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Preservice elementary teachers' performance on tasks involving building, interpreting, and using linear mathematical models based on scientific data as a function of data collection activities

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PRESERVICE ELEMENTARY TEACHERS' PERFORMANCE ON TASKS INVOLVING BUILDING, INTERPRETING, AND USING LINEAR MATHEMATICAL MODELS BASED ON SCIENTIFIC DATA AS A FUNCTION OF DATA COLLECTION ACTIVITIES

by

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APPROVAL PAGE

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BOWMAN, ANITA HILL. Ph.D. Preservice Elementary Teachers' Performance on Tasks Involving Building, Interpreting, and Using Linear Mathematical Models Based on Scientific Data as a Function of Data Collection Activities. (1993)
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A modification of Janvier's "star" model of understanding mathematical function is proposed as a theoretical basis for framing this experimental study of the relationship between preservice elementary teachers' performance on tasks involving building, interpreting, and using linear mathematical models based on physical science data and whether or not the subject participated in data collection tasks. Fifty-two elementary education majors enrolled at a small university in the southeastern region of the United States participated in this experiment by completing two 2-hour workshops and a 50-minute, 36-item posttest. Twenty-seven subjects were randomly assigned to the "data collection" group and 25 to the "no data collection" group. All participants used TI-81 graphing calculators to analyze the relationships between four pairs of variables: (a) total mass of a liquid and its container (Y) versus the volume of liquid used (X), (b) total height from the table top to the water level in a beaker (Y) versus the volume of water in the beaker (X), (c) total mass of coins and the cup containing the coins (Y) versus the number of coins in the cup (X), and (d) the length of a spring (Y) versus the total mass of objects attached to the spring (X). Data analysis via TI-81 calculators included entering data from tables, constructing scatter plots, and determining the least squares linear regression model. For each mathematical model constructed, subjects identified the slope and y-intercept, including units of measure; constructed a contextual interpretation of the slope and y-intercept; and solved verbal problems using the model to predict outcomes.

A series of two-sample t-tests were used to analyze the results obtained on the posttest. Subjects who did not engage in data collection activities scored higher on the posttest and on all 36 individual test items than did the students who engaged in the data collection activities. The results indicate that data collection activities interfere with, rather than enhance, performance on tasks involving building, interpreting, and using linear mathematical models. The results are interpreted within the theoretical framework provided by a pentagonal modification of Janvier's "star" model for understanding mathematical function.

CHAPTER I

INTRODUCTION

Leinhardt, Zaslavsky, and Stein (1990) present a comprehensive research review in which they analyze "interpretation and construction tasks associated with functions and some of their representations: algebraic, tabular, and graphical" (p. 1). In particular, the review focuses on understandings children ages 9 to 14 have of functions and their graphs. One issue raised in the review is the difference in directionality of thinking about functions from mathematical and scientific perspectives.

> The bridge between functions and graphs is . . . interesting because the intellectual landscape, so to speak, looks different from each side of the bridge--if graphs are used to explicate functions, the sense of function (and graph) is quite different from what is presented the other way around. Indeed, part of the problem with graphing as a scientific tool resides in this issue of directionality of thinking about graphs and functions. The mathematical presentation is usually from an algebraic function rule to ordered pairs to a graph, or from a data table of ordered pairs to a graph. The scientific presentation, on the other hand, most often proceeds from observation, to data array, to ordered pairs of data, to selection of axis labels, to scale construction, to graph and (maybe) to function. Often, students who can solve graphing or function problems in mathematics seem to be unable to access their knowledge in science. It is only recently with the insights of cognitive science that we are beginning to learn why the truism holds: Just because learners know something in one way does not mean that they can make immediate use of it from a different perspective or in a different situation. Because of its presence and its distinctive character in both mathematics and science, functions and graphs provides an excellent topic to examine in this respect. (p. 3)

This research project evolved from a recognition that, in general, college students have difficulties transferring knowledge of mathematical function developed in algebra classes to data analysis tasks in a science laboratory. Differences in directionality between mathematical and scientific perspectives may account for part of this difficulty. However, another important factor may be context. Mathematically, the meaning of a function of two variables, expressed either in algebraic formula or graphical form, is generally confined to the context of the Cartesian plane. Within a scientific perspective, a function expresses a mathematical relationship between two real-world variables, and the meaning of a function is generally related to the physical situation from which the function is derived.

For example, consider a linear mathematical function in the form $y = mx + b$: $y = 1.13\,x + 45.0$. Mathematically, the interpretation of the relationship between x and y focuses on what the slope ($m = 1.13$) and the y-intercept ($b = 45.0$) tell us about the graph of the function. Generally, the slope is interpreted as "the rise over the run" or "the change in y with change in x," and the y-intercept is interpreted as "where the line crosses the y-axis." The roles of x and y in the linear equation are rarely articulated. Thus, within a mathematical perspective, students typically interpret the function expressed by the algebraic formula $y = 1.13\,x + 45.0$ in terms of its graph. The expected interpretation is something like the following: "The graph is a straight line which crosses the y-axis at 45.0 and goes up 1.13 units each time it goes one unit to the right."

If this same algebraic formula is considered within the context of a physical situation, different interpretations may result. For example, the algebraic formula

$y = 1.13x + 45.0$ may be the result of a linear regression least-squares fit on a set of mass versus volume data collected in a laboratory. Assume that observations were recorded in a data table according to the following procedure:

1. Approximately 2.5 milliliters of antifreeze was poured into a 25-milliliter graduated cylinder.

2. The graduated cylinder was placed on a balance platform, and the total mass in grams was recorded as a y-value in a data table. The volume was read to the nearest 0.1 milliliter and recorded as the corresponding x-value.

3. More antifreeze was added in approximately 2.5-milliliter aliquots, and the resulting mass and volume were recorded in the data table following each addition.

Assume further that a linear regression fit on the data pairs yielded the equation $y = 1.13x + 45.0$.

Within the given physical situation context, a number of interpretive statements may be written:

1. The total mass (y) is equal to the sum of the mass of the antifreeze in the graduated cylinder (1.13 x) and the mass of the empty graduated cylinder (45.0).

2. The mass of the empty graduated cylinder is 45.0 grams.

3. The total mass increases by 1.13 grams for each 1.0-milliliter increase in volume.

4. One milliliter of antifreeze "weighs" 1.13 grams.

5. The density of antifreeze is 1.13 grams per milliliter.

Considering this function from a scientific perspective gives a different view of the relationship between x and y than that obtained from a mathematical perspective.

Thus, as illustrated by this example, one prominent difference between interpretations of linear functions from mathematical and scientific perspectives lies in the contexts in which the interpretations are framed. The context for a mathematical interpretation is generally the Cartesian plane. Within this graphical space, the focus of interpretation is on what the equation parameters, slope (m) and y-intercept (b), tell us about the graph of the function. Generally, the roles of the variables, x and y, are not emphasized. In contrast, the context for a scientific interpretation is generally the physical situation in which the relationship between the two variables, x and y, is explored. Within this situational space, the focus of interpretation is on the physical relationship between the two variables in question, and the values of the parameters serve to quantify the relationship.

This example also serves as an illustration of the complexity of the function concept. Four factors contribute to this complexity. First, a number of different representations may be used to define a function. Possible representations include algebraic formulas, graphs, tables, sets of ordered pairs, mapping diagrams, situations, and verbal descriptions. A given function "looks" different in each of its representations. Second, a wide variety of processes are involved in translating between pairs of representations. Each translation process may be perceived by students to be separate and unrelated to other translation processes. Third, the concept of function is further complicated because both the symbol systems and the language used in describing a function change according to the representational mode employed. Lastly, no definition of function exists separate from its representations. The concept of function must therefore be constructed inductively by synthesis of numerous and varied instantiations of function.

While the function concept is in many ways the most difficult concept encountered in mathematics, it is also potentially one of the most useful. Thorpe (1989) suggests that we should "not just teach about functions in algebra," but rather, we should "make functions the centerpiece of algebra instruction" (p. 18). As an extension of Thorpe's suggestion, I propose it would be appropriate to make functions globally the centerpiece for mathematics instruction, throughout the K-14 curriculum. This proposition is consistent with the mathematics curriculum reform standards prepared by the National Council of Teachers of Mathematics (NCTM, 1989), and, more particularly, with the standards on function and statistics. Taken together, the sets of standards on function and statistics stress the importance of learning mathematical function from both mathematical and scientific perspectives.

Problems associated with interrelating the aspects of function as presented within a mathematical perspective to the scientific perspective of relationships between real-world variables are due in part to the complexity of the function concept. Since the complex nature of mathematical function cannot be changed, it is important that instruction be designed so students see that the various aspects of what it means to be a function are part of a whole rather than isolated and unrelated ideas. That is, instruction should be designed to help students make connections among various facets of mathematical function. Specifically, for college-level students to make the connections required to bring the various notions of function together into a unified concept of function, instruction might need to be structured so that students are challenged to view the notion of function from both scientific and mathematical perspectives simultaneously.

This study focused on a specific group of college-level students, preservice elementary teachers, and specific content, linear mathematical models based on data collected using a scientific inquiry approach. Treatment activities were designed to provide a rich environment for students to connect notions of linear function previously learned from a mathematical perspective to linear mathematical models developed in scientific contexts. A modification of Janvier's (1987b) "Star" model for function understanding provided a theoretical framework for the study.

CHAPTER II

REVIEW OF THE LITERATURE

Several key ideas were important in framing this study: (a) various conceptualizations of mathematical function, (b) current mathematics and science curriculum reform movements, (c) mathematical function via modeling, applications, and links to science, (d) elementary teacher education, and (e) linear mathematical functions. The organization of this chapter is intended to take the reader from a broad view of mathematical function based on concept definitions and representations to a more limited scientific perspective view to a very narrow view of linear mathematical functions within a scientific perspective, while, at the same time, relating mathematical function to current curricular reform movements in mathematics and science and to associated issues in elementary teacher education. The literature reviewed in this chapter and the theoretical model developed in Chapter III constitute a framework for the study.

The literature on teaching and learning the concept of mathematical function is broad. No attempt is made in this treatment to summarize the literature globally. Instead, this treatment is restricted to consideration of function via mathematical and scientific perspectives and to the implications teaching function from a scientific perspective has on elementary teacher education. The chapter concludes with a rationale for the study.

Mathematical Function

In this section two conceptualizations of mathematical function are discussed. The first is the set-theoretical definition which serves as a cornerstone for the study of mathematical function from a mathematical perspective. The second is a scientific conceptualization of mathematical function. From a scientific perspective, a function is viewed in terms of relationships among variables. Issues related to concept definition and concept image are also included. The section concludes with an introduction to external representations for mathematical function.

Concept Definition and Concept Image

The notion of mathematical function has evolved as mathematicians have defined new systems and discovered new relationships within these systems. Along with these developments, the definition of mathematical function has similarly evolved. In mathematics the generally accepted definition of function, called the Dirichlet-Bourbaki concept of function (Vinner & Dreyfus, 1989), characterizes a function as a correspondence between two non-empty sets A and B that assigns to each element in A one and only one element in B. The beauty of this definition is its inclusiveness. As Tall (1992) notes, "the sets involved may be sets of numbers, or points in n-dimensional space, or geometric shapes, or matrices, or any other type of object, including other functions, and the method of assignment might be through a formula, an iterative or recursive process, a geometric transformation, a list of values, or any serendipitous combination one desires, provided that it satisfies the criterion of assigning elements uniquely" (p. 497).

Even though the Dirichlet-Bourbaki concept definition of function may be considered a good definition from a mathematician's point of view, it may not be a good definition for learners. Several decades ago Poincare (1914) challenged his readers to think about the aspects that make a definition good: "What is a good definition? For the philosopher or the scientist, it is a definition which applies to all the objects to be defined, and applies only to them; it is that which satisfies the rules of logic. But in education it is not that; it is the one that can be understood by the pupils" (p. 117). The Dirichlet-Bourbaki concept definition may be a good definition for mathematicians because it is succinct and all-encompassing, but it may not be helpful for learners because the nuances associated with the definition may not be easily seen and understood. For example, learners may understand this definition when applied to examples expressed as mappings from one set to another, but may not see that the definition also pertains to function examples framed within other representations, such as graphs and algebraic formulas. The Dirichlet-Bourbaki concept definition is rich with meaning, but "unpacking" the definition to reveal a wealth of implicit meanings may be a difficult cognitive task for learners.

Research has shown that students are creative in avoiding the "unpacking" process (Tall & Vinner, 1981; Vinner & Dreyfus, 1989). Although students' first encounter with the concept of function might be via function definition, they generally approach the task of classifying items as examples or nonexamples of function via concept images. A concept image may be composed of mental pictures, properties, mental representations, or contexts of applications. Concept images are shaped through everyday experiences with examples. When the scope of example types is somewhat restricted, students' concept images may actually distort their interpretations of the definition.

What alternatives exist to introducing mathematical function by definition, a practice common in traditional introductory algebra courses? Sfard (1992) suggested two didactic principles related to presenting concepts in structured form: (a) "New concepts should not be introduced in structured terms" and (b) "A structured conception should not be required as long as the student can do without it" (p. 69). Perhaps a key to helping students develop a robust understanding of mathematical function is to design instruction so that students encounter a wide variety of example types before any attempt to define the concept formally. Then the task might become one of "packing" an array of images into a conceptually tight package. It seems reasonable that "packing" known pieces via summarizing is cognitively easier than "unpacking" unknown pieces.

If this "packing" approach is employed, then how and when should instruction in mathematical function begin? From research on both children and Brazilian street people, Resnick (1992) provided what may be a somewhat extreme view that relates directly to this question. She concluded that pre-school children and minimally schooled adults seem to possess substantial amounts of mathematical knowledge. In the case of essentially untutored street people in northeast Brazil who scrounge a living by selling lottery tickets, she found what she considered to be extraordinary arithmetic competence in their flexibility, ability to invent new methods, and the extent to which their problem-solving can be shown to reflect fundamental algebraic principles, such as commutativity, associativity, and distributivity. Analogous results were obtained with children. Stripping away surface details from both groups uncovered a fundamental mathematics structure, the operations of which neither group could reason about nor abstract from. Based on this research, Resnick proposed a reasoning-based arithmetic program which would increase children' trust in their own mathematical abilities by introducing them to formal systems, such as writing mathematical equations, within weeks after starting school, thereby providing them with a basis from which to reason.

Her goal is to "eventually provide all students with true mathematical power--the ability to do algebra, to mathematize a situation, to enter a formal system and then come back out" (p. 43). This same goal may be applied to learning mathematical function. When viewed from a scientific perspective, learning the function concept involves the three steps Resnick identified: mathematizing a situation, entering a formal system, and then coming back out are precisely the steps involved in a scientific inquiry approach to learning science.

Consider a conception of function which is consistent with both Resnick's view and a scientific inquiry approach to science. Sierpinska (1988) suggested that "the most fundamental conception of a function is that of a relationship between variable magnitudes" (p. 572). She later (1992) emphasized the importance of framing students' early experiences with function within the context of relationships between variables.

> Maybe, in teaching, functions should first appear as models of relationships. This is how they came into being in history. They were tools for description and prediction. If we assume that the meaning of a concept lies in the problems and questions that gave birth to it, and we wish that our students grasp the meaning of the notion of function, then this seems to be a quite reasonable claim to make. This does happen in some textbooks. But more often than not the order is reversed: relationships between variable magnitudes are presented as mere illustrations of mathematical functions and the former are so prepared, so idealized that they are almost identical with functions that pretend to be their models. And even if functions do appear as models of some relationship discovered by experience, the latter are idealized to the point of completely distorting their image in the minds of students. Not only discrete sets of data are joined by continuous lines but these points seem to fall exactly on the line. Simplification leads to absurdities such as representing the growth of a population of bacteria in culture by the function $f(n) = 2^n$. This kind of pedagogy may make it difficult for the students to distinguish between relationships discovered by experience and the mathematical models of these. (p. 32)

Sierpinska's view that functions should first appear as models of relationships is consistent with Resnick's goal of true mathematical power for students.

It appears reasonable to suggest that the structure for beginning a study of mathematical function is already in place in the elementary school, specifically within the area of science process skills in the science curriculum. The elementary school science curriculum stresses both the product and the process of science. Science process skills are related to a scientific inquiry approach to teaching and learning science. One part of the scientific inquiry approach is the scientific method. DeBruin (1991) gave a list of typical steps featured in the scientific method: purpose, hypothesis, materials, procedures, collect data from trials and tests, results, and conclusions. In Resnick's terms, purpose, hypothesis, materials, and procedures may be seen as facets of a situation. Collecting data from trials and tests begins a mathematization of the situation. Results may be obtained by using the collected data, in tabular form, to graph and then to fit a mathematical model in the form of an equation. This step constitutes further mathematization and entrance into formal systems of graphing and writing formulas. Conclusions may be obtained by translating the mathematical model into a verbal description of the relationship between the variables under consideration. This verbalization step involves coming back out of formal mathematical systems. Thus, the goal of scientific inquiry, to begin with a situation and to end with a verbal description of the relationship between variables of interest, may be accomplished by way of the steps identified by Resnick. The resulting mathematical model is an example of a fundamental conceptualization of function as described by Sierpinska.

Representational Systems for Mathematical Function

The scientific inquiry approach just described incorporates five external representations of mathematical function: situation, table, graph, algebraic formula, and verbal description. Janvier (1987b) included these representations in his "star" model for function understanding. These representations and processes associated with translating between pairs of representations are crucial to framing this study. External representations for mathematical function are discussed fully in the next chapter. An extension of Janvier's model, presented in Chapter III, constitutes a theoretical framework for the study.

Curriculum Reform

The past decade has seen a tremendous increase in activity toward effecting curriculum reform in both mathematics and science. Collectively, these calls for reform have addressed all levels of formal education from kindergarten through post-baccalaureate studies. In general, movements to reform mathematics curricula have progressed further than the corresponding movements in science. This section focuses on the main aspects of reform movements in both mathematics and science that relate to mathematical function, especially with respect to a scientific inquiry approach to learning.

Mathematics Reform

The National Council of Teachers of Mathematics (NCTM, 1989) published a set of curriculum and evaluation standards designed to establish a broad framework to guide reform in K-12 mathematics. The Standards were developed in response to a growing call for reform in the teaching and learning of mathematics (Conference Board of the Mathematical Sciences, 1983a; Conference Board of the Mathematical Sciences, 1983b; National Commission on Excellence in Education, 1983; National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1983; Romberg, 1984). Central to the Standards is the concept of developing mathematical power in all students, a theme echoed by the National Research Council (NRC, 1989). It is this theme to which Resnick (NSF, 1992) was referring when she described true mathematical power as "the ability to do algebra, to mathematize a situation, to enter a formal system and then come back out" (p. 43).

Several aspects of the Standards are pertinent to this study. At the K-4 level, the Standards point to increased attention to "collection and organization of data" within the probability and statistics strand and "use of variables to express relationships" within the patterns and relationships strand (p. 20). Corresponding changes at the 5-8 level include increased attention to "representing situations verbally, numerically, graphically, geometrically, or symbolically" within the problem solving strand; "connecting mathematics to other subjects and to the world outside the classroom" within the connections strand; "developing and using tables, graphs, and rules to describe situations" and "interpreting among different mathematical representations" within the patterns and functions strand; "using statistical methods to describe, analyze, evaluate, and make decisions" within the statistics strand; and "creating experimental and theoretical models of situations involving probabilities" within the probability strand (p. 70). All of these content areas may be incorporated in a scientific inquiry approach to the study of functions.

Other mathematics reform reports have implicitly supported the notion that instruction needs to be directed toward helping students connect the notions of function from a scientific perspective to the corresponding notions of function from a mathematical perspective. The NRC (1989), in stressing the development of mathematical power for all students, points to mathematics as providing tools for revealing hidden patterns that in turn help us understand the world around us:

> Now much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements, and observations from science; with inference, deduction, and proof; and with mathematical models of natural phenomena, of human behavior, and of social systems. . . . In addition to theorems and theories, mathematics offers distinctive modes of thought which are both versatile and powerful, including modeling, abstraction, optimization, logical analysis, inference from data, and use of symbols. Experience with mathematical modes of thought builds mathematical power--a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives. Mathematics empowers us to understand better the information-laden world in which we live. (pp. 31-32)

This view of mathematics as a toolkit for understanding the world in which we live is strongly reflected in the curriculum standards developed by the NCTM (1989).

Science Reform

According to Shymansky and Kyle (1992), the common goal for reform efforts in science curricula is "to ensure a scientifically literate citizenry for the 21st century" (p. 745). The current reform movement began in response to a variety of reports (National Commission on Excellence in Education, 1983; National Science Board, 1983). More recent reports (National Governors' Association Task Force on Education, 1990; U.S. Department of Education, 1991) have intensified the push to establish a new national set of science curricula. Thus far, a document paralleling the

Curriculum and Evaluation Standards (NCTM, 1989) has not been produced for guiding reform in K-12 science curricula.

Linn (1992) summarized the current status of the reform movement: "Those concerned with science education are united in calling for reform, yet divided in specifying the nature of this reform" (p. 821). A major concern is that students have difficulty applying abstract scientific principles to the complex phenomena they encounter in their lives. There are two schools of thought on how to help students overcome this difficulty. Proponents of the first school take the position that if you want students to know something then you must tell them what you want them to know (Hazen & Trefil, 1991). Proponents of the second school claim that students come to a better understanding of science concepts and the nature of science by grappling with ideas of science themselves. The research base on learning science suggests that instruction is more effective when students are helped to construct ideas by themselves (Collins, Brown, & Newman, 1989; Gabel, 1989), supporting the views of proponents of the second school. It is interesting that this constructivist view, which forms the backbone of mathematics curriculum reform, is still being debated by those working on science curriculum reform.

The constructivist view, applied to science curricula, supports learning about both science concepts and the nature of science simultaneously by active involvement in scientific exploration and experimentation. The constructivists' call for involvement of the learner in doing science is reminiscent of the earlier science reform movement of the 1960s. At the elementary school level, one might identify two main differences. First, the current emphasis is less on having students rediscover known scientific principles and more on providing opportunities for students to create their own understandings of scientific principles by constructing meanings from their experiences.

That is, the emphasis of discovery learning in the 1960s was on science content, while the current constructivist approach focuses on cognitive processes important to learning science. Secondly, the availability of computers and calculators in the classroom has made it feasible for students to manipulate data accumulated through scientific inquiry without knowledge of advanced mathematical concepts required for summarizing data by hand. Thus, at a young age, students might be involved in drawing valid conclusions from data. Participation in the scientific inquiry process also affords elementary school students the opportunity to experience facets of the mathematical function concept years before they begin a formal study of algebra.

Modeling, Applications, and Links to Science

This section summarizes some of Blum and Niss' (1991) major ideas about the state of trends and issues in mathematics instruction related to applied mathematical problem solving, modeling, applications, and links to other subjects. In particular, the issues considered within this section include (a) purposes for and organizations of mathematics instruction, (b) obstacles to including applied mathematical problem solving, modeling, applications, and links to other subjects in mathematics instruction, (c) approaches to including these topics in mathematics instruction, (d) the role of technology, and (e) implications for the elementary school.

It may be helpful at this point to distinguish between the mathematical and scientific perspectives of mathematical function based on instructional objectives within the two disciplines. Within a scientific perspective, the goal of mathematical modeling is to describe the relationships that exist between real-world variables, in hopes that the descriptions will provide a basis for prediction and perhaps even control. Within a school setting, a scientific inquiry approach helps students develop a sense of the nature of science and provides them with the opportunity to actively participate in creating (or re-creating) science content by drawing conclusions from data. Within a mathematical perspective, the school purpose is to develop an understanding of the processes involved in mathematical modeling, including processes associated with various representations of mathematical function.

Mathematics Instruction: Purpose and Organization

Blum and Niss (1991) identified two distinctly different purposes for mathematical instruction.

> 1. To provide students with knowledge and abilities concerning mathematics as a subject in itself.
> 2. To provide students with knowledge and abilities concerning (one or more) other subjects, to which mathematics is supposed to have actual or potential services to offer. (p. 41)

They also defined two organizational frameworks which might be used to frame mathematics instruction.

> 1. Mathematics may be taught as a separate subject, i.e. as an independent organizational unit called "mathematics" or something like that.
> 2. Mathematics may be taught as a part of and integrated in (one or more) other subjects. (p. 41)

They then related the purposes and organizational frameworks using a 2 x 2 matrix, treating integration with other subject areas, inclusively. By restricting the scope of integration with other subjects to science, their treatment reduces to that shown in Figure 1.

Purpose / Organization	Focus on Mathematics	Focus on Science
Mathematics as a Separate Subject	Purpose #1 Organization #1 (a)	Purpose #2 Organization #1 (b)
Mathematics Integrated in Science	Purpose #1 Organization #2 (c)	Purpose #2 Organization #2 (d)

Figure 1. Modification of Blum and Niss' Purpose versus Organization Matrix for Mathematics Instruction

The four situations of mathematics instruction represented in Figure 1 may be related to educational practices. Situation (a) is commonly encountered in school mathematics at elementary, middle, and secondary levels and in university mathematics courses for future mathematicians or mathematics teachers. Situation (b) is encountered in university mathematics courses taught as service courses for future scientists and engineers; mathematics courses in vocational education; and, partly, in school mathematics at elementary, middle, and secondary levels. Situation (c) is rarely found in practice. Situation (d) is encountered in mathematics courses in vocational education and, partly, in university mathematics courses taught as service courses for future scientists and engineers. A scientific inquiry approach, as used in the elementary school, is best described as an example of situation (d) mathematics instruction.

Obstacles to Including Modeling and Application in Mathematics Instruction

Obstacles to including modeling and application in mathematics instruction may be classified according to point of view. From the point of view of instruction, many mathematics teachers, at all levels, are concerned that they do not have enough instructional time to deal with problem solving, modeling, and applications in addition to all the other mathematics content they think important to teach. Furthermore, some teachers contend that applications and connections to other subjects do not belong in the area of mathematics instruction. From the learner's point of view, problem solving, modeling, and applications are far more intellectually demanding than routine mathematical tasks such as calculations. It is particularly difficult to introduce these topics to older students who have already been conditioned to a more procedural approach to learning mathematics. Lastly, from the teacher's point of view, problem solving and applications make planning and implementing instruction more demanding, while, at the same time, requiring more attention to alternative approaches to assessing students' achievement levels. In addition, teachers may not feel comfortable trying to incorporate examples from subjects areas which they have not studied.

Approaches to Including Modeling and Applications

Blum and Niss (1991) classified six types of approaches to teaching problem solving, modeling, and applications within mathematics instruction.

> The separation approach. Instead of including modeling and applications work in the ordinary mathematics courses, such activities are cultivated in separate courses devoted to them. In this way, the "pure" mathematics courses may remain unaffected by the introduction of modeling and applications work in the program as a whole.

The two-compartment approach. The mathematics program is divided into two parts. The first part consists of a usual course in "pure" mathematics, whereas the second one deals with one or more "applied" items, utilizing mathematics established in the first part or earlier.

The islands approach. The mathematics program is divided into several segments each organized according to the two-compartment approach. This means that a "pure" mathematics program is interrupted by "islands" of applicational work, drawing on mathematics developed in the preceding period.

The mixing approach. Frequently in the teaching of mathematics, elements of applications and modeling are invoked to assist the introduction of mathematical concepts etc. Conversely, newly developed mathematical concepts, methods and results are activated towards applicational and modeling situations whenever possible. In this approach, the mathematics to be involved in applications and modeling activities is more or less given from the outset.

The mathematics curriculum integrated approach. Here problems, whether mathematical or applicational, come first and mathematics to deal with them is sought and developed subsequently. In principle the only restriction is that the problems considered lead to mathematics which is relevant to and tractable in the given mathematics curriculum.

The interdisciplinary integrated approach. This approach is largely similar to [the mathematics curriculum integrated approach] but differs from it in that this one operates with a full integration between mathematical and extra-mathematical activities within an interdisciplinary framework where "mathematics" is not organized as a separate subject. (pp. 60-61)

The approaches likely to be used in elementary mathematics instruction are the islands approach, the mixing approach, and the mathematics curriculum integrated approach. An example of the islands approach may be found in elementary mathematics textbook series which "tack on" applications at the end of each instructional unit. Newer textbook series that focus on problem solving using real-world problems use the mixing approach. The mathematics curriculum integrated approach is most likely to be the one used in integrating science and mathematics via a scientific inquiry approach.

The Role of Technology

The availability of computers and calculators for drawing graphs and pictures and for performing numerical and algebraic calculations has opened new possibilities for incorporating modeling, applications, and problem solving in mathematics instruction at all levels. Computers and calculators aid learners in working on more complex applied problems using more realistic data at an earlier age than is possible without technology. Additionally, using technology to handle tedious but routine computational tasks frees the learner to concentrate on the processes of modeling, problem solving, and application. The use of technology potentially leads to an instructional deemphasis on routine computational skills and an emphasis on problem solving abilities such as building, applying, and interpreting models, experimenting, simulating, algorithmic thinking, and performing computational modeling (Blum & Niss, 1991).

Implications for the Elementary School

The adoption of a constructivistic approach to learning implies new directions for elementary school mathematics. From a constructivist's view, mastering elementary school mathematics is not equated with mastering a set of mathematical facts. Instead, the mastering of a set of mathematical processes, all related to problem solving, is viewed as crucial for concept development. Thus, within a constructivist approach, solving problems framed in real-world contexts should form an essential part of mathematics instruction in the elementary school. In particular, investigations of relationships between real-world variables may provide important natural opportunities for developing problem-solving processes.

Because of the current availability of computers and calculators, many processes related to problem solving and mathematical modeling are more accessible to elementary students than ever before. Data collected during investigations of relationships between real-world variables may be analyzed using computers or calculators. In particular, technology may be utilized by students to construct mathematical models even if students do not know all of the underlying mathematical processes involved in the analysis techniques.

A crucial consideration associated with implementing an elementary mathematics curriculum based on problem solving, modeling, and applications is teacher preparation. This approach requires a dramatic shift in preservice and inservice elementary teacher education. Teacher education programs must equip teachers with knowledge, abilities, and experiences they will need to cope with the demands of teaching mathematics using applications, modeling, and problem solving. An even more difficult task for teacher education programs might be to equip teachers with associated attitudes toward mathematics teaching and learning. Elementary teacher education is the topic of the next section.

Elementary Teacher Education

National Reforms

NCTM (1991) addresses issues of preservice and inservice teacher education and teacher support associated with implementation of earlier recommendations (NCTM, 1989). One of the professional standards is entitled "Knowing Mathematics and School Mathematics."

The education of teachers of mathematics should develop their knowledge of the content and discourse of mathematics, including--

- mathematical concepts and procedures and the connections among them;
- multiple representations of mathematical concepts and procedures;
- ways to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality;

and, in addition, develop their perspectives on--

- the nature of mathematics, the contributions of different cultures toward the development of mathematics, and the role of mathematics in culture and society;
- the changes in the nature of mathematics and the way we teach, learn, and do mathematics resulting from the availability of technology;
- school mathematics within the discipline of mathematics;
- the changing nature of school mathematics, its relationships to other school subjects, and its applications in society. (p. 132)

The Mathematical Association of America (MAA) built on the base provided by NRC (1989) and NCTM (1989) in considering changes in teacher preparation (Lietzel, 1991). MAA proposed four sets of standards for teacher preparation: (a) standards common to the preparation of mathematics teachers at all levels, (b) standards for the elementary (K-4) level, (c) standards for the middle grades (5-8) level, and (d) standards for the secondary (9-12) level. For this study only the common standards will be discussed.

There are six common standards: learning mathematical ideas, connecting mathematical ideas, communicating mathematical ideas, building mathematical models, using technology, and developing perspectives. The three standards most pertinent to this study are as follows:

Standard 2: Connecting Mathematical Ideas

The mathematical preparation of teachers must provide experiences in which they:

- develop an understanding of the interrelationships within mathematics and an appreciation of its unity;
- explore the connections that exist between mathematics and other disciplines;
- apply mathematics learned in one context to the solution of problems in other contexts. (p. 3)

Standard 4: Building Mathematical Models

The mathematical preparation of teachers must include experiences that enable, motivate, and encourage them to analyze real-world situations through the use of whatever mathematical ideas or quantitative strategies are available. In particular, they should be able to:

- work with a given model;
- recognize constraints inherent in a given model;
- construct models to analyze real-world settings and use symbols and reasoning in analysis;
- convert among representations (graphical, numerical, symbolic, verbal) that reflect quantitative constraints in a given real-world problem. (p. 6)

Standard 5: Using Technology

The mathematical preparation of teachers must include experiences in which they use calculators and computers:

- as tools to represent mathematical ideas and construct different representations of mathematical concepts;
- to engender a broad array of mathematical modes of thinking through use of powerful computing tools (including function graphers, curve fitters, and symbolic manipulators);
- to develop and use alternate strategies for solving problems. (p. 7)

The three standards incorporate the essential ingredients associated with learning mathematical function within a scientific perspective. If teachers are to implement such instruction in the elementary school, then they must, in some way, develop the associated content knowledge. There are four components of an elementary teacher education program in which students might learn to build, interpret, and use mathematical models based on scientific data: (a) mathematics courses, (b) science courses, (c) mathematics education courses, and (d) science education courses. Traditionally, mathematical modeling is not part of any of these courses. That is, mathematical modeling and mathematical function from a scientific perspective could be placed in all four types of courses, yet generally are omitted from all. A challenge for elementary teacher education programs is to establish a curriculum that incorporates important content on problem solving, applications, and modeling.

Teacher Content Knowledge

Since mathematical modeling has not generally been a part of preservice teacher education, the effects of teachers' limited content knowledge on instruction is an important consideration. Several general studies have delved into the relationship between teacher content knowledge and instruction. Begle (1979) concluded in a review of the literature that "the effects of a teacher's subject matter knowledge . . . seem to be far less powerful than most of us had realized. . . . Our attempts to improve mathematics education would not profit from further studies of teachers" (pp. 54-55). This strand of research remained essentially dormant until Shulman (1986) inspired further work when he identified teacher subject matter knowledge as the "missing paradigm" in research on teaching. More recent research in this area has been based on qualitative research designs. Leinhardt, et al. (1990) expressed the underlying concern of this research in that "limitations on subject matter knowledge might reduce the flexibility and creativity of the teacher as well as create a kind of authoritarianism toward the subject and the student that permits little or no exploration of ideas" (p. 46).

Stein, Baxter, and Leinhardt (1990) concluded that the few studies which have focused on teacher knowledge and elementary mathematics instruction suggest that, even at early grade levels, design and delivery of exemplary lessons demands considerable subject matter expertise.

