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# Evaluating problem solving in the mathematics curriculum: A case study 

Jetton, Janice Hutchinson, Ed.D.<br>The University of North Carolina at Greensboro, 1991

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# EVALUATING PROBLEM SOLVING IN THE 

## MATHEMATICS CURRICULUM:

A CASE STUDY
by
Janice Hutchinson Jetton

A Dissertation Submitted to
the Faculty of the Graduate School at
The University of North Carolina at Greensboro
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of the Requirements of the Degree Doctor of Education

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Approved by:


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## APPROVAL PAGE

This dissertation has been approved by the following committee of the Faculty of the Graduate School at The University of North Carolina at Greensboro.

Dissertation Advisor

$\frac{\text { Way } 21,1991}{\text { Date of ACceptance by Committee }}$
$\frac{\text { May } 21,1991}{\text { Date of Final Oral Examination }}$
(C) 1991 by Janlce Hutchlnson Jetton

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The purpose of thls evaluation case study was to assess the current status of problem solving in a typlcal mathematics curriculum, and to examine the process Involved In the implementation of the recommendations found in NCTM's Standard 1: Mathematlcs as Problems Solving.

Durlng phase 1 , preparing to evaluate, individual Intervlews and self-reporting by teachers indicated that the pre-existing mathematlcs curriculum was not providing problem solving activitles for students on a regular basis. As a result, teachers began an implementation perlod for the recommendations of Standard 1 , during which time students were to engage in problem solving activitles at least once weekly, with increasing frequency as the study progressed until eventually, problem solving was used as part of the normal instructional process. In dolng so, teachers attempted to provide activitles for students which allowed them to view mathematics in a more useful and personal manner. The implementation perlod for Standard 1 requilred nine weeks.

The last phase of the study, a post-implementation perlod, consisted of a second set of Indluldual Interviews, an Attitude Assessment Survey administered to students and follow-up focus group discussions. Results were very
posltive. Students reported that they enjoyed the opportunlty to explore and thlnk for themselves, and Indlcated a bellef that they were learning more mathematics. Teachers indlcated plans to continue problem solving activities on a regular basis and to present new materlal in this manner when posslble. Overall, the Implementation of the recommendations of Standard 1 was successful. In addition, teachers reported numerous factors which they belleve wlll enhance/inhiblt Implementation of the NCTM Standards.

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## CHAPTER I

## INTRODUCTION


#### Abstract

MATHBMATICAL POWER. Thls term denotes an Individual's abllitles to explore, conjecture, and reason logically, as well as the abllity to use a varlety of mathematlical methods effectively to solve nonroutline problems. Thls notion is based on recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communlcation, and notions of context. In addltion, for each individual, mathematical power involves the development of personal self-confidence. The National Councll of Teachers of Mathematics, 1989, page 111 .


Never before has our nation's public school system been the object of such Intense scrutiny. Never before has there been such public demand for accountabllity by schools and teachers. Partlcularly durlng the decade of the 19803 the business community, the general public, and many educators have realized the development of any country depends on the intellectual development of its people (Costa, 1985). An explosion of sclentific and technological knowledge has increased publlc awareness of the importance of mathematlcs education in preparing young people to llve and work in the soclety of the 1990 and Deyond (Davis and Hersh, 1981; Paulos, 1988). The National Councll of Teachers of Mathematlos (1989) In their

## Professional Standards for Teachlng Mathematles has stated that:

> Pupll performance on standardized mathematlcs tests, comparative results on International studles of mathematics education, increasing attrition from the mathematics teachling ranks, and the reasslgnment of teachers not properly qualifled to flll mathematics teaching positlons have ralsed concern about the quality of the mathematics instruction being glven our natlon's youth (page 1 ).

These and other factors have caused many educators to examine the exlsting gap between the reallty of mathematlcs education in schools and classrooms across the continent and the recommended standards of professional practlce for high-quality mathematics education for Amerlcan students.

In a document entitled Curclculum and Evaluation Standards for School Mathematlcs, NCTM (1989) has sald:

Historlcally, the purposes of secondary school mathematics have been to provide students with opportunities to acquire the mathematical knowledge, sklll, and modes of thought needed for dally llfe and effective citizenship, to prepare students for occupations that do not require formal study after graduation, and to prepare students for postsecondary education, partlcularly college (page 123).

However, none of these goals are belng achleved successfully. A new awareness of mathematics education $1 s$ rapldy causing many professionals to conclude that all students need to learn more, and often different, mathematlcs and that the current mathematics curricula must be significantly revised. Research by the National

Research Councll and others has shown that most students cannot learn mathematics effectively by only llstening and Imitating; yet most teachers contlnue to teach mathematlcs just this way. Most teachers teach as they were taught, not as they were taught to teach. Mathematics continues to be primarlly a passive activity: teachers prescribe; students transcribe. Students slmply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Most students gradually construct a view of mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. Practlcing the skllls of mathematlcs often becomes the goal of learning, rather than one of many strategles used by teachers to help students achleve mathematics understandlng. Presentation and repetition help students do well on standardized tests and lower-order skllls, but they are generally ineffectlve as teaching strategles for long-term learning, for hlgher-order thinking, and for versatile problem-solving (Everybody Counts, 1989). Because mathematlcs $1 s$ one of the pillars of education, reform in education must include significant change in the way mathematics is taught and learned. As mathematics and soclety change continuously, so too must mathematlcs education.

The special role of mathematics in education today is approprlately summarlzed by the National Academy Press in

Everybody Counts, 1989, as a consequence of lts unlversal applicability. Mathematlcs is a sclence of pattern and order. The process of "dolng" mathematics ls far more than Just calculation or deduction; it Involves observation of patterns, testing of conjectures, and estimation of results. More speciflcally, The Natlonal Academy Press states that:

> Mathematlcs offers distinctive modes of thought whlch are both versatile and powerful, Including modeling, abstraction, optimization, logical analysis, Inference from data, and use of symbols. Experience with mathematlcal modes of thought builds mathematical power -- a capaclty of mind of increasing value in thls technological age that enables one to read critically, to ldentlfy fallacles, to detect olas, to access risk, and to suggest alternatives. Mathematics empowers us to understand better the information-laden world in whlch we live (page $31-32$ ).

According to the National Research Councll, prlor to the $1980 s$ it had been widely accepted that the learning of mathematics required some special, Innate abllity, whlch most students, partlcularly females and mlnorltles, did not possess (Everybody Counts, 1989). Parents often accepted and even expected their child's poor performance in mathematlcs. In addltion, these parents tended to measure the mathematical needs of today's students by their own experiences and accomplishments. The fact that many adults who never learned mathematlcs had been able to survive and perhaps even succeed wlthout it helped propagate an
attltude of acceptance for poor mathematics performance. However, these attitudes are slowly changling.

The technological advances of the twentleth century have helped transform the fleld of mathematics from one of abstraction Into a profound and powerful part of human culture. The ldeas of mathematics Influence the way we live and the way we work on many different levels. Mathematics can have a practical affect on our llves as we compare prices, calculate rlsks, make more informed consumer cholces, and try to understand the effect of varlous rates of inflation -- all of which can help to improve individual living standards. Mathematics can Impact our professional lives, In applications rangling from theoretical physics to buslness management, slnce lt serves as a prerequlsite for hundreds of careers. Mathematics can affect our civic cholces as soclety debates over such pollcles as tax rates, nuclear deterrence, public health matters, projected population growth, and the many Interactions among the varlous factors of economic growth. Mathematics can even affect our lelsure activitles as ls readly evident by the popularlty of lotteries, sports wagers, and varlous other games of loglc, chance and strategy. Mathematics has become a corner-stone of our present soclety, applicable in almost every aspect of everyday life. Mathematical literacy for all students must become a natlonal goal If we are to prepare today's
students for the twenty-flrst century. Now more than ever, mathematics literacy, mathematical power, must become the educatlonal goal for all students rather than the private domain of a select tew.

OVERVIEW OF THE AREA OF CONCERN

The skills and expertlse of a country's workforce are the foundation of its economlc success. Lately, In our country, this foundation appears too fraglle to withstand the challenge of the 21st century.
National Assessment of Educatlonal Progress, ETS, 1990.

According to recent findings which have been made publlc by the Natlonal Assessment of Educatlonal Progress In The Mathematles Report Card (1990), mathematics instruction consists almost exclusively of teacher explanation, rellance on textbook and chalkboard demonstratlons, regular homework asslgnments, and routlne testing. Their findings Indicated that the only deviation from earlier patterns of Instruction (documented In a 1986 study by the NAEP) were student reports of significant Increases in homework asslgnments and testing. This recent shift may be the only notlceable response to demands for increased academic rigor in the fleld of mathematics. Even though the increased emphasls on sklll development and testing is perhaps warranted, the lack of Innovative Instructlonal approaches and curriculum changes is cause
for concern. The Mathematics Report Card also Indlcates that students say they rarely engage in any activities which would allow them to apply their mathematical skllls In real world situations.

Excessive emphasis on the mechanlcs of mathematics not only inhiblts learning, but also propagates the widespread misconception that the use of mathematical methods leads to a single correct answer (National Research Councll). Mathematics Instruction must not relnforce the common impression that mathematics 13 the product of authorlty, maglc, or wlzardry. The National Academy Press (1989) points out:

Mathematlcs $1 s$ a natural mode of human thought, better sulted to certaln types of problems than to others, yet always subject to conflrmation and checking with other types of analyses. There $1 s$ no place in a proper curriculum for mindless mimlcry mathematlcs (page 44).

The abllity of each individual to use mathematics wherever it arlses in their later lives depends heavily upon the attltudes conveyed toward mathematics in our classrooms. If we expect students to make use of thelr mathematlcs abllity as wage-earners, parents, or cltizens, then steps must be taken to assure that the mathematics curricula In our schools leave a legacy of confidence, clarity, and empowerment, rather than one of mlsunderstandling, apprehenslon, and fear.

The Irony of the current lack of mathematical understanding in our present society is that young chlldren enjoy mathematics and are naturally good at dlscovering patterns and making conjectures (National Research Council). The natural curloslty of a young chlld is a powerful teacher of mathematics. Unfortunately, as chlldren grow and become soclallzed by school, their perceptions of mathematics gradually shift from enthusiasm to apprehension, from confldence to fear. More than half of all students leave mathematics under duress, convinced that only the extremely intelligent can make sense of 1 t. Later, as parents, they pass thls same attltude on to their chlldren. Even more tragic is that some teachers convey this attitude to thelr students.

Contalned in the preface of Eyerybody Counts, The Natlonal Academy Press (1989) comments:

Three of every four Americans stop studying mathematics before completing career or job prerequisites. Most students leave school without sufficlent preparation in mathematics to cope with elther on-the-job demands for problem-solving or college expectations for mathematical Ilteracy. Industry, universities, and the armed forces are thus burdened by extensive and costly demands for remedial education. Our country cannot afford continuling generations of students limlted by lack of mathematical power to second-class status in the soclety in which they live. It cannot afford to weaken lts preeminent position in sclence and technology (page vili).

Even though there $1 s$ no set educational policy for mathematics in the United States, it remalns true,
partlcularly in the fleld of mathematlos, that teachers tend to teach what is in the textbook and students learn only what will be on the test (National Research Councll, 1989). The National Academy Press (1989) states:

In practice, although not in law, we have a national curriculum in mathematics education. It is an "underachleving" curriculum that follows a spiral of almost constant radlus, revlewlng each year so much of the past that little new learning takes place <page 45).

In the past, these standards for mathematics seemed sufficient, if somewhat limited. However, the most recent analyses of school mathematics have concluded that students are not acquiring the skllis and understandings they wlll need for the technology of the future (National Assessment of Educatlonal Progress, ETS). Data from the National Assessment of Educatlonal Progress and from college entrance testing programs reveal a discouraging pattern of mathematics achlevement, partlcularly in important problem-solving and higher-order thinking skills.

There 1 s no shortage of advice on new dlrections for the K-12 mathematles curriculum. The challenge of defining new curriculum priorlties and new standards for teacher performance and student achlevement has attracted attention from a broad range of groups interested in school mathematics (Natlonal Research Council, Natlonal Assessment of Educational Progress, National Councll of Teachers of Mathematics, The National Sclence Board, and others).

Thelf recommendations respond to two generally perceived problems in mathematics education in grades 7 - 12. In a recent artlcle in Educational_Leadershle, Zalman Usiskin (1987) described those problems in the following manner:

The first $l s$ that high school graduates are not learning enough mathematics. And second, the mathematics curriculum has not kept up with changes in mathematics and the ways mathematics is used in business, Industry, and the marketplace (page 31 ).

A mathematics curriculum can no longer afford the luxury of a program whlch ls prescribed for college-preparatory students. Students must prepare now for a world where the benefits and responsibllitles of full cltizenship will require a substantial measure of sklll and understanding in the mathematlcs of sclence and technology. No longer can we settle for a mathematles curriculum that provides its students with only mindless tralning in mechanlcal skllls.

In 1978 the Natlonal Council of Supervisors of Mathematics proposed a list of ten basic skllis in mathematics. These skllls -- problem solving; applying mathematlcs in everyday situations; alertness to the reasonableness of results; estimation and approximation; appropriate computatlonal skllls; basic geometric properties; measurement; use of tables, charts, and graphs; using mathematics to predlct; and computer literacy -reflect new goals for mathematlcs currlcula. These goals
are not simply a matter of style or approach; they constltute a fundamental change in the content of both the elementary and secondary mathematics curricula. The National Council of Teachers of Mathematics In its Curcleulum and Evaluation Standards for School Mathematles (a copy of the NCTM Currlculum and Teachlng Standards appears In Appendix $A$ of this document) has endeavored to create a vision of mathematics education which can help produce those changes. NCTM stated within the mentioned document that:

The fourteen standards developed by NCTM for grades 9 - 12 establish a framework for a core curriculum that reflects the needs of all students, expllcitly recognizing that they will spend their adult lives in a soclety increaslngly dominated by technology and quantitative methods (page 123).

At the very center of this core curriculum is the concept of mathematical problem solving, which should be the focus of school mathematics.

PURPOSE OF THE STUDY

The purpose of this case study, therefore, was to evaluate the current status of problem solving in a mathematics curriculum in a typlcal high school, and to examine the process involved in the implementation of the recommendations found In NCTM's Standard 1: Mathematlcs as

Problem Solving. More speciflcally, the following questions were used to gulde thls program evaluation:

1. To what extent are the recommendations of Standard 1 not belng satlsfled by the current mathematlcs curriculum $\ln$ grades $9-12$ in a specifled hlgh school?
2. What are the changes perceived by teachers to be necessary before the currlculum recommendations found in Standard 1 can be implemented?
3. What are the aspects of current mathematlos education which may inhiblt or enhance the implementation of NCTM's vision for a more relevant and useful mathematlcs curciculum within a typlcal school?

The NCTM Curriculum Standard which was selected by this investlgator to gulde thls program evaluation is:

Standard 1: Mathematlcs as Problem Solving

In grades 9 - 12, the mathematlcs currlculum should Include the refinement and extension of methods of mathematlcal problem solving so that all students can --
--use, with increasing confldence, problem-solving approaches to investlgate and understand mathematlcal content;
--apply Integrated mathematical problem-solving strategies to solve problems from within and outslde mathematles;
--recognize and formulate problems from situations within and outside mathematics;
--apply the process of mathematical modeling to real-world problem situations.

During the evaluation of Standard 1: Mathematics as Problem Solving, this Investigator utllized those Indlcators of quallty for Standard 1 which were developed by the Center for Educational Research and Evaluation at UNC-G for the evaluation of mathematics programs. Those Indlcators are:

Standard 1: The curriculum provides students with the opportunlty to engage in problem solving.
1.1 The curriculum provides students with the opportunity to solve problems on a regular basis.
1.2 The curriculum provides students with the opportunlty to define problems from everyday life as well as mathematlcal sltuatlons.
1.3 The curriculum provides students with the opportunlty to develop and carry out plans to solve a wide varlety of nonroutine problems.

1.4 The curriculum provides students with the
opportunity to look back at the orlginal problems
to verify and interpret their results.

### 1.5 The curriculum provides students with the opportunity to generalize solutions and strategles to other sltuations.

A comparison of the recommendations found in Standard 1 and the five Indlcators of quallty which were used to evaluate problem solving in the mathematics curriculum show they parallel one another in all but one area. The first recommendation of Standard 1 states that the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can use with increasing confidence, problem solving approaches to investigate and understand mathematical content. The quallty Indicators do not address this recommendation. Therefore, thls aspect of Standard 1 was assessed in the following manner: comments found In Indlvidual teacher journals, examples of student work from teacher portfollos, and the examination of the results of a problem solving attitude assessment survey for students. (See Appendlx B for a copy of the survey.)

## SCOPE OF THE STUDY

Only from his actions, his fixed utterances, his effects upon others, can man learn about himself; thus he learns to know himself only by the round-about way of understandlng. What we once were, how we developed and became what we are, we learn from the way in which we acted, the plans whlch we one adopted, the way in which we made ourselves felt in our vocation, from old dead letters, from judgements on which were spoken long ago....we understand ourselves and others only when we transfer our own llved experience into every kind of expression of our own and other people's lives.

Wllhelm Dllthey (1910)

This investigation involved the use of technlques from qualltative or ethnographlc research and has utillzed them In varying degrees as beflts an emergent study. Much of the sclentiflc research whlch has been undertaken has failed to impact upon the realltles of classroom teachers (Hitchcock \& Hughes, 1989; CalkIns, 1985; LaCompte \& Goetz, 1984; McCutcheon, 1981; Paul, 1990; Patton, 1980; Rogers, 1984). Robert Stake (1986) has sald:

The quallty of educational practlce, particularly its teaching and administration, rests largely on intuitive and experiential processes. Some wish that educational practice would be more rational and technical -- but immediate improvement in practice continues to rely largely on experiential understandings (page 46).

If we are to Improve mathematlcs currlcula and Instruction in schools today, we must endeavor to understand the partlcular sltuation, the particular program. Our past efforts to generalize the teaching and learning of
mathematlcs 1 part of the exlsting problem. The intent of this Investigation was to provide a comprehensive understanding of a particular sltuation through which the report readers can draw thelr own generalizations through the combination of previous experience with the new. This Investigation was therefore a case study of the evaluation of problem solving in the existing mathematics curriculum and the attempt to Implement NCTM's standard 1: Mathematics as Problem Solving. The Investlgation attempted to focus upon the congruence of the current mathematics curriculum and the recommendations found in Standard 1 , and examined the process involved in the implementation of the standard into the existing program. The perlod of Investigation was set to cover a time span of less than one semester, and to cease when thls investlgator was able to conclude that Standard 1 had been Implemented or that no additional progress could be made toward implementation of Standard 1 . The Intent of this Investigation was not to determine whether implementation of NCTM's Standards wlll or can brlng about the prevlously stated and much needed reforms in mathematics education. Unfortunately, verlflcation of those types of results were beyond the scope of this study, and perhaps will not be known durling thls decade. However, if Standard 1 is never implemented within the present mathematics currlculum, the question of a resultant mathematics educational reform may
never be addressed at all. Of special interest durlng thls program evaluatlon was the analysis of problems whlch mathematics teachers belleve will result due to the attempted Implementation of Standard 1 . An addltional set of concerns revolved around whether the Standards could be Implemented into the exlsting mathematlcs curriculum, and whether implementation of the standards is even possible given the present classroom condltions and expectations. The study includes comments from teachers concerning thelr perceptions of whether curriculum changes will occur as a result of the investigation. The Investigation attempted to assess the congruence between the current curriculum goals for problem solving and those recommended by Standard 1 and will make recommendations based on those findings. The mathematics program at South Caldwell High School, located In Hudson, North Carolina, was selected for this case study by the investigator.

SIGNIFICANCE OF THE STUDY
Clearly, we know more today about teaching, learning and the development of relevant, useful mathematics currlcula than we did twenty years ago, and yet many of today's students recelve much the same mathematics instruction from exactly the same mathematics curricula as that which was being glven in mathematics classrooms decades ago (Natlonal Research Councll). The 1980s have
been viewed as the decade which reported that schools were not succeeding in their jobs of educating students. It was the decade which demanded quallity education in our public schools across the nation. As a result, we are now experlencing an era of educational reform. The publlc has issued a clear challenge to educators for the next decade: to improve student learning and achlevement, partlcularly student learning and achlevement of hlgher-order thinking skills, such as problem solving in mathematics. There is widespread agreement that major changes are required in mathematics curricula if mathematics programs are to prepare students for the world in whlch they wlll live and work. Considering the current atmosphere in which schools must function and the proliferation of so-called cures proposed by persons lnside and outslde the realm of education, it would seem advisable to look directly at the day to day currlculum practices within typlcal mathematics classrooms. If we can accurately determine the type of curriculum standards teachers are currently using inside thelr classrooms, how closely that curriculum matches the professional views of what a mathematles currlcula should be, and the percelved problems which wlll be encountered if change is allowed to happen, perhaps then wise decisions can be made which will engage students more meaningfully in the study of mathematics. Although this study involves only one standard, thls one standard provides the baslc
framework for all the other standards. The concept of problem solving is the basic foundation for the type of mathematics curriculum proposed by NCTM and others. The depth of this inquiry can be expected to yleld insights which may serve mathematics educators in providing the kind of curriculum standards that support the goals and visions of the NCTM Curriculum and_Evaluation_Standards for School Mathematics.

## CHAPTER II

## LITERATURE REVIEW


#### Abstract

The most important purpose of evaluation is not to prove, but to improve.... We cannot be sure that our goals are worthy unless we can match them to the needs of the people they are intended to serve.


Dandel Stufflebeam, 1985.

There are many reasons for conducting mathematics program evaluation. In general, evaluatlons are needed for the following reasons: to determine effectlveness of exlsting programs, to determlne whether changes are needed In existing programs, to set prioritles and formulate program goals, to develop a program which is sulted to a particular school, and to determine whether a program meets quallty standards. The purpose of thls case study dealing with the evaluation of problem solving is of course the latter. Today we stand on the threshold of the twenty-first century, reallzing mathematlcs education is critlcal to the current generation of students. We also reallze most students do not possess the mathematics proflciency needed to adapt to the technologlcal soclety in which they must live and work. Improving mathematlics performance among our nation's youth wlll require upgrades in the curriculum, corresponding modifications in classroom instruction, and the use of approprlate evaluation results.

The Natlonal Councll of Teachers of Mathematics, In an effort to reform school mathematics, has developed a set of professional standards for the teaching of mathematics. For NCTM, the development of standards as statememts of criteria for excellence in order to produce change was the focus. Schools must reflect the consequences of the current reform movement if our students are to be adequately prepared to live in the twenty-first century. NCTM advocates that the standards should be viewed as facilltators of reform. The purpose of thls inquiry was to assess current curriculum practices with reference to NCTM Standard 1: Mathematics as Problem Solving. The focus of this case study consisted of an evaluation of those speciflc parts of the mathematlcs currlculum whlch provide opportunltles for students to engage in problem solving. During this review of literature relevant to educational evaluation, thls investlgator has attempted to define the structure of the evaluation technlques used in the program evaluation.

A portion of the scholarly literature relevant to this study has been revlewed to galn inslght into seven major areas: (a) mathematics as problem solving, (b) defining the concept of evaluation, (c) planning and evaluation models, (d) assessing the problems of evaluation, (e) planning and conducting evaluation studies, (f) analyzing and

Interpreting evaluation Information, and $\langle g$ ) the cole of evaluation $\ln$ mathematics classrooms.

## Mathematics as Problem Solving

Problems and thelr solutions have always occupled a central place in any school mathematics curriculum, but problem solving has not. "Only recently have mathematics educators accepted the 1 dea that the development of problem solving abllity deserves special attention" (Charles \& Sllver, page 1). It has only been during the last decades that the focus of the teaching of problem solving has shifted from a phllosophy that students should be presented with problems and the rules for solving those particular problems to one which advocates a more general approach to problem solving. Until thls century, It was assumed that the study of mathematics would in some way lmprove an Individual's intelligence or abillty to think. Grube has quoted Plato as saylng that:

> Those who are by nature good at calculation are, as one might say, naturally sharp in every other study, and. those who are slow at it, if they are educated and exerclsed in thls study, nevertheless Improve and become sharper than they were (page 18).

As such, solving problems in the curclculum was simply a ploy to get students to study mathematics. "Problems were a glven element of the mathematics currlculum that contrlbuted, llke all the other elements, to the
development of reasoning power" (Charles \& Sllver, page 10). Mental discipllne theorles during the nineteenth century provided the framework for the ldea that mathematics provided a primary vehlcle for the development of the reasoning faculty for an indlvidual's mind.

Near the beginnling of the twentleth century, the work of Edward L. Thorndike led to signlficant changes in how the study of mathematics was viewed and as a result, he ls generally credlted with refuting the basic notions of mental discipline theory. However, even Thorndlke never completely rejected the ldea of mental discipline. Consequently the early $1900 s$ witnessed two very different ways of looking at people, education, and the school curriculum. The mental disciplinarlans argued that mathematics was a crucial element of the curriculum and that all students could beneflt from the same knowledge and methods of Instruction. Thorndlke, however, provlded the foundation for the Idea which advocated the need to expose different children to different subject matter. Critlcs of mathematics began to feel that most people needed to know no more than sixth grade arlthmetic. The 1930s saw the place of mathematics in the school curriculum come under attack which led to a crisis in mathematics education. On the one hand, crltics were calllng for methods of making mathematics more relevant to real life, whlle mathematics educators were afrald of glving up the former role of
mathematlcs for the sake of applicatlon. Thls crlsis has yet to be resolved. According to Charles and Silver (1988):

The events surrounding the decilne of mental disclpline theory may have set the stage for mathematics educators to begin to give more specific emphasls to the development of problem solving abillty, but the clash of basic ideas about human intelligence, education, and the school curriculum stlll permeates discussions of problem solving <page 13).

The term problem solving is used in many different contexts and has many different meanings. The three most common Interpretations of problem solving are: (1) problem solving as context, (2) problem solving as skill, and (3) problem solving as art.

## Problem Solving as Context

Generally it is agreed that problems and the solving of problems are a means to achleve other valuable ends. Historically, problem solving has held an Important place In the mathematics curriculum because it helped provide justiflcation for the teachling of mathematlcs. If some problems in the curriculum related in some way to real-world experlences, then they served to convince students of the value of mathematics. Problem solving has also been used in an effort to galn student attention and to motivate them to learn new processes or algorithms. The use of puzzles and other problems without any real-world


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connections are used to allow students to have some fun wlth the mathematics they have already learned. Problem solving and discovery techniques can provide a vehicle for learning new concepts and skills. And flnally, problem solving as practice has had the largest influence on the mathematics curriculum as it provides the necessary practice to reinforce skills and concepts.


## Problem_Solving_as_Sklll

Problem solving is considered in some instances as one of a number of skllls to be taught in the school curriculum, rather than as simply a means to achleve other ends. Placing problem solving in a hlerarchy of skllis to be acqulred by studente often leads to a distinction between solving routine and nonroutine problems. As such Charles and Silver (1988) state that nonroutlne problem solving can be:

Characterlzed as a higher level skill to be acquired after sklll at solving routine problems (which, in turn, ls to be acquired after students learn basic mathematical concepts and skllis). This view postpones attention to nonroutine problem solving, and, as a result, only certain students, because they have accomplished the prerequisites, are ever exposed to such problems (page 15).

The lack of exposure to nonroutine problem solving for all students ls a common characteristlc of the mathematics school curriculum $\ln$ classrooms today.

## Problem Solving as Art

The work of George Polya and hls view of problem solving as art provides a more comprehenslve view of problem solving in the school mathematics curriculum. Polya's experlence in learning and teaching mathematics led him to revive the ldea of heuristics sthe art of dlscovery.) Polya belleved that students would understand mathematics much better if they could get some taste of mathematical discovery for themselves.

