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# Teaching an Algebraic Equation to High School Students with Moderate Developmental Disabilities

Bree A. Jimenez
and Diane M. Browder
University of North Carolina at Charlotte

Ginevra R. Courtade
West Virginia University

*Abstract: The purpose of this study was to determine the effect of systematic instruction with a concrete representation on the acquisition of an algebra skill for students with moderate developmental disabilities. Three high school students with moderate developmental disabilities participated in this study. A multiple probe across participants research design was used to evaluate the effectiveness of the treatment. Finally, this study was the first to teach an algebra skill to students with moderate developmental disabilities. Students were successful at learning how to solve an algebraic equation through the use of systematic instruction with a concrete representation, including mastery with generalization across materials and settings.*

Mathematics instruction for students with developmental disabilities has typically focused on functional skills such as money management, telling time, or basic number identification. The ability of students with moderate developmental disabilities to acquire functional math skills like counting or identifying money is well documented (Browder & Grasso, 1999). What is less clear is whether this population can acquire other skills of mathematics that are typically taught in the general curriculum.

General mathematics curriculum has five major strands according to The National Council for Teachers of Mathematics including operations, algebra, geometry, measurement, and data analysis/probability (http://www.nctm.org). The NCTM notes that the amount of coverage given to these components varies across years. For example, students may learn patterns and other pre-algebra content in the early grades, but spend more time on number concepts. Algebra content then becomes a stronger focus at the secondary level. The NCTM also emphasizes that mathematics has processes including problem solving, reasoning and proof, communication about mathematics, making connections, and representation. Most state standards reflect categories of content that parallel these NCTM standards.

Currently there are no studies on teaching abstract problem solving like algebra to students with moderate or severe developmental disabilities (Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, in press). Research on abstract problem solving for students with mild disabilities suggests that students benefit from highly structured, teacher-based instruction (Butler, Miller, Lee, & Pierce, 2001; Kroesbergen & Van Luit, 2003). This instruction may include direct/explicit instruction like modeling and concrete-to-representational-to-abstract sequencing with manipulatives (Witzel, Mercer, & Miller, 2003). For example, Witzel et al. investigated the effect of an explicit concrete-to-representational-to-abstract (CRA) on algebra skill acquisition. To demonstrate the CRA model, classroom teachers were instructed to use physical objects at each step of the algebra equation (i.e. $-1N + 10 = 30$; a minus sign, one coefficient marker, an N, a plus sign, a large stick, an equals line, and three small sticks.) Students were then instructed through the use of pictoral representations, to find the solution, by drawing each step. Finally, the students learned to solve an abstract problem written in Arabic symbols.

Although studies do not yet exist in teach-

Correspondence concerning this article should be addressed to Bree A. Jimenez, Department of Special Education, UNC Charlotte, 9201 University City Blvd., Charlotte, NC 28223. Email: BJimenez@uncc.edu

ing algebra to students with moderate developmental disabilities, concrete representations similar to those used by Witzel et al., (2003) have been effective for teaching functional math skills. For example, Sandknop, Schuster, Wolery, and Cross (1992) used an adaptive device to teach students with moderate mental disabilities to grocery shop by determining the lower dollar amount when buying items. An important difference is that the concrete representation was not faded to Arabic symbols alone. That is, students could continue to use the device as a form of support for price comparisons.

Another difference in mathematics instruction for students with significant cognitive disabilities is that they may need to learn a smaller subset of information with more teaching trials and specific prompting procedures that suppress errors. For example, to learn to choose the amount of money needed for a purchase (Gardill & Browder, 1995) or to round up to the next dollar amount to know how much currency to give for a purchase (Colyer & Collins, 1996), students with moderate mental disabilities received many repeated trials of daily instruction across several weeks.

