Joint Economic Selection of Target Mean and Variance

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Abstract:
This paper considers the problem of selecting the most economical target mean and variance for a continuous production process. In earlier studies, many authors considered the problem of finding an optimal target mean assuming that the variance is known. The problem with this assumption is the difficulty or impossibility of setting a target variance. Taguchi suggested a two-step procedure: first, set the target mean; then, find the smallest variance through redesign or experiment (resetting the level of factors). In this study, three new approaches are suggested for the economic selection of a target variance integrated with a target mean. In the first approach, an expected profit maximization criterion is used to obtain the target mean and variance simultaneously. The example used to illustrate this approach is a filling process where the quality characteristic is assumed to be normally distributed. The containers that are underfilled can be sold in a secondary market at a price of \$$P_L$ per can, those within specification can be sold at a price of \$$P_0$ per can, and those over the upper specification limit can be sold at a price of \$$P_U$ per can. In the second approach, a minimum cost criterion based on the Taguchi loss function is used: first, the processes optimized for the variance; then, an optimal process mean is obtained. In the third approach, an economic model for the selection of the target variance is developed, using both customer and producer costs to minimize societal loss independent of the product quality characteristic distribution.

Keywords: Joint economic selection; Target mean; Target variance; Taguchi loss function

Article:

1 INTRODUCTION

The filling process problem has received considerable attention from researchers in recent years. Springer [19] developed a method for determining the most economic position of a process mean. He considered a manufacturing process in which both upper and lower specification limits were of interest and in which the financial loss due to producing a product above the upper specification limit was not necessarily equal to the loss when producing one below the lower specification limit. He suggested a simple method for determining the optimum target mean to minimize the total cost. The distribution of the product quality characteristic was assumed to be normal. Bettes [2] studied a similar problem with a given lower specification limit; however, he assumed an arbitrary upper specification limit. Furthermore, he assumed undersized and oversized items are reprocessed at a fixed cost. Hunter and Kartha [12] investigated the optimization of a target mean when a lower specification is fixed: each item is inspected to determine whether it satisfies a lower specification limit. If it does, then that item is sold at the regular price; if it does not, the item is sold at a reduced price in a secondary market. Thus, the customer is compensated for poor quality but does not pay extra for excessive quality. A net income function consisting of income from the accepted items, the give-away cost of material in excess of the lower specification, and income from the rejected items was developed. Their study provides a simple procedure for obtaining the optimal process mean. Nelson [17] provided a similar solution to this problem. Bisgaard et al. [3] extended Hunter and Kartha's [12] work to include the selection of the most favorable quality characteristic distribution for the product. Industrial examples of normal, lognormal, and Poisson distributed quality characteristics were provided. Carlsson [4] modified the work of Hunter and Kartha [12] to include both fixed and variable costs. He derived a revenue function whereby the customer pays extra for quality and is compensated for poor quality. Both Hunter and Kartha [12] and Carlsson [4] assume that rejected products are sold on a secondary market. Golhar [8] addressed the problem of finding the most economic setting of a process mean. He modelled a situation where over-filled cans could only be sold in a regular market and where the underfilled cans would be emptied and refilled, with the penalty of extra cost. Golhar and Pollock [9] extended this work to include an upper specification limit and provide solutions for determining both the optimal process mean and the upper specification limit. Schmidt and Pfeifer [18] considered the problem of the economic selection of the target mean and the upper limit for a canning process with limited capacity. Golhar and Pollock [10] studied a canning process to investigate the effect of a reduction in process variance on production cost. These studies may be considered as direct extensions of Springer's work [19].

Montgomery [15] suggested a normally-distributed quality characteristic in which the profit function is related to conformity to upper and lower specifications. The profit obtained by a product below the lower limit is different from that obtained from a product above the upper limit. A common assumption in most of the above studies is that the process variability is known. Schmidt and Pfeifer [18] use Golhar's model [8] to evaluate the economic effects of process variance reduction. Bai and Lee [1] studied a process where the lower specification limit of the quality characteristic is given and where each container is inspected on a surrogate variable correlated with the primary variable. The same type of problem was discussed by Tang and Ho [21]. Lee and Kim [14] extended the model of Bai and Lee [1] by including a controllable upper limit. The current trend is to consider both producers' and customers' costs and incorporate them into manufacturing optimization decisions. Genichi Taguchi's views (see, Kakar [13]) on this aspect of quality control are perhaps the best known. His method has become popular because its premises are simple and easily understood.

