BAUCOM, LAUREN NAOMI. Ph.D. An Exploratory Study on the Identity Work of Secondary Math Teachers and their Engagement with Critical Statistical Literacy. (2022)
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In an era of data saturation, teacher educators have yet to address the problem of how to develop the critical statistical literacy of practicing teachers. This study contributes to our understanding of teaching learning, specifically considering how designed materials can make visible the identity work of practicing secondary mathematics teachers. Exploring how materials designed to develop critical statistical literacy interact with the historicity and knowledge of who a teacher is and is becoming as a critical statistics educator illuminates the critical role that identity plays in teacher learning.

My dissertation focused on the development of critical statistical literacy that occurred in a sample of secondary math teachers who implemented materials designed for a new fourth level math classroom in a southeastern state. I anchored the design materials around exploratory data analysis, a purposefully chosen context of study that allowed for the infusion of critical statistics topics throughout one unit of study in a high school math classroom. Following from the work of Cobb and colleagues (2013), Bakker (2019), and others, my examination of the design-based materials through the lens of the teachers' enactment illuminates how specific key features can develop critical statistical literacy in secondary mathematics teachers. Drawing from Wenger (1998), Sfard and Prusak (2005), Tan and colleagues (2013), and others, my examination of identity work was guided by how one's access to resources can make visible the identity work that is occurring in relationship to the development of critical statistical literacy.

Here, I present my study of teacher development of critical statistical literacy from multiple perspectives, ranging from the details of curricular design, to the identity work made visible through their enactment, to the commonalities and contrasts in their reasoning for their

varied participation. While there is clear evidence of identity work being made visible in the teachers through the enactment of the materials, the interaction between who a teacher was, who they say they are, the resources they have available to them, and who they wish to become yields a range of experiences that provide a rich story for better understanding how to support teacher development of critical statistical literacy. My study pushes for further exploration of how the field can continue to examine the complex interplay of curricular design, teacher identity work, and critical statistics for the betterment of society.

AN EXPLORATORY STUDY ON THE IDENTITY WORK OF SECONDARY MATH TEACHERS AND THEIR ENGAGEMENT WITH CRITICAL STATISTICAL LITERACY

by

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Approved by

Dr. Edna Tan
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DEDICATION

I dedicate this dissertation in memory to my grandmother, Naomi Lewis Brooks, and in honor of my grandfather, MG Leo A. Brooks, Sr., whose encouragement to continue pushed me through many hard days. Their steadfast belief that one's education was of utmost importance motivated me to keep going. I am forever thankful for the love and support they provided me throughout my life and this doctoral journey. May I continue to live into the honor of being your namesake, pave the way for the next generation that comes from your legacy, and make you proud.

APPROVAL PAGE

This dissertation written by Lauren Naomi Baucom has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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'"O give thanks to the Lord, for He is good; For His lovingkindness is everlasting. Then say, "Save us, O God of our salvation, and gather us and deliver us from the nations, to give thanks to Your holy name, and glory in Your praise." Blessed be the Lord, the God of Israel, from everlasting even to everlasting. Then all the people said, "Amen," and praised the Lord.'

-1 Chronicles 16:34-36

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CHAPTER I: INTRODUCTION

A statistical literacy movement has begun as the public recognizes the gravity of one's ability to make data-based decisions and recognize misinformation (Spicer, 2018; Gaissmaier & Gigerenzer, 2008). The responsibility of producing statistically literate citizens by significantly increasing their statistical content knowledge of the general populace has fallen to math teachers. In the last two decades, evidence of how this statistical literacy movement has seeped into public education is demonstrated by the increase in education policies, standards, curricula, and teacher education organizations stressing the need to bring data analysis and statistical literacy into the K-12 classroom (Franklin et al., 2007, National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010, Franklin et al., 2015; NCTM, 2018).

Education policies outlined by the *Guideline for Assessment and Instruction for Statistics Education* (GAISE) Report, Franklin and colleagues (2007) established that the learning of statistics has equal importance in understanding content knowledge and nurturing the statistical literacy one needs in order to be a productive citizen in today's society. However, it was not until the widespread adoption of the Common Core State Standards in 2012 that most math educators were required to teach courses that contained the domain of statistics (National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010). Previous teaching of statistics was only done by those certified to teach college courses at the high school level (e.g., AP Statistics). As a result, the teaching of statistics moved from being the work of the few to the work of many (Franklin et al., 2015).

Within the GAISE report, Franklin and colleagues (2007) argued that statistical literacy is a fundamental right for all people, citing four arguments as evidence for the need to graduate statistically literate high school students: (1) one's citizenship, (2) ability to make personal choices, (3) understanding of scientific findings, and (4) the proximity to statistics in the workplace and professions. The fourth argument of proximity to the workplace and professions is the most often cited for the inclusion of statistical content in mathematics classrooms, appearing in policy documents written by NCTM, such as *Principles and Standards for School Mathematics* (2000) and *Catalyzing Change* (NCTM, 2018), as national organizations claim that more professions use statistics than calculus, thus making it more imperative for students' success in life and work.

While citing this evidence has been particularly effective in pushing educational policy to incorporate standards focused on statistics and data analysis into the K-12 mathematics curriculum, there is ample evidence that suggests that students are still not experiencing a robust statistical education focused on citizenship, the ability to make personal choices, and the understanding of scientific findings has given students a benign experience of statistics, leaving them vulnerable in a data-rich society and, according to the GAISE report, unable to "thrive in the modern world" (Franklin, et al., 2007, p. 4).

There are several reasons for the break in the statistics education pipeline that moves policy and research into practice in the math classroom. For one, the majority of practicing teachers have had no training on how to differentiate statistics instruction for math instruction, and implementation of the CCSS was essentially dropped in their lap without any professional development to support the new content (Franklin, et al., 2015). At the same time, preservice programs are not adequately preparing the next generation of teachers in how teaching statistics differs from teaching mathematics (Shaughnessy, 2007).

There is also the youthfulness of the field of statistics education research to consider, which impacts the practice of teachers in significant ways. One is that the field of statistics education has been less focused on teacher learning of statistical ideas and effective teaching practice as it has been focused on student learning of statistical ideas. There are certainly bodies of research that look at statistical knowledge for teaching (e.g., Groth, 2013; Silverman & Thompson, 2008; Garfield & Ben-Zvi, 2008), yet most of the literature on effective statistics teaching is theoretical, and not empirical (See Bargagliotti et al., 2014; Garfield & Everson, 2009; Gould & Peck, 2004 for counterexamples).

A second impact on teacher practice of the youthfulness of the field of statistics education is the lack of practical, empirical research on how teachers can develop student statistical literacy. Despite the continual call for the development of statistical literacy, there is not an overwhelming consensus among the general populace or the experts in the field of statistics education as to what that includes. How statistical literacy is defined is not the same for everyone. Statistical literacy has been vaguely defined as the "all encompassing goal of statistics instruction" (DelMas, 2002), while researchers attend to naming specific broad goals that must occur within statistical literacy. Rumsey (2002) states that the goals of statistical literacy are twofold: to help students to "understanding statistics well enough to be able to consume the information that they are inundated with on a daily basis, think critically about it, and make good decisions based on that information" and to assist students in developing "research scientist skills". Gal (2002) defined statistical literacy as the relationship between one's ability "to interpret and critically evaluate statistical information" within varying contexts and one's ability "to discuss and communicate" their understanding of the statistical information at hand. Delineating what is in statistical literacy and what is not seems to depend on the eye of the beholder.

Despite varying definitions, there is some agreement as to what statistical literacy should entail, namely the learning of basic vocabulary in context, the comprehension of why data exists and how to generate it, connecting multiple representations, using inference to draw conclusions, and recognizing the omnipresence of variance and its potential to generate conflicting interpretations (DelMas, 2002; Gal, 2002; Garfield and Ben-Zvi, 2007; Rumsey, 2002; Cobb & Moore, 1997). When looking at this list, the majority of what experts say encompasses statistical literacy is a list of skills and specific content genres to understand, rather than an approach to what topics, with what ethics, and for whose gain is desired for the student experience, and there is certainly little support for teachers on how to develop statistical literacy with K-12 students.

Because of its location in math curriculum, looking to the field of mathematics education could shed some light as to how to develop students' statistical literacy and what specific teaching practices do this well. The National Research Council (Kilpatrick, et al., 2001) has clearly defined what is included within mathematical literacy yet has used the terminology of "proficiency" to mean the basic set of skills needed to be seen as mathematically literate and prepared for everyday citizenship (2001). The NRC (Kilpatrick, et al., 2001) has defined mathematical proficiency as containing five components, 1) conceptual understanding, 2) procedural fluency, 3) adaptive reasoning, 4) strategic competence, and 5) a productive disposition. Compared to research on defining statistical literacy, there is a lack of focus on dispositions as tied to the first four components of mathematical proficiency (See Gal (2002) for counterexample).

In more recent findings, mathematical literacy has also been shown to include students' use of their various funds of knowledge to bring understanding to school-based mathematics (NRC, 2004). It has been shown that the ability to see their culture and identity infused within the domain of mathematics is not only a basic skill of someone who is mathematically literate, but also a fundamental right (NCSM & TODOS: Mathematics for ALL, 2016).

Some have termed this infusion of identity and power into the basic skills of mathematical literacy as *critical mathematical literacy*. Gutiérrez (2013) described both the learning dominant mathematics and critical mathematics, with the dominant focused on supporting student access and achievement, and the critical centering student learning on identity and power. Others have taken a Freirian approach to define critical mathematical literacy as the reading and writing of the word and the world with mathematics (Gutstein, 2006; Brantlinger, 2013, Kokka, 2017). Still others claim that critical mathematics is similar to the teaching and learning of mathematics for social justice, or the act of learning mathematics to disrupt socially unjust systems (Bartell, 2013; Gregson, 2012; Gonzalez, 2009). However critical mathematical literacy is defined, what is of key importance among them is the demonstrated necessity of the inclusion of identity work that motivates students to act justly in the learning and teaching of mathematics.

Juxtaposed against the backdrop of mathematics education, the body of literature in statistics education research that infuses identity and power into statistical literacy, or *critical statistical literacy,* is understudied and undertheorized. Some have begun to posit on the inclusion of elements of the critical axis in regard to statistics learning (Gal, 2002; Cobb & Hodge, 2002; Whitaker, 2016; Weiland, 2017; Bailey & McCulloch, 2019). Yet, for the majority, the study of statistics remains a benign, noncritical, objective space focused on the exploration of "signals" (Konold & Pollatsek, 2002) (e.g., calculations such as mean, median, and mode), without a specific agenda towards the dispositions, and identity work that are desired in statistical learners. At the time of this research, there is no known literature that addresses teachers' development of their own critical statistical literacy in an effort to support their work in the development of critical statistical literacy in their students.

Because of this under theorization, it is important to clearly define *critical statistical literacy*, as it is used in this study. For the purposes of this dissertation study, critical statistical literacy is defined as the fundamental right and skill to analyze and evaluate statistical data that shapes one's identity-in-practice for the purposes of disrupting unjust systems by creating agents of power who are able to communicate and discuss their evaluation of this statistical information with the world around them.

The lack of teacher training and support for the recent implementation of the statistics standards combined with the lack of consensus to define statistical literacy is compounded by the lack of practitioner-based research on how to develop even a benign statistical experience in one's students. This lack of teacher support provides evidence that the majority of students are not developing critical statistical literacy, let alone the statistical thinking and reasoning needed to develop healthy skepticism of information, and that inappropriate reasoning of youth and adults about statistical ideas is pervasive, assiduous, and immutable (Lee & Tran, 2015; Garfield & Ahlgren, 1988). Supporting students' skepticism of statistical information is necessary for this current era of data saturation, yet few students have the opportunity to cultivate healthy statistical habits of mind and instead opt for heavy consumption of data as absolute truth (Lee & Tran, 2015; Wilson & Journell, 2011).

What happens in the place of critical statistical literacy in the K-12 math classroom is the pervasive use of benign statistics, or those perpetuating the unjust social systems that whitewash students' mathematics educational experience to be noncritical. A benign statistical educational experience is one in which teachers use stories, mediums (e.g., Skittles, M&Ms, or candy), or datasets (e.g., arm length versus height) to teach statistical concepts or skills that are inconsequential to making visible any power differentials that exist in the world. It is important to clarify what I mean by power differentials, as they come in multiple levels. First, there is the known and often visible power differential that exists between teacher and student in any given classroom. This is to say that the teacher holds considerable power over what the student studies, in what context, in what order, with what examples, and for what length of time (Fiore, 1999; Jamieson & Thomas, 1974). Second, there is the power differential that occurs between a white teacher and a student of color, specifically in the context of math classes. There is considerable evidence demonstrating how race is used as a barrier to keep students of color from accessing upper level math classes (Leonard et al., 2010; Berry et al., 2014, Moses & Cobb, 2001; Stiff & Harvey, 1998; Martin, 2019). With the majority of math teachers being white and the majority of students being students of color, this power differential influences the types of math experiences that students are able to have in whether they can access high quality mathematics instruction and then achieve in those courses.

A third type of power differential comes in whether or not statistics is taught as a tool for social justice in math classrooms. Specifically centering on the experiences of students of color, when teachers choose to use stories, mediums, and datasets that are of no consequence to the students (Koestler, 2012), then these students, and those who desire to be their allies in their fight for justice, are not prepared to understand the ways in which the world uses data for their continual harm and oppression. When students are taught statistics through the sorting of candy only instead of real-world social issues, it impacts students of color differently than it does white students. Most of the statistics that adults consume is not about candy or the frequency of their color in a package. Rather, it is of political nature with grave consequences. Take the novel coronavirus data that we are inundated with daily at the time of this writing. Being able to read news media that portray data is a matter of life and death. But as the virus continues to ravage communities of color, mainly due to the Great Migration in larger cities which is historically derived from the remnants of the era of slavery, the consequences of reading this data are different for people of color and white people. This power differential does considerable harm to students of color and whether or not they are able to engage with the statistics of the world, or develop critical statistical literacy in being able to use statistics as a tool for their own justice (e.g., to demand better healthcare in areas of Black concentration). Throughout this study, when I refer to the benign statistical experiences, I refer to these multiple power differentials occurring in that moment.

A benign statistical educational experience only perpetuates the lack of development of healthy statistical practices, as students are presented with statistical contexts that do not reflect the contexts of the type of critical data they are inundated with in their daily lives (Cobb & Hodge, 2002; Nasir, 2002). Without investigating statistical data focused on critical issues such as race, gender, or class, the study of these benign statistical contexts reproduces statistically illiterate citizens who are incapable of using statistics as a tool to dismantle the very hierarchies that are used to oppress them. And if there is little to no training on how to teach statistics effectively for even the benign experience, there is little evidence of any support for teachers learning to implement critical statistical literacy in their classrooms.

The sociopolitical turn in mathematics education emphasizes that students' mathematics, and therefore statistical teaching and learning, must empower students to identify with and transform their world (Gutiérrez, 2013). It is important that youth have a robust, mirroring, statistical experience in K-12 education in order to engage in and critique the arguments around the statistics that are used to classify them (Bolter & Gromala, 2003; Gutiérrez, 2013). Student identity has been shown to be a critical component of student math learning (Martin, 2010), and a growing body of math education research provides evidence that students' identities matter (Aguirre et al., 2013; Celedon & Ramirez, 2012; Civil, 1998; 2014; Martin, 2010; Wood, 2013, Nasir, 2002, Boaler & Greeno, 2000; Boaler & Staples, 2008). Specifically, Aguirre, Mayfield-Ingram and Martin (2013) "define a mathematics identity as the dispositions and deeply rooted held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the context of their lives" (p. 14). When students' identities are ignored, their math learning becomes a dehumanizing experience, one in which they are not fully seen and heard as learners nor human beings (Gutiérrez, 2018).

Statistical literacy has been shown to be a fruitful space of radical possibility that could prompt a student to "recognize his or her position in society" and "motivate individuals to action" (Frankenstein, 1983, 1989, 1990, 1994; Gutiérrez, 2013), yet less is known about amplifying student identities as the impetus for developing critical statistical literacy (Nasir, 2002; Cobb & Hodge, 2002). Apart from Weiland's (2019) initial theoretical definition of critical statistical literacy through the frame of reading and writing the world and the word with statistics, the field has yet to envision what critical statistical literacy could be.

The infusion of critical statistical literacy into the mathematics classroom is about creating space for students to question how society uses statistics to infer certainty and spread misinformation and how statistics has been used to dehumanize some through the marginalization of some and the normalizing of others (Fendler & Muzzafar, 2008). Critical statistical literacy allows for the recognition that for certain populations, statistics continually reveal deficiency while others seem to patternize towards superiority, and how to wrestle with the mis/representativeness of oneself within data that portrays someone as “less than” and others as better due to demographic variables (e.g., race, gender) that have nothing to do with ability. In order to rehumanize statistics education, students need to be taught about the implications of ethics that occur when a statistic is calculated that predicts a future humanitarian or environmental crisis, and that the right to report is not limited to sexual offenses. The study of critical statistical literacy in the K-12 classroom has the potential to rupture simplistic binaries that rest on the continued use of dehumanizing distributions and reorient students to one another as they see themselves and those around them as real people who are belong in a set of data based on their identity. Allowing students to productively struggle by wrestling with data that can be viewed from different perspectives to tell different stories has the potential to empower students to learn not only about statistics, but also about themselves and the world in which they live.

In order to create the possibility of a K-12 classroom where this type of critical statistical literacy could be developed among citizens, students must have statistics educators who are prepared to use teaching practices that will allow for such a rehumanizing educational experience. The statistics education researchers who have worked to define effective statistics teaching practices have thus far focused on the separation of statistical content knowledge and statistical pedagogical knowledge (Groth, 2013; Cobb & Moore, 1997; Garfield & Ben-Zvi, 2008). The context in which statistics is taught is almost always a component of these theorizations and seen as imperative for the development of statistical literacy. For example, Garfield and Ben-Zvi (2008) state in their second tenet that students should “use real and motivating data sets”. Yet, there is no operationalizing of what it means to be motivating, and to answer the question of "to whom?". With the lack of definition in what is motivating in terms of context, benign statistical contexts that are "engaging" are left up to the device of the teacher (Garfield & Ben-Zvi, 2008). Students studying how long it takes for two Skittles packages to have the same array of candy colors could be "engaging" and motivating to students, if they get to eat the candy. This varies significantly from data sets that are "motivating" students to understand their identity and become agents of power that can disrupt known injustices (Gutiérrez, 2013).

What is motivating to an educator also varies significantly from what is motivating to students. Math teacher identities have been shown to have a significant impact on what curricular resources teachers choose, as math teachers draw upon their own personally powerful math moments in their early math experiences to either reflect or deflect away from their early math experiences (Drake & Sherin, 2006). Teachers also act as gatekeepers for the mathematics learning and mathematics contexts that their students are able to study (NRC, 2004; Gamoran, 1994; Stein et al., 1990; Moses & Cobb, 2001). What we do know is that teachers who ignore the identity of their students or their own identity as math teachers are not fully meeting the diverse needs of their math learners, and the incorporation of identity development in math learning is an effective practice in math teaching (Aguirre et al., 2013; Civil, 2014).

Combining the lack of widespread use of critical statistics steeped in identity development in the K-12 statistics education experience with the lack of consensus on effective teaching practices compounded with the lack of experiences of practicing and preservice teachers with statistics leaves the preparation of statistically literate citizens ill-fated at best, with significant variance from teacher to teacher and classroom to classroom (Groth, 2008; Shaughnessy, 2007; Franklin, et al., 2015).

Teachers hold an extraordinary amount of power in deciding what students will and should experience within their K-12 trajectory toward citizenship (Ingersoll, 1996). There are considerable places in the domain of statistics in K-12 education in which students could experience robust data sets that allow them to analyze how statistics have been used in the past to marginalize some and center others (Fendler & Muzaffar, 2008). Despite data-based metrics such as race, gender, and class being omnipresent in the sorting of students for the purposes of "achievement" in K-12 education, students rarely encounter data investigating critical topics such as race, gender, or class within their statistics lessons. The paradox of students being analyzed using critical statistics, yet not learning about the very statistics that are used to analyze them by their identities perpetuates the cycle of oppression, handcuffing our future citizens with the lack of agency, identity development, and power to invoke real change in our inequitable society. In general, the little research we have on the effective statistical teaching practices in using imploring data that encourages students to look at and recognize their own identity and the identities of those around them, and then act to disrupt systems of injustice as they discover patterns within data that investigates critical issues such as race, gender, or class may help explain why critical statistical illiteracy among prospective and practicing teachers remains pervasive (Weiland, 2019; Bargagliotti et al., 2014; Garfield & Everson, 2009; Gould & Peck, 2004, Shaughnessy, 2007).

Relevance & Research Aim

With the current state of statistics education as described above, it is imperative that teachers of statistics begin to incorporate critical statistical literacy into their instructional practice to end the oppressive cycle of critically statistically illiterate citizens. Without this incorporation, the aim of GAISE to create statistically literate citizens is not obtainable, and it is the marginalized who bear the most cost. Just take the backdrop of the current pandemic as a lived example.

The novel Coronavirus-19 has, at the time of this writing, been the cause of death for 1,000,000 humans in the United States and almost 5 million worldwide. Yet, it has not impacted all subgroups of the population proportionally. Indigenous communities have been ravaged by this virus, infecting these populations almost 3.5 times higher than other racial groups. Yet, there is very little data about the case rates of these communities being shared or even generated. As critical statisticians, we must stop to ask why. Why are the case rates so high, yet so little data is being collected, analyzed, and evaluated to assist this community through this humanitarian crisis? With just a little research, one can find information regarding the inadequate health and social services found in these reservation communities that are subpar compared to the national standard, and the compounding effect of how this virus has impacted a community that faces food insecurity due to the loss of their traditional lands and traditions (UN). The lack of criticality in our evaluation of these statistics means more Indigenous people are dying. The study of this subgroup of the United States population is also set against the backdrop of those on the other end of the spectrum who claim the virus is a hoax. Through the platform of social media, statistics that support misinformation have been overshared and intentionally used to mislead and misguide the general public in misunderstanding the scientific process. Because of our lack of critical statistical literacy as a nation, the pandemic continues to crush both the individually and the systemically vulnerable. The lack of critical statistical literacy has grave consequences for those that society deems outside the margins.

The aim of this work is to contribute to the body of literature on the teaching and learning of critical statistics. This study explored the identity work and negotiations three high school statistics teachers engaged in, as they took up and enacted curricular materials designed to support students' development of critical statistical literacy. I designed these materials to explore, analyze, and evaluate statistical data to investigate critical issues focused on race, gender, and class in order to communicate and discuss the patterns students discover with the world around them. A second aim of the materials is to explore one's identity for the purposes of creating agents of change who are able to disrupt unjust social systems. With the three teacher participants, the scope of this study is to better understand how these curricular materials could support secondary math teachers in their development of critical statistical literacy through the identity work they engage in as they enact the materials.

Most of the research involving critical mathematics and statistics has been conducted through teaching experiments (Frankenstein, 1989; Brantlinger, 2011, 2013; Gutstein, 2003, 2006) where researchers return to the classroom to teach a course for a designated period of time. Situated in practice, these methods have served as an existence proof that critical contexts can be taught within the domains of mathematics, including statistics. However, due to their contextual nature of being conducted by researchers returning to classrooms to teach, these studies are not readily replicable by teachers in traditional classroom settings. Therefore, this study also seeks to use a justice-oriented, design-based research approach to meet the practical and theoretical goals of supporting practicing teachers as critical statistics educators to use curricular resources that are created by researchers, but enacted, modified, and replicated in the secondary math classroom.

With the broad range of statistical concepts and topics, this project will focus on the key statistical topic of exploratory data analysis. Exploratory data analysis has been shown to be fruitful context for the conceptual development of statistical literacy (Cobb & Moore, 1997;

Franklin et al., 2007; Gal, 2002), and identity development (Cobb & Hodge, 2002), making it a potentially effective place for the development of critical statistical literacy. Centered within exploratory data analysis is the omnipresence of variance within the statistical paradigm. Simply put, data varies. Throughout the statistical paradigm, statisticians make choices, often subconscious, that affect the overall outcome of the inference made for a particular question. Different people will ask different questions about a data set. There are multiple ways to sample a population representatively. As samples change, variance is again introduced as the distributions of data samples changes as well. Variance amongst data is what causes the need to calculate different statistics in order to justify patterns within distributions. Determining whether the calculations we make are representative of the original population is based on the inference one is capable of making. The omnipresence of variance that exists within the statistical paradigm as it occurs in exploratory data analysis, and the lack of experience with it for both students and teachers (Cobb & Moore, 1997), provides fertile ground to study the learning of critical statistical literacy.

Parallel to this thinking, our identities also vary. Each of our identities are labeled by social, historical, and geographical constructs that allow humans to be categorized (Fendler & Muzzaffar, 2008). By placing people into bins, statisticians can look for patterns across populations and subpopulations of people to look for known inequities. Yet, the byproduct that comes from binning people by these constructs can also create a dehumanizing experience as people feel they are “just a number”, rather than an individual amidst a data set. For many secondary mathematics students, the exploration of their identity has been a process of becoming, set against the backdrop of their history (Holland & Lave, 2001). For some, they are well aware of the bins that society uses to identify them. For others, it is a new exploration.

Because of the natural variance that exists within each classroom's set of identities, this, too, is a fertile space for the study of learning about statistics through the exploration of data proximal to oneself.

The concept of variance within distributions of samples proves to be a potentially rich site for the development of critical statistical literacy (Weiland, 2019). With a critical lens, the samples that we choose determine the distributions we see and the amount of variance within those distributions. For example, if one chooses to sample the net wealth across a county with multiple zip codes, one may find a fairly normal distribution with moderate, typical variance, indicating that the wealth within that society is normally distributed amongst its citizens. However, should one choose to sample from within each zip code, the data may indicate disparate distributions amongst citizens in different regions. The inferences drawn from each sample have significant consequences. Should a county manager wonder about whether the county has equitable economic wealth, the normal distribution across the county would allow the leaders in government to assure their citizens that this is, indeed, the case, when the reality of that truth differs greatly for citizens across zip codes. While geography seems like the explicit variable understudy in this example, there are underlying lurking variables of race, gender, class, and the intersections between them. The difference in how one chooses to sample demonstrates how data can be used to continually oppress those from marginalized communities.

The purpose of this research is to investigate the overarching question: *How and in what ways do curricular resources make visible the development of critical statistical literacy?* To do so, the scope of this dissertation is focused on secondary math teachers as they are becoming critical statistics educators. I do this by answering the following two research questions:

- *What features of the design did the teachers discuss?*

- *How and in what ways do the materials make visible the identity work in secondary math teachers as critical statistics educators?*

I conjecture that the use of curricular materials focused on investigating critical issues such as race, gender, and class, will provide secondary math teachers an opportunity to develop their critical statistical literacy through the development of their statistical content and pedagogical knowledge but also their identity work as critical statistics educators. I anticipate that secondary math teachers, through comparison to the curricular materials focused on critical statistics, will become aware of their prior use of benign, noncritical statistics and begin to think about their role in perpetuating the oppressive cycle of critically statistically illiterate students in regard to their role as consumers and producers of data, their role as statistics teachers, and their role as teachers of critical statistics. I also conjecture that learning about their students' identities through the curricular materials will provide secondary mathematics teachers to engage in identity work as related to becoming critical statistics educators.

Statement of Positionality

The statistical literacy of minoritized students is compoundingly problematic. Minoritized students are often the subject of statistical analysis, in which their race, gender, and class are used by others from dominant identities to inflict harm and create structures to support the active use of oppression and segregation. These statistics are used to shape our society, inform policy, and determine actual measures in the lives of these citizens. The statistical illiteracy present within not only K-12 schools, but specifically within the subpopulation of Black and brown students perpetuates a system of oppression in which simply not knowing how to analyze data or how data is being used as a tool for harm to situate people of color as less than regurgitates harmful narratives that must be disrupted.

This work is proximal to my identity, as a cisgender, Black and biracial woman of color, who is a mother, raising two daughters of color, one of which attends public school in the southeast region of the United States. As a former math teacher, my own lack of statistical content knowledge kept me from interrogating the statistics used to oppress my Black and brown students. For years, I intentionally leaned into the use of “data-based metrics” without questioning how they were used to continue the dominant narratives of K-12 schooling. I was taught to “trust the numbers” more than my own skin, more than the humanity of the students that sat in the same room with me. My own critical statistical illiteracy and lack of awareness of my role as a consumer and producer of statistics perpetuated the very stereotypes that come with being Black and brown in an education system dominated by whiteness.

As a former math learner, I have distinct memories of being stereotyped because of my racial identity, in being told I was not good enough to be labeled “gifted” in math classes, and that I would not succeed as a math major at a four-year university. Being concerned with the lives of Black and brown children is personal for me, as I recognize how my own historicity did not keep me from using statistics as a tool for harm against my own Black and brown students. Being told I was an anomaly reinstituted a belief in the narratives about Black and brown students being less capable in their mathematical ability. But I am now aware that there are too many of my students and my own experiences for the discourses and narratives to be true, and rather it is the discourses and narratives about Black and brown children that must change.

Therefore, my pursuit of a doctoral degree in mathematics education at a large, minority-serving institution is intentional and purposeful. I am currently pursuing this degree as one who desires to engage in justice-oriented research centered on minoritized students in mathematics. In standing for justice for students like me and mine who have been historically marginalized, I also stand for justice for the educators of those students who experience the inequality of racialization, genderization, and classism. At the intersection of my identity and my dissertation proposal, my goal is to provide students whose identities are often used in statistical analysis to harm them with the knowledge to disrupt that oppressive system. In order to do so, I must be willing to disrupt the system of whiteness that comes with "objective-based", "apolitical" mathematics teaching by designing materials that can be used in opposition to this dominant narrative (Gutiérrez, 2017). I must create materials that support the agitation of secondary math teachers in honoring the full humanity of their students and spur the need to incorporate the investigation of critical topics such as race, gender, or class in their teaching of statistics as it is imperative for their students' citizenship, identity, and ability to thrive in the modern world.

Overview of Key Terms

In this section, I define key terms that will be used throughout this report to clarify any meanings that could possibly have additional definitions in alternative contexts or settings.

- Benign statistics contexts: Benign statistics contexts are those that do not help shape students' identities-in-practice as critical consumers and producers of statistics, but rather perpetuate the systems of oppression by keeping with the "status quo", often in the name of "engagement". An example of a benign statistics context is the use of candy to learn about the statistics content of various graphical displays. Students often appear engaged during lessons that use packages of candy to create frequency tables, for example, but their engagement is based on the fact that candy is provocative for kids. Thus, teachers feel they have successfully engaged students in a meaningful statistics context when the candy is what was meaningful.

- Critical statistical literacy (CSL): CSL is defined as the fundamental right and skill to analyze and evaluate statistical data that shapes one's identity-in-practice for the purposes of disrupting unjust systems by creating agents of power who are able to communicate and discuss their evaluation of this statistical information with the world around them.
- Justice-oriented stance towards statistics and mathematics education: A justice-oriented stance towards statistics and mathematics education is one that believes that the learning and teaching of mathematics should be used as a vehicle for creating a more just society by using the concepts and procedures learned in mathematics class to analyze and evaluate unjust systems used to marginalize some and privilege others in a way that spurs the learners to act.
- Sociopolitical turn in statistics education: I draw upon the work of Gutiérrez (2013) in math education as well as Watts and colleagues (2011) to define sociopolitical turn in statistics education as the turning of one's gaze towards the investigation of critical statistics as it relates to the development of knowledge, identity, and power and spurs one towards critical action from multiple granular levels of the system of education (teachers, students, researchers, curriculum designers, and citizens).
- Statistics or statistical paradigm: The statistical paradigm is a four-step process popularized by the GAISE report (Franklin, et al., 2007). Those steps are (1) Formulate a question, (2) Collect data, (3) Analyze the data, and (4) Interpret the results. The statistical paradigm orients students to statistics as a process of inquiry that is regenerative and based in the asking of questions and is rather different than the common approach to teaching statistics, which is that of calculations. When statistics is taught as a set of calculations, it is essentialized into a list of procedures that one can complete (i.e.,

find the mean, median, and mode). In contrast, the statistical paradigm situates the learning of statistics as a process of data analysis and inquiry that privileges thinking and reasoning over calculating.

Overview of Dissertation

This study will use a traditional dissertation format, organized into six chapters, with a focus on what features of curricular materials support the teacher development of critical statistical literacy, and in what ways do these materials make visible teacher identity work as they are becoming critical statistical educators. This first chapter provides an introduction to the research as well as identifies the problem of focus for the research in this study. The second chapter presents a review of the relevant literature and description of the theoretical and conceptual frameworks used to situate the study. Chapter 3 presents a description of the methods for the research, including a description of the participants, the methods for data collection, and the process of data analysis. Chapters 4 and 5 share findings from the implementation of the materials, organized by research question. Lastly, chapter 6 presents a summary of the findings, implications, and conclusions of the study.

CHAPTER II: LITERATURE REVIEW

This section outlines the bases of literature that support the design of materials that make visible the identity work of secondary mathematics teachers in their development as critical statistical educators. This study holds two views on learning, similar to the work of Sfard (1998): learning as knowledge acquisition and learning as identity development. As each of these perspectives on learning inform the work completed in this study, the literature is organized to reflect these two views. For the perspective on learning as knowledge acquisition, I first describe what the field knows regarding statistical literacy, math literacy, and critical literacy, followed by the intersections of these three knowledge bases. Each of these literacies and the intersections among them inform the work of both the design of the materials as well as the conceptual framework of this study. For the perspective of viewing learning as identity development, I share relevant research from each of these three respective fields regarding the identity development of students and teachers in the math classroom. The identity development of students and teachers is a crucial pillar of the design of the materials, and teacher identity development is the focus of learning for this study. Following, I share my theoretical perspective on identity work as evidence for learning, followed by my conceptual framework in which I describe how the literature bases inform my definition of critical statistical literacy, as well as the identity lens I use to situate the identity work of secondary mathematics teachers in their development of critical statistical literacy.

Statistical Literacy

Over the last two decades, there has been a heightened focus on the development of statistical literacy as the consequences of statistical illiteracy have become more evident to the general public in this era of data saturation. Statistical literacy is a matter of life and death, as it affects one's ability to make informed decisions, recognize misinformation, and be able to "thrive in a modern world" (Franklin et al., 2007, 2015). To develop the statistical literacy of the general populace, the work of significantly increasing the statistical content knowledge has become the responsibility of public educators.

In the last two decades, evidence of how this statistical literacy movement has seeped into public education is demonstrated by the increase in education policies, standards, curricula, and teacher education organizations stressing the need to bring data analysis and statistical literacy into the K-12 classroom (Franklin et al., 2007, National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010, Franklin et al., 2015; NCTM, 2018). This development in the K-12 sector is supported by the growth of the field of statistics education research, which has grown exponentially over the past ten years as evidenced by the number of articles published underneath the domain (Langrall, et al., 2019). A surge of research has surfaced as national organizations such as the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM) have catalyzed the need for the inclusion of statistics in K-12 education by adding it to policy documents that guide national education standards (e.g., NCTM, 1989).

At the focus of many of these documents across the last three decades is the emphasis on defining and importance of statistical literacy. After the NCTM policy document *Curriculum and evaluation standards for school mathematics* included statistics in the big ideas to teach in K-12 mathematics education in 1989, the term surfaced to describe the basic skills one needs to interact with data and statistics in daily life in order to be an informed citizen (Wallman, 1993; Gal, 2002; Rumsey, 2002; Del Mas, 2002). In a 1992 Presidential address for the American Statistical Association (ASA), Wallman (1993) emphasized the importance of developing statistical reasoning and thinking for all members of the population, regardless of background or identity. Wallman's (1993) call was founded in the "misunderstandings, misperceptions, mistrust, and misgivings that people have towards the value of statistics in public and private choices" (as cited in Gal, 2002, p. 2).

Following this initial plea, Watson (1997) developed a hierarchical framework that shared three levels of statistical literacy with increasing sophistication. The first stage is a basic understanding of terminology of statistical and probabilistic concepts, followed by an understanding of this statistical terminology connecting procedures with concepts in context, and finally, a disposition toward approaching statistical information by applying knowledge of statistical content to counter false claims (Watson, 1997). Since then, many have provided definitions for what they feel is included in statistical literacy. Some of those definitions are:

- "People's ability to interpret and critically evaluate statistical information and data-based arguments appearing in diverse media channels, and their ability to discuss their opinions regarding such statistical information" (Gal, 2000).
- "The understanding of statistical language: words, symbols, and terMs. Heing able to interpret graphs and tables. Being able to read and make sense of statistics in the news, media, polls, etc." (Garfield, 1999).
- "... the ability to understand statistical concepts and reason at the most basic level" (Snell, 1999).
- "What statistical ideas will educated people who are not specialists require in the twenty-first century? That is the issue of statistical literacy. What specific concepts and skills will be needed in the context of specific jobs? That is the issue of statistical competence." (Moore, 1998a)

Other researchers have focused their work on trying to measure the development of statistical literacy (e.g., Callingham & Watson, 2005) and looking at ways to support teachers in their work of developing statistical literacy in the K-12 classroom (e.g., Batanero, et al., 2000; Batanero, et al., 2011; Begg & Edwards, 1999; Chick & Pierce, 2008)

Due to a special issue within the *Journal of Statistics Education* in 2002, multiple conceptions of what is and is not statistical literacy emerged, as well as how it can be typified, identified, and delineated (Gal, 2002; Rumsey, 2002; Del Mas, 2002). Becoming "literate" in any field, as Gal (2002) shared, requires the acquisition of basic, skill-based information for the purpose of a full citizenship, not to be confused with mastery or expertise. Yet there is lacking consensus of what skills should be included for all students to master in order to be considered statistically literate. Gal (2002) suggests that part of this lack of consensus comes from the field not mapping statistics concepts taught in schools with the statistical messages that adults encounter in daily life. While there is not a clear consensus from the field about its definition, there is some agreement about what grounds statistical literacy.

Connecting the defining of statistical literacy to K-12 education, Scheaffer and colleagues (1998) shared an extensive review of statistical topics that they suggested should be taught in all schools to all students prior to graduation in order to develop the statistical literacy of all citizens. Gal (2000) shared a specific list of these basic statistical skills that everyone in the population should understand, which are the type of study used, the sample that was selected, the measurements that were made, the statistics that were generated from the data, the graphs (visual displays) that were generated from the data, any probability statements that were made based on the data, claims that were made based on the data, the amount of information that was provided to the consumer, and any limitations of the study. Lists of skills that were the basic requirement for becoming statistically literate were provided by others, as well (e.g., Cobb, 1992; Moore, 1998a, Garfield, 1999).

In each of these lists, what is clear is that statistical literacy has two specific components, one that is based in the acquisition of basic knowledge through the use of key terms, procedures, and concepts, and the other being the disposition that one holds towards the analysis and creation of statistics. Rumsey (2002) named these two components statistical competence and statistical citizenship. Gal (2002), too, separated statistical literacy into two parts, naming one “knowledge elements” and the other “dispositional elements”. Because of his specific attention to both the knowledge acquisition component as well as the dispositional component of statistical literacy, I share a more in-depth review of his seminal piece. To Gal (2002), statistical literacy consisted of,

> (a) people's ability to *interpret* and *critically evaluate* statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to discuss or communicate their reactions to such statistical information, such as their understanding of the meaning of the information their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions. These capabilities and behaviors do not stand on their own but are founded on several interrelated knowledge bases and dispositions… (p. 3, emphasis in original)

In his definition, Gal (2002) listed specific sets of knowledge elements that were building on one’s statistical literacy: literacy skills, statistical knowledge, mathematical knowledge, context knowledge, critical knowledge, and the intersections of these different bases. Statistical literacy incorporates literacy skills as a large majority of statistical information is communicated through written or oral text (Mosenthal & Kirsch, 1998). Gal (2002) used the statistical knowledge base as a space to delineate what specific skills are needed in order to thrive in a data-rich society, to include number sense, understanding variables, interpreting tables and graphs, aspects of planning a survey or experiment, data analysis, relationships between probability and statistics, and inferential reasoning.

Gal (2002) also included the mathematical knowledge base as an element of statistical literacy, viewing statistics as a mathematical science. In order to create statistics, there are certain mathematical procedures that one must understand in order to produce statistics. Yet, the amount of formal mathematical knowledge one must master in order to support statistical literacy has been a question for the field (Cobb & Moore, 1997). As most adults do not take statistics at the collegiate level, for Gal (2002), the amount of mathematical knowledge needed for statistical literacy needed to be restricted to what was necessary for citizens to understand big statistical concepts, such as probability (e.g., precents, chance, and logic), statistical inference (e.g., mean versus median), and variance (e.g., sampling and distributions).

Aligning with Moore (1998a), Gal oriented his knowledge element of context and world knowledge as it relates to how one should approach, and view data as always embedded in some context. This view was also shared by Del Mas (2002), who stated, ‘If a procedure is taught, students should also learn the contexts in which it is applicable and those in which it is not” (p. 2). Because data is generated based on one's need to sense-make around an unknown, the context of this data is the source of meaning for its generation and how it should be interpreted. Contextual and world knowledge also introduces the data consumer to the concept of variation and error, as one must recognize how the same data generated in a different context leads to different results and interpretations around those results (Gal, 2002).

The last knowledge element that Gal (2002) listed was that of critical skills, which was one's ability to recognize that "messages aimed at citizens in general may be shaped by political, commercial, or other agendas which may be absent in statistics classrooms or in empirical enquiry contexts" (p. 15). Here, "critical" skills are tied to one's ability to evaluate data, using a list of "worry questions" to evaluate what the intent is behind a data or statistical message (Gal & Ginsburg, 1994). This focus on critical skills aligns with the work of Frankenstein (1989), as she emphasized the importance of adult learners developing the statistical literacy skills in context. This work also differs from Frankenstein's (1989) approach as she emphasized that this learning of statistics be contexts that mattered for their adult life (e.g., critical topics such as race, gender, class, working compensation, and other proven inequities). In this study, both the need to evaluate data, or use critical thinking skills, as well as the context of data (e.g., critical topics such as race, gender, and class) are crucial components of the way "critical" statistical literacy is defined.

Gal (2002) also attended to the idea that these five different knowledge bases (literacy, math, statistics, context, and critical) would interact with one another based on the individuality of each person's historicity and lived experiences. That is to say that the different knowledge domains "cannot operate independently from each other" (p. 17). For the purpose of this study, this is a crucial point that I agree with, and expand upon that while the knowledge bases cannot operate independently from each other, they also cannot operate independently from the individual, or from the world.

While others have focused primarily on creating a list of elements one needs to be considered statistically competent, Gal (2002) also provided more focus to the dispositional elements that one must acquire in order to become statistically literate.

Regarding the dispositional elements, Gal (2002) shared the need for adults to have a critical stance and beliefs and attitudes towards statistical literacy. For Gal (2002), it was important to separate emotions from beliefs and attitudes, with beliefs and attitudes being somewhat more stable ideations. In regard to attitudes, Gal (2002) shared how attitudes may be tied to intense feelings “that develop through gradual internalization of repeated feelings over time” and exposure to objects, actions, or topics. Of particular interest to this study is his idea on beliefs, or “individually held ideas or opinions, such as about a domain, about oneself, or about a social context” (Gal, 2002, p.18). Gal (2002) also emphasizes the timeline of beliefs in that they are resistant to change but differ based on each individual’s historicity and the learning they have acquired over time. As this study focuses on the identity work of educators, looking at the ways that teachers view themselves, the domain of statistics as it relates to their teaching and themselves, as well as social context through the incorporation of the investigation of critical topics such as race, gender, or class in math class are key pillars that frame the work of this study.

Gal (2002) also shared a second dispositional element of having a critical stance, which he defined as “a questioning attitude towards quantitative messages that may be misleading, one-sided, biased, or incomplete in some way, whether intentionally or unintentionally”. For Gal (2002), both elements of the dispositions imply “a form of action, not just passive interpretation or understanding of statistical or probabilistic information available in a situation”. He expanded this thinking, stating that “it would be hard to describe a person as fully statistically literate if this person does not show the *inclination to activate* the five knowledge bases described earlier or share with others his or her opinions, judgements or alternative interpretations”. (emphasis in original, p. 18). To Gal (2002), action involved taking risks through the sharing of ideas in order to communicate one's thoughts to others, even if one was unfamiliar with the statistical issues being studied.

Lastly, Gal (2002) meshed the two together stating that "adults should develop a *belief in the legitimacy of critical action.*.that it is legitimate to be critical about statistical messages or arguments…[and] to have concerns about any aspect of a reported study, …even if they have not learned much formal statistics or mathematics or do not have access to all the background details need" (p.19).

One central piece about Gal's (2002) study that informed the ideas in the design of the materials and the analysis of this study is attending to the critical as both a knowledge as well as a dispositional element. In doing so, it legitimizes the idea that criticality is something that can be acquired through learning skills, as well as an orientation to the domain of statistics. Both of these different perspectives on criticality cannot be separated from the individual, but are tied to one's identity (e.g., one's historicity, who people say they are, the resources they have available to them, and their lived experiences).

Another pivotal piece of Gal's (2002) work that aligns with this study is the idea that criticality cannot be separated from action. It is important to clarify that, for this study, criticality cannot be separated from taking action towards justice. It is often the case in statistics education that how one defines "critical" and "contextual" are motivated by differing worldviews, causing much variation in the interpretation of what it means for one to have a critical stance towards statistical literacy, or in what contexts should statistical literacy be developed. For many of the authors focused on defining statistical literacy, approaching statistical literacy with a post-positivist or pragmatist worldview dictates the need to forefront a list of skills that should be included in statistical literacy that can be used as a checklist for when it is accomplished. Having such a list is practical, and makes dissemination easier to implement in terms of writing standards and policies. Alternatively, having a transformative worldview to statistical literacy situates the learning as a justice-orientation to the identity of the individual, and that individual to the world.

Statistical literacy as a right to full citizenship has continued to be at the forefront of the movement for the incorporation of statistics into the K-12 education experience (Moses & Cobb, 2001). Access to and achievement in high quality mathematics instruction has been shown to lead to gainful employment opportunities, postsecondary degrees, entrance into upper-level mathematics classes, as well as to increase one's social mobility out of cycles of poverty (Moses & Cobb, 2001). Therefore, without access to and achievement in math courses, including those that include statistics, one is lacking the full spectrum of citizenship.

As was mentioned previously, the *GAISE* report emphasized these two competencies again, in stating that it is the unalienable right of all citizens to understand basic statistical content knowledge and statistical literacy (Franklin et al., 2007). Franklin and colleagues (2007) based its claim of the need for statistical literacy on a four-pronged claim that includes one's (1) citizenship, (2) ability to make personal choices, (3) understanding of scientific findings, and (4) the proximity to statistics in the workplace and professions. This work spearheaded the drive to incorporate statistics education into the Common Core State Standards, which was widely adopted across the United States (National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010), and continues to be a seminal piece for policy documents that demand the development of statistical literacy as a right of all citizens.

In the high school edition of *Catalyzing Change*, NCTM (2018) outlined the "common shared pathway - a progression of courses that all students must take - as part of high school mathematics education", including the domains of Number, Algebra & Functions, Geometry & Measurement, and Statistics & Probability (NCTM, 2018, p.9). Within the domain of Statistics & Probability there are four focal points: Quantitative Literacy, Visualizing and Summarizing Data, Statistical Inference, and Probability. Within each of these four focal points, NCTM distinguishes the essential concepts that students within US schools should experience prior to exiting the K-12 pipeline. Recent studies name quantitative literacy (also referred to as statistical literacy) as an unalienable right for all citizens living in a just democracy (Ernest, 2010). However, this new document meant to guide the work of mathematics teachers across the United States lists nothing of justice or developing the understanding to read, write, and critique one's world using statistical data (Freire, 1970/2005). Rather, the essential concepts are written from the lens of mathematics and statistics as objective and universal, as is the tradition in standards writing (Skovsmose, 1994; Cavanagh, 2005; Moschkovich, 2012), and the organization that drives standards writing, the National Council of Teachers of Mathematics (Martin, 2015). Thus, despite research stating that "students should be able to identify, interpret, evaluate, and critique the mathematics embedded in social, scientific, commercial, and political systems…" (Ernest, 2010, p.72), and policy documents indicating a need to take a social justice stance to dismantle systems of oppression in math education (NCSM & TODOS:Math for all, 2016), it seems as though the lead organization in mathematics education is sending mixed messages about how to make statistical literacy actionable for students and teachers, as these documents from *Catalyzing Change* ignore the social, cultural, and political implications of analyzing data.

Yet, there are examples that have demonstrated how statistical literacy can be a fruitful space in education for stakeholders to take critical action (Frankenstein, 1983, 1989, 1990, 1995, 2001) prompting one to "recognize his or her position in society" and "motivate individuals to action" (Gutiérrez, 2013). It is my worldview that developing one's statistical literacy as defined above is not enough. In order for citizens to be able to act using their statistical literacy to disrupt the systemic injustices that oppress those who have been historically marginalized, statistical literacy requires a sociopolitical turn that empowers students, teachers, and citizens to become agents of change in order to transform their world (Gutiérrez, 2013). Viewing statistics as a mathematical science, I now turn to look at how this sociopolitical turn in mathematics education continues to reorient the K-12 mathematics educational experience to that of justice.

Math Literacy

A determination of the types of mathematical knowledge that is expected of someone who is deemed "mathematically literate" has been a source of debate for several decades. In the era of the Math Wars (Schoenfeld, 2004; Schoen, et al., 1999), education researchers took to two opposing sides, some stating that there was a specific list of skills that denoted one as mathematically literate and understanding the procedures undergirding the list was enough. For the other camp, understanding the concepts behind mathematical ideas was more important than the procedures. In 2000, the National Research Council (NRC) definitively stated how students learn best, in its seminal piece, *How People Learn*. The findings of this work rested on three tenets on how all humans learn, regardless of content domain. Those tenets are a) All knowledge is built from something that is already there; b) Concepts and procedures are equally important. They inform each other, but the order does matter. Teaching procedures first tends to decay overtime, while teaching concepts first tends to stick for longer periods of time; and c) Strategies that link concepts together with schemes create a firmer foundation for future learning. This seminal report sought to end the conversation about how students best learn using unbiased methods and bipartisan investigators from outside the field of education.

In a different report titled *Adding it Up*, the NRC joined with math education researchers Kilpatrick, Swafford, & Findell (2001) specified how students best learn mathematics, naming the five strands of "mathematical proficiency" as

- "Conceptual understanding—comprehension of mathematical concepts, operations, and relations
- Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence—ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p.116)

According to the NRC (Kilpatrick, et al., 2001), these five strands do not operate independently of one another, and together define what it means to be proficient in mathematics. This first attempt to define what research from psychoanalysis and brain researchers says about how the brain best learns math takes the first three concepts, orders them, and then expands upon what it means to be mathematically literate. However, mathematical proficiency in this report is defined from an objective perspective that is devoid of context, identity, and culture, making all individuals, and the ways they learn math, seem exactly the same.

An updated report, *How People Learn 2,* by the NRC (2004) recognized this to not be the case, providing five additional conclusions regarding how people learn effectively. They are:

CONCLUSION 7-1: Effective instruction depends on understanding the complex interplay among learners' prior knowledge, experiences, motivations, interests, and language and cognitive skills; educators' own experiences and cultural influences; and the cultural, social, cognitive, and emotional characteristics of the learning environment.

CONCLUSION 7-2: A disparate body of research points to the importance of engaging the learner in directing his own learning by, for example, providing targeted feedback and support in developing metacognitive skills, challenges that are well matched to the learner's current capacities, and support in setting and pursuing meaningful goals.

CONCLUSION 7-3: A growing body of research supports adopting an asset model of education in which curricula and instructional techniques support all learners in connecting academic learning goals to the learning they do outside of school settings and through which learning experiences and opportunities from various settings are leveraged for each learner.

CONCLUSION 7-4: Purposefully teaching the language and practices specific to particular disciplines, such as science, history, and mathematics, is critical to helping students develop deep understanding in these subjects.

CONCLUSION 7-5: Assessment is a critical tool for advancing and monitoring students' learning in school. When grounded in well-defined models of learning, assessment information can be used to identify and subsequently narrow the gap between current and desired levels of students' learning and performance. (p. 160-161).

This report specified that the first five strands of mathematical proficiency still hold true for how learners best learn mathematics, while recognizing that the "simple accumulation of knowledge is insufficient for tackling sophisticated learning tasks and approaching novel problems and situations" (p.135). By expanding to recognize how the differences in classroom environment, historicity, identity, and culture all shape these five strands of mathematical proficiency, the NRC (2004) clarified why it is important to pay attention to aspects of culture and identity in order to provide every student with an equitable opportunity to learn.

Critical Literacy

Since Freire's (1970/2005) seminal work on critical consciousness, researchers have been incorporating the concepts into their research and design. *Conscientizção*, a term introduced by Freire (1970/2005) is one's ability to perceive social, political, and economic systems, analyze them from one's identity, and use that perception to take action to disrupt known injustices within society. Increasing one's critical consciousness is the developmental process of one's ability to perceive, analyze, and act on known systems that oppress people. For the purposes of this paper, critical literacy is used synonymously with the development of critical consciousness.

The development of critical literacy primarily occurs through learning opportunities, whether they be formal (e.g., in school) or informal (e.g., family, historicity, lived experiences, personal reflection). Research indicates that most people develop their critical literacy by transitioning through stages of moral and cognitive development (Kohlberg, et al., 1983). Researchers have been trying to define and identify these stages of critical consciousness development since the publishing of Freire's (1970/2005) seminal piece. Two such theorists are Gutiérrez (1995) and Watts et al. (2011), who sought to define the incremental steps one takes in critical consciousness development. The components of this developmental process usually include critical social analysis, collective social identity, political self-efficacy, and sociopolitical action (Watts et al., 2011).

The first process in developing critical consciousness is critical social analysis. Also referred to as critical reflection, critical social analysis is the awareness of social inequities, and perceiving the systems that utilize unjust power to create them (Hipolito-Delgado and Lee, 2007). Hopper (1999) defined critical social analysis as,

> Learning to think critically about accepted ways of thinking and feeling, discerning the hidden interests in underlying assumptions and framing notions (whether these be class-, gender-, race/ethnicity- or sect-based). It means learning to see, in the mundane particulars of ordinary lives, how history works, how received ways of thinking and feeling serve to perpetuate existing structures of inequality (p. 13).

Within critical social analysis, scholars have expanded what is embodied in critical reflection in a four-stage process: 1) passive adaptation; 2) emotional engagement; 3) cognitive awakening; and 4) the intention to act (Carlson, Engbretson, and Chamberlain, 2006).

The second component of critical consciousness is collective social identification, or one's ability to identify as a part of the common whole of society (Gutiérrez, 1995). Collective social identification can occur through different narrations of one's identity, such as race, gender, class, language, sexual orientation, immigration status, or other characteristics. As one engages in collective social identification, feelings of solidarity or shared culture bring about a positive sense of power in being defined as a member of a historically oppressed group of people (Cammarota, 2011). This social identification serves as a denouncement of disempowerment or inferiority to others based on one's identity, invoking thoughts of empowerment to act for the good of the collective group (Hipolito-Delgado & Lee, 2007).

Once one has identified themselves with the collective social group, one grows in confidence to be able to invoke change through action to improve one's status in society by challenging known social inequities. Defined as political self-efficacy (Watts et al., 2011), this stage has been shown to be extremely important, yet implicit, and therefore difficult to document

(Watts & Hipolito-Delgado, 2015). Serving as the predecessor to sociopolitical action, political self-efficacy is often defined as the belief that systems can change, motivating the need for action.

The first three stages of critical consciousness development are psychological processes, and often typified as the "problematization" stages (Watts & Hipolito-Delgado, 2015). In other words, one must first problematize socially normalized issues and data and be provoked to act. Freire (1970/2005) believed that action and problematization were reciprocal processes, but that critical reflection preceded critical action. Before one acts to change social inequalities, one must first reflect to know that the social inequalities are unjust. Cyclically, once one chooses to engage in critical action, one gains more understanding of the social inequities that exist (Watts et al., 2011). Sociopolitical (critical) action can be individual or collective, as individuals can join together to challenge known oppressors.

In a systematic review of scholarly articles on the development of critical consciousness, Watts and Hipolito-Delgado (2015) found that across varying fields, program activities all had similar factors of shared values, fostering awareness of socio-political circumstances, encouraging critical questioning, fostering collective identity, and taking sociopolitical action.

The Intersection of Statistical Literacy, Math Literacy, and Critical Literacy

In reviewing the literature on what and who defines statistical literacy, math literacy, and critical literacy, I now turn to review the specific bodies of literature for where these literacies are intertwined. At the intersection of statistical and math literacies, I merge the work of Gal (2002) and the NRC's (2001) five strands of mathematical proficiency to think about the specific math concepts, procedures, strategic competencies, adaptive reasoning skills, and dispositions that undergird the specific learning context of exploratory data analysis, as this is statistical topic in the unit for this dissertation study. At the intersection of critical literacy and math literacy, I review the body of literature on critical mathematics that supports the development of critical consciousness through the domain of mathematics. At the intersection of critical literacy and statistical literacy, I review the empirical studies that have used the domain of statistics to investigate critical topics (e.g., race, gender, and class). At the intersection of all three literacies, I review the two pieces of research that are characterized as critical statistical education.

The Intersection of Statistical and Math Literacies

At the intersection of statistical literacy and math literacy are the specific competencies that one must have from both domains in order to "thrive in the modern world" (Franklin et al., 2007). This design-based study is focused on the identity work of secondary math teachers who enacted a set of curricular materials designed for a unit in a new fourth level math course that used an exploratory data analysis perspective to focus on variance, sampling, and different views on data. In this section, I review the literature on exploratory data analysis, as well as what the field knows about the effective practices for teaching a unit on exploratory data analysis. That is, I seek to review what specific skills and mathematical procedures one must understand in order to understand the statistical concepts within a unit of study on exploratory data analysis, and what does the field know about teaching those skills and procedures.

Exploratory Data Analysis. Historically tied to the work of John Tukey (1977), EDA has five main characteristics: (a) an orientation to data that begins with a question; (b) an emphasis on graphic representations of data; (c) an iterative process of creating and refining conjectures through analysis; (d) use of statistical measures pertinent to answering the question at hand; and (e) the development of one's dispositions of flexibility and skepticism towards the process of inquiry. The overarching goals of EDA is to discover patterns in data and engage in the process of inquiry as a data scientist. The work of a data scientist is similar to that of an investigator and author, as one listens to the patterns that tell different stories within data (Behrens, 1997).

Since the incorporation of statistical concepts into K-12 education standards (NCTM, 1989), providing students the opportunity to act as data scientists through EDA has been at the core of the desired student experience. EDA helps students to orient themselves to statistics as the process of inquiry by providing the opportunity for meaning making and an introduction to inference (Cobb & Moore, 1997; Ben-Zvi, Gil, & Apel, 2007). Because of the iterative nature of revising, one's conjectures, EDA provides students a low-stakes environment in which to test out their ideas without the frontloading of terminology or mastery of upper level statistics concepts (Stanja & Steinbring, 2014). Exploring the different patterns that exist within a set of data provides students an opportunity to be wrong in a way that is meaningful and tied to the process of learning (Cobb & Moore, 1997).

EDA is intrinsically tied to several "big ideas" of statistics, namely variance, distributions, statistical inference, and sampling, which are all key features that students explore in the process of statistical inquiry (Burrill & Biehler, 2011). How all of these topics are connected is often difficult for students (Lavigne & Lajoie, 2007) and teachers (Pfannkuch & Ben-Zvi, 2011) to understand.

In understanding the statistical process of inquiry, students and teachers alike must understand that when they have a question, they must generate data to answer that question. How the data is generated, what data is collected, and the process of cleaning the data all introduce variance into the statistical model. Because of constraints like time and money, data authors often take a sample to answer their question rather than address the entire population. This, too, introduces variance into the model. The sampling method one chooses is an additional source of variance, as are the calculations one selects to try to answer the question. The calculations allow the data authors to make inference and draw conclusions about the question, but it is imperative that students recognize the variance that exists throughout the statistical model. Thus, the interconnectedness of these “big ideas” of statistics, namely variance, distributions, statistical inference, and sampling are key features for students to explore in the process of statistical inquiry (Burrill & Biehler, 2011), and their connections are often difficult for students (Lavigne & Lajoie, 2007) and teachers (Pfannkuch & Ben-Zvi, 2011) to understand.

EDA provides context to immerse students in the omnipresence of variance (Cobb & Moore, 1997), to better understand how the decisions of a data scientist create different stories that can be told with the same set of data. The omnipresence of variance is what defines statistics, making the calculations, solutions, and distributions all dynamic attributes of the chosen datasets, which is one of the key elements that differentiates statistics from mathematics (Cobb & Moore, 1997). For this reason, the concept of variance, how students learn about it, and how teachers teach it has received a significant focus in research in the last two decades (e.g., Cobb & Moore, 1997; Shaughnessy, 2007; Reading & Shaughnessy, 2004; Watson, Kelly, Callingham, & Shaughnessy, 2003).

Research has shown that experiences with EDA are also a productive space to investigate statistical inference. Most introductory statistics courses have mainly focused on studying specific components of statistics, with a central focus on understanding normal distributions. G. Cobb (2007) suggested that a shift towards focusing on statistical inference would allow for a larger body of students to view statistics as a process of inquiry, rather than a set of disconnected skills, formulas, and components. Since then, the field has shifted to have a greater focus on statistical reasoning and thinking and how it is connected to drawing inference.

Statistical inference is the ability to draw conclusions about a population of interest using statistical measures from a sample of data from which the impetus for the statistical analysis was derived. All concepts tied to statistical inference are based on the concept of variance, specifically sampling variation (Langrall, et al., 2019). Within the literature on statistical inference, there are three big ideas, namely reasoning, concepts, and calculations (Harradine, et al., 2011). When students and teachers make statistical inference, they are drawing conclusions about calculated measures based on a sample of a general population. Harradine and colleagues (2011) have stressed the importance of students and teachers having a “population-view” of a sample, recognizing that a sample taken is just one iteration of the many different possible samples that could be taken with which one can draw inference upon.

EDA comes from the work of Konold and colleagues (2004) that describes the various responses students might have in characterizing data. They are (a) data as pointer; (b) data as case value; (c) data as classifier; and (d) data as aggregate. Data as pointer refers to when students think about data as pointing to an event where data was collected. When a student characterizes data as case values, they are isolating a single attribute that they recognize for each individual case. Viewing data as a classifier occurs when students think about how multiple cases can fall under one classifying attribute value. Data as aggregate refers to the student view that occurs when they can unify the characteristics of the data, the data collection process, the individual cases and what classifies all the cases together, with the emerging properties that are used to describe a distribution (Konold et al., 2004). Konold and colleagues (2004) view these as subsuming developments. EDA seems to be a profitable space for the development along this trajectory of viewing data from different perspectives, as it allows students to explore data from multiple vantage points.