Most of the studies relating teacher content knowledge to instruction in the area of mathematical function are based on secondary preservice teachers as subjects (Ebert, 1991; Even, 1989, 1990, 1993; Even & Ball, 1989). In a study of secondary preservice teachers, Ebert (1991) found that knowledge of functions and graphs was incomplete and especially fragile in some areas. From a mathematical perspective, her subjects performed well on tasks dealing with linear functions, but confused exponential and quadratic functions. From a scientific perspective, her subjects displayed weaknesses in attaching verbal descriptions to situational graphs. All of her subjects indicated a degree of uncertainty when confronted with a pair of graphs depicting the same situation, i.e., (a) a position versus time graph and (b) a velocity versus time graph. In addition, for all situations involving linear functions, her subjects did not identify the initial value of the dependent variable as the y-intercept on the graph of the function.

Since secondary mathematics teachers are required to complete an undergraduate degree in mathematics and elementary teachers are generally required to complete no more than six to twelve semester hours of undergraduate mathematics, it is expected that elementary teachers would perform at a lower level than secondary teachers on the function and graphing tasks of Ebert's study. Based on the performance of secondary preservice teachers, there is sufficient reason for concern about the content knowledge of elementary preservice teachers.

One research program has been aimed specifically at describing and analyzing the teaching of functions, graphs, and graphing in the elementary grades (Baxter, Leinhardt, & Stein, 1988; Leinhardt, Stein, & Baxter, 1988; Stein, Baxter, & Leinhardt, 1990). For example, Stein, Baxter, and Leinhardt (1990) investigated the relationship between a teacher's knowledge of functions and graphs and his instructional practice. The single subject was an experienced fifth grade teacher who was recommended to the researchers as an excellent mathematics teacher. His content knowledge in the area of functions and graphs was assessed using a card sorting activity and a subject matter knowledge interview. His instructional practice was assessed using videotapes of 25 lessons he presented to his students on functions and graphing. The results indicated that the teacher's weakness in subject matter knowledge had definite negative effects on instructional practice.

> The results suggest that the teacher's knowledge of functions and graphing was missing several key mathematical ideas and that it was not organized in a manner to provide easily accessible, cross representational understanding of the domain. These limitations were found to relate to a narrowing of his instruction in three ways: the lack of provision of groundwork for future learning in this area, overemphasis of limited truths, and missed opportunities for fostering meaningful connections between key concepts and representations. (p. 639)

The 25-lesson presentation of function and graphing videotaped in this study was based solely on mathematical function from a mathematical perspective. Applied problem solving, applications, and modeling were not incorporated in the unit. In fact, this researcher found no research specifically directed at studying elementary teachers' knowledge of mathematical function from a scientific perspective.

A related area of research may be found in the science education literature. Lederman (1992) reviewed research studies on teachers' conceptions of the nature of science. Most studies focused on secondary science teachers' views about scientific knowledge (e.g., Koulaidis & Ogborn, 1989) and knowledge of the history and philosophy of science (e.g., King, 1991). These studies all indicated that teachers have serious misconceptions about the nature of science. Bloom's (1989) study of preservice elementary teachers' conceptions of the nature of science revealed that preservice teachers (a) believe science is people-centered, with its primary purpose being for the benefit of humankind and (b) are confused concerning the meaning and role of scientific theories. Noticeably absent in the science education literature are studies of the effects of elementary teachers' misconceptions concerning the nature of science on (a) teachers' science process skills and (b) teachers' performances in teaching science process skills to elementary students. Such studies would be expected to include teacher knowledge of procedures in scientific methods of inquiry, including drawing valid conclusions from scientific data via data analysis.

Linear Mathematical Function

This project involved treatment activities on building, interpreting, and using linear mathematical models based on data collected using a scientific inquiry approach. The resulting linear functions are idealized forms of the linear models. The distinction between linear models and linear functions, based on common mathematical, scientific, and statistical presentations is the topic of this section. Also included in this section is a discussion of cognitive obstacles that may inhibit students in attempts to construct connections among the three areas of study.

Mathematical, Scientific, and Statistical Presentations

The slope-intercept form for a linear function is generally given in algebra texts as $y = m x + b$, where m is the slope and b is the y-intercept. Mathematically, a function expressed in algebraic formula notation is a deterministic representation. From a scientific perspective, a function is an idealized summary of scientific data, and is commonly referred to as a mathematical model rather than function in order to emphasize the probabilistic nature of equations derived from scientific data. Statisticians make a distinction between a model and the corresponding function. In cases where a set of two-variable data appears linear, a statistical model may be written in the form $Y_i = a + b X_i + e_i$, where Y represents the dependent variable and X represents the independent variable. In this case e_i represents the deviation of the ith observation (X_i, Y_i) from the idealized "best fit" equation given by $Y = a + b X$. Statisticians refer to this latter equation as the statistical function corresponding to the given model. The differences in definitions of model and function from the three perspectives tend to inhibit students' forming connections among the three subject areas.

The task of connecting facets of mathematical function from mathematics, science, and statistics may be further complicated by differences in notation used in the three disciplines. Also, differences in interpretational emphasis associated with the notational differences may cause problems. For example, consider the linear function (model) discussed in Chapter I: $y = 1.13 x + 45.0$. This equation, as written, is in the usual mathematical form. Mathematically, the emphasis is on what the slope, 1.13, and the y-intercept, 45.0, reveal about the graph of the function in the Cartesian plane. The scientific emphasis is on how the slope and y-intercept are interpreted within the scientific context in which the equation was derived. The statistical emphasis is on the values 1.13 and 45.0 and what they reveal about the usefulness of x in predicting values of y.

The preferred scientific notation for this relationship would be M = 1.13 V + 45.0, where M is the total mass of the graduated cylinder and antifreeze in grams and V is the volume of antifreeze in milliliters. Scientifically, the emphasis is on the variable relationship, as evidenced by renaming the variables y and x as M and V, respectively. The simple act of renaming the variables nudges the interpretation of the equation out of the Cartesian plane and into the physical situation. Using Leinhardt's (1990) term, a shift of "space" (p. 8) has occurred.

A linear mathematical function undergoes another notational and emphasis transformation when moved into the realm of statistics. A linear regression fit of a data set using a TI-81 calculator returns two values, a and b. These values are the parameters of the linear function y = a + b x, an equational form commonly found in elementary statistics textbooks. Two notational changes are of interest here. First, a and b have replaced b and m, respectively, in the mathematical form y = m x + b. Second, a commutative rearrangement has taken place; that is, statistically, the constant term precedes the variable term, while, mathematically, the variable term precedes the constant term. Statistically, the emphasis is on the parameters a and b and what they reveal about the relationship between the variables, x and y, in this case. The b-value is used as a measure of the functional relationship between x and y, often by testing the null hypothesis H_0: $b=0$ against the alternative H_1: $b \neq 0$. If the null hypothesis is not rejected, then it is concluded that inclusion of a first order term in x in the regression model does not help in the prediction of y. If the alternative hypothesis is accepted, then it is concluded that a linear relationship does exist between x and y, and the equation y = a + b x provides a better prediction of y than does the equation y = a.

The differences in notation and emphasis among the mathematical, scientific, and statistical presentations are important. These differences have grown out of three traditions. However, in our attempts to "divide and conquer" we may have succeeded in simply separating rather than conquering. Surely, the greatest irony is that now we find ourselves emphasizing the need to make mathematical connections among areas which we arbitrarily divided. Data analysis provides a context for uniting the mathematical, scientific, and statistical perspectives.

For the purposes of this treatment, consider a data analysis process for linear data that involves four steps. These four steps may be incorporated as an integral part in a scientific inquiry approach to learning elementary school science.

1. Data are collected in a scientific setting in an attempt to determine and describe the relationship between two real-world variables.

2. Data are analyzed using the techniques of statistics in an attempt not only to determine the functional relationship but also to establish a probabilistic statement about the appropriateness of the relationship determined.

3. Algebra is employed to provide a simplified summary of the data in the form of an algebraic function expressed as an algebraic formula. The equation serves as a description of the relationship between the two physical variables of interest.

4. The functional relationship, expressed as an algebraic formula, is translated into a verbal description of the relationship between the two variables.

Basically, this procedure is a more specific form of the steps Resnick (1992) identified: (a) mathematize a situation, (b) enter a formal system, and (c) come back out. A crucial key to implementing these steps in elementary classrooms, as discussed earlier, is teacher preparation. Research is needed to determine the extent of teacher knowledge in this area and to design changes in preservice education programs for the purpose of including a stronger component in mathematical function through mathematical modeling.

Linear Function: A Case Study

Mathematical function, even in the relatively simple case of linear functions, constitutes a complex concept domain. No attempt will be made to review a comprehensive list of research studies. Instead, one study will be used as a illustration of gaps of understanding students may exhibit with linear mathematical models. Schoenfeld, Smith, and Arcavi (in press) documented the complexity involved in one student's evolving understanding of linear functions of the form $y = m x + b$. The subject was a sixteen-year-old high school honor student participating in a special summer calculus class at the University of California - Berkeley. Based on her superior record in high school mathematics classes, it was expected that her understanding of the simplest of mathematical functions, restricted to the commonly presented algebraic formula and graphical representations, would be robust. However, the researchers found serious gaps and misconceptions in her understanding of linear functions. In particular, the student responded to the task of constructing equations corresponding to linear graphs presented in a computer environment as though she believed that three properties are necessary and sufficient to characterize a straight line: slope, y-intercept, and x-intercept. She tried to use the value of the x-intercept as the value of x in constructing the equation corresponding to the graph.

Initially, it seems puzzling that the student would try to incorporate the x-intercept as well as the y-intercept and slope when constructing an equation for a linear graph. Surely, the student was not taught to use the x-intercept in this manner! A common sequence of presentation in a mathematics class is to present the student with an equation in the standard form, $Ax + By = C$ and instruct the student to solve for y and graph the equation. Given the equation $4x - y = -5$, the student is expected to rearrange the equation to obtain $y = 4x + 5$, locate the y-intercept (+5) on the y-axis and mark the point, use the slope (4) to find one or two more points, and draw the straight line passing through the points. But we also teach students to graph an equation like $4x - y = -5$ by determining and plotting the x- and y-intercepts and drawing the line passing through these two points. The similarities and differences in constructing a graph by these two methods, if internalized procedurally rather than conceptually by the student, might account for the observed attempt to use the x-intercept inappropriately. Furthermore, if we consider the student's attempt to use the x-intercept in terms of the emphasis placed on the equation $y = mx + b$ in mathematics instruction, it may be possible to attribute the student's response to over generalization. Mathematically, the emphasis is on the slope and the y-intercept; the variables x and y are largely ignored. Since the student was placed in a situation where she needed to write the equation rather than just determine the value of the slope and y-intercept, she needed to create a meaning for x in the equation.

Rationale for the Study

Elementary school science teachers are expected to teach children science process skills, including the processes associated with a scientific inquiry approach. Traditionally, science methods courses for preservice elementary teachers have stressed incorporating hands-on science experiences in elementary instruction. However, preservice teachers are rarely asked to take the process beyond the data collection step. Therefore, elementary preservice programs have failed to adequately prepare teachers to instruct students in data analysis processes associated with a scientific inquiry approach.

NCTM (1989) stressed a need for increased emphasis on functional relationships, data analysis, and problem solving in the elementary curriculum. Instructionally, these topics may be incorporated within a scientific inquiry approach to teaching science. There are four advantages to integrating science and mathematics instruction in this manner.

1. A major goal of school science, construction of verbal descriptions of relationships between real-world variables, may be facilitated through application of data analysis techniques involving several representations of mathematical function.

2. A major goal of school mathematics, developing understanding of mathematical function and its representations and associated translation processes, may be facilitated by situating instruction within a scientific context.

3. The integration of science and mathematics via a scientific inquiry/data analysis approach may help students develop an understanding of the nature of science.

4. The integration of science and mathematics via a scientific inquiry/data analysis approach may help students appreciate the usefulness of mathematics in exploring our physical world.

Elementary teachers need to experience learning science and mathematics via a scientific inquiry/data analysis approach before they can reasonably be expected to teach elementary children using this approach. The treatment sessions utilized in this study were designed to engage preservice elementary teachers in activities involving building, interpreting, and using linear mathematical models based on sets of scientific data. Specifically, the study was designed to assess the effectiveness of the treatment sessions in helping preservice teachers connect the notion of describing relationships between two variables based on data collected in a physical science setting to what they already knew about linear mathematical functions in the form $y = m x + b$ from the study of algebra.

In addition to assessing preservice teachers' abilities to make connections among various representations of mathematical function, this study was designed to address three other issues. First, the experiment was designed to study the effect of instructional strategy on teacher learning. Basically, the purpose was to determine the effect of engagement in data collection tasks on building, interpreting, and using linear mathematical models. Second, the study explored preservice teachers' reaction to using TI-81 calculators for data analysis. Third, the study was designed to investigate the pentagonal model (described in Chapter III) as a model for framing research in mathematical function.

CHAPTER III

THEORETICAL MODEL FOR THE STUDY

In Chapter I two factors are identified that might explain some of the difficulties students have when attempting to link knowledge of mathematical function developed within a mathematical perspective to data analysis within a scientific perspective. Leinhardt, Zaslavsky, and Stein (1990) suggest that "the issue of directionality of thinking about graphs and functions" (p. 3) is a major factor. The context factor may also be an important issue. In this chapter these two factors are considered within a theoretical framework derived from an extension of Janvier's (1987ab) "star" model for understanding mathematical function.

Representations of Function

Janvier (1987b) suggests that a representation consists of three components, written symbols, real objects, and mental images, along with verbal or language features that serve as links between the three components. Mental images are internal representations, while real objects and written symbols may serve as external representations. Dufour-Janvier, Bednarz, and Belanger (1987) refer to external representations as being "all external symbolic organizations (symbol, schema, diagrams, etc.) that have as their objective to represent externally a certain 'reality'" (p. 109). Dufour-Janvier et al. cite four reasons for the use of external representations:

1. In some cases representations are so closely related to the concept that it is hard to imagine how the concept can be conceived without the representation. For

example, the function concept is highly intertwined with Cartesian graphic representations.

2. Multiple representations provide "concretizations" for the concept in hopes that, through extraction of common properties from diverse representations, the learner will develop the intended mental construct.

3. Certain specific difficulties in learning a concept may be eliminated by the use of alternative representations.

4. Varied representations tend to make mathematics more attractive and interesting to the learner.

Although this research focuses on external representations of mathematical function, it is important to keep in mind that the role of these external representations is to help learners develop strong internal representations of the function concept.

Janvier (1987b) proposes that students construct their own internal representation of mathematical function based on some combination of five common external representations: algebraic formula, table, graph, situation, and verbal description. These five representations are incorporated in his "star" model for understanding mathematical function (Figure 2). Janvier suggests that, taken individually, each representation conceals more about the function concept than it reveals, and he (1987a) further suggests that improved understanding of mathematical function requires focusing on the translations between representations rather than on the representations themselves.

If we include "set of ordered pairs" as another form of a table representation, the mathematical perspective described by Leinhardt et al. (1990), viewed within Janvier's star model, involves algebraic function-to-table and table-to-graph translations. Thus, the mathematical perspective incorporates algebraic formula, table, and graph representations and ignores situation and verbal description representations. These latter two representations are precisely the two that would surface from a discussion of context. Hence, context is a critical factor distinguishing mathematical and scientific perspectives. The scientific perspective, as described by Leinhardt et al. involves situation-to-table, table-to-graph, and possibly graph-to-algebraic formula translations. A more comprehensive view of the scientific perspective requires drawing valid conclusions from data. Within this comprehensive view, the algebraic formula-to-verbal description translation should be added as the fourth translation in the list. Thus, the scientific perspective incorporates all five representations, and focuses on relating the physical situation representation of a function to the corresponding verbal description representation.

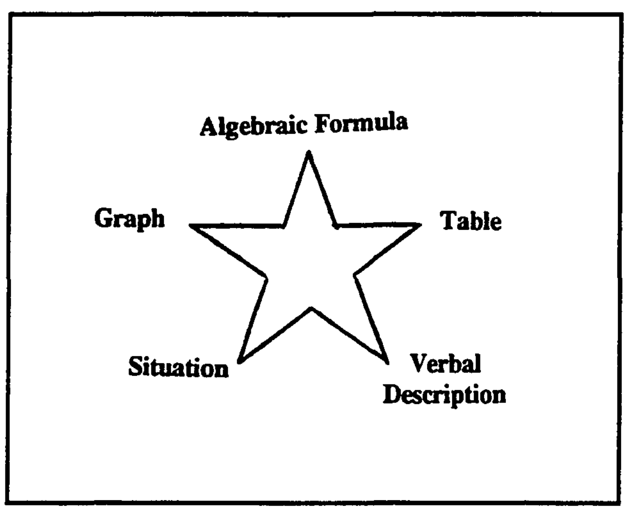

Figure 2: Janvier's "Star" Model for Understanding Mathematical Function

An extension of Janvier's star model is presented in the next section. This extended model, referred to as the "pentagonal" model, incorporates all possible translations between pairs of the five representations identified by Janvier. In addition, the pentagonal model provides a theoretical framework for defining translation processes contained within the translations.

Translations among Representations

A "pentagonal" model may be derived from Janvier's star model by joining adjacent points of the star (Figure 3). In this model the vertices of the pentagon correspond to the five external representations identified by Janvier, with the extension that the interpretation of "table" is expanded to include both mapping diagram and set of ordered pairs representations. The line segments forming the sides and diagonals of the pentagon correspond to the 20 possible one-way translations between pairs of representations. These 20 translations may be identified uniquely as source-to-target translations. For example, Swan's (1985) research on children's abilities to construct qualitative graphs would be viewed within the pentagonal model as a situation-to-graph translation.

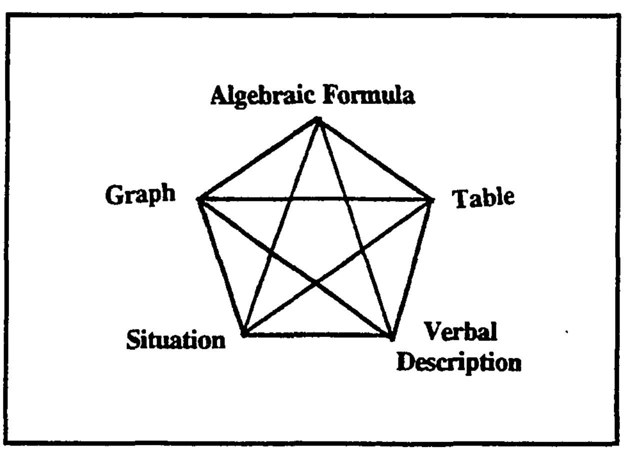

Figure 3: Pentagonal Model of Representations and Translations for the Mathematical Function Concept

Within each source-to-target translation, one or more translation processes may be identified. Janvier (1987a) uses the term translation processes to mean "the psychological processes involved in going from one mode of representation to another" (p. 27). For example, a situation-to-table translation may involve the translation process of measuring. Although Janvier attempted to identify some translation processes, he expressed concern about the tentative nature of the processes he identified. In particular, he found it difficult to define uniquely the process(es) by which a particular translation is accomplished. However, such attempts at identifying translation processes, though incomplete and tentative, may prove useful in allowing for a broader, yet more detailed, view of how representations relate to the development of the function concept. Some translation processes for the source-to-target translations incorporated in the pentagonal model are listed in Figure 4. The processes shown in bold-faced print were identified by Janvier; the other processes are proposed as part of the pentagonal model.

The mathematical and scientific perspectives described by Leinhardt, et al. (1990) may be viewed within the pentagonal model in terms of both the translations and the corresponding translation processes involved. The algebraic formula-to-table translation of the mathematical perspective involves instantiation via computing. The table-to-graph translation involves selecting axes, constructing scales, and plotting. It is important to note that, within a mathematical perspective, notions of measuring and units of measure are not emphasized when selecting axes and constructing scales.

From the scientific perspective, as identified by Leinhardt, et al. (1990), the translations involved are (a) situation-to-table, (b) table-to-graph, and, perhaps, (c) graph-to-algebraic formula. The situation-to-table translation involves defining variables, establishing parameters, and measuring with appropriate measuring

FROM \ TO	SITUATION	VERBAL DESCRIPTION	TABLE	GRAPH	ALGEBRAIC FORMULA
SITUATION		Describing Analyzing	Defining variables Establishing parameters **Measuring**	Identifying Defining axes Approximating scales **Sketching**	Recalling formula **Mathematical modeling** Abstracting
VERBAL DESCRIPTION	Mental imaging Enacting		Instantiating	Identifying Instantiating Defining axes **Sketching**	Identifying Recalling formula Mathematical modeling Symbolizing
TABLE	**Reading** Interpreting Mental imaging	Recognizing patterns Summarizing Generalizing Verbalizing		Selecting axes Constructing scales **Plotting points**	**Fitting** Recognizing patterns Generalizing Symbolizing Abstracting
GRAPH	**Interpreting** Mental imaging Enacting Concretizing	**Interpreting** Mental imaging Verbalizing Abstracting	**Reading off** Interpreting axes Reading scales		**Curve fitting** Recognizing form Generalizing Symbolizing Abstracting
ALGEBRAIC FORMULA	**Parameter recognition** Concretizing Instantiating Mental imaging	Verbalizing Interpreting variables Interpreting parameters	**Computing** Instantiating	**Sketching** Formula recognition Identifying axes Approximating scales	

Figure 4. Processes Associated with Translations in the Pentagonal Model

instruments. The table-to-graph translation is similar to that given from the mathematical perspective in that it involves the same three translation processes. However, this translation within a scientific perspective differs from that within a mathematical perspective because selecting axes and constructing scales within a scientific perspective involves both the units and the magnitudes of the measurements in selecting an appropriate viewing window for the graph. Mathematically, a graph-to-algebraic formula translation is generally accomplished by curve recognition process, if at all. Scientifically, this translation is accomplished by data analysis techniques which may involving either hand-fitting or more sophisticated statistical techniques.

In the next section the ideas of mathematical and scientific perspectives described by Leinhardt, et al. (1990) are expanded to a more encompassing view of mathematical and scientific perspectives of mathematical function. The expansions of the two perspectives have resulted from a classification of tasks typically encountered in teaching mathematical function. The following development is based on using the pentagonal model as a framework for considering how mathematical function is taught.

Approaches to Teaching Mathematical Function

Two distinctly different approaches are traditionally used for developing the function concept: (a) a theoretical approach and (b) a data analysis approach. The theoretical approach is employed in traditional algebra courses. The data analysis approach may be found in elementary, middle, and high school classrooms where data analysis is used as a step in data interpretation. Data analysis is also a crucial component of introductory college physical science courses as well as in statistics courses at all levels. The theoretical and data analysis approaches to teaching mathematical function align well with mathematical and scientific perspectives, respectively. By comparing these approaches within the framework of the pentagonal model, problems associated with connecting representations and developing a comprehensive understanding of the function concept may be seen as inherent in the ways mathematical function is taught.

Theoretical Approach. The theoretical approach is utilized in traditional algebra and calculus courses and is associated with a mathematical perspective. This approach, viewed within the framework of the pentagonal model, consists of six translations: (a) algebraic formula-to-table, (b) table-to-graph, (c) graph-to-algebraic formula, (d) algebraic formula-to-graph, (e) graph-to-table, and (f) table-to-algebraic formula (Figure 5a). The mathematical perspective defined by Leinhardt, et al., (1990) involves two of the six translations identified within the theoretical approach: (a) algebraic formula-to-table and (b) table-to-graph (Figure 5b). The six translations of the theoretical approach are all part of a more comprehensive view of the function concept from a mathematical perspective.

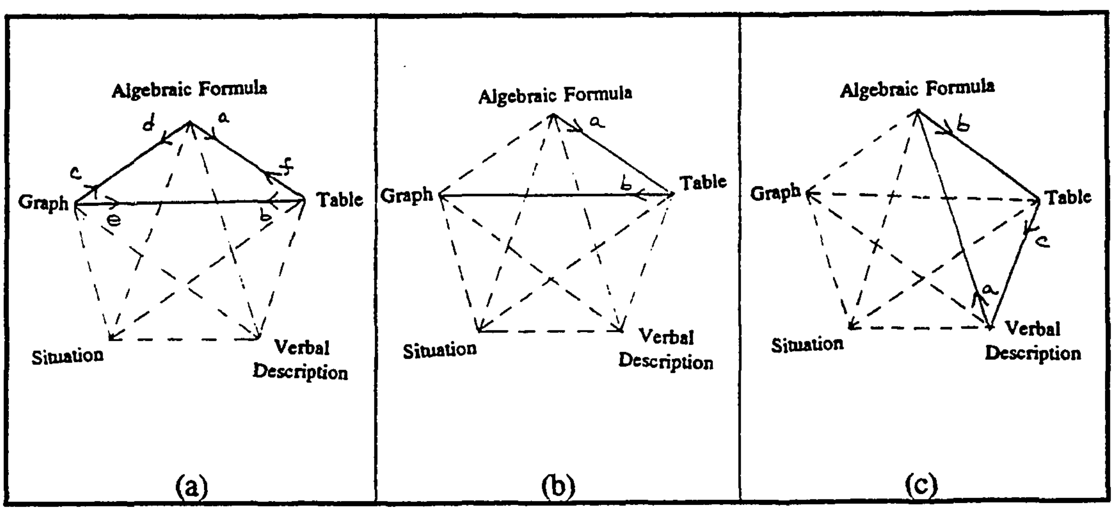

Figure 5. The Function Concept from a Mathematical Perspective. (a) Theoretical View, (b) Leinhardt et al. View, and (c) Applications View

The theoretical introduction to the function concept is by function definition. Modern algebra textbooks typically employ a set-theoretical definition of function. For example, Foster, Winters, Gell, Rath, and Gordon (1992) introduce function with this definition: "A function is a relation in which each element of the domain is paired with exactly one element of the range" (p. 374). Frequently, the function definition is first illustrated with a mapping representation. Subsequent representations may include table, algebraic formula, and graph. Viewed within the context of the pentagonal model, the theoretical approach involves only six of the 20 source-to-target translations incorporated in the model and only three of the representational modes.

The situation and verbal description representations are noticeably missing from the theoretical approach. Consequently, the opportunities for developing language associated with the translation processes of verbalizing and symbolizing are also missing from the theoretical approach. As a partial remedy to this problem, algebra textbooks authors have incorporated a variety of applications in the form of word problems. Often these word problems may be solved quite algorithmically via (a) verbal description-to-algebraic formula, (b) algebraic formula-to-table, and (c) table-to-verbal description translations (Figure 5c).

As an example of an algorithmic solution to a word problem, consider the following example: "One season, Reggie Walker scored 9 more runs than twice the number of runs he batted in. He scored 117 runs that season. How many runs did he bat in?" (Foster, et al., 1992, p. 114). Using a traditional word problem approach the first step to solving this problem is to translate the verbal description into an algebraic formula; for example, if x = the number of runs he batted in and y = the number of runs he scored, the algebraic equation $y = 2x + 9$ may be written. The value of y is known to be 117. This value of y is the y-value of an ordered pair (x, y) or an entry in the table of x, y-values for the function $y = 2x + 9$. By substituting $y = 117$ into the equation $y = 2x + 9$ and solving for x, the x-value of 54 corresponding to $y=117$ in the table representation is found. This substitution/solving process accomplishes the algebraic formula-to-table translation. The value x = 54 in the table is then translated into a verbal description which answers the question asked: "Reggie Walker batted in 54 runs."

Modern problem solving examples may involve numerous variations on the translations involved in this example. However, neither the theoretical approach nor the applications incorporated in teaching function from a mathematical perspective involves algebraic formula-to-verbal description translations. Thus, the mathematical perspective fails to emphasize what may well be a crucial process in function concept development: verbalizing. The verbalizing process involves a translation from symbolic language to verbal interpretation. A discussion of the importance of this process, framed within Kaput's (1987) Symbol Systems Theory, may be found in the Chapter VI.

Data Analysis Approach: NCTM (1989) stresses the importance of incorporating data analysis at all levels within the K-12 mathematics curriculum. However, it is interesting to note that there is a separation of the notions of mathematical function from the notions of data analysis at the K-4 and 5-8 levels. A merging of mathematical function with data analysis is encountered, however, within the "Functions" section at the 9-12 level. "In grades 9-12, the mathematics curriculum should include the continued study of functions so that all students can model real-world phenomena with a variety of functions" (p. 154). Implicit in this standard is the connection of a theoretical treatment of mathematical function incorporated in the mathematical perspective with a data analysis view of mathematical function incorporated in the scientific perspective. This connection can be accomplished via the process of mathematical modeling. A broader view of mathematical function than that implied by NCTM is incorporated in the pentagonal model. In particular, the processes associated with data collection, data analysis, and data interpretation are all incorporated in the pentagonal model for mathematical function.

The data analysis approach constitutes a critical portion of a scientific perspective of mathematical function. The data analysis approach, frequently associated with scientific inquiry methods, involves all five representations defined within the pentagonal model. However, only four of the 20 source-to-target translations are typically incorporated into the data analysis approach: (a) situation-to-table, (b) table-to-graph, (c) graph-to-algebraic formula, and (d) algebraic formula to verbal description (Figure 6). It is important to note that the data analysis approach provides the opportunity for language development in the algebraic formula-to-verbal description translation process of verbalizing--a process missing in the study of function from a mathematical perspective. The scientific perspective also incorporates applications similar to those found within the mathematical perspective (illustrated in Figure 5c).

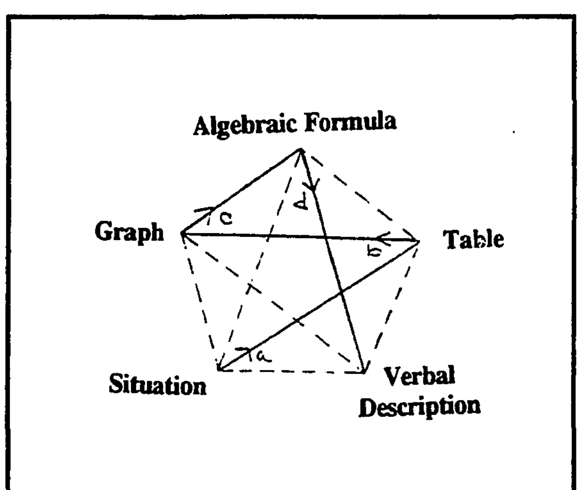

Figure 6. Data Analysis Approach

A Statement of Contrast Between the Two Approaches. From a scientific perspective, the purpose of mathematical modeling is to obtain a succinct expression (algebraic formula and verbal description) of the relationship between two real-world variables. The utility of an algebraic formula derived via the process of mathematical modeling is determined by the accuracy of predictions made with the formula. That is, from a scientific perspective, data analysis is important in helping the scientist better understand the physical world, with underlying goals of prediction and, perhaps, control.

From a mathematical perspective, the purpose of a mathematical model is instantiation. For example, the utility of linear functions of the form $y = m x + b$ is illustrated by examples such as $y = 1.13 x + 45.0$ derived via mathematical modeling from mass versus volume data (Chapter I). Expressed simplistically, examples of mathematical functions derived by data analysis procedures are used by mathematicians to illustrate the importance of studying mathematical theory. However, the processes involved in deriving mathematical models via data analysis are not generally incorporated within the mathematical perspective in K-12 mathematics curricula.

Mathematical Modeling within the Pentagonal Model

Within this treatment, a mathematical model can be defined as an algebraic formula expressing a functional relationship between two real-world variables. The equation encountered in Chapter I, $y = 1.13 x + 45.0$, is a model representing the functional relationship between the total mass of a graduated cylinder containing antifreeze and the volume of antifreeze in the graduated cylinder. Using this definition for mathematical model, the scientific perspective may be defined in terms of mathematical modeling. Mathematical function, studied from the scientific perspective, involves building mathematical models from data, interpreting mathematical models in terms of the original situation, and then using mathematical models to make predictions. The data analysis approach described above incorporates the three translations involved in building a mathematical model (situation-to-table, table-to-graph, and graph-to-algebraic formula) and the translation involved in interpreting a model (algebraic formula-to-verbal description). The translations involved in making predictions using a mathematical model are the same as those identified in Figure 5c.

Students who have studied linear functions in the form $y = m x + b$ from a mathematical perspective have probably developed a sufficient mathematical background to enable them to fit a variety of different data sets based on linear relationships between pairs of physical science variables. If the graph of a data set appears linear, then a "best" line may be drawn through the data points and the mathematical model determined by putting the value of the y-intercept and the value of the slope into the equation $y = m x + b$ in place of b and m, respectively. If the graph of a data set is nonlinear, often a transformation of the data will yield a linear relationship. For example, the pressure versus volume graph of a gas is nonlinear; however, by transforming the data to pressure versus 1/volume, a linear graph may be obtained that can be fit to a $y = m x + b$ equation. More precise curve fitting may be accomplished using computers or calculators to compute least squares regression fits.

This study was based on data analysis involving fitting data that was best described by linear mathematical models. The treatment sessions were designed to help students connect what they already knew about linear mathematical functions from a mathematical perspective to the knowledge they had about mathematical function from a scientific perspective.

CHAPTER IV

METHOD

The first section of this chapter begins with a statement of the purpose for the study. Identification of experimental variables, statement of research questions, and a rationale for the study are also included in this section. The second section describes the pilot study. The pilot study was composed of three parts: (a) a study of the validity of the posttest in its original form, (b) a pilot test of the posttest in its original form, and (c) a pilot test of the treatment activities and the posttest in the final forms. Details of the main body of the study are presented in the last section. This section includes (a) a description of subjects, (b) details of treatment procedures for the two groups, and (c) details of measurement procedures.

Overview of the Study

Purpose

This experiment was designed with primary and secondary purposes. The primary purpose, perhaps more appropriately referred to as a goal, was to test the usefulness of the pentagonal modification of Janvier's "star" model as a model for framing research on designing instruction to increase connections students make among various aspects of the function concept. Parts of the treatment and posttest activities used in the study were viewed, within the pentagonal model, as specifically-defined (a) representations, (b) translations between representations, and (c) translation processes involved in making one-way translations between representations.

The secondary purpose of the study was to determine if the concrete activity of data collection had a measurable effect on subject's performance on tasks involving building, interpreting, and using linear mathematical models. The treatment difference between the two experimental groups may be viewed within the context of the pentagonal model. All subjects participated in tasks in which they had to perform (a) table-to-graph, (b) graph-to-algebraic formula, (c) algebraic formula-to-verbal description, and (d) verbal description-to-algebraic formula translations. Differences in treatments between the two experimental groups were centered around the situation-to-table translation. Subjects in one group (Group A) worked in a laboratory setting. Group A subjects made measurements, constructed data tables, analyzed data to build mathematical models, interpreted the resulting models, and made predictions based on the models. Subjects in the second group (Group B) worked in a classroom setting. Group B subjects were presented with data tables and began directly with data analysis, interpretation, and prediction tasks. The data analysis, interpretation, and prediction tasks were presented to all subjects in identical written formats (Session I and II handouts for Group A subjects may be found in Appendixes A and C, respectively; the corresponding Group B handouts may be found in Appendixes B and D). Thus, subjects in Group A engaged in situation-to-table translation processes while subjects in Group B did not. This difference in treatment between Group A and Group B subjects may be viewed within the pentagonal model as shown in Figure 7.