2 ECONOMIC SELECTION OF TARGET MEAN AND VARIANCE

2.1 Expected Profit Criterion

Assumption

The product quality characteristic x is normally distributed with the density function $f(x)$, mean μ, and variance σ^2. If a product falls within specifications, the selling price is P_0. However, if its quality characteristic is below or above the specification limits, the selling price is P_L or P_U respectively. Let C_m be the material cost per unit of product, and da exp(σ — d) be an exponential function which represents the cost of variance reduction. The problem is to find the joint optimal target mean μ and variance σ^2.

Model Development

The expected profit per item is

$$\begin{aligned} E(\mu,\sigma) = {} & P_0\left[\Phi\left(\frac{U-\mu}{\sigma}\right)-\Phi\left(\frac{L-\mu}{\sigma}\right)\right] \\ & +P_L\Phi\left(\frac{L-\mu}{\sigma}\right)+P_U\left[1-\Phi\left(\frac{U-\mu}{\sigma}\right)\right] \\ & -C_m\mu - a\exp(-d\sigma) \qquad (1) \end{aligned}$$

where $\phi(\bullet)$ and $\Phi(\bullet)$ are the standard normal probability density function and cumulative distribution function, respectively.

Necessary conditions for optimality are

$$\begin{aligned} \frac{\partial E(\mu^*,\sigma^*)}{\partial \mu} = {} & -\phi\left(\frac{U-\mu^*}{\sigma}\right)(P_0-P_U) \\ & +\phi\left(\frac{L-\mu^*}{\sigma^*}\right)(P_0-P_L)-C_m = 0 \quad (2) \end{aligned}$$

and

$$\frac{\partial E(\mu^*, \sigma^*)}{\partial \sigma} = -(P_0 - P_U)\left(\frac{U-\mu^*}{\sigma^*}\right)\phi\left(\frac{U-\mu^*}{\sigma^*}\right)$$
$$+(P_0 - P_L)\left(\frac{L-\mu^*}{\sigma^*}\right)\phi\left(\frac{L-\mu^*}{\sigma^*}\right)$$
$$+ad\exp(-d\sigma^*) = 0 \qquad (3)$$

To obtain μ^* and σ^*, Eqs. (2) and (3) must be solved, simultaneously.

Numerical Example
Let $U = 20$, $L = 15$, $P_0 = 10$, $P_L = 5$, $P_U = 7.75$, $C_m = 0.1$, $a = 2.96$ and $d = 1$.

When the above Eqs. (2) and (3) are solved simultaneously, the optimal solutions of mean µ and variance a2 can be obtained. It is clear from looking at Eqs. (2) and (3) that these are cumbersome and mathematically involved to solve. Thus, an optimization search method was considered which used the gradients of the function, as well as the objective function values (Fletcher and Reeves' [6] gradient method). The unconstrained optimization search method of Hooke and Jeeves [11] was also used which requires only the evaluation of the objective function. Using both search methods, the same optimum solutions were obtained, and these are as follow: $\mu^* = 17.74$ and $\sigma^{2*} = 4.0$.

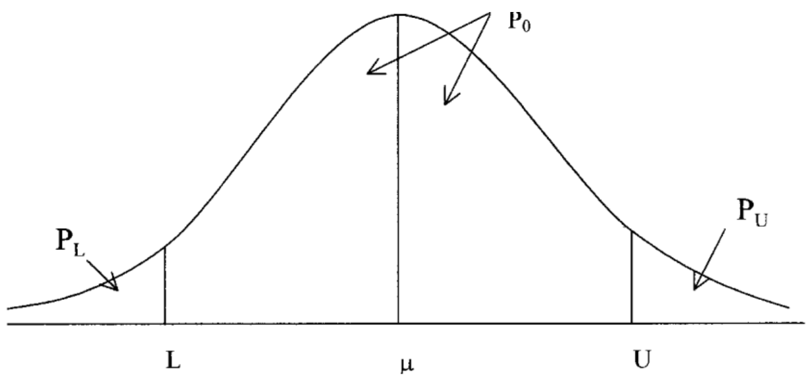

FIGURE 1 Selection of optimal target value.

