Optimal Transmission Range for Wireless Ad Hoc Networks Based on Energy Efficiency

By: Jing Deng, Yunghsiang S. Han, Po-Ning Chen, and Pramod K. Varshney

J. Deng, Y. S. Han, P.-N. Chen, and P. K. Varshney, "Optimal Transmission Range for Wireless Ad Hoc Networks Based on Energy Efficiency," IEEE Transactions on Communications, vol. 55, no. 9, pp. 1772-1782, September 2007. DOI: 10.1109/TCOMM.2007.904395

Made available courtesy of Institute of Electrical and Electronics Engineers: http://www.ieee.org/

*****(c) 2007 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other users, including reprinting/ republishing this material for advertising or promotional purposes, creating new collective works for resale or redistribution to servers or lists, or reuse of any copyrighted components of this work in other works.*****

Abstract
The transmission range that achieves the most economical use of energy in wireless ad hoc networks is studied for uniformly distributed network nodes. By assuming the existence of forwarding neighbors and the knowledge of their locations, the average per-hop packet progress for a transmission range that is universal for all nodes is derived. This progress is then used to identify the optimal per-hop transmission range that gives the maximal energy efficiency. Equipped with this analytical result, the relation between the most energy-economical transmission range and the node density, as well as the path loss exponent, is numerically investigated. It is observed that when the path loss exponent is high (such as four), the optimal transmission ranges are almost identical over the range of node densities that we studied. However, when the path loss exponent is only two, the optimal transmission range decreases noticeably as the node density increases. Simulation results also confirm the optimality of the per-hop transmission range that we found analytically.

Index Terms: wireless ad hoc networks; energy efficiency; optimal transmission range

Article:

I. INTRODUCTION

The research on wireless ad hoc networks has experienced a rapid growth over the last few years. Unique properties of ad hoc networks, such as operation without pre-existing infrastructure, fast deployment, and self-configuration, make them suitable for communication in tactical operations, search and rescue missions, and home networking. While most studies in this area have concentrated on the design of routing protocols, medium access control protocols, and security issues, we investigate the efficiency of energy consumption in wireless ad hoc networks in this work. Due to their portability and fast-deployment in potentially harsh scenarios, nodes in ad hoc networks are usually powered by batteries with finite capacity. It is always desirable to extend the lifetime of ad hoc network nodes without sacrificing their functionality. Thus, the study of energy-efficient mechanisms is of importance.

In wireless ad hoc networks, energy consumption at each node is mainly due to system operation, data processing, and wireless transmission and reception. While there are studies on increasing battery capacity and reducing energy consumption of system operation and data processing, energy consumption economy of radio transceivers has not received as much attention. Such a study is also quite essential for an energy-efficient system design [1]. In some previous work, the radio transmission range of nodes in wireless networks was optimized based on local neighborhood information so that desirable network topologies can be dynamically established with less transmission interference [2]–[6]. In this work, the radio transmission range is considered to be a static system parameter that is determined *a priori*, i.e., during system design, and used throughout the lifetime of a wireless ad hoc network.

When two communicating nodes are not in range of each other in wireless ad hoc networks, they need to rely on multi- hop transmissions. In such a case, packet forwarding, or packet routing, becomes imperative. The selected value of radio transmission range considerably affects network topology and node energy consumption. On the one hand, a large transmission range increases the distance progress of data packets toward their final destinations. This is unfortunately achieved at the expense of high energy consumption per transmission. On the other hand, a short transmission range uses less energy to forward packets to the next hop, but a large number of hops are required for packets to reach their destinations. Thus, there exists an optimum value of the radio transmission range.

There have been some publications [7]–[10] that concentrated on the optimization of radio transmission range in wireless networks. In [7], the optimal transmission radii[1] that maximize the expected packet progress in the desired direction were determined for different transmission protocols in a multi-hop packet radio network with randomly distributed terminals. The optimal transmission radii were expressed in terms of the number of terminals in range. It was found that the optimal transmission radius for slotted ALOHA without capture capability covers on an average eight nearest neighbors in the direction of packet's final destination. The study concentrated on improving system throughput by limiting the transmission interference in a wireless network with heavy traffic load. Energy consumption, however, was not considered in the paper. Similar assumptions were made in [8], which further allowed all nodes to adjust their transmission radii independently at any time. It was found in [8] that a higher throughput could be obtained by transmitting packets to the nearest neighbor in the forward direction. In [9], the authors evaluated the optimum transmission ranges in a packet radio network in the presence of signal fading and shadowing. A distributed position-based self-reconfigurable network protocol that minimizes energy consumption was proposed in [10]. It was shown in [10] that the proposed protocol can stay close to the minimum energy solution when it is applied to mobile networks.

The optimization of transmission range as a system de- sign issue was studied in [11]. The wireless network was assumed to have high node density, and to consist of nodes with relatively low mobility and short transmission range. As justified by the assumption of high node density, the authors further assumed that intermediate routing nodes are always available at the desired location whenever they are needed. Considering nodes without power control capability, the authors argued that the optimal transmission range can be set at the system design stage. Specifically, they showed that the optimal one-hop transmission progressive distance is independent of the physical network topology, the number of transmission sources, and the total transmission distance; and, it only depends on the propagation environment and radio transceiver device parameters.

A similar assumption was made in [12], even though the node density was only considered for the energy consumption of overhearing nodes. They investigated the problem of selecting an energy-efficient transmission power to minimize global energy consumption for ad hoc networks. They concluded that the average neighborhood size is a useful parameter in finding the optimal balance point.

Reference [13] studied the optimal transmission radius that minimizes the settling time for flooding in large scale sensor networks. In the paper, the settling time was evaluated at the time when all the nodes in the network have forwarded the flooded packet. Regional contention and contention delay were then analyzed.

A bit-meter-per-joule metric for energy consumption in wireless ad hoc sensor networks was investigated in [14]. The paper presented a system-level characterization of energy consumption for sensor networks. This study assumed that the sensor network has a relay architecture, and all the traffic is sent from sensor nodes toward a distant base station. Also, it was assumed that the source always chooses, among all relay neighbors, the one that has the lowest bit-meter-per-joule metric to relay its data packets. In the analysis, the power efficiency metric in terms of average watt-per-meter for each radio transmission was first calculated, and was then extended to determine global energy consumption. The analysis showed how the overall energy

[1] We use transmission "range" and "radius" interchangeably in our paper.

consumption varies with transceiver characteristics, node density, data traffic distribution, and base- station location.

In this work, we determine the optimal transmission range that achieves the most economical use of energy under the assumption of uniformly distributed network nodes. Assuming the existence of forwarding neighbors and the knowledge of their locations, we first derive the average per-hop packet progress for a transmission range universal for all nodes, and then use the result to determine the optimal per-hop transmission range that gives the maximal energy efficiency. The relationship between the most energy-economical per-hop transmission range and the node density, as well as the path loss exponent, is then numerically investigated. We observed that the optimal transmission range varies markedly in accordance with the node densities at low path loss exponent values (such as two) but remains nearly constant at high path loss exponent values (such as four). We also found that the node density needs to be extremely high for the result of [11] to be valid, and the optimal transmission radius under low to medium node densities is actually far away from the results reported in [11].

