IN MATHEMATICS EDUCATION.

THE UNIVERSITY OF NORTH CAROLINA AT GREENSBORO, ED.D., 1978

University Microfilms International 300 N. ZEEB ROAD, ANN ARBOR, MI 48106

© 1978

STAVROULA ERIKETTA KOSTAKI

ALL RIGHTS RESERVED

THE MATHEMATICS PROGRAM OF THE GREEK HIGH SCHOOL IN TERMS OF INTERNATIONALLY ACCEPTABLE PATTERNS AND PRACTICES IN MATHEMATICS EDUCATION

by

Stavroula Eriketta Kostaki

A Dissertation Submitted to
the Faculty of the Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Education

Greensboro
1978

Approved by

Lois V. Elinger

Dissertation Adviser

APPROVAL PAGE

This dissertation has been approved by the following committee of the Faculty of the Graduate School at the University of North Carolina at Greensboro.

Dissertation Adviser Lois V. Edinger

Committee Members James B. Macdonald
William A. Love
Fritz Mengert
[illegible]

March 28, 1978
Date of Acceptance by Committee

March 28, 1978
Date of Final Oral Examination

KOSTAKI, STAVROULA ERIKETTA. The Mathematics Program of the Greek High School in Terms of Internationally Acceptable Patterns and Practices in Mathematics Education. (1978) Directed by: Dr. Lois V. Edinger. Pp. 139

The purpose of this study was to examine the mathematics program of the Greek high school in the light of international developments in mathematics education. The program was described in terms of content, textbooks, teaching methodology, and teacher training and qualifications. Analysis of these areas served as the basis for recommending modifications of the mathematics program of the Greek high school in terms of internationally acceptable standards. Data were collected by direct observation of Greek mathematics classes, interviews with Greek educators, and by reviewing literature published by the Greek Ministry of Education.

The content of the Greek mathematics program consists of arithmetic, algebra, geometry, trigonometry, analytic geometry, calculus, and statistics, and is demanding and rigorous. Course content is represented by and limited to the textbooks. Because the course content and textbooks are extensive in scope and depth, they exceed the practical needs of the students. The method of presentation employed by the majority of the Greek mathematics teachers consists of traditional lecture-type approaches. Greek mathematics teachers are graduates of the departments of pure

mathematics but receive no formal professional training in either pedagogics or psychology in their undergraduate program.

In order for the Greek mathematics program to be more in accord with generally accepted international guidelines, the following recommendations were made.

(1) The content should be modified to become more relevant to the needs and interests of non-scientifically bound students.

(2) The study of the history and development of mathematics should be included in the program.

(3) Students should be encouraged to think for themselves instead of relying on memorization.

(4) Attention must be given to the inflexible atmosphere that is present in Greek mathematics classes.

(5) The mathematics teachers should receive formal professional training in addition to their mathematics training.

(6) Mathematics teachers should be given broader opportunities for inservice professional development.

(7) A professional association of teachers of mathematics should be formed.

(8) The roles of the Greek inspectors should be redefined to include the broader aspects of supervision such as: consulting, advising, coordinating, and curriculum leadership.

(9) Textbooks should be improved in terms of presentation and content, physical characteristics, and supplementary materials.

(10) The governmental unit in charge of publishing text-books should be replaced by private publishing firms.

(11) Instructional aids should be adopted for use in the program.

ACKNOWLEDGMENTS

The writer is grateful to her dissertation advisor, Dr. Lois V. Edinger, for the guidance given her in this endeavor. Special thanks are extended to Dr. William P. Love for valuable advice in matters concerning mathematics education. The writer is also obligated to Drs. James B. MacDonald, R. Fritz Mengert, and Marshall Gordon for their full support and assistance.

TABLE OF CONTENTS

Page

CHAPTER

Page

CHAPTER

APPENDICES

CHAPTER I

INTRODUCTION

Statement of the Problem

The purpose of this study was to provide information about the mathematics program of the Greek high school and suggestions for modification in terms of current practices and accepted patterns in mathematics education.

Specifically, the objective was to determine whether the mathematics curriculum of the classical track of the Greek high school met the standards recommended by the international mathematics community in the areas of content and textbooks, teaching methodology, and teacher training and qualifications.

Significance of the Study

In the past decade there have been international efforts for exchange of information, curricula, and approaches in the teaching of mathematics. As a result, mathematics teaching has been the subject of the work of many international conferences such as those organized by the United Nations' Educational, Scientific, and Cultural Organization, (UNESCO): Budapest, Hungary, 1962; Oberwolfach, Germany, 1964; Frascati, Italy, 1964; Utrecht, Holland, 1965; Paris, France, 1971, etc. (Fehr, 1965; "New Trends," 1966)

Under the sponsorship of the Organization for Economic Cooperation and Development (OECD), meetings were held in Zagreb and Dubrovnik, Yugoslavia, 1960; Aarhus, Denmark, 1960; Bologna, Italy, 1961, etc. (Fehr, 1965) There have also been three major International Congresses on Mathematical Education in Lyons, France, 1969, at Exeter, England, 1972, and in Karlsruhe, Germany, 1976. However, Greek participation in any of the international meetings was negligible.

A search of the literature revealed that there has been very little written about the mathematics program of the Greek educational system. Namely, Professor Glavas (1966), director of Teachers' College for Secondary School Teachers in Athens and himself a mathematics educator, described the high school mathematics reform in Greece in an article for the Mathematics Teacher. (Glavas, 1966) He reported that the reform focused on a gradual modification of old programs. It included the preparation of experimental textbooks and new syllabi for certain mathematics courses and an increase in the hours of mathematics teaching per week. Some of the difficulties encountered were related to the inadequate preparation of teachers in methods of teaching or in knowledge of more recent subject matter.

Another educator, G. H. Miller (1964, 1966), made two brief comparative studies of geometry as taught in the fifth and sixth grades of elementary school and in the high schools of Greece. He concluded that in the elementary school the topics taught to the fifth and sixth graders are more advanced than those covered in an American junior high school. On the secondary level, geometry as it existed in the Greek high school placed emphasis on the geometry of the ancient Greeks. Euclidean geometry was taught to all Greek students in grades seven through twelve as opposed to the one-year course for college preparatory students in the United States. In addition to that, all students were instructed in solid geometry during the last two years of high school. This was in contrast with the de-emphasis placed on the subject in the United States. Projective and analytic geometry were also taught to the college bound students.

Dr. Howard Fehr (1965) of Columbia University listed Greece's efforts for change in an article which gave a general overview of the reform of mathematics education around the world. At the time Greece had undertaken two main projects. One was translating into Greek foreign texts which dealt with contemporary high school mathematics. The second project involved modernizing the teaching of mathematics in the schools.

It was therefore considered significant to engage in this study in order to examine Greece's mathematics program and to suggest, if necessary, modifications based on internationally acceptable standards. In addition, this study could serve as a model for other countries to undertake similar tasks; thus, it will contribute to the purposes of comparative education. If the international recommendations are to be effective, it is important that different countries do conduct such studies and examine and evaluate their programs in the light of these recommendations.

Procedure

The method used was a descriptive approach. The writer visited Greece in order to collect data by means of direct observation of mathematics classes, interviews and conferences with persons in key educational positions, and by examining literature published by the Ministry of Education.

Limitations of the Study

The study did not purport to examine each area, i.e., content, textbooks, teaching methodology, and teacher training in depth. However, this was a deliberate choice on the part of the writer who wished to analyze mathematics education in its broader form.

Also, the study was limited to examining mathematics education as only one aspect of the total curriculum.

Organization of Greek Education

The objective of this section is to sketch the present organization of Greek education. The different levels of the educational system are described along with the national aims and government policies for each of these levels.

Before presenting the Greek educational system it is beneficial to mention briefly two factors that influence Greek life in general, and as a result, Greek education. They are the Orthodox faith and the modern Greek language. Dr. T. R. B. Dicks (1971), a lecturer in Geography at the University of Strathclyde, Glasgow, described Greece as being "inhabited by a remarkably homogeneous and fundamentally Greek cultural group" (p. 21). The majority of the population is made up of Greeks while the minorities of Albanians, Jews, and Turks account for less than six percent of the population. Most of the people (94%) are of the Christian Orthodox faith, which is "a symbol of Greek nationality and an essential part of Hellenism" (Dicks, 1971, p. 30). There are also a few Jews, Catholics, Protestants, and Muslims.

The second factor of direct bearing on Greek education is the disagreement on language. The language spoken by 93% of the people is known as modern Greek. It is a direct derivative of the popular Byzantine language, which in turn grew out of the koine that was used throughout the Greek world at the time of Alexander the Great. The spoken form of the modern Greek language, demotiki, has become a rich and forceful literary medium. However, there is also the purist form, katharevousa, which was manufactured, at the beginning of the 19th century to accord more closely with the ancient Greek language. It is used in all official documents, including textbooks.

The above two forces of religion and language tend to induce a spirit of classicism in Greek society. Moreover, they influence education so that "schools in Greece continue to provide an education geared to the past" (Massialas, 1971, p. 194). Combined, these two forces lead to an educational philosophy of conformity and conservatism which does not adapt readily to modifications and change.

The Greek educational system is divided into three components: general, vocational, and higher education. Since education in Greece is a state function and responsibility, all levels fall under the jurisdiction of the Ministry of Education and Religion. Education at all levels is free. Textbooks are also provided free of charge.

The controlling authorities within the Ministry of Education are the Minister, who has supreme executive authority; the Secretary-General, who is the Minister's immediate assistant; the Supreme Council of Education, which is the highest consultative, supervisory, and administrative body for education at all levels except the University level; the Director-General of General Education; the Director-General of Vocational Education; and the Director-General of Higher Education. (Ministry of Education, Athens, 1971)

In order to ensure efficient operation of schools, Greece is organized into districts which come under the supervision of Inspectors-General and Inspectors. The members of these inspectorates are teachers who have been promoted to this special position, and their responsibilities are to supervise and act as consultants.

According to a text prepared by the Ministry of Education, Athens (1971) the broad national aim of Greek education is to "raise the educational level of the population in the shortest possible time, and as a result, the standard of living and level of cultural development" (p. 527).

General Education

General education includes pre-primary, primary, secondary, special education, and adult education schools.

Pre-primary education. Greek pre-school education is designed for children between the ages of three and five. The objective is "the normal development of the physical, mental, and intellectual faculties of the children. Pre-school education is based on play and other appropriate recreational activities" (Ministry of National Education and Religion, 1973, p. 8).

Elementary education. Elementary education, also known as *demotic* education, covers a six year period of studies and presently is compulsory for all children between the ages of five and one half and fourteen years of age. Dicks (1971), however, found that "there is a sixteen percent illegal drop-out of pupils who do not continue their education to the legal school leaving age" (p. 140). Elementary education is coeducational.

Law Decree 129/1967 states that the purpose of elementary education is "to assist the children's physical development and provide them with the necessary basic instruction for promotion to secondary or vocational education" (Ministry of Education, Athens, 1971, p. 527). The same Law Decree describes the Greek elementary school as designed to "instill and have internalized in the student love for the Greek motherland, the Christian Orthodox faith and the moral life" (Massialas, 1971, p. 192).

The elementary school year starts in the latter part of September and ends during the latter part of June. Classes are given six days a week for four to six periods of instruction per day. A period of instruction is approximately forty-five minutes.

The program of studies includes the following subjects: religion, reading and writing of the modern Greek language, arithmetic, elementary geometry, history of Greece, highlights of world history, geography, penmanship, music, arts and crafts, physical education. Curricula and textbooks are determined by the Ministry of Education, whose decisions are mandatory. A complete description of the elementary school program can be found in Appendix A.

Upon completion of the six year program of elementary studies, the student is awarded a certificate, since elementary education may be an exit point for some. Those students planning to attend secondary school are required to take an entrance examination, which mainly consists of two parts: one in arithmetic and the other in composition.

In connection with the campaign to eliminate illiteracy among adults, elementary night schools also exist. According to Legislative Decree 3094/1954 attendance is compulsory for all persons between twelve and twenty years of age who have not completed primary school. (Massialas, 1971)

Primary school teachers are graduates of the Pedagogic Academies (Academies of Education). Their training is of two years duration and includes courses such as religion, Greek language and grammar, Greek history, mathematics, science, pedagogics, psychology, philosophy, sociology, art, physical education, music, administration, home economics, hygiene, elementary agriculture, foreign languages, and student teaching. A detailed description of the above program may be seen in Appendix B.

Secondary education. The text prepared by the Ministry of Education, Athens, (1971) describes the broad national aim of secondary education as providing "pupils with general instruction so that they may either enter immediately into social and professional life or continue their studies in higher institutions" (p. 527). Law Decree 129/1967 states that the curriculum aims at "having students become the carriers of the Hellenic-Christian civilization through the study of selected works...original and translations of the classical Greek and Orthodox Christian scholarship" (Massialas, 1971, p. 192).

Secondary education is known as <u>middle</u> education. The Greek secondary school is a six-year school. This six-year period is subdivided into two three-year cycles, namely the lower cycle and the upper cycle.

In most of the country's high schools, the upper cycle is further divided into two tracks, the <u>classical</u> or theoretical and the <u>practical</u> or scientific one. Students must select one of the two options. However, registration in either section depends on the student's personal choice and is not predetermined by school officials.

The school year starts on October 1st and ends during the first ten days of July. Pupils attend school six days per week for five to seven periods of instruction per day. A period of instruction is approximately fifty minutes. Greek secondary education is not coeducational and therefore boys and girls receive instruction in separate schools, but both curricula are similar.

According to the official syllabus of the Ministry of Education, published in the Official Gazette (1969), the offerings for the lower cycle include courses in religion, modern Greek, ancient Greek, history, mathematics, geography, science, foreign languages, civics, biology, anthropology, hygiene, vocational guidance, physical education, art, music, and home economics.

In addition to the above courses, the upper cycle pupils are instructed in cosmography and Latin. The difference between the two tracks of the upper cycle is that instruction in mathematics and the sciences is increased in hours per week for the students of the scientific track while the ancient Greek load is decreased and Latin is deleted. All programs of study are issued by the Ministry of Education; they include a detailed description of content, and weekly hours of instruction and are mandatory. For a complete description of the secondary school program, see Appendix C.

Major or final examinations for each subject are given twice a year, in February and July. They are prepared and administered by the teachers of the respective courses. Students failing an examination in the July period may be retested at a later date in September prior to the beginning of the new school year.

Upon completion of the high school's program of studies, students are awarded a diploma. Those interested in attending any of the institutions of higher education must successfully pass a series of entrance examinations. These examinations are conducted annually by the different departments and schools of the universities during the month of September.

The secondary school teachers are graduates of one of the Greek universities. Their training lasts four years and includes only courses which pertain to their chosen discipline. Upon completion of their university program of studies, those interested in becoming secondary school teachers apply to the Ministry of Education and are appointed with no previous training in pedagogics, psychology, teaching methodologies, or practice teaching.

At the secondary level there are also schools which specialize in offering courses with concentrations in the areas of economics and maritime studies. There are also night schools.

Another type of school which is closely associated with Greek secondary education is the privately owned <u>frontistirion</u> or tutorial school. These schools specialize in both remedial tutoring for students who are weak in a certain subject and preparatory tutoring for those interested in taking the universities' entrance examinations. Since admission to higher education is competitive many of the students find it necessary to attend the <u>frontistirion</u> for three or more nights a week during the last three years of high school.

<u>Mathematics education in the Greek high school</u>. Mathematics is taught four periods per week to the three lower forms. In the upper cycle it is taught four hours weekly in the classical section and six to seven in the scientific one.

During the first two years, the subject matter consists of review of arithmetic and non-demonstrative geometry. Algebra is introduced at the end of the second year while demonstrative geometry is introduced in the third year.

In the classical section, the subject matter consists of the traditional algebra, plane and solid geometry, and trigonometry. In addition to the above, topics from set theory, vectors, probability and statistics, analytic geometry, analysis and calculus are also included. The syllabus for the scientific section includes the study of all of the above topics but in a more detailed and extended form.

Higher Education

There is a wide range of institutions of higher education in Greece. These include the universities and many professional training schools which are of equivalent status to the universities. All higher institutions are autonomous public bodies under the supervision of the Ministry of Education, and in most cases they are governed by a Rector and the faculty Senate.

The five universities are the National and Kapodistrian University of Athens, which was founded in 1837 by King Otto; the Aristotelian University of Thessaloniki, established in 1925, which is larger and more progressive school than that of Athens; the University of Patras, founded in 1964; the University of Ioannina, established in 1970; and the University of Thraki, established in 1974.

