

A STOCHASTIC DOMINANCE ORDERING OF SCHEDULING RULES

By: [James K. Weeks](#), and Tony R. Wingler

Weeks, J. K. & R, W. T. (1979). A Stochastic Dominance Ordering of Scheduling Rules. *Decision Sciences (Journal of)*, 10 (2), 245-257.

Made available courtesy of Blackwell Publishing Ltd

*** **Note: Figures may be missing from this format of the document**

*** **Note: The definitive version is available at <http://www.blackwell-synergy.com/>**

Abstract:

This paper applies stochastic dominance (SD) preference-ordering criteria to job shop scheduling rules. A simulation model of a hypothetical dual-constrained job shop is used to derive several measures of shop performance for a number of dispatching/due-date scheduling policies. The results presented suggest that previous research conclusions concerning the relative performance of dispatching scheduling rules may need to be reconsidered if production schedulers are risk-averse utility maximizers.

Article:

INTRODUCTION

Preference orderings of alternative production scheduling rules are generally based on the expected values of the distributions of various performance measures. Several performance measures are posited in the shop scheduling literature as surrogate measures for the following three major objectives of any production system: minimizing work-in-process inventory levels, maximizing the utilization of facilities, and maximizing customer service. Measures of job throughput (flow) time and number of jobs in the system are generally used to represent inventory levels, while utilization of resources is usually measured by the percent of time the facilities are in a productive state or the number of jobs completed in a given time frame. Customer service is most often measured by the amount of tardiness (lateness and/or earliness) of jobs or the number of missed job due dates.

The expected values of these measures, or some weighted function of these measures, are used to rank order the performance of various scheduling decision rules. Numerous studies (see, for example, [5], [6], and [7]) indicate the superiority of the shortest-imminent-processing-time dispatching rule in terms of lower mean job flow time and higher facility utilization. The superiority of the job slack or due-date related rules is also well documented in terms of meeting job due dates. Similar conclusions based on estimates of the expected values of other performance measures have been reported in job shop scheduling literature for other operating decision rules and design parameters.

The preference-ordering criteria used in previous shop scheduling studies are based on restrictive implicit assumptions about the decision maker's utility function. Using estimated expected values of performance measures, whether these measures represent single-response or multiple-response measures in the form of cost functions, is appropriate for utility maximization under the restrictive assumption of linear utility functions (where marginal utility is constant). Expected-value criteria may also be appropriate for quadratic or *n*th degree polynomial utility functions if one knows the magnitudes as well as the signs of the *n* derivatives of the utility function [9]. Given the choice, one would always maximize utility by selecting the scheduling rule that results in the lowest expected cost and least risk. As demonstrated by the relative performance of the shortest-imminent-processing-time rule in terms of job flow time, lower expected costs are usually characterized by higher variance of expected cost. Thus, the existence of uncertainty of outcomes of various scheduling rules requires consideration of the decision maker's utility function.

Optimizing expected values of performance measures is appropriate for a constrained set of circumstances. Economic [2], psychological [1], and statistical [15] literature asserts that individuals are risk averse, so one

would not expect production schedulers to be great risk takers. Under conditions of risk aversion, where the utility function is concave, there is decreasing marginal utility of returns or cost reductions. Further, there is no reason to expect that between two alternatives there is an exact offset of the utility effects coming from the higher statistical moments. Given an objective of utility maximization, one would expect unsound decisions to result when preference orderings are based on expected values and inappropriate utility functions.

Preference orderings may be based on a more general efficiency analysis of the entire probability distribution of outcomes of scheduling rules without relying on specific assumptions about an individual's utility function. This approach, referred to as stochastic dominance (SD), provides the individual with ordering rules that have been shown to be theoretically superior to expected-value rules [9]. The usefulness of the SD approach has been demonstrated in the areas of portfolio selection [12] [13] [14], debt-issuance strategies [3], and inventory control [11].