2.2 Expected Loss Criterion

The previous section is a consideration of the problem of simultaneously determining the target mean and target variance for a process. This section describes the problem of simultaneously determining the target mean and target variance for a process in relationship to Taguchi's loss function. According to Taguchi [20], quality is measured by the deviation of a quality characteristic from its target value, even though it remains within the specification limits L and U.

Consider a manufacturing process in which the dimension of the product quality, X, is normally distributed with a mean and variance σ^2. Let the part specification have a target value of m. Thus, Taguchi's loss function is given by

$$L(X) = K(X - m)^2 \qquad (4)$$

The expected loss function is

$$E[L(X)] = KE(X - m)^2$$
$$= K[\sigma^2 + (- m)^2] \qquad (5)$$

Mukhopadhyay and Chakraborty [16] developed a model for determining optimal process variance according to the Taguchi loss function (the M-C model). In their study, a case is considered in which even though the process mean μ is on the target value m, the variance may increase over time. They assume that the quality of the output of the process is symmetrical around the process mean. The loss for onconforming parts, called rejection loss, is denoted by $L_1(\sigma)$; the Taguchi loss for conforming parts is denoted by $L_2(\sigma)$; and the variance reduction cost is denoted by $C_2(\sigma_1, \sigma)$, where σ_1^2 is the current variance and σ^2 is the target variance after the reduction. The reduction in variance from σ_1^2 to σ^2 reduces both the rejection loss, $L_1(\sigma)$, and the Taguchi loss, $L_2(\sigma)$, but at a cost. There must be an optimum variance σ_*^2; any reduction beyond σ_*^2 is not worth the resultant savings.

Mukhopadhyay and Chakraborty [16] propose the following for normally distributed outputs.

PROPOSITION 1 *The rejection loss* L_1 (σ) *is an increasing function of* σ.

PROPOSITION 2 *The Taguchi loss* L_2 (σ) *can increase or decrease depending on the parameters* μ, σ, *and* K.

Mukhopadhyay and Chakraborty [16] assume that the cost of rework for oversized and undersized parts is the same; however, in this paper, the costs of undersized and oversized parts are considered to be different. Therefore, the total cost associated with a nonconforming item for the proposed model is

$$L_1(\sigma) = C_L \int_{-\infty}^{L} (x)dx + C_U \int_{U}^{\infty} f(x)dx \qquad (6)$$

where

$$C_L = P_0 - P_1 \text{ and } C_U = P_0 - P_2$$
$$= C_L \Phi\left(\frac{L-\mu}{\sigma}\right) + C_U\left[1 - \Phi\left(\frac{U-u}{\sigma}\right)\right] \qquad (7)$$

Differentiating with respect to σ results in

$$\frac{\partial L_1(\sigma)}{\partial \sigma} = -C_L\left(\frac{L-\mu}{\sigma}\right)\phi\left(\frac{L-\mu}{\sigma}\right) + C_U\left(\frac{U-\mu}{\sigma}\right)\phi\left(\frac{U-\mu}{\sigma}\right) \qquad (8)$$

Since $\phi(\bullet) > 0$, $L < \mu$ and $U > \mu$, then $dL_1(\sigma)=d\sigma > 0$. It is easy to see that L_1 (σ) is increasing in σ.

Thus, Proposition 1 of the M-C model is applicable to the proposed model.

Under Proposition 2, assuming symmetry and denoting $Z = L - \mu/\sigma$, then

$$L_2\sigma = 2K\sigma^2(Z\phi(Z) + 1 - \Phi(Z) - 1/2)$$

$$= 2K\sigma^2(Z\phi(Z) - \Phi(Z) - 1/2)$$

Differentiation with respect to a results in

$$\frac{dL_2(\sigma)}{d\sigma} = 2K\sigma\big(1 - 2\Phi(Z) + Z\phi(Z)(2 + Z^2)\big) \quad (9)$$

Thus, $dL_2(\sigma)/d\sigma > 0$ if $1 - 2\Phi > -Z\phi(Z)(2 + Z^2)$:

It can easily be verified that the above condition is true only when $Z \leq -1{:}37$, i.e., if $L - \mu^* = \sigma^* \leq -1{:}37$.

From the proposed model described in section 2.1, the results are as follows: $L = 17$; $\mu^* = 17{:}74$, and $\sigma^* = 2{:}00$; substituting these into $L - \mu^* = \sigma^*$ results in exactly Z = −1:37. Therefore, Proposition 2 of the M-C model is also applicable and supports the proposed model.