To Polya, problem solving was an art "like swimmlng, or skiling, or playing the plano," whlch must be learned through imitation and practice. Polya belleved that simply solving problems did very little to lmprove performance, nor did he agree that the study of mathematics contributed by its very nature to one's general level of Intelligence. Polya deflned problem solving as the process of finding the unknown means to a distinctly conceived end. Charles and Sllver (1988) state that Polya:

Recognized that technlques of problem solving need to be illustrated by the teacher, dlscussed with the students, and practiced in an insightful, nonmechanlcal way....He observed that although routine problems can be used to teach students to follow a speciflc procedure or use a definition correctly, only through the judicious use of nonroutine problems can students develop their problem solving ability <page 16).

To Polya, the teacher is the key to providing the right kind of problem for a given class and the proper
amount of help and guldance. Therefore Charles and Sllver have sald that Polya felt:

> No one can program the teaching of problem solving; it remalns an actlvity that requires experlence and judgement. In a sense, problem solving as art gets reduced to problem solving as skill when attempts are made to implement Polya's ldeas by focusing on his steps and putting theminto textbooks (page 17 ).

Certalnly Polya did not provlde a reclpe for maklng all students Into accompllshed problem solvers. However, he did provide us with the baslc lssues of what problem solving is and why we should teach it.

Polya was one of the first mathematlcs educators to advocate the bellef that mathematics in general and problem solving in particular are for all students. It is the alm of this inquiry to promote the notion that problem solving really is for every student. It is important to provide a curriculum which offers all students the opportunlty to develop an abillty to solve problems. In doing so this curriculum must provide a variety of situations and examples whlch can help students link the subject matter of mathematics to the experience of solving meaningful problems.

Defining the concept of evaluation
"Evaluation $1 s$ one of the most widely discussed but little used processes in today's educational system" (Worthen \& Sanders, page 1). Soclety has demanded that
educational systems be held accountable and leglslators have responded by allocating more and more funds for the evaluation of educational programs. However, "desplte these trends toward accountablllty, only a tiny fraction of the educational programs operating at any level have been evaluated in any but the most cursory fashlon, if indeed at all" (Worthen \& Sanders, page 1). Evaluation is a complex process, indeed, even the process of finding an acceptable definition for evaluation seems not only complex, but controversial. At the most general level, evaluation has been defined as "the assessment of merlt" (Popham, 1975). A somewhat more elaborate definition is provided by L. J. Cronbach who defines evaluation as "[the] collection and use of Information to make decisions about an educational program" (Cronbach, 1963). Rlchard Wolf (1979) has sald:

This definition of evaluation, emphasizing the collection and use of information about learner performance, is a distinct lmprovement on the "assessment of merit" definition, but it stlll does not go far enough in saying what evaluation is spage 3).

A more extended definition, supplied by C. E. Beeby, describes evaluation as "the systematlc collection and Interpretation of evidence, leading, as part of the process, to a judgement of value with a view to action" (1975). Implicit In both the Beeby and Cronbach definitions is the distinction that evaluation is
decision-orlented and that 1 ts Intent is to lead to better policles and practices in education.

Ralph W. Tyler ls generally consldered the father of educational evaluation. "In general terms, Tyler considered that evaluation should determine the congruence between performance and objectlves" (Stufflebeam \& ShInkfleld, page 70). Tyler saw the purpose of evaluation as providing a check as to whether the plans for learning experiences actually function to gulde the teacher in producing the desired outcomes. The Tylerlan approach suggested the utilizaton of feedback in educational improvement; however, it has been used almost exclusively to Judge flnal success only. The Tylerian concept of relating outcomes to objectives, gave predominance to a terminal process that yielded information only after the full cycle of the program had occurred. Thls view has continued and is reflected in several current approaches to evaluation.

Edward Suchman, however wrote in 1967 that evaluation should be viewed as a scientiflc process. He stated his bellefs that the same procedures which are used to discover knowledge could be utllized to evaluate the degree of success $1 n$ the application of this knowledge (Stufflebeam \& Shinkfleld). Suchman advocated that program evaluation should conslst essentially of the measurement of success in
reaching the practical objectives of an educational program.

During the last few decades, new deflnations of evaluation have emerged, among whlch are those of Robert Stake, 1967 and Danlel Stufflebeam, 1971. As an alternatlve to the Tylerian definltion, Danlel Stufflebeam redefined evaluation as "the process of providing useful information for decision making" (Stufflebeam, 1966). Stufflebeam reported that evaluation in general was the victim of a great lllness, recognizable by symptoms exhiblted by evaluators at all levels of education and by the dismal quality of their evaluation work. Stufflebeam sees the role of evaluation as a means of sorting out the good from the bad, a method of pointing the way to needed improvements and of helping educators gain a better understanding of thelr field.

Robert Stake has "argued that evaluation's baslc function in education should be to gulde curriculum improvement, not to judge completed, packaged curriculums" (Stufflebeam, page 211). Stake defined evaluation as "an observed value compared to some standard." Stake advised the evaluator to make a comprehenslve statement of what the program is observed to be and to reference the satisfaction and dissatisfaction that appropriately selected people feel toward the program. He views the evaluator as a "truth seeker" and has cautloned that many outcomes, rather than
only those whlch are measurable, testlfy to the worth of an education program.

James Sanders and Blalne Worthen (1987) define evaluation as:

> The determination of the worth of a thing. It includes obtalning information for use in judging the worth of a program, product, procedure, or objective, or the potential utllity of alternatlve approaches designed to attaln specifled objectives (page 19).

Thus it seems an unlimited number of definltions for evaluation exlst, some of which have strong commonalitles. Obviously, the way $\ln$ which one deflnes evaluation has direct impact on the type of evaluation activities conducted. The ultimate role of evaluation must be the determination of merlt or worth. According to Worthen and Sanders (1987):

> Evaluation can play many roles In an educational program: it can aid the developers by providing mastery test data, and lt can provide data to facilltate administration of the program, to name only two. However, the goal of evaluation must always be to provide the answer to an all-limportant question: Does the phenomenon under observation have greater value than its competitors or sufflcient value of itself that it should be malntalned? (page 26).

The definltion of evaluation which was used to gulde this case study is a combination of those offered by Stufflebeam and Stake. The evaluation has endeavored to compare an observed value to a set of standards, then polnt the way toward needed lmprovement and a better


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understanding of the current mathematlcs curriculum. This investigator chose this comblnation of definitions since it implles evaluation should be concerned wlth process rather than simply with outcomes or products. However, the investigation also used Provus' Discrepancy Evaluation definltion (1971) whlch states:


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Program evaluation $1 s$ the process of (1) defining program standards; (2) determining whether a discrepancy exists between some aspect of program performance and the standards governing that aspect of the program; and (3) using the discrepancy information elther to change performance or to change program standards (page 183).


Thls investlgator used Provus' definltion in the assessment of the congruence of the current mathematics curriculum at South Caldwell HIgh School and the criteria of NCTM's Standard 1: Mathematics as Problem Solving.

This case study began as a program evaluation and as such compared current problem solving practlces with those recommended by NCTM. However, it also focused upon the dynamics involved as change occurs in an existing program. Implementation of Standard 1 was expected to be gradua! with problem solving activities belng added in a systematlc manner in the beginnlng, until eventually problem solving becomes an integral part of the mathematics curciculum.
Planning and Evaluation Models
Since there is more than one method of conducting a defenslble educatlonal evaluation, the skllled educational
evaluator should be aware of the varlous alternatlve options for carrylng out that task. There are different evaluation strategles for different educational situations. In choosing a particular evaluation design, the evaluator should consider not only its special features but also the conditions under whlch it wlll be used. Each design has certaln strengths and weaknesses. Knowledge of each of them is important, and adequate provision for dealing with the weakness inherent in a particular design ls critical to the success of an evaluation study.

Immedlately following the enactment of the Elementary and Secondary Education Act of 1965, the development of evaluation models was clearly a fashionable activity. However, as $1 s$ often true, some of the later evaluation models incorporated large portions of previously presented models. Each model was developed as a course of action whlch, lf followed, would lead to more effective evaluation. As such none of the models are truly distinct. Popham (1988) in the book Educational Eyaluation stated that:

No matter what factors one chooses to employ in distingulshing among educational evaluation models, the resulting categories fall to satisfy those who would toss particular models lnto distinctive classlfication cells without overlap (page 22).

Popham has devlsed a five-category set of descriptors for the educational evaluation models currentiy avallable.

Popham descrlbes these categorles as nelther flawless nor mutually exclusive. The five classes of educational evaluation models to be considered by thls evaluator are as follows:

Goal-Attalnment Models
Judgemental Models Emphasizing Inputs Judgemental Models Emphasizing Outputs Decision-Facilltation Models Naturallstlc Models

## Goal-Attalnment Models

"A goal-attainment approach to educatlonal evaluation conceives of evaluation chlefly as the determination of the degree to which an instructional program's goals were achieved" (Popham, page 24). The goal-attalnment concept of educational evaluation is generally associated with the efforts of Ralph W. Tyler. According to Popham (1988):

Tyler's general approach Involves the careful formulation of educational goals according to an analysis of three goal-sources (the student, the soclety, and the subject matter) and two goal-screens (a psychology of learning and a phllosophy of education). The resulting goals are then transformed Into measurable objectives. At the conclusion of an instructional program, measurements of puplls are taken in order to see the degree to which the previously estabilshed goals were achleved (page 25).

Educatlonal goals and the degree to whlch they are achleved constitute the heart of Tyler's evaluation approach.

A more recent varlation of the goal-attalmment model was proposed by Hammond (1969) and includes: (1) lsolating that aspect of the current educational program to be
evaluated, (2) defining the relevant institutional and Instructional varlables, (3) speclfying objectlves in behavioral terms, (4) assessing the behavior described in the objectives, and (5) analyzing goal-attalnment results. Hammond's model goes into greater depth in an effort to determine the factors which mlght be relevant in consldering the degree to which expressed objectlves are achleved.

Metfessel and Michael (1967) offered an elght step goal-attalnment model which includes: (1) involvement of members of the total communlty, (2) construction of broad goals and specific objectives, (3) translation of specific objectives into forms that are commulicable and that facllitate learning, (4) development of measurement Instrumentation, (5) carrying out perladlc measurement, (6) analyzing measurement data, (7) Interpretation of analyzed data, and (8) formulation of recommendations for program change of modifled goals and objectives. The maln thrust of goal-attalnment models is the degree to which prespecifled instructional goals have been achleved.

## Judgemental Models Emphasizing Inputs

Another class of evaluation models includes those in whlch major attention is given to professional judgement. The evaluator's judgement determines how favorable or unfavorable the evaluation turns out to be. Here the evaluator directs his attention toward inputs, or intrinsic
criteria, which can be referred to as process criterla. The Intrinsic features of a textbook might will be its design, lllustration, and use of color. SHow well the student can learn from the book would concern its outputs, Its extrinsic criteria, also referred to as product criterla.) Judgemental approaches to educational evaluation in which the emphasis is on inputs are very common in education, however Popham vlews most of them as too haphazard to be properly classlfied as systematic evaluation. One exception 19 the accreditation model in which an accrediting agency visits a school and, on the basls of prevlously determined criteria, judges a school's program. In most cases, the interest of the accreditation team is directed toward Intrinslc criteria, such as the number and quallty of books in the llbrary, the degree of trainlng of the school's faculty and the physical plant.

Recently, there has been growlng dissatisfaction among educators for thls type of evaluation due to the lack of empirical evidence to confirm and support the final outcomes of the instructional sequence. Consequently, evaluation models that are dominated with a concern for inputs are not often recommended today.

## Judgemental Models Emphasizlog Outputs

There are several approaches to educational evaluation which can be described as judgemental processes in which the primary attention $1 s$ given to outputs, or extrinsic
criteria. The most slgniflcant of these models have been developed by Michael Scriven (1974) and Robert E. Stake (1967). Although Scriven's position has remalned virtually the same, Stake's views have changed conslderably (thls will be discussed later) over the years.

Scrlven's approach to educational evaluation calls for the evaluator to judge a program, attending chlefly to program outputs. He beglns his model with the formative-summative distinction: an evaluator can formatively attempt to improve a still-under-development instructional sequence or he can summatively assess the merits of an already completed instructional sequence. Scriven views evaluation as an assessment of merit. Popham says this of Scriven:

He is particularly dismayed with those who would equate evaluation merely with the degree to whlch goals are achleved. As he points out, "...it is obvious that if the goals aren't worth achieving, then it is uninteresting how well they are achleved" (page 28).

Scriven recommends that evaluators should never simply appralse a program relative to lts goals; Instead evaluators should appralse the goals themselves.

Scriven advocates a comparative orlentation to evaluation, pointing out that decisions regarding educational evaluation typically involve cholces from alternatives which in turn require comparisons of the competitors. It $1 s$ the Job of the educational researcher
to determine which factors lead to a more effective program. Scrlven has also proposed goal-free evaluation. Popham (1988) has sald that Scriven belleved:


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The goal-free evaluator is not concerned with the rhetorlc of the instructional designers regarding what they want to accompllsh, but rather attends to the results accomplished by the designers' programs (page 30).


The chlef advantage of goal-free evaluation $1 s$ that it encourages the evaluator to focus on a wider range of program outcomes than might be possible when the evaluator has been influenced to look for project results related to project alms. A well-designed evaluation, according to Scriven, would contaln both goal-based and goal-free evaluation.

In 1967, Robert E. Stake proposed a system of evaluation often referred to as his Countenance Model. His 1967 conception of evaluation emphasized two activities: description and judgement. Stake distingulshed between descrlptive and judgemental acts of the evaluator according to three phases of an educational program: its antecedent, transaction, and outcome phases. Antecedents, according to Stake, are condltions which exist prior to instruction which may relate to outcomes; transactions constitute the process of instruction; and outcomes are the effects of the Instructional programs. Stake divided descriptive acts through references to what was intended or what was
actually observed; Judgemental acts elther refer to the standards used in reaching judgements or to the actual Judgements themselves. Popham's (1988) account of Stake's view indlcated that:

> He pointed out that when we judge an educational program we engage elther in relative comparlson cone program versus another), absolute comparison (one program versus standards of excellence not associated with any partlcular program), or both relative and absolute comparison. The real payoff in the Countenance Model, of course, was the Judged outputs of the program being evaluated (page 33).

## Decislon-Facllitation Models

Decision-Facilltation Models differ from judgemental models in that the evaluator is less wllling to assess personally the worth of the educational program. They, in essence, collect and present information to someone else, who will then determine worth. One of the best known decision-facilitation evaluation models is the CIPP, an acronym representing the four types of evaluation this model ldentlfles: content, Input, process, and product evaluation. The CIPP Model was designed by Danlel Stufflebeam and Egon Guba (1971) and "Is deeply rooted in Its definition of evaluation: evaluation $1 s$ the process of delineating, obtaining, and providing useful information for Judging decision alternatives" (Popham, page 34). The three major steps in the CIPP model are: (1) dellneating, or a focus on the information requirements of the decision maker, (2) obtaining or the collection, organization and
analysis of information using measurement and statistics, and (3) providing a synthesis of the Information. All three steps Involve Information and how it can best be isolated, gathered and presented to those Individuals who Wlll make decisions. The CIPP model provided the flrst gulde for evaluators who belleved that their primary goal was to ald those who make decislons.

A second decision-facilitation model, offered by Malcolm Provus (1971), $1 s$ the Discrepancy Model, so called due to the particular attention pald to the discrepancies between posited standards and actual performance. The Discrepancy Model consists of flve stages: (1) design, which focuses upon documenting the nature of the program, (2) Installation, or a determination of whether an installed program 13 congruent with lts Installation plans, (3) process, or an assessment of whether enabling objectives are belng achleved, (4) product, or an assessment of whether terminal objectlves are beling achleved, and (5) program comparlson, or a cost-benefit analysis. After performance is compared to standards, the discrepancy information can lead to four alternatives: the program can be ended; the program can proceed unaltered; the program can be altered; or the program standards can be altered.

Naturallstle Models
The final category of educational evaluation models to be considered here is referred to as naturallstlc or qualltative, an evaluation model which places few or no constralnts upon potentlal outputs or those factors which are present in an evaluation at its outset (such as pupil aptitude).

In the 1970s, Robert Stake became somewhat disenchanted with his own highly structured Countenance Model and began to endorse a model he characterized as a responsive evaluation. Popham's (1988) account of Stake says:

He argued that an educational evaluation would be responsive if it "orlents more directly to program activities than to program intents, responds to audience requirements for information, and if the different value perspectlves present are referred to In reporting the success and fallure of the program." Whereas Stake considers most conventional evaluations to be formal, preplanned, objective, and based on prespecified intentions, he views responsive evaluation to be informal, flexible, subjectlve, and based on evolving audlence concerns (page 42).

Ellot Elsner has also developed a model for naturallstic educational evaluation. Elsner's model relies upon the two concepts of educatlonal connolsseurshlp and educational criticism. Connolsseurs are able to appreciate the subtle qualitles of a complex educational phenomena, whlle the critlc serves the role of disclosure. Popham (1988) has sald Elsner belleves:

The educational critlc strlves not only to discern the qualltles constltuting an event or object, but also to render in verbal form what has been experlenced, so that those who do not possess the critic's level of connolsseurship can understand what the critlc has percelved (page 43).

There have been very few guidellnes provided for the Implementation of a connolsseurship model and as a result, this model has been employed mostly by Elsner, his co-workers, and hls students.

An ethnography can be defined as a descrlption of a situation in whlch the bellefs, knowledge, behaviors, and practices of those Involved are deplcted. Therefore ethnographlc educational evaluations are thought to yleld a more meaningful plcture of the educational process. An ethnographic evaluation should be gulded by: (1) phenomenology, or the viewpoints of those being studled, (2) hollsm, or the large plcture rather than detalls and the interrelationship among those under analysis, (3) nonjudgementallsm, where the evaluator avolds making judgements and where biases are made expliclt, and (4) contextualization, in which the evaluator examines Information in its own environment in order to provide an accurate representation.

There are of course other models of educational evaluation and those models could perhaps be classified by varlous other methods. However, the flve categorles discussed here serve to provide a useful set of descriptors
for those models currently avallable for educational evaluation. The model selected by this investlgator for use during this inquiry is a combination of several of the characteristics from Provus' Dlscrepancy Model and those quallties speciflc to an ethnographlc study. The Discrepancy Model provides the necessary framework for a comparison of program performance (ln thls case, the current curriculum problem solving practlces) to NCTM's Standard 1: Mathematics as Problem Solving. The discrepancy information (when found) was gathered and presented along with recommendations to those individuals responslble for program decislons. Through the use of various ethnographic techniques, this investigator has hopefully presented a more accurate description of many of those bellefs and behaviors whlch can affect the actual mathematics curriculum which is now belng used when teaching students in grades 9-12.

Assessing the problems of evaluation
One of the fundamental goals of program evaluation $1 s$ to determine whether a program is dolng what it is Intended to do, whether it $1 s$ meeting its goals. In order to decide whether a goal is being met, one must know what that goal 1s. In other words, "program evaluation actually has two sets of goals: the goals of the evaluation process itself (the research goals) and the goals of the program belng
evaluated (the program goals)" (Moursund, page 9). It is generally agreed that one fundamental goal of evaluation ls to determine whether the stated goals of a program are belng met. $A$ second and equally valid function of evaluation $1 s$ to determine whether the stated goals are the actual goals on which the program is operating, and if these goals are approprlate. Traditionally, an evaluation begins with setting up, or ascertaining, the goals of the program one wishes to evaluate. C. H. Welss (1972) has suggested there are four major problems in determining the real goals to be dealt with in evaluation.

First, the goals of the program being evaluated may be quite hazy and amblguous. Second, even when goals are stated, the list may not be exhaustive; the program often aims toward objectives not included among its "official" goals. Third, most programs are falrly complex, with different parts dolng different things. It ls difficult to declde how the subgoals of each program part interact to accomplish the overall goals of the program. Finally, good evaluative research must be as concerned with why things happen as with whether they happen. This qualitative aspect of evaluation is usually neglected in proportion to the difficulty of carrying it out, but it is a crucial part of evaluation (Moursund, page 12).

The mathematics currlculum goal whlch I attempted to evaluate durlng the course of this inquiry was:

Standard 1: Mathematics as Problem Solving

In grades 9-12, the mathematics curriculum should
Include the reflnement and extension of methods of
mathematlcal problem solving so that all students can --
--use, with increasing confidence, problem-solving approaches to investlgate and understand mathematical content;
--apply Integrated mathematical problem-solving strategles to solve problems from within and outside mathematics;
--recognize and formulate problems from situations within and outside mathematics;
--apply the process of mathematical modeling to real-world problem situations.
Danlel Stufflebeam attributes the "slckness" of
evaluation to five major problems. The first contributor
is that of definition; evaluation can be defined in many
essentlally arbitrary ways, each of which affects the
method of evaluation and perhaps the resulting judgements
and conclusions. Three partlcular deflnitlons have galned
common acceptance: the measurement definition; the
congruence deflnition; and, the judgement definition. Each
definltion has certaln advantages and disadvantages. The
second problem in evaluation ls one of decision making.
Evaluation is an action-related process In the sense that
an action referent $1 s$ implled $\ln$ every evaluation actlvity. Stufflebeam belleves evaluation $1 s$ in difflculty because knowledge of the decision-making process and of the methodologies for relating evaluation to decision making is woefully inadequate. The third major problem area of evaluation concerns values and criterla. Data collection alone does not constlute evaluation; there is always a need to make judgements about the data in terms of some Implicit or explicit value structure. Rather than asking only whether or not objectives are achleved, the question becomes how well they are achleved. The introduction of values creates a number of problems. A fourth major problem area is that of levels. The problem of levels stems from the fact that the evaluator's traditional polnt of focus has been the indlvidual student, the classroom, the school bullding rather than the school district, the state system or the natlonal network. Many evaluators today are faced with problems at hlgher, broader levels. The final major problem area is that of the research model. Perhaps the greatest challenge facing the evaluator is overcoming the ldea that evaluation methodology is ldentlcal to research methodology. Equating the two makes It impossible to meet certaln needs which are served by good evaluation (Stufflebeam, et al.. 1971).

There are several simpler, more obvious problems in program evaluation. These include: cost, limited amounts
of avallable time, lack of interest and commitment on the part of those involved in the evaluation, reluctance or inability to institute change, as well as a general tendency to fear or distrust evaluation results.

Planning and conducting evaluation studies
Before beglnning the evaluation study, the evaluator should flrst consider a needs assessment. "The purpose of a needs assessment is to ldentlfy the goals for which a program should strive, goals that are important to soclety, not currently belng achleved, and potentlally feasible" (Kosecoff \& Flnk, page 27). After completing the needs assessment, the evaluator should begin work on the management of the evaluation studles. Thls activity should begin before the evaluation 1 s Implemented, and continue until the evaluation 13 completed. Every evaluator must learn to establish schedules which monitor the activitles of the evaluation. Any evaluatlon, large or small, must provide Information which wlll accurately descrlbe what the evaluation program 1s, what it does, and how well it does 1t. Kosecoff and Fink (1982) offer a set of evaluation guldelines which can be used to design a new evaluation or to judge the credibility of an evaluation study done by others.

Guldeline 1: An evaluation must ask speciflc questlons or test hypotheses about a program.

Guldeline 2: Limit evaluation questions to those that wlll provide useful Information for the people who expect to act on it.

Guideline 3: Every evaluation should ask questions about outcomes.

Guideline 4: Evaluations of large-scale programs should always ask questlons about costs and generalizabllity.

Guldeline 5: Standards of program merlt should be set for each evaluation question.

Guideline 6: Standards of merit must be set before any data collection begins.

Guldeline 7: Evaluation standards must have scientiflc valldity.

Guideline 8: Select a design sulted to each evaluation question.

Guideline 9: For evaluation questlons dealing with important lssues or large-scale studles, use a design that establlshes causallty.

Guideline 10: For each question, select a sample representative of the population to which the findings wlll be applled.

Guldeline 11: Sample size should be determined by the extent of the effect that is considered to be meaningful.

Guldellne 12: Use Instruments that are rellable, valld, and sulted to the evaluation question.

Guidellne 13: Use more than one method of collecting Information when assessing important 1 ssues.

Guldeline 14: Keep data collection as unobtrusive as possible.

Guldeline 15: Use analysls techniques that are technically sound and sulted to the quallty of the data.

Guldeline 16: Interpret analysis results in terms of the evaluation questions and standards.

Guideline 17: Report techniques and results so they are meaningful to both the layperson and the professional.

Guideline 18: An evaluation report should answer the evaluation questions and explain how each was arrived at.

Guideline 19: Offer recommendations only on those aspects of a program that the evaluation is specifically designed to study -- and then only lf asked to do so.
(Kosecoff \& Fink, page 49-64)

In the design of thls inquiry, this investlgator has endeavored to conform to each of these guldellnes with the exception of guideline 4 and guldeline 9 which do not seem approprlate for this study. Since Guldellnes 16-19 deal
with analysis, Interpretation, and reporting the results of evaluation, they were used to gulde the final stages of this program evaluation.

Analyzing and Interpreting evaluation information
All evaluations accumulate data that need to be analyzed. Kosecoff and Fink have said:

The difference between an efficlent evaluation and an inefficient one is that the former collects and analyzes just what is needed to answer the evaluation questions, whlle the latter may not collect enough relevant data but instead, gathers information that is not really targeted to the program. One way of ensuring efficiency ls to focus on the evaluation question (page 177).

The evaluation questions shape the entire evaluation, and the evaluator should choose analysis methods whlch will answer the questions directly.

The following questions were used to gulde this Inqulry:

1. To what extent is the criterla of Standard 1 being satisfled by the current mathematlcs curriculum in grades $9-12$ ?
2. What are the changes percelved to be necessary before the curriculum recommendations found in Standard 1 can be fully implemented?
3. What are the factors which may inhlbit or

> enhance the implementation of NCTM's vision for a more relevant and useful mathematics curriculum within a typlcal school?

The flrst step in the analysis and interpretation of evaluation information ls the organlzation and summarization of data. The alm ls to give a reader not only the main results of an evaluation study but the full range of findings. There are several analytical technlques which give answers to commonly asked evaluation questions. Those techniques include descriptive statistics, correlations, and regression analysis. Once the analysls has been completed, the evaluator can begin to interpret the results. Some questions typlcally asked concern program merlt, design strategy and sampling procedures, and validity of Information collection and analysis.

Interpreting results also Involves distingulshing between statistlcal and programmatic signiflcance. Statistical signiflcance tells you whether an outcome makes a difference in terms of program goals -- that $1 s$, whether the outcome Justifles the time, and effort. According to Kosecoff and Fink (1982):

[^0]Finally, analytlcal results point to recommendations as to how to improve or certify the effectiveness of a program. The evaluator, however is not always expected to provide those recommendations while those being evaluated certalnly have no obligation to comply with those recommendations.

## THE ROLE OF EVALUATION IN MATHEMATICS CLASSROOMS

The Natlonal Councll of Teachers of Mathematics In their 1961 Yearbook state:


#### Abstract

The evaluation of instruction has been called the quallty control of the education program. It is a means by whlch the quallty of our mathematics programs can be constantly improved. Through evaluation activitles we chart the present achlevement of our students and measure the progress they have made in the desired direction (page 1).


Evaluation is an essentlal part of the mathematlcs curriculum at every level and should guide the Instruction and learnlng of all students. An effective evaluation can serve many purposes, among whlch are: to improve the Instructional program in the school, to enhance the effectiveness of mathematics teachers, to ald the learning of mathematics, and to furnlsh valld data for research. The National Councll of Teachers of Mathematlcs has stated that the evaluation of the instructional program in mathematics has become more important during the last two decades because of these recent developments and pressures:
--New mathematlcs currlcula are belng advocated and tested by experimentation and research.
--New mathematics content is avallable and is belng proposed for inclusion at several levels of instruction.

- New devices and materlals of Instruction, such as computers and calculators, are now avallable to our schools.
--New principles of learning are being emphasized in the presentation of mathematical concepts.
--Soclety $1 s$ demanding greater mathematlcal competence of all citlzens than ever before.
--Natlonal survival may depend upon the development of new mathematical concepts.