Simulations were performed to investigate the applicability of the optimal *per-hop* transmission range we derived to the situation where the energy efficiency of the *entire* path from the originating source node to the final destination is considered. Results showed that the overall energy efficiency almost peaks at the same transmission range as the per-hop energy efficiency. In order to account for the situation that forwarding neighbors may not exist, contrary to the assumption made in our analysis, we have also simulated an extended *connectivity-guaranteed* transmission strategy that allows a network node to increase its initially pre-set transmission range until an appropriate forwarding neighbor appears. We found that the optimal radii of the connectivity-guaranteed transmission strategy, as well as the corresponding maximum energy efficiency, are almost identical to those obtained from the original strategy.

In summary, contrary to the dynamic transmission range employed in [5], [8], and [10], our study determines a single static optimal energy-efficient transmission range for all nodes in the network. Compared with [11], our study does not make the assumption that a relay node that is closest to the destination can always be found; thus, the wireless networks we study do not need to be highly dense. Compared with [14], the network we consider does not have any base station or common receiver, nor do we assume that the destination is far away from the source.

Our paper is organized as follows. The analysis of the single-hop energy-efficient radius is presented in Section II. Analytical and simulation results along with discussions are provided in Section III. In Section IV, we propose an extended connectivity-guaranteed transmission strategy and compare it with our original strategy. Section V concludes our work.

II. ANALYSIS OF FIRST-HOP DISTANCE-ENERGY EFFICIENCY

In this section, we analyze the distance-energy efficiency for the first hop in a wireless ad hoc network with randomly distributed nodes as we consider the snapshot at the time of the *first-hop* transmission, even if a multi-hop transmission is subsequently required for the packet to reach its ultimate destination. Specifically, the *first-hop distance-energy efficiency* is defined as the ratio of *the average progress* of a packet during its first transmission and the *energy consumption* of that transmission. As any intermediate relay transmission can be viewed as a new first-hop transmission for the remaining route, the first-hop distance-energy efficiency should be consistent with the overall distance-energy efficiency of the entire route in a homogeneous environment. (This will be later substantiated by simulations in Section III.)

A. Network Model and Transmission Strategy

Suppose that a source node, S, is located at the center of a circle of radius x, where x is the largest possible distance between S and any destination. In other words, the source node will not send any packet to nodes outside the circle. The destination node D, to which S intends to transmit a data packet, is assumed uniformly distributed over the entire circle.

Due to the limited radio range (or equivalently, limited transmission energy), a packet from its originating source node to its destination node may need to be sequentially routed by a certain number of intermediate nodes, which we term as *routers*. It is assumed that all nodes, including the source node and the intermediate nodes, employ a common transmission radius r. Consequently, direct transmission to the destination occurs only when the destination node is within distance r from the source node.

Any node within the transmission range of a node is called its *neighbor*. We assume that each node knows the locations of all its neighbors and the location of the destination node.[2] Based on this assumption, a transmission strategy can be designed as follows:

(i) The source node S transmits a packet to the destination node D directly, if D is located within distance r from S.
(ii) When the destination node D is outside the transmission range of the source node S, the packet is sent to the neighbor that is closer in distance to the destination node D than the source node S, and that is closest to the destination D among all neighbors.
(iii) Since the source node S knows the locations of all neighbor nodes and the destination node, it will not send out the packet when there does not exist any neighbor satisfying the condition in (ii), and will postpone the transmission until such a neighbor appears.

As pointed out in [12], the selection of transmission radius influences energy consumption and network connectivity. It can be shown that the probability of having no forwarding neighbor is usually negligibly small. Further discussions on the connectivity issue will be provided in Section IV.

The probability that n nodes appear in an area of size A is given by $(pA)^n e^{-pA}/n!$, where p is the density parameter for this two-dimensional Poisson point process [7].[3] The appearance of nodes in any two non-overlapping areas are assumed independent.

The energy consumption corresponding to each transmission can be formulated as [11]:

$$E_t(r) = k_1 r^{\omega} + k_2,$$

where r is the radio transmission range, ω is the path loss exponent, and k_1 and k_2 are parameters determined by the characteristic of the transceiver design and the channel. Let E_r be the energy consumption of receiving, decoding, and processing data packets at the receiver. Note that E_r does not include the energy consumption of the overhearing nodes in the neighborhood of the sender. Inclusion of such extra energy consumption may affect transmission range optimiza- tion. From the above formulations, we infer that the *single-transmission energy consumption* is given by $E_t(r) + E_r$. Throughout this work, we do not count the extra energy consumption due to packet retransmissions similar to [1 1] and [14].

We next determine *the average progress* of a transmitted packet in a single hop.

B. Average Single-Transmission Progress

Denote the distance between the source node S and the destination node D by v. When $v \le r$, direct transmission to the destination node D can be attained; hence, the distance progress of the transmitted packet to the destination node D is v. In the situation where $v > r$, the source node has to locate an appropriate neighbor for

---

[2] This can be achieved by measuring the strength of received signals from neighboring nodes and by the use of locationing service such as geographical location service (GLS) [15] [16].

[3] Note that we have used the definition of node density as the average number of nodes per unit area. An alternative definition is the number of neighbors per node within its transmission radius. Since we focus on the network-wide average, these two definitions would lead to similar results.

subsequent packet routing. In this case, we define the distance progress as the difference between the before-hop distance (between the sender and the destination) and the after-hop distance (between the relay node and the destination) [14]. The distance progress toward the destination node *D* is therefore equal to (v — z), where z is the distance between the first-hop router *T* and the destination node *D* (cf. Fig. 1).

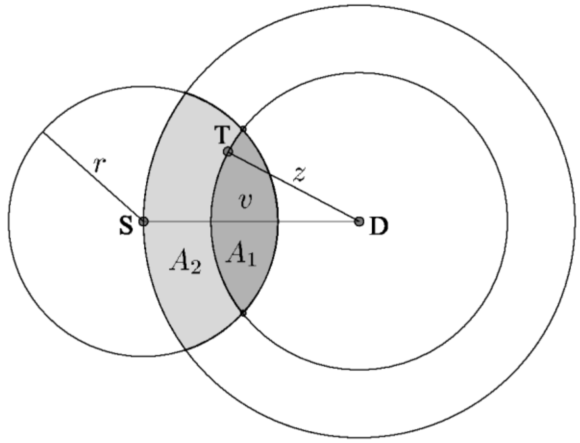

Fig. 1. Illustration of a relaying node.

Denote by $\boldsymbol{P}$ the random variable corresponding to the distance progress for a single transmission. Let $\boldsymbol{V}$ and $\boldsymbol{Z}$ be respectively the random variables corresponding to v and z discussed above. Define a new random variable $\boldsymbol{H}$ as:

$$\boldsymbol{H} = \begin{cases} 1, & \text{if a neighbor satisfying (ii)in the transmission strategy;} \\ 0, & \text{otherwise} \end{cases}$$

With the above notations, the problem of finding the average single-transmission progress is equivalent to the derivation of the expected value of $\boldsymbol{P}$, where

$$\boldsymbol{P} = \begin{cases} \boldsymbol{V}, & if\ V \le r; \\ & (cf.\ transmission\ strategy\ step\ (i)) \\ \boldsymbol{V} - \boldsymbol{Z} & if\ \boldsymbol{V} > r \cap \boldsymbol{H} = 1; \\ & (cf.\ transmission\ strategy\ step\ (ii)) \\ 0, & if\ \boldsymbol{V} > r \cap \boldsymbol{H} = 1; \\ & (cf.\ transmission\ strategy\ step\ (iii)) \end{cases} \tag{1}$$

Note that, if ($\boldsymbol{V} \le r$) ∪ ($\boldsymbol{V} > r \cap \boldsymbol{H} = 1$) is false, no transmission will take place according to transmission strategy step (iii); hence, no energy is consumed, and the progress is zero. It can, therefore, be easily verified that $E[\boldsymbol{P}] = E[\boldsymbol{P}|(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)]$.