Research is needed to determine if strategies such as concrete representations and systematic prompting with repeated daily trials of instruction make it possible for students with moderate disabilities to acquire other strands of mathematics for full access to the general curriculum. This research is especially important given the requirements of No Child Left Behind (2002) that all students show adequate yearly progress in mathematics. Although students with moderate developmental disabilities may be working towards alternate achievement standards, these must be linked in meaningful ways to the academic content for their grade level (U.S. Department of Education, 2005, p. 26.) For secondary students, the grade level emphasis of this academic content is abstract strands of mathematics such as algebra and geometry. Aside from the requirements of federal policy, students may also benefit from reasoning skills embedded in this content.

The purpose of this study was to determine the effect of systematic instruction with a concrete representation on the acquisition of an algebra skill for students with moderate developmental disabilities. Specifically, the target was to solve a simple linear algebraic equation (e.g., $3 + X = 5$).

## Method

### *Participants and Setting*

The study was conducted in a public high school in an urban setting in a self contained class for students with moderate developmental disabilities. All training was conducted by the classroom teacher. To be included in this study, participants had to be able to count to nine and identify numbers one through nine. Three students were recruited who met these eligibility criteria. The participants ranged in age from 15 to 17 years and were verbal. Their full-scale IQ scores on the Wechsler Intelligence Scale (WISC-R) ranged from 41–49 with a mean of 45. Concurrent with the study, the participants were receiving ongoing instruction in functional math including reading numbers found in their daily routine (e.g., sports scores) and using money (e.g., to purchase lunch or a soda).

### *Materials*

The materials used in this study were teacher made, and consisted of a number chart, a strip with an algebraic equation without numbers ( $\_\_\_ + X = \_\_\_$ ), a green object to use as a place marker, a red object to use as a place marker, erasable pens, and a variety of manipulatives to use for counting (wooden shapes, spoons, pens, pencils, paperclips and colored markers). The number chart had the numbers 1–9 which had been laminated and made removable with Velcro fasteners.

### *Dependent Variable*

To select a target skill for intervention, the teacher consulted with an algebra teacher at the high school. Together they identified solving simple linear equations to be a foundation to learning algebra. The research team then identified the steps of solving an equation as a task analysis (see Table 1). The dependent measure was the number of correct steps the student completed independently on the task analysis to solve for X in an algebraic equa-

TABLE 1

**Data sheet used in probes and instruction showing the task analysis for solving a simple linear equation.**

| **Student:** *Jack*<br>**Academic Component:**<br>☒ Math | | **Task: Student will complete simple algebraic addition equation.**<br>**Example: 3+x=7** | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Steps:** | **Date:** ⟶ | Probe 10/2 | Teach 10/5 | | | | | |
| 1. Student points to sum on equation (e.g., 7).<br>*How many (spoons) do you need?* | | + | + | | | | | |
| 2. Moves red marker to sum on chart (at 7) | | + | + | | | | | |
| 3. Counts number of items in container and finds this known number on equation (3)<br>*How many spoons do you already have?* | | + | + | | | | | |
| 4. Moves the green marker to known number on chart (at 3) | | 0 | V | | | | | |
| 5. Count to the sum with materials (from 3 to 7)<br>*How many more spoons will you need to get?* | | 0 | M | | | | | |
| 6. Selects the number counted (4) | | 0 | M | | | | | |
| 7. Puts correct number in for x in for formula (4) | | 0 | + | | | | | |
| 8.Puts correct number needed in container (4 items) | | 0 | V | | | | | |
| 9. Solves for x (writes 4 for x=4) | | 0 | V | | | | | |
| **Total Independently Correct:** | | 3 | 4 | | | | | |

**+ Independent Correct**
0 **No response (probes)**
– **Error**
**V verbal prompt**
**M model prompt**

tion. During baseline, each student was given one demonstration of how to solve for x, and then asked to do so. No prompts or feedback were given on the student's performance. The number of correct responses was graphed and baseline probes were repeated daily until a stable trend was noted. Once students began intervention, data was collected on the first algebraic equation the student used after the zero delay was faded. Only independent correct responses were graphed and used for data analysis. Steps completed out of order were considered correct if they did not impact achieving the correct answer.