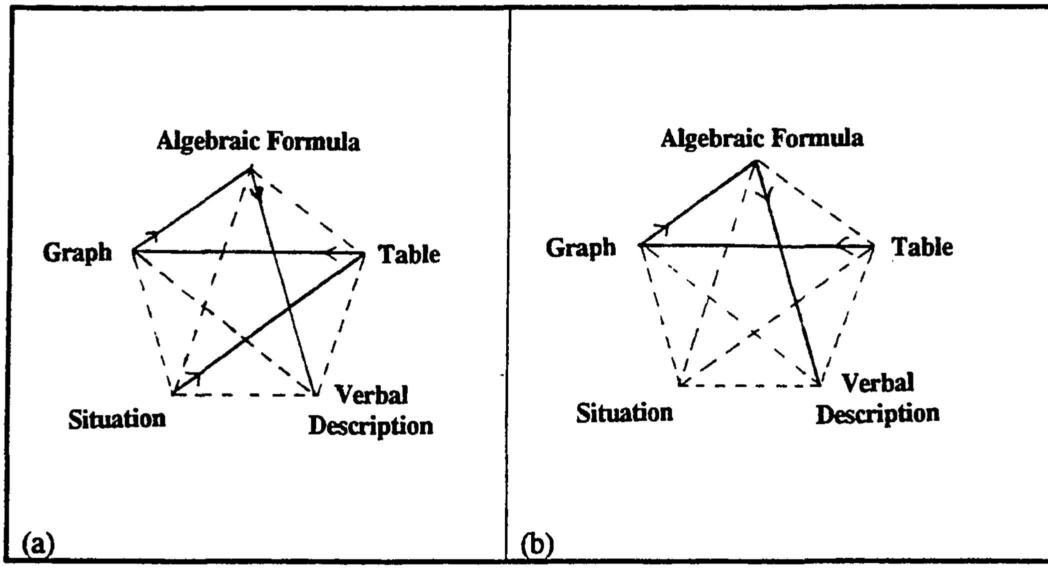

Figure 7. Treatment Difference Viewed Within the Pentagonal Model.
(a) Group A and (b) Group B.

Identification of Variables

The major independent variable in this study was subject's treatment group. Group A subjects experienced the instructional unit on mathematical modeling of scientific data beginning with data collection. Group B subjects experienced the instructional unit beginning with data analysis. In addition there were 17 variables of secondary interest.

1. CLASS. The subjects in this study were students enrolled in one of two university courses. The first class was the first semester of a two-semester, freshman-level course specifically designed for elementary education majors. This class is referred to as MATH1 throughout this report. The second class was a one-semester, senior-level, mathematics methods course required for all elementary education majors. This class is referred to as ED4 throughout this report. The ED4 students were also co-registered in a one-semester, senior-level, science methods course.

2. HSR. High school rank in class (percentile ranking).

3. HGPA. High school grade point average.

4. MSAT. Score on mathematical portion of Scholastic Aptitude Test (SAT).

5. VSAT. Score on verbal portion of SAT.

6. TSWE. Score on Test of Standard Written English.

7. HMHRS. Total number of credits in high school mathematics.

8. HSHRS. Total number of credits in high school science.

9. HMGPA. Grade point average in high school mathematics courses.

10. HSGPA. Grade point average in high school science courses.

11. PGPA. Predicted grade point average, as calculated by the University's Admissions Department.

12. CGPA. Current college grade point average.

13. CHRS. Total number of semester hours completed in college.

14. CMHRS. Total number of semester hours in college mathematics courses.

15. CSHRS. Total number of semester hours in college science courses.

16. CMGPA. Grade point average in college mathematics courses.

17. CSGPA. Grade point average in college science courses.

The major dependent variable was score on the posttest (POST). The posttest was a 36 item test of subjects' performances on tasks involving building, interpreting, and using mathematical models (Appendix E). The posttest was constructed specifically for this study. Details on the development of this posttest are discussed in "The Pilot Study" section of this chapter. A subject's score on the posttest, POST, was the total number of points obtained on the posttest, where the score on each individual item was assigned as 0.0, 0.5, or 1.0 points. Thus, possible values for POST ranged from 0 to 36 points. Other variables considered were subscores based on subsets of questions on the posttest. In addition to the individual item scores (P1 through P36), 16 subscores were considered.

1. DATASETS. Combined score on items involving building, interpreting, and using mathematical models given data tables (sum of item scores P1 through P12, corresponding to Problems 1 and 2). These problems required translations within the pentagonal model analogous to those completed by both groups during treatment sessions.

2. VERBAL. Combined score on items involving building, interpreting, and using mathematical models given verbal descriptions (sum of item scores P13 through P24, corresponding to Problems 3 and 4). These problems involved building models from verbal descriptions. The tasks involving interpreting and using the models were presented in the same way as given in the treatment sessions handouts. The first item in each problem may be viewed, within the pentagonal model, as involving a verbal description-to-algebraic formula translation.

3. ALGEBRAIC. Combined score on items involving interpreting and using mathematical models given the models as algebraic formulas (sum of item scores P25 through P36, corresponding to Problems 5 and 6). Within these problems, the tasks involving interpreting and using models were presented in the same way as was done during the treatment sessions.

4. BUILD1. Combined score on items involving building mathematical models given data tables and the values of a and b derived from the tables using a TI-81 calculator (P1 + P7).

5. BUILD2. Combined score on items involving building mathematical models given verbal descriptions (P13 + P19).

6. BUILD. Combined score on items involving building mathematical models (BUILD1 + BUILD2).

7. SLOPE1: Combined score on items involving identifying magnitudes and units for slopes in mathematical models (P2 + P8 + P14 + P20 + P25 + P31).

8. INTERCEPT1. Combined score on items involving identifying magnitudes and units for y-intercepts in mathematical models (P4 + P10 + P16 + P22 + P27 + P33).

9. SLOPE2: Combined score on items involving writing physical interpretations of slopes in mathematical models (P3 + P9 + P15 + P21 + P26 + P32).

10. INTERCEPT2. Combined score on items involving writing physical interpretations of y-intercepts in mathematical models (P5 + P11 + P17 + P23 + P28 + P34).

11. INTERPRET. Combined score on items involving identifying and interpreting slopes and y-intercepts in mathematical models (SLOPE1 + INTERCEPT1 + SLOPE2 + INTERCEPT2).

12. USEY. Combined score on items involving using mathematical models to predict values of y given values of x (P6 + P12 + P18 + P24 + P29 + P35).

13. USEX. Combined score on items involving using mathematical models to predict values of x given values of y (P30 + P36).

14. USE. Combined score on items involving using mathematical models to predict the value of one variable given the value of the other variable (USEY + USEX).

15. FAMILIAR. Combined score on items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions (sum of P1-P6, P13-P18, and P25-P30).

16. UNFAMILIAR. Combined score on items involving building, interpreting, and using mathematical models based on physical contexts different from the contexts utilized during treatment sessions (sum of P7-P12, P19-P24, and P31-P36).

Another set of 12 dependent variables considered in this study (S1-S12) were student responses on the 12 Likert-type items on the Workshop Evaluation Form (Appendix F). Since this scale was not central to the hypotheses, no attempt was made to validate this form.

Research Questions

1. Will the mean posttest scores (POST) for the two treatment groups differ?

2. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models given data tables (DATASETS)?

3. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models given verbal descriptions (VERBAL)?

4. Will treatment group mean scores differ on posttest items involving interpreting and using mathematical models given the models as algebraic formulas (ALGEBRAIC)?

5. Will treatment group mean scores differ on posttest items involving building mathematical models (BUILD and subsets BUILD1 and BUILD2)?

6. Will treatment group mean scores differ on posttest items involving interpreting mathematical models (INTERPRET and subsets SLOPE1, INTERCEPT1, SLOPE2, and INTERCEPT2)?

7. Will treatment group mean scores differ on posttest items involving using mathematical models (USE and subsets USEY and USEX)?

8. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions (FAMILIAR)?

9. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models based on physical contexts different from the contexts utilized during treatment sessions (UNFAMILIAR)?

Rationale for the Study

During treatment sessions, subjects in Group B began the exploration of each x,y relationship with a data table and proceeded with tasks involving building, interpreting, and using a mathematical model derived from the data. Subjects in Group A began each exploration by collecting data and constructing a data table. Once the data table had been constructed, Group A subjects proceeded with tasks involving building, interpreting, and using a mathematical model derived from the data. Basically, the question of interest in this study was "Do collecting data and constructing a data table affect subjects' performances on tasks involving building, interpreting, and using mathematical models?"

The tasks involving building, interpreting, and using mathematical models were presented to all subjects in exactly the same written form. However, subjects in the two treatment groups completed the tasks in different learning environments. Group B subjects worked in pairs within a classroom setting. Following a filler activity, they began the modeling tasks by building a mathematical model from a given set of data. During the first session, the filler activity was viewing a video on the importance of measurement in chemistry. In the second session Group B subjects worked with their partners to create a list of pairs of real-world variables appropriate for elementary school children to use in exploring relationships. Group A subjects were in a laboratory setting. They worked in pairs to make measurements using rulers, electronic balances, and graduated cylinders.

Group A subjects might be expected to have benefited from data collection activities in two major ways:

1. The data collection activities provided situational experiences in the scientific context that might have been internalized in the form of mental images. If so, the mental images may have been retrieved by Group A subjects while they were engaged in tasks involving building, interpreting, and using mathematical models based on the data they collected. Further, the mental images also may have been retrieved by Group A subjects during the posttest.

2. Discussions between partners during the data collection phase might have contributed to each subject's development of language associated with the scientific situation. If so, language development during the data collection phase might have facilitated interpretation and prediction tasks during treatment sessions and on the posttest.

If Group A subjects did form mental images and develop situation-specific language during the data collection phase, it would be expected that the two groups might perform differently on tasks involving building, interpreting, and using mathematical models. If differences in group mean scores on the posttest and other subscores of the posttest specified in the research questions exist, then it would be important to address how the development of strong mental images and situational language affect performance on tasks involving building, interpreting, and using mathematical models.

The tasks completed by subjects may be considered within the context of the pentagonal model. The tasks for subjects in the two treatment groups may be characterized by the translations that are involved in completing the tasks. One possible set of translations for the two treatment groups is given in Figure 8. In this figure the translations are identical except Group B subjects did not participate in data collection tasks.

If participation in the situation-to-table translation had no effect on the processes involved in completing the other six translations shown in Figure 8, then the group mean performances of the two groups on building, interpreting, and using mathematical models would be expected to be approximately the same. That is, equivalent group performances on the posttest and subtests of the posttest would provide evidence that the translations, and corresponding translation processes, used by the two groups in building, interpreting, and predicting tasks could be the same.

If the situation-to-table translation has an effect on the processes involved in completing the other six translations, then the performance of the two groups on building, interpreting, and using mathematical models will likely be different. As suggested earlier, any differences in group performance would be consistent with the hypothesis of differences in mental image and situational language development within the two groups during treatment sessions. Differences in group performance, if they exist, might indicate one of two situations. The more moderate interpretation would be that the two groups completed the tasks via the same translations, but with some variation in translation processes. That is, the tasks might have been completed by the two groups using the same representations and translations between representations, in the same order, but involving some variation in translation processes.

Tasks	Translations: Group A	Translations: Group B
Data Collection	Situation-to-Table	N/A
Building a Model	Table-to-Graph Graph-to-Algebraic Formula	Table-to-Graph Graph-to-Algebraic Formula
Interpreting a Model	Algebraic Formula-to-Verbal Description	Algebraic Formula-to-Verbal Description
Using a Model	Verbal Description-to-Algebraic Formula Algebraic Formula-to-Table Table-to-Verbal Description	Verbal Description-to-Algebraic Formula Algebraic Formula-to-Table Table-to-Verbal Description

Figure 8. Set of Possible Translation Processes Used by Groups A and B

A more extreme interpretation of group differences that might occur on the posttests and subsets of the posttest involves a variation in the translations used by the two groups. For example, subjects in Group A might be expected to incorporate another sequence of translations in the prediction tasks due, perhaps, to mental images and situational language development. That is, for Group A subjects, the prediction tasks may proceed through (a) verbal description-to-situation, (b) situation-to-algebraic formula, (c) algebraic formula-to-table, and (d) table-to-verbal description translations. This example is only one of several that might be proposed.

This discussion serves as an indication of the complexity involved in a study of the effects of just one planned translation difference between the two groups. The study was designed to answer the major "first" question: Do the two groups perform differently on the posttest? If the answer to this question is yes, however, deeper analysis of the posttest results by subscores might provide some indication of the nature of these differences.

The Pilot Study

Validation of the Posttest

A panel of nine mathematicians, scientists, and mathematics educators provided input on the validity of the posttest. Each person who agreed to participate in the validation process received an envelope containing a copy of the original 45-item posttest (Appendix G), and the TI-81 key stroke reference sheet (Appendix H). The validators commented on (a) how well the posttest content paralleled the objectives in content and process and (b) how balanced the posttest and treatment activities were in concepts and procedures. The comments of the nine validators were used to refine both the posttest and the treatment activity sheets.

Pilot Testing of the Original Posttest

Initially, a 9-problem, 45-item posttest was constructed to be administered during a 2-hour time period. A copy of the original posttest is given in Appendix G.

This posttest was administered to 17 students enrolled in a sophomore-level, Natural Science class at the same university attended by subjects in the experimental study.

It was difficult to find a group of students reasonably similar to the subjects in the study to use in piloting the posttest, because students who had a sufficient background in doing data analysis with a TI-81 calculator were not readily available. The closest matching group found was the Natural Science class. The students in this class were elementary education majors, most of whom were sophomores who had completed MATH1 as freshmen. In order to prepare the Natural Science students to take the posttest, a 60-minute workshop was conducted on (a) using TI-81 calculators to build linear mathematical models, (b) writing situational interpretations of the models built, and (c) using the models built to make predictions.

One week before the workshop was conducted in this class, the Natural Science students had completed a laboratory assignment in which they placed a centimeter ruler beside an inch ruler and recorded a set of corresponding values off each scale. Initially, they began by aligning the zeros on each scale. Then they repeated the experiment, this time aligning the zero on the inch ruler with the 4.0 cm mark on the centimeter ruler. They graphed, by hand, each set of data on the same set of axes. In the final portion of the laboratory they identified the slope and y-intercept and interpreted the meaning of each. The slope they obtained was approximately 2.5 centimeters per inch, which the students recognized as the conversion factor for changing length in inches to length in centimeters. They found that the y-intercept in the first case was approximately 0 centimeters and in the second case was approximately 4.0 centimeters.

During their morning class one week later, I began the workshop with a set of data analogous to the data they had obtained and analyzed in the laboratory the previous week. This example was used to introduce the students to data analysis using the TI-81 calculator. I then lead them through building mathematical models for two other sets of data: (a) number of pennies versus mass of cup and pennies and (b) mass versus volume of liquid. Students then completed tasks involving identifying the slope and y-intercept in each model, including the units of each, and interpreting each within its specific scientific context. The same afternoon the students completed the posttest as a laboratory exercise during their regularly scheduled two-hour laboratory period. This constituted the first pilot test of the posttest.

Pilot of Treatment Activities and Final Form of Posttest

A class of ten Physics students were used to pilot both the Group A treatment materials (Appendixes A and C) and the posttest in its final form (Appendix E). During a regularly scheduled 3-hour laboratory period, each student worked alone to collect all sets of data as described in the two sets of activity sheets. Five days later the final form of the posttest was administered to the students during their regularly scheduled 50-minute class period. Observations made during the laboratory session and posttest results were used to evaluate the treatment and posttest materials.

The Experimental Study

Subjects

Students at a small church-affiliated university in the southeastern region of the United States participated in this study. The subjects in the experimental treatment portion of this study were elementary education majors enrolled in one of two required courses. One course, MATH1, is the first of two 3-semester-hour courses in mathematics designed for and restricted to elementary education majors. The second course, ED4, is a senior-level, 3-semester-hour, elementary mathematics methods course. All 28 students enrolled in ED4 participated in the study as a required course activity. Project participation for MATH1 students was on a volunteer basis, with each participant receiving extra credit in the course for participating. Twenty-seven of the 38 students enrolled in MATH1 participated fully in the project. Of the remaining 11 students, (a) one did not sign up to participate due to scheduling problems, (b) one had already participated as part of the pilot testing of the posttest instrument, (c) one was not attending class, (d) five attended the first session but failed to show up at the second session, citing car problems and changes in work schedules as the reasons, and (e) three originally signed up to participate but failed to show at either session, providing no reasons for this. Three MATH1 subjects who participated fully in the project were not included in the analyses because they worked without a partner during the second session. In all, a total of 55 subjects participated fully in the project, and 52 subjects were included in the analyses.

ED4 students participated during regularly scheduled class time. These students were randomly assigned to treatment groups. The 14 students assigned to each treatment group were then randomly paired to form seven experimental pairs in each group. Each pair was treated as an experimental unit throughout the experiment.

Because students in MATH1 had to participate in the project outside of regularly scheduled class time, adjustments in the randomization procedure had to be made to accommodate individual schedules. Initially, students were asked to check one or more of three time slots when they could attend sessions, or, if none of the times would work, to suggest other times. One of the three proposed time slots was dropped because no students chose it as the only time when they could participate. Students who checked only one time slot were automatically assigned to that period. Students who checked both were randomly assigned to one of the periods. Some adjustments were made to provide some balance to the number of subjects assigned to each group. A new time slot was created to accommodate students who could not attend one of the proposed times. One student, who could not attend at a time corresponding to that for any other student, was dropped from the subject list. Within each time slot students were randomly paired. Then a treatment group label was randomly assigned to each time slot. Initially, this process resulted in 21 students being assigned to Treatment Group A and 16 students to Treatment Group B. In the end, a total of 27 of these students fully participated, 16 in Treatment Group A and 11 in Treatment Group B. In the final analysis these subjects were treated as 6 experimental "pairs" in Treatment Group A and 5 experimental "pairs" in Treatment Group B. The "pairs" referred to for MATH1 students, in the final analysis, were actually composed of 9 pairs and 2 triples (Treatment Group A: 1 triple, 5 pairs; Treatment Group B: 1 triple, 4 pairs).

Overall, of the 52 subjects included in the analysis portion of the study, 13 experimental pairs were assigned to Treatment Group A and 12 experimental pairs were assigned to Treatment Group B. Seven experimental pairs in each treatment group were ED4 students. The majority of subjects from the MATH1 class were traditional first-semester freshmen, while those in the ED4 class were mostly seniors, scheduled to complete student teaching the following semester, graduating immediately thereafter. In all, 48 females and 4 males who participated in this treatment portion of this project were included in the analysis phase.

A note on statistical power: A priori power calculations were conducted. These calculations were based on the overall posttest score (POST) using standard deviations of 5.0 for groups A and B, a Type I error rate of .05, a Type II error rate of .20, and an assumption of equal numbers of experimental units in Groups A and B. It was determined that a statistical power of .80 would result if (1) the posttest mean group difference was 5 points and 17 experimental pairs were in each group, (2) the posttest mean group difference was 7 points and 9 experimental pairs were in each group, or (3) the posttest mean group difference was 9 points and 6 experimental pairs were in each group. Based on these calculations it appeared reasonable to proceed with the study using the group sizes discussed above. Final analysis of experimental results determined that the standard deviations for the groups were between 4.6 and 5.7 and the difference in sample group means was 7.1 points. Thus the actual statistical power exceeds .80.

Procedure

The experiments chosen for the treatment sessions were carefully constructed to involve (a) simple measurements, using common measuring instruments, which elementary school children could make, and (b) linear mathematical models, with little error variation, where both the slope and y-intercept have simple and clearly-recognizable physical interpretations. Time spent on data analysis, interpretation, and prediction activities was controlled to be the same for subjects in each treatment groups. Since subjects were randomly assigned to the two treatment groups, the major difference in the two groups was that subjects in Group A collected data before analysis, whereas Group B subjects worked on, presumably, non-interfering activities for a time period equivalent to the time Group A subjects spent collecting data. During the first session, the filler activity was viewing a video on the importance of measurement in chemistry. In the second session Group B subjects worked with their partners to create a list of pairs of real-world variables appropriate for elementary school children to use in exploring relationships.

Each subject participated in two 2-hour treatment sessions. Both treatment groups of ED4 subjects were scheduled to treatment sessions during the same time slots on a Friday and the following Monday. The posttest and workshop evaluation was administered to all ED4 students on the following Wednesday. Thus, for ED4 students, their involvement in the experimental portion of the project spanned five days.

MATH1 students participated in the treatment sessions outside of regularly scheduled class time, but all MATH1 students completed the posttest and workshop evaluation during a regularly schedule MATH1 period. Since students had to participate in treatment sessions "on their own time," treatment sessions were scheduled a week apart with the posttest and workshop evaluation scheduled the following week. For MATH1 students, their involvement in the experimental portion of the project spanned between 13 and 16 days.

Treatment Session I. At the beginning of the first session, subjects in both groups received the same 25-minute introduction to the project. An outline of this introduction may be found in Appendix I. During this introduction, subjects agreed to participate in the project by signing a consent form (Appendix J). Following the introduction, subjects in Group A participated in a 25-minute key-punching lesson on using the TI-81 calculator to analyze the antifreeze data given in the first data table on the activity sheet (Appendix A). They then spent the next 25 minutes collecting mass/volume and height/volume data as described on the activity sheets for Session I. During the corresponding 50 minutes, Group B subjects viewed a 25 minute video entitled "Measurement: The Foundation of Chemistry" and then participated in the same lesson on using the TI-81 calculator for data analysis. The video presents a general overview of the role of measurement in science but does not include details of measuring techniques or specific examples of making measurements utilizing the measuring instruments used by Group A subjects during the treatment sessions. Both treatment groups then spent the remaining 45 minutes of the session analyzing the other two sets of data. A copy of the activity sheets for Group B, Session I may be found in Appendix B.

During this first session, the first two sets of data focused on the relationship between the volume of liquid in a graduated cylinder and the total mass of the graduated cylinder and its contents. The third set of data focused on the relationship between the volume of water in a beaker and the height of the water surface from the table top.

In the initial experiment subjects determined the relationship between the volume of a liquid in a 25-milliliter graduated cylinder (X) and the total mass of liquid plus graduated cylinder (Y). A table of data derived by using antifreeze as the liquid was given to subjects in both treatment groups. This data set was used during an introductory lesson on data analysis using a TI-81 calculator. A reference sheet (Appendix H) that summarizes the key strokes needed to do data analysis on the TI-81 was distributed to subjects. Based on the data analysis of the antifreeze data, students arrived at the mathematical model $y = 1.13 x + 45.0$. They concluded that the slope (1.13 grams per milliliter) corresponds to the density of antifreeze and the y-intercept (45.0 grams) corresponds to the mass of the empty graduated cylinder. Subjects in both groups then worked in their experimental pairs to analyze a second, analogous set of data based on rubbing alcohol. During this experiment, subjects in Group A determined the volume of rubbing alcohol by reading the scale on a 25-milliliter graduated cylinder. The corresponding mass of the graduated cylinder and rubbing alcohol contained within it was determined by reading the digital output of an electronic balance which had been tared with nothing on the balance pan. The y-intercept in this case is the mass of the empty graduated cylinder. The slope is the density of the rubbing alcohol in units of grams per milliliter.

The relationship between mass and volume of a liquid was chosen as the initial relationship because it is perhaps the most fundamental of relationships encountered in elementary physical science. Physical science is defined to be the study of matter. Matter is defined to be anything that has mass and volume. Therefore, the most fundamental relationship of matter is that of mass and volume, and, experimentally, the easiest group of mass/volume relationships to investigate is that of liquids. Typically, students are taught this relationship by definition rather than exploration. That is, students are instructed that "density is mass per unit volume" or "density is the mass of an object divided by the volume of the object" or "D = m/V." By including this experiment, preservice teachers had the opportunity to explore a way of helping elementary school students develop the density concept concretely, thereby avoiding the memorizing of definitions and formulas.

The second experiment included during the first treatment session focused on the relationship between the volume of water poured into a 500-milliliter (X) beaker and the height of the water surface from the table top (Y). In this case the measurements done by Group A subjects involved reading the scale on a 50-milliliter graduated cylinder to determine the volume added each time and then measuring the distance from the table top to the water surface using a centimeter ruler with millimeters as the smallest graduation. Because a beaker is cylindrical in shape, the rate of change of height with volume is constant. Therefore, the resulting mathematical model is linear. The slope in this model defines how the height from table top to water surface is dependent upon the amount of water added to the beaker and is expressed in units of centimeters per milliliter. The y-intercept corresponds to the thickness of the glass forming the bottom of the beaker.

Treatment Session II. It was anticipated that the first session might be confusing to the subjects because (a) the use of a TI-81 calculator for data analysis was unfamiliar to them, (b) they had a limited understanding of linear mathematical functions, (c) they had had little opportunity, if any, to build mathematical models from data sets, (d) their experiences with interpreting slopes and y-intercepts had been limited to mathematical interpretations, and (e) their experiences with science concepts had been based more on the products of science than on the processes of scientific inquiry. Therefore, the second session began with a 20-minute review of the scientific inquiry process, linear functions from a mathematical perspective, and interpretation and prediction based on the linear model determined by the analysis of the antifreeze data during the first session. An outline of the Session II introduction may be found in Appendix K.

Following the introduction, subjects in Group A then spent the next 20 minutes collecting total mass/number of pennies, total mass/number of nickels, and length/mass data as described in Session II activity sheets (Appendix C). They then spent 60 minutes building, interpreting, and using linear mathematical models derived from the three data sets just collected. During the remaining 20 minutes, the subjects went back over their activity sheets from Session I, completing and correcting parts, as necessary, and discussing, within the experimental pairs, how all four experiments were related mathematically. Immediately after the review session, Group B students spent the next 60 minutes building, interpreting, and using the linear mathematical models derived from the three data sets given to them in the activity sheets (Appendix D). For the next 20 minutes, the Group B subjects went back over their activity sheets from Session I, as described above for Group A. During the remaining 20 minutes, Group B students worked together, as randomly paired, to list as many pairs of science variables as they could think of which would be possible for elementary school children to study by a scientific inquiry method.

The first experiment in Session II was based on the relationship between the number of pennies placed in a plastic cup (X) and the total mass of the pennies and the cup (Y). In this case the slope corresponds to the average mass of a penny and the y-intercept gives the value for the mass of the empty plastic cup. Subjects then built, interpreted, and used a second linear model based on analogous data obtained from an experiment involving nickels instead of pennies.

The final experiment of the treatment sessions involved a simple, yet important physical concept: the elongation of a spring. This experiment, as typically conducted in physics classes, involves the determination of the characteristic spring constant for the given spring, where the spring constant is expressed in units of Newtons per centimeter and is represented by the variable k in the Hooke's Law equation $F = kx$. In the Hooke's Law equation F represents the force attached to the spring and x represents the change in length of the spring due to the force exerted on the spring. Conceptually, it is rather difficult to study spring elongation in the Hooke's law form because (a) it involves a mass to force conversion, (b) change in length rather than total length of the spring is considered, and (c) determining the force (dependent variable) as a function of the elongation (independent variable) is experimentally more difficult than determining the length of the spring as a function of the attached mass. By treating the total length of the spring (X) as the dependent variable and the mass of the objects attached to the spring (Y) as the independent variable, the experiment becomes conceptually simple and appropriate for upper elementary school children to conduct.

The length of the spring was determined by Group A subjects by reading the scale value, corresponding to the last coil of the spring, on the meter stick that had been mounted behind the spring such that the 0.0 centimeter level lined up with the first coil of the spring. Since the known mass was engraved on each object, the mass of the attached objects was obtained by adding the individual masses of the hanger and the weights. The resulting slope is expressed in units of centimeters per gram. The slope corresponds to the increase in total length of the spring which is due to an increase of one gram in the total mass attached to the spring. The y-intercept corresponds to the length of the spring, expressed in units of centimeters, when no objects are attached.

Measures

Posttest and Workshop Evaluation. The posttest was administered to each subject during a regularly scheduled class period for ED4 or MATH1. Individually, subjects completed the posttest (Appendix E) and workshop evaluation form (Appendix F) during the 50-minute time period. No student was allowed to leave early. Each student was given a TI-81 calculator to use while completing the posttest.

The workshop evaluation form was designed to ascertain subjects' perceptions of the project. In particular, the workshop evaluation was conducted to determine if there was an overall difference in the way subjects in the two treatment groups perceived the project with respect to importance of content, usefulness to preservice teachers, and level to which they enjoyed participation in the project.

Scores on the posttest were obtained by a blind, double grading procedure. This scoring technique was used because the open-ended nature of the posttest items made scoring results dependent on graders' interpretations of subjects' responses. The two graders worked from a common set of criteria for scoring the posttest items holistically. A score of 1.0, 0.5, or 0.0 was assigned for each item. The posttest score was the sum of all 36 item scores. Thus, the maximum score was 36 points. Responses on the project evaluation form were tallied and group means and standard deviations were calculated.

Background Information from Student Records. After the faculty members teaching MATH1 and ED4 had agreed to have their students participate in the project and their respective department heads had approved the project, the Academic Dean of the University granted permission to access student records. Following an initial discussion, a letter was submitted to the Dean defining the complete details of the study. A copy of this letter, with all identifying information omitted, may be found in Appendix L. The Dean granted permission to access student records according to the guidelines outlined in the letter.

Subjects signed the "Consent to Act as a Human Subject" forms (Appendix J) immediately after the introduction portion of the first treatment session, thereby, granting me permission to access their academic records. The consent forms were then delivered to the Office of the Registrar where they were filed in each subject's academic folder as the background information was obtained from the folder and recorded on a Student Background Information Sheet (Appendix M). Transcript information was obtained directly from the computerized database.

Post-Treatment Interviews. Six subjects were selected from the ED4 class to participate in the interview phase of this project. One subject selected was a Group A subject who scored very low on the posttest. Two other Group A subjects were interviewed: (a) one who scored approximately at the mean Group A posttest score and (b) one who scored very high on the posttest. The three subjects from Group B who were interviewed were selected in the same manner. The interviews were conducted seven to eight weeks after the posttest was administered. These interviews were conducted to provide additional insights into differences in group responses to treatment sessions and the posttest. The six subjects were each paid ten dollars for participating in an individual 30-minute interview.

Statistical Measures. Two-sample t-tests were conducted on the overall posttest score, each individual item score (P1 through P36), and on 16 subscores of the posttest (as defined earlier in this chapter). In a similar manner, two-sample t-tests were conducted for the 16 background variables and 12 evaluation form responses. A measure of the reliability of the posttest was obtained by determining Cronbach coefficient alpha values for the posttest and for five subsets of the posttest: DATASETS, VERBAL, ALGEBRAIC, FAMILIAR, and UNFAMILIAR. Several repeated-measures MANOVAs were run on sets of subscores.

CHAPTER V

RESULTS

There were three distinct phases in this study. During the first phase, the posttest was validated, both the original and final forms of the posttest were pilot tested, and the treatment activities were piloted. The second phase was the treatment and posttest phase. All treatment sessions and measures used for analysis were incorporated in this phase. During the last phase, six subjects, three from each treatment group were interviewed in an attempt to obtain a clearer picture of subjects' thought processes during the treatment and posttest phase. The results of each phase of the study are summarized within this chapter.

The Pilot Study

Validity of the Posttest

In general, members of the validation team reported a high degree of validity in the posttest. All nine agreed that the test content strongly paralleled the objectives in both content and process. In response to the question "Do the test and the treatment activities emphasize the same concepts and procedures in approximately the same proportions?" five validators commented on minor problems with lack of parallelism between the treatment activity sheets and the posttest.

Most of these concerns were addressed by revising the treatment activity sheets. Three members of the validation team were concerned that the last problem on the test required that subjects deal with a concept not presented in the treatment sessions: the y-intercept lies outside the domain of the model and therefore has no physical interpretation. This question was omitted from the final posttest.

Pilot Testing of Posttest: Original Form

The mean score on the posttest obtained by the 17 Natural Science students who piloted the initial form of the posttest was 16.53 (52% correct), based on only the 32 items common to both the initial and final forms of the posttest. The 13 items on the original posttest which did not appear on the final form of the posttest were not considered in this analysis. During this first pilot study, students were allowed two hours to complete the posttest. Students were asked to record their starting and stopping times. The mean required time was 82 minutes with a standard deviation of 14 minutes. This information was used to revise the posttest so that subjects in the experimental groups could complete the test in 50 minutes. Three complete problems and a total of 13 items were omitted from the initial posttest. In addition, the initial posttest contained three items that required the use of the TI-81 calculator to fit the model. The final posttest did not require any data analysis directly using the TI-81 calculator. The four new items on the final form of the posttest were added to increase the number of items requiring predictions using mathematical models. The raw data and detailed mean posttest results for this pilot study may be found in Appendix N.

Pilot of Treatment Activities and Final Form of the Posttest

The Group A treatment activities and the posttest in final form were piloted by 10 Physics students. Students in this group were given the activity sheets and asked to follow the directions and answer the questions. The students completed the activities with no difficulties. The mean score on the posttest for this group was 30.9 (86%).

The raw data and detailed mean posttest results for this pilot study may be found in Appendix O.

This second pilot study was used mainly to gauge the time required for students to complete the revised form of the posttest. During the regularly scheduled 3-hour laboratory period, the Physics students completed the activities planned for subjects in Treatment Group A, Sessions I and II. Five days later, during a 50-minute class period, the students completed the posttest. The time required for the students to complete the test was between 15 and 40 minutes, with a mean time of 26 minutes. This result indicated that the 50-minute time interval for administering the posttest to the experimental groups should be adequate.

The Experimental Study

The major results of interest are those related to the nine research questions. However, analysis of the background data provides a check of the effectiveness of the randomization procedures used to assign subjects to treatment groups. In addition, analysis of responses on the workshop evaluation form provides a check of perceptional and attitudinal differences between subjects in the two treatment groups. The background and evaluation form analyses are important for eliminating the possibility that any observed group differences on the posttest were due to background and perceptional and attitudinal differences within the two groups.