Evaluation becomes even more Indlspensable when we commlt ourselves to the task of having each pupil achleve his optimum potentlal in mathematics. It can serve to improve the effectiveness of instruction in many different ways. Evaluation can establish levels of learning and locate a student at a level sultable for his current status in mathematics. It can help to provide data whlch can be used In the selection of materials, modes of instruction, and the organization and content of curriculum goals. Evaluation can help students learn mathematlcs more effectlvely by providing insight into how students learn as well as what motivates them to learn.

The Assoclation for Supervision and Curriculum Development stated in 1967 that:

Accurate assessment of educational outcomes is essentlal for sound planning and effective stimulation of growth in our educational structure. Assessment has always been an integral aspect of currlculum development and ls a major responslbillty of curriculum workers. Thls responsibility is especially critical in a time of awakened publlc concern, massive federal commltment and widespread professional reappralsal of our educatlonal endeavors (page $v$ ).

These comments are Just as valld today. Evaluation, according to the ASCD is feedback -- feedback whlch conditions what happens next in a school, or classroom. Thus the test of a good evaluation depends upon whether it satisfles the following basic criterla: (1) Evaluation must facllitate self-evaluation; (2) Evaluation must encompass every objective of the school; (3) Evaluation must facilitate learning and teachlng; (4) Evaluation must produce records approprlate to the purposes for which records are essential; and (5) Evaluation must provide continulng feedback into the larger questions of curriculum development and educational pollcy. It seems unlikely that any school system will ever devise a program of evaluation that wlll meet all flve of these crlteria. However, what we must realize $1 s$ that evaluation involves much more than measurement. According to The Association for Supervision and Curriculum Development (1967):

If the evaluation $1 s$ of sufflcient scope and lf it is handled through an interactlve process, it has thls clarlfylng, renewing effect upon learners, teachers, higher educational offlcers, and public allke. It helps everybody. who is Involved to think more clearly about what he is after and how he is getting along. In the long run, then, a hlgh-quallty evaluation program is the surest guarantee a learner, a teacher, or a school system can have of the abllity constantly to envision valld objectives, plan for their achlevement, look successes and fallures in the eye, and develop new plans as these are needed (page 9).

Thomas L. Good and Bruce J. Blddle (1988) argue that evaluatlon has the capacity for generating empirically-based insights concerning the causes, conduct, and consequences of teaching, and that those insights can be used by educators to inform the decislons they make when planning or evaluating innovations in schools. Those insights can help educators to resist the enthuslasms of vendors who are trying to sell an educational product. They can lead educators to understand why certaln teaching strategles are effective with some groups of pupils and Ineffective with others. And finally, they can provide information useful for antlcipating, measuring, or interpreting the outcomes of Innovations. In short, Good and Biddle argue that schools and educators make more senslble decisions, that resources are saved and mathematics education 1 s possibly improved, when the normal processes of educational innovation are supplemented by the insights arising from evaluation (Grouws \& Cooney, page 120).
During this examination of relevant evaluation literature, several program evaluations were reviewed. However none were found whlch matched exactly the type of program evaluation which this Investlgator planned to attempt. One mathematics program evaluation of interest was done in 1985, by the Anne Arundel County public schools, in Annapolis, Maryland. The model which was designed to evaluate curriculum programs, provided for the evaluation of three phases of a program: the curriculum; implementation of the curriculum; and students' performance, attltudes, and later success. The model provided a comprehenslve view of a program whlch went beyond the scope of evaluation models previously used. Within each phase of the model, a series of broad research questions were generated to guide the evaluation design. The evaluation resulted in a set of specific program recommendations and signlficant program changes whlch are ongolng and are monltored annually. These are some of the questions which were asked concerning the mathematics curclalum:
--Does the mathematlcs curriculum reflect current trends in curriculum development and current research in learning and Instruction?
--Does the mathematics curriculum match students' cognltive development at each grade and age level?
--Does the mathematics curriculum meet students'
diverse needs and characterlstics?
--Are the mathematics content, skills, and learning outcomes approprlately sequenced and balanced?
--Is the amount of time devoted to mathematics Instruction, kindergarten through grade twelve, sufflcient and balanced with other content areas?
--Are the inservice opportunltles avallable to teachers sufficient to Insure that the mathematics curriculum ls implemented to the fullest?

The questions were answered by collecting information from seven sources; consultants, a survey of current students, survey of former students, survey of school staff, high school achlevement data in mathematics, and other data previously collected which was avallable to the school system.

Another study was done by Eugene Muller at Columbia University and was an evaluation of a Science/Mathematics glfted education program for junlor high students. The maln focus of the study pertalned to the math, sclence, and computer sclence performance of the 7 th grade class, entering the fall of 1983. The evaluation was directed at determinlng the cognltive and affective changes that would indlcate student growth and posl ve effects of the program, and determining what aspect of the program could be Improved.

Of major Interest to thls investlgator was a study done in 1980-82 by the International Assoclation for the Evaluation of Educatlonal Achlevement. In the study, known as the Second International Mathematics Study or SIMS, detailed information was obtalned on the content of the implemented mathematics curriculum, what mathematics was actually taught by the teachers, and how that mathematics was taught. Eleven countrles participated in SIMS, and in the United States, students in approxlmately 500 mathematics classrooms in about 250 public and private schools randomly selected from across the country were tested at the end of the 1981-81 school year. The SIMS study is based on a model that views the curriculum as Intended, as implemented, and as attained. Consequently, patterns of achlevement may be examlned agalnst a background of detailed information on the content of the curriculum both as intended to be taught and as actually taught. Such detalled curricular data may be useful to curriculum supervisors and evaluators, for example, as they assess present curricula, plan new programs and seek to document the extent to which curricular innovation has taken place. The kinds of data which may be obtalned from SIMS replications include: a.) background data of a great varlety, including characterlstlcs of schools, teachers and students; b.) curricular content data concerning what toplcs are in the curriculum for each target population;
and $c$.$) teacher coverage data, concerning whlch students$ recelve what course coverage.

Much of the evaluation in the United States involves testing of general intellectual development or aptitude which is often used as criteria for school achlevement or effectiveness. SIMS, by contrast, focuses on the mathematical content of the curriculum, as found in the syllabus or textbook, as taught by the teacher and as learned by the student. SIMS, more that any other program evaluation revlewed by thls investlgator, most nearly approaches the type of educational evaluation which was performed durling thls inquiry.

Current mathematics Instruction has become stagnant; the cholces now being made by mathematics educators will affect an entire generation of students, not only in determining what mathematics they will learn, but also how they will learn and perhaps, more importantly, how much they will learn. Evaluation is the means we use to discover where we stand on the path between present experience and the desired objectlve. Effectlve evaluatlon can yield a better understanding of the learning of mathematlcs and can provide more adequate models for the Improvement of its instruction. Only then can we hope to move forward in our attempts in mathematics education.
Evaluation $1 s$ a compllcated, sometimes palnful process. However, it holds great promise for providing educators with badly needed information which can be used to improve the process of education. When used properly, evaluation can have a profound lmpact on the fleld of education. This case study began as a program evaluation and compared pre-existing problem solving practices for an existing curriculum with those recommended by NCTM. However, it also focused upon the dynamlcs involved as change occurred within an existing mathematics program.

Evaluation research, not a new but nevertheless an increasingly robust enterprise, can have a major impact on social problems. Whlle it would be foollsh to argue that all the deflciencles of current programs or all the political and conceptual problems can be swept away by evaluation studies, the adequate assessment of existing and innovative programs can be a vital force in directing social change and improving the lives and the environments of communlty members.

Francls Caro, 1971

CHAPTER III
METHODOLOGY

> Evaluation is not a search for cause and effect, an inventory of present status, or a prediction of future success. It is something of all of these but only as they contribute to understanding substance, function, and worth. Robert E. Stake \& Terry Denny, 1969

In 1989 The National Council of Teachers of Mathematics wrote the following:

The fourteen standards developed by NCTM for grades 9 through 12 establish a framework for a core curriculum that reflects the needs of all students, explicitly recognizing that they will spend their adult lives in a society increasingly dominated by technology and qualitatlve methods (page 123).

At the very center of thls core curriculum $1 s$ the concept of mathematical problem solving, which should be the focus of school mathematics.

Therefore, the purpose of thls emergent case study was to evaluate the current status of problem solving in a typlcal mathematics curriculum, and to examine the process Involved in the implementation of the recommendations found in NCTM's Standard 1: Mathematics as Problem Solving. The Intent of this investigation was not to determine whether Implementation of NCTM's Standards wlll or can bring about the desired reform in mathematics education.

Unfortunately, that type of conclusion $1 s$ beyond the scope of this study, and perhaps will not be known during this decade. Rather, the main purpose of thls investlgation was to determine whether Implementation can take place and how teachers react to percelved change within the mathematics curriculum. More speciflcaliy, the following questions were used to gulde the inquiry:

1. To what extent are the recommendations of Standard 1 not being satlsfled by the current mathematics curriculum in grades 9-12 in a specified high school?
2. What are the changes percelved by teachers to be necessary before the curriculum recommendations found in Standard 1 can be implemented?
3. What are the aspects of current mathematics education which may inhiblt or enhance the implementation of NCTM's vision for a more relevant and useful mathematlcs curriculum within a typical school?

The NCTM Curriculum and Teaching Standard which has been selected by this investigator to gulde this program evaluation 1s:

Standard 1: Mathematics as Problem Solving

In grades 9-12, the mathematlcs currlculum should Include the refinement and extension of methods of mathematlcal problem solving so that all students can --
--use, with increasing confldence, problem-solving approaches to investigate and understand mathematlcal content;
--apply integrated mathematlcal problem-solving strategles to solve problems from within and outside mathematics;
--recognize and formulate problems from situations within and outside mathematlcs;
--apply the process of mathematical modelling to real-world problem sltuations.

During the evaluation of Standard 1: Mathematics as Problem Solving, thls investigator utllized those Indlcators of quallty for Standard 1 whlch were developed by the Center for Educatlonal Research and Evaluation at UNC-G for the evaluation of mathematics programs. Those Indlcators are:

Standard 1: The curriculum provides students with the opportunity to engage in problem solving.

> 1.1 The currlculum provides students with the opportunlty to solve problems on a regular basis.


#### Abstract

1.2 The curriculum provides students with the opportunity to define problems from everyday life as well as mathematlcal situations.


> 1.3 The curriculum provides students with the opportunity to develop and carry out plans to solve a wide varlety of nonroutine problems.


#### Abstract

1.4 The curriculum provides students with the opportunlty to look back at the orlginal problems to verify and interpret their results.


> 1.5 The curriculum provides students with the opportunlty to generallze solutlons and strategles to other situations.

An assessment of the abllity of students to use with Increasing confidence, problem solving approaches to investigate and understand mathematical content was completed in the following manner: comments found in Individual teacher Journals, examples of student work from the teacher portfollos, and the examination of the results of a problem solving attitude assessment survey for students.

The procedures for this study are discussed under three major headings: (1) evaluation setting, (2) evaluation plan, and (3) data analysls.

## Evaluation Setting

This investigator has obtalned permission from the adminlstrative office of the Caldwell County Schools to conduct thls study. South Caldwell High School (SCHS) is a rural high school, located in the southern part of Caldwell County. Bullt to accommodate the consolidation of two smaller community high schools, SCHS, a modern attractlve facllity, opened Its doors In 1977. Wlth a building capaclty of 1100 and a current enrollment of 1141 , South is experiencing overcrowding and all the Inherent problems caused by too many students and not enough space.

The mathematics department at SCHS conslsts of nine faculty members, three males and six females, with seven of the nine each having more than flfteen years teaching experlence. As one af the nine members of the South Caldwell mathematics faculty, thls Investlgator was a participant-observer in this study. As such, this investigator not only partlcipated in all activities Involved with this study while observing the processes Involved, but addltionally when necessary, assumed the role of facilator. However, Information from this investigator was not included in data collection, accumulated data
results, or data analysis as a precautionary effort not to Influence the outcome of the study. Therefore data was gathered from the remaining eight mathematics teachers only. The current mathematics enrollment at South Caldwell is 895 students, excluding those students in special education classes. The course selection in mathematics at South $1 s$ qulte dlverse and ranges from General Math to College Calculus. Students at SCHS typlcally score twenty points above the state SAT average of 440 , and complled a 1990 average score of 462 on mathematics with $41 \%$ of all senlors partlcipating. However, slnce the average SAT score in North Carolina is well below the National average, SAT scores have been targeted by the school's Senate Blll 2 committee as an area for needed improvement.

The eight members of the South Caldwell High School mathematics faculty were asked and all agreed to participate in the evaluation of the current mathematics curriculum with regards to the NCTM Curriculum and Teaching Standards of school mathematics in grades 9-12. They were made aware that the standard which was selected for currlculum assessment was Standard 1: Mathematics as Problem Solving.

## Evaluation Plan

The evaluation phase consisted of four categories: (1) preparlng to evaluate; (2) program evaluation, or the actual assessment of problem solving opportunities within
the present curriculum, including recommendations for Change; (3) implementation of Standard 1; and, (4) post Implementation. Procedures for gatherlng data are based on several sources which discuss ethnographic research (LeCompte \& Goetz, 1980; Bogdan \& Bllken, 1982; Fetterman, 1988; Patton, 1980). Data sources consisted of a qualitative/quantltative mix as thls program evaluation endeavored to document both quantltative and qualitative program outcomes.

1. Preparing to Evaluate

Change is not only difflcult, but often impossible. If change is to occur in mathematics education today, then we must understand those factors which could enhance or Inhiblt such Innovation. Therefore, $I$ began my investigation using qualitative data to construct an accurate plcture of the South Caldwell mathematlcs teachers and their impressions of both the present mathematics curriculum and those recommendations presented by the NCTM Standards. Thls investlgator utillzed a questionnalre to collect background Informatlon from each teacher (such as: years of experlence, educational background, personal definition of curriculum, etc.), while teacher impressions and opinions toward the NCTM. Standards were obtained through an open-ended interview. The focus of these questions was to determine teacher attitudes toward the
process of change. These are the questions whlch were answered:

1. What are your personal feellngs regarding the teaching profession?
2. What are your personal opinions concerning the status of mathematics education today?
3. How do you respond to outside demands for change?
4. How do you determine the Individual course curriculum for each of your classes?
5. What is your initial response to the NCTM Currlculum and Teachlng Standards?
6. What factors do you personally feel will enhance/ Innlbit the Implementation of the NCTM Curriculum and Teaching Standards?

I believe this qualitative data collection helped create a complete picture of the views and attitudes of the participants. (A follow-up interview was used to determine teacher impressions after the implementation of Standard 1.)

Since the purpose of thls case study was to evaluate the current status of problem solving in the South Caldwell High School curriculum and to examine the degree of congruence between this curriculum and the recommendations
for problem solving found in NCTM's Standard 1 , it was necessary for the teachers involved in the study be famillar with those Currlculum and Teachlng Standards and the method whlch were scheduled to be used to evaluate those recommendations of Standard 1 . Since seven of the elght teachers were not famlllar with the NCTM Standards, this Investigator held a focus group work session of all elght mathematlcs teachers involved, during which each of the fourteen Standards were discussed in the following manner: percelved Importance within the present mathematics curciculum; methods whlch indlvidual teachers could use for implementation; and, changes perceived to be necessary before complete implementation might be achieved. This meeting served to famlliarlze teachers with the Standards and to generate an informal comparison of the pre-existing curriculum and the type of instruction advocated by the Standards. A complete explanation of the results of the focus group dlscussion summarlzing teacher comments concerning all fourteen Standards can be found in Appendix C.

Since several terms found in the quality indlcators which were belng used during the evaluation have varlous interpretations, a second focus group was held for the purpose of determining consensus deflnitions for the following terms: curriculum, problem solving, on a regular basis, mathematlcs in everyday life, and nonroutine
problems. These definitions were used for the duration of the case study. Following the formulation of the definitions, each Individual teacher was asked to keep a Journal for the duration of the evaluation period, in which they were asked to document thelr impressions of Standard Implementation, and their corresponding vlews toward the NCTM Standards and the evaluation in general. The journal should have begun with their initial reaction to the Standards and should have concluded with their reflections of the Standards and the evaluation, once the evaluation was completed.

## 2. Program Evaluation

The assessment of the congruence of the pre-exlsting curriculum and NCTM Standard 1: Mathematics as Problem Solving, was done in the following manner:

Individual teachers were asked to complete a checkllst (see table 1), which conslsted of detalled self-reporting of whether the pre-existing curriculum allowed students the opportunity to engage in solving a variety of routine and nonroutine problems on a regular basls, whether the problems define everyday life, whether the students verlfy and interpret their results, and whether students generallze strategles to other sltuations. On this inltial checklist, Indlvidual teachers responded on a likert scale from 1 to 5, where: $1=$ never, $2=$ seldom, $3=$ occasionally, $4=$ frequently, and $5=$ on a regular basis.

TABLE 1

## Individual Curriculum Inventory Checklist

To be completed at the beginning and at the end of thls program evaluation.

Teacher: $\qquad$
Standard 1: The curriculum provides students with the opportunity to engage in problem solving.

Use a scale from 1 to 5
Where: $1=$ never, $2=$ seldom, $3=$ occasionally, $4=$ frequently, and $5=$ on a regular basis.


Individual checklists were returned to the evaluator, who used the data to complle a group proflle (see table 2) which Indicated by an average of those responses to each of the individual questions on the checkllst the degree to which the mathematics curriculum was used to provide problem solving activitles for students. The group profile sheet was used to indlcated the congruence of the original mathematics curriculum and Standard 1.

Following the completion of the curriculum inventory and the group proflle, the evaluator conducted a third focus group with the eight members of the mathematics faculty. The group discussed existing problem solving practlces, recommendations for change, and began worklng together to develop the necessary strategies which might gulde them toward congruence with the recommendations of Standard 1. The dlscussions of the focus group were taped and transcribed. The following questlons were answered:

1. Does the current curriculum provide students with the opportunlty to engage in problem solving?
2. How often do students engage in problem solving? --Is it on a weekly basls? dally basis? etc.
3. Is there a varlety of nonroutine problems?
4. Do students generalize solutions and strategles?
5. What recommendatlons were made as a result of the

TABLE 2

Group Profile For Curriculum Inventory
Scale: 1 to 5
Where: $1=$ never, $2=$ seldom, $3=$ occasionally, $4=$ frequently, and $5=$ on a regular basis.

STANDARD 1: The curriculum provides students with the opportunity to engage in problem solving.


Group Proflle for Curriculum Inventory?
3. Implementation of Standard 1

This phase of the program evaluation consisted of the attempt to implement Standard 1 . When the group profile checklist indicated that problem solving was not being done In the current curriculum on a regular basis, recommendations for change were made. As soon as recommendatlons were made, teachers were asked to provide students in each of their classes with problem solving activitles. All eight teachers began an implementation period, during which time they were asked to complete weekly checklists, detalling problem solving activities which were completed each week, to malntain individual Journals, detalling reaction to each problem solving activity, and to maintain a portfolio of student work, containing one dated example of each problem solving exercise. (A copy of the weekly checkllst for teachers can be found in Table 3.) Teachers were asked to answer 'yes' or 'no' to each indlcator on the weekly checklist since for the short time interval Involved during any speciflc week, they elther satisfled each Individual indlcator or they did not.

At the end of each week, teachers met with the evaluator in weekly focus groups in order to determine the amount of progress belng made toward the implementation of

TABLE 3

## Weekly Individual Checklist

Teacher: $\qquad$

Standard 1: The curriculum provides students with the opportunity to engage in problem solving.


Standard 1. The evaluator charted progress on graphs with a horizontal axis variable of 'time,' given in weeks and a vertical axis varlable of 'number of positive responses' given on the weekly indlvidual checklists during the week in question. The evaluator used the examples of student work found in the portfollo to verlfy the data found on the weekly individual checklist. This phase of the program evaluation was set to continue for an indefinlte period of time (not to exceed one semester) and to cease when this evaluator was able to conclude that no addltional progress could be made toward the implementation of Standard 1: Mathematics as Problem Solving. The criteria avallable for use in making such a determlnation were:

1. Standard 1 has been implemented and has become a continuing aspect of the mathematics curriculum at SCHS.
2. Standard 1 has been implemented as completely as $1 s$ possible under exlsting condltions and curriculum expectations at SCHS.
3. Weekly graphs Indlcate the number of Indicators with positlve responses for problem solving activitles have ceased to increase, or have actually begun to decrease.
4. Teacher journals and weekly graphs indlcate

Implementation of Standard 1 ls not possible.

An assessment of the abllity of students to use, with Increasing confidence, problem-solving approaches to Investlgate and understand mathematical content was done using the following information: data from the indlvidual teacher journals, examples of problems from the portfollos contalning student work, and the results from a survey used to assess student attltudes toward problem solving.
4. Post Implementation

The last phase of data collection began with another serles of indlvidual intervlews. These are the questions which were asked:

1. What are you present perceptlons of the NCTM Standards In general, and Standard 1 In partlcular?
2. Have your perceptions of problem solving changed during this study?
3. Can mathematics education be improved by implementation of the NCTM Standards?
4. What factors will inhlbit the implementation of the type of mathematics curriculum advocated by the NCTM Standards?
5. Can the NCTM Standards be implemented Into the
present mathematlos curriculum?
6. What were the benefits / llabilities of thls program evaluation?
7. Wlll thls program evaluation impact the mathematics curriculum at SCHS?

Teachers were asked to complete the Individual journals, detalling their impressions of the evaluation period, the frequency of problem-solving activities, student reaction to each activity, along with recommendations and planning strategies.

The post implementation perlod concluded with a final focus group of all partlcipating teachers held for the purpose of discussing the perceived success of the attempt to Implement Standard 1 Into the existing curciculum. The followlng questlons were answered:

1. Was the implementation of Standard 1 successful and complete?
2. If the implementation of Standard 1 was not complete, what were the Inhiblting factors?
3. How did students react to the change in curriculum?
4. What problems were encountered during the attempt to implement Standard 1?
5. Did teacher perception of Standard 1 change during
the course of thls study?

This program evaluation was set to adhere to the proposed evaluation activity time-table found in Table 4 and Table 5. As Indicated in the tables, the flrst four weeks of the evaluation were used for Individual interviews, formulation of definltions to gulde the study, and assimilation of data collected during the interviews. Implementation of Standard 1 began during week five and was set to continue for an indefinite period, not to exceed one semester. The last phase of the program evaluation lasted four addltional weeks and was used to perform the second Individual interviews, to hold one last focus group, and to analyze all data which had been collected.

## Data Analysis

The purpose of thls case study was to evaluate the current status of problem solving in a mathematics curriculum in a typlcal high school, and to examine the process involved in the implementation of the recommendations found in NCTM's Standard 1: Mathematics as Problem Solving. The following questions were used to guide this program evaluation:

1. To what extent are the recommendations of Standard 1 not being satisfied by the current mathematics curriculum in grades 9 - 12 In a specified high school?

TABLE 4
Proposed Evaluation Activity Time-Table \#1
Preparing to evaluate:
WEEX ONE - Indlvidual Interviews.
FOUR: Focus Group, formulation of definitions.
Assimilation of information from interviews. Focus Group, discussion of NCTM Standards.

WEEK FOUR: Curriculum assessment through individual checkllsts. Teachers maintaln individual journal. First group proflle for curriculum inventory. Focus Group, discussion of Curriculum Inventory Profile, including recommendations for change.

Implementation of Standard 1:
WEEK FIVE: Teachers begin implementation period.
Teachers maintain indlvidual journal.
Teachers maintain portfolio of student work.
Teachers complete weekly checkllst.
Evaluator completes problem solving progress graph.
Weekly Focus Group.
WEEK SIX: Implementation period continues.
Teachers maintain indivldual journal.
Teachers maintaln portfollo of student work.
Teachers complete weekly checkllst.
Evaluator completes problem solving progress graph. Weekly Focus Group.

WEEK...X: Implementation period ends.
Teachers maintain Individual journal.
Teachers complete second curriculum inventory checklist.
Return checklist and portfolio of student work to
evaluator.
Second group profile for curriculum inventory.
Evaluator completes problem solving progress graph.
Teachers adminlster Problem Solving Student Attltude
Assessment Survey.
Weekly Focus Group.
Post Implementation:
LAST FOUR- Second individual interview.
WEEKS: Final Focus group.
Examination of completed journals and portfollos of student work.

TABLE 5

## Proposed Evaluation Activity Time-Table \#2

```
\begin{tabular}{rlllll} 
Preparlng & & & Post \\
To evaluate & Implementation period & Implementation \\
& & & & \\
WEEK & WEEK & WEEK & WEEK... & WEEK & LAST 4 \\
\(1-4\) & 5 & 6 & 7 & \(X\) & WEEKS
\end{tabular}
Individual
interviews: X
Focus groups (2): \(X\)
First
Curriculum X
Inventory:
Group profile
for curriculum
Inventory: X
Individual
Journals: X X X X X
Standard
Implementation: X X X
Weekly checklists: X X X X
Portfollo of
student work: X X X
Weekly graphs: X X X
Second curriculum
Inventory checklist: X
Second group profile for
curriculum inventory checklist: X
Second individual Intervlew: \(X\)
Final focus group: X
Student Attitude Assessment Survey: X
Examination of data : X
```

2. What are the changes perceived by teachers to be necessary before the curriculum recommendations found in Standard 1 can be implemented?
3. What are the aspects of current mathematics education which may inhlblt or enhance the implementation of NCTM's vision for a more relevant and useful mathematics curriculum in a typlcal school?

This investlgator attempted to determine the Inherent reasons for or agalnst, as well as the types of changes necessary for, the implementation of a revised curriculum through individual teacher interviews, before and after the attempted implementation of Standard 1 , and through weekly focus groups of all teachers involved in this study. All Individual intervlews and focus group discussions were taped and transcribed, whlle all tapes, notes and documentation from particlpant observation, interviews and focus groups were reviewed for common attitudes, blases, or interpretations concerning NCTM's Standard 1 and its implementation. Each Individual journal was reviewed for commonallties as well.

The first group profile checkilst (see Table 2) for curriculum Inventory was used to indlcate the degree to which the pre-existing curriculum satisfled the recommendations of Standard 1 . Since $5=$ on a regular
basis, if the group average from all elght teachers for each of the five indicators for problem solving was found to be at least 4.5, thls evaluator will have concluded that the mathematics currlculum under study satlsfled the recommendations of Standard 1. (A discrepancy between the pre-existing curriculum and the recommendations of Standard 1 would have been indicated by any group average less than 4.5.) If the recommendations of Standard 1 were not belng satlsfled, the evaluator wlll have assessed the amount of discrepancy using the average of all elght responses for each Indlcator, and will have endeavored to direct appropriate curriculum changes according to the recommendations made by the focus group.

During the implementation period, weekly individual checklists (see Table 3) were used to assess the amount of progress beling made toward implementation of Standard 1. Individual journals and portfollos contalning student work were used to verify the self-reporting by teachers on each weekly checklist. Implementation of Standard 1 progress was indlcated on weekly graphs and on a cumulative weekly progress graph.

Following the implementation perlod, a second group profile checklist sthe same checklist was used for the flrst and second group proflle for currlculum inventory, see Table 2) was used to assess the success of the attempt to implement the recommendations of Standard 1 into the
existing mathematics curriculum. If the group average from all elght teachers for each of the flve Indlcators for problem solving was found to be at least 4.5, this evaluator will have concluded that the attempted implementation of Standard 1 Into the existing curriculum was successful. A focus group wlll have been held to discuss the successful or falled attempt to implement Standard 1.

Triangulation procedures were used in both the qualltative and quantitative data (Fetterman, 1989; Worthen \& Sanders, 1987; Lincoln \& Guba, 1985; Bogdan \& Bilken, 1982), as there are at least three sources of information for each type of data collected as Indlcated in Table 6. At the end of each week, this investigator examined each teacher checklist in order to determine which indlcators were being satisfled. When any teacher responded 'yes' to an Indicator, verification and triangulation was done through the examination of dated examples of student work, comblned with entrles from individual teacher journals for evidence which supported each response. Findings from the triangulation procedures were discussed during the weekly focus groups.

## SUMMARY

The abillty to use mathematlcs skllls in general and mathematical problem solving, mathematical reasoning and

TABLE 6

## Trianoulation of Data

## DATA COLLECTION

Curriculum assessment for problem solving.

