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Effects of integrated writing on attitude and algebra performance of high school students

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EFFECTS OF INTEGRATED WRITING ON ATTITUDE
AND ALGEBRA PERFORMANCE OF
HIGH SCHOOL STUDENTS

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APPROVAL PAGE

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This study investigated effects of a program integrating writing activities in a regular Algebra II curriculum on students' mathematics achievement and attitudes in writing and mathematics. The sample involved 68 students, 34 in both experimental and control groups, from four classes of regular Algebra II students at a private school in the southeastern United States. The treatment spanned four chapters in the text and 12 weeks. Both groups received the same instruction using a basic Algebra text. Writing activities were integrated within the experimental group's lessons. Data were collected in several ways. Each student was given a preliminary algebra test and writing and mathematics attitude scales. Students then completed appropriate chapter tests. After two chapter tests, the students were given the midtest. At the conclusion of the study, students were given the posttest and the writing and mathematics attitude scales. Following each chapter test, the midtest, and the posttest, students explained in writing how they solved two preselected items. These writing samples were scored holistically.

Analysis of covariance was used to analyze mathematics achievement data and writing sample data, and t-tests to analyze the attitude scales. Results were mixed. For the mathematics achievement data, there was no significant difference between the groups for the midtest, posttest, and the first and third chapter tests.

However, on the second and fourth chapter tests and the average of the chapter tests, the experimental group performed significantly better than the control. For the writing sample data, there was no significant difference between the groups for the midtest or the posttest. However, on all chapter tests, the experimental group performed significantly higher than the control group. For the attitude scales, there was no significant difference either between the two groups before or following the study or between the attitudes of each group before or following the study. From the data collected, it appears that the use of writing-to-learn mathematics can be a valuable tool for learning mathematics.

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CHAPTER I
INTRODUCTION

Schoenfeld (1992) describes the purpose of mathematics education as learning to think mathematically. To think mathematically "means (a) developing a mathematical point of view - valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing the competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making" (p. 335). In 1989, the National Council of Teachers of Mathematics (NCTM) placed communication as a high priority for grades K-12 stating that the "mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can: reflect upon and clarify their thinking about mathematical ideas and relationships; and express mathematical ideas orally and in writing" (NCTM, 1989, p. 140). Recognizing a related concern, in 1980 NCTM recommended that "problem solving be the focus of school mathematics in the 1980's" (NCTM, 1980, p. 2). Then in 1989, NCTM strongly endorsed the 1980 recommendation. "The development of each student's ability to solve problems is essential if he or she is to become a productive citizen" (NCTM, 1989, p. 6).

The use of writing-to-learn in mathematics classes is one way teachers can implement both communication and problem solving goals. Emig (1977), Vygotsky (1962), Odell (1980), and Imscher (1979) all emphasize the important link between writing and learning. They believe that writing in a content area can encourage students to analyze, compare facts, and synthesize material. "Writing helps tie down ideas and make connections between old and new concepts" (Vygotsky, 1962, p. 92).

Many mathematics educators agree that writing should become a part of the daily routine of every mathematics class. Johnson (1983) suggests that, if students can write clearly about mathematical concepts, they can then probably understand them. Bell and Bell (1985) state that writing has been an effective and practical tool for teaching mathematics problem solving. Asking students to write about a process or a problem requires them to clarify their thoughts. This writing procedure then becomes an integral part of the thought process. Writing forces students to become active rather than passive learners, and thus they are more likely to be actively involved in "constructing" their own knowledge.

This study investigated the effects of implementing an integrated, experimenter-designed writing program within an existing basic text of Algebra II. This program consisted of specifically formulated lessons in writing designed to enhance the students' understanding of topics studied.

Hypotheses

The primary hypothesis was that students receiving the integrated writing program in Algebra II within the framework of a course based on an existing basic text would exhibit greater achievement than students receiving the regular instruction in the basic text. The secondary hypothesis was that students receiving the integrated writing program would develop more positive attitudes towards mathematics and writing.

Importance

Knowing whether the hypotheses are true is important because such knowledge will add support to the developing body of research which already strongly suggests that writing is of intrinsic value in learning mathematics. As mathematics educators, we cannot overlook the possibility that writing may be a tool that will enable students to better understand and more effectively relate to their mathematics learning.

Although research in writing-to-learn has been carried out primarily on the college level, more research is needed on the secondary level to ascertain if writing as a way to learn is effective. Certainly, the implications of college-level studies would suggest that similar results would accrue from similar programs of writing-to-learn mathematics, especially when used in classes of bright and motivated high school students.

CHAPTER II
REVIEW OF LITERATURE

The topic of writing to learn mathematics is of major concern and will, accordingly, be discussed first. Thereafter, a general discussion of research in mathematics education will follow, considering in particular the topics of metacognition, problem solving, and algebra. A brief discussion of the role of affect in learning mathematics will conclude the review of related literature.

Writing-to-Learn Mathematics

Writing-to-learn mathematics has been an important area of research for over two decades. Over twenty years ago, Bruner (1968) stated that writing and mathematics were "devices for ordering thoughts about things and thoughts about thoughts" (p. 112). Vygotsky (1962) viewed the relationship between language and thought as "a dialectical one," that is, as a process in which language and thought work together to transform each other into a final presentation. He believes that writing helps tie down ideas so that connections can be made between old and new concepts. Both Vygotsky and, in a later book, Bruner (1986) view writing and other forms of communication as contributors to learning in that writing and thought are intertwined. In recent years, writing in content areas has been used as both a learning aid for the student and as an instructional tool for the teacher.

The movement for writing across the curriculum has focused mainly on improving the quality of writing. This is a different objective than that of the movement for writing-in-content. The objective of writing-in-content is to focus the student's thinking on better understanding of the subject matter. Writing-in-content is writing to learn. Both writing and learning are thought of as "meaning-making processes that involve the learner in actively building connections between what she's learning and what is already known" (Mayher, Lester, & Pradl, 1983, p. 78).

> Writing's capacity to place the learner at the center of her own learning can and should make writing an important facilitator of learning anything that involves language. Writing that involves language choice requires each writer to find her own words to express whatever is being learned. Such a process may initially serve to reveal more gaps than mastery of a particular subject, but even that can be of immense diagnostic value for teacher and learner alike. And as the process is repeated, real and lasting mastery of the subject and its technical vocabulary is achieved. (Mayher et al., 1983, p. 79)

Support for writing-in-content has been developing for over ten years. Many educators are of the opinion that writing-in-content encourages learning and thinking and that there is an important link between writing and learning. Fulwiler (1986) states that "writing is the specific activity that promotes independent thought" (p. 25). Pearce (1984), together with Bell and Bell (1985), advocates that writing-in-content will encourage learning and thinking.

Both Emig (1977) and Farrell (1978) claim that writing-in-content can help students to analyze and synthesize material. According to Emig (1977), writing has certain advantages in the learning process: the students can be actively involved in structuring their own meanings, can move at their own pace, and can be given immediate feedback. Kenyon (1988) states that writing can be used effectively in the mathematics classroom for the acquisition of knowledge because, in the writing process, students begin to organize their old and new knowledge and concepts and thus synthesize this information into a structure that becomes a part of their own knowledge.

Others (Flaningam & Warriner, 1987; Haley-James, 1982; Hartman, 1989; Lehr, 1980; Sanders, 1991; Smith & Bean, 1980) have also advocated writing-in-content as a tool for helping students learn, stating that, since writing can lead to better understanding of concepts, it should be used as a regular instructional technique. Kennedy (1980) states that writing about a topic requires students to think about the topic, focus on important concepts, and make those concepts their own, thus constructing their own knowledge. Herrington (1981) states that writing forces the students to see new relationships and invent new ideas and thus to communicate better. In support of these views that writing can be used as a tool for learning, educators have studied the effects of various types of writing activities on learning in general and learning mathematics in particular.

Types of Writing Activities

Many types of writing activities, ranging from journals to free writing to in-class writings to term papers, have been used in mathematics instruction. Britten, Burgess, Martin, McLeod, and Rosen (1975) classified written language into transactional and expressive. Transactional writing, the purpose of which is to communicate to an audience in order to instruct or persuade them, is used in summaries, reports, essays, projects, or notetaking. Expressive writing, the purpose of which is for the writer to explore what he or she thinks, knows, or feels, is usually freewriting or journal writing.

Pearce and Davidson (1988), who are strong advocates of writing-to-learn mathematics, argue that "writing activities can be used to facilitate most aspects of students' mathematical development, from concept development to problem-solving abilities" (p. 493). Pearce and Davidson expressed disappointment, however, with the results of two studies they completed in junior high schools. In one study involving junior-high school mathematics classes, they classified the writing activities into five categories: direct use of language, which included copying and transcribing information; linguistic translation, which included translation from mathematical symbols to words; summarizing and interpreting; applied use of language, which included applying a mathematical idea to a problem situation; and creative use of language. They collected data on the amount, kinds, and uses of writing by using structured interviews and by studying lesson plans and students' work of 31 junior high school teachers and their students. They found that the junior high school teachers made very little use of writing in their mathematics classes. The students' writing was primarily note taking.

Concern about this lack of writing prompted a later study of junior high school mathematics texts. Davidson and Pearce (1988) examined five commonly used junior high mathematics texts to determine the types and number of writing activities that were included. The texts contained very few writing activities and little or no instructional support for writing.

Expressive writing. Journal writing is probably the most common type of expressive writing used in mathematics classes, and it has been researched more than any other type of writing to learn. At various grade levels, including college, many experiments have been conducted using journal writing in mathematics classes (Buerk, 1986; Burton, 1986; Mayher et al., 1983; Mett, 1987; Nahrgang & Peterson, 1986; Powell, 1986; Stempien & Borasi, 1985; Vukovich, 1985; Watson, 1980). Most of these studies on the use of journal writing have focused on the cognitive and affective benefits the students may derive directly from writing in mathematics, and most of the evidence has been anecdotal. Little attention has been paid in these studies to how the teachers were affected or how the student-teacher relationship was affected from reading the journals.

Borasi and Rose (1989), in their comprehensive study of journal writing in a college mathematics class, investigated both the conceptual and empirical components of writing in journals. They concluded that, when students write about mathematics topics or concepts or the process of doing mathematics, the students increase their learning of mathematical content and improve their learning and problem-solving skills. Borasi and Rose found that, when the students wrote about feelings and attitudes, it had a positive therapeutic effect. The teachers, from reading the journals, were able to complete better evaluations of the students and thus provide better remediation of individual students and make improvements in the teaching of the course. Through the student-teacher dialogue with the journal, a more supportive atmosphere was created in the classroom and more individualized teaching occurred.

Powell analyzed the freewritings and journal entries of one first-semester freshman, Lopez, enrolled in a developmental computation class (Powell & Lopez, 1989). Freewriting, according to Powell, is thinking aloud on paper by writing down whatever comes in your mind. Powell found that writing was a heuristic tool that could be used to generate knowledge and learning. He concluded that journals were a very effective way for dialogue to occur between the student and the teacher and that these dialogues served the added purpose of reassuring the student that his concerns were being acknowledged. Powell concluded, furthermore, that writing caused the student to reflect critically about the mathematics he was learning. Such critical reflection created an active rather than a passive student. Writing provided the student the opportunity to work with mathematical concepts and terminology in his own language and on his own terms. Powell found that, as the student became more involved in the use of language, he became more involved in constructing and reconstructing meaning and making sense out of mathematics. Reflecting critically also gave the student a sense of accomplishment as the student felt more in control of his learning. Powell further concluded that, by acquiring greater control of his learning, the student felt more capable of understanding and doing mathematics. Powell finally concluded that writing supported the cognitive and metacognitive acts of exploring relationships, making meaning, and manipulating thoughts.

Marwine (1989) integrated informal writing in his classes of freshman college students and in workshops, finding it to be a valuable addition for checking on quality of communication and for providing ways of enhancing that quality for both students and teachers. By reading the students' writing, he found that he was able to determine the students' views on the material and thus was better able to communicate with the students. In his responses to the students, Marwine always responded in generous and encouraging words; his purpose was not to tell them what they had missed, but to help them discover that on their own. Marwine found that writing was a useful tool for learning for most students if it was used often enough for the students to discover its benefits.

Transactional writing. Transactional writing requires students to think and organize their thoughts before they write. Bell and Bell (1985) have successfully used transactional writing as an effective tool for teaching mathematical problem solving. They stress that the writing activities need to be designed carefully in order to enhance the mathematical skills being taught.

Miller and England (1989) studied the effects of using writing on attitudes and skills in first and second year algebra classes. The primary emphases of the study were to investigate what teachers could learn about their students' understanding of mathematics by reading the students' responses to in-class impromptu writing prompts and to determine whether the teacher's instructional practices were influenced by reading students' responses. Three algebra teachers from a large metropolitan high school and two university professors, one a mathematics educator and one a writing specialist, took part in the year-long, qualitatively-designed research study. The prompts the teachers used elicited both expressive and transactional types of writings. The university professors provided lists and explanations of writing prompts, and the teachers allowed the students to write for a minimum of five minutes at least four out of every five instructional days. The teachers spent some time each week reviewing the students' writing and then provided the university team their own writings, which reflected the teachers' dominant impressions drawn from the students' writings. Miller and England used both affective and subject-oriented prompts and reached several conclusions. First, the students seemed to write more when addressing their writings to a friend or another teacher. Second, students' writing improved with time. Third, students found longer and more detailed prompts frustrating. Fourth, students thought of the writing as a means by which the teacher could discover ways to help them better. The teachers involved in this study found the students' writings valuable in determining individual needs. Through the writings, the teachers found that they were able to identify misconceptions quickly, which allowed for immediate reteaching. In addition, the teachers thought the continuous dialogue they developed with the students as the students wrote and the teachers responded to the students' writings contributed positively to the teaching and learning processes. The experienced teachers in the group said that they felt that the less experienced teacher learned more about teaching and students in one year than they themselves had learned in several years.

Birken (1989) reported on her experiences with writing in college mathematics classes for seven years. Because of the passivity of the students in traditional lecture classes, she used various types of transactional and expressive writing assignments in attempts to make them more actively involved in their learning. Her students commented that they developed a deeper understanding, improved clarity, and had better retention of concepts after writing. They also stated that they had never before thought about what they did or why they did it while they were working mathematical problems.

Birken also found that she learned much about the students' mathematical misconceptions which enabled her to pinpoint exactly where they had gone wrong and thus to help them redirect their thinking.

Ganguli (1989), in a study involving 125 university students in developmental algebra classes, investigated the effects of integrating writing into the regular classes. The study addressed the questions of whether the integration of writing improved the students' performance in terms of their final grade and whether the integrated writing decreased valuable time for the actual instruction in mathematics. The results reveal that the writing assignments did not conflict with the essential instruction and that the experimental section did do significantly better than the control section on their final grades.

Taylor's study (cited in Abel & Abel, 1988) found that college students who wrote an essay explaining a specific statistical technique did better on a traditional computational test than did their peers who had only the traditional computational problems. He concluded that writing may force the students to become more involved in their learning and thus be able to understand the topics better.

Student-teacher interaction

In the area of student-teacher interaction, Kennedy (1985), in his experiences with having middle school students write letters to him during the last few minutes of class, found that the teacher needs to establish an atmosphere of trust in order for the students to risk the exploratory and personal aspects of writing to learn. He had the students write about what they did or did not understand or were just wondering about. With an atmosphere of trust, he found that the students were more willing to acknowledge their lack of understanding in a letter to him than in a classroom. He found that he learned more about his students from reading their letters than he did in many days of observing them in the class.

In a study of relative effectiveness of writing assignments in an elementary algebra class for college students, Hirsch and King (1983) studied 83 students placed in four sections. The students in each section were randomly assigned to either an experimental or control group. The experimental group was given 15 assignments requiring written responses to conceptual questions and some mathematics problems, while the control group was given 15 assignments covering mathematics problems. So that the teachers of each section would not know to which group the students were assigned, all completed papers and assignments were distributed folded with the student's name on the outside. The papers were collected at the beginning of each class and new assignments were distributed at the end of class. The assignments were returned to the investigators, who graded them in time to be returned at the next class. Since the homework counted as only a small part of the total grade, the grading was done primarily for the purpose of determining which students were participating in the study. The teachers were instructed not to give any directions on the writing assignments. Of the 83 original students in the study, only 26 turned in 10 of the 15 assignments and were considered participants in the study. An analysis of the results of the final exam found no significant differences between the groups. Hirsch and King found that writing assignments used without teacher feedback were no more effective than the traditional instruction. They concluded that, for writing to be effective, it has to be integrated into the mathematics class and the teachers have to be involved in the process and in responding to the students. They also found that the experimental group's performance on the final exam was not harmed by their doing fewer traditional mathematics problems.

Language and Concept Development

As a natural concomitant of such studies, there has also developed considerable interest in and research on language and concept development. Theories concerning the relationship between concept development and language (Inhelder & Piaget, 1964; Siegel, 1971, 1982) in general agree that language has a strong influence on concept development. Research on learning and elaboration (Mayer, 1980) has demonstrated that language activities such as explaining and writing encourage students to relate new information to their current knowledge. Without this knowledge integration, learning remains at a rote level (Mayer, 1980). Wittrock (1974) found that, when specific instructional activities are used to encourage the students to generate their own language and ideas, comprehension is increased.

Bradley (1990), concerned about students' understanding of the meaning of mathematical concepts and their inability to communicate in mathematics, analyzed the relationship between mathematics language facility and mathematics achievement in 200 junior high-school students. Students were administered mathematics achievement tests and mathematics language tests that were developed for the study. The mathematics achievement test was designed to measure both procedural and conceptual knowledge. The mathematics language test was designed to measure mathematical vocabulary, understanding of mathematical terms in context, and use of mathematical language in written explanation of procedures. She found that the students who were high in either procedural knowledge or language facility, but not both, were not significantly higher in conceptual knowledge than the students who were low in both mathematics procedure and language. However, the students who were high in both procedural knowledge and language facility were significantly higher in conceptual knowledge. She found that the combination of high language facility and high procedural knowledge in mathematics contributed significantly to conceptual achievement for the students in the average classes. Her study demonstrated that mathematics language skills, together with procedural skills, were related to conceptual understanding.

Moreover, the students needed both procedural knowledge and language facility in mathematics for an acceptable level of conceptual achievement. Her findings further demonstrated that, without language facility, procedural knowledge alone did not predict conceptual understanding. The study found a positive relationship between students' ability to communicate mathematically and their level of meaningful learning. She recommended incorporating mathematical language activities, both written and oral, involving explanations of solutions of problems and relationships between old and new topics, into current teaching practices.

In a study of ninth graders, McCabe (1981) found, as had earlier researchers (Amsden, 1964; Peltz, 1973; Tatham, 1970; Williams, 1968), that students comprehended mathematical material better when the language patterns used by both the teacher and the text approximated the syntax of the students. Researchers of language processing (Doctorow, Wittrock, & Marks, 1978; Jenkins, Heliotis, Haynes, Stein, & Beck, 1986; Jenkins, Heliotis, Stein, & Haynes, 1987) have found that students who restated their reading in their own words demonstrated substantial improvements in comprehension. This restatement of their reading made it more personal and meaningful for the students, and thus their comprehension was increased. These researchers of language processing and its relation to mathematical problem-solving skills have regularly noted that students try to increase their comprehension and facilitate their problem solving by translating the problem into personal and meaningful vocabulary and structure. Transforming the text by writing and other language activities such as drawing, drama, and other communication systems has been recommended by several researchers (Grumet, 1985; Rowe & Harste, 1986).