The universities are composed of different schools, which in turn are divided into departments. As an example, the different schools and departments that comprise the University of Thessaloniki may be seen in Appendix D. All schools confer degrees, and many have graduate programs leading to graduate diplomas.

Some of the professional schools are the National Metsovion Polytechnical School of Athens, the Highest School of Economic and Business Studies of Athens, the Pantios Highest School of Political Science of Athens, the Highest Agricultural School of Athens, the Highest Industrial School of Piraeus and that of Thessaloniki, etc.

Admission to a particular school or university depends on the candidate's performance on the entrance examinations. Since these institutions accept only a limited number of students, the entrance examinations are highly competitive. Dicks (1971) estimated that only three percent of those graduating from high school are admitted to higher education.

Technical and Vocational Education

In a prepared text the Ministry of Education, Athens (1971), lists three kinds of vocational and technical schools: (1) those which admit students upon graduation from elementary school and lead into qualification as a skilled worker; (2) those which train specialists and technicians and accept students upon completion of the lower cycle of secondary education; and (3) those that are designed to train sub-engineers, who must first graduate from a secondary school and then successfully pass an examination.

In summary, the basic characteristics of the Greek educational system are: (1) the six-year elementary school, six-year high school and the institutions of higher learning; (2) admission from one level of education to the next is by entrance examination; (3) mathematics is taught to all Greek high school students for the total of the six years; (4) the content of these courses consists of the traditional subject matter and is presented in the traditional lecture type manner; (5) the mathematics teachers are graduates of the departments of pure mathematics and receive no formal training in pedagogy.

Organization for the Remainder of the Study

The second chapter contains an analysis of the current mathematics program in the Greek high school. The third chapter is devoted to a review of related literature in terms of recent international developments in secondary school mathematics. The fourth chapter analyzes the Greek mathematics program based on criteria identified in Chapter III. The final chapter includes a summary of the key points of the study and recommendations for modification of the Greek mathematics program based on internationally acceptable policies.

CHAPTER II

MATHEMATICS EDUCATION IN THE CLASSICAL TRACK OF THE GREEK HIGH SCHOOL

Introduction

The chapter contains an analysis of the mathematics program in the classical track of the Greek high school. The program is described in detail in terms of: (1) content and textbooks - based on the official syllabus and the approved texts; (2) teacher training and qualifications - based on the catalogues of the Greek universities; and (3) methodology - based on direct observations, interviews, and literature published by the Ministry of Education.

Mathematics, along with ancient and modern Greek, is considered to be one of the most essential subjects of Greek secondary education. According to the official syllabus published by the Ministry of Education, the purpose of teaching mathematics is "to become aware, and stimulate the A PRIORI geometrical capacity that the human mind possesses" ("Syllabus of Courses," 1969, p. 1638). This is done within the context of geometrical problems that arise from everyday experience.

Specifically, the purpose is:

(a) to recognize precisely the properties of geometrical figures;

(b) to learn different ways of arithmetical reasoning; (c) to provide students with practice in formulating statements of any kind in a clear and concise manner, and in proving these statements according to mathematical models, in order to avoid misunderstandings and confusion. ("Syllabus of Courses," 1969, p. 1639)

Content

First Year

In the first year, which corresponds to the American 7th grade, the subject matter consists of arithmetic and non-demonstrative geometry. Arithmetic is taught three hours per week in the first semester and two hours per week in the second semester while geometry is taught one hour weekly in the first and two hours weekly in the second semester.

The arithmetic course content is mainly drawn from the theory of set operations and the "theory of numbers." The set operations part focuses on the basic ideas and definitions of a set, a subset, the null set, 1-1 correspondence, union and intersection of sets, etc.

The section on "number theory" examines extensively the four operations (addition, subtraction, multiplication, and division) and their properties (closure, uniqueness, commutative, associative, identity) on the set of whole numbers, on the set of the positive rational numbers, and the set of decimals.

Powers of the form $\underline{a^n}$, where $\underline{n}$ is a natural number equal to or greater than 2, and $\underline{a}$ is a whole number, are also studied as well as the properties of these powers. The concept of divisibility is reviewed along with that of the G.C.D. and L.C.M.

Geometry is first introduced by way of visualizing and examining simple geometrical solids and considering the different geometrical concepts and shapes that result from such an examination. Specifically, the topics covered are the following: lines, angles, symmetry, perpendicular lines, the circle and its parts, triangles, and simple constructions.

Second Year

The program for the second year includes topics from arithmetic, algebra, and non-demonstrative geometry. Arithmetic primarily consists of a review of the previous year's material and leads to the introduction of algebra. Geometry is also a continuation of the first year's subject matter.

Arithmetic is taught two hours per week. The topics are mainly from the theory of set operations. It is recommended by the Ministry of Education that this review be done in four to five lesson periods.

Instruction in algebra begins upon completion of the arithmetic review and is presented as a continuation of and expansion on the arithmetic. It begins with a thorough examination of the set of rational numbers, the four operations on this set and their properties. Powers of rational numbers with whole numbers as exponents and the operations with powers are now studied formally and in depth. The solution of first degree equations and inequalities in one unknown as well as their graphical solutions are also taught. The course concludes with a study of ratio and proportions and their applications.

Geometry is taught both semesters for two hours per week. It deals with the following topics: inscribed angles, circles inscribed in or circumscribed about a triangle, Theorem of Thales, similar triangles, area of different polygons, measurements on a circle, Pythagorean Theorem, etc. Concepts from the geometry of space include the basic ideas of solid geometry such as relative positions of lines and planes, dihedral and polyhedral angles, area and volume of the different polyhedra, the cylinder and the sphere.

Third Year

The third year, which corresponds to the American 9th grade, is devoted to algebra and demonstrative or theoretical geometry. Both are taught twice a week during the entire year.

The algebra course is subdivided into five parts. The first consists of topics taken from elementary symbolic logic such as the ideas of implication and logical equivalence, quantifiers, Cartesian products of finite sets, binary relations, equivalence relations, order relations, mappings, and functions.

The second part is mainly the study of algebraic expressions, polynomials, and systems of first degree equations and inequalities. Elements of vector theory are included in the third part. Elementary descriptive statistics make up the fourth section of the course. It deals with collecting and reporting data, diagrams, and measures of central tendency. The topics of the last part are taken from trigonometry. They include trigonometric numbers of the acute angle, trigonometry of the right triangle and the use of the trigonometric tables.

Geometry is approached axiomatically as opposed to the non-demonstrative methods of the previous years. At first the student becomes familiar with both the deductive and inductive methods of scientific inquiry, and the concepts of axioms, theorems, and corollaries in geometry, along with the history and the purpose of geometry.

As part of the geometry of the plane, the different types of lines and surfaces are examined. Angles are defined as ordered pairs of two half-lines with a common origin, and theorems involving angle relationships are proven. Parallel lines and Euclid's fifth postulate are also studied. Triangle congruency is another topic of extreme importance and the course culminates with methods of solving problems of simple constructions and loci.

Fourth Year

This year's program includes algebra, geometry and trigonometry. The latter is at this time formally studied as a separate subject for one hour per week.

Algebra is taught twice per week. It includes the following topics from the theory of logical propositions: simple propositions, complex propositions, connectives and truth tables. To develop these topics, the teachers are directed to use examples mainly from the area of mathematics and follow them with mathematical applications.

The concepts chosen from vector theory are orthogonal and orthonormal systems of reference, sum, difference, and ratio of two vectors, and product of a vector and a real number. Solution of first degree systems of equations in two unknowns are now taught by means of determinants. The study of real numbers is completed with the introduction of irrational numbers.

Complex numbers are also studied. This is followed by second degree equations in one unknown. Quadratics of the form $\underline{ax}^2 + \underline{bx} + \underline{c}$ are examined in depth and in terms of the discriminant, the nature of the roots, sign of the quadratic, etc. Second degree inequalities and systems of second degree equations follow. The course ends with topics selected from the area of statistics such as measures of central tendency, measures of dispersion, and the idea of correlation.

Geometry is taught twice per week during the first semester and once during the second. It includes a quick review of the third year's material and new topics from plane geometry such as Thales' Theorem, the Pythagorean Theorem and its applications, circle of Appolonius, similar polygons, power of a point with respect to a circle, loci, and constructions.

Trigonometry this year deals with directed circles and arcs, the unit circle, trigonometric or circular functions in general, trigonometric numbers for arcs of 45°, 30°, and 60°, theorems of sines and cosines and their applications, and trigonometric identities.

Fifth Year

Analytic geometry is introduced this year as part of the algebra course and algebra, geometry and trigonometry are continued. Algebra and analytic geometry are taught two hours per week during the first semester and one hour per week during the second. The topics studied are absolute value theory in general and absolute value equations, properties and limits of series, progressions, logarithms, exponential and logarithmic equations, elements of combinatorics, elements of probability. Analytic geometry mainly involves the equations of lines and their graphs in the Cartesian coordinate system and solutions of equations and systems of equations by graphing.

Geometry is taught two hours per week during the entire year. The section on plane geometry involves theorems on regular polygons and their properties, theorems about the circumference and the area of a circle, etc. Solid geometry concentrates on planes and lines, parallelism theorems, theorems on the intersection of planes and lines, dihedral, trihedral, and polyhedral angles, and loci.

Trigonometry is taught once per week during the second semester only. It involves expressing the trigonometric functions of arc ($a \pm b$), known as angle ($a \pm b$), in terms of the trigonometric functions of arc *a* and arc *b*; trigonometric identities in general and their properties; and the logarithms of the different trigonometric functions of a given arc.

Sixth Year

The sixth year program consists of algebra and elements of analysis, geometry and trigonometry. Algebra-analysis is taught for two hours per week while geometry and trigonometry are each taught for one hour per week. The algebra part deals with the basic definitions and properties of groups, rings, fields. All these are covered in four lessons. Analysis focuses on functions of the real variable, increasing and decreasing functions, maxima and minima of a fuction, etc. Functions of the form $y = ax^2$, $y = ax^2 + bx + c$, $y = a/x$, $y = (ax + b) / (cx + d)$, where the coefficients are real numbers and x takes real values, are studied and graphed. Functions which result from the relations: $x^2 + y^2 = r^2$, $(x - a)^2 + (y - b)^2 = r^2$, $b^2 x^2 + a^2 y^2 = a^2 b^2$ are also analyzed and graphed. The course culminates with the derivative of a function, L'Hospital's rule, the indefinite integral, and applications from geometry and mechanics.

Geometry is mainly that of space and the main topics are polyhedra (prisms, pyramids, etc.), symmetry in space, theorems on the cone and those on the sphere. And trigonometry is basically the study of trigonometric equations, inequalities, and inverse circular functions, solutions of triangles and applications.

In summary, the mathematics courses taught in the classical track of the Greek high school are extensive and detailed in both scope and depth of content.

Textbooks

It is characteristic of Greek education to display strong conformity between curriculum content, course content, and textbook content to the extent that any one of them can be replaced by either of the other two. As a result of that, curricula, syllabi, and manuals are synonymous and interchangeable.

The Ministry of Education formulates the different curricula and chooses all textbook adoptions with its decisions being mandatory. In searching the literature, it was found that there are eight mathematics textbooks currently used by students in years one through six of the classical track. They are as follows:

Mathematics -
1st Year of the Gymnasion - Used in year 1

Mathematics -
2nd Year of the Gymnasion - Used in year 2

Mathematics -
3rd Year of the Gymnasion (Vol. 1) - Used in year 3

Mathematics -
4th Year of the Gymnasion (Vol. 1) - Used in year 4

Mathematics -
5th Year of the Gymnasion (Vol. 1) - Used in year 5

Algebra - Used in years 4 - 6

Theoretical Geometry - Used in years 3 - 6

Trigonometry - Used in years 4 - 6

Textbooks, like programs of study, are common for all Greek students and are distributed free of charge to those in the public schools. The books are of the paperback form, which presents problems since some of them are used for periods of three to four years, two to three times per week. Six of the eight books contain tables of contents which are placed in the back of the book, but none has an index.

The printing is rather small. Bold face type setting is used for definitions, new terms, and statements of theorems thus compensating for the absence of glossaries and vocabularies. The general format of the books is unattractive and has the forbidding aspect of the old traditional textbooks.

The language used is that of the katharevousa, which adds to the rigid tone that permeates through the books since it is not the language of everyday conversation. There are no answers to any problems except for those that have been worked as examples. Absent also are diagnostic or progress tests. The only review summaries in the whole series are found at the end of the first two chapters in the second year's book. However, chapters in some of the books contain a special section with exercises representative of the whole chapter.

There are no illustrations except the necessary figures and diagrams. In all of the eight books, there are only three pictures of great mathematicians, namely those of Euclid, Ipparhos, and Viete. Also, two to three pages in the Algebra, the Theoretical Geometry, and the Trigonometry texts are devoted to brief historical notes and comments on the significance of each of these three branches of mathematics. Nowhere in the books are to be found any motivational or enrichment materials. Neither additional materials for student use (workbooks, programmed booklets, tapes, etc.), nor materials for the teacher (tests, teachers' editions, etc.), accompany any of the above textbooks.

Teaching Methodology

According to official publications of the Ministry of Education, the teaching method most often used, with the necessary adaptations according to each subject, proceeds as follows:

> (1) stimulation of the interest of the class;
>
> (2) presentation of the new teaching item;
>
> (3) deepening analysis of the item in discussion;
>
> (4) acquisition and assimilation by relating the new item to the modern standards, to such degree the maturity of the class permits. (Ministry of National Education and Religion, 1973, p. 13)

It is common for the mathematics inspectors to issue circulars containing information on the teaching of mathematics. For example, C. Christoforithis (1971, 1972), Inspector of the 5th and 6th Educational Districts, distributed the following general guidelines about methodology to the teachers of his area. He recommended that the period of instruction be divided into two stages:

Stage 1 - exploration and induction of a learning environment;

Stage 2 - the actual teaching stage, which is further subdivided into:

- (a) presentation of the new material (5 minutes);
- (b) elaboration and completion (30 minutes);
- (c) expression (5 minutes).

During stage one, approximately 10 minutes, the previous day's homework is checked and any difficulties or questions the students might have are answered. It is also suggested that this is the proper time to stimulate the students' curiosity for the presentation of the new material and make them aware of the lack of knowledge. This is done by giving the students an appropriate problem that is concrete and complex but yet within their capabilities. They are asked to work independently in their notebooks while the teacher circulates about the room observing their work. A student is then asked to write on the board both the problem and the solution. The new topic is thus introduced and the students have assisted in stating the problem under consideration.

Presenting the new concept is considered to be the most crucial part of the whole teaching act. It is difficult and important, since the purpose of the presentation is to cover the surface of the new topic and also tie it in with previous learnings.

Elaborating on the newly presented concept is the formal aspect of the instructional period. This is the time to explore different situations pertaining to the topic under consideration, to prove, to generalize, to abstract and formulate hypotheses, to consider applications, etc. The purpose is to help the student assimilate the material, add to his experience, and maintain his interest.

According to the circular, the instruction continues with the brief stage of expression. This is the period of evaluating and assessing the degree of understanding achieved. It is done by questioning and by solving more problems. It also calls for a review of the highlights.

In concluding the instructional period, the students are assigned homework problems and the corresponding paragraphs in the text. It is strongly urged that the homework consists of three types of problems: the first is a direct application of the day's presentation; the second is a complement to it; and the third prepares for the new ideas to be discussed in the following lesson.

The circular defined the type of instruction that follows the above guidelines as guided discovery. It was further mentioned that this type of instruction lends itself either to the deductive or the inductive approach. However, the guidelines suggested that the inductive approach is more successful with younger students since it contributes toward increasing the students' interest and comprehension.

Teacher Training and Qualifications

The mathematics teachers of the Greek high school are graduates of the mathematics departments of the Greek universities. Their training lasts four years. However, for the completion of all work and the award of the diploma, five years are needed in most cases. During the four-year period they receive instruction mainly in mathematics and a foreign language while physics, chemistry, pedagogics and psychology can be chosen as electives. The mathematics courses include general mathematics, calculus, topology, algebra, complex functions, real functions, analysis, linear and differential geometry, probability and statistics, number theory, computer programming, etc. As an example, the program of studies of the Department of Mathematics at the University of Thessaloniki can be seen in Appendix E. It is representative of the programs of the other universities.

Upon graduation from the university those interested in teaching petition the Ministry of Education for appointment to one of the country's high schools. The newly appointed teachers receive no training in pedagogy prior to actual teaching.