Self-reporting by teachers.
Dated examples of student work.

Individual Journals.
EVIDENCE

Teachers Impressions, views Individual Interviews. and attitudes toward Standard 1. Individual journals.

Increased student confidence Portfollo of student work.
in using problem solving.

> Follow-up interviews.

Individual journals.

Survey for the assessment of
student attltudes toward
problem solving.
decision making in partlcular, are creating a new vision for mathematics instruction and learning in today's classrooms. New ways of instruction and new curriculum concepts must be explored whlch will provide solutions to persistent problems and which will ultimately allow all students to become mathematlcally powerful. Although the field of mathematics has changed dramatlcally during the last three decades, the mathematlcs currlculum of today does not reflect those changes. School mathematics has become an entlty which has very little to do with what is Important in mathematics today. For students, mathematics can open doors to careers; however today, more than any other subject, mathematics filters students from hundreds of professional careers. There are many posslble steps to improving mathematlcs teaching and learning in today's schools. Mathematics educators at all levels have a responsibility to invest the time and energy necessary to find ways to communlcate the excitement and usefulness of mathematics to young people, and to devise programs which wlll help all students persevere in the learning of mathematics. We need experimentation and carefully done follow-up evaluations of new and innovative curricula for mathematics. Perhaps then we will be prepared to choose the approprlate path to reform in mathematics education for the future.

## CHAPTER IV

RESULTS
In education, we do not march steadlly and
unhesltatingly forward. We repeat not only
errors of the past, but also the
successes-usually without knowing we are
repeating ourselves. But worse, we regularly
find that the procedures that falled at some time
ln the past are successful at a later date, and
the procedures that were successful no longer
succeed.
Stephen S. Willoughby, 1990

The focus of thls Inqulry was to assess the current status of problem solving in a mathematics curriculum in a typical high school, and to examine the process involved in the Implementation of the recommendatlons found In Standard 1: Mathematics as Problem Solving, developed by the National Councll of Teachers of Mathematics in their Teaching and Curciculum Standards for Hlah Schoal Mathematics. The procedures for this study are discussed under three major headings: (1) the evaluation setting, which will describe the school, the students who attend thls school, and the individual mathematics teachers whlch were selected for particlpation $\ln$ thls case study, (2) the evaluation plan, which will detall the methods of data collection used to document teacher reaction to Standard 1 , the degree of congruence between the pre-existing
curriculum problem solving opportunltles and those recommendations found In Standard 1 , and the actual implementation of Standard 1 Into that exlsting curciculum, and (3) post implementation, which will assess the attempt to implement Standard 1.

## The Evaluation Setting

The elements of the evaluation setting will be discussed in the following manner: the school, the teachers, and the students.

## The School

South Caldwell High School was selected by the investigator as the site for this emergent case study and permission was obtalned from the adminlstrative office of the Caldwell County Schools to examine the current status of problem solving in the Caldwell County High School curriculum. One of three county high schools, South Caldwell lies nestled among the rolling hills of the southern end of Caldwell County. The bullding itself, located six miles from Lenoir off Highway 321 , was designed to blend with and reflect the mountalnous terraln visible to the north. Sltuated on one hundred acres of beautiful country, the facllity boasts of 186,700 square feet, and $1 s$ the largest educational complex within the county. South Caldwell is a one-bullding, three level complex, with the academic center of the school located on the upper level.

On the east side, the math-science loft contains classroom centers, Blology labs, a computer lab and separate qulet and project labs. Language and Social Studies occupy the west loft. Teacher's offlces and work spaces are clustered within these lofts offering separate space for small group discussions and private Interviews. Students may be Involved in activitles outslde the loft areas without Interference with classes which are in progress around the perimeter. The Media Center and theater are located at opposite ends of the third floor.

The second floor houses each of the vocational areas, while facilitles for the performing arts occupy the first floor. Both academic and vocational areas feature the semi-open classroom concept deslgned around the central gymnasium and combination commons-cafeterla area. Carpet and air-conditioning add to the beauty and comfort of the complex. (A floor plan of SCHS is Included in Appendix D of this manuscript.)

Bullt to accommodate the consolldation of two smaller community hlgh schools, South Caldwell opened its doors in August of 1977 to a student body of grades 10 - 12 . Uniting two communities and bonding two student populations, South became a cinderella school, rising up quickly to achleve academic and athletic honors and awards durling its first year of existence. Settilng down to the business of schooling, South Caldwell spent the next twelve
years improving and reflning its academic programs while expanding and supplementing the exlsting athletlc programs and facilltles. However, 1989 would mean drastlc changes for students and faculty, as the local school board voted to lnclude the ninth grade at South. With a bullding capacity of 1100 students and a current enrollment of 1141, South Caldwell is llterally bursting at 1 ts seams, and is currently experiencing all the Inherent problems caused by too many students and not enough space.

## The Teachers

The mathematics department at SCHS consists of nine faculty members, three males and slx females. As Indicated earller, thls investigator is one of the nine mathematics teachers. As a particlpant observer, this investlgator took part in all activities involved with this study. However, Information from this investlgator was not included in elther data collection, data results, or data analysls. Therefore, data was gathered and complled from the remaining elght teachers only. Identified in this study as Teacher \# 1, Teacher \# 2, etc., the following interview account for each teacher will help provide Insight into the background of each in areas such education, experience, and teaching attltudes. The following questions were used to guide and direct each Individual intervlew:

1. What are your personal feelings regarding the teaching profession?
2. What are your personal opinions concerning the the status of mathematics education today?
3. How do you respond to outside demands for change?
4. How do you determine the indlvidual course curciculum for each of your classes?
5. What is your initial response to the NCTM Currlculum and Teaching Standards?
6. What factors do you personally feel wlll enhance/ inhibit the implementation of the NCTM Curriculum and Teaching Standards?

An account of each Individual Interview 13 provided for each of the elght teachers followed by a summary of their combined responses to the six questlons.

Teacher \# 1:
Years of teaching experlence: 19
Highest educatlonal degree: BS+
Certification: Mathematics
Current Teachlng Assignment: Geometry; Alg I, Part 2;
Consumer Math
Teaching attltudes: "I like teaching and would select it as my career choice agaln today. I enjoy my job, yet I
feel overwhelmed and overworked by too much paper shuffling. Wouldn't it be great to have an alde! I spend very little time worrying about all the socletal demands for reform in mathematics education. I resent those people outside the classroom who think they have all the answers to all the problems In the classroom today. I try to gear my teaching methods to demand my students strive to excel In all areas of math. I belleve the current mathematics program needs enhancement, perhaps requiring three years of math before graduation. Inadequate teachers need to be replaced. The curriculum which I teach each day is determined by end-of-course tests and the textbook. I belleve Senate Bill 2, end-of-course testing and scholarshlps for math teachers are all efforts to improve mathematics equcation, and $I$ agree with the reasoning behind all three; they just don't seem to be working. I'm only slightly famillar with the NCTM Standards. But I do not belleve they can be implemented into the present curriculum; they need explanation and slmplification before teachers attempt to implement them. It's difflcult to know exactly what some of the Standards really mean. Besldes, words won't cure the lack of mathematics knowledge; good teachers will. Teachers should work together and be Involved in the development of new teaching techniques. I would change the way I teach if it would Improve education, but not just to ralse test scores. But first I'd need to
know how to change. Teachers are not maglclans; change has to be a cooperative effort, with everyone involved and willing to go the extra mile."

Teacher \# 2:
Years of teaching experience: 8
Highest educational degree: BS
Certiflcation: Mathematics and Blology
Current Teaching Assignment: Algebra I; Geometry;
Business Math
Teaching attltudes: "I like helping students succeed. I really love math -- it's the only subject $I$ would ever teach. However, due to the pressure teachers recelve from the public and the lack of respect from students I would make a different career cholce today. I agree that reform is needed in mathematics education, but we can't do it all in high school. Change will have to occur slowly and will need to begln in the first grade. Students should not be passed on to the next grade untll they can demonstrate a mastery of basic skllls. Rlght now I spend so much time revlewing concepts students should already know that I barely have tlme to teach the baslos. Extra materlal ls out of the question. My major focus each year is to finlsh the textbook; end-of-course testing requires it to be. I simply don't have time for extra topics whlch could beneflt my students. I have no objection to end-of-course testing, as long as it lsn't used to reflect the quality of teaching
a student recelves. Longer school days and school years slmply are not the answer. Students and teachers would only get discouraged. Parents, students, the publlc and teachers must all work together in order to have good education. I'm willing to try almost anything to help my students learn more. But a radical change in teaching styles would probably cause confuslon. The present public opinion of teachers is hard for me to handle. Educators are criticized by people who have no ldea what the publlc school system ls like. Students have not been taught to value learning; they just want to make good grades. There are no quick-fix solutions for the problems in today's schools, including the NCTM Standards. Basically, I resent outsiders who want to change the way I teach without knowing anythlng about it. Teachers know what problems exist and their input should be part of the solution. The NCTM standards look good on paper, but implementation is another matter."

Teacher \# 3:
Years of teaching experlence: 12
Highest educatlonal degree: BS
Certiflcation: Mathematics
Current Teaching Assignment: Algebra I; Computer Prog;
General Math
Teaching attltudes: "I truly enjoy the varlety of teaching since no two days are ever exactly alike. I like teaching
math and I llke having the freedom to control, for the most part, what I do in the classroom. I wish there was less paper work involved so teachers could spend more time teaching. I get upset when I think about the way the teaching profession is percelved by the general public. I agree that some change in mathematics education is necessary, but $I$ believe much of the reform should occur in early grades, with more time spent on the basics. At present, mathematics education seems adequate for the higher level and lower level students, but average students are belng totally left out; they're the forgotten majorlty. When I plan the curriculum which I teach inside my classroom, I depend on three things: end-of-course tests, sequencing presented in the textbook, and my own experlence. I thlnk end-of-course testing is of no value and in some ways seems to hurt the overall math program. I often feel that I need to rush through certaln topics just to get to the end of the book. I don't really see a longer school day or year as a solution, because students and teachers tend to burnout. I think part of "what's wrong" with mathematics education today has more to do with attitude than actual education. Students simply do not value knowledge. The NCTM Standards don't address that problem. I had never heard of the Standards before thls study began. I belleve the Standards probably could be Implemented Into the present curriculum, but not without
teachers who are willing to make the effort and textbooks that parallel the type of instruction implled by the Standards. I have no falth in those persons outside education who always seem to know Just how to flx every problem. I would gladly make changes in the way I teach if I could be assured the students would benefit, but not to improve test scores alone. Overall, I belleve the NCTM Standards are very ldealistic and as such, it would be difficult to include them in the current curriculum. Before teachers can use these Standards as curriculum guides by which to teach, they first have to understand them. That in itself may be a huge task."

Teacher \# 4:
Years of teachling experlence: 19
Highest educational degree: BS
Certlfication: Mathematlcs
Current Teachlng Asslgment: Geometry, AG; Algebra I, AG; Alg I, Part 1; Alg III/Trig

Teaching attitudes: "I like working with students and belng able to watch them grow-up, mature and develop thelr own personalities. I enjoy working in the field of mathematics and teaching it most of the time, even though It can be a difflcult subject to teach. I get angry when teachers are given the blame for all the things wrong in education. Overall I agree that the mathematics education currently being received by most students is minimal. I
belleve lack of knowledge in mathematlcs can be traced back to the early grades where students were elther unsuccessful or became unconcerned. Students in high school not only do not know the basics, they also do not know how to thlnk. The curriculum i use in my classroom is determined for the most part by state guldellnes and end-of-course tests. We should be using end-of-course testing to Insure that minimum requirements are belng satisfled at each level of mathematics; however, currently they seem to serve no purpose. There is no quick-flx for today's educational problems; most of the problems in schools are simply a reflection of the problems in soclety. I thlnk it's time the public realized that the schools can't solve every problem, that most teachers are dedicated and handle a difflcult job quite well, in splte of outside interference. I wouldn't make radical changes in the way I teach; I feel more comfortable with the ldea of slow, gradual change. I really had no knowledge of the NCTM Standards before this study and I'm not really that comfortable with them. If I thought I had to implement all those standards into the present curriculum, first I'd panic. Then I'd ask how to do 1 t, because I wouldn't know where to start."

## Teacher \# 5:

Years of teaching experience: 26
Highest educational degree: BS
Certiflcation: Mathematlcs and Chemistry

Current Teaching Asslgnment: Tech Math; Alg I, Part 2;
Algebra I
Teaching attltudes: "I like associating with young people and the variety of teaching five different groups of students each day. There seems to be a lot of pressure belng placed on math teachers for students to perform well on SAT and college placement tests. It's hard to keep motivating yourself to do a good job when there are so few signs of appreclation from adminlstrators, parents, and community members. I wish more parents cared about and understood what was best for their child in the long run. I belleve some of the demands for reform in mathematics education are justifled, mostly in the classes for average students. I belleve we have to begin in the lower grades with more emphasis on basic skllls and problem solving. I thlnk math teachers have to start giving more examples of problems which require deductive thlnking skills. The curciculum I use to teach my classes however is determined by the state guldelines, end-of-course tests, and the book. End-of-course testing in theory should improve mathematics education, but in reallty it hasn't. I'm not really that familiar with the NCTM Standards, but I belleve they probably could be phased Into the present curriculum over a long period of time. However, I don't belleve the Standards alone can cure the present lack of mathematics knowledge among our youths. There are many factors other
than currlculum that affect a student's success or lack of it. There are no quick-fix programs, and $I$ won't change the way $I$ teach unless $I$ 'm convinced it will help the students."

Teacher \# 6:
Years of teaching experlence: 27
Highest educational degree: BS+
Certification: Mathematlcs, Dept Chairman
Current Teaching Assignment: Alg II/ Trig; Consumer Math Teaching attltudes: "I enjoy worklng with young people who are motivated and enthusiastic. I like the concreteness of mathematics and belleve it is the best subject to teach. We could improve education $100 \%$ if we were able to get rid of about $80 \%$ of all administrators and support personnel -these people spend all day thlnking up more busy work for teachers to do, rather than the one thlng they need to do -- teach! I do not respond to public demands for educational reform. Inslde my classroom, I do what I feel should be done. Most schools have an outdated mathematles curriculum and low standards of expectation for achlevement. I believe we do need to make mathematics more relevant to the needs of all students through periodic updates and the reassessment of needs. Higher performance standards must be established and enforced at all grade levels. The curriculum I teach in my classes is determined by course objectives and the sequencing determined by the
textbook. End-of-course testing is a good concept, but rlght now the standards are too low. Most of the other attempts presently being made by the state to upgrade mathematics education are a waste of time and money. I'm not familiar with all the NCTM Standards, but I belleve a few of the Standards could be Implemented by Individual teachers under present conditions by just dolng lt. However most of the Standards would requlre further teacher training and the development of approprlate materlals. Some of the Standards are good and certalnly some might lmprove mathematics education, but they certainly are not a total cure for all the mathematical educational '111s.' Teachers working alone will never be able to implement all the Standards. Full Implementation would require a total commitment from all levels of education. Teachers would require further educational training, new materials and textbooks would need to be adopted, greater parental and public support would be needed; all of which require time and money. People who think they know how to provide quick solutions to the problems in education are not realistic and are just a pain to contend with. A goal of higher test scores would never be enough to make me change the way I teach. Under the present conditions of mass education, I would greatly question the wisdom of any declsion to alter the present curriculum and methods of Instruction."

Teacher \# 7:
Years of teaching experlence: 20
Highest educational degree: Ed. S.
Certification: Mathematics, AG
Current Teaching Assignment: Alg II, AG; Computer Prog; Adv. Math; Alg I, Part 1

Teaching attltudes: "I enjoy watching students grow academically and seeing their faces 'light-up' when they finally understand a difflcult concept. I enjoy the order and structure of mathematics. I wish the public would realize that teaching is a demanding job and that there are no short-cuts in education. I find it difficult to deal with the public's attitude toward education and with students who don't want to learn. I agree that some reform is needed in mathematics education, but not for the sake of improved SAT scores. State Department offlclals and other professionals may have some sound ideas about how to Improve the mathematlcs education of our students, but these ldeas never reach the indlvidual teacher. Most mathematics teachers have never even seen a copy of the NCTM Standards. Education is hard work and requires commitment from teachers, students, and parents. Inside my classroom, curriculum is determined by the Basic Education program, the textbook, and my own personal experience. Right now, end-of-course testing represents nothing more than minlmum competency. I feel only slightly famllar
with the NCTM Standards. Certalnly not all standards can be reached by all high school students. It seems to me that the Standards are directed more toward college-bound students rather than toward all students. The implementation of the Standards could possibly improve mathematics education, but certainly not solve all the problems. There are many things that would add interest to the classroom so that students could see how mathematics is used, but taking the time to do these things takes time away from covering the book and course objectlves. Also before most teachers, including myself, could explore many of the Standards with students, more education and training would be required. I'm slck and tired of all the talk about what's wrong with education. I belleve those Involved in education at all levels need to work together to declde the best strategles for improvement. I will not respond to demands for higher test scores and will not change the way I teach unless I know it will improve education. I agree with the idea of spending less time drllling concepts that calculators can handle and more time on problem solving, but I would panic if I thought I had to implement all those Standards without any training or materlal."

Teacher \# 8:
Years of teaching experlence: 16
Hlghest educational degree: BS+

Certiflcation: Mathematlcs and Middle School Sclence Current Teaching Assignment: Alg I; Consumer Math;

Geometry
Teaching attitudes: "I enjoy getting involved with students and the thrlll of seeing that 'light bulb' go off when they understand something for the first time. I find mathematics difficult to teach for two reasons: most students are afrald of math and public perception of mathematics is very negative. Most students have to overcome thelr 'fear' before they can learn, which is not easy when their parents and the media relay a message that It's okay to be dumb in math class -- since no one understands it anyway. I'm all for reform in mathematics education, but it must begin in the early grades. The type of instruction presently used in mathematics classes lends itself to memorization rather than understanding; quantity as opposed to quallty is stressed. Mathematics education In high school has, in theory, developed into a serles of classes for a select few where the average student is discouraged from taking math. Geometry has been labeled as too theoretical for most students. The mathematics curriculum of today is almost completely determined by end-of-course tests and by state adopted textbooks. Teaching for end-of-course testing leaves no time for equally important 'extra' material. I was famlliar with the NCTM Standards prior to thls study and belleve that


#### Abstract

implementation will be slow and difflcult. Rlght now, educators need a time interval devoted to improvement, without belng pressured about test scores. We need to concentrate on critical thlnking skills; of course some of the other aspects of the curriculum would suffer, and test scores might even deciline. I won't change the way I teach just for the sake of test scores. However, if we can find a way to better prepare the students for the future, then, yes I'll do whatever it takes. Right now there's so much to do and so much pressure from the outside that teachers have no time to plan or explore new methods of teaching. I belleve it was the quick-flx people in education who put us where we are today, in quick sand. If you want me to implement the NCTM Standards, then give me suggestlons and methods, then the time necessary to plan and do 1 .."


Teacher response to each of the questions asked can be summarized as follows:

Q 1. What are your personal feellngs regarding the teaching profession?

A 1. The greatest pleasure of the teaching profession is the opportunlty to work with young, motlvated, Interested students. One of the biggest thrills In life is seelng a young person grow and develop before your very eyes, or being able to make a difference in someone's life. The teaching
profession ls difflcult, and most of the general public have no Ideal of the pressure involved in being a dedicated, caring teacher. But despite those nuge negatlves, the actual 'teachlng' aspect of the profession $1 s$ very enjoyable.

Q 2. What are your personal opinlons concerning the status of mathematics education today?

A 2. The majorlty of students are leaving high school with minimal skills in mathematlcs. Students know the requirements for graduation and are encouraged by counselors, parents, and frlends to stop taklng math as soon as they meet those requirements; many students have 'math avoidance.' The curriculum is outdated and neither relevant nor interesting to students of today who are accustomed to the instant results and gratification of calculators and computers. Standards of expectation are low. Mathematics is still belng taught as a memori-zation-type skill, with drlll and practlce as a common lnstructional process.

Q 3. How do you respond to outslde demands for change?

A 3. Some reforms in mathematics are warranted. However, untll the Individual teacher is seen as a part of the solution, rather than a part of the
problem, outside demands will produce very little change and no lasting results. Teachers resent people outside education (the classroom) who are quick to polnt out existing deficlencies without providing solutions and the approprlate tools necessary to achleve them.

Q 4. How do you determine the Individual course curriculum for each of your classes?

A 4. Teachers tend to use the textbook, and personal experience almost exclusively to determine the curriculum in any individual course. End of Course testing has also come to play a major role in determining course curriculum, followed by sequenced courses whlch require a certain amount of materlal to be covered. Teachers sometimes use the outlines which are provided by the Baslc Education program. However, in reallty, nothing influences what a teacher teaches as much as the 'next section in the textbook.'

Q 5. What is your initial reaction to the NCTM Curriculum and Teaching Standards?

A 5. The NCTM Standards alone will not change mathematics instruction; dedicated, educated,
wllllng and Informed teachers wlll.

Q 6. What factors do you personally belleve wlll enhance/inhlbit the implementation of the NCTM Curriculum and Teaching Standards?
6. Those factors which mlght enhance NCTM Standard implementation:
--Students take more responsibillty for their own learning.
--Learning mathematics $1 s$ more relevant to the Individual student.
--The study of mathematlcs becomes less stressful for students and teachers.
--Students gain strength as problem solvers and Independent thinkers.

Those factors which might inhlbit NCTM Standard implementation:
--TIme.
--Class size.
--Lack of appropriate teacher tralning.
--Lack of appropriate materlals.
--Money.
--Lack of general agreement on how to 'fix' mathematics education.
--Lack of planning time.
--Parents and students who are not ready to
accept change or new trends in education.
--Inabllity of observers to evaluate teachers who act as facllltators of learnlng.
--Teachers who cannot or will not change their ideas and methods of teaching.

Table 7 provides a summary of selected data for the elght South Caldwell math teachers.

Following the first set of Individual interviews, a large amount of personal data had been collected for the elght mathematics teachers who were participants in this case study. From the number of years of teaching experience, it was evident that thls mathematics faculty was well established, with all but two members having at least fifteen years of teaching experlence. These teachers had long since developed their own Individual teaching styles and were reluctant to make drastic program changes at the suggestion of outside influences. Speclfically, only the teacher with the fewest years of experience celght years) indicated a willingness to change, with seven of the elght teachers stating they would not change thelr teaching methods for the sake of improving test scores alone. However, all elght teachers indlcated they were willing to change if they had some ldeal that students would beneflt. All eight of these teachers indicated they enjoyed being around young people, that they liked their jobs, and that thelr greatest pleasures came from seeing students learn

TABLE 7

## Summary of Selected Data for the SCHS Mathematics Faculty SEX

```
# of male math faculty3
```

\# of female math faculty ..... 5
Years of teaching experlence
5-10 ..... 1
11 - 15 ..... 1
16 - 20 ..... 4
26-30 ..... 2
Highest educational degree
BS - Teachlng ..... 7
Ed. Speciallst ..... 1
Career selection
Would choose teaching as career today ..... 5
Would not choose teachlng as career today ..... 3

Would make radical changes in their teaching methods for the sake of improving test scores
willing ..... 1
unwllling ..... 7

Prior knowledge of NCTM Standards
Had prior knowledge 1
Had slight knowledge 2
Had no knowledge 5

Belleved the NCTM Standards (or some other method) would provide a 'qulck-fix' for the present lack of mathematics knowledge among today's youth.
Yes
0
No
8

## TABLE 7 (Continued)

Membership in Professional Organizations

| Organization | Membershlp |  |
| :--- | :--- | :--- |
| NCAE | Yes | 6 |
|  | No | 2 |
| NCTM | Yes | 1 |
|  | No | 7 |
| NCCTM | Yes | 3 |
|  | No | 5 |
| Other | Yes | 0 |
|  | No | 8 |

and achleve. This faculty was truly concerned about the quality of education their students recelve, but were discouraged because of publlc opinion and lack of respect for their profession. Seven of the elght teachers did not believe that the implementation of the NCTM Standards alone can 'cure' or improve the present lack of mathematics knowledge among today's youth. They stated a bellef that only dedicated, educated, wllling and informed teachers will improve mathematics education. These teachers all expressed a degree of resentment for those indlviduals outside education who spend a lot of time talking about all the things wrong with education without supplying the methods and materials necessary to Improve them. Only one of the elght teachers indicated having knowledge of the NCTM Curciculum and Teaching Standards and recommendations prlor to the onset of this study, and then only as a result of information received during continuing college course work. They belleve education is not likely to improve until teachers understand what they should do to make those improvements.

The elght members of the South Caldwell HIgh School mathematics faculty all agreed to particlpate in the evaluation of the current mathematics curriculum with regards to the NCTM Currlculum and Teaching Standards of school mathematics in grades 9-12. This investigator has found them all to be extremely cooperative and receptive to
any suggestion whlch might improve mathematlcs instruction and education at SCHS.

The Students
The students who attend South Caldwell are typlcally from middle to upper-income famllies. On the average, less than $1 \%$ of the student body is composed of students from minorlty groups. Students typlcally score at least twenty points above the state mathematics SAT average of 440 , and compiled an average mathematlcs score in 1990 of 462 with 41\% of all senlor students partlcipatlng. More than 65\% of all graduating seniors continue their education, elther in four year colleges, two year community colleges, or in vocational training. Students at SCHS average $10-15$ points above the state average on the mathematlcs competency test. One of every three students do not reside with both natural parents.

The current mathematics enrollment at South Caldwell is 895 in regular classes and 29 in special education, for a total of 924 of the 1141 students who attend. The course selection in mathematics is quite diverse and ranges from General Math to College Calculus. Tables 8 and 9 offer detailed information concerning the course offerings and student enroliments in the various mathematics classes for the 1990-91 school year.

TABLE 8

## Mathematics Course Encollment for 1990-91

Course Title \# of students enrolled
General Math ..... 77
Consumer Math ..... 88
Algebra I, Part I ..... 103
Algebra I, Part II ..... 56
Algebra I ..... 174
Algebra I - AG ..... 11
Algebra II ..... 50
Tech Math ..... 64
Geometry ..... 109
Explo Geometry - AG ..... 12
Algebra II \& Trig ..... 63
Exp Algebra II - AG ..... 26
Algebra III \& Trig ..... 10
Advanced Math ..... 15
Computer Programming ..... 22
Precalculus-Calculus ..... 15
Math 1 - E ..... 7
Math 2 - E ..... 1
Math 3 - E ..... 4
Math 1 - L ..... 11
Math 2 - L ..... 6

TABLE 9
Mathematics Course Enrollment Percentages for 1990-91

|  | \# STUDENTS | \# STUDENTS | \% OF |
| :--- | :---: | :---: | :---: |
| CLASS | ENROLLED IN | IN CLASS | STUDENTS |
| LEVEL | MATH CLASSES | LEVEL | ENROLLED |


| 9 TH GRADE | 342 | 349 | $97.99 \%$ |
| ---: | :---: | :---: | :---: |
| 10 TH GRADE | 261 | 278 | $93.9 \%$ |
| 11 TH GRADE | 208 | 273 | $75.4 \%$ |
| 12 TH GRADE | 113 | 241 | $46.9 \%$ |
| TOTALS | 924 | 1141 | $80.9 \%$ |

## Evaluation Plan


#### Abstract

The evaluation and case study conslsted of three phases: (1) preparing to evaluate: (2) a program evaluation, which included the assessment of problem solving opportunities within the pre-existing curriculum, recommendations for change required to satisfy Standard 1 ; and the attempted implementation of those recommendations; and, (3) post implementation period, including the assessment of the attempt to Implement Standard 1. For the convenlence of the reader, the actual tlme perlod for the evaluatlon plan $1 s$ shown in Tables 10 and 11.