3 ECONOMIC SELECTION OF TARGET VARIANCE, INDEPENDENT OF PRODUCT QUALITY DISTRIBUTION

There is a wide variety of optimization and variation reduction problems; one of these is the determination of the target value, and another is uniformity around the target value according to the Deming thought line. Optimization can be described as cooperation between the producer and the customer that results in reduction of variability, independent of the product quality characteristic distribution. Chen [5] states that reducing process variations can lead to increased customer satisfaction, but may result in higher costs to the manufacturer. On the other hand, larger allowable variations will adversely affect quality. The goal in design is to minimize the total societal loss, including the costs to the customer and the producer.

Each production set-up is unique, and it is unrealistic to expect that a general equation that holds true in all instances can be derived. Intuition, however, suggests that a reduction in variance at any point in time can be accomplished by increasing production costs. This approach appears to conflict with the generally-held belief that variance reductions are accompanied by decreasing costs, for example, reductions in scrap rates. Overall decreases in production costs are a long-term effect that are accomplished by increased short-term expenditures in such things as new equipment, training, and=or increased inspection. Eventually, these costs are recoupled through increased efficiency, but over the short term they must be borne by the organization and should be factored into quality decisions. In the majority of instances, this improved performance may be characterized as a decrease in the variability of production. However, managers must balance purchase and operating costs against reduced process variation.

It was assumed that producer costs increase non-linearly with a decrease in variability and can be represented by

$$M(\sigma^2) = \frac{v}{1+\sigma^2} \text{ where } v > 0 \qquad (10)$$

This equation describes a non-linear function that has a maximum, v, and a minimum that tends to zero as the variance increases. The possibility of zero variance is best explained with an example derived from the canning problem discussed in Section 2. A producer who wishes to sell cans weighing exactly one kilogram can do so if he is willing to inspect all cans and reject those not meeting this criterion. If the scale used is sufficiently accurate, any deviations will be so small that they will be ignored by the customer. Thus, the producer achieves variance that effectively is zero at a cost of v. Of course, the cost of such an achievement is passed on to the consumer.

The customer's cost of variance can also be expressed as a non-linear increasing function. This cost is represented by

$$C(\sigma^2) = m_1 + m_2\sigma + m_3(\sigma^2 + \text{bias}^2) \qquad (11)$$

Eq. (11) is an example of the Taguchi loss function (see, Taguchi et al. [20]) because nominal is the best when the process mean is centered on the target value. In this case, m1 and m2 equal zero, and $m_3 = k$ such that

$$C(\sigma^2) = E[L(x)] = k(\sigma^2 + \text{bias}^2) \qquad (12)$$

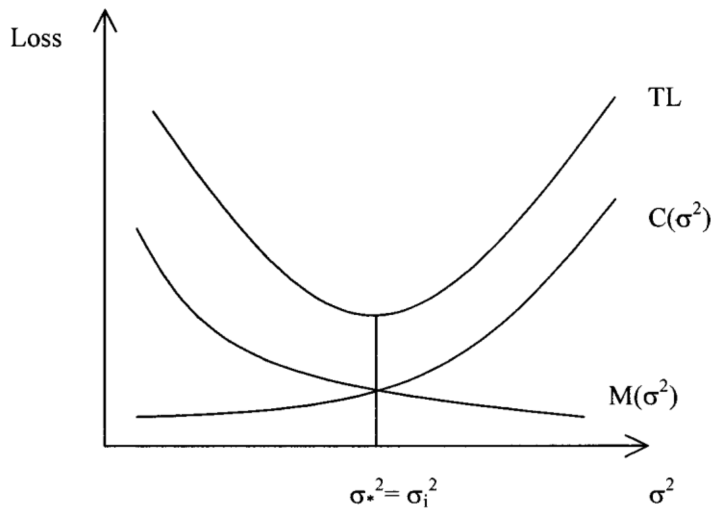

FIGURE 2 Economic selection of target variance.

According to Taguchi's loss function, the variance can be optimized, assuming that the process mean is on target (i.e., no process bias). Traditionally, reducing variations in product quality characteristics can lead to higher customer satisfaction. However, this reduction may also require higher-grade parts, materials, and production facilities, each of which results in a higher production cost. On the other hand, larger variations adversely affect quality and decrease the competitiveness of the manufacturer. Thus, an optimization must be established. The objective function of this model represents a societal loss TL, which includes a cost of variance to the customer $C(\sigma^2)$; the cost of the variance to the producer $M(\sigma^2)$ includes costs for the upgrading or for the tightening of the tolerance limits. Figures 2, 3, and 4 depict both the producer's and customer's costs and their sum, i.e., total cost in a general format. The total cost TL includes the cost of variance for both parties.