Using an EDA perspective, considerable research has been done recently regarding the ways in which students develop reasoning around informal statistical inference when exploring data. Much of this research is focused on working with undergraduate students, as they use informal statistical inference without the use of formulas and probability distributions to build towards formal statistical inference over time (Bakker & Derry, 2011; Garfield & Ben-Zvi, 2008; Ben-Zvi, et al., 2012). Others have shown how valuable it can be to also bring informal statistical inference into the K-12 classroom to support student learning of making data-based claims with evidence while communicating one's level of uncertainty (Makar & Rubin, 2009; Gil & Ben-Zvi, 2011; Pfannkuch, 2006, 2011; Pfannkuch & Horring, 2005; Pfannkuch & Reading, 2006b, Wild & Pfannkuch, 1999; Burgess, 2008; Paparistodemou, et al., 2008). Makar and Rubin (2009) found that by providing a context-rich task for students to engage in, they were able to draw informal inference from a sample of data towards a population. In their work with grade 6 students, Gil and Ben-Zvi (2011) found that by beginning with an EDA perspective and immersing students in a rich context, students were able to explore patterns within the data in order to draw informal inferences about the population in question. In each of these studies, informal statistical inference was embedded in a context-rich task that allowed for the exploration of data, which increased student engagement as well as statistical content knowledge and skills. In supporting the work of teachers in using EDA as a foundation for the development of informal statistical inference, Burgess (2011) found that teachers valued the context-rich investigations that allowed students to explore data, but that they still tended to focus on a procedural, calculation-centered view, rather than a process of inquiry view of statistics.

Several researchers have tried to define informal statistical inference, with some commonalities (Rossman, 2008; Garfield & Ben-Zvi, 2008; Makar & Rubin, 2009; Zieffler, et al., 2008). In general, informal statistical inference can be derived to three main objectives, that students are: “(1) making claims or predictions beyond the given data while (2) using the data as evidence for any claims that are made and (3) acknowledging that there is uncertainty in any claims or predictions” (Langrall, et al., 2019).

In summary, EDA allows students to play with statistics in real contexts even without the development of procedural fluency in basic statistics skills. Approaching statistical inquiry through informal statistical inference has been shown to support powerful connections between probabilistic and statistical thinking (Pratt & Ainley, 2008), even for students in elementary grade levels (Fielding-Wells & Makar, 2015; Makar & Fielding-Wells, 2011; Konold & Kazak, 2008). With the widespread use of technology in K-12 schools, new software such as TinkerPlots 2 (Konold & Miller, 2011) and CODAP (The Concord Consortium, 2014) allow students to make visual connections to big statistical ideas through the use of interactive applets and simulations. The use of technology continues to prove consequential in helping learners draw connections across the big ideas in statistics by providing opportunities to visualize the interconnectedness of variance, distributions, statistical inference, and sampling (Pfannkuch et al., 2011; Wild, et al., 2011).

Yet, research has shown that students’ experiences of learning statistics in schools have remained largely procedural with vast misconceptions about viewing statistics as being driven by a set of calculations one must perform rather than discovering patterns (Cobb, 2004; Cobb & Moore,1997). Harradine and colleagues (2011) have shown that calculations receive a significant amount of instructional time in K-12 classrooms, leaving students with less understanding of reasoning and concepts and an unbalanced approach to understanding statistical inference and EDA. Viewing statistics from a procedural point of view inhibits one’s ability to understand the concept for the study of statistics in general, or the orientation to statistics as a process of inquiry (Cobb, 2004). Researchers have well established the disadvantages that come from developing one's understanding of statistics procedurally (e.g., Ben-Zvi, 2000; Biehler et al., 2013; Wassong, Frischemeier, Fischer, Hochmuth, & Bender, 2014).

Effective Statistics Teaching Practices. One reason for the continuation of this persistently ineffective view of statistics is lack of teacher preparation, knowledge, and understanding of viewing statistics from a conceptual perspective in a learning environment in which students can experience EDA (Langrall, et al., 2017). Research has shown that statistics is very difficult to teach (e.g., Pfannkuch & Ben-Zvi, 2011; Bakker & Derry, 2011), and that many educators have the same misconceptions as their students (Batanero et al., 2011). Studies have also shown that teachers' experiences with statistics are few, yet mastery is still required of those who have often had little teacher preparation courses on statistics teaching methods and little professional development on statistics in their inservice teaching (Shaughnessy, 2008). For this reason, many teachers choose not to teach statistics even though it is a part of their curriculum standards (Shaughnessy, 2008).

Studies have shown that the role of the teacher is pivotal in helping students develop statistical thinking (Ben-Zvi, 2004; Noll & Shaughnessy, 2012), and that the way that teachers approach statistical concepts with their students greatly increases or diminishes the complexity of some of the big ideas in statistics (Bakker & Derry, 2011). Ben-Zvi (2004) shared that much of student progression through complex statistics thinking, such as the understanding of the omnipresence variance, is greatly dependent on the teacher's ability to help their students adopt a statistical model of thinking without specifically telling them what to do or how to think (Langrall et. al, 2019). He shared that instruction should involve "appropriate teacher guidance, peer work and interactions, and more importantly, ongoing cycles of experiences with realistic problem situations" (Ben-Zvi, 2004, p. 60).

Adopting a holistic approach to teaching statistics is an essential orientation to developing students' statistical literacy. Specifically, Bakker and Derry (2011) looked at the consequences of teaching with an inferential versus a referential frame of statistics. In an inferential frame, instruction is focused on connecting big ideas of statistics informally by drawing on the complexities of their interconnectedness. A referential frame focuses on teaching statistics concepts in isolation, often referring to additional concepts without specifically showing their interrelationships (Bakker, 2014). Bakker and Derry (2011) argue that a more holistic, inferential frame to teaching statistics allows for more coherence across curriculum, which avoids the "atomistic" approaches to statistical thinking and learning that breeds confusion. Researchers have stressed how EDA continues to be a fruitful site for the effective teaching of statistics to develop one's statistical literacy (Bakker & Derry, 2011; Ben-Zvi, 2004; Garfield, Ben-Zvi, & Apel, 2007). But what does effective statistics teaching look like, who is it effective for, and who gets to decide?

Statistics Knowledge for Teaching. It has been shown that math teaching requires distinct forms of knowledge (Hill, Sleep, Lewis, & Ball, 2007). Following the work of Shulman (1987) which differentiated subject matter knowledge from pedagogical content knowledge, Hill and colleagues (2007) defined mathematical knowledge for teaching (MKT) that consists of two types of knowledge broken down into subcategories: subject matter knowledge, which includes common content knowledge, specialized content knowledge, and horizon content knowledge, and pedagogical content knowledge (PCK), which encompasses knowledge of content and curriculum, knowledge of content and students, knowledge of content and teaching) (Hill, et al., 2007).

Continuing to view statistics as a mathematical science, statistics education researchers have used this frame as a model to try to define statistical knowledge for teaching (SKT). Research has shown that there is a direct connection between a teacher's understanding of statistics and their students' learning of the same concepts (Groth & Bergner, 2005). Specific research within subject matter knowledge has found that teachers often oversimplify statistical concepts to mere procedural, nonstochastic understandings (Liu & Thompson, 2007; Lee & Hollebrands, 2011), and that a teacher with a robust MKT does not preclude a robust SKT (Hannigan, Gill, and Leavy, 2013).

Despite some overlaps in the domains of mathematics and statistics, the fact that MKT and SKT do not overlap exactly by simply interchanging statistical content knowledge for math content knowledge has made mapping SKT more difficult. For that reason, most of the research on statistical knowledge for teaching has centered on common content knowledge rather than specialized content knowledge (Langrall, et al., 2019), defined by Hill and colleagues (2007) specialized content knowledge as "including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (p. 377).

While there are many similarities in math and statistics, the differences between the two fields make it difficult to map teacher knowledge one-to-one (Contreras, et al., 2011; Groth, 2007). Several researchers have attempted to modify the original MKT model to fit SKT (Burgess, 2011; Wild & Pfannkuch, 1999; Simon, 2006; Silverman & Thompson, 2008). Silverman and Thompson (2008) tried to combine the work of Hill and colleagues (2007) with

Simon's (2006) construct of key developmental understandings (KDUs), or specific conceptual understandings that a teacher must hold conceptually of statistics. They shared the following,

> Teachers who develop KDUs of particular mathematical ideas can do impressive mathematics with regard to those ideas, but it is not necessarily true that their understandings are powerful pedagogically; It is possible for a teacher to have a KDU and be unaware of its utility as a theme around which productive classroom conversations can be organized. Developing MKT, then, involves transforming these personal KDUs of a particular mathematical concept to an understanding of: (1) how this KDU could empower their students' learning of related ideas; (2) actions a teacher might take to support students' development of it and reasons why those actions might work (p. 502)

Others have diverged from the model completely, with the justification that SKT and MKT are not congruent (Groth, 2013; Garfield & Ben-Zvi, 2008).

In one such example, Groth (2013) combined the work of Simon (2006) on key developmental understandings (KDUs) as well as the Piagetian driven concept of decentering (Silverman & Thompson, 2008) to create a framework defining effective statistics teaching. In doing so, Groth (2013) was able to characterize the critical developmental stages of teachers' subject matter knowledge and pedagogical content knowledge to conceptualize the development of effective statistics teaching through what he called "pedagogically powerful ideas" (p.138). For Groth (2013), pedagogically powerful ideas are derived from personally powerful ideas that one has with statistics. In the work of teaching, teachers must decenter themselves and the powerful experiences they have had personally with statistics in order to consider alternative perspectives that will support the learning of their students (Groth, 2013). While this work has been grounding for the field of statistics in regard to characterizing effective teaching, there is still much to learn about the developmental process that teachers undergo as they move from personal to pedagogically powerful ideas, and how their identity influences this development.

What is not in Groth's model (2013) for SKT is space for critical statistics educators in either a critical approach to which one views the impetus for teaching statistics or the acknowledgement of the specialized content knowledge associated with the teaching of statistics that investigates critical issues such as race, gender, or class. The effective pedagogical practices that are used to facilitate conversations around data that motivate students to disrupt known systemic injustices also lacks space in current models of effective statistics teaching.

In a different model that attempts to characterize the work of effective statistics teaching, Garfield and Ben-Zvi (2008) proposed a six-tenant framework describing a teacher's development. The six tenants are:

1. Focuses on developing central statistical ideas rather than on presenting [a] set of tools and procedures.
2. Uses real and motivating data sets to engage students in making and testing conjectures.
3. Uses classroom activities to support the development of students' reasoning.
4. Integrates the use of appropriate technological tools that allow students to test their conjectures, explore and analyze data, and develop their statistical reasoning.
5. Promotes classroom discourse that includes statistical arguments and sustained exchanges that focus on significant statistical ideas.
6. Uses assessment to learn what students know and to monitor the development of their statistical learning as well as to evaluate instructional plans and progress. (Garfield & Ben-Zvi, 2008, p. 48)

These principles put the work of effective statistics teaching into tangible practice.

Of particular interest in this study are principles 2 and 5. The mention of “real and motivating data” (principle 2) and “significant statistical ideas (principle 5) are ideas that resonate with the development of critical statistical educators, but the principles lack distinction towards the inclusion of identity and power. For whom is the data real and motivating? What statistical ideas are significant? While the ambiguity allows for teacher autonomy to define these based on their students, this amount of autonomy in choosing contexts could lead educators to choose data that is contrived, benign, and noncritical for class exploration, perpetuating the cycle of critical statistical illiteracy. Teachers could also center themselves in their choice of data, choosing data that is motivating for them rather than their students. Combined with the decentering work of Groth (2013) and Silverman and Thompson (2008), this is a *personally* powerful idea regarding statistics, not a *pedagogically* powerful idea. What is clear in all of the models (Groth, 2013; Silverman & Thompson, 2008; Simon, 2006; Garfield & Ben-Zvi, 2008) is that there is a lack of criticality and focus on identity and power in defining the characteristics of effective statistics teaching, which means there is more work to be done.

The Intersection of Critical Literacy and Math Literacy: Critical Mathematics

In 2005, NCTM Research Committee devoted an entire issue to discussing the implications for equity in mathematics education. In the forward, the editors share that, “the main issue for us is how mathematics education research can contribute to understanding the causes and effects of inequity, as well as the strategies that effectively reduce undesirable inequities of experience and achievement in mathematics education” (p. 94). Since then, mathematics education has begun to problematize how mathematics education perpetuates injustices and how to eradicate known injustices of the world through the teaching and learning of mathematics in K-12 education. This has been called the “sociopolitical turn” of mathematics education (Gutiérrez, 2013). In taking this turn, mathematics educators and education researchers are reorienting themselves to mathematics as a tool that can be used to wield justice rather than oppression as it has been documented to do in the past (Moses & Cobb, 2001). This act of transforming mathematics learning to address wicked problems of the world through issues of identity, agency, power, and culture through the learning of mathematics has been termed many things, some of which are critical mathematics, teaching math for social justice, and social justice mathematics. While there have been documented nuances between each of these terms (Stinson & Wager, 2013; Stinson, Wager, & Leonard, 2012; Gates & Jorgensen, 2009;), for the purposes of this review, they will be used interchangeably.

Critical mathematics has been defined differently by many, mainly due to a difference in theoretical perspective. Gutstein (2010) defines critical mathematics as the “teaching and learning of mathematics for social justice” (p.455). Using a Freirean conceptual perspective, Gutstein (2010) posits that critical mathematics involves reading and writing the world and the word with mathematics by using “mathematics to comprehend and change the world—and through the process, deepen their knowledge of both mathematics and their social reality” (p. 455). Viewing mathematics as a verb, the act of *doing mathematics* means doing mathematics in an effort to eradicate the world of oppression (d’Ambrosio, 1985, 2001).

In a seminal piece, Gutiérrez (2013) problematizes what is defined as “effective” teaching by introducing a dominant and critical dimension to the equity work of mathematics education. Within each axis, there are two dimensions. The dominant axis is defined by "access" and "achievement". Gutiérrez (2013) suggests that it is not enough for all students to have access to high-quality mathematics education as is defined by rich tasks (Franke, Kazemi, and Battey, 2007; Hiebert, 1997; Hiebert & Grouws, 2007), but that they must also be able to achieve in the education system. Research has documented how mathematics education acts as a gatekeeper for many students, with filters that indicate racial, linguistic, and socioeconomic segregation (Moses & Cobb, 2001; Stiff & Harvey, 1998; Martin, 2019; Martin, Gholson, & Leonard, 2010). Due to some of the earlier research the objectified what it means to be mathematically proficient (Kilpatrick, et al., 2001), mathematics “ability” has often been interpreted as speed and depth of conceptual understanding, and these factors have been used to determine which students are deserving of high-quality mathematics teaching and learning experiences (Moses & Cobb, 2001; Stiff & Harvey, 1998; Martin, 2019). More recent research asks that math education expand to provide every student an equitable opportunity to learn to develop their mathematical proficiency (Ladson-Billings, 2006).

Along the critical axis lies the dimensions of "identity" and "power". Gutiérrez (2013) suggests that a mathematical learning experience that is devoid of one’s identity, or one that indicates to students that their identity does not belong, is oppressive. Rather, students deserve a mathematics learning experience that provides them with “windows and mirrors” (Bolter & Gromala, 2003). Windows provide students opportunities to look into the world to view others’ experiences who may be unlike them and find similarities and differences within those experiences, while mirrors provide learners with the opportunity to see that mathematics comes from, belongs to, and is successfully done by people with similar identities to them. In providing windows and mirrors through the learning of mathematics, students are learning *critical mathematics* (Gutiérrez, 2013).

Other critical math pedagogues have described the need for students to be able to center and decenter themselves through the differences that exist in the world (Skovsmose, 1994, 2005;

Stinson & Wager, 2008). Skovsmose posits that social justice is just one aspect of critical mathematics, but that students who end their education trajectory should also have had the experience of different people, cultures, and contexts in which they can learn meaningful math. In decentering one's experience with math to contextualize it as being just one experience, students (and teachers) are able to expand their view on the world (Skovsmose, 2005)

These foundational pieces are just two examples for how the mathematics education field has oriented itself to defining what it means to take a critical lens towards teaching the content domain of mathematics. Other mathematics education researchers have demonstrated the importance of this type of robust mathematics education that shapes students into citizens who can read, write, critique, and change our world (e.g., Brantlinger, 2011, 2013, 2014; Frankenstein, 1983, 1989, 1990, 1994, 2001; Gutiérrez, 2007, 2013, 2018; Gutstein, 2003, 2006, 2016; McGee & Hostetler, 2014; Skovsmose, 1994, 2005; Tate, 1995, 2005; Tate & Rousseau, 2002), and provided examples (McGee & Hostetler, 2014; Berry, et al., 2020). While scholars debate the difference in vocabulary between critical mathematics, teaching math for social justice, or ethnomathematics (Gates & Jorgensen, 2009; Stinson & Wager, 2013), the common emphasis in all of these terms involves students using mathematics to understand a real issue of critical importance, and actively working to transform our society into one that is more just. This change may be actionable, but the transformation could also occur within the mind, as students develop their critical consciousness to gain a wider perspective of the world (Freire, 1970; Freire & Macedo, 2003).

To say that this sociopolitical turn has occurred in mathematics with ease, or even that it has, would be a farce. In 2016 and 2020, two national organizations, the National Council of Supervisors of Mathematics and TODOS: Mathematics for ALL, issued joint position statements calling for teachers to recognize this call for including linguistically, culturally, socially, and historically meaningful and motivating activities in their mathematics teaching. This call requires teachers to recognize "the roles power, privilege, and oppression play in the current unjust system of mathematics education - and in society as a whole…" (p.1).

Moving the work of critical mathematics out of the realm of theory and into practice has proven difficult, despite efforts of policy and large-scale organizations calling for its infusion (Aguirre, et al., 2017). Evidence of critical mathematics in everyday teaching and learning of the K-12 education experience is sparse (Gutstein, 2006). There are existence proofs of critical mathematics being taught in K-12 classrooms, mainly by self-studies conducted by education researchers who return to the classroom to demonstrate that it is possible (e.g., Gutstein, 2003, 2006, 2010; Brantlinger, 2011, 2013; Terry, 2009; Varley Gutiérrez, 2009; Lam, 2012; Turner, 2003; Gregson, 2013). Across these studies, the research provides evidence that students can learn mathematics concepts and procedures as outlined by the aforementioned policy documents while simultaneously learning about their role in a sociopolitical world. Yet, many challenges remain for incorporating critical mathematics into the mainstream classroom.

One challenge noted in the literature is that there is a specific content knowledge and prior experiences needed to feel knowledgeable enough to educate students well on issues of social justice (e.g., Bartell, 2013; Foote, 2010; Brantlinger, 2011), and this takes a significant amount of time to prepare teachers. Additional challenges are that of balancing the institutional alignment for whether teaching with this lens is accepted or desired by students, parents, administrators, and other stakeholders within the community (Gregson, 2012; Brantlinger, 2011; Turner, 2003), the inordinate amount of time required to preparing lessons of this context (Brantlinger, 2011), and navigating the complexities and conflicting messages of school, district, state, and national level curriculum mandates on what should be enacted within math classrooms (Brantlinger, 2011; Gregson, 2012).

The call from national organizations has initiated some teacher preparation programs to incorporate teaching mathematics with a critical lens into their preparatory programs for preservice teachers in hopes of creating a larger body of educators who are oriented to their role of advocacy prior to entering the profession (de Freitas, 2008; Gonzalez, 2009). Despite these advances (e.g., Martin, 2015; Stinson & Bullock, 2012; Nasir, et al., 2017), the normalized existence of a liberating mathematics curriculum that honors Black and brown students has yet to be realized (Martin, 2007, 2010, 2015, 2019).

The Intersection of Critical Literacy and Statistical Literacy

Until recently, the existence of critical statistics has mainly existed as practical examples within the subset of the critical mathematics literature. Scholars have cited the use of statistical concepts and ideas as examples in lessons that focus on the investigation of real-world, critical topics (e.g., race, gender, and class) as potential spaces for provoking the need for critical statistics (Frankenstein, 1983, 1989, 1990, 1994, 2001; Gutstein, 2003, 2006; McGee & Hostetler, 2014; Tate, 1995, 2005). Powell and Frankenstein (1987) defined their work as "ethnomathematics", in which adult learners learn the content of statistics by investigating the intersection of their identity and the world around them. Through her knowledge of students, Frankenstein (1995) tried to identify political topics that would be of interest to her remedial math students who were working adults seeking a degree through night classes. Many of her examples were based on using data to explore situations of injustice within the contexts of unemployment, military spending, taxation, economic policy, and wage gaps (Frankenstein, 1995).

Other scholars have suggested the domain of statistics as a potential cite for helping students understand their role in a sociopolitical world (e.g., Stinson, Bidwell, & Powell, 2012; McGee & Hostetler, 2014; Skovsmose, 1994, 2005; Cobb & Hodge, 2002; Nasir, 2002). Aligned to the critical axis of identity and power in math learning (Gutiérrez, 2013), some of these examples incorporate student identity into statistics instruction with a transformative worldview. Cobb and Hodge (2002) conducted a classroom design experiment with 11 eighth grade students across a 14-week period. The design experiment was focused on assisting students with analyzing bivariate data that would be meaningful and motivating to students, stating that they intentionally chose data sets that students “viewed as realistic for purposes that they considered legitimate” (p. 1). The researchers cited examples of datasets such as using carbon dioxide levels to investigate global warming, and salaries aggregated by gender to look at pay inequality. For Cobb and Hodge (2002), the identity orientation they were most concerned about was students seeing themselves as “doers of statistics” (p. 5), as it related to the world around them, using statistics as a window and a mirror (Bolter & Gromola, 2003) to view their statistics identity. What they found was that by bringing the investigation of critical statistics into the classroom, it provided opportunity for equitable experiences as students were able to draw from the different communities of practice in which they participate. Blurring the boundary of the statistics that occurs in the formal setting of math classrooms with the statistics that students experience outside of the math classroom was nonnormative for students and provided them opportunities to speak from their different bases of knowledge, which created a more diverse, inclusive, and equitable environment in their math classrooms (Cobb & Hodge, 2002).

The work of Nasir (2000a, 2000b, 2002) provides evidence for how a student’s identity, specifically racial, gendered, and cultural identities, are intertwined with their mathematics learning. Studying the context of basketball, Nasir (1996) investigated how middle school and high school boys' used their statistical thinking and reasoning to compare the quality of basketball players. Specifically looking at the statistical content knowledge mediated by one's identity, Nasir (2002) shared, "In basketball, players' mathematical goals shifted from understanding basic statistics involving counts in middle school to calculating relatively complex statistics with percentages and averages in high school" (p. 237).

The work of Cobb and Hodge (2002) and Nasir (2002) provide evidence that statistical thinking, reasoning, and learning is fundamentally linked to a student's identity, and provide promising avenues for the inclusion of critical statistical literacy development in the K-12 classroom centered on justice for Black and brown children.

At the Intersection of Critical, Statistical, and Math Literacy: Critical Statistical Literacy

Only very recently have statistics education scholars begun to call for the development of "critical" statistical literacy and define it as taking a justice approach to viewing data that leads to sociopolitical action via the teaching and investigation of critical statistics topics such as race, gender, and class (Watson & Callingham, 2003; Weiland, 2019; Bailey & McCulloch, 2019).

Watson and Callingham (2003) briefly mentioned critical statistical literacy in the findings of their survey of 600+ students in grades 5 through 10 as they developed a learning trajectory for the development of statistical literacy. Their findings suggest high school students need opportunities "to question critically statistical claims from media sources or other real-world contexts in order to develop the analytical habits of mind that are needed to respond critically to quantitative claims" (Watson & Callingham, 2003, p. 133). While the focus of the survey was not on critical statistical literacy or students' understanding or experiences with it, their work had implications for the types of sources that are brought into the math classroom with which students learn statistics, namely those found in the media and scientific articles.

Weiland (2019) provided a theoretical framework for critical statistical literacy, characterizing it as an emancipatory right, using Freire and Macedo's (1970/2005) frame of reading and writing the word and world. Paralleling the work of Gutstein (2006) in the domain of mathematics, Weiland (2019) defined critical statistical literacy as one's ability to read and write the word and the world with statistics. He defines reading the world through four components: 1) Understanding the language of and use of symbols within the domain of statistics to become aware of systemic injustices within society; 2) Analyzing and challenging the use of statistics when used to create and reinforce inequitable structures in society; 3) understanding one's identity and how it influences one's ability to interpret data; and 4) evaluating the statistical process when presented with data, recognizing the potential bias that comes with the author's position or lens.

Weiland (2019) also characterized what constitutes writing the word and the world with statistics in 14 elements. Each of these 14 elements can be mapped to the concept of sociopolitical action within Freire's (1970/2005) framework for critical consciousness development, as students learn how to change the world in which they exist to establish a more just society.

Bailey and McCulloch's (2019) empirical study of undergraduate students' development of CSL is one of the easiest empirical investigations of developing critical statistical literacy through active citizenry, defined as "the ability to utilize critical statistical literacy to become more socially aware of inequities, fight against inequities, and become an active, informed citizen" (p. 377). Using Gal's (2002) definition of statistical literacy as a conceptual framework and focusing on the dispositional elements, Bailey and McCulloch (2019) examined the development of critical statistical literacy through active citizenry in eight undergraduate students enrolled in an introductory statistics course at a four-year university through the use of analysis of online discussion board posts across one semester. Through students' written responses to prompts, Bailey and McCulloch (2019) found that students in the study engaged in their use of critical statistical literacy to "increase awareness of inequity, question methodology, acknowledge conflicting conclusions and identify bias, and develop a desire for more information" (p. 378).

Each of these studies serve as lampposts to the statistical education research community, calling for more heightened focus on the types of data that students should experience in their statistical education, as well as the motivation for why these experiences are meaningful, valuable, and consequential to learners of statistics. This small collection of literature focuses on the need for students to engage with critically sourced data, data from the news and media, and data that engages them with the statistics of the world. It also provides evidence of the ability to develop critical statistical literacy in a classroom setting, linked to one's sociopolitical action, or "active citizenry". It is clear that these three articles also are stating that what statistical data students engage with is important, and differs from the nonconsequential, pseudo contextual, benign statistics typically found in the K-12 classroom.

Despite research defining statistical literacy suggesting that one needs a critical stance and critical and contextual knowledge to be considered statistically literate (Gal, 2002; Rumsey, 2002), and a specific call for secondary mathematics students to have a robust critical statistics experience (Watson & Callingham, 2003), there is a paucity of research in the field of statistics education that takes up the investigation of critical topics such as race, gender, or class in the K-

12 math classroom for the purposes of disrupting known systemic injustices for populations of people who have been marginalized through the weaponized tactics of oppression. There are several reasons why this may be the case.

One reason could be the assumption by researchers that by taking a “critical stance” (Gal, 2002), and leaning on context being a driver of data, that the statistics education field has already taken this sociopolitical turn to incorporate ideas of identity and power into the teaching and learning. This stance has been proven to be a false claim as most K-12 teachers of statistics still teach from a referential, skill-based frame that limits statistics to a set of procedures or calculations stripped of context, rather than a process of inquiry, nullifying the idea that context will take care of the issue of criticality (Bakker & Derry, 2011; Shaughnessy, 2007).

Another reason could be that the definition of “critical” varies from researcher to researcher, and the lens for critical that is justice-oriented is undertheorized. For some, the term critical is viewed as similar to “critical thinking”. Critical thinking in statistics would be one’s ability to evaluate statistical information with a lens for skepticism and “worry questions” (Lee & Tran, 2015; Gal, 2002). Critical could also be defined as being gravely important in regard to time, context, or situation. But, again, who gets to decide when something is critical (or motivating)? (Garfield and Ben-Zvi, 2006). Because of these varying definitions on the word “critical”, it could be that the field has steered away from the justice-oriented definition of critical as it aligns to the body of literature in critical consciousness development as the field is focused on defining the knowledge elements of what statistical literacy is, rather than considering the dispositions and approaches one has towards the world and how this changes the ways that one interacts critically with statistics. Whatever the case, the paucity of literature focusing on the defining of, development of, and existence of critical statistical literacy in K-12 math education is an issue that should be addressed by the field with theoretical and empirical research.

Identity Development

Having reviewed the various literacies and their intersections that support viewing learning as knowledge acquisition, I now turn to review the literature that holds learning as identity development. The review of the literature above provided evidence for how identity is linked to one's learning of statistics and mathematics. Identity development is one of the key commitments in the materials designed for the development of critical statistical literacy in this study, and math teacher identity work is the lens for learning used for analysis. Therefore, I will briefly review the literature on student identity work in mathematics classrooms and math teacher identity work through curriculum enactment.

Student Math Identity

A growing body of research indicates that a student's math learning is connected to who they are becoming through their participation in the mathematics classroom (Boaler, 2002; Nasir, 2002; Sfard, 2002; Sfard and Prusak, 2005; Aguirre, Mayfield-Ingram, & Martin, 2013; Esmonde, 2009). Most of these bodies of literature stem from the seminal work of Lave and Wenger (1991), and others who view identity as a function of participation in different communities. One such community is that of the mathematics classroom in which students and teachers engage in the authorship of their identities-in-practice through the sharing of narratives and performances of those narratives. Based on this view, identities are dynamic, situated, formed through talk in everyday experiences (Antaki & Widdicombe, 1998) that occur across space (Gee, 2001) and time (Lemke, 2000). These narrations and performances do not occur in isolated vacuums, but instead are set against the backdrop of historical, institutional, systemic, and personal struggles directed by each individual's lived experiences (Juzwik, 2006; Tan et al., 2013; Holland and Lave, 2001).

As mathematics has been shown to act as a gatekeeper that operates as a weapon of oppression against students of color, females, students in poverty, and the intersection of these social identities (Moses & Cobb, 2001; Gholson, 2016; Berry, et al., 2014; Stiff & Harvey, 1998), how a student's identity is authored and shaped in the knowing, doing, talking, and being that occurs within a mathematics classroom has grave implications for their development of critical statistical literacy as well as their ability to "thrive in the modern world" (Franklin et al., 2007). Mathematics learning experiences are often racialized, engendered, and classist, as the subject area has been labeled a "high status discipline" that can garner cultural capital for individuals who are capable of completing courses in the upper tertiary of mathematics trajectories (Apple, 1999; Leonard, 2008; Martin, 2007). These types of experiences have led Martin (2000) to define mathematics identity as one's "(a) ability to do mathematics, (b) the significance of mathematical knowledge, (c) the opportunities and barriers to enter mathematics fields, and (d) the motivation and persistence needed to obtain mathematics knowledge" (p. 19). Developing a positive mathematics identity is a high risk/reward for minoritized and marginalized students (Martin, 2010; Leonard, 2008), as identity that comes from "being good at math" affords one social and cultural capital (Tate & Rousseau, 2002). There is an abundance of research that has demonstrated that students' math identities are shaped by their race and gender and the intersection of these two identity markers, particularly in STEM fields (e.g., Stiff & Harvey, 1998; Joseph, et al., 2020; Berry, et al., 2013; Morton, et al., 2020; Esmonde, 2014; Langer-Osuna, 2017; Gholson, 2016).

Boaler and Staples (2008) studied how different teaching methods produced different mathematical identities in students. In one school, students were taught using didactic instructional mathematics methods, which students saw as a set of skills to acquire. Students interviewed saw mathematics as something that occurs primarily in school and completely separated from their out-of-school lived experiences. In contrast, students in a different school were taught through rich tasks and problem solving. Students in this school saw mathematics as the work of the everyday, and some saw themselves as "doers of mathematics" (Boaler, 2002). Boaler (2002) suggested that mathematics learning occurs in a triangular relationship between knowledge acquisition, identity, and practice.

Esmonde (2009, 2014), using Lave and Wenger's (1991) framework for situated learning, helped explain how engaging in the set of mathematical practices that come from engaging in communities of practice like mathematics classrooms creates opportunity for learning through the speaking, acting, and interacting that occurs through its members. She stated, "mathematics learning has come to be conceptualized as learning to participate in mathematical practices - that is, students learn to construct representations, make arguments, reason about mathematical objects, explain their thinking, construct proofs, and so on" (p. 1011). Esmonde's (2009) work also validated students as participants in multiple communities of practice, some of which are within school and some that move beyond the classroom context. Students' identities are being authored in the moment by more than their participation in the math classroom community in which they are physically present, but also against the backdrop of their participation in multiple communities that are interacting with each other to present the identities-in-practice that are being performed in the mathematics classroom. In her later work, Esmonde (2014) investigated the patterns that occur when affluent students learn social justice mathematics and found that student's identities as coming from affluent children influenced their ability to engage with the social justice topics, providing another instance of how students are operating on their participation in multiple communities of practice simultaneously while they engage in math learning.

Specifically turning to how student identity can be incorporated into curricular design, research in the field of math education has begun to view culturally relevant, culturally responsive, and culturally sustaining pedagogical practices as ways to incorporate and honor student identity in the lessons taught in the daily classroom. For all of these pedagogical practices, educators cannot simply trade out "toast for tortillas" and call it a day, but rather seek to understand the cultures they are trying to integrate into their instructional practices in meaningful, authentic ways for students.

These three pedagogies are some of the more current aims at integrating student identity into the mathematics curriculum, and research has shown that they all create meaningful opportunities for learning, specifically for Black and brown students (Gholson, 2016; Leonard, et al., 2010; Hunter et al., 2018). As classrooms are communities of practice, teachers are also engaged in identity work through the use of narratives and performances as they author their identities in practice. In this next section, I review what the literature discusses about math teacher identity work, specifically as it relates to curricular decisions.

Math Teacher Identity Work

As participants in mathematics classroom communities of practice (Esmonde, 2009; Wenger, 1998), mathematics educators author their identities-in-practice against the historical backdrop of being a "high status discipline" (Ahlquist, 2001) whose STEM-based curriculum is related to future economic success in a capitalist society. As such, math teachers can act as gatekeepers for students' future success (Moses & Cobb, 2001), yet a prevailing oppressive narrative is that of mathematics teaching being objective and apolitical (Apple, 1999; Martin, 2019). Because of the social and cultural capital that can be acquired through school mathematics, the act of math teaching and learning plays a pivotal role in the social structuring of students' citizenship and livelihood (de Freitas, 2008). As the enactors of the teaching and learning of mathematics, investigating math teacher identity helps to understand how a teachers' "sense of self as well as their knowledge, beliefs, interests, dispositions, and orientations toward their work" (Drake, Spillane, and Hufferd-Ackles, 2001, p.2) influence the authorship of their own identities as well as the other participants in the learning community (Aguirre, Mayfield-Ingram, & Martin, 2013).

Like their students, teachers also have multiple communities in which they participate, and they are authoring their identities-in-practice in reference to these multiple communities at every present moment. Focusing on their math teacher identity - "an identity that consists of knowledge and lived experiences, interweaving to inform teaching views, dispositions, and practices to help children learn mathematics" (Drake, Spillane, & Hufferd-Ackles; 2001; Gresalfi and Cobb; 2011, as quoted in Aguirre, Mayfield-Ingram, and Martin, 2013), researchers can make sense of how math teacher professional identities impact their teaching practice.

Math teachers were once math students, and their prior experience as mathematics learners impacts their math teacher identity. Aguirre, Mayfield-Ingram, and Martin (2013) shared cases of K-8 math educators who narrated stories from their mathematics experience that provided powerful examples of how their narrated identities-in-practice as math learners authored their embodied identities-in-practice as math teachers. For many of the teachers in their study, the teachers experienced mathematics as racialized, engendered, and classist, and these chosen narratives that the teachers shared were examples of what they wanted to keep their students from experiencing, thus impacting their mathematics teacher identity (Aguirre, Mayfield-Ingram, & Martin, 2013).

Additional studies have focused on the identity authorship that occurs among math teachers when they are provoked to incorporate social justice into their enacted curriculum (Gonzalez, 2009; Leonard, et al., 2010). Teaching for social justice requires the narration of multiple identities within one community - that is the teacher's personal identity, math teaching identity, and identity as an agent of political change (Gonzalez, 2009). Teachers often navigate authoring these identities as social justice educators against the historical-institutional backdrop of their school systems not aligning with their critical stance towards teaching mathematics.

In a study of seven secondary mathematics teachers, Gonzalez (2009) studied how the teachers' participation in a professional development and community of practice that was focused on teaching mathematics for social justice changed their teaching practices and helped to shape their teacher identities. Through their participation, teachers understood how teaching mathematics for social justice could empower their students to invoke change in their communities, but few were willing to enact their learning for fear of retribution from their administrators, parents, and other stakeholders (Gonzalez, 2009). The key factor of an empowering school system caused one teacher to leave her school assignment in search of a school that would support the teaching of mathematics for social justice, rather than constantly feeling the tension of the historical-institutional backdrop of her changing identity as an agent of political change (Gonzalez, 2009).

Math teacher identity influences the curricular and pedagogical choices they make in the classroom (Remillard, 2005; Stein, et al., 2007; Drake & Sherrin, 2006). From the literature on curriculum enactment, teachers and the curriculum they teach are in a participatory relationship set against a sociopolitical context that causes variations in planned and enacted curricular materials based on teacher historicity, identity, beliefs, and more (Remillard, 2005). In the current era of reform-mandated curricular materials, teachers are given little autonomy about the curricular materials made available to them, yet Stein and colleagues (2007) have demonstrated key differences between an intended curricula, enacted curricula, and student learning outcomes based on teacher choices. The dynamic process of transformation between an intended and enacted curriculum is in constant flux, as intention is not always realtiy and the teacher's prior experiences as a student (the teacher's student learning), as a teacher in practice (a prior enacted curriculum), as well as the students' beliefs and knowledge (the students' learning) all have an impact on the teacher's intended curriculum (Hofer, 1986).

Looking specifically at the literature focused on math teacher identity related to curriculum enactment, Drake and Sherin (2006) compared two teachers of similar backgrounds, but different historicity with mathematics and found that their classroom practices were closely related to their identities in three specific ways. First, math teachers view their work with their students through the lens of their personal, early experiences with math. Second, a teacher's recent experiences with mathematics as a learner impact their ability to make curricular choices, such as adaptations and implementation choices. Lastly, the narratives that teachers share about how their identities intersect with mathematics, especially those concerning their close relationships with family members, are pivotal for understanding the choices teachers make in enacting and adapting curriculum (Drake & Sherin, 2006). More succinctly, teachers narrate about their identities-in-practice as they are related to their historicity, lived experiences, and their experiences with mathematics as an adult learner (Drake & Sherin, 2006).

Sfard and Prusak (2005) suggest that "'identities may be defined as collections of stories about persons or, more specifically, as those narratives about individuals that are *reifying*, *endorsable*, and *significant''* (p. 16). Their framework suggests that identity work may be detected through the telling of narratives through text and discourse. The work of da Ponte and Chapman (2008) claims that teacher identity work is complex, built on the levels of both individual and community. Because mathematics knowledge in teaching can be viewed as situated, social, and distributed (Stein et al., 1998; Hodgen, 2011), becoming a mathematics teacher involves a process of identity change.

While research on understanding the identity work of mathematics teachers continues to increase, there is a paucity of research in statistics education regarding the identity work that occurs for educators in relationship to the situated, social, and distributed knowledge in teaching statistics. One exception is that of Whitaker (2016), who examined 12 Advanced Placement Statistics educators and their narrated identities as both mathematics and statistics teachers. His findings suggest math teachers who are charged with teaching statistics are not just learning a new content domain, they are also engaging in the process of identity authorship as they learn who they are as statistics educators. Thus, the identity of a statistics educator is tied to their specialized content knowledge. The second major finding of this study was that the identity authorship of this group of educators was associated with their ability to connect to a network of other statistics teachers. That is to say, that math teachers who predominantly teach statistics often feel as though they are in isolation, and being connected to a larger social, situated community of practice assists them in authoring their identity as statistics educators (Whitaker, 2016). While Whitaker's (2016) work attends to the teaching experience and level of the statistics teachers, it does not attend to how their identities or historicity, specifically their race, gender, or class, influence their specialized content knowledge.

Having reviewed the literature based on viewing learning as the acquisition of knowledge of the specific literacies of statistics, mathematics, critical consciousness and the intersections between them, as well as the literature that views learning as identity development for both students and teachers, a significant gap in the field of statistics education has been identified in the lack of empirical research focused on the development of critical statistical literacy in secondary mathematics teachers through their engagement in identity work as critical statistics educators. For multiple reasons cited above, critical statistical literacy remains undertheorized and under-researched. In the tangential field of mathematics education, there is promising research that demonstrates how materials can shape a math teacher's identity work through the sharing of narratives. With their own specialized content knowledge, statistics teachers engage in their own identity work that is situated, social, and contextual. Could it also be the case that critical statistics educators engage in identity work as they author their identities through the sharing of narratives in relationship with the available resources that investigate critical topics such as race, gender, or class and the curricular and pedagogical choices they make in the classroom?

Theoretical Perspective: Making Visible the Identity Work of Critical Statistics Educators

For the purposes of this study, learning is situated in practice and intimately related to one's identity. To Lave and Wenger (1991), "learning is an integral part of generative social practice in the lived-in world", and "learning and a sense of identity are inseparable: They are the same phenomenon" (p. 35. 115). In other words, individuals learn through the process of being and doing in community. Our participation in these particular communities authors our identities, and through this participation, we become different people. Lave and Wenger (1991) called this situated identity learning our “identities-in-practice” to reinforce the idea that our identities are changing moment by moment and negotiated through our participation in these particular communities. Lave and Wenger (1991) link learning and identity authorship by sharing that “Who you are becoming shapes crucially and fundamentally what you ‘know’” (p. 53).

In this study, a mathematics classroom is viewed as a community of practice, in which the participants are students and teachers (Wenger, 1998; Esmonde, 2009). While both students and teachers engage in the practice of identity authorship through their participation in these situated learning communities, the focus of this dissertation is on that of statistics teachers’ identities-in-practice. Work in mathematics education on the authorship of math teacher identity has shown that teachers’ identities-in-practice are formed from their early math experiences (Aguirre, Mayfield-Ingram, and Martin, 2013; Drake & Sherin, 2006), their recent experiences as a learner, their experiences related to the subject domain and their close relationships with other people, like family (Drake & Sherin, 2006), as well as their current perception of themselves as a math teacher and learner (Sherin & Drake, 2009), and their historical experience as a math teacher as it relates to curricular resources (Drake, 2002). Many statistics teachers were not previously statistics learners and have had few opportunities to experience statistics as an adult learner (Shaughnessy, 2007). With the widespread adoption of CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), many educators have a negative historical experience of having to implement reform curricular materials that they were not prepared to teach (Elmore, 1997), and their perception of themselves as a statistics teacher and learner is steeped in their lack of statistical content knowledge and pedagogical knowledge for teaching statistics (Shaughnessy, 2007). The rare cases of statistics teachers with strong belonging in their identity as statistics educators are those who have taught Advanced Placement courses in statistics over a long period of time (Whitaker, 2016), none of whom are the focus of this study. The work of the above bodies of literature demonstrates how important a teacher's identity is to their math identity, yet there is not as much focus on how this influences a statistics teacher's identity, or more specifically important to this study, a critical statistics teacher identity.

Yet, there is enough evidence against the backdrop of math teacher identity literature and critical math teacher identity to know that critical statistics educators are authoring their identities in relationship to available resources (Sfard & Prusak, 2005) in the situated learning context of their math classrooms (Esmonde, 2014), in connection with the historicity of who they have been, who they say they are, and who they want to become (Holland & Lave, 2001), as statistics educators.

My Study

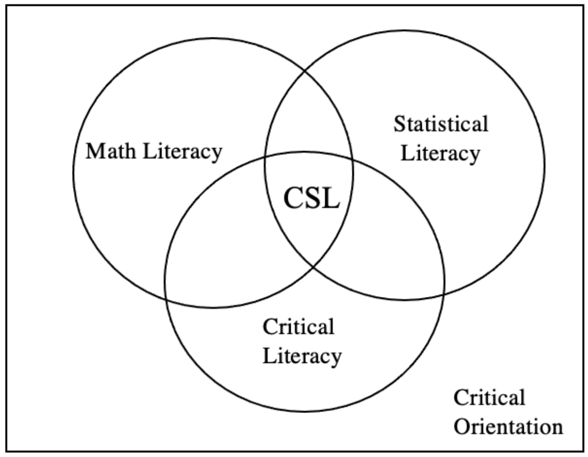
My dissertation aims to examine statistics teachers' identities-in-practice, specifically their narrated identities-in-practice through the "telling stories" they share (Sfard & Prusak, 2005) to better understand how the availability of resources centered on the development of critical statistical literacy makes visible the identity work that teachers do as critical statistical educators. Specifically, I focus on how the narratives that teachers share regarding their a) role as consumers and producers of data; b) their perspective on the purpose of teaching, and c) their perspective on the purpose of teaching critical statistics matter for their identity development as critical statistics educators.

Conceptual Framework: Critical Statistical Literacy

When considering teachers' development of critical statistical literacy, I drew from the three bodies that encompass critical statistical literacy: math literacy, statistical literacy, and critical literacy, as well as the intersectional literacies between them (See Figure 2.1). I include math literacy because, since the widespread adoption of the CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), a significant portion of the teaching of statistics occurs in math classrooMs. Hecause math teachers are the ones charged with teaching statistics content, the way that they define math literacy most likely interacts with how they define statistical literacy, as well as critical statistical literacy. Yet, I believe that the work of teachers in developing critical statistical literacy along these three domains has remained untroubled. As Foucault stated, "People know what they do; frequently they know why they do what they do; but what they don't know is what what they do does" (1964). This could be the case with critical statistical literacy; that teachers know that they teach math and statistics, and they know they have a purpose for teaching statistics, yet they have not troubled for themselves how their teaching of benign statistics perpetuates cycles of critical statistical literacy, specifically harming their students of color.

For math literacy, I draw from the two major reports from the National Research Council (2001; 2004) that include the five elements that define mathematical proficiency (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition), as well as five conclusions about learning focused on culture, lived experiences, and identity (2004).

Figure 2.1 Conceptual Framework for Critical Statistical Literacy



Following the work of Drake and Sherin (2006) as well as Aguirre and colleagues (2013) on math teacher identity, a teacher's mathematical proficiency as well as their culture, and lived experiences, specifically with the domain of mathematics, will influence their identity work as math teachers, as well as the curricular choices they make.

While there are several empirical works that theorize about statistical literacy (Rumsey, 2002; delMas, 2002), I chose to focus on Gal's framework (2002) for statistical literacy because it contains both the knowledge elements and the dispositional elements. The five knowledge elements (e.g., literacy skills, statistical knowledge, mathematical content knowledge, context knowledge, and critical knowledge) and two dispositional elements (e.g., critical stance and beliefs) create a collective set of skills, embodiments, and approaches towards the study of statistics in order for one to thrive in the modern data-rich world. Gal's model (2002) attends to critical knowledge, a skill that can be acquired through learning opportunities, as well as a critical stance, or an orientation towards the way one views data through their lens of the world as it motivates one to action. For Gal (2002), one could not be statistically literate without the ability to act on the data one consumes and produces in a way that it changes our view of the world. This idea maps on well to that of critical literacy.

For critical literacy, I pull from the work of Freire (1970/2005) and his development of *conscientizção*, one's ability to perceive social, political, and economic systems, analyze them from one's identity, and use that perception to take action to disrupt known injustices within society. For teachers who are developing their identity as critical statistical educators, one's ability to perceive social, political, and economic systems through the investigation of statistical data focused on critical topics such as race, gender, and class, would develop one's critical literacy.

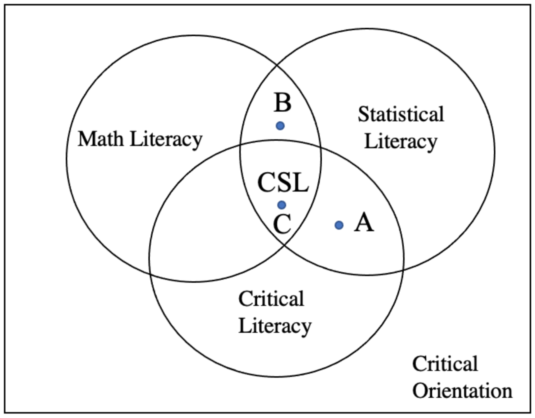
Because these three literacies interact with one another, I draw from the field of critical mathematics at the intersection of math literacy and critical literacy. From the literature on critical mathematics, I draw on the work of Gutiérrez (2002, 2013) that defines critical mathematics with dual goals of helping learners succeed in the system of education through access and achievement, as well as helping learners succeed in mathematics through identity authorship and empowerment. "Critical mathematics… takes students' cultural identities and builds mathematics around them in such ways that doing mathematics necessarily takes up social and political issues in society, especially highlighting the perspectives of marginalized groups" (Gutiérrez, 2002, p. 151). Like Gutiérrez, I also center the mathematical experiences on communities that have been historically marginalized, although for me this is true for teachers as well.

I draw from the body of literature on statistics knowledge for teaching at the intersection of math literacy and statistical literacy, as the work on defining MKT (Hill, et al., 2007) influences the work to define SKT (Groth, 2013; Simon, 2006; Silverman & Thompson, 2006). Specifically looking at the work of Groth (2013) and Silverman and Thompson (2006), I focus on the idea of decentering oneself to move from a personally powerful idea to a pedagogically powerful idea in the learning of statistics content. First, teachers have powerful, personal experiences with data, as both consumers and producers, that impacts the identity work they do as statistics educators. For teachers, especially those who have not had adequate training in the effective practices of statistics teaching and how it differs from mathematics (Shaughnessy, 2007), decentering these powerful experiences to consider the experiences of others is what creates a pedagogically powerful idea. This concept maps on to the critical consciousness literature as one must consider the experiences of others in order to disrupt social and political systems. This also maps well onto the body of literature in critical mathematics. As the body of math teachers is predominantly white and female, decentering their own experiences with statistics to become aware of the experiences of those who have been historically marginalized is the work of teaching critical mathematics.

Like Gal (2002), I include in my conceptual framework both a critical literacy as well as a critical orientation, to honor the historicity that teachers, particularly teachers of color, may bring to the work they do as educators in these three literacy areas.

As each of these literacies has been shown in the theoretical framework to align to the literature on math teacher identity work, I use this same lens for identity development to indicate how I view the three elements of critical statistical literacy that are in focus for this study. While there are many other elements that one could investigate for evidence of the development of critical statistical literacy, I am choosing these three elements because of their location at the intersections of the literacies described above (See Figure 2.2).

Figure 2.2 Elements of CSL



A: Role as consumers and producers of data

B: Perspective on the purpose for teaching statistics

C: Perspective on the purpose of teaching critical statistics

To investigate the ways that curricular materials make visible the identity work of critical statistics educators, I am specifically looking for evidence of identity work in three areas: their role as consumers and producers of data; their perspective on the purpose of teaching statistics education; and their perspective in teaching critical statistics curriculum.

Consumers and Producers of Data

Every teacher is, first, a human being in a data-rich society. Inasmuch, all people consume and produce data simply by existing in the world during this current era. What is less apparent, and therefore a focus of this study, is how one's critical literacy, or development of critical consciousness influences the data one consumes and produces, impacting one's statistical literacy. If an educator actively chooses to study data that is benign and noncritical in their math classroom, does this impact one's specialized content knowledge to understand statistics?

Conversely, what is also unknown is how one's statistical literacy through the data one consumes and produces, influences the development of one's critical consciousness. If an educator consumes and produces a significant amount of data in their role as a statistics teacher, will they, overtime, become more aware of the social, political, historical struggles, through the investigation of patterns in critical data?

Both of these questions situate a teacher's role as both a consumer and producer at the intersection of their critical and statistical literacies.

Looking at the role of consumer and producer through the lens of identity. All people are consumers and producers of data through their everyday experiences (Antaki & Widdicombe, 1998) as citizens across space (Gee, 2001) and time (Lemke, 2000). What citizens know about statistics is shaped by who they are and who they are is shaped by what data they consume. Today's society is data saturated (Franklin et al., 2007), meaning our bodies and identities consume data daily in our everyday experiences. Specifically, teachers of statistics, due to their specialized content knowledge, consume data in different ways than other citizens. Their consumption of data is formed by who they say they are as math and statistics teachers and learners.

All citizens are also producers of data, and we do so in several ways. First, our beings, or specific attributes about ourselves, are used as data daily. For example, when one uses their cellphone, the applications on the cell phone collect data about our race, gender, age, location, likes, dislikes, and favorite stops. In agreeing to allow these applications to collect data, we are, in part, agreeing to produce data that uses our attributes as data points. Therefore, simply by owning a cellular device, we are complicit in the collection, analysis, and production of data.

Secondly, by being who we say we are, we are producing or reproducing statistics as they align with our identity. As math teachers, we assign grades to our students, thus producing data that decides whether or not a student is able to gain access to the next course in our sequence of studies. In doing so, we produce data through our identities as math teachers. And, as these statistics can often be used as a barrier for students, particularly Black and brown students, to gaining entry to the next course, we also reproduce statistics that can often be used for harm, as it is tied to our math teacher identity.

Our education system is also in a data-driven era that requires educators in the K-12 system to produce data to justify claims of student achievement based on their instruction and student placement in upper-level math courses. As gatekeepers who produce this type of data that often yield to the segregation of students based on race, gender, and class, math teachers engage in the reproduction of statistics, as they are the ones that choose who is allowed to belong in mathematics and to whom mathematics belongs. Therefore, all statistics teachers are both consumers and producers of data, through their everyday experiences as citizens and in their work as educators, and the data that they consume and produce shapes their identities-in-practice as statistics educators.

The Purpose for Teaching Statistics

The teaching of statistics currently occurs predominantly in mathematics classrooms through the instructional practices of mathematics teachers (i.e., CCSSM, National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010). Therefore, a teacher's perspective on the purpose for teaching statistics may be embodied in their perspective on the purpose for teaching mathematics. Research has indicated that a teacher's specialized content knowledge and pedagogical content knowledge *in mathematics* undergirds a math teacher's identity work, yet less is known if a math teacher's specialized content knowledge and pedagogical content knowledge *in mathematics* undergirds a teacher's identity work as a statistics educator. Investigating what teachers think about the purpose for teaching statistics may illuminate how these two domains interact in a teacher's identity work.

Looking at the purpose for teaching through the lens of identity. The second point of investigation is related to the way that teachers view their purpose for teaching statistics. For many math teachers, the purpose for teaching statistics is complex, tied to reform initiatives of mathematics (e.g., CCSSM), their specialized content knowledge and pedagogical knowledge of statistics, and, as the teaching of statistics is situated in math classrooms, in relationship with their purpose for teaching mathematics. Identity work occurs between our narrated identities-in-practice and our performed identities-in-practice. As teachers of statistics narrate their identities-in-practice as statistics educators through the sharing of particular events and experiences that describe who we say we are (Goffman, 1959), these "telling identities" help to shape our trajectories for who we hope to become by selectively choosing which stories to share about who we say that we are (Sfard & Prusak, 2005). Math teacher identities have been shown to be informed by the interweaving of knowledge and lived experiences, informing both teaching views and practices to help children learn mathematics (Gresalfi & Cobb, 2011). Therefore, in teachers narrating about who they say they are as statistics teachers, they are choosing which stories to share, which are informed by the knowledge and lived experiences they hold, which permeate their views on teaching and their teaching practices to help children learn statistics.

For example, one statistics educator may view themselves as a mathematics teacher that is required to teach statistics because of reform initiatives, such as CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Their lack of prior experiences as a statistics learner shapes how they narrate their identity-in-practice, as well as their embodied identity-in-practice through the choice they make to skip statistics instruction completely in their classroom, viewing themselves as "a math teacher, not a statistics teacher".

Another statistics educator may have a depth of content and pedagogical knowledge in statistics because of their prior experience of working in a STEM-related field, and they use the curriculum mandated standards as a genesis for an expanded unit of study in their algebra-centric course.

A third statistics educator, having a critical lens towards mathematics teaching and learning, sees the curriculum mandated standards as an opportunity to provoke students to learn about and disrupt known injustices through the exploration of data centered on race, gender, or class. All of these examples of educators have a particular purpose for teaching statistics that is connected to their identities as it is tied to their historicity and specialized content knowledge and pedagogical content knowledge in statistics. Each of these statistics educators have a narrated identity-in-practice that tells themselves and others who they say they are, which impacts their choices in how they view the purpose of teaching statistics which informs their teaching views and practices in their everyday work in math classrooms.

The Purpose of Teaching Critical Statistics

It has been argued that statistical literacy is an unalienable right of all citizens needed in order to be full participants in society (Gal, 2002; Franklin et al., 2007). Thus, one's inability to acquiesce statistical thinking and reasoning acts as a gatekeeper to future success in life and work (Franklin et al., 2007; Apple, 1999). Statistics have also been shown to cause immutable harm against marginalized and minoritized populations, but they also have the ability to render justice through the investigation of statistical data on critical topics such as race, gender, or class (Fendler & Muzzaffar, 2008). By making visible the systemic injustices that are used to marginalize some and center others, statistics education has the possibility to be a site for making agents of change through the development of critical statistical literacy.

The teaching of critical statistics rarely occurs in math classrooms (Franklin, et al., 2015), and therefore there is much to learn about the identity work of critical statistics educators. What I believe is that the way that teachers think about each of the underlying literacies influences the identity work and motivation for a teacher's purpose for teaching critical statistics. Looking for evidence of when teachers talk about their perspective on the purpose of teaching critical statistics may help to elucidate how these different literacies are intertwined and in what ways they each inform the identity work that a critical statistics educator engages in.

Looking at the purpose for teaching critical statistics through the lens of identity. The last point of investigation in relationship to statistics teacher identity work is how they view the purpose for teaching critical statistics. The situated communities that we participate in do not operate in benign, siloed worlds, but rather across networks of institutional, historical, and personal relations (Lave & Wenger, 1991). Therefore, our identity authorships are not just occurring in the moment but are also set against an historical backdrop of both institutional and personal struggles (Lave and Wenger, 1991; Lave, 1996; Holland & Lave, 2001; Tan et al., 2013). Through prior research on math teacher identity, research has shown that teachers make curricular choices based on their students, particularly viewing their students through the lens of their early mathematical experiences (Drake & Sherin, 2006). For many math teachers, these early math experiences were racialized, engendered, and classist (Aguirre, Mayfield-Ingram, and Martin, 2013). Therefore, part of math teacher identity work occurs through how one views mathematics as it pertains to race, gender, and class.

This, too, holds for statistics educators, as their statistics teacher identities are set against the historical-institutional and historical-political backdrop of the past use of statistics for harm, the gatekeeping of statistics as a high-status discipline, and the reproduction of critically statistically illiterate students who do not have the conceptual knowledge or statistical skills to disrupt the unjust systems that operate in the world to oppress people who have been sent to the margins.

The purpose of teaching critical statistics is liberation (Freire, 1970/2005) through the intentional action (Gal, 2002) to disrupt the known injustices of the world through the investigation of data on critical topics such as race, gender, and class. With the majority of teachers being white, nonhispanic, and female (US Department of Education, 2020) the majority of teachers' personal struggles and lack of history in recognizing how their social identities intersect with the situated communities of practice in their math classroom leave their students, particularly their students of color, vulnerable in a data rich society that continues to use data to marginalize, gatekeep, harm, and oppress them from experiencing what it means to "thrive in the modern world" (Franklin, et al., 2015). As the more powered others in their math classrooms, statistics teachers must recognize how their racial, gendered, and classist identities influence what they see is the purpose for teaching critical statistics and the practices they use to teach it.

Statistics educators must recognize the position of power they hold as educators of a "high status discipline" (Ahlquist, 2001). Similar to that of mathematics educators, statistics educators teach a content that renders cultural and social capital to its learners in the form of future success and employment in STEM-related fields (Apple, 1999), but also in the ability to make data-based decisions regarding one's health, employment, science, and politics, all of which can be considered matters of life and death (Franklin, et al., 2015). Statistics teachers must recognize their position as gatekeepers, and acknowledge the "unjust system of mathematics education, its legacy in segregation, and other forms of institutional systems of oppression, and the hard work needed to change it" (p.1) (NCSM & TODOS: Mathematics for ALL, 2016).

In regard to historical-institutional struggles, statistics educators must acknowledge the unjust use of statistics in the past to marginalize some and normalize others (Fendler & Muzaffar, 2008). They must recognize the inordinate amount of harm they cause when they choose to not teach their students statistics based on the examination of identity and power, as students are left vulnerable in a world that will use statistics to control the narratives on who is valued and who is not. Statistics educators must also reckon with the use of benign statistics contexts rather than those centered on justice as it reproduces students who become citizens who are critically statistically illiterate and cannot disrupt known injustices due to their lack of statistical thinking and reasoning.

Defining Critical Statistical Literacy

Drawing from all of the literacies as well as the intersections among them, I conceptualize my definition of critical statistical literacy that guides this study. Critical statistical literacy is the fundamental right and skill to analyze and evaluate statistical data that shapes one's identities-in-practice for the purposes of disrupting unjust systems by creating agents of power who are able to communicate and discuss their evaluation of this statistical information with the world around them. As the purpose of this study is to better understand the development of critical statistical literacy in teachers, this conceptual framework seeks to highlight the relationships between the different literacies as it makes visible the identity work of the educators in regard to a) their role as consumers and producers of data; b) their perspective on the purpose of teaching statistics, and c) their perspective on the purpose of teaching critical statistics. The creation of the conceptual framework was informed by the research questions, the review of the literature, the theoretical framework focused on statistics teacher identity work, and the historicity and lived experiences of the research as a female scholar of color and former educator in a STEM-field. Addressing these inequities through the teaching of critical statistics is a meaningful endeavor to me. I have experienced statistics used as a weapon against former students, acting as a gatekeeper that kept them from gaining full participation in our local community. It is my prerogative as a researcher and statistics teacher that the teaching and learning of statistics be a cite to problematize the use of statistics to oppress, and instead be used as the impetus for freedom.

CHAPTER III: METHODS

In three phases, this dissertation study explored how design resources made visible the identity work of three secondary mathematics teachers in the process of becoming critical statistical educators. In this chapter, I describe the methodology used in this study as I begin to investigate the overarching question: *How and in what ways do curricular resources make visible the development of critical statistical literacy?*

Rationale for Methodology

To begin investigating this research question, I used a design-based research approach (Cobb et al., 2003; Cobb et al., 2013; Bakker, 2019; McKenny & Reeves, 2013; Kelly, 2009) to study the identity work of secondary mathematics teachers through the enactment of designed curricular materials of an exploratory data analysis unit with a justice-orientation in a fourth level math classroom. Design-based research requires the engineering of learning opportunities since the opportunity will not naturally occur in practice (Cobb et al., 2003). This study sought to manufacture a learning experience to prompt identity work for teachers as critical statistics educators through the use of provided resources focused on developing critical statistical literacy in one unit of study in a new fourth level high school math course. A review of the literature demonstrated that the majority of teachers will not create and/or are not using resources centered on the investigation of critical statistics that balances both the learning of statistics content with the exploration of critical topics such as race, gender, or class (Shaughnessy, 2007, Franklin et al., 2015; Gutstein, 2006; Brantlinger, 2013; Gutiérrez, 2013; NCTM & TODOS, 2016, 2020). Therefore, explicitly designing for the learning experience and studying how the enactment of the resources make visible the identity work of the secondary math teachers as critical statistical educators through a design-based approach was appropriate for this study.