Many educators have been concerned with and studied the interaction between written mathematics texts, written problems, and language processing. Hollander (1990), who studied the problem-solving skills of sixth graders working on problems, found that students who rewrote problems demonstrated improvements in comprehension of the problems. Similarly, Johnson (1983) found that when students rewrote problems in their own words, they could focus on key words and relationships. Abraham (1983) and Zimmerman (1989), in separate studies involving elementary school students, found it was beneficial for students to read information or written problems and restate the concepts or the problems in their own words. They found that mathematics books are written in language that is hard for the students to understand. Encouraging the students to write the problems in their own words increased comprehension. Some researchers (Reutzel, 1983; Van de Walle, 1987; Wirtz & Kahr, 1982; Zimmerman, 1989) have found that having elementary school students create their own written problems helped them make connections between the concrete and the abstract. Bray (1988) found that having elementary school students create and solve written problems helped them in interpreting and solving previously written problems.

Several educators have suggested language-related procedures for making students more aware of their thought processes while solving problems. Whimbey and Lochhead (1980) suggested verbalization, a technique that requires students to report on the thinking process during problem solving. Kenyon (1988) recommended writing down the thoughts and procedures in each step of the problem-solving process, so that the writing process becomes a vital part of the thought process. By writing about the problem, he contended, students have to clarify their thoughts about the problem and how they will approach it.

Summary of Writing-to-Learn

Before continuing to the important topic of metacognition, it will be helpful to review the more important findings regarding writing-to-learn mathematics. It will be recalled that, with learning to think mathematically as a primary purpose of mathematics education, increased emphasis has been placed in recent years on having the mathematics curriculum include the development of language so that students can communicate mathematical ideas. In the forefront of research in this area has been that on writing-to-learn mathematics, which is an outgrowth of the movement for writing-in-content.

Writing-in-content is writing to learn. Educators believe such writing encourages learning and thinking. They further claim that writing-in-content helps students develop the abilities to analyze and synthesize. In light of these beliefs, pertinent research has been done on types of writing, student-teacher interaction, and language and concept development.

Many types of writing activities, ranging from journals to free writing to term papers have been the subjects of research. While many students' writing activities in mathematics are limited to notetaking, others have been found to profit markedly from both transactional writing and expressive writing. Researchers have observed that the writing process changed the students from passive learners to active thinker-participants. It was also observed that students had better understanding and retention when they wrote about mathematical processes and problems.

Through regular reading of student writings, teachers were able to better observe and remedy problems and misconceptions in the learning process, a benefit which the students also appreciated. However, it was found that, for writing to be effective, it has to be integrated into the mathematics class and the teacher has to be involved in the process and responsive to the students.

Research on language and concept development has shown that language has a strong influence on concept development. For instance, when students generate their own language and ideas, comprehension is increased. Research has also demonstrated that mathematics language skills, combined with procedural skills, are related to conceptual understanding. Researchers in these areas consistently recommend incorporating mathematical language activities into current teaching practices.

Metacognition

When one considers writing as writing about a process or thinking about thinking, it may fall under the category of metacognition. Although researchers differ in their definitions of metacognition, the term generally refers to the thoughts and knowledge one has about one's own thoughts. According to Flavell (1976), metacognition "refers to one's own knowledge concerning one's own cognitive processes and products and anything related to them" (p. 232). In the process of writing, students gather and organize old and new concepts, knowledge, and strategies, and then synthesize the information to construct their own meanings (Nahrgang & Peterson, 1986). As students write, they enhance their executive abilities and thus their metacognition skills.

Schoenfeld (1980, 1983) and Silver (1982) used the term metacognition in a way similar to earlier scholars to describe the knowledge individuals have about their own thought processes. Developing his concepts from such definitions, Schoenfeld (1985) described expert problem solvers as "vigilant managers." He says that they are always mindful watchers of their own procedures, who try to be accurate and efficient. The novice, on the other hand, seems to get sidetracked by details that do not add to the solution of the problem. Considerable research (Schoenfeld, 1980; Silver, Branca, & Adams, 1980; Wittrock, 1974) suggests not only that novice problem solvers can benefit from being taught metacognition skills, but that they can also be taught to be conscious of their thinking processes and thus be more effective and efficient problem solvers. Palincsar and Brown (1987) and Deshler and Schumaker (1986), in working with adolescent students who had academic difficulties in mathematics, found that a focus on metacognitive instruction was appropriate and effective in identifying and enlisting strategies to promote and monitor learning.

Such concerns for the development of strategies are important in themselves, but they also lead naturally to research on problem solving. In the process of writing, students gather and organize old and new concepts, knowledge, and strategies, and then synthesize the information to construct meaning. Problem solving is a process by which people use previously acquired knowledge and skills to attempt to find resolutions, not immediately apparent, to situations that confront them. The same metacognition skills which allow the student to perform the self-monitoring skills in writing would seem to be those involved in problem solving. In fact, Kenyon (1988) describes writing as problem solving.

Problem Solving

Some of the first studies on the processes used in problem solving were done by Kilpatrick (1969), who started with the strategies or heuristics outlined by Polya (1945). Using clinical interview research methods, Kilpatrick studied the mental processes students used to solve problems. Following these efforts, Kantowski (1977) also concentrated on process and used clinical observation to study ninth-grade geometry students' problem-solving strategies. Her study showed an increased use of heuristics as problem solving developed. She also noted the need for reliable instruments for measuring processes.

Krutetskii (1968/76) claimed that good and poor problem solvers differ not only in solution performance but also in recall of problem information and structure. His findings suggest that a difficulty for poor problem solvers may lie in their ability to notice structure before solving a problem. He found that the poor problem solver had difficulties in distinguishing between relevant and irrelevant data and also in generalizing across a wide range of mathematically similar problems. Bright (1977), concerned about the significance of generalizing from Krutetskii's research, pointed out that much of Krutetskii's research was done on gifted children. However, Silver (1979), in working with average eighth-grade students, found results similar to those of Krutetskii.

Marked progress has been made since the late seventies in research regarding problem-solving process. It has been determined that expert problem solvers use heuristic devices more frequently than do novice problem solvers (Larkin, 1977; Larkin, McDermott, Simon & Simon, 1980; Schoenfeld, 1980; Simon & Simon, 1978). Furthermore, it has been shown (Schoenfeld, 1979, 1980) that these devices can be successfully taught to the novice problem solver.

Berger (1984) studied analytical geometry students as experts and first-year algebra students as novices. He found that the expert problem solver was able to identify the structure of the problem quickly, whereas the novice was unable to see relationships inherent in the structure of the problem. In additional research with the first-year algebra students, he found that instruction focusing on the structure of problems was successful in improving performance. His research pointed to the direct teaching of heuristic skills and the identifying of similarities in categories of word problems in algebra.

Hardiman (1988), in her research with problem solvers in physics, found that novices and experts attend to different types of information when classifying problems according to solution similarity. Novices appear to classify problems mainly with respect to surface similarity, such as objects involved in the problem, whereas experts classify problems on the basis of solution principle. Hardiman also found that it benefited students to categorize problems, a process which pointed the novice problem solver toward attending to the structure rather than the surface details of the problem. In summary, the research on process in problem solving has demonstrated that the expert problem solvers recall structure and use heuristics. Novice problem solvers have difficulty distinguishing between relevant and irrelevant material and also have difficulty generalizing.

In other research involving novice problem solvers, several educators working with adolescent students with learning disabilities also have had positive results with teaching these students specific strategies for problem solving. Deshler and Schumaker (1986) found that adolescent students with learning disabilities could benefit from direct instruction in developing strategies in problem solving. Fleischner, Nuzum, and Marzola (1987) devised an instructional program to teach arithmetic problem-solving skills to students with learning disabilities. These educators devised their instructional program by using not only the information-processing theory of Newell and Simon (1972) as a conceptual framework within which to analyze the complex array of cognitive processes essential to solving problems, but also task analysis to analyze the skills necessary to solve problems. The experimental group of fifth and sixth-grade students classified as learning disabled was first taught the specific strategies necessary to solve word problems and then performed significantly better on story problems than the control group that had not been given the instruction on strategies.

Although much research in mathematics has focused on process in solving problems, another important area of research is that of the organization of knowledge in long-term memory. Researchers in information processing agree that the concepts in long-term memory are structured (Anderson & Bower, 1973). Studies in science (Chi, Glaser & Rees, 1982; Stewart, 1980), as well as in mathematics (Geeslin & Shavelson, 1975; Greeno, 1976; Stewart, 1980), have focused on mapping concepts in long-term memory. These studies gained information on the ways individual students organize and think about related concepts, discovering that concepts stored in chunks can be used more efficiently in problem solving.

Many studies have compared memory differences between novice and expert problem solvers (Chase & Simon, 1973; Chi et al., 1982; Larkin, et al.,1980; Simon & Simon, 1978), and have demonstrated that experts tend to organize their knowledge into larger chunks and that these chunks are organized on higher-order principles. Experts put more structure into the information they learn (Chi et al., 1982). Novices tend to store their knowledge in isolated bits, or to sort their knowledge by surface characteristics.

Expert and novice problem solvers, moreover, differ as to the ways in which they attend to a problem. Expert problem solvers in general attend to a problem's structure; they can be considered to be field independent. Novices in general attend to the surface details of a problem; they can be considered to be field dependent (Krutetskii, 1968/1976; Silver, 1979). Expert problem solvers, then, are able to put more structure into what they store, to pay more attention to structure, and to carefully watch their procedures. The novice problem solvers pay more attention to detail and lack the ability to focus on structure.

Algebra

The purpose of this study was to determine the effects of an integrated writing program on the attitudes and algebra performance of high school students. Therefore, the considerable research heretofore discussed on writing-to-learn, types of writing assignments, student-teacher interaction, language and concept development, metacognition, and problem solving must ultimately be related to algebra. Accordingly, a brief review of pertinent research on algebra is necessary.

There is evidence that algebra students pass through some of the same stages of development that the system of algebra itself proceeded through historically, from the use of ordinary language descriptions for solving problems, to a use of abbreviations for unknown quantities, and finally to symbolic algebra, with the use of letters to stand for certain quantities (Harper, 1987). According to Wheeler (1989), high school algebra is derived overtly from arithmetic; algebra is presented as a generalization of arithmetic. However, algebra pedagogy is not consistent with arithmetic pedagogy and there are many signals sent to learners that algebra has its own rules, which are not at all the same as those of arithmetic. A number of research studies have dealt with the continuities and discontinuities between arithmetic and algebra.

One of the differences between arithmetic and algebra is the interpretation given to signs and symbols. For example, Vergnaud (1984, 1988) found that arithmetic students read the equal sign as "it gives," whereas in algebra the student must respect the symmetric and transitive character of the sign. Kieran (1981) observed the reluctance of students to accept statements such as

$4 + 3 = 6 + 1$ as an additional signal that the algebra student had trouble accepting the equal sign as a symbol of equivalency.

Arithmetic students usually solve problems informally either primitively, that is, based on instinctive knowledge, or intuitively, that is, tied closely with early experiences in arithmetic (Booth, 1984). Therefore, the beginning algebra student has difficulty solving problems with formal mathematical models, that is by setting up an equation symbolizing the relationships presented in the problem. In setting up the equation for a word problem in algebra, students have to think exactly the opposite of the way they did in arithmetic. In arithmetic, students think of the operation they need to solve the problem. In algebra, students must represent the problem situation in an equation. In arithmetic, students do not have to make explicit the procedures they use in solving problems since the procedures they use are often informal and hard to symbolize. In algebra, they have difficulty in representing formal mathematical methods. Moreover, solving equations requires procedures not used in arithmetic. In arithmetic, students are used to equations in which the operations are set up to give the answer, whereas in algebra the equation is set up to describe the relationships between the variables. Writing activities could be used to assist the students in this transition.

The procedure for solving an equation in algebra is based on the principal of conserving equality, which is new to the Algebra I student and is still a problem with some Algebra II students, especially when they are working with complex expressions and irrational and complex numbers. Problems of the type $x + a = b$, $ax = b$, and $ax + b = c$ can easily be solved by arithmetic. The difficulty occurs when students are faced with problems with variables on both sides of the equation, such as, $ax + b = cx + d$. Mevarech and Yitschak (1983) found that even some students in college had a poor understanding of equivalency and of the meaning of the equal sign even though they could successfully solve different types of equations.

Although algebra instruction includes formal solutions to equations, students come from arithmetic being able to solve equations intuitively or by trial and error. Whitman (1976) and Petitto (1979) researched the relationship between solving equations intuitively and formally. Whitman taught 156 seventh grade students in one of three ways. She taught one group intuitive techniques. Another group she taught formal techniques. Finally, her last group was taught intuitive techniques followed by formal techniques. Whitman found that students who learned to solve equations only intuitively performed better at solving equations than those who learned both ways. The students who were taught the formal techniques performed worse on solving equations than those taught both techniques. She concluded that formal techniques may get in the way of the students' intuitive ability to solve equations. Petitto, however, found that students who used a combination of both intuitive and formal methods were more successful in solving equations than those students who used only one of these processes.

Other studies related to equations (Kieran, 1989; Wagner, 1984) point to the difficulty algebra students have with the structure of equations. Students have difficulty perceiving multi-term expressions as a single unit. For example, students do not perceive that the surface structure of $4(2r + 1) + 7 = 35$ is the same as $4x + 7 = 35$. They also have difficulty with operations and their inverses. For example, students are not always aware that $x + 4 = 7$ and $x = 7 - 4$ are equivalent and have the same solution (Kieran, 1984).

The use of letters in arithmetic as measurement labels often interferes with students' understanding of a variable as used in algebra. Some studies have dealt with the confusion various levels of algebra students have regarding the use of letters to stand for specific unknowns. Some students continue to use "label interpretation" of literal terms even in situations where it is not appropriate. Clement (1982) and Clement, Lochhead, and Monk (1981), for example, presented a group of university engineering students with the "students and professors problem":

> Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.

Of the students studied, 37% answered incorrectly. Of the students who answered incorrectly, 68% represented the problem as $6S = P$. They used the labels S and P literally, translating the statement as 6 times $S = P$, unaware that the number of students had to be equal to 6 times the number of professors. In a similar study, Mevarech and Yitschak (1983) found 38% of 150 college students they tested answered that, in the equation $3k = m$, k is greater than m.

Kucheman (1981) has systematically studied the various interpretations that algebra students assign to letters. He evaluated 3,000 British students, 13 to 15-years-old, and found that the majority of the students considered letters in expressions and equations as objects; few were able to consider letters as specific unknowns and fewer still considered them as generalized numbers or variables.

Another problem that algebra students have is in accepting an expression as an answer. Chalouh and Herscovics (1988) reported that students believed that algebraic expressions were incomplete and tried to set them equal to something. Wagner, Rachlin, and Jensen (1984) found that students tried to add "= 0" to expressions. Kieran found similar results when she asked students to assign a meaning to "a" in the expression "a + 3" because the expression was not set equal to something.

The concept of function is usually introduced to the algebra student in the chapter of the text dealing with equations of lines or linear functions. Most students have experienced the concept of function on an intuitive level in earlier years, but many have difficulty making the transition from their intuitive ideas about function to the concept of function as used in mathematics. The concept of function has been described by many mathematics educators as one of the most important in modern mathematics. Much research has been done on this topic, for example, Thomas (1975), Karplus (1979), Dreyfus and Eisenberg (1982), Pointe (1984), Sfard (1989) and Bowman (1993). Three major problem areas have been found as a result of this research. There is a discrepancy between the mathematical definition of the concept that the student is given and the mental conceptual image that the student actually uses. There are difficulties in interpreting information from graphs and in graphically visualizing aspects of the functions. Finally, students have difficulty going beyond thinking of a function as a rule and thinking of it as an object. With the use of writing-to-learn, the students may be able to relate and synthesize these various aspects of function and construct a clearer meaning of the concept for themselves.

Affect and Mathematics

Most of the research in mathematics learning and teaching has centered on cognitive factors; there has been, however, an increasing interest in the role of affect in the learning of mathematics. This interest goes back to the early 1900's, when the Yerkes-Dodson law introduced the idea that the relationship between stress and learning efficiency was curvilinear (Yerkes & Dodson, 1908); that is, learning efficiency was strong at intermediate levels of stress and weak at high and low levels of stress. In a more recent study concerning affect, Easterbrook (1959) hypothesized that a person's capacity for learning is increasingly impaired by increasing emotional intensity.

Psychologists have used the word "affect" in a variety of ways. The Encyclopedia of Psychology describes affect as "a wide range of concepts and phenomena, including feelings, emotions, moods, motivations, and certain drives and instincts" (Corsini, 1984, p. 32). Using this definition, some examples of affect would be fear, anger, joy, hate, pride, and anxiety. Affect is sometimes used as a synonym for emotion, feeling, and mood. Among educators, affect has been used to describe attitudes, feelings, emotions, preferences, and values.

In the area of mathematics education, many educators and cognitive psychologists (Grouws & Cramer, 1989; Hart, 1989; Lester, Garofalo & Kroll, 1989; Mandler, 1984) suggest that beliefs, attitudes, and emotions are particularly important factors in research on affect. McLeod (1989) describes beliefs, attitudes, and emotions as terms that express the range of affect involved in learning mathematics. He views attitudes and beliefs as relatively stable attributes that are built up over a long period of time, whereas emotional responses are more likely to change quickly and can be developed quickly. As a means of better understanding the important concept of affect, the constructs of attitudes, beliefs, and emotions will be further examined.

Attitudes

Psychologists have generally defined attitude as a predisposition to respond in a favorable or unfavorable way with respect to a given object (Rajecki, 1982). Mathematics educators have generally defined attitude less clearly than psychologists. In the 1960's and 1970's, most mathematics educators limited mathematics attitude to liking or disliking mathematics. Scales were developed to measure the degree to which the students liked or disliked mathematics. A few scales also included certain items that dealt with anxiety aroused by mathematics. Later scales designed by mathematics educators (Crosswhite, 1972; Fenema & Sherman, 1976; Sandmard, 1980) to measure more specific components of attitudes showed a multidimensional view of attitude toward mathematics. One such scale measured the usefulness of mathematics at the present and in the future. Another scale was designed to measure a student's self-concept and confidence with respect to mathematics.

Hart (1989), in examining the attitude scales used by mathematics educators, found that attitude meant any number of perceptions students have about mathematics, themselves, their parents, or their teachers. McLeod defined attitude as "positive or negative feelings of moderate intensity and reasonable intensity" (McLeod, 1989, p. 249). McLeod also suggested that attitudes toward mathematics appear to be developed by a repeated emotional reaction to mathematics being automatized or by a previously existing attitude being assigned to a new task.

