ADEOYE, SULAIMAN O., Ph.D. Deaf and Hard of Hearing College Students' Cognitive Strategies for Equal Sharing Problems. (2021)
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Deaf and hard of hearing (DHH) students' performance on fraction story problems is a cause for concern given that knowledge of fractions in the elementary grades is essential for learning Algebra in secondary school and advanced mathematics in college. Using grounded theory, the current study investigated DHH college students' cognitive strategies for solving equal sharing story problems presented to them in two distinct conditions: Interpreted and Co-constructed. Students watched the American Sign Language (ASL) renditions in pre-recorded videos of the English version of the equal sharing story problems in the interpreted condition. In the co-constructed tasks, the researcher and each participant co-constructed equal sharing story problems. Thirteen DHH college students who were at least 18 years old participated in the study. Data were collected through Think Aloud Protocol and interviews in which students explained their strategies for solving six interpreted and four co-constructed equal sharing mathematical tasks. Data were analyzed through coding and constant comparison analyses. Findings of the study indicated DHH college students used a broad range of cognitive strategies similar to the existing framework on students' cognitive strategies for equal sharing. In particular, the study yielded four broad themes (a) No-Link to Context (NLC) defined as students who used the wrong values or operations or who saw the problem as unsolvable; (b) Non-Anticipatory Coordination (NAC), defined as students who failed to pre-coordinate the number of individuals with the number of items being shared from the onset of the sharing activity; (c) Emergent Anticipatory Coordination (EAC) defined as students who pre-coordinated the number of shares with the number of items being shared right from the onset of the sharing activity, but they shared one item or group of items at a time; and (d) Anticipatory Coordination (AC) defined as students who used the long division operation or multiplicative $\frac{a}{b}$ operation. In addition to these four broad cognitive strategies, this study identified emerging strategies such as executive function skills, fraction conversion, and efficacy of the two conditions based on students' comments. Implications for practice and recommendations for research are discussed.

DEAF AND HARD OF HEARING COLLEGE STUDENTS' COGNITIVE STRATEGIES

FOR EQUAL SHARING PROBLEMS

by

Sulaiman O. Adeoye

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Approved by

____

Dr. Shaqwana Freeman-Green
Committee Chair

DEDICATION

I dedicated this dissertation to:

My Almighty God who I served and who has been gracious to me in every life endeavor from day one.

My beloved late mother of blessed memory, Alhaja Safuratu Lawal Adeoye who had the foresight to re-enroll in a mainstream school after I became Deaf.

My affectionate late father, Alhaji Lawal Adigun Adeoye, for his love, moral, and financial commitments toward my upbringing and education.

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My siblings from the same mother, Mrs. Ramota Azeez, Alhaji Moshood Adeoye, Alhaji Lateef Adeoye, Ismaila Adeoye, Yidiatu Adeoye, and Moriliatu Adeoye who did not have the opportunity to attend college or university.

My children, who I anticipate, will break additional ceiling glasses of which I am the pacesetter.

APPROVAL PAGE

This dissertation written by Sulaiman O. Adeoye has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

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CHAPTER I: INTRODUCTION

Background of the Problem

This dissertation investigated the cognitive abilities of Deaf and Hard of Hearing Students (DHH) as they engaged in mathematical problem-solving. Notably, this study examined the cognitive strategies of DHH college students using language, specifically on a mathematics task, as they engaged in several equal sharing fraction elicitation tasks. First, information about DHH students and fractions is presented, followed by a discussion of student characteristics, and finally, the theoretical lenses of the study.

DHH Students

Many DHH students are English language learners, and they differ from the general hearing population and other students with disabilities. There are many differences within the deaf community itself (Ferrell et al., 2014). For example, within the deaf community, DHH students vary in their degree of hearing loss (from mild to profound) and vary by culture, language, and education (Lomas et al., 2017). Culturally, approximately 40% of DHH children attending educational programs in the United States are from different ethnic, linguistic, and cultural backgrounds (Foster & Kinuthia, 2003; Sass-Lehrer et al.,1997). Gallaudet Research Institute (GRI, 2011) found that only 46.6% of the Deaf students are White; the rest of the Deaf students (53.4%) are from diverse ethnic and racial backgrounds.

DHH students also vary in their communication options. These include spoken, sign, and written oral language. For example, they can use English, American Sign Language (ASL), and simultaneous communication method where English words and grammar are represented using ASL signs (Ferrell et al., 2014). Linguistically, most DHH children (95%) are born to hearing parents, the majority of whom use a spoken language and have difficulty communicating with their DHH children (Morgan, 2015).

Educationally, the experience of DHH students varies by where they received their education as compared to students from the majority. Some attended general education settings while others attended special boarding or day schools for DHH students or were placed in self-contained classes in mainstream settings (Ferrell et al., 2014). Within educational settings, DHH students who are White may face significant delays in written literacy, social and academic achievement, and DHH students who are culturally and linguistically diverse may face even more challenges (Cannon & Luckner, 2016).