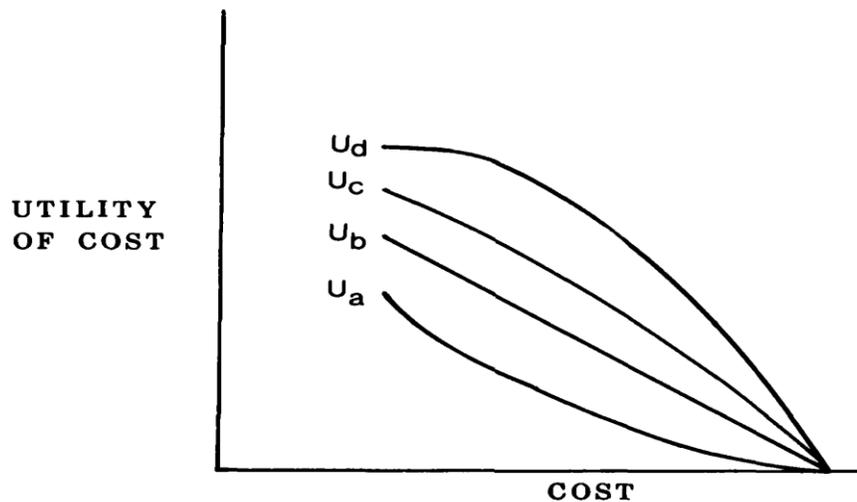
The SD ordering rules assume that individuals will select a production scheduling rule that maximizes expected utility. The rule employed depends upon the form of the utility function. These rules may then be used to divide all alternative scheduling methods into two mutually exclusive sets: an efficient set of undominated alternatives and an inefficient set of dominated alternatives. The next section presents two SD rules and relates these rules to the appropriate utility function.

In the Methodology section an exemplary job shop system is used to illustrate how the SD methodology may be used in job shop scheduling to choose among alternative scheduling rules. An application of the SD ordering rules to the results of the exemplary job shop is presented in the Experimental Results section. Finally, the conclusions of this research and suggestions for future research in this area are presented.

STOCHASTIC DOMINANCE ORDERING RULES

This section presents the notation and definitions required to develop SD rules. Two types of utility functions are considered as subsets of the general utility function, $U = h(x)$, where x represents the costs of an alternative scheduling rule. The first type of utility function $U_1 = \{UEC^\circ, U' < 0\}$, is the most general case, where C° is the set of all functions having continuous first-order conditions and U' represents the first derivative. This utility function is assumed to be non-increasing. A more restricted type of utility function may be represented by $U_2 = \{UEC^\circ, U' < 0, U'' \leq 0\}$, where U'' is the second derivative.¹ This utility function restricts the previous set of utility functions to those functions that are concave from below.

FIGURE 1
Utility-of-Cost Functions



¹These conditions are more commonly stated in returns space where x represents returns of an alternative and $U = h(x)$ represents the general utility function of returns. In returns space the two types of utility functions are represented by $U_1 = \{UEC^\circ, U' > 0\}$ and $U_2 = \{UEC^\circ, U' > 0, U'' \leq 0\}$. Since costs may be viewed as foregone revenues, utility-of-cost functions are mirror images of utility-of-returns functions. Thus, the direction of the inequality condition for U' is reversed to reflect the inverse relationship between utility and cost.

Figure 1 shows the two types of utility functions expressed in terms of cost. The utility-of-cost curves U_1 , through U_2 , satisfy the most general requirement that $U' < 0$. The utility-of-cost curves U_3 , through U_4 , represent a subset of the general class where $U'' \leq 0$.

In the most general case where $U' < 0$, no restrictions are placed on the decision maker's utility function beyond the reasonable assumption that it is non-increasing with respect to cost. This assumption is appropriate for risk seekers as well as risk averters since the utility-of-cost function can be either convex or concave. An implicit assumption is that decision makers behave consistently in that less cost is preferred to more cost.

A more restrictive assumption concerning the shape of the utility function must be added to reflect risk-averse behavior. Since risk aversion is characterized by concave utility functions, the second derivative of the utility function must be less than or equal to zero. The condition $U'' \leq 0$, which describes the functions through U_3 , shown in Figure 1, reflects the non-increasing (diminishing) marginal utility of cost reductions associated with risk-averse decision making.

For any decision maker whose utility-of-cost function meets the general requirement of U_1 , $U'(C) < 0$, independent of their attitude toward risk, policy X will be preferable to policy Y if and only if the expected utility of cost of X, $EU_X(C)$, is greater than the expected utility of cost of Y, $EU_Y(C)$. That is, if $EU_X(C) > EU_Y(C)$, policy X dominates policy Y. Policy Y can be relegated to the nonefficient group of scheduling policies.