Those with ten to fifteen years of teaching experience are given an opportunity for professional training. They are invited, upon successfully passing a series of examinations, to attend the Teachers' College for Secondary Education. This is a two-year in-service training, leading to a diploma, and is referred to as leadership training. From the mathematics teachers competing, eighteen are chosen every year. They attended classes in both pedagogy and mathematics. Specifically, the courses are as follows: introduction to philosophy, theory of education, educational psychology, principles of teaching, psychology of individual differences, guidance, secondary school administration, civics-sociology, and foreign languages.

The mathematics courses include set theory and number systems, probability and statistics, analysis, topology, foundations of geometry and elements of symbolic logic, vectors, matrices and linear algebra, history of mathematics, mathematics methodology, practice teaching and critique.

While attending the institute the participants receive their regular salary, plus a 25% supplement and travel expenses to and from their hometown.

In general, Greece's mathematics program for the classical track of the high school is demanding and rigorous. Course content is truly represented and fully included in the textbooks. However, it is presented in a rather rigid and authoritative manner. This is also true of the strategies and techniques that the majority of the Greek mathematics teachers employ. Of course, this comes as a direct result of the fact that Greek teachers have an excellent subject matter mastery but no knowledge of, or training in the area of pedagogy.

Since Greece is part of the international community and the country's broad goal is the development in all areas as well as the attainment of standards similar to those of other advanced countries, it will be fitting to examine what is being done around the world in mathematics education.

CHAPTER III

RECENT INTERNATIONAL DEVELOPMENTS IN SECONDARY SCHOOL MATHEMATICS

Introduction

In this chapter special attention will be given to reports growing out of international conferences. In addition, the basic features of mathematics programs in the United States and several other countries will be examined. The preparation of mathematics teachers in countries other than Greece will be discussed, and literature and research findings pertaining to teaching methodology and textbooks will be reviewed.

In 1959 the OECD organized a seminar on the Reform of Mathematics Education which was held at Royaumont, France. (Fehr, 1965) This meeting, along with technical developments such as the flight of Sputnik, precipitated an international movement which aimed at making significant changes in mathematics education. Mathematicians, educators, and psychologists joined forces, both within and between countries, and became active in the reform of the content and of the methods of teaching mathematics.

Organizations such as UNESCO and OECD, in an effort to foster the improvement of mathematics education throughout the world, have sponsored many international meetings and have published several volumes on the teaching of mathematics.

This writer will concentrate on reports of the most recent international meetings, since they express current thinking. As an example, the trends in mathematics education that emerged from the 2nd International Congress on Mathematical Education, held in August, 1972, at Exeter, England will be discussed. Reports prepared by the International Commission on Mathematical Instruction (ICMI) and published by UNESCO will be reviewed, as will be recommendations made by leaders in mathematics education.

Content

The content of mathematics courses has been tremendously affected by the efforts to modernize school mathematics. One of the main characteristics of today's programs is a tendency to teach in high school topics that were studied at the university two decades ago.

In examining the content of the different mathematics programs, one notices commonalities. Outstanding are the introduction of certain topics which have been widely accepted and the use of unifying notions which serve as a foundation to all mathematics. Topics from set theory, abstract algebra, logic, and probability and statistics occupy prominent place in today's high school curricula. Traditional topics which only emphasized manipulations and acquisition of skills are now incorporated into activities that lead to new concepts. In most cases, the organization of the content is done in a spiral form, that is, returning to the same topic at different levels and with varying degree of depth and detail.

Clear-cut divisions between arithmetic, algebra, and geometry as distinctly separate branches of mathematics are de-emphasized. Areas of mathematics are now integrated and unified by means of fundamental concepts. These concepts include the ideas of sets, relations, mappings, and functions.

In many countries at the present time, set theory serves as the core of all mathematics and is taught to elementary school children, because it is believed to enhance communication in mathematics. For example, the study of structures, the study of probability, etc. depend on set language and manipulations.

Relations and mappings provide opportunities not only for the coverage of some traditional topics in algebra but also for certain topics to be seen in a new prospective, i.e., trigonometry by means of wrapping functions.

Another unifying aspect is the notion of mathematical structures. To that effect, structures such as groups, fields, rings, and vector spaces are essential. The use of structures and their properties helps bridge the gaps between the different branches. For example, it is possible now for algebra to be seen as an extension and a development of arithmetic. However, the delegates of the 2nd International Congress on Mathematical Education warned about the introduction of "the rigorous axiomatic definition of structures before it is possible to make useful deductions which have relevant applications for the pupils" (Howson, 1973, p. 25).

Another feature is the use of precise language, rigorous proofs, and great reliance on the use of symbolism and notation. The vocabulary of logic and notions, such as logic of sets, relations, propositions, truth tables, tautologies, and quantifiers, are found to be very useful for proofs in general.

In most programs, geometry is presented in terms of geometrical transformations (mappings), or vectors, or coordinate systems instead of the classical Euclidean approach of synthetic axiomatics. Emphasis is also placed on non-Euclidean spaces.

Geometry now lends itself to the study of structures such as groups of transformations. The proponents of transformational geometry who were present at the 2nd International Congress at Exeter explained their position by stating that the students both like and find easy this kind of approach to geometry, and the teachers seem to easily adapt to it. (Howson, 1973) By making available and exposing the student to different approaches, the teacher allows the students to choose the one best suited for a problem.

In addition to these different approaches, there are also different geometry programs based on the degree of axiomatization within a certain approach. ("New Trends," 1972) There are programs which use no axiomatics at any level. Then, there are programs that emphasize axiomatic methods from the very introduction of the subject in the lower levels, and the ones which begin with an intuitive and descriptive method in the first years and move to a complete axiomatization during the higher grades of high school. Greece follows the third position, as it has been described in Chapter II.

The use of models, schemas, diagrams or traditional graphs and the use of color is encouraged in the teaching of geometry.

The axiomatic approach, which was a characteristic of geometry only, and absent from algebra or arithmetic is now being extended throughout mathematics. However, geometry is still considered to be the best means to expose students to axiomatics and deductive reasoning.

Probability and statistics are widely recommended and tried out. And the uses of probability and statistics in industry, economics, and other areas are emphasized. It is recommended to use descriptive statistics collected by the students, such as height, weight, etc., at the beginning, make an analysis of these statistics, and that should lead to the deduction of conclusions. Probability, too, is first taught by the use of intuitive and operational experiments and then by theoretical problems. In teaching the above, the availability of instructional aids, field trips, and coordination with other subjects is greatly emphasized.

Generally, it is strongly urged for mathematics teachers to coordinate and cooperate with other subject matter teachers such as science, art, music, geography, economics, etc. in order to cover areas of applications and show the use of mathematics as a tool. It is also considered important to expose the students to the construction of mathematical models for problems taken from the everyday world. Applications could serve as motivational material.

The advent of computer science has had an impact on mathematics education. Today computers are being introduced and used in the schools either as management tools or as motivational aids. In terms of managing the classroom business (Computer Managed Instruction - C.M.I.), the computer administers and scores tests, gives assignments, collects data, etc. Computers can also be used to assist instruction (Computer Assisted Instruction - C.A.I.) in a tutorial or reinforcement capacity or as motivational devices to stimulate the learning of mathematical ideas.

The early introduction of computer science into the school programs was among others recommended by the participants of the World Conference on Computer Education, held in Amsterdam, in August, 1970. In addition to computers, calculating machines are beginning to occupy an eminent place in the schools. However, the merits and demerits of using calculators have not yet been resolved and established. (Atchison, 1973)

Calculus topics such as continuity, limits, differentiation, integration, differential equations and their applications are now taught in many high schools. This is partly due to the growing role of science in society. ("New Trends," 1972) It is possible, however, for "too much rigor too early to spoil the pupils' confidence" (Howson, 1973, p. 26).

The 2nd International Congress voiced concern over the absence and neglect of courses in the history of mathematics on the high school level. These topics are ideal for the study of relationships both within mathematics and between mathematics and culture in general. However, there seems to be a shortage of suitable teaching aids, texts, and properly trained teachers in this area. (Howson, 1973)

Mathematics Programs In The United States

Following are some of the highlights of certain programs in use in the United States taken from the report made by the Committee on the Analysis of Experimental Programs ("An Analysis of," 1963) and from reviews of new programs which are published in The Mathematics Teacher. The Committee undertook the project of examining several existing programs with the primary purpose to be of assistance to "teachers and school systems in their consideration of program changes" ("An Analysis of," 1963, p. 1).

Eight items were identified as being essential in the consideration of any program. These were the degree of emphasis on social applications, the appropriate placement of topics at particular levels, the degree of emphasis on mathematical structures, the degree of formality of vocabulary, the method used in presenting the material, the relationship between concept formation and skill acquisition, the degree of rigor found in proofs, and the availability of means of evaluation for both the program itself and the progress of the students engaged in it. ("An Analysis of," 1963)

School mathematics study group - (SMSG). In 1958 the president of the American Mathematical Association appointed a group of mathematicians and educators who came to be known as The School Mathematics Study Group (SMSG). Under the leadership of Dr. E. Begle, they directed their efforts to the improvement of the teaching of mathematics in the schools. The group believed that effective and intelligent citizenship requires a certain degree of understanding of the role of mathematics in society, and it is important for mathematics to be so taught "that students will be able in later life to learn new mathematical skills which the future will demand of many of them" ("An Analysis of," 1963, p. 33).

They were primarily concerned with attracting and training "more of those students who were capable of studying mathematics with profit" ("An Analysis of," 1963, p. 33).

Therefore, the emphasis was on the introduction of more subject matter at an earlier age. The program consisted of Mathematics for the Elementary School (Books K-3, and Grades 4-6), Mathematics for Junior High School (Vol. I & II), First Course in Algebra, Geometry, Geometry with Coordinates, Intermediate Mathematics, Elementary Functions, Introduction to Matrix Algebra, Introduction to Secondary School Mathematics (Volumes I & II), Introduction to Algebra, and Analytic Geometry. The program also included programmed materials and a series of supplementary and enrichment materials.

In an analysis of the program, it was apparent that the placement of topics was in appropriate sequence in general. The structure of mathematics was definitely emphasized except possibly in the book Introduction to Algebra. Although the language was precise, the texts were on the reading level of the students, and definitions were clear. Rather sophisticated language was used in Elementary Functions and Introduction to Matrix Algebra.

The methodological approach used was that of presentation or lecture-discussion type, although there were instances when discovery-oriented techniques were used such as in the book First Course in Algebra. The Geometry volume also provided more opportunities for student participation.

On the evidence presented in the report, there was balance between concepts and skills except in the case of

Elementary Functions and Intermediate Mathematics. Little emphasis was placed on formal proofs in Volumes I & II of the Junior High School Mathematics and Introduction to Algebra. A few proofs were present in First Year Algebra. However, the most advanced level books used complete and rigorous proofs.

There were sufficient applications in Mathematics for the Junior High School and applications taken from physical situations in Geometry. The books were accompanied by teachers' commentaries which contained review items, test questions, and elaborations on certain difficult topics.

University of Illinois committee on school mathematics (UICSM). The UICSM project was started in 1951 under the direction of the late Professor Max Beberman. The series consisted of eleven units to be used in grades 9 - 12 or even as early as 8 - 12. They were Units 1-4 - High School Mathematics; Unit 5 - Functions and Relations; Unit 6 - Geometry; Unit 7 - Mathematical Induction; Unit 8 - Sequences; Unit 9 - Elementary Functions: Powers, Exponentials, and Logarithms; Unit 10 - Circular Functions and Trigonometry; and Unit 11 - Complex Numbers.

The differences between UICSM materials and others was not in the choice of topics as such, but in the approaches and techniques used. Topics were not placed on the basis of grade levels but on the basis of background or experience levels. The study of structures was essential, and the vocabulary was extremely precise. It is important to note the distinction made between number and numeral and the use of the terms pro-numeral and numerical variable. Number means the thing itself, numeral denotes the name for a thing, and pro-numeral is a place holder for a numeral. To follow the notation and definitions used in a specific unit required familiarity with previous units.

Emphasis was on discovery on the part of the student, and for that purpose many exploration exercises were used. Formation of concepts was slightly favored over acquisition of skills. Proofs were emphasized and were often rigorous. In addition to the teachers' commentaries, there were newsletters to aid teachers in the handling of various topics, provide suggestions for the effective use of discovery, and sample test questions.

Madison project. In addition to the SMGG and UICSM Projects, there are several other, such as the Madison Project, which originated under the direction of Robert Davis. It was discovery oriented and used an informal, conversational approach. There were numerous examples and illustrations to encourage the students to think the solution for themselves.

Cambridge conference on school mathematics - (CCSM). In 1963 a group of distinguished mathematicians and scientists met in Cambridge, Massachusetts in order to establish goals for mathematics education. The results of these meetings were published in the report Goals for School Mathematics (1963), also known as the Cambridge Report. They suggested the following to be used as broad goals for mathematics education: (1) acquisition of skills by means of integrating drill into problems that lead to new concepts; (2) parallel development of geometry and arithmetic-algebra; (3) familiarizing the student with mathematics by means of a spiral curriculum; (4) building self-confidence; it was stated that "even modestly endowed students can recreate a large part of mathematics if they can remember a few basic ideas" ("Goals for School," 1963, p. 36). Students must be convinced that they can rely on their own analytical thinking; (5) precise language, notation and symbolism as necessary for communication with precision; (6) balance between pure and applied mathematics; and (7) understanding both the power and the limitations of mathematics.

Among other items, they drafted two proposals for grades 7-12 which follow different approaches. It is important to note that all work in arithmetic and intuitive geometry is completed in grades K-6, and as a result students following the Cambridge recommendations will cover an additional three years of college-level work by the end of their high school education.

The two outlines for grades 7-12 were as follows:

Plan A

Grades 7 & 8	- Algebra
	- Probability
Grade 9	- Geometry
Grade 10	- Geometry, Topology, Algebra
	- Linear Algebra
Grades 11 & 12	- Analysis

Plan B

Grades 7 & 8	- Algebra and Geometry
	- Probability
Grade 9	- Introduction to Calculus
	- Algebra and Geometry
Grade 10	- Analysis, Probability and Algebra
Grades 11 & 12	- Analysis

The algebra in Plan A began with a review of the real numbers and proceeded with the study of the ring of polynomials over a field and their algebra, and complex numbers. The corresponding course in Plan B emphasized polynomial functions instead of the ring approach, and complex numbers were introduced as order pairs of real numbers instead of as residue classes in the ring of polynomials mod $(x^2 + 1)$.

Both geometry courses used the synthetic approach up to the Pythagorean theorem and similarity. Beyond that, the geometry was treated in terms of motions of Euclidean space, transformations, and matrices, which led to the study of linear algebra.

Plan B introduced calculus in the 9th grade in the nature of heuristic approaches. Both plans included a complete and extensive study of calculus in the 11th and 12th grades. Probability was also studied by intuitive methods at first, and more advanced cases with the use of calculus were encountered at a later time.

Although it was not the primary purpose of the Conference to engage in a project, it was later considered important to develop some classroom materials and test them in a limited number of schools.

Comprehensive school mathematics program - (CSMP). The Comprehensive School Mathematics Program, directed by Burt Kaufman, was an experimental project influenced by the recommendations of the Cambridge Conference. (Exum, 1973) Its overall purpose was to individualize mathematics programs to fit the needs and abilities of each student. The project included two programs one for the elementary school and the other for the secondary school.

The secondary school program was developed as a series of books entitled Elements of Mathematics. The emphasis was on sound mathematical content, presented in an enjoyable manner. The series was geared to well-motivated and highly verbal high school students. However, CSMP is now in the process of developing materials for use by all students.

Before teachers were permitted to teach one of the CSMP courses, they attended special summer-school workshops.

Secondary school mathematics curriculum improvement study -(SSMCIS). The project originated at Teachers' College, Columbia University, under the leadership of Professor Howard Fehr in 1965. (Fehr, 1966b) It was a unified mathematics program designed for the upper 15 or 20 percent of the students. The developers believed that with modifications in terms of more applications and less abstraction it could be used by the majority of high school students. However, questions were raised concerning the program's applicability for more than the top five percent of the students.

The goals of the project were the development of mathematical thinking, the transmission of information and skills, and the development of the capacity to apply mathematics in other disciplines.