Interrater Reliability

The posttests were blind scored. Each posttest was scored by two raters. Differences in item score assignments by the two raters are detailed in Appendix P. Each rater assigned scores for 1872 items (52 subjects x 36 items per subject). Of the

1872 items scored, the raters agreed on 1803 items. This corresponds to an interrater reliability of 96.3%. Forty-four discrepancies (2.2%) were accounted for by misapplication of established scoring criteria. These changes were made during discussions between the raters. Only 30 items (1.5%) were scored differently because of rater interpretation differences. Scores on these 30 items were adjusted by mutual agreement of the raters.

Reliability of the Posttest

A measure of the reliability of the posttest was obtained by analysis of the posttest results. Scores for all subjects who participated in the experimental portion of this study were pooled for this analysis. Cronbach coefficient alpha values were determined for the posttest and for five subsets of the posttest, corresponding to the subscores DATASETS, VERBAL, ALGEBRAIC, FAMILIAR, and UNFAMILIAR. The Cronbach coefficient alpha values for the posttest and each subset are given in Table 1. These values indicate that the internal consistency reliabilities for the posttest and the subsets of the posttest are very high.

Table 1

Cronbach Coefficient Alpha Values for the Posttest and Five Subsets of the Posttest

Items	Cronbach Coefficient Alpha
POST	.964
DATASETS	.885
VERBAL	.940
ALGEBRAIC	.915
FAMILIAR	.948
UNFAMILIAR	.919

Background and Workshop Evaluation Form Analyses

Background Analyses. Analyses of background variables by group, based on the 13 experimental pairs in Treatment Group A and the 12 experimental pairs in Treatment Group B, were conducted on 16 background variables. The variables analyzed were high school rank (HSR), high school grade point average (HGPA), score on the mathematical portion of the SAT (MSAT), score on the verbal portion of the SAT (VSAT), score on the Test of Standard Written English (TSWE), total number of course credits in high school mathematics (HMHRS), total number of course credits in high school science (HSHRS), grade point average in high school mathematics courses (HMGPA), grade point average in high school science courses (HSGPA), college predicted grade point average (PGPA), current college grade point average (CGPA), total number of semester hours completed in college (CHRS), total number of semester hours completed in college mathematics courses (CMHRS), total number of semester hours completed in college science courses (CSHRS), grade point average in college mathematics courses (CMGPA), and grade point average in college science courses (CSGPA). The group means, two-sample t-statistics, and corresponding p-values are given in Appendix Q. The means are slightly higher for Group A than for Group B on four background variables: (a) total number of course credits in high school mathematics, (b) current college grade point average, (c) total number of semester hours completed in college science courses, and (d) grade point average in college science courses. Group B means are slightly higher on the remaining 12 background variables. However, the 16 p-values range from .36 to .98, indicating that all differences between the groups were not significant.

Workshop Evaluation Form Analyses. Response analyses by group, based on the 16 experimental pairs in Treatment Group A and the 12 experimental pairs in

Treatment Group B, were conducted on the 12 response items presented on the workshop evaluation form (Appendix F). The group means, two-sample t-statistics, and corresponding p-values are given in Appendix R. The means are higher for Group A than for Group B on four items and higher for Group B on the remaining eight items. The item p-values range from .12 to .96, indicating that differences between the groups were not significant.

Based on the results of the background and the workshop evaluation form analyses, the groups appear to be comparable. That is, the analyses indicate that differences in group mean scores on the posttest and various subsets of the posttest are not due to differences in subjects' academic backgrounds or subjects' perceptional and attitudinal differences. Therefore, it is reasonable to consider differences in group mean scores as being due to treatment effects.

Posttest Analyses

The raw data by individual subjects may be found in Appendix S, and the raw data by experimental pairs may be found in Appendix T. The posttest results are summarized in Table 2 and Table 3. Two-sample t-tests by treatment group were conducted on the posttest, on 16 subsets of the posttest, and on each individual test item. The variable definitions for the subscores corresponding to the subsets of the posttest considered are given in Chapter IV.

Research Question #1. Will the mean scores for the two treatment groups on the posttest (POST) differ? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on the overall posttest ($p = .0023$).

Table 2

Posttest Results by Item: Group Means, t-Statistics, and p-Values

Item	Group A Mean	SD_A	Group B Mean	SD_B	t-Value	p-Value
P1	.820	.240	.958	.144	-1.72	.0989
P2	.506	.265	.736	.181	-2.51	.0194
P3	.269	.260	.451	.356	-1.47	.1550
P4	.532	.282	.771	.225	-2.33	.0290
P5	.455	.346	.590	.212	-1.17	.2560
P6	.340	.265	.556	.228	-2.17	.0402
P7	.782	.249	.917	.195	-1.50	.1479
P8	.391	.191	.646	.129	-3.88	.0008
P9	.186	.181	.479	.310	-2.92	.0077
P10	.558	.291	.729	.198	-1.71	.1017
P11	.077	.188	.125	.226	-.58	.5674
P12	.442	.423	.694	.407	-1.52	.1431
P13	.436	.351	.653	.261	-1.74	.0947
P14	.391	.260	.625	.272	-2.20	.0381
P15	.410	.237	.694	.274	-2.78	.0106
P16	.532	.242	.764	.200	-2.60	.0161
P17	.494	.222	.611	.239	-1.28	.2150
P18	.455	.315	.764	.273	-2.61	.0156
P19	.218	.249	.417	.289	-1.85	.0776
P20	.321	.240	.535	.267	-2.11	.0458
P21	.353	.330	.563	.264	-1.75	.0940
P22	.365	.300	.479	.291	-.96	.3462
P23	.295	.304	.375	.225	-.74	.4649
P24	.237	.347	.472	.316	-1.77	.0908
P25	.429	.183	.625	.199	-2.56	.0176
P26	.154	.217	.438	.304	-2.70	.0128
P27	.500	.306	.688	.241	-1.69	.0144
P28	.237	.240	.444	.237	-2.17	.0405
P29	.333	.373	.660	.220	-2.64	.0148
P30	.218	.249	.576	.265	-3.49	.0020
P31	.372	.217	.583	.163	-2.74	.0117
P32	.077	.188	.229	.249	-1.73	.0962
P33	.442	.208	.465	.202	-.28	.7824
P34	.077	.188	.083	.195	-.08	.9339
P35	.256	.251	.472	285	-2.01	.0560
P36	.218	.227	.410	.260	-1.97	.0611

Table 3

Posttest Results by Selected Subtests: Group Means, t-Statistics, and p-Values (df = 23)

Subtest (# Items)	Group A Mean (% Correct)	SD_A	Group B Mean (% Correct)	SD_B	t-value	p-value
POST (36)	13.18 (36.61)	5.65	20.28 (57.33)	4.63	-3.42	.0023
DATASETS (12)	5.36 (44.66)	2.05	7.65 (63.77)	1.65	-3.06	.0055
VERBAL (12)	4.51 (37.55)	2.39	6.95 (57.93)	2.18	-2.66	.0140
ALGEBRAIC (12)	3.31 (27.62)	1.98	5.67 (47.28)	1.57	-3.28	.0033
BUILD1 (2)	1.60 (80.13)	.42	1.88 (93.75)	.31	-1.82	.0813
BUILD2 (2)	.65 (32.69)	.51	1.07 (53.47)	.44	-2.16	.0417
BUILD (4)	2.26 (56.41)	.80	2.94 (73.61)	.63	-2.36	.0269
SLOPE1 (6)	2.41 (40.17)	1.00	3.75 (62.50)	.87	-3.56	.0017
INTERCEPT1 (6)	2.93 (48.82)	1.24	3.90 (64.93)	1.06	-2.09	.0482
SLOPE2 (6)	1.45 (24.15)	.92	2.85 (47.57)	1.27	-3.19	.0041
INTERCEPT2 (6)	1.63 (27.24)	.87	2.23 (37.15)	.74	-1.83	.0801
INTERPRET (24)	8.42 (35.10)	3.33	12.73 (53.04)	3.51	-3.14	.0045
USEY (6)	2.06 (34.40)	1.54	3.62 (60.30)	1.02	-2.94	.0073
USEX (2)	.44 (21.79)	.44	.99 (49.31)	.26	-3.75	.0010
USE (8)	2.50 (31.25)	1.91	4.60 (57.55)	1.30	-3.31	.0031
FAMILIAR (18)	7.51 (41.74)	3.19	11.60 (64.47)	2.38	-3.61	.0015
UNFAMILIAR (18)	5.67 (31.48)	2.59	8.67 (48.19)	2.68	-2.85	.0091

Research Question #2. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models given data tables (DATASETS)? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest (p = .0055).

Research Question #3. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models given verbal descriptions (VERBAL)? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest (p = .0140).

Research Question #4. Will treatment group mean scores differ on posttest items involving interpreting and using mathematical models given the models as algebraic formulas (ALGEBRAIC)? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0033$).

Research Question #5. Will treatment group mean scores differ on posttest items involving building mathematical models (BUILD)? Results: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0269$).

In order to provide further detail, this score was separated into two subscores: (a) BUILD1, the score on items involving building mathematical models given data tables and the values of a and b derived from the tables using a TI-81 calculator and (b) BUILD2, the score on items involving building mathematical models from verbal descriptions. The t-test results for BUILD1 ($p = .0813$) and BUILD2 ($p = .0417$) indicate that the two groups scored similarly on tasks involving building mathematical models given data tables while Group B scored significantly higher than Group A on items involving building mathematical models from verbal descriptions.

Research Question #6. Will treatment group mean scores differ on posttest items involving interpreting mathematical models (INTERPRET)? Results: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0045$).

In order to provide further detail, this score was separated into four subscores: (a) SLOPE1, the score on items involving identifying magnitudes and units for slopes in mathematical models, (b) SLOPE2, the score on items involving writing physical interpretations of slopes in mathematical models, (c) INTERCEPT1, the score on items involving identifying magnitudes and units for y-intercepts in mathematical models, and (d) INTERCEPT2, the score on items involving writing physical interpretations of y-intercepts in mathematical models. The t-test results for SLOPE1 ($p = .0017$), SLOPE2 ($p = .0041$), INTERCEPT1 ($p = .0482$), and INTERCEPT2 ($p = .0801$) indicate that Group B scored significantly higher than Group A on three of the four types of model interpretation items. The two groups scored similarly on items involving writing physical interpretations of y-intercepts in mathematical models. The difference between group performance was greater for tasks involving identifying and interpreting slopes than for tasks involving identifying and interpreting y-intercepts.

Research Question #7. Will treatment group mean scores differ on posttest items involving using mathematical models (USE)? Results: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0031$).

In order to provide further detail, this score was separated into two subscores: (a) USEY, the score on items involving using mathematical models to predict values of y given values of x and (b) USEX, the score on items involving using mathematical models to predict values of x given values of y. The t-test results for USEY ($p = .0073$) and USEX ($p = .0010$) indicate that Group B scored significantly higher than Group A on both types of prediction items. The difference between group performances was greater for tasks involving predicting x values given y than for tasks involving predicting y values given x.

Research Question #8. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions (FAMILIAR)? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0015$).

Research Question #9. Will treatment group mean scores differ on posttest items involving building, interpreting, and using mathematical models based on physical contexts different from the contexts utilized during treatment sessions (UNFAMILIAR)? Result: As noted in Table 3, the data support the conclusion that Group B scored significantly higher than Group A on this subtest ($p = .0091$).

Additional Comments. The individual item mean scores displayed in Table 2 show that Group B scored higher than Group A on each of the 36 test items. The difference between group means is significant at an $\alpha = .05$ level on 17 of the 36 items. The item-by-item comparison of group means is represented graphically in Figure 9.

Figure 10 displays the mean differences between groups in terms of percent of correctly answered items when posttest items are grouped into three sets of items: (a) items involving building, interpreting, and using mathematical models given data sets (DATASETS), (b) items involving building, interpreting, and using mathematical models given verbal descriptions (VERBAL), and (c) items involving interpreting and using mathematical models given the models as algebraic formulas (ALGEBRAIC). The differences between group means on these subscores (DATASETS, 19.1%; VERBAL, 20.4%; ALGEBRAIC, 19.7%) are essentially equal. Both groups scored better on tasks that began with data sets than on tasks that began with verbal descriptions. Both groups scored lowest on tasks that began with models expressed as algebraic formulas. A Wilks' lambda value of .99 ($F = .046$; $df = 2, 22$; $p = .96$) for testing two-way interactions was obtained when a repeated measures MANOVA was run. There is no indication of group-by-subscore interaction.

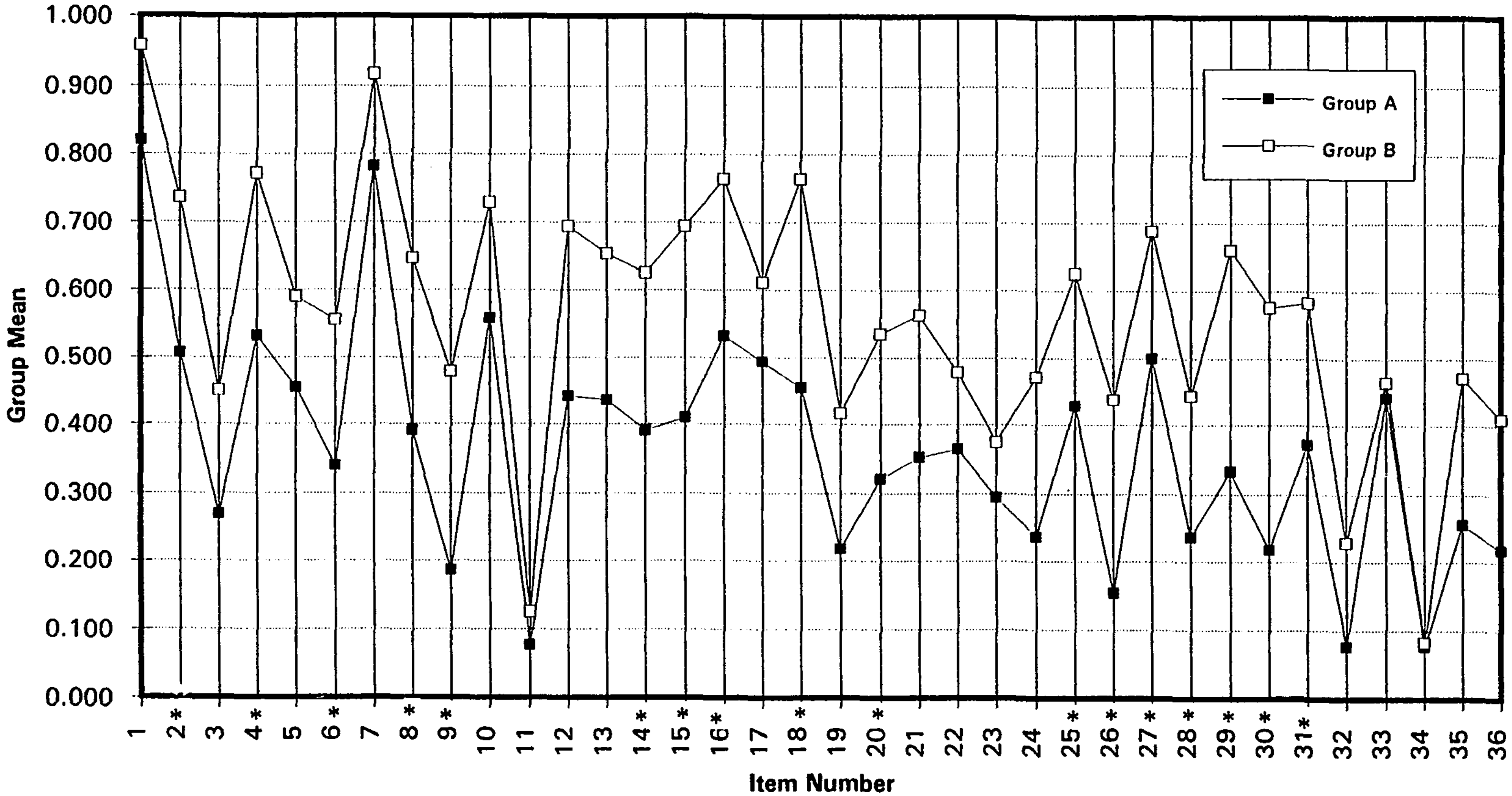

Figure 9. Item-by-Item Comparison of Group Means
(* indicates item group difference is significant at α = .05 level)

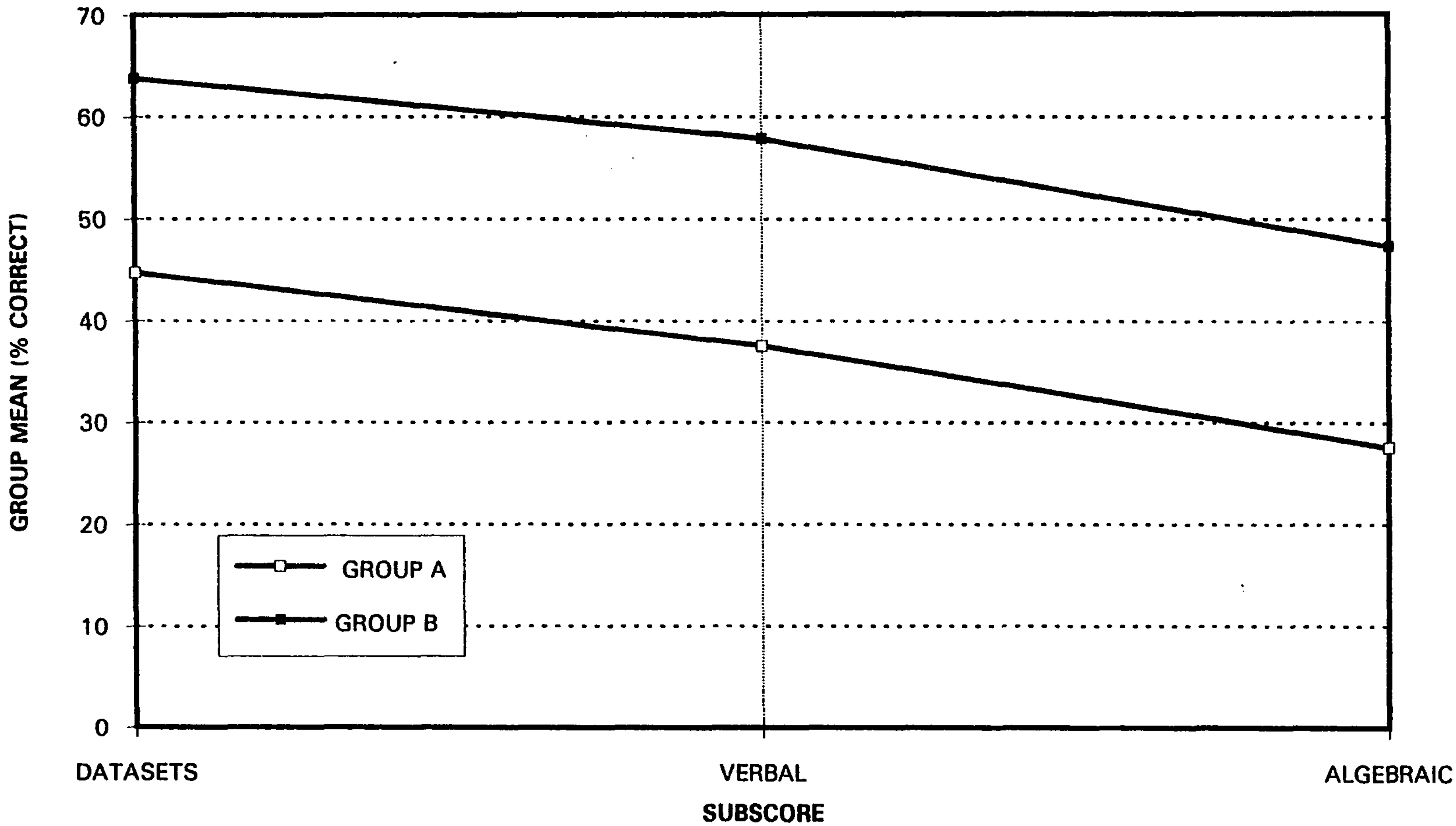

Figure 10. Group-by-Subscore Interaction Plot. Subscores: starting with data sets (DATASETS), verbal descriptions (VERBAL), and algebraic formulas (ALGEBRAIC)

Figure 11 displays the mean differences between groups in terms of percent of correctly answered items when posttest items are grouped by task type into building (BUILD), interpreting (INTERPRET), and predicting (USE) tasks. The differences between group means in building models (17.2%) and interpreting models (17.9%) are essentially equal. The group mean differences are notably higher (26.3%) for making predictions based on models. Group A scored higher on building tasks (56.4%) than on interpreting and predicting tasks. Group A scored approximately the same on interpreting (35.1%) and predicting (31.3%) tasks. The results were similar for Group B. Group B scored higher on building tasks (73.6%) than on interpreting and predicting tasks. Group B scored approximately the same on interpreting (53.0%) and predicting (57.6%) tasks. However, Group A scored better on interpreting tasks than on predicting tasks (3.8%), while Group B scored better on predicting tasks than on interpreting tasks (4.5%). A Wilks' lambda value of .92 ($F = .963$; $df = 2, 22$; $p = .40$) for testing two-way interactions was obtained when a repeated measures MANOVA was run. There is no indication of group-by-subscore interaction.

Three additional MANOVAs were run on subscores of BUILD, INTERPRET, and USE, respectively. Figure 12 displays the mean differences between groups in terms of percent of correctly answered items when BUILD items are grouped by task type into BUILD1 and BUILD2 tasks. Both groups scored higher on tasks involving building mathematical models from data tables (BUILD1: Group A, 80.1%; Group B, 93.8%) than on tasks involving building mathematical models from verbal descriptions (BUILD2: Group A, 32.7; Group B, 53.5%). The within-group difference on the two types of building tasks was greater for Group A (47.4%) than for Group B (40.2%). A Wilks' lambda value of .97 ($F = .600$; $df = 1, 22$; $p = .45$) for testing

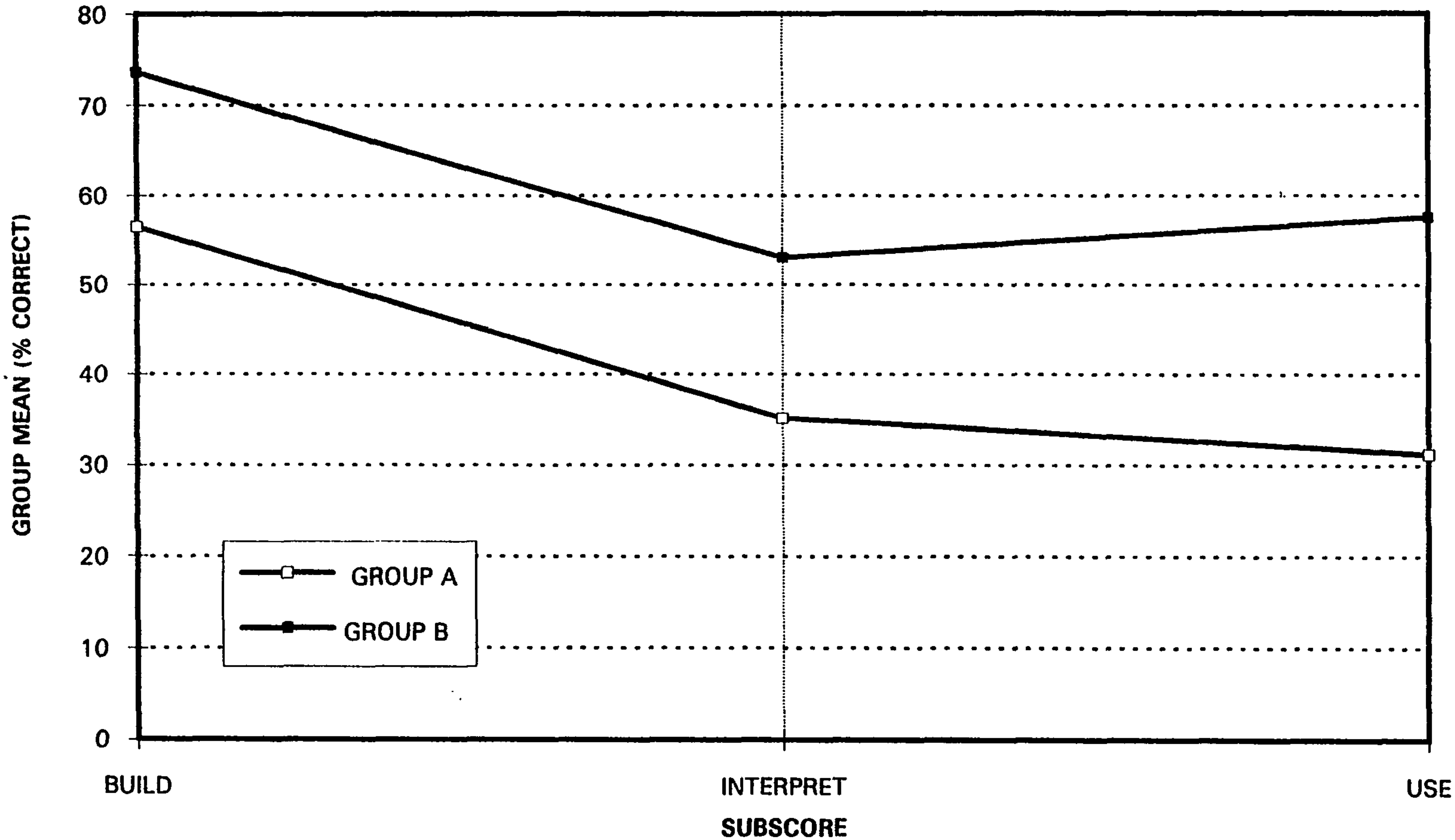

Figure 11. Group-by-Subscore Interaction Plot. Subscores: building models (BUILD), interpreting models (INTERPRET), and using models (USE).

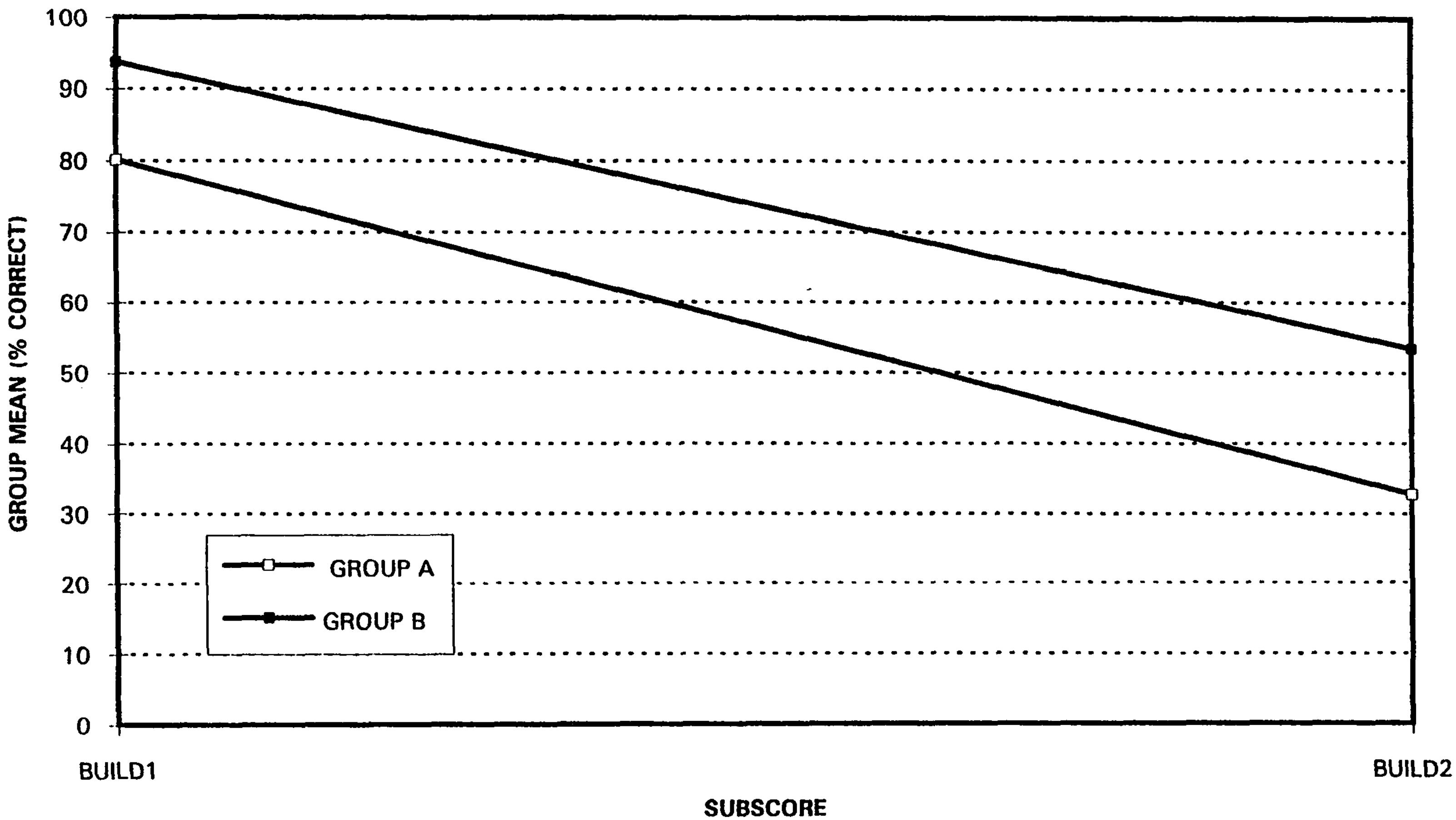

Figure 12. Group-by-Subscore Interaction Plot. Subscores: building models from data sets (BUILD1) and building models from verbal descriptions (BUILD2).

two-way interactions was obtained when a repeated measures MANOVA was run. There is no indication of the presence of a BUILD1*BUILD2 interaction.

Figure 13 displays the mean differences between groups in terms of percent of correctly answered items when INTERPRET items are grouped by task type into SLOPE1, INTERCEPT1, SLOPE2, and INTERCEPT2 tasks. The four subscores of were grouped into identification (SLOPE1 and INTERCEPT1) and interpretation (SLOPE2 and INTERCEPT2) tasks. Three Wilks' lambda values of interest were obtained by the repeated measures MANOVA analysis. A Wilks' lambda value of .99 (F = .27; df = 1, 23; p = .61) for testing the two-way interactions between SLOPE1 and INTERCEPT1 was obtained. This result indicated the absence of a SLOPE1*INTERCEPT1 interaction effect. A Wilks' lambda value of .80 (F = 5.60; df = 1, 23; p = .03) for testing the two-way interactions between SLOPE2 and INTERCEPT2 was obtained. This result indicates the presence of a SLOPE2*INTERCEPT2 interaction effect. A Wilks' lambda value of .95 (F = 1.26; df = 1, 23; p = .27) for testing the two-way interactions between identification tasks (SLOPE1 and INTERCEPT1) and interpretations tasks (SLOPE2 and INTERCEPT2) was obtained. This result indicated the absence of two-way interaction effects between identification and interpretation tasks.

Figure 14 displays the mean differences between groups in terms of percent of correctly answered items when USE items are grouped by task type into USEY and USEX tasks. Both groups scored higher on tasks involving using mathematical models to predict values of y given values of x (USEY: Group A, 34.4%; Group B, 60.3%) than on tasks involving using mathematical models to predict values of x given values of y (USEX: Group A, 21.8%; Group B, 49.3%). The within-group difference on the two types of predicting tasks was slightly greater for Group A (12.6%) than for

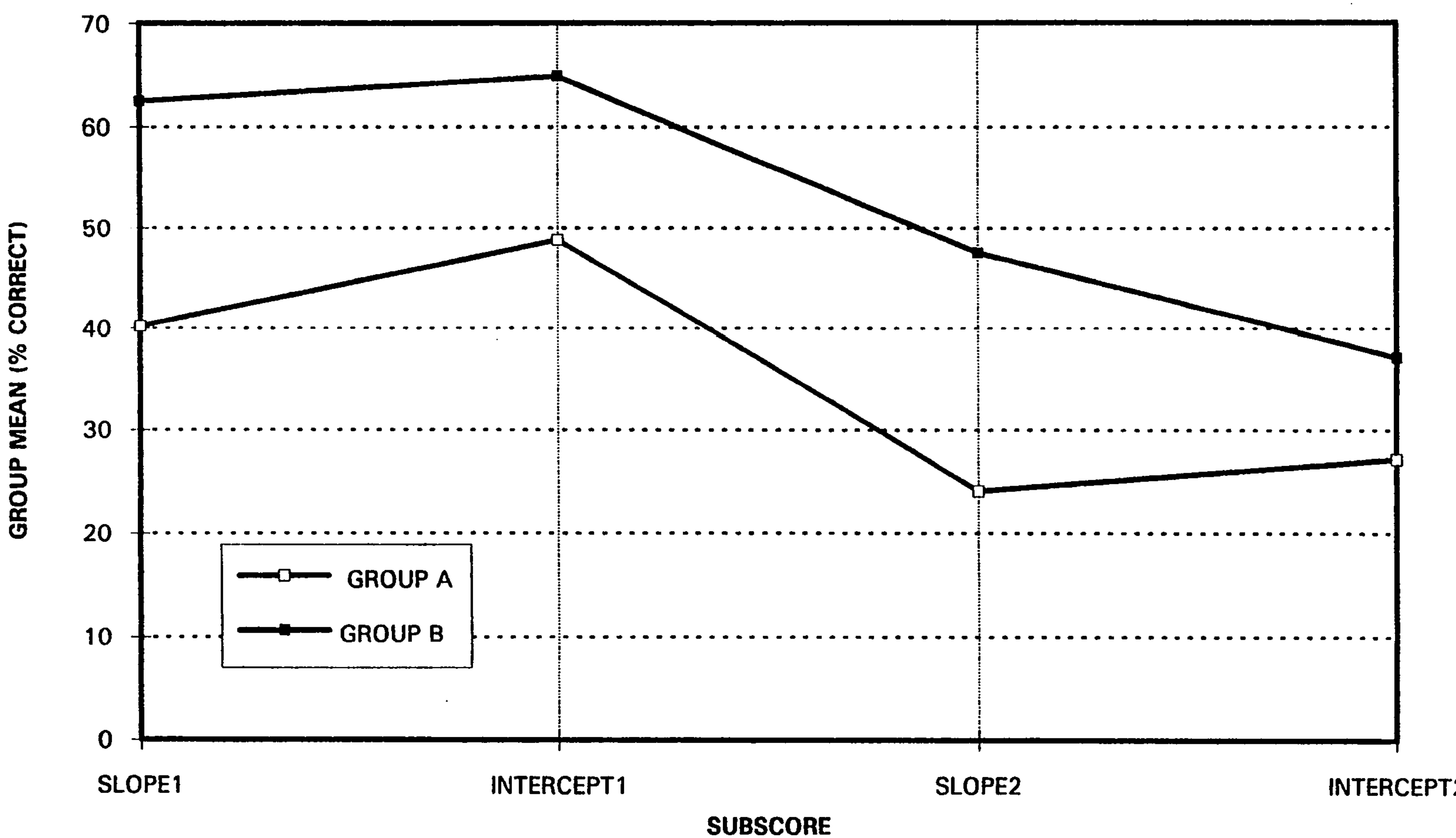

Figure 13. Group-by-Subscore Interaction Plot. Subscores: identifying magnitude and units for slope (SLOPE1), identifying magnitude and units for y-intercept (INTERCEPT1), writing physical interpretation of slope (SLOPE2), and writing physical interpretation of y-intercept (INTERCEPT2).

