
**Reference List**

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ABSTRACT: This review of *Class Notes for Advanced Atonal Music Theory* by Robert D. Morris (Lebanon, N.H.: Frog Peak Music, 2001) describes the book’s organization, content, and applicability for pedagogical and research uses.

[1] In *Class Notes for Advanced Atonal Music Theory*, Robert D. Morris (2001) has gathered and interpreted much of the work done in atonal and twelve-tone theory since his book *Composition with Pitch-Classes* (Morris 1987; hereafter, *CwPC*). The result is a masterful pedagogical and research text by one of our most prolific and respected composer/theorists. Morris covers such now-standard topics as contour theory, transpositional combination, Klumpenhouwer networks (K-nets), generalized interval systems, and pc/order-number isomorphisms. The book also introduces new and previously unpublished material by Morris.

[2] The genesis of *Class Notes* is in lecture notes and handouts prepared by Morris for use in a graduate theory course at the Eastman School of Music. After teaching the course successfully for many years, Morris assembled the materials in a form suitable for pedagogy, research, and composition. While the text is not a composition treatise in the vein of *CwPC*, there is much here that composers may find of use.

[3] In 1995, I was one of many students and colleagues of Morris’s who studied the text, used it in class, and offered suggestions for its improvement. *Class Notes* has benefited from over six
years of such suggestions, as well as fine editing by Dora Hanninen. The difference between the present text and its earlier incarnations is striking. The current, mature version of Class Notes is a model of concision and consistency.

[4] Class Notes is a two-volume spiral-bound book. Volume 1 consists of six chapters of text; Volume 2 consists of Examples, Tables, Appendices, a Glossary, and Works Cited. The layout of the two volumes is clear, and alternating between them presents no special difficulties. The text is organized with "two types of readers in mind: those reading the text beginning to end, versus those reading individual sections or chapters. For this reason the chapters are not arranged in an altogether sequential manner, and their tone and difficulty vary somewhat" (p. x). While the latter point holds true, the consistent presentational format smoothes out what otherwise may have been significant differences in expository density.

[5] Chapter 1, "Basic Terms and Concepts," isolates the terminology that runs through the entire text, recalling in format and content the opening chapter of David Lewin’s book, Generalized Musical Intervals and Transformations (Lewin 1987, hereafter, GMIT). Morris begins by defining Cartesian products, functions, and three types of relations: equivalence, similarity, and partial ordering. From this, the concept of hierarchy follows, defined in a value-free manner as a "partial ordering that can be represented as a connected graph without loops" (p. 9). This allows Morris to characterize some twelve-tone music as hierarchic (p. ix). The remainder of Chapter 1 is devoted to the study of groups. Morris’s lucid exposition permits immediate connections to various topics introduced by other authors, including Lewin’s simply transitive and direct product groups, permutation groups in work by Daniel Harrison (1988) and Henry Klumpenhouwer (1991), and affine groups in work by Paul Lansky (1973). The material in Chapter 1 will also prove useful for persons who wish to read CwPC and GMIT.

[6] Chapter 2, "Topics in Contour-Space and Pitch-Space," is explicitly based on Morris (1993, 1995). I read it as an update to Chapter 2 of CwPC. Here Morris lays out the fundamentals of contour- and pitch-spaces, incorporating research on these topics published after CwPC. The chapter is rife with potential for analytic application. For example, Morris’s theory of pitch-sets will interest analysts working with Ligeti, Varèse and other composers whose music is often characterized as inhabiting non-modular pitch-spaces. Additionally, similarity relations among pitch-sets that are discussed here finesse many of the problems attendant with traditional pcset similarity, thus also inviting analytic application.

[7] In Chapter 3, "Relating Pc Entities," the material on equivalence relations, similarity relations, and partial orderings presented in Chapter 1 is expanded into three self-standing sections. In the equivalence relations section, various set-group (SG) systems are posited as ways of determining sc equivalence. With one exception, each SG determines its sc members by way of a canonical group—a set of operations that exhibits group structure. The discussion of canonical groups revisits Morris (1982) and recalls a passage from Lewin’s GMIT (p. 105). The first SG system, SG1, uses pc transposition only. SG1 bisects the Forte scs that do not have TnI invariance. For example, SG1 divides Forte’s 3-11[037] into two scs: 3-11a(037) and 3-11b(047). Such distinctions are especially relevant to analysts who may question the musical validity of inversionsal equivalence. SG2 uses transposition and inversion (Tn and TnI). SG3 adds the multiplicative ("cycle of fifths") operations TnM and TnMI to those in SG2 for a total of
forty-eight twelve-tone operators (TTOs). Two variations on SG3 are presented as well: SG3a, which employs Tn and TnM, and SG3b, which employs Tn and TnMI. SG iv determines sc equivalence on the basis of identical ic vectors; this erases some of the Z relations in SG2. SG iv has no canonical group, instead declaring sc equivalence on the basis of shared ic vectors. Two last SG’s are SGH, which employs a TTO subgroup H as its canonical group, and SGXH, which employs both H and a nonstandard operator X. The motivation for positing these SG systems is musical and context-sensitive in nature: a given composition or repertory may be populated by pcsets that relate most convincingly under a given SG, which may not be the commonplace SG2.