A necessary condition for one policy to be preferred to (or dominate) a second policy is that the expected cost of the preferred policy be less than the expected cost of the second. No such systematic relationship can be established with respect to the variance (or other dispersion measures) of cost. Whether policies with high (low) variance are preferred to policies with low (high) variance depends on the risk-taking preference of the decision maker.

For the general condition U_1 , the efficiency criterion is designed to find the efficient (undominated) set of policies for risk takers as well as risk averters. Since the attitude toward risk is not specified, only the expected value and not higher moments of the costs of alternative policies is required to develop the general efficiency criterion. For the risk-averse type of utility function U_2 , the entire distribution, not just the first moment, is required to develop an efficiency criterion.

The efficiency criterion based on the conditions specified for U_1 enables an initial screening of policies and the elimination of policies that are inefficient for all decision makers. The stronger assumption of risk aversion set forth in U_2 enables a more sensitive, but less general, screening of policies so that the efficient subset for risk averters can be expected to be smaller than the efficient set determined under U_1 .

When an alternative is preferable to another alternative under U_1 , it is called first-degree stochastic dominance (FSD). Similarly, when an alternative is preferred under the more restrictive conditions of U_2 , it is called second-degree stochastic dominance (SSD). The ordering rules for these two situations are defined as follows:

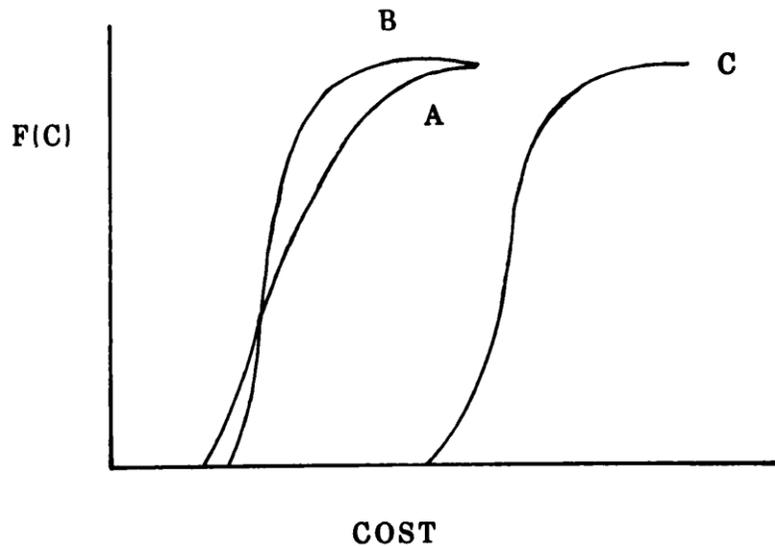
FSD Rule: For any two distributions f and g , g is said to dominate f by FSD if $G(X_i) \leq F(X_i)$ for all X , $E X$ where X , represent cost flows, with the strict inequality holding for at least one i . Thus, the cumulative distribution of g must lie completely to the left of the cumulative distribution of f .

SSD Rule: For any two distributions f and g , g is said to dominate f by SSD if $\int_{-\infty}^{\infty} [G(X_i) - F(X_i)] dX_i \leq 0$ with the strict inequality holding for at least one i . Thus, over the entire range of costs the cumulative area between the two distributions always remains negative.² Risk averters would always choose alternatives that dominate by second degree.

Figure 2 presents a graphical example of the ordering rules. The curves a, b, and c represent the cumulative cost distributions of three alternative scheduling policies. Since the cumulative distributions of a and b lie below that

of c, policies a and b dominate policy c by FSD because they have a lower level of expected cost and a higher level of expected utility of cost.

FIGURE 2
Cumulative Cost Distributions of
Alternative Scheduling Policies a, b, and c



When the more restrictive ordering rule for SSD is applied to the FSD efficient set, policy d dominates policy a because the cumulative difference between b and a is negative over the range of costs. Risk-averse decision makers will prefer policy b because it provides a greater utility of costs.