We now proceed to derive the expectation of $\boldsymbol{P}$ given that ($\boldsymbol{V} < r$) ∪ ($\boldsymbol{V} > r \cap \boldsymbol{H} = 1$) is true. Observe that:

$$\begin{aligned} & \Pr\{\boldsymbol{P} > p|(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\} \\ &= \frac{\Pr\{(\boldsymbol{P} > p) \cap [(\boldsymbol{V} < r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)]\}}{\Pr\{(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\}} \\ &= \frac{\Pr\{(\boldsymbol{P} > p \cap \boldsymbol{V} \le r) \cup ((\boldsymbol{P} > p) \cap (\boldsymbol{V} > r \cap \boldsymbol{H} = 1))\}}{\Pr\{(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\}} \\ &= \frac{\Pr\ \{\boldsymbol{P} > p \cap \boldsymbol{V} \le r\} + \Pr\{\boldsymbol{P} > p \cap \boldsymbol{V} > r \cap \boldsymbol{H} = 1\}}{\Pr\{\boldsymbol{V} \le r\} + \Pr\{\boldsymbol{V} > r \cap \boldsymbol{H} = 1\}} \end{aligned}$$

where the last step follows from the fact that both the events in the numerator and the denominator are mutually exclusive. Substituting (1) into the above expression yields:

$$\begin{aligned}&\Pr\{\boldsymbol{P} > p|(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\}\\ &= \frac{\Pr\{p < \boldsymbol{V} \le r\} + \Pr\{\boldsymbol{V} — \boldsymbol{Z} > p \cap \boldsymbol{V} > r \cap \boldsymbol{H} = 1\}}{\Pr\{\boldsymbol{V} < r\} + \Pr\{\boldsymbol{V} > r \cap \boldsymbol{H} = 1\}}\end{aligned} \tag{2}$$

The statistics specified in subsection II-A then immediately implies:

$$\Pr\{p < \boldsymbol{V} \le r\} = \frac{(r^2 - \mathrm{p}^2)}{x^2} 1\{p < r\}$$

and

$$\Pr\{V \le r\} = \frac{r^2}{x^2}, \tag{3}$$

where $1\{\cdot\}$ denotes the set indicator function and x represents the largest possible distance between the source and any destination. It remains to determine $\Pr\{\boldsymbol{V} > r \cap \boldsymbol{H} = 1\}$ and $\Pr\{\boldsymbol{V} — \boldsymbol{Z} > p \cap \boldsymbol{V} > r \cap \boldsymbol{H} = 1\}$.

Let A_{SD} denote the area of the overlapping region between the circle centered at S with radius r and the circle centered at D with radius v, i.e., the shaded region in Fig. 1. We can divide A_{SD} into two regions by the circle centered at D with radius z. The areas of these two regions are respectively denoted as A_1 and A_2 as shown in Fig. 1. Then

$$\begin{aligned}&\Pr\{\boldsymbol{V} > r \cap \boldsymbol{H} = 1\}\\ &= \int_0^x \Pr(\boldsymbol{V} > r \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\, dP_{\boldsymbol{V}}(v)\\ &= \int_r^x \Pr\{\text{at least one neighbor exists in area } A_{SD}\}\, dP_{\boldsymbol{V}}(v) &(4)\\ &= \int_r^x \left(1 - e^{-pA_{SD}(v,r)}\right)\frac{2v}{x^2} dv\\ &= 1 - \frac{r^2}{x^2} - \frac{2}{x^2}\int_r^x ve^{-pA_{SD}(v,r)} dv, &(5)\end{aligned}$$

where the lower integration limit is r in (4) because of the condition of $\boldsymbol{V} > r$, $P_{\boldsymbol{V}}(v) = Pr(\boldsymbol{V} \le v)$, and

$$\begin{aligned}&A_{SD}(v,r) = r^2 \cos^{-1}\left(1 - \frac{r^2}{2v^2}\right)\\ &-\frac{1}{2} r\sqrt{(2v + r)(2v - r)}.\end{aligned} \tag{6}$$

This completes the determination of PrIV > r n H = 11. From Fig. 1, we have:

$$\begin{aligned}&\Pr\{\boldsymbol{Z} \ge z \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\\ &= \begin{cases} \Pr\{H = 1|V = v\} & if\ z \le v - r;\\ \Pr\{no\ neighbors\ in\ A_1 \\ and\ at\ least\ one\ in\ A_2\} & if\ v - r < z < v;\\ 0, & if\ z \ge v.\end{cases}\end{aligned}$$

By the independence of node appearance in non-overlapping regions, the above expression for v — r < z < v can be re-written as:

$$\begin{aligned}&\Pr\{\boldsymbol{Z} \geq z \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\\ &= \Pr\{\text{no neighbors in } A_1\} \Pr\{\text{at least one in } A_2\}\\ &= e^{-pA_1(z,v,r)}\left(1 - e^{-pA_2(z,v,r)}\right)\\ &= e^{-pA_1(z,v,r)} - e^{-pA_{SD}(v,r)},\end{aligned}$$

where

$$\begin{aligned}&A_1(z,v,r)\\ &= r^2 \cos^{-1}\left(\frac{r^2+v^2-z^2}{2rv}\right) + z^2 \cos^{-1}\frac{z^2+v^2-r^2}{2vx}\\ &-\frac{1}{2}\sqrt{(r+v+z)(v+z-r)(r+v-z)(r+z-v)}.\end{aligned} \tag{7}$$

Therefore,

$$\begin{aligned}&\Pr\{\boldsymbol{Z} < z \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\\ &= \Pr\{\boldsymbol{H} = 1|\boldsymbol{V} = v\} - \Pr\{\boldsymbol{Z} > z \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\\ &\begin{cases}0, & if\ z \leq v - r;\\ 1 - e^{-pA_1(z,v,r)}, & if\ v - r < z < v;\\ 1 - e^{-pA_{SD}(v,r)}, & if\ z \geq v,\end{cases}\end{aligned} \tag{8}$$

where we have used $\Pr\{\boldsymbol{H} = 1|\boldsymbol{V} = v\} = 1 — e^{-pA_{SD}(v,r)}$ in the above derivation.

Using (8), we obtain:

$$\begin{aligned}&\Pr\{\boldsymbol{V} - \boldsymbol{Z} > p \cap \boldsymbol{V} > r \cap \boldsymbol{H} = 1\}\\ &= \int_r^x \Pr\{\boldsymbol{Z} < v - p \cap \boldsymbol{H} = 1|\boldsymbol{V} = v\}\, dP_{\boldsymbol{V}}(v)\\ &= \int_r^x \Pr\{Z < v - p \cap H = 1|V = v\}\frac{2v}{x^2} dv\\ &= \left(1 - \frac{r^2}{x^2} - \frac{2}{x^2}\int_r^x ve^{-pA_1(v-p,p,r)}dv\right)1\{p < r\}\end{aligned} \tag{9}$$

This completes the determination of $\Pr\{\boldsymbol{V} — \boldsymbol{Z} > p \cap \boldsymbol{V} > r \cap \boldsymbol{H} = 1\}$.