Preparing to evaluate

Since the purpose of this inquiry was to assess the current status of problem solving in the SCHS curriculum and to examine the degree of congruence between this curriculum and the recommendations for problem solving found In NCTM's Standard 1, teachers had to be familiar with those Curriculum and Teaching Standards and the method whlch were to be used to assess those recommendations of Standard 1. Since seven of the eight teachers were not famlliar with the NCTM Standards, thls investlgator held a focus group of all elght mathematics teachers, durling whlch each of the fourteen Standards were dlscussed in the following manner: percelved degree of Importance within the present mathematics curriculum; suggested methods whlch

TABLE 10

## Evaluation Activity Time-Table \#1

Preparing to evaluate:

```
WEEK ONE - Individual Interviews.
    FOUR: Focus Group, formulation of definitions.
    Assimllation of Information from interviews.
    Focus Group, dlscussion of NCTM Standards.
```

WEEK FOUR: Curriculum assessment through individual checklists.
Teachers malntain indlvidual journal.
First group proflle for curriculum Inventory.
Focus Group, discussion of Curriculum Inventory Proflle,
including recommendations for change.
Implementation of Standard 1:
WEEX FIVE: Teachers begin implementation perlod.
Teachers maintain indlvidual journal.
Teachers maintain portfollo of student work.
Teachers complete weekly checkllst.
Evaluator completes problem solving progress graph.
Weekly Focus Group.
WEEK SIX: Implementation period continues.
Teachers maintaln Individual journal.
Teachers maintaln portfollo of student work.
Teachers complete weekly checklist.
Evaluator completes problem solving progress graph.
Weekly Focus Group.
WEEK... : Implementation perlod ends.
THIRTEEN Teachers maintain Indlvidual Journal.
Teachers complete second curriculum Inventory checklist.
Return checkllst and portfolio of student work to
evaluator.
Second group proflle for curriculum inventory.
Evaluator completes problem solving progress graph.
Teachers administer Problem Solving Student Attlitude
Assessment Survey.
Weekly Focus Group.
Post Implementation:
WEEKS ... : Second Indlvidual Interview.
FOURTEEN -- Final Focus group.
SEVENTEEN Examination of completed journals and portfollos of
student work.

## TABLE 11

## Evaluation Activity Time-Table \#2

|  | Preparing 1 To evaluatel | Implementation period |  |  |  | Post <br> Implementation <br> WEEK |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | WEEK I | WEEK | WEEK | WEEK | WEEK |  |  |
| ACTIVITY | 1-4 \| | 5 | 6 | 7 | 13 | 1 | 14-17 |
| Individual |  |  |  |  |  |  |  |
| Interviews: | X |  |  |  |  |  |  |
| Focus groups (2): $X$ |  |  |  |  |  |  |  |
| First |  |  |  |  |  |  |  |
| Curriculum | X |  |  |  |  |  |  |
| Inventory: |  |  |  |  |  |  |  |
| Group proflle |  |  |  |  |  |  |  |
| Inventory: | $x$ |  |  |  |  |  |  |
| Individual |  |  |  |  |  |  |  |
| Standard |  |  |  |  |  |  |  |
| Implementation: |  | X | $X$ | $x$ | X |  |  |
| Weekly checklists: |  | X | X | $X$ | X |  |  |
| Portfollo of |  |  |  |  |  |  |  |
| Weekly graphs: |  | X | X | X | $x$ |  |  |
| Second curriculum |  |  |  |  |  |  |  |
| inventory checklist: X |  |  |  |  |  |  |  |
| Second group proflle for curriculum inventory checklist: |  |  |  |  |  |  |  |
| Second Individual Interview: X |  |  |  |  |  |  |  |
| Final focus group: X |  |  |  |  |  |  |  |
| Student Attitude Assessment Survey: X |  |  |  |  |  |  |  |
| Examination of dat |  |  |  |  |  |  | $X$ |


#### Abstract

Individual teachers might use for implementation; and, changes perceived to be necessary before complete implementation would be achleved. SSee Appendix $C$ for $a$ complete discussion of all fourteen Standards.) This meeting served to famillarize teachers with the Standards and to generate an informal comparison of the pre-exlsting curriculum and the type of Instruction advocated by the Standards. The results of the dlscussion for Standard 1 are as follows:


Standard 1: Mathematics as Problem Solving
a. Importance in curriculum: vitally important. Mathematics ls problem solving, therefore if students are not learning problem solving, they are not learning mathematics.
b. How to implement: Teachers can begin implementation through a serles of exercises, where students are introduced to a variety of problems, including both routine and nonroutlne problems with routlne and nonroutine methods for finding solutions. Students should be taught to view mathematics in a more personal and relevant manner, and to learn to generalize solutions to different problems in mathematics and in everyday life. Teachers should gradually increase the frequency for problem solving activities and make every effort to incorporate problem solving strategles into appropriate teaching
methods.
c. Changes required for full implementation (where problem solving is part of the mathematlcs curriculum): Teachers must realize the importance of problem solving in the mathematles curriculum. Second, teachers belleved that before they could teach most toplcs from a problem solving approach, they would require extensive re-tralning. There should be less emphasis placed on end-of-course testing and less pressure to cover all the pages In the text. Flnally, and perhaps most important, there must be development and provision of new textbooks with appropriate materlals which emphasize mathematlcs through a problem solving approach, slnce most textbooks currently emphaslze drlll and practice.

During this focus group where each of the fourteen curriculum and teaching standards were discussed, it was apparent that a llst of standards alone is not very helpful to teachers. Several of the standards set forth clear, important goals, yet a method of implementation and subsequent assessment of that implementation were difficult for these teachers to formulate. Without exception, upon discussion of each of the fourteen standards, teachers stated that implementation could only occur when teachers were retrained and were provided with the new textbooks and
materlals necessary to supplement instruction. Time, money, materials, training and resistance to change were cited as inhibitors to the implementation of the NCTM Standards.

Teachers were then given the five indicators of quality which were developed for the evaluation of problem solving within the mathematics curriculum. Since several terms used within these indlcators can be interpreted in various ways, a second focus group was held for the purpose of defining for use in this study the following terms:

```
curriculum
problem solving
regular basls
nonroutIne problems
mathematics from everyday llfe
```

Table 12 lists the definitions whlch were used for the duration of this study.

At first, teachers were eager to formulate their own definitions for the terms being used during the study. The Investlgator had elected to use a focus group for the development of these definitions in order to allow those teachers involved to gain ownership of and involvement in the study and this seemed to work well. However, later during this study, it was evident that some of those definitions created problems for the teachers who were attempting to use them. At that time, the teachers Indicated they felt Inadequate to Interpret and define those terms used by NCTM and stated a bellef that thls

TABLE 12

## Definltions Which Were Used During The Study

1. CURRICULUM -- any activity which occurs as a result of attending a particular school.

In a classroom -- the curriculum is an operational plan for instruction and includes what students need to know, how they will learn and what the teacher will do to help students to develop their knowledge.
2. PROBLEM SOLVING -- any attempt, as well as the process involved, to find an unknown solution for an existing question.

Problem solving Includes the ability to :

1. define the problem
2. formulate a plan
3. use various techniques appropriate for the problem 4. verify results
4. REGULAR BASIS -- at least once weekly, in the beginning with increasing frequency as the study continues.
5. NONROUTINE PROBLEM -- any problem not normally found in a particular course; problems for which the students are not taught speciflc solution methods; problems which require investigation and organization, rather than the use of a particular math sklll. SSee Appendix E for examples of nonroutine problems.)
6. MATHEMATICS IN EVERYDAY LIFE -- any mathematics which encourages the development of independent and organized thinking abillty. (See Appendix $F$ for examples of mathematics problems from everyday life.)
should have been done by professionals (NCTM) In order to assure the widespread use by all teachers of one definition for each of those terms. A revlew of teacher comments and the study in general has caused thls Investlgator to belleve those definltions should have been provided for teachers rather than allowing them to be developed.

Following the formulation of definitions, this Investlgator determlned that the "preparing to evaluate" stage was complete. The program evaluation began at this time.

Program evaluation
The assessment of the congruence of problem solving in the current curriculum and the recommendations of NCTM's Standard 1: Mathematics as Problem Solving was done in the following manner. Each of the eight mathematics teachers were asked to complete a curriculum Inventory checklist. The checklist allowed teachers to glve detailed self-reporting of whether the pre-existing curriculum allowed students the opportunity to engage in solving a varlety of routine problems and nonroutlne problems on a regular basis (at least once weekly), whether the problems define everyday life, whether the students verify and interpret their results, and whether students generalize strategies to other situations. The eight teachers responded, using a likert scale from 1 to 5 , where: $1=$ never, $2=$ seldom, $3=$ occasionally, $4=$ frequently, and
$5=$ on a regular basis. Individual checklists were returned to the investlgator, who used the data to compile a group proflle for curriculum inventory (see Table 13), indicating by an average the degree to which the mathematics curriculum was used to provide problem solving activitles for students. The group proflle sheet was used to determine the congruence of the original mathematics curriculum and Standard 1 recommendations.

Using the information from the group proflle for curriculum inventory (see table 13), thls investigator conducted a focus group with the members of the mathematics faculty. (Throughout the course of thls study, the focus group meetings evalved into a very helpful and powerful activity. For it was during the focus groups that the teachers involved were glven flrst time opportunitles to examine and think about concepts in new and different ways. Teachers used the other members of the group to clarify and redefine thelr own ldeas and suggestlons. When problems came up, teachers found solutlons together, causing them to form a strong sense of sharlng and comradeshlp. This bonding among teachers was a very unexpected, yet very positive and pleasant aspect of the implementation period.) The group discussed the findings for the initlal problem solving practlces, which indlcated the opportunity to solve problems in the exlsting curriculum was between 1 and 2 with $1=$ never and $2=$ seldom. All teachers agreed that

TABLE 13

## Eirst Group Profile for Curriculum Inventory Results

STANDARD 1: The curriculum provides students with the opportunity to engage in problem solving.

Scale: 1 to 5 where:
1 = never; 2 = seldom; 3 = occasionally;
4 = frequently; and $5=$ on a regular basis

typically problem solving opportunlties were reserved for the routine word problems which usually occur at the end of a chapter in the book. Often, these problems were consldered to be 'extra' and as such, were many times simply omltted due to lack of time and pressure to cover the next chapter. The following questions summarize the questions whlch were discussed durling the focus group.
Q. 1. Does the current curriculum provide students with the opportunity to engage in problem solving on a regular basis?
A. 1. No. With an average score of 1.925 for all Indlators for the assessment of Standard 1 , it was concluded that problem solving opportunitles were between seldom and never.
Q. 2. How often do students engage in problem solving: --is it on a weekly basls? dally basis? etc.
A. 2. Teachers indicated that problem solving activities were usually reserved for application problems (word problems) whlch are found at the end of a section or chapter in the textbook. Teachers are so busy covering textbook material which might be included on end of course testing, they seldom have time for additional activitles. Thus problem solving activities
might occur once a month, and perhaps not even then, since many teachers vlew these problems as 'extra' exercises.

Q 3. Is there a varlety of nonroutine problems included in the problem solving activitles?

A 3. No. Problem solving activitles are reserved for the routine problems (age, coln, DRT, mixture, etc.) whlch are usually found in textbooks.

Q 4. Do students generallze solutlons and strategles?

A 4. No. Students generally walt to be taught speciflc methods or strategles for each set of problems, then attempt to solve all similar problems using that method.

Q 5. What type or recommendations were made?

A 5. The following recommendations were made:
--Teachers were to begin implementation of problem solving activities on a regular basls (at least once weekly in the beginning, with Increasing frequency as the study progressed), thus allowing students to gradually become accustomed to this type of activity.
--Teachers were to define problems from

> everyday llfe as well as from mathematical situations so students would begin to see the relevance of mathematics.
> --Teachers were to introduce nonroutine problems into the mathematlcs curriculum.
> --Teachers would encourage students to verify and interpret thelr results.
> --Teachers would provide students with the opportunlty to generallze thelr results and strategles to other situations.
> --Teachers would begin to use problem solving strategles and methods to introduce new topics whenever possible.

By allowing students the opportunlty to vlew mathematles in a more relevant and logical manner, the math faculty at SCHS hoped to improve thinking skills, reduce mathematics anxiety, and gradually lmprove mathematlcs Instruction for all students enrolled in mathematics classes. At this time, using the above recommendations, teachers began the implementation period for Standard 1.

## Implementation of Standard 1

This phase of the program evaluation consisted of the attempt to implement the quallty Indlcators for Standard 1. Since the group profile curriculum inventory indlcated a clear discrepancy (an average score of less than 4.5 for all indicators on the group proflle for curriculum

Inventory, see Table 13) between the criterla of the pre-exlsting curriculum and the recommendations of Standard 1, slx recommendations for change were made. Teachers began an indefinlte implementation perlod, during which time they were asked to adhere to the previously mentioned recommendations. Teachers were asked to complete weekly checklists, detalling the problem solving activitles which were completed each week and the degree of compliance with the above recommendations. In addition they were asked to maintain a portfollo of student work for veriflcation, and Individual journals in which they were to detall the frequency of problem solving exercises, student reaction, teacher reaction, and future plans. A copy of a completed weekly checkilst from Teacher \# 1 for week one is given in Table 14.

The implementation of Standard 1 began slowly, with the first week showing very little progress toward satlsfying all five indicators. The easlest indicator to implement and the first to recelve elght positive responses was the first indicator -- providing students with the opportunlty to solve problems on a regular basls. Teachers indlcated this was the easiest indlcators to satlsfy since all they had to do was find an appropriate activity. Provlding students with nonroutine problems (indlcator 3) was the second indlcator to be satlsfled, with providing problems from everyday life third (indicator 2). Allowing

TABLE 14
Weekly Individual Checkllat for Week One

Teacher: \# 1 Week: \#1

Standard 1: The curriculum provides students with the opportunity to engage in problem solving.

students to verify and interpret thelr results <Indicator 4) was the fourth Indicator to be satlsfled and allowing students to generallze solutions (Indlcator 5) was last.

At the end of each week, individual teachers met with the evaluator in order to discuss the progress being made toward standard implementation. Using a tabulation of the checklists, the evaluator displayed progress on both weekly charts and a cumulative graph, plotting the total number of positive responses from all elght teachers as to the number of Indicators which were implemented during each week. Teachers were also asked to maintain a portfolio of student work, contalning one dated example of each problem solving exercise completed. These examples were used to verlfy the information indicated by the weekly individual checklists. Figure 1 Indicates the tabulation of data found on the elght indlvidual weekly checklists for week one. For example, all elght teachers began to provide problem solving opportunitles at least once during the week. Two teachers provided problems from everyday life as well as from mathematical situations. Four teachers provided opportunlty for students to define and carry out plans to solve a variety of nonroutine problems. Four teachers allowed students to verify and Interpret thelr results, and two of the eight teachers provided opportunity for students to generallze solutions and strategies to other situations. When all eight teachers had responded 'yes' to all five


Indicators, a focus group was held to determine whether Standard 1 had been successfully implemented. The findings for each week are displayed in Flgures 1-9, followed by the cumulative weekly progress graph (see Figure 10). This graph shows for each week, the combined total from all elght teachers, the number of 'yes' responses on the weekly individual checklists. The quallty Indicators for Standard 1 were considered satisfled when the cumulative graph reached a score of 40 (all elght teachers responded yes to all five indicators.

At the end of the first week, the investlgator examined the weekly checklists for each teacher along with the indlvidual journals and the portfollos of student work. All elght teachers had begun gradually, adding only one problem solving activity to their regular instructional process. Four of the eight teachers had elected to have thelr students work together in small groups, usually two to four students, while the other four had students working independently. The types of problems given to students ranged from serious problems (solving Pascal's triangle), to problems which seemed more entertalning like the following:

```
Simon is deslgning a number trlangle to quiz his
classmates. If he continues the pattern below, what
wlll the sum of the numbers be in the tenth row?
```

$$
\begin{array}{llllll} 
& & & & 1 & \\
& & 3 & 5 & & \\
& & & & & \\
& 7 & & 11 & & \\
13 & 15 & 17 & 15
\end{array}
$$

Teachers who began with less serlous problems reported extremely positlve results from their students, but even the more serious problems generated enthuslasm from students who seemed to welcome the change. Since only two teachers had Indlcated they were providing problems from everyday life as well as mathematlcal sltuations (indlcator 2), this investigator examined the problems solving activities presented by these two teachers. One of the problems stated:

A fireman was standing on the middle rung of a ladder, spraying water into a burning bullding. As the blaze lessened, he climbed up 3 rungs. A sudden flare sent him down 7 rungs. When the fire died down, he climbed up 9 rungs. When the fire was finally out, he climbed the remaining 4 rungs to the top of the ladder. How many rungs were on the ladder?

At this point, the investigator reminded teachers of their definition of mathematics from everyday life ssee table 10). In reality, each of the problem solving activitles from all elght teachers were meant to encourage the development of independent and organized thinking abllity (Indicator 2). However, teachers seemed to be looking for
only consumer type problems rather than problems which satisfled thelr own deflnltion. (Examples of problems from everyday life can be found in Appendlx F.) Thls contlnued to be a problem for teachers (particularly teacher \#4 and teacher \#5) for the first four weeks of the study. The last area of concern for week one concerned indicator 3 . While all problem solving activities were accurately classifled as nonroutine (according to the definition found on table 10), there was no varlety for week one since teachers began with only one exercise during this week.

After week two, an examination of the checkllsts, the portfolios, and the journals indicated teachers were beginning to remember their definltion of problems from everyday life, however this investigator decided to discuss this problem during the weekly focus group after week three since there continued to be a discrepancy between the checklist responses and the actual problem solving activitles belng done by the students. After week three students were beglnnlng to ask for addltlonal problems, and seven of the elght teachers had lncreased the frequency of activities and were providing at least two activities per week during week four. After week three there was quite a variety of problems being done as most of the activities involved multiple problems. Students were being asked to verify and interpret their results by developing their own formulas or patterns which would generate solutions for any



Eloure $\widehat{3}$. Progress Toward Implementation of Indicators for
Standard 1: Mathematics as Problem Solving. Week Three.


Fioure 4. Progress Toward Implementation of Indicators for
Standard 1: Mathematics as Problem Solving. Week Four.


#### Abstract

situation rather than a speciflc one. Only one teacher (teacher \#6) contlnued to offer only one activity per week. Teachers who began the activities as opportunltles for students to galn extra credit had, by the end of week three, begun to use problem solving actlvitles as homework grades for students. After three weeks, teachers Indlcated the need for additlonal materials, since all problem solving activities were belng taken elther from resource material teachers had previousiy, material purchased speciflcally for thls study, or material developed by the teacher. During week three, teacher \#7 quoted one student as saying, "I love thls stuff," and was planning to use problem solving to introduce the concept of the distance formula the following day to see if any of the students could derive the formula on their own. (It worked!)


The results from the weekly individual checklists for week two, three and four appear $\ln$ Figures 2 - 4. By the end of week four, triangulation procedures consisting of an examination of the weekly checklists, the portfolios, and the Indlvidual journals Indlcated there had been significant progress made in the attempted Implementation of Standard 1. Seven of the elght teachers were providing problem solving at least two times a week, with two teachers offering some type of activity three to four times per week (some of these activities were done during class
in group work, some were done by students Independently outside class). Only one teacher (teacher \#6) continued to provide only one activity per week. The Investlgator remlnded this teacher that 'on a regular basis' had been defined to be once weekly in the beginning, with increasing frequency as the study progressed. The teacher indicated lack of available time and lack of approprlate materials as problems which were inhiblting an increase in frequency. The discussion of problems from everyday life during the last focus group had helped teachers realize they were satisfying this indicator with most actlvities. Four of the elght teachers were having difflculty allowing students to generalize solutions and strategies to other situations. During the weekly focus group, one teacher indicated she felt there was confusion as to what this indlcator really meant. Therefore, the remainder of the meeting was devoted to a discussion of how to satisfy this indicator. Teacher \#7 suggested using an actlvity much like the following:

A cevian is a segment drawn from a vertex of a triangle to the opposite side. How many triangles are produced when:
a). 10 cevians are drawn
b). 20 cevians are drawn
c). $n$ cevlans are drawn

There was some discussion concerning whether even though thls activity allowed students the opportunity to
generallze, was it in fact actually generalization to other sltuations. After much disagreement, all elght teachers agreed this activity satisfied Indlcator 5. During week four, six of the eight teachers indicated positive responses to Indicator 4, however triangulation produced evidence to support only three of those responses. This too was discussed during the focus group. During the focus group discussion, it was discovered that some of the teachers had misinterpreted and were misusing the true meaning of Indicator 4 sstudents have the opportunity to verify and interpret their results). Three of the eight teachers had interpreted "verlfylng results" as allowing students to check their answers. The focus group discussed the meaning of verlfy and eventually agreed that the true purpose of the Indicator was to allow students to sample data, to analyze and make predictions on the basis of their sample, to make conjectures, to discuss and valldate their conclusions, and to prepare arguments to convince others of those concluslons. Students should be analyze their own thinking, rather than depending upon the teacher to tell them whether they are right or wrong. Teachers should stress the problem-solving process, not just the right answer(s). The group determined that some of the problem solving activities should allow students to experience problems with too much or not enough information, in addition to problems with no solution or ones that have
multiple solutions, each with different consequences (examples can be found in Appendlx E and F). Students then would be asked to verlfy results, interpret solutions, and question whether a solution makes sense. Such experiences would serve to develop student confidence in using mathematics. This focus group discussion was very productive. From thls point in the program evaluation, teachers reported that they had a new and very clear picture of what they needed to do and how to accompllsh their objectives. Progress in the implementation of Standard 1 (as indicated in Figures 5-9) supports this implication.

The results from the weekly individual checklists for weeks five through nine appear in Figures 5-9. An examination of all data from week flve produced results similar to week four. Teacher \#6 continued to provide only one activity per week, with all other teachers contlnuing to offer activities at least three times per week. Week five produced all positive responses, from every teacher to every indlcator, except indicator 5. Evidence from the portfollos of student work and the Individual teacher journals supported every response on the Individual checklists, with the possible exception of teacher \#6 who had not Increased the frequency of actlvitles. Teachers indlcated a need for addltional materials and a desire to learn new methods of using problem solving to Introduce new


## KEY

Indicator 1: The curriculum provides opportunity for students to solve problems on a regular basis.

Indicator 2: The curriculum provides opportunlty for students to deflne problems from every day life as well as from mathematical situations.

Indicator 3: The curriculum provides opportunity for students to define and carry out plans to solve a varlety of nonroutine problems.

Indicator 4: The curriculum provides opportunity for students to verify and interpret their results.

Indicator 5: The curriculum provides opportunity for students to generallze solutions and strategles to other situations.

Fioure 5. Progress Toward Implementation of Indlcators for
Standard 1: Mathematics as Problem Solving. Week Five.

WEEK SIX


## KEY

Indicator 1: The curriculum provides opportunity for students to solve problems on a regular basis.

Indlcator 2: The curriculum provides opportunlty for students to define problems from every day life as well as from mathematical situatlons.

Indicator 3: The curriculum provides opportunity for students to define and carry out plans to solve a varlety of nonroutine problems.

Indicator 4: The curriculum provides opportunity for students to verify and interpret thelr results.

Indicator 5: The curriculum provides opportunlty for students to generalize solutions and strategles to other situations.

Eloure 6. Progress Toward Implementation of Indlcators for
Standard 1: Mathematics as Problem Solving. Week Six.



materials and a desire to learn new methods of using problem solving to introduce new concepts. Three of the elght teachers had begun to work together, sharing ldeas which might help them develop thelr own strategies. They, however, indicated that time was an inhibiting factor.

Week six produced across the board positive responses to all indicators from all elght teachers. Evidence from the portfollos and the indlvidual journals supported all responses, except those for Indicator 4. Examples of student work verlfled a positlve response for seven of the eight teachers, with no evidence of students being allowed to verlfy and interpret thelr results during the activity provided by teacher \#5. Two teachers indicated they had used problem solving strategies to introduce new topics during this week.

After four stralght weeks of similar results (week six through nine), the weekly focus group was used to discuss whether the teachers involved in the study felt Standard 1 had been fully implemented. All elght teachers agreed that even though they believed they could accurately respond 'yes' to all five Indicators, they were not $100 \%$ sure they had begun teachlng in the manner advocated by NCTM in the Currlculum and Teachlna Standards. The teachers stated a bellef that any attempted implementation of Standard 1 would be limited by the definitions used during the Implementation perlod. (These teachers did not necessarily
feel their definitions were poor, they just lacked confldence in their abllity to formulate their own guide for Standard Implementation.) It was agreed that if teachers are to adhere to NCTM's own vision for Standard Implementation, then much work remalns to be done by NCTM and other professionals. Each aspect of the Standards must be clearly defined and stated precisely in order to avoid misinterpretation, misrepresentation, and misgulded instruction. (For example, how does NCTM define the actual term problem solving, which is basic to any attempt to implement Standard 1 , or how would NCTM assess increasing student confidence in using problem solving approaches to investigate and understand mathematical content?) All eight teachers indicated that only after much refinement of the Standards can a true comparison be made between actual teaching practices and those advocated by NCTM. At this time all elght teachers indicated that given the current curriculum requirements, the present textbooks, and the availabllity of appropriate materlals, Standard 1 had been implemented as completely as possible. Therefore given these llmitations, it was concluded that full Implementation of Standard 1 where problem solving is used for instruction is not possible at this time at SCHS, and that implementation of Standard 1 into the present curriculum can only be done on a limited basis.

During the implementation period, all elght teachers
 various classes. Thus problem solving activitles were provided in all courses, ranging from General and Consumer Mathematics to Advanced Math and Calculus. Teachers indicated the problem solving activities were well received by all levels of students, even eventually those in General and Consumer Math. Many of the lower level students reacted negatively at first, indicating they "just couldn't do word problems." However, a few weeks into the study, teachers were writing in their journals that even those students were responding in a more positlve manner. Whlle many students in General and Consumer Math remalned adamant throughout the study that they really did not like word problems, (they continuously referred to the problem solving activities as word problems) several students began to indicate that they did in fact have the ability to do these problems and that this type of experience was probably helping them become better in mathematics. One General Math student stated, "You know, for the first time ever, $I$ kinda like coming to math class. Sometimes, it's even Interesting."

Triangulation of Data
An examination of the individual checklists, the examples of student work, and the Individual teacher
journals after week nine yielded the following data for each teacher involved in the study.

Teacher \#1:
Beginning slowly, this teacher had spent three weeks providing one problem solving actlvity per week for students. Thus problems and nonroutlne problems were belng done on a regular basis, however, students were not generallzing solutions and strategles. Even though this teacher said problems were not being provided from everyday life as well as mathematlcal sltuatlons, they in fact were. The investigator discussed this discrepancy with the teacher. During week four, problem solving activities were increased to two per week, allowing students much more variety in their problem solving attempts. During week five, problem solving activities increased to three per week. During week six, teacher \#1 responded 'yes' to all flve indicators for Standard 1 and continued to do so for the remaining three weeks, however, there was little evidence to support a positive response to indicator 5 . Teacher \#1 indlcated a deslre to contlnue problem solving activities after the study concluded, stating however, that the number of actlvitles would probably decrease to two per week. Most activitles were assigned as extra credit exercises, with students recelving feed back one day after the due date.

Teacher \#2:
Problem solving activitles from teacher \#2 were being done in class, with students working in small groups. By selecting the first problems quite carefully, teacher \#2 found that by the end of week two, these activities had created quite an interest among his students; during week three, the frequency for activities Increased to two per week, with three activities provided during week four. Teacher \#2 stated that time became an inhibiting factor at this point, and chose to continue with three activities per week for the remalnder of the study. Most of the problems provided by teacher \#2 were much like the following:

If three clocks were purchased and all set at the same time, how long would it take them to again read the same time if one clock lost 1 minute per day, one clock galned 1 minute per day, while the third clock kept perfect time?

Students were also asked to find the date on which this would occur. Teacher \#2 Indicated in the journal that almost every student worked hard and was quite successful during the activitles, even after the problems became much harder. It was difficult for thls investlgator to determine whether students were allowed to verify their results until week four, but there was strong evidence that students were generalizing previous strategles to new sltuations two weeks before the teacher Indicated such.

Teacher \#2 responded 'yes' to all five Indlcators after week four with evidence to support those responses easily obtained from the individual journal and portfollo of student work.