[8] The second section of Chapter 3, on partial orderings, includes especially useful material on lattice representations of maxpoint sets and transpositional combination sets, as well as Morris’s complement union property (CUP). Morris defines CUP as follows: "Given pcsets S, T, and V, such that S is a member of SC(X), T is a member of SC(Y), and V is a member of SC(Z), if S and T share no pcs, and the union of S and T is V, for all S, T, and V, then SC(Z) has CUP" (p. 71). (I have substituted the words "is a member of" for the inclusion symbol, "share no pcs" for the intersection and null set symbols, and "the union of" for the union symbol.) For example, 6-14[013458] has CUP: any two non-overlapping members of 3-6[024] and 3-12[048] will form a member of 6-14. Morris also discusses the complement intersection property (CIP) and CUP pairs. A CUP pair is a CUP shared by two scs. For instance, any two non-overlapping members of ic3 and ic6 will form a member of 4-15[0146] or 4-29[0137]. While Morris does not state it explicitly, it is important to note that CUP assumes SG2, and that the hexachords in Appendix A that are shown to possess CUP have been determined in that way. In a different SG, different scs will have CUP. For instance, the aforementioned 4-15 and 4-29 are CUP pairs in SG2; in SG3, however, they merge into a single sc that has CUP. The analytic usefulness of CUP, CIP, and CUP pairs is considerable. Numerous passages in the post-tonal literature can be understood as the combinations of a pcset of a sc X with most or all of the non-intersecting pcsets of a sc Y to form members of a single, larger sc Z; this is the CUP relation. Other passages of similar bent may be modeled by CUP pairs. There are also open research questions involving CUP. For instance, any all-combinatorial hexachord can be formed by transpositional combination but none has CUP, a somewhat surprising fact given the symmetry that TC and CUP scs and their subsets typically exhibit.

[9] The final section of Chapter 3 covers similarity relations among scs. The section, while explicitly based on Morris (1980), engages more recent work on similarity relations as well. A fine discussion of aural similarity (pp. 73-5) deals with three main problems that arise with the use of these relations: diversity (there are many types of listeners), type-token (there are conflicts between measuring similarity among pcsets versus the scs they belong to), and realization (there are many ways to realize a pcset in pitch, timbre, etc.). An interesting connection between similarity relations and the folding operation original with Jonathan Bernard (1987) closes the chapter.

[10] Chapter 4, "Aspects of TTOs," revisits material in Chapter 4 of CwPC and Morris (1982, 1990), then extends it to cover more recent constructs. Here Morris proposes an alternate labeling system for TTOs. All TTOs take the form TnMm, where m (multiplication) = 1, 5, 7, or 11. M1 is the identity operator (the traditional T0), M5 is M (the traditional T0M), M7 is MI (the traditional T0MI), and M11 is I (the traditional T0I). For example, the traditional T0 is notated as
T0M1, and the traditional T5I is notated as T5M11. The TnMm labeling format is essential for the work carried out in this chapter, which places the M operations on equal footing with the T and I operators.

[11] After presenting the formalisms necessary for the concatenation of TTOs, Morris introduces Lewin’s simply transitive networks, which in turn sets the stage for a discussion of generalized interval systems in Chapter 5. The discussion of context-sensitive operations is of pertinence to all analysts of atonal and twelve-tone music, even undergraduates encountering hexachordal combinatoriality for the first time. Consider the opening row of Schoenberg’s Fourth Quartet, T2(P) = <219A5340876B>. The hexachordally combinatorial partner of T2(P) is T9I(P) = <780B4659123A>. Although T9I(P) is the T7I transform of T2(P), all combinatorial row pairs in the piece do not relate by T7I. There are two solutions to this problem. The first involves Morris’s Theorem 4.7b (p. 93), which calculates the transposition of inversion operators via the formula H K H-1 (the inverse of H). Let us determine the combinatorial partner of T0(P) with this formula. T0(P) is the TA transform of the opening T2(P). The opening T9I(P) is "transposed" by TA via TA T9I T2 = T7 I T2 = T7 IT2 = T7 TAI = T5I. T0(P) and T5I(P) are thus hexachordally combinatorial. The second solution to the problem, a context-sensitive operation Jn (Lewin 1993), accomplishes the same task with less overhead. As Morris states, "keeping track of transpositional subscripts is painful" (p. 98), and the Jn operators accomplish this more easily, with the additional advantage of commuting with the Tn operators.