METHODOLOGY

The operational usefulness of the SD approach to preference ordering alternative production-scheduling rules is dependent on (a) the ability to identify the appropriate criteria for performance and model the relationship between these criteria and alternative rules, (b) the availability of sufficient data points to apply the SD ordering rules presented in the previous section, and (c) the computational requirements of the SD ordering rules.

Research Methodology

Because of the complexity inherent in most scheduling systems, computer simulation was chosen as the research methodology of this study. Direct experimentation with a system is likely to be too costly, while analytical methods are not sufficiently powerful for most systems to relate system performance and alternative scheduling rules. Most investment applications of SD use past-period returns to preference order alternative investments [12] [13] [14]. Computer simulation models have been used to analyze the sensitivity of SD orderings for changes in sample sizes [10].

A computer simulation model of a hypothetical dual-constrained job shop is used as an exemplary job shop system. A detailed description of the simulation model employed may be found in Weeks [17]. The shop consists of four work centers of different machine types and four laborers. Each work center contains two identical machines and all laborers are equally efficient in operating any machine. Job interarrival and service times are stochastically determined using the negative exponential distribution. The expected times are selected to yield a labor utilization of approximately 90 percent. A mixed job shop scheme is used to randomly generate job routings.

A laborer is eligible for reassignment when he completes servicing a job. All laborers eligible for reassignment are transferred to the work center with the job in queue that has been in the system the longest period of time.

No jobs are in the shop, all laborers and machines are idle, and one laborer is assigned to each of the four work centers at the start of each simulation run. The first 1000 time units are discarded to establish an approximate

steady-state condition. After the equilibrium period, each run is segmented into equal sequential time blocks to obtain multiple measures of performance. For each experiment in this study the same sequence of pseudo-random numbers is used to generate job and machine service characteristics. Therefore, the same set of jobs is used in all cases. A particular job arrives at the same time and receives the same routing and processing times for all cases, although for each experimental condition different sets of jobs are used across the different time blocks.

A scheduling policy for the shop is comprised of two decisions: (1) dispatching rules to determine the job selected for processing from the queue at a work center and (2) due-date assignment rules. The two dispatching rules used in this study and identified by the variable q are the shortest-imminent-operation-time rule ($q = 1$) and the least-slack-per-operation-remaining rule ($q = 2$). Total work content rules used to assign due dates to arriving jobs are identified by the variable d . The due dates are assigned with the following equation:

$$Dd_i = ta_i + Ktp_i \tag{1}$$

where dd_i = due date of job i ,
 ta_i = time of arrival of job i ,
 tp_i = processing time of job i , and
 $K = 7.0, 3.0, 1.5$ for $d = 1, 2, 3$, respectively.

The six scheduling policies representing all possible combinations of the decision rules are defined in Table 1. General results of these dispatching and due-date rules have been widely reported in previous research [5] [6] [18] and provide a basis for comparing the results of this study.

TABLE 1
Definition of Scheduling Policies

Scheduling Policy	Decision Rules	
	Dispatching q	Due Date d
1	1	1
2	1	2
3	1	3
4	2	1
5	2	2
6	2	3

Several criteria are used to measure the shop performance of these rules. Single-response measures such as job flow time, job lateness, job earliness, job missed due dates, and labor utilization are used to compare previous research conclusions based on expected-value criterion rules with those based on SD ordering rules.

Research Design

An important research design decision is the manner in which observations are to be collected. Comparing scheduling policies based on expected values of performance requires an estimation of the standard error of the estimate to construct confidence intervals. Since simulation data are usually autocorrelated, the conventional methods for computing interval estimates for expected values are not appropriate. The recommended procedures for determining sample sizes and blocking simulated data observations have revolved around estimating or removing the correlation in the data. Various blocking procedures have been recommended [4] [8] to compute uncorrelated block means that may then be used as the basic observations.

Similarly, the estimation of the probability distributions required for SD tests is usually based on samples from populations. Research in this area [10] indicates that the power of the SD-ordering rules is improved with a larger number of observations (sample size), each of which is collected over longer periods (block sizes) of time.