Having a justice-orientation centered on making visible the oppression of marginalized groups, this project used a social design experiment approach towards design-based research (Gutiérrez & Jurow, 2016). Social design experiments take the foundation of design-based experiments that are "organized around a commitment to transforming the educational and social circumstances of members of nondominant communities as a means of promoting social equity and learning" (Gutiérrez & Jurow, 2016, p. 566). Social design experiments also seek to disrupt the known issues of power between researcher-participant relationships through the use of the process of co-construction (Gutiérrez & Jurow, 2016). A review of the literature also revealed that, while examples exist, math teachers, specifically those who are charged with teaching statistics, who are committed to transforming educational spaces by promoting socially equitable learning opportunities for their students, are hard to find. A social design experiment is appropriate to answer the research questions of this study because sought to expose and disrupt the perpetuating use of benign statistics in high school mathematics classrooms through the exploration of statistical data centered on identity development. In providing curricular resources centered on the development of critical statistical literacy to educators, the larger goal of this study is to engineer the opportunity for fundamental social transformation for marginalized populations of students and teachers (Gutiérrez & Jurow, 2016).

The goal of this project is to not only conduct research with a justice-orientation towards the content, but also with regard to the participants. Therefore, an additional goal of this study was to disrupt the traditional notions between the researcher and "the researched" by taking a participatory approach in design (Vakil et al., 2016). Shedding the traditional role of researcher as sole expert, this study sought to frame the teachers as partners in the learning process. While the researcher created the initial version of the design, teachers had full autonomy to modify, extend and adapt the lessons. The goal of implementing the lessons was not teacher compliance, but rather an agreement to embody the spirit of the designs. In taking up the ideas in these lessons and implementing the lessons in their spaces, they joined with me in creating a curriculum that is practical for a larger body of educators.

Design-based research also uses phases of design in order to study the innovation process through the different iterations (Cobb et al., 2003). This study was conducted in three phases. In Phase I, I designed the materials for one unit of study for a new fourth level mathematics course in a southeastern state. In Phase II, the materials were reviewed by experts in the field of critical statistical literacy and the new fourth level math course, and then piloted by a statistics teacher who was also a standards author of the new fourth level math course. The data collected by the review of the experts and the pilot data were used to refine the original materials for use in Phase III. In Phase III, three high school math teachers implemented the refined materials in their fourth level math classes. The focus of this dissertation is on the Phase III retroactive analysis of the teacher implementation of the refined and piloted designed materials. Through the engineering of the learning opportunity, the justice-orientation to the design, the use of multiple phases, and the iterative analysis of conjectures, the use of a designed-based research approach was appropriate to answer the research questions of this Phase III-focused dissertation study.

Elements of the Design

Phases of the Design

To better understand the design-based research approach, I now describe the phases of the design in more detail. This dissertation study was conducted in three phases. In the first phase, I designed the materials for one unit of study for a new fourth level mathematics course in a southeastern state. The standards of the fourth level math course focus on exploratory data analysis and the understanding of the omnipresence of variance throughout the statistical paradigm. To complement the exploratory nature of exploratory data analysis, the design materials, called Critical Statistical Literacy (CSL) materials, were also created to center on the exploration of one's identity through the use of data sets focused on social identity demographics (e.g., race, gender, class). As the population of the earth and any classroom has a large amount of variability in regard to social identities, this, too, was a fertile space for creating design materials for an exploratory data analysis unit. The materials encompassed six lessons across an approximate two-week learning period that culminated in an exploratory data analysis student-led project based on a question of interest about each student's identity. The lessons were designed for a flexible learning environment (e.g., virtual, hybrid, or face-to-face) due to the ever changing setting of pandemic-era classrooms. The materials provided to the teachers included lesson plans and curricular resources (e.g., PowerPoints, Desmos dashboards, databases, media sources, and discussion boards). Further detailed description of the materials is provided later in this chapter.

Phase II served as a review and pilot of the resources. The original lessons were shared with three experts: two critical statistics educators (CSE) and one fourth level mathematics standards writer. These experts reviewed the materials, provided suggested edits based on their teaching experience and knowledge of the literature. Some of the suggestions made by the reviewers were taken up in an initial round of changes to the original design of the curricular materials. For example, one CSE reviewed Lesson 5 and suggested a more critical questioning period for students regarding the lack of data for U.S. territories and what this might mean for the populations impacted by COVID-19. The standard writer also suggested that students review the statistical paradigm prior to engaging in the Lesson 6 unit-summary project. Both of these suggestions were incorporated into the second iteration of the CSL materials. In another suggestion, a CSE suggested that students should begin exploring large datasets in Lesson 1. This suggestion was not incorporated into the curricular design because I did not want students to rush towards abstraction with a focus on data as aggregate (Konold, et al. 2004), but rather wanted students to build towards this view through intentional investigation of viewing themselves as data points within sets of data.

Following the first redesign, the lessons were shared with one statistics teacher who enacted the lessons in two first-year introductory statistics courses at a four-year university. The educator implemented the lessons, made modifications and extensions to meet the needs of her students, reflected on her experience with the activities, and shared her observations and reflections with the researcher. Her observations and reflections, as well as the appropriate literature, were used to refine the designed materials for use in Phase III. For example, the pilot teacher shared in her post-interview how she referred to "data as story" throughout the unit to provide students with insight into understanding that data can be used to tell different stories based on the data author's perspective. This was such a grounding suggestion that the materials were redesigned to incorporate "data as story" as a key feature throughout the unit in helping students reflect as they explored different data sets.

In the original design, I shared a lesson overview as well as a Google presentation with the pilot teacher. She chose to transform several of the Google presentations in to Desmos activities. This, too, was an important contribution to the format of the lessons. I chose to create all of the lessons in the Desmos platform and provide teachers with both the Google presentations and Desmos-based activity links in order to ease facilitation of the curricular materials based on teacher preference.

In Phase III, three high school math teachers implemented the refined materials in their fourth level math classes in the spring semester of 2021. Teachers were asked to engage with the materials first as a learner, record their initial anticipations of how they planned to enact the lessons with their students as well as how students would engage with each lesson, document the modifications and extensions made, enact the lessons, and then reflect on their experiences. Through their reflections, modifications, and narrations about the curriculum, I made additional changes to the designed lessons so that they could be used by a larger body of educators. As this is the focus of this dissertation study, a secondary goal for the analysis presented is to inform the next iteration of the CSL materials for larger distribution and implementation.

Phase I: Description of the Design

Because the materials aim to change the social infrastructure of the classroom through the intentional implementation of outside resources not designed by the enacting teacher (Bielaczyc, 2006), it is important to describe the task structure used within the design (Sandoval, 2014).

I was prompted to create these materials after reviewing the standards for the new fourth level math course, as well as the materials designed by a statewide collaborative group of math educators. The existing materials for the new fourth level math course were pulled from collective resources from prior or advanced placement courses teaching statistics, as well as outside resources that seemed to align with the standards. In reviewing the materials, I did not think that the resources met the aim of the standards writers, and they were lacking the development of critical statistical literacy in students as all of the data sets explored within the unit were noncritical and benign (e.g., gross income for Pixar vs DreamWorks movies). This lack of criticality in these resources left students vulnerable as this was the last formal learning experience of statistics prior to exiting the K-12 education system.

To address this vulnerability, I designed all of the lessons to teach the four standards of a new fourth level math course that address exploratory data analysis, which are the following:

SP.1: Create statistical investigations to make sense of real-world phenomenon.

SP.1.1: Construct statistical questions to guide explorations of data in context.

SP.1.2: Design sample surveys and comparative experiments using sampling methods and to collect and analyze data to answer a statistical question.

SP.1.3: Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (i.e. spreadsheets, dynamic data analysis tools) to determine: types of variables that are in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.

SP.1.4: Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomenon.

While the materials addressed all of the standards, they also were designed to specifically center on standard SP.1.4 by providing students opportunities to view, describe, analyze, and make inferences about "non-standard data visualizations", with an explicit focus on visualizations that come from the media and scientific papers that discuss "real-world phenomenon". Employing this standard, I specifically incorporated a non-standard data visualization into each lesson that centered on studying a "real-world phenomenon" such as the systemic patterns caused by race, gender, or class. All of the visualizations came from media sources or scientific papers, and I used pedagogical routines (e.g., Notice & Wonder (Ray-Riek, 2013), Rough Draft Thinking (Jansen, 2020) to prompt students to make sense of these phenomena. In doing so, I intertwined the development of critical statistical literacy into each lesson.

The materials included lesson plans that followed a Launch, Explore, Discuss, Summarize model of instruction (Shroyer, 1984) to center classroom instruction on students' exploration of data, rather than the didactic model of students watching the teacher model how to explore data. Each lesson incorporated both whole group and individual exploration of data, understanding that by using oneself as a data point may provide moments of privacy the students would rather keep private. For this reason, I used the anonymize feature within the technology tool to hide students' names from each other and from their teachers. While this design feature and the reason behind it were described within the teacher facilitation guide, teachers were given autonomy and agency to modify or extend the lessons to best meet the needs of their class.

The tools were designed using an online technology dashboard (e.g., Desmos, 2011) for all lessons due to the unknown structure of classes because of the pandemic. The dashboards provided flexibility for the different participants in various instructional classroom settings during the pandemic period (face-to-face, virtual only, or hybrid).

Key Features of the Materials

Bakker (2019) discussed how the term "design principles" can have various meanings as they stem from different knowledge bases. For Bakker (2019), design principles are be based on "values, ethical norms, criterion, guidelines, heuristic, advice, or predictions". In the case of these key features, I drew on my review of the literature, my conceptual framework, my lived experiences as a Black math educator, and my values for yearning for a just society. By operationalizing all of these knowledge bases, I was able to distinguish what I was committed to including in the design of lessons that would prompt critical statistical literacy development. They are:

1. The use of a familiar routine (Garfield & Ben-Zvi, 2009; Ray-Riek, 2013; Lampert et al., 2010).
2. Using identity as data (LSA Learning Initiative, 2017a; Cobb & Hodge, 2002).
3. Exploring data as relevant to self and the world (Konold & Higgins, 2003; Konold, et al., 2015; Russell, et al., 2002; Harradine, et al., 2011; Pratt et al., 2008; Haldar, et al., 2018)
4. Viewing statistics as a process of inquiry rather than a calculation (McGatha, Cobb, & McClain, 2002)
5. Data as Truth versus Story (Cobb & Moore, 1997)
6. Use of technology, articles, and the news as learning sources (Gal, 2002)
7. Acknowledging and Accessing Multiple Knowledge Bases (Gal, 2002)
8. Exploring Real, Messy data (Garfield & Ben-Zvi, 2008; Doctorow, 2008; Finzer, 2013)
9. Use of technology tools (Konold & Miller, 2011; CODAP (The Concord Consortium, 2014)

Table 3.1 ***List of Design Key Features***

Design Key Features
DKF 1: Familiar Routine
DKF2: Using Identity as Data
DKF3: Exploring data as relevant to self and the world
DKF4: Statistics as a process rather than a calculation
DKF5: Data as Truth/Story
DKF6: Use of technology, articles, news as sources

DKF7: Acknowledging and Accessing Multiple Knowledge Bases

DKF8: Exploring Real, Messy Data

DKF9: Use of technology tools

In this next section, I describe these five key features in detail and how they relate to the literature.

Feature 1: The use of familiar routine. Brown and Benken (2009) found that teachers who feel as though their content knowledge is threatened often sink back into the roles of passive learners and return to familiar forms of teaching. Other research has shown that reducing the complexity of a task can support teacher learning (Lampert et al., 2010). Therefore, I incorporated the consistent use of a learning routine (e.g., Notice & Wonder, Ray-Riek, 2013) throughout each of the six lessons to reduce the complexity of the task. The use of common pedagogical structures were incorporated into the lessons to decrease cognitive load to support student and teacher engagement and learning through the materials. When looking at common pedagogical structures that align with the statistical paradigm (See Chapter 1), I wanted to incorporate a mathematics learning routine that would highlight the desire for students to create conjectures that they would revisit overtime as they analyzed data with more precision. The Notice and Wonder mathematics learning routine has been shown to be one such routine that allows students to explore their initial conjectures, although it has mainly been used with the domains of algebra and functions in mathematics so far (Ray-Riek, 2013; Rumack & Huinker, 2019). Thus, a key feature of the CSL lessons was to incorporate the mathematics learning routine of noticing and wondering in each statistics lesson.

At the beginning of each lesson, students look at a non-standard visualization of data, then they are asked, "What do you notice? What do you wonder?". Out of their noticings and wonderings, students are asked to write conjectures about their analysis of the data as they understand it thus far. In the first lesson, I made it clear that this was a "rough draft" conjecture (Jansen, 2020), and they would need to refine it through the process of inspection and data collection. Through the lesson, students are encouraged to revisit their conjectures as they test their hypotheses by attending to precision in their analysis. As students wondered about the non-standard visualizations, the materials encouraged students to ask statistical questions that could be answered by the data set and those that could not to help students recognize the limitations of data to answer specific questions. The incorporation of this routine into all six of the lessons was also an element of pedagogical design. By using the Notice & Wonder routine (Ray-Riek, 2013), students are asked to make a conjecture about an interesting data set before analyzing it deeply, and then test those conjectures through the process of inquiry, which has been shown to be an effective statistical teaching practice (Garfield & Ben-Zvi, 2009). The use of this routine also "promotes classroom discourse that includes the making of statistical arguments…that focus on significant statistical ideas." (Garfield & Ben-Zvi, 2008, p.48). Therefore, asking teachers to conduct the Notice & Wonder routine provided space for students to engage in the process of inquiry without being told what to think first.

Feature 2: Using identity as data. The second key feature of the materials is correlated to the nature of the resources used and how they intersect with one's identity. In the process of designing the lessons, I wanted students to use who they and the world says they are through their social identity markers as a source for data collection to encourage identity authorship (Martin, 2000; Nasir, 2002; Cobb & Hodge, 2002). As students use their identity markers as a source for data collection, they are exploring on two-dimensions - exploration of the data and exploration of one's identity. In one of the lessons, students completed a Social Identity Wheel

(LSA Inclusive Teaching Initiative, 2017a) in order to name the different identity markers that society uses to define them. This served as a prompt for finding data that is or is not representative of oneself. In other lessons, students were asked to look at large data sets (SP1.2) that used identity markers as variables. While there are many different identity variables that one could use as a source of data, I specifically wanted students to focus on their racial, engendered, and class identity markers because of my positionality and historicity as a Black math educator, as well as data from the literature that suggests that this type of mathematical study leads to identity authorship in students (Aguirre, Mayfield-Ingram, & Martin, 2013; McGee & Hostetler, 2014; Nasir, 2002). I believed that the inspection of data based on racial, engendered, or class variables would allow students to explore data from a systemic, global space as well as an individual, local space that would prove fertile for the development of critical statistical literacy.

Feature 3: Exploring data as relevant to self and the world. The third key feature of the materials was a focus on viewing data from multiple levels of abstraction. There is a plethora of data that demonstrates the importance of learners of statistics holding multiple views of data simultaneously (Konold & Higgins, 2003; Russell, et al. 2002; Konold et al., 2015, Harradine, et al, 2011; Pratt, et al., 2008; Haldar et al., 2018). Researchers have shown that when there is a rush to an abstracted view of data as a distribution, it leaves students with the idea that data is "over there" (Konold & Higgins, 2003; Russell, et al. 2002; Konold et al., 2015) found that one of the major challenges for teachers is to help students hold both the big picture of data while simultaneously holding the parts. In their study, Harradine and colleagues (2011) investigated how curricular design that forefronts a population view of a sample supports learning about the concept of variance as it is related to sampling. In the development of statistical literacy, students must understand that a distribution is made up of individual data points, and that individual data points make up a distribution. This finding is even more complex in the development of critical statistical literacy as students must hold the distribution view, the individual data point perspective, as well as the critical nature of data pointing to a systemic injustice that impacts real, individual people. While the development of statistical literacy typically rushes to the abstraction and holds the view of the distribution as more important than the individual, it was imperative for the development of critical statistical literacy that students be able to abstract to the larger systemic injustices, while also contrast to have empathy for the individuals that are impacted by these system wide injustices. Attention to this feature was therefore critical for the design.

To assist with this known issue, I designed the materials to first focus on data as a case of the individual (i.e., Who am I? I am a member of the data set) which presses students to see themselves as an individual within the data set while also holding the context of what the distribution is about (Konold & Higgins, 2003; Konold et al., 2015). Once students see themselves as an element of the data, they begin to acknowledge the representativeness of the statistical calculations that come from analysis (i.e., I am/am not a part of that group). Furthering this exploration, students also explored subsets of the population by grouping individuals through purposeful sampling on identity markers that were similar/dissimilar to themselves. This provided the tension for abstraction to the distribution, while being an individual that was part of a subset of the larger population.

Lastly, looking at the distribution of a population after recognizing oneself as a value in the data set allows students to see themselves as members of the larger sample (Konold & Higgins, 2003). The same is true when students are *not* represented within a distribution. In asking students to hold both the big picture and individual context of critical data, we are asking students to empathize with themselves, others like them, and others not like them.

Feature 4: Viewing statistics as a process of inquiry rather than calculational procedure. The next key feature of the materials was the emphasis on statistics as a process of inquiry rather than a calculation or procedure (McGatha, Cobb, & McClain, 2002; delMas, 2002; Franklin, et al., 2015). It has been shown that when students' ideations of statistics are based on "doing something with numbers" rather than thinking of statistics as a process of inquiry, they lose connection to data sets and the conceptual understanding behind the need for a calculation or procedure (McGatha, Cobb, & McClain, 2002). In many K-12 statistical experiences, students are left thinking that the study of statistics can be essentialized to the calculations of mean, median, and mode, rather than the process of inquiry that begins by asking a question. With a process of inquiry view of statistics, students learn that the study of statistics always starts with a question, and that question drives what type of calculations one can make, the type of inference one can make, the conclusions one can draw, and whether or not the answer can be answered in a data set. There is no set list of calculations that should be ascribed to any data set, rather the reasoning and thinking that comes from asking a question should drive the type of calculations one completes (McGatha, Cobb, & McClain, 2002).

In order to address this known issue, the statistical paradigm was introduced in the first lesson, in an effort to reorient students to statistics as a process of inquiry. At the end of this lesson, students are asked to reflect on how this view of statistics was different than their previous experiences in prior math courses, further prompting them to recognize the intentional shift. Throughout the remaining five lessons in the unit, students were reminded of the fact that data analysis is a process of inquiry, not a calculation (Cobb, 2004). In lesson 2, the materials do not just introduce sampling terminology, they also engage students in the act of sampling. The act of sampling helps students recognize that choices are made that introduce variation, and these choices are more than just being able to differentiate between the sampling strategies.

Literature has shown that it is beneficial to student learning to have students complete the entire statistical paradigm during an exploratory data analysis unit (Ben-Zvi, Gil, & Apel, 2007; Stanja & Steinbring, 2014). Therefore, the last two lessons in the unit were designed for students to engage in the statistical process of inquiry and “act as statisticians”. Lesson 5 asks students to complete the entire statistical process on a given data set, beginning with asking a question that they believe is worth answering (e.g. Is the novel COVID-19 infecting different racial groups disproportionately in the United States ?), determine the population of focus (e.g. all cases of COVID-19 in the United States), finding or generating a data set that can be used to answer their question (e.g. a dataset with this information can be found at *https://covidtracking.com/race)*, choosing a meaningful sample (e.g. the dataset is broken down by states; one could choose to sample the entire data set to include only those states in the southeast), calculating valuable statistics that will help answer the question (e.g. total number of cases by race in each state), making inferences from the statistics (e.g. compare the total number of cases by race in each state to the demographics information for each state), and then revisiting the original question to see what new questions arose from the process (e.g. are there patterns to certain subgroups that should be explored further such as the populations of Indigenous people?).

Having students engage in the process of statistics with their own questions has proven consequential (Stanja & Steinbring, 2014). Therefore, lesson 6 asks students to conduct the process of the statistical paradigm on a question of their choosing. Students must ask their own question, find their own dataset, choose their own statistics, make their own inference, and answer their own questions.

Feature 5: Data as truth versus story. The fifth key feature of the materials seeks to problematize for students what is meant by "data as truth" and reframe it to "data as story". The current data saturation era leads many people to believe that all arguments that use statistics should be accepted as truth; that when data is presented, the inferences made cannot be debated and must be taken as fact (Velleman, 2008; Wilson & Journell, 2011; López-Beltrán, 2006). Literature has shown how viewing data as story or connecting data to storytelling can assist early statistical thinkers to draw conclusions and make informal statistical inference (López-Beltrán, 2006). Because data is written and analyzed by data authors, the lack of criticality to question who gets to define data as truth and for whom is it truthful is a crucial component of the CSL materials.

To combat this epistemological myth, the materials were created to ask students to question "what is truth?" by having students analyze the same data set from different perspectives, with different statistical questions in mind, and scaffolding them to see that one can get different results from the same data set by telling a different story with the data. Specifically, lessons 3, 4, and 5 were designed to help students recognize that what one person says is truth based on data could vary from what another says is truth, often through the process of sampling and the use of bias. Similarly, this key feature helps students recognize that the data itself from multiple perspectives can tell different stories. This key feature is tied to the concept of the omnipresence of variance, yet different than it was originally described by Cobb and Moore (1997). While the omnipresence of variance is part of what allows students to recognize and question the epistemological myth, the materials were designed to show that there is variance in the story that can be told with the data.

For example, in lesson 4, students were asked to read a short excerpt from a scientific article about COVID-19 infections in school and how they impact community spread. After reading the article excerpt, students were asked to read three headlines from newspaper articles taken from real newspapers that cited the scientific article as a source but had vastly different versions of the story being told. The scientific article states that "community COVID rates are strongly correlated with in-person and hybrid models of school, yet these effects do not persist when we allow the effects of school modality to differ". One headline that sourced this article said, "In school classes don't contribute to the community spread of COVID", while another said, "New Study cautiously suggests schools don't increase spread of coronavirus", and a third stated, "Are schools safe? A growing body of evidence suggests that, with the right measures, they contribute little to the spread of virus". Students were asked to think critically about how the data in the scientific article is being used to tell different stories in the newspaper articles, and to question which one is true. In doing so, students were prompted to see how every data has multiple stories that can be told and asked to question "what is truth" when it comes to news articles.

Feature 6: Use of technology, scientific articles, and news as sources of learning. When statistical literacy was first being defined by the field at the turn of the century, Gal (2002) noted that one of the key challenges with incorporating statistical literacy into the K-12 classroom was that the sources of curriculum for teaching statistics that were needed to develop statistical literacy were very different than the ones that were normally employed. Specifically, Gal (2002) shared, To better cover all knowledge bases supporting statistical literacy, topics and skills that are not normally stressed in regular statistics modules or introductory courses, for lack of time or teacher preparation may have to be addressed. Some examples are:

- understanding results from polls, samples, and experiments *as reported in newspapers or other media channels,*
- understanding probabilistic aspects of statements about risk and side effects *as reported in newspapers or other media channels*
- learning about styles, conventions, and biases in journalistic reporting or advertisements
- familiarity with “worry questions”, coupled with experience in applying them to real examples (such as one-sided messages, misleading graphs), or seeing someone else, e.g., a teacher, model their application
- development of a critical stance and supporting beliefs, including positive beliefs and attitudes about the domain (usefulness of statistical investigation) and oneself. (p.21)

Despite this specific acknowledgement of the need to incorporate news, media, and other sources to prompt the learning of statistical ideas, the curricular resources and instructional methods typically used two decades later still do not cover many of these knowledge bases.

To address this issue, I specifically used a variety of sources from news articles, scientific articles, and technology sources (e.g., social media posts from Twitter, Facebook, Reddit, TikTok or Tumblr) in the design of the CSL materials. The use of these various resources was also supported by the standards for the new fourth level course, specifically in SP 1.4 which charged students with being able to… “interpret non-standard data visualizations *from the media or scientific papers* to make sense of real-world phenomenon” {emphasis added by author}. I chose to incorporate this design feature because, similar to how Garfield and Ben-Zvi (2008) suggested that the data sets students use should be real and motivating, I believe that the prompts we use for the learning of statistical contexts, too, should be real and motivating and come from sources that students use regularly.

For example, in lesson 2, I asked students to read a short paragraph from a scientific article. Students were then given three newspaper headlines that cited the scientific article as a piece of evidence for their claims about the spread of the coronavirus in schools. Students were then asked to Notice and Wonder (DKF1) about the similarity and difference among the articles and the stances they took towards school opening. This prompt for learning allowed students to question and conjecture about how news headlines distort the evidence presented in scientific articles to tell a specific story (DKF5) and prompted students to think about the big statistical idea of bias.

Feature 7: Accessing prior knowledge of statistical ideas. In the seminal work on defining statistical literacy, Gal (2002) described six key knowledge bases that must be acknowledged: literacy skills, statistical knowledge base, mathematical knowledge base, context/world knowledge base, critical skills, and the interaction between each of these knowledge bases. What I appreciate about Gal's (2002) distinction in this definition is recognizing that one's historicity and prior knowledge matter for the ways in which learning can occur. The literature on funds of knowledge in the field of math education (Civil, 1998; 2014) also points to the importance of recognizing student historicity and prior knowledge bases as crucial components of curricular design.

To address this, I designed the lessons to pull from multiple sources of student knowledge. In lesson 2, I asked the students to use their literacy skills to read a scientific article.

In lesson 1, I asked students to use their prior statistical knowledge base to think about the difference in the levels of obtrusion for participants between surveys, experiments, and observations, and how this may make collecting data and answering questions involving the existence of systemic injustices difficult. In lesson 5, I asked students to use their context/real world knowledge base to think about a complex data set that was missing crucial information for marginalized populations of people. Due to the nature of the CSL materials, students were asked to use their critical knowledge base throughout the entirety of the unit.

Feature 8: Exploring real, "messy" data. The eighth key feature focuses on exploring real, messy data. The literature that defines statistical literacy clearly indicated the importance of "using real and motivating data sets to engage students…" (Garfield & Ben-Zvi, 2008, p.48). Yet, it does not specifically address who gets to define what is "real and motivating". In this study, it was important that the students be able to explore data that made them aware of "real" systemic injustices and be "motivated" to act upon their awareness. Simultaneously, it was also important to provide autonomy and agency for students to define what was real and motivating to them in the choice of a dataset to explore and move this from the teacher to the student.

When students explore real data in classrooms, there is still the possibility of the data being clean and neat. Real data is often "messy" (Doctorow, 2008), with gaps and holes, that require the data author to make decisions which introduces variance throughout the data analysis process. Literature has shown that students need access to complex data sets and to watch analysis of these large data sets be modeled in real-time in the classroom (Doctorow, 2008; Finzer, 2013).

To incorporate the exploration of real, messy data, I chose the globally-relevant, real and motivating topic of the novel coronavirus as the thematic context throughout the unit. In all six lessons, students were asked to explore data sets about the coronavirus from different perspectives. For example, in Lesson 4, students were asked to explore a large dataset that contains people's opinions on mask wearing as it relates to their race, gender, and geographical location. In lesson 6, students were asked to find their own data set to explore to assure student autonomy in what was real and motivating to them.

I also designed for the specific incorporation of a messy data set. In lesson 5, students were asked to look at a large data set (SP1.3) tracking COVID-19 cases by race and state (*https://covidtracking.com/race*). This data is messy; it's missing data for U.S. territories and certain racial subgroups, and some states did not disaggregate the data by race but only provided the total number of cases each day. I used the messiness of the data to prompt students in the lesson to think about the decisions they would need to make in order to continue on with the exploration of the data (e.g., What do you notice and wonder about the cases in Oklahoma? Why do you think there is no data for Indigenous people in Oklahoma?). I asked them to keep a "variance log", as they made choices to clean up the data to emphasize the omnipresence of variance throughout the statistical process of inquiry (Cobb & Moore, 1997).

Feature 9: Use of technology tools. The last key feature focuses on the use of technology tools to explore statistical concepts. The conceptual understanding of exploratory data analysis has been shown to be improved through the use of technology tools that emphasize statistical inquiry (Konold & Miller, 2011). With recent technologies, such as TinkerPlots 2 (Konold & Miller, 2011), CODAP (The Concord Consortium, 2014), and Desmos Classroom (Desmos, 2011), students have been able to make important connections using applets and software packages that create simulations. Technology tools have been shown to offer students the ability to explore and examine statistical inference informally without the need for formal statistical knowledge and skills (Stanja & Steinbring, 2014). This literature base aligned with the standards for the new course, specifically SP 1.3 that asked students to be able to… "organize large datasets of real-world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology…" [emphasis added by author].

To incorporate technology as a tool that enhanced the learning experience, I utilized the Desmos platform that allowed students to interactively engage with the data for all six of the lessons. In Lesson 5 and 6, I designed for the exploration of a large data set using the technology software CODAP. For example, in Lesson 5, students are asked to upload data on COVID-19 cases by race (*https://covidtracking.com/race*) into CODAP and then use the software to begin to explore the data set for patterns and trends in the data. The interactivity of the software allowed for students to pull different variables (e.g., racial subgroups, geographical regions, date ranges) with ease to populate distributions to visualize what stories could be told from the large dataset.

Anecdotally, not many teachers I have worked with have used CODAP previously, but many of them had used Desmos. I anticipated that CODAP might cause some issues with teacher implementation because of lack of experience in using the platform. To address this, in the facilitation guide for Lesson 5 and 6, I provided teachers with several tutorials for their own learning utilizing a different data set on roller coasters. I also offered to meet with teachers for a one-on-one tutorial explaining how to use CODAP to simulate data and calculate statistical values.

The description of the key features is used to highlight how critical statistical literacy was interwoven throughout the materials. In this next section, I highlight how each of the six lessons within the exploratory data analysis unit incorporate these nine key features by providing a detailed description of each lesson.

Description of Materials

The design materials were created to develop critical statistical literacy by infusing the current knowledge of developing statistical literacy and the ideas from the conceptual framework described in Chapter 2 into the individual lesson design and the overarching unit design through the incorporation of the key features mentioned above. The materials needed to develop critical statistical literacy that would make teacher identity work visible, and also be a practical set of materials that would be easy to use. Literature has shown that, when asking teachers to implement something complex into their practice, it is important to simplify other aspects to assure the most probable chance of implementation (Lampert et al., 2010). For this reason, I used a similar lesson plan format that teachers would have seen from previous curricular materials provided for the new fourth level math course, and a teaching facilitation guide for each lesson for one unit of study focused on the statistics content of Exploratory Data Analysis (See Appendix A and B). For each lesson, there were three learning goals: statistical, interpersonal, and critical. Each of those learning goals are described for each lesson as well. For the purposes of brevity, I offer a table that summarizes each learning goal and the design key features incorporated into each lesson (Table 3.2). For a more detailed description of each of the six lessons, highlighting which key features were present across the unit of study, and how they connect to the literature and my conceptual framework, please refer to Appendix A.

Table 3.2. The statistical, interpersonal, and critical learning goal and key features present for each lesson

Lessons, Learning Goals, and Design Features

Lesson Title	Statistical Learning Goal	Interpersonal Learning Goal	Critical Learning Goal	Design Key Features Present
What & Why of Statistics	To understand the statistical process, how variance and bias are introduced to the statistical process, the types of studies and process of sampling that are often used in the statistical process	To understand how variance in the statistical process introduces bias	To understand how statistics can be obtrusive in the lives of those from whom data is being collected	1, 4, 5
Being Skeptical	To recognize the purposes of and differences among statistical surveys, observational studies, and experiments	Beginning to develop a healthy skepticism of data presented based on the type of design used	To recognize that, for many statistical questions that impact real systems and policies, the surveys, observational studies, and experiments we use are distorted versions of the data we need to answer the statistical question of interest	1, 4, 5
What does data say about me?	To begin thinking about how the design of an experiment affects the results and to introduce key vocabulary that will be used throughout the unit (e.g., sample, population, representative, well-designed experiment, and the concept of error)	To understand that not all statistics about someone or others is representative of the entire population	To make students aware of how statistics are being used to represent a population of people for which they or others identify	2, 3, 5
Different People, Different Results	To understand how sampling methods affect the results of an experiment, survey, or observation and to understand why randomization is a component of a well-designed experiment	To understand that not all statistics about someone is representative of the entire population	To build criticality in digesting statistics shared without enough information and to understand how sampling methods can be used for harm to misrepresent a population of people	1, 2, 3, 4, 5
Same Data, Different Conclusions	To explore a given data set with at least 200 entries and experience exploratory data analysis in a scientific context	To understand that our initial view of data may change with more information	To recognize how the novel COVID-19 virus is disproportionately impacting communities of color	1, 4, 5
Exploring Meaningful Data	To explore a data set with at least 200 entries, experience exploratory data analysis in a scientific context, and complete the statistical paradigm with a dataset of one's choosing	To create a compelling statistical question that is meaningful to the student, and to find a way to answer the question using the statistical method	Dependent on the student's choice of a question to explore, but not limited to the struggle to find data based on critical contexts and the potential empowerment and powerlessness of consequential findings	1, 2, 3, 4, 5

Conjectures & Hypothetical Learning Trajectory

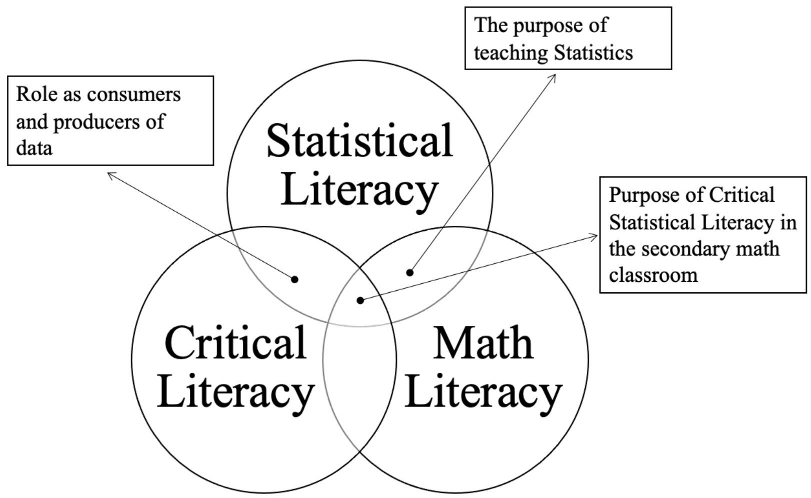
An additional feature of design-based research is the use of conjectures to assist in the process of theorizing. By conjecturing about what might happen before the implementation of a design, the researcher has the ability to test out different hypotheses to determine which ones are supported. If, through the process of implementation, a conjecture is proven valid, then there is support for the underlying theory (Confrey & Lachance, 2000; Sandoval, 2004).

A high-level conjecture guiding this study was that curricular resources created to develop critical statistical literacy would make visible the identity work of teachers as they develop as critical statistical educators. To test this conjecture, I designed materials that explored data that used identity demographics (e.g., race, gender, class, etc.) throughout one unit of study containing six lessons for a new fourth level mathematics course in a southeastern state.

Sfard & Prusak (2005) suggested that the resources available to someone can prompt identity work. Recalling my conceptual framework, this dissertation study is informed by the bases of literature from statistical, math, and critical literacy and the intersections among them. By using these curricular resources, I conjectured that teachers would engage in identity work as critical statistics educators that made them inspect a) their role as consumers and producers of data, b) the purpose of teaching statistics, and c) their perspective of critical statistics in the secondary math classroom.

Being a consumer of data refers to how one might interpret and make use of statistics. This element of statistical literacy is quite common in the literature (e.g., Watson, 2002; Garfield, 2002; Rumsey, 2002). Yet, an additional, less prominent component of critical statistical literacy is understanding one's position as a producer of statistics. All people are creators of statistics, whether through the generation of data sets, or by being a point within a dataset themselves. This position of being a producer of statistics is severely untroubled in the literature as it relates to the historical harm done by past and current producers of statistics.

Figure 3.1 ***Elements of Critical Statistical Literacy***



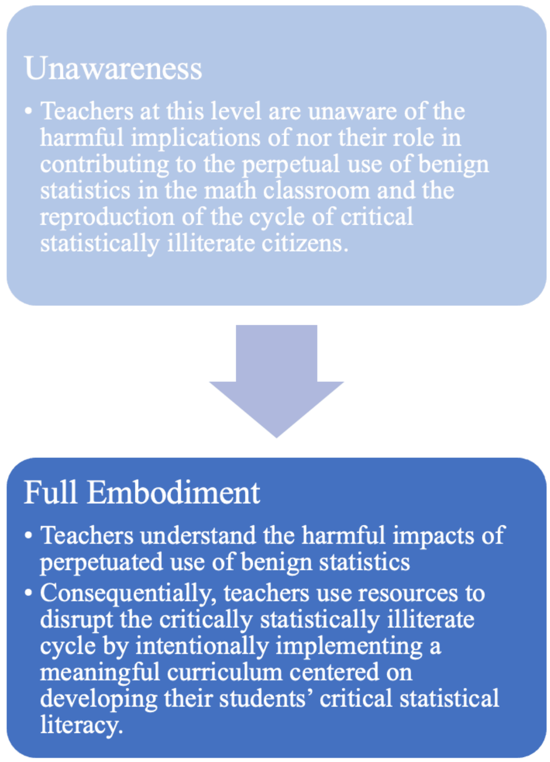
Educators sit in a unique position as they engage in the production of statistics from multiple tiers in the structure of society. From a societal position, teachers are citizens, and all citizens engage in the production of statistics. From a pedagogical position, teachers make conscious choices about the statistics that they teach within their math classroom in terms of content and framing. From a systemic position, through their actions as a teacher, teachers contribute to the production of statistics that maintain the perpetual cycle of critically statistically illiterate students. Therefore, one conjecture of the study is that teachers would become aware of their multi-tiered role as both consumers and producers in the system of education and society at large.

An additional conjecture was that the resources would make visible the identity work for critical statistics educators in making sense of the purpose of teaching statistics. In recognizing their role as consumers and producers, teachers could be prompted to ground their purpose in teaching statistics in similar ways that the GAISE framework (2007) has shared, to end the cycles of statistical illiteracy.

The last conjecture suggested that teachers who use these materials would recognize the difference in developing statistical literacy and critical statistical literacy in their students. That is to say, that teachers would differentiate that the context in which one learns statistics through data exploration matters, and the context should point students to understanding, exploring, and acting on findings from data sets that make visible the systemic injustices that plague our society. This was different than the GAISE framework (2007), as it suggested that simply studying data in “real world” or “meaningful” contexts was not enough to end the cycle of critical statistical illiteracy as it will continue to center whiteness in the teaching of statistics by not pointing students and teachers to seeing the systemic injustices that are evident in data.

With this lens, an additional conjecture was that through the availability of these resources, teachers would become aware of the benignity of previous statistics content taught in the high school math classroom, and be motivated to incorporate content in their statistics courses that focused on developing critical statistical literacy in their students to disrupt the cycle of critical statistical illiteracy. To accompany this last conjecture, I developed a hypothetical learning trajectory (HLT) of the possible evolution of the teachers’ thinking and understanding of becoming a critical statistics educator by enacting these materials. As defined by Simon (1995), the HLT has three parts: the learning goal, the learning activities, and the learning process. In this case, the learning goal was for teachers to fully embody their identity as critical statistics educators who are aware of the harmful acts of benign statistics in the secondary math classroom and make intentional curricular choices to disrupt the cycle of critical statistical literacy with their pedagogy. To move towards this goal, teachers enacted the designed curriculum centered on critical statistical literacy. In doing so, they would begin the process of becoming from unawareness to full embodiment.

Figure 3.2. ***Hypothetical Learning Trajectory that describes the development of critical statistical literacy in secondary math educators.***



A math teacher who sees themselves as fully embodying the identity of a critical statistical educator would understand the harmful acts of perpetual use of benign statistics in the classroom, and consequentially choose resources that instead disrupted this benignity and developed students' critical statistical literacy. A math teacher who talks about who they are or the resources they use as a critical statistics educator defines a critical statistically literate student as one who, through the use of curricular materials, is also aware of the systemic injustices in the world through the inspection of statistics focused on issues of inequality. This is in comparison to the normalized use of benign statistical resources which perpetuate harm, the cycle of critical statistical illiteracy, and the continued data-rich tropes that oppress Black and brown citizens from experiencing a full democratic citizenship.

From the critical consciousness literature (Gutiérrez, 1995; Watts et al. 2011; Hipolito-Delgado & Lee, 2007), I recognize that there are more nuanced levels in between unawareness and full embodiment. The hope is that, through careful analysis, I would be able to better understand and articulate this movement and development and what features of the CSL materials that make the identity work visible with specific orientation to critical statistics.

Methods

Research Questions

My overarching research question is: *How and in what ways do curricular resources created to develop critical statistical literacy make visible the identity work of teachers as they are becoming critical statistical educators?* Here, I use my overarching conjecture to focus my study on how the use of the materials centered on developing CSL creates opportunities for identity work for secondary mathematics teachers as critical statistics educators. Therefore, my specific research questions are:

- *What features of the design did the teachers discuss?*
- *How and in what ways do the materials make visible the identity work in secondary math teachers as critical statistics educators?*

Context and Participants

Context of the Study. This study took place in three secondary mathematics classrooms with teachers who were instructed to teach a new fourth level math course in a southeastern state. In 2018, teachers and education researchers from across the state came together to decide what content should be included in a capstone course for the three integrated high school math courses required for graduation fulfillment. This was the second semester of the course being offered across the state, after a standards adoption process that occurred from 2018-2020. The standards, having been written by state educators, were not aligned to a particular commercially available curriculum, and there was no textbook provided for the course. Therefore, a collaborative group of math educators and math education researchers (NC2ML) worked collectively to create a set of resources to aid teachers in the implementation of standards-aligned materials for the course.

This study took place beginning in January 2021 through June 2021. During this time, teachers were six months into teaching during a global pandemic due to the novel Coronavirus-19. Throughout the 2020-2021 school year, schools had switched teaching format several times, from virtual to hybrid to full face-to-face instruction. All participating teachers were required to wear face coverings while teaching with students present.

Participant Recruitment. Participant recruitment for this study involved convenience sampling (Patton, 2002) of teachers known to be teaching the new fourth level math course within two districts for recruitment. Teachers were conveniently sampled through two methods. A group of teachers were attending a regional professional development to support the teaching of the new fourth level math course. Teachers within this professional development were invited to continue working with the facilitator to develop additional lessons for the new course. Mr. G was recruited through this method, and taught in District A. The second recruitment method was via an invitation by a math curriculum director in District B. The math curriculum director emailed teachers involved in a district-level professional development focused on creating lessons using culturally relevant practices. Five teachers volunteered to participate from District B. Two teachers from this set of five were chosen to teach the lessons, based on their pacing and whether or not they had already taught the unit of focus.

The two districts were both in the same state in the southeastern region of the United States and were close in proximity, but differed in size (See Table 3.3). District A had 26 schools, 5 of which were high schools, while District B had 134 schools, 29 of which were high schools. The demographics data for all three of the individual schools of the teacher participants differed from the district demographic data (See Tables 3.3 & 3.4), with the population of Black and Hispanic students being larger than the population of White students. Similarly, all three schools were classified as Title 1, with two of the schools classified as Magnet Schools. Both districts supported the implementation of the materials focused on critical statistical literacy, through an LEA approval. District B had begun discussing the implementation of instruction that was centered on culturally relevant and sustaining practices in all courses, including mathematics, yet conversations with educators and curriculum directors in this district suggested implementation had not occurred or was not widespread at the time of this study.

Table 3.3. District Demographic Data

		District A	**District B**
Student race and ethnicity classifications	White	59.9%	32.1%
	Black	19.4%	40.7%
	Hispanic	14.0%	15.8%
	Asian	0.5%	6.5%
	American Indian/Alaska Native	0.3%	0.4%
	Native Hawaiian/Pacific Islander	0.1%	0.1%
	Two or More Races	5.7%	4.2%
Student Gender	Female	48.5%	48.7%
	Male	51.5%	51.3%
English Language Learners		4.4%	8.4%

Each of the three secondary mathematics educators who participated in this study was assigned to teach the new fourth level math course during the Spring semester of 2021, which occurred during a year of teaching greatly affected by the novel Coronavirus-19 global pandemic. All three of the teachers were using a virtual setting at the beginning of their Exploratory Data Analysis unit, and all three of them switched to a hybrid setting, with a portion of the students being virtual, and the remainder of the students being face-to-face, but socially distanced from one another.

Table 3.4. ***School Demographic Data***

		Mr. G's School	**Ms. H's School**	**Ms. J's School**
Total Enrollment		690	1,094	1,453
Student race and ethnicity classifications	White	32.3%	22.9%	27.9%
	Black	42.0%	37.4%	38.3%
	Hispanic	18.0%	25.8%	18.0%
	Asian	0.4%	8.8%	11.6%
	American Indian/Alaska Native	0.6%	0.6%	0.5%
	Native Hawaiian/Pacific Islander	0.1%	0.0%	0.1%
	Two or More Races	6.5%	4.5%	3.6%
Student Gender	Female	48.0%	47.6%	49.9%
	Male	52%	52.4%	50.1%
English Language Learners		4.3%	10.3%	11.6%
Title 1 Classification		x	x	x
Magnet Classification			x	x

Collectively, the three teachers had an average 10.3 years of experience teaching mathematics, with 1 receiving undergraduate training in education, and two entering teaching through alternative licensure programs. The participants had an average of 9 hours of undergraduate hours in mathematics courses, and 3 hours of undergraduate hours in statistics courses. None of the teachers were National Board of Standards for Teaching certified, nor were any of the teachers certified by the college board to teach Advanced Placement (AP) Statistics. The teachers shared that they had an average of .5 hours of professional development in statistics content or statistics teaching in the last five years. Both teachers in District B were enrolled in Culturally Responsive Teaching professional development provided by the district that was centered on intertwining student identity and culture into the mathematics curriculum. The professional development had just begun when this research study took place. The following is a description of each of the three teachers and the way they described themselves and their school populations.

Mr. G*:* Mr. G self-identified as “a white male who loves sports”, who coached both the baseball and football teams at his school. He taught in a rural district at one of the five high schools within the district. He described the population at his school as “pretty diverse” stating, “We're, like, 40% white, 40% Black and 20% other”. He self-selected to join a region-wide professional learning team that focused on statistics content knowledge for teachers for the new fourth level math course. He was recruited to join the research study out of his participation in the professional learning team, of which the researcher was also the facilitator.

Ms. H: Ms. H self-identified as a “Black female” who viewed teaching mathematics as her second career. With a Bachelor’s of Science in engineering and a Master’s degree in information technology, she shared that she switched careers into teaching mathematics because of her desire to see young children flourish in STEM. She stated that she has taught all of the courses offered in the standard course of study, except those that require Advanced Placement training. She taught in a rural school in an urban district and described the student population as “pretty multicultural”, stating, “It's a good mix. So, we have roughly, I want to say, maybe 30% African American and somewhere between 25 and 28% of Hispanic and also somewhere around that number for Caucasian backgrounds. And then the rest would be other”. Ms. H self-selected to join a Culturally Responsive Teaching Task Force professional development through her district, in which they were actively recruiting and training teachers to incorporate culturally responsive teaching into the various content areas, but specifically STEM domains. She was recruited out of the participant list for this professional development.

Ms. J: Ms. J self-identified as a “Black, female, mother” who views her students as “all her children”. Having worked in “the STEM field of accounting” for fifteen years, Ms. J joined the education field through lateral entry and a recruitment program targeted at mathematics educators. She had been teaching for 14 years, 13 of which had been at her current site placement. She taught in an urban school in an urban district and described her school population as such, “The majority of our students are Black students, but of course, we have white students. We have a large Hispanic population, and we have a lot of Korean students as well. And so, we have a pretty large EL population”. She also shared that her school struggles with racial segregation among honors and standard level math courses. She said, “You'll see a class that has no Black kids and then you'll go to another class, and it's, you know, from the classes 100% Black to classes 100% White. And you're like, am I in the same building? What is the timeframe? you know, is this 1952?”. Ms. J also self-selected to join the Culturally Responsive

Teaching Task Force professional development through her district, and was recruited out of her participation through this learning opportunity.

Data Sources and Collection

Methodologically, the present study was informed by the work of Brantlinger (2011, 2013) and Gutstein (2006) who investigated students' reactions to social justice math lessons using multiple data sources from focus students, such as interviews, student work, and observations. As the focus of this dissertation is on teachers and not students, I used a similar frame for the collection of data, specifically interviews, teacher notes, teacher-provided student work, and teacher reflections. Due to the pandemic and the varied formats of the teacher participants' classrooms, I was unable to observe classroom implementation of the CSL materials.

All three teachers enacted the CSL materials for 12-15 instructional days of the spring term, similar to the percentage of instructional time that Gutstein (2006) devoted to his lessons on social justice mathematics with middle school students in his earlier work. I interviewed the three teacher participants one-on-one, twice throughout the unit of study, once before and once after implementation, but also provided for modes of communication during the implementation should the teachers want to discuss a particular lesson or the provided materials. Teachers 1 and 2 both reached out for a mid-unit meeting, and these meetings were included in the data collection. These multiple interviews as well as the additional modes of communication allowed for the development of rapport and relationship building (Seidman, 2006, p. 21), the learning of teacher historicity, and the ability to analyze potential change in teacher identity work across the two chronological points of time. Interview data collected totaled 341 minutes, of which included the three post-interviews (52 - 120 minutes) that occurred within two weeks of the completion of the enactment of the unit. For analysis of research question 1, all the data were used equivalently. In phase 2, interview data were primary, and the other data sources were secondary.

I also collected each teacher participant's lesson anticipations, modifications, and daily reflections throughout their implementation of the CSL materials. Informed by the social justice orientation of design-based research (Gutiérrez & Jurrow, 2016), I asked the participants which format was best for them for reflecting on the lessons. Mr. G chose to use a video format, in which he recorded his anticipations and modifications prior to the enactment, and then a video of his reflection post-enactment. Ms. H and 3 preferred a written format for their reflection, so I created a questionnaire (e.g., Google Form) to be completed daily by the teachers pre- and post-enactment. Providing the teachers with choice for their anticipation and reflections varied the data collected (transcripts versus text), yet it assisted with the development of rapport and relationships with the educators who I would later interview (Seidman, 2006).

While this study is not focused on students or student development of critical statistical literacy, any student work provided by the teachers was collected for future research. A letter was sent home with students addressed to their parents or guardians prior to the beginning of the unit of study to alert them that their student work may be viewed by an outsider of the classroom environment. Parents and students were given the opportunity to not have their student work viewed for research purposes. All students and parents chose to all their data to be viewed and collected. All of the data collected was supported through an IRB approval as well as a local LEA approval in both districts. All data was stored using student and teacher pseudonyms to protect both student and teacher identity.

In this section, I share the research questions that guided this study, and describe the sources of data used to answer each research question. Table 3.5 presents a crosswalk of data sources with research questions.

Table 3.5. ***Crosswalk of Research Question and Data Sources.***

Research Question	**Data Sources**
RQ1: *What features of the design did the teachers discuss and with what prevalence?*	• Teacher Anticipation Notes • Teacher Daily Reflections • Teacher Post Interview
RQ2: *How and in what ways do the designed materials make visible the identity work in the teachers as critical statistics educators?*	• Pre-Interview • Post-Interview • Initial Demographics Questionnaire • School Data • Teacher Modified Lessons & Anticipations • Teacher Provided Sample of Student Work • Teacher Daily Reflections • Researcher memos • Teacher profiles

Research Question 1: To answer my first research question which focused on the materials as a study of the design, I collected multiple sources of data as is recommended for design-based research studies (Cobb, et al., 2003; Bakker, 2019). Teachers were asked to preview the lessons as learners, then document their anticipations and modifications pre-implementation, and document a reflection post-implementation for each of the six lessons. After teachers completed teaching the unit, they were interviewed within two weeks about the materials, the implementation of the materials, and their ideas about teaching critical statistical literacy in the math classroom (See Appendix C for interview questions). For the post-interview, teachers were asked to bring student artifacts with them to demonstrate moments throughout the unit in which they felt students had the opportunity to learn statistical literacy, student identity development, and critical statistical literacy. Additional questions were sometimes posed to teachers to elicit their thinking on specific key features of the materials, but not all questions were posed to all participants. This is because each teacher discussed specific key features differently, and the questions asked to each teacher were based on which key features they discussed.

These data collected will help me refine the design principles mentioned above and provide evidence for which key features serve the purpose intended, solidifying my conjecture that access to resources can develop critical statistical literacy in secondary math educators.

Research Question 2: To answer the second research question, I used the same data sources as in question 1, with the addition of the pre-interviews. Here, I collected multiple sources of data as recommended for a case study analysis (Stake, 1995, pp.107-116; Yin, 1984, pp.84-95). Teachers were interviewed prior to seeing the materials to collect initial demographic data, as well as their initial ideas about teaching critical statistical literacy in the math classroom (See Appendix C). This initial interview served as a baseline for their perspectives on teaching critical statistical literacy, but also allowed for the development of rapport and relationships (Seidman, 2006, p.21). Additional questions were sometimes posed to teachers to elicit their narratives about incorporating critical statistical literacy into their math classroom, but not all questions were posed to all participants. Because identity work is evidenced by narrations, I asked different questions to different participants to probe further into their narratives and gather additional details to understand how their individual historicity was intertwined in their narratives. I also asked teachers to expand upon their own narratives often to help me explore the more nuanced levels of the hypothetical learning trajectory (HLT) described earlier in this chapter. At times, when teachers discussed something at the intersection of the statistical literacy, critical literacy, or math literacy circles, I probed further to gain more insight into their perspectives on a) their role as producers and consumers of data, b) the purpose of teaching statistics, and c) the purpose of developing critical statistical literacy in their students. All of these questioning techniques within the interview were focused on helping me answer the research question by making visible through their narratives and our conversation the identity work that each teacher was doing as a critical statistical educator.

Because I was analyzing some of the data while the teachers were enacting the curriculum as is recommended in design-based research (Cobb, et al., 2003), I wrote analytic memos during the pre-interview and as the teachers' anticipations and reflections were shared with me. Following Ely's (1991) recommendation for memo writing, I wrote after each interview or new piece of data was collected in a file organized for each teacher, as well as one that was overarching across the teachers as I noticed patterns in their enactment. These memos were then used as a source of data to triangulate the other data sources.

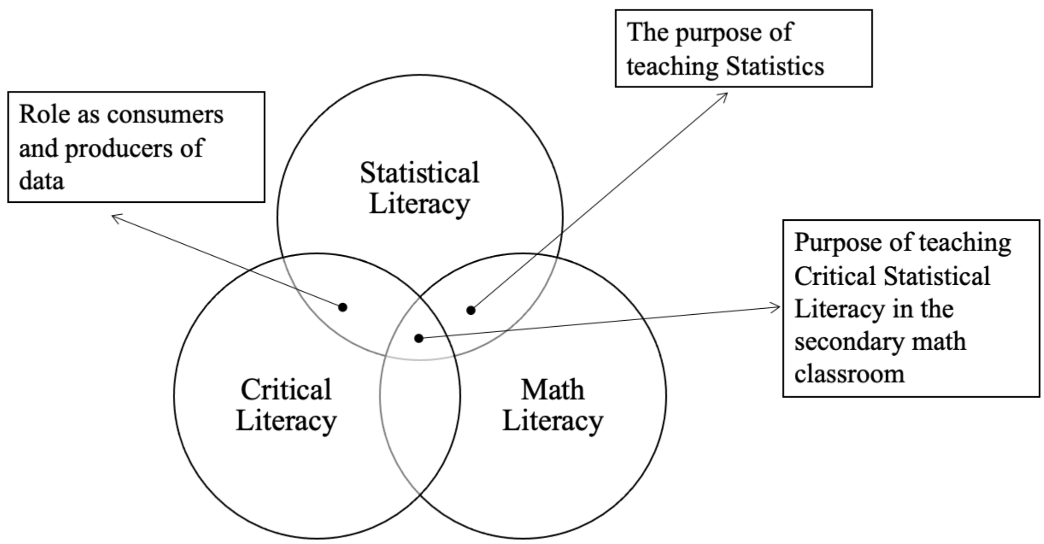
Data Analysis

Overview. A variety of qualitative methods were used to analyze the data collected to answer the two research questions guiding this study (Charmaz, 2006; Strauss & Corbin, 1997; Miles & Huberman, 1996). Analysis used both theory-based and data-based approaches, and data were analyzed both iteratively and retroactively, as is common in DBR studies (Cobb et al., 2003; Bakker, 2019). Theoretically, my analysis was grounded in my conceptual framework and the intersection of the three literacy domains.

Because narrative identities-in-practice are collections of stories (Sfard & Prusak, 2005), my theory-based analysis was focused on looking for evidence of how teachers shared stories at the intersection of the three literacy domains tied to my conceptual framework (Figure 3.3).

While there are many characteristics that could be looked for within the intersections of these literacy domains, I wanted to look for evidence of these specific instances with respect to: (a) each teacher's perception of themselves as a consumer and producer of statistics (b) each teacher's perception of themselves as a statistics teacher, and (c) each teacher's perception of themselves as a critical statistics educator.

Figure 3.3. ***Diagram of conceptual framework with specific evidence within the overlaps highlighted.***



At the intersection of critical and statistical literacy, I was interested in how teachers discussed their role as consumers and producers of data. Because of their position in the education system, teachers are both consumers of data (e.g., end of course test data) and producers of data (e.g. grades used for recommendations). Outside of the system of education, they also engage in the consumption and production of data as everyday citizens. Considering the way they discussed their roles from both within and outside of the system of education met the criteria of overlapping the critical and statistical literacy domain.

At the intersection of statistical literacy and math literacy, I was interested in how teachers talked about the purpose of teaching statistics. Specifically, I wanted to see if there was evidence in their narrations for the way they viewed statistics as either a procedural list of calculations or a process of inquiry that informed how to approach data (Cobb & Moore, 1997; Cobb, 2004), and if this view influenced the way they taught statistics to their students. I was also interested in how they shared through their narrations the way they differentiate math literacy and statistics literacy, and how this philosophy of teaching influenced their pedagogy.

At the intersection of all three literacy domains, I was interested in knowing how teachers narrated about the purpose of teaching critical statistics in the secondary math classroom. Specifically, I wanted to see if there was evidence of a shift in their narrations between their pre- and post-interviews, indicating that there was some identity work occurring for them as critical statistics educators. I looked for evidence to see how they discussed the resources, and if the resources prompted any identity work. The intersection of math literacy and critical literacy was not taken up in this study, as it did not align with the standards of the course for this unit.

I also used data-based analysis guided by my hypothetical learning trajectory. Throughout my analysis, I looked for evidence of developing critical statistical literacy for each of the teachers, as access to resources prompted identity work (Sfard & Prusak, 2005). I looked for instances when the teachers discussed critical statistical literacy between the two chronological data points, and then looked for similarities and differences between their narrated stories to better understand their learning along the trajectory.

Iterative Analysis Phase. First, I began analyzing data iteratively, analyzing new data continuously as the teachers shared their experiences to compare to previously collected data to the pre-interview data. All interviews were audio- and video-recorded and transcribed through a transcription software program, Otter.ai. Interviews, analytic memos, teacher anticipations and reflections were coded using a grounded theory approach (Strauss & Corbin, 1997; Glasser & Strauss, 1967).

I began analyzing the transcribed data through open coding and recoding, focusing on the pre-interviews since the teachers had not yet begun to enact the curriculum. I developed pattern codes for each teacher, and then used group codes as themes emerged (Miles & Huberman, 1994, p.69).

Because I wanted to make sure the narrations teachers shared were captured in totality, I determined that the idea unit was an appropriate unit of measure to code the two transcripts. An idea unit was defined as a distinct shift in focus or change in topic (Jacobs et al., 1997). For example, when asked "what is the purpose of teaching statistics?", Mr. G shared the following narration,

> Providing… the opportunity through like examples and things to, to have those learning experiences where they can take what they've learned…Because that's the biggest question I get..as a math teacher… "when am I ever going to use this?" So I think if we can incorporate some things that they can actually use after they graduate, then they'll be a little bit more apt to be able to actually want to complete the lesson.

Here, the teacher was sharing a narration on how examples in the classroom needed to be attached futuristically to students' lives after graduation. It was important to code for the entirety of this narration to assure that Mr. G's whole idea about graduation was connected to his initial thought.

Constant comparison involves developing grounded theory from the beginning of data collection (Krathwhol, 1998, p.260), which aligns with the iterative analysis timeframe of DBR study analysis. Using a constant comparison method, I was able to code and recode as themes emerged that either supported or refuted the developing theory related to the design conjectures, learning conjectures, and HLT. Analytic memos were written to help document the iterations on emerging themes by creating a spreadsheet of notes with quotes from the transcribed pre-interview data, then adding additional quotes from their initial demographics questionnaire. I, then, created profiles for each of the three teachers that culled data from across the interview data, school data, and initial demographics questionnaire. These pre-profiles helped engage in cross case analysis across the three teachers' experiences prior to their enactment of the CSL materials. This initial iterative analysis was used to create the profiles of each teacher as well as provide a robust, rich description of their initial narration of their identity-in-practice as critical statistics educators.

Retroactive Analysis Phase. During retroactive analysis, I first used the key features from the design phase as *a priori* codes to analyze the teacher post-interviews, reflections, and anticipations. My analysis of the post-interviews was guided by the question: “What design key features of the materials did teachers discuss and with what prevalence?” The purpose of this guiding question was to help me look for instances when the teachers explicitly discussed a key feature that was intentionally incorporated into the design of the lessons. First, I transcribed the lesson reflections and post-interviews. Next, I read the transcripts and watched the videos for instances when teachers mentioned a specific design key feature.

As teachers discussed a key feature, they would share an idea. I utilized a qualitative data analysis coding software, Atlas.ti, to code the post-interview transcripts, teacher notes, and daily reflections for instances that mentioned a specific key feature (e.g., how the Notice & Wonder routine helped students make conjectures about data). This same process was completed for each of the three teachers' post-interviews, daily lesson reflections, and anticipations. Third, I noted the prevalence of each of the key features across all of the transcripts, identifying the frequency of each one mentioned.

Next, I continued to use Atlas.ti to organize all of the mentions of each key feature into its own document, and coded these new documents to look for patterns in the ways the teachers discussed the most prevalent key features. I, again, used analytic memos to help document emerging, conflicting, and revised themes that arose from the data. The use of Atlas.ti, the qualitative data analysis software, helped to triangulate data by organizing multiple data sources and storing codes. Google Documents, a word processor, was used to store memos and create profiles for each teacher. Documentation of codes and analytic memos using Atlas.ti and Google Document helped to attend to reliability, providing a clear path of analysis that would allow for the possible replication of this study by another researcher that would lend the same results.