Most mathematics educators would like to see their students have positive attitudes toward mathematics, especially in order for their students to achieve greater success in mathematics. Several perspectives on relationships and combinations of relationships between attitude and achievement in mathematics have been proposed (Hart, 1989). One perspective is that positive attitudes will improve students' abilities to learn. Another perspective is that attitudes are an important educational outcome, regardless of the impact on student learning. Another perspective is that positive attitudes are fostered by increasing the level of understanding in mathematics. Some educators hypothesize a causal connection between attitude and achievement in mathematics. Some propose that attitude is the causal factor and some propose that achievement is the causal factor. According to Hart (1989), research has not specified a general causal relationship between attitude and achievement in mathematics.

Beliefs

Social psychologists have been interested in attitudes and affect, while cognitive psychologists have been interested in beliefs. "Beliefs are nonobservable theoretical entities postulated to account for certain observable relations in human behavior" (Colby, 1973, p. 254). Two types of categories of beliefs have been related to mathematics students: beliefs about mathematics, and students' and teachers' beliefs about themselves. Beliefs about mathematics as a subject initially have little affective component, while beliefs about self and about the individual's relationship to mathematics have a very strong affective component. Beliefs about self are related to self-confidence and self-concept.

Recognizing the importance of the study of beliefs, the National Assessment of Educational Progress (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988) has included "affect items" in their assessment. The results have shown, for example, that students believe mathematics is important, difficult, and based on rules.

Related to the preceding studies is the work of Schoenfeld (1985), who has been interested in students' beliefs concerning the ways mathematics can be useful to them and the ways these beliefs limit students' understandings of and performance in mathematics. Both Schoenfeld (1985) and Silver (1985) have investigated the belief systems of students as a way of researching problem solving. Schoenfeld (1885) and Silver (1985) found that college level students' beliefs about mathematics may hinder students' performance in solving non-routine problems. Further, if students develop the belief that all mathematics problems can be worked in a short duration of time, the students may be unwilling to spend the necessary time to solve a more difficult problem.

An additional important area of research on beliefs in mathematics has come from the research on gender-related differences in mathematics. Fennema and Sherman (1976), for example, found that males more than females generally believe mathematics is useful. Research on gender-related differences (Meyer & Fennema, 1988; Reyes, 1984) shows that males tend to be more confident than females even when the females have better reason, based on performance, to be more confident. Males, moreover, are more likely to attribute their success to mathematical ability, while females are more likely to attribute their failure to lack of ability. Females tend to attribute their success to extra effort, while males tend to attribute their failure to lack of effort (Fennema & Peterson, 1985; Meyer & Fennema, 1988; Reyes, 1984). Furthermore, participation in mathematically related careers appears to reflect these gender-related differences in attribution.

Additional research concerning beliefs about self has come from that on causal attributions of failure and success (Weiner, 1986). Weiner found that a student who failed to solve a problem attributed this failure to uncontrollable, external causes, such as the difficulty of the problem. By comparison, a student who succeeded in solving a problem attributed the success to controllable, internal causes, such as personal effort.

In addition, several studies of mathematics and mathematical problem solving have been concerned with confidence of the student. For instance, Charles and Lester (1982) studied the effects of direct instruction in problem solving in Grades 5 and 7. Even though affective factors were not the main concern of the study, Charles and

Lester found that both the students and the teachers responded with positive changes in confidence. Silver (1982), in working with junior high school students, found that a strong relationship between the affective factors of confidence and willingness to persist had a substantial effect on the problem solver. Schoenfeld (1983), in working with college level students, discussed the effects of confidence and attitude, emphasizing that attitudes toward mathematics and confidence about mathematics are very important in students' managerial decisions and that students' belief systems have a great bearing on the ways they manage their cognitive resources.

Although there has been general research on teachers' beliefs, more research is needed on teachers' beliefs concerning mathematics (Cooney, Grouws, & Jones, 1988). Thompson (1984), in studying three junior high school mathematics teachers, examined the degree of consistency between what teachers say they perceive mathematics to be and what they actually do in the classroom. "Much of the contrast in the teacher's instructional emphasis may be explained by differences in their prevailing views of mathematics" (p. 119). One of the teachers in the study viewed mathematics as made up of an accumulation of rules and skills and thus taught in a prescriptive manner emphasizing teacher demonstrations of rules and procedures. Another teacher viewed mathematics as a subject of logically interrelated topics and thus emphasized the mathematical meaning of concepts and the logic of mathematical procedures. The third teacher had a problem solving view of mathematics and thus emphasized activities that would engage students in generating mathematics.

In a related study, Sowder (1989) considered the influence of affective factors on computational estimation performance of preservice teachers. She found that the good estimators were people with strong mathematics self-concepts, who attributed their success to ability and valued mental computation. The poor estimators had low mathematics self-concepts, attributed their failure to task difficulty or too little time, and did not value mental computation.

Emotion

A number of important research studies have considered the effects of emotion. For instance, researchers have not only defined the term but have attempted to study various relationships of emotions to student activities. According to Corsins (1984), emotions are high-energy states of mind that give rise to feelings. Research on affect in mathematics education, however, has not attended very much to emotion. Rather, research on affect has primarily looked at factors that are fairly stable and can be measured by questionnaires and scales. Another reason for this dearth of research on emotion may be that there has been a lack of theoretical framework within which to interpret the role of emotion in mathematics learning (McLeod, 1989). To remedy this need, McLeod recommends applying Mandler's (1984) theory on mathematical problem solving as a basis for that theoretical framework. To appreciate this recommendation, it is necessary to briefly consider Mandler's work.

Mandler (1975), a cognitive theorist, states that a major source of emotion comes from the interruption of a person's plans or planned behavior. This interruption results in physiological arousal of the person, followed by an evaluation of the meaning of the interruption interpreted as an emotion such as surprise, joy, frustration, or anger. These interruptions can also be thought of as blockages. Since problem solving is usually defined as a task in which the answer is not immediately obvious, the student may feel an interruption or blockage in plans and thus be faced with either a positive or negative emotion.

Buxton (1981), working in areas of research similar to Mandler, studied adults who described their emotional reaction to mathematical tasks as "panic." He found that their reports of panic were associated with such a high degree of physiological arousal that it disrupted their ability to concentrate on the task at hand. He described this emotional reaction as fear, anxiety, and embarrassment, as well as panic. In another study, focused primarily on cognitive factors, Wagner, Rachlin, and Jensen (1984) found that algebra students who became stuck on a problem would get upset and try any response, rational or irrational, to get past the blockage. In consideration of such student emotional reactions, McLeod (1989) questions whether the curriculum changes for the eighties, with emphasis on problem solving for all, acknowledges the affective characteristics of emotion with which the students have to contend.

In research related to that above, Silver (1982), in studying junior high school students, and Schoenfeld (1983), in studying college students, found that helping problem solvers reflect on their own cognitive processes helps them to bring to consciousness an awareness of their own emotional reactions to problem solving. By increasing students' awareness of these emotional influences, they gain greater control over their cognitive processes. The use of writing-to-learn may help students to become increasingly aware of their cognitive processes and their emotional reactions to mathematics.

One type of emotion that has been studied is anxiety, which can be defined as "a subjective feeling of tension, apprehension, and worry, set off by a particular combination of cognitive, emotional, physiological, and behavioral cues" (Benner, 1985, p. 65). Mathematics educators have had varying views of what anxiety is in relation to mathematics. Some view it as negative feelings or dislike for mathematics; others view it as a fear of mathematics. Spielberger (1972) divides anxiety into two types: state and trait anxiety. State anxiety is of short duration and is an acute reaction to some perceived threat. Trait anxiety is a long-term habitual emotional response to events of life. Educators categorize mathematics anxiety as state anxiety (Byrd, 1982). However, since a student's emotional state during problem solving may vary from positive to negative, it is likely that a student whose experiences have always been negative may have developed a permanent negative anxiety. Thus, in Spielberger's (1972) definition of types of anxiety, this kind of negative experience could develop into a trait anxiety. Buxton (1981) argues that this type of long-term negative reaction to mathematics is common, referring to it as a panic response.

In a related study of field-dependent and field-independent learners, Hadfield and Maddux (1978), using a sample of 481 students enrolled in mathematics classes at a large suburban high school, studied the relationship of these two cognitive styles, mathematics anxiety level, and mathematics achievement. They found that field-dependent students had significantly higher mathematics anxiety than did field-independent students.

One might expect the preceding studies of students' emotional states to have included measures of physiological change, but it has been unusual for research in mathematics learning to do so. Gentry and Underhill (1987), in their study of female college students, however, did use physical measurements of muscle tension and questionnaires to measure anxiety. Their study found little correlation between the two measures of anxiety.

Mathematics educators, according to Hart (1989), in considering beliefs, attitudes, and emotions, find them to be characteristics of individual students that are identifiable and related to the students' scores of mathematics achievement. They also can be used to predict the students' decisions to enroll in various mathematics classes and the students' differences in mathematical achievement related to race and gender. These beliefs, attitudes and emotions have been researched primarily by questionnaires and scales, and occasionally by individual student interviews. These processes need both to continue and to be better integrated in more research on mathematics learning (McLeod, 1989).

Summary

According to Resnick (1987), research has shown that learning does not happen by passive absorption alone; students approach learning with a background of knowledge, take in the new information, and construct their own meanings. Instructional methods, such as writing-to-learn, have the potential to make passive students into active students, actively constructing their own knowledge (Kenyon, 1988). Through the writing process, students gather and organize old and new knowledge and synthesize it into their own structure of knowledge (Nahrgang & Peterson, 1986). "As students write down, reflect on, and react to their thoughts and ideas, they enhance their executive problem-solving skills" (Kenyon, 1988, p. 8). Thus, self-monitoring or metacognition skills are improved.

Schoenfeld (1980) and Silver, Branca, and Adams (1980) suggest that instruction that emphasizes the metacognitive aspects of problem solving can be effective in helping students develop awarenesses of their own thought processes. There is strong evidence that, by making students aware of their own thought processes, learning can be enhanced. Silver (1982) and Schoenfeld (1983) have found that helping problem solvers reflect on their cognitive processes also helps to bring the students to an awareness of their emotional reactions to problem solving. Writing as an instructional tool has the potential to achieve that.

According to Connolly (1989), writing-to-learn is reflective and questioning. "The basic purpose is to help students become independent, active learners by creating for themselves the language essential to their personal understanding" (p. 6). Writing activities can be used to involve the students actively in analytical thinking and reflecting on their own learning (Miller & England, 1989). By writing and reflecting on their own mathematical experiences, the students become active learners. Thus, writing can have a significant impact on learners' cognition and metacognition (Powell & Lopez, 1989). One of the most important concepts for mathematics education has been expressed in the following statement: "Now we are beginning to realize that writing is not just the end product of learning; it is a process by which learning takes place" (Griffin, 1983, p. 121).

Bell and Bell (1985) recommend that all mathematics teachers use writing as a part of the daily routine. Hirsch and King (1983) recommend integrating writing into the mathematics class. Ganguli (1989), in research with college algebra students, found that an integrated writing program was more effective than the traditional program. He suggests that the integration of writing into mathematics instruction deserves much more attention, and that studies which examine the effects of such programs are needed.

The research that has been reviewed is consistently enthusiastic about the potential value of writing to learn mathematics. Existing formal research and informal studies done by teachers from elementary school to college consistently point toward both the advantages of using writing in the mathematics class and the need for further research in this area. Thus, this study was designed to investigate the effects of implementing an integrated writing program within a basic text of Algebra II. The primary research hypothesis was that students receiving the integrated writing program in Algebra II within the framework of the basic text would exhibit greater achievement than those students receiving the regular instruction in the basic text. The secondary research hypothesis was that those students receiving the integrated writing program would develop more positive attitudes about mathematics and about writing.

CHAPTER III
METHOD

The purpose of this study was to investigate whether a program integrating writing activities into a regular Algebra II curriculum would have a positive effect on the students' mathematics achievement and on the students' attitudes toward mathematics and writing.

Subjects

The sample for this study was selected primarily from the tenth and eleventh-grade students enrolled in the regular Algebra II classes at a private school in the southeastern part of the United States. The school is a private college-preparatory school with limited enrollment. It does not reflect the socio-economic conditions of the region in that the students are primarily from middle to upper-class families. The average student has Algebra I in the eighth grade, Geometry in the ninth grade, and Algebra II in the tenth grade. In some cases, as a result of entering the school after eighth grade or of not mastering the material in earlier years, the student may be in the eleventh grade when enrolled in Algebra II. There were four sections of Algebra II for the 1992-93 school year.

The enrollment in each section was approximately 15-20 students. The four classes assigned to the experimenter were used for this study. Two were used as experimental sections and two as controls. Due to normal attrition, a total of 68 students, 34 students in the experimental and 34 in the control sections, participated in the study. The two classes receiving the treatment were randomly selected by rolling a die. The two classes with the smaller number on the die were determined to be the experimental group. Table 1, on page 49, lists the demographics of the students involved in the study.

Experimental Design

This was a single-variable group design, involving one independent variable. Both the control and the experimental groups used the basic Algebra II text (Foster, Winters, Gordon, Rath, & Gell, 1992). The experimental group received a specifically designed integrated-writing program. The control group used the regular lesson format.

Since the classes receiving the treatment could be randomly selected, but the experimenter had no control over the procedure used to place the students in each class, this study was conducted as a quasi-experimental study with a non-equivalent control-group design. Pretests were given to all groups to determine if there were

Table 1

Demographics of Students Involved in the Study

Categories	Experimental Group N = 34	Control Group N = 34
Sex:		
Male	19	15
Female	15	19
Grade:		
Ninth	1	0
Tenth	26	26
Eleventh	7	8
American Students:		
Caucasian	26	31
African American	6	0
International Students:		
Irish	0	1
French	1	0
Spanish	1	0
Egyptian	0	1
Japanese	0	1

initial differences in the groups. The experimental groups received the treatment. All groups were administered regular end-of-chapter tests, a test midway through the experiment, and a posttest. Following the collection of each test, the students were asked to explain in writing how they had worked two specific predetermined problems. In addition, the students completed both a writing and a mathematics attitude scale at the beginning and conclusion of the study. Analysis of covariance (ANCOVA) was used to analyze the achievement data and t-tests were used to analyze the attitude data.

The primary research hypothesis was that students receiving the integrated writing program in Algebra II within the framework of the basic text would exhibit greater achievement than those students receiving the regular instruction in the basic text. The secondary research hypothesis was that those students receiving the integrated writing program would develop more positive attitudes in mathematics and in writing. All hypotheses were tested at the .05 level of significancy.

There were several major limitations to the study. A primary limitation was that the sample did not reflect the general population. An additional limitation was the size of the sample studied. As mentioned above, since the members of each class were preselected, a quasi-experimental research design had to be used. With this design, there were several threats to internal validity. Statistical regression may have been a problem, since there is a tendency for scores to regress toward the mean. Another threat to internal validity may have been selection interaction with preformed groups since the experimenter had no control of the ways the groups were formed. Another possible threat to internal validity may have been that the experimental group had an advantage over the control group because they were given instructions in writing. In order to control for this possible threat and to demonstrate that this threat was not pertinent, the experimenter administered a writing survey to all of the students in the study and all of the teachers at the school for the purpose of determining that all the students were actively involved in writing activities. Since the school operated on a six-day fully rotating cycle, no group was advantaged by having their class meet at a certain time each day.

Threats to external validity may include pretest-treatment interaction since the subjects may have reacted differently to the treatment because they were pretested. Experimenter effects may have also been a concern since the experimenter was directly involved in conducting the study. To control for this effect, the experimenter taped one lesson in each class in each of the four chapters covered to ascertain that the same, or very similar, instruction was given in all classes. Other possibly threats to external validity may have been the Hawthorne and John Henry effects. The Hawthorne effect may have been a threat since the subjects' behavior may have been affected just because they knew they were involved in the study. The John Henry effect may have been a factor since the control group may have felt in competition with the experimental group and thus tried to outperform the experimental group.

Material

The pretests used were the Sequential Test of Educational Progress III (STEP III) (ETS, 1980) end-of-course Algebra test, the Attitude Toward Mathematics Scale (Suydam & Crawford, 1986), and the Florida Writing Attitude Scale (O'Neal, Guttinger, & Morris, 1984).

STEP III has been described by reviewers Floden and Shanahan (1985) as technically sound and well designed. The tests were designed by Educational Testing Service (ETS). According to Floden and Shanahan, great lengths were taken to make the tests valid. Panels of teachers, administrators, and professors were assembled to study and determine the content of the tests. The tests were normed by using a stratified sample of the national student population in grades 1 through 12, with approximately 100 schools and 800 students sampled at each grade level in the fall and in the spring. The reliability of the Algebra test was reported to be .75. The Algebra test contained 30 items and the raw score error of measurement was reported as 2.5. This test was selected as a pretest because of its historical ties with the Comprehensive Testing Program (CTP) tests which have been used by the experimenter's school and were used for one of the posttests in this study. Research by ETS has demonstrated equating links between the STEP III and the CTP I and CTP II.

The Attitude Toward Mathematics Scale contained 26 items designed to ascertain, in a relatively short amount of time, how strongly the student likes or dislikes mathematics (Suydam, 1986). The reliability has been reported as ranging from .88 to .96 (Suydam, 1986). In determining the validity of the scale, the authors combined items developed for other scales with those developed by other mathematics education faculty, submitted them to a panel of judges, administered them to a group of preservice teachers, and revised them. The scale was then administered to groups of preservice teachers and to students, totaling approximately 3000.

The Florida Writing Attitude Scale was developed as a part of the Florida Writing Project. The Likert-type 25-item scale was designed for use with students from grade 6 to 12. The internal consistency of .83 was judged to be adequate for research purposes (O'Neal, 1984). Test-retest reliability has been reported at .80 for grade 10. In determining the validity of the scale, the authors modified items taken from other writing scales, revised them, and administered them to the approximately 600 students in the project.

The midtest and posttests used were constructed by the experimenter by combining questions taken from the Educational Records Bureau's (ERB) Comprehensive Testing Program II (CTP II), Level V, Form C, Algebra section, the Educational Records Bureau's (ERB) Modern Second Year Algebra Test, Form Y, and the multiple-choice forms of the chapter tests constructed by the publishers of the text.

ERB is a nonprofit organization of approximately 900 independent and suburban school members in the United States and overseas. The Comprehensive Testing Program II (CTP II) and the Modern Second Year Algebra Test consist of carefully constructed and standardized achievement tests. The tests were developed by the Educational Testing Service (ETS) of Princeton, New Jersey, working closely with ERB. The goal in the development of the tests was to tailor the test to meet the needs of the member schools. Content validity was achieved by a systematic test development process in which the member schools were actively involved by surveys, reviews, and evaluations in determining content of the tests (ERB Technical Information, 1983). Extensive norming was conducted, which resulted in norms being established for both independent and suburban schools. In addition, national norms were established by equating the test to a standardized achievement test, Sequential Test of Educational Progress (ETS, 1980). The internal reliability was determined by using Kuder-Richardson formula (KR20). The KR20 estimate for the algebra test was .88. The scaled-score standard error of measurement was reported as 2.2.