Since an underlying objective of this study is to compare the performance rankings of scheduling rules using SD rules with those using traditional expected-value criteria, the sample size and blocking procedure were designed to satisfy the above considerations. Ideally, a large number of block means, each representing a large number of data observations, should be collected, but constrained resources (computer time for this study) do not usually permit collecting such large samples.

For each experiment (scheduling policy) in this study, a sample size of forty block means, each collected over a time period of 1000 time units, is used to estimate the parameters and probability distributions required to compare the scheduling rules. Based on previous research [17] a block size of 1000 time units was selected to provide independent (zero-correlated) block means. A sample size of forty block observations was selected as a compromise between the substantial simulation costs of large sample sizes and the attendant increase in precision in estimating the true probability distribution resulting from large sample sizes.

Six experiments representing all possible combinations of the dispatching and due-date assignment rules are performed for this study. For each experiment, block means of the single-resource measures are computed as the basic data observations to be used in the expected-value and SD-ordering rules.

Computational Feasibility of SD Rules

Expected-value ordering rules may be efficiently performed with pocket calculators or any number of software packages. SD-rule programs, however, are not widely available and are quite cumbersome to compute manually. The computation requirements increase exponentially as either the number of scheduling policies or sample size increases. The algorithm used to make the comparisons required to apply the SD rules is described by Porter, Wart, and Ferguson [14]. Execution times of 1.4 seconds were typical for stochastic dominance comparisons in this study.

TABLE 2
Experimental Results

Scheduling Policies	Mean Flow Time		Mean Lateness		Mean Earliness		Mean Missed Due Dates		Mean Labor Utilization	
	\bar{X}	S.E.	\bar{X}	S.E.	\bar{X}	S.E.	\bar{X}	S.E.	\bar{X}	S.E.
1	9.12	.46	.46	.11	16.63	.31	17.08	.23	.9020	.0030
2	9.12	.46	2.02	.30	3.73	.15	5.75	.19	.9020	.0030
3	9.12	.46	4.20	.42	.50	.03	4.70	.40	.9020	.0030
4	9.67	.46	.32	.07	15.95	.36	17.27	.30	.9020	.0030
5	9.11	.43	1.70	.27	3.43	.15	5.13	.16	.9020	.0030
6	9.22	.45	4.24	.42	.46	.03	4.68	.40	.9024	.0030

EXPERIMENTAL RESULTS

The results in terms of performance means and standard errors for the six experiments are shown in Table 2. Analysis of variance and multiple comparison techniques were employed to identify the best scheduling policies based on expected-value ordering criteria. Two-way analyses of variance were performed to test the homogeneity of the various scheduling policies. As indicated by the analysis of variance statistics shown in Table 3, the null hypotheses that predicted lateness, earliness, and missed due dates are equal for each of the policies can be rejected.

TABLE 3
Analysis of Variance of Experimental Results

Factor	F-Level				
	Mean Flow Time	Mean Lateness	Mean Earliness	Mean Missed Due Dates	Mean Labor Utilization
Dispatching (q)	.32	.33	4.04*	4.10*	.00
Due Date (d)	.21	84.33**	3116.98**	1037.99**	.00
Interaction (q × d)	.21	.18	1.08	.98	.00

*Significant beyond the .05 level

**Significant beyond the .01 level

TABLE 4
Tukey-B Multiple-Range Tests of Scheduling Policies*

Mean Flow Time	Mean Lateness	Mean Earliness	Mean Missed Due Dates	Mean Labor Utilization
5 } 1 } 2 } 3 } 6 } 4 }	4 } 1 } 5 } 2 } 3 } 6 }	6 } 3 } 5 } 2 } 4 } 1 }	6 } 3 } 5 } 2 } 4 } 1 }	6 } 1 } 2 } 3 } 4 } 5 }

*The numbers in the body of the tables are used to identify the six scheduling policies. Homogeneous subsets (at the .05 level of significance) of scheduling policies are connected by solid lines. The policies are ranked in descending order.