In the second phase of retroactive analysis, data were analyzed using a constant comparison method methodologically informed by the work of Bakker (2019) (Strauss & Corbin, 1997) to yield more theoretical insights about the teachers' identity work. Since one's identity is dependent upon the resources that one has access to (Sfard & Prusak, 2005), the purpose of using this constant comparative method was to help me compare instances in which the designed materials made visible the identity work that occurred between the two chronological narrated identities-in-practice. I analyzed the during and post-enactment data sources in isolation from the pre-enactment data sources, using analytic memos to document emerging, conflicting, and revised themes. I then created a post-profile for each teacher by creating a spreadsheet and document of quotes from their during and post-enactment data sources. These two profiles helped me engage in cross case comparison across the three teachers

(Stake, 1995), as I looked for patterns in their identity work made visible through the enactment of the CSL materials. Using the constant comparison method with two chronological data points (pre- and post-profiles) and multiple sources of information that provided a rich context, individual case analysis was completed for each teacher, and cross-case analysis was then used to look for similarities and variances amidst the teacher participants in their pre- and post-profiles narrated identities-in-practice through their enactment of the CSL materials.

A case study is "an exploration of a 'bounded system' or a case (or multiple cases) over time through detailed, in-depth data collection involving multiple sources of information rich in context" (Creswell, 1998, p. 61). Yin (1984) suggests six sources of information for case study data analysis: documentation, archival records, interviews, direct observations, participant observation, and physical artifacts (pp. 84-95). For this portion of the analysis, I used the following data sources: all student work completed in the Desmos dashboard within the designed lessons, publicly available records of school and student demographics, interview data with each teacher, teacher observation data in their daily and unit reflections, and any notes the teachers created during their unit, as well as the pre-, mid- (if requested by the teacher), and post-interviews. Due to the pandemic, direct observations nor video recordings of the classroom sessions were not available.

Addressing Validity and Reliability

Statement of Positionality

Pillow (2003) situated reflexivity as a methodological tool in which researchers can begin to reflect, question, and legitimize their qualitative research practices. Additionally, Glesne (2015) stated that researchers begin their reflexivity by acknowledging and reflecting on their personal experiences, beliefs, and identity categories that influence their implementation, interpretations, and their final product. As stated in Chapter 1, I am a Black and biracial mother of two daughters of color, who is concerned with the math education of students of color. As a former math learner and teacher and current math education researcher, I am aware of how my identity influences my justice-orientation towards the design of the materials, as well as the investigation of teacher learning for critical statistical literacy. I see my lived experiences as an asset upon which I draw as the curriculum designer and researcher of this project, as I seek to disrupt the status quo statistical education experience of Black and brown students in the secondary mathematics classroom.

Validity and Reliability

In design-based research studies, researchers must attend to research quality of validity and reliability through replicability, adaptability, trustworthiness, and clarity (Bakker, 2019). To address replicability, I utilized multiple data sources of evidence to triangulate the data. For example, to investigate the frequency and use of key features discussed by the teachers, I analyzed interview data, teacher daily reflections, and anticipations. In addition, I created a chain of evidence, by using qualitative data analysis software that documents the methodological procedures and evidence found. I addressed adaptability through the use of multiple resources to provide a rich and robust explanation of the context (e.g., teacher profiles, school profiles, student classroom rosters), and providing the rationale for the selection of the three teacher participants.

To attend to trustworthiness, I organized and documented all steps taken in a research journal, including all memos, documents, and transcribed interviews in Atlas.ti. I also kept a conjecture journal in which I documented the iterations on the conjectures made based on new data sources. This documentation and organization facilitates possible replication of the study by other researchers.

Lastly, I attended to clarity by being mindful of my potential bias as the designer of the curriculum and my desire as a Black math teacher for critical statistical literacy to occur in K-12 classrooms. I utilized a journal that documented my comments and feelings throughout my analysis (Bogdan & Biklen, 2003, p.151). This allowed me to separate my feelings and reactions to teacher enactment of the CSL materials. I also chose to engage in reflective journaling, writing memos about my notices and wonderings, fears and excitement after each design or analysis session (Paulus, Lester, & Dempster, 2013), to “untangle the personal from the theoretical” (Kleinsasser, 2000).

CHAPTER IV: KEY FEATURES OF CRITICAL STATISTICAL LITERACY

The primary goal of the first level of analysis was to look for the existence of the key features identified in the designed curriculum, as it was enacted and communicated by the teachers. I sought to develop: 1) an understanding about the prevalence of the different key features in the teachers' discussion; 2) an understanding of how the teachers discussed the different key features, and 3) how the key features made identity work that the educators might be undertaking, visible. In this chapter, I specifically focus on answering the following research question: *What features of the design did the teachers discuss?*

Prevalence of Key Features

All nine of the key features were mentioned explicitly by the teachers in the data sources that were coded, yet which ones were mentioned with what frequency differed by teacher. Ms. J's interviews were considerably longer than the other two participants (e.g. 52 minutes for Mr. G versus 120 minutes for Ms. J). Her longer interview meant she had substantially more mentions of the key features than the other two peers. To counterbalance the bias in this data, I utilized weighted averages to determine which key features were most prevalent across the three participants. Of the nine key features, two features were mentioned more than the others, that being the key feature of *using identity as data* (84/363 idea units, $\bar{x} = 27.9$), and the key feature of *use of articles, news, and media as content examples* (67/363 quotations coded, $\bar{x} = 22.3$). Two other key features were mentioned with relatively high frequency, that of *exploring data as relevant to self and the world* (53/363, $\bar{x} = 17.6$) and *exploring real, messy data* (48/363, $\bar{x} = 15.9$). There was generally less mention of the other five key features of *(DKF1) the use of a familiar teaching routine, (DKF4) viewing statistics as a process rather than a calculation, (DKF5) viewing data as truth or as a story, (DKF7) acknowledging and accessing multiple prior* *knowledge bases,* and *(DKF9) the use of technology tools.* See Table 4.1 for a summary of the prevalence of the key features mentioned across all teachers in the post-interviews, reflections, and teacher notes.

Table 4.1 ***Prevalence of Design Key Features***

Design Key Features	**Number of Quotations N=363 (%)**	**Weighted Average, $\bar{x}$**
DKF 1: Familiar Routine	16 (4.4%)	5.328
DKF2: Using Identity as Data	84 (23.1%)	27.972
DKF3: Exploring data as relevant to self and the world	53 (14.6%)	17.649
DKF4: Statistics as a process rather than a calculation	27 (7.4%)	8.991
DKF5: Data as Truth/Story	22 (6.1%)	7.326
DKF6: Use of technology, articles, news as sources for learning statistical content	67 (18.5%)	22.311
DKF7: Acknowledging and Accessing Multiple Knowledge Bases	16 (4.4%)	5.328
DKF8: Exploring Real, Messy Data	48 (13.2%)	15.984
DKF9: Use of technology tools	30 (8.3%)	6.66

Looking at the prevalence of the design key features mentioned by the teachers allowed me to understand which key features resonated with the teachers, and to determine which key features to focus on to look for distinctions between the ways that teachers discussed them. It is not surprising to see that *using identity as data* and *using technology, scientific articles, news,* *and media as sources for learning statistical content* were the most prevalent key features discussed as these were the focal points of the CSL materials and its alignment to the standards. It is also not surprising that *exploring real, messy data* was prevalent amongst the teachers' discussions, as this was the focal point of the unit. It is somewhat surprising to see teachers mention with relatively high frequency the attention to *exploring data as relevant to self and the world* as something that resonated in their discussion, as it was not central to the standards of the course or the focus of the unit.

The prevalence of each key feature varied by individual teacher and aligned with the criticality of the design key features. Briefly, I revisit the criticality of the different design key features. In Table 4.1, DKF 2, 3, and 6 are highlighted in dark blue to demonstrate that as the designer, there was an intentional incorporation of a layer of criticality within the elements of the lessons that included these design key features. While DKF6 is not necessarily critical outside of the context of this study, as the designer, I intentionally selected news articles, scientific articles, and data sources that focused on systemic injustices (e.g., COVID-19 cases by race as tied to health care), as defined by this study (See p. 20, and 84 for definition of criticality and critical statistical literacy used in this study). DKF 5 & 8 are both colored lightly blue, as the lens of criticality for these features was dependent on the teacher's enactment of the designed lessons and the students' selection of data within those designed lessons. One finding that is evident in the table is that the teachers collectively focused their discussion on the three design key features that were specifically interlaced with a lens of criticality, that is DKF 2, 3, and 6, as these were the design key features with the highest total frequency and weighted average throughout the discussions. Additionally, Ms. H & Ms. J each discussed these three critical design key features more than the other 6. In comparison, Mr. G discussed DKF8, *exploring real, messy data,* with the most frequency, which again, the level of criticality of this key feature was dependent on the enactment of the teacher and the students' selection. His second and third most prevalent discussed key features were DKF 2 & 6, which were intentionally designed to incorporate a lens of criticality into the lesson designs. Mr. G's fourth most prevalent key feature was DKF 4, *viewing statistics as a process rather than a calculation,* and Ms. H's was DKF9, *the use of technology tools.* See Table 4.2 for a complete summary of the prevalence of the key features mentioned, separated by teacher participants.

Table 4.2 ***Prevalence of Design Key Features Noticed by Teachers***

Design Key Features	**Mr. G N=131**	**Ms. H N=97**	**Ms. J N=135**	**Weighted Average**
DKF 1: Familiar Routine	7 (5.3%)	1 (1.0%)	8 (5.9%)	5.328
DKF2: Using Identity as Data	24 (18.3%)	17 (17.5%)	43 (31.9%)	27.972
DKF3: Exploring data as relevant to self and the world	15 (11.5%)	14 (14.4%)	24 (17.8%)	17.649
DKF4: Statistics as a process rather than a calculation	16 (12.2%)	9 (9.3%)	2 (1.5%)	8.991
DKF5: Data as Truth/Story	6 (4.6%)	7 (7.2%)	9 (6.7%)	7.326
DKF6: Use of technology, articles, news as sources for learning statistical content	21 (16.0%)	21 (21.7%)	25 (18.5%)	22.311
DKF7: Acknowledging and Accessing Multiple Knowledge Bases	3 (2.3%)	5 (5.2%)	8 (5.9%)	5.328
DKF8: Exploring Real, Messy Data	27 (20.6%)	11 (11.3%)	10 (7.4%)	15.984

DKF9: Use of technology tools	2 (9.2%)	12 (12.4%)	6 (4.4%)	6.66

In the sections that follow, I focus on the four most prevalent key features discussed by teachers, as denoted by their weighted averages, and describe what the teachers noticed regarding each feature, drawing from their post interviews, teacher notes, and daily reflections. Next, I provide a cross-case comparison of Mr. G, Ms. H, and Ms. J. The sections are organized by order of overall prevalence based on Table 4.1. Below, I have organized the four most prevalent key features and their subthemes (See Table 4.3).

Table 4.3 Most Prevalent Key Features & Subthemes

Design Key Feature	Subthemes
DKF2: Identity as a Data Source	• Teacher learning about their students' identities • Meta-analysis of student identity data
DKF6: Scientific & newspaper articles, and social media as sources for learning statistical content	• Development of healthy skepticism • Student awareness of current events
DKF3: Exploring data relevant to self and the world.	• (Mis)representation • Prompting learning about others • Teacher learning
DKF8: Exploring Real, Messy Data	• Exploring in math class • Losing control • Personal, nontrivial, non-contrived data sets

Design Key Feature: Using identity as data

Using identity as data was the most prevalent key feature based on the number of ideas coded (N=84). This prevalence varied by individual teacher. Ms. J highlighted this key feature in 31.9% of the ideas mentioned, making it her most frequently discussed topic, while it was the second most discussed topic for both Mr. G (18.3%) and Ms. H (17.5%). As each teacher discussed using identity as data, I listened for which specific social identities they mentioned. While social identity is a fluid construct that cannot be segmented into compartmentalized components, in this instance I coded for the individual reference of every specific social identity marker (e.g., pertaining to race, gender, sexual orientation, class, age). As an example, Mr. G shared how his students chose and then explored data sets in Lesson 6. Mr. G stated:

> The girl that I said was Asian American, she took the COVID data, and kind of, I guess, looked more into it, but specifically, more to see how it affected Asian Americans than looking at all the different races and ethnicities.

In this example, Mr. G discussed the social identities of race and gender. Each of those account for one instance below in Table 4.4.

There were also several instances in which teachers mentioned other types of social identities, such as whether a student was an athlete, medical status, or whether they played video games. As an example, Ms. J discussed the identity of one student who played video games. Ms. J stated, "Like I had a kid, that's a gamer, so I guess, and that's part of his social identity; being a gamer, right?" Overall, Ms. J had 49 instances of specific social identities, with race (22) being the most frequent, while Mr. G had 18 mentions of a specific social identity, with race being the most frequent (9). Both Mr. G and Ms. J discussed a wide array of social identities that their students used as data sources. In comparison, Ms. H had 5 mentions of specific social identities, and mentioned age with the most frequency. The one instance in which she described someone using the social identity markers of race and gender was in describing herself as a model to her students in the way they should engage with researching articles about their social identities in Lesson 3 (*What Does Data Say About Me?).* Ms. H stated,

> I would share different things like that. I told them, of course, how I would search if I was going to Pew Research to find something that interests me, you know. Of course, mine was very high level. It was just like black women in STEM. And I said, because, you know, "obviously, I teach math, I have an engineering degree. So that's the stuff that just resonates with me." Just to see what kind of articles I would choose.

When discussing *using identity as data,* two themes arose from the data that point to the development of critical statistical literacy for the teachers. Specifically, teachers discussed learning about their students' identities in more depth, and there was a common practice of meta-analyzing their students' analysis of their social identities. In other words, teachers were often looking for patterns across their students' responses in regard to their social identities. In these next sections, I provide examples of each of these two themes and how they each relate to the teacher development of critical statistical literacy.

Table 4.4 ***Social identities discussed by each teacher***

Sources	Mr. G	Ms. H	Ms. J
Age	2	2	6
Race	9	1	22
Gender	1	1	8
Sexuality	1	0	3
Class	1	0	1
Religion	2	0	7
Citizenship Status	0	0	1
Physical, Emotional, Developmental (Dis)ability	0	0	2
Other	2	1	1

Total	**18**	**5**	**49**

Teacher learning about their students' identities. Over the course of our post-interviews, Mr. G, Ms. H, and Ms. J each shared moments in which their students' interactions with the designed lessons prompted moments of learning about their students that they did not know previously. Mr. G shared, "I mean you know… it kind of shows me…what they were interested in. I like that aspect of it". Ms. J shared more instances in which the students' sharing their social identities through the designed lessons allowed her to learn more about her students. Ms. J shared,

> I never read this until, I just made sure that they were doing stuff and I went through, but it wasn't until, you know, after class when I had to do the reflection for you that I went and looked at their responses. And also it was anonymized. So even if they you know, happen to have looked at, you know, something, whatever. Um, so I think, because it was it was, it was guided, but it wasn't directly. Right? So, I think that I felt far enough removed, that it was okay that I wasn't telling them what to do or what to put. And also, I think they felt comfortable, but that anything was okay. You know, because we have some fear of some Christian stuff, we have some Muslim stuff, we have some atheist stuff, you know, we have bisexual, you know, we have... And that was the other thing, um, that was very enlightening to me that not only did we have this whole realm of identities, if you would, but that they were willing to express that.

Ms. J went on to share a more in-depth detail in which she learned more about a student than she had previously. She discussed how one of her students participated in the class virtually, and a parent reached out to let her know that the student was enjoying being a member of her class. In

Lesson 3 (*What Does Data Say About Me?)*, when the student shared their social identity wheel with Ms. J, she shared what she learned saying,

> She loves your class. And a couple of people have said that, and I'm like, I've never seen this kid and my class is virtual, and I'm just like, wow, that's crazy. But, um, and, you know, so I didn't, I never knew that she was, you know, Asian American. I never knew any of that. The only thing I knew was that, you know, her dad said that she was having some anxiety because of being virtual and because everything that was happening. But then when I saw this, that added another tier to that, I was like, Wow, that's a part of it as well. You know what I'm saying?

Ms. J shared here that she did not know the race of the student prior to the student's engagement with Lesson 3 (*What Does Data Say About Me?)*, and through the student's interactions with the lesson, the teacher learned about the student's racial social identity marker.

Ms. H shared similar results. In one part of the post-interview, Ms. H shared about a student who researched articles regarding teen abortion. Ms. H shared,

> And I can't remember which student this was, but I had one that dealt with the situation dealing with abortion and he was a male. And so, he had a really, really strong opinion about that, because of somebody close to him. So that was interesting, too, that again, you know how I told you they were sharing things that I, you know, I was just interested and mind blowing that they felt comfortable enough to tell me about something like that. But yeah, cuz he had this really strong opinion about it. And then I don't know if this was the student, because he went on to write about it a little bit later. But, um, he was on the end, and, you know, didn't favor [abortion]. So, he didn't see himself in this statistic.

Here, Ms. H shared how a student openly expressed their opinion about abortion by interacting with the designed lessons. Ms. H went on to say,

> And, and it also, I guess, it also made them feel, like, more open to share with me certain things that they probably would have never shared. Like, so now, also, I think it might have helped to, you know, even build a real, you know, a closer relationship with me as the teacher. So, you know, as teachers, we always want to try to, you know, build a relationship with our students, and just kind of understanding where they're coming from, and their background. And so, that gave them an opportunity to actually do that, so not only did it help with, you know, the actual lessons itself, but it just created this trusting relationship as well. And, you know, the students that really, really participated, I believe, had a better appreciation for me as their teacher, and just advice and things.

The findings indicate that the lessons were educative for the participating teachers, not only in terms of statistical content knowledge or critical content knowledge, but also in the realm of knowledge of their students. Mr. G, Ms. H, and Ms. J all provided explicit examples in which they shared that they learned something about their students that they did not know prior to their enactment of the designed lessons.

Additionally, both Ms. H and Ms. J expressed how viewing their students as individuals with intersecting identities provided more introspection for them as a teacher. Ms. J more specifically addressed this claim by sharing, “Because I pride myself on being able to get to know and to identify with my students. And I think this has helped me to realize, like we've said before, how many tiers and how many levels there are to them.” Similarly, Ms. H shared, “…so it, you know, created again, a whole I mean, you know, a whole nother instead of just being a math teacher, it's like, Hey, this is a teacher who cares, you know, saying so, you know, who cares about our opinion, that was just like this lesson for this unit period was interesting to see it kind of foster and strengthen the relationships between me and my students, so. Not only was it statistics, but it was "hey, you know, it's a chance to be myself, and have the teacher there who's not going to judge me but listen to me and understand me”. Both teachers shared how the learning opportunity through the engagement with the lessons provided opportunity for the deepening of relationships with their students by furthering their view of them as individuals with multiple identities.

Meta-analysis of student identity data. In their post interview, both Mr. G and Ms. J shared patterns that they noticed in the ways that their students used their identity as data within the enacted lessons. Mr. G was sharing about the students going to search for articles that aligned with the social identities they described for themselves within the Social Identity Wheel (Lesson 3: *What Does Data Say About Me?*). Mr. G shared,

> And so, like I noticed, so this girl Ann Howard, she, and she talked, I guess she's like Asian American. So, this seemed like, like the stuff were kind of analyzing things that had to do with race, it seemed like she was really interested in it because she was kind of seeing where she fit in to the data. And then the I sent this, because I thought it was interesting that, so we have Ann Howard who's a, she's an Asian American. And then Elijah is a black male. And then there was like three other white students. So, I thought it was interesting that they like researched stuff about their race or ethnicity, where the white students didn't. Like none of the statistics or articles that they looked up had anything to do with their race or ethnicity. So that is interesting. Yeah, I remember thinking that was kind of interesting when I was looking through that. So that's why I snipped that...They all, like the three other students that I was talking about, they all had

> like some article that talked about millennials. Like they were different articles where…like this one was millennials living with their family. This was marriage rates. That was that it was something specifically about millennials, even though I guess technically they're not millennials.

When asked about this observation, Mr. G extended with, "I don't know. I think probably, I mean, probably them being white, they probably don't, I mean, they just, it's not really, I guess talked about or focused on as much." Here, Mr. G was able to analyze the patterns he viewed on students' social identities and interpret the reason for the lack of racial identification due to their whiteness and their lack of attention on their racial identity as being central to how society viewed them.

As she looked over the data in the same lesson, Ms. J looked at her roster of students, and shared,

> And in the slides, that whichever case that was whichever lesson that was where you asked about how they most identify and what they think they're most perceived as, that that spoke to that directly, the ones that chose age are going to be your white students, and the ones that chose race are going to be your children of color.

Mr. G and Ms. J both noticed patterns regarding the social identity of race and the way it influenced their students' choices of articles they searched for. In both classes, the teachers noticed that the white students gravitated towards the social identity of age, where their students of color focused on the social identity of race in their research of data. In the case of Mr. G, he noticed that the students researched articles about millennials, "even though I guess technically they're not millennials".

Ms. J went on to share a story about a student in her class and a discussion that occurred among the whole class. Ms. J shared,

> So, after the fact, Aiden and I had a conversation, Aiden is a white male. And so, when we were doing this, he was like, "Well, what am I supposed to do?" And it was so funny, because a couple of kids were like, "Everything is about you, Aiden!", like, not being facetious, but like, it's easy to find stuff. And he and I had a conversation because in that class, I only have three kids that come in that class: Aiden, whose a white male, Natasha who is an African American female, and Alejandro, who is Asian or Hispanic…and male. Oh, so we can have candid conversations. But anyway, oh, Aiden was like, "[Ms. J], it really was hard for me to find articles." He's like, "I really don't think about how I identify." And I was like, interesting. Wow.

Describing the patterns that occurred within students' social identities did not only occur in reflection for Ms. J, but also with her students in her classroom. Ms. J extended her analysis to share that she believed her white students gravitated to the social identity of age because they struggled to identify racially, as a result of not thinking about it often and not recognizing their whiteness as race. For her, the incorporation of the using identity as data provided insight into the experiences of her white students and her students of color. The different ways that these two subgroups of her students discussed their social identities mattered for her, especially as she recalled the experience of a white, female math teacher colleague in a professional development. She saw the incorporation of identity as data as an opportunity for her to learn about her students' identities, and a way for her to help her students learn about their racial identity, specifically.

These findings suggest that by engaging in the act of meta-analysis of their students' use of identity as data, teachers were prompted to conduct their own critical statistical analysis as they looked for patterns amidst their students' research and findings. In short, the CSL lessons that used identity as data prompted the teachers to develop their critical statistical literacy. Mr. G demonstrates this learning in a scenario in which he is sharing what a student wrote as they engaged with the lesson, but then expands on his own idea on how the social identity of race and statistics intertwine.

> I thought this student's response was kind of compelling, talking about race and ethnicity. This student put that, "race is not a conclusive thing and it's not like, conclusive, and fluid, basically it's just kind of describing it. Some people just don't fit into a specific box when it comes to race, or ethnicity." So, I thought that was an interesting comment from that student. When we come down to statistics it's helpful or, everything seems to come out more kind of cut and dry if we can put people in really specific boxes. But when things come to race, and kind of how we describe people, people don't always fit into nice, neat boxes like we would want in a math problem.

Here, Mr. G recognized the tension that exists between using statistics to bifurcate people based on a social identity that does not fit into "nice, neat boxes". This recognition came through the student work sample. In this instance, the student's admonition that statistics can be used to harm certain groups, especially racialized groups of people, prompted an opportunity for the teacher to engage in the learning of critical statistics. Critical statistics that use variables such as race can be both helpful in assisting with the recognition of instances of oppression, and harmful as it dehumanizes individuals through the slicing of their identity into compartmentalized components that are not the realistic manifestation of the person as a human being.

Design Key Feature: Scientific, newspaper articles, and social media as sources for learning statistical content

All three teachers discussed the key feature of using *scientific & newspaper articles and social media as sources for learning statistical content* in their discussion, and it was the second most prevalent feature mentioned (71 out of 341 instances, 20.8%), and was consistently the first or second most key feature discussed across the three teacher participants (Mr. G - Second, Ms. H - First, Ms. J - Second). Looking first at the content of the idea units coded, the teachers were often discussing the content of a specific source (e.g., Pew Research, COVID-19 graphic, Tweet from Twitter, etc.) or activity within the designed lessons (e.g., Lesson 2: comparing scientific articles and newspaper headlines). This was true for 40 out of the 71 instances coded for DKF6. See Table 4.5 for a summary of the sources discussed by teachers.

Table 4.5 ***Sources discussed by each teacher***

Sources	Mr. G	Ms. H	Ms. J	**Total**
Scientific articles	7	5	7	**19**
Newspaper articles & headlines	15	5	11	**31**
Twitter content	2	1	1	**4**
Pew Research	1	10	5	**16**
COVID-19 nonstandard visualization	9	5	11	**25**

(*Note: Teachers often discussed multiple sources within one idea unit, therefore the codes are not mutually exclusive and do not sum to the total number of ideas coded (N=71))

Comparing the discussion of the teachers of the designed lessons in regard to key feature 6, the teachers consistently discussed the three ideas of the development of healthy skepticism in their students and themselves, the incorporation of real, relevant and current material, and how the designed lessons prompted them to notice their students' awareness of current events and the state of the world. In these next sections, I share more about each of these more commonly discussed themes and how they relate to the development of critical statistical literacy.

Development of healthy skepticism. Within the idea units coded for DKF6, teachers discussed how the use of scientific & newspaper articles and social media as sources for learning statistics content prompted the development of healthy skepticism in their students. In describing a discussion that occurred with the students who present for face-to-face instruction, Mr. G shared,

> I feel like with some of my face to face students, I remember them having…some discussion about…how… people may not necessarily read headlines of articles, but I guess like the clickbait type of thing, like YouTube videos or things along those lines, where it's the title or basically,...the headline isn't always lined up with the content of the video, like people just put titles on it, just to get you to click on it.

Similarly, Ms. J shared about her students' reactions to the sources included in the designed lessons. Ms. J stated,

> That just the some of the reactions from the students was just that it's kind of hard to trust headlines and think, especially their, I don't know how often their age is on Facebook, whether they use Facebook or not, but it seems like they should be pretty good at reading, like fake news by now….I think that through the process and through these lessons, I think they're definitely starting to get a better understanding of, you know, why it's important to be skeptical of things they see in the media. It seems like I'm starting to hear that a little bit more in their conversations and when they were doing their own research.

The data also suggest that this development of healthy skepticism was not limited to their students, but also occurred within themselves. In her reflection on Lesson 2 *(Being Skeptical)*, Ms. J shared,

> It made me think about how people simply read a headline and "run with the story." I loved the way that the 3 headlines were all associated with the same article…. It made me feel skeptical of headlines and articles I read and graphics that I observe.

The evidence indicates that the key feature of incorporating sources from scientific and news articles and social media was not only generative for the development of healthy skepticism in students, but also for the educators.

Incorporation of real, relevant, and current material. In reference to the use of scientific and newspaper articles as sources for learning statistics content, teachers often mentioned how the sources were real, relevant, and current. In her notes on Lesson 2 *(Being Skeptical)*, Ms. J shared, “I appreciated how the lesson was grounded in current, real topics. My perception: Current events come to math class and stay for a while.” With a lens towards equity, we have to ask to whom these sources were real, relevant and current. Teachers consistently indicated that the sources incorporated were real, relevant, and current to themselves and their students. Ms. J shared, “I think the students (and myself as a student) perceived the tasks as interesting because of the real connections being made” (parenthetical citation included in original).

There is a plethora of literature that supports the inclusion of real world topics in math class (e.g., Frankenstein, 1998; Gainsburg, 2008; Goldin, 2008), and things that are relevant and motivating to students (Garfield & Ben-Zvi, 2008), that are also current topics of interest (Klein, 1993; Wilson & Journell, 2011; Watson & Callingham, 2003), but the findings here suggest that the nexus of these three areas supports the opportunity for learning for teachers as critical statistics learners and educators. However, finding real, relevant, and current material requires a constant, iterative search for content, which was a tension that immediately presented itself. Similarly, Ms. H shared,

> Because, I don't know, to me, if we have something that's current, I think it's more engaging. And with this being, you know, COVID being the latest thing and keeping them out of school. You know, it was a big topic. There's lots of things that came out there. So just wondering where it would go from here. But yeah, we could still look at COVID and then see what happens, maybe post COVID…overall, I just like being able to use current and relevant material.

What is current now will soon expire. Since all current materials have an expiration date, this causes the need for teachers to constantly search for new materials that replace ones that have become obsolete.

There is also evidence that the use of scientific & newspaper articles, and social media as sources being real, relevant, and current prompted the learning of critical statistics thinking for teachers. Mr. G discussed the realization that the context that students will be using math content outside of the classroom is a political space and bringing that political space into the classroom felt less foreign when realizing what he was preparing them to be able to do and experience. Mr. G stated,

> I mean, to me, the kind of the cool part about the lesson is, the lessons throughout the unit is… them being able to use math…outside of the classroom…for them to be able to read an article …or look at the graphs…that are in the article and be able to…use something that they actually learned in math class to kind of make sense of what politics is talking

> about, or what the news and media we're talking about. So that's, for me, the kind of cool part about the unit.

Mr. G shares his acknowledgement that we are not preparing students to use mathematics in a benign vacuum that is disjoint from the polarized reality of the systems of the world. Thus, it is imperative that the “math outside the classroom” be invited to take up space and be intertwined with the math inside the classroom. This development of criticality in Mr. G’s consciousness is an example of how the realness, relevance, and timeliness of the scientific & news articles and social media sources incorporated into the designed lessons prompts the development of critical statistical literacy.

Student awareness of current events. When discussing the use of varied media sources within the designed lessons, Ms. J consistently spoke to her recognition of the students’ awareness of current issues and events. In one example, Ms. J was quoting a student’s response to a question regarding a nonstandard visualization for a graphic displaying COVID-19 data. She shared,

> There's usually a pattern because of things happening on the outside. So, I think cases rose in July and are steady going down. I think this is because of everyone doing their part.” So that you know, things like that made me think, okay, they're listening, they're paying attention to what's happening in the world.

In another instance, Ms. J shared about a student who was answering questions about news and scientific articles found on Pew Research. Ms. J shared,

> Oh, look at it, look at this one with Erin right here. For the report that was representative for me, I noticed it talked about things my parents were worried about. Like, you know, it's just like, wow, you know. For kids to know that their parents are worried and then to

> have the wherewithal to make the connection between what their parents are worried about and what was in this article, and then to share that…You start to see, you know, how…they're bringing the statistics, which they learned through the critical portion, which is the news and what's happening and relating it to their interpersonal life. So, it's like full circle, that they're doing it and I'm just like, oh, wow, this is, this is really something.

Ms. J was able to see and name the elements and purpose of the design: for the students to be able to experience statistics through the use of real, relevant, and current sourced data in order to develop their critical statistical literacy and think about their world. In naming this out loud, she recognized that this opportunity for student learning prompted something within herself as an educator. For her, this became the purpose of learning statistics.

Design Key Feature: Exploring data relevant to self and the world.

The third most prevalent key feature mentioned by teachers was that of *exploring data relevant to self and the world.* In coding idea units, there were several examples of *exploring data relevant to self and the world* that occurred when teachers discussed the concept of representation of a population. This is similar to the work for Konold and colleagues (2004) and the three perspectives students take to viewing data as aggregate: data as pointers, case values, and classifiers, and to the work of Rezvi and colleagues (2020), Gutiérrez (2013), Secada (1989) that focuses on providing students with windows and mirrors as sources of identification in math learning. Here, I use the language of interpolation to discuss aggregating oneself within the data (data as classifiers and mirror) and extrapolation (data as aggregate and window). As the third most prevalent key feature, teachers explicitly referred to *exploring data relevant to self and the world* in 53 of the 363 idea units coded (14.6%). Transversely, the three educators spoke to how the CSL lessons encouraged the discussion of the topic of mis/representation, prompting students' desire to learn about others, and provided an opportunity for teacher learning. In these next sections, I share examples of each of these topics and how they support the learning of critical statistical learning in the teachers.

Mis/representation. The purpose of incorporating the key feature *exploring data relevant to self and the world* into the designed lessons was to encourage students to recognize themselves as a data point within a larger data set, to wrestle with the concept of mis/representation through statistics both individually and as a window to view others, and to think about how intentional sampling can be used to harm a population, specifically populations that have been marginalized. The concept of mis/representation was a common topic discussed by the teachers when coding for this key feature. Out of the 53 idea units that discussed the *exploring data relevant to self and the world* key feature, 34 of them addressed the idea of mis/representation. In sharing some of their students' responses on how it feels to be misrepresented by a statistic, teachers shared dynamic viewpoints. Ms. J quoted one student, sharing,

> "It feels bad. I want to be properly represented in a statistic. Yeah, it's important that the correct information goes out. And if your survey is bad, it can harm a group." That was a good point.

In comparison, Mr. G shared a student's response to the same question, stating,

> I thought one of the student's answers to this question was kind of interesting. It says, "How does it feel to be misrepresented?", and their answer to the question was, that they were only one person, and that they didn't really feel like that if they were misrepresented that it was that big of a deal, because it was coming from such a large group of people.

> So, to me, it kind of understood that some people are going to be misrepresented. And then, in other words, the sample is not always going to represent the whole population. And there's a kind of error in that.

Mr. G shared an example in which the idea of the omnipresence of variance exists as the student recognized that there would be and should be some error within a data set, which will lead to misrepresentation of the individual data points. While personal, the idea of misrepresentation here is used to introduce the statistical concept of error and variance. Ms. J chose to highlight an example in which the misrepresentation can cause harm to the larger population, attending to the critical nature of statistics. Both of these contrasting views are important narratives that were prompted by the key feature of *exploring data relevant to self and the world.*

Prompting learning about others. An additional theme teachers discussed regarding DKF3 was that it provided a window for students to think about individuals and groups of people that were not like them. Teachers highlighted different scenarios that extended from students like them, to students they knew, and to whole ethnic groups of people. In sharing about Lesson 3 *(What Does Data Say About Me?*), Ms. H discussed a student's process for the topics they were researching. Ms. H shared,

> I had one student who…looked at articles about teens…and how depression is high in that demographic. And it was just like...this is your group, your peers…I don't think she was part of that statistic. But she knew of…friends or other people that fit that, you know, description, and it was in her age group. And it was just interesting to hear that student's point of view. And like, yeah, I can see that or I, you know, it doesn't describe me in particular, but I do have friends, or I know of people and other things like that.

Similarly, Mr. G shared an example of a student who looked at data for their unit project on incarcerations based on over policing. He shared,

> And then I had the one student who was looking at the incarceration data. I feel like he said he identified with it because he was interested. He just put, "because it was a problem...that they were over policed or underfunded…He said, I see myself in the data because I know friends that could be imprisoned for something small that wouldn't happen if they had better opportunities.

Ms. J expanded on why she thought the designed lessons prompted this kind of student thinking. She stated,

> But even with that, I think that sometimes students…feel comfortable with people, and sometimes they don't. But I think with it being in this medium, that it was just an opportunity for them to express themselves, because while you're talking about yourself, you're not really talking about yourself. You know what I'm saying? So, um, and it's also a learning experience, you know, that you're learning about other people like you… I think this whole thing about different groups of people comes into play more. They think about it more so after that social identity, after they've seen those different pieces, that broadens their horizon to think about more people more often.

For all three of the teachers, *exploring data relevant to self and the world* provided students an opportunity to think about others like them because "you're not really talking about yourself…you're learning about other people like you". The findings suggest that the difference between using yourself as a data point and moving to thinking about others like you as a data point resonated with teachers. Ms. H also shared how she thought the *exploring data relevant to self and the world* key feature led students to have more empathy for those not like them, including herself as their teacher. Ms. J's expansion also suggests that there is an interaction occurring between DKF2 (*using identity as data*) and DKF3 (*exploring data relevant to self and the world*).

Teacher learning. As has been similar across the other design key features, teachers also shared moments in which they themselves engaged in the learning process through the specific key features. In Lesson 5 as the students were exploring the COVID data disaggregated by race, Ms. J shared how a student's response prompted her to think about the purpose behind disaggregating. She shared,

> And Tina says this over and over, "It shouldn't be by race at all, only by how many people are in each state". And she said that. It was a theme it seemed with her. And it was interesting, because I started to think, Well, you know what. I know why they're disaggregating the data that way. But, um, it was just interesting.

Teachers also shared examples of how students were able to move outside of themselves as data points to hold a more global view of data. In an example from Lesson 2 *(Being Skeptical)*in which the students were thinking about the different types of studies that could be conducted to gather information from a population of interest. Ms. J shared,

> That's something else that she mentioned before about lack of access. She mentioned that also when we were looking at, you know, the data when it was done by racial group and those sorts of things. That was something that came up as well that people in certain areas or people in certain ethnic groups may or may not have as much internet access. So, when you're taking these polls, your numbers may or may not be as accurate. And some of them also said that may have accounted for some of the holes in the data. Access. Yeah. And again, I'm looking at them knowing about access.

For Ms. J, the designed lessons presented an opportunity for her students to demonstrate their understanding of the systemic barriers due to the lack of a common utility, internet access. It was more than just the student's view, or people with a similar identity to the student. It was an issue that was raised about an "entire ethnic group". Ms. J shares her surprise when she says, "I'm looking at them knowing about access", demonstrating her knowledge of critical statistical literacy as she points out that the concept of access is bigger than one individual self, but a more global, world issue that affects entire populations.

Moving further, Ms. J also shared an example of when she applied the learning from the designed lessons to lead a student through a learning experience based on his race and the way he discussed sampling at his school. Ms. J discussed a scenario that occurred a few days after Lesson 4, after students discussed the concepts of sampling, misrepresentation, and the global perspective of data. Ms. J shared,

> And a couple of days later...he said something about, "I don't really like the white kids at this school." And I said, "Aiden, you cannot make blanket statements about whole groups of people". And he was like, "well, didn't say all white people. I said, just the white people at the school". I said, "Well, that would include you". And then he said, "Well, I mean, the rich white kids", and I was like, "so you narrow it. Now you're starting to narrow it down", I said, "But realistically, you're only talking about certain people". So, you can't say 'all of the white kids at the school'". But anyway, so…it was interesting when he was, you know, we were doing this because I never thought about it like that.

Here, Ms. J demonstrates an application of the learning of critical statistical literacy. In sharing that lessons prompted her to think about the way data is disaggregated and for what purpose, specifically in a racialized context, she shared how *exploring data relevant to self and* *the world* provided an opportunity for her to think globally about the concept of race and sampling. Afterwards, she led a student through the same learning experience she had herself, helping the student see themselves within a dataset, using the fact that the student was a data point within the data set to help them disaggregate the data, and finally describe the sample of students they were discussing with more precision. In this scenario, the findings indicate that the key feature of *exploring data relevant to self and the world* not only prompted the teacher to develop critical statistical literacy, but also impacted the development of critical statistical literacy in their students by sharing their own learning.

Design Key Feature: Exploring real, messy data

Discussing *exploring real, messy data*, teachers made reference to this key feature 48 out of 363 (13.2%) idea units coded, making it the fourth most prevalent feature mentioned. Looking across the teachers' experiences with the designed lessons, when discussing *exploring real, messy data,* teachers commonly shared the idea that exploring and researching in math class felt important and different than in their past statistics class experiences, that there was a difference in the nontrivial, noncontrived data sets used in the lessons, and there was a common tension between student autonomy to choose data and teachers feeling a loss of control in students' freedom of choice.

Exploring in math class. Aligning with the title of the unit, one of the key features of the design was to have students explore data and research consistently throughout the unit. Teachers were hyper-aware of the amount of exploring that students were asked to do within the lessons. In thinking about the unit overall, Ms. H shared,

> It's an opportunity for them to really think about themselves, and then being able to like, actually, research in math class, you know, do something like that. It was different. And

now, you know, I think it gave them a different perspective of statistics, and how they can look at them, and see how they fit in the overall picture.

Similarly, reflecting across the unit, Mr. G shared his perspective on how exploring data was a valuable experience for student learning. He shared,

> I'll definitely like this kind of ending the lesson and then being able to kind of have time set aside to explore their own data and come up with their own statistical question and kind of go through this process themselves. I think that it seems like that'd be very helpful from a student's perspective, to be able to, instead of just saying, "Here's a data set. We're going to go through it", that they actually have to go through the process of collecting data on something that they're interested in, and then kind of seeing where that data leads them. And telling the story of that data, and going through that process on their own.

When coding for the sub-theme of exploring and researching in math class, this code consistently ha a co-occurrence with DKF1, or teachers sharing that statistics should be viewed as s a process rather than a calculation. In the previous quotes, both teachers mention the process of exploring the data set and how it "gave [students] a different perspective on statistics". Ms. H shared more in depth how exploring in class felt different than her previous statistics classes, by saying:

> Before I did a lot more with calculations and stuff like that, you know, with the Five Number Summary. But now it's like, Okay…you want to go to that next level, which I think this takes it to a whole 'nother level as far as being able to analyze...But this was definitely a different approach. Because before I spent more time with calculating things versus exploring, you know, what I'm saying this one really does hit that exploring piece, where you're looking at data and actually exploring it, you know, not just calculating…So

> before more of the calculation, this is taking it going into that conceptual and analyzing, and, you know, really exploring, whereas before, I don't feel like they were really…they looked at some things, but they didn't really dig into it or explore it the way that this did. And that's what the unit says, you know, exploratory data analysis. We need to be able to explore some things. And I really think that's why I had liked this, because I like them going out to, you know, Where do these statistics come from? You got Pew Research, you've got all these other sites and databases, where people are just collecting information all the time, right? And then they put these numbers out there for you.

Here, the findings indicate that having students explore real, messy data was correlated to understanding that statistics is a process, and more than just a set of calculations. For teachers, this demonstrated a shift in pedagogical knowledge of teaching statistics and placed value on exploration of real data with students. As was mentioned previously, the critical lens of this key feature was dependent on the data sets that teachers and students chose to explore. Therefore, this is an example in which the strengthening of the teachers' pedagogical knowledge for teaching statistics undergirded the opportunity for the development of critical statistical literacy. Opportunities for more explicit development of critical statistical literacy will be explored in the next two sections.

Losing control. In juxtaposition to the value of exploring data in math class, teachers also discussed the tension between allowing students to choose their own data, and teachers sensing a loss of control within the classroom. Ms. H expressed tension in students exploring data and not being able to see what students were doing. Ms. H shared,

> I think the hardest part was I couldn't see what some of them were doing. You know what I mean? And so, you know, I did have students asking questions, so I knew, you know, at

> least [the ones] that were remote. So, I knew they were doing it, but some of them. I just wasn't very comfortable with some of the other ones.

Mr. G shared how, due to the context of the pandemic and the flexibility of not having tests, he allowed himself, and therefore his students, more time to explore than he normally has in the past:

> Sometimes, especially with the statistics stuff, I just kind of like everything to be nice and clean. So those kind of little refreshing, especially this year, specifically, I didn't really have to worry about like a timeline of getting every topic in, so I have a few more days to kind of work with I wasn't like I wasn't like stressed about them spending two days on something instead of like worrying typically try to like just motor through it. So, I had a little bit more freedom. And I enjoyed just the different aspects of the lessons versus kind of how I normally teach those topics.

In sharing that he prefers the data to be "nice and clean", he suggests that real, messy data takes more time to explore. He later shared an example, stating,

> Finding data and being able to organize it is something that's just very time consuming. This was like the goal for day one, but a lot of my students were still doing this into day two and were kind of finally finishing it into day two. And that was something we want them to kind of struggle through and be able to do and understand that process on their own. But as a teacher it is definitely frustrating to see how long that process takes in class. Specifically, I had one student in class, who, his question is kind of similar to the COVID data in that, he was looking at incarceration data, and was looking at, or was interested in looking at incarcerations by race to see if there was any proportions that were disproportionate. But in doing that, it took him like a whole class period to transfer

> the data from the websites that he was getting the data from for each state. Because you want to look at all 50 states. And it just took him the whole class period just to input the data. So that was a great question, very good data to kind of look at. Again, just kind of frustrating as a teacher to see how long that process takes, but interesting.

Teachers also mentioned that allowing students to explore real, messy data is a process that takes time that teachers and students are often not afforded. Due to the restrictions of pacing as determined by testing, teachers often feel pressed when the process of exploration takes more time than they have allotted for in their pacing. Similarly, teachers may not trust their students to explore datasets of their own choosing. Both of these scenarios suggest that in allowing students to *explore real, messy data* of their choosing, that teachers felt a loss of control over time and actions of their students.

Exploring personal, nontrivial, noncontrived data sets. Within the idea units coded for *exploring real, messy data*, teachers also shared about student empowerment that occurred when they were given the autonomy to choose what data to explore, and how this varied from previous teaching scenarios in which students were given data sets chosen by the teacher. Ms. H shared how the research and datasets being personal to the students was a driver for engagement and learning. Ms. H shared,

> Them going out and looking at the Pew Research. And then I could see, this is where I tailored it some too you know, they had to go in and look at the actual reports that went behind that research. So, they could see what kinds of graphics were used, like, how many people were actually studied, and just where all the data was collected and gathered and then having to, you know, assess it and say, “Hey, okay, this is, you know,

interesting, this is what I found. Um, yeah, I kind of see myself in this data". So, I really, really felt like it was personal, you know, they could be personal with it.

Similarly, Ms. J shared how her students initially struggled to think of ideas for them to research and explore.

> …and so, at first it was a lot of, "I don't know what to do, I don't know, what to do, what about" and I was like, you need to choose something that matters to you, something that you care about. And then I even share with them that sometimes, you know, it's easier, someone gives you a topic, because you don't have to figure out one, but it's more interesting if you do something that matters to you. Um, and that definitely happened for the ones who did it. And really, delved into it, they did stuff that mattered to them. And that was relevant.

Both teachers shared how the data being personal engaged students in productive struggle. Because this experience felt "different" and more "personal", students initially struggled with the idea of finding their own topics and datasets to research, as these items are typically selected and chosen by the teacher. But in both scenarios, the teachers suggest that after the initial struggle of choosing a topic, the students engaged more because it "mattered to them".

From the meta-level of teacher learning, Mr. G reflected on past pedagogical choices in choosing datasets for students. Mr. G shared,

> So, I feel I mean, it's something I've always tried to do, but it's more of a, I don't want to use the word, like fake, but it's not as, like I am making it up for them. They're not having the choice to kind of go explore something that they're personally interested in…So yeah, it's more just like, like, typically when I try to pick stuff that relates to the math content that actually has some like real world application, it's more of like I'm saying, Here is this

> thing that we're going to do. And they just kind of view it as like, “Oh, this is just another math lesson”, not like, you actually get to pick something that you're interested in while you're learning about the math. To me, that's different.

Here, we see Mr. G wrestle with the fact that in past statistical experiences, students were not given the opportunity to choose data that mattered to them, resulting in what felt like “fake”, “real world applications”. This aligns with the research that discusses how canned real-world applications can often be devoid of the real world, in which data is real, messy, and something that students care about. For all three teachers, providing students the opportunity to choose data that was personal, nontrivial, and noncontrived was more “real” than previous real-world applications. Here, the development of critical statistical literacy for the teachers is less about the context of the data, and more about the pedagogical development of what occurred for the teachers. The teachers are sharing that the student empowerment that they experienced through the designed lessons was humanizing for both their students and themselves.

To further illustrate this, I share two robust examples shared by Ms. J and Mr. G in which the autonomy to choose one’s data set created student empowerment through the exploration of real, messy data.

Mr. G shared an example of a student who chose to research types of offenses in prisons across the state in which she takes residence. Mr. G shared her process,

> She looked at [state redacted] prisons and jails. She got the statistics by race. I'm pretty sure she got it from the [state redacted] governmental website, whatever website that was with them based on the proportions. So, this is what she observed. Like this is what the data actually said. And then based on the proportions of people who live in [state redacted], this is what the data should be if the proportions were the same, this is what

> should have happened…I was mainly just excited that she was able to use some math to talk about because like she, she knew she wanted to talk about this somehow….She definitely had more questions you know, like, "It'd be interesting to actually figure out like, why, like, what causes this?" And then I think she was…specifically focused on drug, like instances where they were incarcerated, because of some like drug use, or whatever something to do with drugs, somehow that website let her filter out certain things…That kind of led her to just explore some more into the data, the website we were looking at wasn't very user friendly. This like, kind of said that several times getting real data. And being able to actually look at real data is kind of difficult sometimes. So, it definitely was for that, but we were able to use some of it anyway.

Later in the interview, I asked Mr. G how it felt navigating this exploration with this student. He shared,

> Oh, well I think with anything, like, the more you like, find out about something, the more you realize that you don't know about certain things. I feel like that's how I am especially about like, I mean, I personally do not know, like, why that is, so I feel like I just I mean, I just don't know.

Mr. G shared how his student using statistics to explore "real" data and answer a "real" question mattered to her and to him. In exploring, the student and the teacher are led to the question of, "Why are the racial proportions of individuals within incarcerated institutions not aligned with the racial proportions within a state?". Through the exploration, the student is led to additional questions that spur more exploration and research to answer this new question. In relaying her process, the teacher shared his excitement but also tension in using statistics to confirm an institutional structure that was causing harm to a minoritized group of people. While the act of finding something statistically left the student and teacher feeling empowered and "excited", they are both left with the tension of how to act on the information they found.

Similarly, Ms. J shared a story of a student who

> So, Tamara has AP art. And so, for art, they had to do like this whole project thing...So when this came up, she's like "I don't know what to do", I was like, "Tamara, don't recreate the wheel". And so, she's like, "I can do something with hair?". I was like, yeah, yeah, yeah! So, this is her, this is her topic: Black Girls Relax from Relaxers". And when I opened it, I was like, Oh, no she didn't! "Relax from relaxers". So, um yeah, I thought that was cute. Um, so in recent years, how much more have black women transitioned to natural hair rather than relaxed. And then why she chose the topic: Because I have natural hair. But I also transitioned from relaxed hair to natural in 2018. I love my hair and care a lot about it. I enjoy styling it and watching it grow since I cut it all off. So, she did the big chop and you know, so then she did some research on hair chemicals and relaxers, some information on hair care products. Um, some statistics, she has a graph here, her story, and then her conclusion and then her reflection. More women prefer their natural hair over adding chemicals. Because as a Black person, I find that my naturally curly hair is an important part of my, what?!, identity. (Laughing with excitement) I was like, Oh my goodness! I see my story because I transitioned from relaxed to natural hair before. And so, I'm just like yes, yes, yes, yes, yes!

Ms. J helped Tamara navigate the choice of topics to research by reminding her that she was already exploring a topic in another class, AP art. By bridging this content divide for her student, the student is able to use what they have learned in another class to explore the statistics behind their work. In this case, the autonomy of choosing data that mattered to Tamara was empowering not only to the student, but also to the teacher. Ms. J was overwhelmed by her student's choice and the way she expressed her identity through the exploratory data project within the designed lessons.

In each of these cases, Mr. G and Ms. J shared how *exploring real, messy data* brought about findings for their students that were personal, nontrivial, and noncontrived. In the case of Mr. G, the students' choice to look at data that was unfamiliar to him led him to consider several questions. One question was statistical: how can statistics be used as a tool for the dismantling of structures of oppression, in this instance the institution of incarceration. A second question was that of action: what to do when the statistical findings support the existence of systems that support racial injustice. In the case of Ms. J, the teacher is led to the question, "What happens when students are allowed to bring their identity into the statistics learning experience?" In each case, the teachers' questioning demonstrates their development of critical statistical literacy.

CHAPTER V: TEACHER IDENTITY WORK AS CRITICAL STATISTICS EDUCATORS

Having analyzed the ways in which the teachers discussed the design key features, I now turn my focus to the identity work of the secondary math teachers made visible by the CSL materials. Because identities are in constant negotiation with the social, political, historical, and cultural settings around them *and* the resources afforded to individuals, this study seeks to understand how resources, such as the CSL materials in this study, can promote shifts in the identity work that teachers engage in as they navigate their identities-in-practice as critical statistics educators and how their identities-in-practice are connected to how they frame critical statistical literacy.

As Sfard and Prusak (2005) suggest, 'identities may be defined as collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable, and significant'' (p. 16). Therefore, looking at the shifts in discourse and narratives of teachers in pre- and post-enactment provides evidence that the resources available to the teachers made visible their ongoing identity work as critical statistics educators.

In this second phase of analysis, I used the existing data sources to explore moments in which identity work was made visible by the teachers as critical statistical educators. Looking specifically at the pre-and post-enactment materials, I compared moments in the teachers' two chronological narrative identities-in-practices, looking for moments of continuity and disjuncture between their pre- and post-enactment, which would serve as evidence for the identity work of the teachers prompted by the CSL materials. The first part of this chapter presents each teacher as a case, first by sharing an introduction based on their historicity shared in their pre-interviews, then their pre-enactment narrative identity-in-practice. Following these two sections, I share a post-enactment narrative identity-in-practice, which uses constant comparison between the two chronological moments in time to bring clarity to moments of disjuncture and continuity. After all three teachers have been analyzed, the second portion of this chapter provides a comparison across the three cases. In the following sections, I share findings in which I specifically explored the research question:

In what ways do the CSL materials make visible the identity work in the teachers as critical statistics educators?

My analysis showed that teacher historicity mattered for the identity work that was made visible by the teachers enacting the resources in regard to their identities-in-practice as critical statistics educators. The analysis also showed that the teachers' statistical content knowledge, statistical pedagogical knowledge, and critical knowledge also mattered in regard to the identity work that was made visible by the use of the resources. The dynamic relationship between the teachers' statistical knowledge for teaching and their historicity proved to be a powerful combination for making visible the identity work of the teachers as critical statistical educators occurring through the enactment of the resources.

To further illustrate the identity work that was made visible through the enactment of the resources, at the end of each case, I share the shifts made visible in each teacher's view of critical statistics as informed by the three sources of literacy defined in my conceptual framework: math literacy, statistical literacy, and critical literacy.

Teacher Identity Made Visible Through CSL Resources

In this next section, I present each teacher as a case to illustrate how resources can prompt identity work in teachers as critical statistics educators, and how each teacher's response to the design materials is uniquely based on their historicity and their statistical content knowledge, statistical knowledge for teaching, and critical content knowledge. Each case begins with an introduction to the teacher, then their pre-enactment narrated identities-in-practice are shared. This is followed by examples of continuity or disjuncture in their post-enactment narrated identities-in-practice. All quotes in the introduction and pre-enactment identity-in-practice come from the pre-interview or demographics questionnaire, while the post enactment identity-in-practice has quotes from both the pre-interview and post-interviews. Lastly, for each teacher, I discussed the changes in their overall narrated identities-in-practice in how they discuss math literacy, critical literacy, and statistical literacy in regard to becoming a critical statistics educator. At the end of the three cases, I conduct a cross-case analysis, in which I look more closely at the similarities and differences in the identity work made visible as critical statistics educators as it relates to the design of the resources.

Introducing Mr. G.

School and Historical Contexts. Mr. G worked in the same school for the entirety of his teaching career (N=10). His school served a culturally and racially diverse student population that was primarily working class and middle-income families in a semi-rural town, with a significant population of students whose families migrated for work. Due to his previous classes' success on end-of-course exams, Mr. G had been frequently tasked with teaching courses with intensified pressure and expectations associated with high-stakes testing that held consequences for the entirety of the school and community (e.g., School Grades). Like many teachers, Mr. G held many roles and responsibilities within his high school. Mr. G had worked simultaneously as both a coach and a high school mathematics teacher for ten years, since he began teaching. He coached football and baseball year-round, and his baseball team had just won a conference championship in the spring semester that the research was conducted.

In previous years, Mr. G had participated in professional development for a mathematics reform-oriented curriculum that focused on student exploration of mathematics topics and the simultaneous development of conceptual and procedural fluency. Mr. G mentioned that this professional development shifted his thinking and work as a teacher. Because of the success he has in the professional development, he moved his instruction from lecture-style and teacher-centered and began modeling all of his lessons on the methods learned in this week-long professional development, which used a Launch-Explore-Discuss model (Shroyer, 1984; Van de Walle et al., 2016) that allowed students to explore and build connections across mathematical concepts and procedures.

Pedagogical orientation. In this pandemic year, Mr. G was constantly changing his environment and pedagogical methods to attend to his students' needs, and amend his teaching methods to fit the next new learning environment. Because he was a teacher that held discussions in class and asked questions to his students, teaching in a virtual environment created a barrier to his teaching style and he described his disappointment with losing the student discussion that he had often associated with his teaching. In response to the pandemic, Mr. G chose to try to "flip the classroom", and provide students with recorded videos of him explaining concepts or modeling procedures to encourage participation, engagement, and student success. Mr. G knew that students' schedules were hectic and fluctuated due to the different home lives that each of his students had. Some students had quiet spaces to learn, while others had to help their siblings with their online schoolwork during the day before they could think about their own. While he did not prefer this method to face-to-face teaching and saw the virtual environment as a barrier to the discussion-based teaching he had become accustomed to, Mr. G chose this method of recording videos so that students would have the choice to watch the videos live or on their own whenever it made sense for them.

Recruitment. Simultaneously, Mr. G also chose to participate in a year-long professional learning community based on a new fourth-level math course that he was assigned to teach in both the fall and spring semester. This professional development focused on the development of critical statistical literacy in teachers and students as it related to the standards aligned to the new, fourth-level course in which half of the standards covered statistics content. Within the fourth level math course professional development, Mr. G articulated a strong belief in the need for students to develop critical statistical literacy and his desire to use some of the pedagogical practices discussed in the professional learning community. Within the professional learning community, the teachers engaged with some of the designed materials as a learner (specifically Lessons 1, 3, and 5) as well as other lessons with a critical focus that centered on other non-statistics standards within the new fourth level math course. The teachers were prompted to reflect on how those three lessons were different from those used in their past teaching experiences of statistics, and compare the lessons to the standards in the new fourth level math course. Mr. G was recruited as a candidate for this study because I, as the facilitator of the PD, mentioned that I had created an entire unit of designed materials focused on exploratory data analysis, which was a large portion of the new fourth level math course, and Mr. G said he would be interested in implementing them if they were similar to the ones we completed in the professional learning community.

Mr. G's Narrated Identity-in-Practice, Pre-Enactment.

View of Statistics. Mr. G's critical statistics educator identities-in-practice were encompassed in his identity as a mathematics teacher. For him, teaching statistics was a component of teaching mathematics, and therefore centered on the view of statistics as a set of mathematical calculations, rather than a process or understanding. In describing his previous statistics lessons, Mr. G shared,

> Last year, we measured the forearm length of each student in our class, and then also their height in inches. And we tested for correlation, I guess. Then we did other things like, test for like outliers. And we just went through all the statistics that we could think of through that task.

The way that Mr. G stated “all the statistics that we could think of” suggests a calculation-centered view of statistics focused on the “signals”, that is more procedural than conceptual (Konold & Pollatsek, 2004). Specifically, Mr. G predominantly views the field of statistics as a long checklist of specific calculations (e.g., mean, median, mode, outliers) that are not connected to answering a specific question, making inference based on data, or taking action based on the answer to the question.

View of Critical Statistics. When specifically asked about the role of statistics education, Mr. G narrated,

> For one, I mean, there's a math component, definitely to learn certain math components that go with statistics. But then also, bigger picture wise to be able to just understand basically things that are put out by the media as they get older, especially when we have years, like we've had the past year about, whether it's presidential elections, or COVID data will definitely be, I mean, some of those graphs I'll look at, I don't even understand as a math teacher. So, I can't imagine what a normal, everyday person who hates math thinks when they see certain graphs and tables.

Here, Mr. G attended to the role of statistics as providing a potentially beneficial learning space for students to learn about critical issues that occurred in the media, with both political and social contexts. Mr. G further explained how important it was for students to have examples that allow students to see beyond the classroom. Mr. G shared,

> I guess, the opportunity through, like, examples and things to, to have those learning experiences where they can take what they've learned…Because that's the biggest question I get..as a math teacher… "when am I ever going to use this?" So I think if we can incorporate some things that they can actually use after they graduate, then they'll be a little bit more apt to be able to actually want to complete the lesson.

Centering the learning of statistics on "real world" contexts satisfied, in Mr. G's mind, the student question of "When am I ever going to use this?". For Mr. G, the real-world context was an incentive for students to become engaged in their own learning, because the context was relevant and happened outside of school. While Mr. G talked about incorporating "real world" contexts into statistics that merged the "big picture" and the "math component", he was clear in the interview that this was not a part of his current teaching practice.

Later, Mr. G shared how easy it was for him and other teachers to get "caught in the trap" of attending to test questions due to the pressures of high stakes testing, rather than focusing on the learning opportunities necessary to produce productive citizens. He stated,

> Mainly as a teacher, sometimes we get kind of get caught in this trap of like, trying to get the things in and we know, eventually we're trying to lead them to be able to answer, like, some type of test question. Not actually like to be able to use it after they graduate.

For Mr. G, the purpose of teaching statistics was sometimes blurred between creating a meaningful educational experience that impacted students' future lives post-graduation with the faux goal of achieving high test scores on end of course exams.

When asked specifically about including topics in statistics teaching on systemic barriers such as race, gender, or class, Mr. G discussed how context within his teaching was based on his comfort, which was typically sports. Mr. G said,

> I'm definitely a person that avoids conflict in any way, especially trying to resolve that in my classroom. So I would definitely take things, assignments and tasks where students will be more likely to [talk about critical statistics topics] and just to avoid that in general…. I guess the task for me is more that I'm completing a math assignment, not something else. Typically, I fall into sports, like I pick stuff that has to do with sports, which might leave out people not interested in sports a lot of times.

Even though Mr. G discussed the need for students to experience statistics in context that allowed them to make sense of the world they would live in upon exiting high school, here he suggests that he avoids the topics of race, gender, and class because it detracted from the focus of the lesson being "a math assignment". Simultaneously, he defaults to a context of comfort with sports over his stated understanding of student need to engage with statistics of both political and social context. He was clear that this was a cognizant choice that centers on his discomfort of holding discussions with students on issues that could create conflict in the classroom. While Mr. G's classroom pedagogy on a discourse-centered instruction in which he orchestrates conversations with students around math topics, to Mr. G the previous skills learned in the math PD do not translate to being able to orchestrate discussions with students around statistics topics that look at data sets centered on interrogating systems of injustice based on race, gender, or class.

Mr. G's Narrated Identity-in-Practice, Post-Enactment. In the post-enactment interview, Mr. G shared several narrations that provide evidence for the CSL materials acting as a resource that made visible the identity work of Mr. G as a critical statistics educator. Specifically, Mr. G shared about the desire for predictability, the avoidance of controversial topics, and challenging the benign notion of statistics.

Desire for predictability in an era of accountability. Mr. G narrated in both his pre- and post-enactment interviews about the predictability that nice, neat datasets provide. For Mr. G, the predictability of nice, neat datasets was enticing and comfortable. In his pre-enactment interview, Mr. G stated,

> I don't really have any experience in organizing big datasets like that. Typically, the datasets we use in class are like really dry. Like, they're…nice and neat and fit towards…standardized test questions…I don't really have much experience working with the large datasets like the kind in the new math curriculum.