Teacher \#3
Teacher \#3 began the problem solving activitles by providing one activity per week for three weeks. Students were excited and enjoyed the change. Teacher \#3 began week one by having the students work the problems independently, then allowed time for the teacher and students to demonstrate their own varlous methods of solving the problems. Thls worked well, as students elected to use those methods on later problems. Durlng week four, teacher \#3 Increased the frequency for activities, providing two per week during class, with at least one additional activity being done by students independently as extra credit or homework. Teacher \#3 responded 'yes' to all five indicators during week flve sand for the remalnder of the study), however, once again there was little evidence of students verlfying thelr results. During week six, problem solving was used to introduce the concept of simplifylng square roots. Students used calculators to verlfy their results. From week six untll the end of the study, journal entrles and student work supported all positive responses found on the weekly checklists.

## Teacher \#4

For the first three weeks of the study, weekly checklists from teacher \#4 were identical; only indicator 1 recelved a positive response. (Teacher \#4 was one of three teachers who had difficulty realizing that mathematics from everyday life lncluded more than consumer type problems involving shopping, household expenses, etc. She also had difficulty getting students to generalize solutions and strategles.) Even though the activities provided during week two and three were in fact nonroutine problems from everyday life, these indicators were not answered in a positive manner on the weekly checklist. The weekly discussion during the focus group for week 3 helped her reallze these indlcators were being satisfled. Teacher \#4 provided two problem solving activities during week three; activitles were provided three times weekly for the remaining weeks of the study. Again time and available materlals were inhiblting factors. Teacher \#4 responded 'yes' to all flve indlcators for the first time during week six; Indicator five was the most difflcult to achieve. Teacher \#4 was overly cautlous in her responses, as data from the journal entries and the portfolio of student work supported all positive responses earlier than actually glven.

Teacher \#5
Problems solving activities were provided once weekly for a period of three weeks, with two actlvities during week four. Even though they were well recelved by the students who asked for more problems, teacher \#5 found it difficult to justify lost instructional time. This investigator found no evidence to support a positive response to indicator 3 during week two, three and four. <Students were not being provided with a varlety of nonroutine problems.) A focus group discussion during which other teachers gave examples of the varlous benefits students seemed to be gettlng from the activitles convinced him to gradually increase the frequency. SThe fact that other teachers were also willing to share their own materials was added incentive.) During week flve, teacher \#5 increased the frequency to three activities per week, with one activity done in class and two outside class. This seemed to work well. Teacher \#5 was another of the group who satisfled indicator 2 each week after week two (he too kept looking for consumer type problems), however, he did not respond 'yes' on that indicator untll week five. Teacher \#5 responded 'yes' to all five indicators during week number six, however there was never any evidence of students generalizing solutions and strategles to other situations.

## Teacher \#6

Problem solving actlvitles began during week one, and continued for the remainder of the study, on a weekly basis. Teacher \#6 was steadfast in hls bellef that while problem solving is beneflcial, other teaching strategies are just as worthwhlle and should continue as usual. Students of teacher \#6 were enthuslastic and very receptive to the change, and even though he plans to continue problem solving after the study concludes, the frequency will remain once weekly. Thls of course, falled to satisfy the definition of 'on a regular basis,' consequently a positive response to indlcator 1 was not really warranted after week two or three. Teacher \#6, however continued to respond in a positive manner to indicator 1 on each of his weekly checklists, and justlfled his response by stating that implementation of Standard 1 could only be done on a "limited basis" at this time due to the present curriculum and time constralnts imposed by course gulde lines and end of course testlng. This was one of the major reasons why the implementation of Standard 1 was finally classifled as belng successful on "limited" basis at the end of this study. Journal entrles and the portfolio of student work indicated a wise selection of activitles however, and justifled all positive responses to Indlcators 2 - 5 after week four.

## Teacher \#7

Teacher \#7 provided one problem solving activity during week one, two during week two, and three during week three. Activitles began with students being divided into small groups of 2 - 4 according to abllity. Students Immediately responded well. By week four teacher \#7 had used problem solving to introduce the concept of distance in one class and compound interest in another. For the weeks remaining in the study, the frequency of activltles varled between three and four per week. Teacher \#7 responded 'yes' to all five indlcators during week five, however, an examination of the data found in the portfollo of student work along with journal entrles supported all positive responses during week four. Teacher \#7 and her students enjoyed using problem solving as often as possible. Of all elght teachers involved in thls study, teacher \#7 came the closest to full implementation of Standard 1.

## Teacher \#8

Teacher \#8 had been using problem solving activities once or twice weekly in all her classes as extra credit exercises throughout the year. Therefore, during the first two weeks of implementation, teacher \#8 simply continued as usual. During week three, the frequency was increased to three activitles per week. Week four brought an increase In activities to four per week. Evidence found in student
work indicated they were generalizing strategies during week three, however, a 'yes' response to this indicator did not come untll week four, with week five recelving another 'no' response. Teacher \#8 responded 'yes' to all five indicators during week four and then agaln during week six through nine. Examples of student work and Journal entrles verified checklist entrles. Teacher \#8 expressed concern that she was unable to teach all material from a problem solving approach. However, Journal entrles of the reactions and attitudes of students caused this investigator to conclude that her student were developing thinking skills which would help them become mathematically functional in society.

Concluding the implementation period
The attempt to implement Standard 1 into the existing curriculum had been in progress for nine weeks when this investigator began to examine the exlsting data for patterns which would suggest a conclusions as to whether Standard 1 had been implemented. The crlterla whlch was to be used in making such a determination were:

1. Standard 1 has been implemented and has become a continuing aspect of the mathematics currlculum at SCHS.
2. Standard 1 has been 1 mplemented as completely as is possible under existing condltions and
curriculum expectations at SCHS.
3. Weekly graphs show that the number of Indlcators with positive responses for problem solving activitles have ceased to increase, or have actually begun to decrease.
4. Teacher journals and weekly graphs Indlcate implementation of Standard 1 is not possible.

The first data source to be examined by this investigator dealt with whether the revised curriculum was providing problem solving activities for students on a regular basis. Table 15 indicates the frequency of problem solving activltles for each of the elght teachers for each week of the nine week implementation perlod. In every case except one (teacher \#6), teachers were providing problem solving on a regular basis, Increasing the number of activitles from one per week to at least three. Teacher \#6 did create cause for concern, and even though he failed to Increase the frequency of problem solving activities during the course of thls study, his students had formerly received no opportunities to engage in problem solving. Thus once per week was indeed a great improvement for teacher \#6 after all. This investlgator concluded that Standard 1 had been implemented as completely as possible glven present conditions, teacher attltudes, and curriculum

TABLE 15

Frequency of Problem Solving Activities Provided by Each
Teacher During the Implementation Perlod

WEEKNUMBER

expectations at SCHS. Therefore, thls aspect of Standard 1 was satisfled on a limlted basls.

The investlgator then began to re-examine the data from the weekly checklists, indlvidual journals, and portfollios of student work. For the most part, six of the elght teachers had been overly cautious when submitting their weekly Individual checklists. Examination of individual journals and portfolios of student work often indlcated teachers were not responding 'yes' to several indicators as soon as they should. The exceptions to this have already been mentioned. Teachers approached the problem solving activities with a 'hopeful' attitude; students accepted them quickly, welcoming the opportunity to learn mathematics in a less boring, more meaningful manner. A cumulative weekly progress graph containing data from each of the nine weeks of the implementation perlod appears in Figure 10.

Since all eight teachers had given across the board positlve responses for each of the five indlcators for four straight weeks, thls investlgator made the decision to administer the assessment for congruence between the revised curriculum and the recommendations of Standard 1. Using the same likert scale checklist for curriculum Inventory which began the initial step of the evaluation phase, teachers were asked to indicate the frequency of problem solving opportunities for students in the revised

Eloure 10. Cumulative Weekly Progress Graph, Showing Combined Number Of 'YES' Responses From All Elght Checkllsts.
curriculum. A compllation of the results on a second group proflle checkllst, shown in Table 16, Indicated an average of $5.0<5=0$ a regular basls) for all five quallty Indlcators of Standard 1 .

At thls time the implementation perlod seemed complete. However, a comparlson of Standard 1 and the Indlcators of quallty belng used to assess standard 1 resulted in a small discrepancy. The recommendations of Standard 1 and the Indlcators of quallty whlch were used to evaluate problem solving in the mathematics curriculum parallel one another in all but one area. Standard 1 recommends that the mathematics curriculum should include the reflnement and extension of methods of mathematical problem solving so that all students can use with Increasing confldence, problem solving approaches to Investigate and understand mathematlcal content. The quality indicators do not address this recommendation. Therefore, this aspect of Standard 1 was assessed in the following manner: comments found in the Individual teacher Journals, examples of students work from the teacher portfollos, and the examination of the results of a problem solving attitude assessment survey whlch was glven to students.

Individual teacher journals were examlned by this Investigator. Teachers Indlcated that students began the problem solving activitles with a degree of apprehension.

TABLE 16

## Second_Group Proflle for Curriculum_Inventory Regults

SCALE: 1 to 5 where:
1 = never, $2=$ seldom, $3=$ occasionally,
$4=$ frequently, and $5=$ on a regular basls
STANDARD 1: The curriculum provides students with the opportunity to engage in problem solving.


Students were reluctant to try new methods and walted for examples and teacher input. However, by the thlrd week, students seemed more comfortable and relaxed, Indlcating a desire to do more problems. By the end of the seventh week, students on the whole were eager to begln the activitles, did not ask for, or want, teacher assistance, and showed increased competence and success in attalning correct answers. Teachers also reported in thelr journals that the unexpected and extremely positive response from the students served to re-enforce thelr own positive attltudes toward the problem solving activitles. All but teacher \#6 reported additional motivation to try new methods of instruction as a direct result of student enthusiasm and interest.

The portfollos of student work were examined. Problem solving actlvitles began at flrst with slmple activities, rapldly becoming more Involved and more sophisticated, with the last problems of the portfollo becoming quite complicated and Involved. Teachers Indlcated that students, for the most part became more successful at solving problems correctly as the study progressed, even though the difflculty of the problems Increased.

Lastly, a Problem Solving Attltude Assessment was administered to students involved in thls study. Results of the survey can be found In Table 17. Teachers were curious to determine student reaction to the change in

## TABLE 17

## Problem Solving Student Attitude Assessment Survey Results

1. I belleve the problem solving actlvities in whlch I have partlclpated wlll improve my mathematlcs abllity.
$\qquad$ AGREE
12\% DISAGREE
2. I enjoy finding different methods for solving problems.
$\qquad$ AGREE $\qquad$ DISAGREE
3. If I had a cholce, I would not continue the problem solving actlvitles.
20\% AGREE 80\% DISAGREE
4. I belleve the problem solving activitles are a waste of time.

10\% AGREE
90\%
DISAGREE
5. I would rather the teacher just do the sections in the book.

17\% AGREE 83\% DISAGREE
6. I would like the teacher to use a problem solving approach when teaching.
74\% AGREE 26\% DISAGREE
7. I belleve working a wide varlety of problems wlll help improve my confldence in my abllity to solve problems.

89\% AGREE $11 \%$ DISAGREE
8. Working with different types of problems will not help my mathematlcs abllity.

14\% AGREE 86\% DISAGREE
9. Having experience in a wide variety of problem solving will help me attempt problems which I do not know how to solve.

92\%
AGREE
8* DISAGREE

## TABLE 17 (ContInued)

10. Partlclpating in the problem solving actlvitles has helped me to reallze i have to abllity to solve varlous problems.

78\% AGREE 22\% DISAGREE
11. I do not like problem solving.

31\% AGREE 69\% DISAGREE
12. I would rather the teacher just told me how to do the problems.

29\% AGREE 21\% DISAGREE
13. I feel better about my abllity to solve problems slnce the problem solving activities.

78\% AGREE 22\% DISAGREE
14. Problem solving has improved my abllity to think in a loglcal manner.
80\% AGREE 20\% DISAGREE
15. Because of the problem solving activitles, I am more confldent about my abllity to use different strategles to find a solution for problems.

78\% AGREE 22\% DISAGREE
curriculum. The questions were formulated by the teachers particlpating in this study using a mixture of positive and negatlve statements in order to avold 'patterned responses. Students were simply given the questionnaire, asked to read and answer each question with no further Instruction or discussion. A total of 864 or $96.5 \%$ of the students involved In the study responded <31 students were absent; surveys were not administered to speclal education students since they were not participating in the study).

Questions \#7, \#9, \#10, \#13, \#14, and \#15 were deslgned speciflcally to determine whether students had percelved an Increase in thelr confldence to use problem solving approaches to investlgate and understand mathematical concepts. In question \#7, 89\% of students indicated a bellef that problem solving would help Improve Indlvidual confldence to solve problems. Questlon \#10 with a 78\% positive response indlcated that students have increased confldence in thelr abllity to solve varlous problems. Question \#9, \#10, \#13, \#14 and \#15 all indlcate a positive response greater than 75\%. An analysis of the responses for each of these questlons led thls Investlgator, and the teachers lnvolved, to conclude that more than three-quarters of the students partlcipating had assumed Increasing confldence in their use of problem solving strategles to understand mathematlos <there is an average of $83 \%$ positlve responses for the comblned six questions).

The remalning questlons were designed to determine whether the partlcipating students liked the problem solving activitles, and wished to contlnue with them, or whether they would simply prefer teachers to use their regular methods of Instruction. Agaln results and attltudes toward problem solving were very positlve. Only 69\% of students admitted they actually liked problem solving equestion \#11), however 88\% belleve It wlll Improve their mathematlcs abillty (questlon \#1) and only $20 \%$ of students would choose not to continue using problem solving techniques (question \#3).

A flnal examination of the student survey results revealed that students in the more advanced classes were typlally more positive in their attltudes toward the problem solving activitles. However, even those students from the General Math classes Indlcated eagerness to continue with the activitles and a bellef that they were learning more useful mathematics from the problem solving activitles than they typlcally reported learning from worksheets and drill. It was also observed by thls Investlgator that the positive attltudes of students and teachers concerning the problem solving activities were directly related to one another.

After an analysis of all three Information sources (checkllsts, journals, and portfollos of student work), this investlgator was able to conclude that Standard 1 had
been implemented, with certaln pre-specifled limltations. Those limitations specify implementation would begin gradually with students engaging in problem solving activitles at least once weekly, increasing in frequency as the curriculum allowed and eventually belng used as an instructional practice. Teacher \#6 falled to satisfy the true spirit of Standard 1 implementation, however problem solving actlvitles had Increased from 0 activitles to one per week. It was not posslble to persuade him to increase the frequency beyond one activity per week. Resistance to change by some teachers will be a major concern which will be difflcult to control during Implementation of any type of program or instructional change. As a result, at this tlme, the implementation perlod ceased.

## Post Implementation Period

The last phase of the study consisted of an analysis of the attempt to implement Standard 1 into the exlsting currlculum. The last four weeks of the study began with a second set of Individual interviews. There follows a brief proflle of each teacher's comments in response to the followlng questions:

1. What are your present perceptlons of the NCTM Standards In general, and Standard 1 In partlcular?
2. Have your perceptions of problem solving changed durlng thls study?
3. Can mathematics education be improved by Implementation of the NCTM Standards?
4. What factors will inhlbit the implementation of the type of mathematics currlculum advocated by the NCTM Standards?
5. Can the NCTM Standards be implemented into the present mathematics curriculum?
6. What were the beneflts/liabllitles of thls program evaluation?
7. Will thls program evaluation impact the mathematlcs currlculum at SCHS?

## Teacher \# 1

"I belleve the goals of the NCTM Standards are good, but there are too many factors involved to Implement the entire set of Standards at once. I think the Standards could help improve mathematics education, but i still belleve mathematics improvement has to start with Interested, quallfied, wllling teachers. On that topic, I have not changed my mind. But I have seen improvement in my students after working with the Indicators of Standard 1. I dldn't expect thls study to change the way I teach,
but it has. I think I can help my students more by making them think, rather than just concentrating on covering the next page in the book. Rlght now the blggest factors which will inhibit Standard Implementation are textbooks and qualified instructors. There was a lot of positive reaction from my students to the problem solving activitles; they responded well to my efforts to change teaching methods. When $I$ give them an opportunity to find their own solutions and strategles, they're much more Interested and involved. I thlnk that's great. I wlll contlnue to use problem solving as an instructional method whenever possible, however finding approprlate materials lsn't easy. It was difflcult to show students how to generalize solutions and strategles sometimes. I think overall, we're making progress."

Teacher \# 2
"I belleve the recommendations of Standard 1 are reallstic and as a result, they can be implemented into the present mathematlcs curriculum. However, once again I think lack of time will be a problem. Some of the other Standards look good on paper, but are not reallstic. Teachers could never implement them all In the present curriculum. My students truly enjoyed the problem solving actlvitles, and since there has been such positlve results, I plan to contlnue providing them with problem solving opportunltles after this study concludes. I started out
glving problems once a week for a couple of weeks. Then I Increased the frequency. I've even used problem solving to Introduce a couple of topics. That's hard, but now the kids expect lt. I've seen many of my students actually 'think' for the flrst time. I feel I have learned a lot from this study. First of all, I am now aware of the NCTM Standards and recommendations, whlch is a benefit. Second, I now view problem solving in a different light; before, I considered problem solving to be Just the word problems at the end of the chapter. Last, I've learned that covering the next section in the book may not be the best way to teach my students. I found lt difficult to find appropriate problem solving materlal and that may be a problem for the future. It was hard to provide students with the opportunlty to generallze solutions and strategles. Sometimes $I$ found $1 t$ difflcult to draw a connection between mathematical situations and mathematics from everyday life. In order for me to teach mathematlcs in the manner advocated by the NCTM Standards, someone must elaborate on some of the standards and what they actually mean. But most of all I thlnk teachers need to revlew the Standards and recommendatlons and provide input and clariflcation based on practical experlence. As to the results of thls study, I'd say, Implementation of Standard 1 was a success."

Teacher \# 3
"I belleve the NCTM Standards are very Idealistlc and very difflcult to lmplement in a typlcal high school curriculum. However, I'm glad we trled Standard 1 and I've been quite surprised by the results. I would resist the Idea of full Implementation of all fourteen Standards at once though. I just don't think you could do it and stlll cover all the basics these klds have to know also. I already see thls lack of baslc knowledge now and lack of traditional Instruction would make it worse. I think teachers themselves will be the major obstacle for Standard Implementation; they Just reslst change of any sort. Personally, I belleve the Standards don't stand a chance of implementation untll our soclety gets away from thelr flxation on standardlzed test scores. I'm surprlsed to say that I wlll continue with the problem solving exercises after this study concludes. My students demand lt. They truly look forward to the activlties and because of their excitement, $I$ was able to increase the frequency right away. I can fit lots of examples Into my Pascal class without any real problem. It's hard to find good activities for all levels, but they're out there if you dig. The blggest beneflt of using the problem solving strategles for me had to be students who were using their reasoning abllity for the flrst tlme. Another blg plus has been that students don't seem afrald of 'word' problems
anymore. The only real llabllitles are the time factor and the lack of avallable quallty problems. For me the hardest part of the Standard to lmplement was generalizing solutions and strategles. I belleve mathematics instruction wlll improve at SCHS as a result of this study. We just have to make a consclous effort to malntaln the progress we've made. I'm not really sure $I$ have the abllity to implement all the Standards. Before $I$ have to try, I hope someone wlll provide the tralning and the materlals to help me."

## Teacher \# 4

"I think implementing Standard 1 has been a realistic goal and has been very beneflcial to most of my students. Some of the other standards would be much harder to attempt, and many of them I don't understand well enough, at this time, to even try. Durlng the course of this study, I have at least become famillar with the Standards (I wasn't famlllar with them at all before). I see the problem solving strategles as a definite way to improve mathematics education. It teaches students to reason logically and use strategles of thelr own to solve problems, rather than walting to imitate the teacher. However, some courses already have so much materlal whlch has to be covered, that it is difflcult to get it all done. It's golng to be a problem finding enough time to do everythlng. With so much emphasis on end-of-course
testing, and the pressure to produce high scores on them, It wlll be extremely hard to concentrate on what's really best for the students. However, I do plan to continue with the problem solving activities, and to use problem solving to Introduce new material when 1 t seems possible. My students welcomed the change and actually looked forward to working on strategles of thelr own. The hardest thing for me to implement was getting students to understand how to generalize strategles and solutions. I really thlnk the Standards would be easier to implement if students had a better math background. Then new materlals and new concepts could be presented rather than using so much time to revlew old material again and again. I think that's one reason students find math so boring and consequently, unstimulating. I really would like to take a course whlch would help me teach in a manner more consist with the recommendations of the Standards."

## Teacher \# 5

"The Standards sound good, but they certainly can't be Implemented across the board. Standard 1, unlike several others, was quite realistlc and reasonable. I belleve Implementation of some of the Standards could help Improve mathematics education eventually. I wlll continue the problem solving activitles after this study ends. I've seen a blg change in some of my students; many have asked for more problems, have seemed more motivated, and have
actually shown slgns of thlaking on their own. I believe though that in order to be truly successful, teachers must start problem solving in early grades and expand and Intenslfy gradually untll mathematics $1 s$ taught routinely In a problem solving manner. Right now, time is a major concern. How do I continue to cover all those pages in the book, while $I$ also instruct more slowly using problem solving? I wlll certalnly try, because I realize students are not satisfled with the old memorlzation technlques elther. The hard part is finding approprlate materials. Most textbooks don't even come close to problem solving technlques; I'd like to see one that did. It would be nlee to be able to have a staff development class on how to teach from a problem solving perspectlve."

Teacher \# 6
"I see problem solving as a meanlngful activity, but I think lf we try to use this as the total approach to mathematics education 1 t would be placlng too much emphasis on a slingle facet of mathematics. Concepts and skills need development also. Overall, however the Standards are worthwhile and with proper lmplementation could definitely Improve mathematics education. However, lack of appropriate materials and teacher tralning wlll be serlous problems for Standard Implementation. At first my students were quite reluctant to try anything different. They walted for me to gulde them, prod them, and glve them
hints. Now they work independently, eagerly, not wanting my guldance. At first $I$ was skeptlcal about problem solving actlvities; but now lt's working beautlfully. This evaluation gave students a chance to see appllcations of mathematics beyond the normal scope of the course, which was deflnltely a plus. Materlals for thls type of actlvity are in falrly short supply, however $I$ wlll continue to provide problem solving opportunities for my students on a regular basis. The hardest Indlcator for me to Implement was the last one -- providing students the opportunity to generallze solutions. Defining problems from everyday life was difflcult in the $h l$ gher math classes. Thls study has caused me to be more alert to the need to lnvolve students in situations requiring logical reasoning and realistic thought processes. While I would not teach any toplc consclously tryling to Incorporate these or any other set of standards, I will use problem solving techniques when approprlate."

Teacher \# 7
"Standard 1 ls not only reallstlc, it should be an essential part of our curriculum. However, measuring a student's abllity to thlnk critlcally may be the difficult part of implementing the Standard. My perception of this Standard has certalniy changed during the course of this study. I belleve I now have a clearer understanding of what actually constitutes problem solving. Also I now
belleve Implementation of Standard 1 can Improve mathematics instruction whlch I had reservations about before the study. I was surprlsed at my students' reactions to the problem solving activitles; they truly enjoyed dolng them. It will make lt more difflcult to cover the requlred material for end-of-course testing, but I plan to continue offering problem solving activitles to my classes. I started out slowly, but the students caught on quickly. They worked hard to find their own solutions and strategles. One day $I$ Just walked Into class, gave them three polnts and told them to flnd a way to determine the distance between them. It felt great, for the kIds and myself, when they came up with the distance formula. Another of my classes flgured out the formula for compound interest. I plan to attempt similar methods anytime I can. I have always tried to get my students to think and to show divergent methods for solving problems, however I feel that now I wlll be more aware to allowing them to discover thelr own methods and solutions. I belleve I am capable of teaching mathematics in the manner advocated by NCTM, but I would certalnly appreclate a course or a book which could help provide approprlate materlals. Even though lt's always dlfficult to measure the success in any attempt to Improve critlcal thinking skllls, I feel the implementation of Standard 1 may eventually help our students in ways we can't even be aware."

Teacher \# 8
"The NCTM Standards are positive and necessary. However, I feel that in order to glve more attention to them, we wlll have to relleve some of the pressure on the end-of-course testing for a couple of years. Problem solving, like all the Standards, needs to be Introduced gradually, allowing students time to galn skllis before all, or most, ldeas in mathematics can be taught using this approach. I belleve thls study has helped do exactly that. I started out slowly with the problem solving, but now my students are nooked. I try to teach using problem solving technlques whenever possible. End-of-course testing and Senate Bill \#2 requirements will inhiblt implementation of NCTM Standards. If we try to 1 mplement the Standards, at first $I$ thlnk test scores wlll go down. But in the long run a generation of 'thinkers' wlll be produced and that can only help society. The most difflcult aspect of Standard 1 to implement is allowing students to generalize to other situations. I am still not teaching from a problem solving approach. I need more planning time to revamp an entlre subject area into a problem solving approach. I'd like to take some courses which might help; I'd also like to see good materlal made avallable. But as always on education, patlence pays off. Wlth the proper help and a little time, we wlll improve mathematics
education through problem solving and the other NCTM Standards."

Following the second serles of individual interviews, It was noted by this investigator that the teachers in this program evaluation had not changed their vlews of the NCTM Standards in general, however, thelr Impressions of Standard 1 were much more favorable. All elght teachers seemed genulnely surprised and pleased by the response of thelr students; none of them had honestly expected to see a difference in thelr teachlng attltudes and methods. However, all elght teachers indlcated a desire to continue with the problem solving activitles in an effort to help create an attitude among students that mathematics is reasonable, Interesting, and useful. The general attltude among the elght teachers toward the usefulness of this type of study was a posltive one. They stated a bellef that this study had helped them see a need to change their teaching styles; something which would not have happened otherwlse. Finally, all elght teachers Indlcated a bellef that the Implementation of Standard 1 into the present currlculum (even though it was not a full implementation) would gradually improve mathematics education at South Caldwell High School. The teacher responses to the intervlew questions can be summarized as follows:

Q 1. What are your present perceptions of the NCTM

## Standards In general, and Standard 1 In particular?

A 1. Standard $11 s$ worthwhlle and wlth work and dedication can be implemented at least on a limited basis Into the present curriculum. Indlvidual teachers however must be wlliling to change for the sake of Improving mathematics education. By following the recommendations of Standard 1, teachers can help students organlze and develop thelr ablllty to reason and thlnk and provide them the opportunity to increase their confldence in thelr own abllity to use mathematics. Implementation of all fourteen Standards, however would be difflcult and would necessltate drastlc change, somethlng most educators would resist.

Q 2. Have your perceptlons of problem solving changed during this study?

A 2. Yes. Most teachers have a concept of problem solving as working the word problems at the end of each chapter. Thls study has helped teachers realize that problem solving can be any activity which allows students to attempt to find an unknown solution to an existing question. Teachers also stated a bellef that the use
of problem solving strategles would improve mathematlcs instruction at SCHS.

Q 3. Can mathematics education be improved by the implementation of the NCTM Standards?

A 3. Probably yes. However, time and materlals wlll be inhlbiting factors whlch wlll be difflcult to overcome.

Q 4. What factors will inhibit the implementation of the type of mathematics curriculum advocated by the NCTM Standards?

A 4. Teachers will interpret the Standards in varying ways.

Educators at all levels will be resistant to change.

The assessment of student progress would be more difflcult.

Teacher evaluation would be more difflcult. Local school unlts will need increased funding In order to supply the necessary materials. Teachers would require massive retralning. As long as teachers feel accountable for and continue to teach toward End of Course Tests, the recommendations found in the NCTM Standards wlll not be taken serlously.

Q 5. Can the NCTM Standards be Implemented Into the present mathematics curriculum?

A 5. Perhaps, but only partlally. Untll teachers at all levels are glven an opportunity to learn new methods and technlques of Instruction and understand completely the recommendations of each Standard, Implementation wlll be only on a limited basis at best.

0 6. What were the benefits / llablllties of this program evaluation?