Since these null hypotheses can be rejected, further analysis of the treatment differences is necessary. One-way analyses of variance, linear contrasts, and multiple comparisons of the results shown in Table 2 are used to rank order the significantly different policies. A priori linear contrasts and a posteriori contrasts using Tukey-B [19] multiple-range tests were performed to test the null hypothesis of no significant difference between the scheduling policies. As indicated by the results shown in Table 4, the expected-value (EV) preference ordering of the scheduling policies depends on the performance measure under consideration. For the performance measures mean flow time and mean labor utilization, there is no significant difference among the six policies. Therefore, all six policies constitute the EV-efficient set for these performance measures. For mean lateness and mean earliness, the EV-efficient sets are comprised of the policies employing the loose ($K = 7.0$) and tight ($K = 1.5$) due dates, respectively. Similarly, the EV-efficient set for mean missed due dates includes the policies employing the tighter ($K = 1.5$ and 3.0) due dates.

An SD preference ordering of the experimental results is shown in Table 4. The undominated policies constitute the efficient sets of policies from which any individual belonging to a risk-preference class will choose. Since $E_1(X) < E_R(X)$ is a necessary condition for both FSD and SSD, the SD-efficient sets will not usually include as many alternatives as the EV-efficient sets. As indicated by the results shown in Table 5, the SD-efficient sets are subsets of the EV-efficient sets. Assuming a utility function characterized by diminishing marginal utility, the policies deleted from the EV-efficient sets by the SSD rule must be inconsistent with maximizing expected utility. That is, using EV one might select either policy 1 or policy 4 as the best scheduling policy when only policy 4 is the best in terms of mean lateness for the risk-averse utility maximizer. Therefore, using FSD may eliminate some alternative policies that are inefficient to all individuals, while SSD will allow a substantially more sensitive selection of policies efficient for risk averters. For the performance measure mean flow time, selecting from policies using the shortest-imminent-operation-time (SOT) versus those using the first-come, first-served (FCFS) dispatching rule is likely to result in an example of the above problem. Previously cited

research [5] demonstrates that the SOT rule results in a lower mean and a higher variance of flow time than the FCFS rule.

TABLE 5
Stochastic Dominance Ordering of Scheduling Policies*

Performance Measure	Undominated Scheduling Policies	
	FSD	SSD
Mean Flow Time	1, 2, 3, 5, 6	5
Mean Lateness	1, 4	4
Mean Earliness	6	6
Mean Missed Due Dates	3, 5, 6	5, 6
Mean Labor Utilization	6	6

*The numbers in the body of the table are used to identify the six scheduling policies.

Therefore, the SOT policies may constitute the EV-efficient set, but would not necessarily dominate the FCFS policies by SSD.

Interestingly, the policies employing the shortest-imminent-operation-time rule are not in the SSD-efficient sets for any of the performance measures. A risk-averse production scheduler, therefore, would never use this dispatching rule. Whether these results are peculiar to the exemplary system used for this study or apply in general is now being investigated. Preliminary results indicate that for larger, more complex shops, the SOT rules constitute the EV- and SSD-efficient sets for flow time and utilization performance measures. For shops of intermediate complexity, the SOT rules as well as the least-slack-per-operation-remaining results may constitute the EV- and SSD-efficient sets depending on the work-content multiple used in assigning due dates.

CONCLUSIONS

Previous research in production-scheduling literature has ranked scheduling policies based on expected-value (EV) criteria models. The shortcomings of EV-preference orderings in terms of utility maximization for risk averters are noted and alternative general efficiency stochastic dominance (SD) rules are described to preference-order scheduling policies. First-degree (FSD) and second-degree (SSD) rules are presented and applied (along with EV rules) to the simulation results of a hypothetical dual-constrained job shop.

The results indicate that SD rules provide a more sensitive selection among alternative policies than do EV rules. Further, the findings indicate that earlier research conclusions concerning the relative performance of dispatching rules may need to be reconsidered if one accepts the notion of individual production schedulers being risk-averse utility maximizers. For this study, the renowned shortest-imminent-operation-time dispatching rule was not found to be an efficient rule for any of the following performance measures: mean flow time, mean lateness, mean earliness, mean missed due dates, or mean utilization.

It is suggested that future research be concerned with identifying efficient and inefficient sets of scheduling policies. The specific policy chosen as the best from the efficient set would then depend on the individual decision maker's risk-taking preference or utility function. Several areas of further research are readily apparent before an SD approach is feasible. Of primary importance is the development of a formal theory, or at least some experience-based guidelines, for estimating cumulative probability density functions. Questions such as the following must be investigated:

- How many observations are required?
- How long a time period should be used to generate each observation?
- How is sample size affected by the nature of the underlying distribution?