The topics were presented by a spiral approach with emphasis on the unifying concepts of sets, relations, functions, and structures. The algebra part of the program was based on an extensive study of structures so the student could undertake rigorous abstract and linear algebra courses in college. Linear algebra included the study of matrices, vector spaces as ordered n-tuples, etc.

Geometry did not follow the usual synthetic Euclidean treatment, but it was presented as a study of spaces related to algebraic structures such as vector spaces, etc. Mirrors, paper folding, and drawings were used to teach geometry in terms of transformations of the plane.

Trigonometry stressed circular functions of real numbers and their applications to periodic phenomena. Calculus was approached intuitively.

Probability and statistics were also part of the program. Computer programming using the BASIC language integrated with the other courses when appropriate. Emphasis was placed on the applications of mathematics in both the physical and the behavioral sciences. All courses included teachers' commentaries.

In addition to the United States based projects, there were mathematics curricular projects developed in several other countries.

International Curriculum Projects

Britain. Two of the better known British curriculum projects are the School Mathematics Project (SMP) and the Midlands Mathematical Experiment (MME). (Campbell, 1969) SMP included both traditional and modern topics, the sequence of which was not divided into algebra, geometry, etc., since the British consider it advantageous to include all areas of mathematics in a single year's work. The axiomatic approach was not over-emphasized. The properties were introduced but there were no formal proofs given. Even in geometry, theorems were often not proven. Calculus was introduced at an early stage. However, this was done by intuitive explanations and not by formal proofs. According to Campbell (1969), SMP reverses the role of the teacher and the text. It is the text that guides the students through the difficult parts by means of experiences that could lead to discovery and leaves the summarizing to the teachers.

MME differed significantly from SMP in its approach to geometry. The former used an approach in terms of vectors while the latter was mainly concerned with motion, transformations, and topological aspects.

U.S.S.R. According to Professors Maslova and Markushevitz (1969), the mathematics program of the Soviet Union is a more conservative one than other countries'. Its broad objectives are the development of mental abilities and the acquisition of mathematical knowledge helpful for the study of other subjects and for future life. Unifying concepts are gradually but cautiously introduced. Soviet educators believe that a much broader and contemporary symbolism should not be required knowledge for all students; it should be required only of those who have decided to devote themselves to further study of mathematics. The emphasis is placed on computations and applications, and the average student spends approximately 1,832 hours in mathematics classes between the fourth and the tenth school year.

Scandinavia. Sweden and Denmark both have very rigorous mathematics programs through the study of differential and integral calculus. In addition, the Danish program includes the study of non-Euclidean geometries (Servais & Varga, 1971).

Japan. According to Fehr (1966a), in Japanese high schools, mathematics is a required subject for all students in grades 7 - 10. After that, the study of mathematics is still compulsory, but the courses chosen depend on the specific track the student is following. The tracks available include (1) the scientific; (2) the technical or vocational; (3) the non-scientific/liberal arts; and (4) music and fine arts. The different tracks must take a prescribed combination of mathematics courses.

In summary, the following are basic features of the content of most contemporary mathematics programs: (1) acceleration of mathematics learning based on an earlier introduction of topics in the lower grades; (2) introduction of new topics and removal of other thought to be of less significance; (3) emphasis on ideas and concepts which unify and interrelate the different branches of mathematics; (4) emphasis on rigor, precise language and symbolism; and (5) renewed interest in the application of mathematics, modeling, and the introduction and use of computers in the schools.

Textbooks

Textbook selection is crucial since the text, in essence, presents the content of the course. Therefore, in 1965 the National Council of Teachers of Mathematics in the United States published a report to aid evaluators of mathematics textbooks. ("Aids for the Evaluation," 1965)

This was a list of criteria which were presented in the form of questions which related to presentation and content, the physical characteristics of a textbook, and the services provided by the publisher.

The section on presentation and content included the items on:

Structure - Does each new topic presented fit into the whole structure of mathematics? How does it relate to it? What is its scope and depth?

Rigor - What is the nature of the development of the arguments? What kinds of justifications are used in proof? Is the level of rigor appropriate for the maturity of the students?

Vocabulary - Is the vocabulary and reading level appropriate for the students? Are ideas restated in different ways rather than by exact repetition?

Defined and undefined terms - Are there distinctions made between defined and undefined terms? Is the terminology used appropriate for students at this stage?

Correctness - Is the text free of statements most mathematicians would regard as errors?

Theorems and proofs - How are major generalizations used? By what arguments are the formal conclusions established? Is frequent occurrence of a generalization regarded as a substitute for proof?

Generalizations - Are opportunities to generalize provided?

Ordering - Is the sequence between and within topics arranged in a spiral form?

Tests, exercises and reviews - Are they adequate and self-evaluating? Are problems stated clearly? Are there any challenging exercises with the levels of difficulty being identified? Do review exercises point to further instructional needs?

Illustrative examples - Do they clarify and reinforce the presentation of concepts?

Teachability - Can the text be used by students on their own? Is the book's approach compatible with that of the teacher?

Optional topics - Can they be deleted with no loss of continuity?

The section on the physical characteristics of a book and publishers' services focused on:

General format - Is the layout and construction both attractive and functional in regard to book size, quality of paper and binding, type of print? Is color being used?

Index and references - Is the table of contents detailed? Is a glossary of symbols and definitions provided? Do they facilitate referral to ideas?

Usability - Is the book free of typographical errors? Is the purchase of additional materials necessary?

Publisher's services - Are the sales and promotional materials informative? Does the publisher include supplementary materials such as tapes, etc.?

Teacher's manuals - Do they clarify the text and are of help to the teacher? Do they include diagnostic and follow-up tests, etc.?

At the Superior Pedagogic Institute in Belgium, Professor W. Servais, drew on contributions from various countries and pointed out some of the features which distinguish the new textbooks from the older ones. (Servais & Varga, 1971) He stated that school mathematics books were becoming more attractive. Two or more colors were frequently used for figures and diagrams. The style was of a more familiar and individual tone instead of the general and impersonal of the older ones. Books often included humorous drawings which, as Dr. Servais pointed out, "show that although mathematics is a serious science, there is no reason to present it glumly" (Servais & Varga, 1971, p. 118). Sections on the pioneers of mathematics pointed out the fact that mathematics was created by people.

Photographs helped illustrate the role played by mathematics in the arts, sciences, industry.

He also mentioned the fact that texts were now written with greater rigor. Definitions were more precise; proofs were stricter and stripped of nonessentials. A new type of textbook was also being published in the form of books and booklets written to be read by the pupils themselves.

Textbooks are valuable aids to instruction. However, they are not the only aid since one text cannot be expected to fulfill the needs of every student in the class. A search of the literature ("New Trends," 1970; Fitzerald & Vance, 1970), revealed that, in addition to the traditional texts, there is an increasing number of supplementary instructional materials. These can be used by either the pupils or the teacher alone or both in order to improve the quality of instruction. These supporting materials come in the form of programmed materials, audio-visuals (tapes and/or filmstrips), demonstration apparatus, manipulatives, projectuals and transparencies, games and puzzles, activity packages, kits, calculators, mini-computers.

In summary, in terms of textbooks, emphasis is on (1) correctness of content and precision of language and definitions; (2) logical organization of material so that mathematical structures are clarified and emphasized; (3) many methods of presentation characterized by flexibility for adapting to certain situations, with the majority of textbooks being discovery oriented; and (4) introduction of numerous teaching aids to supplement instruction.

Teaching Methodology

According to the report published by UNESCO, one of the current trends in mathematics education is the support given to pupil-centered instructional approaches as opposed to the traditional teacher-dominated ones. ("New Trends," 1972) As a result there is a gradual change of focus from the expository methods of teacher-lecturing and student-memorizing and repeating to learning by active participation through doing and using. However, expository teaching is still used in many countries because "curricula are heavily crowded and students must pass official examinations which determine their getting jobs or entering the universities" ("New Trends," 1972, p. 100).

Professor Polya (1965) said that the "best way to learn anything is to discover it by yourself" (p. 103). The discovery methods which are used today in some countries, United States, England, etc., form a continuum ranging from directed or guided discovery of the Socratic dialectic form to pure discovery of a heuristic and incidental nature.

These discovery-oriented techniques suggest that the students should first engage in preliminary exploratory activities. At the next step clarification and formalization of what has been done takes place. The sequence concludes with formulating problems, establishing properties, and building and testing systems of relationships.

The above steps are in accord with what the research of Dienes (1960) and Polya (1963) has pointed out that in order to learn mathematics the human mind goes through a set of stages which together make a cycle. When, or sometimes before, one cycle is completed, another may begin.

Intuition plays an important role in this kind of active approach to learning. Ideas are first treated in a non-rigorous, natural manner and later they are translated into formal mathematics. This delayed verbalization is often beneficial to the slower students. Of course, approaches such as that can only flourish in a non-threatening environment where the fear of failure or riducule is minimized.

School systems that use these new teaching approaches also have a new description of the role of the teacher. Instead of being regarded as the fountain of all knowledge, the teacher now becomes "a guide and a counselor whose main task is to stimulate the pupils' activity and learning" ("New Trends," 1972, p. 102).

Another characteristic of the student-centered methods is a growing emphasis on individual differences and ways to provide for such differences. Tamas Varga, an expert in the field of mathematics teaching, strongly believes that in "No other subject does a disregard for these differences influence so severely the efficiency of teaching" (Servais & Varga, 1971, p. 25).

Students may work individually or in small groups or they may be grouped on the basis of interest and future occupation rather than ability. This is common practice in the form of tracking the students of higher grades into scientific and non-scientific forms as done in Japan, Greece, etc. In order to better achieve differentiated instruction, a variety of means and media is used.

In conclusion, although expository type teaching is still evident in some mathematics programs, the majority of the programs are pupil-centered and use discovery and inquiry-oriented methods.

Teacher Training and Qualifications

The success of any mathematics program is directly related to the teachers who participate in it. Recommendations and guidelines growing out of reform efforts are of little value if the teachers are neither capable nor willing to follow them. Therefore, both the initial training and the continuous training of the mathematics teacher is of great importance.

Dr. Cecil Beeby, a British scholar writing about the problems of education in developing countries, postulated that an educational system must of necessity pass through the four stages described by the following model:

Stage I - the "Dame School" stage, at which the teachers are neither trained nor educated.

Stage II - the stage of formalism, at which the teachers are trained but poorly educated. This stage is characterized by the highly organized state of the classroom, the rigid syllabus, the fixed textbook, and the emphasis placed on inspection. It is the stage found in England around 1900.

Stage III - the stage of transition, at which teachers are trained and better educated but still lack full professional competence. The aims are little different from Stage II, but the syllabus and textbooks are less restrictive. Teaching is still formal and there is little in the classroom to cater to the emotional and creative life of the child.

Stage IV - the stage of meaning, at which teachers are well trained and well educated. Meaning and understanding are now stressed, individual differences are catered for and the teacher is involved in the assessment of the pupils. He may now be so confident as to reject any curriculum but his own. (Griffiths & Howson, 1974, p. 63)

From a search of literature (Fehr, 1969), (Servais, 1971), it was found that mathematics teachers over the world receive their training at the universities for periods of four to six years. This consists almost entirely of mathematics courses and some courses in an allied science and leads to a degree equivalent to the M.A. of the American universities. Their pedagogical training is in most cases of less significance, and it usually consists of one year of psychology and education courses either at the university or special Teachers' College. In most of the countries it is not required of the teachers to undertake any in-service or up-grading work.

Fehr (1969), in reviewing teacher training around the world mentioned that Germany has a very intensive program for training mathematics teachers. The complete study is of seven years duration with five of these years devoted strictly to the study of mathematics. Upon successfully completing the mathematics portion of this training, which also includes the writing of a thesis in addition to oral and written comprehensive examinations, the prospective teachers enter a two-year period of professional training. Of these, the first year is an internship in a high school, and the second consists of participation in seminars which combine both theory and practice. When the entire course of study in completed and certification by the State has been achieved, the teachers are given tenured positions and salaries equivalent to those of doctors, engineers, lawyers, etc. Holland has also a program very similar to that of Germany.

The UNESCO International Symposium on School Mathematics, held in Budapest, recommended that the teachers receive more advanced and thorough training in both mathematics and pedagogics. (Servais & Varga, 1971) It was suggested that a prospective teacher should be well versed in courses such as set theory and logic, abstract algebra, geometry and topology, analysis, probability and statistics, and the history of mathematical thought. It was also recommended that they take a few courses in one other science subject such as physics, chemistry, biology, mathematical economics, or psychology. This enables the future teacher to realize the usefulness of mathematics by studying interesting applications and also to cooperate with colleagues in these other fields.

The report of the Budapest Symposium pointed out that there exist two types of teachers: those who have an excellent preparation in mathematics but are practically ignorant in educational theory and methodology and a second category for whom the reverse is true. And neither one is acceptable or satisfactory. Therefore, there should be a harmonious balance between mathematical and professional training.

The Symposium participants suggested that professional training should include work in educational psychology and especially in the areas of human development from childhood to adulthood, with emphasis on adolescence. This will give the teacher an understanding of the attitudes and the resources of that age group.

Behavior theories and theories of personality should also be included in order for the teacher to be able "to adapt teaching to individual pupils rather than merely lecturing to an abstract pupil" (Servais & Varga 1971, p. 240). Furthermore, theories of learning need to be studied for the purpose of understanding the different cognitive aspects of learning. In addition, the prospective teacher must be aware of the importance of the emotional climate in a learning situation and the different theories of motivation.

The teacher's methodological training aims to acquaint the individual with the different teaching strategies. The Budapest Symposium emphasized the pupil-centered approaches over the traditional teacher-centered methods. A rather long period of practice teaching was also recommended for the prospective teacher.

Teacher training does not terminate with this initial pre-service experience. Further training is essential. In countries advanced in teacher education, this takes the form of seminars, workshops, formal courses, T.V. programs designed for teachers, etc. In general, the teacher should keep abreast of any research bearing on mathematics education. Belonging to professional organizations on the local, national and/or international level is encouraged. It is interesting to note that the literature (Servais & Varga, 1971) indicated that student teachers should be taught by professors who "display in their own teaching what they require of the future teachers" (p. 244).

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America published its own recommendations on course content for the training of teachers of mathematics. ("A Compendium of," 1971) The CUPM report contains the following minimal considerations. The mathematics training should include three courses in calculus and one in real analysis; two algebra courses (abstract and linear); two geometry courses covering Euclidean, non-Euclidean geometries and also in terms of a linear algebra approach; two probability and statistics courses covering both intuitive and more advanced topics; and one course in the applications of mathematics, with special emphasis on mathematical models in the physical and social sciences.

The Committee further recommended that the teachers should also be familiar with at least one computer language such as FORTRAN, BASIC, etc. Electives should be chosen from the areas of real and complex analysis, topology, number theory, numerical analysis, foundations of mathematics, and logic and linguistics. However, the above report dealt mainly with mathematics content preparation and not with the aspect of professional training.

The National Council of Teachers of Mathematics Guidelines for the Preparation of Teachers of Mathematics (1973) suggested that the prospective teachers must have a sound background in mathematics--more than they will be asked to teach--and in the philosophical, historical, and psychological aspects of education. They should also be familiar with the applications of mathematics in other areas and the role that mathematics plays in today's world. Knowledge of human development was emphasized, too. Courses in learning theories, cultural foundations of education, and philosophies of teaching should be accompanied with concurrent clinical and laboratory experiences as early as the second year of university study. Practice teaching should develop as an extension of these activities. They also recommended that it will be beneficial for the mathematical training to include the use of a computer language, the study of probability and statistics with applications, and courses in linear and abstract algebra among others.

The American Association's for the Advancement of Science (AAAS) Guidelines and Standards for the Education of Secondary School Teachers of Science and Mathematics (1971) are in agreement with all others previously mentioned. Guideline #1 named humane interest, respect for individuals and fostering an open mind, as qualifications of a teacher. Guidelines #6 and #7 recommended a series of courses in analysis, algebra, geometry, probability and statistics, and computer knowledge. Guidelines #2 and #8 emphasized the importance of applications and the building of mathematical models, respectively. Guideline #10 suggested that the teacher should have a good background in the nature of learning and the nature of the learner. Guideline #11 referred to the training of the future teacher in selecting, adopting, and evaluating both materials and teaching strategies. Guideline #12 emphasized continuous learning and professional growth for the teacher.

In conclusion, the AAAS report stated that "most importantly, the teacher's preparation should foster an open and humane mind, and encourage versatility, motivation, and confidence for continuing pursuit of self-development" ("Guidelines and Standards," 1971, p. 51).