A 6. There were several benefits. Students enjoyed the opportunlty to think, and to reason logically. They seemed much more receptlve to maklng attempts to solve new problems without walting for teacher Instruction and guldance. Students also seemed more receptlve to and less afraid of typlcal word problems. Teachers became more aware of problem solving strategies and new technlques for teaching. There seemed to be an overall posltive effect on students and teachers. The blggest llablllty was the one concerning time spent away from material in the book. Another was the unavallabllity of appropriate materials.

Q 7. Wlll this program evaluation impact the mathematics curriculum at SCHS?

A 7. Definitely. Teachers wlll continue to provide problem solving activitles for students after the study concludes. In addition, teachers will continue to look for methods of introducing toplcs through the use of problem solving actlvities.

The study concluded with one last focus group. Due to the overall positive response to problem solving activitles by students and teachers, all elght teachers plan to continue problem solving activities and hope to find methods of Introducing new topics using problem solving techniques. The actual results from the implementation of Standard 1 are not easily measured, but the teachers involved in this study have indicated that students are learning to thlnk and inquire in new and different ways, and as a result mathematics instruction has been affected In a positlve manner. The questions whlch were answered during the last focus group can be summarlzed as follows:
Q. 1. Was the implementation of Standard 1 successful and complete?

A 1. Teachers classlfled the attempt to implement Standard 1 as belng very successful, even though
at thls time the implementation remalns limlted. Both students and teachers indlcated a bellef that mathematlics education could be Improved by the continuation of the problem solving activitles. Those teachers Involyed in thls study have indlcated also that they began the implementation period with an attitude of acceptance, thinklng problem solving activitles would cease when the implementation period was complete. However, teachers now plan to continue using problem solving strategies whenever possible. The Implementation of Standard 1 Into the mathematics curriculum at SCHS not only produced lasting instructional changes, it has also helped create new attltudes toward and Interest for mathematics among students and teachers.

Q 2. If the implementation of Standard 1 was not complete, what were the inhibiting factors?

A 2. Teachers Indlcated the Implementation of Standard 1 was not complete, since the introduction of new materlal from a problem solving perspective remalned very difflcult and as yet was not a common occurrence in mathematlcs classes at SCHS. Before problem solving can become a routine
method of Instruction, teachers wlll need to adopt a new role in their classrooms-a role as a facllitator of knowledge rather than as the source of and dispenser of all Information. Before this can be done, teachers will need new materlals, additional tralning, more freedom to decide what and how to teach, more tlme and smaller classes, all combined with the public and professional support necessary to get the job done. A present lack of all these Items helped inhibit the implementation of Standard 1 Into the present curriculum.

Q 3. How did students react to the change in currlculum?
3. An examination of the results of the problem solving assessment survey whlch was adminlstered to 96.5\% of the students involved in this study show a very positive response by students to the problem solving activities. Teachers have also indicated that many students made comments that for the first time ever, math class was interesting and even fun. Students requested an increase in the number of activitles, and even suggested using thelr own time to work on special projects. Student

# reaction was Indeed not only very positive, but very surprlsing and gratifylng. 

Q 4. What problems were encountered during the
Implementation of Standard 1 ?

A 4. The major problems teachers expressed repeatedly pertalned to lack of approprlate materlals and an Inabllity to develop thelr own. There were also major concerns over the amount of time spent away from the textbook and whether thls would result in lower scores on end of course testing. In addltion, teachers felt uncomfortable with the lack of preclsion and direction from NCTM regarding implementation of thelr Standards. Recommendatlons for Improvement are fine, but teachers need to know how to put them into practlce.

Q 5. Dld teacher perception of Standard 1 change durlng the course of the study?

A 5. Deflnitely. First, none of the teachers involved in this study expected to make any lasting changes in their methods of instruction. Implementation of Standard 1 into the SCHS currlculum has resulted in a re-assessment of Individual teachlng styles by at least seven of


#### Abstract

the elght teachers partlclpating. Second, problem solving is a term famlliar to every mathematics teacher; it's a toplc teachers commonly belleve is covered in math class every day. This study helped teachers at SCHS to understand that not all mathematics $1 s$ true problem solving and that students can beneflt from more than one type of Instruction.


Following the final focus group, this investigator utllized the remainder of the post implementation period to examine and syntheslze the data whlch had been collected. After a four week post implementation period, thls evaluation case study offlcially came to an end, however teachers at SCHS continue to use the beneflts of this study In an effort to Improve the mathematics curriculum and thelr own methods of Instruction.

## CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS


Thls chapter contalns four sections. First, a summary of the study 1 s presented. The second section gives the concluslons of the research. Section three sites limitations of the study. Finally, the chapter concludes with recommendatlons for further study.

## Summary of the Study

The purpose of this inquiry was to assess the current status of problem solving in a mathematics curriculum in a typlcal high school, and to sxamine the process Involved In the implementation of the recommendations found in Standard 1: Mathematics as Problem Solving, which was developed by NCTM In the Teaching and Currlculum Standards of Hlah School Mathematlcs. More speciflcally, the following questions were used to gulde this program evaluation:

1. To what extent $1 s$ the crlterla of Standard 1 not belng satlsfled by the current mathematlcs currlculum $\ln$ grades $9-12 \ln$ a specifled high school?
2. What are the changes percelved by teachers to be necessary before the curriculum standards found In Standard 1 can be fully Implemented?
3. What are the factors which may inhlbit or enhance the implementation of NCTM's vision for a more relevant and useful mathematics curriculum within a specified school?

The NCTM Curriculum Standard whlch was selected by this investlgator to gulde this evaluation case study 1 s:

Standard 1: Mathematics as Problem Solving

In grades 9 - 12, the mathematlcs curriculum should include the refinement and extension of methods of mathematlcal problem solving so that all students can --
--use, with increasing confldence, problem-solving approaches to investlgate and understand mathematical content;

# --apply integrated mathematical problem-solving strategles to solve problems from within and outslde mathematles; 

--recognize and formulate problems from situations within and outside mathematics;
--apply the process of mathematlcal modeling to real-world problem situatlons.

During the evaluation of Standard 1: Mathematics as Problem Solving, thls investlgator utllized those Indlcators of quallty for Standard 1 whlch were developed by the Center for Educatlonal Research and Evaluation at UNC-G for the evaluation of mathematics programs. Those Indlcators are:

> Standard 1: The curriculum provides students with the opportunity to engage In problem solving.
> 1.1 The curriculum provides gtudents with the opportunlty to solve problems on a regular basis.
1.2 The curriculum provides students with the opportunlty to define problems from everyday life as well as mathematlcal sltuatlons.
1.3 The curriculum provides students with the opportunlty to develop and carry out plans to solve a wide varlety of nonroutine problems.
1.4 The currlculum provides students with the opportunity to look back at the orlginal problems to verlfy and Interpret their results.
1.5 The curriculum provides students with the opportunlty to generallze solutions and strategles to other situations.

A comparlson of the recommendations of Standard 1 and the five indicators of quallty which were used to evaluate problem solving in the mathematics curriculum show they parallel one another in all but one area. The first recommendation of Standard 1 states that the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can use with increasing confidence, problem solving approaches to lnvestigate and understand mathematical content. The quallty Indicators do not address this recommendation. Therefore, this aspect of Standard 1 was assessed in the following manner: comments found in the individual teacher journals, examples of student work from the teacher portfollos, and the examination of the results of a problem solving attltude assessment survey adminlstered to students.

South Caldwell High School was selected as the site for this program evaluation with all nine members of the mathematlcs department particlpating. As one of those nlne teachers, thls investigator was a participant in all activities involving this case study. However, personal reflections and data from thls Investlgator has not been included in the data collection or data analysis which documents results from the remaining elght mathematics teachers only.

The study consisted of three phases: preparing to evaluate, program evaluation, and a post implementation perlod which was used to examine the attempt to implement Standard 1. The first phase began with a serles of individual interviews, during which teacher background, educational views, attltudes concerning the status of the current mathematics instruction and outside demands for reform, as well as knowledge of the NCTM Curciculum and Teaching Standards were discussed. A focus group of all elght teachers was held, durlng whlch time standard 1 was discussed in the following manner: importance within the mathematics curciculum; methods which might ease implementation; and, changes perceived to be necessary to achleve complete implementation. Teachers were given the five Indlcators of quallty which were developed for the evaluation of problem solving within the mathematics curriculum. Since several terms used within these

Indlcators can be interpreted differently, a second focus group was held to define for use in this study the following terms: curriculum, problem solving, on a regular basis, nonroutine problems, and mathematlcs for everyday life.

The evaluation phase of the study began after four weeks of preparation, beginning with the assessment of the congruence of problem solving in the pre-existing currlculum and the recommendations of Standard 1. Each of the elght mathematics teachers were asked to complete a likert scale checklist for curriculum lnventory detalling the status of problem solving opportunitles for students in the pre-existing curriculum. Individual checklist were returned to the Investlgator, who used the data to complle a group proflle, indicating by an average the degree to which the mathematics curriculum was used to provide problem solving activities for students. This group profile for all elght teachers indlcated the opportunlty to solve problems in the pre-existing curriculum with an average of 1.925 was some where between 1 (never) and 2 (seldom). Since the group proflle checkilst indlcated a clear discrepancy between the criterla of the pre-existing curriculum and the recommendations of Standard 1 , six recommendatlons for change were made:

1. Begin implementation of problem solving activitles on a regular basis <at least once weekly In the
beglnnlng, then with lncreasing frequency as the study progressed).
2. Introduce nonroutlne problems Into the mathematics curriculum.
3. Encourage students to verlfy and Interpret their results.
4. Provide students opportunlty to generallze results.
5. Deflne problems from every day llfe as well as from mathematical sltuations.
6. Make every effort to utllize problem solving strategles to introduce new material when approprlate.

Teachers began an Indefinlte Implementation perlod, during which time they were asked to adnere to the above recommendations. Teachers were asked to complete weekly checkilsts, detalling the problem solving activitles which were completed each week and the degree of compliance with the above recommendatlons. A portfollo of dated student work was used to verlfy results, along with individual teacher Journals which recorded teacher concerns and perceptions of each activity. After a period of slx weeks, all elght teachers were able to respond in a positive manner to each of the five indlcators found on the weekly checklist and after four weeks of across the board positlve responses, a second assessment of the congruence between the curriculum and the criterla of Standard 1 was
completed. Using the same likert scale checklist for curriculum inventory which began the initlal step of the evaluation phase, teachers were asked to indicate the frequency of problem solving opportunlties for students in the revised curriculum. A compilation of the results on a second group proflle checkilst indicated an average of 5.0 ( $5=$ on a regular basis) for all five indicators for Standard 1. After an examination of the data from the Individual Journals, the portfollos of student work, and the Problem Solving Student Attitude Assessment results, this investigator was able to conclude that Standard 1 had been implemented. At this time the implementation period concluded.

The last phase of thls study conslsted of an examinatlon of the attempt to Implement Standard 1 Into the exlsting curriculum. The last four weeks of the study began with a second set of individual interviews during which the questions asked were:

1. What are your present perceptions of the NCTM Standards In general, and Standard 1 In particular?
2. Have your perceptions of problem solving changed during thls study?
3. Can mathematics education be improved by the implementation of the NCTM Standards?
4. What factors wlll inhlblt the Implementation of the type of mathematics curriculum advocated by the NCTM Standards?
5. Can the NCTM Standards be Implemented Into the present mathematics curriculum?
6. What were the beneflts / liabllitles of thls program evaluatlon?
7. Wlll this program evaluation impact the mathematics currlculum at SCHS?

The study concluded with one last focus group of the elght teachers, during which teacher reactions to the implementation were discussed as well as plans for future problem solving activitles. Due to the overall positive response to problem solving activitles by students and teachers, all elght teachers plan to continue problem solving on a regular basis. Actual results from the Implementation of standard 1 wlll probably never be conclusively known, but the teachers involved in this study Indicated that students were learning to 'thlnk' and 'Inquire' In new and different ways, and as a result mathematics instruction has been affected in a positive manner.

Conclusions of the Research
Data from the research support the following conclusions to the questions which were used to gulde thls evaluation case study.

1. To what extent are the recommendations of Standard 1 not being satisfled by the current mathematics curriculum in grades 9 - 12 In a specifled high school?

Conclusion: The recommendations of Standard 1 are not belng satisfled by a typlcal mathematics curriculum. Data from teachers indlcated an average of 1.925 on the First Proflle for Curriculum Inventory. Wlth 1 = never and $2=$ seldom, thls data indlcated that students were participating in problem solving on a very limited basis. Data from teacher intervlews Indicated that teachers contlnue to teach mathematics in the manner it was taught decades ago -- teachers prescribe; students transcribe. Students continue to learn mathematics from imltation, lectures, worksheets and routine homework. Problem solving is an activity in whlch many students never cor very seldom) have an opportunlty to engage.
2. What are the changes percelved by teachers to
be necessary before the currlculum recommendations found in Standard 1 can be Implemented?

Conclusion: Teachers must flrst recognize the need for change. Second, teachers must be given the opportunity to change. This wlll include providing them with new materlals, approprlate tralning, smaller classes, more tlme, less pressure to produce $h / g h$ test score averages, the freedom to make cholces on what and how to teach, and the public and professlonal support to requlred for each of these. And last, but perhaps most important, teachers must not resist change.
3. What are the aspects of current mathematlos education which may inhiblt or enhance the implementation of $\mathrm{NCTM}^{\prime}$ s vision for a more relevant and useful mathematlcs curriculum within a typlcal school?

Conclusion:
Those aspects which may enhance NCTM Standard implementation:
--Students take more responsibllity for thelr own learning.
--Learning mathematics becomes more relevant to the Individual student.
--The study of mathematics becomes less stressful for students as they reallze mathematics can be interesting and stimulating.
--Students gain strength as problem solvers and Independent thinkers.

Those aspects which may Inhiblt NCTM Standard implementation:
--Time.
--Class slze.
--Lack of appropriate teacher tralning.
--Lack of materlals.
--Lack of general agreement on how to 'flx' mathematics education.
--Lack of planning time.
--Parents and students who are not ready to accept change or new trends in education.
--Pressure to produce high test score averages, and to cover textbook material first.
--Difflculty in student assessment.
--Teacher evaluation.
--Teachers who resist change.

Conclusions from this study have also been derlved from each of its three phases. During the preparing to evaluate phase, data from the Individual Interviews support the following conclusions:
--Teachers resist pressure from outside influences.
--Teachers in general are not famillar with NCTM Standards.
--Teachers will not make radlcal changes in mathematlcs Instruction, but are willing to make slow, gradual change for the sake of improved Instruction.
--Teachers do not belleve a set of Standards can cure the present lack of mathematlcs knowledge among the nation's youth. They belleve educatlonal lmprovement will only come lf parents, teachers, students, professlonals, and the general public joln together to find workable solutions.
--Teachers do not feel motivated to try new technlques which have excluded the expertise of teachers during thelr formulation.
--Teachers currently feel pressured to teach to end-of-course tests, and to cover each section in the textbook. Problem solving $1 s$ viewed as an 'extra' activity and as such is often excluded from the curriculum.
--The NCTM Standards are considered to be vague and easily mis-interpreted. Teachers wlll need further explanation and clarlflcation before implementation wlll be attemptable.

- Implementation of the NCTM Standards wlll require masslve teacher re-tralning comblned with the adoption of appropriate materials and textbooks.

All elght teachers involved in this case study agree that reform is needed in the present mathematics curriculum, yet there is widespread disagreement as to which course of action will best accomplish thls reform. Two teachers Indicated a need for back to baslcs, with the necessary skills belng stressed in earller grades, in much the same manner as reading skills. Two teachers indlcated a need for addltional and more relevant mathematics courses at every grade level for average students. They stated that much time and effort is spent on advanced mathematics Instruction for the top ten percent and remedial Instruction the lower twenty percent of students, yet there is almost no effort toward instruction of, and very few courses designed specifically for, the majority of students who are consldered average. Teachers indlcated that many of these students simply avold mathematics, particularly during high school and later become one of the many Individuals who are unprepared mathematically to function In soclety today. Two other teachers indlcated a bellef that the apparent lack of adequate education among youth in general is a direct result of the bellefs and attitudes propagated in our present soclety. Teachers say students are encouraged to memorize rather than learn, and to
belleve that test scores are more important than learning and retention, and that grades, rather than abllity, determine one's future. These teachers belleve education will improve only when the vlews so prevalent in soclety change.

Only one of the elght teachers involved in thls study had previous knowledge of the NCTM Standards, and then only because of college course work encountered whlle working on a Master's Degree. The other seven teachers had never seen a copy of the fourteen Curciculum and Teaching Standards for High School Mathematics. These teachers stated that If they are not kept informed and made aware of recommendations for improved instruction, that these recommendations were not likely to be implemented. Professional organizations and state offlcials should find a way of keeping teachers well informed of all developments and curriculum updates. SThls investlgator has also concluded that teachers should join thelr professional organizations and should take the initiative to find out what innovations are taking place in their field. This was based on the fact that only one of elght teachers in this hlgh school belonged to NCTM. See Table 7.) Teachers report that if they are not involved, it seems unlikely that instruction will improve. It $1 s$ also understandable why none of these teachers seemed too concerned by outside Influences who demand change in the present educational
system. None of the elght teachers involved in this case study respond to demands for higher test scores and Indeed are not convinced that test scores are accurate Indlcators of mathematics instruction. They are concerned for the progress of thelr students and are willing to develop new teachlng methods if students can benefit from them.

While none of the elght teachers recognized the NCTM Standards as a cure-all for mathematics education, each of them Indlcated a wlllingness to attempt a slow, gradual Implementation of the Standards. However, they feel many of the Standards wlll need to be clarlfied and defined, otherwlse many teachers will assume they already satlsfy most, if not all, of the recommendations. They agree that they teach math today in much the same manner as they were taught twenty to thlicty years ago, and any drastic change wlll require extensive re-tralning and the development of new teaching techniques. Currently teachers report that they simply do not have the time to develop on their own the type of material which would allow them to teach mathematics in the manner advocated by the Standards.

All elght teachers indicated that presently they have a tendency to teach toward the end-of-course tests and that they feel obligated to cover each section in the textbook. This too is understandable, due to the current emphasis on test scores and the inevitable comparisons of local test score averages with state and national averages.

Implementation of the Standards wlll slow down those teachers who seem proud of the number of pages covered in the textbook each year.

Finally, teachers indlcated a bellef that educatlonal trends are cyclic and that problem solving is not a new method of mathematics instruction. During the pre-evaluation interview, Teacher \# 8 produced a copy of the preface to a book titled General Mathematics: A Problem Solving Approach. The preface addressed the revision and refinement of a former text with the new text seemingly contalning all the features and criterla contalned In NCTM's Standard 1. Yet the surprising fact was not the suggestion of a problem solving approach, but a copyright date of 1965 . Thls seemed to support these teachers' bellefs that educational trends are repetlitive over a cycle of about twenty-flve years. Thls mathematics faculty $1 s$ experienced and has seen varlous reform movements come and go, most of whlch have made no signiflcant impact toward improving mathematics education. As such, thls mathematics faculty reported that they are not yet convinced that the fourteen Curciculum and Teaching Standards are not just another quick-fix remedy which may or may not work. These teachers state that they would like to be a part of the process when solutions are suggested for Improving mathematics education. They have Indicated a belleve they have the knowledge and experlence necessary to
make wise cholces and a responsibillty to make those cholces carefully.

Data from the evaluation phase of this inquiry supports the following conclusions:
--The recommendations from NCTM's Standard 1 are not currently belng satisfled by the typical mathematics curriculum.
--Problem solving actlvitles are generally reserved for end of the chapter routine problems, perhaps once a month or less often.
--Students are typically taught specific. methods for solving these routine problems.
--Standard 1 can be implemented Into the existing mathematics curriculum, with certaln pre-specified conditions. Those condltions specify implementation will begin as 'add on' activitles, which will increase in frequency and gradually become part of the normal Instructlonal process and eventually an integral part of the curriculum.
--Allowing students to generallze solutions was the most difficult aspect of Standard 1 to Implement, followed closely by defining mathematics from everyday life as well as mathematical situations. Very few textbooks address elther concept and material ls llmited.
--Teachers wlll require the provision of new materlals, since lack of tlme will prevent them from developing their own.

When thls case study began, problem solving 'on a regular basis' was defined to be at least once weekly. As such, problem solving opportunltles began as 'add-on' activities which were done in addition to the regular mathematics instruction. Teachers in essence were not changing thelr methods of teaching, they were slmply adding problem solving activitles which they previously had excluded. They began by introducing new types of problems, some with routine solutions, others with multiple solutions, and stlll others with possibly no solution at all. Teachers attempted to provide activities for students which would allow them to view mathematics in a more useful and personal manner. Students who often think of mathematics as an exercise in memorlzation were encouraged to simply reason through a sltuation in a logical, systematic and organized manner. Results were surprising. Journals which were kept by Individual teachers indicated that students enjoyed the change and looked forward to the exercises. Students began to find their own methods for solutions and were able to show others why certain strategles work and others falled. The frequency of the problem solving activitles began to increase steadly. By the end of the implementation period, teachers were
beginning to search for methods which would allow them to Introduce and teach new toplcs through a problem solving orientation. This however proved to be difficult. Whlle some topics lend themselves to problem solving strategles, most topics seem to revert back to teacher lecture methods, while students copy and imitate. This fact was discouraging. However, all eight teachers indicated a desire to continue offering problem solving opportunities to their students on a regular basis, and to continue searchlng for methods which would allow them to use problem solving as an instructional method. All elght teachers Indlcated positive results from the implementation of Standard 1.

Post Implementation
Data from the last phase of thls case study was obtalned from a second serles of Individual Interviews and a final focus group. According to the information obtalned, this investigator was able to make the following concluslons:
--The Implementation of Standard 1 was successful with certaln pre-specifled limitation. Those limitations specify that implementation would begin gradually, with students engaging in problem solving actlvities at least once weekly, increasing in frequency as the curriculum
allowed and eventually belng used as an instructional practlce.
--Teacher perception of Standard 1 became more positive during the course of this study.
--Students became more willing to attempt solutions for problems without walting for teacher instruction.
--The major factor which will enhance or inhlbit the implementation of the Standards will be the willingness or reluctance of the teachers themselves.
--Students enjoyed the opportunlty to explore and think for themselves.
--Teachers plan to continue problem solving activities on a regular basis.
--Teachers expect to be more aware of the manner in which new materlal $1 s$ presented and wlll try to use problem solving techniques to do so when possible.
--Teachers want textbooks and renewal courses which will help them Implement the Standards in the manner advocated by NCTM.

All data collected from teachers during the initial Intervlews indicated that even though they were wllling to try Implementation of Standard 1 , they were not expecting much in terms of results and they were far from convinced that their curriculum and subsequent instruction would undergo any signiflcant change. This investigator developed a distinct Impression that problem solving
opportunitles would slmply be extra activltles for the duration of thls study and would then cease. The most surprising and gratifylng aspect of the entire Investigation was the realization that teachers and students Indicated a desire to continue problem solving on a regular basis, after the study was completed. Even more surprising was the indication by teachers of the intent to use problem solving as an instructional tool whenever possible. During the last focus group, thls investigator asked teachers to share ldeas which would allow them to present new topics using problem solving techniques. Seven of the teachers responded with well thought out examples, while only Teacher \# 6 Indlcated that time constralnts would not allow him to develop his own teaching materials.

Each of the elght teachers Indlcated that providing students the opportunlty to generallze solutions and strategles to other situations was the most difflcult Indlcator of Standard 1 for them to implement; it was also the last of the five indicators to receive across the board positive responses on the weekly checklists. Once agaln, lack of avallable materlal seemed to be a problem.

The teachers involved in this study belleve the NCTM Standards wlll be difflcult to 1 mplement Into the existing curriculum. They would like to see an Implementation period, during whlch time test scores and student achlevement comparisons would be eliminated as Indlcators
of educational success. Teachers Indlcated a deslre to see mathematics instruction emphasize learning rather than memorization, quallty rather than quantlity. They ask for the time and training required to do both; only then will the reality of mathematics education resemble the professional views of what mathematics education should be.

The final conclusions of thls study pertain to the question of instructional techniques. Why do teachers continue to teach mathematics using the same methods of Instruction whlch were used decades ago, when they apparently see the need and value of teaching in the manner advocated by NCTM? There appears to be three major factors which propagate the use of outdated modes of Instruction: time; materlals; and teacher evaluatlon.

The decade of the 1980 s brought increased demand for Improvement in public education. Many individuals advocated a back to basics approach to Instruction, where students are presented with more materlal and more toplcs, and are expected to retain more information thus becoming more knowledgeable. As a matter of efficlency, teachers learned that the quickest way to cover new materlal was the 'teacher presentation, student imitation' method. As more and more teachlng became expected of our educators, teachers report less and less time became avallable to them to experiment, to try new techniques, and to involve the student in his own learning. At the same time, teachers
have indicated there is strong evidence to show that students concluded that the most efflcient way to deal with the Increase in information was the 'memorlzation' method, where they learn what they need for the upcoming test, then discard old information and replace it with new. Teachers report that outside pressures have caused them to become more concerned with finishing the textbook than with providing students the opportunlty to reason, to think, and to make sense of what they are expected to learn. Therefore, teachers have indlcated that students are just not learning all they are expected to learn. Teachers say the ldea of minlmum competency and end-of-course testing is forcing them into a frantic cycle which demands maximum output and no varlation in teaching methods.

The second reason which causes teachers to continue using outdated methods of instruction concerns available materlals. State adopted textbooks offer no new teaching technlques, no problem solving strategies for the Introduction of new materlal and no new suggestions for teachers who wish to Improve or change their Instructional methods. Again, teachers report that they do not have the time, nor perhaps the abllity or inclination to develop their own materials. Textbooks are wrltten to utllize the concept of the economy of tlme and the plethora of materlals presented.

The last reason teachers continue to use old methods of lnstruction pertalns to teacher evaluation. Many states have adopted the concept of a 'six-step' lesson plan whlch Is used when observing and evaluating teachers. Teacher lecture methods or teacher presentation methods readily lend themselves to thls type of lesson plan, while problem solving strategies in which the student is responsible for much of hls own learning do not. Teachers have indlcated a bellef that it seems easy for an adminlstrator, or other observer, to determine the effectiveness of a teacher who is presenting material for his students to 'learn' but another matter entirely for that observer to determine the effectlveness of a teacher who serves malnly as a guide or a catalyst for learning. Throughout the course of this study, all eight teachers repeatedly expressed concerns pertalning to each of the three factors just mentioned (there were fewer concerns about teacher evaluation than the other two factors, however it remalns a valld concern). Teachers say they are belng forced, more or less, to use the familiar teacher lecture method of Instruction.

The following suggestions or recommendations can be made as a result of the conclusions of this study:

1. Since one of the major concerns of the elght teachers involved in this study was lack of approprlate tralning, thls investlgator will request that the adminjstrative offlce conslder providing a
serles of workshops andor staff development classes which will address this concern.
2. A textbook committee should be appointed for the purpose of seeking out and identlfying appropriate materlals and making them avallable to teachers.
3. Some type of network system should be developed in order to inform and involve teachers in local and national curriculum planning and reform.
4. The National Council of Teachers of Mathematics should provide training for teachers which would enable them to begin using the Curriculum and Teaching Standards, in the manner in which they were intended.
5. End-of-course testing should cease along with state and local comparisons of other test score averages for an undetermined perlod of time to allow teachers the freedom and opportunlty to implement teaching styles simllar to those recommended by the Standards, without teachers having to deal with the criticism which could result from any Initlal decrease In test score averages.
6. All definltions to be used during the course of Standard implementation should be developed and fully explalned by professionals in order to avold misuse or misinterpretation.