It was decided that there would be 30 items on both the midtest and posttest. Sixteen of the items covering material in Chapters 2-5, four from each chapter, would be common to both tests. The remaining fourteen items on each test would be items testing the material on Chapters 2 and 3 on the midtest and Chapters 4 and 5 on the posttest. From analyzing the test items in both ERB tests, it was evident that there were not sufficient problems to cover the material included in the chapters. There were only four items covering the material in Chapter 3 and no items covering Chapter 4. This material had to be tested using items from the multiple-choice forms of the chapter tests provided by the publishers of the text.

Additional data were collected by administering the free choice forms of the chapter tests at the end of each chapter. In order to control for experimenter bias and subjectivity in the correction of the chapter tests, the tests were corrected in the following manner. To give more objectivity to this correction process, all tests for each chapter were mixed together and corrected one problem at a time with a predetermined scale for determining any partial credit. The students' names were not noted until the correction process was complete.

At the conclusion of the study, the two aforementioned attitude scales were also administered. Following completion of the end of the chapter algebra tests, the midtest, posttest and collection of all the papers, the students were asked to explain in writing for two preselected items why their answers were correct and how they arrived at those answers. These items were selected to reflect the major ideas of the chapters.

The material used for the study was the Algebra II text (Foster, et al., 1992), together with specifically designed writing assignments that were integrated with the lesson for the experimental groups. The integrated writing assignments, together with the lesson plans for both the experimental and control groups, are included in the appendix.

Procedure

The four Algebra II classes at the school assigned to the experimenter were used for this study. The classes receiving the treatment were randomly selected. The pretests were administered to all the classes. During the study, all regular Algebra II students received instruction in Algebra using the Algebra II text (Foster, et al., 1992). The control groups received regular instruction as guided by this text; the experimental groups received regular instruction with an integrated program emphasizing language skills. The integrated program consisted of specifically designed activities involving writing-in-content for the purpose of making the students more actively involved in their own learning process and possibly assisting them in the "construction" of their own knowledge.

The program of intervention consisted of specifically designed, expressive or transactional writing assignments for each lesson. The writing assignments were developed to enhance the students' understanding of the concepts being studied and to help the students in seeing the structure of the content. The assignments, both in class and outside of class, resulted in fewer problems being assigned from the text so as not to burden the experimental group with excessive homework. The students at the school are usually assigned 45 to 60 minutes of homework for every class meeting for each subject.

In the beginning of the treatment, the students were given many prompts to help guide them through their writing assignments. The students were asked specific questions to help direct their writing. As they became more skilled and more comfortable with their writing in mathematics, the prompts diminished. For example, the first few times they were asked to summarize, they were given an outline of topics to include. As their skills improved, these prompts were faded. These assignments were evaluated daily for correction and feedback.

The Chapter Tests were administered at the end of each chapter, the midtest at the end of two chapters, and the posttest at the conclusion of the experiment. The experiment was conducted over four chapters of the text, which corresponded to twelve weeks of the school year. The chapters included: linear relations and functions; systems of equations and inequalities; matrices; and polynomials.

Permission was requested from and granted by both the experimental and the control groups and their parents or guardians to take part in the study. The school administration was supportive of the study.

CHAPTER IV
RESULTS

This study investigated the effects of implementing an integrated, experimenter-designed writing program within an existing basic text of Algebra II. Subjects came from the four sections of regular Algebra II taught by the experimenter at a private school in the southeastern part of the United States during the 1992-93 school year. The enrollment in each section was approximately 15-20 students. For this study, two sections were used as the experimental group and two as the control group.

Due to normal attrition, a total of 68 students, 34 in the experimental group and 34 in the control group, participated in the study. Partial data collected from the following students were not used in the study: one student who had been hospitalized since early in the study; one student who returned to Algebra I; one Korean student new to the country with limited English; and four students who changed from Honors Algebra II to regular Algebra II and joined the classes at different points in the study.

Data for the study were collected in several ways. (See Chapter 3 for details.) At the beginning of the study, each student was given the preliminary algebra test, the writing attitude scale, and the mathematics attitude scale. During the study, the students were given the appropriate chapter tests. After completing the first two of four chapters, the students were given the midtest. At the conclusion of the study, the students were given the posttest, the writing attitude scale, and the mathematics attitude scale. After papers had been collected for each of the chapter tests, the midtest, and the posttest, the students were asked to explain in writing how they worked two preselected items. These writing sample items were scored using a modified holistic method, which is described in Appendix E.

Analysis of Data

Means and standard deviations for the pretest, midtest, posttest, individual chapter tests, and chapter test average are presented in Table 2. Means and standard deviations for the writing samples are presented in Table 3. Means and standard deviations of the attitude scales are presented in Table 4.

The primary hypothesis was that students receiving the integrated writing program in Algebra II within the framework of the basic text would exhibit greater achievement than those students receiving the regular instruction in the basic text. The secondary hypothesis was that those students receiving the integrated writing program would develop more positive attitudes in mathematics and in writing. All hypotheses were tested at the .05 level of significance.

Table 2

Descriptive Statistics for Mathematics Achievement Tests

		Experimental Group (N = 34)		Control Group (N = 34)	
Tests	Maximum Score	Mean	SD	Mean	SD
Pretest	30	14.88	3.18	14.71	4.40
Midtest	30	13.29	2.76	12.38	3.83
Posttest	30	15.68	3.24	14.38	2.79
Chapter 2 Test	100	83.47	11.11	78.59	12.09
Chapter 3 Test	100	82.44	15.02	73.97	12.09
Chapter 4 Test	100	93.62	10.77	90.85	9.37
Chapter 5 Test	100	84.97	9.24	77.26	10.38
Chapter Test average	100	86.12	8.70	80.38	8.04

Table 3

Descriptive Statistics for Writing Samples

Samples	Maximum Score	Experimental Group (N = 34) Mean	SD	Control Group (N = 34) Mean	SD
Midtest	10	3.82	2.33	3.44	2.11
Posttest	10	7.18	2.12	6.35	2.73
Mid/posttest total	20	11.00	3.50	9.79	3.79
Chapter 2 Test	10	7.09	2.50	5.77	1.67
Chapter 3 Test	10	6.53	2.94	4.03	3.19
Chapter 4 Test	10	7.35	1.95	5.59	2.23
Chapter 5 Test	10	6.85	2.56	4.59	2.38
Chapter Tests total	40	27.77	7.65	19.94	6.93
All Tests total	60	38.77	10.01	29.74	9.69

Table 4

Descriptive Statistics for Attitude Scales

	Experimental Group (N = 34)		Control Group (N = 34)	
Scales	Mean	SD	Mean	SD
Writing-Pre	80.68	12.96	77.88	12.93
Writing-Post	80.08	11.61	78.94	12.84
Mathematics-Pre	84.91	13.84	79.85	16.37
Mathematics-Post	85.29	16.78	80.15	16.03

For the mathematics achievement test data and the writing sample data, analysis of covariance (ANCOVA) was used to test the differences between the groups. The scores on the achievement tests and writing samples were used as the dependent variables and the pretest was used as the covariate. There were 16 separate ANCOVAS. Table 5 lists the results of the ANCOVA for the midtest and Table 6 lists results of the ANCOVA for the posttest.

Table 5

Analysis of Covariance Between Groups for the Algebra Midtest

Source	df	F	p
Treatment	1	1.639	0.205
Covariate	1	20.258	0.000
Error	65		

Table 6

Analysis of Covariance Between Groups for the Algebra Posttest

Source	df	F	p
Treatment	1	3.216	0.078
Covariate	1	12.177	0.001
Error	65		

Tables 7-10 list the results of the ANCOVAs for the four chapter tests used in this study. Table 11 lists the results of the ANCOVA for the average of the chapter tests.

Table 7

Analysis of Covariance Between Groups for the Chapter 2 Test

Source	df	F	p
Treatment	1	3.188	0.079
Covariate	1	4.038	0.049
Error	65		

Table 8

Analysis of Covariance Between Groups for the Chapter 3 Test

Source	df	F	p
Treatment	1	5.586	0.021
Covariate	1	1.450	0.233
Error	65		

Table 9

Analysis of Covariance Between Groups for the Chapter 4 Test

Source	df	F	p
Treatment	1	1.401	0.241
Covariate	1	0.277	0.635
Error	65		

Table 10

Analysis of Covariance Between Groups for the Chapter 5 Test

Source	df	F	p
Treatment	1	9.568	0.003
Covariate	1	1.523	0.222
Error	65		

Table 11

Analysis of Covariance Between Groups for the Average of Chapter Tests

Source	df	F	p
Treatment	1	8.121	0.006
Covariate	1	2.260	0.138
Error	65		

As can be seen from Tables 5 and 6, no significant difference was found between the experimental group and the control group for the midtest and posttest.

The results of the ANCOVAs on the individual chapter tests (Tables 7-10) are mixed. On the Chapter 2 and Chapter 4 tests, no significant difference was found. On the Chapter 3 and Chapter 5 tests, the experimental group performed significantly higher than the control group. The results of the ANCOVA for the average of the chapter test (Table 11) indicate that the experimental group performed significantly higher than the control group. The results of the ANCOVAs for writing samples are listed in Tables 12-20. Again, the pretest was used as the covariate.

Table 12

Analysis of Covariance Between Groups for the Midtest Writing Sample Scores

Source	df	F	p
Treatment	1	0.543	0.464
Covariate	1	6.041	0.017
Error	65		

Table 13

Analysis of Covariance Between Groups for the Posttest Writing Sample Scores

Source	df	F	p
Treatment	1	1.903	0.172
Covariate	1	0.157	0.693
Error	65		

Table 14

Analysis of Covariance Between Groups for the Sum of the Midtest and Posttest Writing Sample Scores

Source	df	F	p
Treatment	1	1.916	0.171
Covariate	1	3.014	0.087
Error	65		

Table 15

Analysis of Covariance Between Groups for the Chapter 2 Test Writing Sample Scores

Source	df	F	p
Treatment	1	6.587	0.013
Covariate	1	1.101	0.298
Error	65		

Table 16

Analysis of Covariance Between Groups for the Chapter 3 Test Writing Sample Scores

Source	df	F	p
Treatment	1	11.481	0.001
Covariate	1	1.932	0.169
Error	65		

Table 17

Analysis of Covariance Between Groups for the Chapter 4 Test Writing Sample Scores

Source	df	F	p
Treatment	1	11.880	0.001
Covariate	1	0.075	0.784
Error	65		

Table 18

Analysis of Covariance Between Groups for the Chapter 5 Test Writing Sample Scores

Source	df	F	p
Treatment	1	14.161	0.000
Covariate	1	0.370	0.545
Error	65		

Table 19

Analysis of Covariance Between Groups for the Sum of the Chapter Tests Writing Sample Scores

Source	df	F	p
Treatment	1	19.533	0.000
Covariate	1	1.071	0.305
Error	65		

Table 20

Analysis of Covariance Between Groups for the Sum of the Writing Sample Scores

Source	df	F	p
Treatment	1	14.478	0.000
Covariate	1	1.985	0.305
Error	65		

As can be seen from Tables 12-20, no significant difference was found between the experimental and control group on midtest, posttest, or combined mid and posttest. For the individual chapter tests, the combined chapter tests, and the total writing samples, the experimental group performed significantly higher than the control group.

The attitude scales were analyzed in several ways. Paired samples t-tests were used to compare each group's pre and post attitudes in both writing and mathematics. Independent samples t-tests were used to compare the pre and post attitudes between groups for both writing and mathematics. Table 21 lists the results of the paired samples t-tests and Table 22 lists the results of the independent samples t-tests.

Table 21

Paired Samples T-tests Comparing Pre and Post Attitudes of Each Group in Writing and Mathematics

		Pre Attitude	Post Attitude	t	p
			Experimental Group		
Writing					
	Mean	80.676	80.088	-0.364	0.718
	SD	12.963	11.613		
Mathematics					
	Mean	84.735	85.294	0.336	0.739
	SD	13.940	16.781		
			Control Group		
Writing					
	Mean	77.882	78.941	0.705	0.486
	SD	12.935	12.839		
Mathematics					
	Mean	79.853	80.324	0.293	0.771
	SD	16.365	16.071		

Table 22

Independent Samples T-test Comparing Experimental and Control Groups' Attitudes Before and After the Experiment

Attitude Scale		Experimental	Control	t	p
		Writing			
Pre:	Mean	80.676	77.882	0.890	0.377
	SD	12.963	12.935		
Post:	Mean	80.088	78.941	0.386	0.700
	SD	11.613	12.839		
		Mathematics			
Pre:	Mean	84.735	79.853	1.324	0.190
	SD	13.940	16.365		
Post:	Mean	85.294	80.324	1.247	0.217
	SD	16.781	16.071		

As can be seen from Table 21, no significant difference was found between the pre and post attitudes of either groups and as can be seen from Table 22, no significant difference was found between the attitudes of the experimental group and the control group.

CHAPTER V
SUMMARY AND CONCLUSIONS

According to Resnick (1987), research has shown that learning does not happen by passive absorption alone; students approach learning with a background of knowledge, take in the new information, and construct their own meanings. Instructional methods, such as writing-to-learn, have the potential to make passive students into active students, who can productively construct their own knowledge (Kenyon, 1988). Both formal research and informal studies have consistently pointed toward both the advantages of using writing in the mathematics class and the need for further research in this area.

Summary of the Study

This study investigated the effects of implementing an integrated, experimenter-designed writing program within an existing basic text for Algebra II. The study took place at a private school in the southeastern part of the United States during the 1992-93 school year. The experimenter taught the four sections of Algebra II used for this study. Enrollment in each section was approximately 15-20 students. For this study, two sections were used as the experimental group and two as the control group. Sections were randomly assigned to treatment groups. Due to normal attrition, a total of 68 students, 34 in the experimental group and 34 in the control group, participated in the study.

Both the control and experimental group received the same instruction using a basic Algebra II text. Writing activities, both transactional and expressive, were integrated within the experimental group's lessons. Data for the study were collected in several ways. At the beginning of the study, each student was given a preliminary algebra test, a writing attitude scale, and a mathematics attitude scale. During the study, students completed the appropriate chapter tests. After completing study of the first two of four chapters, students were given the midtest. At the conclusion of the study, students were given the posttest, the writing attitude scale, and the mathematics attitude scale. Following each of the chapter tests, the midtest, and the posttest, students were asked to explain in writing how they worked two preselected items. These explanations were scored holistically.

The primary hypothesis was that students receiving the integrated writing program in Algebra II within the framework of the basic text would exhibit greater achievement than those students receiving the regular instruction in the basic text. The secondary hypothesis was that those students receiving the integrated writing program would develop more positive attitudes towards mathematics and writing. All hypotheses were tested at the .05 level of significance.

For the mathematics achievement test data and the writing sample data, analysis of covariance (ANCOVA) was used to test the differences between the groups. The scores on the achievement tests and writing samples were used as the dependent variables and the pretest was used as the covariate.

The results for the algebra performance tests were mixed. For the midtest, posttest, Chapter 2 test, and Chapter 4 test, no significant difference was found between the experimental group and the control group. On the Chapter 3, Chapter 5, and Chapter Test average, the experimental group performed significantly higher than the control group.

ANCOVA was also used to analyze the data on the writing samples. Again, the results were mixed. No significant difference was found between the experimental and control group on midtest, posttest, or combined mid and posttest. For the individual chapter tests, the combined chapter tests, and the total writing samples the experimental group performed significantly higher than the control group.

The attitude scales were analyzed in several ways. Paired samples t-tests were used to compare each group's pre and post attitudes in both writing and mathematics. Independent samples t-tests were used to compare the pre and post attitudes between groups for both writing and mathematics. No significant differences were found between the pre and post attitudes of writing or mathematics for either group. No significant differences were found between the experimental and control groups.

Conclusions

Although there was insufficient evidence to support the hypothesis, the results of the midtest and posttest show a trend. The difference between the means of the experimental group and the control group grew from .17 on the pretest, to .91 on the midtest, to 1.30 on the posttest. The pretest, midtest, and posttest each had a maximum score of 30. The test statistic was 1.639 for the midtest and 3.216 for the posttest, which approached the critical value of 4.0. This trend in the data suggests that a longer study might have resulted in the experimental group performing significantly higher than the control group on an achievement measure.

The chapter test data also demonstrated a distinctive trend; the differences between the means of the experimental group and control group were 4.88 on the Chapter 2 test, 8.47 on the Chapter 3 test, and 7.71 on the Chapter 5 test. The maximum score on the chapter tests was 100. The test statistic was 3.188 for Chapter 2, 5.586 for Chapter 3, and 9.568 for Chapter 5. The data, both from the algebra tests and the writing samples, suggest that the writing activities had a cumulative positive effect on the mathematical achievement of the experimental group and that with a study of longer duration, the hypothesis might have been supported.

However, one cannot discount that many factors may have been involved in this widening of differences between the groups. Since the students were aware of the purpose of the study, for example, the experimental groups' achievement may have been influenced by the Hawthorne effect.

The smaller difference in the means for the Chapter 4 test, 2.77, was probably the result of a chapter test that was too easy for all the classes, and thus there was a ceiling effect. Chapter 4 was on matrices, which all the students enjoyed and on which they felt very much at ease with their understandings of the concepts. The means for both groups on this test were substantially higher than on other tests.

For the writing sample data, the differences between the means of the experimental and control groups also demonstrated a trend. On the midtest the difference was 0.38, and on the posttest the difference was 0.83. In the chapter tests, the differences between the means were 1.32 on Chapter 2 writing samples, 2.50 on Chapter 3 writing samples, 1.76 on Chapter 4 writing samples, and 2.26 on Chapter 5 writing samples. The test statistic for the writing samples also increased from 0.543 on the midtest to 1.903 on the posttest. On the chapter tests writing samples, the test statistic increased from 6.587 on Chapter 2, to 11.481 on Chapter 3, to 11.880 on Chapter 4 , and finally to 14.161 on Chapter 5. This trend suggests a potential cumulative effect for the writing samples.

It is possible that the writing-to-learn activities were more effective on the chapter tests than on the mid and posttests because the material covered by the chapter tests more closely correlated with the material the students were currently studying and writing about. In addition, the free-response type format of the chapter tests, as opposed to the multiple-choice type format of the midtest and posttest, required the students to work out the problems and show their work, which forced them to be more mindful of their procedures. By comparison, the midtest and posttest, which had been constructed by selecting items primarily from ERB Algebra tests and supplemented by multiple-choice items from the basic text's evaluation manual, resembled standardized achievement tests. These multiple-choice items were written with traditional instruction in mind, and might have been biased toward the control group who had not been concentrating on explaining their procedures.

These differences in the types of tests may also be related to the differences in the data collected from the writing samples. In the standardized-type format of multiple-choice items, the students were not forced to describe the process they used to arrive at an answer and at times resorted to guessing. In such cases, they may have had a more difficult time later in explaining their answers. Another factor involved in the difference between the results of the writing samples from the midtest/posttest and chapter tests may have been that the writings the students had been involved in for each chapter were so recent that they may have felt more secure with expressing their thoughts. By comparison, the writing sample items on the midtest/posttest may have been less recent and the students may have been less sure of their explanations. The writing samples for the midtest were especially low because the second problem they had to respond to was a linear-programming problem at the end of the test. Many of the students did not get that far on the test and since the problem was more difficult than many others, some opted not to do it.