Following his enactment of the CSL materials, Mr. G shared the following:

> Sometimes, especially with the statistics stuff, I just kind of like everything to be nice and clean. So, this was kind of a little refreshing, especially this year, specifically. I didn't really have to worry about a timeline of getting every topic in so I have a few more days to kind of work with. I wasn't stressed about them spending two days on something instead of worrying, typically trying to just motor through it. So, I had a little bit more freedom. And I enjoyed just the different aspects of the lessons versus kind of how I normally teach those topics.

While he recognized that some of the datasets he chose (e.g., sports statistics) were contrived and benign and could often be seen as "really dry", these types of datasets often mimicked the ones that students would encounter in high stakes standardized testing, providing a potential benefit to helping the school's accountability score. Yet, the new fourth level math standards provided some tension with the use of these predictable, nice, neat datasets as it specifically asked for teachers to use large datasets that contained at least 200 entries, making them more cumbersome and difficult to organize.

The new course also did not have an end-of-course exam that counted towards the school's accountability. Mr. G described the materials as "refreshing" since he was not "stressed" or "worrying" about the pressure of testing accountability. The incorporation of large datasets that are not canned, benign, neat, or clean in the CSL materials made visible the identity work that Mr. G was doing in regard to teaching a course with no high-stakes testing. With the course not being test-score oriented, Mr. G felt more freedom to allow the students in his class to experience the process of statistics without feeling rushed because of the need to cover more topics on the test. But, for Mr. G, part of his identity as a math teacher is to prepare students to test well, which he is valued for at his school. It is interesting how this identity translated into a course with no high stakes-test, yet he still felt pressured to "just motor through it".

Later in the post-interview, Mr. G reflected about a teaching scenario that occurred with a student during *Lesson 6: Exploring Data Meaningful To Me.* He shared,

> Finding data and being able to organize it is something that's just very time consuming. [Finding, collecting, and organizing data] was the goal for day one, but a lot of my students were still doing this into day two and were kind of finally finishing it into day two. And that was something we want them to kind of struggle through and be able to do and understand that process on their own. But as a teacher it is definitely frustrating to see how long that process takes in class. Specifically, I had one student in class, who he, his question is kind of similar to the COVID data in that, he was looking at incarceration data, and was looking at, or was interested in looking at incarcerations by race to see if

> there was any kind of the same thing we did with the COVID data, if there was any, look at the proportions and see if there was any proportions that were disproportionate. But in doing that, it took him like a whole class period to transfer the data from the websites that he was getting the data from for each state. Because you want to look at all 50 states. And it just took him the whole class period just to input the data. So that was a great question, very good data to kind of look at. Again, just kind of frustrating as a teacher to see how long that process takes, but interesting,

Here, Mr. G narrated about navigating the “frustrating” tension between the desire for students to explore real, messy data with the time and process it takes to do such an exploration. While Mr. G certainly saw the value of students going through the statistical process to develop conceptual understanding, it seems that the messiness of the data inhibited him as a teacher, even without the pressures of standardized testing within the course. The pressure of pacing students to move through the statistical process at a standardized pace reified the identity work that canned and clean datasets serve the teacher’s desire to remain in control of the time it takes for students to learn.

Within this same scenario also brings the context of the datasets into focus. Mr. G mentioned here how interesting the student’s question was, but that the collection process for finding the data was difficult and frustrating. In the pre-enactment interview, Mr. G was discussing *Lesson 5: Looking at COVID data by race* and the idea of having students use a different set of benign data to allow for an initial exploration using the technology tool CODAP. This different dataset was shared as a learning tool for teachers to get comfortable with using CODAP. The dataset was about different roller coasters at different amusement parks with various characteristics (e.g., height, age, material composition) about the different coasters. I am labeling this as benign, compared to my definition of critical statistics, in that the dataset does nothing to get students to think about systems of injustice occurring within the world. Mr. G shared the following:

> I almost might want to give them an experience, something that's not quite so many things to look through at first. I think that might be a good place for me and my students to start, just to get some experience with it before we look at a little more complicated dataset with different, other stuff.

The findings suggest that the context of the data focused on race seemed more "complicated" and "different" to Mr. G than the benign dataset he would use as the introduction. In the post-interview, he discussed this idea again, by reflecting on a student's response to this same activity.

> I thought this student's response was kind of compelling, talking about race and ethnicity. This student put that, "race is not a conclusive thing and it's not like, conclusive, and fluid, basically it's just kind of describing it. Some people just don't fit into a specific box when it comes to race, or ethnicity." So, I thought that was an interesting comment from that student. When we come down to statistics it's helpful or, everything seems to come out more kind of cut and dry if we can put people in really specific boxes. But when things come to race, and kind of how we describe people, people don't always fit into nice, neat boxes like we would want in a math problem.

One possible explanation was that, for Mr. G, that the inclusion of race as a variable within the dataset made it messy and unpredictable. The social identity of race not being easily disaggregated into "nice, neat boxes" reified for Mr. G the idea that using social identities as a data created a lack of precision within the problem that was unlike the ones he was comfortable using in math class. By prioritizing the precision of the calculations over the need for students to explore datasets with critical contexts, Mr. G solidifies the need to stay away from topics such as race, gender and class, because to him, they are not suited for the type of math problems that he believes belong in the learning environment. For Mr. G, the incorporation of topics of race, gender, and class in the math class went against his perspectives on the purpose of teaching statistics.

Avoidance of "Conflict". Mr. G also narrated across both the pre- and post-enactment interviews about his need to avoid conflict within the classroom, and how this avoidance of conflict was a driver for the decisions he made in regard to choosing and teaching statistics topics. In his pre-enactment interview, Mr. G narrated about how he avoids any tasks that bring the study of critical topics into the math classroom, and instead chooses topics related to sports. As a successful coach at the school, Mr. G recognized that the topic of sports often leaves students out of the conversation, but to him, the topic of sports is safe and avoids conflict.

In his reflection on Lesson 3 *(What Does Data Say About Me?)*, Mr. G demonstrated this avoidance of conflict as he modified the lesson to better suit his identity-in-practice as a teacher. Mr. G shared,

> I would say probably the biggest thing was kind of the introduction or a warm up activity. Um, the "Who are you the, what social groups do you identify with?" I'll say, just in general, as a teacher, well, the two, I would definitely kind of shy away from even mentioning even though they can identify on their own if they want to the too, I would probably stay away from the most as a teacher, especially on a typical year when I have a full class of students in front of me are the religious affiliations and then also the sexual orientation are the two that I would probably definitely avoid intentionally speaking

> about. When I went through it on zoom, I just kind of mentioned the age, race, I think I mentioned like, personally, for me, I mentioned like job title or family, like, so I just kind of gave them an example of the identify as like a teacher and a parent, the most kind of things I identify with. And that was the example that I used in the lesson, when I presented it.

While Mr. G recognized that students may have different comfort levels with the exploration of their identities, he did not feel comfortable asking students to complete the activity as it was written. He specifically highlighted that the virtual environment assuaged this tension some, but that in a year when his students were all face to face, he would avoid this activity entirely. Rather than focus on some of the more critical social identities for himself, he modeled the activity focused on the benign social identities such as job title and family status (e.g., "I am a teacher and a parent"). For Mr. G, the focus on social identities of students or himself was uncomfortable and had the potential to create conflict within the classroom. This impacted him in three specific ways of interest: a) Mr. G did not feel comfortable consuming data about other people or himself that involved their social identity markers, b) because he did not feel it was part of his purpose of teaching statistics in the K-12 math classroom, which meant c) Mr. G was uncomfortable discussing systemic injustices tied to one's social identity marker in the math classroom.

In the post-interview, Mr. G continued to endorse this idea that social identities were best left outside the classroom to avoid conflict. He stated,

> But I mean, obviously, being a white male, I feel like I don't really have a lot of ground to stand on sometimes when I talk about race, like in just bringing it up in a lesson or something like that.

Mr. G felt constrained by his social identity of being a white male in what conversations he could and could not bring into the math classroom. Because of his identity as a white male, he did not feel as though he could go against the socially constructed norms that existed in the culture of his classroom or school in order to discuss topics about race. His school population was diverse with over 60% of the student body being represented by students of color, and to him his position as a white male meant he did not have expertise in discussing race. In this way, he used his whiteness as a form of colorblindness, inferring that white was not also a race and that he did not have enough skin in the game in order to be knowledgeable about racial identities.

In reflecting on the designed materials and their incorporation of critical statistics topics within the unit, Mr. G shared how bringing critical topics into the math classroom made him feel like the conversation could potentially get out of control because of the lack of predictability of what a student might say. Mr. G shared,

> I guess the only thing that kind of scares me is like, you don't really know what a student will say…I guess that's the only thing that really scares me about, you know, I'm just picturing certain students and the things that they say out loud. So there’s kind of like two sides. I definitely would..uh...kind of feel more confident in approaching those types of lessons, but then, like, you know, in the back of my head, I'm kind of like, I gotta be ready to kind of control where the conversation goes.

While Mr. G was initially excited about implementing the CSL materials because of their use of critical topics that would make students use their statistical inquiry skills to better understand issues of politics and society that they would encounter outside of the classroom post-graduation, evidence from his post-interview suggests that his enactment of the materials solidified for him the need to avoid the teaching of these topics to keep the conversations within the classroom under control and the classroom environment free from conflict. While his enactment of the materials made him feel more confident in teaching lessons with critical topics, it is unclear as to whether or not the value of having students develop critical statistical literacy was worth it for the potential conflict that it may bring into the classroom space. For Mr. G, there is some evidence that the materials reified the desire for clean, benign datasets that avoid conflict over real, messy data about systemic injustices. In this way, Mr. G's purpose for teaching statistics interacted with his development as a critical statistics educator.

Challenging the notion of benign statistics. The design materials also mediated a shift in Mr. G's identity-in-practice as a critical statistics educator by challenging the notion of benign statistics. In the pre-interview, Mr. G narrated about how imperative it was for all citizens to consume data well. He shared,

> For one, I mean, there's a math component, definitely to learn certain math components that go with statistics. But then also, bigger picture wise to be able to just understand basically things that are put out by the media as they get older. Especially when we have years like we've had the past year. Whether it's presidential elections, or COVID data will definitely be, I mean, some of those graphs I'll look at, I don't even understand as a math teacher. So I can't imagine what a normal, everyday person who hates math thinks when they see certain graphs and tables.

He shared how he understands that being able to consume data through the reading of graphs and tables is a matter of life and death. He understood that data outside of the classroom was not benign, and the implications for citizens not being able to consume data with accuracy could cause harm. But within the classroom, he was more focused on teaching procedures rather than understanding. Despite having the grounding PD experience that focused on teaching mathematics with a conceptual knowledge building lens rather than one that is procedural, this teaching style did not translate into his statistics teaching. He simultaneously held a view for citizens to consume data well outside the classroom with the procedural teaching of benign data within the classroom but had not yet troubled the relationship between those two data points or his role within the system of creating critical statistical illiterate citizens.

Throughout the post-interview, Mr. G shared several instances in which he was challenging the notion that statistics can be benign. For him, this came in two forms: through the view of data as story (DKF5), and through the use of data focused on critical topics such as race, gender, or class.

For Mr. G, the design materials presented the idea that data has an author, and the author can tell a story using the data from a particular vantage point. In framing data this way, statistics is no longer harmless and has the potential to harm others from the story being told. In his reflection of Lesson 3 *(What Does Data Say About Me?)*, Mr. G shared the following,

> The thing we've kind of been talking about throughout all these lessons is the variance that occurs throughout each process and the story that is told by the person collecting the data…Like is it representative? Is the person collecting data telling the story of the data or are they trying to tell their own story based on selecting a certain sample? Or looking at kind of where they got their data from, is it representative of the actual population? So, I think those are really good questions for the students to think about throughout this lesson.

For Mr. G, the idea of data as story helped to trouble the benign notion of statistics and shape the idea that the story written about data can be used to harm others. This was evident in the post-interview in which Mr. G shared a narration about a student who was doing a project to learn more about the populations racially disproportionately incarcerated compared to the citizens of the state. The student found results that clearly supported that Black and brown populations were incarcerated as much as 15x more than white populations. I asked Mr. G how he was helping this student navigate the tensions of finding evidence of this type of injustice in her data analysis, and he shared, “I mean, she definitely had, like questions or like, you know…It'd be interesting to actually figure out like, why, like, what causes this?” For Mr. G, this instance with this student was a powerful one that helped to challenge the benign use of statistics in schools. While he recognized the need for all citizens to be critical users of statistics outside of school in regard to their democracy and livelihood, here he had evidence that looking at meaningful data that allowed students to find evidence of systemic barriers caused by race, gender, or class prompted real change in the way the student and he viewed the system of incarceration. His problem, though, was in knowing how to act. At this point, the analysis was complete, but the thinking had just begun on what to do about the new data. Later in the post-interview, I asked Mr. G about his knowledge of systemic barriers caused by race, gender, or class, and he stated, “I definitely know it exists, but I just don't know, like, why, or what can be done, or, like, what could change.”

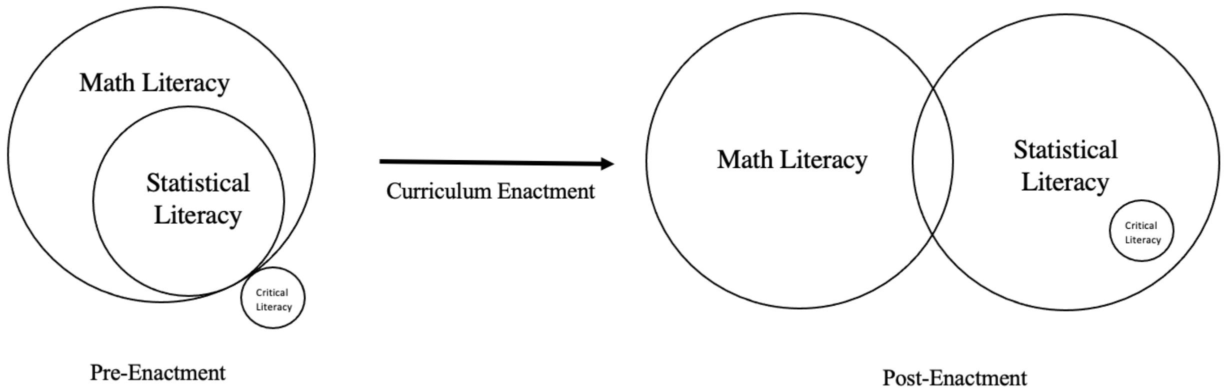
The CSL materials made visible the identity work of Mr. Gas a critical statistics educator by prompting him to action, and by changing the lens we use to look at data. Part of the action for Mr. G was in engaging with the CSL materials in his classroom. For him, this was part of taking action towards critical statistics as an educator. Along the spectrum of development, Mr. G engaged with the materials and concepts of the CSL lessons, yet paused at how to act when the lessons show evidence of the existence of a systemic injustice. The action did not move into self-learning about the system, injustices, or social action to dismantle such systems to disrupt the harm caused to the people oppressed by the system. By viewing the story being told with data, data is no longer benign. By using critical contexts within the school building and math classroom, learning is no longer benign. For him, this challenged what the purpose for teaching and learning statistics could be like and therefore his development as a critical statistics educator.

Identity Work Made Visible for Mr. G.

For Mr. G, his perspective of critical statistical literacy was disjoint from math literacy and statistical literacy, except in the case of his own consumption of data (see Figure 5.1). This is represented in the figure by the concentric circle of critical literacy only touching statistical literacy and math literacy in exactly one point. In addition, Mr. G saw statistical literacy as a subset of math literacy, as something that occurred within the courses taught in the math classroom.

Post-enactment, the resources made visible the identity work that Mr. G engaged in to recognize how statistical literacy is both distinct and related to math literacy. Simultaneously, the resources made visible Mr. G's identity work to recognize how statistical literacy provided an opportunity for the value of teaching critical literacy to his students. However, the resources also prompted some discomfort with the idea of using data sets, media sources, or distributions that centered on studying the existing injustices caused by race, gender, or class. This discomfort provided evidence for Mr. G to limit the amount of lessons focused on critical literacy, and to completely bound them within statistical literacy. To Mr. G, critical literacy could be taught in one example as an absolute subset of statistical literacy, often as a counterexample for why statistics focused on variables such as race, gender, or class should not be studied.

Figure 5.1 ***Mr. G's shifted perspectives of critical statistical literacy components as evidenced by identity work made visible by resources.***



The Case of Ms. H.

School and Historical Context. Ms. H spent 13 years in a STEM industry before entering the teaching profession. As a former electrical engineer, Ms. H entered the teaching profession through an alternative licensure pathway. In her five years of teaching, she had taught almost all of the courses offered in the mathematics standard course of study, except those that required additional licensure (e.g., Advanced Placement courses). In the time that she began teaching, she had enrolled and finished a Master's in Information Technology. She was not seeking the graduate degree to help her transition to another job, but rather to use the information to help her keep abreast of new technologies that she could incorporate into her teaching.

While she began her teaching career at a school in an urban setting, she shifted to a second school site that had a specific STEM focus on health and technology with the hopes of using her previous skills in the STEM industry to impact her students' learning experiences. Because of her background and school setting, Ms. H shared that her reason for participating in this study revolved around access to resources. She shared,

> I'm always looking for resources. Plain and simple. I'm always looking for resources to use and ways to try to make that part a little bit easier for me, instead of always trying to

search for something, then, you know, being able to take something that somebody else has, and maybe tweak it to how, well maybe not even tweak it, but just find a way to make it mine so that that way, I can teach it in the way that I want to teach it. And that's I mean, I guess that's the way it is, when you're given resources, you can look at them. And you know, look at the notes and then decide what students you have and how to best go about giving it to them or teaching it to them.

With all of the courses that Ms. H has taught, she has been given many curricular resources that often did not fit the needs of her student population. For Ms. H, teaching her students math well meant figuring out how to reach her students. whether it be a pedagogical routine, fascinating real-world context, interesting video, or a nurturing teacher relationship. She was willing to try “anything to make it make sense”.

With her participation in the culturally relevant PD, she was just beginning to think about a new method, creating lessons “based off of the experiences of my students and their backgrounds”. Ms. H shared that designing materials that were culturally relevant was more difficult than initially described. Ms. H shared how, in the workshop, creating the lessons just seemed like a checklist, but the facilitators forgot that the people in the workshop would have to do some identity work first to understand the cultures and backgrounds of the students they were serving. Ms. H shared, “We just discovered that it's a whole lot more that we’ve got to dig into because it's a lot of layers to be able to reach our students better.”

Pedagogical orientation. Nevertheless, Ms. H was committed to learning how to implement culturally relevant lessons in the high school math classroom because she hoped it would be the straw that worked to “reach” her students. Consistently, Ms. H reiterated her commitment to creating a math learning environment for her students through her own labor and pursuit of learning through any avenue possible (e.g., Master's degree, CRP Workshop, Participation in this study).

Recruitment. This semester, Ms. H had been tasked with teaching the new fourth level math course to students who had been tracked to the remedial level of math courses for the entirety of their K-12 schooling. The majority of the students in the course had an individualized education plan that required accommodations for learning and state testing and had typically been taught below grade level content. The class was also taught in hybrid, with a portion of students being online and a portion of the students face-to-face. Teaching math to the population of students online had been quite a challenge for Ms. H, and she worried about whether or not they understood the content. The virtual environment she utilized did not provide many opportunities for feedback, and she talked about how teaching during the pandemic had been a challenge for her teaching style. Yet, she chose to participate in this study because she hoped it would provide her with resources that would work for her student population during this difficult season of teaching.

Ms. H Pre-Enactment Narrated Identity-in-Practice.

View of Statistics. Ms. H's identity-in-practice as a critical statistics educator was steeped in her prior experience with statistics as an electrical engineer. Her historicity as a Black woman in a STEM field for 13 years informed her views of what statistics was and why it was important for students to learn and experience. When asked about the purpose of teaching mathematics, Ms. H shared,

> I'm always talking about how when I think about math, I'm thinking about being able to think through processes and problems. So anytime we're given a problem, there's certain steps and things that we have to take or may need to take based on what we're given. So,

> can you process things? And can you troubleshoot? Like go back, if something doesn't seem quite right in your answer, because, you know, you may have an idea of what things should look like for certain types of problems…So, and if you're trying to, I'm just doing an example trying to show growth, and something comes back, and it's not showing growth, and then you know you've done something wrong in your process and you need to go back and tweak it. But it's developing processing skills.

To Ms. H, the purpose of teaching mathematics was about developing skills and processes that would lead to the ability to question, reason, troubleshoot, reflect, and redesign. While there was a heavy focus on the procedures one must master, there was also strategic competence and adaptive reasoning skills in thinking of ways to approach a problem (Kilpatrick, et al., 2001). Conceptual understanding of the problem or how it relates to one's identity or citizenship are not the focus for Ms. H.

Specifically, within the context of statistics, Ms. H shared that she had used statistics and the processes of statistical analysis within her former career daily. Her Master's degree also had a focus on learning how to analyze statistical data using various technologies. She shared that she understood statistics quite a bit, and was very confident in her knowledge of statistics, but she had never had professional development or training on how to teach statistics to students. She viewed the learning of statistics for her students as a key skill they needed to learn before graduating, sharing,

> I think it helps with thinking, because instead of taking everything that they see as fact, I would hope they would want to look at it and maybe dissect a little further. Like, 'do I want to believe what I see or do I want to take a snapshot, and really dig into what they're saying, and then kind of draw conclusions on what to believe.' It helps with decision

> making. You know, in everyday life to me, do I take everything that somebody tells me as fact, or do I dig a little deeper? Just being able to think about what they're really saying...developing those thinking skills.

View of Critical Statistics. Developing the ability to question, making data-based decisions, and becoming a healthy skeptic (Lee & Tran, 2015) were of primary importance to Ms. H. But the context that students learned how to develop these "skills" was less of a focus. When asked about her previous teaching experiences with lessons designed to help students analyze systemic barriers caused by race, gender, or class, she shared how she focused on asking students questions rather than helping them name the injustices in particular. She stated,

> So, I didn't necessarily say what some of the barriers could be with the data. I kind of did a high-level thing, like I…would emphasize how your statistical data is only as good as the information that you're collecting. So, what are your sources? Or even if you're sharing statistical data, and where did it come from? Who's being looked at?...You know, does it represent a good set of the population? …But I didn't really dig as deep into that.

For Ms. H, being a critical statistics educator was about developing "critical" thinking skills to be able to question all things with a healthy level of skepticism (Lee & Tran, 2015), with a heavy focus on collecting good data using quality procedures. The context was not as important as developing the skill to question, since the ability to question could be transferred to whatever data set one studied. For Ms. H, she was mostly concerned with helping her students become producers of good quality data, regardless of contextual nature. Ms. H's purpose for teaching statistics was akin to this "high level thing", or a focus on getting her students to ask questions about the data and their authors, to understand the procedures taken to produce quality data, which means that, prior to her enactment, Ms. H had not entertained the idea of having students look at data sets that centered on systemic injustices. When we discussed incorporating critical data sets that studied systemic injustices into the curriculum, she realized that she had never thought about doing so previously, and she was excited about trying to do so.

Ms. H's Narrated Identity-in-Practice, Post-Enactment.

In the post-enactment interview, Ms. H shared several narrations that provide evidence for the design materials acting as a resource that shifted Ms. H's narrated identities-in-practice as a critical statistics educator. Specifically, Ms. H shared about personalizing the process and differentiating "critical" vs critical thinking.

Post-Enactment View of Statistics. In the pre-interview, Ms. H shared several narratives about how she viewed statistics and math learning as a process, or a skill to be acquired. While Cobb and Moore (1997) described viewing statistics as a conceptual process or paradigm as the goal for learners, in the pre-interview, Ms. H was mostly focused on the procedural set of skills that a student must acquire or master before being labeled as "successful" with the content, as determined by end of grades testing. One of the reasons that Ms. H cited for her procedural view of statistics was because of the pacing of the course. In the pre-interview she shared,

> And that's the sad part about it again, because [statistics] gets the short end of the stick…And it gets like, oh, gosh, okay, we just got to show you this. This is how you do it. All right. We're done…because it gets pushed all the way to the end. That's one reason why I said I felt like, I might need to preload a few things, because you *should* see sampling in Math 3 or understand the concept of sampling. But this year, I'm not gonna lie. I didn't get there. I didn't get there last year, either. So, you know, it's just one of those things where you try to improvise and catch them up, so to speak.

For Ms. H, it was hard to teach statistics in any way that did not center calculations because there was no time to teach it conceptually. To her, teaching conceptually meant it took more time away from the other standards of the course, which were more heavily tested and had a greater impact on the school's accountability scores. Because of this pressure, Ms. H often did not teach the statistics in other courses at all or resolved to teach it to her students in a quick, procedural, calculation-centered manner. Again, what is interesting here is that the new fourth level math course does not have an end of course exam, yet this constraint still presses on Ms. H to perform, but it is not clear for what or whom she is performing.

A significant shift in the language of Ms. H in the post-interview was redefining her calculation-centered view of the processes in statistics to one that was focused on viewing statistics as a process of inquiry (Cobb & Moore, 1997), as she began to see the value in teaching students to look beyond statistical measure to the broader context and situation from which data are drawn (Ben-Zvi, Gil, & Apel, 2007). In the post-interview, she indicated this by sharing,

> Before I did a lot more with calculations and stuff like that, you know, with the 5 number Summary. But now it's like, Okay, I'm taking it…to that next level, which I think this takes it to a whole 'nother level as far as like being able to analyze…. But…this was definitely a different approach. Because before I spent more time with calculating things, versus exploring, you know, what I'm saying this one really does hit that exploring piece, where you're looking at data and actually exploring it, not just calculating.

Here, Ms. H shared how the resources acted as a prompt to view statistics as a verb rather than a noun, moving her perspective of statistics from a list of calculations to model for students to a process of inquiry that one holds when authoring, reading, and interpreting data. Ms. H began to take on Moore's (1998) view that the context of data should motivate procedures, and now students needed to do more than just calculate a statistical measure but instead understand the context and why the statistical measure was a valuable calculation to do in the first place. In this quotation, we see the language of Ms. H move from “calculations” to “calculating”. Students were ‘exploring' data, actively searching data, but there was no specific checklist of calculations to prescribe to a data set before having looked at it. To Ms. H, this was a significant change in what she expected her students to be able to do in the past with statistics.

Throughout the post-interview, Ms. H expressed how personal the CSL materials were in making the statistics relevant and conceptual. Ms. H shared,

> They were going out and looking at Pew Research. And they could see what kinds of graphics were used, like, how many people were actually studied, and just where all the data was collected and gathered and then having to, you know, assess it and say, ‘Hey, okay, this is, you know, interesting, this is what I found. Um, yeah, I kind of see myself in this data.’ So, I really, really felt like it was personal, you know, they could be personal with it. And, and the studies, of course, that they were looking at did involve large amounts of data. And you know, how you can take that data and display it in different kinds of ways, too.

In this narration, Ms. H shares how the CSL materials made their learning personal to the students, and that it mattered for how they learned statistics. She also emphasizes that the CSL materials were aligned to the new fourth level math standards that include the analysis of large data sets. Rather than focus on the specific calculations that students used within the lessons, the materials helped Ms. H focus on one of her teaching goals in statistics: to help students question statistics they encountered in the world (e.g., “what kind of graphics were used, how many people were studied, …where all the data was collected and gathered [from], [and how] to assess it”). Ms. H reiterated this by sharing,

> It's an opportunity for them to really think about themselves, and then being able to like, actually, research in math class, you know, do something a little different. And now, you know, I think it gave them a different perspective of statistics, and how they can look at them, and see how they fit in the overall picture.

To Ms. H, the focus on identity as a data source (DK2) flipped the student's view of statistics for how they would hopefully look at data with skepticism moving forward. In stating this, it also demonstrates that the CSL materials changed the view of statistics for Ms. H as well, acting as a prompt for her own learning. Her view moved from a focus on calculations to one in which the context of the data drove the need for procedures and mathematical calculations (Moore, 1998a). By viewing oneself in the statistics, it allowed students and Ms. H to see the process of statistics, as described by Cobb and Moore (1997), as well as personalizing the statistics to see oneself within them. For Ms. H, her purpose for teaching statistics was tied to her own learning about the role of the production of statistics, which impacted her development as a critical statistics educator.

View of Critical Statistics. A second shift for Ms. H was in how she defined "critical" in regard to statistics. In the pre-interview, when asked about her role as a math teacher, she shared that it involved helping students develop "critical thinking skills", specifically discussing this in the way one approaches statistical information. She shared, "either you're gonna take something, and believe it or maybe you should just think about what you see and dig a little deeper." Ms. H's definition of "critical" was not about the inspection of systems of injustice or bringing those into the classroom. Her definition centered on the careful evaluative and analytic process that one uses to approach information, which differs from the definition of critical, as defined in the framework of this study.

Throughout the post-interview, Ms. H wavered between these two definitions of critical. When asked to think of specific instances when students learned about statistics in critical contexts that taught them about the systemic injustices based by race, gender, or class, Ms. H shared samples of instances when students were evaluating headlines and analyzing data. In sharing these things, she made statements like, “Oh, this person had a really strong opinion”, or “So I mean, some of them were more critical than others”. Also, within the post-interview, Ms. H shared the following:

> You know, I think it made them think. And, again, this whole unit, I think, gave them the opportunity to see that statistics are more than just the numbers that are shown. But there's the story behind it as well. So, um, being able to understand…that you got to make informed decisions based off of more than just looking at the numbers but what goes on behind the numbers and how those numbers came to be.

Here, the definition of critical starts to shift as she mentions the need to focus on “how those numbers came to be”. Later in the post-interview, when asked about how she felt the designed materials shifted her students’ view of the world, Ms. H reoriented to the evaluative definition of critical, the analytic process one uses to prepare an argument, sharing, “This involves some critical thinking, or had students really thinking about how they felt about what they saw, and just being able to, again, intertwine that in some of the other areas of math.”

When specifically asked if students discussed race, gender, or class, Ms. H shared one narration that occurred during the unit.

> Some of them were like, ‘why does race even matter?’ You know, that kind of stuff came up…It was just interesting to see their opinions on how they felt it was going in with COVID and wearing masks and not wearing masks and how some were like, ‘Well, yeah,

because folks don't believe in wearing masks'. It was just, it was interesting, because, you know, as they were putting in some of the stuff, I would read the comments. And, you know, we would kind of discuss some of those things that were coming up…Overall, you know, just having some relevant topics of interest for them, I think helped with this particular unit.

At the end of the interview, I asked Ms. H what she thought the purpose of statistics was in mathematics education. Her response is below:

> So, I guess, you know, when you think about statistics, it does have that mathematical piece to it. But to me, it just embodies so much more than math. But for us, or in the math classroom, I think it requires us just digging, instead of just taking the numbers at face value, being able to figure out where the numbers came from to begin with. So going back into the data that was collected, how it was collected, and how it can influence the numbers that you actually see. So, what the math that's produced from it, or from the data that's collected. And then just interpreting and seeing how it can be interpreted in different ways. I mean, everybody can take numbers and tell a different story. So, to me, it's almost like mathematical storytime.

Here again, we see Ms. H blurring the evaluative and justice-oriented definitions of critical, the difference between the "critical" thinking one uses to prepare an argument versus the "critical" consciousness one uses to view the world and the systems within the world that are created for the oppression and decentering of some. She focuses on the need to "dig" into numbers, evaluating sources, but she also focuses on how data can cause harm through its different interpretations. She leans on the idea that data can tell specific stories from the viewpoint of different stakeholders. The evidence also shows her awareness that the statistics in this unit was “so much more than math”.

Because of her participation in the culturally responsive teaching workshop, I asked how she felt the designed materials aligned with what she was learning. She shared with me,

> It really, to me, hits that culturally responsive teaching. Like, ‘Hey, this is about me, how do I see myself in this? How can I insert myself into the math classroom and add value?’ So that's one reason why I did like this unit. I was like ‘hey, this kind of is this culturally responsive teaching, that we're trying to focus on in giving the students the opportunity to be engaged with math’. Like, you know, ‘this is me being a part of the math lesson, instead of just being taught procedure’.

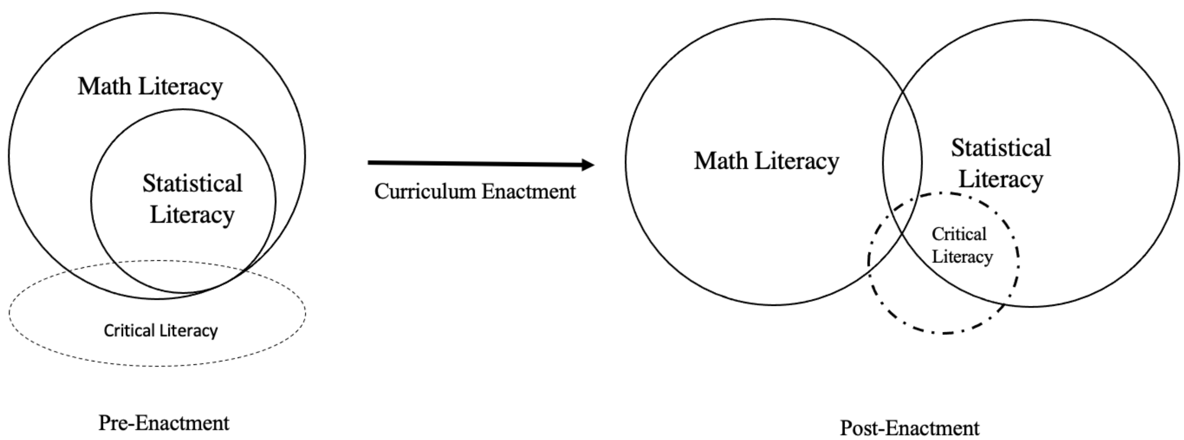
Ms. H recognized the value of intertwining student identity and culture into math lessons. The designed materials provided an example of what culturally responsive teaching could look like, as defined by Ms. H and her learning through the workshop. To her, this meant inserting oneself into the math lesson, and recognizing that this adds value. She, then, further reiterated how this helped to personalize the learning of statistics, and switched focus from procedural to conceptual.

The evidence here demonstrates that Ms. H is in the process of using multiple definitions for the word “critical”, in regard to becoming a “critical statistics educator”. This blurred definition demonstrates a lack of evidence that she took up teaching statistics using the concepts of race, gender, and class specifically, while also recognizing how focusing on them with students adds value to the lessons and design. By using the evaluative definition of critical focused on critical thinking, she reiterates for herself a procedural view of teaching statistics that asks students to question, but not specifically about contexts such as race, gender, or class. This allows her to stay within the frames of a benign statistics education for students, covering procedures and calculations, and occasionally spurring healthy skepticism, while not focusing too much on the systems that we should be skeptics of, staying “high level” rather than discussing the topics outright. This aligns with her view of the purpose of teaching statistics is to create students who know how to calculate statistics, while also being cognizant of the data they produce. The interview provides evidence that her desire is to get students to question their world, but bringing that world into the classroom made her uncomfortable. So, she teeters back and forth on how she wants to define critical and whether it aligns with the critical statistics educator she wants to become. Thus, her purpose for teaching statistics is tied to her development as a CSE.

Identity Work Made Visible for Ms. H

Because of her historicity as a former engineer, Ms. H’s initial view of the different literacies held statistical literacy as a subset of math literacy. Statistical literacy was learned by going through a list of procedures or calculations which were all mathematical formulas to follow. Her view of critical literacy was mostly focused on the evaluative definition of critical, as in “critical thinking”. The dotted line in Figure 5.2 is used to represent the different definition of “critical” used by Ms. H and the author. To Ms. H, critical thinking belonged in the math classroom, while critical contexts did not. Critical thinking was a 21st Century skill that could be developed through reasoning and should be developed in the classroom; critical contexts are one instance where critical thinking could be used, but students should use their critical thinking skills on critical contexts post-graduation. Post-enactment, Ms. H viewed statistical literacy and math literacy as differing, as evidenced by her sharing how statistics was more than just numbers and math. Her view of statistical literacy grew to include a conceptual focus on understanding the statistical paradigm rather than one centered on calculations. However, her lack of statistical knowledge for teaching impacted her view of statistical literacy and critical literacy. While she saw value in teaching critical literacy as defined by the inspection of systemic barriers, she continued to think of critical literacy as an evaluation or the incorporation of "critical thinking". The segmented line here represents the continuation of the prior view of "critical thinking" while also the addition of the new definition of critical literacy.

Figure 5.2. ***Ms. H's shifted perspectives of critical statistical literacy components as evidenced by identity work made visible by resources.***



Introducing Ms. J.

School and Historical Context. Teaching was the second career for Ms. J. Formerly an accountant, Ms. J began her teaching career through an alternative licensure program that allowed her to teach while gaining the education credits required. When she began her career as a math educator, the field was desperate for math teachers. To recruit professionals to alternative licensure programs, local counties would provide stipends, free tuition vouchers to local universities, incentives, and paid professional development. Ms. J was recruited through such a program and described her entrance into teaching as one that made sense for her family both financially and relationally, as the schedule of a 9-5 job was not conducive with their family schedule. While she taught her first year in a neighboring county, Ms. J moved to a second school site during her second year of teaching and has been at the same school for the last 13 years. The alternative licensure programs were specifically targeted to recruit teachers to teach in the schools that struggled with teacher retention, which were also the schools with large populations of students living in poverty and students of color. As a Black woman, Ms. J enjoyed teaching in a school with Black and brown students, as it allowed her “to be real” with her kids, colleagues, and herself. Her school is culturally diverse, and she enjoys learning about the cultures of her students, and sharing about her culture with her students. Since her students consistently score well on end of course exams, Ms. J is often charged with teaching students who have been consistently tracked into the remedial level courses. To her, teaching students who have been consistently labeled as remedial is a challenge she enjoys, as she wants to help her students “rise above” the situation and find a way to succeed. Like other Black educators, Ms. J views education as a financial, social, and emotional ladder for her students to achieve more than the cycle of poverty that many of them exist within.

Critical Context. Because this population of students can often require additional labor, mentoring, guidance, parental contact, and emotional support due to their history with math placement, Ms. J requests to teach at least one class of students who have been placed on an accelerated track to counter the load. Ms. J has noticed that the racial makeup of the school is dissimilar to the population of students placed in the accelerated classes. Ms. J shared,

> Our population is the majority Black and you'll see a class that has no Black kids. And then you'll go to another class, and it's, you know, from the classes 100% Black to classes 100% white. And you're like, am I in the same building? What is the timeframe? Is this 1952?

She recognizes that segregation is a current issue that is ushered through the education system, and it is one of the reasons she continually chooses to teach students who look like her.

Pedagogical Orientation. Ms. J was working in a hybrid classroom, with a portion of her students online and the remainder attending class face-to-face. She shared that attendance and engagement during the pandemic teaching period had been difficult, but that it also brought about many real conversations with her students who came to class daily. She shared that her teaching style before the pandemic was centered on making sure the kids understood the math by whatever means necessary. To her, this meant individualized, modeled, and whole group instruction. Her classroom was not specifically centered around the use of tasks, although she occasionally did “activities” that were shared by her fellow PLC members.

Recruitment. Ms. J had recently joined a yearlong, district-led professional development opportunity focused on creating lessons that were culturally sustaining and relevant. She’d recently completed the Social Identity Wheel (LSA Initiative, 2017a), and was enjoying learning about her own social identity and thinking about ways she could bring identity work into the classroom. Ms. J shared several times that bringing culturally relevant pedagogy into the mathematics classroom was hard work, and she had not seen any examples previously that had done it well, and she had not really done much of it herself. She shared,

> And you know, I can say when that whole culturally relevant thing came up, I was like, You know what, this is something I'm going to do. Sometimes we know we're doing problems, and I'm like, you know, I'm doing it because I have to do it. But I want to say, how could I do this so that it would be more relevant? And sometimes I'd be honest with you, Lauren, I don't know. Something's you just got to know, because they say you got to know them. But if we could make it more pertinent for the kids, then I'm all for it.

To Ms. J, most of the mathematics she teaches is something someone else has told her that students need to know. She teaches math topics to prepare students for career and college, but not necessarily to engage with the ideas of world. Yet, she shared that she was open to the idea of doing so, but was not sure how to get started. She was recruited to this study through her participation in this professional development, as the facilitator of that PD, who was not the author, thought she would be someone who would be willing to try to implement the designed materials and enjoy doing so.

Ms. J Pre-Enactment Narrated Identity-in-Practice.

View of Statistics. Ms. J shared that her knowledge of statistics and teaching statistics was lower than what she wanted it to be, stating,

> I don't normally teach statistics. And the statistics that we have is more, it's like margin of error, and then there's some vocabulary talking about different types of sampling and things of that nature. So that's pretty much my extent of teaching statistics.

Ms. J held a procedural, calculation-centered view of statistics (Cobb & Moore, 1997). In the pre-interview, when asked about teaching statistics, she listed specific types of calculations one may do in a course rather than the concepts behind why one may want to calculate them. Ms. J also shared that in many instances of her previous courses that included statistics standards, her pacing guide placed statistics as the last unit of study and she ran out of time to teach the content, and never taught it at all. She mentioned that this was true for several courses, not just the new math four content. While she took statistics classes in college to prepare her for her accounting career, she had not received specific professional development on statistics teaching since she began.

View of Critical Statistics. Ms. J's identity as a critical statistics educator was centered on the way she viewed herself as a critical person. In her pre-interview, Ms. J shared that she wished she knew more about statistics, teaching statistics, or critical statistics and math teaching, but that her knowledge of systemic barriers caused by race, gender, or class was "much higher". For Ms. J, her knowledge of these injustices came from her lived experiences (Anzaldua, 1987). She had a deep knowledge of critical topics because of the skin she was in as a Black woman and mother, and these identities were centralized in the way she viewed her teaching. She talked about these identities as informed the way she taught her students, who she referred to as her "kinfolk" and how her identities allowed her "to be real" with them. She had a deep appreciation for her students' cultures and heritage and found that her students' identities were how she created relationships with them. She did so by attending cultural fairs or festivals that she was invited to by her students. She was always willing to try something her students brought to share with her, whether it be food or costume or dance of cultural importance. She did this because wanted her students to have respect for others and for themselves. For Ms. J, creating a classroom environment in which every student was comfortable being themselves was of utmost importance. To her, this worldview was connected to her mathematics teaching because she felt like students needed "to get out of that mindset that everybody does the same thing the same way all the time." For Ms. J, her *way of being* informed who she thought she was as a critical statistics educator. While she clearly embraced criticality in her life, she did not do so in her math pedagogy.

Ms. J shared that she had not incorporated critical topics into her math teaching, except in the content area of finances and economics in one course that she taught previously. In that fourth level math course, Ms. J created a unit of study focused on teaching students how to be successful with their finances to gain wealth by using the concepts of integers in bank accounts, exponential functions when looking at compounding rates of interest, piecewise functions by looking at tax brackets for the rich and poor, and the stock market crash of 2008. Ms. J became very animated when discussing this unit of study, and how her students' engagement with the content fueled her as a teacher. To her, teaching this unit was more than just helping her students understand the financial world that they were about to enter, it was about helping her students gain the cultural capital of wealth by talking to them about money. She specifically addressed that her goal was to help students who were stuck in the system of poverty find ways to use their money to get out of the cycle of poverty, with a hyper focus on the Black and brown students in her class. She discussed passionately about how she had never learned about creating financial wealth as a youth and she saw that lack of education around economics as a clear tool of oppression for students who looked like her. Thus, her one instance of critical mathematics teaching came in a sector that she 1) previously was employed, 2) had a specific association with as a Black youth, and 3) made clear connections to systemic injustices for minoritized groups of people, and 4) saw her knowledge base and classroom teaching as an opportunity to dismantle this specific financially-based oppression.

Ms. J's identity-in-practice as a critical statistics educator fully embodied what Lave and Wenger (1991) shared that "who you are becoming shapes crucially and fundamentally what you 'know' " (p.53). Ms. J's lack of professional development and knowledge on critical statistics topics shaped her view of statistics, the way it should be taught, or whether it should be taught at all. She did not "know" statistics or teaching statistics outside of doing calculations devoid of context. Yet, she "knows" what it means to be a Black woman in the United States, and the injustices that come from racial, gender, and class barriers. Ms. J viewed teaching and learning math as a means towards social mobility, in which she viewed herself as a catalyst for social change for many of her minoritized students by being an example of a person of color who was successful in the field of mathematics. She saw her identity and lived experiences as a way to lift up others through her math teaching, not through critical pedagogy, but by her critical being. Because of her participation in the culturally relevant math PD, this study piqued her interest about what it would be like to teach using critical pedagogy in math class, so she could continue "to be real" with her students amidst her math teaching.

Ms. J's Narrated Identity-in-Practice, Post-Enactment.

In the post-enactment interview, Ms. J shared several narrations that provide evidence for the CSL materials acting as a resource that made Ms. J's identity work visible. Specifically, Ms. J shared about how the materials taught students about more than just the numbers and the opportunities she saw to bring critical conversations into the classroom.

More than Just the Numbers. **In** the pre-interview, when asked about incorporating critical contexts into math lessons, Ms. J shared,

> You're asking me that about such lessons in my pure math classes…. But a lot of times, those are just pure math. And it's more difficult to do those, especially when you're, you know, trying to make sure that you're on track with your pacing.

In her pre-interview, Ms. J discussed her view of statistics, mathematics, and her purpose for teaching them at length. She considered her math classroom as a place in which critical contexts existed because of the people that were inside the space, including herself. It seemed that her own historicity drove the way she viewed the world and how she thought about mathematics, but it did not change the mathematics context or content she taught, making it a space of "pure math". This pure math space was held in tension with her worldview, fueled by the accountability that comes with the era of high stakes testing, such as pacing to state-created standards that the teacher has no power to change. In her 15 years of teaching, she had ideas about what she wished the math curriculum could do, about the courses she wished existed, about the pathways she wished the students could take, but she continued to follow the guidelines given to her. She had become content with being a Black woman in a math space, and that her existence was enough exposure to criticality for her students.

During the post-interview, Ms. J reiterated multiple times her excitement and zeal about what her students were sharing about themselves, about the world, and about others. She had taken copious notes to discuss with me, wanting to highlight every moment in which her students discussed something “more than just the numbers”, as she termed it. In preparing for the post-interview, I asked her to bring 3 samples to discuss; she brought over 50 student work samples (compared to Mr. G and Ms. H who each brought 3 examples). Her interview lasted almost two hours, as she shared every minute detail of her students’ interactions with the lesson, and she never lost an ounce of passion as she discussed each one. Ms. J expressed several times that she “felt on fire”, that “something had changed” for her, and she “wanted more”. It was as if something had turned on for her that was not previously there.

In trying to describe what was different about these designed materials compared to ones she has used in the past, Ms. J shared,

> Well, you know, it really, it was eye opening to me, because, you know, I'm a math teacher. And so normally I give students math problems. And normally math problems have one solution. Sometimes there are different ways to arrive at the solution. I'm always excited when students find different ways. Or, you know, if we're working through a task, and some people see it from a different perspective, but this is something

> that I never see. You know, this is more something that happens in the Social Studies, or an English class where you have this level of thinking, and this level of exposure into yourself. Like this, I've never done this.

Ms. J described how the design of the lessons allowed her to facilitate the lesson from the view as an outsider while still feeling like she could share about herself.

> I just asked them how they identify. You know what I'm saying? Like, I'm not giving, I'm asking…And then I just let them figure out who they were, and write it on paper…I was just facilitating…And then I also told how I identified. And I said, for example, you know, "I would look for articles about you know, whatever." Um, And still, I'm on the outside, and then it was on them to go and find things.

To Ms. J, the pedagogical design of the lessons was just as important as the content. She suggested that, in some critical lessons, there is tension between the math content and the critical content as one is generally prioritized. In this scenario, rather than forcing students to discuss critical content, students were instead asked to use themselves as data points to discuss things that mattered to them (Konold et al., 2004). The critical content and the statistics content worked harmoniously together, allowing the teacher to not be indicted for giving a specific data set, but rather just asking students to explore data sets they cared about.

Ms. J mentioned how in this current climate of teachers being attacked for teaching lessons that discuss race or sexual orientation (Goldstein & Saul, 2022), critical teachers must be exceptionally careful to protect themselves and their students. Ms. J suggested that, "But maybe if we can just tiptoe…", math teachers could use pedagogical structures and routines that focused on the discussion rather than the specific contexts, allowing for math instruction focused on critical topics, as the designed materials did here. To her, this was something she was willing to risk to continue to bring this type of learning into her classroom. Ms. J shared,

> How can I do more things like this? Not necessarily to this level, but have things that we can do more than just the numbers. I want to challenge us to try to do at least one culturally relevant assignment, if you will, per month. Um, and, and when I say that, I mean, by purposely doing it. Not that it just kind of happens to be. And obviously, you know, in some subject areas, it will be easier than others. Math is a little more difficult.

While she was fully committed to incorporating more lessons using critical contexts into her math curriculum due to the responses she saw from her students, Ms. J was still undecided on defining these lessons as "mathematics". She recognized that creating these types of lessons was "a little more difficult" than other content areas. There were several times in the post-interview where she discussed with great excitement incorporating "these types of lessons" within her courses, sharing, "I was thinking about doing things like this, that are kind of outside of the math, but they're part of human learning." Despite her enthusiasm, she still classified these lessons as different and "outside of the math" curriculum that she was accustomed to working with. To her, these designed materials were "more than just numbers" which was more than what she was tasked with teaching with the math classroom.

For Ms. J, the CSL materials provided evidence of something she had been longing for but did not think was possible. In doing so, the materials created a discontinuity for Ms. J in who she thought she was as a critical statistics educator. Prior to using the materials, she felt bounded by the curriculum given, accepting her "passive role" (Freire, 1970/2005) as an educator who does not get to decide what content students learn in her classroom space. She held a mentality similar to that described by Gutiérrez, in wanting to teach her students how to "play the game to change the game" (2017) by teaching "pure mathematics" with the hope of impromptu conversations that would allow her to share her worldview with her students. After using the materials, she saw an opportunity to incorporate her identity into the content of the classroom in a way that made her feel liberated, as she “abandoned the educational goal of deposit-making and replace[d] it with the posing of problems of human beings in relations with the world” (Freire, 2018, p.6). She felt the freedom to dream about what else she could change, what else she could do and discuss with her students, all while being “real”. In some ways, her discussion of the materials provides evidence of solidifying narrated identities-of-practice for Ms. J, as it solidified who she sees herself becoming with the necessary resources that let her know that this narrated identity is possible. At the same time, the materials presented a discontinuity for Ms. J, as she recognized that teaching her students to play the game to change the game from a passive role was not who she says she is or wanted to be as a math teacher. For Ms. J, the act of “doing” critical statistics with her students was identity work that helped her author who she is becoming. For Ms. J, her development as a critical statistics educator drove her purpose for teaching statistics.

*Opportunistic Teaching Moments***.** For Ms. J, the designed materials also prompted an opportunity to teach her students about critical contexts *within* the math classroom. When asked in the pre-interview what she thought the role of the math teacher was, Ms. J shared,

> To teach how to, to teach students how to solve problems. And all problems aren't math, of course. But I think if students can learn to, how to approach problems, that prepares them for lots of things. And when I say that, I mean, mathematically, but I also mean critically, to be able to think through things and reason.

She knew she wanted her students to apply their math knowledge to help them view the world critically, but to her, those applications would come later, in the future, outside of the math classroom where she and her students interacted. Her job was to “prepare them”, to help them “to be able to think through things and reason”, but not necessarily have them become prepared or think through and reason with things within the math classroom space.

In the post-interview, Ms. J shared multiple narrations about specific moments that occurred with her students that came to school for face-to-face instruction. In one such instance, Ms. J shared about her student Aiden, a white male, who struggled to find articles in Lesson 3 *(What Does Data Say About Me?)* that related to his identity. She shared how during the lesson, the designed materials presented an opportunistic teaching moment to have a conversation about how white Americans are less aware of their racial identity. Ms. J shared,

> Oh, so we can have candid conversations. But anyway, oh, Aiden was like, "[Ms. J], it really was hard for me to find articles." He's like, "I really don't think about how I identify." And I was like, interesting. Wow.

Within the same narration, Ms. J shared about a fellow teacher in her district who was a white, middle-aged, female who had a similar experience to recognizing her whiteness as part of her identity as her student. She stated,

> You know, it was so ironic, because I know, even in the other workshops, that there was a teacher who said, that, you know, over this last year with everything that has happened, that she has realized she never knew her [racial identity]. She said [in the workshop], ‘I never think about the fact that I am a white female’, and it kind of came up, but then with Aiden, it was just like, wow, yeah, yeah.

To Ms. J, she drew a connection between her interaction with the white female teacher in her culturally relevant professional development workshop and her white male student, Aiden. She saw this moment of similar struggle to recognize his whiteness as an opportunistic teaching moment to support Aiden in his journey to learn about his identity. In reflecting about the lesson, she mentioned, “And again, I think that when someone asks them about their identity going forward, they will be much.... It'll be a much easier conversation as far as them knowing [who they are] and being able to say ‘I'm this, this, this and this’.

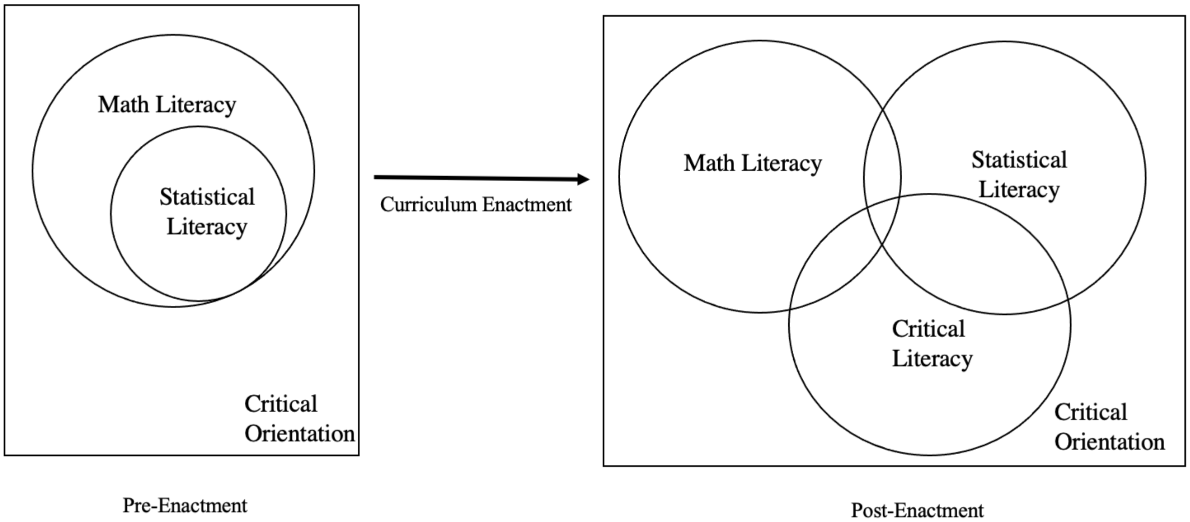
The CSL materials provided a moment of discontinuity in the identity work of Ms. J as a critical statistics educator. Rather than “prepare” students to learn about their identities after they matriculate out of the K-12 education system, Ms. J saw how the CSL materials provided the opportunity to teach students about these elements *now.* She connected the privilege of her middle-aged colleague to not have to be aware of her race with the future privilege of her student, and saw that as a reason to remove the barrier for her student inside her math classroom. In reflecting, she named the purpose for doing so clearly, in hopes that her students would be able to easily identify themselves in the future. For Ms. J, the materials provided these opportunistic moments of teaching, which reified for Ms. J the reason why the “prepare and wait” model of teaching mathematics devoid of critical contexts can be harmful for students. Ms. J’s purpose for teaching was, again, tied to her development as a critical statistics educator.

Identity Work Made Visible for Ms. J.

Ms. J’s initial perspective of critical statistical literacy viewed statistical literacy as a subset of math literacy, as she also held a procedural view of statistics. Ms. J was clear that she did not have a lot of prior experience with statistics or teaching statistics in her pre-interview, and that despite standards for teaching statistics having been in courses she was charged with teaching for 10 years, she rarely taught the content because of her lack of statistical knowledge for teaching. To Ms. J, statistical literacy could only be taught as a list of calculations to memorize rather than concepts to understand. Simultaneously, Ms. J’s critical orientation was based entirely in her historicity as a Black, female, mother who was previously employed as an accountant. Her identity as Black person provided Ms. J the lens to see systemic injustices based on race, while her identity as a female helped her to see systemic injustices based on gender, and her identity as a mother connected her view of the treatment of her students with the treatment of her own children. She approached her work as a mathematics teacher in reference to her historicity, as a way of being. Yet, critical literacy was not a source for teaching math nor statistical literacy. Ms. J had never used data based on race, gender, or class, or other "critical contexts" to teach math or statistics content before.

Post enactment, Ms. J began to view math and statistical literacy as literacies that were not mutually exclusive. This shift in perspective was made visible as she discussed how the resources showed her that statistics was "more than just the numbers" to her. She began to view statistics as a concept that was different from mathematics because of its ability to inspect and enact systemic injustices using specifically chosen data sets. In her post-enactment, Ms. J also shared how she wanted to teach using critical contexts but did not have examples and did not know how. Critical literacy shifted for Ms. J from only a way of being in the math classroom to a source of data one could use to create resources for teaching math and statistical literacy. In this, critical literacy became a both/and for Ms. J, as it informed who she said she was as a Black math teacher and also the content she shared with her students.

Figure 5.3. ***Ms. J's shifted perspectives of critical statistical literacy components as evidenced by identity work made visible by resources.***



Cross Case Analysis: Patterns in Identity Work of Critical Statistical Educators.

Above, I described each teacher as an individual case to show how their identity work was made visible by their use of the resources. In this section, I now use these individual cases to complete a cross case analysis that looks at the patterns across the three teachers' experiences with the resources to show how their historicity and teacher knowledges impacted how they developed as critical statistical educators, with specific focus on: (a) each teacher's perception of themselves as a consumer and producer of statistics (b) each teacher's perception of themselves as a statistics teacher and the purpose of teaching statistics, and (c) each teacher's perception of themselves as a critical statistics educator.

For the purposes of this paper, I again refer to the conceptual framework that centers on math literacy, statistical literacy, and critical literacy, and provide a brief review. Math literacy is akin to math knowledge for teaching (Hill et al., 2007), in which teachers have the math content knowledge as well as pedagogical methods for teaching mathematics to their students. As there has been considerable work in clarifying the differences and recognizing the tension between math and statistical literacy (Moore & Cobb, 2000), statistical literacy incorporates the statistical ideas and concepts needed for the consumption and production of data, including the mathematical procedures to carry out these ideas, and the statistical knowledge for teaching students these same big ideas. Critical literacy refers to one's ability to read and write the world, as well as view yourself within that world. At the intersection of these three literacies lies critical statistical literacy, in which a citizen must be able to understand the statistical ideas in order to consume and produce data that occur around themselves in the world using mathematical procedures to carry out statistical calculations driven by context in order to become aware of the systemic injustices within the world, and act to disrupt them for the betterment of those people marginalized through oppression.

In Table 5.1, I share salient quotes from each teacher's pre- and post-enactment that demonstrate the identity work that was made visible by the use of the resources. I shared quotes that embodied the identity work related to the following three perspectives: (a) each teacher's perception of themselves as a consumer and producer of statistics (b) each teacher's perception of themselves as a statistics teacher, and (c) each teacher's perception of themselves as a critical statistics educator. The quotes in the table demonstrate how each teacher's identity work centered on their historicity with statistics, with teaching statistics, and with critical statistics.

Table 5.1 ***Pre- and Post-enactment quotes that embody the identity work made visible by each teacher***

	Perspective of Statistics *Pre-Enactment*	Perspective of Statistics *Post-Enactment*	Perspective of Self as Statistics Educator *Pre-Enactment*	Perspective of Self as Statistics Educator *Post-Enactment*	Perspective of Self as Critical Statistics Educator *Pre-Enactment*	Perspective of Self as Critical Statistics Educator *Post-Enactment*
Mr. G	For one, I mean, there's a math component, definitely to learn certain math components that go with statistics. But then also, bigger picture wise to be able to just understand basically things that are put out by the media as they get older.	To kind of just prepare students to be able to understand and try to think of the word I'm trying to use to interpret things, like the media and other things that are kind of presented out to them, but they're not just kind of blindly accepting things that are presented to them."	I'm just trying to pick something that might pique their interest…. Typically, I fall into sports, like I pick stuff that has to do with sports, which might leave out people not interested in sports a lot of times."	Um, I mean, just overall, as a math teacher, I've tried to, like use, like real life things. But like I said, That's more of like, math or sports or something very, like surface level. So I guess this definitely kind of challenges me to look at bigger topics that would just have an actual meaning outside of just getting the math task done.	I mean, I don't know how much that comes up in the math class. I know you, if you're looking at, like real life data definitely presents an opportunity. …we don't really get into it that much.	Yeah, I mean, like, I definitely know [systemic barriers] exists, but I just don't know, like, why or …what could change, but I guess through this unit definitely presented some different opportunities to be able to like use those topics to also present math topics kinda intertwined in those.
Ms. H	I still think about being able to think through processes and problems. So anytime we're given a problem, there's certain steps and things that we have to take or may need to take based on what we're given.	So I guess, you know, when you think about statistics, it does have that mathematical piece to it. But to me, it just embodies, of course, so much more than math.	"…Being able to see statistics in action. And where you would use it, and that kind of thing."	"...In the math classroom, I think it requires us just digging, instead of just taking the numbers at face value, being able to figure out where the numbers came from to begin with. So going back into the data that was collected, how it was collected, and how it can influence the numbers that you actually see. So what's the math that's produced from it	I think it helps with thinking, because instead of taking everything that they see, as fact, they can…maybe dissect a little further… It helps with decision making…Just developing those thinking skills like, either you're gonna take something and believe it, Or maybe you should just think about what you see and dig a little deeper."	"So I guess, you know, when you think about statistics, it does have that mathematical piece to it. But to me, it just embodies, of course, so much more than math…Everybody can take numbers and tell a different story.
Ms. J	I think the purpose of learning statistics is to be able to understand how numbers work. Because I think statistics is like it's real… And all problems aren't math, of course. But I think if students can learn to, how to approach problems, that prepares them for lots of things. And when I say that, I mean, mathematically, but I also mean critically, to be able to think through things and reason."	I think overall, in addition to learning the statistics content, which was the intent, …this was much more real and much more relevant. And so they were able to take that and move forward with it, especially when they needed to categorize those different tests, so in addition to the statistics, …but the critical content, I think, really was eye opening for them, as well as the interpersonal content piece.	"You're asking me that about such lessons in my pure math classes….But a lot of times, those are just pure math….I think the purpose of learning statistics is to be able to understand how numbers work. Because I think statistics is like it's real. "	The purpose of teaching statistics is so that you know how to gather, analyze, and interpret data.	The other thing is that, it can help give you perspective. Because when people start throwing statistics, you have to think about it as far as how they did it. Because a lot of times you can get data that's skewed. And so I think that's interesting as well when kids start to see, well, who said it and who did that study? And those types of things. I think it helps them to understand numbers in everyday life.	It hasn't really changed my philosophy; it's more cemented it. Because I pride myself on being able to get to know and to identify with my students. And I think this has helped me to realize, like we've said before, how many tiers and how many levels there are to them…it helps me to realize even more so that students want to embrace who they are."