Summary

Mathematics education on a world-wide basis is undergoing changes and is still in a formative stage. However, it is beneficial to the individual countries to be aware of and consult international practices and trends and also present their own ideas and findings. This exchange of information will contribute to the improvement of mathematics programs. Therefore, as the next step in this study, the writer will contrast and compare Greek mathematics education with internationally acceptable trends.

CHAPTER IV

ANALYSIS OF THE GREEK MATHEMATICS PROGRAM

Introduction

The purpose of this chapter is to evaluate the Greek mathematics program described in Chapter II in terms of criteria identified in Chapter III in the areas of content, textbooks, teaching methodology, and teacher preparation. To this point 'content' and 'methodology' have been discussed under separate sections primarily for the purpose of organization and presentation. However, in this chapter they will be considered under the same heading.

Content and Teaching Methodology

It was previously stated that the Committee on the Analysis of Experimental Mathematics Programs listed eight factors which influence any mathematics program and merit careful consideration when assessing one. The factors were: applications of mathematics; placement of topics; emphasis on mathematical structure; vocabulary; method of content presentation; concept-skill balance; rigor; and evaluation. In addition, most contemporary mathematics programs were characterized by an acceleration of learning, the introduction of new topics, and an emphasis on unifying concepts and ideas.

Essential though, to the success or failure of any mathematics program are the instructional methods and techniques used for presenting the content. Contemporary teaching approaches were found to be inquiry oriented as opposed to expository ones, with the instruction centering on actively participating students. In the light of all the above criteria the mathematics program of the classical track of the Greek high school may be analyzed as follows.

Factors Influencing the Content of the Greek Mathematics Program

Applications of Mathematics. The program of the classical track does not include opportunities for the application of mathematics to problems arising from real life situations. The units on solution of first and second degree equations and systems of equations contain the traditional problems about age, money, numbers, motion, etc. For example, the book for the second year, entitled *Mathematics - 2nd Year of the Gymnasion*, includes a section on verbal problems that can be solved by means of a first degree equation in one unknown. The section consists of eight problems which have been worked out by the authors, and thirteen problems to be solved by the students.

However, most of the problems are not relevant to the needs and interests of 13-year old students, as it can be concluded by examining the following examples:

(1) A faucet running water fills up a pond in 4 hours. Another faucet does the same in 12 hours. In how many hours will the pond be filled if both faucets are running simultaneously? (Grafakou, Diakaki & Mantzara, 1972, p. 97)

(2) From what number must we subtract 13 times its 1/21 in order to get a number which is 4 less than twice its 1/7? (Grafakou, Diakaki & Mantzara, 1972, p. 99)

In the same book the chapter on ratio, and direct and direct and indirect variation has a section on applications of these concepts in solving verbal problems about business and investments, mixtures, arithmetic and geometric means.

In addition to the above type of problems the book for the third year, Mathematics - 3rd Year of the Gymnasion includes a section on verbal problems which can be solved by a system of equations in two unknowns. For example, the students are given problems such as:

> Today Peter is 8 years older than his brother John. In 6 years their ages will be in the ratio of 11:9. Find their ages. (Bousgou & Tamvakli, 1972, p. 145)

The Algebra book, in addition to problems similar to the ones mentioned above, includes problems which are direct applications of an operation. For example, immediately following the section on multiplication and division of algebraic fractions the students are given the following problem:

> A man has 5k drachmas. From this amount he first spends the one third, then the 1/7, and last the 4/9 of the original amount. How much does he have left? (Sakellariou, 1970, p. 99)

This type of problem seems to be aimed at a higher cognitive level than that of application, as opposed to the lower one of comprehension if it were simply given in the form $5k - (1/3)k - (1/7)k - (4/9)k = ?$

In another chapter of the same book, logarithms are applied in solving compound interest problems. An example is:

> A man loaned 150,000 drachmas to another man, with a 4% yearly compounded interest. How much money will he receive in 6 years? (Sakellariou, 1970, p. 297)

The solution is demonstrated in the book, as follows:

> Let $\underline{S}$ be the unknown; $\underline{a} = 50,000$; $\underline{n} = 6$; $\underline{t} = 0.04$
> By substituting, we have $S = 150,000 \cdot (1.04^6)$.
> Taking logarithms of the equal sides we have:
> $\text{Log } S = \text{Log } (150,00) + 6 \cdot \text{Log } (1.04)$.
> From the logarithmic tables we have that:
> $\text{Log } 150,000 = 5.17609$;
> and $6 \cdot \text{Log } (1.04) = 6 \cdot (0.1703) = 0.10218$, from where we get $\text{Log } S = 5.27827$ and therefore $S = 189,787$. Thus he will receive 189,787 drachmas in six years. (Sakellariou, 1970, p. 297)

The book for the fourth year, Mathematics - 4th Year of the Gymnasion, is the only book which includes verbal problems based on second degree equations in one unknown. As an example, on page 257 there are problems such as:

> Of what number one more than the sum of three times its square and twice the number, equals 86?
>
> (Vavaletskou & Bousgou, 1971, p. 257)

Characteristic of the whole program is the absence of word problems as applications of inequalities.

Lack of relevancy and unrealistic situations are the weaknesses of most of the verbal problems of the program. For instance, the authors often use in the problems measurement units which have been obsolete since 1950, as in the case of the problems referring to the oka as a unit for measuring mass instead of the kilogram. (Sakellariou, 1970, p. 117) The same evidence of unrealistic use of figures is found in another problem in the same book which refers to a person's annual salary as being 6,000 drachmas, while one's monthly salary will be more than this amount today. (Sakellariou, 1970, p. 117) Not only are the problems not relevant to the needs of the students but they are also stated in awkward language that it is often difficult to translate from Greek into a mathematical sentence.

Placement of topics. In this area the program is more nearly in line with the evaluative criteria. The spiral approach to the placement of topics is a feature of the Greek mathematics curriculum. Ideas are revisited at different levels and with differing degrees of difficulty. For example, regarding the concepts of powers and exponents, on the introductory level, the book for the second year focusses on the properties of powers which have rational numbers as the base and integers as exponents. The level of difficulty is of the form "$(-1/2)^{-3} = ?$; $(-8)^2 \cdot (-4)^3 = ?$" (Grafakou, Diakaki & Mantzara, 1972, p. 80).

In the third year book which may be considered intermediate level, these ideas are reviewed but the examples now are of a more complex nature, i.e., "$(5x^3y^4/2x^{-2})^{-2} = ?$" (Bousgou & Tamvakli, 1972, p. 64). In addition, the concepts of roots and radicals are introduced with the main emphasis being on square roots. An example is "$\sqrt{8} + \sqrt{50} - \sqrt{98} = ?$" (Bousgou & Tamvakli, 1972, p. 64).

In the fourth year book, on a more advanced level, students review the basic ideas of powers and radicals and they expand their knowledge of the subject by being introduced to powers with rational exponents. It is at this point that the students are given the formal definition:

$a^{m/n} = \sqrt[n]{a^m}$, if $m/n>0$ and $a \neq 0$; and

$1/\sqrt[n]{a^m}$, if $m/n<0$ and $a \neq 0$. (Vavaletskou & Bousgou, 1971, p. 125)

The properties of exponents are now generalized and emphasis is placed on the fact that any radical may be written as a power with a rational number as its exponent.

In general, the material seems to be placed in the accepted sequence according to other curricula. However, the depth of investigation of topics is quite demanding for non-scientifically bound students. The material is heavy with what this writer feels are unnecessary technicalities. This may result from the importance the Greek educational system attaches to the knowledge of facts and details. For example, the fourth year students are required to know the following specific cases on rationalizing denominators:

$$A = \frac{k}{\sqrt[n]{a} + \sqrt[n]{b}} \quad \text{where } a,\ b,\ \varepsilon\ R^{+}.$$

<u>Case I</u>: If $n = 2m + 1$, $m\ \varepsilon\ N$.

$$A = \frac{k}{\sqrt[n]{a} + \sqrt[n]{b}} = \frac{k}{a - b} \cdot \frac{a - b}{\sqrt[n]{a} + \sqrt[n]{b}} = \frac{k}{a - b} \cdot \frac{(\sqrt[n]{a})^{n} - (\sqrt[n]{b})^{n}}{\sqrt[n]{a} + \sqrt[n]{b}} =$$

$$= \frac{k}{a + b} \cdot (\sqrt[n]{a^{n-1}} - \sqrt[n]{a^{n-2}b} + \sqrt[n]{a^{n-3}b^{2}} - \ldots + \sqrt[n]{b^{n-1}}) =$$

<u>Case II</u>: If $n = 2m$, $m\ \varepsilon\ N$.

$$A = \frac{k}{\sqrt[n]{a} + \sqrt[n]{b}} = \frac{k}{a - b} \cdot \frac{a - b}{\sqrt[n]{a} + \sqrt[n]{b}} = \frac{k}{a - b} \cdot \frac{(\sqrt[n]{a})^{n} - (\sqrt[n]{b})^{n}}{\sqrt[n]{a} + \sqrt[n]{b}} =$$

$$= \frac{k}{a + b} \cdot (\sqrt[n]{a^{n-1}} - \sqrt[n]{a^{n-2}b} + \sqrt[n]{a^{n-3}b^{2}} - \ldots - \sqrt[n]{b^{n-1}}).$$

(Vavaletskou & Bousgou, 1971, p. 123)

It is this writer's opinion that the case $A = \frac{k}{\sqrt{a} + \sqrt{b}}$; a, b ε R would have been sufficient.

Structure. In terms of structure the Greek mathematics program stresses the development of the different sets of numbers with emphasis on building a new set from the previous ones. As an example, on page 20 in the second year book, $Q_o^+ = \{0, \ldots, 1/2, \ldots, 3/4, \ldots, 1, \ldots, 2, \ldots\}$ is the set of 'rational numbers of arithmetic' (known as the positive rational numbers) and is defined as the union of the set of 'integers' (known as whole numbers) and of the set of the non-integral quotients of integers divided by natural numbers. Thus, $Q_o{}^+ = N_o \cup \{x|x$ non-integral quotient of an integer divided by a natural number$\}$, or $Q_o{}^+ = \{x|x = a/b$, when $a \varepsilon N_o$, $b \varepsilon N$ and a/b is in simplest form. (Grafakou, Diakaki & Mantzara, 1972)

As the next step, on page 25, the positive rational numbers are introduced as $Q^+ = \{\ldots, +1/2, \ldots, +1, \ldots +2, \ldots\}$, and the negative rational numbers as the set $Q^- = \{\ldots, -1/2, \ldots, -1, \ldots -2, \ldots\}$, and finally the set of 'real and rational numbers' (known as rational numbers) is given as $Q = Q^- \cup \{0\} \cup Q^+$. (Grafakou, Diakaki & Mantzara, 1972)

In the book for the third year, the set of irrational numbers is introduced because of a "... need to expand the set of rational numbers by creating new numbers which must be called irrational and which will be so constructed as to cure the weaknesses of the set of rational numbers" (Bousgou & Tamvakli, 1972, p. 55). The set is denoted by Aρ. Finally, the Real numbers are introduced as $R = A\rho \cup Q$ or R_e.

The properties of closure, uniqueness, commutativity, associativity, and identity, under the operations of multiplication and addition, are identified and formally labelled in considering all of the above sets. The existence of additive and multiplicative inverses is not discussed as a separate property. Instead, a statement such as the sum of two 'opposite numbers' (the term used for additive inverses) equals zero replaces the additive inverse property. The different number sets and their respective properties are emphasized, but algebraic structures such as groups, rings, fields are not discussed formally. Specifically, the algebra material is that of the classical algebra, i.e., operations with expressions and solving equations, but it is presented by means of the symbolism of sets and logic.

Vocabulary. The vocabulary and symbols which are used throughout the whole program are formal. As an example, A_B^C denotes the complement of a set A in reference to a universal set B. $\underset{U}{C}A$ also represents the complement of a set A with respect to the universal set U. However, two different notations for the same concept may be confusing to students when first exposed to them. Another example of formality may be considered the definition of an infinite set given in terms of 1-1 correspondence with the set of the natural numbers, as seen in the book for the third year. (Bousgou & Tamvakli, 1972, p. 11)

In addition to the precise language, emphasis is placed on the symbolism and language of both set theory and logic. Symbols such as the logical operators for disjunction ($\vee$), conjunction ($\wedge$), and negation ($\sim$); the universal quantifier ($\forall$) for all x, the existential quantifier ($\exists$) for some x there exists, the equivalence ($\leftrightarrow$) and implication ($\rightarrow$) symbols, the symbols for 'is a member of' ($\in$), the empty set (ϕ), subset ($\subset$), union ($\cup$), and intersection ($\cap$) are heavily used throughout the program.

Method of presentation. The method of presentation is of an expository nature with no attempt to include opportunities for any form of discovery. This can be seen in the following examples.

In the Theoretical Geometry book, the following theorem is given.

Compare the two triangles ABC and DEZ, if they have AB = DE, AC = DZ, $\angle A = \angle D$.

Proof: Suppose that DEZ is superimposed on ABC so that the side DE is put on top of AB with the vertex D on A. We easily conclude that the straight line DZ will coincide with straight line AC and vertex A with vertex C. As a result EZ will coincide with BC and triangle DEZ with ABC. Thus, if two triangles have two sides equal, one for one, and the in-between angles equal, the triangles are equal. (Nikolaou, 1970, p. 59)

(Figure is not included in the presentation.)

In the third year book, the section on operations with algebraic polynomials is presented as follows:

Addition of Polynomials - Since every polynomial is the sum of its terms, the addition of polynomials is addition of sums. Thus we have: to add polynomials we form a polynomial which contains all the terms of the given polynomials and these only. It is natural in the sum of polynomials to collect like terms and to write the polynomial in a combined form. (Bousgou & Tamvakli, 1972, p. 83)

Examples of addition of polynomials follow, the students are then given a list of four properties of addition of polynomials namely, commutative, associative, identity, and existence of inverses. The next concept introduced is subtraction of polynomials, done in the following manner:

> Subtraction of Polynomials - Subtraction of a polynomial B from a polynomial A is called the addition to A of the opposite of B. (At this point one example is presented and the exposition continues). From the previous examples we conclude that in every polynomial sum, in order to find its final form, we eliminate the parentheses and combine like terms. In eliminating parentheses we notice that: (1) If before the parentheses there is the sign + (or no sign) the terms of the parentheses remain as are; (2) if before it there is the symbol -, the terms change to their opposites. (Bousgou & Tamvakli, 1972, p. 83)

Topics are often developed quickly and some of the explanations seem confusing or misleading. For example, in the second year book, $|-6| + |-3| = |-9|$ or $|(-6) + (-3)| = |-6| + |-3|$ is first presented, and then the general cases $|a + b| = |a| + |b|$ if both numbers have like signs, and $|a + b| < |a| + |b|$ if the numbers are of different signs, are distinguished. (Grafakou, Diakaki & Mantzara, 1972, p. 39) This can be misleading to students when they are at first introduced to the concept of absolute value.

Balance of Skills and Concepts. Although basic mathematical concepts are developed, acquisition of skills is given greater emphasis. As previously mentioned the algebra component of the program for the lower grades consists of the traditional algebra which focuses on operations with expressions and solution of equations. Similarly, trigonometry emphasizes operations with trigonometric expressions and solution of triangles.

All the books place emphasis on rules and definitions, which appear in heavy black letters, and on procedures as opposed to understanding the underlying concepts. For example, in the second year book, the authors presented four problems dealing with powers and exponents, namely "$(-3)^2 = 3^2$ (positive); $(-2)^3 = -2^3$ (negative); $(-2/3)^4 = (2/3)^4$ (positive); $(-4)^5 = -4^5$ (negative)" and as the next step the student is given the rule: "thus if a negative number is raised to an even power the result is positive, if raised to an odd power the result is negative" (Grafakou, Diakaki & Mantzara, 1972, p. 81). It is the opinion of this writer that approaches of that kind are skill oriented and involve little understanding of the concept of a power.