In the analysis of the attitude scales, no significant differences were found between the experimental group and the control group. In addition, there was no significant differences found between the pre attitude and post attitude of either group. A brief background of these classes may help to explain these results and the stability of these measurements. Since the experimenter had taught the majority of the students the year before either in Honors or Regular Geometry, the classroom routine was not changed substantially. Furthermore, the majority of the students were well motivated and were from homes in which education is valued highly. The students were used to a demanding curriculum requiring a large amount of writing. An informal survey of both the students and the teachers substantiated the amount of writing done by the students. The students and the teachers estimated that, on average, the students spend approximately 5 hours per day in writing activities. The 5-hour estimate for the teachers was arrived at by totaling the time spent on writing as estimated by each department. The 5-hour estimate for the students was arrived at by averaging their individually estimated times. The faculty's estimates ranged from the student spending 10 minutes per day on writing in the mathematics department to 75 minutes per day in the history department. The students' estimates ranged from spending 2 to 9 hours per day on writing activities. The results of these surveys are included in Appendix G.

Personal Reflections

The students in all classes were excited to be included in this study and all cooperated willingly. Because the experimenter had taught most of the students the previous year, there was an environment of trust already established, so the students were comfortable expressing their concerns and questions to the teacher. In the experimenter's judgment, in this atmosphere of open expression, an important additional advantage of using writing in the mathematics class was the regular feedback it gave to the teacher from all of the students. The teacher had the opportunity daily to have students' reaction to the teaching procedures, so there was constant adjustment to the needs of the students. Thus, misconceptions and confusions that were obvious from reading the students' writings could be used as a signal to modify instruction for all the students. For example, early in the study, it was obvious from their writings that at least two students were confused about reversing inequality signs when solving inequalities. The students thought that, if there were a negative sign anywhere in the inequality, the inequality had to be reversed. Had it not been for the regular writing activities, this confusion might not have been noted for some time. Another misconception that was discovered from the writings was a confusion about slope. One student thought that the slope of a line either perpendicular or parallel to a given line was the negative reciprocal. Another student thought that, since parallel lines were inconsistent, two lines had to be perpendicular to be classified as independent and consistent. One other student confused slope and equation, stating, "The equation of a vertical or horizontal line is undefined and 0."

Closely related to the advantages of feedback was the improved communication between student and teacher which resulted from the use of writing in the mathematics class. This was especially beneficial in this study for the quiet student and, more particularly, for the quiet struggling student. One such student, in reflecting back about the writings she had done, said:

> The writing exercises did help me because they help me communicate with you and it let you know how I felt about the material. When I had to write down what I learned it helped to reinforce the material and my understanding of it.

From reading the writings of students, the experimenter gained insights into their difficulties. For example, there were several struggling students whose difficulties were far better (or perhaps, for the first time) understood from reading their writings than had been possible in working with them in individual oral sessions or in interviews during the previous year. From grading these students' quizzes and tests, it would have appeared that they knew very little of the mathematics. However, from reading their writings, it was obvious they knew much more about the subject than showed on tests. It was usually minor flaws in their reasoning or simple misconceptions that were causing them trouble. One such student wrote the following summarizing the chapter on matrices:

> I learned that a matrix is a system of row and columns. It varies in dimensions. It can be 3 X 2, 2 X 2, 3 X 3, 1 X 3, etc. I also learned that two matrices are equal only if they have the same dimensions and their corresponding elements are equal. I learned how to multiply scalar multiplication and to add matrices.

The above dialogue, together with correct illustrations of all the procedures, continued as the student summarized the rest of the chapter. One other student who is not always as successful on tests as he would like to be wrote the following about linear equations:

> We have come to find that when given two points, you can substitute into the formula $\frac{y_2 - y_1}{x_2 - x_1}$ the coordinates for the points. Lines that are parallel are going to have the same slope, but perpendicular lines have the negative reciprocal slope. We can determine the equation of a line when given one point and the slope by putting the given numbers into the $y = mx + b$ formula.

The following examples of the students' reflections on the mathematical content they were studying demonstrated, in part, their understandings of the concepts.

> A linear equation is an equation whose graph is a line. A linear equation is identifiable by its containing one or two variables with no variable having an exponent other than one.
>
> To find the slope, you take the difference in y over the difference in x. That equals the slope. Then, to find the y-intercept, you take a set of points and the slope and put them into the equation $y = mx + b$, then figure out "b" which represents the y-intercept. Once you have done all that, just place the values into the equations; $y = mx + b$ and $Ax + By = C$.
>
> In graphing a line, solve the equation for y. So for example, if you have the equation $3y = -2x - 6$, divide the equation by 3. Now you have $y = -(2/3)x - 2$. Now to graph it, you would start at -2 on the y-axis and go over 3 and down 2 and connect the points.
>
> To find out if the lines are perpendicular, parallel, or neither, you only need the slope. If the first slope is 3/5 and the second slope is - 5/3, so the product is -1, so they are perpendicular.
>
> Some special functions are direct variation, constant, and identity functions. A linear function in the form of $y = mx + b$ where $b = 0$ and m is not = 0, is called a direct variation. If $b = 0$ and $m = 1$, then it is an identity function. If $m = 0$, it is a constant function.
>
> Lines that intersect have different slopes and are consistent and independent. There is only one solution for lines that intersect. Lines that coincide have the same slope and the same intercepts. These lines are consistent and dependent and have infinite solutions. Lines that are parallel have the same slope and different intercepts. They are inconsistent and have no solution.
>
> In this chapter, we have learned several ways to solve systems of equations, some hard and some easy. We learned to solve by graphing. If you graph the two equations, then you could find

where they intersect. For solving systems of equations by substitution, you would solve one of the equations for one of the variables, then you could solve the rest by plain algebra. For elimination, you get one of the variables to cancel out, then you can solve for the other. Next you just substitute to find the first variable.

To solve three equations with three variables, I took two of the equations and eliminated the z, then I chose one of the equations I had already used and used the one I had not used. I eliminated the z again and had two equations for x and y. I then eliminated again for x and solved for y. I then substituted for y and got what x equaled. I then substituted for x and y in the original equation and solved for z.

To solve a linear programming problem, you must first define the variables. You must incorporate these into inequalities. You would then be able to graph these and then you should be able to see the feasibility region. It will look like some sort of polygon. You will then need to record the coordinates of the vertices that make this polygon. You will then need to determine the maximum or minimum expression. Then substitute the coordinates into the expression and find the answer.

When you have two matrices and you need to combine them through multiplication, first you have to make sure the matrices can be multiplied. The first matrix (A) must have the same number of columns as the second matrix (B) has rows. Next you multiply the first row of A by each column of B. Then you add each product of the first number of the first column with the second number of the second column. Now you do the same as with the first row, but with the second row. After all the steps are completed you should have a new matrix.

To evaluate the three by three determinant, I used the diagonal method and copied the first two columns of the matrix to the right side. I drew diagonal lines going from top to bottom, multiplied each row, then added them together. I then drew diagonal lines going from bottom to top, multiplied them together, and then subtracted them.

> To solve a system of equation as a matrix equation, the first thing I did was write the system as a matrix equation. Then I found the inverse of the first part and multiplied both sides of the equation by it. I checked my answer by substitution to make sure it was right.

> To multiply three binomials together, you take the first two factors and multiply. What you get from that, you multiply by the next factor. Then you combine alike terms.

The first five of these quotes of the students' reflections on the mathematical content demonstrate that the students had a sufficient basic understanding of the concepts to define and identify a linear equation, to find the slope and the equation of a line given two points, to graph a line when in slope-intercept form, to determine if two line are parallel, perpendicular, or neither, and to identify some special linear functions.

The next three quotes pertaining to systems of linear equations demonstrated that the students also had a good understanding of the terms inconsistent, consistent and dependent, and consistent and independent, and of the processes used to solve systems of two and three equations. The quote on linear programming demonstrated that the student was aware of the process and complex procedures necessary to solve problems of this type.

The three quotes on matrices and determinants demonstrated the students' understandings of the complex processes used in some matrix operations and in evaluating three-by-three determinants.

The final quote, which explains the process used to multiply three binomials together, shows a good understanding of this process.

In writings such as these, the students are forced to use mathematical terminology and to be precise with their explanations. By taking a process and explaining it in their own words, the students make the particular process a part of their own knowledge and skills. Furthemore, reading writings such as these gave the experimenter one additional way to evaluate the students' understanding. Studying the students' explanations as well as their algebraic and numerical work gave the experimenter a much clearer picture of the concepts the students were, or were not, understanding.

The following quotes are the reflections of two students on the relationship between two chapters studied.

> I feel that the relationship between chapters 2 and 3 is very evident and needed. Without the information and tactics that are learned in chapter 2, there wouldn't be anyway that someone could fully understand chapter 3. Concepts like the coordinate system, linear equations, slopes and intercepts are vital to the understanding of chapter 3. An example of this would be the following. To be able to do Linear Programming, you must be able to use many things learned in chapter 2.

> Chapter 2 is a section that's purpose is not to show us ways to solve problems; rather its purpose is to show us techniques that can be used to solve equations. This chapter is like giving us the raw materials to make up an engine, but not showing us how to put it together. Chapter 3 does not serve the same purpose. Chapter 3 explicitly teaches us how to solve problems. With the "raw materials" we learned in Chapter 2, we are now able to

put it all together to solve problems.

The preceding two quotes were examples of the type of increase in metacognitive skills that the practice of writing in the mathematics class encouraged. Both of these quotes were from boys who have matured greatly as students this year as they have taken increased responsibility for their own learning.

Several students who had worked very hard in mathematics, both in geometry last year and in algebra this year, without being as successful as they would have liked, were convinced that the writing activities helped them. Two of these students were, by nature, very quiet, so that their writings were especially helpful to the teacher. One of these, who otherwise might never have had the willingness to so respond, said:

> I think I have done well, considering that I am not very good in math. I feel that all of the writing we did helped me and by my explaining the problem, it gave me more of an understanding for it. Then again, writing is just a technique I learn by well.

Another student stated:

> The writing has helped me a great deal in understanding the problems.

A third, an energetic student who is very good at asking questions in class and at extra-help sessions and is determined to master the material, said:

> The writing our class did during the 4 chapters in the book, in my point of view, helped me <u>extremely</u>. The writing helped me to understand the material too.

One other category of the writing activities that the experimenter found useful was that in which the students were to express their feelings or thoughts about certain topics. This personal written communication between the teacher and student was very beneficial to both. Insights into the concerns the students were having provided the teacher with valuable information to use in addressing all the classes. The following responses were typical answers to a question asking about concerns or problems with the current chapter:

> This chapter (Matrices) has been relatively easy for me until the inverses came into play. I definitely need extra help on that.

> I don't like graphing especially when there are 4 or 5 lines to graph. It gets confusing. I think Cramer's Rule will be helpful later, but it takes too much time.

> This has been the best chapter so far for me. Not only is it the easiest, but I understand it, and it's fun.

> I like this chapter in the sense of it is pretty neat. I do not like how you have to know more than one way to do things. The last couple of chapters have slammed me because they were impossible.

> I think this chapter is very hard. We need to spend a lot of time on this chapter and take it slow.

> I think this chapter is fun and Cramer's Rule is interesting.

> This chapter has its ups and downs. I like everything except for the evaluation of matrices. I am scared for the test.

It was obvious to the experimenter that as the students were progressing, they were becoming more aware of their particular learning style. The students wrote the following about their studying for a test:

> I felt that I studied well and knew the material thoroughly but failed to apply myself on the test.
>
> I was not happy at all with the way I studied for the test. I would have done more problems and looked over the sections I had difficulty with, if I could do it again.
>
> I think my strategy was good and effective. But if I were to change it I would spend more time on understanding the individual sections.
>
> I felt I studied effectively, but I waited to the last minute to start studying.
>
> I reviewed problems with a friend the day before. I would study 3 days before and go to the Math Center on problems I needed help with.
>
> I studied for the last test, both as we went along and the night before. As we covered the sections, I read the information before doing the problems. Then the night before I went over the chapter test. I feel that my strategies were good, but as a way to help myself, I think I could do more problems when I don't understand one of that type.
>
> For the last test, my buddy and I got together to study. We went over the review, one section at a time. If there was something we did not understand, we would stay working on it or ask for help. The next morning, we went to the Math Center and cleared up

> everything we couldn't clear up ourselves. This was an effective way to study for the test.
>
> I thought my strategies were good but I didn't do well on the test so obviously they weren't. I have no idea what else to do. Before the test I know everything, but during the test my mind gets blank and I get nervous.

In considering the advantages the writing program had, it would be a mistake not to mention two individual students who were directly and positively affected by the writing. One of these students had been having difficulty with test taking in most of his classes for several years. His teachers, advisor, and parents had all been concerned. During discussions in his classes, both mathematics and others, it was obvious that he was comfortable with and knowledgeable about the material. Then he would take the tests and not do well. From reading his writings, it was obvious to this researcher and then to others that he had a very strong understanding of the concepts. In fact, his writings showed a depth of understanding equal to or even surpassing the strongest students in the class. With this additional insight into this student, his advisor and teachers were better equipped to work on the problems of how to assist him in his test taking skills.

The other student that should be mentioned is a particularly shy young man, who had not said a word out loud in mathematics class for the past year and a half. The classroom environment has always been relaxed and non-threatening, and the students have been encouraged to ask questions and not to feel that any question is too trivial to ask. In other words, the classroom has not been a formal one in which a student would fear making a mistake or asking a "dumb question." This very quiet student wrote some of the best, well-organized, and most insightful papers that were received. By reading his writings, the teacher was finally able to realize what a talented student he was. For a year and a half, the teacher had been trying to break through to this young man, to communicate to him and to get him to communicate in return. Since the writing exercises, this student has ventured out of his quietness to come on several occasions to the Math Center to ask the teacher questions of concern to himself, out loud, although very quietly.

Several weeks following the close of the study, as the students were writing comments on their first semester in Algebra II, they were all asked to reflect on the writing they had done and to express their thoughts thereon. All but 3 out of the 34 stated confidently that the writings helped them. Some typical comments were:

> I feel that writing did help me learn Algebra better. When I wrote about a certain topic in math, I found out if I understood it or not.
>
> I feel that the writing helped me organize what we were learning,
>
> The writing assignments keep the ideas going in my head. It also allows me to review the chapter before the test.
>
> I think the writing assignments helped alot. It's been said that writing stuff down helps you alot, so if you have to remember a process, writing it down in the assignments will help.
>
> The writing assignments helped me to see what exactly I didn't know or wasn't sure of.

> Since we stopped writing regularly, it has been hard for me to organize my thoughts.
>
> Most of my high test scores came when we were in the experiment. Writing definitely helped me to organize my thoughts.
>
> The writing helped me and when we decreased the writing, I stopped doing as well on tests and quizzes. Please continue writing.

Implications

From conducting and being a part of this study, it has become clear to the experimenter that the use of writing in the mathematics class has many advantages both for the students and the teacher. A primary advantage is communication--communication about learning and communication about feelings. The second standard advocated by the NCTM is that students should learn "to communicate mathematical ideas so that all students can: reflect on and clarify their thinking about mathematical ideas and relationships" (NCTM, 1989, p. 140). By having students explain their understandings, insights were obtained into the ways they were learning or not learning and teaching strategies could be developed or altered. By having students confide their concerns, insights could be obtained into their feelings and these could be addressed.

The benefits to the students were many. As the students were writing, they were actively involved in constructing their own knowledge and making it more personal. They were able to organize and to reflect on what they were learning. For many, this organization and reflection increased their knowledge of mathematics. Many students in both the experimental and control group realized as they wrote their explanation to a writing sample item that they had made a mistake when initially working the problem. By taking the time to write about the problem, the students understood the problem better.

The writing also benefited the students therapeutically as they expressed their feelings, attitudes, and concerns. The students felt that the teacher cared about them individually and was concerned about their needs as a person as well as their needs regarding mathematics.

The benefits to the teacher were also numerous. By reading the students' writings, the teacher gained insights into the students' understandings and misunderstandings. Immediate feedback could be given to the classes in the form of reteaching or reexplanations, paying special attention to topics that had been misunderstood. The teacher was better able to evaluate the students' progress by reading their writings. By only looking at a quiz or a test, the teacher had not been able to evaluate as effectively what the student did or did not understand.

By reading the students' expressions of concern, the teacher was better able to address the needs of the students. If the students were concerned about the pace of the class, frustrated with their understanding of a certain concept, or annoyed by a person sitting next to them, their concerns were made known through their writings and thus could be addressed.

Based on the data collected from this study, it seems that the students in the experimental group performed somewhat better than the control group on the chapter tests and the writing samples from the chapter tests. Furthermore, the data suggest that when students are tested on the material they are taught and tested in a similar format, they improve by using writing as a tool for learning. The trend in the data suggests that the experimental group might have performed significantly higher than the control group on the posttest had the experiment lasted longer. From the data collected, it would appear obvious that the use of writing-to-learn mathematics is potentially a valuable tool for both the students and teachers and that it should not be overlooked by the mathematics teacher.

Although the attitude scales did not demonstrate a significant change for the experimental group, 91% of the students in the experimental group reported that they enthusiastically supported the writing activities. Interestingly, of the three students who did not believe the writing helped, two have been diagnosed as having some serious learning difficulties. This suggests that further study of students with various learning difficulties may be necessary to determine if writing is beneficial to them. Interestingly, the two students diagnosed as learning disabled scored one and two standard deviations below the mean on the preliminary writing attitude scale. Following the study, they scored only one and two points below the mean. Both of these students scored one standard deviation below the mean for both the preliminary and final mathematics attitude scale. The third student scored two standard deviations below the mean for both the preliminary and final writing attitude scale. On both the preliminary and final mathematics attitude scale, he scored nearly one deviation above the mean. The results of this study suggest that writing-to-learn mathematics can be a valuable tool for both students and teachers of mathematics, but more research is needed to determine the effectiveness of the procedure for some students.

Recommendations for Further Research

Since this study was conducted with only one teacher and with a small sample that may not reflect the general population, it would be desirable to have this study replicated with a larger and more representative sample and with more than one teacher participating. Since the trend in the data suggests that a significant difference between the groups on the posttest may have been obtained had the study been of longer duration, replicating this study for a longer period of time appears advisable. Since much of the previously published research has been done with college level students and since the second standard advocated by the NCTM on increasing communication in mathematics is for grades K-12 (NCTM, 1989), it would also be good to see a similar study effected using other mathematics classes and other grade levels, especially lower levels.

Also, as mentioned above, since the particular students in this study who had been diagnosed as learning disabled did not believe writing helped them, research in writing-to-learn could be carried out with students with learning disabilities. It could be that, for students with some learning difficulties, the burden of writing may compound the students' problems and prove to be an additional obstacle for them. Future studies of writing-to-learn mathematics could well consider having paragraph or essay writing as a regular part of mathematical testing in combination with traditional problem solving.