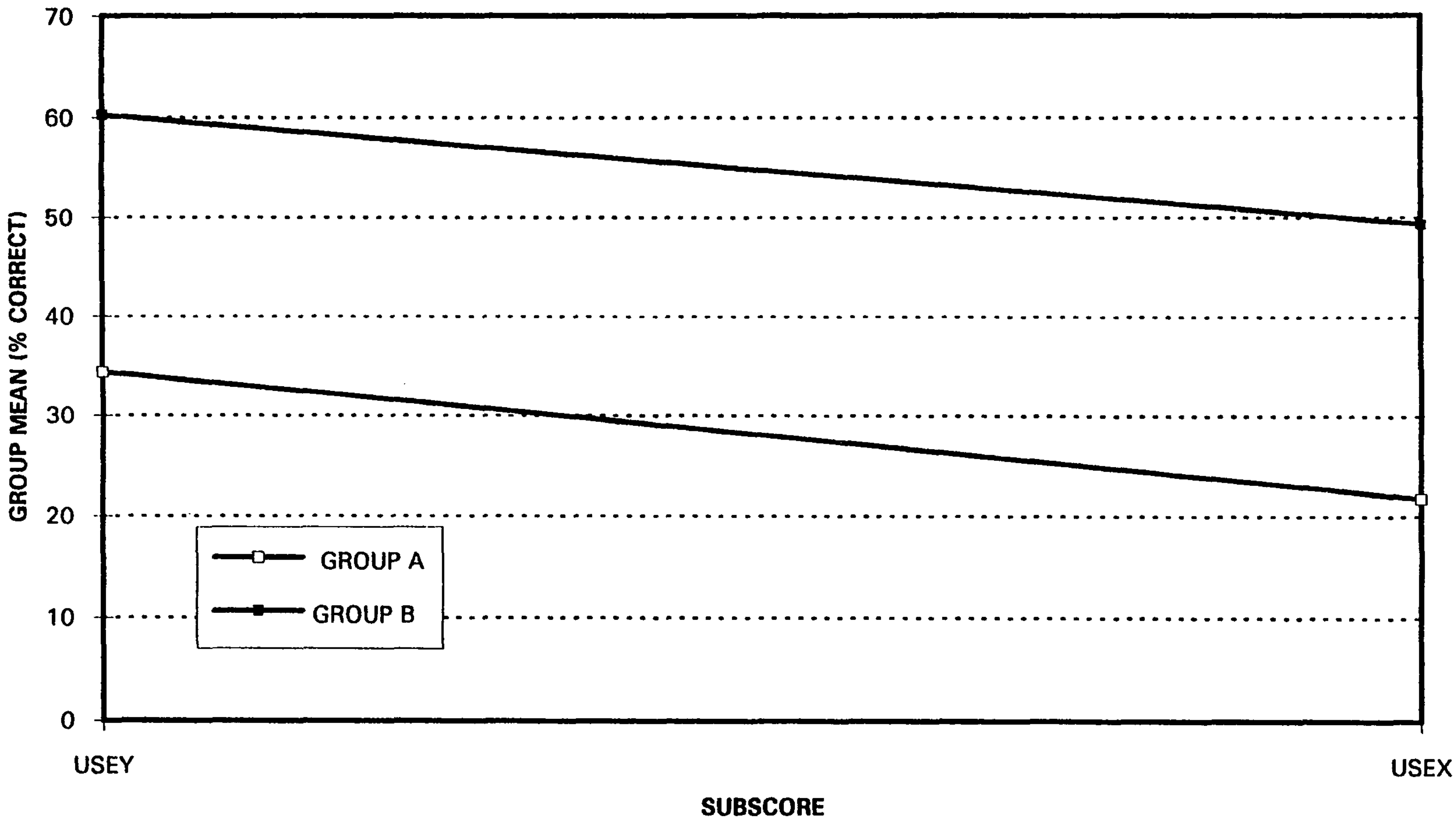

Figure 14. Group-by-Subscore Interaction Plot. Subscores: predicting values of y given value of x (USEY) and predicting values of x given value of y (USEX).

Group B (11.0%). A Wilks' lambda value of .99 ($F = .058$; $df = 1, 23$; $p = .81$) for testing two-interactions was obtained when a repeated measures MANOVA was run. There is no indication of the presence of a USEY*USEX interaction.

Figure 15 displays the mean differences between groups in terms of percent of correctly answered items when posttest items are grouped by task type into (a) items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions (FAMILIAR) and (b) items involving building, interpreting, and using models based on physical contexts different from the contexts utilized during treatment sessions (UNFAMILIAR). Both groups scored higher on the set of items in familiar contexts than on the set in unfamiliar contexts. The differences between group means on these two subscores (FAMILIAR, 22.7%; UNFAMILIAR, 16.7%) are moderately different. That is, while Group B scored higher than Group A on both sets of items, the difference between group scores was less for unfamiliar contexts than for familiar contexts. A Wilks' lambda value of .90 ($F = 2.43$; $df = 1, 23$; $p = .13$) was obtained when a repeated measures MANOVA was run. There is no indication of a group-by-subscore interaction.

Summary. Group B scored higher than Group A ($\alpha = .05$) on (a) the overall posttest (POST); (b) items involving building, interpreting, and using mathematical models given data tables (DATASETS); (c) items involving building, interpreting, and using mathematical models given verbal descriptions (VERBAL); (d) items involving interpreting and using mathematical models given the models as algebraic formulas (ALGEBRAIC); (e) items involving building mathematical models (BUILD); (f) items involving interpreting mathematical models (INTERPRET); (g) items involving using mathematical models (USE); (h) items involving the same physical contexts utilized during treatment sessions (FAMILIAR); (i) items involving physical contexts different

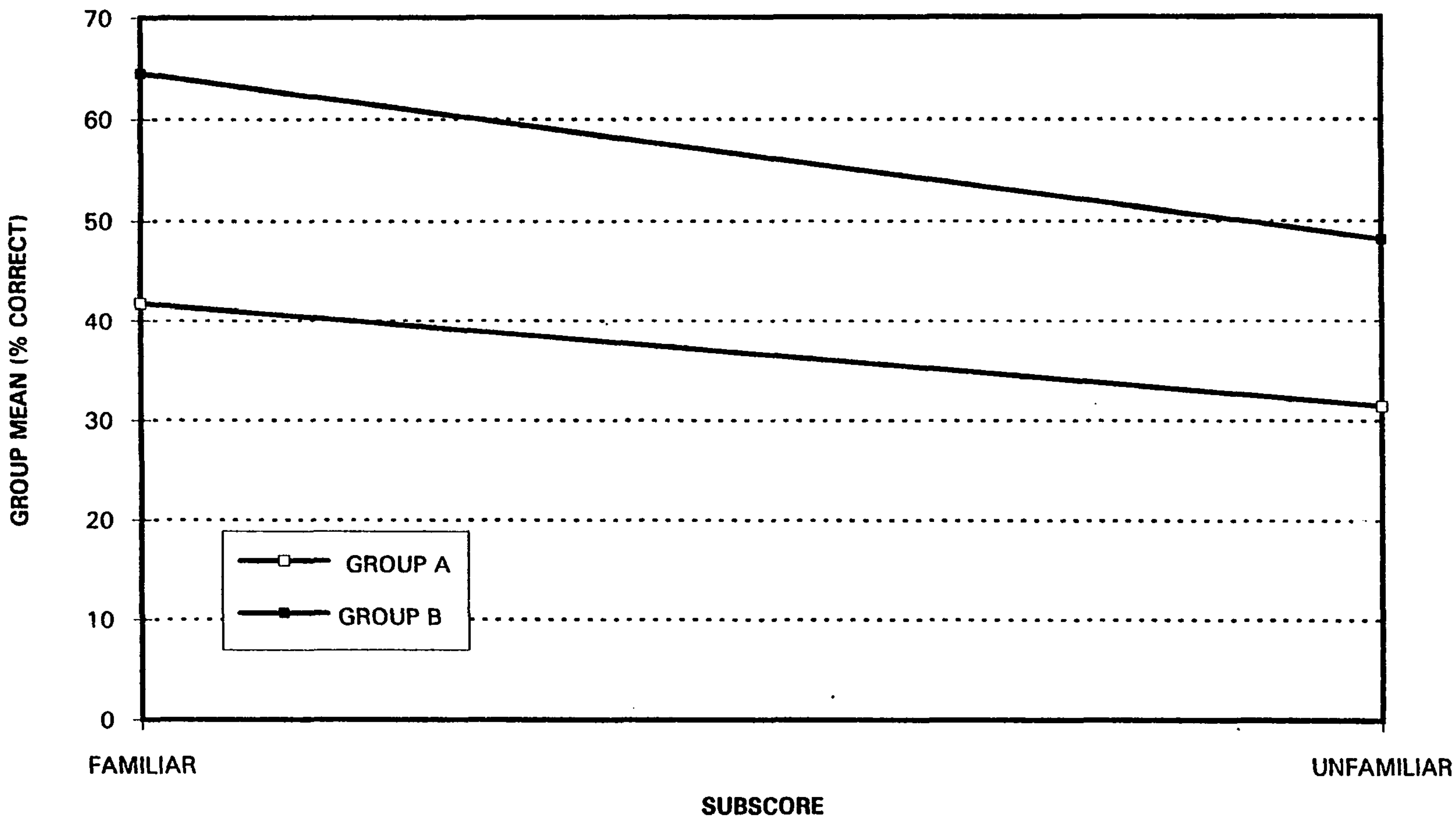

Figure 15. Group-by-Subscore Interaction Plot. Subscores: familiar contexts (FAMILIAR) and unfamiliar contexts (UNFAMILIAR).

from the contexts utilized during treatment sessions (UNFAMILIAR); and (j) 17 of the 36 individual posttest items (P1 through P36). Both groups scored higher on tasks that began with data sets than on tasks that began with verbal descriptions or algebraic formulas, and both groups scored higher on items framed in the same physical contexts utilized during treatment sessions than on items framed in physical contexts different from the contexts utilized during treatment sessions. The only interaction effect indicated is a group interaction effect between interpreting the y-intercept and interpreting the slope.

The Individual Interviews

Individual interviews of six subjects were conducted in an attempt to gain some insight into factors that might explain why, overall, subjects in Group B outscored subjects in Group A on all 36 posttest items. Interview questions were geared toward pinpointing differences in thinking between subjects in the two treatment groups. In particular, the key element of interest from the interviews centered around the differences in internal and external representations used by the subjects during building, interpreting, and using mathematical models activities, as revealed by the subjects themselves. In this section the six subjects interviewed are identified as (a) LOW-A, a Group A subject who scored low on the posttest, (b) LOW-B, a Group B subject who scored low on the posttest, (c) MEAN-A, a Group A subject whose posttest score was near the Group A mean, (d) MEAN-B, a Group B subject whose posttest score was near the Group B mean, (e) HI-A, a Group A subject who scored high on the posttest, and (f) HI-B, a Group B subject who scored high on the posttest.

A comparison of interview comments by LOW-A and LOW-B indicate that these two subjects had essentially no understanding of mathematical modeling and little sense of the purpose of the workshop. Both subjects assumed the role of spectator while their partners did the activities. However, there appeared to be an important difference in the thinking expressed by the two. LOW-A was trying to make sense of the workshop content and felt that if the workshop had lasted longer, she would have understood the content.

> [Interpreting the slope and y-intercept] were equally hard, probably because I just didn't have enough time to sit and think about it. . . . I was kind of lost from the beginning . . . everything [was] occurring at a pace that I thought was fast for something of this nature. . . . [I was] nervous about just not knowing what was going on at certain points. I was sitting there wondering "is everybody getting this or is it just me." As [my partner] was doing [the activity], there were certain things that I understood--that made sense. At some later point in time I would enjoy going back . . . to see exactly what it was that we were doing, or that I should have been doing; what are the points that I missed that I could key in on and make it all make sense. (LOW-A)

LOW-B seemed to be concerned only with finishing the workshop activities and the posttest; making sense out of the experience did not seem to concern her. She also expressed no concerns about needing more time to learn the material.

> [The workshop] was worthwhile but I was kind of bored. It was interesting the way the calculator worked. . . . I didn't like the rest of it. I didn't really want to be there taking the posttest, and I didn't want to be in the project. The first page [of the posttest] was easy. On the second page I didn't know if I thought it was easy because I knew how to do it or because I didn't know how to do it. I finished [the posttest] in a short amount of time. If I didn't know the answer I was just writing it down so I didn't know if I was putting down right or wrong answers. Problem #5 was hard. . . . Before [in the workshop] we had the data and we just put in the numbers in the calculator and got the equation. In this problem we were just given the equation. I just didn't know where to start the problem. In the problems I didn't know what to do except just try to match it

> up but I didn't know what x was or what y was, so I really didn't know how to match it up. (LOW-B)

MEAN-A and MEAN-B seemed to understand more about the mathematical modeling content in the workshop. MEAN-A seemed to struggle hard to develop a conceptual understanding of mathematical modeling. She kept reflecting on what she had done, trying to figure out why she had done it that way. She also referred back to the data tables when making predictions, and she found that having a partner to discuss answers with was helpful.

> A lot of times I would be doing the experiment and not actually thinking about what I was doing. And after I had done it, I went back and thought "Why did I do this?" That's what seemed difficult to me. [I had trouble interpreting models because] I really wasn't thinking back to what I did when collecting the data when I was trying to interpret the models. I wasn't going back and saying "Well, this is why I did it and how it would go into the model." I think it was hard to pull that together. Actually doing something was easy, but trying to figure out why you did it and what would go here, I thought that was hard. If I had not collected the data, I think it would have been harder to go back to the table and figure out what the actual answer was. By coming up with the table ourselves, . . . I thought it seemed easy to go back and know how to plug values into the problem. . . . A lot of times we went back to the table instead of the equation [when solving a problem]. . . . [My partner and I] talked about [the questions] a lot and then we came to a conclusion of what we thought [the answers] would be. I would suggest something and she would suggest something, and then we would just decide which one we thought was right. It was definitely helpful to talk with each other. (MEAN-A)

MEAN-B seemed especially concerned about her need for mental images of the situations being modeled. She also seemed to focus on the procedural patterns. In particular, she solved the prediction problems using the models, without considering values in the data tables.

> Using the calculator was fine because we had the [reference] sheet you gave us. . . . We talked about [the interpretations and problems]. There were areas I remembered what went where and what it meant and there were areas [my partner] remembered. The important thing was seeing a pattern. . . . The second session was easier. I don't know if it was [easier] because it was coins and I could see it in my mind or maybe the little introduction you gave us. And I can understand about the length of the spring. I can see the spring length increasing as the weight increases. I think that made it easier. Even though I didn't get to see it physically, I could see it in my head. [On the posttest] I remember problem #4--I remember reading it but I don't think I even attempted it. I remember the rate of three inches per minute and trying to make sense. I remember thinking about it and trying to see in my head how [the water level] would rise three inches. (MEAN-B)

The issues of mental imaging, language development, and procedural versus conceptual understanding were even more prominent in the HI-A and HI-B interviews. During the interview, HI-A focused on "making sense" of the activities and how data collection helped her "see" the relationships clearly. Apparently, HI-A and her partner did not work collaboratively on the activities.

> I thought [interpreting the models] was easy because it made sense. . . . One unit over the other unit makes sense once you look at the graph. I've been through calculus in high school and calculus in college. I am very comfortable with y = m x + b. I had never thought about slope and y-intercept in terms of a physical interpretation. I had only thought about what they mean in terms of the Cartesian plane. I guess I kind of knew [about physical interpretations] all along but never said it like that. . . . Gathering the data helped me. I need a picture. I need something I can see before I can actually use it. . . . Interpreting the model was also easier because I had collected the data. Working with the units I think that made it easier. It made sense. . . . Looking at a graph now that I have not had any experience with, I think I could interpret what the points mean in a real setting. . . . [On the posttest] I went through and did the ones I was familiar with first and saved the unfamiliar ones to last. I did them in the order 1, 3, 5, 2, 6, and 4. With #4 I had trouble creating a mental picture. I had to have a picture. I had trouble drawing it on the test. Building the model was the problem. The rest of it didn't give me any

> problems. . . . I think that working in pairs, one partner has the tendency to let the other partner do everything. I think that's why I feel comfortable doing this--because I did all the work. I didn't mind. I am an independent person. But I think [my partner] probably didn't benefit from that at all. She told me "I'm glad you are my partner or I wouldn't be able to do this." I benefited but she was at a loss. (HI-A)

HI-B's comments indicated that she was focusing on procedures rather than concepts. She stressed both her need for mental images and her problems with putting relationships into words.

> When I was doing [the activities] I kept wondering if I was doing it correctly. Is this right? Is this what we are supposed to get? I remember thinking as I went out the door [at the end of the first session] that I really wasn't certain that I had done the correct procedures. I was still having problems in the second session--I was working with procedures I was still uncertain about. I had trouble especially with the spring problems. I had a problem with inserting the information into the equation. I switched [the variables] around. Now as I look back on the activities we did in the workshop, it looks fairly easy. I am more familiar with the processes I need to do. . . . [On the posttest] the fish tank problem, #4, gave me the most difficulty. I remember having a question about the distance, the level. I had problems visualizing the problem. The problem was with building the model. . . . The problem with the physical interpretation wasn't that I didn't know what it meant. I just didn't know how to put it down in my own words. (HI-B)

Collectively, the interviews pointed to the importance of several representations involved in building, interpreting, and using mathematical models. Constructing a mental image (internal representation) of the associated situation seemed crucial to interpreting data. Language development also seemed important. HI-B's comment about knowing what the slope and y-intercept meant in the physical situation but not knowing how to put it down in her own words especially pointed to the importance of language development. The external representations of table, graph, and model (equation) also seemed important. Group B subjects had problems plugging into the model--they were not sure whether to put the given value in the place of x or y in the model. This indicated a procedural approach to solving word problems. Group A subjects tended to refer back to the data table that they had constructed to determine an approximate answer and to reflect on the variables and their units before using the model to calculate an answer. Then they looked back at the tables and see if their answers were reasonable. Graphical representations of the functional relationships were especially helpful to the subjects when they were trying to identify slopes and y-intercepts and the corresponding units.

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CHAPTER VI

CONCLUSIONS

Summary

Methodology

Fifty-two preservice elementary teachers completed two, 2-hour workshop sessions designed to help them connect mathematical and scientific perspectives of linear mathematical function using a scientific inquiry approach. The subjects were assigned to one of 25 experimental pairs. Subjects in 13 pairs (Group A) collected data in a laboratory setting before beginning tasks involving building, interpreting, and using mathematical models. Subjects in 12 pairs (Group B) were given data sets, analogous to those collected by the Group A subjects, and used the data sets to complete the same set of data analysis, interpretation, and prediction tasks as Group A. All subjects completed a 50-minute, 36-item posttest and a 12-item workshop evaluation form. Values of 16 background variables for each subject were collected from university records.

During the workshop sessions, four relationships between pairs of variables were studied: (a) total mass of a liquid and its container versus volume of liquid in the container, (b) total height from the table top to the water level in a beaker versus the volume of water in the beaker, (c) total mass of coins and cup versus the number of coins in the cup, and (d) the length of a spring versus the total mass of objects attached to the spring. Eighteen posttest items were based on relationships studied in the workshop sessions. The remaining posttest items were based on different, but analogous, relationships.

For each pair, responses on each posttest and workshop evaluation form item were averaged and used as an experimental pair response value. Pair values on each of the background variables were obtained similarly. Pair responses on each posttest and workshop evaluation form item were averaged by treatment group to obtain Group A and Group B means for each item. Group A and Group B means on background variables were obtained similarly. Two-sample t-tests by experimental group were conducted, individually, for posttest and workshop evaluation form items and background variables.

Analyses of background variables and workshop evaluation form responses were used to determine if group differences on posttest items could be attributed either to group differences in academic background or to group differences in perceptions and attitudes pertaining to the workshop, respectively. The posttest responses were further analyzed by grouping posttest items into overall posttest and subtests. Overall posttest group means (POST) and 16 subscore group means (DATASETS, VERBAL, ALGEBRAIC, BUILD1, BUILD2, BUILD, SLOPE1, INTERCEPT1, SLOPE2, INTERCEPT2, INTERPRET, USEY, USEX, USE, FAMILIAR, UNFAMILIAR) were analyzed.

Results

Group mean responses on the 12 workshop evaluation form items did not differ significantly (p-values ranged from .12 to .96). Similarly, group mean values on the 16 background variables were not significantly different (p-values ranged from .36 to .98). The analyses of workshop evaluation form responses and background variables indicated that posttest differences could not be accounted for by differences in academic preparation, attitudes toward the project, or perceptions of the project.

Collectively, Group B subjects scored higher than Group A subjects on the posttest, on all 16 subsets of the posttest considered in this analysis, and on all 36 individual test items. Inferentially, these differences in group means were significant, at an $\alpha = .05$ level, on the posttest, on 14 of the 16 subsets of the posttest considered in this analysis, and on 17 of the 36 individual posttest items. In terms of the research questions, the mean posttest scores for Group B were significantly higher than the mean scores for Group A on each of the following:

1. the overall posttest (POST; $p = .0023$);
2. items involving building, interpreting, and using mathematical models given data tables (DATASETS; $p = .0055$);
3. items involving building, interpreting, and using mathematical models given verbal descriptions (VERBAL; $p = .0140$);
4. items involving interpreting and using mathematical models given models as algebraic formulas (ALGEBRAIC; $p = .0033$);
5. items involving building mathematical models (BUILD; $p = .0269$);
6. items involving interpreting mathematical models (INTERPRET; $p = .0045$);
7. items involving using mathematical models (USE; $p = .0031$);
8. items involving building, interpreting, and using mathematical models based on the same physical contexts utilized during treatment sessions (FAMILIAR; $p = .0015$); and

9. items involving building, interpreting, and using mathematical models based on physical contexts different from the contexts utilized during treatment sessions (UNFAMILIAR; p = .0091).

Group-by-subscore interactions were investigated using the technique of repeated measures MANOVA, based on percentage scores, for the subscore groupings (a) DATASETS, VERBAL, and ALGEBRAIC; (b) BUILD, INTERPRET, and USE; and (c) FAMILIAR AND UNFAMILIAR. A Wilks' lambda value of .99 (F = .046; df = 2, 22; p = .9552) indicated that there was not a significant group-by-subscore interaction for the subscores DATASETS, VERBAL, and ALGEBRAIC. A Wilks' lambda value of .92 (F = .96; df = 2, 22; p = .3971) indicated that there was not a significant group-by-subscore interaction for the subscores BUILD, INTERPRET, and USE. A Wilks' lambda value of .90 (F = 2.43; df = 1, 23; p = .1326) indicated that there was not a significant group-by-subscore interaction for the subscores FAMILIAR and UNFAMILIAR.

Additional repeated measures MANOVA analyses were run on subscores of BUILD, INTERPRET, and USE. For the two subscores of BUILD, a Wilks' lambda value of .97 (F = .60; df = 1, 22; p = .4464) indicated that there was not a significant group-by subscore interaction for the subscores BUILD1 and BUILD2. For the four subscores of INTERPRET,

(1) a Wilks' lambda value of .99 (F = .27; df = 1, 23; p = .6073) for testing the two-way interactions between SLOPE1 and INTERCEPT1 indicated that there was not a significant group-by-subscore interaction for the subscores SLOPE1 and INTERCEPT1;

(2) a Wilks' lambda value of .80 (F = 5.60; df = 1, 23; p = .0268) for testing the two-way interactions between SLOPE2 and INTERCEPT2 indicated that there was

a significant group-by-subscore interaction for the subscores SLOPE2 and INTERCEPT2; and

(3) a Wilks' lambda value of .95 ($F = 1.26$; $df = 1, 23$; $p = .2737$) for testing the two-way interactions between identification tasks (SLOPE1 and INTERCEPT1) and interpretation tasks (SLOPE2 and INTERCEPT2) indicated the absence of two-way group-by-subscore interaction effects between identification and interpretation tasks. For the two subscores of USE, a Wilks' lambda value of .99 ($F = .06$; $df = 1, 23$; $p = .8116$) indicated that there was not a significant group-by subscore interaction for the subscores USEY and USEX.

Discussion

The primary goal of this study was to test the usefulness of the pentagonal modification of Janvier's "star" model as a model for framing research focusing on the relationship between instructional modes and connections students make among various aspects of the function concept. The two instructional modes employed in this study involved (a) hands-on data collection followed by data analysis and (b) data analysis only. Viewed within the context of the pentagonal model, Group A participated in tasks associated with direct situation-to-table translations while Group B subjects were not involved in tasks requiring this translation. Other than situation-to-table translations associated with data collection tasks, both groups were involved in the same tasks requiring building mathematical models from data sets (table-to-algebraic formula translations), identifying and interpreting slopes and y-intercepts based on mathematical models expressed as algebraic formulas (algebraic formula-to-verbal description translations), and using mathematical models to make predictions (verbal description-to-algebraic formula, algebraic formula-to-table, and table-to-verbal description translations). Figure 16 illustrates the direct translation difference between experimental groups, framed within the pentagonal model.

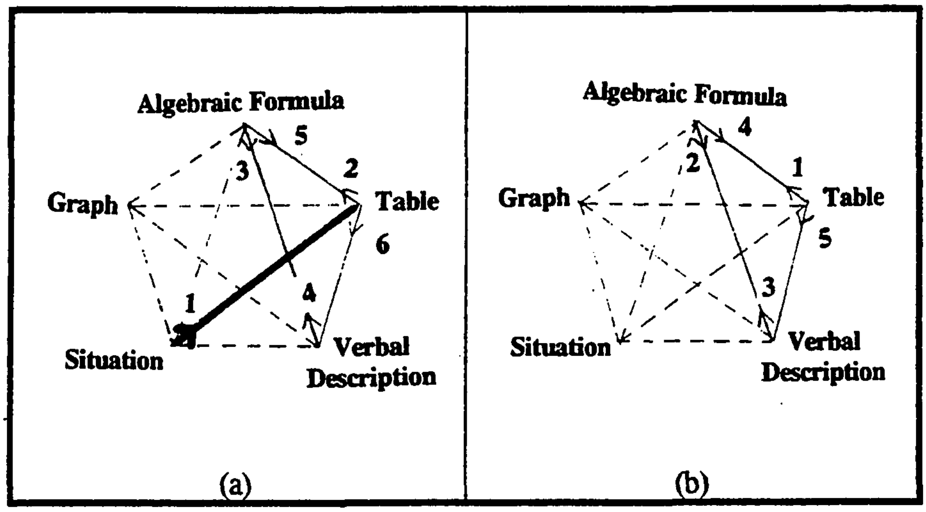

Figure 16: Direct Translations for (a) Group A and (b) Group B.

Basically, this study was designed to address one fundamental question: Does participation in the processes involved in collecting scientific data affect elementary preservice teachers' performance on tasks involving building, interpreting, and using mathematical models based on the data? The results of this study support a conclusion that performance on these tasks is affected by instructional mode. In particular, the results indicate that, under the conditions employed in this study, involvement in data collection tasks adversely affects performance on tasks involving building, interpreting, and using mathematical models. This conclusion is supported by group mean score differences on the overall posttest and all 16 subsets of the posttest analyzed in this study. The absence of interaction effects in all, but one, subscore comparisons indicates that the performance differences by group are not different for different types of tasks. Thus, the results of this study support the conclusion that performance on tasks involving building, interpreting, and using linear mathematical models is dependent on instructional mode, where subjects less involved with the situation representation and situation-to-table translations (Group B) exhibit a higher performance on mathematical modeling tasks than those who are more involved with these representations and translations (Group A). Subscore analyses of group main effect support analogous conclusions.

In the next three sections subscore main effects, treatment group main effects, and group-by-subscore interactions are considered separately. First, subscore main effects are considered within the framework of the pentagonal model and Kaput's (1987) Symbol Theory. Second, an explanation of group differences based on conceptual versus procedural learning and treatment time limitations is proposed. Lastly, the one group-by-subscore interaction effect revealed by repeated measures MANOVA analysis is used to consider possible group differences in completing interpretation tasks.

Subscore Main Effects

When subscores DATASETS, VERBAL, and ALGEBRAIC were analyzed, it was found that Group B outscored Group A on all three subscores (see Figure 10, page 87). For both treatment groups, the mean scores were higher for problems that began with building mathematical models from data sets (table-to-algebraic formula translation) than for problems that began with building mathematical models from verbal descriptions (verbal description-to-algebraic formula translations). The lowest scores in this set resulted on problems that began with the mathematical model given as an algebraic formula. In this latter case no data were available to subjects for reference and no verbal description of the model was provided. An illustration detailing the differences in starting points among the three problem sets, framed within the pentagonal model, is given in Figure 17.

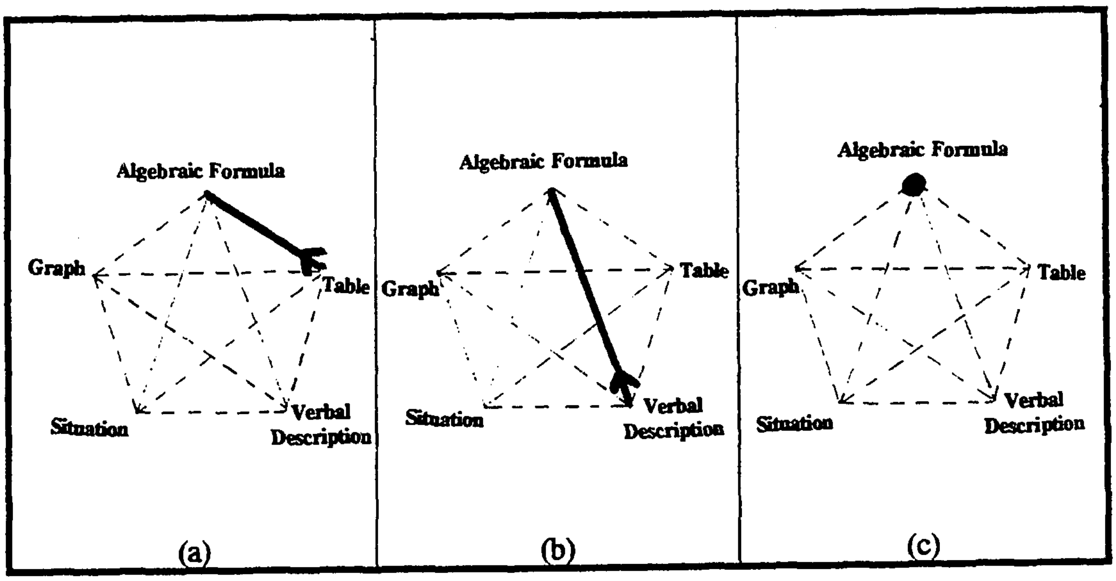

Figure 17. Subscore Differences. (a) Building Mathematical Models from Data Sets; (b) Building Mathematical Models from Verbal Descriptions; (c) Given Mathematical Models as Algebraic Formulas

An explanation of the within-group differences in performance on these three sets of tasks might be based on Kaput's Symbol Theory. Kaput (1987) identified two types of symbol use in mathematics: (a) reading and encoding and (b) elaboration. Elaboration is classified into two forms: syntactic and semantic. Kaput associates syntactic elaboration with procedural knowledge because both procedural knowledge and syntactic elaboration are based on direct manipulation of immediate symbolic representations. In contrast, semantic elaboration involves using the features of the reference field of the symbol system rather than using its symbol scheme syntax. For the tasks corresponding to the ALGEBRAIC subscore, the reference field is expected to be the algebraic formula. Without reference to the corresponding situation and verbal description, the subject might be forced to rely on syntactic elaboration within an abstract reference field. During the treatment sessions, all mathematical models were built from data sets. Thus, subjects in both treatment groups had been taught to answer questions using mathematical models where the physical situation or data table served as the reference field. The VERBAL subscore was based on students' building of mathematical models from verbal descriptions. Since the language used in the verbal descriptions might serve to conjure up mental images of the situation, subjects might be expected to use the actual situation as the reference field, although perhaps not as effectively as they were able to do from the data sets. Thus, viewed within Kaput's Symbol Theory, the subscore order, DATASETS > VERBAL > ALGEBRAIC, appears reasonable. However, this explanation of within-group differences does not shed light on the between-group differences measured in this experiment.

Similar group results were obtained for the subscores BUILD, INTERPRET, and USE. Subjects in both treatment groups were expect to have had few experiences, if any, with building and interpreting mathematical models prior to participating in the experimental sessions. However, since using mathematical models to make predictions is a component of most algebra and physical science classes, all subjects were expected to have experiences performing prediction tasks. Although Group B outscored Group A on all three of these subscores, both groups scored higher on BUILD than on either INTERPRET or USE tasks. Performance on both INTERPRET and USE tasks were approximately the same within both treatment groups (Figure 11, page 89). The results seem reasonable when the associated tasks are viewed within the pentagonal model. Building mathematical models is expected to involve only a single table-to-algebraic formula translation, as illustrated in Figure 17a, or a verbal description-to-algebraic formula translation (with perhaps some mental reference to the situation), as illustrated in Figure 17b. Interpreting mathematical models is expected to be more complicated. The associated direct translation is an algebraic formula-to-verbal description translation. However, it is expected that the physical situation provides the reference field for the symbols of the algebraic formula. Thus, interpretation tasks might be expected to involve algebraic formula-to-situation and situation-to-verbal description translations (Figure 18a). Tasks within the subscore USE, though more familiar to students at the beginning of the sessions, proved to be more difficult than the building and comparable in difficulty to interpretation tasks. This result seems reasonable when prediction tasks are viewed within the pentagonal model. Prediction tasks are expected to involve the physical situation reference field. Thus, it is expected that subjects might complete prediction tasks by way of a series of translations: (a) verbal description-to-situation, (b) situation-to-algebraic formula, (c) algebraic formula-to-table, and (d) table-to-situation, and (e) situation-to-verbal description (Figure 18b). The complexity of prediction tasks would indicate that students would not perform as well on these tasks as on building tasks.