> In the fourth year book, the students are given: absolute value or measure of a real number is the 'number of arithmetic' that is derived from it if the sign is deleted. Therefore, the absolute value

of +4 is 4 and the absolute value of −4 is again 4 and is denoted by $|+4| = 4$ and $|-4| = 4$. And since the positive numbers are identical with the 'numbers of arithmetic,' we have $4 = +4$ and thus, $|+4| = +4$ and $|-4| = +4 = -(-4)$. Thus, we could say in a stricter sense that the absolute value of a real number or a real expression a is called the number a itself if it is positive or zero, or its opposite −a, if the number is negative. (Vavaletskou & Bousgou, 1971, p. 107)

The students are then given five properties about absolute values namely,

(1) if $a \varepsilon R$, $|a| = |-a|$

(2) if $a \varepsilon R$, $-|a| \leq a \leq |a|$

(3) if $a \varepsilon R$ and $n \varepsilon R$, $|a|^{2n} = a^{2n}$

(4) if $a \varepsilon R^+$ and $n \varepsilon R$, $|a|^{2n+1} = a^{2n+1}$

(5) if $a > 0$, $x \varepsilon R$, and $|x| \leq a$, then $-a \leq x \leq a$

Proof: If $x \varepsilon R_o^+ \rightarrow |x| = x$ and since $|x| \leq a$, it is $x \leq a$ and thus $-a \leq x \leq a$, because $|x| \leq a \rightarrow a \geq 0$. If $x \varepsilon R^- \rightarrow |x| = -x$ and since $|x| \leq a$, then $-x \leq a$ or $x \leq a$. Therefore, $-a \leq x \leq a$, since $a \geq 0$. (Vavaletskou & Bousgou, 1971, p. 108)

In the opinion of the writer this method of presenting the idea of absolute value is skill oriented, too. It emphasizes memorization of facts void of true understanding of the concept. The concept of absolute value involves the notion of distance, and can best be introduced by using the number line, and by having the students, guided by the book, demonstrate specific cases such as $|x| \leq 2$, etc., which will lead to the generalization of $|x| \leq a$.

Rigor. Proofs follow a continuum from rationalizations, to sophisticated rationalizations, to rigorous proofs. An example of rationalization, is found in the second year book when students are instructed to measure the central angle and its corresponding inscribed angle by using a protractor or by superimposing one on the other, in order to establish the relationship that exists between the two. Specifically, it says that "if we measure or use transparent paper we will conclude that the central angle is twice the size of the inscribed angle" (Grafakou, Diakaki & Mantzara, 1972, p. 164).

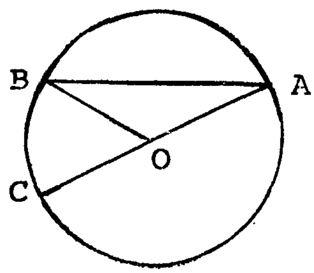
Next, as a sophisticated rationalization, this result is justified and proven in a more formal manner:

> Given circle (O, R), the inscribed angle <BAC, and its corresponding central angle <BOC.
>
> Proof: The central angle <BOC is the exterior angle of the isosceles triangle AOB. Therefore <BOC = <BAC + <ABO and since <ABO = <BAC, we have that <BOC = 2 · <BAC. Therefore <BAC = ½ · <BOC. Thus the inscribed angle <BAC is half of the central angle <BOC. (Grafakou, Diakaki & Mantzara, 1972, p. 165)

The same theorem, in the book for the third year, is presented without the manipulation of concrete materials and the proof is as follows: "Obviously, <w = <A + <B. Since <A = <B, it follows that <w = 2 · <A and <A = <w/2" (Bousgou & Tamvakli, 1972, p. 116).

The idea of moving from concrete to abstract is desirable. However, the general proofs that were cited above lack clarity and elegance. A student who is exposed to proof for the first time might find it beneficial if a different format were to be followed, similar to the one found in many United States programs. The proof could be given in two columns one for the statements and the opposite column for the justification of each step. For example,

circle (O, R)	given
central angle <BOC	given
inscribed angle <BAC	given
<BOC = <BAC + <ABO	exterior angle of ΔBOA
<ABO = <BAC	ΔBOA isosceles; OB & OA radii
<BOC = <BAC + <BAC	substitution
<BOC = 2 · <BAC	substitution
<BAC = ½ · <BOC	q. e. d.



An example of a more formal and rigorous proof, in the book for the fifth year entitled Mathematics - 5th Year of the Gymnasion, is the theorem:

If $g(x) \mid f(x) \rightarrow g(x) \mid f(x) \cdot s(x)$, for every polynomial $s(x) \in R(x)$.

Proof: Because $g(x) \mid f(x)$ we have $f(x) \equiv g(x) \cdot q(x)$, thus $f(x) \cdot s(x) \equiv g(x) \cdot \{q(x) \cdot s(x)\} = g(x) \cdot q_1(x)$, where $q_1(x) = q(x) \cdot s(x)$, that is $g(x) \mid f(x) \cdot s(x)$. (Ntziora, 1970, p. 82)

Another rigorous proof is found in the book for the fourth year where the commutative property for the union of two sets is proven. That is:

If $A = \{x \in U \mid P(x)\}$, and $B = \{x \in U \mid q(x)\}$, and $A \cup B = \{x \in U \mid P(x) \vee q(x)\}$, prove that $A \cup B = B \cup A$.

Proof: $A \cup B = \{x \in U \mid x \in A \vee x \in B\} =$

$= \{x \in U \mid x \in B \vee x \in A\}$, because

$p \vee q \leftrightarrow q \vee p = B \cup A$.

(Vavaletskou & Bousgou, 1971, p. 39)

Evaluation. There are no standardized tests to measure the results of the program, and the only formal measures of student progress are periodic recitations, and the semester examinations which consist of solving two out of three problems pertaining to the material covered during that semester. There are also no provisions for the evaluation of the program itself. The writer believes that no real efforts for refinement have been made in the case of two of the textbooks, namely that of the Algebra, and Theoretical Geometry, which have been used for the past two decades since they do not include any of the later features, that is, new terminology, symbolism, set theory, etc.

Other Factors Considered

In examining the content of the Greek mathematics program, it is evident that some attempts have been made to conform with contemporary trends in mathematics education in books other than that of Algebra and Theoretical Geometry. Topics from set theory, probability, statistics, vector algebra, etc., have been included in the program on all levels since 1971. Logic is formally introduced in the fourth year of the high school, which corresponds to the sophomore year in the United States high school. Unifying concepts such as those of function and set have been introduced and used throughout the program, and consideration has been given to the use of language and symbolism.

Critical Analysis of the Greek Mathematics Program

The writer is obligated at this point to offer comments which are critical not only of the Greek program alone but of contemporary mathematics education in general. The study of mathematics is not meant for just a few individuals who are gifted with special abilities. It is the right of all to be exposed and acquainted with this type of knowledge. Although the kind of mathematics taught today purports to be for everyone and is required of almost everybody, in reality it is written with the prospective scientist or mathematician in mind.

It is true that in today's technological societies there is great demand for people with good background in mathematics. However, instruction of this type should not be made the primary goal of liberal or general education programs such as the classical track of the Greek high school. Its students, in the future, will engage in non-technical or non-science oriented occupations. All they need and want to have is a view of and an appreciation for mathematics instead of an esoteric or pseudoesoteric knowledge of it. Therefore, dispensing with subject matter which is irrelevant to the needs will indeed raise the standards and quality of their education, as it will not result in a loss but it will allow for a more realistic alternative. It is detrimental to require students to memorize complex formulas and reproduce elaborate proofs. Teaching the algorithm for synthetic division of polynomials, for example, when students lack the essential understanding of division is literally 'putting the cart before the horse'.

The mathematics programs of Greece, and especially that of the classical track of the high school, are unduly extensive and detailed; and, as a result, a great quantity of material is covered in too short a time at the expense of quality. Because students are given limited chances to truly 'digest' the material, they may be able to produce answers to problems by faithfully following rules, but they have little if any understanding of the principles involved. Learning under these conditions becomes superficial and void of true understanding and leads to increased reliance on memorization and automation of mathematical thinking. It is, therefore, pedagogically unsound to try and present in a matter of a few lessons ideas and concepts for which mathematicians have spent many years both to develop and to comprehend.

The Greek students passively listen to the teacher and repeat what they have heard with little chance for original or independent thinking. However, true understanding of concepts involves active participation in the knowledge getting process. It does not mean verbatim repetition of what the teacher or the text says. It calls for finding different ways of proof, searching for both examples and counter-examples, or just being intuitively aware.

Although one of the goals of mathematics teaching is to ensure understanding the 'why' as well as the 'how' of mathematics and to encourage creative and independent thought, the way it is often presented penalizes those who are not strong in abstract thinking. Too much emphasis is placed on abstraction before the student has enough concrete experimentations to make abstractions meaningful.

The so-called modern topics such as sets, logic, algebraic structures, etc., constitute material which might be unnecessary and irrelevant to the needs of the students. Knowing the properties which describe a group or a field does not ensure the ability to perform operations with the elements belonging to these structures. (Kline, 1958)

Rigor in proofs is another weak point of present-day mathematics programs. As Morris Kline (1958) put it very aptly, "the capacity to appreciate rigor must be developed and depends on the students' mathematical maturity" (p. 423). Although rigorous approaches appear elegant to the trained mathematician, nevertheless, they are artificial and meaningless to many students. Students may often be intuitively aware of a concept, and emphasis on rigorous proofs clouds and confuses the issue instead of clarifying it.

Intuition, being the essence of mathematics, comes before any formalized, abstract, or rigorous experience. Ideas must first be investigated intuitively in terms of concrete, physical or geometrical situations and when need arises for checking the intuitive meaning, formality and rigor may be introduced to an extent that is necessary and beneficial for the student. It is true, for example, that students as early as the lower grades of elementary school have an intuitive understanding of the commutative property, i.e., 2 + 3 = 3 + 2, or (a) · (b) = (b) · (a). It is, therefore, unnecessary and cumbersome to subject the students to a formal and rigorous proof in abstract terms.

Closely related to the role of intuition in mathematics learning is the misconception about mathematics being of a deductive nature. Mathematics, in reality, is created inductively, although the finished product is reported deductively. The mathematician experiences various problematic situations, formulates a problem, guesses the outcome, and then sets to prove or disprove it and generalize. It is after this process is completed that mathematics can be presented in a deductive manner.

Too much emphasis on content has caused mathematics teachers to lose sight of the students. Mathematics educators are reluctant to accept that mathematics is only one part of the pupil's total educational experience. Thus, mathematics teaching as it exists in many countries, and in Greece, disregards the fact that the student is an experiencing human being in whom the balanced interplay of physical, psychoemotional, and intellectual forces produces a dynamically functioning entity.

It is unfortunate that many students, in Greece and elsewhere, dislike mathematics and are even afraid of it.

This is partially related to the fact that they are forced to learn a certain body of knowledge which school systems or Ministries of Education feel they ought to know. The objectives often center around preparation for more advanced mathematics courses, or preparation for future life, or receiving a passing grade; little effort is made to arouse genuine interest. As a result, and in self-defense, the students either rebel or withdraw and these symptoms are often erroneously interpreted as 'laziness' or 'low mental ability'.

Mathematics educators have been in most cases unable or unwilling to capitalize on intrinsic motivation which springs from within a person and is not externally imposed. This type of motivation is the kind that leads to personal fulfillment, enjoyment, and significant learning. The writer feels that it is professionally unethical and dehumanizing to impose on students objectives predetermined by some external agency, which are of no relevance or have no personal meaning to the individual.

Therefore, the writer wishes to offer the idea of mathematics as a liberating activity. Self-realizing and liberating mathematical encounters are unique and personalized in nature as opposed to standardized and stereotyped ones. They emanate from the individual's action upon and exploration of the environment. Learning mathematics results from organizing (integrating) these experiences rather than from acquiring specific factual knowledge. The student is thus filled with a feeling of satisfaction that promotes personal growth and development.

Mathematical experiences associated with exploration of the environment may deal with aspects of an aesthetic or sociocultural relevance. In that case, the goal is to understand or appreciate the environment or the parts of it which surround an individual. As an example, consider the computer and the significant role it plays in today's society. In that capacity the individual is certainly affected by more mathematics but does not necessarily use more mathematics. One may wish to gain understandings of how computers influence aspects of one's life, how they make it easier, etc. However, in order to do so, it is not necessary to learn computer programming or any of the other technical aspects about computers unless one so desires.

On the other hand, acting upon the environment has more of a technical and specialized meaning. It may be argued that acting upon the environment could create problems for a person who wishes to know something of an advanced and specialized nature but who lacks the prerequisite understandings. The writer firmly believes that if this experience is of great importance to the individual, he will seek to remedy all deficiencies and difficulties that may be encountered. It should be noted though that both kinds of mathematical experiences may overlap at times and that they are personalized in nature.

Mathematics teaching as observed in Greece by this writer discourages the students from going on tangents of personal meaning and relevance; it provides for no alternatives, and it demands conformity. Mathematics teachers seem to underestimate the students' ability to choose wisely and take responsibility for their choices and disregard the fact that individuals pursuing interests of personal concern are not only active participants but also directors of their own activities. To require students to study mathematics for mathematics' sake, or as a mental exercise, or for future utility only is questionable. To allow, however, for student-defined learning expectations does not imply a non-structured environment similar to that of some progressive movements. The school is still responsible "to stimulate the students' awareness, to respond to their growing awareness with help and suggestions, and to initiate opportunities which support the students' learning in the areas they have chosen" (MacDonald, Wolfson & Zaret, 1973, p. 21).

Learning situations in which interest and motivation are kept high call for a new description of the teacher's role, of the teaching methodology, and of the classroom environment in general. Kaye (1969) stated the emphasis should shift from teaching to learning and to learning how to learn. Subject matter should be fitted to the students' needs. Content, time, and achievement should be regarded as variable instead of constants. Instruction should aim in providing starting points for new experiences and explorations, as opposed to end results which are behaviorally described and poorly evaluated by means of external measures only. Interesting and familiar situations could serve as beginnings for investigations with the students given freedom and responsibility to explore, discover, learn, and appreciate. They can choose the topic to be investigated, the media of expression, the pace and the sources to be used. (Biggs & MacLean, 1969)

The task facing the teacher is to create a non-threatening climate for the facilitation of knowledge. Carl Rogers (1961) stated that to achieve such a goal the teacher needs to be congruent; that is, a real person in his interacting with the students; he should not play a role. The teacher must also possess unconditional positive regard for the students, accept them as they are, empathize with them, and be able to encourage personal work and expression of the students. The teacher must be ready to offer, not impose, guidance and direction when needed and be willing to learn and grow along with the students. Once this atmosphere of mutual respect and trust has been established, then the stage is set for the kind of "learning which makes a difference, which pervades the person and his actions, and provides with a sense of release and a thrust for forward movement" (Rogers, 1961, p. 281).

A tense atmosphere is evident in many Greek mathematics classes. The relationship between students and teachers is that of adversaries. Teachers are perceived by the students as the enemy because they act in an extremely authoritative manner. There exists a "confusion of the teacher's personality and what he considers to be due to it, with the process of learning itself. Students are not encouraged to exercise their intellect apart from memory and their role is to accept the instructor's authority and passively admire his performance" (Campbell, 1968, p. 385).

The objective should be to change the attitudes of teachers and students toward each other. The education of teachers could serve as an appropriate beginning. If they themselves experience open, active, and personalized learning during their mathematics and professional training, they will develop a strong preference for this type of schooling and will practice it in their classroom.

It is important to impress on the prospective teacher that mathematics teaching is not based on following prescription-type approaches for given situations. On the contrary, eclectic selection of methods and means best suited to the needs of the students and the teacher in a given situation is the key to successful teaching-learning experiences.

The future teacher should also be aware that keeping communication lines open to the students, their colleagues, the administration, and the community in which the school exists is of great value. They will both evaluate and invite evaluation on the ongoing program, the students' progress, and their own performance. The spirit prevailing these assessments should not be one of negative criticism but of a constructive analysis of the state of affairs which will serve as the starting point for further growth and development.

Textbooks

The idea that the textbook is the curriculum and vice-versa has been criticized by educators (Bassler & Kolb, 1971; Forbes, 1970) who believe that an attempt to use a text as the curriculum is likely to fail. This is, however, standard procedure for the Greek mathematics program. Textbooks strictly conform to the curricular syllabi and teachers are overly concerned with covering the contents of the book from page to page.

Contrary to claims made by the Ministry of Education, Athens (1971) that a choice of text is available, there is only one approved book for each mathematics subject, and teachers are not allowed any selection. It is, however, unrealistic to expect one book to meet the needs of all the students in a country, and to be compatible with the teaching styles of many teachers.