It might prove interesting to explore the relationships between students with a positive attitude toward writing and their achievement in a writing-to-learn mathematics program and students with a negative attitude toward writing and their achievement in the same program. Similarly, a study involving students with positive and negative attitudes in mathematics could be explored. Instead of using previously written attitude scales, instruments could be specifically constructed to measure affective factors the experimenters considered related to the study.

From the previously published research and from this study, the experimenter has concluded that writing-to-learn mathematics is a tool that can be used effectively for most students. Further research is necessary to understand more precisely what components of writing have the greatest influence on achievement.

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Appendix A

Algebra II Lesson Plans
for
Integrated Writing Study

Algebra II Lesson Plans
for
Integrated Writing Study

Introduction

The basic text used for this study was Algebra II (Foster, Winters, Gordon, Rath, & Gell, 1992). To control for experimenter bias, the format of the lessons, except for the inclusion of the writing activities, generally followed the lessons as presented by the publishers. The study was conducted over Chapters 2 through 5 of the text.

The following general guidelines were adopted. The students, in both the experimental and control groups, were required to have a loose-leaf notebook with the following sections: dictionary, notes, classwork, homework, and tests-quizzes. All assignments were labeled with name, date, and type of assignment. All assignments were collected daily for immediate feedback. Specific writing assignments were constructed for each lesson.

Chapter 2
Linear Relations and Functions

Introduction

In this chapter the students studied the connection between an algebraic and a geometric representation of a function. The students began by graphing relations and identifying those that were functions. Next, they graphed linear equations from a table of ordered pairs, identified the slope and intercepts, and used these to group other linear equations. The students wrote various linear equations, including those for lines parallel and perpendicular to given equations. They concluded the study of the chapter by graphing linear inequalities.

The chapter objectives were to:

-----Identify different types of relations and functions.

-----Graph relations and functions on the coordinate plane.

-----Graph inequalities on the coordinate plane.

-----Solve applications of equations and inequalities.

Lesson 2-1: Relations and Functions

Objectives:

-----Graph a relation, state its domain and range, and determine if the relation is a function.

-----Find the values of functions for given elements of the domain.

Introductory Class Assignment:

The Five-Minute Check was used.

Control Group: Students were assigned all five problems.

Experimental Group: Students were asked to do problems 1, 3, and 5 and explain in writing how they did problems 1 and 5.

Corrected and discussed.

Previous Assignment:

Tests of Chapter 1 were distributed, reviewed, checked, and discussed.

New Lesson Introduction:

A discussion of relations followed. Many examples were used to develop the concept, including those suggested by the students and the following:

Number of books read for summer reading and the number of pages read.

Number of meals eaten and number of calories.

Number of items purchased at lunch and the cost of lunch.

A discussion of function as a type of relation followed. Many examples were used to develop the concept of function, including those suggested by the students and the following:

Pulling a drapery cord and the draperies opening.

Number of items, each of the same price, purchased and the total cost.

Number of hours traveled at a constant speed and distance covered.

A review of graphing on a coordinate plane, the terminology of domain and range, and function notation followed. Using the above examples of relations and functions, the data were written as ordered pairs and graphed. Guidelines for determining when relations are functions were developed.

<u>Closing Class Assignment:</u>

The Guided Practice in the assignment for the lesson was completed.

<u>Homework Assignment:</u>

Pages 56-57, numbers 18-44.

Lesson 2-2 and 2-4 Combined

Lesson 2-2: Linear Functions

Objectives:

-----Identify equations that are linear and graph them.

-----Write linear equations in standard form.

Lesson 2-4: Slopes and Intercepts

Objectives:

-----Determine the slope and intercepts of a line.

-----Use slope and intercepts to graph a linear equation.

-----Determine if two lines are parallel, perpendicular, or neither.

Introductory Class Assignment:

Control Group: Students completed the Five-Minute Check.

Experimental Group: Students were asked to describe in writing two relations, one that was and one that was not a function.

Classes checked and discussed their work.

Previous Assignment:

Classes reviewed, checked, and discussed.

Classes reviewed relations and functions.

New Lesson Introduction:

Discussed linear equations; definitions of independent and dependent variables; standard form; definition of slope; parallel and perpendicular lines.

Graphing calculators were used.

Closing Class Assignment:

Control Group: Students did Guided Practice together as a class.

Experimental Group: Students were asked to write about what they understood the most and the least about the lesson.

Homework Assignment:

Page 62, numbers 11-13

Page 63, numbers 37-40

Page 71, numbers 33-47

Control Group: Assignment as above.

Experimental Group: On page 71, did only the odd problems and explained in writing the procedure they used.

NOTE: Sections 2, 4, and 5 were review from both Algebra II and Geometry. Most of the students had had considerable experience with the material.

Lesson 2.5 Writing Linear Equations

Objectives:

-----Write the slope-intercept form of an equation, given the slope and a point, or two points.

-----Write the standard form of an equation, given the slope and a point, or two points.

-----Write the equation of a line that is parallel or perpendicular to the graph of a given equation.

Introductory Class Assignment:

All classes did the Five-Minute Check.

Control Group: Students did all the problems.

Experimental Group: Students did problems 2 and 3 and explained in writing their procedures.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Discussed slope-intercept form of the equation of a line.

Discussed finding the equation of a line given slope and one point, two points, the x and y intercepts.

Had students work on the board on various problems to check for understanding.

Closing Class Assignment:

Students discussed number 45 on page 71.

Control Group: Students discussed verbally as a class.

Experimental Group: Students discussed in writing.

Homework Assignment:

Control Group: Page 77, numbers 16-37

Experimental Group: Page 77, numbers 25-37, explained in writing

Lesson 2-6: Scatter Plots and Prediction

Objectives:

-----Draw a scatter plot and find the prediction equation.

-----Solve problems using prediction equations.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students wrote to an Algebra I student and explained how to graph a line, given the equation in slope-intercept form.

Previous Assignment:

Reviewed, checked, and discussed.

All problems were done on the board by the students.

Students used graphing calculators to check work.

New Lesson Introduction:

Discussed scatter plots; used overhead graphs.

Class worked in groups on problems 9-13 on pages 83-84.

Closing Class Assignment:

Worked in groups on assignment.

Homework Assignment:

Finish problems not completed in class.

Lesson 2-7: Special Functions

Objective:

-----Identify and graph special functions (direct variations, constant, absolute value, and greatest integer).

Introductory Class Assignment:

Control Group: Students did Five-Minute Check on the board in groups.

Experimental Group: Students were asked to write their comments on the last lesson on scatter plots.

Previous Assignment:

Reviewed, checked, and discussed.

Shared results of assignment and worked on the statistical functions on the graphing calculator.

New Lesson Introduction:

Discussed direct variation, identity, and constant functions.

Discussed graphing absolute value and greatest integer

Used many examples so students could see pattern and discover the relationships.

Used the graphing calculators.

Closing Class Assignment:

Worked on graphing calculators.

Homework Assignment:

Control Group: Pages 89-90, numbers 13-34.

Experimental Group: Pages 89-90, numbers 13-33 odd.

Students wrote a summary page of the functions studied, describing each and giving an example of each.

Lesson 2-8: Graphing Linear Inequalities

Objectives:

-----Draw a graph of inequalities in two variables.

-----Write an inequality to solve problems.

Introductory Class Assignment:

Quiz: Given two points, find the slope, y-intercept, slope-intercept form and standard form of the equation of the line.

Control Group: Students did the quiz as discussed above.

Experimental Group: Students did the quiz and explained in writing how they did each step.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Discussed graphing inequalities.

Closing Class Assignment:

Did Guided Practice.

Homework Assignment:

Pages 94-95, numbers 13-23.

Control Group: Students started reviewing for the test of Chapter 2.

Experimental Group: Students started reviewing for the test of Chapter 2. There were given an outline as a guide to write a summary of linear equations that was due the day of the test.

Review for Test of Chapter 2

Two days were spent on review for the test. The first day was a conclusion of Section 2-8. The students were assigned the Chapter Test on page 99 for homework. The second day was spent in groups, with the **Control Group** working on the Chapter Review on pages 97-98, and with the **Experimental Group** working on Chapter Summaries.

Chapter 3
Systems of Equations and Inequalities

Introduction

In this chapter the students reviewed and extended their understanding of equations and inequalities as developed in Chapter 2 by examining and solving systems of equations. Systems of equations were solved by both graphing and using algebraic methods. Determinants were introduced and systems were solved using Cramer's Rule. The students learned to find the maximum and minimum values of a function over a region using linear-programming techniques. Finally, the chapter concluded with graphing and solving systems of equations in three variables.

The chapter objectives were to:

-----Solve systems of equations in two or three variables.

-----Solve systems of inequalities.

-----Use linear programming to find maximum and minimum values of functions.

-----Graph linear equations in space.

Lesson 3-1: Graphing Systems of Equations

Objective:

-----Solve a system of equations by graphing.

Introductory Class Assignment:

Returned and discussed test of Chapter 2.

Control Group: Students worked in pairs correcting tests.

Experimental Group: Students wrote for five minutes explaining how they studied for the test and whether they might have done it differently.

Previous Assignment:

Reviewed, checked, and discussed.

Test of Chapter 2 discussed.

New Lesson Introduction:

Discussed systems of equations.

Used graphing calculators to graph systems and discuss possibilities.

Discussed terminology- consistent, dependent or independent, and inconsistent.

Closing Class Assignment:

Used graphing calculators to start homework assignment.

Homework Assignment:

Control Group: Pages 110-111, numbers 7-35 odd.

Experimental Group: Pages 110-111, numbers 19-35 odd and wrote a summary of the lesson.

Students had the option of doing their homework in the Math Center where they could use the graphing calculators.

Lesson 3-2: Solving Systems of Equations Algebraically

Objectives:

-----Use substitution method to solve a system of equations.

-----Use the elimination method to solve a system of equations.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students did the odd problems on the Five-Minute Check and explained their solutions.

Previous Assignment:

Reviewed, checked, and discussed.

Used the graphing calculators to check problems.

New Lesson Introduction:

Discussed solving systems of equations algebraically using elimination and substitution.

Closing Class Assignment:

None

Homework Assignment:

Control Group: Page 115, numbers 11-37 odd.

Experimental Group: Page 115, numbers 13-37 odd. Students wrote their opinion of the graphing calculator as a tool for graphing.

Lesson 3-3: Cramer's Rule

Objectives:

-----Find the value of a second-order determinant.

-----Solve a system of equations using Cramer's Rule.

Introductory Class Assignment:

Control Group: Students did Five-Minute Check on the board.

Experimental Group: Students did numbers 1 and 3 of the Five-Minute Check and explained their procedures.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Discussed Cramer's Rule.

Closing Class Assignment:

Control Group: Students started homework assignment.

Experimental Group: Students wrote about how they felt about the procedures we had used to solve systems of equations.

Homework Assignment:

Page 121, numbers 19-47.

Lesson 3-4: Graphing Systems of Inequalities

Objectives:

-----Graph a system of inequalities.

-----Solve a system of inequalities.

Introductory Class Assignment:

Quiz: Used numbers 26 and 30 from page 121

Control Group: Students also did number 28 from page 121.

Experimental Group: Students explained their procedures on the quiz.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Used reteaching example in the text to introduce this section on graphing systems of equations.

Closing Class Assignment:

None

Homework Assignment:

Control Group: Pages 124-125, numbers 21-39.

Experimental Group: Pages 124-125, numbers 21-31. Students wrote a summary of ways to solve systems of equations, including graphing, substitution, elimination and Cramer's Rule.

Lesson 3-6: Linear Programming

Objective:

-----Find the maximum and minimum values of a function over a region using linear-programming techniques.

Introductory Class Assignment:

Quiz:

Control Group: Page 121, numbers 22 and 28

Experimental Group: Page 121, number 28. Students explained the procedure.

Previous Assignment:

Reviewed, checked, and discussed.

Control Group: Students reviewed graphing systems.

Experimental Group: Students reviewed graphing systems and wrote their thoughts on this chapter so far.

New Lesson Introduction:

Introduced Linear Programming with various examples. Concentrated on the graphing aspects in this section.

Closing Class Assignment:

Students worked together graphing systems.

Homework Assignment:

Page 132-133, numbers 21-37 odd.

Lesson 3-7: Applications of Linear Programming

Objective:

----- Solve problems involving maximum and minimum values using linear-programming techniques.

Introductory Class Assignment:

Because it was the end of the first quarter, all students wrote comments about their progress.

Previous Assignment:

Reviewed, checked, and discussed.

Discussed homework. Students put all the problems on the board.

New Lesson Introduction:

Discussed Examples 1 and 2 in the text.

Closing Class Assignment:

Students worked in groups on numbers 4-11 on page 136.

Homework Assignment:

Pages 137-138, numbers 13, 15, and 17.

Lesson 3-8: Graphing Equations in Three Variables

Objectives:

-----Determine the octant in which a point in space is located.

-----Graph linear equations in space and determine the intercepts and traces.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students wrote discussing their problems or concerns about Linear Programming.

Previous Assignment:

Reviewed, checked, and discussed.

Discussed in detail with the use of the overhead graphs the three homework problems.

New Lesson Introduction:

Introduced equations of planes, graphing in three dimensions, octants, x, y, z intercepts, traces, and finding equations of planes.

Closing Class Assignment:

Students worked on Guided Practice

Homework Assignment:

Pages 142-143, numbers 13-29 odd.

Lesson 3-9: Solving Systems of Equations in Three Variables

Objective:

-----Solve a system of equations in three variables.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students explained in their own words how to do a Linear Programming problem.

Previous Assignment:

Reviewed, checked, and discussed.

Discussed the problems by having the students put them on the board.

New Lesson Introduction:

Introduced solving three equations with three variables.

Closing Class Assignment:

Control Group: Students did the Guided Practice.

Experimental Group: Students wrote a note to an absent classmate explaining how to solve a set of three equations with three variables.

Homework Assignment:

Pages 147-148, numbers 17-31 odd.

Review for Chapter Test:

Two days were spent on review for the test. The first day was a conclusion of Section 3-9. The students were assigned the Chapter Test on page 153 for homework. The second day was spent in groups with the **Control Group** working on the Chapter Review on pages 150-152 while the **Experimental Group** wrote a paper analyzing Chapters 2 and 3, specifying how the chapters were related and how they differed.

Chapter 4

Matrices

Introduction

In this chapter matrices were introduced by having the students create a matrix, perform scalar multiplication on it, and then add matrices. Determinants were then related to matrices, and students were next led to discover the numerous applications of matrices to real life and to transformational geometry. Students were thereafter introduced to using Cramer's Rule to solve systems of equations in three variables.

The Chapter objectives were:

-----Create matrices to represent data and algebraic expressions.

-----Perform operations with matrices.

-----Use matrices to achieve transformations of geometric figures.

-----Use matrices to solve systems of equations.

Lesson 4-2: An Introduction to Matrices

Objectives:

----- Create a matrix and name it using its dimensions.

-----Perform scalar multiplication on a matrix.

-----Add matrices.

-----Find unknown values in equal matrices.

Introductory Class Assignment:

Control Group: Discussed the Chapter 3 Test and the strategies they used to study for the test.

Experimental Group: Discussed the Chapter 3 Test and wrote about the studying strategies they used to prepare for the test.

Previous Assignment:

None

New Lesson Introduction:

Introduced matrices, equal matrices, scalar multiplication, and addition and subtraction of matrices.

Closing Class Assignment:

Started working on their homework.

Homework Assignment:

Control Group: Pages 165-166, numbers 11-31 odd

Experimental Group: Pages 165-166, numbers 19-31. students took notes on the new concepts in this section, writing definitions in their own words.

Lesson 4-3: Matrices and Determinants

Objectives:

-----Evaluate the determinant of a 3 X 3 matrix.

-----Find the area of a triangle given the coordinates of its vertices.

Introductory Class Assignment:

All did the Five-Minute Check

Previous Assignment:

Control Group: Students reviewed, checked, and discussed.

Experimental Group: Students reviewed, checked, and discussed. Wrote a letter to Amanda explaining what a matrix is and how to do scalar multiplication, addition and subtraction with matrices.

New Lesson Introduction:

Reviewed evaluation of a two-by-two determinant

Introduced evaluation of a three-by-three determinant using expansion by minors and diagonals

Closing Class Assignment:

Control Group: Students did the Guided Practice.

Experimental Group: Students wrote telling how they felt about matrices.

Homework Assignment::

Pages 171-172, numbers 13-35 odd.

Lesson 4-4: Multiplication of Matrices

Objective:

-----Multiply two matrices and interpret the results.

Introductory Class Assignment:

Quiz: Board problems on equal matrices, scalar multiplication, addition and subtraction of matrices.

Previous Assignment:

Students reviewed, checked, and discussed evaluation of three-by-three determinants.

New Lesson Introduction:

Introduced multiplication of matrices using various examples.

Closing Class Assignment:

Control Group: Students did guided practice.

Experimental Group: Students explained in writing how to evaluate a three-by-three determinant using expansion by minors and diagonals.

Homework Assignment:

Pages 176-177, numbers 13-35 odd.

Lesson 4-5: Identity and Inverse Matrices

Objectives:

-----Write the identity matrix for any matrix.

-----Find the inverse matrix for a 2 X 2 matrix.

Introductory Class Assignment:

Board Quiz on evaluating a three-by-three determinant.

Previous Assignment:

Reviewed, checked, and discussed by students working problems on the board.

New Lesson Introduction:

Introduced Identities and Inverses of matrices.

Closing Class Assignment:

Control Group: Students did Guided Practice.

Experimental Group: Students explained in writing how to multiply matrices and why matrix multiplication was not commutative.

Homework Assignment:

Pages 182-183, numbers 11-25 odd.

Lesson 4-6: Using Inverse Matrices

Objective:

------Write a system of linear equations as a matrix and use the inverse to solve the system.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students wrote about the concerns they had about multiplication of matrices.

Previous Assignment:

Reviewed, checked, and discussed by having the students work in pairs putting the problems on the board.

New Lesson Introduction:

Introduced using inverses to solve equations.

Closing Class Assignment:

Control Group: Students started working together in pairs on their homework.

Experimental Group: Students explained in writing the relationships between real numbers and matrices in regard to identities and inverses.

Homework Assignment:

Pages 188-189, numbers 8-19 all.

Lesson 4-7: Using Cramer's Rule

Objective:

-----Use Cramer's Rule to solve a system of linear equations in three variables.

Introductory Class Assignment:

Board Quiz on multiplication of matrices and identities, and inverses of matrices.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Introduced Cramer's Rule for three equations and three variables.

Closing Class Assignment:

All did Guided Practice.

Homework Assignment:

Pages 192-193, numbers 9-23 odd.

Review for Chapter 4 Test:

Two days were spent on review for the test. The first day was a conclusion of Section 4-7. The students were assigned the Chapter Test on page 205 for homework. The second day was spent in groups, with the **Control Group** working on the Chapter Review on pages 202-204, while the **Experimental Group** wrote a paper summarizing Chapter 4.