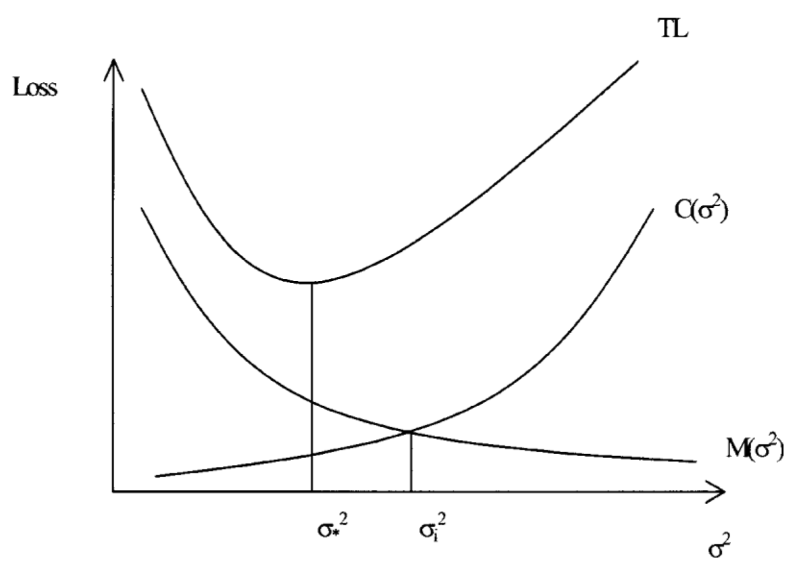

FIGURE 3 Minimum less than crossover.

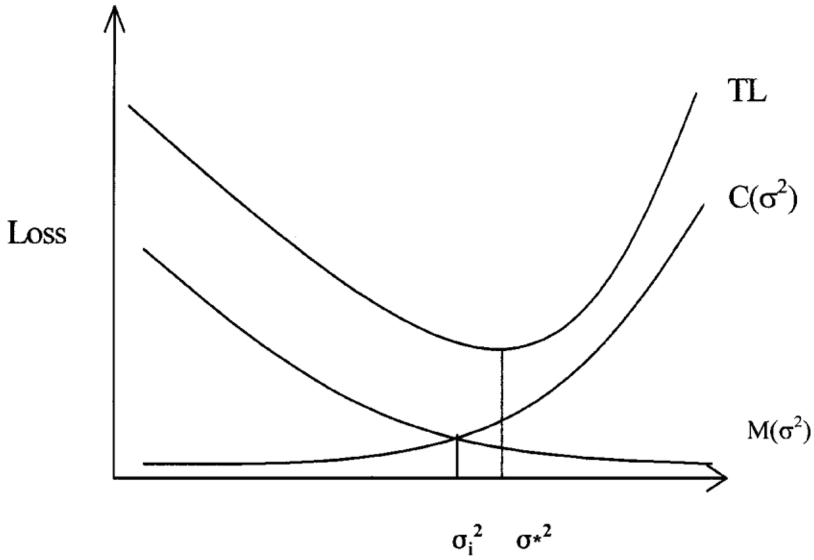

FIGURE 4 Minimum greater than crossover.

If σ_i^2 is defined as the variance at the intersection of the two cost functions, $C(\sigma^2)$ and $M(\sigma^2)$, and σ_*^2 is defined as the minimum of the total cost TL, there are three possible outcomes:

Outcome I, $\sigma_*^2 = \sigma_i^2$;
Outcome II, $\sigma_*^2 = \sigma_i^2$;
or Outcome III, $\sigma_*^2 = \sigma_i^2$.

In the case of Outcome I, the point at which the producer's and customer's costs are equal determines the optimal variance. When the crossover point is not the minimum of the total cost, a situation is created in which bargaining between the producer and the consumer is possible.