In conversations with Greek teachers, it was noted by this writer that the lack of textbook selection is partially attributed to the fact that books are distributed free to public school students. This, the teachers felt, makes it necessary for control to be exercised by a central authority. It is true, however, that books in the United States are given to students without charge and yet school systems are able to choose their own books. Similarly, in England individual teachers are given the opportunity to select their textbooks. (Griffiths & Howson, 1974)

Greek mathematics textbooks, when examined in the light of the NCTM guidelines for evaluating textbooks, as listed in Chapter III, fall short in a number of areas. In terms of presentation and content, the existing mathematics texts do not meet the needs of the classical track students. Although the topics presented fit into the whole structure leading from one concept to the next, the scope and depth of the presentation is overly extended. This may have an adverse affect on non-mathematically bound students, and it results in a waste of effort and energy. The time spent on unnecessary and technical details could be beneficially used for activities of relevance and interest to the students, not strictly in the area of mathematics. Similar arguments may be used for the overemphasis on rigor which is evident in the Greek texts.

Attention must be given to the language used for writing the texts. The pure form of katharevousa creates an austere and rigid mood of presentation. Its high formality contributes to making mathematics an alien subject separated from the everyday lives of many of the students.

The General Assembly of Teachers during its July 1974 meeting stated, in a memorandum submitted to the Ministry of Education, that the katharevousa be replaced as the language of writing textbooks. ("The High School," 1974). Furthermore, the assembly voiced concern for the poor quality of the books, stating that it is imperative for projects to be undertaken by experienced educators for the writing of new books. These books should reflect the Greek reality as opposed to the haphazard efforts to tranlate foreign books which do not represent the background and interest of the Greek students.

There are also several other items which are in contrast with accepted policies in mathematics education. For example, Greek textbooks do not include answers to any of the homework problems. Cummulative review sections are scarce, and there are no chapter tests. In many of the books there are no tables of contents or indexes, and the ones available are not detailed. Glossaries of symbols and definitions are also absent. The textbooks do not include any material in the form of motivational or enrichment topics. There are no teachers' editions to accompany the students' texts. Supplementary materials such as workbooks, transparencies, audio-visual aids (cassettes, filmstrips, etc.) are not available.

Teacher Training and Qualifications

As it has been stated in Chapter III the C. Beeby model of the development of an educational system lists four stages through which every system must pass. They are the stage of the "Dame School", the stage of formalism, the stage of transition, and the stage of meaning. (Griffiths & Howson, 1974, p. 63) It is the opinion of this writer that the preparation of most teachers in the Greek system and specifically in its mathematics component falls between stages II and III, being closer to stage II, of the Beeby model. Greek mathematics teachers are well trained in the area of mathematics but they are poorly prepared professionally. The majority of the mathematics professors in the Greek universities are interested in transmitting large amounts of abstract and theoretical knowledge, but they consider preparation in the areas of pedagogics and psychology not essential. The consensus among Greek mathematics faculties is that a sound mathematics background is necessary and sufficient preparation for teaching the subject. It is misleading, however, to assume that a good mathematician will also make a good teacher of mathematics.

The mathematics content taught at the Greek universities is highly advanced and does not correspond to the subject matter the teachers will be asked to teach in the high school. Thus, according to Professor Glavas (1966), the new teachers are faced with making modifications of the different mathematical ideas in order to adjust them to the level of the high school students. However, their efforts do not always meet with success because of the lack of professional background. They do not know how to take an abstract and advanced idea to a lower level of sophistication and present it in a simpler but yet mathematically correct way.

It has always intrigued the writer that in Greece, as in many other countries, physicians receive extensive training in both theory and practice, while teachers, who daily influence the cognitive and psychoemotional growth of developing pupils, are given no professional training. It is as dangerous for incompetent teachers to influence the lives of young pupils as it is for a medical doctor to make an incorrect diagnosis and prescribe the wrong medicine.

The first International Congress on Mathematical Education, held in Lyon, France in 1969, resolved that mathematics education is an evolving science and as such it should be given a place in the mathematics departments of the universities with "appropriate academic qualifications available" (Hlavaty, 1970, p. 321).

In order to establish teacher preparation programs similiar to those in existence in United States schools, it will be necessary to reorganize the present university system in Greece. Under the present system there are no provisions for any course work in education or psychology during the four years of undergraduate work. It will also require a change in attitude towards the 'education' part of mathematics education. However, the Greek system lends itself to programs similar to those in existence in Germany and Holland, as described in Chapter III, with minor modifications.

Pre-service training is crucial since in most cases it is difficult to correct bad foundations once they have been acquired. Further professional development, however, is a must in a rapidly growing science such as mathematics education. The only form of actual in-service training available to the Greek mathematics teacher is the two-year institute in leadership training. Although it is a step in the right direction, to limit such institutes to eighteen participants at a time is like a drop of water in an ocean.

The question that arises is what becomes of the remaining thousands of teachers who either do not have the desire to participate in the competitive examinations for these institutes or those unfortunate ones who in spite of their willingness to participate were not among the chosen few. They are either allowed to remain apathetic and mediocre at the expense of the students they teach, or they are denied a chance for further development. Such practices are contrary to the recommendations made by the different International Congresses of Mathematics. These directives stress the need to give mathematics teachers ample opportunities to pursue further professional development.

The different conferences organized under the auspices of the Greek Mathematical Society are valuable since they contribute to further professional growth. However, the emphasis of these meetings is on mathematics content rather than pedagogics. It is also true that the high school teachers are sometimes unable to attend the conferences because the Ministry of Education does not deem it appropriate to relieve them of their duties for two to three days in order to participate in the activities of the meetings.

The lectures and model lessons which are occasionally sponsored by the district inspectors are also of value and assistance to the teachers. The occurrence, however, of such meetings depends on the leadership qualities of the inspector. It is true, however, that several of the inspectors are preoccupied with administrative chores rather than the supervision and advising of the teachers. They seem to be primarily concerned with compiling files of individual performance reports and negative criticisms instead of providing the teachers with constructive suggestions upon which they can build and improve.

Summary

An analysis of the Greek mathematics program in the light of internationally acceptable policies and trends revealed that the program does not measure up to many of these guidelines. It is closest to some of the policies dealing with content and farthest from the ones regarding textbooks, teaching methodology and teacher training.

In the Greek mathematics program course content is represented and limited to the textbooks. Because the course content and textbooks are extensive in scope and depth, they exceed the needs of the students. Also, textbooks fall short in terms of presentation, physical characteristics and supplementary materials.

The teaching method used by the majority of the Greek mathematics teachers consists of traditional lecture-type approaches. In general, an inflexible atmosphere is present in Greek mathematics classes. This may be due to the fact that Greek mathematics teachers receive no professional training in their undergraduate program.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This study provided information about the mathematics program of the classical track of the Greek high school. The areas considered were content and textbooks, teaching methodology, and teacher training and qualifications. Data were collected by direct observation of Greek mathematics classes, interviews with Greek educators, and by reviewing literature published by the Greek Ministry of Education. The program was examined in the light of recent international developments in mathematics education. The following standards were used for evaluating the Greek mathematics program:

Content

(1) acceleration of mathematics learning

(2) introduction of new topics and deletion of others

(3) emphasis on unifying concepts, on rigor, on vocabulary and symbolism

(4) increased use of computers and emphasis on applications

Textbooks

(1) emphasis on correctness of content and logical organization of material

(2) flexibility of presentation of content, accompanied by discovery-oriented techniques

(3) increased use and production of supplementary teaching aids

Teaching Methodology

(1) emphasis on pupil-centered approaches in combination with inquiry and discovery-oriented strategies

Teacher Training and Qualifications

(1) emphasis on thorough preparation in mathematics, pedagogy, and educational psychology

(2) emphasis on studies in a related field

(3) emphasis on a long period of practice teaching

(4) emphasis on in-service professional development

Conclusions

The Greek mathematics program was examined in light of the above standards and the conclusion reached was that it does not measure up to many of the above criteria. It is closest to some of the guidelines regarding content and farthest from the ones dealing with textbooks, teaching methodology, and teacher training.

Specifically, it was found that in terms of content the program was subjected to some alterations in agreement with internationally acceptable policies. The changes primarily involved the introduction of topics, new terminology, and new symbolism. The material is placed in a sequence which is in agreement with practices in mathematics education. However, the material is extensive in both scope and depth of content and does not meet the needs of the student population for which it is intended.

It was also found that in the Greek program, there is only one approved text for each mathematics subject. As in the case of content, the textbooks are overly extended and detailed and thus not suited to the needs of the non-scientifically bound students. The physical characteristics and general format of the textbooks render them inefficient and unattractive. There are no aids or materials to supplement instruction.

A critical review of the teaching methods used in the majority of the country's schools led to the conclusion that Greek mathematics instruction is based on traditional teacher-centered approaches of <u>tell them and drill them</u>. This may be accounted for by the fact that Greek mathematics teachers receive no formal professional training in either pedagogy or educational psychology.

Recommendations

These recommendations are proposed with full knowledge of the current Greek educational philosophy which is not conducive to change, but relies on conformity. Although implementation may not be possible at the moment, the recommendations are presented in the spirit of offering suggestions useful in the future, when changes may come.

In order for the Greek mathematics program to be more in accord with generally accepted international guidelines, certain changes need to be made. The following recommendations, while growing out of an analysis of Greek mathematics education and directed toward changes in that program have implications also for school mathematics in the United States and elsewhere.

(1) The content of the mathematics program of the classical track of the Greek high school, should be modified to become relevant to the needs and interests of non-scientifically bound students. This may be accomplished by deleting subject matter which is complex or technical in nature, such as elements of analysis, and certain advanced topics in algebra or trigonometry. Time could then be spent on examining elementary concepts from a new standpoint, i.e., an emphasis on the why of understanding as well as the how of skills. For example,

it may be beneficial for students to gain understandings and insights of the idea of the numeration system, the concept of base of numeration, the concept of place value; to be able to explain why the different algorithms are performed as they are; and to be able to analyze the differences and similarities of the number systems and their properties.

(2) A study of the history and development of mathematics should be included in the Greek mathematics program. Experiences of this type will not only facilitate comprehension of mathematical concepts per se, but will aid in understanding the cultural significance of mathematics and its relationship to other disciplines and to society in general.

(3) Opportunities should be given for students to think for themselves and to create their own mathematics instead of relying on memorization of rules and procedures. It is through independent and original thinking that one may experience the beauty of mathematics and come to appreciate its powers and limitations.

In order for Greek students to develop independent and original thinking, they must first experience a variety of mathematical situations, especially on an informal and concrete level. These concrete experiences play a key role in subsequent mathematical learning and must precede any formalized or abstract treatment of ideas and concepts.

It must also be kept in mind that there may be instances when the students do not reach the highest level of abstraction, but instead they may operate on a semi-concrete or even a concrete level as the terminal goal.

Greek students must be encouraged to 'mess around' (explore, investigate, experiment) and 'play' with ideas, to 'check out' hunches, to use 'trial and error' approaches in solving problems. It is of outmost importance that students be allowed to make mistakes, for they can serve as true learning experiences.

(4) Attention must be given to the atmosphere that is present in Greek mathematics classes. If students are to participate actively in the learning environment, changes must be made in the Greek school. It is recommended that the center of instruction shift from the teacher to the students. This will enable Greek mathematics teachers to become facilitators of knowledge, guiding and aiding the students, when necessary, in their efforts to gain understandings. A re-assessment of the role of the teacher should result in a non-threatening type of classroom climate as opposed to the authoritative and rigid one which is prevailing now.

(5) In order for Greek mathematics teachers to be able to cope with diverse or difficult situations and to be sensitive to the needs of the pupils, they must have

the proper training. The writer recommends that upon completion of their mathematics training at the university, the prospective teachers undergo a two-year professional training period. A year of courses such as cultural foundations of education, educational psychology, learning theories of mathematics, methods for teaching mathematics, etc., must precede that of the internship. The courses should not focus on factual knowledge only, but they must be coordinated with associated clinical experiences which will be preparatory to the second year's internship. Similarly, the practicum should include seminars and workshops which grow out of classroom situations, so that the whole program may be brought into focus. Most importantly, future teachers must be taught by professors who believe in and practice the kind of teaching they expect of their students and who are professionally capable of advising and guiding student teachers in their training.

(6) It must be kept in mind that professional development begins with the pre-service training and must be continued during the in-service years. It is, therefore, suggested that Greece adopt a system of mandatory and periodic up-grading. This will provide opportunities for all teachers, especially those whose mathematical or pedagogical knowledge and practices are outmoded, to either up-grade their competencies or leave the profession, since their presence is more of a liability than an asset

to education. This in-service professional development may take the form of formal course study, active seminar participation, engaging in projects or research pertaining to mathematics education, or a combination of these. The above activities provide means for keeping informed about recent developments and practices.

(7) It is further recommended that professional teachers' associations be formed as opposed to the present day union-like ones. These groups could be patterned after the United States' National Council of Teachers of Mathematics or the British Association of Mathematics Teachers.

(8) It would also be beneficial if the roles of the Greek inspectors were to be redefined to include the broader aspects of supervision since they are in a unique position to function as curriculum leaders, consultants, advisors, and coordinators. In general, Greek teacher education should aim in attracting the most promising and qualified candidates. It is unfortunate that presently many of the mathematics graduates choose to teach either at the privately owned schools or at the frontistirion, which offer higher salaried positions.

(9) Textbook quality is another area in which the Greek program falls short of accepted trends and policies. It is this writer's opinion that Greek textbooks should adhere to the curriculum only in terms of presenting the

basic course content and must be flexible otherwise to adapt to different situations. It is imperative that there be a number of books available for each mathematics subject, with the final selection and adoption being the responsibility of the mathematics teachers of every school. Teachers who are actively engaged in teaching and who have insights as to the specific needs of their particular students are the only ones in a position to choose the most appropriate book; not some administrators in the Ministry of Education.

(10) It is recommended that the government unit in charge of publishing the approved textbooks, which also has the monopoly on school distribution, be replaced by private publishing firms. This will provide healthy competition, which will lead to textbooks of better quality than the existing ones.

(11) A factor affecting the quality of the textbooks is the language, katharevousa, in which they are written. The writer feels that it will be an improvement if the demotiki were to be adopted as the language of all levels of instruction. The demotiki, being the everyday spoken language of the people, lends itself to a more personal and individual tone in contrast to the artificiality and rigidity of the katharevousa.

(12) There are several other items which the writer believes will contribute to better textbook quality. It is recommended that books include answers to at least some

of the problems, i.e. even or odd, thus giving the students immediate feedback on their work. Cummulative review sections should be included at regular intervals as a check on the attainment of previous goals. It would also prove beneficial to the students if chapter tests were included. It is advised that detailed tables of content and indexes be included in each textbook. This will facilitate easy referal to the topics and ideas encountered in the book. Glossaries of symbols and definitions will also be valuable.

In countries advanced in educational policies, textbooks are accompanied by teachers' editions and commentaries. Greek mathematics teachers are likely to benefit from such editions since in addition to the regular content of the student text, these books include hints on teaching strategies or approaches for certain topics, clarifications and background notes on difficult concepts, sample tests, etc.

It is suggested that the textbooks include motivational and enrichment topics. These may be in the form of historical notes, references, ideas for projects, research topics, applications, puzzles, etc. which will make studying mathematics more interesting and appealing to the students.

(13) It is further recommended that the Greek mathematics program incorporate the use of supplementary materials. These aids may be used in conjunction with the text and in

support of the total instruction program. They may vary in form from audio, to visual, to manipulative and tactile. It is often thought that materials of that nature are very expensive. However, many of them can be made by the teachers themselves instead of purchased commercially. For example, a teacher may plan and organize topics or units on cassette tapes to be used by the students in a supplementary or tutorial capacity. Such presentations may be accompanied by teacher-made workbooks, transparencies, etc. Of course, projects of this nature are time consuming and may be avoided by teachers who are already overburdened by red tape, clerical duties, etc., as in the case of the Greek teachers. However, they lend themselves as excellent projects for summer in-service workshops.

The revision of the present books will require time since a good product must be field tested and revised before its final form. However, the fact that the Greek teachers themselves have voiced concern for the poor quality of textbooks and have petitioned the Ministry of Education to undertake revision projects may be considered as a positive step.

BIBLIOGRAPHY

Aids for the evaluation of mathematics textbooks. The Mathematics Teacher, 1965, 58, 467-473.

An analysis of new mathematics programs. Washington, D. C.: National Council of Teachers of Mathematics, 1963.

Atchison, W. F. The impact of computer science education on the curriculum. The Mathematics Teacher, 1973, 66, 7; 81-83.

Bassler, O. C. & Kolb, J. R. Learning to teach secondary school mathematics. Scranton: Intext Educational Publishers, 1971.