Chapter 5
Polynomials

Introduction

This chapter reviewed and extended the student's knowledge of operations on monomials and polynomials. The opening lessons increased the students' operational skills with monomials. Thereafter, addition, subtraction, and multiplication of polynomials were stressed. The students then were introduced to various methods of factoring. The chapter concluded with methods for dividing polynomials, including synthetic division.

The chapter objectives were to:

-----Multiply monomials.

-----Factor polynomials.

-----Divide polynomials.

Lesson 5-1: Monomials; and Lesson 5-2: Dividing Monomials

Objectives:

-----Multiply monomials and powers of monomials.

-----Divide monomials.

Introductory Class Assignment:

Control Group: Students discussed the test of Chapter 4 and whether they thought they might have studied in a different way.

Experimental Group: Students discussed the test of Chapter 4 and then wrote about their feelings regarding the test and the ways they studied for the test.

Previous Assignment:

None

New Lesson Introduction:

Reviewed monomials and division of monomials.

Closing Class Assignment:

Control Group: Students worked together in groups on their homework assignment.

Experimental Group: Students were asked to write about the following topic: "As we are getting close to the end of the first semester in Algebra II, have you learned more about yourself and how you learn mathematics?"

Homework Assignment:

Page 213, numbers 25-39 odd and page 218, numbers 17-51 odd.

Lesson 5-4: Polynomials

Objectives:

-----Add polynomials.

-----Subtract polynomials.

-----Multiply polynomials.

Introductory Class Assignment:

Control Group: Quiz: page 213, numbers 32-40 even and page 218, numbers 36, 38, and 40.

Experimental Group: Quiz: page 218, numbers 38, 44, and 46 and students explained in writing how they did each problem.

Previous Assignment:

Reviewed, checked, and discussed by having students go to the board in groups and work specific problems.

New Lesson Introduction:

Introduced polynomials, degree, addition, subtraction, multiplication, and division.

Closing Class Assignment:

Control Group: Students did the Guided Practice.

Experimental Group: Students wrote for five minutes on everything they knew about monomials.

Homework Assignment:

Pages 226-227, numbers 19-67 odd.

Lesson 5-5: Factoring; Day 1

Objective:

-----Factor polynomials.

Introductory Class Assignment:

Control Group: Students did the Five-Minute Check.

Experimental Group: Students explained to a student who had been absent the day before how to multiply binomials and trinomials.

Previous Assignment:

Reviewed, checked, and discussed problems in groups

.

New Lesson Introduction:

Introduced simple factoring using patterns using supplementary materials.

Closing Class Assignment:

Control Group: Students worked together in groups on homework.

Experimental Group: Students wrote for five minutes about Chapter 5 and whether they had any concerns.

Homework Assignment:

Supplementary sheets on factoring.

Lesson 5-5: Factoring; Day 2

Introductory Class Assignment:

Board Quiz on factoring.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Introduced more difficult factoring, including difference of two squares, sum and difference of two cubes, factoring by grouping.

Closing Class Assignment:

Students started working together in groups on the homework

Homework Assignment:

Supplementary sheets and page 223, numbers 27-51 odd.

Lesson 5-6: Dividing Polynomials

Objective:

-----Divide polynomials, using factoring and long division.

Introductory Class Assignment:

Board Quiz on factoring.

Previous Assignment:

Reviewed, checked, and discussed.

New Lesson Introduction:

Introduced dividing polynomials.

Closing Class Assignment:

Control Group: Students did the Guided Practice.

Experimental Group: Students were asked to explain to a friend how to factor numbers 36 and 38 on page 233.

Homework Assignment:

Page 239, numbers 15-33 odd.

Lesson 5-7: Synthetic Division

Objective:

-----Divide polynomials, using synthetic division.

Introductory Class Assignment:

Control Group: Students did Five-Minute Check.

Experimental Group: Students did numbers 1 and 3 of the Five-Minute Check and explained their work.

Previous Assignment:

Reviewed, checked, and discussed by having pairs of students work on problems on the board.

New Lesson Introduction:

Introduced dividing polynomials, using synthetic division.

Closing Class Assignment:

Control Group: Students did the Guided Practice.

Experimental Group: Students wrote a paragraph comparing long division and synthetic division.

Homework Assignment:

Page 244, numbers 11-27 odd.

<u>Review for Chapter 5 Test:</u>

Two days were spent on review for the test. The first day was a conclusion of Section 5-7. The students were assigned the Chapter Test on page 249 for homework. The second day was spent in groups, with the **Control Group** working on the Chapter Review on pages 246-248, while the **Experimental Group** wrote a paper summarizing Chapter 5.

Appendix B

Permission to copy Pretest,
Pretest, Midtest, and Posttest

EDUCATIONAL TESTING SERVICE PRINCETON, N.J. 08541

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August 4, 1992

Ms. Rebecca Kasparek
5701 Glen Forest Drive
Charlotte, NC 28226

Dear Ms. Kasparek:

This is in reference to your letter of July 14, 1992, requesting permission to include the STEP End-Of-Course Tests - Form Y Algebra/Geometry in your dissertation. We understand that you will be using the test to do a study on the effects of an integrated writing-in-content program on algebra students as part of your dissertation at the University of North Carolina at Greensboro.

Educational Testing Service is pleased to grant this permission, which is nonexclusive and royalty free, for the use described above. Please place this test in an appendix to the dissertation, and do not make the appendix available to University Microfilms, Inc. Our copyright notice, of course, must remain intact followed by the words "Reproduced by permission."

Enclosed is the STEP End-Of-Course Tests - Form Y and related test materials which you will need for your research. These materials are on loan to you and must be returned to the attention of Gerri Szakacs - Room 230K, immediately after reproduction.

If these arrangements are satisfactory, please sign both copies of this letter and return one copy to me for our records.

Sincerely,

Anne F. Marcantonio

Anne F. Marcantonio
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Administrator

ACCEPTED AND AGREED TO:

Name____________________

Date____________________

PRETEST
FOR
INTEGRATED WRITING STUDY

MIDTEST
FOR
INTEGRATED WRITING STUDY

1. $\dfrac{k}{k + k^2} =$

A) $\dfrac{1}{1 + k}$

B) $\dfrac{1}{k^2}$

C) $\dfrac{1}{1 + k^2}$

D) $1 + \dfrac{1}{k}$

$$\begin{cases} Ax + By = 2 \\ x + y = 1 \end{cases}$$

2. The linear equations above have more than one common solution if which of the following is true?

A) $A = B = 1$
B) $A = B = 2$
C) $A = 1$ and $B = 2$
D) $A = 2$ and $B = 0$

3. What is an equation of the line passing through the point $(0, -1)$ and parallel to the line $y = x$?

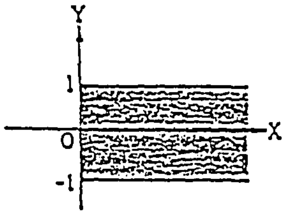
A) $y = -x + 1$
B) $y = -x - 1$
C) $y = x + 1$
D) $y = x - 1$

4. $$\begin{cases} x - 8y = 2 \\ x + 4v = 5 \end{cases}$$

What is the value of x that satisfies the system of equations above?

A) 0
B) 2
C) 4
D) 6

5.



The figure above is the graph of which of the following sets?

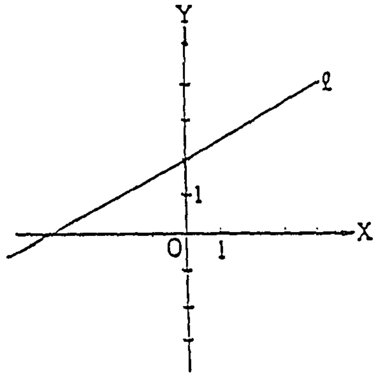
A) $\{(x, y) | y \geqq 0 \text{ and } -1 \leqq x \leqq 1\}$

B) $\{(x, y) | x \geqq 0 \text{ and } -1 \leqq y \leqq 1\}$

C) $\{(x, y) | x \geqq -1 \text{ and } y \leqq 1\}$

D) $\{(x, y) | y \geqq -1 \text{ and } y \leqq 1\}$

6.



What is the slope of line ℓ above?

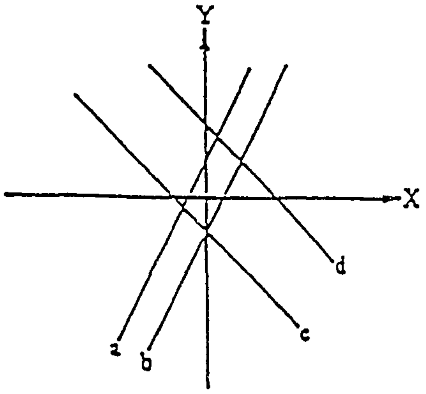
A) 2

B) $\dfrac{1}{2}$

C) $-\dfrac{1}{2}$

D) -2

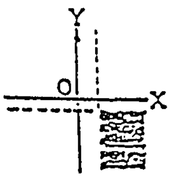
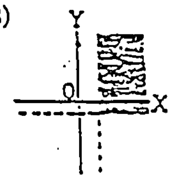
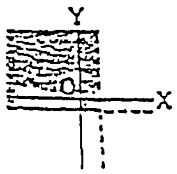
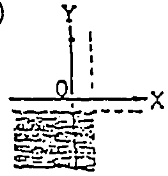
7.



In the figure above, which line could be the graph of the equation $y = 2x - 1$?

A) a
B) b
C) c
D) d

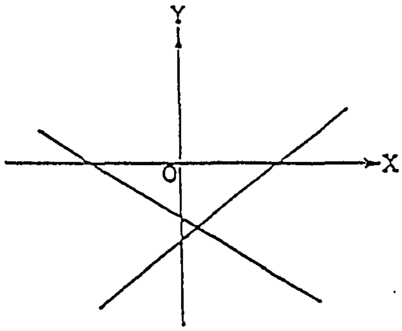
8. In which of the following could the shaded region represent $\{(x, y): x > 2 \text{ and } y < -1\}$?

A)

B)

C)

D)


9. The graph of which of the following has y-intercept 5 ?

A) $y = 5x$
B) $y = 5x + 1$
C) $y = x - 5$
D) $y = x + 5$

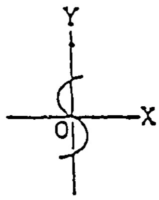
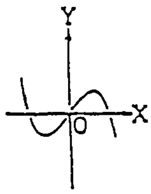
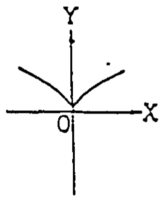
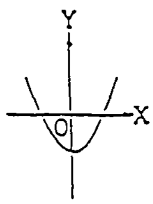
10.



In the figure above, which of the following (x, y) pairs could be the common solution of the two linear equations represented on the graph?

A) $(1, -4)$
B) $(1, 4)$
C) $(-1, -4)$
D) $(4, 1)$

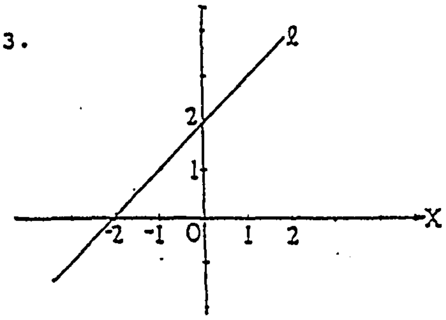
11. All of the following represent graphs of functions EXCEPT

A)

B)

C)

D)


12. If $f(x) = 2(x + 1)$, then $f(3) =$

A) 4
B) 6
C) 7
D) 8

13.



In the figure above, line ℓ is defined by the equation

A) $y = 2$
B) $y = x$
C) $y = x + 2$
D) $y = x - 2$

14. Simplify $5(x-1)^2 - (2x-3)(2x+3)$

a. $x^2 - 10x - 4$
b. $x^2 - 10x + 4$
c. $x^2 - 10x + 14$
d. $x^2 - 10x - 14$
e. $x^2 - 2x + 10$

15. $8x^3 - 27$ can be expressed in factored form as

a. $(2x-3)(2x-3)(2x-3)$
b. $(8x+3)(x-3)(x+3)$
c. $(8x-3)(x^2+9)$
d. $(2x-3)(4x^2-6x+9)$
e. $(2x-3)(4x^2+6x+9)$

16. If the x-intercept of a line is (-4) and the y-intercept is (-3) then an equation of the line is

a. $4x + 3y + 9 = 0$
b. $3x + 4y + 3 = 0$
c. $-3x - 4y + 12 = 0$
d. $4x + 3y + 12 = 0$
e. $3x + 4y + 12 = 0$

17. $4x^3 - 4x^2 - x + 1$ can be expressed in factored form as:

a. $(4x^2+1)(x-1)$
b. $(2x+1)(2x-1)(x+1)$
c. $(4x^2-1)(x+1)$
d. $(2x+1)(2x-1)(x-1)$
e. $(x^2+1)(4x-1)$

18. A is the point $(2, 1)$, B is the point $(6, -5)$. The slope of a line perpendicular to line AB is:

a. $-$[illegible]
b. [illegible]
c. [illegible]
d. $-$[illegible]
e. -1

19. Which of the following equations determines a line through the point $(3, 0)$ parallel to the line $2y = x$?

a. $y = 2x - 6$
b. $y = -2x + 6$
c. $y = \frac{1}{2}x - \frac{3}{2}$
d. $y = \frac{1}{2}x + \frac{3}{2}$
e. $y = \frac{1}{2}x$

20. Which expression represents y in the system at the right?

$$4x - 12 = 2y$$
$$3x + 2y = 18$$

A. $\dfrac{\begin{vmatrix} 12 & -2 \\ 18 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 3 & 2 \end{vmatrix}}$ B. $\dfrac{\begin{vmatrix} 4 & 12 \\ 3 & 18 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 3 & 2 \end{vmatrix}}$ C. $\dfrac{\begin{vmatrix} 4 & 2 \\ 3 & 18 \end{vmatrix}}{\begin{vmatrix} 4 & -12 \\ 3 & 2 \end{vmatrix}}$ D. $\dfrac{\begin{vmatrix} 2 & -12 \\ 18 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -12 \\ 3 & 2 \end{vmatrix}}$

21. What does x equal in the solution of the system at the right?

$$x + y = 2z = 10$$
$$x - y - 4z = 6$$
$$2x + y + 3z = 2$$

A. 0 B. −1 C. 1 D. 2

22. Given $0 \leq x \leq 4$, $0 \leq y \leq 3$, and $y \leq -x + 5$, find the maximum value of $f(x, y) = 2x + 3y$.

A. 8 B. 9 C. 11 D. 13

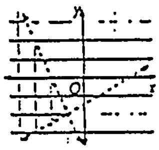
23. Which system of inequalities is shown by the graph at the right?

A. $y > \frac{1}{2}x - \frac{3}{2}$ and $y > -2x - 4$

B. $y < \frac{1}{2}x - \frac{3}{2}$ and $y > -2x - 4$

C. $y > \frac{1}{2}x - \frac{3}{2}$ and $y < -2x - 4$

D. $y < \frac{1}{2}x - \frac{3}{2}$ and $y < -2x - 4$



24. A farmer has 15 days in which to plant corn and beans in a 300-acre field. The corn can be planted at a rate of 30 acres per day and the beans at a rate of 15 acres per day. If corn profits are \$35 per acre and bean profits are \$42 per acre, how many acres of beans should the farmer plant to maximize the profit?

A. 225 acres B. 150 acres C. 300 acres D. 100 acres

25. If the system $\begin{cases} 3x - y = 1 \\ 2x + y = 4 \end{cases}$ is written as a matrix equation, by which matrix could you multiply both sides to obtain the solution?

A. $\begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$ B. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ C. $\begin{bmatrix} \frac{3}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$ D. $\begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} \end{bmatrix}$

26. For what value of k does the system at the right not have a unique solution?

$$\begin{aligned} x - 2y + kz &= -1 \\ x + 2z &= 4 \\ 3x - 3y + 8z &= 2 \end{aligned}$$

A. 3 B. −3 C. 0 D. −9

27. What does x equal in the solution of the system at the right?

$$\begin{aligned} \frac{x}{3} + \frac{y}{4} &= 5 \\ \frac{x}{6} - \frac{y}{12} &= 0 \end{aligned}$$

A. −6 B. −12 C. 12 D. 6

28. The augmented matrix for a system is $\begin{bmatrix} 0 & 2 & 4 \\ 1 & 0 & 2 \end{bmatrix}$. What is the solution?

A. (4, 2) B. (2, 2) C. (2, 1) D. (2, 4)

29. Find the area of a triangle with vertices with coordinates (−2, 6), (1, 5) and (6, −3).

A. $\frac{19}{2}$ B. $-\frac{19}{2}$ C. 19 D. −19

30. For which system are there infinitely many solutions?

A. $3x + 5y = 7$, $6x + 10y = 14$ B. $x - 5y = 7$, $2x = 10y + 15$ C. $3x - y = 8$, $3x + y = 10$ D. $2x - y = 1$, $y = 2x - 3$

MID-TEST ANSWER SHEET

Algebra Test

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	29 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	30 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

POSTTEST
FOR
INTEGRATED WRITING STUDY

1. $\begin{cases} Ax + By = 2 \\ x + y = 1 \end{cases}$

The linear equations above have more than one common solution if which of the following is true?

A) $A = B = 1$
B) $A = B = 2$
C) $A = 1$ and $B = 2$
D) $A = 2$ and $B = 0$

2. What is an equation of the line passing through the point $(0, -1)$ and parallel to the line $y = x$?

A) $y = -x + 1$
B) $y = -x - 1$
C) $y = x + 1$
D) $y = x - 1$

3. $\dfrac{k}{k + k^2} =$

A) $\dfrac{1}{1 + k}$

B) $\dfrac{1}{k^2}$

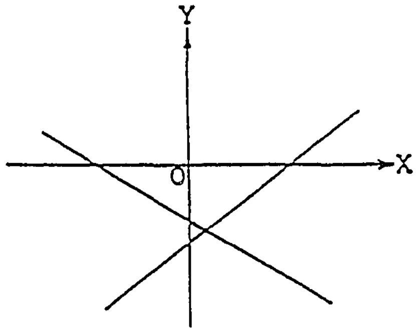
C) $\dfrac{1}{1 + k^2}$

D) $1 + \dfrac{1}{k}$

4. Which of the following is a factor of $k^2 + 7k - 18$?

A) $k - 2$
B) $k + 2$
C) $k - 9$
D) $k - 6$

5.



In the figure above, which of the following (x, y) pairs could be the common solution of the two linear equations represented on the graph?

A) $(1, -4)$
B) $(1, 4)$
C) $(-1, -4)$
D) $(4, 1)$

6. $(x + 2y)(2x - y) =$

A) $2x^2 - 2y^2$
B) $2x^2 + 3xy - 2y^2$
C) $2x^2 + 3xy + 2y^2$
D) $2x^2 + 5xy + 2y^2$

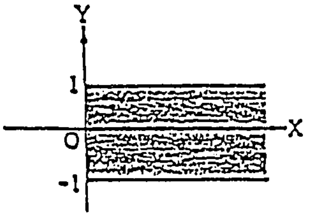
7. $(3x - 1)^2 =$

A) $9x + 1$
B) $9x^2 + 1$
C) $9x^2 - 3x + 1$
D) $9x^2 - 6x + 1$

8. If $x \neq 0$, then $\dfrac{15x^3 + 3x}{3x} =$

A) $15x^3$
B) $15x^3 + 1$
C) $5x^2 + 3x$
D) $5x^2 + 1$

9.