Finally, substituting (3), (5), and (9) into (2), we obtain that for $p > 0$,

$$\begin{aligned}&\Pr\{\boldsymbol{P} > p|(\boldsymbol{V} < r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)\}\\ &= \left(\frac{x^2 - p^2 - 2\int_r^x ve^{-pA_1(v-p,v,r)}dv}{x^2 - 2\int_r^x ve^{-pA_{SD}(v,r)}dv}\right)1\{p < r\}.\end{aligned}$$

The expected value of $\boldsymbol{P}$ given the validity of $(\boldsymbol{V} < r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)$ is then equal to [17, Eq. (21.9)]:

$$
\begin{aligned}
& E[\boldsymbol{P}|(\boldsymbol{V} < r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)] \\
&= \int_0^{\infty} \Pr\{\boldsymbol{P} > p|(V \le r) \cup (V > r \cap H = 1)\}\, dp \\
&= \frac{1}{x^2 - 2\int_r^x v e^{-pA_{SD}(v,r)} dv} \\
&\cdot \int_0^r \left[x^2 - p^2 - 2\int_r^x v e^{-pA_1(v-p,v,r)} dv \right] dp \\
&= \frac{3x^2 r - r^3 - 6\int_0^r \int_r^x v e^{-pA_1(v-p,v,r)} dv dp}{3\left(x^2 - 2\int_r^x v e^{-pA_{SD}(v,r)} dv\right)},
\end{aligned}
$$

and the single-transmission distance-energy efficiency $e(r)$ is given by:

$$
\begin{aligned}
& e(r) \\
&= \frac{E[\boldsymbol{P}|(\boldsymbol{V} \le r) \cup (\boldsymbol{V} > r \cap \boldsymbol{H} = 1)]}{k_1 r^{\omega} + k_2 + E_r} \\
&= \frac{3x^2 r - r^3 - 6\int_0^r \int_r^x v e^{-pA_1(v-p,v,r)} dv dp}{3(k_1 r^{\omega} + k_2 + E_r)(x^2 - 2\int_r^x v e^{-pA_{SD}(v,r)} dv}
\end{aligned}
$$

By re-formulating $A_1(z, v, r) = r^2 \overline{A}_1(z/r, v/r)$ and $A_{SD}(v, r) = r^2 \overline{A}_{SD}(v/r)$, and defining $\overline{z} = z/r$ and $\overline{v} = v/r$, where

$$
\begin{aligned}
& \overline{A}_1(\overline{z}, \overline{v}) \\
&= \cos^{-1}\left(\frac{1 + \overline{v}^2 - \overline{z}^2}{2\overline{v}}\right) + \overline{z}^2 \cos^{-1}\left(\frac{\overline{z}^2 + \overline{v}^2 - 1}{2\overline{v}\overline{z}}\right) \\
&- \frac{1}{2}\sqrt{(1 + \overline{v} + \overline{z})(\overline{v} + \overline{z} - 1)(1 + \overline{v} - \overline{z})(1 + \overline{z} - \overline{v})}
\end{aligned} \tag{11}
$$

and

$$
\begin{aligned}
& \overline{A}_{SD}(\overline{v}) = \cos^{-1}\left(\frac{1}{2\overline{v}}\right) + \overline{v}^2 \cos^{-1}\left(1 - \frac{1}{2\overline{v}^2}\right) \\
&- \frac{1}{2}\sqrt{(2\overline{v} + 1)(2\overline{v} - 1)}
\end{aligned} \tag{12}
$$

we can simplify the single-transmission distance-energy efficiency $e(r)$ to:

$$
e(r) = \frac{r}{3k_1(r^{\omega} + k_0)} \cdot \frac{g_1(r)}{g_2(r)}, \tag{13}
$$

where

$$
\begin{aligned}
k_0 &= (k_2 + E_r)/k_1, \\
g_1(r) &= 1 + 2\int_0^1 \int_1^{x/r} \overline{v}\left(1 - e^{-pr^2 \overline{A}_1(\overline{v} - \overline{p}, \overline{v})}\right) d\overline{v} d\overline{p},
\end{aligned}
$$

and

$$g_2(r) = 1 + 2\int_1^{x/r} \bar{v}\left(1 - e^{-pr^2\bar{A}_{SD}(\bar{v})}\right) d\bar{v}$$

C. Optimum Transmission Radius in High Density Networks

In high density networks, i.e., when ρ is considerably large, we can approximate

$$1 - e^{-pr^2\bar{A}_1(\bar{v}-\bar{p},\bar{v})} \approx 1 \text{ and } 1 - e^{-pr^2\bar{A}_{SD}(\bar{v})} \approx 1,$$

and obtain

$$\frac{g_1(r)}{g_2(r)} \approx \frac{2 + 6\int_0^1 \int_1^{x/r} \bar{v}d\bar{v}d\bar{p}}{1 + 2\int_1^{x/r} \bar{v}d\bar{v}} = 3 - \frac{r^2}{x^2}.$$

Therefore,

$$e(r) = \frac{r}{3k_1(r^\omega + k_0)} \cdot \frac{g_1(r)}{g_2(r)} \approx \frac{3x^2r - r^3}{3k_1x^2(r^\omega + k_0)},$$

which gives maximum value at some positive r satisfying

$$(\omega - 3)r^{\omega+2} - 3(\omega - 1)x^2r^\omega - 3k_0r^2 + 3k_0x^2 = 0. \qquad (14)$$

For the special case of $\omega = 2$, (14) reduces to

$$r^4 + (3x^2 + 3k_0)r^2 - 3k_0x^2 = 0.$$

Thus, the optimal r for $\omega = 2$ is equal to:

$$\sqrt{\frac{-(3x^2 + 3k_0) + \sqrt{(3x^2 + 3k_0)^2 + 12k_0x^2}}{2}}.$$

We depict the optimal transmission range for high density networks in Fig. 2 for x ranging from 40 to 200 *m* and $k_0 = 222.56\ m^2$.[4] It can be observed from this figure that the optimal transmission range goes from 13.74 to 14.86 *m* while x is changed from 40 to 200 *m*. This suggests that under high node density, x is not a dominant factor in the determination of the optimal transmission range when the path loss exponent ω is 2.