### *Experimental Design*

A multiple-probe across participants, single-subject design was selected to evaluate the effectiveness of the task analytic instruction with the concrete representation on the acquisition of an algebraic equation. First, all participants were given the baseline probe. When all students exhibited a stable baseline, the first student received the intervention. When the first student neared mastery all students were probed to be sure treatment diffusion had not occurred. The second student then received intervention and when he neared mastery all students were probed again. The third student then received the intervention. The use of a multiple probe design rather than a baseline prevented repeated exposure to a skill the student had not yet been taught.

*Intervention*

A multi-component intervention was used that included: a) a concrete representation of solving a simple linear equation, b) task-analytic instruction on the steps to solve the equation multiple trials for learning, and c) systematic prompting with fading to promote errorless learning. The concrete representation can be seen in the steps shown in Table 1. The students learned to solve the algebraic equation by using a poster with the equation, manipulatives to represent numbers in equation, markers to keep their place, and a number line to count out the solution. During generalization probes, the problem was presented like a job task with various new manipulatives (e.g. spoons, pencils, pens, paperclips, candy). For example, how many spoons do you have? I need you to have 9 spoons, how many more do you need? Solve for x (see Table 1 for timing of the questions in the steps of the task analysis). In addition to the concrete representation, the students were taught the problem solving skill using task analytic instruction for each step in sequence. The students learned to first find the sum, use an object to mark it, then find how many they had, etc.

For each step of the task analysis the teacher used systematic prompting with feedback. The prompts were introduced with time delay to promote errorless learning (Snell & Brown, 2006). On the first session of the intervention the students were given a zero second time delay for a verbal and a model prompt on each step (i.e., the teacher showed and told the student each step and the student then imitated the response). On the second day of the intervention the students were given a zero second time delay for the verbal prompt and a four second time delay for the model prompt (i.e., the teacher told the student what to do, but then waited to see if the student could do it with only a verbal direction before demonstrating it). On the third and subsequent days, the teacher waited the four seconds before giving a verbal direction and then four more seconds, if needed, before modeling the step. If correct, the student received praise on pre-selected steps (i.e., steps 2, 5, 8, & 9). Once students could perform all steps, this praise was faded by dropping one praise statement each day for three days until praise only occurred at the end of the equation. If the student began to make an error, the teacher interrupted the response and gave the next prompt. Different numbers (sums to 9) were used each day for the equation so students could not memorize the answer. The teacher recorded data at the end of the first trial of solving an equation (after all nine steps). No data were recorded on the first two days of intervention because it was not possible for the student to make an independent response on those days (i.e., an immediate, zero delay prompt was used those days to promote errorless learning).

*Inter-rater reliability.* Another member of the research team observed the classroom teacher on approximately every third session to collect inter-rater reliability and procedural fidelity data. Inter-rater reliability was computed as step-by-step agreement and was 100% for all sessions. Procedural fidelity was assessed by scoring if each step was taught using the correct order of prompts, timing of prompts, timing of praise, and interruption of errors. Procedural fidelity was computed as the percentage of steps for which all of elements were correct and ranged from 89% to 100% with a mean of 98.9%.