A comparison of the USEY and USEX subscores revealed that students did not perform as well on tasks involving predicting an x value given the y value as they did on tasks involving predicting a y value given the x value (Figure 14, page 93). Differences in these two tasks involve processes within the algebraic formula representation. In order to predict a y value given the x value, the subject is expected to substitute the value of x in place of x in the algebraic formula and compute the corresponding y value directly. However, in order to predict an x value given the y value, the subject is expected to algebraically rearrange the equation in addition to substituting the value of y in place of y in the algebraic formula. The additional rearrangement of the algebraic formula required in USEX tasks is expected to make USEX tasks more difficult than the USEY tasks, especially for students with weaker algebra skills.

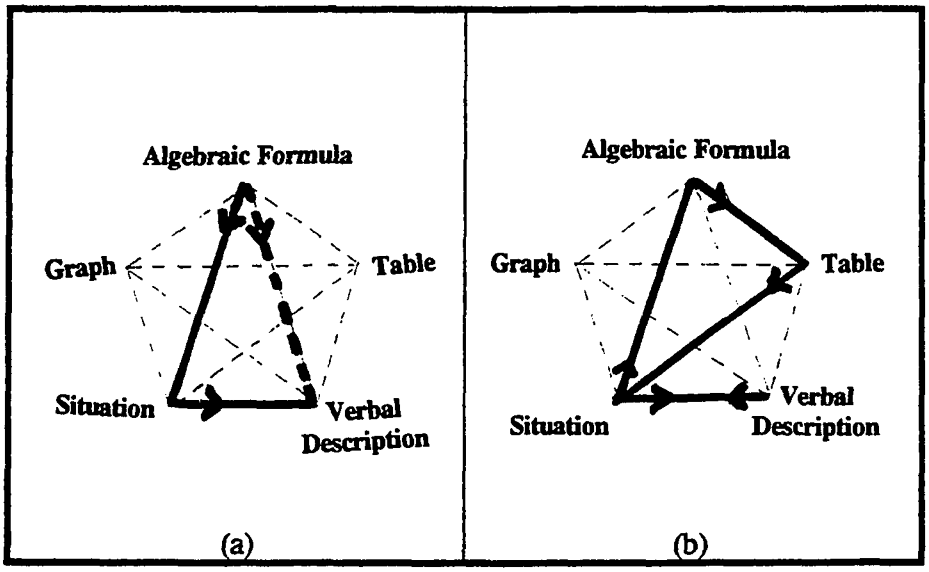

Figure 18. Translations Involved in (a) Interpretation Tasks and (b) Prediction Tasks.

The within-group differences on subscores FAMILIAR and UNFAMILIAR (Figure 15, page 95) are consistent with research findings in a huge body of transfer of learning studies within the discipline of cognitive science. Basically, this research documents limitations in human abilities to transfer knowledge learned in one context to new contexts. Thus, in terms of this study, subjects are expected to perform better on tasks in a familiar context than on analogous tasks in new contexts.

Subscore main effects, viewed within the pentagonal model and related to Kaput's Symbol Theory, were discussed in this section. Although this consideration of subscore main effects details possible translations and translation processes involved in the mathematical modeling tasks of this study, the subscore main effects do not shed light upon observed group differences on the various tasks. Treatment group main effects is the subject of the next section.

Group Main Effect

In terms of the research questions presented in Chapter IV, the most important results of this study to consider are the group main effects on the posttest and all subscores of the posttest. The importance of hands-on experiences in developing understandings of mathematical and scientific concepts forms the backbone of constructivists' calls for mathematics and science curricula reforms. Yet, on first examination, the results of this study might be interpreted as providing evidence that hands-on experiences might actually inhibit learning. Should it be concluded, based on the results of this study, that hands-on experiences are not important in understanding mathematical function in the context of relationships between real-world variables? Or might alternative explanations account for the group performance differences observed in this study?

A plausible explanation for the observed group differences might involve a combination of group differences in type of learning involved during treatment sessions and treatment time limitations. That is, if Group A subjects were learning conceptually and Group B subjects were learning procedurally, the measured differences might be accounted for by differences associated with conceptual verses procedural learning.

The proposition that subjects in the two groups were learning differently is supported by comments made by subjects during the interviews.

Group A:

LOW-A: There were certain things that I understood--that made sense.

MEAN-A: I think it was hard to pull [what we did in the experiment and why we did it] together. ...

HI-A: I thought [interpreting the models] was easy because it made sense. ... Working with the units ... made it easier. It made sense.

Group B:

LOW-B: [On the posttest] I didn't know if I was putting down right or wrong answers. ... [In the workshop] we had the data and we just put in the numbers in the calculator and got the equation. ... [On the posttest when we were given the equation], I just didn't know where to start the problem. [In the application problems] I didn't know what to do except just try to match it up but I didn't know what x was or what y was, so I really didn't know how to match it up.

MEAN-B: Using the calculator was fine because we had the [reference] sheet you gave us. ... The important thing was seeing a pattern.

HI-B: When I was doing [the activities] I kept wondering if I was doing it correctly. Is this right? Is this what we were are supposed to get? I remember thinking as I went out the door [at the end of the first session] that I really wasn't certain that I had done the right procedures. I was still having problems in the second session--I was working with procedures I was still uncertain about. ... Now as I look back on the activities we did in the workshop, it looks fairly easy. I am more familiar with the processes I need to do.

The procedural versus conceptual learning argument is also consistent with the limitations of time involved in this study. If, as the interviews indicated, Group B subjects were concerned with learning the correct procedures which in turn would lead them to the correct answers, while Group A subjects were trying to understand what they were doing and why they were doing it, a plausible explanation of the results might be that it simply takes longer to develop an understanding of concepts than it does to develop skill in carrying out procedures. Assimilation of new ideas and approaches is expected to involve the establishment of an equilibrium between existing internal knowledge structures and the new information. This process might simply require a longer adjustment period than afforded by the time-frame of the study. In addition, unlike the Group B subjects, Group A subjects participated in a whole set of data collection activities. Thus, Group A subjects might have faced a more difficult assimilation task due to the additional number of pieces that they had to fit together mentally. If this explanation is correct, then increasing treatment time might be expected to result in Group A outscoring Group B.

Group-by-Subscore Interactions

The one group-by-subscore interaction effect suggested by the MANOVA analyses was for subscores SLOPE2 and INTERCEPT2. These subscores are associated with constructing interpretations of slopes and y-intercepts within the physical context (situation) of the relationship being explored. Viewed within the pentagonal model, these interpretations involve direct algebraic formula-to-verbal description translations. Considering the number of comparisons involved in analyzing posttest and subscore results, all at a testwise Type I error rate of $\alpha = .05$, it is entirely possible that this one group-by-subscore interaction detected during the MANOVA analysis was obtained by chance rather than being due to real group differences. However, if the result is indicative of real group differences, what explanation might be proposed to account for these observed differences?

First, consider the observed group differences. Group B scored higher on interpreting slopes (SLOPE2) than on interpreting y-intercepts (INTERCEPT2), and

Group A scored higher on interpreting y-intercepts than on interpreting slopes. Alternatively, the group mean difference was less for tasks involving interpreting y-intercepts than for tasks involving interpreting slopes (Figure 13, page 92).

Next, consider differences in the two types of interpretation tasks. The interpretation of y-intercepts in mathematical models might be considered to be a more straight-forward task, conceptually, than interpretation of slopes. In order to interpret a y-intercept it is necessary to consider what is known about the y value of the mathematical model when the x value is 0. For example, consider the experiment discussed in Chapter I. In that experiment the relationship between the total mass of a graduated cylinder containing antifreeze and the volume of antifreeze in the graduated cylinder was determined. The relationship was represented with the mathematical model $y = 1.13\,x + 45.0$. The y-intercept, 45.0 grams, corresponds to the total mass of graduated cylinder and antifreeze when 0 milliliters of antifreeze is in the graduated cylinder. Alternatively, the y-intercept corresponds to the mass of the empty graduated cylinder. The interpretation of slope is, conceptually, more complicated than the interpretation of y-intercept because slope is a ratio that focuses directly on the relationship between the two physical variables being investigated. In the case of the mass versus volume experiment the slope, 1.13 grams per milliliter, expresses the ratio between the mass of antifreeze in the graduated cylinder and the volume of antifreeze in the graduated cylinder.

Lastly, consider what explanation might account for the observed group differences. Since Group A subjects completed data collection tasks, which involved completing processes within the situation representation and the situation-to-table translation, and interpretation tasks are expected to rely on the situation reference field, it might be expected that Group A subjects would perform better than Group B subjects on both types of interpretation tasks. This is clearly not the case. This group main effect was considered in the previous section. For now, consider just the interaction effect. Since the group mean difference is less for interpreting y-intercepts than for interpreting slopes, it might be proposed that Group A and Group B are approaching the same skill level on this interpretation task. The larger group mean difference for tasks involving interpreting slopes than tasks involving interpreting y-intercepts might be accounted for by Group B approaching the tasks in a procedural way and Group A approaching the tasks in a conceptual way. An important consideration in this argument is the higher performance of Group B subjects on slope interpretation tasks than on y-intercept interpretation tasks. Although interpreting y-intercepts is conceptually easier than interpreting slopes, interpreting slopes might be procedurally easier than interpreting y-intercepts since slopes may be interpreted procedually using the word template: The y-variable increases/decreases (a given number of) units for each 1 unit increase in the x-variable. Thus, the group-by-subscore interaction effect for the subscores SLOPE2 and INTERCEPT2 appear to provide additional support for the conjecture that group main effects observed in this study are due to group differences in type of learning, conceptual for Group A versus procedural for Group B.

Concluding Remarks

The results of this study indicate that learning is different when subjects participate in data collection prior to completing tasks involving building, interpreting, and using mathematical models than when subjects complete data analysis tasks without data collection. This difference was reflected uniformly in the overall posttest and in all 16 subscores considered. At this point, it seems reasonable to propose that the group differences noted in this study might be related to differences in type of understanding being developed: more conceptual for Group A and more procedural for Group B.

The primary goal of this study was to test the usefulness of the pentagonal modification of Janvier's "star" model as a model for framing research on designing instruction to increase connections students make among various aspects of the function concept. This study serves as an example of designing and interpreting research within the framework of the pentagonal model. The concept of mathematical function is a complex concept that might be viewed differently in (a) each of its representations and (b) each task involving translations between representations. Viewing representations, translations, and translation processes within the framework of the pentagonal model provides a way to connect various aspects of the mathematical function concept. Making such connections seems crucial, not only for the student, but for teachers, curriculum specialists, and researchers. Making connections among so many, apparently diverse, ideas is not an easy task. Therefore, it seems important that instruction be specifically designed to increase the probability that students will develop multiple connections. The pentagonal model has proved helpful in detailing possible paths of translations between representations associated with the posttest items. It appears that the pentagonal model is helpful in identifying the "thought paths of the mind." This study indicates that the pentagonal model provides a framework for mapping the internal thought processes associated with learning the concept of mathematical function.

Implications for Educators

Data analysis is an important subset of the probability and statistics strand described in the K-12 Standards (NCTM, 1989). An important goal of data analysis is the construction of verbal descriptions of relationships between real-world variables, especially when integrating mathematics with other subject matter areas. The results of this study indicate that a data analysis approach to teaching mathematical function might result in conceptual learning if students are involved in hands-on activities of data collection. Since the data collection component is often omitted within mathematics classes on data analysis, students might fail to develop a conceptual understanding of mathematical function unless current instructional practices are changed.

Conceptual learning is expected to take longer than procedural learning. Instruction focused at fully developing all five representations and 20 associated translations within a given example and a given context might prove more helpful in developing student understanding of mathematical function than a piecemeal exposure to parts of a large number of examples in many contexts. In fact, development of conceptual knowledge as flexibility-in-translating-non algorithmically-between-representations might be expected to lead to an abstraction of the function concept from its multiple representations.

Recommendations for Further Research

The next question which needs to be answered is "Will the overall posttest results for the two treatment groups reverse if the treatment time is increased?" In particular, it would be interesting to study, quantitatively, the effects of doubling, tripling, and quadrupling the treatment time. A large group-by-time interaction term would lend strong support to the conceptual versus procedural learning argument proposed in the discussion section.

In addition, a more detailed view of group differences might be obtained via a qualitative study. During the treatment sessions, an experimental pair might be asked to complete the same set of activities used in this study. As the pair works together on the tasks involving building, interpreting, and using linear mathematical models, the conversations could be audio taped. An interviewer could be present to probe thought processes of the pair. Analysis of the audio tapes within the pentagonal model might provide a detailed picture of group learning differences, somewhat akin to the microgenetic analysis conducted by Schoenfeld et al. (in press).

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Appendix A

Activity Sheets: Treatment Group A, Session I

Team: ______________________

____

BUILDING AND USING MATHEMATICAL MODELS
Session I

PART I: The Relationship between Mass and Volume of a Liquid

A. Materials: 100-mL beaker, 25-mL graduated cylinder, medicine dropper, laboratory balance, rubbing alcohol, and a TI-81 graphing calculator

B. Procedure

1. Add approximately 2.5 mL of rubbing alcohol to a clean, dry 25-mL graduated cylinder using a clean medicine dropper.

2. Read the actual volume to nearest 0.1 mL and record the measurement in the first data table.

3. Zero the balance.

4. Place the graduated cylinder containing rubbing alcohol on the balance platform.

5. Determine the mass of the graduated cylinder and contents, and record the measurement in the first data table.

6. Without removing the graduated cylinder from the balance platform, add *another* approximately 2.5 mL rubbing alcohol to the graduated cylinder. There will now be approximately 5 mL of liquid in the graduated cylinder.

7. Measure and record the total volume and total mass in the table.

8. Add another approximately 2.5 mL of rubbing alcohol to the graduated cylinder. Measure and record volume and mass values.

9. Repeat step 8 until the total volume reaches approximately 25 mL.

NOTE: A similar experiment has been done for you using antifreeze as the liquid. The resulting measurements are given in the second table.

C. Data Tables

Rubbing Alcohol Data

Volume of Rubbing Alcohol (milliliters)	Total Mass, Graduated Cylinder + Liquid (grams)

Antifreeze Data

Volume of Antifreeze (milliliters)	Total Mass, Graduated Cylinder + Liquid (grams)
2.4	47.7
5.2	50.9
7.6	53.6
10.0	56.4
12.2	58.7
15.2	62.1
18.0	65.4
19.8	67.4
22.4	70.3
24.7	73.0

D. Modeling

1. Using your TI-81 calculator to fit the **rubbing alcohol** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the volume of the liquid. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Using your TI-81 calculator to fit the **antifreeze** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the volume of the liquid. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

7. What is the value of the slope? [Include units!]

8. In your own words, give a physical interpretation of the slope.

9. What is the value of the y-intercept? [Include units!]

10. In your own words, give a physical interpretation of the y-intercept.

E. Using the Models

1. What is the mass of 16.2 mL of rubbing alcohol?

2. What is the volume of 35.7 grams of antifreeze?

PART II: The Relationship between Height of Liquid in a Cylindrical Container and Volume of Liquid Added

A. Materials: 500-mL beaker, 1.0-L beaker, 100 mL graduated cylinder, ruler, and a TI-81 graphing calculator

B. Procedure

1. Add tap water to a 1.0-L beaker until the beaker is approximately two-thirds full.

2. Pour approximately 50 mL of water from the 1.0-L beaker into a clean, dry 100-mL graduated cylinder.

3. Read the actual volume of water in the graduated cylinder to the nearest 0.5 mL, and record the measurement in the data table under the **Volume Added** column.

4. Pour the water from the graduated cylinder into a clean, dry 500-mL beaker. Measure the height of water in the 500-mL beaker *from the table-top*. Record this value in the **Height** column of the data table.

5. Add another approximately 50-mL portion of water to the graduated cylinder, record the actual volume in the data table, and add this water to the 500-mL beaker. Measure and record this new value for water height in the data table.

6. Repeat step 5 until you have eight data points in the table.

7. Fill in the **Total Volume** column of the data table.

C. Data Tables

Height versus Volume Data

Volume Added (milliliters)	Total Volume (milliliters	Height (centimeters)

D. Modeling

1. Using your TI-81 calculator to fit the **volume versus height** data to an equation, determine the linear mathematical model describing how the height of water in the beaker is dependent on the total volume of water in the beaker. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

E. Using the Model

1. What would be the height of water in the beaker if a total volume of 365 mL of water were added to the beaker?

2. What total volume would correspond to a height of 8.50 cm?

Appendix B

Activity Sheets: Treatment Group B, Session I

Team: ______________________

____

BUILDING AND USING MATHEMATICAL MODELS
Session I

PART I: The Relationship between Mass and Volume of a Liquid

A. Situation

Four fifth-grade students worked together in a cooperative group to make mass and volume measurements on two different liquids. The values for the measurements they made are listed in the following two tables.

Rubbing Alcohol Data

Volume of Rubbing Alcohol (milliliters)	Total Mass, Graduated Cylinder + Liquid (grams)
2.6	46.9
4.9	49.0
7.7	51.5
9.8	53.4
12.5	55.8
15.1	58.0
17.5	60.2
20.1	62.4
22.4	64.5
24.9	66.7

Antifreeze Data

Volume of Antifreeze (milliliters)	Total Mass, Graduated Cylinder + Liquid (grams)
2.4	47.7
5.2	50.9
7.6	53.6
10.0	56.4
12.2	58.7
15.2	62.1
18.0	65.4
19.8	67.4
22.4	70.3
24.7	73.0

B. Modeling

1. Using your TI-81 calculator to fit the **rubbing alcohol** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the volume of the liquid. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Using your TI-81 calculator to fit the **antifreeze** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the volume of the liquid. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

7. What is the value of the slope? [Include units!]

8. In your own words, give a physical interpretation of the slope.

9. What is the value of the y-intercept? [Include units!]

10. In your own words, give a physical interpretation of the y-intercept.

C. Using the Models

1. What is the mass of 16.2 mL of rubbing alcohol?

2. What is the volume of 35.7 grams of antifreeze?

PART II: The Relationship between Height of Liquid in a Cylindrical Container and Volume of Liquid Added

A. Situation

Another group of four fifth-grade students worked together in a cooperative group to make measurements in an investigation of the relationship between the height of water in a beaker and the volume of water added. The values for the measurements they made are listed in the following table.

Height versus Volume Data

Volume Added (milliliters)	Total Volume (milliliters	Height (centimeters)
50.0		1.35
54.5		2.95
48.0		4.10
51.5		5.45
50.0		6.60
54.0		7.95
49.0		9.15
49.0		10.30

B. Modeling

1. Using your TI-81 calculator to fit the **volume versus height** data to an equation, determine the linear mathematical model describing how the height of water in the beaker is dependent on the total volume of water in the beaker. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

C. Using the Model

1. What would be the height of water in the beaker if a total volume of 365 mL of water were added to the beaker?

2. What total volume would correspond to a height of 8.50 cm?

Appendix C

Activity Sheets: Treatment Group A, Session II

Team: ______________________

____

BUILDING AND USING MATHEMATICAL MODELS
Session II

PART I: The Relationship between Mass and Number of Coins

A. Materials: plastic cup, 20 pennies, 20 nickels, balance, and TI-81 graphing calculator

B. Procedure

1. Zero the balance.
2. Place two pennies in the plastic cup.
3. Place the plastic cup containing the two pennies on the balance platform.
4. Determine the mass of the cup and contents, and record the measurement in the first data table.
5. Add two more pennies to the cup and determine the mass of the cup and contents. Record the mass in the data table.
6. Repeat step 5 until there are a total of twenty pennies in the plastic cup.
7. Repeat the entire procedure using nickels instead of pennies. Record the data in the second table.

C. Data Tables

Pennies

Number of Pennies in the Cup	Total Mass (grams)
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	

Nickels

Number of Nickels in the Cup	Total Mass (grams)
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	

D. Modeling

1. Using your TI-81 calculator to fit the **pennies** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the number of pennies in the cup. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Using your TI-81 calculator to fit the **nickels** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the number of nickels in the cup. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

7. What is the value of the slope? [Include units!]

8. In your own words, give a physical interpretation of the slope.

9. What is the value of the y-intercept? [Include units!]

10. In your own words, give a physical interpretation of the y-intercept.

E. Using the Models

1. What is the mass of 15 pennies?

2. What is the mass of the cup plus 19 nickels?

PART II: The Relationship between Length of a Spring and Attached Mass

A. Materials: 1 - harmonic oscillator spring, clamps, meter stick, 1 - 50 gram hanger, 1 - 50 gram disk, 4 - 100 gram disks, and a TI-81 graphing calculator.

B. Procedure

1. The apparatus for this set of measurements has been set-up for you. To begin with the measurements attach the 50-gram hanger to the loop at the bottom of the spring.

2. Read the length of the spring as the reading on the meter stick corresponding to the bottom coil of the spring. Record this length, to the nearest 0.1 cm, in the table on the next page next to 50 grams total mass.

3. Place the 50-gram disk on the hanger. At this point you have attached a total of 100 grams to the spring. Record the length of the spring now.

4. Replace the 50-gram disk on the hanger with a 100-gram disk. At this point you have attached a total of 150 grams to the spring. Record the length of the spring.

5. Continue increasing the mass attached to the spring by 50 grams each time until you have attached a total mass of 500 grams, recording the length of the spring after each 50-gram increase.

C. Data Tables

Length versus Mass Data

Total Mass Attached to Spring (grams)	Length of Spring (centimeters)
50	
100	
150	
200	
250	
300	
350	
400	
450	
500	

D. Modeling

1. Using your TI-81 calculator to fit the **length versus mass** data to an equation, determine the linear mathematical model describing how the length of the spring is dependent on the total mass attached to the spring. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

E. Using the Model

1. What would be the length of the spring if a total mass of 280 grams was attached to the spring?

2. What total mass would need to be attached to the spring in order to stretch the spring to a total length of 60.0 centimeters?

Appendix D

Activity Sheets: Treatment Group B, Session II

Team: ______________________

____

BUILDING AND USING MATHEMATICAL MODELS
Session II

PART I: The Relationship between Mass and Number of Coins

A. Situation

Four third-grade students worked together in a cooperative group to measure the mass of differing numbers of pennies and nickels. They began by placing two pennies in a cup, putting the cup on a balance platform, and recording the mass. They then added two more pennies to the cup and recorded the new mass. This process of adding two more pennies and recording the mass was continued until 20 pennies were in the cup. The students then followed the same procedure with nickels. The data they collected is given in the following two tables.

Pennies

Number of Pennies in the Cup	Total Mass (grams)
2	12.968
4	17.980
6	23.066
8	28.131
10	33.710
12	38.709
14	43.76
16	49.27
18	54.26
20	59.86

Nickels

Number of Nickels in the Cup	Total Mass (grams)
2	17.963
4	27.805
6	37.808
8	47.72
10	57.75
12	67.67
14	77.69
16	87.70
18	97.74
20	107.86

B. Modeling

1. Using your TI-81 calculator to fit the **pennies** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the number of pennies in the cup. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Using your TI-81 calculator to fit the **nickels** data to an equation, determine the linear mathematical model describing how the total mass is dependent on the number of nickels in the cup. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

7. What is the value of the slope? [Include units!]

8. In your own words, give a physical interpretation of the slope.

9. What is the value of the y-intercept? [Include units!]

10. In your own words, give a physical interpretation of the y-intercept.

C. Using the Models

1. What is the mass of 15 pennies?

2. What is the mass of the cup plus 19 nickels?

PART II: The Relationship between Length of a Spring and Attached Mass

A. Situation

Another 4-member cooperative group of third graders worked together to investigate how the length of a spring depends on the mass attached to the spring. The values for the measurements they made are listed in the following table.

Length versus Mass Data

Total Mass Attached to Spring (grams)	Length of Spring (centimeters)
50	33.5
100	38.8
150	43.8
200	48.2
250	53.4
300	58.7
350	64.0
400	69.1
450	74.3
500	79.5

B. Modeling

1. Using your TI-81 calculator to fit the **length versus mass** data to an equation, determine the linear mathematical model describing how the length of the spring is dependent on the total mass attached to the spring. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

C. Using the Model

1. What would be the length of the spring if a total mass of 280 grams was attached to the spring?

2. What total mass would need to be attached to the spring in order to stretch the spring to a total length of 60.0 centimeters?

Appendix E

Posttest: Final Form

PROBLEM 1

A group of fourth grade students conducted an experiment to determine how the mass of a liquid is dependent on the volume of the liquid. They poured some of an unknown liquid into an empty graduated cylinder and then placed the cylinder on the balance platform. They recorded the volume of the liquid and the total mass. They continued to add more of the liquid, recording volume and total mass after each addition, until they obtained the following data table.

Volume of the Liquid (mL)	Total Mass (grams)
5.1	**49.1**
9.7	**51.8**
14.9	**55.2**
20.1	**59.2**
24.8	**61.7**

1. Using a TI-81 calculator to fit the model, the values obtained were **a** = **45.6** and **b** = **0.655**. The variables were identified as:
   X = volume of the liquid (in mL)
   Y = total mass (in grams)
   Write the mathematical model (equation):

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Use the mathematical model you determined in #1 of this problem to answer the following question: **What is the mass of 17.9 mL of the unknown liquid?**

PROBLEM 2

A group of sixth-grade students conducted an experiment to determine how the volume a gas occupies depends on the temperature of the gas. The pressure on the gas was constant throughout the experiment. They obtained the following data table.

Temperature (^{o}C)	Volume of the Gas (mL)
51.5	**58.1**
68.9	**60.1**
80.1	**62.1**
87.9	**64.1**
93.1	**66.0**

1. Using a TI-81 calculator to fit the model, the values obtained were a = **48.0** and **b = 0.184**. The variables were identified as:
   X = the temperature of the gas (in ^{o}C)
   Y = the volume of the gas (in mL)
   Write the mathematical model (equation):

2. What is the value of the slope? [Include units]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Use the mathematical model you determined in #1 of this problem to answer the following question: **What is the volume of the gas at 75°C?**

PROBLEM 3

A group of third-grade students were learning how to make mass measurements. They conducted a very simple experiment. They put a cup on the balance and determined the mass. The value of this first measurement was 15.8 grams. Then they placed a quarter into the cup and recorded the mass. They continued placing more quarters into the cup and recording the total mass. They discovered that the total mass increased by an average of 12.6 grams each time they put in another quarter.

1. Write a mathematical model (equation) to describe how the total mass is dependent on the number of quarters in the cup. Identify the variables as:
   X = number of quarters in the cup (in quarters)
   Y = total mass (in grams)
   Write the mathematical model (equation):

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Use the mathematical model you determined in #1 of the problem to answer the following question: **What is the mass of the cup plus 25 quarters?**

PROBLEM 4

A rectangular fish tank is setting on a 27-inch-high stand. The tank is 36 inches tall and is being filled with water. The water is rising in the tank at a rate of 3 inches per minute.

1. Write a mathematical model (equation) to describe how the total distance from floor to water level in the tank is dependent on the number of minutes water has been flowing into the tank. Identify the variables as:
   X = time water has been flowing into the tank (in minutes)
   Y = total distance from the floor to water level in the tank (in inches)
   Write the mathematical model (equation):

2. What is the value of the slope? [Include units]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

6. Use the mathematical model you determined in #1 of this problem to answer the following question: **What is the total distance from floor to water level in the tank when water has flowed into the tank for 2.75 minutes?**

PROBLEM 5

When a group of sixth-grade students analyzed data they obtained by measuring the length of a spring when various masses were attached, they obtained the mathematical model:

$$Y = 0.107 X + 15.2$$

where

X = mass attached to the spring (in grams)
Y = length of the spring (in cm)

1. What is the value of the slope? [Include units!]

2. In your own words, give a physical interpretation of the slope.

3. What is the value of the y-intercept? [Include units!]

4. In your own words, give a physical interpretation of the y-intercept.

5. Use the mathematical model to answer the following question: **What is the length of the spring when a mass of 43 grams is attached?**

6. Use the mathematical model to answer the following question: **What mass must be attached to the spring for the length of the spring to be 19.3 cm long?**

PROBLEM 6

The mathematical model expressing the relationship between the Kelvin (^{o}K) and Fahrenheit (^{o}F) temperature scales is

$$\mathbf{Y = 1.80\,X - 459.4}$$

where

X = the temperature of the object being measured (in ^{o}K)
Y = the temperature of the object being measured (in ^{o}F)

1. What is the value of the slope? [Include units!]

2. In your own words, give a physical interpretation of the slope.

3. What is the value of the y-intercept? [Include units!]

4. In your own words, give a physical interpretation of the y-intercept.

5. Use the mathematical model to determine the Fahrenheit temperature equivalent to $300^{o}K$.

6. The normal body temperature for humans is considered to be $98.6^{o}F$. Determine normal human body temperature on the Kelvin scale.

Appendix F

Workshop Evaluation Form

DATA ANALYSIS USING THE TI-81 CALCULATOR
Workshop Evaluation

As a elementary school teacher you will participate regularly in inservice workshops similar to the preservice workshop you have just completed. At the conclusion of each workshop you will be asked to evaluate the workshop. This instrument has been developed to determine the value of this workshop *from your perspective*. Please circle the number to the right of each item which best describes your level of agreement or disagreement with the given statement.

Scale:

1 - Strongly disagree
2 - Disagree
3 - Neutral (neither agree nor disagree)
4 - Agree
5 - Strongly agree

This workshop helped me develop a stronger understanding of how to interpret scientific data.	1	2	3	4	5
This workshop helped me develop a stronger understanding of how to help elementary school children interpret scientific data.	1	2	3	4	5
This workshop helped me develop more confidence in exploring relationships between pairs of variables using a scientific inquiry approach.	1	2	3	4	5
This workshop helped me develop more confidence in helping elementary school children explore relationships between pairs of variables using a scientific inquiry approach.	1	2	3	4	5
It is important for elementary school teachers who teach science and mathematics to know how to use a TI-81 calculator to interpret data collected in experiments.	1	2	3	4	5

It is important for elementary school teachers to know how to use computers and calculators in teaching mathematics and science.	1 2 3 4 5
It is important for elementary school teachers to know how to build mathematical models from scientific data.	1 2 3 4 5
It is important for elementary school teachers to know how to use mathematical models which have been derived from scientific data.	1 2 3 4 5
It is important to integrate language with science and mathematics in order to develop an understanding of science concepts.	1 2 3 4 5
It is important for preservice elementary teachers to analyze data tables which they develop by making measurements.	1 2 3 4 5
This workshop was worthwhile.	1 2 3 4 5
I enjoyed participating in this workshop.	1 2 3 4 5

Please make any additional comments which might help me improve this workshop for future participants.

THANK YOU FOR PARTICIPATING IN THIS WORKSHOP AND COMPLETING THIS EVALUATION SHEET

Appendix G

Posttest: Original Form

PROBLEM 1

A group of fourth grade students conducted an experiment to determine how the mass of a liquid is dependent on the volume of the liquid. They placed an empty graduated cylinder on a balance pan and added some of the liquid. Then they recorded the volume of the liquid and the total mass. They continued to add more of the liquid, recording volume and total mass after each addition, until they obtained the following data table.

Volume of the Liquid (mL)	Total Mass (grams)
5.1	**49.1**
9.7	**51.8**
14.9	**55.2**
20.1	**59.2**
24.8	**61.7**

1. Using your TI-81 calculator to fit the data to an equation, determine the linear mathematical model describing how the total mass is dependent on the volume of the liquid. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 2

A group of sixth-grade students conducted an experiment to determine how the volume a gas occupies depends on the temperature of the gas. The pressure on the gas was constant throughout the experiment. They obtained the following data table.

Temperature (^{o}C)	Volume of the Gas (mL)
51.5	**58.1**
68.9	**60.1**
80.1	**62.1**
87.9	**64.1**
93.1	**66.0**

1. Using your TI-81 calculator to fit the data to an equation, determine the linear mathematical model describing how the volume the gas occupies is dependent on the temperature of the gas. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 3

Wind-chill factor is a combination of the actual temperature and wind speed. The wind makes it feel colder that it really is. Below are the wind-chill Fahrenheit temperatures when the wind speed is 10 miles per hour.

Actual Temperature (^{O}F)	Wind-Chill Temperature at 10 mph (^{O}F)
40	**28**
30	**16**
20	**3**
10	**-9**
0	**-22**
-10	**-34**
-20	**-46**
-30	**-58**

1. Using your TI-81 calculator to fit the data to an equation, determine the linear mathematical model describing how the wind-chill temperature at 10 mph is dependent on the actual temperature. Be sure to identify the variables in the equation.

a = ______
b = ______
r = ______

2. What is the value of the slope? [Include units]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 4

A group of third-grade students were learning how to make mass measurements. They conducted a very simple experiment. They put a cup on the balance and determined the mass. The value of this first measurement was 15.8 grams. Then they placed a quarter into the cup and recorded the mass. They continued placing more quarters into the cup and recording the total mass. They discovered that the total mass increased by an average of 12.6 grams each time they put in another quarter.

1. Write an algebraic formula to describe how the total mass is dependent on the number of quarters in the cup. Be sure to identify the variables.

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 5

A rectangular fish.tank is setting on a 27-inch-high stand. The tank is 36 inches tall and is being filled with water. The water is rising in the tank at a rate of 3 inches per minute.

1. Draw a diagram illustrating this situation. Write an algebraic formula to describe how the total distance from floor to water level in the tank is dependent on the number of minutes water has been added to the tank.

2. What is the value of the slope? [Include units]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 6

Stapleton International Airport is located in Denver, approximately 12 miles from Mile High Stadium. Assume that the elevation above sea level is the same for the airport as for the stadium --- 5,280 feet. An airplane taking off from the airport is ascending at the rate of 1500 feet per minute. Assume that the plane continues at the same rate of ascent until it reaches an elevation of 20,000 feet above sea level.

1. Write an algebraic formula to describe how the plane's elevation above sea level is dependent on the number of minutes it has been ascending. Be sure to identify the variables.

2. What is the value of the slope? [Include units!]

3. In your own words, give a physical interpretation of the slope.

4. What is the value of the y-intercept? [Include units!]

5. In your own words, give a physical interpretation of the y-intercept.

PROBLEM 7

When a group of fourth-grade students analyzed data they obtained by measuring the length of a spring when various masses were attached, they obtained the mathematical formula:

$$L = 0.107\,M + 15.2$$

where L = length of the spring in centimeters and M = mass attached to the spring in grams.

1. What is the value of the slope? [Include units!]

2. In your own words, give a physical interpretation of the slope.

3. What is the value of the y-intercept? [Include units!]

4. In your own words, give a physical interpretation of the y-intercept.

5. Use the mathematical model to determine the length of the spring when a mass of 43 grams is attached.

6. What mass must be attached for the length of the spring to be 19.3 cm long?

PROBLEM 8

The mathematical model, expressed as an algebraic formula, for converting Kelvin (^oK) to Fahrenheit (^oF) temperature is

$$F = (9/5)\,K - 459.4$$

1. What is the value of the slope? [Include units!]

2. In your own words, give a physical interpretation of the slope.

3. What is the value of the y-intercept? [Include units!]

4. In your own words, give a physical interpretation of the y-intercept.

5. Use the mathematical model to determine the Fahrenheit temperature equivalent to 300^oK.

6. The normal body temperature for humans is considered to be 98.6^oF. Determine normal body temperature on the Kelvin scale.