3.1 Unequal Minimum and Crossover Point

An interesting dilemma occurs when the minimum of the total costs and crossover point are at different values of sigma, as shown in Figures 3 and 4. Redefining the Taguchi quality loss as the sum of the costs to both the consumer and producer, and not just the customers' costs (as is often done in the literature), creates a situation in which to minimize this cost both parties must operate at variances that clearly are not optimal for either of them. One of the major techniques of total quality management is producer/consumer cooperation. Clearly, neither party wishes to gain at the expense of the other because the trust necessary for this management technique would be destroyed. This situation is modeled in terms of cooperative game theory in which both players work together to achieve a result that is mutually beneficial.

The problem is one of choosing whose interest takes precedence and determining a method of compensation (*i.e.*, transfer utility) for the penalized party. If the total costs are minimized as shown in Figure 3 a situation arises where the producer can compensate the customer with the savings from the increased operating variance. The converse is true in Figure 4. Gitlow et al. [7], discussing a similar situation using the Taguchi loss function with the emphasis on the target mean, conclude that the total cost should subsume that of the players, i.e., the manufacturer and customer. Unfortunately, they do not discuss a method of compensating the penalized party. In general, this situation can be defined as follows. Let the cost of variance to the producer and the consumer be $M(\sigma^2)$ and $C(\sigma^2)$, respectively. $M(\sigma^2)$ is a monotonically decreasing function such that

$$M'(\sigma^2) < 0 \ \forall \sigma^2 \qquad (13)$$

and $C(\sigma^2)$ is a monotonically increasing function such that

$$C^{'}(\sigma^2) > 0 \ \forall\sigma^2 \qquad (14)$$

Then, the following lemma is a sufficient condition to provide for an intersection of these two costs.

LEMMA 1 *If at* $\sigma^2 = 0$, $C(\sigma^2) < M(\sigma^2)$, *then there exists a* σ_i^2 *such that*

$$\mathrm{M}(\sigma_i^2) = \mathrm{C}(\sigma_i^2) \qquad (15)$$

If society's cost, defined as $T(\sigma^2) = M(\sigma^2) + C(\sigma^2)$, is to have a minimum, then, at this point

$$T^{'}(\sigma^2) = 0 \qquad (16)$$

and

$$T^{''}(\sigma^2) > 0 \qquad (17)$$

which implies

$$\frac{\partial}{\partial\sigma^2}[M(\sigma^2) + C(\sigma^2)] = 0 \qquad (18)$$

Also implying that, at the optimal variable σ_*^2,

$$\frac{\partial[M(\sigma_*^2)]}{\partial\sigma^2} = -\frac{\partial[C(\sigma^2)]}{\partial\sigma^2} \qquad (19)$$

i.e.,

$$M^{'}(\sigma_*^2) = -C^{'}(\sigma_*^2) \qquad (20)$$

Thus, σ_i^2 is obtained by solving Eq. (15), and σ_*^2 is obtained by solving Eq. (20). As previously mentioned, σ_i^2, the intersection variance, does not have to equal σ_*^2, the minimum of the total costs.

It can be verified that if $M(\sigma^2)$ and $\mathrm{C}(\sigma^2)$ are given by (10) and (12), respectively, then (15) reduces to

$$\frac{v}{1+\sigma_i^2} = k\sigma_i^2 \qquad (21)$$

i.e.,

$$\sigma_i^2 = \frac{\sqrt{k^2+4vk} - k}{2k} \qquad (22)$$

Similarly, (20) implies that at σ^2 1/4 σ_*^2,

$$\frac{\partial}{\partial\sigma^2}\left(\frac{1}{1+\sigma^2}\right) = -\frac{\partial}{\partial\sigma^2}(k\sigma^2) \qquad (23)$$

Hence,

$$\frac{v}{(1+\sigma_*^2)^2} = k \qquad (24)$$

and

$$\sigma_* = \sqrt{\frac{v}{k}} - 1 \qquad (25)$$

Thus, for Outcome I to occur (*i.e.*, for σ_*^2 to be equal to σ_i^2), (22) and (25) must be equal. In other words

$$\sqrt{\frac{v}{k}} - 1 = \frac{\sqrt{k^2 + 2vk} - k}{2k} \; or$$
$$2\sqrt{vk} - \sqrt{k^2 + 2vk} - k = 0 \qquad (26)$$

Hence, Outcome II will occur if

$$2\sqrt{vk} - \sqrt{k^2 + 2vk} - k < 0 \qquad (27)$$

and Outcome III will occur if

$$2\sqrt{vk} - \sqrt{k^2 + 2vk} - k > 0 \qquad (28)$$

If side payments are not allowed or there is no mutually acceptable form of compensation and both parties are equally empowered, then σ_i^2, is the solution of this problem, regardless of whether or not σ_i^2, equals σ_*^2. On the other hand, if a suitable transfer utility is agreed upon, then σ_*^2 becomes the optimal solution to the negotiation process because of its lower total cost.