**Results**

Figure 1 shows the number of steps correct for the three participants in baseline and intervention for the solution of an algebraic equation. No data were taken on the students during the first two sessions following the intervention because a zero time delay was being used to facilitate instruction. Jack got no steps correct in baseline. Despite frequent ab-

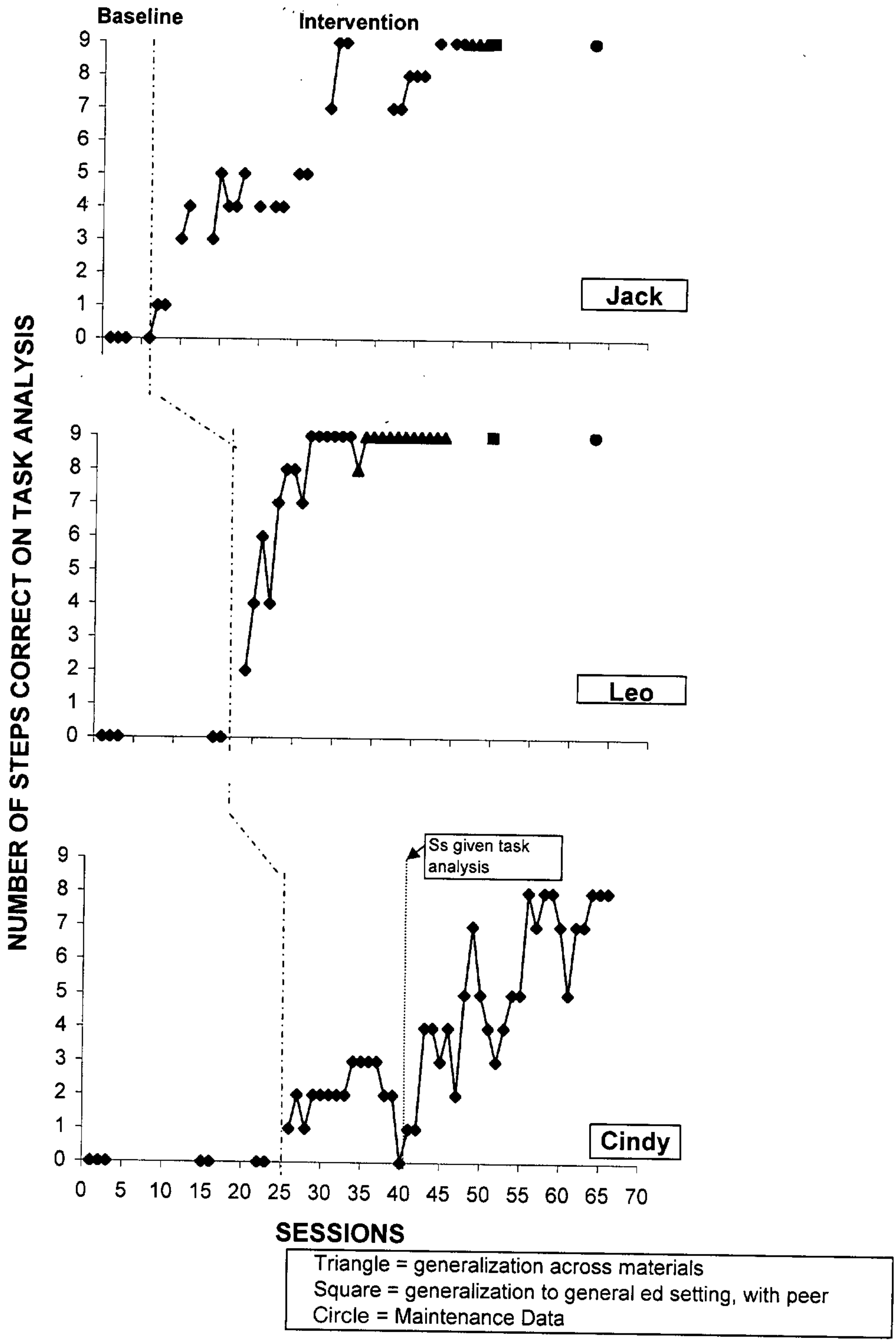


**Figure 1.** Number of steps correct in algebra task analysis for participant 1, 2, and 3. Breaks in data paths indicate absence from session. The square indicates a generalization session in a typical high school algebra class.

sences, he learned all nine steps in 17 lessons. Leo had no steps correct during baseline and learned all the steps in nine lessons. Cindy also had no steps correct in baseline. Cindy did not show the same rate of acceleration as the first two students after intervention and at lesson 15 was given the task-analysis to help her self-monitor the steps to be completed. Cindy required an additional modification after the 29th lesson, due to errors. A 15-second time delay was built in prior to the first step to allow Cindy time to focus on the task analysis. Cindy was able to learn eight out of the nine steps by lesson 31. Jack and Leo maintained correct completion of all steps on the follow-up probes.