## LImitatlons of the Study

There are three primary limitations of this case study. The first limitation arises from the fact that this Inquiry made an attempt to implement only one of the fourteen Curriculum and Teachlng Standards for Hiah School Mathematics. Therefore, it would not be prudent to speculate on the degree of success for implementation of the remaining thlrteen Standards. Several of those Standards seemed unclear when discussed by the teachers Involved in this study, whlle problem solving is a concept with which most mathematics teachers feel comfortable. Teachers expressed concern with more than one of the remaining Standards, indlcating doubt for successful Implementation and uncertalnty for rellable methods of assessment. However, the concept of problem solving forms the basic framework for the type of mathematlcs curriculum proposed by NCTM and others. In order to improve student learnlng and achlevement, partlcularly student learning and achlevement of higher-order thlnklng skllls, the development of problem solving skills must become a priority. Thus Standard 1 was selected as the foundation of this inquiry. A second reason for the selection of only one of the fourteen Standards for the duration of thls study pertalns to teachers and thelr attltudes toward Change. Teachers seem willing to make gradual curriculum change and for the most part not only feel uncomfortable
with the concept of radical change, they also feel students adapt best to slow methodical change. In the opinion of thls Investigator, any attempt to implement more than one Standard would have created not only added resistance, but perhaps an impossible task with disastrous results.

The second limitation of thls study pertains to the time period involved. In any study of thls type, it would be preferable to contlnue the investlgation over a longer perlod of time, perhaps one to two years. An extended period of time would allow a true test of Standard 1 implementation and would allow the investlgation of long-term permanent results, and whether the change in mathematics instruction was accompanied by the desired increase in student knowledge. It would be interesting to re-evaluate this mathematics program in a year or two and assess the problem solving opportunlties avallable to students at that time in comparlson to those avallable at the conclusion of thls study.

The third and final limitation of this study pertains to generalizabllity. In any evaluation study, it is desirable that most results and outcomes can be generallzed to other similar situations. The findings for this study were based on one school and consequently on one mathematics curriculum. Therefore the results should not be generalized to any other existing mathematics curriculum or program. A similar program evaluation replicated in a
different locatlon with different teachers, students and problem solving activitles would perhaps produce dramatically different results. Thus this Investigator encourages extreme caution in any attempt to generallze the conclusions from thls study.

Recommendations for Further Study
Listed below are several recommendations for further study.

1. Since this program evaluation was done in only one hlgh school with only elght mathematics teachers, thls study should be replicated in other schools in order to compare the results of this study.
2. Since there were virtually no minorlty students in thls sample, the study should be repllcated in a school with a larger minorlty sample in order to compare the results.
3. The entire study should be repllcated in a different school system and/or geographlc location to compare results with a different population.
4. The entlre study should be replicated using a dlfferent NCTM Standard, or more than one Standard in order to compare the results.
5. The entire study should be repllcated over a longer period of time, allowing the investigation of permanent long-term results.

Change is needed in the way all children learn mathematics. As a matter of equlty, we should stop lgnoring 90 percent of our population when we teach mathematics. Equally important for soclety, we cannot hope for the solution of the problems that will face us in the 2ist century if we fall to educate all chlldren to the limlt of their capacity. In a world that is becoming steadily more quantitatlve, we must provide better mathematlcs education, for everyone, from Kindergarten through graduate school.

The actlvities suggested here require more work on the part of authors, teachers, and pupils. But activities that are meaningful to the students are more likely to be remembered and more likely to leave the learners with a feeling that mathematics is useful and worth learning. Surely that is better than having students belleve that mathematics is a subject they are required to learn to satlsfy other people, and that it should be put out of their minds as soon as posslble.

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APPENDICES
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## Appendlx A

## NCTM Currlculum and Teaching Standards for Hiah School Mathematles

## STANDARD 1: MATHEMATICS AS PROBLEM SOLVING

In grades 9-12, the mathematics curciculum should include the reflnement and extension of methods of mathematical problem solving so that all students --
--use, with increasing confidence, problem-solving approaches to investlgate and understand mathematical content;
--apply Integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
--recognize and formulate problems from sltuations within and outslde mathematics;
--apply the process of mathematlcal modeling to real-world problem situations.

## STANDARD 2: MATHEMATICS AS COMMUNICATION

In grades 9-12, the mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ldeas so that all students can--
--reflect upon and clarify thelr thlnking about mathematical ideas and relationships;
--formulate mathematical definitions and express generallzations dlscovered through investlgatlons;
--express mathematical ideas orally and in writing;
-read written presentations of mathematics with understanding;
--ask clarifying and extending questions related to mathematics they have read or heard about;
--appreclate the economy, power, and elegance of mathematical notation and its role in the development of mathematlcal ldeas.

## STANDARD 3: MATHEMATICS AS REASONING

In grades 9-12, the mathematics currlculum should include numerous and varled experlences that reinforce and extend logical reasoning skllls so that all students can--
--make and test conjectures;
--formulate counterexamples;
--follow logical arguments;
--Judge the valldlty of arguments;
--construct slmple valld arguments;
and so that, in addition, college-intending students can-
--construct proofs for mathematical assertions, Including Indirect proofs and proofs by mathematical Induction.

## STANDARD 4: MATHEMATICAL CONNECTIONS

In grades 9-12, the mathematlcs curriculum should include investigation of the connections and interplay among various mathematlcal topics and thelr applications so that all students can--
--recognize equivalent representations of the same concept;
--relate procedures in one representation to procedures in an equivalent representation,
--use and value the connections among mathematical toples;
--use and value the connections between mathematics and other dlsciplines.

STANDARD 5: ALGEBRA

In grades 9-12, the mathematlos curriculum should include the continued study of algebralc concepts and methods so that all students can--
--represent situations that Involve varlable quantitles with expressions, equations, Inequallties, and matrices;
--use tables and graphs as tools to interpret expressions, equations, and inequalities;
--operate on expresslons and matrlces, and solve equatlons and inequalitles;
--appreciate the power of mathematical abstraction and symbolism;
and so that, in addition, college-intendlng students can--
--use matrlces to solve llnear systems;
--demonstrate technical facillty with algebralc transformations, including techniques based on the theory of equations.

STANDARD 6: FUNCTIONS

In grades 9-12, the mathematics curriculum should include the continued study of functions so that all students can----model real-world phenomena with a varlety of functions;
--represent and analyze relationships using tables, verbal rules, equations, and graphs;
--translate among tabular, symbolic, and graphical representations of functions;
--recognize that a varlety of problem sltuatlons can be modeled by the same type of function;
--analyze the effects of parameter changes on the graphs of functions;
and so that, in addition, college-Intending students can--
--understand operations on, and the general properties and behavlor of, classes of functions.

STANDARD 7: GEOMETRY FROM A SYNTHETIC PERSPECTIVE

In grades 9-12, the mathematics curriculum should Include the continued study of the geometry of two and three dimensions so that all students can--
--Interpret and draw three-dimensional objects;
--represent problem situations with geometric models and apply propertles of flgures;
--classify figures in terms of congruence and similarlty and apply these relationshlps;
--deduce properties of, and relationships between, flgures from given assumptions;
and so that, in addition, college-intending students can--
--develop an understanding of an axiomatic system through investigating and comparing various geometries.

## STANDARD 8: GEOMETRY FROM AN ALGEBRAIC PERSPECTIVE

In grades 9-12, the mathematics curriculum should include the study of the geometry of two and three dimensions from an algebralc polnt of view so that all students can--
--translate between synthetic and coordinate representations;
--deduce properties of flgures using transformations and using coordinates;
--Identify congruent and slmilar flgures using transformations;
--analyze propertles of Euclldean transformations and relate translations to vectors;
and so that, in addition, college-Intending students can--
--deduce propertles of figures using vectors;
--apply transformatlons, coordinates, and vectors in problem solving.

## STANDARD 9: TRIGONOMETRY

In grades 9-12, the mathematlcs curriculum should include the study of trigonometry so that all students can--
--apply trigonometry to problem sltuations involving trlangles;
--explore periodic real-world phenomena using the sine and cosine functions;
and so that, in addition, college-intending students can--
--understand the connectlons between trigonometric and clrcular functions;
--use circular functlons to model periodic real-world phenomena;
--apply general graphing techniques to trigonometric Identltles;
solve trigonometrlc equations and verlfy trigonometrlc Identltles;
understand the connections between trigonometric functions and polar coordinates, complex numbers, and serles.

## STANDARD 10: STATISTICS

In grades 9-12, the mathematics curriculum should include the continued study of data analysls and statistics so that all students can--
--construct and draw inferences from charts, tables, and graphs that summarlze data from real-world sltuations;
--use curve fitting to predict from data;
--understand and apply measures of central tendency, varlabllity, and correlation;
--understand sampling and recognize its role in statistlcal claims;
--design a statistical experiment to study a problem, conduct the experiment, and interpret and communlcate the outcomes;
--analyze the effects of data transformations on measures of central tendency and varlabllity;
and so that, In addition, college-Intending students can--
--transform data to ald in data interpretation and prediction;
--test hypotheses uslng appropriate statistlcs.

## STANDARD 11: PROBABILITY

In grades 9-12, the mathematics curriculum should include the continued study of probabillty so that all students can--

```
--use experimental or theoretical probabllity, as
approprlate, to represent and solve problems involving
uncertalnty;
--use simulations to estlmate probabillties;
--understand the concept of random varlable;
--create and lnterpret discrete probablllty
distributions;
```

--describe, in general terms, the normal curve and use Its propertles to answer questions about sets of data that are assumed to be normally distributed;
and so that, in addition, college-intending students can--
--apply the concept of a random varlable to generate and Interpret probabllity distributions including binomial, uniform, normal, and chl square.

## STANDARD 12: DISCRETE MATHEMATICS

In grades 9-12, the mathematics curriculum should include toplcs from discrete mathematics so that all students can----represent problem situations using discrete structures such as finite graphs, matrlces, sequences, and recurrence relatlons;
--represent and analyze finite graphs using matrices;
--develop and analyze algorlthms;
--solve enumeration and finite probabllity problems;
and so that, in addition, college-intending students can--
--represent and solve problems using linear programing and dlfference equations;
-- Investigate problem situations that arlse in connection with computer valldation and the application of algorlthms.

STANDARD 13: CONCEPTUAL UNDERPINNINGS OF CALCULUS

In grades 9-12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerlcal perspective so that all students can--
--determine maximum and minimum polnts of a graph and interpret the results in problem sltuations;
--Investlgate limlting processes by examinlng inflnlte sequences and serles and areas under curves;
and so that, In addition, college-intending students can--
--understand the conceptual foundations of limlt, the area under a curve, the rate of change, and the slope of a tangent line, and thelr applicatlons in other disciplines;
--analyze the graphs of polynomial, rational, radical, and transcendental functions.

STANDARD 14: MATHEMATICAL STRUCTURE

In grades 9-12, the mathematics curriculum should include the study of mathematical structure so that all students can--
--compare and contrast the real number system and its varlous sub-systems with regard to their structural characterlstlos;
--understand the loglc of algebralc procedures;
--appreclate that seemingly different mathematical systems my be essentially the same;
and so that, in addition, college-intending students can--
--develop the complex number system and demonstrate facility with its operations;
--prove elementary theorems within varlous mathematical structures, such as groups and fields;
--develop an understanding of the nature and purpose of axlomatic systems.

## Appendix B <br> Problem Solving Attitude Assessment Survey

1. I belleve the problem solving activitles in whlch I have participated will improve my mathematics abllity.
$\qquad$ AGREE $\qquad$ DISAGREE
2. I enjoy finding different methods for solving problems.
$\qquad$ AGREE $\qquad$ DISAGREE
3. If I had a choice, I would not contlnue the problem solving activities.
$\qquad$
4. I belleve the problem solving actlvitles are a waste of time.
_ AGREE DISAGREE
5. I would rather the teacher just do the sections In the book.
_ AGREE DISAGREE
6. I would like the teacher to use a problem solving approach when teaching.
$\qquad$
7. I belleve working a wlde varlety of problems will help improve my confldence in my abllity to solve problems.
_ AGREE DISAGREE
8. Working with different types of problems will not help my mathematics abllity.
_ AGREE
_ DISAGREE
9. Having experience in a wide varlety of problem solving will help me attempt problems whlch I do not know how to solve.

AGREE
10. Particlpating in the problem solving activitles has helped me to reallze $I$ have to abllity to solve various problems.

A AGREE
DISAGREE
11. I do not like problem solving.
_ AGREE
—_ DISAGREE
12. I would rather the teacher just told me how to do the problems.
_ AGREE
DISAGREE
13. I feel better about my abllity to solve problems since the problem solving activities.
— AGREE DISAGREE
14. Problem solving has improved my ability to think in a logical manner.
—_ AGREE DISAGREE
15. Because of the problem solving activitles, I am more confident about my abillty to use different strategies to find a solution for problems.
$\qquad$
$\qquad$ DISAGREE

# Appendix C <br> Results of Focus Group Discussion for the Fourteen NCTM <br> Curriculum and Teaching Standards 

Standard 1: Mathematics as Problem Solving
a. Importance in curriculum: vitally important. Mathematics is problem solving, therefore if students are not learning problem solving, they are not learning mathematics.
b. How to implement: Teachers can begin Implementation through a serles of exercises, where students are Introduced to a variety of problems, Including both nonroutine problems and nonroutine methods for finding solutions. Students should be taught to view mathematlcs in a more personal and relevant manner, and to learn to generallze solutions to different problems in mathematics and In everyday life. Teachers should gradually Increase the frequency for problem solving actlvitles and make every effort to Incorporate problem solving strategies into appropriate teaching methods.
c. Changes required for full implementation (where problem solving is part of the mathematics currlculum): Teachers must reallze the Importance of problem solving in the mathematlcs curriculum.

Second, teachers belleved that before they could teach most topics from a problem solving approach,


#### Abstract

they would require extensive re-tralning. There should be less emphasis placed on end-of-course testing and less pressure to cover all the pages In the text. Finally, and perhaps most important, there must be development and provision of new textbooks with approprlate materlals whlch emphasize mathematlcs through a problem solving approach, since most textbooks currently emphasize drlll and practlce.


Standard 2: Mathematics as Communlcation
a. Importance in curciculum: Important. Students should be able to express mathematical Ideas, both orally and In writing. They should be able to read mathematics content with comprehension. When glving explanations and discussions of problems, appropriate modes of communication would allow them to convey to others what they actually mean without pointing and gestering in meaningless ways.
b. How to Implement: Allow students to give oral explanations and instructions for working problems. Have students research and actually present some topics to the class. Introduce mathematics vocabulary llsts.
c. Changes required for full implementation: Teachers need to become more passive in their classrooms, allowing students the opportunity to dlscuss mathematlcs toplcs.

Standard 3: Mathematics as Reasoning
a. Importance in currlculum: very important. The abllity to reason allows students to make sense of the world around them, to make good Judgements, and to become better cltizens. Mathematlcally, the abillty to reason is the basis of all problem solving; it enables students to hypothesize, conjecture, and to formulate solutlons and strategles.
b. How to implement: This can begin in the mathematics curriculum as one facet of problem solving and then continue into formal proof through Algebra I, Geometry, etc. Teachers should allow students the opportunlty and time to think through mathematlcal sltuations before supplying a solution. Allow student to develop their own theorles and to discuss these during class time.
c. Changes required for full implementation:

Teachers should allow more 'think time.'

More minutes per class perlod would be helpful. Teachers require addltional tralning and the development of new attitudes toward their own role in the classroom. Eliminate standardized testing in the mathematics classroom, and begin tests whlch require discussion and the formation of conclusions.

Standard 4: Mathematical Connections
a. Importance in curriculum: Important.

Students should use and value the relationshlps and connections among the various mathematical topics. It is also necessary that students reallze mathematlcal relatlonshlps are applicable In other disclplines. They should be able to apply thelr mathematical knowledge to situations In the real world.
b. How to implement: Teachers should use examples from business, social studles, science, physics, drafting, and all other disciplines. It is Important to allow students to view mathematics as useful and appllcable in all areas.
c. Changes required for full implementation:

Teachers need more time to plan and develop lessons relative to other disclplines. Course requirements
would need to focus more on Inter-dlsclplinary applicatlons. Teachers must flrst understand the standard and belleve in its importance. New textbooks would need to be written and made avallable.

Standard 5: Algebra
a. Importance In curriculum: extremely Important. Algebra provides the framework and language through which most mathematlcs is communlcated. Therefore, algebra $1 s$ an Important processing tool for applying mathematics in many disciplines. All students should have a proflciency in algebra.
b. How to implement: Begin a Pre-Algebra course for all students in the seventh grade, with Algebra I offered in the elght grade for advanced students, and In the ninth for all others.
c. Changes required for full implementation: Students and parents must realize the importance of Algebra proficiency. Counselors should stop advising students to avold Algebra courses.

Standard 6: Functions
a. Importance In curriculum: important.

An understanding of functions allows students to conceptuallze the relationships and correspondence between the elements of two sets. A study of functlons begins with simple arithmetic operations and should continue through the study of mathematlcs.
b. How to implement: Teachers should establish a strong conceptual foundation before the formal notation and language of functlons are presented. The study of functions should begin with those relationships which exist in the student's own world. The use of graphs in depleting data is also a useful method of showing the relationships of functions.
c. Changes required for full implementation: The concept of functlons and the relatlonshlps between numbers should be introduced to students very early in arithmetic. New materials and teacher re-trainling would be essential.

Standard 7: Geometry from a Synthetlc Perspective
a. Importance in curriculum: very important.

Students must have an understanding of shapes and their properties, with an emphasis on their applicabllity in human activity.
b. How to lmplement: Teachers should use examples of how geometry is used in recreations, in practical tasks, in the sclences, and In the arts. Students should have the opportunity to visuallze and work with three-dimensional flgures. Teachers should use physical models and other real-world objects to help students develop a geometric intuition.
c. Changes required for full implementation: Student must develop a strong foundation in the K - 8 programs. Teachers should focus on more than deductive reasoning and proof. Teachers must be able to visualize, provide plctorial representation and application of geometric ideas, and to answer questions about natural and physical phenomena. This will require teacher training and new materlals.

Standard 8: Geometry from an Algebralc Perspectlve
a. Importance in currlculum: somewhat important. This standard was difflcult for thls group of teachers to actually visuallze. Transformations are not usually consldered by most teachers to be of great Importance.
b. How to implement: Other than continuing the methods currently belng used, there were no
suggest 1 ons.
c. Changes required for full implementation: Teachers did not know.

Standard 9: Trigonometry
a. Importance in curriculum: very important.

Trigonometry is based on the study of triangles. Many real-world problems require the solution of triangles. All students should apply trigonometric methods to practical situations involving triangles.
b. How to implement: Using calculators, trigonometry should be introduced to students at much earller ages. Continue current curriculum practices. Other strategies not known.
c. Changes required for full implementation: Offer teacher workshops and develop new materials.

Standard 10: Statistics
a. Importance in curriculum: Increasingly important. Collecting and representing data are activities of major importance in today's soclety. Knowledge in statistlcs allows students to test hypotheses and to draw Inferences.
b. How to Implement: Students should be exposed to data analysis in grades $K-8$, and should be encouraged to apply statistical tools to other academic subjects such as English, social studies, and blology, as well as athletles and other out of school activlties.
c. Changes required for full implementation: Massive re-tralning of teachers in all subject areas and at all grade levels. Development of new materlals.

Standard 11: Probablllty
a. Importance in curriculum: somewhat important. Probabllity provides the methods for dealing with uncertalnty and for lnterpreting predictions based on uncertalnty. Students should know how to make Informed observations about the llkelihood of events, and to judge the valldity of statistlcal clalms. Although probability provides useful models for solutions of problems in physics, medicine and economics, many problems in dally living can also be better understood using probabllity.
b. How to implement: Not readlly known.
c. Changes required for full implementation:

Teacher training and development of approprlate materlals.

Standard 12: Discrete Mathematics
a. Importance In curriculum: somewhat Important Discrete mathematics is a relatively new term and as such was not fully understood by this group of teachers.
b. How to lmplement: Not known.
c. Changes required for full implementation: Extensive teacher training along with the development of approprlate teaching units and materials.

Standard 13: Conceptual Underpinnings of Calculus
a. Importance in curriculum: important. Today, methods of calculus are applled increasingly In the social and blological sclences and in business as well. Students should appreclate the value of calculus in the improvement of the world's economic status.
b. How to lmplement: Teachers should provide students an opportunity to informally explore some of the central ldeas of calculus, while introducing and
answerlng questions about real-world phenomena.
c. Changes requlred for full implementation: Develop a course for teachers which would allow them to experience those recommendations found in section b. above.

Standard 14: Mathematlcal Structure
a. Importance in curriculum: somewhat important An awareness of the broad structure of the principles of mathematics provides them with a framework which facllitates long-term retention.
b. How to Implement: Allow students the opportunlty to understand the idea of structure through the observation of the common properties of systems that seem on the surface to be quite dissimilar. How this could be done is not readily known.
c. Changes required for full implementation: Teacher re-training, along with the development of relevant and appropriate materlals. New textbooks would be helpful.

## Appendix D

## South Caldwell Hioh Schoal Floor Plan




## Appendlx E

## Examples of Nonroutlae Froblems

Example 1: Suppose that one-third of the population smokes, that 60\% of all smokers are female. What is the fraction of the population represented by smoking males?

Example 2: Suppose a dress is sold at full price on Monday. The dress is marked down 30\% on Tuesday, then marked down 20\% from Tuesday's price on Wednesday. If the final price $1 s$ \$28, the what was Monday's price?

Example 3: If one-thlrd of the alr in a contalner is removed with each stroke of a vacuum pump, what fraction of the orlginal amount of alr remalns in the container after 5 strokes?

Example 4: Mr. Kato had some paperback books that he no longer wanted. Rather than throw them away, he put them in a box and brought them to school for his students. At the end of each class perlod, he let the students in that class take a fraction of the books that were still left in the box. He told flrst period to take $1 / 6$ of the books; second period to take $1 / 5$ of the remalning books; third period to take $1 / 4$ of those that remalned; fourth period to take $1 / 3$ and fifth perlod to take $1 / 2$ of the remalning books. Thls left 14 books for the sixth period, who took all 14 books. How many books did Mr. Kato start with?

Example 5: Find the sum of

$$
1+3+5+7+9+11+\ldots+(n-2)+n
$$

Example 6: A cake shaped like a cube falls into a vat of chocolate creme. It is then cut into one inch squares. What are the dimensions of the cake if there are 384 pleces with one side frosted? What are the dimenslons if there are $n$ pleces with one slde frosted?

Example 7: Two friends, Al and Bob, and their dog, spent their vacation in the Malne woods. One day Al went on a walk, alone, while Bob, followed him an hour later, accompanled by the dog. He ordered the dog to follow Al's trall. When the dog reached Al, Al sent hlm back to Bob, and so on. The dog ran to and fro between the two friends untll Bob caught up with Al, who happened to be a slow walker. Indeed Al was making no more than $1 / 2$ miles an hour, while Bob made 3. The dog's speed was 6 miles an hour. Now, what is the distance the dog ran to and fro untll Bob caught up with Al? We may presume that the dog lost no time playing with his two masters or hunting rabbits.

Example 8: Find the product of
$(1-1 / 2)(1-1 / 3)(1-1 / 4) \ldots(1-1 / 98)(1-1 / 99)(1-1 / 100)$

Example 9: I have a robot. It is not very smart. There are two buttons on the machlne. The first causes the robot
to take one step and the second causes the robot to take two steps. How many sequences of button pushes will cause the robot to take 12 steps? How about 20 steps?

Example 10: The new high school has just been completed. There are 1000 lockers in the school and they have been numbered 1 to 1000. During recess, the students decide to try an experiment. When recess $1 s$ over, each student will walk into the school one at a time. The first student wlll open all the locker doors. The second student wlll close all the locker doors with even numbers. the third student will change all the locker doors with numbers that are multiples of three. The fourth student wlll change the position of all locker doors numbered with multiples of four: The fifth student wlll change the position of the lockers that are multiples of flve, and so on. After 1000 students have entered the school, which locker doors will be open?

## Appendix F <br> Examples of Mathematics Problems from Evervday Life

Example 1: How many handshakes will occur at a party if every one of the 15 guests shakes hands wlth each of the others?

Example 2: Jean 1 s attending a two week (14 day) summer camp. Her parents gave her $\$ 50$ for 'Incidental' expenses for the entire two weeks. After three days at camp she had spent ${ }^{\text {15 }}$. Assuming she continues to spend at the same rate, will her budget of $\$ 50$ hold out for the 5 wo weeks? If not, when will she run out of money and how much more will she need?

Example 3: A farmer wishes to buy a plece of land that is adjacent to his farm. The real estate agent tells him that the plot is triangular in shape, with sides of 20,75 and 45 meters. The land will cost only $\$ 5.58$ a square meter. How much should the farmer pay for the plece of land?

Example 4: Six people, let's call them $A, B, C, C, E$ and F, have witnessed a burglary and are only too willing to let the police know what the burglar -- who by the way, managed to escape -- looked like. But you know how eyewitnesses' accounts go; the descrlptions of the criminal differed in every important polnt, partlcularly with regard
to the color of his halr and eyes, the color of his sult and probable age.

This is the testimony the pollce sot from these six witnesses:

Hair Eyes Sult Age
A brown blue grey 34
B blond black dark blue 30
C red brown dark brown 34
D black blue not dark brown 30
E brown black grey 28
F blond brown dark blue 32
Through these contradictory reports the police finally got their man and compared his real appearance with the six descriptions. They found that each of the six witnesses had made three erroneous statements and that each of the four questions had been answered correctly at least once. What did the burglar really look like?

Example 5: An office manager must assign offlces to six staff members. The avallable offlces, numbered 1 - 6 consecutively, are arranged in a row, and are separated by six-foot high dividers. Therefore, volces, sounds, and cigarette smoke readlly pass from each offlce to those on either side. Miss Braun's work requires her to speak on the telephone frequently throughout the day. Mr. White and Mr. Black often talk to one another in thelr work, and prefer to have adjacent offlces. Miss Green, the senlor,
employee $1 s$ entltled to Offlce 5 , whlch has the largest window. Mr. Parker needs sllence in the offlce(s) adjacent to his own. Mr. Allen, Mr. White, and Mr. Parker all smoke. Miss Green $1 s$ allergic to tobacco smoke and must have non-smokers in the office(s) adjacent to her own. Unless otherwise specifled, all employees maintain sllence while in their offices. Find the best locations for each Individual's offlce.

Example 6: Plan the food for a group party. What quantitles should you get? What $1 s$ the total cost? Don't forget the cost of ltems such as ice, napkins, paper plates, cups, etc. Decide how much to charge each person?

Example 7: During the census, a man told the census-taker that he had three chlldren. When asked thelr ages he replled, "The product of their ages 13 72. The sum of their ages is my house number." The census-taker turned, ran outside to look at the house number dlsplayed over the door. He then re-entered the house and said, "Using the Information you have given me, I cannot tell their ages." The man then said, "I should have told you that the oldest likes angel food cake." Hearing thls, the census-taker promptly wrote down the ages of the three children. What did he write?

Example 8: Six gamblers play a remarkable game of chance. The game ltself is rather primitive but the loser is in a
bad spot. He is supposed to double the pool of each of the other five gamblers. Altogether, they play six games and by chance each of the men loses Just once. When the men later counted how much is left to each of them, they discover that each owns exactly \$64. How much had each of them when they started?

Example 9: The Smith family, which conslsts of Mr. and Mrs. Smlth, thelr son, Mr. Smith's sister, and Mrs. Smith's father, has for years dominated the community life of Plainsville. At the present time the flve members of the family hold among themselves the positions of grocer, lawyer, postmaster, preacher, and teacher in the little town. The lawyer and the teacher are not blood relatives. The grocer is younger than her sister-ln-law but older than the teacher. The preacher, who won this letter playing football in college, is older than the postmaster. What position does each member of the family hold?

Example 10: According to the U. S. Department of Agriculture, the amount shown for each of the following foods ylelds 20 grams of protein. Check current prices in the community and then decide whlch of these foods is the most economical source of protein.
$21 / 3$ ounces, center-cut pork chop
1 1/3 cups, whole mllk
$31 / 2$ hot dogs

4 1/2 tablespoons, peanut butter
3 ounces, ground beef
$31 / 3$ ounces, cured ham
3 large eggs
9 slices of white enrlched bread


[^0]:    Statistical slgniflcance and programmatic slgniflcance are analogous to rellabillty and validity. Like rellability, statistical signiflcance is a measure of preclsion; like valldity, programatic slgnlficance ls a measure of efflcacy and cogency (page 187).