[12] Chapter 4 closes with a far-reaching generalization of K-nets. By employing the concept of operator spaces, Morris is able to develop algorithms that preserve the node/arrow content of any K-net. An operator space is a complete statement of a T1, T5, T7, and/or T11 cycle. The cycles are labeled C1, C5, C7, and C11. Below is an example of a four-row operator space:

\[
\begin{align*}
C1 & = 0 1 2 3 4 5 6 7 8 9 A B \\
C5 & = 0 5 A 3 8 1 6 B 4 9 2 7 \\
C7 & = 0 7 2 9 4 B 6 1 8 3 A 5 \\
C11 & = 0 B A 9 8 7 6 5 4 3 2 1
\end{align*}
\]

The relation between operator cycles and K-nets also engages the familiar Stravinskian rotation operation rn(X), which places the last n elements of X first. Consider the two-row operator space shown below:

\[
\begin{align*}
C1 & = 0 1 2 3 4 5 6 7 8 9 A B \\
r2 & C11 = 2 1 0 B A 9 8 7 6 5 4 3
\end{align*}
\]

The pcs of the operator cycles model the pcs of a K-net, while TTOs relate pcs. Operator spaces and their rotations can also form George Perle’s (1977) cyclic sets (p. 98). In Perle’s terminology, the vertical dyads of the two-row operator space shown above form "sum 2 dyads"-pcs whose integers sum to 2 modulo 12.

[13] Chapter 5, "The TTO Group: Its Subgroups and Supergroups," revisits topics in Morris (1982, 1987, 1990), but with different terminology and in greater depth. The first section of the chapter considers subgroups of the TTO group. The orbits of these subgroups—the pc mappings under the operations of the subgroup—are put to practical use in demonstrations of pc-to-pc
designs that comport with the arrays in $CwPC$. The section closes by outlining the automorphisms and isomorphisms of these subgroups, along with the conjugacy classes they sort into, by way of an interesting compositional application.

[14] The second section of Chapter 5 considers the TTO group as a subgroup of a supergroup $N$. $N$ is defined by five operations, alpha through epsilon, which engage material in Mead (1988-89) and Morris (1982). Entire SGs may relate via the nonstandard operations in a given $N$. The chapter concludes with a discussion of Lewin’s generalized interval systems, which Morris offers "as an invitation to begin reading Lewin (1987)."

[15] Chapter 6, "Twelve-Tone Topics," discusses types of rows and their relations, pc binary relations, partitions and mosaics, and pc/order-number isomorphisms. Row types covered include all-combinatorial, all-interval, ten-trichord, supersaturated set-type, and multiple order-number function. Row-classes are then covered; these are to rows what scs are to pcsets. Lewin’s protocol pairs (equivalent to the Cartesian product of a set $S$) and partially-ordered sets (1976, 1987) are instances of pc binary relations, which form the third section of the chapter. The section on Latin squares (pp. 182-84) is among the few published musical discussions of the topic that does not take Webern’s music as its starting point. Such discussions are typically analytic in nature; Morris instead demonstrates the ability of Latin squares to undergird combinatoriality and mosaics.

[16] To be sure, the material in Class Notes for Advanced Atonal Music Theory is genuinely advanced. For pedagogical purposes, Class Notes is best suited to a research seminar or second semester graduate course in atonal theory. As Morris states (p. x), students with little or no background will be better served by Rahn (1980), Straus (2000), or Morris (1991). A second aspect of Class Notes that may present difficulties to some readers is the absence of musical examples. My experience with Class Notes, however, has been that the absence of musical examples promotes the use of Morris’s material in analytic work without prejudice. For instance, supersaturated set-type rows crop up in Elliott Carter’s Changes, a non-twelve-tone piece; instances of CUP and pitch-scs occur throughout Schoenberg’s Suite, Opus 25; and partial orderings, which Morris implicitly presents as manifested in temporal order, can model vertical sonorities as well.

[17] As a research guide, Class Notes for Advanced Atonal Music Theory is a state-of-the-art contribution to the field of atonal music theory. It is informed by the most current work in the field, its organization and manner of presentation are exemplary, and the text is nearly free of significant typographical errors. Few music texts enjoy the luxury of a six-year gestation period during which drafts are test-run in class after class of graduate students; Class Notes for Advanced Atonal Music Theory bears all the positive marks of such a text.

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