Jack was able to generalize the steps of the problem solving procedure across materials. The last four days of data included different daily materials (e.g., spoons, pencils, pens, candy). Jack also generalized the skill to a typical high school algebra class (indicated on graph with a square). Leo was also able to generalize the steps of the procedure across materials (final day's data) and to a typical high school algebra class (square). For the generalization session, both students joined a general curriculum algebraic thinking classroom with high school peers who were nondisabled. When it was their turn, Jack and Leo were each successful in the completion of an algebra equation on the front board with the use of their manipulatives and a peer who stood with them but did not give the answers.

Total percentages of errors during the steps of the procedure were as follows: 6% for Jack, 5% for Leo and 5% for Cindy.

**Discussion**

All three students were able to master the concrete representation of an algebraic equation. For Jack, progress was influenced by his frequent absences. For Cindy, some additional modifications were needed in which she followed the task analysis for herself and had more time to begin responding on her own. Given that these students were able to learn to solve a simple linear equation, it is important to consider the components of intervention that may have contributed to this outcome.

First, the algebra problem was presented as a concrete problem (needing more supplies in a job task) with visual cues (the equation board) and concrete manipulatives (objects to count; numbers to select). In prior research on algebra, concrete manipulatives have also been used. For example, Witzel et al. (2003) used an explicit concrete-to-representational-abstract (CRA) sequence of instruction to teach students with mild disabilities to solve algebra equations. In 1999, Allsopp taught secondary students with disabilities in inclusive settings how to solve beginning algebra problems using modeling, manipulatives, and mnemonics. An important difference is that in studies with students with mild disabilities, concrete manipulatives are used in an introductory phase of instruction and then faded. In contrast, students in the current study continued to rely on manipulatives to solve the linear equations. Whether or not the students would have been able to fade to Arabic symbols alone with more instructional time (e.g., a full school year) is unknown. In contrast, fading these materials would not necessarily be the goal if they are viewed as a form of assistive technology the student needs for support during mathematics. Also, these manipulatives gave the algebra lesson a meaningful context for the students by making it similar to other job tasks the students had performed.

A second component of the intervention that may have influenced learning was the use of task analytic instruction. There are numerous studies demonstrating the effectiveness of task analytic instruction for students with moderate to severe developmental disabilities. For example, Gast, Winterling, Wolery, and Farmer (1992) taught first aid skills to students with moderate disabilities using a backward chaining procedure. The next-dollar strategy for making purchases (Coyler & Collins, 1996) and banking skills (Donnell & Ferguson, 1989) have also been successfully taught to a similar population using task analytic instruction. Extending this effective methodology to academic instruction may be an important key to making this content accessible. In using a task analysis for functional skills, the instructor typically defines the chain of responses needed to complete the activity. For example, paying for an item may begin with noting the price, computing the amount needed, and counting out the correct number of bills. Other steps may be taught such as selection of the item, placing it on the conveyor belt, pocketing change, and so on. For academic tasks, the chain of responding also needs to be defined. For example, in solving for a linear equation, the first step was to point to the sum, mark the sum, then count the items already on hand, and so on. By teaching the problem solving as a chain of responding, the completion of one response can serve as a natural cue to begin the next response in the chain. Some students may need help to learn to complete the entire chain since academic routines may not be as self evident as functional ones. For example, Cindy was not acquiring the problem solving as a chain of responding at first. She would perform a step and wait on the teacher to set up the next step. By giving Cindy a copy of the task analysis to use for self-instruction and delaying the teacher's verbal prompt longer, Cindy began to anticipate what to do next after completing the prior step.