Being a Consumer and Producer of Statistics

In looking across the identity work of the three cases in regard to the teachers recognizing their role as both consumers and producers of statistics, there were similarities and differences. All three teachers were aware that they were consumers of statistics, yet less cognizant of themselves as producers of statistics. Their awareness was largely dependent on each teacher's historicity and their knowledge.

Mr. G shared how the context of a data saturated year filled with graphics from COVID-19 and the presidential election in his personal life provided impetus for him wanting to make sure his students could leave understanding how to absorb large amounts of statistical data without taking it as absolute truth, and how this was prompted by *his* need to understand the large amounts of data he was absorbing daily. Ms. H and Ms. J also shared similar sentiments regarding how the current political and health crisis climate heightened their awareness of their above average consumption of statistical data. Therefore, their awareness of their roles as consumers of data was largely tied to their statistical literacy, with their acknowledgement of how their ability to read graphs and tables had assisted them in this data-heavy year. Their role as consumers was also tied to their critical statistical literacy, or their ability to view the world and engage with statistics that discussed systemic injustices. Mr. G demonstrated this relationship best by sharing,

> When we have years like we've had the past year…whether it's presidential elections, or COVID data…I mean, some of those graphs I'll look at, I don't even understand as a math teacher. So I can't imagine what a normal, everyday person who hates math thinks when they see certain graphs and tables.

The statistics they described that they were consuming were not benign statistics, but ones of great peril that impacted the daily lives of the citizens of our country. Their consumption was rooted in their knowledge of critical statistics, and the way they consumed these statistics was rooted in their historicity.

For example, Mr. G's historicity as a privileged white male in society directed his primary consumption of the COVID/presidential data into "complicated statistical content" only. Yet these statistics he was consuming illustrated serious power imbalances, specifically, in terms of COVID, to marginalized populations of people. By filing this in complicated statistical content, Mr. G missed the critical nature, the life-or-death nature of the statistics he was reading, for the populations in question, keeping him from seeing the connection to the systemic injustices present in the statistics he consumed.

There was less awareness from the three teachers of their role in being producers of statistics, and this awareness was also dependent on their historicity. As a former engineer, Ms. H shared how she often used statistics in her work. She drew from her lived experiences to know that the production of statistics *was* important for her at one time, but was less aware of her role in producing statistics *now*. She knew that her production of statistics as an engineer hinged on her ability to collect quality data. Her historicity then matches her purpose for teaching statistics, which was tied greatly in helping her students recognize their role as producers of data. She wanted her students to "think critically" about how data was collected, and to know that the quality of a statistic rested on the procedures used to collect and create it. This was her reason for wanting to make sure her students "mastered" the procedures for calculating different statistical measures, so that they could be sure of the quality of the data they were producing. This also aligns with her self-expressed confidence in her statistical content knowledge, which was high and she attributed to her "statistical background" (see Table 5.2).

Ms. H did not discuss her role as a producer of statistics within the system of education, the quality of data analysis she created in her work as a teacher, how her production of data impacted the future lives of students, or how her identity as a Black woman teaching in a STEM content was tied to the production of statistics.

Ms. J also had prior job experience that produced statistics daily, yet she expressed a lower self-confidence in statistical content knowledge. A significant portion of the narrations around being producers of statistics involved their past use of statistics without recognizing their present involvement in being a person who actively produces data, either based on their identity or being a teacher in the system of education that produces data on students daily.

From the narrations shared, Mr. G was unaware of his active role in the creation of data. While he specifically shared many narrations of his coaching experience and how he tied this into his teaching, he did not share any narrations about being a coach and his role in producing data. The one example in the data of connecting with the production of data for Mr. G came from his shared narrations around "his" test scores from previous courses, and how these metrics were used to decide which courses he would teach the following year.

Table 5.2 Self-Expressed Confidence Levels by Teacher

Specific Teacher Knowledges	**Mr. G** *Pre/Post*	**Ms. H** *Pre/Post*	**Ms. J** *Pre/Post*
Statistical Content Knowledge (SCK)	3.5/4	4/4	2.5/3
Pedagogical Knowledge for Teaching Statistics (PKTS)	4/4	3/3	2/2.5
Knowledge of Critical Statistics (KCS)	3/2	3/4	4/4

Ms. J shared openly about her role as a producer of statistics as connected to her being an educator. Specifically, Ms. J referenced how statistics were produced by teachers that tracked students into classrooms that were segregated by race, class, and gender. She was aware of her role as an educator in being someone who often produced these statistics that perpetuated the segregated system of education, and her knowledge spurred her to action to try to use statistics to benefit her students of color by providing entry into accelerated math classes that they did not have access to. She also recognized how being a producer of statistics was tied to her everyday actions as a human being, but specifically as a teacher, that contributed to the unjust practices that are often collected and reflected in statistical data. It was evident in her post-interview that she knew that being a Black, female math teacher made her a statistic and her everyday work of teaching children in her skin produced statistics tied to her identity. For Ms. J, her role as a producer of statistics was related to her knowledge of critical statistics, as she recognized how data was being weaponized against her Black and brown students as a gatekeeper for accelerated math courses, as they were against her as a student. For her, this was related to her historicity as a Black woman, as she identified with her students who were being oppressed by this harmful use of statistics.

The Purpose of Teaching Statistics

For all three participants, the role of being a consumer of statistics was related to their perspective for the purpose of teaching statistics. For Mr. G, his goal was to "to prepare students to be able to understand and try to …interpret things that like the media and other things that are presented to them, so they're not just blindly accepting things that are presented to them." For him, his goal for teaching statistics was to help his students become critical consumers of data. Ms. H also shared this same sentiment, that her view of the importance of statistics was in assuring that her students knew how to question and reason with the statistics shared with them using healthy skepticism (Lee & Tran, 2015). Ms. J also wanted to help her students develop healthy skepticism, but with a specific purpose of teaching her Black and brown students about the systems used to marginalize them, and to teach her white students that there should be no racial center. She felt the purpose of teaching all mathematics, but especially statistics, was in helping students see the world.

Across all three cases, the teachers grasped the significance of the correlation between being able to digest statistics evaluatively and one's full-embodied citizenship as described in the GAISE framework (Franklin et al., 2007). Through each teacher's narration there was an understanding of the gravity of students learning statistics in order to be able to "thrive in the world" (p. 2) that they would soon experience. They all knew they wanted students to be able to consume data well. They knew that students would soon be reading data about science that would impact their health, in their workplace that would impact their professions, about identity specific data that would impact their personal choices, and data that would impact their citizenship. It was their own experiences with consuming data that drove all three teachers to engage with the materials, so their pedagogical knowledge for teaching statistics was tied to their own consumption of data. Mr. G, Ms. H, and Ms. J wanted to be sure their students had access to the statistical tools and habits of mind (Lee & Tran, 2015) they would need to engage in a world that would use data to convince them of falsities without inspection.

Despite all three teachers comprehending the seriousness of students being able to understand statistics that were complex in nature and rooted in topics that centered on systemic injustices, all three teachers did not recognize their role in reproducing critically statistically illiterate students, and how this was tied to their historicity as educators. Ms. H and Ms. J both shared how they often did not teach statistics at all in their courses due to their lack of confidence in their pedagogical and/or content knowledge to teach it well, which aligns with the findings of Shaughnessy (2007) that shared that many teachers do not teach statistics as it is tied to their lack of confidence in statistics, teaching statistics, or available resources. Both teachers discussed the pressure of pacing as a reason to push the content out of the course, leaving statistics to the end and focusing on other content areas. Mr. G expressed that he had more confidence in both his pedagogical knowledge for teaching statistics due to his experience in teaching statistics through many courses. While his pedagogical skills allowed for the production of statistically literate students, he was unaware of his role in producing critical statistically illiterate students who were unprepared to consume the data of the world. Therefore, the teachers' historicity as data consumers and producers was tied to their pedagogical knowledge for teaching statistics as well as their critical statistical il/literacy.

Becoming a Critical Statistics Educator

For all three teachers, their view of themselves as consumers and producers of data as well as their purpose for teaching statistics was tied to their identity work in becoming critical statistics educators. However, there was a significant amount of variability among the teachers in being able to use their personally powerful moments as consumers and producers of data as a motivator to change their pedagogical practice. They all wanted to create critical consumers. They recognized their role as educators in that process. Prior to the resources, none of the teachers changed their pedagogical approaches to help their students become the critical consumers they desired for society, and their reasons for doing so varied based on their differing understanding of their knowledge of critical statistics, awareness of the interaction between their knowledge of critical statistics and their consumption and production of statistical data, and their purpose for teaching.

Both Mr. G and Ms. H reduced lessons due to their own discomfort. For Mr. G, his discomfort was centered on the need to avoid the perceived conflict that comes with discussing non-benign contexts such as race, gender, or class. Mr. G shared that this was due to his identity in being a white male. Here, his historicity as a white male caused him to whitewash the curricular design that allowed students to explore their own identity, and in doing so he made the critical lessons benign. This aligns with the work of Silverman and Thompson (2006), as Mr. G is not able to decenter his own historicity to recognize the pedagogically powerful idea in having students explore and investigate critical data. By not decentering to create space for the pedagogically powerful idea of investigating critical data, Mr. G inadvertently stifled his own identity work and the identity work of his students as well. He expressed that he lacked confidence in his knowledge of critical statistics, and his self-expressed confidence even decreased post-enactment. This provides evidence that some of the materials did not support the identity work of Mr. G in developing his identity-in-practice as a CSE.

When asked about this, he shared about the student who used a chi-squared test of homogeneity to look at incarceration rates, stating, "I really like the investigation with the chi squared, like, I mean, I really like it, but I personally do not know, like, why that is, so I feel like I just… I mean, I just don't know." Mr. G and his student has done some significant statistical analysis that unearthed evidence of a systemic injustice. For him, this was a personally powerful moment with statistics (Silverman & Thompson, 2006; Groth, 2013), and it may have also been for his student. He was uncomfortable questioning or reasoning about why the data displayed the patterns or to make inferences around the pattern to name the reason for the existence of it as the injustice of racism. His discomfort with making inferences seems to imply his lack of consumption of this type of data, which is tied to his historicity as a white male. Yet, because his student chose the data, he did not have the ability to not engage with what the data showed. This seemed to provide a moment of dissonance for Mr. G, as he was unable to decenter his own historicity as a white male and the types of data he normally consumed and produced. At the same time, there is not evidence in the data that Mr. G was unable to decenter his own perspective on this personally powerful moment to draw towards a pedagogically powerful idea (Groth, 2013; Silverman & Thompson, 2006).

Ms. H felt discomfort with the resources in a different way. Ms. H shared that she amended the lessons by redacting some questions and by modeling the exploration of data for her students rather than letting them explore on their own. Specifically, Ms. H shared how she was uncomfortable with her students using the technology tool, CODAP, to explore data. She stated that this reason was due to her students being in an "inclusion" class that contained a majority of learners who were labeled as Exceptional Children (EC). She said, "I just didn't feel comfortable enough with some of them to actually use the CODAP themselves."

For Ms. H, truncating lesson five's initial design meant that students were no longer the ones exploring the data, and it moved the lesson from dialogic and student-centered to didactic and teacher-centered. Ms. H also changed the database that the students used in CODAP to one that was more benign about the characteristics of roller coasters rather than the one included in the lesson that focused on COVID-19 data by race and geographical location. For her, the discomfort came in her lack of pedagogical knowledge for teaching statistics, or perhaps her technological pedagogical content knowledge. Her lack of confidence in teaching statistics with technology and the population of students being labeled EC in her class caused her to diminish the criticality of the lessons and the student experience with those lessons. It is unclear in the data as to whether or not the modeled exploration was a personally powerful idea for Ms. H, as it seems that her discomfort with the technology tool overshadowed her ability to explore the data in a meaningfully powerful way. For Ms. H, the incorporation of the CODAP technology did not support the identity work of Ms. H as a critical statistics educator.

In both of these cases, the teachers' historicity prompted a reduction of the curricular design and also the possible identity work for them as critical statistics educators. In reducing the lessons due to their discomfort, the teachers created learning opportunities that were more benign in comparison to the original design. In both cases, there is evidence that the materials did not show linear development of critical statistical literacy in teachers, but that there were some components of the design that caused the educators to not develop as far in their identity work as CSEs. Yet, the cases diverge in creating personally powerful moments for the teachers with statistical data. For Mr. G, the moment was personally powerful, although possibly negative. For Ms. H, the data is inconclusive on whether the CODAP teaching moment was personally powerful. Neither experience with the materials left teachers able to move these moments into pedagogically powerful ideas that developed their critical statistical literacy (Groth, 2013; Silverman & Thompson, 2006).

In contrast, Ms. J used her own knowledge directly relevant to critical statistics as a source for her lacking confidence in statistical content knowledge and pedagogical knowledge for teaching statistics. In the post-interview, Ms. J shared how she was unfamiliar with CODAP, and she was uncomfortable with the resource herself, but she was determined to help her students explore the COVID data by race. She reached out to her colleagues for help with the technology to assure that her students would receive the fully robust experience of exploring the data and be able to use the tool successfully to surfacing the patterns that pointed towards systemic injustices within the data set. Ms. J contacted me for a one-on-one tutorial, and she practiced on her own with the roller coaster data set prior to teaching her students using the one on COVID-19 by race. For Ms. J, she was motivated by the personally powerful moments with data that she wanted for herself and for her students, and this drove her to gather resources to ensure she could support the pedagogically powerful idea of using CODAP to explore critical data.

Similarly, Ms. J also shared how she was not sure how Lesson 3 (What Does Data Say About Me) would go or how to teach it, but she wanted her students to experience the fullness of the complexity of exploring their identities with data. She shared how she had recently completed the social identity wheel in her CRP professional learning group, and it had been a personally powerful moment for her as a learner and a colleague. She shared how for her it was interesting to see herself through the eyes of the world using so many different lenses, and how one of her colleagues struggled with naming their racial identity in being white. This was a personally powerful moment for her in regard to her identity development, and she was able to use this moment to decenter her own experience to recognize the social identity wheel as a pedagogically powerful idea she wanted to share with her students. In doing so, Ms. J expanded her confidence in her pedagogical knowledge for teaching statistics and her statistical content knowledge as it related to her historicity, as well as her recent experiences as an identity learner (Aguirre, Mayfield-Ingram, & Martin, 2013).

CHAPTER VI: DISCUSSION & CONCLUSION

The present study examined the identity work made visible of three secondary mathematics teachers who enacted a curriculum designed to develop critical statistical literacy in a new fourth-level math course. Through iterative and retroactive analysis, I investigated differences in two chronological moments in time in which the teacher educators shared narrations about their identities-in-practice as critical statistical educators, as well as how and in what ways they discussed the key features of the materials that reverberated from their experience.

Through the design process, I committed to designing materials that incorporated nine key features that I deduced from the literature as well as from my own lived experiences as a Black female mathematician and educator that would support the development of critical statistical literacy. In my first analysis, I examined which of these key features resonated with teachers, and how they discussed them. Findings indicated that the specific key features that prompted the most discussion were (a) using identity markers as data, (b) exploring data relevant to self and the world, (c) the use of scientific and newspaper articles, as well as additional media as sources for learning, and (d) the exploration of real, "messy" data (Doctorow, 2008; Langrall, et al., 2019), three of which were the components most centered on making apparent known systemic injustices through analysis of data informed by the teacher's a) role as a consumer and producer, b) perspective on the purpose of teaching statistics; and c) perspective on the purpose of teaching critical statistics. Findings also suggest that, for these educators, these key features correlated with their development of critical statistical literacy by impacting specific elements of their statistical literacy, critical literacy, math literacy, and therefore critical statistical literacy, as it relates to their historicity.

In the second phase of analysis, I examined two chronological narrated identities-in-practice for each educator, pre- and post-enactment to look for evidence of identity work made visible through the enactment of the curricular materials. Findings suggest that teacher historicity and their knowledge bases mattered for their development as critical statistics educators as it relates to their understanding of their (a) role as consumers and producers of data, (b) purpose for teaching statistics, (c) purpose for teaching critical statistics. These findings align with the body of literature on math teacher identity work to show that statistics teacher identities are 1) related to their math teacher identities, 2) based on their experiences with statistics (Drake & Sherin, 2006), 3) racialized, engendered, and classist (Aguirre, Mayfield-Ingram, & Martin, 2013), and 4) inform their views on teaching and the pedagogical choices they make for their students (Aguirre, Mayfield-Ingram, & Martin, 2013), through the lens of these three elements.

The findings of these chapters add to the literature found in previous studies that find the development of critical consciousness in math classes to be successful (Gutstein, 2006; Yang, 2009) and unsuccessful (Brantlinger, 2011, 2013; Esmonde, 2014) by paying specific attention to the historicity and lived experiences of math teachers' identity work (Esmonde, 2014; Kokka, 2017). This study also specifically addresses the identity work of statistics teachers (Whitaker, 2016) as critical statistics educators, serving as an existence proof of the need for statistics education research to take a socio-political turn (Gutiérrez, 2013) that embraces criticality in the learning and teaching of statistics (NCTM & TODOS, 2020) by studying the enactment of materials designed for the development of critical statistics with teachers as they investigate issues related to race, gender, and class with their students (Gutiérrez, 2002). Consistent with research on curricular use, my findings suggest that teachers, and the sociopolitical backdrop they are in, are a critical filter for how designers' intentions get implemented in classrooms (Remillard, 2005; Drake & Sherrin, 2006; Stein, et al., 2007).

This study is one of the few to investigate full-time teachers' implementation of critical statistics rather than focusing on work in after school programs (e.g., Turner et al., 2009), part time teachers (e.g., Gutstein, 2006), self-studies (e.g., Brantlinger, 2011), or other domains other than statistics such as Teaching Mathematics for Social Justice (TMfSJ) (Bartell, 2013; Gregson, 2013; Esmonde, 2014), while expanding the field of critical statistics from theoretical perspectives (Weiland, 2019) or undergraduate students (Bailey & McCulloch, 2019) to teachers. Lastly, these findings answer the call of the GAISE and SET frameworks (Franklin et al. 2007; Franklin et al., 2015) as an example of a statistics curriculum that centers on the democratic citizenship based on national and international recommendations and expands on this recommendation to center this curriculum on justice through the investigation of systemic injustices related to race, gender, and class (Frankenstein, 1989; Gal, 2002; Gutiérrez, 2013).

Contributions of this Study

Contributions to the Field of Statistics Education.

Data as a product of social systems. One contribution of this study expands the work of Konold and colleagues (2004) in the different perspectives one can take of data. In the original study, Konold and colleagues discuss four perspectives of data that students may have, each building from the previous: data as pointer; data as case value; data as classifier; and data as aggregate. In many statistics lessons, students are rushed to view data as aggregate, assuming a strong understanding of the underlying levels.

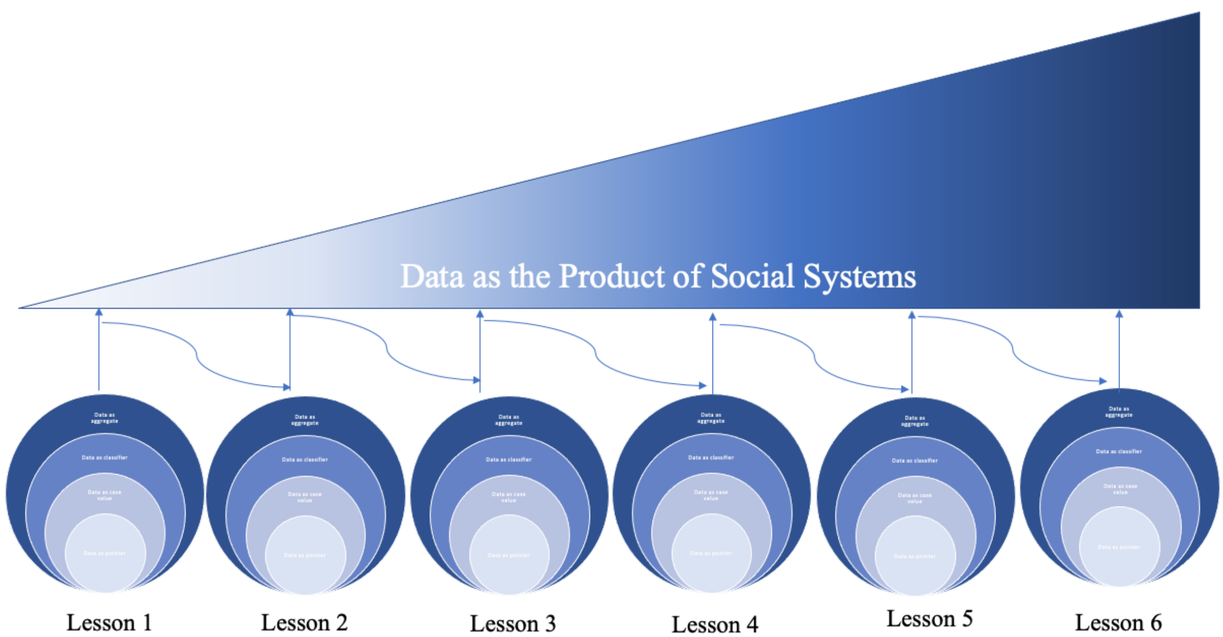
In this design, I intentionally wove these four different levels throughout each of the six lessons, attending to the different ways that students could use their identities as ways to view the different perspectives of data. In doing so, my hope was that students would not be rushed to abstraction; rather, they would be supported to view the four perspectives of data simultaneously, holding the tension between what it means to view data at all four levels.

For example, in Lesson 3 (*What Does Data Say About Me?*), the materials ask students to use their identity as data. This asked students to collect data using a Social Identity Wheel which served as a data collection event (e.g., *data as pointer*). Then, students used the identity information from the Social Identity Wheel to "provide information about the value of some attribute for each individual case" (Konold, et al., 2004, p. 6), (e.g., *data as case values)*. Next, they researched statistical studies based on specific attributes in their identity data, pushing them to view *data as classifiers*. Later in the lesson, students view distributions of data based on these statistical measures, focusing on viewing *data as aggregate*. Towards the end of the lesson, students are asked to return to view *data as classifiers*, to consider representativeness as a concept of a statistical study. Lastly, students are asked to reflect through viewing *data as a case value* to think about what it feels like to be misrepresented by a statistic, being an individual with a common characteristic within a dataset.

Meandering back and forth among the four different perspectives of data allowed the learner to entertain that the different perspectives of data tell different stories. Specifically, identifying or not identifying as a member of a dataset prompted students to consider not only themselves, but others like them and not like them as they imagined themselves as a case within a distribution.

There was also intentional design in expanding students' view of data to include a fifth perspective, *data as a product of social systems.* Konold and colleagues (2004) discussed *data as aggregate* "as a unity with emergent properties such as shape and center" (p. 6), focused on the statistical measures of a distribution. *Data as a product of social systems* prompts students to look across multiple instances of *data as aggregate* to draw conclusions across a sequence of experiences with data centered on the investigation of the same social system (e.g., COVID-19 disproportionately impacted people of color) (See Figure 6.1).

Figure 6.1 ***Data as a product of social systems***



By intentionally designing the curriculum so that students would return to the same conclusion from a different data perspective each time, the goal was to have learners begin to draw their own conclusions regarding the existence of social systemic injustices from the different perspectives on data and their multiple investigations of the system using different datasets. As students continued throughout the unit, the work of one perspective of data from a previous lesson strengthened the association with the conclusion, expanding the learner's view to encapsulate more and more perspectives of data on the same topic, but always attending to the existence of the systemic injustice. Ms. J picked up on this, and shared the following:

> Having them focus on their own identity first, because you made them pay attention to it then, um, because before, you know, they may have been due to do, but you have to focus in on your identity. And so once I'm focused in on it, and when I'm moving forward, I'm thinking about it more…I think this whole thing about different groups of people comes into play more. They think about it more so after that social identity, after they've seen those different pieces, that broadens their horizon to think about more people more often.

Focusing the perspective on *data as a product of social systems* in Lesson 3 (*What Does Data Say About Me?)* intensified the focus in subsequent lessons. By asking students to first think about themselves (*data as case value*), students focused on who they are as a data point, imagining themselves as a case value in a larger distribution. Because the data is an attribute of their identity, they are more attuned to continuing to hold the case value view of data throughout the lessons, referring back to themselves and their own perspectives on being an individual within a data set. Constantly comparing themselves to the distribution provides cognitive conflict, as the learner searches for invariance. The resolution of this cognitive conflict is a source of learning which is tied to their identity, and hopefully a prompt to act on the systemic injustice under investigation.

Slowly over the course of multiple lessons focused on the investigation of the same social system, students begin to shift the spotlight on themselves to consider the viewpoint of others, other people with identity markers not like them that may see a different story from the same data. This "decentering" (Silverman & Thompson, 2006) that is occurring shifts the focus of self compared to the aggregate to a simultaneous focus of group compared to the aggregate *and* self to group. Across their experiences with data sets drawing from the same social system, students begin to broaden their horizon to consider more and more people, and to recognize the systemic nature of data and the multiple perspectives it can hold simultaneously.

This decentering approach to data perspectives provided the opportunity for statistical inquiry, to ask questions about the multiple perspectives and distributions of data, and for the learner to draw their own conclusions without being rushed to abstraction or inference without justification and evidence. In the investigation of critical issues such as inequities based on race, gender, or class, allowing students to draw their own inferences through careful, repeated investigation was a necessary design commitment to not "push an agenda" or "indoctrinate" children, but rather provide a holistic view on data from which one can draw conclusions for themselves.

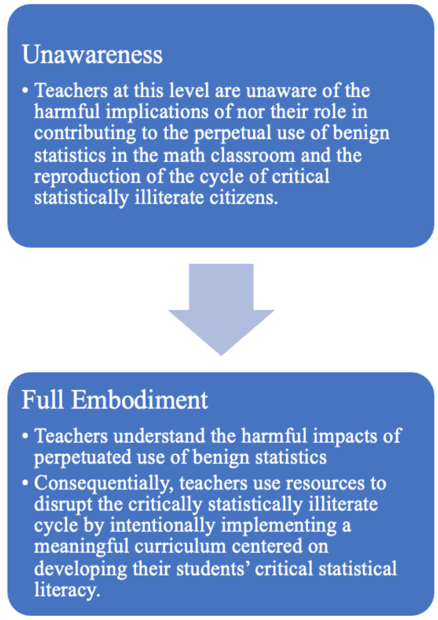
Using the *data as the product of social systems* perspective also connects two important bodies of literature for critical statistics teacher development of critical consciousness. First, as teachers investigate *data as the product of social systems* alongside their students they go through this same process of "decentering" that occurs as one moves from personally powerful ideas and experiences with statistics to pedagogically powerful ideas (Silverman & Thompson, 2006; Groth, 2013). Groth (2013) shared his perspective on this decentering saying, "Decentering allows teachers to understand potential student difficulties with content, providing a basis for developing and selecting strategies likely to support students' learning" (p. 26). As students take in multiple distributions of data from the same social system, teachers support their students' learning by helping their students recognize that distributions of data are a result of larger systems, which can be viewed from multiple perspectives based on one's identity and historicity.

This inclusion of *data as a product of social systems* is also tied to the development of critical consciousness literature (Gutiérrez, 1995; Cammarota, 2011; Hipolito-Delgado & Lee, 2007). As one develops their critical consciousness through the process of personal reflection, one moves towards collective social identification (Cammarota, 2011; Hipolito-Delgado & Lee, 2007). As teachers support their students' learning of systemic injustices through decentering their personally powerful ideas to pedagogically powerful ideas (Silverman & Thompson, 2006), they, too, develop their critical consciousness as critical statistics educators towards collective social identification with their class and the larger social groups with which they are connected. A teacher who has viewed data about the existence of a social system from the sole source of their identity is prompted to consider viewing systems from alternative perspectives, that of their students' identities. This is a source of cognitive dissonance, which must be resolved, as they compare the distributions understudy to their group of students as well as to themselves. How I conjecture that a teacher learns through these moments of cognitive dissonance is the next contribution of this study discussed.

Contributions to the Field of Critical Statistics Education.

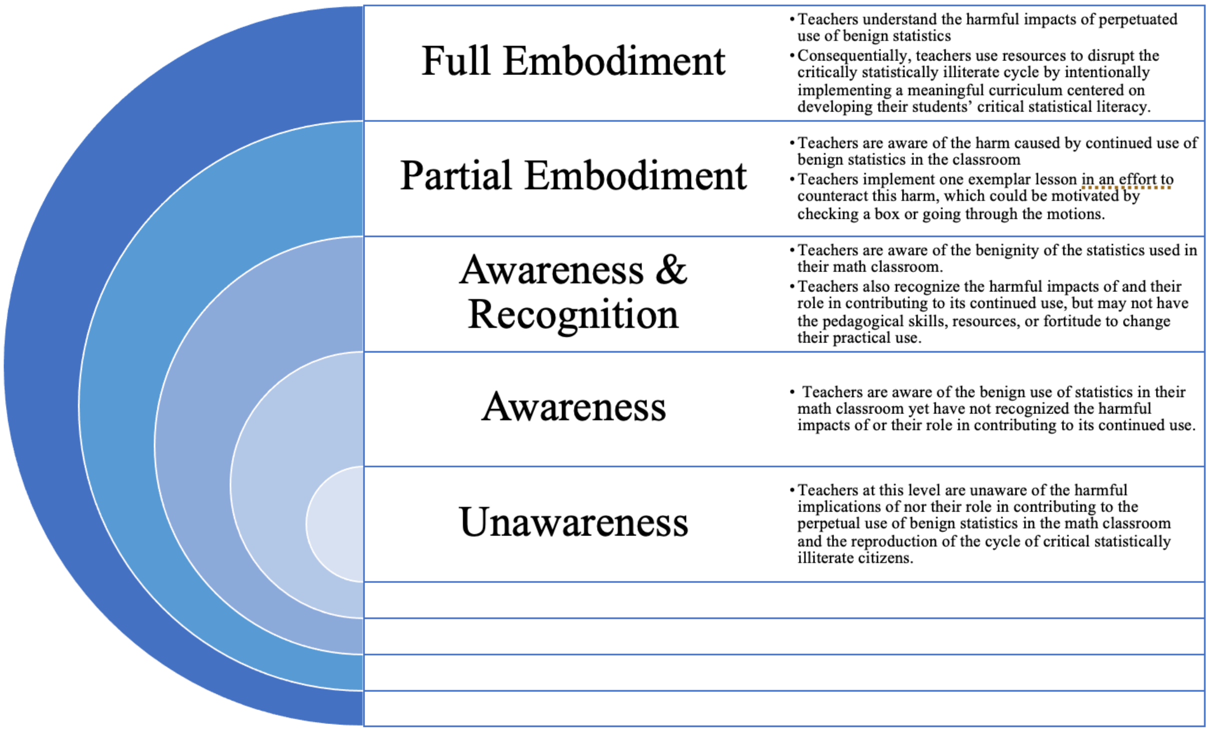
Conjectured Hypothetical Learning Trajectory. One goal of this design-based study was to develop theory regarding the development of critical statistical literacy of the secondary math teachers through the enactment of the CSL materials. My initial conjecture was that teachers would develop on a spectrum, based on their initial point of beginning prior to enactment and develop towards full embodiment.

Figure 6.2 Initial Hypothetical learning trajectory for critical statistics educators



Here, I provide a more developed conjecture through a more detailed and nuanced Hypothetical Learning Trajectory (HLT) regarding the spectrum of development as evidenced by the three teachers in this study.

Figure 6.3. ***Hypothetical Learning Trajectory describing the potential path of development for critical statistics educators through their enactment of available CSL materials***



The first level of the HLT towards becoming a critical statistics educator is unawareness. At this level, secondary math teachers are unaware of the harmful implications of their role in contributing to the perpetual use of benign statistics in the math classroom through the reproduction of critical statistically illiterate students and citizens. In this study, prior to the professional development on CSL, Mr. G began at a level of unawareness. For Mr. G, he had not troubled how his identity as a white male teacher who focused only on sports for statistical investigations reproduced critical statistically illiterate students.

The second level of the HLT is awareness, indicating that teachers are aware of the benignity of the statistics they teach in the math classroom. Ms. H began at a level of awareness. Due to her engagement in the Culturally Relevant Pedagogy Professional Development, she was aware of the homogeneity in her statistics lesson from previous years. Her hyperfocus on the procedural calculations due to her historicity as an engineer kept her from connecting how students studying statistics in this way would keep from developing the critical statistical literacy she desired them to have. Ms. H was unable to connect the difference in resources to the reproduction of critical statistically illiterate students through their continued practices of "frontloading procedures".

The third level of the HLT towards becoming a CSE is awareness and recognition. At this level, teachers both become aware of the benignity of their own lessons and recognize the harmful implications for the reproduction of critical statistically illiterate students. At the beginning of this study, Ms. J was both aware of the lack of criticality of her statistics lessons and recognized through the racial demographics of the different math classes in her school how her role as a teacher perpetuated educational injustice within her own school. Teachers who are at this third level have the desire to change, yet, due to some constraint (e.g., pedagogical knowledge, content knowledge, critical statistics knowledge, historicity, opposing environment, resources, or fortitude), the teacher does not make any curricular changes. For Ms. J, the interaction between her lack of statistical content knowledge, lack of statistical knowledge for teaching, and lack of empowerment to deviate from the provided curriculum from her local educational agency kept her from making these types of curricular changes.

The fourth level of the HLT is partial embodiment. A teacher who partially embodies development of being a critical statistical educator level has awareness and recognition and feels the need to act on their curricular choices to disrupt the reproductive cycle of critical statistically illiterate students and citizens. For some teachers, this need to act is achieved through alternating one lesson in a semester that investigates issues such as race, gender, or class, without connecting the work of teaching critical statistics to students to the reproductive cycle of critical statistical illiteracy. At the beginning of enactment, Mr. G moved to partial embodiment. After engaging in our professional development, he was moved to exchange his lessons from previous iterations of the fourth level math course to ones of more consequence, which he stated was his motivation for joining this study.

Ms. H developed to partial embodiment by the end of the enactment period. She was willing to exchange some noncritical lessons for ones like those she experienced in the CSL materials. Yet, she still felt the need to "go back and frontload" more procedural, statistical calculations. For Ms. H, her purpose for teaching statistics and lack of statistical knowledge for teaching constrained her from full embodiment.

The learning goal for critical statistics educators is full embodiment. Teachers who fully embody becoming a critical statistics educator are aware of the harmful acts of benign statistics, and consequentially amend their teaching of statistics to disrupt the reproduction of critical statistical illiteracy through their intentional choices of curricular materials that center on the development of critical statistical literacy in their students. There was some evidence that after the enactment of the CSL materials that Ms. J had developed to full embodiment. The proximity of the resources to who she said she wanted to be, or her narrated identity-in-practice, prompted for her a shift in her view of teaching, recognizing not only her role in the reproduction of critical statistical literacy in her students, but also her power to choose to teach her students critical statistics. She desired to teach her students about systemic issues and had an example from which to draw upon, spurring her to act on her curricular choices thus far. She was insistent in the interview of needing the resources to be public so that she could share them with colleagues across her school, district, and country, and continue to use them in her own classroom in the future.

A teacher's development on the HLT is nonlinear and not unidimensional. That is, a teacher may move towards partial embodiment due to the proximity and availability of resources, yet folded back to awareness. This was the case with Mr. G. While he moved to partial embodiment as he engaged with the CSL resources, at the end of the unit, he was unsure as to whether or not he would continue to investigate critical issues such as race, gender, or class in his statistics teaching. Having seen the CSL materials, he was aware of the lack of criticality in his prior lessons, but had not connected how his continued use of his prior materials perpetuated the cycle of critical statistical illiteracy. For him, the materials prompted both development towards and away from full embodiment as his identity and historicity were in tension with what it meant to become a critical statistical educator, the inclusion of discomfort and conflict in the classroom.

An additional contribution of this study tied to the HLT was found in how and in what ways teachers took up and made sense of the key features of the CSL materials and how it was tied to their identities as people. Specifically, there were significant implications between who they are, who they say they are, and who they are becoming in how they each consume and re/produce statistics. All three teachers had the desire for every student to be able to critically consume data, yet there were contextual barriers that kept them from incorporating non-benign statistics into their curricular designs. These findings align with the work of Shaughnessy (2007) and Brantlinger (2013) in that expecting teachers to bear the load of curricular design is an insurmountable weight, even for teachers with great desire and pedagogical skills. If statistics education is to take a sociopolitical turn, the weight of creating curricular resources that investigate issues such as race, gender, or class, must be distributed to those who support teachers in their work. This is not to say that teachers cannot create lessons that develop critical statistical literacy, but expecting them to do the majority of curricular design as well as the work of is irresponsible of curricular designers, teacher educators, and teacher education researchers.

In the work to disrupt the cycles of critical statistical illiteracy in the K-12 education pipeline, the reproduction of statistics as status quo has different timelines of grave consequence. For example, for Mr. G, as a "white male without a lot of ground to stand on" when it comes to talking about race, engaging with the materials was a major step in his development in the short term, yet long term, his development towards full embodiment could be very slow, as he must work against his own discomfort through the investigation of critical issues like race in math class. He may understand personally powerful ideas regarding statistics about race, gender, or class *now*, but not be able to support his students' learning of them at this moment. In not being able to "decenter" (Silverman & Thompson, 2006) his personal experiences into pedagogically powerful ones, there may not be enough time to disrupt the cycle of critical statistical illiteracy for his students (Cammarota, 2011; Hipolito-Delgado & Lee, 2007)

Short term, the length of time of his development is directly related to his reproduction of students who are critically statistically illiterate, as his students now are not able to benefit from his personally powerful ideas regarding systemic injustices. Long term, as Mr. G continues to take small incremental steps in his development, he will move further along the spectrum of embodiment, but possibly not far enough along to not reproduce statistics that perpetuate learning environments that allow only certain demographics of students (e.g., white, middle to upper class students) to be able to take upper-level math courses to even be able to investigate statistics in this way. Across the timeline of Mr. G's career as an educator, he may never reach full embodiment, continuing the process of gatekeeping of critical statistical literacy from the students under his tutelage.

Juxtaposing his development to that of Ms. J, who was already consuming statistics regarding the systemic injustices related to the demographics in students in upper-level math courses, her timeline for development towards full embodiment could be very different. As was evident through her post-interview and reflection, she developed quite rapidly towards full embodiment in just two weeks from first viewing the CSL materials until the post-interview, and at the end of those 14 days was aware of her role in disrupting the reproduction of critically statistically illiterate students and determined to do more. In the three student samples she shared of the final project for the unit, she was overcome by emotion at what students shared regarding their learning about themselves, others, and their world, as she repeatedly exclaimed, “Look at my kids!”. She was overjoyed by some of the inferences students were able to make through their investigation of issues like racial injustice, gender inequality, and the rights of the dis/abled. Part of this joy came from knowing she was producing students who were critically statistically literate and that she had played a role in disrupting the cycle of illiteracy.

Contributions to the work of Curriculum Design in Current Times.

At the time of this writing, the department of education in Florida released a list of textbooks and curriculum that are banned from use in math classrooms for teaching “prohibited topics” (Goldstein & Saul, 2022). Spearheaded by Governor DeSantis and Education Commissioner Richard Cocoran, this attack on math education has been framed as a parents’ right to choose, as Cocoran stated that every parent should expect “a world-class education without the fear of indoctrination or exposure to dangerous and divisive concepts in our classrooms” (Goldstein & Saul, 2022). Without a shadow of a doubt, the curriculum designed in this study would be added to this list for its central focus of having students investigate issues of race and systemic injustices as tied to the novel coronavirus. One interesting observation from this study came from the post-interview with Ms. J, in which she referenced, not the materials themselves, but the curriculum design as a way to “tiptoe” around the dog whistle of critical race anti-theorists by asking kids how they identify. By bringing identity into the math classroom, curricular materials provide the space for students to bring up race if this is an identity marker of interest they would like to explore more deeply. For others, their age, gender, or sexual orientation may be an identity they wish to explore more. By not explicitly asking students, “What is your race?” or “Why does your racial identity matter?”, the curricular materials open the door for teachers to talk about and investigate racial identity in their classrooms. Curriculum designers who want to support teachers, like those in Florida, who wish to teach students statistics through the investigation of critical issues, but need to be cautious due to the threat of lost livelihood may find value in this portion of the study.

Implications

When Dr. King said that the “arc of the moral universe bends towards justice”, he did not mean that the universe would bend towards justice on its own accord, without the work of many. The arc of a moral universe only bends because we, as those within the universe, work together to push it towards justice. With this in mind, what does a sociopolitical turn in statistics education look like and how does it bend the arc of the moral universe towards justice? A sociopolitical turn will not occur simply because there are theorizations that suggest the field should do so. There must be teachers willing to engage in this work through participation in professional learning and the implementation of curricular resources. There must be teacher educators who support in-service teachers through professional development focused on the development of critical statistical literacy, as well as teacher preparatory programs that prepare math teachers to enter the field as critical statistics educators. There must be curriculum designers to design curriculum with teachers and students in mind so that statistics can be used as a tool for justice. And yes, the field needs more theoretical perspectives on critical statistical literacy. It also needs more research that is focused on testing these theories in real classrooms, with real teachers and real students. I discuss the implications of this study through these granular lenses of how the field of statistics education can take a sociopolitical turn towards justice.

Implications for Teachers

For many educators, our early math experiences, our math identities as learners, and the powerful statistical moments of our past as consumers and producers of data shape us and are part of the reason we choose to teach, hoping to either reflect or deflect students towards a similar experience to our own. But, it is also our responsibility as statistics teachers to use these personally powerful moments to recognize the different stories that can be told with data, and to decenter ourselves in order to provide the robust experiences that our students need in order to develop critical statistical literacy. Without this act of decentering, we remain personally troubled by issues of the world, but inactive as agents of change that can do real work in dismantling systemic injustice for the next generation. We must recognize the power we hold in what our students experience as well as the bias that comes with the choices we make based on our historicity and math identities.

Particularly for teachers of students of color and students who wish to be their allies in racial justice, the work here has implications for the type of gatekeeping that we do as educators in creating a barrier from the experiences that our students need in order to be able to thrive in the modern world and seek justice against the statistics that are often used for their harm. If our classrooms are supposed to be imaginative rehearsals for the world, what does a benign statistical experience help them rehearse? If, as educators, we deny them the ability to rehearse their knowledge of statistics in a way that empowers them to seek justice and act, what are we preparing them for? When we feel discomfort about teaching a topic, we should first ask ourselves what is the source of our discomfort, and is this discomfort going to deny my students some skill they will need for the betterment of their life. Using the process of decentering through critical reflection may assist us as we do the work of engaging students in conversations that are different than the ones we had as math learners or engaging with data that shares a different story than the one we experienced through our historicity.

Second, there is real progress that can be made in the development of critical statistical literacy for students by carefully choosing data sets that allow students to draw from the same social systems across a unit of study. For teachers wanting to develop students' critical statistical literacy, implications from this study indicate that focusing on one social system, rather than a variety, proved to support students in investigating, conjecturing, and drawing their own conclusions about the existence of systemic injustices based on race. Specifically, when looking across the six lessons, by focusing on the disproportionate rate of infection of the novel coronavirus for BIPOC, students were able to explore the data and discover patterns that allowed them to draw their own conclusions. For some students, this propelled them to ask additional questions on their own as to why the disproportionate pattern they found existed (e.g., like with Mr. G's student who discovered the disproportionate rate of Indigenous people infected with COVID-19, and then looked up an article as to why).

Implications for Curricular Designers of Critical Materials.

This study has implications for curricular designers of critical statistical literacy materials, especially with the insights on the salience of teachers' identity work described in the section above. First, the use of identity as data was a productive way to prompt the development of critical statistical literacy, first in teachers, and then as filtered through the nature of teachers' identity development, possibly in students through how teachers then enacted the curriculum. Prompting students to think about themselves as a data point within a distribution that investigated critical issues on race supported their thinking about the way the world views them, causing a moment of personal reflection. Based on what the teachers' shared, This seemed to provide opportunities for learning about different perspectives on data (Konold et al., 2004) as well as the development of critical consciousness (Gutiérrez, 1995; Cammarotta, 2011), meeting mutually beneficial goals of critical statistical literacy.

The processes of becoming a critical statistics teacher (for teachers) and a critical statistics learner (for students) are dialectic in nature. As teachers watched their students engage with these ideas from different perspectives of data, they, too, began to draw conclusions regarding the existence of social systems. This was a powerful experience for some of the teachers in this study to figure out how to bring these types of contexts into the math classroom, again, developing their own critical statistical literacy. Future work should look at other key features that are prolific in developing critical statistical literacy and the dialectic nature of consequential learning for both teachers and their students.

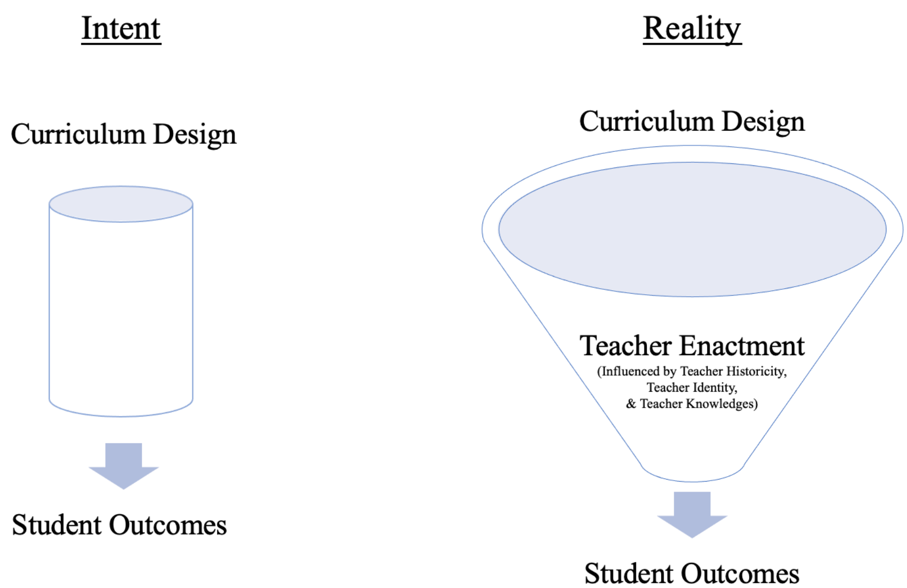
Second, exploratory data analysis provided a productive context to investigate data on critical topics as it allowed for the conjecturing of ideas without the pushing of a particular narrative. Providing students the opportunity to discover their own patterns that they saw within the data was an effective method to allow space for questioning, even past the questions that the teachers posed to them. The more students questioned and conjectured, the more conclusions they were able to draw on their own, regardless of teacher prompting. This was particularly important regarding the facilitation of the unit by teachers. One issue with the designing of curricular materials that focus on critical topics is the facilitation by teachers other than the designer who may cause harm to students through their teaching techniques that differ from the original intent. By focusing the unit on exploratory data analysis, the work of teachers was to help students discover patterns within the data they chose, rather than tell them the patterns they should see. This kept teachers from pushing a particular agenda with the data, and allowed the students to again, draw their own conclusions. In this current wave of resistance to teaching critical topics (e.g., Goldstein & Saul, 2022), future research should consider the teaching practices and techniques that allow for the investigation of critical topics through student-initiated exploration of data.

Following the expansion of the work of Konold and colleagues (2004) and the perspectives of data, designers should attend to the perspectives of data that they privilege, and use them as mechanisms for statistical inquiry as well as the development of critical consciousness. Statistical curriculum designers tend to rush towards abstraction, quickly moving to view data as aggregate without properly problematizing the other perspectives of data that undergird these views. This study demonstrates how developing the different perspectives of data without rushing to view *data as aggregate* provided beneficial moments of cognitive dissonance that were also able to be used as prompts for the development of critical consciousness. Future studies are needed to further understand how these different perspectives on data make visible the identity work of students and teachers as they develop critical statistical literacy.

Building on the work of Groth (2013) and Silverman and Thompson (2006), curriculum designers must consider how to design materials that draw a teacher to consciousness of their role as a consumer and producer of data in a way that helps them name their personally powerful moments with statistics. This must also happen in conjunction with designing materials that allow teachers to move past their personally powerful moments to recognize the importance of pedagogically powerful ideas that decenter their personal experiences with statistics to include the experiences that Black and brown students must have in order to disrupt the systems that are built to oppress them.

Lastly, as curriculum designers we often forefront the student experience and student outcomes in the design to determine if the materials are successful or not at creating a particular learning experience. Too often, we ignore the teacher implementation and enactment of the materials as part of the goal of the design, believing that the most important piece is the student outcomes. Focusing only on the student outcomes in curricular design will not produce the sociopolitical turn that we are looking for in statistics and math education (Gutiérrez, 2013). There is a plethora of literature that demonstrates how important the teacher is in determining the student experience of curricular materials (See Remillard, 2006; Stein, et al., 2007; Drake & Sherin, 2006; Aguirre, Mayfield-Ingram, & Martin, 2013 for pertinent examples), yet less attention is given to the opportunities for teacher learning through their enactment of the curricular resources. We typically jump to student outcomes and then blame teachers when student outcomes are not reproducible by curriculum across contexts. Perhaps the first step is to work with teachers before they implement the materials. Teachers act as a critical filter of the curriculum design (Figure 6.4), as they adapt, modify, and use their power as educators to choose what their students will experience, which is tied to their historicity and math and statistics teacher identities.

Figure 6.4 Intent versus Reality of Curriculum Design



For example, in lesson 5 of this unit, I provided teachers with a benign dataset on roller coasters to use for their own introduction to CODAP. I did not foresee how teachers would use this dataset in place of the one I intended students to use, to explore COVID data by race and geography. But teachers did replace the dataset. Had I considered teacher learning and implementation in the curriculum design phase, I would have provided a more critical data set for teachers to use for their own learning, which could have provided some students a more robust, critical statistical education experience.

Implications for Researchers.

The literature base for critical statistical literacy is under-theorized. The work in this study expands the literature on critical statistical literacy, adding a lens for math teacher identity work. This study seeks to contextualize some instances of where, with whom, in what spaces, and across what timelines this work could potentially occur, but this is just one instance of the combinations of such. While there are instances of theorizations about how to define critical statistical literacy (e.g., Weiland, 2019), for this study, it was important to draw from different bases of literature (e.g., critical mathematics, critical consciousness, statistical literacy) and my own lived experiences. Future work could center theorizations of critical statistical literacy from other critical theoretical frameworks, such as Critical Race Theory (Ladson-Billings & Tate, 1995), Black Excellence (Leonard & Martin, 2013), Queer Pedagogy (Waid & Turner, 2021; Pennell, 2020), Intersectionality (Collins, 2019; Joseph, 2020) or QuantCrit (Frank, et al., 2021).

There are many places in the literature that would support critical statistics researchers in agitating benign statistical ideas towards ones that are more critical. One such instance in this study expands on the work of Konold and colleagues (2004). The different perspectives of data was a productive cite for statistics education research to take a sociopolitical turn (Gutiérrez, 2013) as it provided students with windows and mirrors (Bolter & Gromola, 2003) to make sense of their own experiences, the experiences of others in regard to issues such as race, gender, or class (Gutiérrez, 2002), as well as learn meaningful statistics content in context (Konold & colleagues, 2004; Harradine, et al., 2011). Investigating instances where statistics is capable of being used as a tool to dismantle systems of oppression through the acquisition of critical statistical knowledge is a worthy endeavor for the field to take up.

Implications for Teacher Educators.

This same work also has implications for teacher educators. Teacher preparation programs predominantly do not focus more than one class, if that, in differentiating statistics and mathematics teaching practices (Shaughnessy, 2007). There is a healthy body of literature that demonstrates math teacher educators supporting the work of pre-service and practicing teachers in recognizing the experiences of others in mathematics, yet extremely less so in statistics education research. Due to the location of statistics being taught in math classrooms, math teacher educators must learn to support teachers' development of critical statistical literacy through this decentering as they move from personally to pedagogically powerful ideas (Silverman & Thompson, 2006; Groth, 2013).

Lastly, the literature has shown that one of the many issues for developing statistical literacy in the next generation of thinkers is the lack of professional development opportunities for teachers in the domain of statistics (Shaughnessy, 2007; Franklin, et al., 2015). The implications of this study show that materials designed for the development of critical statistical literacy served as an opportunity for professional learning without formal professional development, yet with some limitations. More opportunities for professional development and learning how to facilitate "critical conversations" could provide support for some teachers who find the investigation of critical topics interesting, but fear the conflict that could arise in the dialogue. Future research should look at specific ways to support teachers through professional development to facilitate critical conversations in math classes with the goal of developing critical statistical literacy.

Limitations & Future Research

In this section, I discuss the limitations of the knowledge generated from this study, and how this informs future research.

Due to the pandemic, I was not able to observe the teachers' enactments of the CSL materials to provide a more in-depth understanding of the teachers' development of critical statistical literacy. Observing the interactions that occurred between teachers and students as the CSL materials were enacted could have provided additional opportunities for teacher learning and understanding. For example, if I had recordings of the interactions that occurred between Ms. J, Aiden, and the other classmates when he shared it was difficult for him to find studies about his identity because he was white and he did not think about it very much, I would have been able to dig deeper into how this conversation impacted Ms. J's development as a critical statistics educator in prompting the need for this type of investigation in classroom settings. This would have allowed for the use of other methods, such as video stimulated recall, to prompt the teacher to critically reflect on the in-the-moment actions of the classroom observation as they were tied to their identity work. What insights could be learned in understanding the overlaps and disjunctures that occur between a teacher's narrated identity in practice and their embodied (performed) identity-in-practice as it relates to becoming a critical statistics educator? How does who teachers say they are impact what they do in the classroom? Future research could stem from the work of Tan and colleagues (2013) in investigating how a teacher's narrated identity-in-practice aligns with their embodied identity in practice, and how it is negotiated against a sociopolitical and sociohistorical backdrop of the contextual learning environment.

Drawing from the work of Esmonde (2014), Aguirre, Mayfield-Ingram, and Martin (2013), and Drake and Sherin (2006), teachers of statistics were first statistics learners, and this informs their views and practice on teaching. Combining this with the work of Silverman and Thompson (2006) and Groth (2013), teachers, like all people, have had early, personally powerful interactions with statistics that inform who they are, who they want to be, and who they are becoming as statistics teachers. The inclusion of the teachers' stories and early, powerful, experiences with statistics and mathematics from their youth could provide insight into how one's early and/or powerful experiences with statistics as a consumer and producer are tied to their statistics teacher identities and who they are becoming. Whether they have had significant formal learning opportunities in statistics (Shaughnessy, 2007) does not negate their consumption and production of data as citizens throughout their lives in our data-rich society. For example, it would have been compelling to know the stories around Ms. H's background as a Black, woman engineer to better understand what was solidified in her statistics identity and how that impacts her statistics teaching identity and practice now. What stories might teachers share about their personally powerful statistical moments from their early experiences with consuming and producing data? When these moments are critical in nature (e.g., regarding one's race, gender, or class), how do teachers draw upon these personally powerful moments to influence their statistics pedagogy? What are the instances when teachers are not able to decenter their personally powerful moments to pedagogically powerful ones, and how does this align with their historicity and identity as teachers and learners of statistics? What ought to be the role of curricular design in prompting teachers to move from personally to pedagogically powerful ideas? Future research should attend to statistics teacher identity as it relates to their early stories and narratives around being consumers and producers of data, and how this impacts their purpose for teaching (critical) statistics.

Additionally, looking at the interaction between the development of critical statistical literacy in students and their teachers' identity work could have provided additional insight regarding the negotiation of one's identity within the situated community of practice of the math classroom. While teachers were asked to bring student work samples with them to the interview, observing and interviewing students regarding their perspective of their learning of critical statistical literacy could have provided an impetus for learning for teachers as they listen to their students discuss why they feel learning critical statistics is personally important, possibly prompting further development of critical statistical literacy for teachers. For Ms. J, when she shared her students' projects regarding data on Black hair care, gender pay inequality, and misdiagnosis of ADHD in Mexican children, she was overwhelmed with emotion and pride about what her students were able to discover and communicate using statistical information. For her, her student learning was a driver for her own development as a teacher of critical statistics. Investigating student's development of critical statistical literacy in tandem with and catalyst for teacher development of critical statistical literacy is a promising area for future research.

The field of critical statistics education may benefit from additional empirical research to investigate teachers' enactment of CSL materials as it relates to the specific knowledges mentioned by Gal (2002) or those encompassed in Groth's (2008) statistics knowledge for teaching. The findings in this study suggest that there are additional knowledge bases that are not currently recognized within these models that have important implications for how teachers learn and teach statistics concepts, namely one's critical statistical knowledge for teaching (CSKT). A teacher's level of critical consciousness and historicity with such knowledge play a key role for their development of critical statistical literacy. A teacher's orientation towards critical statistics impacts their specialized content knowledge as well as their pedagogical knowledge for teaching, and their technological pedagogical content knowledge (TPACK) (Kurt, 2016). The use of additional research tools (e.g., measures to assess CSKT and its components as well as critical consciousness in teachers) could have provided additional insight in the learning acquisition of teachers in specific subsets of their statistical knowledge for teaching and critical consciousness development. For example, Ms. H and Ms. J had different reactions to using new technology tool (e.g., CODAP) to explore data about COVID-19 by race in the CSL materials. One teacher felt discomfort with the new tool, and the other sought additional resources (e.g., colleagues, tutorials, contact with researcher, individual learning sessions) to be sure that her students got to explore the data. It could be possible that Ms. J extended her TPACK as it relates to her CSKT, based on her desire for her students to develop critical statistical literacy. Research tools that measure a teachers' critical consciousness as it relates to their statistical content knowledge, pedagogical knowledge, or knowledge of systemic injustices could prove helpful for better understanding the development of critical statistical literacy in secondary math educators. How does one evaluate the critical statistical literacy of a secondary mathematics teacher? What knowledge bases are informed by one's critical consciousness and historicity of critical pedagogy? What are the expansive learning outcomes that should be prioritized when we focus on teachers' critical statistical literacy? The idea of creating a measure for Critical Statistical Knowledge for Teaching (CSKT) is one that should be taken up with care and caution. The use of such a measure may assist researchers in understanding the relationship between one's critical consciousness knowledge for teaching and one's statistical knowledge for teaching, but it could also provide evidence that teachers who have a desire to agitate the status quo of statistics education exist.

Due to the use of volunteer sampling focused on teachers willing to participate, all of the school settings within this study were demographically similar (e.g., diverse, predominantly students of color). While I looked at the demographics data of the students in the schools and classes that participated, I did not attend to the organizational structure or school climate of the teacher's settings to understand the environment that the teachers were in at their school site, and how the environment may have hindered or facilitated a teacher's ability to enact CSL materials. How might the identity work of a teacher in different school settings differ from those in this study? What does it look like to support the development of critical statistical literacy of a teacher who is in a micro/macro aggressive, supportive, or privileged environment? Future research should investigate ways that a teachers' classroom, school, district, and state climate facilitate or inhibit one's curricular choices related to the development of critical statistical literacy.

In the field of curriculum design, there is always the question of uptake and scalability. As a curriculum designer for the last 10 years, I have been reminded numerous times of why it is important to design curriculum for the majority, namely white, female teachers (US Department of Education, 2020). We design curriculum in this way to increase the possible number of students who can be exposed to math content that is meaningful (Borko, 2004). Yet, doing so, because of the demographics of the majority, means centering whiteness. As evidenced previously, with the potential long-term timelines of grave consequence of slow developing teachers, it plagues the question: What does it mean to center the design of curriculum for those who want to be disruptive of the status quo of how statistics is taught rather than centering the majority? How does the design of curriculum change when we center the teachers that have a developed critical consciousness in understanding that statistics and statistics education is political? If scalability was not the end desire, but instead uptake and development of CSL were centered, who would we be designing for and how would this impact the design? By not compromising on my commitment to center Black and brownness as excellence, I wonder what institutional support and partnerships are necessary to support the scalability without recentering the materials back to being benign, noncritical, and centered on whiteness. Future research should address how the design features of curriculum change when the focus is on scalability versus agitation of the status quo statistics experience.

Lastly, the work of this study seems to suggest that the knowledge bases on which critical consciousness can emerge are often not the same knowledge bases that are commonly used in math canon, but rather those found in the context of social studies (e.g., redlining, election mapping, incarceration rates, health disparities, COVID-19 data, etc.). Math teachers are trained to pull resources from particular knowledge bases, most of which have been whitewashed of the level of criticality needed to develop critical statistical literacy and "thrive in the modern world" (Franklin et al., 2007). Statistics is embedded and driven by context, while one of the primary goals of mathematics is to strip context away to look for and make use of structure (National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010, CCSS.MATH.PRACTICE.MP7). Despite literature differentiating statistics from mathematics which should trouble the resources teachers draw from to be "real and motivating" (Garfield & Ben-Zvi, 2006), due to its location in the math classroom, the study of statistics has inherited this whitewashed approach to statistics that strips away all criticality from the possible contexts that could be brought into the K-12 math classroom. Math educators turn to doing statistics with Skittles in their classrooms because they have been trained to do so, bounded by their own experiences as math learners (Drake & Sherin, 2006) as well as their lack of training to do anything else (Shaughnessy, 2007), and asking them to include critical contexts in their work is nonnormative and antithetical to their historicity and identities as math teachers. For all three teachers in this study, they recognized that their personal consumption of data was political in nature outside of their classroom, but this did not translate prior to enactment to any change in their teaching practice. This could be because they are entrenched in these types of silos with artificial boundaries to believe that a benign experience is the kind of mathematics that students need to learn in order to engage in a world that varies in its use of mathematics and statistics. Teachers need support to discuss the implications of pulling resources from these "apolitical", "objective" bases of knowledge. What does it look like to support teachers in recognizing the disjuncture between the statistics taught in the math classroom and the statistics of the world? How can math education researchers support the agitation of math teachers to differentiate the contextual nature of statistics from the structural nature of mathematics? What professional learning experiences are needed to support teachers in this aspect of their critical statistical literacy? How teachers can be agitated to see this disjuncture is a profitable area of future research in the field of critical statistical literacy.

My future research agenda in this section is tied to my positionality as a justice-oriented math and statistics education researcher who is concerned with the humanity of Black and brown students and teachers. I need the research in the field, including my own, to move in these directions because this is who I am, who I say I am, and who I am becoming.

Conclusion

The present study indicates that the availability of CSL materials may provide secondary math teachers with the opportunity to engage in identity work as critical statistics educators. The development of critical statistical literacy in teachers is one promising strategy that may help to disrupt the reproduction of critically statistically illiterate citizens as they learn how to "thrive in the modern world" (Franklin et al., 2007) by acting justly to create a world that is worth thriving in.