For an environment with $\omega = 3$, (14) becomes

$$2x^2r^3 + k_0r^2 - k_0x^2 = 0$$

The real solution r for the above equation is equal to:

$$-\frac{k_0}{6x^2} + \frac{k_0^2}{6x^2\left(-k_0^3 + 54k_0x^6 + 6k_0x^3\sqrt{81x^6 - 3k_0^2}\right)^{1/3}}$$

---

[4] We have used the same parameter values as in [11].

$$+\frac{\left(-k_0^3+54k_0x^6+6k_0x^3\sqrt{81x^6-3k_0^2}\right)^{1/3}}{6x^2}$$

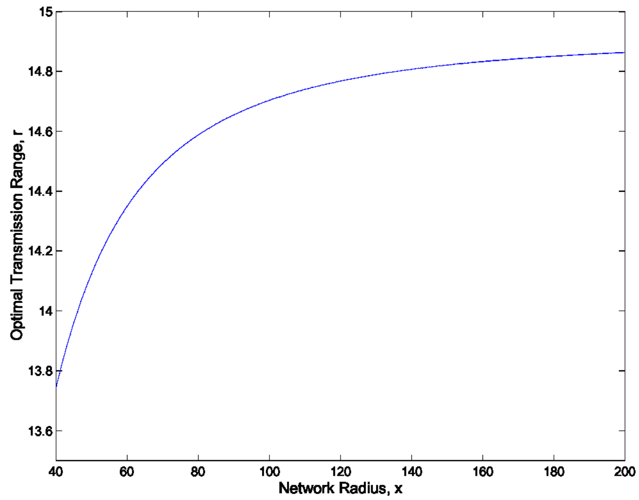

Fig. 2. Optimal transmission range of high density networks (path loss exponent $\omega = 2$).

Again, the optimal transmission range for high density networks at ω = 3 is depicted in Fig. 3 for x ranging from 40 to 200 m and k_0 = 222.56 m 3. From this figure, we conclude that the optimal transmission range goes from 4.78686 to 4.80901 m while x is changed from 40 to 200 m. This again suggests that, for a moderately large x, optimal transmission range is quite insensitive to the values of network radius, x.

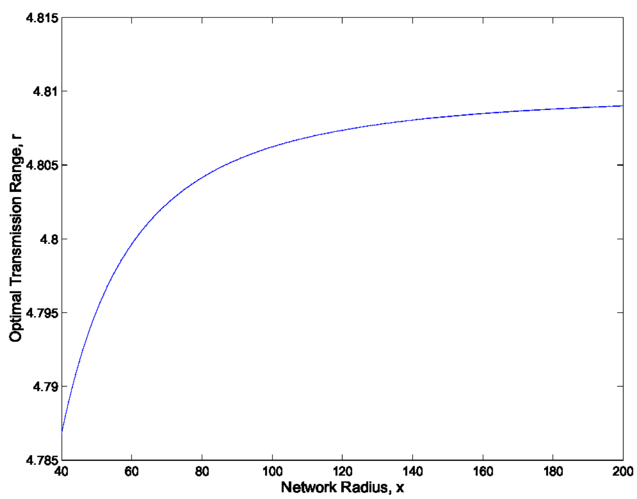

Fig. 3. Optimal transmission range of high density networks (path loss exponent $\omega = 3$).

III. ANALYTICAL AND SIMULATION RESULTS

Analytically evaluated distance-energy efficiency and simulation results for its verification are summarized in this section. Quantities k_1 and $k_2 + E_r$ are assumed to be 6.6319 x 10^{-5} and 1.476 x 10^{-2}, respectively, unless specified otherwise.[5]

[5] These parameters are chosen the same as in [11] for the purpose of comparison. Systems with different types of hardware will generally lead to different set of optimum transmission ranges, as shown in Fig. 10.

A. Analytical Results

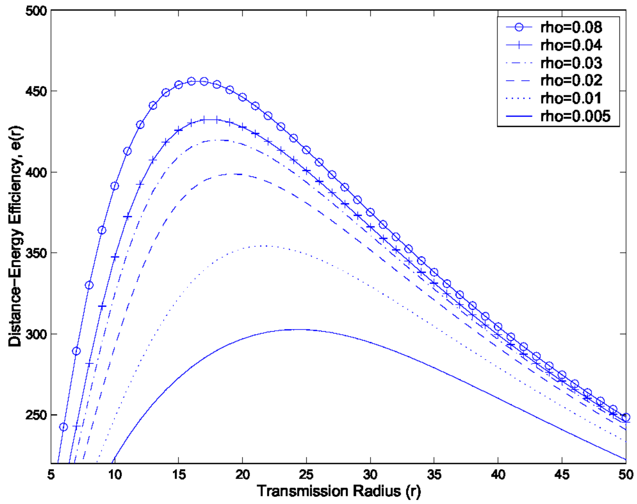

Fig. 4. Distance-energy efficiency ($\omega = 2$).

Figure 4 compares the analytically obtained first-hop distance-energy efficiencies calculated by (13) for different node densities. The network coverage area is assumed to be a circle with radius 100 meters (i.e., x = 100). The path loss exponent ω is assumed to be 2. Node density varies from 0.005 to 0.04, which corresponds to 157 to 1256 nodes on an average in a circle with radius 100 meters.

It can be observed from Fig. 4 that the first-hop distance- energy efficiency improves initially for small r, and then degrades after r exceeds a certain value. Figure 4 also shows that the distance-energy efficiency in a network with higher node density is higher. The explanation of this result is that the probability of finding relay nodes closer to the final destination is higher when there are more nodes in the network. Thus, each hop makes more progress toward the final destination, thereby improving the distance-energy efficiency.

Additionally, we observe from Fig. 4 that the optimal transmission range (r^*) changes for different node densities. When the node density is 0.005, the optimal transmission range is around 25 meters. It reduces to 17 meters when p reaches 0.04. Such a decrease in r^* with an increasing p is due to the increase in relative first-hop progress with respect to the radio transmission range; therefore, a smaller transmission range achieves better energy efficiency when p is larger. Note that this observation regarding r^* agrees with that found in [11] under a strong assumption that a source node can always find a neighbor at the required location to forward its data packet, which is valid only in networks with very high node density. Our analysis, however, shows that the same conclusion is reached for networks with low to medium node density.

Figure 5 compares the analytical first-hop energy-distance efficiencies for different node densities for a larger path loss exponent ω = 4. It shows that, when ω = 4, the optimum transmission range remains around 3 meters for all node densities p lying between 0.005 and 0.04. Therefore, the node density has little effect on the optimal transmission radius when signals encounter serious attenuation. Notably, due to a more rapid signal attenuation, this optimal transmission radius is markedly smaller than 17-25 meters obtained at ω = 2. This result confirms that the optimal transmission range can be set at the system design stage.

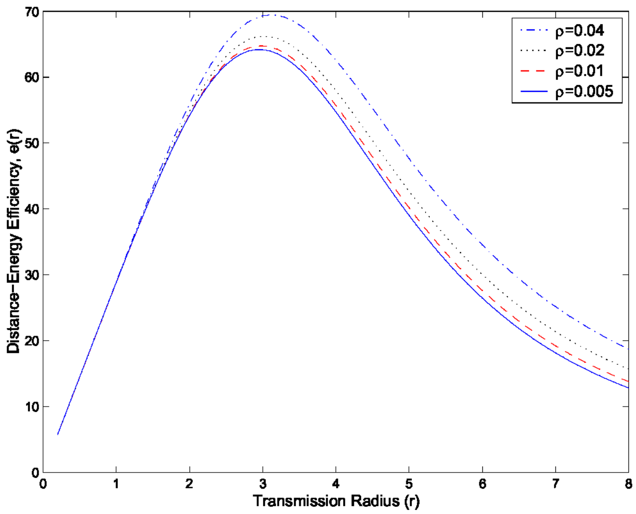

Fig. 5. Distance-energy efficiency ($\omega = 4$).