The third component of the algebra intervention was the use of time delay, an "errorless" learning strategy. As noted in the results, the students acquired the skill with an overall low error rate. Time delay has been found to be effective for this population in teaching functional academics like sight words. For example, Gast, Wolery, Morris, Doyle, and Meyer (1990) found that constant time delay procedure was effective for elementary students with moderate disabilities to read sight words. Miracle, Collins, Schuster, and Grisham-Brown (2001) also used a constant time delay procedure to teach sight words to students with moderate disabilities at the secondary level and that the intervention could be implemented by peers as well as teachers. Determining that an intervention can be implemented by peers may increase opportunities for students to learn mathematics in general education contexts. In the current study, students first learned to solve the linear equation and then applied it to an activity in a typical high school algebra classes.

A limitation of the current study is that the contribution of each of these individual components to the overall effectiveness of the intervention is unknown. Future research is needed to determine if the students could perform the skill without the concrete manipulative, or after it was faded. For example, could they learn to do linear equations found in a textbook format typical of general education if given task analytic instruction with systematic prompting? Also, it is important to determine if skills that do not lend themselves to task analysis might be acquired with a concrete representation and systematic prompting. In contrast, the combination of components created an effective strategy that worked for all three students.

A second limitation is that Cindy required the additional modification of self instruction. The potential impact of self instruction on the first two participants is unknown. For example, a treatment package that included self instruction might have decreased the time needed to acquire the skill or promoted additional generalization (e.g., to equations using other computations like subtraction). Future research might begin with self-instruction for all participants to incur these and other potential benefits (Agran, King-Sears, Wehmeyer, & Copeland, 2003.)

Future research is needed not only on the individual components of the intervention and the use of self-instruction, but also on the application of this method to other advanced level mathematics skills. As noted, there are five strands of mathematics. Within a strand like algebra, each grade level has numerous topics and objectives for student learning. Much more research is needed to explore the extent to which this population can achieve skills from the full breadth of the general curriculum.

Second, research is needed on the social validation of the outcomes achieved. In this study, it could be argued that the students did not actually learn algebraic reasoning, but a step-by-step process to derive an answer. While this may be true, it may also illustrate the concept of alternative achievement that might be part of alternate assessment for showing adequate yearly progress for students with significant cognitive disabilities. In alternate achievement, the depth, breadth or complexity of the grade level content is reduced (U.S. Department of Education, 2005.) Because this concept of alternate achievement of academic content is new, input from students, their families, and other stakeholders is needed about the importance they ascribe to this level and type of skill acquisition. Some may propose that the criteria of functional use in daily living should be used to validate the skill acquisition. It could be argued that this is a higher standard than expected for nondisabled students who take algebra! In contrast, it is important not to assume that students with moderate or severe disabilities will apply skills unless generalization has been demonstrated through instruction and assessment. In the current study, two students showed some generalization across types of materials and settings. All three also express an ongoing interest in the lessons by requesting the lessons at the beginning of each day (*Do I get to do algebra today?*) and telling friends and family outside of class that they were "taking algebra." Such outcomes need to be documented in future research through stakeholder evaluations including: a) whether the skill learned retains the original concept (general educator), b) the skill acquired has value to the student and his or her family, and c) the skill is used in real life contexts.

In summary, this study was the first to teach an algebra skill to students with moderate developmental disabilities. Students were successful at learning how to solve an algebraic equation through the use of systematic instruction with a concrete representation. They showed mastery with generalization across materials and settings. This research contributes to the much needed world of general curriculum access to grade-level standards in the field of mathematics for high school students with moderate developmental disabilities. Continued research on high school mathematics curriculum instruction is needed to assist students gain access to grade-appropriate standards.

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