Additionally, there is the need to re-evaluate past production-scheduling literature in light of possible erroneous conclusions drawn concerning the preference ordering of various dispatching, labor assignment, due-date assignment, and other operating decision rules.

Notes:

²For a more detailed discussion and formal proof of the optimality of the SD ordering rules, see [91 and [161.

REFERENCES

- [1] Belovicz, M. W., and F. E. Finch. "A Critical Analysis of the Risk Shift Phenomenon." *Organizational Behavior and Human Performance*, Vol. 6 (1971), pp. 150-168.
- [2] Bernoulli, D. "Exposition on a New Theory on the Measurement of Risk." *Econometrica* (January 1954), pp. 23-36.
- [3] Brooks, L. D. "Stochastic Dominance Tests for Selecting Acceptable Debt Issuance Strategies." *Management Science*, Vol. 22, No. 4 (December 1975), pp. 477-486.
- [4] Conway, R. W. "Some Tactical Problems in Digital Simulation." *Management Science*, Vol. 10, No. 1 (October 1963), pp. 47-61.
- [5] Conway, R. W. "Priority Dispatching and Job Lateness in a Job Shop." *Journal of Industrial Engineering*, Vol. 16 (July-August 1965), pp. 228-237.
- [6] Conway, R. W.; W. L. Maxwell; and L. W. Miller. *Theory of Scheduling*. Reading, Mass.: Addison-Wesley Publishing Co., 1967.
- [7] Day, J. E., and M. P. Hottenstein. "Review of Sequencing Research." *Naval Research Logistics Quarterly*, Vol. 17, No. 1 (March 1970), pp. 11-39.
- [8] Fishman, G. S. *Concepts and Methods in Discrete Event Digital Simulation*. New York: John Wiley & Sons, 1973.
- [9] Hadar, J., and W. Russell. "Rules for Ordering Uncertain Prospects." *The American Economic Review*, Vol. 59 (March 1969), pp. 25-34.
- [10] Johnson, K. H., and R. C. Burgess. "The Effects of Sample Sizes on the Accuracy of EV and SSD Efficiency Criteria." *Journal of Financial and Quantitative Analysis* (September 1975), pp. 813-820.
- [11] Karlin, S. "Dynamic Inventory Policy with Varying Stochastic Demands." *Management Science*, Vol. 6 (April 1960), pp. 231-258.
- [12] Levy, H., and G. Hanoch. "Relative Effectiveness of Efficiency Criteria for Portfolio Selection." *Journal of Financial and Quantitative Analysis*, Vol. 5 (March 1970), pp. 63-76.
- [13] Porter, R. B. "An Empirical Comparison of Stochastic Dominance and Mean-Variance Portfolio Choice Criteria." *Journal of Financial and Quantitative Analysis*, Vol. 8 (September 1973), pp. 587-608.
- [14] Porter, R. B.; J. R. Wart; and D. L. Ferguson. "Efficient Algorithms for Conducting Stochastic Dominance Tests on Large numbers of Portfolios." *Journal of Financial and Quantitative Analysis*, Vol. 8 (January 1973).
- [15] Pratt, J. W.; H. Raiffa; and R. Schlaifer. "The Foundation of Decisions Under Uncertainty: An Elementary Exposition." *American Statistical Association Journal* (June 1964), pp. 353-375.
- [16] Quirk, J. P., and R. Saposnik. "Admissibility and Measurable Utility Functions." *Review of Economic Studies*, Vol. 29 (February 1962), pp. 140-146.
- [17] Weeks, J. K. "A Simulation Study of Due-Date Assignment Policies in a Dual-Constrained Job Shop." Columbia: University of South Carolina, 1974. (Ph.D. dissertation.)
- [18] Weeks, J. K., and J. S. Fryer. "A Simulation Study of Operating Policies in a Hypothetical Dual-Constrained Job Shop." *Management Science*, Vol. 22, No. 12 (August 1976), pp. 1362-1371.
- [19] Winer, B. J. *Statistical Principles in Experimental Design*. New York: McGraw-Hill Book Co., 1962.