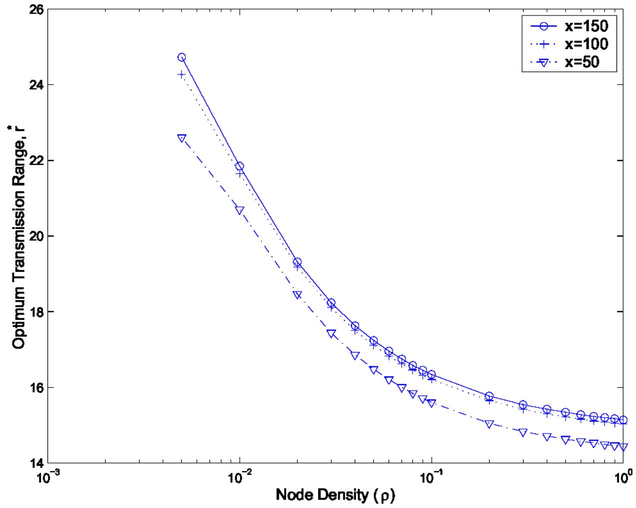

Fig. 6. Distance-energy efficiency for different network radius, x ($\omega = 2$).

Figure 6 compares the analytical results in (13) for different network radii. The path loss exponent w is assumed to be 2 in this figure. This figure clearly illustrates that the optimal trans-mission range decreases as *p* grows. The optimal transmission range, r^*, increases slightly as the network radius, *x*, increases. Figure 7 shows this trend more clearly. We can also see from Fig. 7 that for the same amount of increase in *x*, the increment in optimal transmission radius is larger when node density *p* is smaller. The analytical results from Figs. 6 and 7 clearly show that the optimal transmission radius is no longer 15 m for low node density. The optimal transmission radius is close to 15 m only when the node density is extremely high, e.g., p = 1. 0. Therefore, the results presented in [1 1] are applicable only for very high node density networks. Detailed discussion on simulation results will be presented in Section III-B.

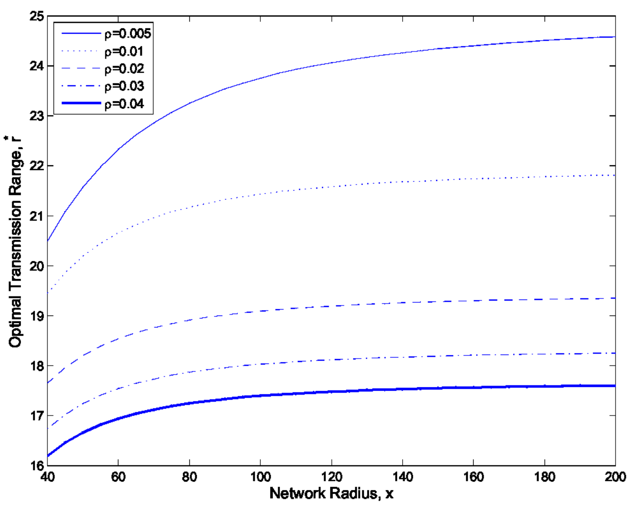

Fig. 7. Optimal transmission range for different node densities ($\omega = 2$).

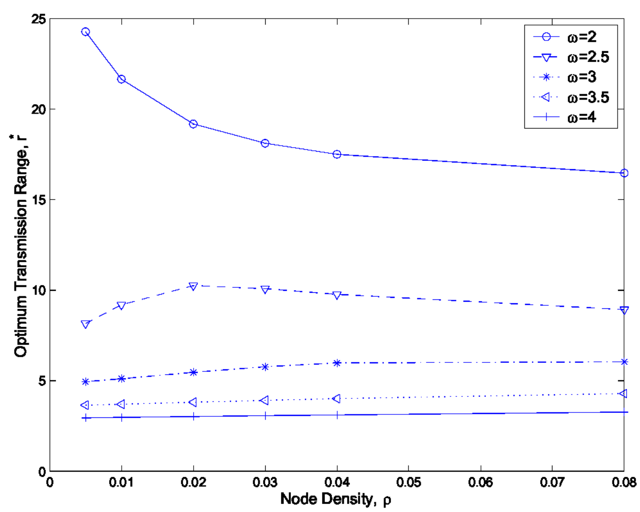

Fig. 8. Optimum transmission range for different ω.

In Fig. 8, we compare the optimal transmission range that maximizes the distance-energy efficiency, *e*(*r*), for different node densities and path loss exponents. When ω is 2, a decrease in optimum transmission range is observed with an increase in node density. A direct interpretation is that, as *p* decreases, it is less likely to find a neighbor node to relay data packets efficiently toward the final destinations; thus, the optimum transmission range increases as *p* decreases. This interpretation, however, is not applicable to other values of ω.

For instance, when $\omega = 4$, the optimum transmission range remains relatively flat for different node densities. This can be explained with the prohibitively high energy consumption of increasing the transmission range when path loss exponent is high. When path loss exponent ω lies between 2 and 4, the trend of r^* as a function of p becomes less predictable, which is due to the combined effects of the two factors discussed above.

We can view the curves in Fig. 8 in the context of number of neighbors, m, seen by the sender. For a fixed transmitted power, both the node density and the path loss exponent have similar effects on m. In fact, m increases linearly with node density; it increases exponentially as path loss exponent decreases. When the path loss exponent is small (such as 2) and node density increases, the optimum transmission range is shifted to a smaller value as more neighbors are seen. As ω increases, the benefit of increasing r to increase m diminishes.

B. Simulation Results

Simulations (programs written in C language) have been performed to verify our analytical results. In our simulations, the network nodes are distributed in a circular region according to a two-dimensional Poisson distribution. The circle is centered at (0, 0) with radius x ranging from 50 to 150 meters. The source node is fixed at (0, 0), while destination nodes are randomly chosen in the circle. The source node transmits packets to the selected destination node in accordance with our transmission strategy. We measured the average first- hop distance-energy efficiency of each pair of source and destination. All the results presented are the average of 500 runs, each of which selects 100 destinations randomly.

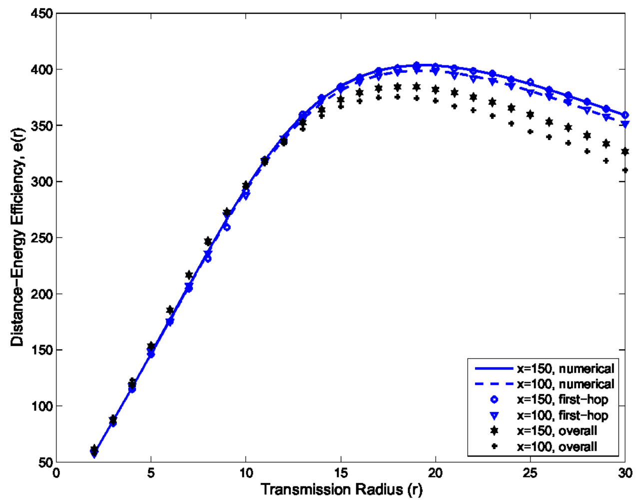

Fig. 9. Distance-energy efficiency for $\rho = 0.02$ and $\omega = 2$.

In Fig. 9, we compare numerical results on first-hop distance-energy efficiency with the simulation results under the conditions of $p = 0.02$ and $\omega = 2$. The simulation results of first-hop distance-energy efficiency match with our numerical results quite well. As shown in this figure, the optimal transmission range that maximizes first-hop distance-energy efficiency is about 18 meters. As indicated in Fig. 9, the value of x only mildly affects the first-hop distance-energy efficiency in the ranges of r that we have simulated.