Becker, J. P. Note on the first international congress on mathematical education. The Mathematics Teacher, 1970, 63, 318-319.

Biggs, E. E. & MacLean, J. R. Freedom to learn: an active learning approach to mathematics. Ontario: Addison-Wesley, 1969.

Bousgou, G. & Tamvakli, J. [Mathematics - 3rd year of the gymnasion (vol. 1).] Athens: Organization for Publishing Textbooks, 1972.

Campbell, J. K. & Sherrard, P. Modern Greece. New York: Praeger, 1968.

Campbell, L. G. Mathematics in the British secondary school: curriculum projects. The Mathematics Teacher. 1969, 62, 329-333.

[Catalog of the Aristotelian University of Thessaloniki.] Thessaloniki, Greece, 1976.

Christoforithis, C. [Circular to the mathematics teachers of the 5th and 6th educational districts.] Thessaloniki, Greece, February 11, 1971.

Christoforithis, C. [Circular to the mathematics teachers of the 5th and 6th educational districts.] Thessaloniki, Greece, February 1, 1972.

Christoforithis, C. [Circular to the mathematics teachers of the 5th and 6th educational districts.] Thessaloniki, Greece, September 18, 1972.

A compendium of CUPM recommendations (vol. 1). Committee on the Undergraduate Program in Mathematics. Mathematical Association of America, 1971.

Dendrinou-Antonakaki, K. D. Greek education: reorganization of the administrative structure. New York: Bureau of Publications Teachers College, Columbia University, 1955.

Dicks, T. R. B. The Greeks: how they live and work. New York: Praeger Publishers, 1971.

Dienes, Z. P. Building up mathematics. London: Hutchinson, 1960.

Exum, J. E. CSMP today. The Mathematics Teacher, 1973, 66, 538-539.

Fehr, H. F. The education of mathematics teachers in other countries. The Mathematics Teacher, 1969, 62, 48-56.

Fehr, H. F. The mathematics program in Japanese secondary schools. The Mathematics Teacher, 1966, 59, 577-583. (a)

Fehr, H. F. Reform in mathematics education around the world. The Mathematics Teacher, 1965, 58, 37-44.

Fehr, H. F. A unified mathematics program for grades 7-12. The Mathematics Teacher, 1966, 59, 463. (b)

Fitzerald, W. M. & Vance, I. E. Other media and systems. In The teaching of secondary school mathematics. 33rd Yearbook. Washington, D. C.: National Council of Teachers of Mathematics, 1970.

Forbes, J. E. Textbooks and supplementary materials. In The teaching of secondary school mathematics. 33rd Yearbook. Washington, D. C.: National Council of Teachers of Mathematics, 1970.

Gibb, G. The computer: a facilitator in management and instruction. The Mathematics Teacher, 1973, 66, 6; 78-80.

Glavas, C. B. High school mathematics reform in Greece. The Mathematics Teacher, 1966, 59, 671-673.

Goals for school mathematics. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton-Mifflin, 1963.

Grafakou, G., Diakaki, K. & Mantzara, S. [Mathematics - 2nd year of the gymnasion.] Athens: Organization for Publishing Textbooks, 1972.

Greece. In The international yearbook of education (vol. 30). Geneva: UNESCO, 1968.

Griffiths, H. B. & Howson, A. G. Mathematics: society and curricula. London: Cambridge University Press, 1974.

Guidelines and standards for the education of secondary school teachers of science and mathematics. Washington, D. C.: American Association for the Advancement of Science & National Association of State Directors of Teacher Education and Certification, 1971.

Guidelines for the preparation of teachers of mathematics. Reston, Va.: National Council of Teachers of Mathematics, 1973.

[The high school teachers recommend.] [Macedonia,] July 6, 1974, p. 1.

Hlavaty, J. The first international congress on mathematics education. The Mathematics Teacher, 1970, 63, 318-319.

Howson, A. G. (Ed.) Developments in mathematical education. Proceedings of the second international congress on mathematics education. London: Cambridge University Press, 1973.

Katsarlinou, S. & Baiba, M. [Mathematics - 1st year of the gymnasion.] Athens, Organization for Publishing Textbooks, 1974.

Kaye, G. A. A new role for the teacher. The Arithmetic Teacher, 1969, 16, 39-47.

Kazamias, A. M. Greece. Comparative Education Review, 1967, 11, 331-347. Plans and policies for educational reform in Greece.

Kazamias, A. M. & Massialas, B. G. Tradition and change in education: a comparative study. New York: Prentice - Hall, 1965.

Kline, M. The ancients versus the moderns, a new battle of the books. The Mathematics Teacher, 1958, 51, 418-427.

MacDonald, J. B., Wolfson, B. J. & Zaret, E. Reschooling society: a conceptual model. Washington, D. C.: Association for Supervision and Curriculum Development, 1973.

Maslova, G. G. & Markushevitz, A. I. |Mathematics in the schools of the U.S.S.R.| (B. R. Vogeli, trans.). The Mathematics Teacher, 1969, 62, 231-239.

Massialas, B. G. Greece. In The encyclopedia of education (vol. 4). The Macmillan Co. & The Free Press, 1971.

Miller, G. H. Geometry in the Greek elementary school. The Arithmetic Teacher, 1964, 11, 85-88.

Miller, G. H. Geometry in the secondary school of Greece: a comparative study. School, Science and Mathematics, 1966, 66, 817-822.

Ministry of Education, Athens. Greece. In The world survey of education (vol. 5). Paris: UNESCO, 1971.

Ministry of National Education and Religion. Education in Greece. Athens: National Printing Office, 1973.

New trends in mathematics teaching (vol. 1). Paris: UNESCO, 1966.

New trends in mathematics teaching (vol. 2). Paris: UNESCO, 1970.

New trends in mathematics teaching (vol. 3). Paris: UNESCO, 1972.

Nikolaou. N. D. [Theoretical geometry.] Athens: Organization for Publishing Textbooks, 1970.

Nikolaou, N. D. [Trigonometry.] Athens: Organization for Publishing Textbooks, 1967.

Ntziora, E. V. [Mathematics - 5th year of the gymnasion (vol. 1).] Athens: Organization for Publishing Textbooks, 1970.

Papanikolaou, C. G. [Euclidean Geometry.] Athens: Organization for Publishing Textbooks, 1975.

Polya, G. Mathematical discovery; on understanding, learning, and teaching problem solving (vol. 2). New York: Wiley, 1965.

Polya, G. On learning, teaching and learning teaching. The American Mathematical Monthly, 1963, 70, 605-619.

[Proceedings of the mathematics conference.] Crete: Greek Mathematical Society, 1960.

[Proceedings of the mathematics conference.] Ioannina: Greek Mathematical Society, 1962.

[Proceedings of the mathematics conference.] Thessaloniki: Greek Mathematical Society, 1964.

[Proceedings of the mathematics conference.] Patra: Greek Mathematical Society, 1971.

[Program of courses in the Greek high school.] [Official Gazette,] November 10, 1969, pp. 1638-1644.

[Program of courses in the pedagogic academies.] [Official: Gazette,] May 31, 1972, pp. 617-618.

Rogers, C. R. On becoming a person; a therapist's view of psychotherapy. Boston: Houghton-Mifflin, 1961.

Sakellariou, N. [Algebra.] Athens: Organization for Publishing Textbooks, 1970.

Servais, W. & Varga, T. (Eds.). Teaching school mathematics; a UNESCO source book. Middlesex, England: Penguin, 1971.

[Supplement to the catalog; the School of Mathematics and Physics of the Aristotelian University of Thessaloniki.] Thessaloniki, Greece, 1972.

Vavaletskou, T. & Bousgou, G. [Mathematics - 4th year of the gymnasion (vol.1).] Athens: Organization for Publishing Textbooks, 1971.

APPENDIX A

PROGRAM OF COURSES IN THE GREEK ELEMENTARY SCHOOL

Program of Courses in the Greek Elementary School

Courses	First Year	Second Year
Religion	2*	2
Greek Language	9	9
Study of the Enviroment	4	6
Arithmetic - Geometry	3	3
Art	3	3
Music	1½	1½
Physical Education	1½	1½
Total hours of instruction per week	24	26

* Numbers indicate hours of instruction per week

Program of Courses in the Greek Elementary School

Courses	Third Year	Fourth Year
Religion	2*	2
Greek Language	10	10
History	2	2
Physics - Chemistry; Hygiene	3	3
Geography	3	3
Arithmetic - Geometry	4	4
Art	4	4
Music	2	2
Physical Education	2	2
Total hours of instruction per week	32	32

* Numbers indicate hours of instruction per week

Program of Courses in the Greek Elementary School

Courses	Fifth Year	Sixth Year
Religion	3	3
Greek Language	9	9
History	2	2
Physics - Chemistry; Hygiene	4	4
Geography	2	2
Arithmetic - Geometry	5	5
Civics	-	1
Art	4	3
Music	2	2
Physical Education	2	2
Total hours of instruction per week	33	33

* Numbers indicate hours of instruction per week

APPENDIX B

PROGRAM OF COURSES IN THE PEDAGOGIC ACADEMIES

Program of Courses in the Pedagogic Academies

Courses	First Year	Second Year
Religion	2*	1
Greek language and grammar; speech	2	2
Greek history and civilization; archaelogical and historical places of Greece	2	2
Mathematics	2	2
Physical science	3	4
Pedagogy		
General pedagogics	2	-
General teaching methods	2	-
Special teaching methods	1(a)	2
Teaching the slow learner	-	1
Child development	-	1
Psychology		
General and pedagogic psychology	2	2
Human development (emphasis on the child)	2	-
Psychology of individual differences	-	2
Elements of introductory philosophy	2	-
Sociology and civics	-	1

Courses	First Year	Second Year
Art	3	3
Physical education (gymnastics; competition; folk dancing)	2	3
Music (theory; instrumental; ecclesiastical; voice)	3	3
Organization and administration of education	-	1
Home economics and family education		
(men and women)	1	-
(women only)	1	1
Hygiene (personal; social; school; first aid)	1	-
Foreign languages (English, French; or German)	2	2
Elements of agriculture and agricultural economics	1	1(b)
Practice-teaching experiences (hours determined by the Directors of the Academies)		

* Numbers indicate hours of instruction per week

(a) Second semester only

(b) Men only

APPENDIX C

PROGRAMS OF COURSES IN THE GREEK HIGH SCHOOL

Program of Courses in the Lower Cycle of the Greek High School

Courses	First Year	Second Year	Third Year
Religion	2*	2	2
Modern Greek language and grammar	4	4	4
Ancient Greek language and grammar	5	6	6
History	3	3	3
Civics	-	-	1(a)
Vocational awareness	-	-	1(b)
Mathematics	4	4	4
Physical science	2	3	3
Biology	-	-	1
Anthropology	-	1	-
Foreign languages	3	3	3
Hygiene; first aid	-	1	-
Physical education (boys)	3	3	3
Physical education (girls)	3	3	3
Art	1	1	1
Music	1	1	1
Home Economics (girls only)	2	1	1
Geography	2	1	2

	First Year	Second Year	Third Year
Total hours of instruction per week (boys)	30	33	34
Total hours of instruction per week (girls)	32	34	35

* Numbers indicate hours of instruction per week

(a) - first semester only

(b) - second semester only

Program of Courses in the Classical Track

of the Greek High School

Courses	Fourth Year	Fifth Year	Sixth Year
Religion	3*	2	2
Modern Greek language and grammar	4	4	4
Ancient Greek language and grammar	7	7	8
History	3	3	3
Elements of philosophy; psychology; logic	-	2	2
Civics	-	-	1
Mathematics	4	4	4
Cosmography	-	-	1
Physical science	3	3	3
Biology	-	-	1
Anthropology	-	1	-
Foreign languages	2	2	2
Latin language and grammar	3	3	2
Hygiene; first aid	-	1	-
Physical education (boys)	3	3	3
Physical education (girls)	3	3	3
Art	1	-	-
Music	1	-	-

Courses	Fourth Year	Fifth Year	Sixth Year
Home economics (girls only)	1	1	-
Geography	1	1	-
Total hours of instruction per week (boys)	35	36	36
Total hours of instruction per week (girls)	36	37	36

* Numbers indicate hours of instruction per week

Program of Courses in the Scientific Track

of the Greek High School

Courses	Fourth Year	Fifth Year	Sixth Year
Religion	3*	2	2
Modern Greek language and grammar	4	4	4
Ancient Greek language and grammar	6	5	4
History	3	3	2
Elements of philosophy; psychology; logic	-	2	2
Civics	-	-	1
Mathematics	6	6	7
Cosmography	-	-	1
Physical science	5	5	6
Biology	-	-	1
Anthropology	-	1	-
Foreign languages	2	2	2
Hygiene; first aid	-	1	-
Physical education (boys)	3	3	3
Physical education (girls)	3	3	3
Art	1	1	1
Music	1	-	-
Home economics (girls only)	1	1	-
Geography	1	1	-

	Fourth Year	Fifth Year	Sixth Year
Total hours of instruction per week (boys)	35	36	36
Total hours of instruction per week (girls)	36	37	36

* Numbers indicate hours of instruction per week

APPENDIX D

SCHOOLS AND DEPARTMENTS OF THE UNIVERSITY OF THESSALONIKI

Schools and Departments of the University of Thessaloniki

School of Theology

Department of Theology

Department of Pastoral Theology

School of Philosophy

Department of Classics

Department of Byzantine and Modern Greek

Department of History

Department of Art and Archaeology

Department of Philosophy

Department of Psychology and Pedagogy

Department of Linguistics

Institute of Foreign Languages and Literatures

Department of English

Department of French

Department of German

Department of Italian

School of Physics and Mathematics

Department of Physics

Department of Mathematics

Department of Chemistry

Department of Biological Sciences

Department of Pharmacy

<u>School of Law and Economics</u>

Department of Law

Section of Jurisprudence

Section of Public Law

Section of Political Science

Department of Economics

<u>School of Agriculture and Forestry</u>

Department of Agriculture

Department of Forestry

<u>School of Technology</u>

Department of Civil Engineering

Department of Architecture

Department of Surveying and Topography

Department of Mechanics and Electrical Engineering

Department of Chemical Engineering

<u>School of Medicine</u>

<u>School of Veterinary Medicine</u>

<u>School of Dentistry</u>

APPENDIX E

PROGRAM OF COURSES IN THE DEPARTMENT OF MATHEMATICS AT THE UNIVERSITY OF THESSALONIKI

Program of Courses in the Department of Mathematics at the University of Thessaloniki

Courses	Lecture	Tutorial	Laboratory
First Year			
Required courses			
202 General mathematics I	2*	2	-
203 General mathematics II	2	2	-
204 General mathematics III	2	2	-
102 Physics	4	-	-
- Foreign language	3	-	-
Elective courses (choose any two)			
- Pedagogics	2	-	-
- Introduction to philosophy	2	-	-
- Psychology	2	-	-
401 Crystallography	2	-	-
307 Chemistry	2	-	2
Second Year			
Required courses			
211 General mathematics I	2	2	-
212 General mathematics II	3	2	-
213 Topology	2	2	-

Courses	Lecture	Tutorial	Laboratory
Second Year			
Required courses			
214 Linear geometry	2	2	-
- Foreign language	2	-	-
Elective courses (choose any two)			
104 Electricity and electromagnetic theory	3	2	-
210 Differential forms	2	1	-
404 Meteorology	2	2	-
215 Electronic computers and programming I	2	2	-
Third Year			
Required courses			
209 Algebra	2	2	-
220 Complex functions	2	2	-
221 Theory of real functions	2	2	-
218 Arithmetical analysis	2	3	-
- Foreign language	2	-	-
Elective courses (choose any two)			
112 Astronomy	4	2	-
108 Optics	3	2	-

Courses	Lecture	Tutorial	Laboratory
Third Year			
Elective courses (choose any two)			
219 Electronic computers and programming II	2	2	-
411 Climatology	2	-	-
222 Topology	2	2	-
109 Theoretical mechanics	3	2	-
Fourth Year			
Required courses			
225 Differential geometry I	2	2	-
220 Complex functions	2	2	-
224 Theory of probability and statistics	2	2	-
226 Theory of numbers	2	1	-
Elective courses (choose any two)			
228 Differential geometry II	2	1	-
229 Functional analysis	2	1	-
230 Algebra	2	1	-
123 Mechanics	2	1	-
120 Astronomy	2	1	-
125 Theoretical physics	1	1	-

* Numbers indicate hour of instruction per week