The figure above is the graph of which of the following sets?

A) $\{(x, y) | y \geqq 0 \text{ and } -1 \leqq x \leqq 1\}$

B) $\{(x, y) | x \geqq 0 \text{ and } -1 \leqq y \leqq 1\}$

C) $\{(x, y) | x \geqq -1 \text{ and } y \leqq 1\}$

D) $\{(x, y) | y \geqq -1 \text{ and } y \leqq 1\}$

10. The length of a rectangular floor is 1 meter more than twice the width of the floor. If the area of the floor is 36 square meters, then the length of the floor, in meters, is

A) 4.5
B) 6
C) 9
D) 18

11.

$$\begin{cases} x - 8y = 2 \\ x + 4y = 5 \end{cases}$$

What is the value of x that satisfies the system of equations above?

A) 0
B) 2
C) 4
D) 6

12. $9x^2 - 100 =$

A) $(3x - 10)(3x - 10)$
B) $(3x + 10)(3x - 10)$
C) $(9x - 10)(x - 10)$
D) $9x(x - 100)$

13. If $x > 1$ and $\frac{x^n}{x^2} = x^3$ for all x, then n =

A) 1
B) 2
C) 5
D) 6

14. Which of the following equations determines a line through the point (3, 0) parallel to the line $2y = x$?

a. $y = 2x - 6$
b. $y = -2x + 6$
c. $y = \frac{1}{2}x - \frac{3}{2}$
d. $y = \frac{1}{2}x + \frac{3}{2}$
e. $y = \frac{1}{2}x$

15. If the x-intercept of a line is (−4) and the y-intercept is (−3) then an equation of the line is

a. $4x + 3y + 9 = 0$
b. $3x + 4y + 3 = 0$
c. $-3x - 4y + 12 = 0$
d. $4x + 3y + 12 = 0$
e. $3x + 4y + 12 = 0$

16. A is the point (2, 1), B is the point (6, −5). The slope of a line perpendicular to line AB is:

a. $-\frac{2}{3}$
b. $\frac{2}{3}$
c. $\frac{3}{2}$
d. $-\frac{3}{2}$
e. -1

17. $4x^3 - 4x^2 - x + 1$ can be expressed in factored form as:

a. $(4x^2 + 1)(x - 1)$
b. $(2x + 1)(2x - 1)(x + 1)$
c. $(4x^2 - 1)(x + 1)$
d. $(2x + 1)(2x - 1)(x - 1)$
e. $(x^2 + 1)(4x - 1)$

18. Simplify $5(x-1)^2 - (2x-3)(2x+3)$

a. $x^2 - 10x - 4$
b. $x^2 - 10x + 4$
c. $x^2 - 10x + 14$
d. $x^2 - 10x - 14$
e. $x^2 - 2x + 10$

19. If the system $\begin{cases} 3x - y = 1 \\ 2x + y = 4 \end{cases}$ is written as a matrix equation, by which matrix could you multiply both sides to obtain the solution?

A. $\begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix}$ B. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ C. $\begin{bmatrix} \frac{3}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$ D. $\begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} \end{bmatrix}$

20. Find the area of a triangle with vertices with coordinates $(-2, 6)$, $(1, 5)$ and $(6, -3)$.

A. $\frac{19}{2}$ B. $-\frac{19}{2}$ C. 19 D. -19

21. For what value of k does the system at the right not have a unique solution?

$$\begin{aligned} x - 2y + kz &= -1 \\ x + 2z &= 4 \\ 3x - 3y + 8z &= 2 \end{aligned}$$

A. 3 B. -3 C. 0 D. -9

22. Find $\begin{bmatrix} 1 & 11 \\ -4 & -9 \end{bmatrix} - \begin{bmatrix} 3 & -5 \\ 8 & -6 \end{bmatrix}$.

A. $\begin{bmatrix} 4 & 6 \\ 4 & -15 \end{bmatrix}$ B. $\begin{bmatrix} 14 \\ -15 \end{bmatrix}$ C. $\begin{bmatrix} -7 & 17 \\ -7 & -4 \end{bmatrix}$ D. $\begin{bmatrix} -2 & 16 \\ -12 & -3 \end{bmatrix}$

23. The augmented matrix for a system is $\begin{bmatrix} 0 & 2 & 4 \\ 1 & 0 & 2 \end{bmatrix}$. What is the solution?

A. (4, 2) B. (2, 2) C. (2, 1) D. (2, 4)

24. $8x^3 - 27$ can be expressed in factored form as

a. $(2x-3)(2x-3)(2x-3)$

b. $(8x+3)(x-3)(x+3)$

c. $(8x-3)(x^2+9)$

d. $(2x-3)(4x^2-6x+9)$

e. $(2x-3)(4x^2+6x+9)$

25. Which is the identity for a 3 × 3 matrix?

A. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

26. What is the value of $\begin{vmatrix} -1 & 3 & 2 \\ 4 & -2 & 1 \\ 3 & -3 & -4 \end{vmatrix}$?

A. 34 B. −86 C. −34 D. 86

27. Which augmented matrix is not equivalent to the augmented matrix at the right? $\begin{bmatrix} 3 & -1 & 4 \\ 4 & 2 & -2 \end{bmatrix}$

A. $\begin{bmatrix} 3 & -1 & 4 \\ 7 & 1 & 2 \end{bmatrix}$ B. $\begin{bmatrix} 3 & -1 & 4 \\ 2 & 1 & -1 \end{bmatrix}$ C. $\begin{bmatrix} 4 & 2 & -2 \\ 3 & -1 & 4 \end{bmatrix}$ D. All are equivalent.

28. Find $-\frac{2}{3}\begin{bmatrix} -1 & 6 & 12 \\ 9 & -3 & 15 \end{bmatrix}$.

A. $\begin{bmatrix} \frac{2}{3} & -4 & -8 \\ -6 & 2 & -10 \end{bmatrix}$ B. $\begin{bmatrix} \frac{2}{3} & -4 & -8 \\ 9 & -3 & 15 \end{bmatrix}$ C. $\begin{bmatrix} -\frac{2}{3} & 4 & 8 \\ 6 & -2 & -10 \end{bmatrix}$ D. $\begin{bmatrix} 2 & -12 & -24 \\ 27 & -9 & 45 \end{bmatrix}$

29. Find the first row of the inverse of $\begin{bmatrix} 3 & 4 \\ -2 & 4 \end{bmatrix}$.

A. $\begin{bmatrix} -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$ B. $\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} \end{bmatrix}$ C. $\frac{1}{20}$ D. $\begin{bmatrix} \frac{3}{20} & \frac{1}{5} \end{bmatrix}$

30. Find the first row of $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 7 \\ 6 & -2 \end{bmatrix}$.

A. [18 8] B. [0 −21] C. [−18 20] D. [0 21]

POST-TEST ANSWER SHEET

Writing Attitude

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	

Mathematics Attitude

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	

Algebra Test

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	12 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	22 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	14 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	24 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
6 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	16 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	26 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
7 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	27 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	18 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	28 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
9 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	29 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
10 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	20 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	30 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Appendix C

Attitude Inventories

Attitudes Toward Mathematics

(Scale Form B)

Marilyn N. Suydam and Cecil R. Trueblood
The Pennsylvania State University

This is to find out how you feel about mathematics. You are to read each statement carefully and decide how <u>you</u> feel about it. Then indicate your feeling on the answer sheet by marking:

A - if you strongly agree
B - if you agree
C - if your feeling is neutral
D - if you disagree
E - if you strongly disagree

1. Mathematics often makes me feel angry.
2. I usually feel happy when doing mathematics problems.
3. I think my mind works well when doing mathematics problems.
4. When I can't figure out a problem, I feel as though I am lost in a mass of words and numbers and can't find my way out.
5. I avoid mathematics because I am not very good with numbers.
6. Mathematics is an interesting subject.
7. My mind goes blank and I am unable to think clearly when working mathematics problems.
8. I feel sure of myself when doing mathematics.
9. I sometimes feel like running away from my mathematics problems.
10. When I hear the word mathematics, I have a feeling of dislike.
11. I am afraid of mathematics.
12. Mathematics is fun.
13. I like anything with numbers in it.
14. Mathematics problems often scare me.
15. I usually feel calm when doing mathematics problems.
16. I feel good toward mathematics.
17. Mathematics tests always seem difficult.
18. I think about mathematics problems outside of class and like to work them out.
19. Trying to work mathematics problems makes me nervous.
20. I have always liked mathematics.
21. I would rather do anything else than do mathematics.
22. Mathematics is easy for me.
23. I dread mathematics.
24. I feel especially capable when doing mathematics problems.
25. Mathematics class makes me look for ways of using mathematics to solve problems.
26. Time drags in a mathematics lesson.

FLORIDA WRITING PROJECT STUDENT SURVEY

Directions: Indicate your agreement or disagreement with the following comments by marking your answers in pencil on the computer sheet. DO NOT WRITE ON THIS PAGE.

A - if you strongly agree
B - if you agree
C - if your feeling is neutral
D - if you disagree
E - if you strongly disagree

1. I write for relaxation or as a hobby.
2. I have to force myself to write.
3. Writing is one of the activities I like least in school.
4. I have difficulty beginning a writing assignment.
5. I am a good writer.
6. Good writers spend more time than poor writers in revising their work.
7. I share my writing with others.
8. I revise my writing to make it better.
9. The teacher is the most important audience for what I write in school.
10. In general, I like school.
11. I save my writing.
12. I write notes to my family and friends.
13. I write letters.
14. I am proud of at least one piece of writing I have written during the last year.
15. I am sometimes able to write about things that are hard for me to say.
16. I keep a journal or a diary.
17. I enjoy reading.
18. I have good ideas, but I can't put them down on paper.
19. I make too many mechanical errors when I write.
20. At least one teacher I have had during my years in school has told me that I am a good writer.
21. In class, I share what I write with other students.
22. I am embarrassed by my writing.
23. I have many stories I would like to tell in writing.
24. Writing will probably be a part of the job I plan to hold in the future.
25. Writing is an important way for me to express my feelings.

Florida Writing Project - H.I. Guttinger, C.M. Morris, 1983

Appendix D

Writing Sample Items from Tests

NAME ____

Find the slope-intercept form of an equation for each graph described.

23. passes through $(-1, -5)$ and $(3, 2)$

For each pair of equations, determine if the lines are parallel, perpendicular, or neither.

29. $y = \frac{3}{5}x$ and $y - 8 = -\frac{5}{3}x$

NAME ______________________________

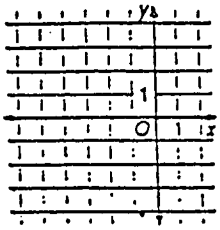
13. Graph the system of inequalities. Find the vertices of the polygon formed. Find the maximum and minimum values of $f(x, y) = 2x - y$.

$y \leq \frac{1}{2}x + 3$

$y \geq 3x - 4$

$x \leq 2$

$y > 0$

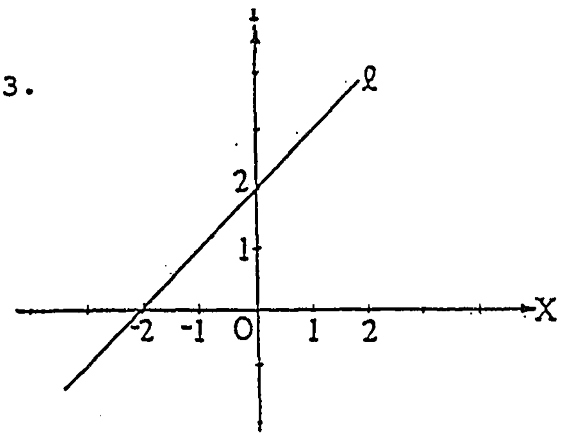


16. $x + 2y - z = -7$

$2x - 2y - z = 6$

$x + y - 2z = -6$

NAME ______________________________

13. 

In the figure above, line ℓ is defined by the equation

A) $y = 2$
B) $y = x$
C) $y = x + 2$
D) $y = x - 2$

24. A farmer has 15 days in which to plant corn and beans in a 300-acre field. The corn can be planted at a rate of 30 acres per day and the beans at a rate of 15 acres per day. If corn profits are \$35 per acre and bean profits are \$42 per acre, how many acres of beans should the farmer plant to maximize the profit?

A. 225 acres B. 150 acres C. 300 acres D. 100 acres

NAME ____________________________________

12. $9x^2 - 100 =$

A) $(3x - 10)(3x - 10)$
B) $(3x \div 10)(3x - 10)$
C) $(9x - 10)(x - 10)$
D) $9x(x - 100)$

26. What is the value of $\begin{vmatrix} -1 & 3 & 2 \\ 4 & -2 & 1 \\ 3 & -3 & -4 \end{vmatrix}$?

A. 34 B. -86 C. -34 D. 86

NAME ____________________

9. $(m - 3)(m + 2)(m - 4)$

Divide using long division.

21. $(m^3 - 3m^2 - 18m + 40) \div (m + 4)$.

NAME ____

10. Evaluate $\begin{vmatrix} 4 & 0 & -1 \\ 5 & 3 & 6 \\ -2 & -5 & 2 \end{vmatrix}$.

15. Write the system below as a matrix equation. Then use the inverse to solve.

$4x - 2y = -6$
$3x + y = -7$

Appendix E

Scoring Procedure for Writing Samples

SCORING SCALE FOR WRITING SAMPLES

For the points listed at the left, the explanations have one of the following characteristics.

0 Points: No explanation given.

The student stated that they guessed.

The problem was simply recopied, but no explanation was given.

1 Point: There was a start toward explaining the problem that reflected some understanding, but the approach either did not lead or would not have led to a correct solution.

The student started with an inappropriate strategy that did not work and then gave up.

2 Points: The student used an inappropriate strategy and got an incorrect answer, but the work showed some understanding of the problem.

An appropriate strategy was used but it either was not carried out far enough or it was implemented incorrectly.

The correct answer is given, but the explanation given would not describe the process necessary to obtain that solution.

3 Points: The student implemented a solution strategy that could have led to the correct solution, but the student either misunderstood part of the problem or ignored a condition of the problem.

Appropriate solution strategies were properly applied, but either the student answered the problem incorrectly for no apparent reason or no answer was given.

4 Points: The student made an error in carrying out the appropriate solution strategy. However, this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather seems to be a coping or computational error.

Appropriate strategies were selected and implemented, the correct answer was given, but the explanation was brief.

5 Points: Appropriate strategies were selected and implemented, the correct answer was given, and the explanation was detailed and precise.

Appendix F

Samples of Writing Surveys

WRITING SURVEY

What classes do you teach?

Class 1:____

Class 2:____

Class 3:____

Class 4:____

Class 5:____

Is writing a component of your classes? If so, in what ways?

In your classes, how much time each day do you estimate your students should be spending on writing activities?

Class 1:____

Class 2:____

Class 3:____

Class 4:____

Class 5:____

WRITING SURVEY

What classes do you take?

Class 1:____

Class 2:____

Class 3:____

Class 4:____

Class 5:____

Class 6:____

Class 7:____

Do you do any writing in any of your classes? If so, what kind?

Class 1:____

Class 2:____

Class 3:____

Class 4:____

Class 5:____

Class 6:____

Class 7:____

In your classes, how much time each day do you estimate you spend on writing activities?

Class 1:____

Class 2:____

Class 3:____

Class 4:____

Class 5:____

Class 6:____

Class 7:____

Appendix G

Summary of Results of Writing Surveys

Results of Faculty Writing Survey

Department	Estimated Time Students Spend Per Day in Writing Activities*
History	75 Minutes
English	60 Minutes
English as a Second Language	60 Minutes
Science	50 Minutes
Foreign Language	30 Minutes
Fine Arts	15 Minutes
Mathematics	10 Minutes

* This estimate is an average per department and includes in-class and out-of-class writing such as note-taking, quizzes, essays, lab reports, etc.

Results of Algebra II Student Writing Survey

Department	Estimated Time Students Spend Per Day in Writing Activities*
History	80 Minutes
English	60 Minutes
English as a Second Language	30 Minutes
Science	70 Minutes
Foreign Language	25 Minutes
Fine Arts	15 Minutes
Mathematics: Algebra II	
Control Group	10 Minutes
Experimental Group	25 Minutes

Estimated Total Number of Hours Each Day Writing

Range:	2-9 Hours
Median:	4 Hours
Mode:	4 Hours
Mean:	5.2 Hours

* This estimate is an average per department and includes in-class and out-of-class writing such as note-taking, quizzes, essays, lab reports, etc.

Appendix H

Permission Forms

923025

UNIVERSITY OF NORTH CAROLINA AT GREENSBORO

Institutional Review Board
Notification Form

DATE: August 11, 1992

PROJECT TITLE: Effects of Integrated Writing on Attitude and Algebra Achievement of High School Algebra Students

PRINCIPAL INVESTIGATOR: Rebecca Kasprzak / George Bright

SCHOOL/COLLEGE: ED DEPARTMENT: CSS

ACTION TAKEN:

___ Exempt

_X_ Expedited Review

___ Full IRB Review

DISPOSITION OF APPLICATION:

_X_ Approved

___ Disapproved

MODIFICATIONS/COMMENTS:

[signature]
IRB Chair/Designee

Approval of research is valid for one year unless otherwise indicated. If your research goes beyond one year, the project must be reviewed prior to continuation.

September 9, 1992

Dear Algebra II Parents:

Some of you may know that I have been working on my Ph.D. in Mathematics Education. At this point, I have completed my course work, my written and oral examinations, and just last week my committee approved my dissertation proposal.

My primary interest, and much of my course work, has been in studying ways to better serve the individual needs and learning styles of all my students. For many years, I have been studying the interaction between mathematics and the language activities of writing and reading. For this reason, I have designed my dissertation study to investigate whether writing activities within the context of the Algebra program will improve students' attitudes and performance in Algebra. I will teach all four classes with the same text and instruction, except that two of the classes will be asked to do various constructive writing activities. At the completion of the study, I will use the results to improve methodology in all my classes.

On Tuesday, September 15, at 7 P.M., I will have an informal meeting in the Upper School Dining Hall to explain my study in more detail and to answer any questions you may have. Please either bring your consent form to the September 15 meeting or mail it to me in the stamped self-addressed envelope by September 15.

Thank you for your assistance and for the pleasure of being able to work with your young students, many for the second year. Anytime you have a question or concern, please feel free to call me either at home (543-6309) or at school (366-1241).

Sincerely,

Rebecca Kasparek

Rebecca Kasparek

Consent Form

Student's Name: ______________________________

Date of Consent: ______________________________

Having read the letter describing Rebecca Kasparek's proposed study and having asked any questions I may have, I consent for my child to participate in the Doctoral study entitled: Effects of Integrated Writing on Attitude and Algebra Performance of High School Algebra Students

____
Parent(s)/Guardian Signature