The proof that σ_*^2 is always superior or equal to σ_*^2 as a target variance (if side payments are allowed) is as follows:

$$T(\sigma_*^2) \leq T(\sigma_i^2) \qquad (29)$$

and

$$M(\sigma_i^2) - C(\sigma_*^2) \leq M(\sigma_i^2) + C(\sigma_i^2) \qquad (30)$$

thus

$$[M(\sigma_i^2) - M(\sigma_*^2)] - [C(\sigma_*^2) - C(\sigma_i^2)] \geq 0 \quad (31)$$

The gain to one party in a move from σ_i^2 to σ_*^2 is equal to or greater than the loss to the other. In the instance of Outcome II or III, a portion of this gain can be used to induce the penalized party to move to this variance.

For Outcome II, $\sigma_*^2 < \sigma_i^2$,

$$M(\sigma_*^2) > M(\sigma_i^2), \qquad (32)$$

and

$$C(\sigma_*^2) < C(\sigma_i^2). \qquad (33)$$

Since $T(\sigma_i^2)$ is less than $T(\sigma_*^2)$, the reduction in total cost is $T(\sigma_i^2) - T(\sigma_*^2)$. This reduction in cost can be shared in some fashion between the producer and the customer by transferring some of the customer's savings to the producer as an incentive for the producer to move to the target variance of σ_*^2.

If Outcome III occurs, then $\sigma_*^2 > \sigma_i^2$. The producer should offer a part of the savings of $T(\sigma_*^2) - T(\sigma_i^2)$ to the customer as an incentive to move to the target variance.

Thus, it is easy to see that if Outcomes II or III occur, negotiation between the producer and the customer, along with a transfer of savings from one party to the other, can lead to a target variance of σ_*^2, which minimizes the total cost to society. The need for such negotiation is further highlighted by the following result.

LEMMA 2 *If both parameters v and k are chosen from two independent random variables, then the probability of occurrence of Outcome I is zero.*

Proof Assume that v and k are chosen from two independent random variables, V and K, where density functions, $f_v(v)$ and $f_k(k)$, respectively, are defined on the two sets, S_v, and S_k. Then, Outcome I will occur only if the event associated with (26) occurs. The probability of this happening is.

$$\iint_{S_1} f_v(v) f_k(k) dv\, dk$$

where S_1, is the set of values of v and k in the set (S_v x S_k) that satisfies (25). However, the set S is a one-dimensional manifold and is a subset of the 2-dimensional set (S_v x S_k). Hence, the above integral would yield zero area under the joint distribution of v and k in the set S_1. Therefore, there is zero chance that Outcome I will occur.

The importance of Lemma 2 is best understood by noting that it accents the need for cooperation between the producer and the customer in almost all circumstances. Given the reasonable assumption that the producer and customer cost function parameters occur randomly and independently, it is highly unlikely that Outcome I will ever occur. Therefore, one would expect Outcomes II or III to occur, necessitating the need for negotiation between the two parties. As shown above, this negotiation can lead to the selection of a target variance that is "globally optimal" in the sense that it minimizes the societal cost; furthermore, both parties are better off with this negotiated solution. If such a negotiation does not take place, then globally optimal target variances can still be enforced through other mechanisms. These mechanisms are discussed in Section 3.2.

3.2 Producer/Consumer Motivation for Minimum Selection

If the total cost of variance is ignored, the intersection of the producer's cost and the consumer's cost will provide the target variance necessary for determining the optimal mean. If the total cost of variance is not ignored, the previous section proves that there are funds, transfer utility, available to facilitate the movement from the intersection variance, σ_i^2, to the minimum variance, σ_*^2. The question, then, becomes one of will: Is there a mechanism, either market or internal, that forces or encourages this movement?

As shown in the next section, if the producer and consumer are part of the same organization, total cost consideration becomes paramount. In this instance, it would be illogical for the parties to ignore the minimum of the total cost. Any deviation from this point results in a loss that must be borne by the company and, ultimately, by the parties themselves. This threat of loss provides the impetues to move to minimum variance.