REFERENCES

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The Impact of Identity in K-8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices*. The National Council of Teachers of Mathematics, Inc., Reston: VA.

Aguirre, J., Herbel-Eisenmann, B., Celedon-Pattichis, S., Civil, M., Wilkerson, T., Stephan, M., ... & Clements, D. H. (2017). Equity within mathematics education research as a political act: Moving from choice to intentional collective professional responsibility. *Journal for Research in Mathematics Education*, 48(2), 124-147.

Ahlquist, R. (2001). Critical multicultural mathematics curriculum: Multiple connections through the lenses of race, ethnicity, gender, and social class. In J.E. Jacobs, J.R. Becker, & G.F. Glimer (Eds.), Changing the faces of mathematics: Perspectives on gender (pp. 25–36). Reston, VA: NCTM Publishing.

Antaki, C., & Widdicombe, S. (Eds.). (1998). *Identities in talk*. Sage.

Anzaldúa, G. (1987). Borderlands/La frontera: The new mestiza.

Apple, M. (1999). Power, meaning, and identity: Essays in critical educational studies. New York: Peter Lang.

Bailey, N., & McCulloch, A.W. (2019). Undergraduates' expression of critical statistical literacy. In S. Otten, A. G. Candela, Z. de Araujo, C. Haines, & C. Munter (Eds.), *Proceedings of the 41st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 375-379). St. Louis, MO: University of Missouri.

Bakker, A. (2014). Implications of technology on what students need to know about statistics. In T. Wassong, D. Frischemeier, P. R. Fischer, R. Hochmuth, & P. Bender (Eds.), Mit

Werkzeugen Mathematik und Stochastik lernen–Using tools for learning mathematics and statistics (pp. 143–152).

Bakker, A. (2019). Design research in education: A practical guide for early career researchers. Routledge.

Bakker, A., & Derry, J. (2011). Lessons from inferentialism for statistics education. *Mathematical Thinking and Learning*, 13(1–2), 5–26. doi:10.1080/10986065.2011.538293

Bargagliotti, A., Anderson, C., Casey, S., Everson, M., Franklin, C., Gould, R., ... & Watkins, A. (2014, July). Project-SET materials for the teaching and learning of sampling variability and regression. In Sustainability in statistics education. *Proceedings of the Ninth International Conference on Teaching Statistics* (ICOTS9).

Bartell, T. G. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education*, *44*(1), 129-163.

Batanero, C., Garfield, J. B., Ottaviani, M. G., & Truran, J. (2000). Research in statistical education: some priority questions. *Statistics Education Research Newsletter*, 1(2), 2–6. Online: www.stat.auckland.ac.nz/~iase/serj/

Batanero, C., Burrill, G., & Reading, C. (Eds.). (2011). Teaching statistics in school mathematics— Challenges for teaching and teacher education: A joint ICMI/IASE study. Dordrecht, The Netherlands: Springer.

Begg, A., & Edwards, R. (1999). Teachers' ideas about teaching statistics. Proceedings of the 1999 Combined Conference of the Australian Association for Research in Education and the New Zealand Association for Research in Education. Melbourne: Australian

Association for Research in Education & New Zealand Association for Research in Education. Online:www.aare.edu.au/99pap/

Behrens, J. T. (1997). Principles and procedures of exploratory data analysis. *Psychological Methods*, *2*(2), 131.

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. *Mathematical Thinking and Learning*, 2(1–2), 127–155. doi:10.1207/S15327833MTL0202_6

Ben-Zvi, D. (2004). Reasoning about variability in comparing distributions. *Statistics Education Research Journal*, 3(2), 42–63.

Ben-Zvi, D., Gil, E., & Apel, N. (2007). What is hidden beyond the data? Helping young students to reason and argue about some wider universe. *Paper presented at the Fifth International Research Forum on Statistical Reasoning, Thinking, and Literacy (SRTL-5)*, University of Warwick, United Kingdom.

Ben-Zvi, D., Aridor, K., Makar, K., & Bakker, A. (2012). Students' emergent articulations of uncertainty while making informal statistical inferences. *ZDM—The International Journal on Mathematics Education*, 44(7), 913–925. doi:10.1007 /s11858-012-0420-3

Berry III, R. Q., Ellis, M., & Hughes, S. (2014). Examining a history of failed reforms and recent stories of success: Mathematics education and Black learners of mathematics in the United States. *Race Ethnicity and Education*, *17*(4), 540-568.

Berry III, R. Q., Conway IV, B. M., Lawler, B. R., & Staley, J. W. (2020). *High school mathematics lessons to explore, understand, and respond to social injustice*. Corwin Press.

Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2013). Technology for enhancing statistical reasoning at the school level. In K. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 643–690). New York, NY: Springer.

Bielaczyc, K. (2006). Designing social infrastructure: Critical issues in creating learning environments with technology. *Journal of the Learning Sciences*, 15, 301–329.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education*, 33(4), 239–258.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. *Multiple perspectives on mathematics teaching and learning*, 1, 171-200.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record*, *110*(3), 608-645.

Bogdan, R. C., and S. K. Biklen. 2003. Qualitative research for education: An introduction to theories and methods. 4th ed. Boston: Allyn & Bacon

Bolter, J. D., & Gromala, D. (2003). Windows and mirrors. *HOW BIAS SPREADS*, 144.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational researcher*, *33*(8), 3-15.

Brantlinger, A. (2011). Rethinking critical mathematics: A comparative analysis of critical, reform, and traditional geometry instructional texts. *Educational Studies in Mathematics*, 78(3), 395-411.

Brantlinger, A. (2013). Between politics and equations: Teaching critical mathematics in a remedial secondary classroom. *American Educational Research Journal*, 50(5), 1050-1080.

Brantlinger, A. (2014). Critical mathematics discourse in a high school classroom: Examining patterns of student engagement and resistance. *Educational Studies in Mathematics*, 85(2), 201-220.

Brown, N., & Benken, B. M. (2009). So when do we teach mathematics? Vital elements of professional development for high school mathematics teachers in an urban context. *Teacher Education Quarterly*, 36(3), 55-73.

Burgess, T. (2008). Teacher knowledge for teaching statistics through investigations. *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study. Proceedings of the ICMI Study*, 18.

Burgess, T. (2011). Teacher knowledge of and for statistical investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 259–270). Dordrecht, The Netherlands: Springer.

Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 57–69). Dordrecht, The Netherlands: Springer.

Callingham, R., & Watson, J. (2005). Measuring statistical literacy. *Journal of Applied Measurment,* 6(1), 19–47.

Cammarota, J. (2011). From hopelessness to hope: Social justice pedagogy in urban education and youth development. *Urban Education*, 46(4), 828-844.

Carlson, E. D., Engebretson, J., & Chamberlain, R. M. (2006). Photovoice as a social process of critical consciousness. *Qualitative health research*, 16(6), 836-852.

Cavanagh, M. (2005). Working mathematically: The role of graphics calculators. In *Making mathematics vital: Proceedings of the 20th biennial conference of the Australian Association of Mathematics Teachers* (pp. 80-86).

Celedon-Pattichis, S., & Ramirez, N. G. (Eds.). (2012). *Beyond good teaching: Advancing mathematics education for ELLs*. Reston, VA: National Council of Teachers of Mathematics.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Sage.

Chick, H. L., & Pierce, R. U. (2008). Teaching statistics at the primary school level: Beliefs, affordances, and pedagogical content knowledge. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008). *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study*. Dordrecht, The Netherlands: Springer.

Civil, M. (1998). Bridging In-School Mathematics and Out-of-School Mathematics. *Paper presented at the Annual Meeting of the American Educational Research Association,* San Diego, CA.

Civil, M. (2014). Why Should Mathematics Educators Learn from and about Latina/o Students' In-School and Out-of-School Experiences?. *Journal of Urban Mathematics Education*, 7(2).

Cobb, G. (1992). Teaching statistics. *Heeding the call for change: Suggestions for curricular action*, 22, 3-43.

Cobb, G. W. (2007). The introductory statistics course: A Ptolemaic curriculum? *Technology Innovations in Statistics Education*, 1(1). Retrieved from http://escholarship.org/uc /item/6hb3k0nz

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American mathematical monthly*, 104(9), 801-823.

Cobb, P. (2004). Mathematics, literacies, and identity. *Reading Research Quarterly*, 39(3), 333-337.

Cobb, P., & Hodge, L. (2002). Learning, identity, and statistical data analysis. *In Sixth International Conference on Teaching Statistics (ICOTS6)*, Cape Town, South Africa.

Cobb, P., Confrey, J. diSessa, A., Lehrer, R., Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13.

Cobb, P., Jackson, K., Smith, T., Sorum, M., & Henrick, E. (2013). Design research with educational systems: Investigating and supporting improvements in the quality of mathematics teaching and learning at scale. In B. J. Fishman, W. R. Penuel, A.-R. Allen & B. H. Cheng (Eds.), *Design based implementation research: Theories, methods, and exemplars*. National Society for the Study of Education Yearbook (Vol. 112, Issue 2, pp. 320-349). New York: Teachers College.

Collins, P. H. (2019). Intersectionality as critical social theory. Duke University Press.

Confrey, J., & Lachance, A. (2000). Transformative Teaching Experiments through Conjecture-Driven Research Design.

Contreras, J. M., Batanero, C., Díaz, C., & Fernandes, J. A. (2011, February). Prospective teachers' common and specialized knowledge in a probability task. *Paper presented at the Seventh Congress of the European Society for Research in Mathematics Education*

(CERME-7), Rzeszów, Poland. Retrieved from http://www.cerme7.univ.rzeszow.pl/WG/5 /CERME_Contreras-Batanero.pdf

Creswell, J. W. (1998). Qualitative inquiry and research design: Choosing among five traditions. Thousand Oaks, CA: Sage Publications.

d'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of Mathematics*, 5(1), 44-48.

d'Ambrosio, U. (2001). In My Opinion: What Is Ethnomathematics, and How Can It Help Children in Schools?. *Teaching children mathematics*, 7(6), 308-310.

da Ponte, J. P., & Chapman, O. (2008). 11 Preservice mathematics teachers' knowledge and development. *Handbook of international research in mathematics education*, 223.

de Freitas, E. (2008). Troubling teacher identity: Preparing mathematics teachers to teach for diversity. *Teaching education*, 19(1), 43-55.

Desmos. (2011, May 10). *Desmos Classroom*. Desmos Classroom Activities. Retrieved October 18, 2020, from https://teacher.desmos.com/

DelMas, R. C. (2002). Statistical literacy, reasoning, and thinking: A commentary. *Journal of Statistics Education*, 10(2).

Doctorow, C. (2008). Big data: Welcome to the petacentre.Nature, 455(7209), 16–21. doi:10.1038/455016a

Drake, C. (2002). Experience counts: Career stage and teachers' responses to mathematics education reform. *Educational policy*, *16*(2), 311-337.

Drake, C., & Sherin, M. G. (2006). Practicing change: Curriculum adaptation and teacher narrative in the context of mathematics education reform. *Curriculum inquiry*, *36*(2), 153-187.

Drake, C., Spillane, J. P., & Hufferd-Ackles, K. (2001). Storied identities: Teacher learning and subject-matter context. *Journal of Curriculum Studies*, 33(1), 1-23.

Elmore, R. F. (1997). The politics of education reform. *Issues in Science and Technology*, *14*(1), 41-49.

Ely, M. (1991). Doing qualitative research: Circles within circles (Vol. 3). Psychology Press.

Ernest, P. (2010). The scope and limits of critical mathematics education. *In Critical mathematics education: Past, present and future* (pp. 65-87). Brill Sense.

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. *Review of Educational Research*, 79(2), 1008-1043.

Esmonde, I. (2014). "Nobody's rich and nobody's poor… it sounds good, but it's actually not": Affluent students learning mathematics and social justice. *Journal of the Learning Sciences*, *23*(3), 348-391.

Fendler, L., & Muzaffar, I. (2008). The history of the bell curve: Sorting and the idea of normal. *Educational Theory*, 58(1), 63-82.

Fielding-Wells, J., & Makar, K. (2015). Inferring to a model: Using inquiry-based argumentation to challenge young children's expectations of equally likely outcomes. In A. Zieffler & E. Fry (Eds.), *Reasoning about uncertainty: Learning and teaching informal inferential reasoning* (pp. 1–28). Minneapolis, MN: Catalyst Press.

Finzer, W. (2013). The data science education dilemma. Technology Innovations in Statistics Education, 7(2). Retrieved from http://escholarship.org/uc/item/7gv0q9dc

Fiore, G. (1999). Math-abused students: are we prepared to teach them?. *The Mathematics Teacher*, *92*(5), 403-406.

Foote, M. Q. (2010). *Mathematics teaching & learning in K-12: Equity and professional development.* New York, NY: Palgrave.

Foucault, M. (1964). Madness and Civilisation: A History of Insanity in the Age of Civilisation.

Frank, T. J., Powell, M. G., View, J. L., Lee, C., Bradley, J. A., & Williams, A. (2021). Exploring racialized factors to understand why Black mathematics teachers consider leaving the profession. *Educational Researcher*, *50*(6), 381-391.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. *Second handbook of research on mathematics teaching and learning*, 1(1), 225-256. Charlotte, NC: Information Age Publishing.

Frankenstein, M. (1983). Critical mathematics education: An application of Paulo Freire's epistemology. *Journal of Education*, 315-339.

Frankenstein, M. (1989). *Relearning mathematics: A different third R--radical math (s)*. Free Association Books.

Frankenstein, M. (1990). Incorporating race, gender, and class issues into a critical mathematical literacy curriculum. *The Journal of Negro Education*, 59(3), 336-347.

Frankenstein, M. (1994). Understanding the politics of mathematical knowledge as an integral part of becoming critically numerate. *Radical Statistics*, 56, 22-40.

Frankenstein, M. (2001). Reading the world with math: Goals for a critical mathematical literacy curriculum. *The Australian Association of Mathematics Teachers Inc*., 53.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education* (GAISE) report. Alexandria: American Statistical Association.

Franklin, C., Bargagliotti, A., Case, C., Kader, G., Scheaffer, R., & Spangler, D. (2015). *Statistical Education of Teachers*. American Statistical Association.

Freire, P. (1970). *Pedagogy of the oppressed* (MB Ramos, Trans.). *New York: Continuum*, *2007*.

Freire, P. (2018). The banking concept of education. In *Thinking about schools* (pp. 117-127). Routledge.

Freire, P., & Macedo, D. (2003). Rethinking literacy: A dialogue. In A. Darder, M. Baltodano, & R. Torres (Eds*.), The critical pedagogy reader* (pp. 354–365). New York, London: Routledge Falmer.

Freire, P., & Macedo, D. (2005). *Literacy: Reading the word and the world*. New York, NY: Taylor and Francis.

Gainsburg, J. (2008). Real-world connections in secondary mathematics teaching. *Journal of Mathematics Teacher Education*, *11*(3), 199-219.

Gaissmaier, W., & Gigerenzer, G. (2008). Statistical illiteracy undermines informed shared decision making. *Zeitschrift für Evidenz, Fortbildung und Qualität im Gesundheitswesen*, 102(7), 411-413.

Gal, I., & Ginsburg, L. (1994). The role of beliefs and attitudes in learning statistics: Towards an assessment framework. *Journal of Statistics Education*, 2(2). Online: www.amstat.org/ publications/jse/

Gal, I. (ed.), (2000), *Adult Numeracy Development: Theory, Research, Practice,* Cresskill, NJ: Hampton Press.

Gal, I. (2002). Adults' statistical literacy: Meanings, components, responsibilities. *International statistical review*, 70(1), 1-25.

Gamoran, A. (1993). Alternative uses of ability grouping in secondary schools: Can we bring high-quality instruction to low-ability classes?. *American journal of education*, 102(1), 1-22.

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education*, 44-63.

Garfield, J. (1999), “Thinking about Statistical Reasoning, Thinking, and Literacy,” *Paper presented at First Annual Roundtable on Statistical Thinking, Reasoning, and Literacy* (STRL-1).

Garfield J., & Ben-Zvi, D. (Eds.). (2005). Reasoning about variation [Special section]. *Statistics Education Research Journal*, 4(1).

Garfield, J., & Ben-Zvi, D. (2007). How students learn statistics revisited: A current review of research on teaching and learning statistics. *International statistical review*, 75(3), 372-396.

Garfield, J. B., & Ben-Zvi, D. (with Chance, B., Medina, E.,Roseth, C., & Zieffler, A.). (2008). *Developing students' statistical reasoning: Connecting research and teaching practice*. Dordrecht, The Netherlands: Springer.

Garfield, J., & Ben-Zvi, D. (2009). Helping students develop statistical reasoning: Implementing a statistical reasoning learning environment. *Teaching Statistics*, 31(3), 72–77.Gay, G. (2010). Culturally responsive teaching: Theory, research, and practice. Teachers College Press.

Garfield, J., & Everson, M. (2009). Preparing teachers of statistics: A graduate course for future teachers. *Journal of Statistics Education*, 17(2).

Gates, P., & Jorgensen, R. (2009). Foregrounding social justice in mathematics teacher education. *Journal of Mathematics Teacher Education*, 12(3), 161-170.

Gattuso, L., & Pannone, M. A. (2002). Teachers' training in a statistics teaching experiment. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics. Cape Town, South Africa: International Statistical Institute and International Association for Statistics Education.* Online: www.stat.auckland.ac.nz/~iase/publications

Gee, J. P. (2018). Reading as situated language: A sociocognitive perspective. In *Theoretical models and processes of literacy* (pp. 105-117). Routledge.

Gholson, M. L. (2016). Clean corners and algebra: A critical examination of the constructed invisibility of black girls and women in mathematics. *Journal of Negro Education*, *85*(3), 290-301.

Gil, E., & Ben-Zvi, D. (2011). Explanations and context in the emergence of students' informal inferential reasoning. Mathematical Thinking and Learning, 13(1–2), 87–108. doi:10.1080/10986065.2011.538295

Glasser, B. G., & Strauss, A. L. (1967). The development of grounded theory. *Chicago, IL: Alden*.

Glesne, C. (2015). *Becoming qualitative researchers, the 5th edition*. New York and Boston: Pearson.

Goffman, E. (1959). *The presentation of self in everyday life*. Garden City,NY: Doubleday.

Goldin, G. A. (2008). Perspective on representation in mathematical learning and problem solving. *Handbook of international research in mathematics education*, 176-201.

Goldstein, D. & Saul, S. (2022, May 7). Florida releases reviews that led to rejection of math textbooks. *The New York Times.* https://www.nytimes.com/2022/05/07/us/politics/florida-rejected-math-textbooks.html

Gonzalez, L. (2009). Teaching Mathematics for Social Justice: Reflections on a Community of Practice for Urban High School Mathematics Teacher. *CUNY Academic Works*. York College, NY.

Gould, R., & Peck, R. (2004). Preparing secondary mathematics educators to teach statistics. *Curricular Development in Statistics Education: International Association for Statistical Education*, 278-283.

Gregson, S. A. (2012). *The equity practice of secondary mathematics teachers in a school committed to college preparation, community connection, and social justice* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3600676)

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education*, *42*(3), 270-304.

Groth, R. E. (2007). Research commentary: toward a conceptualization of statistical knowledge for teaching. *Journal for research in Mathematics Education,* 38(5), 427-437.

Groth, R. E. (2008). Assessing teachers' discourse about the Pre-K-12 Guidelines for assessment and instruction in statistics education (GAISE). *Statistics Education Research Journal*, 7(1).

Groth, R. E. (2013). Characterizing key developmental understandings and pedagogically powerful ideas within a statistical knowledge for teaching framework. *Mathematical Thinking and Learning*, 15(2), 121-145.

Groth, R., & Bergner, J. (2005). Pre-service elementary school teachers' metaphors for the concept of statistical sample. *Statistics Education Research Journal*, 4(2), 27–42.

Gutiérrez, K. D., & Jurow, A. S. (2016). Social design experiments: Toward equity by design. *Journal of the Learning Sciences*, 25(4), 565-598.

Gutiérrez, L. M. (1995). Understanding the empowerment process: Does consciousness make a difference?. *Social work research*, 19(4), 229-237.

Gutiérrez, R. (2007). (Re)Defining equity: The importance of a critical perspective. In N. S. Nasir & P. Cobb. (Eds*.). Improving access to mathematics: Diversity and equity in the classroom*. (pp. 37c50). New York: Teachers College Press.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for research in mathematics education*, 44(1), 37-68.

Gutiérrez, R. (2018). When mathematics teacher educators come under attack. *Mathematics Teacher Educator*, 6(2), 68-74.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics education*, 34(1), 37-73.

Gutstein, E. (2006). " The Real World As We Have Seen It": Latino/a Parents' Voices On Teaching Mathematics For Social Justice. *Mathematical Thinking and Learning*, 8(3), 331-358.

Gutstein, E. (2010). Critical multicultural approaches to mathematics education in urban, K-12 classrooms. *In Critical Multiculturalism* (pp. 133-144). Routledge.

Gutstein, E. R. (2016). "Our issues, our people—Math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education*, 47(5), 454-504.

Gutstein, E., Fey, J. T., Heid, M. K., DeLoach-Johnson, I., Middleton, J. A., Larson, M., ... & Tunis, H. (2005). Equity in School Mathematics Education: How Can Research Contribute?: NCTM Research Committee. *Journal for Research in Mathematics Education*, *36*(2), 92-100.

Haldar, L. C., Wong, N., Heller, J. I., & Konold, C. (2018). Students Making Sense of Multi-Level Data. *Technology Innovations in Statistics Education*, 11(1).

Hannigan, A., Gill, O., & Leavy, A. M. (2013). An investigation of prospective secondary mathematics teachers' conceptual knowledge of and attitudes towards statistics. *Journal of Mathematics Teacher Education*, 16(6), 427–449. doi:10.1007 /s10857-013-9246-3

Harradine, A., Batanero, C., & Rossman, A. (2011). Students and teachers' knowledge of sampling and inference. In C. Batanero, G. Burrill, & C. Reading (Eds.), Teaching statis-tics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study (pp. 235–246). Dordrecht, The Netherlands: Springer.

Hiebert, J. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912..

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. *Second handbook of research on mathematics teaching and learning*, 1(1), 371-404. Charlotte, NC: Information Age Publishing.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge. *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*, 112.

Hipolito-Delgado, C. P., & Lee, C. C. (2007). Empowerment theory for the professional school counselor: A manifesto for what really matters. *Professional School Counseling*, 10(4), 2156759X0701000401.

Hodgen, J. (2011). Knowing and identity: A situated theory of mathematics knowledge in teaching. In *Mathematical knowledge in teaching* (pp. 27-42). Springer, Dordrecht.

Holland, D. & Lave, J. (Eds.), (2001). *History in person: Enduring struggles, contentious practice, intimate identities.* Albuquerque: School of American Research Press.

Hopper, R. (1999). Going public about social interaction. *Research on Language & Social Interaction*, 32(1-2), 77-84.

Hunter, R., & Hunter, J. (2018). Opening the space for all students to engage in mathematical practices within collaborative inquiry and argumentation. In *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 1-21). Brill Sense.

Ingersoll, R. M. (1996). Teachers' decision-making power and school conflict. *Sociology of education,* 159-176.

Jacobs, J. K., Yoshida, M., Stigler, J. W., & Fernandez, C. (1997). Japanese and American teachers' evaluations of mathematics lessons: A new technique for exploring beliefs. *The Journal of Mathematical Behavior*, *16*(1), 7-24.

Jamieson, D. W., & Thomas, K. W. (1974). Power and conflict in the student-teacher relationship. *The Journal of Applied Behavioral Science*, *10*(3), 321-336.

Jansen, A. (2020). *Rough draft math: Revising to learn*. Stenhouse Publishers.

Joseph, N. M. (Ed.). (2020). *Understanding the Intersections of Race, Gender, and Gifted Education: An Anthology by and about Talented Black Girls and Women in STEM*. IAP.

Juzwik, M. M. (2006). Situating Narrative-Minded Research: A Commentary on Anna Sfard and Anna Prusak's" Telling Identities". *Educational researcher*, *35*(9), 13-21.

Kelly, A. (2009). Design-based research in education: Yes, but is it methodological? *The Journal of Learning Sciences*, 13(1), 115 – 128.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics* (Vol. 2101). J. Kilpatrick, & National research council (Eds.). Washington, DC: National Academy Press.

Klein, M. F. (1993). Mathematics as current events. *The Mathematics Teacher*, *86*(2), 114-116.

Kleinsasser, A. M. (2000). Researchers, reflexivity, and good data: Writing to unlearn. *Theory into practice*, *39*(3), 155-162.

Koestler, C. (2012). Beyond apples, puppy dogs, and ice cream: Preparing teachers to teach mathematics for equity and social justice. in *Teaching mathematics for social justice: Conversations with educators*, 81-97.

Kohlberg, L., Levine, C., & Hewer, A. (1983). Moral stages: A current formulation and a response to critics.

Kokka, K. (2017). *Social justice mathematics: Pedagogy of the oppressed or pedagogy of the privileged? A comparative case study of students of historically marginalized and privileged backgrounds* (Doctoral dissertation).

Konold, C., & Higgins, T. (2003). Reasoning about data. A research companion to principles and standards for school mathematics, J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), A research companion to Principles and Standards for School Mathematics (pp. 193-215). Reston, VA: National Council of Teachers of Mathematics.

Konold, C., Higgins, T., Russell, S. J., & Khalil, K. (2015). Data seen through different lenses. *Educational Studies in Mathematics, 88*(3), 305-325.

Konold, C., & Kazak, S. (2008). Reconnecting data and chance. *Technology Innovations in Statistics Education,* 2(1), 1–37. Retrieved from http://www.escholarship.org/uc /item/38p7c94v

Konold, C., & Miller, C. D. (2011*). TinkerPlots v.2.0: Dynamic data exploration* [Computer software]. Emeryville, CA: Key Curriculum Press.

Konold, C., & Pollatsek, A. (2002). Data analysis as the search for signals in noisy processes. *Journal for research in mathematics education*, *33*(4), 259-289.

Krathwohl, D. R. (1998). Methods of educational and social science: An integrated approach.

Kurt, G. (2016). Technological pedagogical content knowledge (TPACK) development of preservice middle school mathematics teachers in statistics teaching: A microteaching lesson study.

Ladson-Billings, G. (2006). From the achievement gap to the education debt: Understanding achievement in US schools. *Educational researcher*, *35*(7), 3-12.

Ladson-Billings, G., & Tate, W. F. (1995). Toward a critical race theory of education. *Teachers college record*, *97*(1), 47-68.

Lam, M. S. M. (2012). *A case study of social justice mathematics: The experiences of secondary students and preservice teachers in mathematics teaching and learning*. Queen's University (Canada).

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In *Instructional explanations in the disciplines* (pp. 129-141). Springer, Boston, MA.

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics Education*, *48*(3), 237-247.

Langrall, C., Makar, K., Nilsson, P., Shaughnessy, J. (2017) Teaching and Learning Probability and Statistics: An Integrated Perspective. In Cai, J., *Compendium for Research in Mathematics Education*. National Council of Teachers of Mathematics. 1906 Association Drive, Reston, VA 20191.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, culture, and activity*, *3*(3), 149-164.

Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. Cambridge: Cambridge University Press.

Lavigne, N. C., & Lajoie, S. P. (2007). Statistical reasoning of middle school children engaged in survey inquiry. *Contemporary Educational Psychology*, 32(4), 630–666. doi:10.1016 /j.cedpsych.2006.09.00

Lee, H. S., & Hollebrands, K. F. (2011). Characterising and developing teachers' knowledge for teaching statistics with technology. In *Teaching statistics in school mathematics-challenges for teaching and teacher education* (pp. 359-369). Springer, Dordrecht.

Lee, H. S., & Tran, D. (2015). Statistical habits of mind. Teaching statistics through data investigations MOOC-Ed, *Friday Institute for Educational Innovatio*n: NC State University, Raleigh, NC. Retrieved from http://info. mooced. org. s3. amaxonaws. com/tsdi1/Unit, 202.

Lemke, J. L. (2000). Across the scales of time: Artifacts, activities, and meanings in ecosocial systems. *Mind, culture, and activity*, *7*(4), 273-290.

Leonard, J. (2008). *BLACK LIVES MATTER A Context for Teaching Mathematics for Social Justice: strategies for teachers and students.* Routledge.

Leonard, J., Brooks, W., Barnes-Johnson, J., & Berry III, R. Q. (2010). The nuances and complexities of teaching mathematics for cultural relevance and social justice. *Journal of Teacher Education*, 61(3), 261-270.

Leonard, J., & Martin, D. B. (Eds.). (2013). *The brilliance of Black children in mathematics*. IAP.

Liu, Y., & Thompson, P. (2007). Teachers' understandings of probability. *Cognition & Instruction*, 25(2/3), 113–160. doi:10.1080/07370000701301117

López-Beltrán, C. (2006). Storytelling, statistics and hereditary thought: the narrative support of early statistics. *Studies in History and Philosophy of Science Part C: Studies in History and Philosophy of Biological and Biomedical Sciences*, *37*(1), 41-58.

LSA Inclusive Teaching Initiative. (2017a). Social identity wheel. Retrieved from https://sites.lsa.umich.edu/inclusive-teaching/2017/08/16/social-identity-wheel/

Makar, K., & Fielding-Wells, J. (2011). Teaching teachers to teach statistical investigations. In C. Batanero, G., Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics— Challenges for teaching and teacher education: A joint ICMI/ IASE study* (pp. 347–358). Dordrecht, The Netherlands: Springer.

Makar, K., & Rubin, A. (2009). A framework for thinking about informal statistical inference. Statistics Education Research Journal, 8(1), 82–105.

Martin, D. B. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency.* Routledge.

Martin, D. (2007). Mathematics learning and participation in African American context: The co-construction of identity in two intersecting realms of experience. *Diversity, equity, and access to mathematical ideas*, 146-158.

Martin, D. B. (Ed.). (2010). *Mathematics teaching, learning, and liberation in the lives of Black children*. Routledge.

Martin, D. B. (2015). The collective Black and principles to actions. *Journal of Urban Mathematics Education*, *8*(1).

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education*, 22(4), 459-478.

Martin, D. B., Gholson, M. L., & Leonard, J. (2010). Mathematics as gatekeeper: Power and privilege in the production of knowledge. *Journal of Urban Mathematics Education*, 3(2), 12-24.

McGatha, M., Cobb, P., & McClain, K. (2002). An analysis of students' initial statistical understandings: Developing a conjectured learning trajectory. *The Journal of Mathematical Behavior,* 21(3), 339-355.

McGee, E. O., & Hostetler, A. L. (2014). Historicizing mathematics and mathematizing social studies for social justice: A call for integration. *Equity & Excellence in Education*, 47(2), 208-229.

McKenney, S., & Reeves, T. C. (2013). Systematic review of design-based research progress: Is a little knowledge a dangerous thing? *Educational Researcher*, 42(2), 97–100. http://doi.org/10.3102/0013189X12463781

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Moore, D. S. (1998a), *Shaping Statistics for Success in the 21st Century: A Panel Discussion*, Kansas State University Technical Report II-98-1.

Moore, D. S., & Cobb, G. W. (2000). Statistics and mathematics: Tension and cooperation. *The American Mathematical Monthly*, *107*(7), 615-630.

Morton, C., McMillan, D. T., & Harrison-Jones, W. (2020). Black Girls and Mathematics Learning. In *Oxford Research Encyclopedia of Education*.

Mosenthal, P. B., & Kirsch, I. S. (1998). A new measure for assessing document complexity: The PMOSE/IKIRSCH document readability formula. *Journal of Adolescent & Adult Literacy*, *41*(8), 638-657.

Moses, R., & Cobb, C.E. (2002). *Radical equations: Civil rights from Mississippi to the Algebra Project*. Beacon Press.

Moschkovich, J. (2012). Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. *Commissioned papers on language and literacy issues in the Common Core State Standards and Next Generation Science Standards*, 94, 17.

Nasir, N. (1996). *Statistics in practice: African-American youth in the play of basketball.* Unpublished master's thesis, University of California, Los Angeles.

Nasir, N. (2000a). *All money ain't good money: The development of strategy in the game of dominoes.* Unpublished doctoral dissertation, University of California, Los Angeles.

Nasir, N. (2000b). Points ain't everything: Emergent goals and average and percent understandings in the play of basketball among African-American students. *Anthropology & Education Quarterly*, 31, 283–305.

Nasir, N. I. S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical thinking and learning*, 4(2-3), 213-247.

Nasir, N. I. S., McKinney de Royston, M., O'Connor, K., & Wischnia, S. (2017). Knowing about racial stereotypes versus believing them. *Urban education*, 52(4), 491-524.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council, 1989.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (2016). *Providing Opportunities for Students with Exceptional Promise.* NCTM Position Statement. Reston, VA: NCTM. http://www.nctm.org/Standards-and-Positions/Position-Statements/Providing-Opportunitiesfor-Students-with-Exceptional-Promise/

National Council of Teachers of Mathematics (NCTM). (2018). *Catalyzing Change in High School Mathematics: Initiating Critical Conversations*. Reston, VA: NCTM.

National Council of Supervisors of Mathematics and TODOS: Mathematics for ALL. (2016). *Mathematics education through the lens of social justice: Acknowledgement, actions, and* *accountability*. A joint position statement. Retrieved from http://www.todosmath.org/assets/docs2016/2016Enews/3.pospaper16_wtodos_8pp.pdf.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics.

National Research Council. (2000). *How people learn: Brain, mind, experience, and school: Expanded edition*. National Academies Press.

National Research Council. (2004). *How students learn: History, mathematics, and science in the classroom.* National Academies Press.

Paparistodemou, E., & Meletiou-Mavrotheris, M. (2008). Developing young students' informal inference skills in data analysis. *Statistics Education Research Journal*, 7(2), 83–106

Patton, M. Q. (2002). Two decades of developments in qualitative inquiry: A personal, experiential perspective. *Qualitative social work*, *1*(3), 261-283.

Paulus, T., Lester, J., & Dempster, P. (2013). *Digital tools for qualitative research*. Sage.

Pennell, S. M. (2020). Queer Theory/Pedagogy and Social Justice Education. *Handbook on Promoting Social Justice in Education*, 2291-2311.

Pfannkuch, M. (2006). Comparing box plot distributions: A teacher's reasoning. *Statistics Education Research Journal*, 5(2), 27–45. Online: www.stat.auckland.ac.nz/~iase/publication.

Pfannkuch, M. (2011). The role of context in developing informal statistical inferential reasoning: A case study. Mathematical Thinking and Learning, 13(1–2), 27–46. doi:10.1080/10986065.2011.538302

Pfannkuch, M., & Ben-Zvi, D. (2011) Developing teachers' statistical thinking. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics—Challenges* *for teaching and teacher education: A joint ICMI/IASE study* (pp. 323–333). Dordrecht, The Netherlands: Springer.

Pfannkuch, M., & Horring, M. (2005). Developing statistical thinking in a secondary school: A collaborative curriculum development. In G. Burrill & M. Camden (Eds.), *Curricular Development in Statistics Education International Association for Statistical Education 2004 Roundtable* (pp. 163–173). Voorburg, The Netherlands: International Statistical Institute.

Pfannkuch, M., & Reading, C. (2006b). Reasoning about distribution: A complex process. *Statistics Education Research Journal*, 5(2), 4–9.

Pfannkuch, M., Regan, M., Wild, C., Budgett, S., Forbes, S., Harraway, J., & Parsonage, R. (2011). Inference and the introductory statistics course. *International Journal of Mathematical Education* in Science and Technology, 42(7), 903–913. doi:10.1080/0020739X.2011.604732

Pillow, W. (2003). Confession, catharsis, or cure? Rethinking the uses of reflexivity as methodological power in qualitative research. *International journal of qualitative studies in education*, 16(2), 175-196.

Powell, A. B., & Frankenstein, M. (Eds.). (1997). *Ethnomathematics: Challenging Eurocentrism in mathematics education*. Suny Press.

Pratt, D., & Ainley, J. (Eds.). (2008). Informal inferential reasoning [Special issue]. *Statistics Education Research Journal*, 7(2).

Pratt, D., Johnston-Wilder, P., Ainley, J., & Mason, J. (2008). Local and global thinking in statistical inference. *Statistics Education Research Journal*, 7(2).

Ray-Riek, M. (2013). *Powerful problem solving: Activities for sense making with the mathematical practices.* Heinemann.

Reading, C., & Shaughnessy, M. (2004). Reasoning about variation. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 201–226). Dordrecht, The Netherlands: Kluwer.

Rezvi, S., Han, A., & Larnell, G. V. (2020). Mathematical mirrors, windows, and sliding glass doors: Young adult texts as sites for identifying with mathematics. *Journal of Adolescent & Adult Literacy*, *63*(5), 589-592.

Rossman, A. (2008). Reasoning about informal statistical inference: One statistician's view. Statistics Education Research Journal, 7(2), 5–19.

Rumsey, D. J. (2002). Statistical literacy as a goal for introductory statistics courses. *Journal of statistics education*, 10(3).

Rumack, A. M., & Huinker, D. (2019). Capturing mathematical curiosity with notice and wonder. *Mathematics Teaching in the Middle School*, 24(7), 394-399.

Russell, S. J., Schifter, D., & Bastable, V. (2002). *Developing mathematical ideas: Working with data.* Parsippancy, NJ: Dale Seymour.

Sandoval, W. (2014). Conjecture mapping: An approach to systematic educational design research. *The Journal of Learning Sciences*, 1-19.

Secada, W. G. (1989). Agenda setting, enlightened self-interest, and equity in mathematics education. *Peabody Journal of Education*, *66*(2), 22-56.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational researcher*, *27*(2), 4-13.

Sfard, A. (2002). *Telling identities: Conceptualizing diversity in terms of identity*. Paper presented at the Fifth Congress of the International Society for Cultural Research and Activity Theory, Amsterdam, The Netherlands.

Sfard, A., & Prusak, A. (2005). Telling identities: In search ofan analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34, 14–22. Swarat,

Shaughnessy, M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 957–1010). Charlotte, NC: Information Age Publishing.

Scheaffer, R. L., Watkins, A. E., & Landwehr, J. M. (1998). What every high-school graduate should know about statistics. *Reflections on statistics: Learning, teaching, and assessment in grades K-12*, 3-31.

Schoen, H. L., Fey, J. T., Hirsch, C. R., & Coxford, A. F. (1999). Issues and options in the math wars. Phi Delta Kappan, 80(6), 444.

Schoenfeld, A. H. (2004). The math wars. Educational policy, 18(1), 253-286.

Seidman, I. (2006). *Interviewing as qualitative research: A guide for researchers in education and the social sciences*. Teachers college press.

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies*, *41*(4), 467-500.

Shroyer, J. (1984). The LES instructional model: Launch-explore-summarize. In *Honors Teachers Workshop of Middle Grade Mathematics Proceedings* (pp. 101-114).

Shulman, L. S. (1987). *Knowledge and teaching: Foundations of the new reform.* Harvard

Silverman, J., & Thompson, P.W. (2008). Toward a framework for the development of mathematical knowledge for teaching. Journal of Mathematics Teacher Education, 11, 499- 511.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26(2), 114–145.

Simon, M. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning*, 8(4), 359–371. doi:10.1207/s15327833mt10804_1

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education.* New York, NY: Springer, Science+Business Media.

Skovsmose, O. (2005). Meaning in mathematics education. In *Meaning in mathematics education* (pp. 83-100). Springer, New York, NY.

Snell, L. (1999), "Using Chance media to Promote Statistical Literacy," *Paper presented at the 1999 Joint Statistical Meetings*, Dallas, TX.

Spicer, R. N. (2018). Lies, damn lies, alternative facts, fake news, propaganda, pinocchios, pants on fire, disinformation, misinformation, post-truth, data, and statistics. In *Free Speech and False Speech* (pp. 1-31). Palgrave Macmillan, Cham.

Stake, R. E. (1995). *The art of case study research*. sage.

Stanja, J., & Steinbring, H. (2014). The epistemological charac- ter of visual semiotic means used in elementary stochastics learning. In T. Wassong, D. Frischemeier, P. R. Fischer, R. Hochmuth, & P. Bender (Eds.), *Mit Werkzeugen Mathematik und Stochastik lernen–Using tools for learning mathematics and statistics* (pp. 223–235). Wiesbaden, Germany: Springer Fachmedien Wiesbaden.

Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject-matter knowledge and elementary instruction: A case from functions and graphing. *American Educational Research Journal,* 27(4), 639-663.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics teaching in the middle school*, *3*(4), 268-275.

Stiff, L., & Harvey, W. B. (1988). On the education of Black children in mathematics. *Journal of Black Studies,* 19(2), 190-203.

Stinson, D. W., & Bullock, E. C. (2012). Critical postmodern theory in mathematics education research: A praxis of uncertainty. *Educational Studies in Mathematics*, 80(1), 41-55.

Stinson, D. W., Bidwell, C. R., & Powell, G. C. (2012). Critical pedagogy and teaching mathematics for social justice. *The International Journal of Critical Pedagogy*, 4(1).

Stinson, D. W., Wager, A., & Leonard, J. (2012). *Teaching mathematics for social justice: Conversations with educators.* Reston, VA: National Council of Teachers of Mathematics.

Stinson, D. W., & Wager, A. A. (2013). Teaching Mathematics for Social Justice: Conversations with Educators–A Symposium.

Strauss, A., & Corbin, J. M. (1997). *Grounded theory in practice*. Sage.

Tan, E., Calabrese Barton, A., Kang, H., & O'Neill, T. (2013). Desiring a career in STEM-related fields: How middle school girls articulate and negotiate identities-in-practice in science. *Journal of Research in Science Teaching,* 50(10), 1143-1179.

Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory Into Practice*, 34(3), 166-173.

Tate, W. F., & Celia Rousseau Anderson. (2002). "Access and Opportunity: The Political and Social Context of Mathematics Education." In English, L.D & Kirshner, D. (Eds.) *Handbook of International Research in Mathematics Education*, (pp. 271–300). Mahwah, NJ: Lawrence Erlbaum Associates.

Tate, W. F. (2005). Race, retrenchment, and the reform of school mathematics. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers* (pp. 31-20). Milwaukee, WI: Rethinking Schools.

Terry, C. L. M. (2009). *An exploration of the impact of critical math literacies and alternative schooling spaces on the identity development of high school-aged Black males in South Los Angeles* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3394895)

The Concord Consortium. (2014). *CODAP*. Retrieved October 18, 2020, from https://codap.concord.org/

Tukey, J. W. (1977). *Exploratory data analysis* (Vol. 2, pp. 131-160).

Turner, E. E. (2003). *Critical mathematical agency: Urban middle school students engage in mathematics to investigate, critique, and act upon their world* (Doctoral dissertation). Available from ProQuest Dissertations and Theses database. (UMI No. 3120307)

U.S. Department of Education. (2020). Institute of Education Sciences, National Center for Education Statistics.

United Nations. (2021). Covid-19 and Indigenous Peoples for Indigenous Peoples. United Nations. Retrieved November 4, 2021, from https://www.un.org/development/desa/indigenouspeoples/covid-19.html.

Vakil, S., McKinneym de Royston, M., Suad Nasir, N. I., & Kirshner, B. (2016). Rethinking race and power in design-based research: Reflections from the field. *Cognition and Instruction*, 34(3), 194-209.

Van de Walle, John A., Sandra Folk, Karen S. Karp, and Jennifer M. Bay-Williams. 2010. *Elementary and Middle School Mathematics: Teaching Developmentally.* Upper Saddle River, NJ: Pearson Education.

Varley Gutiérrez, M. (2009). “I thought this U.S. place was supposed to be about freedom”: Young Latinas engage in mathematics and social change to save their school. *Rethinking Schools, 24*(2), 36–39.

Velleman, P. F. (2008). Truth, damn truth, and statistics. *Journal of Statistics Education*, *16*(2).

Waid, B. E., & Turner, K. H. (2021). Inqu [ee] ry across the Curriculum. *English Journal*, *110*(3), 82-88.

Wallman, K. K. (1993). Enhancing statistical literacy: Enriching our society. *Journal of the American Statistical Association*, 88(421), 1–8.

Wassong, T. Frischemeier, D., Fischer, P. R., Hochmuth, R., & Bender, P. (Eds.). (2014). *Mit Werkzeugen Mathematik und Stochastik lernen–Using tools for learning mathematics and statistics*. Wiesbaden, Germany: Springer Fachmedien Wiesbaden.

Watson, J. (1997). Assessing statistical literacy through the use of media surveys. In I. Gal & J. Garfield, (Eds.), *The assessment challenge in statistics education* (pp. 107–121). Amsterdam, The Netherlands: International Statistical Institute/IOS Press.

Watson, J. (2002). Discussion: Statistical literacy before adulthood. *International Statistical Review*, 70(1), 26– 30.

Watson, J., & Callingham, R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal*, 2(2), 3–46.

Watson, J. M., Callingham, R. A., & Kelly, B. A. (2007). Students' appreciation of expectation and variation as a foundation for statistical understanding. *Mathematical Thinking and Learning*, 9(2), 83–130. doi:10.1080/10986060709336812

Watts, R. J., Diemer, M. A., & Voight, A. M. (2011). Critical consciousness: Current status and future directions. *New directions for child and adolescent development,* 2011(134), 43-57.

Watts, R. J., & Hipolito-Delgado, C. P. (2015). Thinking ourselves to liberation?: Advancing sociopolitical action in critical consciousness. The Urban Review, 47(5), 847-867.

Weiland, T. (2019). Critical mathematics education and statistics education: Possibilities for transforming the school mathematics curriculum. In *Topics and trends in current statistics education research* (pp. 391-411). Springer, Cham.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press

Whitaker, D. (2016). The Development of a Professional Statistics Teaching Identity. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, 67(3), 221–248.

Wild, C. J., Pfannkuch, M., Regan, M., & Horton, N. J. (2011). Towards more accessible conceptions of statistical inference. *Journal of the Royal Statistical Society: Series A*, 174(2), 247–295. doi:10.1111/j.1467-985X.2010.00678.x

Wilson, P. H., Journell, W. (2011). Lies, damn lies, and statistics: Uncovering the truth behind polling data. *Social Studies Research and Practice*, 6(1), 169-180.

Yang, K. W. (2009). Mathematics, critical literacy, and youth participatory action research. *New directions for youth development*, *2009*(123), 99-118.

Yin, R. K. (1984). Case study research: Design and methods. Sage publications.

Zieffler, A., Garfield, J., delMas, R., & Reading, C. (2008). A framework to support research on informal inferential reasoning. Statistics Education Research Journal, 7(2), 40–58. Retrieved from http://www.stat.auckland.ac.nz/~iase/serj/SERJ7%282%29_Zieffler.pdf

APPENDIX A: DESCRIPTION OF MATERIALS

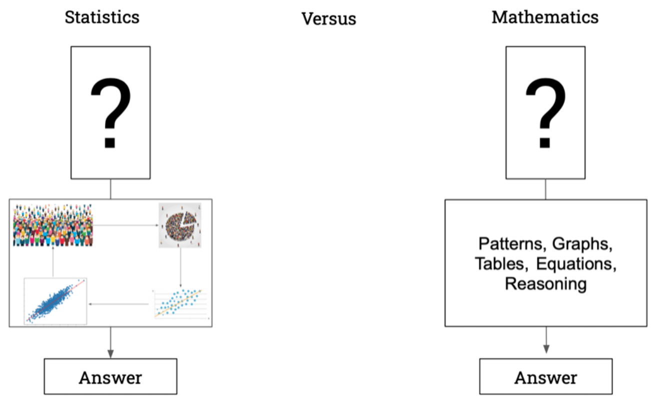
Critical Statistical Literacy Unit: Exploratory Data Analysis (6 Lessons)

Lesson 1: What & Why of Statistics: (DKF: 1, 4, 5)

The statistical learning goal for this lesson is to understand the statistical process, how variance and bias are introduced to the statistical process, the types of studies and processes of sampling that are often used in the statistical process (Cobb, 2004; McGatha, Cobb, McClain, 2002). The lesson has an interpersonal goal of understanding how variance in the statistical process introduces bias (Lee & Tran, 2015). The critical learning goal of this lesson is to develop understanding of how statistics can be obtrusive in the lives of those from whom data is being collected (Bell, 1995; Fendler & Muzaffar, 2008).

In this initial lesson to the unit on Exploratory Data Analysis, students are introduced to the study of statistics as the process of inquiry (Cobb, 2004). The lesson is launched by looking at a non-standard visualization of data (SP.1.4) based on the context of the novel Coronavirus-19. Students are asked to make a conjecture based on the visualization using the Notice & Wonder routine (DKF1) (McGatha, Cobb, McClain, 2002). Then, students are asked to compare and contrast the similarities and differences between mathematics and statistics as a field of study (Lesh, Galbraith, Haines, & Hurford, 2013; Stillman, Kaiser, Blum, & Brown, 2013).

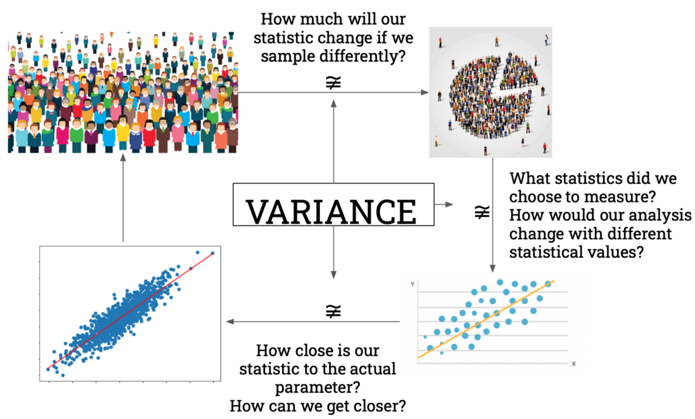
The similarities shared are that both statistics and mathematics begin with asking a question and looking for an answer, yet the process of achieving the answer is different.



Students are also introduced to the idea of statistical questions differing from mathematical questions, noting that statistical questions often ask us about the context of a problem, rather than a calculation. Literature has shown that differentiating between statistical methods and mathematical methods is a pivotal feature of student learning (Ben Zvi & Garfield, 2008), so I designed the materials to address this issue from the beginning of the unit.

Students then are formally introduced to the statistical paradigm, in which the omnipresence of variance throughout the statistical paradigm was foregrounded for students (see Figure 4) (Cobb & Moore, 1997). The lesson continues to highlight the omnipresence of variance throughout the statistical process and the reorientation to the process of inquiry as the study of statistics rather than calculating a specific statistic (DKF4). Researchers have demonstrated the value in having students wrestle with the omnipresence of variance throughout the statistical paradigm (Cobb & Moore, 1997; Engel, 2010), yet most examples have used benign datasets to do so (e.g., Friel, O'Connor, & Mamer, 2006; Konold, Higgins, Russel, & Khalil, 2015; Makar, 2014). In this lesson, the students are asked to consider the omnipresence of data and the statistical process of inquiry through the critical context of the novel COVID-19 virus as it relates to various populations by race, gender, class, and geographical location.

At the end of the lesson, students are asked to revisit their conjecture given additional information. Students are asked again to Notice & Wonder (DKF1) how additional sampling added to their understanding and provided opportunity for more questioning, thus reinforcing the omnipresence of variance, and the study of statistics as a process that is self-perpetuating (DKF4). In this last revision of their conjecture, students are asked to make sense of how two people could come to different conclusions when analyzing the same data, and to question which statement is true (DKF5) (Garfield & Ben-Zvi, 2009). The lesson ends with students completing a matching activity with the different components of studies found in the news and media (SP.1.4) with each containing an initial question, population, sample, and non-standard visualization in a critical context.



Lesson 2: Being Skeptical (DKF 1, 4, 5)

In the second lesson of the unit, the statistical learning goal is for students to recognize the purposes of and differences among statistical surveys, observational studies, and experiments, and what learning is facilitated or hindered by the use of each study type (Watson, 2002). The interpersonal learning goal of this lesson is to begin to develop a healthy skepticism of data presented based on the type of design used (Lee and Tran, 2015). The critical goal of this lesson is to recognize that for many statistical questions that impact real systems and policies, the surveys, observational studies, and experiments we use are distorted versions of the data we need in order to answer the statistical question of interest (Fendler & Muzaffar, 2008; Bell, 1995). In other words, sometimes statisticians conduct a study that does not allow them to solve or answer a question about systemic injustice, but rather provides data that answers the wrong question that convolutes the existence of the injustice at all.

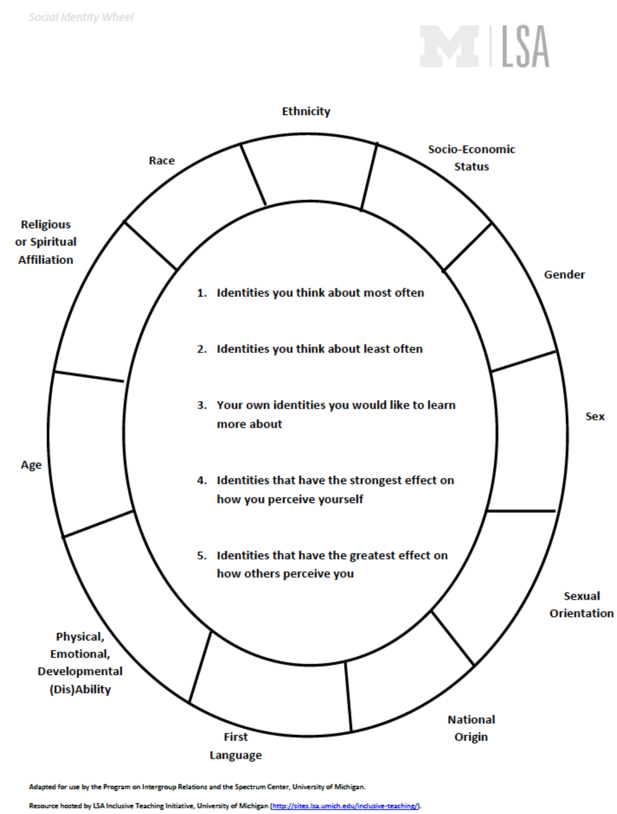
Students begin this lesson using the Notice & Wonder routine to develop a conjecture about a non-standard visualization (SP.1.4; DKF1). Students are then presented with four headlines that were associated with the non-standard visualization in the news from different media sources and asked to consider the story the data visualization was trying to share, and “what is the truth?” (DKF5). After the launch, students were given vocabulary definitions for survey, experiment, and observational study, and asked to think of a way to study police brutality via the variables race, gender, class, and zip code with each of the study types. Students were then asked to explore chosen studies found in the news and media and classify them by study type, and whether the study was good or bad, and more or less obtrusive to participants. While other research has pointed to the benefit of having students evaluate study designs (e.g., Garfield, 2003), this lesson specifically asks students to consider the people being studied. This draws attention to the people the study may harm rather than the benign practice of labeling a study as good or bad based on nontrivial characteristics. In doing so, it asks students to consider their position as consumers and producers of statistics.

The lesson ends by asking students to write a study design that is an observational study, experiment, and survey for someone who wants to study unemployment, and then consider the constraints of studying that information. They are reminded of the statistical paradigm and how constraints can introduce variance into the process of inquiry because they are studying human beings (DKF5). This, again, asks students to handle statistical production with care, a key aspect of developing one's critical statistical literacy. Lastly, the lesson summarizes some key takeaways and emphasizes the new vocabulary of the lesson.

Lesson 3: What does data say about me? (DKF 2, 3, 5)

In this lesson, the statistical learning goal is for students to begin thinking about how the design of an experiment affects the results and to introduce key vocabulary that will be used throughout the unit (e.g., sample, population, representative, well-designed experiment, and the concept of error) (Shaughnessy, 2007). The interpersonal goal of this lesson is to understand that not all statistics about someone or others is representative of the entire population (Makar & Rubin, 2009). The critical goal of lesson 3 is to make students aware of how statistics are being used to mis/represent a population of people for which they or others identify and complicate how data can be used to harm populations of people with false narratives (Fendler & Muzzafar, 2008).

This lesson begins by asking students to use the Social Identity Wheel to think about the different identities that society uses to label each person (LSA Inclusive Teaching Initiative, 2019) (see Figure 3). Students are asked to think about these different social identities individually, and then asked to determine which of their identities fall into the categories of most thought of, least thought of, would you like to know more about, strongest effect on how they perceive themselves, and greatest effect of how others perceive them (LSA Inclusive Teaching Initiative, 2019). The use of this identity wheel allows students to use their identity markers as a source of data collection for exploration (DKF2). Students are then asked to visit a database website, such as Pewresearch.org, and explore different databases that align with some of their identity markers (DKF 2). In this part of the lesson, students are focused on seeing themselves as an individual data point within a distribution (DKF3), and holding both the big picture and individual data view simultaneously (Konold and Higgins, 2003; Russell, et al., 2003).



Students are asked to find three statistics within their exploration and to determine whether the statistic found for the population is representative of them (DKF 2). They are then asked to look across the statistics they found and see "what is the story being told about you?" with regard to that particular social identity marker. In this portion of the lesson, students are being asked to hold the tension of being an individual data point within a population (Konold and Higgins, 2003; Russell,et al., 2003) (DKF2).

Next, students are asked to focus on one study that they found that is representative of themselves and their view of the world and one study that is not representative of themselves and their view of the world. They are asked to describe the type of data collected (quantitative or categorical), to look at the sampling methods for each study, to look at the variables of interest, and to connect back to the statistical question that initiated the data collection. They then evaluated each research study overall based on the design of the study. To summarize, students are asked what it feels like to be misrepresented by data, and why that is important to remember in regard to others when it comes to trusting statistics without question (DKF5). Compared to other research studies that ask students to hold two views of data simultaneously (e.g., Konold and Higgins, 2003), this activity differs as it specifically asks students to reflect on themselves and their identities first, and then reflect as a member of the group with whom they share identity markers. This simultaneous view was designed to cause a moment of interpersonal inspection and a critical connection to understanding that statistics can be used for harm through misrepresentation, and that data on samples of populations inadequately represent whole populations of people (i.e Black males who have been incarcerated).

Lesson 4: Different People, Different Results (DKF 1, 2, 3, 4, 5)

In this lesson, the statistical learning goal is to understand how sampling methods affect the results of an experiment, survey, or observation and to understand why randomization is a component of a well-designed experiment (Pfannkuch et al., 2011). The interpersonal learning goal of lesson 4 is to understand that not all statistics about someone is representative of the entire population (Makar & Rubin, 2009). Lesson 4's critical goal is to build criticality in digesting statistics shared without enough information and to understand how sampling methods can be used for harm to misrepresent a population of people (Fendler & Muzzafar, 2008).

This lesson is launched by looking at a tweet as text that discusses the concept of spanking children (see Figure 4). The tweet discusses the statistical context of sampling and why it is a powerful tool for using data as information.

Stacey Patton
@DrStaceyPatton

I tell pro-hitters: "Over 1500 studies on over 160,000 kids around the world show spanking is bad."

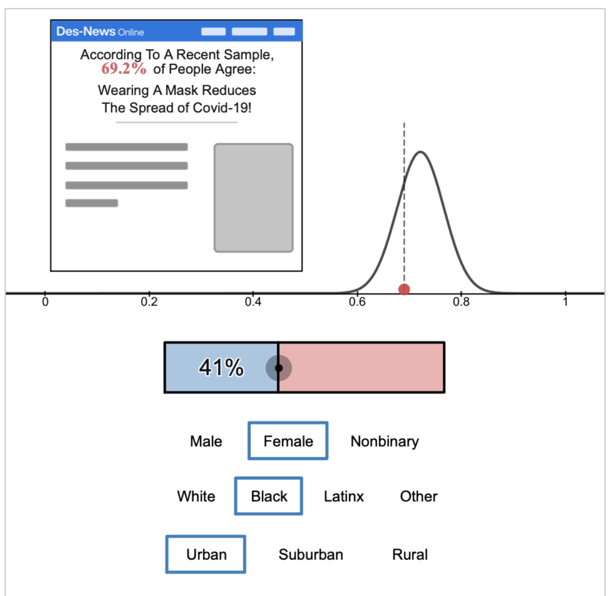
Them: "I was whupped. I turned out fine. You can make data say what you want."

Me: "And here you are using a sample size of one to argue hitting isn't bad for kids or society."

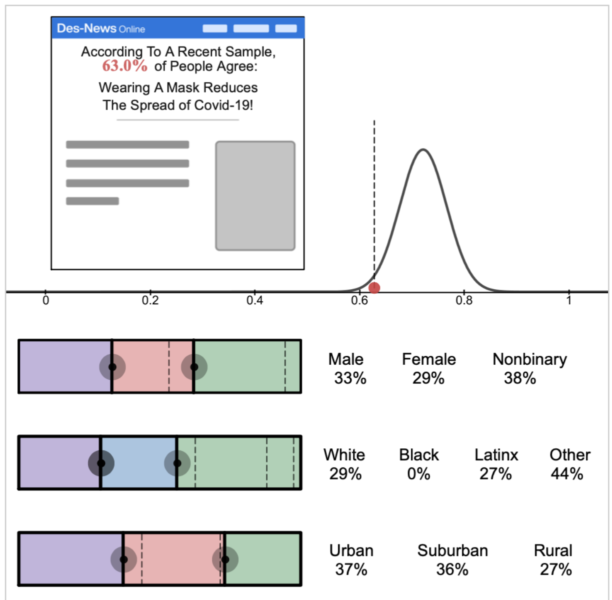
Students are asked to read the text, and respond to questions about the information within the tweet. Students are then given an excerpt from a research study on the novel Coronavirus-19 and school attendance. Students are asked to read the study and analyze it for the components that have been discussed thus far (statistical question, population, sample taken, sampling methods, type of data and study) (Garfield, 2002). Then, the students are given three news headlines that used the study as a source of data in pushing a particular story. The Notice & Wonder routine is used to get students to compare and contrast the excerpt of the study with the news headlines provided (DKF1). Students are asked to respond to what the potential consequences are for someone who reads the news headlines, but not the article that it is sourced from (DKF5).

The exploration portion of this lesson begins by asking students for some identity markers (gender, race, and geographic location) (DKF2). They are then asked a yes/no question of "Does wearing a mask reduce the spread of covid-19?". Students' data and response is anonymously added to a database using these variables. On the next slide, a news headline appears that says, "According to a recent survey, 61.2% of people agree: Wearing a mask reduces the spread of COVID-19", with 61.2% being the calculated proportion mean for a sample of the population in the database. Students are asked to respond to the question of "Do you trust this headline? Why or why not?" (DKF5) (Lee & Tran, 2015). Throughout this investigation sequence, students recognize themselves as members of the distribution (Konold & Higgins, 2003). They are able to see themselves as a case of the data, as a member of the aggregate and disaggregate, and then able to recognize the mis/representativeness that occurs as the data is sampled (DKF3) (Russell et al., 2003).

Next, students focus on how sampling can change the distribution mean. Students are able to sample the database multiple times to see how the distribution mean changes, and to make sense of how the headline is changing each time (Shaughnessy, 2007). Students may choose to sample the population to match their identity markers or to sample a population that aligns a social identity for someone unlike them (DKF2).