In addition, we simulated the overall distance-energy efficiency (not just the first-hop, but the entire path from the originating source node to the final destination). As illustrated in Fig. 9, the first-hop distance-energy efficiency and the overall distance-energy efficiency are approximately the same when r is small. Their difference is more noticeable when r becomes larger. The first-hop distance-energy efficiency is slightly larger than the overall distance-energy efficiency. This is anticipated as the last hop is usually not as efficient as all other hops.[6] In general, a larger x results in a little higher overall distance-energy efficiency. Such a slight

[6] Based on a uniform selection of traffic destinations, the expected value of the last hop progress is approximately $r/2$, which is smaller than the average hop progress when an adequate number of nodes are present.

difference could be due to the increase in the number of hops for a larger x. From our simulations, the average number of hops for packets to reach the final destination is larger for a larger x; so the influence of the small last hop progress is less significant.

We present several other simulation results in Figs. 10 and 11. We use Fig. 10 to demonstrate the effect of energy consumption characteristics on the distance-energy efficiency and optimal transmission range. In Fig. 10, the value of k_1 is chosen as 20 x 6.6319 x 10^{-5} = 1.3264 x 10^{-3}. Such a large k_1 represents transceiver hardwares that use relatively larger portion of energy for packet transmission. As shown in Fig. 10, the peak of the distance-energy efficiency shifts to lower range of r when k_1 is larger. The optimal transmission range becomes roughly 4 for such a k_1 value.

In Fig. 11, we show the simulation results and numerical results for $\omega = 4$. With a higher path loss exponent, the energy consumption of each hop increases quickly as r increases. Therefore, the benefit of increasing the transmission range diminishes quickly. Based on Fig. 11, the optimal transmission range is around 3. The similarity in shape in Figs. 10 and 11 implies that an increase of either the transmission energy consumption characteristics, k_1, or path loss exponent, ω, has similar effects on the distance-energy efficiency and optimal transmission range.

IV. AN EXTENDED TRANSMISSION STRATEGY

Although the probability of no forwarding neighbors is fairly small when $p\pi r^2$ is moderately large,[7] its occurrence can still disconnect the link between the source and the destination nodes. In order to account for the relatively rare situation that forwarding neighbors may not exist, we have simulated an extended transmission strategy, which allows the network nodes to increase their initially pre-set transmission range until an appropriate forwarding neighbor is found. The extended transmission strategy is the same as our original transmission strategy in Section II-A except rule (iii):

> (iii') When a node cannot find a forwarding node using the pre-set transmission radius based on rule (ii), it increases its transmission radius until such a forwarding node appears.

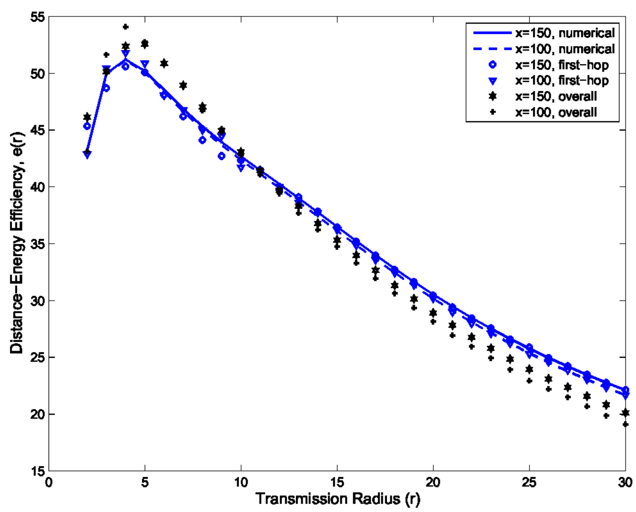

Fig. 10. Distance-energy efficiency for $\rho = 0.02$ and $\omega = 2$, k_1 in this figure is 20 times the value in all other figures (i.e., $k_1 = 1.3264 \times 10^{-3}$).

---

[7] It can be shown that this probability is upper bounded by $e^{-\left(\frac{2}{3}-\frac{\sqrt{3}}{2\pi}\right)N(p)}$, where $N(p) = p\pi r^2$ is the average number of nodes within transmission range r.

It is interesting to evaluate the distance-energy efficiency of the extended transmission strategy for the first-hop as well as for the entire path. The simulation results are summarized in Fig. 12. The node density p is 0.02 and the path loss exponent ω is 2. It can be observed that the extended transmission strategy and the original transmission strategy perform almost the same in terms of energy-distance efficiency when the transmission radius is large. This is an anticipated result since the probability of no forwarding neighbors is negligibly small for a large transmission radius. When the transmission radius is small, however, the distance-energy efficiency of the extended transmission strategy becomes markedly better than that of the original transmission strategy because the actual transmission range is increased more frequently as no forwarding neighbor exists. Nevertheless, the optimal transmission ranges, as well as the resulting maximum distance-energy efficiencies, are almost identical for both original and extended transmission strategies. We therefore conclude that the energy-economic radius derived in this paper, when used in a static manner, is a reasonable approach for the design of energy efficient wireless ad hoc networks.

V. CONCLUDING REMARKS

The radio transmission range as a system parameter affects the energy consumption economy of wireless ad hoc networks. On the one hand, a large transmission range increases the expected progress of a data packet toward its final destination at the expense of a higher energy consumption per transmission. On the other hand, a short transmission range consumes less per-transmission energy, but requires a larger number of hops for a data packet to reach its destination.

Based on the underlying device energy consumption model and a two-dimensional Poisson node distribution, we have pro- posed an analytical model to investigate the optimal value of the radio transmission range. The optimal transmission range for a location-aware transmission strategy is then determined. Our analysis shows that the optimum transmission radius is influenced more by the node density than the network coverage area. It is observed that when the path loss exponent is four, the optimal transmission ranges are almost identical over the range of node densities we studied. However, the optimal transmission range decreases noticeably as the node density increases when the path loss exponent is only two. Our results can be used to determine suitable radio transmission power for wireless ad hoc networks or wireless sensor networks in the pre-deployment phase.

Compared with other methods that also assume network nodes having adjustable transmission power (and thus trans- mission range) [5], our technique will not lead to unidirectional links and requires little maintenance once the common optimal transmission power is identified. This ensures the practicality of our technique. The examination of the connectivity-guaranteed transmission strategy that allows a sender to extend its transmission range to force the appearance of a forwarding node, further confirms the applicability of our analysis.

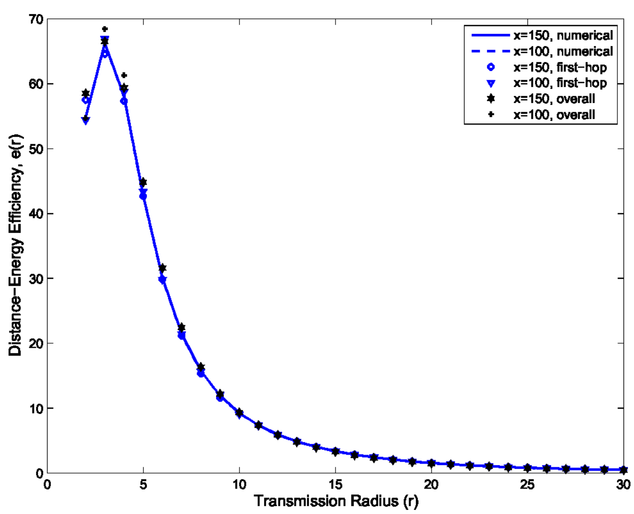

Fig. 11. Distance-energy efficiency for $\rho = 0.02$ and $\omega = 4$.