If the parties are separate entities, however, this impetus no longer exists. Since the loss in efficiency caused by operating at a point other than the minimum variance is borne by the market as a whole, the motivation to move to the minimum of the total cost curve is effectively eliminated. Unfortunately, there is no direct means for the market to push the participants towards the optimal point. Although the capitalist system tends to eliminate inefficiencies through a process akin to natural selection, it usually does so only in "perfectly competitive" markets. Given that a large number of producers and consumers is necessary to make negotiation attractive, it is unlikely that free market forces will apply. As a result, three possible mechanisms for the total cost curve take precedence. The first involves outside interference or regulation. Consider the case where product variance is directly related to an important societal value, for example, the level of pollution. In this case, it is conceivable that government regulation could force the producer and the consumer to the minimum of the total cost curve through legal or financial means.

The second method is more circumspect. Although neither party wants to move to the minimum variance, both consider the possible outcomes of inaction. Markets tend to be protected from new entrants through entry barriers caused by the potential entrant's lack of knowledge of the current operating environment, likely resulting in higher operating costs. However, if the possible savings of variance optimization are great enough, potential entrants could offset their increased operating costs by making an educated guess as to the value of the minimum variance. The room for acceptable error in this approximation increases with the magnitude of the savings available. Thus, if current producers and consumers wish to protect their positions in the market, then eliminating this inducement to outside organizations is rational.

The third method involves outright purchase of one of the parties by the other in what is termed "vertical integration". The purchaser realizes the savings offered by the increased variance efficiency. The implementation of this study's proposed model for variance selection using the motivation of reduced organizational variance costs is outlined in the next section.

3.3 Implementation

The implementation of this model presupposes a producer and consumer who, although not necessarily well-versed in the techniques of quality management, have some familiarity with it. This is not to say that using this model relieves participants of the responsibility of quality management; it doesn't. Corporate quality implementation failures are often the result of management's underestimation of the level of commitment required.

Issues of quality management aside, assume that a company decides to implement this model in order to determine the optimal mean and variance levels of a number of products or quality characteristics. Since the determination of the target variance is independent of the underlying distribution of the quality characteristic, its determination is the logical place to start.

Ideally, but not necessarily, the various departments should be set up as profit centers so that each is aware of its expenses. The task of quantifying the cost of variance may not be easy because of the high number of factors that influence this expense. This difficulty is reduced, however, if both the manufacturer and the consumer assume that Eq. (10) and (11) represent the general behavior of these costs. For the producer, there is only one point that needs to be defined to specify the cost of the variance function. If the cost associated with any variance is known, then the cost of zero variance, v, can be determined using Eq. (10). Unfortunately, the task is not as simple for the consumer: Eq. (11) needs three cost-of-variance points to completely define it. The complexity is reduced, however, by using Eq. (5), which is the Taguchi loss function. In this case, one point is sufficient for quantifying the consumer's cost of variance.

Once both parties agree on a target variance (after analysis and prioritization of individual costs), the producer determines a target mean. Depending on the quality characteristic probability distribution and the nature of the tolerance limits (whether upper, lower, or both), a mathematical treatment can be chosen. The producer uses the agreed-upon variance in the applicable equations. In addition, if the producer follows the Taguchi philosophy of quality control, tolerance limits can be tightened based on what is learned from the consumer in the same manner as discussed earlier in Section 3.1. As shown in Section 3.1, the variance chosen may not be optimal for both parties; it may be imposed by an outside body, in this case the organization as a whole. If this is the case, then some method of compensation for the penalized party must be determined and incorporated into the transfer pricing system that accompanies the set-up of profit centers within an organization. If the consumer and producer are separate business entities, the method of compensation (i.e., transfer utility) would, in all likelihood, be monetary.

As in most models of quality management, the expectations of both the producer and consumer may be the most important feature. Understanding the impact of variance or quality decisions on consumer and producer costs is an important step for any organization and may itself be sufficient justification for the use of this model.

CONCLUSION

In this paper, three approaches for determining optimal target mean and target variance are considered. The first approach considers the problem of determining optimal settings for the mean and variance of a process so that the total profit is maximum. In the second approach, the variance is optimized based on Taguchi's loss function. However, in both of these approaches it is assumed that the product quality characteristic is normally distributed, whereas in the third approach the variance is optimized based on producer and customer costs that are independent of product quality distribution.

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