PROBLEM 9

During the summer, as the temperature gets over $80^{o}F$, the chickens on a chicken farm drink more water. This behavior is modeled by the equation

$$W = 25T - 1250$$

where W = number of gallons of water drunk per hour and T = Fahrenheit temperature ($T \geq 80^{o}$).

1. What is the value of the slope? [Include units!]

2. In your own words, give a physical interpretation of the slope.

3. What is the value of the y-intercept? [Include units!]

4. In your own words, give a physical interpretation of the y-intercept.

5. How many gallons of water are used in an hour when the temperature is $110^{o}F$?

6. According to this model, what is the minimum amount of water the chickens will drink?

Appendix H

Reference Sheet: Key Stroke Summary for Data Analysis Using a TI-81 Calculator

DATA ANALYSIS USING TI-81 CALCULATORS

1.	Clearing the memory:	**2nd, RESET** **2:Reset**
2.	Making the screen lighter:	**2nd, Hold down "down arrow" until light**
3.	Entering data:	**2nd, STAT, DATA, 1:Edit, enter all data pairs**
4.	Setting the range:	**RANGE, enter appropriate min and max for x and y**
5.	Plotting the Scatterplot:	**2nd, STAT, DRAW, 2:Scatter, ENTER**
4.	Linear regression analysis:	**2nd, STAT, CALC, 2:LinReg, ENTER**

Note: Three equations appear; for example: $a = 5$, $b = 2$, and $r = 0.95$. Interpretation: $Y = bX + a$ is the linear equation and the correlation coefficient is $r=0.95$.

5.	Plotting the "best fit" equation with the scatter plot:	**Y=, at :Y_1= press VARS, choose LR, choose 4:RegEQ, 2nd, STAT, DRAW, 2:Scatter, ENTER**

Note: How close are the data points obtained to the best-fit line?

Extra Information:

1.	To delete a data point:	**2nd, STAT, DATA, 1:Edit** **Place cursor on = at x value for point to delete** **DEL**
2.	To insert a data point:	**2nd, STAT, DATA, 1:Edit** **Place cursor on = at x value after insert point** **INS, Enter x and y value for new point**
3.	To plot a second set of data without erasing first graph:	**2nd, STAT, DATA, 2:ClrStat, 2nd, STAT, DATA, 1:Edit, enter data, 2nd, STAT, CALC, 2:LinReg, ENTER, Y=, at :Y2= press VARS, choose LR, choose 4:RegEQ, GRAPH**

Appendix I

Outline: Session I Introduction

I. Operational Definitions

A. Mathematics - set of tools for understanding our world
B. Science - study of our world for understanding and prediction

II. Example of where science and mathematics come together:

Scientific Inquiry Method
A. Purpose
B. Hypotheses
C. Materials
D. Procedures
E. Collect Data from Trials and Tests*
F. Results*
G. Conclusions*

III. Science is a process "Doing" (as well as Content "Knowledge" and Attitudes "Feelings and Values") Process Skills:

A. Observing
B. Classifying
C. Using space/time relations
D. Using numbers
E. Communicating*
F. Measuring*
G. Inferring
H. Predicting*
I. Interpretating data*
J. Controlling variables
K. Defining operationally
L. Formulating hypotheses
M. Experimenting*
N. Formulating models*

IV. What will we be doing in the two sessions?

A. Collecting simple data to study the relationship between two real-world variables

B. Session I
1. "Matter is anything which has mass and volume" What is the relationship between M and V?
2. What is the relationship between height and volume?
3. Using TI-81 graphing calculators to analyze data and fit models
4. Using models to make predictions

C. Session II: Two more similar experiments

V. Research Component

A. How to structure elementary education major so that it best prepares students to teach elementary science and mathematics

B. Standards for all teachers of mathematics (MAA) - formulating models and using technology - refer to important documents

C. Procedures
1. Work with assigned partner each time.
2. Do not discuss with any other person or pair.
3. Focus on communicating (connecting science + math + language) - convince each other
4. Sign consent form

VI. Review of linear mathematical functions

A. Equation: $y = mx + b$ (Example: $y = 2x + 5$)

B. Graph

C. Slope: Rise/run; (change in y)/(change in x)

D. y-intercept: value of y when $x=0$

Appendix J

Form: Consent to Act as a Human Subject

CONSENT TO ACT AS A HUMAN SUBJECT

PARTICIPANT'S NAME ______________________________

DATE OF CONSENT ______________________________

PROJECT TITLE:

Data Analysis in Physical Science using TI-81 Graphing Calculators

DESCRIPTION AND EXPLANATION OF PROCEDURES:

As a participant in this study, you will participate in two 2-hour workshop sessions. During each session you will work with a partner. The task in each session will be to analyze sets of scientific data using a TI-81 calculator. Emphasis will be placed on drawing valid conclusions from data. You will take a test during a third 1-hour session. The purpose of the test is to assess the effectiveness of the workshop. In order to consider the effects of differences in students' scientific and mathematical backgrounds, I will need to access the following information from your file in the Office of the Registrar: list of high school and college mathematics and science courses taken (including grades), SAT scores, high school and college GPA, and high school class rank.

POTENTIAL BENEFITS:

The biggest benefit to you, as a participant in this study, is the experience of analyzing data using technology. By this experience of drawing valid conclusions from scientific data, you will be better prepared to guide elementary school students in conducting class and individual science experiments.

COMPENSATION/TREATMENT FOR INJURY: **None**

CONSENT: I have been satisfactorily informed about the procedures described above and the possible risks and benefits of the project, and I agree to participate in this project. Any questions that I have about the procedures have been answered. I understand that this project and this consent form follow federal regulations guaranteeing my right to privacy. If I have any questions about this, I will call the Office of the Registrar.

I understand that I am free to withdraw my consent to participate in the project at any time without penalty or prejudice. In addition, I will not be identified by name as a participant in this project.

Any new information that might develop during the project will be provided to me if that information might affect my willingness to participate in the project.

______________________________ ______________________________

Subject's Signature Witness to Signature

Appendix K

Outline: Session II Review

I. Scientific Inquiry Method - Focusing particularly on last three steps

   A. Data Tables from Measurements

   B. Results based on Data

   C. Conclusions based on Results

II. Algebra: Linear Equations in Slope-Intercept Form

   A. Equation: $y = mx + b$ (Example: $y = 3x - 5$)

   B. Graph

   C. Slope: Meaning?

   D. y-Intercept: Meaning

III. Science: Linear Mathematical Model in Slope-Intercept Form

   A. General Equation: $Y = bX + a$ (TI-81: a is y-intercept; b is slope)

   B. Specific Equation for Antifreeze Data: $Y = 1.13\,X + 45.0$
   Variables:

      1. Y = total mass of antifreeze and graduated cylinder (grams)
      2. X = volume of antifreeze added (mL)

   C. Graph (sketch; label axes)

D. Slope

1. Identification: b = 1.13 grams per milliliter
2. Interpretation: "The total mass of the graduated cylinder with contents increases 1.13 grams for each one milliliter of antifreeze added; "one milliliter of antifreeze has a mass of 1.13 grams;" "the density of antifreeze is 1.13 g/mL."

E. y-Intercept

1. Identification: a = 45.0 grams
2. Interpretation: "The mass of the empty graduated cylinder is 45.0 grams."

F. Predictions based on the model

1. Meaning of terms in the model

   a. "1.13 X" represents the mass of the antifreeze in the graduated cylinder
   b. "45.0" represents the mass of the antifreeze; dependent on amount of antifreeze poured into the graduated cylinder
   c. "Y" represents the total mass; sum of the mass of antifreeze and the mass of the graduated cylinder

2. Reduced model:

   a. What is the mass of 16.2 mL of rubbing alcohol?
      Y = 1.13 X (find Y)
   b. What is the volume of 35.7 grams of antifreeze?
      Y = 1.13 X (find X)

3. Full model:

   If 21.2 mL of antifreeze were added to the graduated cylinder, what would the total mass be?
   Y = 1.13 X + 45.0 (find Y)

Appendix L

Letter: Request for Administrative Permission to Access Subjects' Academic Records

Dear < >:

On Monday and Wednesday of this week I spoke with < >, < >, < >, and < > about offering a workshop for < > elementary education majors on formulating mathematical models, using TI-81 graphing calculators to analyze science data. They have expressed interest in having students currently enrolled in MATH1 and ED4 participate in such a workshop. We are currently in the process of scheduling workshop sessions for these students.

This workshop has been designed specifically to help preservice elementary education teachers connect the concept of mathematical function with the scientific inquiry method. Currently this area generally "falls through the cracks" among mathematics, science, mathematics methods, and science methods courses required in the elementary education major. Ideas developed during the workshop are based on recent calls for educational reform from a variety of national organizations: National Science Teachers Association (NSTA), National Research Council (NRC), National Council of Teachers of Mathematics (NCTM), and Mathematical Association of America (MAA).

This workshop sequence has been designed as part of an experimental research project to be completed in partial fulfillment of my Ph.D. degree in Mathematics Education. Basically, participating students will be assigned to one of two treatment groups. Treatment Group A will build and use mathematical models based on data that they collect. Treatment Group B will build and use mathematical models based on data that is provided to them. The research project is designed to investigate the importance of data collection to the data analysis process for understanding relationships between real world variables. Students who complete the two 2-hour treatment sessions will take a posttest. Posttest results will be analyzed to determine if there is a difference in mean scores for Treatment Group A and Treatment Group B.

I would also like to analyze the posttest results on several background variables: SAT scores, high school class rank, high school GPA, college GPA, high school mathematics courses taken and grades, high school science courses taken and grades, college mathematics courses taken and grades, and college science courses taken and grades. I hereby request that I be allowed to gather this data under the supervision of the Registrar within the following guidelines:

(1) Individual students participating in this project will not be identified in my dissertation, papers presented for publication resulting from this research, or in any other written documents or oral presentations. Once data has been collected, all identifying marks (name and social security number) will be

permanently removed from project files. At the end of this study all files will be destroyed.

(2) The institution will not be identified in my dissertation, papers presented for publication resulting from this research, or in any other written documents or oral presentations.

(3) A student's records will not be accessed without the written consent of the student. All participating students will sign a form granting me permission to use specified information from their academic file. A copy of the signed "Consent to Act as a Human Subject" form will be placed in my individual participant's file and the original form will be given to the Registrar, to be filed as the Registrar deems appropriate.

(4) The Registrar will verify that all aspects of Public Law 93-380, titled Family Education Rights and Privacy Act (1974), and all university guidelines on the release of student records have been complied with in releasing student records for the purpose of this research project.

(5) All data collection from students' files will done via procedures established by the Registrar and under his supervision. Data collection will be done at times that will not interfere with the regular operational functions of the Office of the Registrar.

Thank you for your consideration of this request. I have attached a document providing more information about the experimental project. If you should require additional information, please call me at < >.

Sincerely,

< >

Project Title: Data Analysis Involving Linear Mathematical Models of Physical Science Phenomena as a Means of Strengthening Function Knowledge of Preservice Elementary Teachers

Project Goal: The goal of this project is to determine the effect completing data collection tasks based on physical science variables has on subjects' abilities to perform data interpretation tasks. The data interpretation tasks to be measured are (a) building linear mathematical models, (b) interpreting mathematical models, and (c) making predictions from mathematical models of physical phenomena.

Procedures: Subjects will be randomly assigned to one of two treatment groups. Subjects in Group A will complete two physical science workshop assignments which involve collecting data. They will then use the data they have collected to complete data interpretation tasks: (a) building linear mathematical models, (b) interpreting mathematical models, and (c) making predictions from mathematical models. For subjects in Group B, activities related to the scientific inquiry method and teaching children using a scientific inquiry method will be substituted in place of the data collection portion of Group A's treatment. Group B subjects will be given data sets analogous to the ones collected by the Group A subjects, and Group B subjects will complete data interpretation tasks identical to those done by Group A subjects. All subjects will then complete a posttest. In order to consider the effects of differences in student's scientific and mathematical backgrounds, I will need to access the following information from student's academic folders: list of high school and college mathematics and science courses taken (including grades), high school and college GPA, high school rank, and SAT scores.

Data Gathering Tool: The data gathering tool is a researcher-constructed test of content knowledge. The test is designed to assess subjects' abilities to build and interpret mathematical models beginning with a set of data, build and interpret mathematical models beginning with a verbal description, and interpret mathematical models and make predictions beginning with an algebraic formula.

Number of Subjects:_ The experimental design calls for 64 subjects, with 32 in each of the two treatment groups. Twenty-eight potential subjects are currently enrolled in ED4. Thirty-six potential subjects are currently enrolled in MATH1.

How long will the procedures take? Each subject will participate in two 2-hour treatment sessions and one 50-minute testing session.

Benefits: The elementary preservice teachers involved in this study will learn data analysis in a form consistent with teaching children via a scientific inquiry method. This will prepare them for helping children bridge the gap between the concrete experiences of observation and data collection in science and the more abstract experiences of drawing conclusions from data, which in turn may lead children to a better understanding of science. Early positive experiences with science might result in more students pursuing scientific careers to fill ever-increasing needs in our technological society.

Confidentially of Data: Confidentially of data will be maintained by assigning each subject a reference code number. As soon as all data is gathered for a particular subject, the subject's name and other identifying information will be removed from the subject's file folder. From that point on the only file identification will be the reference code number. All files will be kept by the researcher in a secured area. No one will have access to the files except the researcher. As soon as the data is no longer needed the files will be destroyed.

Appendix M

Form: Student Background Information Sheet

STUDENT BACKGROUND INFORMATION SHEET
Research Project: Mathematical Modeling in Physical Science

Name: ______________________________ SSN: ______________

Participant Code: __________

****

Information from High School Records:

SAT Scores: ________ Verbal ________ Mathematical

High School GPA: ________

High School Class Rank: ________ of ________

Mathematics and Science Courses Completed and Course Grades:

Mathematics	Grade

Science	Grade

****

Information from College Records (including Transfer Credits):

Most recent College GPA: ________

Mathematics and Science Courses Completed and Course Grades:

Mathematics	Grade

Science	Grade

Appendix N

Pilot Test #1: Posttest Item Data and Posttest Item Means

Table N-1
Posttest Item Data for Natural Science Students

Code	P1	P2	P3	P4	P5	P7	P8	P9	P10	P11
NS1	0.5	1.0	0.0	1.0	0.0	0.5	1.0	1.0	0.5	0.0
NS2	1.0	1.0	0.0	0.5	0.0	1.0	1.0	0.0	0.5	0.0
NS3	1.0	1.0	1.0	1.0	1.0	0.5	0.5	1.0	1.0	0.0
NS4	0.5	0.5	0.0	0.5	0.0	0.5	0.5	0.5	0.5	0.0
NS5	1.0	1.0	1.0	0.5	0.0	0.5	1.0	0.5	0.5	0.0
NS6	0.5	0.5	0.0	0.5	0.0	0.5	0.5	0.0	0.5	0.0
NS7	0.5	1.0	0.0	0.5	0.0	0.5	0.5	0.0	0.5	0.0
NS8	1.0	1.0	0.5	1.0	0.0	1.0	0.5	0.0	0.5	0.0
NS9	1.0	1.0	0.0	0.5	1.0	1.0	1.0	0.0	0.5	0.0
NS10	1.0	1.0	1.0	1.0	0.0	1.0	1.0	0.5	1.0	0.0
NS11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
NS12	0.5	1.0	0.5	1.0	0.0	0.5	1.0	0.0	1.0	0.0
NS13	0.5	1.0	0.0	0.5	0.0	0.5	1.0	0.0	0.5	0.0
NS14	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0
NS15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0
NS16	0.5	1.0	0.5	1.0	0.0	0.5	1.0	0.5	1.0	0.0
NS17	1.0	1.0	0.0	0.5	0.0	1.0	0.5	0.5	1.0	0.0

Code	P13	P14	P15	P16	P17	P19	P20	P21	P22	P23	P25
NS1	0.5	0.5	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.5
NS2	1.0	1.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.5
NS3	0.5	0.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	0.5	1.0
NS4	1.0	0.5	0.0	1.0	0.0	0.0	0.0	1.0	0.0	0.0	0.5
NS5	1.0	1.0	0.5	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.5
NS6	1.0	1.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	1.0
NS7	1.0	0.5	0.5	1.0	1.0	0.0	0.5	0.5	0.0	0.0	0.0
NS8	1.0	0.5	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5
NS9	0.5	0.5	1.0	0.0	1.0	0.0	0.0	1.0	0.0	0.0	0.5
NS10	1.0	0.5	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0
NS11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5
NS12	1.0	1.0	0.0	1.0	0.0	1.0	1.0	0.0	1.0	1.0	0.5
NS13	0.0	0.0	0.0	0.0	0.0	0.5	0.5	0.0	0.0	0.0	0.5
NS14	0.5	1.0	1.0	0.0	1.0	0.5	1.0	1.0	0.0	0.0	0.5
NS15	1.0	0.5	0.5	1.0	1.0	0.5	1.0	1.0	0.0	0.0	1.0
NS16	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0	0.0	0.5
NS17	1.0	0.5	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table N-1 (continued)
Posttest Item Data for Natural Science Students

Code	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36
NS1	0.5	1.0	0.5	1.0	0.5	1.0	1.0	0.5	0.0	1.0	0.0
NS2	0.0	0.5	0.0	0.0	0.0	1.0	0.0	0.5	0.0	0.0	0.0
NS3	0.5	0.5	0.0	1.0	1.0	0.5	0.0	0.5	0.0	1.0	1.0
NS4	0.0	0.5	0.0	0.5	0.0	0.5	0.0	0.5	0.0	0.0	0.0
NS5	0.0	0.5	0.0	1.0	1.0	1.0	0.0	0.5	0.0	1.0	1.0
NS6	0.0	0.5	0.0	0.0	0.0	0.5	0.0	0.5	0.0	0.0	0.0
NS7	0.5	0.0	0.0	0.0	0.0	0.5	0.0	0.5	0.0	1.0	0.0
NS8	0.0	0.5	0.0	1.0	0.0	0.5	0.0	1.0	0.0	0.0	0.0
NS9	0.0	0.5	0.0	1.0	0.0	0.5	0.0	0.5	0.0	0.0	1.0
NS10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
NS11	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0
NS12	0.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0	0.0	1.0	1.0
NS13	0.0	1.0	0.0	0.0	0.0	0.5	0.0	0.5	0.0	0.0	0.0
NS14	1.0	0.5	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.5
NS15	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	0.0	1.0	1.0
NS16	1.0	0.5	0.0	1.0	1.0	0.5	0.5	0.5	0.0	1.0	1.0
NS17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table N-2
Posttest Means by Item for Natural Science Students

Question(s)	Mean
P1	.79
P2	.94
P3	.44
P4	.76
P5	.26
P7	.74
P8	.82
P9	.44
P10	.74
P11	.06
P13	.82
P14	.65
P15	.44
P16	.65
P17	.38
P19	.32
P20	.53
P21	.50
P22	.29
P23	.21
P25	.56
P26	.38
P27	.62
P28	.26
P29	.59
P30	.44
P31	.68
P32	.29
P33	.65
P34	.18
P35	.59
P36	.50

NOTE: The initial version of the posttest (Appendix G) piloted with Natural Science students consisted of 45 questions, 32 of which were included in the final version of the posttest. The means given in the tables of this Appendix are only for the questions common to both versions of the posttest. Questions labeled P6, P12, P18, and P24 are found on the final posttest (Appendix E) but not on the initial version.

Appendix O

Pilot Test #2: Posttest Item Data and Posttest Item Means

Table O-1
Posttest Item Data for Physics Students

Subject	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
PHY1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5
PHY4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0
PHY5	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0
PHY6	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0
PHY7	1.0	1.0	0.5	1.0	1.0	0.5	1.0	1.0	0.5	0.0	1.0	1.0
PHY8	0.0	1.0	1.0	1.0	1.0	0.5	0.0	1.0	1.0	1.0	0.0	1.0
PHY9	0.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	0.0	1.0
PHY10	0.0	0.5	0.0	1.0	0.0	0.5	0.0	0.5	0.0	0.5	0.0	1.0

Subject	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24
PHY1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY6	0.5	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0
PHY8	0.0	1.0	1.0	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0
PHY9	0.0	1.0	0.0	1.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0	1.0
PHY10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	0.0	1.0

Subject	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36
PHY1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY4	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5
PHY6	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PHY7	1.0	1.0	1.0	0.5	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0
PHY8	1.0	1.0	1.0	0.5	1.0	1.0	1.0	1.0	0.5	0.5	0.0	0.0
PHY9	1.0	1.0	0.5	1.0	1.0	1.0	1.0	1.0	0.5	0.0	0.0	0.0
PHY10	0.5	0.0	1.0	0.0	1.0	0.0	0.5	0.0	0.5	0.0	1.0	0.0

Table O-2
Posttest Means by Item for Physics Students

Item	Mean
P1	.70
P2	.95
P3	.85
P4	1.00
P5	.90
P6	.75
P7	.70
P8	.95
P9	.85
P10	.85
P11	.60
P12	.95
P13	.65
P14	.90
P15	.80
P16	.85
P17	.90
P18	.90
P19	.75
P20	1.00
P21	.90
P22	1.00
P23	.85
P24	1.00
P25	.95
P26	.90
P27	.95
P28	.75
P29	1.00
P30	.90
P31	.95
P32	.85
P33	.85
P34	.75
P35	.80
P36	.65

NOTE: The final version of the posttest (Appendix E) was piloted with Physics students.

Appendix P

Interrater Reliability

The posttests were blind scored. Each posttest was scored by two raters. The following table details the scoring differences between the two raters. For each subject a summary of scoring differences is given by test item. The "Agreed" column gives the number of items on which the scores assigned by the two raters were the same. The "Fixed" column gives the number of items on which the scores assigned by the two raters differed because one or both of the raters made an error in following the established scoring criteria.. The "Compromised" column gives the number of items on which the scores assigned by the two raters differed because the raters interpreted the correctness of the subject response differently. Both the Fixed and Compromise columns also list the items which were scored differently and, in parenthesis, the item difference between the two assigned scores. After the scoring was completed by the two raters, the raters compared assigned scores, item-by-item, and made adjustments to obtain one set of item scores per subject.

Table P
Details of Posttest Raters' Differences by Subject

Subject	Agreed	Fixed	Compromised
1-A11	34	2 [2.6(.5),5.6(.5)]	0
2-A12	34	2 [5.6(.5),6.6(.5)]	0
3-A21	35	0	1 [3.5(1)]
4-A22	33	3 [1.4(.5),2.4(.5),5.2(.5)]	0
5-A31	36	0	0
6-A32	36	0	0
7-A41	36	0	0
8-A42	33	2 [4.6(.5),6.1(.5)]	1 [2.3(.5)]
9-A51	33	2 [3.2(.5),5.3(.5)]	1 [3.3(.5)]
10-A52	36	0	0
11-A61	36	0	0
12-A62	34	1 [4.4(.5)]	1 [4.5(.5)]
13-A71	36	0	0
14-A72	33	1 [4.4(.5)]	2 [6.5(.5),6.6(.5)]
15-B11	35	1 [2.6(.5)]	0
16-B12	35	1 [2.6(.5)]	0
17-B21	36	0	0
18-B22	35	0	1 [1.5(.5)]
19-B31	36	0	0
20-B32	34	1 [2.4(.5)]	1 [1.3(.5)]
21-B41	35	0	1 [5.6(.5)]

Table P (continued)
Details of Posttest Raters' Differences by Subject

Subject	Agreed	Fixed	Compromised
22-B42	34	1 [4.4(.5)]	1 [2.3(.5)]
23-B51	35	1 [5.6(1)]	0
24-B52	35	1 [2.5(.5)]	0
25-B61	34	1 [5.5(.5)]	1 [6.6(.5)]
26-B62	36	0	0
27-B71	34	2 [4.4(.5),5.1(.5)]	0
28-B72	33	3 [1.6(.5),4.4(.5),6.6(.5)]	0
29-T11	35	1 [4.4(.5)]	0
30-T12	36	0	0
31-T13	35	0	1 [1.3(.5)]
32-T22	34	1 [4.4(.5)]	1 [4.5(1)]
33-T31	36	0	0
34-M21	35	1 [1.4(.5)]	0
35-M22	35	0	1 [1.3(.5)]
36-M41	34	2 [5.1(1),6.3(.5)]	0
37-M42	36	0	0
38-M51	35	1 [2.1(.5)]	0
39-M52	34	1 [4.4(.5)]	1 [1.1(1)]
40-M61	36	0	0
41-M62	36	0	0
42-W11	36	0	0
43-W12	31	3 [1.2(.5),2.2(.5),4.4(.5)]	2 [1.6(.5),4.3(.5)]
44-W31	34	0	2 [5.2(.5),6.6(.5)]
45-W22	36	0	0
46-W31	35	0	1 [1.3(.5)]
47-W32	34	0	2 [6.5(.5),6.6(.5)]
48-W41	32	3 [1.4(.5),2.4(.5),5.3(.5)]	1 [6.2(.5)]
49-W42	36	0	0
50-W51	31	4 [1.4(.5),2.2(.5),5.1(.5),5.3(.5)]	1 [6.6(.5)]
51-W52	34	0	2 [3.3(.5),4.6(.5)]
52-W53	35	0	1 [6.6(.5)]
Totals	1803	44	30
Total Percent	96.3%	2.2%	1.5%

Summary. Of the 1872 items scored by each, the raters agreed on 1803 items. This corresponds to an Interrater Reliability of 96.3%. Forty-four of the remaining 69 scoring discrepancies (2.2%) were accounted for by misapplication of established scoring criteria. Only 27 items (1.4%) were scored differently because of rater interpretation differences.

Appendix Q

Analysis of Background Data by Experimental Pair

Treatment Sample Sizes:

$n_A = 13$	Maximum value; may be lower due to missing data
$n_B = 12$	Maximum value; may be lower due to missing data

Variable Identification:

HSR	High school rank in class (percentile)
HGPA	High school grade point average
MSAT	Score on mathematical portion of SAT
VSAT	Score on verbal portion of SAT
TSWE	Score on Test of Standard Written English
HMHRS	Total number of credits in high school math
HSHRS	Total number of credits in high school science
HMGPA	Grade point average in high school mathematics courses
HSGPA	Grade point average in high school science courses
PGPA	Predicted grade point average
CGPA	Current college grade point average
CHRS	Total number of semester hours completed in college
CMHRS	Total number of semester hours in college mathematics courses
CSHRS	Total number of semester hours in college science courses
CMGPA	Grade point average in college mathematics courses
CSGPA	Grade point average in college science courses

Table Q
t-test Results for Background Data

Variable	n_A	Group A Mean	n_B	Group B Mean	df	t-Value	p-Value
HSR	13	61.94	11	62.57	22	-.10	.9209
HGPA	13	2.83	11	2.88	22	-.28	.7857
MSAT	13	41.09	11	42.73	22	-.71	.4858
VSAT	13	37.68	11	38.06	22	-.18	.8561
TSWE	13	38.73	11	39.57	22	-.26	.7999
HMHRS	13	3.59	12	3.36	23	.89	.3826
HSHRS	13	2.87	12	2.90	23	-.12	.9059
HMGPA	13	2.32	12	2.39	23	-.29	.7764
HSGPA	13	2.41	12	2.54	23	-.56	.5823
PGPA	9	2.42	10	2.42	17	-.03	.9742
CGPA	10	3.01	10	2.86	18	.59	.5654
CHRS	10	73.50	10	89.45	18	-.93	.3638
CMHRS	9	6.39	10	7.10	17	-.61	.5524
CSHRS	9	8.44	10	7.10	17	.60	.5536
CMGPA	8	2.78	9	2.80	15	-.03	.9745
CSGPA	7	2.61	9	2.60	14	.02	.9882

Appendix R

Analysis of Workshop Evaluation Form Results

Treatment Sample Sizes:

$n_A = 13$
$n_B = 12$

Table R
t-test Results for Workshop Evaluation Form Responses (df = 23)

Statement	Group A Mean	Group B Mean	t-Value	p-Value
S1	3.09	3.51	-1.46	.1579
S2	2.87	2.72	.48	.6375
S3	3.21	2.75	1.59	.1263
S4	2.87	2.89	-.05	.9586
S5	3.67	3.57	.26	.7959
S6	4.42	4.58	-.60	.5563
S7	3.91	4.13	-.78	.4421
S8	3.80	4.13	-1.27	.2151
S9	3.94	4.21	-.79	.4350
S10	3.54	3.61	-.30	.7661
S11	3.24	3.17	.25	.8082
S12	3.12	3.40	-.97	.3401

Appendix S

Data by Individual Subject

Table S-1
Background Data by Individual Suibject

SUBJECT	TREAT	HSR	HGPA	MSAT	VSAT	TSWE	HMHRS	HSHRS	HMGPA
A11	1	93.5	3.79	65	40	45	5.0	4.0	3.60
A12	1	71.0	2.87	34	31	33	3.0	2.0	2.50
A21	1	61.2	2.71	39	30	29	4.0	3.0	2.63
A22	1	66.2	–	–	–	–	3.0	2.0	2.00
A31	1	61.9	2.81	42	37	36	4.0	4.0	2.13
A32	1	–	–	–	–	–	–	–	–
A41	1	40.1	2.47	38	41	47	4.0	4.0	2.00
A42	1	90.0	4.00	33	30	34	4.0	3.0	3.50
A51	1	93.6	4.00	47	41	–	4.0	3.0	3.50
A52	1	28.0	2.64	33	49	53	3.0	4.0	1.67
A61	1	62.5	2.87	–	–	–	2.0	1.0	2.0
A62	1	23.8	2.08	37	37	38	4.0	2.0	1.80
A71	1	38.8	2.43	28	32	–	3.0	2.0	1.33
A72	1	71.3	3.17	42	34	46	5.0	3.0	2.40
B11	2	–	–	–	–	–	–	–	–
B12	2	38.2	2.60	38	37	47	3.5	3.0	1.71
B21	2	89.6	3.68	46	36	43	4.0	2.0	3.67
B22	2	92.5	–	34	33	42	3.0	3.0	2.00
B31	2	–	–	–	–	–	–	–	–
B32	2	–	–	–	–	–	2.0	2.0	3.50
B41	2	56.2	2.64	41	35	38	4.0	3.0	1.75
B42	2	–	–	40	34	32	4.0	3.0	2.63
B51	2	69.6	2.71	31	39	45	4.0	3.0	2.00
B52	2	99.8	3.96	53	43	50	4.0	3.0	3.75
B61	2	40.2	2.06	41	31	33	3.0	3.0	1.00
B62	2	–	–	–	–	–	–	–	–
B71	2	49.1	2.61	41	40	31	3.0	4.0	1.33
B72	2	65.5	2.80	36	46	48	4.0	4.0	2.50
M21	1	59.3	3.12	45	40	36	4.0	2.0	2.50
M22	1	81.6	–	37	39	42	3.0	2.0	2.33
M41	1	63.6	2.61	27	30	30	4.0	2.0	2.50
M42	1	22.2	2.05	39	32	34	4.0	3.0	2.50
M51	1	39.1	1.86	42	28	27	4.0	3.0	1.60
M52	1	65.7	2.79	32	36	–	4.0	4.0	2.75
M61	1	91.9	3.43	65	42	50	4.0	3.0	3.25
M62	1	54.7	2.90	48	42	44	4.0	4.0	2.50
T11	1	93.8	3.70	37	45	41	3.0	3.0	2.67
T12	1	37.0	0.57	34	45	42	4.0	4.0	2.00
T13	1	37.3	3.03	45	43	43	4.0	3.0	2.63
T21	1	–	–	–	–	–	0.0	1.0	0.00
T22	1	78.3	2.73	50	48	35	3.0	3.0	2.33
W11	2	54.1	2.99	42	37	–	3.0	2.0	3.67
W12	2	69.0	2.92	47	35	42	3.0	3.0	3.00
W21	2	87.1	3.39	40	34	40	3.0	3.0	2.67
W22	2	77.3	3.60	56	54	57	4.0	3.0	3.63
W31	2	42.5	2.12	36	31	40	4.0	2.0	2.50
W32	2	87.9	3.52	51	42	43	4.0	3.0	2.75
W41	2	75.0	3.10	52	33	28	3.0	2.0	2.33
W42	2	61.9	2.66	46	52	50	3.0	4.0	1.33
W51	2	71.3	3.43	39	34	42	4.0	4.0	2.75
W52	2	11.2	1.40	57	49	38	2.0	2.0	1.33
W53	2	47.1	2.60	39	33	44	4.0	4.0	1.20