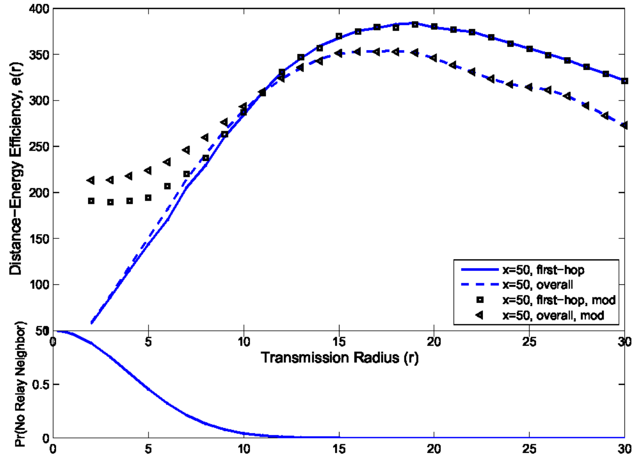

Fig. 12. Distance-energy efficiency for $\rho = 0.02$ and $\omega = 2$. The sub-figure below depicts the probability of no relay neighbor in the range of transmission radius. This probability is calculated as $E_V[P_0(v,r)] = \int_r^x e^{-\rho A_{SD}(v,r)}(2v/x^2)dv$ [18]. The optimal transmission radii respectively for first-hop, first-hop (mod), overall, overall (mod) are 16, 16, 18, and 18, which result in the energy-efficiencies of 381, 376, 353, and 352, respectively.

In determining the distance-energy efficiency, this work assumed that the transmission power $E_t(r)$ is an increasing function of transmission range r, and is given by $k_1 r^{\omega} + k_2$ [1 1]. However, in some systems, the same transmission power may result in different *effective* transmission ranges due to the use of different code punctuation or modulation schemes. An example is the implementation of IEEE 802.11a, where data rates of 1 Mbps and 54 Mbps result in quite different *effective* transmission ranges even with the same transmission power. Therefore, a node can effectively increase its transmission range by reducing the data rate without changing its transmission power, contrary to fixing the data rate and adapting the transmission range by dynamically adjusting the transmission power. It would be interesting to consider such a data-rate adaptive possibility to conserve energy. Furthermore, the interference among multiple traffic flows may affect the transmission range optimization by introducing packet collisions and retransmissions. We leave it as our future work.

ACKNOWLEDGMENT

The work of Deng and Han was partially performed dur- ing their visit to the CASE Center and Dept. of Electrical Engineering and Computer Science at Syracuse University, USA. J. Deng's work was supported in part by Louisiana Board of Regents RCS grant LEQSF (2005-08)-RD-A-43. It is also partly supported by National Science Council of Taiwan, R.O.C., under grants NSC 90-2213-E-260-007 and NSC 91- 2213-E-260-021, and by Chung-Shan Institute of Science & Technology, Taiwan, R.O.C., under grant XC93B95P

We would also like to express special thanks to Prof. Y. Fang and all the anonymous reviewers for their valuable comments.

References

[1] L. M. Feeney and M. Nilsson, "Investigating the energy consumption of a wireless network interface in an ad hoc networking environment," in *Proc. IEEE INFOCOM*, April 2001, pp. 1548-1557.
[2] L. Hu, "Topology control for multihop packet radio networks," *IEEE Trans. on Commun.*, vol. COM-41, no. 10, pp. 1474–1481, October 1993.
[3] N. Bambos, "Toward power-sensitive network architectures in wireless communications: concepts, issues, and design aspects," *IEEE Personal Communications*, vol. 5, pp. 50–59, June 1998.
[4] M. Sanchez, P. Manzoni, and Z. J. Haas, "Determination of critical transmission range in ad hoc networks," in *Multiaccess Mohility and Teletraffic for Wireless Communications 1999 Workshop*, October 1999, pp. 293–304.

[5] R. Ramanathan and R. Rosales-Hain, “Topology control of multihop wireless networks using transmit power adjustment,” in *Proc. IEEE INFOCOM*, March 2000, pp. 404–413.
[6] R. Wattenhofer, L. Li, P. Bahl, and Y.-M. Wang, “Distributed topology control for power efficient operation in multihop wireless ad hoc networks,” in *Proc. IEEE INFOCOM*, April 2001, pp. 13 88–1397.
[7] H. Takagi and L. Kleinrock, “Optimal transmission ranges for randomly distributed packet radio terminals,” *IEEE Trans. on Commun.*, vol. COM-32, no. 3, pp. 246–257, March 1984.
[8] T.-C. Hou and V. O. K. Li, “Transmission range control in multihop packet radio networks,” *IEEE Trans. on Commun.*, vol. COM-34, no. 1, pp. 38–44, January 1986.
[9] M. Zorzi and S. Pupolin, “Optimum transmission ranges in multihop packet radio networks in the presence of fading,” *IEEE Trans. on Commun.*, vol. COM-43, no. 7, pp. 2201–2205, July 1995.
[10] V. Rodoplu and T. H. Meng, “Minimum energy mobile wireless networks,” *IEEE Journals on Selected Areas of Communications*, vol. 17, pp. 1333–1344, August 1999.
[11] P. Chen, B. O’Dea, and E. Callaway, “Energy efficient system design with optimum transmission range for wireless ad hoc networks,” in *Proc. IEEE ICC*, 2002, pp. 945–952.
[12] Y. Chen, E. G. Sirer, and S. B. Wicker, “On selection of optimal transmission power for ad hoc networks,” in *Proc. of the 36th Hawaii International Conference on System Sciences (HICSS ’03)*, January 2003, pp. 300–309.
[13] M. Zuniga and B. Krishnamachari, “Optimal transmission radius for flooding in large scale sensor networks,” in *Proc. of the 23rd International Conference on Distrihuted Computing Systems Workshops*, May 2003, pp. 697–702.
[14] J. L. Gao, “Analysis of energy consumption for ad hoc wireless sensor networks using a bit-meter-per-joule metric,” IPN Progress Report 42- 150, August 2002.
[15] J. Li, J. Jannotti, D. S. J. De Couto, D. R. Karger, and R. Morris, “A scalable location service for geographical ad hoc routing,” in *Proc. of the sixth annual international conference on Mohile computing and networking (MobiCom)*, August 2000, pp. 120–130.
[16] R. Jain, A. Puri, and R. Sengupta, “Geographical routing using partial information for wireless ad hoc networks,” *IEEE Personal Communications*, vol. 8, pp. 48–57, February 2001.
[17] P. Billingsley, *Prohahility and Measure*, New York, NY: John Wiley and Sons, 1995.
[18] J. Deng, Y. S. Han, P.-N. Chen, and P. K. Varshney, “Optimum transmission range for wireless ad hoc networks,” in *IEEE Wireless Communications and Networking Conference (WCNC ’04)*, Atlanta, GA, USA, March 21-25 2004, vol. 2, pp. 1024–1029.