Next, sliders are introduced that are categorized by the initial identity markers of race, gender, and geographic location, with a focus on the question of "How does the composition of the sample effect the headline?". Students can move the sliders to watch the proportion mean and the news headline change in real time as who is sampled changes. Throughout this exploration, students are asked to reorient their idea to the concept of statistics as process of inquiry rather than calculation (Cobb & Moore, 1997). By recalculating the mean multiple times, students are able to see how the calculation of the mean for the sample is not the purpose of the study of statistics, but rather understanding the purpose of sampling itself (DK4) (Shaughnessy, 2007).



Next, students are given a challenge to choose a sample that creates a specific outcome, (e.g., as far from the population parameter as possible). Students are then introduced to the idea of random sampling from an ethical perspective (Fendler & Muzaffar, 2008; Bell, 1995). Students are given a similar screen, but asked to randomly sample the population and to explore what happens to the population parameter. Using the Notice & Wonder routine (DKF1), students are asked to conjecture through observation about the purpose of random sampling. To summarize, students are challenged to explain why random sampling provides an ethical approach to sampling methods or to explain how sampling methods could be used to bias the story the data presents (DK5).

Lesson 5: Same Data, Different Conclusions (DKF 1, 4, 5)

The learning goals of this lesson are to explore a data set with at least 200 entries and experience exploratory data analysis in a scientific context (statistical) (Finzer, 2013; Ben-

Zvi, 2000; Biehler et al., 2013; Wassong, Frischemeier, Fischer, Hochmuth, & Bender, 2014), to understand that our initial view of data may change with more information (interpersonal), and to recognize how the novel COVID-19 virus is disproportionately impacting communities of color (critical). The lesson begins by students looking at two non-standard visualization graphs (SP.1.4) from the New York Times *What's Going On In This Graph?* website and using the Notice & Wonder routine (DKF1) to conjecture about the nature of the data set, which is per capita COVID-19 cases by county. Students are then asked to look at another non-standard visualization from The Marshall Project that displays the distribution of COVID-19 cases by race for the United States. Students again notice and wonder to make conjectures about the nature of the data and what story is being shared (DKF1, 5).

After the launch, students are asked to explore a large data set and to "act as a statistician" as they move through the statistical paradigm (DKF4) (Finzer, 2013). This lesson asks students to utilize a new technology tool, CODAP, to explore the large dataset (Finzer & Konold, 2009). The dataset includes the number of COVID-19 cases distributed by race, ethnicity, and state (DKF2) from March of 2020 through the date of the lesson, and includes entries for all states and territories of the United States. Students are asked to answer the statistical question of, "Is COVID-19 disproportionately affecting some races more than others?". Students begin their exploration by observing the file and making sense of the missing data entries (e.g., Why do you think we have no data from certain states?). Students are asked to make decisions in order to sample their data set meaningfully (Shaughnessy, 2007). Students are asked to build graphs and calculate statistics that will help them answer the question (Finzer and Konold, 2009). The data in the file is in numerical format for number of cases, and at some point in the lesson, the idea is introduced that the file must be manipulated to percentages in order to answer the question clearly (Konold and Higgins, 2003). To continue to connect students to the critical goal of this EDA, these questions are posed to students: “Why do you think it is so difficult to find data to answer these kinds of questions? Why isn’t the data provided in a format that would allow us to answer the question easily?”. The teacher facilitation guide suggests to teachers to consider probing students further by asking them to think about hard questions and why data isn’t readily available (Fendler & Muzzafar, 2008).

A transposed version of the file is then given to students to continue exploring. Students are asked to look up racial statistics for the state of North Carolina to determine if the COVID-19 virus is affecting those in our state disproportionately by race and to make inferences based on the proportions within the file and the known proportions in the state, all while asking themselves, what is the story that is within the data and what is the truth? (DKF5). At the end, students are prompted to think of questions that arose while exploring the database. These questions could be about themselves, their proximity to the data (e.g., Am I within this data set? Do I know someone within this dataset?), or the populations impacted. Throughout the investigation, students complete the entire statistical paradigm, including the reiteration of the perpetuating process of new questions arising from the process of exploration (DKF4) (Cobb, 2007).

Lesson 6: Exploring Data Meaningful To Me (DKF 1, 2, 3, 4, 5)

The culminating mathematical goal of this unit is for students to complete the entire statistical paradigm, from creating a statistical question that is meaningful for them to answer, identifying a population, generating a data set to explore, answering the question by calculating a chosen statistic, and making inference back towards the population in question (Cobb, 2007). Research has demonstrated the power in having students explore a question of their own interest (Finzer, 2013; Ben-Zvi, 2000; Biehler et al., 2013; Wassong, Frischemeier, Fischer, Hochmuth, & Bender, 2014). While the previous lesson asked students to explore a data set provided to them, in this lesson, students will work to find a data set that interests them (DKF4). The interpersonal learning goal is to create a compelling statistical question that is meaningful to the student, and to find a way to answer the question using the statistical method (Finzer and Konold, 2009), with the critical learning goal being correlated to the student's choice of a question to explore, but not limited to the struggle to find data based on critical contexts and the potential empowerment and powerlessness of consequential findings.

The teaching facilitation guide suggests that teachers review the statistical paradigm, the omnipresence of variance, and bias within the model before beginning the project. The goal of this lesson is for students to "act as statisticians" with data that they decide what question is meaningful, rather than being told that data should be meaningful to them (DK2) (Ben-Zvi, Gil, and Apel, 2007). Students begin by asking a statistical question, then searching for a dataset that could be used to answer that question. Students then explore the database to search for meaning in their answer and consider what statistics might be meaningful in answering their question. Students draw conclusions from their results, and ask additional questions that arise because of their exploration (Ben-Zvi, Gil, and Apel, 2007). Throughout this process, students are asked to journal regarding the variance that arises because of their choices, from the dataset they choose to their sampling method and the statistics they choose to calculate. I chose to ask students to write a variance journey as it is often not obvious to students that they made a choice (Cobb and Moore, 1997). In the end, students are asked to write about their findings, but also their experience with exploring meaningful data. The focusing question for their reflection is "What is the story you wanted to share with the data? What is the truth?” (DK5). How they summarize their learning is left up to the student and teacher, but a Google presentation was suggested.

APPENDIX B: UNIT OVERVIEW & LESSON DESCRIPTION [REDACTED]

<table>
<tr><td>Unit Title</td><td>Exploratory Data Analysis</td><td>Pacing (Block and yearlong)</td><td>11 Block
22 Yearlong</td></tr>
<tr><td colspan="4">Content Standards</td></tr>
<tr><td colspan="4">NC.M4.SP.1.1 Construct statistical questions to guide explorations of data in context.
NC.M4.SP.1.2 Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question.
NC.M4.SP.1.3   Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (e.g.,, spreadsheets, dynamic data analysis tools) to determine: types of variables in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.
NC.M4.SP.1.4 Interpret non-standard data visualizations from the media or scientific papers to make sense of real world phenomenon.</td></tr>
<tr><td colspan="4">Unpacking of the Standards</td></tr>
<tr><td colspan="2">Knowledge
Students will know…</td><td colspan="2">Skills
Students will be able to…</td></tr>
<tr><td colspan="2">● The differences between categorical and quantitative data.
● The differences between discrete, continuous, nominal, and ordinal variables.
● How to organize data using statistical software (CODAP).
● How to describe and analyze categorical data.
● How to describe and analyze quantitative data.
● How to read a box and whisker plot, dot plot, histogram, segmented bar graph and scatter plot.
● That mean and median are measures of center.
● That standard deviation, IQR and Range are measures of spread/variability.
● What makes a question “Statistical” rather than simply a reference question.
● Calling a question “Statistical” is different from classifying as Quantitative or Categorical.
● How to organize a data set on CODAP.
● What makes a statistical question.</td><td colspan="2">● Organize data using statistical software (CODAP).
● Describe a data set by the type of data.
● Classify variables as categorical or quantitative.
● Make and interpret bar charts and/or segmented bar charts for categorical data.
● Classify categorical variables as ordinal or non-ordinal.
● Calculate frequencies and relative frequencies for categorical data.
● Make and interpret box plots, histograms, bar charts, scatterplots, dot plats.
● Classify quantitative variables as continuous or discrete.
● Calculate the mean, median, minimum, maximum, range, quartile 1, quartile 3, interquartile range (IQR), and standard deviation.
● Determine the difference between numerical and quantitative data.
● Decide an appropriate graphical representation for a set of data.
● Use technology to create a graphical representation.</td></tr>
</table>

● How to use data to answer a statistical question. **Key Vocabulary:** Categorical data, quantitative data, discrete variables, continuous variables, nominal variables, ordinal variables, mean, median, mode, numerical displays, graphical displays, frequencies, box plots, histograms, bar charts, scatterplots, dot plots, mean, median, minimum, maximum, range, quartile 1, quartile 3, interquartile range (IQR), and standard deviation, Variability, Population, Attribute, Data, Statistical, Reference, observational study, survey, experiment, experimental group, control group, placebo, random sample, random assignment, Population, Sample, Parameter, Statistic, Survey, Simple Random Sample, Systematic Random Sample, Stratified Random Sample, Cluster Random Sample, Convenience Sampling, Volunteer Sampling, Bias, Undercoverage, Response Bias, Non-Response Bias	● Create numerical summaries of a large dataset. ● Use numerical and graphical summaries to make conclusions about a a dataset. ● Identify unusual features of a graphical representation (including outliers). ● Distinguish between statistical questions and other types of questions. ● Understand the concept of variability within a data set. ● Select a single attribute and create an appropriate display to share. ● Create appropriate statistical questions and be able to provide a counter-example. ● Distinguish between observational studies, surveys, and experiments. ● Explain why random selection is an important consideration in observational studies and surveys. ● Explain why random assignment is an important consideration in experiments. ● Design an experiment based on random sampling to investigate the phenomenon of a real-world context. ● Describe the population, sample, parameter and statistic for given studies. ● Determine the type of sampling used given a scenario. ● Determine which type of sampling should be used to answer statistical questions. ● Determine if any bias and what type is present in a study. ● Design a study to answer a statistical question. ● Identify the type of study being used and explain when to use each type.
Interpersonal Goals	
Students will know...	**Students will be able to...**
● Not all statistics are representative of the population of interest, and this misrepresentation can occur to them as well. ● That our initial view of data may change with more information. ● How to be a healthy skeptic of statistical data, and ask questions to critique the study by type of design, sampling method, and inference made to be better consumers of	● Create a compelling statistical question that is meaningful to them and to find a way to answer that question using the statistical process. ● Critically digest data with a lens for healthy skepticism.

data.	
Critical Goals	
Students will know...	**Students will be able to...**
• How variance and bias can be used to misrepresent a population of people. • How to critique and ingest statistical studies with healthy skepticism • That statistics often impact real systems and policies, that impact the lives of real people.	• Describe how the decisions they make create variation and bias in the statistical process. • Critically digest statistics information. • Recognize biased versions of data in the media and news and how it can be used to portray a particular image of a population of people.
Links to Lessons	
Lesson Topic (Suggested Pacing)	**Standard Number(s)**
The What & Why of Statistics (1 day)	NC.M4.SP.1.1, NC.M4.SP.1.4
Being Skeptical (1 day)	NC.M4.SP.1.1, NC.M4.SP.1.2, NC.M4.SP.1.4
What does data say about me? (1 - 2 days)	NC.M4.SP.1.1, NC.M4.SP.1.4
Different People. Different Perspectives on Data (1 day)	NC.M4.SP.1.1, NC.M4.SP.1.2, NC.M4.SP.1.3
The Same Data can Say Different Things (2 days)	NC.M4.SP.1.1, NC.M4.SP.1.3, NC.M4.SP.1.4
Exploring Data Meaningful To Me (2 - 3 days)	NC.M4.SP.1.1, NC.M4.SP.1.2, NC.M4.SP.1.3, & NC.M4.SP.1.4

Lesson 1: The What & Why of Statistics

	The What & Why of Statistics
Goals	Mathematical: To understand the statistical process, how variance and bias are introduced to the statistical process, the types of studies and process of sampling that are often used in the statistical process Interpersonal: To understand how variance in the statistical process introduces bias Critical: To understand how statistics can be obtrusive in the lives of those from whom data is being collected.
Standards Covered	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.4** Interpret non-standard data visualizations from the media or scientific papers to make sense of real world phenomenon.
Standards Description	In this lesson, students are introduced to the statistical process. They make sense of how statistics is different from mathematics, and how variance and bias are introduced into the statistical process. They also begin to form their first statistical questions (NC.M4.SP.1.1) by reviewing non-standard data visualizations that are based on the contexts of real world phenomenon in the media and scientific papers ((NC.M4.SP.1.4).
Warm Up	This lesson has been created into a Desmos Link. Notes for each individual slide are in the Image Descriptions on each page.
Lesson Exploration	This lesson has been created into a Desmos Link. Notes for each individual slide are in the Image Descriptions on each page, but additional suggestions for the lesson are below. In this lesson, students are introduced to statistics, and how it differentiates from mathematics as numerical study. Students are asked to make conjectures about graphs, to create statistical questions, and to think about how bias and variation are introduced through the statistical paradigm. One of the main goals of this lesson is to help students see that variance is at the

	foundation of the study of statistics, which it is not for mathematics. Variance is new for students, and we should expect learning in this first lesson to be developmental. Slides 2 - 3: Ask students to make a conjecture. Students may not know what the word “conjecture” means. Tell them, “A conjecture is a hypothesis, or an estimated guess about what will happen next.” Ask them to make a conjecture based on the graph, even though they lived through what happened next. What does the graph make you predict? Then, share some of the student responses with the rest of the group. Be sure to use the anonymize feature. Note: If students struggle to make a conjecture, possibly use breakout rooms or small groups to help students group-think to create a conjecture collaboratively. Slide 4 - 7: These slides help students see the difference between math and statistics. It is important to share with students that most of the numerical learning they have done so far has been about calculations. Statistics is about the story and finding truth in data. Slides 8 - 9: I would suggest using a Rough Draft Format for this section. Let students know that they will write several drafts of their question and edit it along the way. Ask students to write a statistical question and a mathematical question about the graph on the screen. Then, choose one or two to look at as a group. If the statistical question is asking about a calculation, remind students that we want a question that points to the larger story of the graph, or one that is seeking the truth from the data. Ask the students to go back and edit their statistical question to make it reflect the story/truth. You may do this for several rounds until you find an example of a question that you think fits the graph well. Students will return to

edit their questions on Slide 18. You may choose to do several rounds of editing at slides 7 - 8 or wait until Slide 18.

A few examples of statistical questions:

- How is COVID-19 affecting the United States?
- Is COVID-19 surging in the United States?
- Should United States citizens be concerned about opening schools in September?

A few examples of mathematical questions:

- When has COVID-19 cases been the highest/lowest in the United States?
- What does the 7-day moving average help us predict about COVID-19 cases in the United States?

Slide 21 : Have students work in pairs to think about the obtrusiveness of each study. Then, return to whole group discussion and ask students:

- What were some ways that you could tell a study was more obtrusive than others?
- Can you think of a reason for why we may need more obtrusive studies? Less obtrusive studies?

Slides 23 - 24: These slides are used to introduce the concept of sampling. Here, we want students to notice that when we intentionally sampled (NY, NJ, & CT were NOT randomly sampled), we learned something different than the original graph suggested. Sampling is a GOOD thing - it acts like a window into the data that allows us to see farther than we could see with the big picture.

Slide 25 - 26: Pace these two slides together so students can go back and forth. We want students to be able to use the definition to make sense of bias introduced in sampling. After

	students order their sampling methods, ask them how they decided which ones were more biased than others. Slide 27: Have students work in pairs to organize the card sort. When the majority of teams are finished, have a whole group discussion with the following questions: ● How did you organize the information in the card sort? ● Which of the graphs did you find interesting? Why? ● What's one thing you learned today about statistics that you did not know?

Lesson 2: Being Skeptical

Lesson 2	**Being Skeptical**
Goals	Mathematical: To recognize the purposes of and differences among statistical surveys, observational studies, and experiments Interpersonal: To begin to develop a healthy skepticism of data presented based on the type of design used Critical: To recognize that, for many statistical questions that impact real systems and policies, the surveys, observational studies, and experiments we use are distorted versions of the data we need to answer the statistical question of interest.
Standards Covered	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.2** Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question. **NC.M4.SP.1.4** Interpret non-standard data visualizations from the media or scientific papers to make sense of real world phenomenon.
Standard Description	Students begin this lesson by making sense of a non-standard visualization for COVID-19 cases, a real-world phenomenon found in the media and scientific papers (NC.M4.SP.1.4). Students then distinguish between the different types of statistical studies (Survey, observational study, experiment) (NC.M4.SP.1.2) that could take place by thinking through the constraints of each type of study for a particular context. Students then read statistical questions (NC.M4.SP.1.1) and decide the type of study and whether a study described is well designed. Students finish by reading about the Tuskegee Medical Project, and having a discussion.
Warm Up	All of the this lesson is enclosed in the following [Desmos Link]() Display the image and statements for all students to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think

time, and then 1 minute to discuss the things they notice and wonder with their partner, followed by a whole-class discussion.

The following were three headlines associated with the above graph:

1. As of March 29, COVID-19 cases are decreasing by 26%.
2. 80% of doctors suggest wearing a cloth mask.
3. North Carolina is doing 30% better with COVID-19.

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image or statement. After all responses have been recorded without commentary or editing, ask students, “Is there anything on this list that you are wondering about?” Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

	For headline 1, ask, "How does having the whole graph tell a different story than just having the part of the graph from March 27 to March 29?" (In the whole graph, you can see that the number of cases is increasing as represented in the graph, and you could find other 12-year periods in which the slope is negative, positive, or close to zero.) If time allows, follow up by sharing the following graph of the same data, "How does the information on this graph tell a different story? Explain your reasoning." (The cumulative graph keeps up with the total number of cases, showing an exponential growth in cases by day. The cumulative graph does not show any negative trend, since there are more cases being added each day, sharing a different story of what is happening.) Here are some questions for discussion: ● "Why is the second headline potentially misleading?" (It does not mention what type of masks they are recommending or if they are recommending on type over another. For example, some physicians may recommend an N-95 mask over a cloth mask and this statistic would not include them.) ● "Why is the third headline misleading?" (It does not mention what the state of North Carolina is better than and how that is measured. For example, is North Carolina better than surrounding states? Better than the nation? Better than other countries? At controlling the virus? At testing for the virus? At minimizing the number of deaths from the virus?)
Lesson Exploration	All of the this lesson is enclosed in the following Desmos Link Arrange students in groups of 2. Display these statements for all to see:

	Survey - In a survey, the researcher records information collected from the survey participants, but does not influence the population in any ways to attempt to change their answers. Observational Study - In an observational study, the researcher observes the subjects and measures variables, but does not influence the population in any way or attempt to intervene in the study. Experimental Study - In an experimental study, the researcher is purposely attempting to influence the results with the goal to determine what effect a particular treatment has on the outcome. Researchers take measurements of the sample population, then manipulate the sample population in some manner. After the manipulation, the researchers re-measure, using the same procedures to determine if the manipulation possibly changed the measurements. Students should then be prompted to consider the following research question: A researcher wants to study connections between police brutality in the United States and race, gender, class, and zip code in order to prevent future acts of violence against citizens. Think of a way to study this question through a: ● Survey ● Observational Study ● Experimental Study Students should work in pairs to think of ways to answer the statistical question with each study type. As they design, ask them to consider concerns for time, money, resources, and ethics.

	Do not ask pairs to share their studies out loud. Ask questions like: ● What made designing a study about police brutality difficult? ● Which constraint did you run into as you were thinking of your designs? (Time? Money? Resources? Ethics?) ● Studying sensitive content that has concern for ethics can be very difficult. How can looking at historical data help? How can it not help? ○ Looking at historical data helps show that the patterns exist, but may not help patterns from recurring in the future. (Student Handout Below) One partner reads the statistical question and study design to the other partner. The reader should then sort the item into the type of study described: Survey Observational study Experimental study The other partner should decide whether the study is good or bad and explain their reasoning. After the discussion, switch roles and have the other partner read the next statistical question and study design aloud. Take turns until no questions remain. Identify students who sort any of the studies incorrectly.
Lesson Closure	The purpose of this discussion is for students to classify each study by study type, and to informally critique the design of each study. For previously identified students, ask, "What misconception did you have that caused you to sort one of the studies the way you did?" (I thought the unemployment study was experimental because it seemed like an experiment. I

	realized it was observational because all the researcher did was observe people going into a building.)
	
	Here are some questions for discussion:
	
	● "For one of the studies, how did you determine what type of study it was?" (I decided that the missing school example in which the letter sent home was a survey because the participants had to respond to a question.)
	● "What were some reasons that you said some of the studies were good?" (There was random assignment in some of the designs. Some of the questions asked in surveys asked about the facts. Some of the studies had a large sample size.)
	● "What were some reasons that you said some of the studies were bad?" (The sample size was small. The questions seemed to lead the people being surveyed to give a particular answer. Some of the experiments did not test for what the research question was asking.)
	● "What is important to think about when designing a study?" (It is important to think about whether or not the design will answer the question being asked, and whether you are influencing the results of the study by the way you ask the questions.)
	● "How did you determine if a study was observational or experimental?" (I looked for the researcher influencing a variable for the subjects in the study. If this was due to the design of the study, then I knew it was an experiment.)
	● "A survey is a specific type of observational study. How did you determine if a situation should be called a survey or observational study?" (In a survey, the researcher asks the participants directly to respond to a set of questions. In a general observational study, the researcher does not need to ask questions of the participants.)
	● There are two studies out of the 8 that are the real processes that occur to answer the statistical question of interest. Which do you think they are? Why
	    ○ Question 4 is the current way that the unemployment rate is calculated. When we hear the unemployment rate in the news and media, it is always an underestimate of the real unemployment rate due to the filtering of senior citizens, seniors in high school, and those who are not categorized to be "actively" looking for work. In order to receive unemployment assistance, a citizen must provide documentation that they have applied for a certain number of jobs each week and that they continue to unsuccessfully find work. Share the information above with students. Ask, "How is this different than what you may have previously believed about receiving unemployment assistance?"
	    ○ Question 6 is one of the ways that statistics has been used in the past in ways that are misleading. While there is a lot of information about less females being

	in upper level math courses, like AP Calculus, there are a lot of compounding factors for why the population is that way. A few are: ■ historically, AP Calculus courses were limited to only male students, and female students were not encouraged to enroll. ■ Female students who consistently see upper level math courses filled with male students begin to believe they may not be able to do math at high levels. ■ Mathematical ability is not the only factor used when recommending students for upper level math courses, leaving room for bias to enter.
Lesson Summary	There are many things a researcher should consider when collecting data about a question they are interested in. How the subjects of the study are selected as well as the details of how the study is conducted are very important in getting useful data to answer the question at hand. In particular, the researcher should consider: ● selecting subjects that are representative of the larger population ● how subjects are selected for a study or assigned to groups within a study ● making sure that the question does not lead subjects to answer a certain way ● making sure that data is collected and analyzed fairly ● using a sample that is large enough to detect differences in the presence of variability ● collecting data directly related to the question being asked ● Whether the data can be collected in an ethical way that does not harm participants Without directly addressing these concerns, the data collected might lead to misleading conclusions. Three common types of studies are *surveys*, *observational studies*, and *experimental studies*. ● A survey is a set of questions given to people to seek their responses. ● An observational study collects data without influencing the subjects directly. ● An experimental study collects data by directly influencing something to determine how another thing is changed.

This task is adapted from Illustrative Mathematics, Algebra 2, Unit 7, Lesson 1 Task [“Being Skeptical”](#).

Student Response

Sample response:

1. Survey. The design is okay, although some parents may not remember all the reasons their child missed school, or may be unwilling to return their answers to the school.
2. Observational study. The design is bad since it only uses a single elementary school. That school may have had a breakout of a contagious disease among its students which would not represent all students well. It also does not include older students who often miss school for different reasons than elementary school students.
3. Observational study. The design is bad. The observation does not address the entire population of the United States. The researcher could have chosen a day when the numbers were low or high. The local office may not be representative of the entire population.
4. Observational study. The design is bad. The observation is not representative of the entire population of the United States since it eliminates certain groups of people.
5. Experimental study. The design is bad. The students who take the test are only in one math class, and math ability is not determined by the class one is in. Also, gender is a social construct, and does not fall succinctly into two categories, as defined by the World Health Organization. This means the sample is not representative of the entire population of students as it erases non-binary gendered students.
6. Survey. The design is bad. The participants are chosen at random, so there will be a better chance of including different types of people. The data is also collected from a large national database which reduces local bias. Final grades in the course do not reflect math ability; instead they reflect whether a student completed the work a teacher asked them to do.
7. Survey. The design is not good. The question places unfair emphasis on voters paying extra money for something that seems small, like flowers. The question does not directly answer what the research question is asking and seems like it would lead people surveyed to answer “no.”
8. 8. Survey. The design is okay. The question asks about the tax in a way that does not encourage one answer or the other.

Student Handout

Being Skeptical

One partner reads the statistical question and study design to the other partner. The reader should then sort the item into the type of study described:

- Survey
- Observational study
- Experimental study

The other partner should decide whether the study is well or poorly designed and explain their reasoning. After the discussion, switch roles and have the other partner read the next statistical question and study design aloud. Take turns until no questions remain.

1. Why are students missing so much school in the district? A district administrator selects 300 student names at random from the enrollment list and sends a letter to each student's home. The letter includes a page to be returned to their school signed by a parent or guardian. The page asks, "How many days has your student missed school this year?" and "What are the reasons for missing school on those days?"

2. Why are students missing so much school in the district? A district administrator chooses one of the elementary schools in the district and asks the principal to provide information about the number of absences and the excuse notes provided to the school.

3. What is the current unemployment rate in the United States? A researcher goes to the local unemployment office and counts the number of people who enter the building for 4 hours. The researcher then compares the number of people counted to the total population in the district.

4. What is the current unemployment rate in the United States? A researcher collects data from a national database on unemployment. The researcher then uses the data to filter out senior citizens, those under the age of 18, and those who are not "actively" looking for work. The remainder of the data is counted towards unemployment against the general population.

5. Is there a link between a student's gender and a student's math ability? A statistician gathers all of the AP Calculus students in a state, selects 50 males and 50 females at random, then gives each student a Calculus test. The scores of each gender are calculated and used to answer the question.

6. Is there a link between a student's gender and a student's math ability? A statistician looks at the historical data for AP Calculus enrollment and the genders of the students in the class, as is collected by the Department of Education. The statistician randomly selects 500 students to find their final grade in the course. This information is used to compare males and females math ability.

7. Do voters in the district favor a sales tax increase of 1% to fund the parks and recreation department? A politician sends a letter to 300 voters in the district asking, "Would you pay extra money on your essential groceries to hire more government workers to plant flowers around the town?"

8. Do voters in the district favor a sales tax increase of 1% to fund the parks and recreation department? A politician sends a letter to 300 voters selected at random in the district asking, "Would you be in favor of a 1% increase in sales tax to fund the parks and recreation department in town?"

Ready for more?

A Math 4 student wants to rethink how unemployment is calculated and measured in their area.

1. Ask a statistical question about this topic that can be answered with a survey.
2. Ask a statistical question about this topic that can be answered with an observational study.
3. Ask a statistical question about this topic that can be answered with an experimental study.

There are many things a researcher should consider when collecting data about a question they are interested in. How the subjects of the study are selected as well as the details of how the study is conducted are very important in getting useful data to answer the question at hand. In particular, the researcher should consider:

- selecting subjects that are representative of the larger population
- how subjects are selected for a study or assigned to groups within a study
- making sure that the question does not lead subjects to answer a certain way
- making sure that data is collected and analyzed fairly
- using a sample that is large enough to detect differences in the presence of variability
- collecting data directly related to the question being asked

- Whether the data can be collected ethically without harming participants

Without directly addressing these concerns, the data collected might lead to misleading conclusions. Three common types of studies are *surveys*, *observational studies*, and *experimental studies*.

- A survey is a set of questions given to people to seek their responses.
- An observational study collects data without influencing the subjects directly.
- An experimental study collects data by directly influencing something to determine how another thing is changed.

Lesson 3: What Does Data Say About Me?

Lesson 3	What Does Data Say About Me?
Goals	Mathematical Goals: To get students to begin thinking about how the design of an experiment affects the results; To introduce key vocabulary that will be used throughout the unit (e.g., sample, population, representative, well-designed experiment, and the concept of error). Interpersonal Goals: To understand that not all statistics about me and others are representative of the entire population Critical Goals: To make students aware of how statistics are being used to represent a population of people for which they or others identify.
Standards Covered	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.4** Interpret non-standard data visualizations from the media or scientific papers to make sense of real world phenomenon.
Standards Description	In this lesson, students begin to explore what data says about who they are in the media and in science (NC.M4.SP.1.4). Students first navigate through their social identity wheel to recognize the different ways that society labels who they are. They then use this data to explore different aspects of their social identities through statistics. In doing so, students observe different non-standard visualizations found in the media and scientific papers (NC.M4.SP.1.4), while also making sense of the statistical questions that were asked to collect that data (NC.M4.SP.1.1).
Warm Up	Students should fill out the Social Identity Wheel handout as the warm up today. They should answer the five questions about their identity memberships individually. They should also be assured that this information will remain anonymous and not be shared. It is for thinking purposes only. For this reason, you should think through how you'd like this information to be recorded. A desmos link has been created for this lesson, but students may feel more secure if they are allowed to do this on paper, or just to think silently without writing anything down. For this purpose, a Google Slides presentation has also been prepared if an alternative method is preferred.
Task	*Exploring Data Already Written* *A Desmos Link has been created for this lesson. Due to the sensitive nature of the identities of students, it is imperative that if the lesson is used via Desmos, that the students are anonymized the entire time and their answers are not shared with their peers. For that reason, the multitude of this lesson is individual, except the end when students are asked to reflect as a whole group. To understand the sensitive nature of this lesson, it is suggested that you try the lesson out as a student learner. You can do so here.*

	Students will use the pewresearch.org website to explore data that has been collected that closely matches with their social identity, as they researched on the Social Identity Wheel. Students can use the search bar to type in terms such as White, Male, Gen Z, Middle Class, Christian, or College Bound. It is best to type in one term at a time. Students can explore the results that are returned based on the different identities they place in the search bar. Students may also browse through the home section to look at "Topics" that interest them, such as Death Penalty, Health Care, Science and Innovation. Students should be directed to create an infographic using Google Slides or PowerPoint that displays a statistic that is related to their identity. The infographic should include some statistics that align and some that do not align with how they see themselves. Allow them to be as creative as they'd like by adding pictures or images that help tell the story of the statistics they find. If you choose to use the Desmos Link, instruct students to create a google slide and share it with you. They should then use the student handout or continue in the Desmos link to think through the statistics they find.
Lesson Closure	Below are some questions for whole group discussion: A Google Slide presentation has been prepared to help share the questions with students ● Do you think that the way a researcher samples a population is important? Why or why not? ● What does it feel like to be misrepresented by a statistic? ● Why is it important for a sample to be representative of a population? ● Statistics are always reported with some amount of error in case the sample is not 100% representative of the total population. Does knowing that there is error within the statistic change how you feel? Why or why not? ● A well-designed experiment is critical to making sure there is as little error as possible in the question of interest. What are some elements that you think are important for a well-designed experiment? Why are they important?
Lesson Summary	Many times, the data collected for an experiment, survey, or observation is different from reality for members of the population. A well-designed experiment helps this to occur less often, although the results from an experiment should always be expected to vary. A researcher has many things to consider when collecting data about a question they are interested in studying. Without a careful design, an experiment could misrepresent the population as a whole and draw misleading conclusions from the study.

Exploratory Data Analysis Unit

Lesson 3- What Does Data Say About Me?

Student Handout

1. **Create an infographic about statistics that you find on [pewresearch.org] that align with how you identify (e.g., Black, Female, Christian, Gen Z, Hetereosexual). You can also browse the "Topics" section to look at subjects you are interested in. Find at least one report that displays a categorical and quantitative variable.**

2. **A "statistic" is a piece of data or information that is found from a study. For instance, in the report on Marriage for Millennials, "**three-in-ten Millennials live with a spouse and child compared with 40% of Gen Xers at a comparable age". **Based on the reports from Pew Research that you studied, what are some statistics you found? List at least three statistics, with at least one categorical and quantitative variable.**

1.

2.

3.

3. **A statistic is said to be *representative* of a population when the attributes of the sample represent the population with only a small degree of error. Are the statistics you found above true for you, or in other words, are the statistics you found *representative* of you being within the population? Why or why not?**

4. **The purpose of statistics is often to tell a story or share the truth about the data that is being explored.**

   What is the story being told for each of the statistics you found? What do you think the truth is?

5. **Decide whether each of the above statistics was measured using a categorical or quantitative variable. Then describe how each statistic was displayed (i.e. Using a bar chart, pie chart, histogram, percentages, etc.)**

1.

2.

3.

6. **Are there any themes among categorical and quantitative variable displays?**

7. **Explain how the different types of displays help explain the story being told by the statistics.**
8. **An experiment is a study in which a researcher tries to understand the relationship between different variables. While we often think of the word experiment as changing something, an experiment can also be conducting a survey like the ones on the Pew Research website. Choose one Pew Research report that is representative for you and one that is not. Go to the report and click on the "How We Did This" section using the "+" icon. Explore how the researchers conducted their experiments. Jot down any important decisions the researchers made.**

Representative	**Not Representative**
• • • •	• • • •

9. **It is often *very difficult* to survey an entire group of people mainly due to time and money constraints. To ask every human in the world about their opinion on a topic requires someone to conduct the experiment who must be paid and the time to ask. This is why statisticians choose to *sample* from the population of interest to answer their questions. How did the statisticians sample the population in the reports you explored? Do you feel as though the sampling method used is satisfactory enough to say that it represents the population in question? Why or why not?**

Representative	**Not Representative**

10. **In a well-designed experiment, the statistician controls certain variables while studying others. This allows them to look at the differences among the variables they want to know more about. What did the statisticians control in your experiments? What did statisticians allow or expect to change?**

Representative	Not Representative

11. **Analyze the experiment as a whole and the results of the experiment. What are the positives? What decisions would you make differently? What would you change so that the experiment is more representative of the total population?**

Lesson 4: Same Data, Different Results

Lesson 4	**Same Data, Different Results**
Goals	Mathematical Goals: To understand how sampling methods affect the results of an experiment, survey, or observation; To understand why randomization is a component of a well-designed experiment. Interpersonal Goals: To understand that not all statistics about me and others are representative of the entire population Critical Goals: To build criticality in digesting statistics shared without enough information; To understand how sampling methods can be used to misrepresent a population of people.
Standards Covered	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.2** Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question. **NC.M4.SP.1.3** Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (e.g.,, spreadsheets, dynamic data analysis tools) to determine: types of variables in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.
Standards Description	In this lesson, students continue to make sense of variance within a data set by looking a the different constructions of statistical questions (NC.M4.SP.1.1). Students observe how the sampling method NC.M4.SP.1.2) of statistical study introduces variance and bias by participating in a survey that is >200 entries (NC.M4.SP.1.3). Students use sliders to disproportionately sample the data set, and observe how this changes the news headline that may be given based on the statistics that could be calculated from the data set (NC.M4.SP.1.2). They, then, begin to draw conclusions about how randomizations does not always mean equal proportions, but does assure that the population is better represented in the sample (NC.M4.SP.1.2).
Warm Up	Share the following study excerpt with students. A [Google Slide Presentation]() has been prepared to help: Using district and county level data from Michigan and Washington, we investigate how the instructional modality in public K-12 schools – in-person, hybrid, or remote – in the wake of the COVID-19 pandemic influences spread of COVID in the wider community. We find that community COVID rates are strongly positively correlated with in-person and hybrid forms of schooling in simple, naïve regression models. But for an important exception described above in Section 5 and again summarized below, these findings do not persist when we allow the effects of school modality to differ according to the level of community spread in the population and add covariates that account for various district and community factors, such as mask wearing and political preferences. These general findings hold up under a variety of OLS and fixed effects model specifications. The important exception is that we *do* find some evidence that in-person modality is associated with increased COVID spread in communities with relatively high pre-existing levels of COVID. In Michigan, for instance, districts offering an in-person instructional modality show

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Ask them to respond to the following questions:

What is the statistic being shared in the excerpt?
Is the variable quantitative or qualitative?
What statistical question was asked in the experiments?

Then, project these three newspaper headlines on the screen:
In-person classes don’t contribute to community spread of COVID-19, report shows
New Study Cautiously Suggests Schools Don’t Increase Spread of Coronavirus
Education
Are schools safe? A growing body of evidence suggests that, with the right measures, they contribute little to virus spread.
These are real newspaper headlines written based on the above study.

How is the information in the excerpt different from what is portrayed in the headlines?
Is the information in the headlines false? Justify your reasoning.
What are the potential consequences to the reader who only reads the newspaper but not the study it came from?

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<td>Task</td>
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Sampling Methods Task
Use the following Desmos Activity Builder link to access this task:
Within the Activity, students will be asked their opinion about the following question:

Do masks reduce the spread of the COVID-19 virus?

Within the activity, the students will be sampled for their answers and a statistic will be revealed to the class. The class will then look at whether or not the statistic is representative of the class as a whole, looking at the summary screen in histogram/boxplot form.
Note: It is important to use the anonymize feature in Desmos for this activity so that students can freely share their opinion without feeling ostracized from the class for expressing an opinion that may be different from the larger group.
For additional tasks, look at the Sampling Lesson from Illustrative Mathematics
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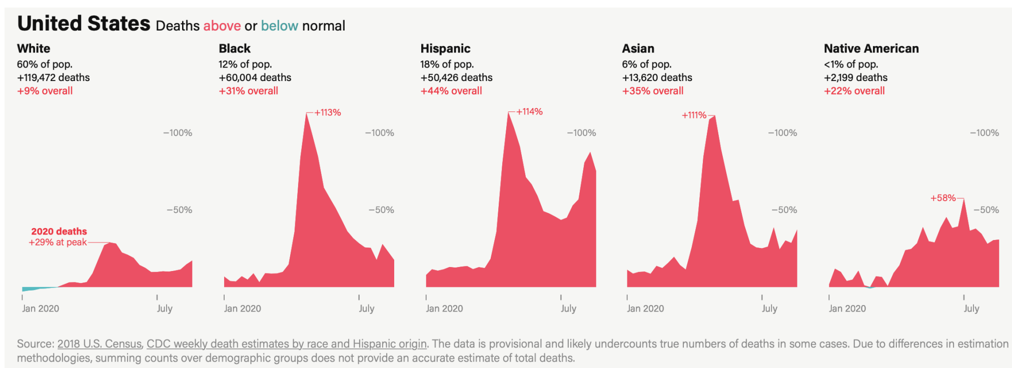
Lesson Closure	Below are some questions for discussion: ● How can the sampling strategy a researcher chooses change the conclusions of the study? ○ Is this normal for statistics or is this dangerous? ● Return to the image from the beginning of class. What do you think Dr. Patton meant when she said, “You can make data say what you want.”? How does poor sampling methods help with this? ● Your old baseball coach shares the following statistic on Facebook: “70% of North Carolinians believe blue lives matter.” What information might you want to know before believing this statistic? ● Does random sampling always mean that the sample is representative of the population? Why or why not?
Lesson Summary	***Adapted from Illustrative Mathematics, Randomness in Groups, Algebra 2, Lesson 3*** So how do we get a representative sample? The best way is to let chance select the sample. For example, you might randomly select 25 different times throughout the day to remove the next bag of trail mix from the conveyor belt and count its pretzels. Using a process based on chance, in which each individual in the population is equally likely to be selected, is called random selection of the sample. In experimental studies, it is often necessary to assign the individual participants in the sample to one or more groups. It is also best to assign individuals to groups using a random process. For example, say that you were studying the effect of students turning off electronic devices while doing homework. After a representative sample is selected, you need to assign the individuals in the sample to two groups: one group makes no changes to the conditions by which they normally do homework, and another group that turns off electronic devices while doing homework for the duration of the study. Examples of assignment processes that are *not* random include: ● Assigning students whose names start with A–L to one group and M–Z to the other group ● Assigning students who play a musical instrument to one group and the rest to the other group ● Asking for volunteers to be part of the group that turns off electronic devices In order to assign individuals randomly to groups, every individual must have an equal chance of being assigned to either group. Examples of assignment processes that *are* random include: ● Writing each participant’s name on a slip of paper and mixing the slips well in a bag. Drawing half of the names from the bag and assigning these participants to one group, and the rest to the other group. ● Flipping a coin for each participant, and placing them in one group if the result is heads and the other group if the result is tails. ● Getting a list of participants and numbering the list. Using a random number

	generator to select participants for one group. When subjects are not assigned to experimental groups using a random process, other factors may influence the results from the experimental study so that the data does not answer the initial question. In this example, if the groups are split by volunteering, the impact of turning off the devices may be impacted by similar traits by the subjects who volunteer, such as their not using electronic devices much already or having a personality that is willing to volunteer to try something new. These traits may influence the results so the data from the experimental study does not accurately address the question about the impact of electronic devices on student homework.

Lesson 5: Same Data; Different Conclusions

Lesson 5	**Same Data; Different Conclusions**
Goals	Mathematical: To explore a data set with at least 200 entries and experience exploratory data analysis in a scientific context. Interpersonal: To understand that our initial view of data may chance with more information. Critical: To recognize how COVID-19 virus is disproportionately impacting communities of color.
Standards Covered	**NC.M4.SP.1.1:** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.3:** Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (i.e. spreadsheets, dynamic data analysis tools) to determine: types of variables that are in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data. **NC.M4.SP.1.4:** Interpret non-standard data visualizations from the media or scientific papers to make sense of real-world phenomenon.
Standards Description	In this lesson, students will construct statistical questions (NC.M4.SP.1.1) that guide their exploration through a data set of more than 200 entries (NC.M4.SP.1.3). Students continue to make sense of the statistical process by understanding how sampling can lead to different conclusions (NC.M4.SP.1.3), and why it is important to ask meaningful statistical questions (NC.M4.SP.1.1). In the warm up, students will make meaningful of a non-standard visualization (NC.M4.SP.1.4), and they will also work to create their own non-standard visualization through their own statistical exploration in the data set on COVID-19 cases in the United States, a real-world phenomenon that is occurring in both the media and science (NC.M4.SP.1.4).
Warm Up	A Google Slides Presentation has been prepared to help. Part 1: https://www.nytimes.com/2020/09/10/learning/whats-going-on-in-this-graph-covid-19-cases-in-america.html?searchResultPosition=2 Project the two graph images from the NY Times for your students. Ask them the following questions: ● What do you notice? ● What do you wonder? ● What impact does this topic have on you and your community?

	● What's going on in these maps? Write a catchy headline that captures the maps' main idea. Part 2: Project the image from The Marshall Project for your students. Ask them the following questions: ● What do you notice? ● What do you wonder? ● What statistical question is this graphic trying to answer? ● Whose voices are missing in this graph? ● What are some questions that you would like to know about the source of data for this graphic? 
Task	CODAP is a program that runs through a web browser. It sometimes can make other tabs crash. Since Zoom also runs through a web browser, you may advise students to close all other tabs and browsers before using CODAP. Students will explore the provided data files using CODAP to look for patterns and trends among the data. The data set on COVID cases by race came from https://covidtracking.com/race. To help students get acquainted with CODAP, you may want to first use the Roller Coaster Data. To get there go to codap.concord.org, click Try CODAP in the upper right hand corner. Then choose "Open or Browse Existing Files". In the search bar, type Roller Coaster, and find the file. The Text Window that appears on the far right of the screen walks students through how to explore data in CODAP. Students should use the student task guide to assist them in their exploration, but general exploration of patterns is encouraged to give students an understanding for how statistical analysis works. This lesson has been built in Desmos for your convenience. This task is broken up over two lessons to allow students to (1) engage in the exploration portion of exploratory analysis by looking for patterns, trends, and relationships within the

	given data; and (2) analyze the data given for ethical considerations, and use additional variables to highlight key findings. For the first part, students will look at the COVID_19_By_Race Data file and on the second part, students will use the file COVID_19_By_Race_By_State data file.
Lesson Closure	Below are some questions for discussion after Part 2 is complete: ● What were your initial conjectures about the data after Part 1? ● Did you think that the virus was impacting different communities disproportionately? ● How did the data being presented in the Number of Cases prevent you from seeing the patterns by race? ● Some news outlets have portrayed the COVID-19 data as impacting everyone equally, and not a racial matter. Some have said that using race as a variable to look at COVID-19 data is wrong. What would you say to people who think that way? ● Whose voices were missing from this data analysis? ● What are you still wondering about after viewing this data set? What questions do you still have that we should continue to explore?
Lesson Summary	Statisticians are often asked to explore a data set to make predictions. Sometimes, these predictions can have a huge impact on government policy, funding, or citizens' daily lives. When statisticians begin looking at data, they have to be sure that the data they have will actually answer the question they are interested in. It may take additional information in order to answer the statistical questions we want to know more about. This is part of the ethics of statistics. Reporting trends and patterns on too little data presents a narrow view and often only part of the picture, which can be very damaging. In this case, presenting total case counts can lead society to believe that COVID-19 is impacting all communities equally, but this is not the case, and the data supports these findings.

Exploratory Data Analysis Unit

Lesson 5 - Same Data; Different Conclusions: Part 1

Student Task Sheet

1. **Using your web browser, navigate to https://codap.concord.org/ (you can also just Google CODAP).**
2. **Select "Try CODAP"**
3. **Select "Open Document or Browse Examples"**
4. **Choose the file provided COVID_19_By_Race.csv. You can do this by finding the location of the file on your computer, or by dragging the file into the square that pops up. This data file contains the number of cases by race and by state for the COVID-19 pandemic, as well as demographic data for each state.**

5. **Exploratory Data Analysis is just that:** *exploratory.* **Begin your exploration by looking at the titles of each column and making sense of the data in the file. What is the data about? Without doing any statistical analysis (making graphs or charts), what do you notice about the data? What do you wonder?**

6. **There are several rows that have missing data. What are some initial conjectures as to why the rows are empty? Are there any patterns to the rows that are empty? Why might we not know the COVID-19 data for this group?**

7. **It is not uncommon for statisticians to work with data that needs to be cleaned because data entries are missing. What is important in this process is to note why data entries are thrown out of the analysis, and to justify the reason for cleaning the data in this way. What would you suggest in this case: should the empty rows be thrown out or is the data useful somehow? Justify your reasoning for keeping the data in or throwing the data out.**

8. **You may also notice that the data on cases counts for COVID-19 are broken down by race. What do we gain and lose by categorizing the data in this way?**

9. **A statistical question is one that allows for data to be collected and will have *variability* in the data. Statistical questions also follow these criterion:**

- **The variable(s) of interest is/are clear and available.**
- **The population of interest is clear.**
- **The intent is clear.**
- **The question can be answered with the data.**
- **The question is one that is worth investigating – is interesting, has a purpose.**
- **The question allows for analysis to be made of the whole group.**

Based on your initial exploration, what is a statistical question you think the data may be able to help you answer?

10. **Oftentimes, researchers are given data and asked to explore for patterns, trends, and relationships. Explore the data in this file for approximately 15 minutes, looking for patterns, trends, and relationships among the different variables. Write down what you find. To give you some ideas of places to explore, you may try the following processes you've learned from previous courses:**

- ☐ **Build a histogram of one variable of interest(if the data is normally distributed). Why do you think the data is normal?**

- ☐ **Build a boxplot of one variable of interest (if the data is skewed left or right). Why do you think the data is skewed?**

- ☐ **Find the five number summary for a set of data. What do each of the statistics mean in context for the variable you chose?**

- ☐ **Find the mean and standard deviation for a variable in question. What do the values mean in context for the variable you chose?**

- ☐ **Compare two variables. Find the least squares regression (or line of best fit) for the data. What two variables did you choose and why? What does the rate of change tell you about the two variables? What does the y-intercept tell you about the two variables?**

11. Based on your initial exploration, what are some ideas that you are beginning to generate about the data? What patterns, trends, and relationships did you notice? What are some questions that you wonder about?

Ideas	Notice	Wonder
• •	• •	• •

12. If you begin to notice a pattern, you can continue to search for that pattern among other variables to see if your conjecture holds true. For instance, you could drag the states to the y-axis and the total number of cases to the x-axis. What do you notice? Then you could compare the states to the different races. Do the patterns change? Why would the patterns change?

- **Total Cases vs. States**
- **White Cases vs. States:**
- **Black Cases vs. States:**
- **LatinX Cases vs. Total Cases:**

- **Asian Cases vs. Total Cases:**
- **American Indian & Alaskan Native (AIAN) Cases vs. Total Cases:**
- **Non-Hispanic Pacific Islander Cases vs. Total Cases:**
- **Multiracial Cases vs. Total Cases:**
- **Other Cases vs. Total Cases:**
- **Unknown Cases vs. Total Cases:**

13. A conjecture is an initial idea that we believe will hold true if we continue our investigation. Statisticians who begin to explore data start to develop conjectures, or ideas that they believe will hold firm even with more exploration.

What is a conjecture that you believe will hold true for all of the different relationships modeled above?

14. Conduct a similar process for another set of variables of your choosing as you did in Question 11. What are the patterns and trends in the data? What is an initial conjecture you can make based on your exploratory analysis?

Exploratory Data Analysis Unit

Lesson 3 - Same Data; Different Conclusions: Part 2

Student Task Sheet

Yesterday, we focused on exploring a data set.

1. **In your own words, what is exploratory data analysis?**

2. **In the last lesson, we looked for case patterns among the different races by states. Build several graphs to review the patterns for North Carolina: states vs total cases; states vs White cases, states vs Black cases, and states vs Ethniticity_Hispanic. How does the state of North Carolina compare to other states? Are there more cases of COVID-19 among Hispanic North Carolinians, Black North Carolinians, or White North Carolinians? Why do you think that is?**

3. **At times, when statisticians are exploring, they need additional data to understand why the pattern exists. Go to Google and type "Population Demographics for North Carolina". Click on the Census.Gov website to gather more information about the racial demographics of North Carolinians.**
    - **North Carolinians are ______% White.**
    - **North Carolinians are ______% Black.**
    - **North Carolinians are ______% Asian.**
    - **North Carolinians are ______% Alaskan or Native American.**
    - **North Carolinians are ______% Hispanic or LatinX.**
    - **North Carolinians are ______% Multiracial (two or more races).**

4. **You'll notice that the demographics data from the US Census website uses different definitions to define races. What do these differences tell you about the data authors (the people who created the files)? What do these differences tell us about the way we define race in society?**

5. **Compare these demographics with the patterns you found in question 2. What do you notice about the data you found and the demographics for the state? How do they align?**

6. **One statistical question that this data may help us answer is, "Is COVID-19 impacting different racial groups disproportionately?". But looking at the number of cases alone will not help us answer that question. In order to answer that question, we need to compare the cases of each state and race by the demographics data for that state. For example, if there are states where the population has a majority of White people, but there are more cases for Hispanic or Black people in that state, we could see that the virus is impacting different races disproportionately. Import the new file COVID_19_By_Race_And_State. What's different in this file compared to the other file?**

7. **This data is identical to the previous file except the data has been transposed to show *relative frequencies*. Begin your exploration of the relative frequencies by building graphs between the Case Percentages by Racial Populations and the states column. Focus on the state of North Carolina. What patterns do you notice? What do you wonder?**

8. **To help explore data, statisticians will often look at a common statistic (or value that is used to measure a trend or pattern) across multiple variables. Create and record a *statistic* to help make sense of the data for North Carolina. Be consistent in the statistic you use for each demographic (You can choose the mean, median, maximum, minimum, etc, but use the same one for every category). Record the statistic for each demographic variable.**

   - **Total:**
   - **White:**
   - **Black:**

- **Hispanic:**
- **Asian:**
- **Alaskan or Native American:**
- **Pacific Islander:**
- **Multiracial:**
- **Other:**
- **Unknown:**

9. **For statisticians, the choice of a statistic is important, and must be justified so it can later be explained to others who view the results they find. What statistic did you choose to measure? Why did you make this choice?**

10. **For the state of North Carolina, are there specific subgroups that are being disproportionately (***more or less than expected***) infected by the COVID-19 virus? What evidence do you have to support your claim?**

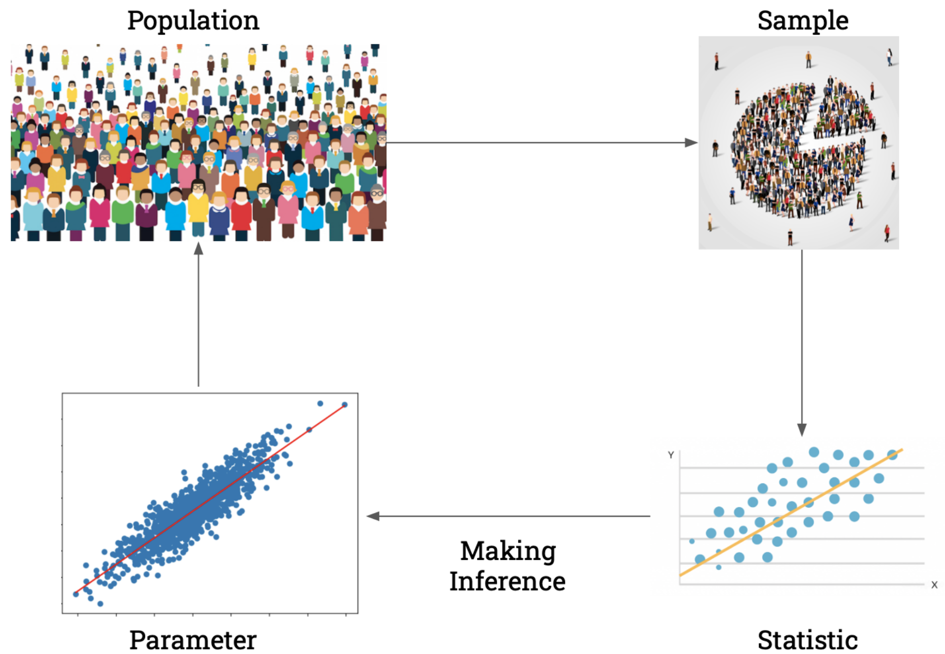
11. **Read The Marshall Project's Article on the Disproportional Impact of COVID-19 on different communities.**

12. **At the bottom of the article, the data on cases is broken up by percentages by state. Choose another state to explore. Look up the demographics proportions for that state. Are there specific subgroups that are being disproportionately (***more or less than expected***) infected by the COVID-19 virus in _______________? Use statistical data to support your claim.**

13. **How have our conclusions about this data set changed across our exploration?**

Lesson 6: Exploring Data Meaningful To Me

Culminating Unit Lesson	**Exploring Data Meaningful To Me**
Goals	Mathematical: The culminating mathematical goal of this unit is for students to complete the entire statistical paradigm, from creating a statistical question that is meaningful for them to answer, identifying a population, generating a data set to explore, answering the question by calculating a chosen statistic, and making inference back towards the population in question. Interpersonal: To create a compelling statistical question that is meaningful to the student, and to find a way to answer the question using the statistical method. Critical: The criticality of this lesson is correlated to the student's choice of a question to explore.
Standards Covered & Description	**NC.M4.SP.1.1** Construct statistical questions to guide explorations of data in context. **NC.M4.SP.1.2** Design sample surveys and comparative experiments using sampling methods to collect and analyze data to answer a statistical question. **NC.M4.SP.1.3** Organize large datasets of real world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (e.g.,, spreadsheets, dynamic data analysis tools) to determine: types of variables in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data. **NC.M4.SP.1.4** Interpret non-standard data visualizations from the media or scientific papers to make sense of real world phenomenon.
Warm Up	Review the statistical paradigm briefly.



Task

The goal of this lesson is to serve as a summative assessment for the unit as well as for students to seek out the truth in a data set using the critical skills they have learned in this unit. Students have a goal of searching for the truth in a set of data and telling/sharing the story that exists from their perspective, as they have done in previous lessons.

Students will move through the entire statistical paradigm to explore data with a compelling, statistical question that is meaningful to them.

- Remind students that exploring data begins with a compelling, statistical question that is meaningful.
- After determining a meaningful statistical question, students should identify the population from whom they need to collect data.
- Students should either find data from a preexisting data set, or decide if a survey, experiment, or observation would be appropriate to gather their own data.
    - Try to help students find data with at least 200 entries in the dataset.
    - If students choose to create a survey, help them figure out how to obtain information from 200 people.
- After finding a data set that will allow them to answer their statistical question, students should begin exploring the data, and begin calculating statistics to help them begin to answer their question.
- Students should then begin to make inference back to their population of interest.
- During this process, students should be encouraged to take notes on the decisions they make so they can understand their thought process. This helps to reinforce how variability appeared within their process. A few places to note:
    - How did students choose their data set?
    - How did students choose whether a survey, observation, or experiment would be best to answer the question?
    - How representative is the sample taken of the data set to the population of

	interest? ◦ How did students decide what statistics to calculate? ◦ How close to the population parameter is the statistic? The final product is dependent on the student's data. It could be a presentation or meaningful graphic that displays the summary of findings and a written response explaining their statistical process, and how they made decisions. Each student's project should answer the following questions: • What is the story told through this data? • What is the truth? The final product should also include a written reflection on the following questions: • Why was this statistical question important for you to explore? • How do you see yourself within the data? How did that change your exploration? • In your opinion, can we trust statistics?
Potential Compelling Moments	• Some students may struggle to find data to answer their statistical question. ◦ It is ok if students edit their statistical question as they find data sets. Help them understand the need to document the decision to change questions and why. ◦ Help students hold the tension of the data being difficult to find. ▪ Is there a reason why you think the data is difficult to find? ▪ What data are you finding regarding the question? Why doesn't the data answer the question you are asking? ▪ Would it be possible to survey or observe people to answer the question? ◦ Here are a few places to point students to look for meaningful data sets to help answer their questions: ◦ Gender Inequality: data.un.org ◦ Education: ocrdata.ed.gov ◦ Pew Research ◦ Incarceration Data: https://www.sentencingproject.org ◦ Racial inequality: The Marshall Project ◦ COVID Tracking Data: covidtracking.com ◦ Political Data: https://fivethirtyeight.com ◦ General Data Sets: ▪ https://www.gapminder.org ▪ https://www.kaggle.com • Some students may struggle to realize they have made a decision that introduced variability into their statistical process. Ask students questions such as the following to spur their thinking: ◦ Where did you get your data source from? Were there other data sources you could have used? ◦ What does the data source tell you about the data authors? ◦ Did you sample your data set? If so, what was your sampling method? How did you make that decision?

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<li style="list-style-type:circle">How did you decide what variables in the datasource to use to answer your question?
<li style="list-style-type:circle">What statistical measure did you decide to calculate? Why?
<li style="list-style-type:circle">How close to the population parameter is the statistic you calculated?
Some students may struggle as they explore data that is close to their identity. Ask students questions such as the following:
<li style="list-style-type:circle">How do you see yourself in the data?
<li style="list-style-type:circle">How is your identity different from what the data shows? The same?
<li style="list-style-type:circle">What parts of your identity are not found in the data?
<li style="list-style-type:circle">How does knowing this data exists change your perspective on your future?
<li style="list-style-type:circle">Do you have to live into the data or do you see ways to exist outside of the population?
Note: It was suggested by previous teachers to hold small conferences with students as they work on this project to check in with them about where they are and have meaningful discussion about the variances that are introduced because of their decisions. Make sure to plan for space to meet the individual needs of students here to assure learning is occurring.
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<tr><td>Lesson Closure</td><td>
Here are some questions for discussion:
How has your understanding of statistics changed since the beginning of the unit?
In your opinion, can we trust statistics?
What is valuable about statistics?
Why is it important to understand bias and have a healthy skepticism?
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APPENDIX C: RESEARCH TOOLS

Interview Protocols

Teacher Pre-Implementation Interview

This is intended to be a semi-structured interview that will be recorded for analysis at a later date.

- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your statistical content knowledge?
- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your pedagogical knowledge for teaching statistics?
- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your knowledge of systemic barriers caused by race, gender, or class?
- What is the role of a math teacher?
- Why should we teach statistics to high school students? Or Why is statistical important to learn?
- Talk to me about a time when you feel like something was "unveiled" in education, when you were awakened to something you originally could not see. What was the event? How were you awakened? How did your teaching change afterwards? (If a particular student case is discussed, ask for a systemic example)
- In the past, what contexts have you used to teach statistics?
- How are the tasks in this unit the same as/different than the ones you have used in the past?
- Describe a lesson you have taught in the past that covers critical statistics content.
- After looking through the tasks for this unit, what would you say are the major learning goals?
- After looking through the tasks for this unit, what do you anticipate are some compelling moments that will occur with students?
- After looking through the tasks for this unit, what do you anticipate will be some potential barriers to instruction?
- After looking through the tasks of this unit, do you anticipate making any modifications to the tasks? If so, for what purpose?
- After looking through the tasks in this unit, what teaching strategies do you feel you will use to best facilitate the tasks?
- What is your motivation for using these lessons with your students?

Teacher Post-Implementation Interview

This is intended to be a semi-structured interview that will be recorded for analysis at a later date.

Semi-structured interview

- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your statistical content knowledge?
- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your pedagogical knowledge for teaching statistics?
- On a scale of 1 - 5, with 1 being not knowledgeable, how would you rate your knowledge of systemic barriers caused by race, gender, or class?
- As the lead learner in your classroom, what is something you learned about statistics through this unit?
- As the lead learner in your classroom, what is something you learned about the teaching of statistics through this unit?
- As the lead learner in your classroom, what is something you learned about systemic barriers caused by race, gender, or class through this unit?
- Listen to this clip of students discussing their experience in learning statistics through a context embedded in real news and real data. Reflect on what you hear.
- Teachers will be asked to bring evidence (student work, recalled conversation, etc) to show that their students developed statistical literacy, specifically the concept of variance and then asked to explain their selection.
- Teachers will be asked to bring evidence (student work, recalled conversation, etc) to show that their students developed critical statistical literacy and asked to explain their selection.
- How do you see students' identities shaping how they interacted with the lessons?
- How have you changed in your practice (if any)?

Daily Teacher Reflection Prompts

This feedback form is to be shared with teachers to be completed at the end of each day throughout the unit of study.

- What did you anticipate would occur in this task with your students? (Try to write this part down before)?
- What adaptations did you make from the lesson plan? Explain any reasons for the adaptations.
- What were the compelling moments that occurred during the lesson? (If you can remember specific quotes from students or approximate times in the class, that would be helpful!)
- Drag the scale to show the balance between critical and statistical thinking in the task.
- What teaching strategies assisted with the facilitation of this task? Explain how they were helpful.
- If you were to teach this task again, what would you do differently? What would you do the same?
- Reflect on the task as a learner (from the student perspective) and as a teacher.
- Share any additional information about the task, learning goals, modifications, facilitation, or teaching strategies here.