Table S-1 (continued)
Background Data by Individual Subject

SUBJECT	TREAT	HSGPA	PGPA	CGPA	CHRS	CMHRS	CSHRS	CMGPA	CSGPA
A11	1	3.75	–	2.88	103	6.0	12.0	4.00	4.00
A12	1	2.50	2.52	2.54	99	6.0	8.0	3.00	2.50
A21	1	2.33	–	2.98	122	6.0	8.0	3.50	2.00
A22	1	2.00	–	3.22	90	6.0	14.0	2.50	–
A31	1	2.63	2.15	2.54	85	9.0	8.0	2.50	2.00
A32	1	–	–	2.87	99	6.0	4.0	1.50	2.00
A41	1	2.00	–	3.13	90	9.0	12.0	3.00	3.00
A42	1	3.67	–	3.35	101	9.0	12.0	3.00	3.00
A51	1	3.67	–	3.68	89	9.0	22.0	3.50	–
A52	1	2.50	2.13	3.20	84	9.0	8.0	3.00	3.00
A61	1	2.00	–	2.90	93	6.0	8.0	2.00	2.00
A62	1	2.50	–	3.23	108	9.0	8.0	3.00	2.00
A71	1	2.00	–	3.06	115	9.0	12.0	3.00	3.00
A72	1	3.33	–	3.33	88	0.0	0.0	–	–
B11	2	–	–	4.00	106	12.0	7.0	4.00	–
B12	2	3.00	2.40	2.45	91	6.0	8.0	3.00	1.00
B21	2	3.50	–	3.61	106	7.0	5.0	4.00	–
B22	2	4.00	–	4.00	136	6.0	34.0	4.00	4.00
B31	2	–	–	3.34	139	15.0	7.0	3.00	3.00
B32	2	3.00	–	4.00	101	9.0	5.0	4.00	–
B41	2	2.67	2.18	3.07	97	6.0	8.0	4.00	2.50
B42	2	2.67	2.35	2.90	101	12.0	4.0	3.00	3.00
B51	2	2.00	2.23	2.53	89	6.0	8.0	1.67	1.50
B52	2	4.00	3.35	3.97	97	6.0	8.0	4.00	4.00
B61	2	2.00	1.91	2.61	105	9.0	8.0	3.00	2.00
B62	2	–	–	3.20	182	6.0	12.0	4.00	3.33
B71	2	2.50	2.34	2.64	99	9.0	8.0	2.67	1.50
B72	2	2.50	2.39	3.10	92	9.0	8.0	2.00	3.00
M21	1	2.00	2.79	–	–	–	–	–	–
M22	1	2.00	2.56	2.76	25	2.0	0.0	2.00	–
M41	1	2.00	2.28	–	–	6.0	8.0	–	–
M42	1	2.00	1.94	–	–	–	–	–	–
M51	1	2.00	1.63	–	–	–	–	–	–
M52	1	2.50	2.35	–	–	–	–	–	–
M61	1	2.67	3.18	–	–	–	–	–	–
M62	1	2.50	2.59	–	–	–	–	–	–
T11	1	3.00	3.09	3.50	6.0	–	–	–	–
T12	1	2.00	2.36	–	–	–	–	–	
T13	1	2.50	2.28	–	–	–	–	–	–
T21	1	0.50	–	1.91	21	–	–	–	–
T22	1	2.00	2.71	–	–	–	–	–	–
W11	2	1.50	2.43	–	–	–	–	–	–
W12	2	2.33	2.49	2.12	26	3.0	3.0	2.00	3.00
W21	2	1.67	2.35	2.04	52	3.0	3.0	–	2.00
W22	2	3.83	3.30	–	–	–	–	–	–
W31	2	2.00	1.85	1.70	46	6.0	0.0	0.00	–
W32	2	3.00	3.18	–	–	–	–	–	–
W41	2	2.00	2.61	–	–	–	–	–	–
W42	2	1.75	2.50	–	–	–	–	–	–
W51	2	1.57	2.10	–	–	–	–	–	–
W52	2	.67	2.20	–	–	–	–	–	–
W53	2	2.25	2.07	–	–	–	–	–	–

Table S-2
<u>Evaluation Form Data by Individual Subject</u>

SUBJECT	TREAT	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
A11	1	4	3	4	3	4	4	4	4	4	4	4	4
A12	1	2	3	2	2	4	4	4	4	4	4	1	2
A21	1	3	3	2	2	2	4	4	3	4	4	3	3
A22	1	2	4	2	3	5	5	4	4	5	4	4	4
A31	1	4	3	3	3	5	5	5	5	5	4	3	3
A32	1	3	4	3	4	3	5	5	5	5	3	3	3
A41	1	2	2	2	3	1	1	2	2	1	2	2	2
A42	1	3	2	3	3	2	4	3	3	3	3	3	2
A51	1	3	3	3	3	3	3	3	3	4	3	3	3
A52	1	4	4	4	3	3	5	5	5	5	5	5	4
A61	1	3	3	3	3	4	4	4	3	5	3	2	2
A62	1	3	3	4	3	2	5	4	4	4	4	3	3
A71	1	3	3	3	3	5	5	5	5	5	3	3	3
A72	1	4	4	4	4	5	5	5	4	5	4	4	4
B11	2	3	2	3	3	4	5	3	4	4	3	3	3
B12	2	4	2	2	2	5	4	4	4	4	2	2	1
B21	2	4	3	3	4	3	5	4	4	4	4	4	5
B22	2	4	4	5	5	5	5	5	5	5	4	4	4
B31	2	3	4	2	2	4	5	2	2	4	2	4	4
B32	2	4	3	2	3	4	5	3	4	4	4	3	3
B41	2	4	5	4	5	4	5	5	5	5	5	4	4
B42	2	4	4	4	4	5	5	5	5	5	5	4	4
B51	2	1	1	1	1	3	4	4	4	5	3	1	2
B52	2	–	–	–	–	–	–	–	–	–	–	–	–
B61	2	3	3	2	2	2	5	4	4	4	4	2	3
B62	2	–	–	–	–	–	–	–	–	–	–	–	–
B71	2	3	3	2	3	2	1	3	3	2	3	3	3
B72	2	4	2	4	2	5	5	5	5	5	4	4	4
M21	1	2	3	4	4	4	4	4	4	4	4	2	3
M22	1	4	1	4	2	3	5	2	2	3	2	4	3
M41	1	2	3	3	3	5	5	4	4	3	3	4	4
M42	1	2	1	4	2	3	5	3	3	3	4	2	3
M51	1	3	3	4	4	5	5	5	5	5	5	4	3
M52	1	–	–	–	–	–	–	–	–	–	–	–	–
M61	1	2	2	3	3	3	4	5	4	3	3	3	2
M62	1	3	2	4	3	2	5	3	3	–	2	3	2
T11	1	4	4	3	3	4	5	5	4	5	4	4	4
T12	1	3	2	3	3	5	5	5	5	5	5	3	4
T13	1	4	4	2	4	5	5	3	4	4	3	4	4
T21	1	3	2	2	1	5	5	3	5	4	3	4	5
T22	1	4	4	4	4	5	5	4	5	5	5	4	1
W11	2	5	2	4	2	5	5	5	4	5	4	4	3
W12	2	3	2	2	2	3	5	5	5	5	3	3	4
W21	2	4	3	2	2	3	5	4	3	3	3	3	4
W22	2	4	3	3	3	4	5	5	5	5	5	4	4
W31	2	4	3	3	3	4	4	4	3	4	3	4	4
W32	2	4	3	3	3	5	5	5	5	5	5	5	5
W41	2	4	2	2	4	3	3	3	4	3	4	3	3
W42	2	4	2	4	4	3	5	5	5	3	3	3	3
W51	2	3	3	2	4	3	5	4	4	3	3	2	3
W52	2	4	2	3	3	2	5	3	3	4	4	3	3
W53	2	4	3	4	4	2	5	5	5	5	5	4	4

Table S-3
Posttest Item Data by Individual Subject

SUBJECT	TREAT	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
A11	1	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	.0	1.0	.0	1.0
A12	1	1.0	1.0	.0	1.0	1.0	.5	1.0	.5	.0	1.0	.0	1.0
A21	1	1.0	1.0	.0	1.0	.5	.5	1.0	1.0	.0	1.0	.0	1.0
A22	1	.0	.0	.0	.5	.0	.5	.0	.0	.0	.5	.0	1.0
A31	1	1.0	1.0	.5	.5	.0	.0	1.0	1.0	.5	.5	.0	.0
A32	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
A41	1	1.0	.5	.5	.0	1.0	1.0	1.0	.5	.0	1.0	.0	.0
A42	1	1.0	.5	.5	.5	.5	.0	1.0	.5	.5	.5	.0	.5
A51	1	1.0	.5	.5	1.0	1.0	.5	1.0	.5	.0	1.0	.0	1.0
A52	1	1.0	1.0	.5	1.0	1.0	.5	1.0	.0	.5	1.0	.0	1.0
A61	1	1.0	.5	.0	.5	.0	.0	1.0	.5	.0	.5	.0	.0
A62	1	1.0	1.0	.0	.5	.0	.0	1:0	.5	.0	.5	.0	1.0
A71	1	1.0	.0	.0	.0	.0	.5	1.0	.0	.0	.0	.0	1.0
A72	1	1.0	1.0	.0	1.0	.5	1.0	1.0	.5	.5	.5	1.0	.5
B11	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0
B12	2	1.0	.5	1.0	.0	.0	.0	1.0	.5	.0	.5	.0	1.0
B21	2	1.0	.5	.0	1.0	.0	.5	1.0	.5	.0	1.0	.0	1.0
B22	2	1.0	1.0	.0	1.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0
B31	2	1.0	.5	.0	1.0	.0	.0	1.0	.5	.5	1.0	.0	.0
B32	2	1.0	1.0	.0	.5	1.0	.5	.0	.5	.0	.0	.0	.0
B41	2	1.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0	.5	.0	.5
B42	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	.0	.5
B51	2	1.0	.0	.0	.0	.0	.5	1.0	.0	.5	.0	.0	1.0
B52	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0
B61	2	1.0	1.0	.0	.5	.0	.0	1.0	.5	.0	1.0	.0	1.0
B62	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0
B71	2	.0	.5	.0	1.0	.0	.0	.0	.5	.0	1.0	.0	1.0
B72	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.5	.0	1.0
M21	1	.0	.0	.5	.0	.0	.5	1.0	.5	.5	1.0	.0	.0
M22	1	1.0	.5	.0	1.0	1.0	.0	1.0	.5	.0	.5	.0	.0
M41	1	1.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0
M42	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
M51	1	1.0	.5	.5	.5	.0	.5	.0	.0	.0	.0	.0	.0
M52	1	1.0	1.0	1.0	.5	1.0	.0	1.0	.5	.0	1.0	.0	.0
M61	1	1.0	.5	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0	.5
M62	1	1.0	.0	.0	.5	.0	.5	1.0	.0	.0	.0	.0	1.0
T11	1	1.0	.5	.0	1.0	1.0	.0	1.0	.5	.5	.5	.0	.0
T12	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T13	1	1.0	.5	.0	1.0	1.0	.5	1.0	.5	.0	1.0	.0	.0
T21	1	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T22	1	1.0	1.0	.5	1.0	1.0	.5	1.0	1.0	1.0	1.0	.0	1.0
W11	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	.0	.0
W12	2	1.0	.5	.0	1.0	1.0	.0	1.0	.0	.0	.5	.0	.0
W21	2	1.0	.5	.0	.5	.5	.5	1.0	.5	.0	.5	.0	1.0
W22	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
W31	2	1.0	.5	1.0	1.0	.0	.5	1.0	.5	1.0	1.0	.0	1.0
W32	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0
W41	2	1.0	1.0	.0	.0	.0	.0	1.0	.5	.0	.0	.0	.0
W42	2	1.0	.5	.0	1.0	1.0	.1.0	1.0	.5	1.0	1.0	.0	1.0
W51	2	1.0	.0	.0	.5	.0	.5	1.0	.5	.0	.5	.0	.0
W52	2	1.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	1.0
W53	2	1.0	1.0	.5	1.0	1.0	.0	1.0	1.0	.0	1.0	.0	.0

Table S-3 (continued)
Posttest Item Data by Individual Subject

SUBJECT	TREAT	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24
A11	1	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0
A12	1	1.0	.5	.0	1.0	.0	1.0	.0	.0	.0	.0	.0	.0
A21	1	.0	.0	.0	.5	.5	1.0	.0	.0	.0	.0	.0	.0
A22	1	.0	.0	.0	.5	.0	.0	.0	.0	.0	.5	.0	.0
A31	1	1.0	.5	.5	1.0	1.0	.0	.0	.5	.0	.0	.0	.0
A32	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
A41	1	.0	.5	.5	.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0
A42	1	.0	.0	.0	.5	.0	.0	.0	.0	.0	.0	.0	.5
A51	1	.0	.5	.5	.5	1.0	.5	1.0	.5	.0	1.0	1.0	1.0
A52	1	1.0	1.0	1.0	1.0	1.0	1.0	.0	.0	1.0	1.0	1.0	1.0
A61	1	.0	.0	.0	.0	.0	.0	.0	.5	.5	.0	.0	.0
A62	1	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	1.0	.5	.0	.0
A71	1	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0
A72	1	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	.0	.5	.0	.0
B11	2	1.0	1.0	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
B12	2	.0	1.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0
B21	2	.0	.0	1.0	.5	.0	1.0	1.0	.5	.5	1.0	.0	1.0
B22	2	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B31	2	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0
B32	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B41	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B42	2	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	1.0	.5	.0	.0
B51	2	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	1.0
B52	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0	1.0
B61	2	1.0	.5	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0
B62	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B71	2	.0	.0	.0	.5	.0	.0	.0	.0	.0	.5	.0	.0
B72	2	1.0	.5	1.0	1.0	1.0	1.0	.0	1.0	1.0	.5	.0	.0
M21	1	.0	.0	.5	.0	.0	.0	.0	.0	.0	.0	.0	.0
M22	1	.0	.5	.5	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
M41	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0
M42	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
M51	1	1.0	.5	.5	.5	.0	.0	1.0	.5	1.0	1.0	.5	.0
M52	1	1.0	1.0	1.0	1.0	1.0	.0	.0	.5	1.0	.5	.0	.0
M61	1	1.0	.5	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	.5	1.0
M62	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T11	1	1.0	.5	.0	1.0	1.0	1.0	.0	.0	.0	.5	.0	.0
T12	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T13	1	1.0	.5	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0	1.0
T21	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T22	1	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	1.0	.5	1.0	.0
W11	2	1.0	1.0	1.0	1.0	1.0	.5	1.0	1.0	1.0	1.0	1.0	1.0
W12	2	1.0	1.0	1.0	1.0	.0	.0	.0	1.0	1.0	.5	.0	.0
W21	2	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
W22	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
W31	2	1.0	.5	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
W32	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
W41	2	.0	.0	.0	.5	.0	.5	.0	.0	.0	.0	.0	.0
W42	2	1.0	1.0	1.0	1.0	1.0	1.0	.0	.5	1.0	.0	.0	.0
W51	2	.0	.5	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0
W52	2	.0	.0	.5	.0	.0	1.0	.0	.0	.0	.0	.0	.5
W53	2	1.0	1.0	.5	1.0	1.0	1.0	.0	.5	.0	.0	.0	.0

Table S-3 (continued)
Posttest Item Data by Individual Subject

SUBJECT	TREAT	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36
A11	1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	1.0	1.0
A12	1	.5	.0	1.0	.0	1.0	.0	.5	.0	.5	.0	.5	.0
A21	1	1.0	.0	1.0	1.0	.0	.0	1.0	.0	1.0	.0	.0	.0
A22	1	.5	.5	.5	.0	.0	.0	.5	.0	.5	.0	.0	.0
A31	1	.5	.0	.5	.0	.0	.0	.5	.0	.5	.0	.0	.0
A32	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
A41	1	.5	.0	1.0	1.0	.0	.0	.5	.0	1.0	.0	.0	.0
A42	1	.5	.0	1.0	.0	.0	.0	.5	.0	.5	.0	.5	.0
A51	1	.5	.0	.5	.0	1.0	1.0	.5	.0	.5	.0	1.0	.5
A52	1	.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
A61	1	.5	.0	.5	.0	.0	.0	.5	.0	.5	.0	.0	.0
A62	1	.5	.0	1.0	.5	1.0	.0	.5	.0	.5	.0	1.0	1.0
A71	1	.0	.0	.0	.0	1.0	1.0	.0	.0	.0	.0	.0	.0
A72	1	.5	.5	1.0	1.0	1.0	.0	.5	.0	1.0	1.0	.5	.5
B11	2	.5	.0	1.0	1.0	1.0	1.0	.5	.0	.0	.0	.0	.0
B12	2	1.0	1.0	.5	.0	1.0	.5	.5	.0	.5	.0	.0	.0
B21	2	.5	.0	1.0	.0	.0	.0	.5	.0	.5	.0	.0	.0
B22	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B31	2	.0	.0	.0	.0	.0	.0	.5	.0	.5	.0	.0	.0
B32	2	.5	1.0	1.0	.5	1.0	1.0	.5	.0	.0	.0	.0	.0
B41	2	1.0	1.0	1.0	1.0	1.0	.0	1.0	1.0	.5	.0	.5	.5
B42	2	1.0	.5	1.0	1.0	.0	.0	.5	.0	.5	.0	1.0	1.0
B51	2	.0	.0	.0	.0	1.0	1.0	.0	.0	.0	.0	.0	.0
B52	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
B61	2	.5	.0	.5	.0	.5	1.0	.5	.0	.5	.0	.5	.0
B62	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.0	.0	.0
B71	2	.0	.0	1.0	.5	.0	.0	.5	.0	.5	.0	.0	.0
B72	2	1.0	1.0	.5	.5	1.0	1.0	.5	.0	.5	.0	1.0	1.0
M21	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
M22	1	.5	.0	.5	.0	.0	.0	.5	.0	.5	.0	.0	.0
M41	1	1.0	.0	.0	.0	.0	.0	.0	.0	.5	.0	.0	.0
M42	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
M51	1	.5	.0	.5	.0	.0	.0	.5	.0	.5	.0	.0	.0
M52	1	.0	.0	.0	.0	.0	..0	.0	.0	.0	.0	.0	.0
M61	1	.5	1.0	1.0	1.0	1.0	1.0	.5	.0	1.0	1.0	.5	1.0
M62	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T11	1	.5	.0	1.0	.0	.0	.0	.5	.0	1.0	.0	.0	.0
T12	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T13	1	.5	.0	.5	1.0	1.0	1.0	.5	.0	.5	.0	1.0	1.0
T21	1	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
T22	1	1.0	1.0	1.0	.0	1.0	1.0	1.0	1.0	.5	.0	1.0	1.0
W11	2	.5	.0	.5	.0	1.0	1.0	.5	.0	.5	.0	1.0	1.0
W12	2	.5	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
W21	2	.5	.0	.5	.0	1.0	.0	.5	.0	.5	.0	1.0	.0
W22	2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.5	.0	1.0	1.0
W31	2	.5	1.0	1.0	.0	.0	.0	.5	.0	.5	.0	.0	.0
W32	2	1.0	1.0	1.0	.5	1.0	1.0	1.0	.5	1.0	.0	1.0	1.0
W41	2	.5	.0	.0	.0	.0	.0	.5	.0	.0	.0	.0	.0
W42	2	.5	.0	1.0	1.0	1.0	1.0	.5	.0	.5	.0	1.0	1.0
W51	2	.5	.0	.5	.0	1.0	1.0	.5	.0	.0	.0	1.0	.5
W52	2	.0	.0	.0	.0	.0	.0	.0	.0	.5	.0	.0	1.0
W53	2	1.0	.0	1.0	1.0	1.0	1.0	1.0	.0	.5	.0	1.0	.5

Appendix T

Data by Experimental Pair

Table T-1
Background Data by Pair

PAIR	TREAT	HSR	HGPA	MSAT	VSAT	TSWE	HMHRS	HSHRS	HMGPA
A1	1	82.2	3.33	49.5	35.5	39.0	4.00	3.00	3.05
A2	1	63.7	2.71	39.0	30.0	29.0	3.50	2.50	2.32
A3	1	61.9	2.81	42.0	37.0	36.0	4.00	4.00	2.13
A4	1	65.0	3.24	35.5	35.5	40.5	4.00	3.50	2.75
A5	1	60.8	3.32	40.0	45.0	53.0	3.50	3.50	2.58
A6	1	43.2	2.48	37.0	37.0	38.0	3.00	1.50	1.90
A7	1	55.0	2.80	35.0	33.0	46.0	4.00	2.50	1.86
B1	2	38.2	2.60	38.0	37.0	47.0	3.50	3.00	1.71
B2	2	91.0	3.68	40.0	34.5	42.5	3.50	2.50	2.84
B3	2	---	---	---	---	---	2.00	2.00	3.50
B4	2	56.2	2.64	40.5	34.5	35.0	4.00	3.00	2.19
B5	2	84.7	3.34	42.0	41.0	47.5	4.00	3.00	2.88
B6	2	40.2	2.06	41.0	31.0	33.0	3.00	3.00	1.00
B7	2	57.3	2.70	38.5	43.0	17.9	3.50	4.00	1.92
M2	1	70.4	3.12	41.0	39.5	39.0	3.50	2.00	2.42
M4	1	42.9	2.33	33.0	31.0	32.0	4.00	2.50	2.50
M5	1	52.4	2.32	37.0	32.0	27.0	4.00	3.50	2.18
M6	1	73.3	3.16	56.5	42.0	47.0	4.00	3.50	2.88
T1	1	56.0	2.43	38.7	44.3	42.0	3.67	3.33	2.43
T2	1	78.3	2.73	50.0	48.0	35.0	1.50	2.00	1.16
W1	2	61.6	2.96	44.5	36.0	42.0	3.00	2.50	3.34
W2	2	82.2	3.50	48.0	44.0	48.5	3.50	3.00	3.15
W3	2	65.2	2.82	43.5	36.5	41.5	4.00	2.50	2.62
W4	2	68.4	2.88	49.0	42.5	39.0	3.00	3.00	1.83
W5	2	43.2	2.48	45.0	38.7	41.3	3.33	3.33	1.76

Table T-1 (continued)
Background Data by Pair

PAIR	TREAT	HSGPA	PGPA	CGPA	CHRS	CMHRS	CSHRS	CMGPA	CSGPA
A1	1	3.12	2.52	3.21	101	6.0	10.0	3.50	3.25
A2	1	2.16	---	3.10	106	6.0	11.0	3.00	2.00
A3	1	2.63	2.15	2.70	92	7.5	6.0	2.00	2.00
A4	1	2.84	---	3.24	96	9.0	12.0	3.00	3.00
A5	1	3.08	2.13	3.44	86	9.0	15.0	3.25	3.00
A6	1	2.25	---	3.06	100	7.5	8.0	2.50	2.00
A7	1	2.66	---	3.20	102	4.5	6.0	3.00	3.00
B1	2	3.00	2.40	3.22	98	9.0	7.5	3.50	1.00
B2	2	3.75	---	3.80	121	6.5	19.5	4.00	4.00
B3	2	3.00	---	3.67	120	12.0	6.0	3.50	3.00
B4	2	2.67	2.26	2.98	99	9.0	6.0	3.50	2.75
B5	2	3.00	2.79	3.25	93	6.0	8.0	2.84	2.75
B6	2	2.00	1.91	2.90	144	7.5	10.0	3.50	2.66
B7	2	2.50	2.36	2.87	96	9.0	8.0	2.34	2.25
M2	1	2.00	2.68	2.76	25	2.0	0.0	2.00	---
M4	1	2.00	2.11	---	---	6.0	8.0	---	---
M5	1	2.25	1.99	---	---	---	---	---	---
M6	1	2.58	2.88	---	---	---	---	---	---
T1	1	2.50	2.58	3.50	6	---	---	---	---
T2	1	1.25	2.71	1.91	21	---	---	---	---
W1	2	1.92	2.46	2.12	26	3.0	3.0	2.00	3.00
W2	2	2.75	2.82	2.04	52	3.0	3.0	---	2.00
W3	2	2.50	2.52	1.70	46	6.0	---	---	---
W4	2	1.88	2.56	---	---	---	---	---	---
W5	2	1.50	2.12	---	---	---	---	---	---

Table T-2
Evaluation Form Data by Pair

PAIR	TREAT	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
A1	1	3.0	3.0	3.0	2.5	4.0	4.0	4.0	4.0	4.0	4.0	2.5	3.0
A2	1	2.5	3.5	2.0	2.5	3.5	4.5	4.0	3.5	4.5	4.0	3.5	3.5
A3	1	3.5	3.5	3.0	3.5	4.0	5.0	5.0	5.0	5.0	3.5	3.0	3.0
A4	1	2.5	2.0	2.5	3.0	1.5	2.5	2.5	2.5	2.0	2.5	2.5	2.0
A5	1	3.5	3.5	3.5	3.0	3.0	4.0	4 0	4.0	4.5	4.0	4.0	3.5
A6	1	3.0	3.0	3.5	3.0	3.0	4.5	4.0	3.5	4.5	3.5	2.5	2.5
A7	1	3.5	3.5	3.5	3.5	5.0	5.0	5.0	4.5	5.0	3.5	3.5	3.5
B1	2	3.5	2.0	2.5	2.5	4.5	4.5	3.5	4.0	4.0	2.5	2.5	2.0
B2	2	4.0	3.5	4.0	4.5	4.0	5.0	4.5	4.5	4.5	4.0	4.0	4.5
B3	2	3.5	3.5	2.0	2.5	4.0	5.0	2.5	3.0	4.0	3.0	3.5	3.5
B4	2	4.0	4.5	4.0	4.5	4.5	5.0	5.0	5.0	5.0	5.0	4.0	4.0
B5	2	1.0	1.0	1.0	1.0	3.0	4.0	4.0	4.0	5.0	3.0	1.0	2.0
B6	2	3.0	3.0	2.0	2.0	2.0	5.0	4.0	4.0	4.0	4.0	2.0	3.0
B7	2	3.5	2.5	3.0	2.5	3.5	3.0	4.0	4.0	3.5	3.5	3.5	3.5
M2	1	3.0	2.0	4.0	3.0	3.5	4.5	3.0	3.0	3.5	3.0	3.0	3.0
M4	1	2.0	2.0	3.5	2.5	4.0	5.0	3.5	3.5	3.0	3.5	3.0	3.5
M5	1	3.0	3.0	4.0	4.0	5.0	5.0	5.0	5.0	5.0	5.0	4.0	3.0
M6	1	2.5	2.0	3.5	3.0	2.5	4.5	4.0	3.5	2.0	2.5	3.0	2.0
T1	1	3.7	3.3	2.7	3.3	4.7	5.0	4.3	4.3	4.7	4.0	3.7	4.0
T2	1	3.5	3.0	3.0	2.5	5.0	5.0	3.5	5.0	4.5	4.0	4.0	3.0
W1	2	4.0	2.0	3.0	2.0	4.0	5.0	5.0	4.5	5.0	3.5	3.5	3.5
W2	2	4.0	3.0	2.5	2.5	3.5	5.0	4.5	4.0	4.0	4.0	3.5	4.0
W3	2	4.0	3.0	3.0	3.0	4.5	4.5	4.5	4.0	4.5	4.0	4.5	4.5
W4	2	4.0	2.0	3.0	4.0	3.0	4.0	4.0	4.5	3.0	3.5	3.0	3.0
W5	2	3.7	2.7	3.0	3.7	2.3	5.0	4.0	4.0	4.0	3.3	3.0	3.3

Table T-3
Posttest Item Data by Pair

PAIR	TREAT	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
A1	1	1.00	1.00	0.50	1.00	1.00	0.50	1.00	0.75	0.00	1.00	0.00	1.00
A2	1	0.50	0.50	0.00	0.75	0.25	0.50	0.50	0.50	0.00	0.75	0.00	1.00
A3	1	0.50	0.50	0.25	0.25	0.00	0.00	0.50	0.50	0.25	0.25	0.00	0.00
A4	1	1.00	0.50	0.50	0.25	0.75	0.50	1.00	0.50	0.25	0.75	0.00	0.25
A5	1	1.00	0.75	0.50	1.00	1.00	0.50	1.00	0.25	0.25	1.00	0.00	1.00
A6	1	1.00	0.75	0.00	0.50	0.00	0.00	1.00	0.50	0.00	0.50	0.00	0.50
A7	1	1.00	0.50	0.00	0.50	0.25	0.75	1.00	0.25	0.25	0.25	0.50	0.75
B1	2	1.00	0.75	1.00	0.50	0.50	0.50	1.00	0.75	0.50	0.75	0.00	1.00
B2	2	1.00	0.75	0.00	1.00	0.50	0.50	1.00	0.75	0.50	1.00	0.50	1.00
B3	2	1.00	0.75	0.00	0.75	0.50	0.25	0.50	0.50	0.25	0.50	0.00	0.00
B4	2	1.00	1.00	0.75	1.00	1.00	1.00	1.00	0.75	1.00	0.75	0.00	0.50
B5	2	1.00	0.50	0.50	0.50	0.50	0.75	1.00	0.50	0.50	0.50	0.50	1.00
B6	2	1.00	1.00	0.50	0.75	0.50	0.50	1.00	0.75	0.50	1.00	0.00	1.00
B7	2	0.50	0.75	0.50	1.00	0.50	0.50	0.50	0.75	0.50	0.75	0.00	1.00
M2	1	0.50	0.25	0.25	0.50	0.50	0.25	1.00	0.50	0.25	0.75	0.00	0.00
M4	1	0.50	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00
M5	1	1.00	0.75	0.75	0.50	0.50	0.25	0.50	0.25	0.00	0.50	0.00	0.00
M6	1	1.00	0.25	0.50	0.50	0.50	0.75	1.00	0.25	0.50	0.50	0.50	0.75
T1	1	0.67	0.33	0.00	0.67	0.67	0.17	0.67	0.33	0.17	0.50	0.00	0.00
T2	1	1.00	0.50	0.25	0.50	0.50	0.25	0.50	0.50	0.50	0.50	0.00	0.50
W1	2	1.00	0.75	0.50	1.00	1.00	0.50	1.00	0.50	0.00	0.75	0.00	0.00
W2	2	1.00	0.75	0.50	0.75	0.75	0.75	1.00	0.75	0.50	0.75	0.50	1.00
W3	2	1.00	0.75	1.00	1.00	0.50	0.75	1.00	0.75	1.00	1.00	0.00	1.00
W4	2	1.00	0.75	0.00	0.50	0.50	0.50	1.00	0.50	0.50	0.50	0.00	0.50
W5	2	1.00	0.33	0.17	0.50	0.33	0.17	1.00	0.50	0.00	0.50	0.00	0.33

Table T-3 (continued)
Posttest Item Data by Pair

PAIR	TREAT	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22	P23	P24
A1	1	1.00	0.75	0.50	1.00	0.50	0.75	0.50	0.50	0.50	0.50	0.50	0.50
A2	1	0.00	0.00	0.00	0.50	0.25	0.50	0.00	0.00	0.00	0.25	0.00	0.00
A3	1	0.50	0.25	0.25	0.50	0.50	0.00	0.00	0.25	0.00	0.00	0.00	0.00
A4	1	0.00	0.25	0.25	0.25	0.50	0.25	0.50	0.50	0.50	0.50	0.50	0.75
A5	1	0.50	0.75	0.75	0.75	1.00	0.75	0.50	0.25	0.50	1.00	1.00	1.00
A6	1	0.50	0.50	0.50	0.50	0.50	0.50	0.00	0.75	0.75	0.25	0.00	0.00
A7	1	0.50	0.50	0.50	0.50	0.50	1.00	0.00	0.50	0.00	0.25	0.00	0.00
B1	2	0.50	1.00	1.00	0.75	1.00	1.00	0.50	0.50	0.50	0.25	0.50	0.50
B2	2	0.50	0.25	1.00	0.75	0.50	1.00	1.00	0.75	0.75	1.00	0.50	1.00
B3	2	0.50	0.50	0.50	0.50	0.50	1.00	0.50	0.50	0.50	0.50	0.50	0.50
B4	2	1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	0.75	0.50	0.50
B5	2	0.50	0.50	0.50	0.50	0.50	1.00	0.50	0.25	0.50	0.50	0.50	1.00
B6	2	1.00	0.75	0.50	1.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
B7	2	0.50	0.25	0.50	0.75	0.50	0.50	0.00	0.50	0.50	0.50	0.00	0.00
M2	1	0.00	0.25	0.50	0.50	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00
M4	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00
M5	1	1.00	0.75	0.75	0.75	0.50	0.00	0.50	0.50	1.00	0.75	0.25	0.00
M6	1	0.50	0.25	0.50	0.50	0.50	0.50	0.50	0.25	0.50	0.50	0.25	0.50
T1	1	0.67	0.33	0.33	0.67	0.67	0.67	0.33	0.17	0.33	0.50	0.33	0.33
T2	1	0.50	0.50	0.50	0.50	0.50	0.50	0.00	0.50	0.50	0.25	0.50	0.00
W1	2	1.00	1.00	1.00	1.00	0.50	0.25	0.50	1.00	1.00	0.75	0.50	0.50
W2	2	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
W3	2	1.00	0.75	1.00	1.00	1.00	1.00	0.50	0.50	0.50	0.50	0.50	0.50
W4	2	0.50	0.50	0.50	0.75	0.50	0.75	0.00	0.25	0.50	0.00	0.00	0.00
W5	2	0.33	0.50	0.33	0.67	0.33	0.67	0.00	0.17	0.00	0.00	0.00	0.17

Table T-3 (continued)
Posttest Item Data by Pair

PAIR	TREAT	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34	P35	P36
A1	1	0.75	0.50	1.00	0.50	1.00	0.50	0.75	0.50	0.75	0.00	0.75	0.50
A2	1	0.75	0.25	0.75	0.50	0.00	0.00	0.75	0.00	0.75	0.00	0.00	0.00
A3	1	0.25	0.00	0.25	0.00	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00
A4	1	0.50	0.00	1.00	0.50	0.00	0.00	0.50	0.00	0.75	0.00	0.25	0.00
A5	1	0.50	0.00	0.25	0.00	0.50	0.50	0.25	0.00	0.25	0.00	0.50	0.25
A6	1	0.50	0.00	0.75	0.25	0.50	0.00	0.50	0.00	0.50	0.00	0.50	0.50
A7	1	0.25	0.25	0.50	0.50	1.00	0.50	0.25	0.00	0.50	0.50	0.25	0.25
B1	2	0.75	0.50	0.75	0.50	1.00	0.75	0.50	0.00	0.25	0.00	0.00	0.00
B2	2	0.75	0.50	1.00	0.50	0.50	0.50	0.75	0.50	0.75	0.50	0.50	0.50
B3	2	0.25	0.50	0.50	0.25	0.50	0.50	0.50	0.00	0.25	0.00	0.00	0.00
B4	2	1.00	0.75	1.00	1.00	0.50	0.00	0.75	0.50	0.50	0.00	0.75	0.75
B5	2	0.50	0.50	0.50	0.50	1.00	1.00	0.50	0.50	0.50	0.50	0.50	0.50
B6	2	0.75	0.50	0.75	0.50	0.75	1.00	0.75	0.50	0.75	0.00	0.25	0.00
B7	2	0.50	0.50	0.75	0.50	0.50	0.50	0.50	0.00	0.50	0.00	0.50	0.50
M2	1	0.25	0.00	0.25	0.00	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00
M4	1	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00
M5	1	0.25	0.00	0.25	0.00	0.00	0.00	0.25	0.00	0.25	0.00	0.00	0.00
M6	1	0.25	0.50	0.50	0.50	0.50	0.50	0.25	0.00	0.50	0.50	0.25	0.50
T1	1	0.33	0.00	0.50	0.33	0.33	0.33	0.33	0.00	0.50	0.00	0.33	0.33
T2	1	0.50	0.50	0.50	0.00	0.50	0.50	0.50	0.50	0.25	0.00	0.50	0.50
W1	2	0.50	0.00	0.25	0.00	0.50	0.50	0.25	0.00	0.25	0.00	0.50	0.50
W2	2	0.75	0.50	0.75	0.50	1.00	0.50	0.75	0.50	0.50	0.00	1.00	0.50
W3	2	0.75	1.00	1.00	0.25	0.50	0.50	0.75	0.25	0.75	0.00	0.50	0.50
W4	2	0.50	0.00	0.50	0.50	0.50	0.50	0.50	0.00	0.25	0.00	0.50	0.50
W5	2	0.50	0.00	0.50	0.33	0.67	0.67	0.50	0.00	0.33	0.00	0.67	0.67