Other factors may impact the development of DHH children. These include their intelligence, family socioeconomic status, community resources, and quality of the K-12 educational program. Other specific factors consist of a) type of hearing loss experienced, b) on-set of the loss, c) age of identification, d) provision of early intervention services, e) quality and quantity of early intervention services (quality and quantity), f) utilization of assistive technology, g) language used in the home, and h) presence/absence of additional disabilities among others (Ferrell et al., 2014; Lomas et al., 2017). Similar to hearing students, DHH students experienced difficulties in mathematics and specifically fractions (e.g., Mousley & Kelly, 2018; Mousley & Kurz, 2016; Titus, 1995). These varying factors may influence the social, cognitive, and academic outcomes of DHH students.

DHH Students and Cognition

Historically, hearing students are viewed as more intelligent and abstract thinkers than DHH students (Loma et al., 2017; Moores, 2001). However, recent findings from research studies have countered this view (Morgan, 2015). DHH and hearing students' cognitive abilities are not significantly different than their hearing peers (Loma et al., 2017; Morgan, 2015). Yet DHH students, in comparison to hearing children, continue to experience consistent educational achievement gaps across grade levels and content areas, including significant and pervasive delays in literacy and mathematics.

Several factors may cause a delay in the abilities of DHH students, such as access to appropriate and equal educational opportunities, the student's level of motivation, and the teacher's classroom communication abilities (Bull et al., 2005). These factors, rather than the lack of cognitive skills, have been attributed to the academic achievement gap in Deaf students (Bull). Bull and colleagues considered these factors as mere speculation and called for scholarly research on the cognitive abilities of DHH children as it is tied to their language learning.

Importance of Fractions

Fraction knowledge is a foundational skill taught and learned in early grades. Success in this foundation skill supports children's success in learning more advanced mathematics (e.g., algebra and geometry) in later grades. In fact, in one study, knowledge of fractions at ten years of age-predicted algebra knowledge in high school after the authors controlled for family education, income, IQ, and knowledge of whole number arithmetic (Hansen et al., 2015). In addition, knowledge of fractions is required for pursuing science-related courses (e.g., biology, physics, chemistry, engineering; Hurst & Cordes, 2017).

Fractions are used to compare the prices of goods or services in everyday life situations (Fuchs et al., 2013). Fractions are used to calculate mortgage rates (Hurst & Cordes, 2017; Tsai & Li, 2017). Knowledge of fractions is also essential to gainful employment in STEM (e.g., nursing and pharmacy) and non-STEM related careers (Lortie-Forgues et al., 2015).

English-Speaking Students' Performance in Fractions

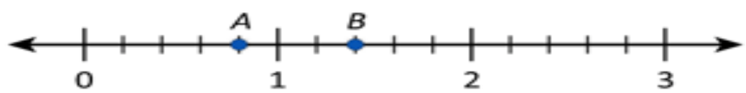
Despite the importance of fractions, many students and adults in the U.S. find fraction concepts challenging to learn (Hunt & Empson, 2015; Siegler et al., 2013). These difficulties cut across fraction concepts and grade levels (Fazio & Siegler, 2011; Hunt & Empson, 2015; Hunt et al., 2016). For example, one way to understand the fraction abilities of students is through the results of the National Assessment of Educational Progress (NAEP). The NAEP assessment (i.e., The Nation's Report Card) evaluated representative samples of 4th (i.e., 9-year-old), 8th (i.e., 13-year-old), and 12th (i.e.,17-year-old) graders and compared their performance across states. While the test provided a national snapshot of student performance in 4th, 8th, and 12th grades, it did not track the performance of individual students over time, so the results of the test should be interpreted with caution.

In the 2017 NAEP test results involving fraction comparisons, 4th-grade students were asked to decide which of the following fractions $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{6}$, $\frac{4}{6}$, $\frac{2}{8}$, and $\frac{4}{8}$ is less than, equal to, or greater than $\frac{1}{2}$. Thirty-two percent (32%) of the students got the answer correct, twenty percent (20%) of the students got the answer partially correct, and forty-seven percent (47%) of students who took the assessment got the answer incorrect (NAEP, 2017). The result showed that many 4th grade students who are supposed to be proficient in fraction comparison tasks struggled.

Similarly, the 2007 NAEP assessment data revealed difficulty among 8th graders who took a different fraction comparison task in the. Students were asked to arrange the three fractions $\frac{2}{7}$, $\frac{5}{9}$, and $\frac{1}{2}$ in ascending order of magnitudes. Approximately 50% of students who took the test answered incorrectly (Hamdan & Gunderson, 2017), suggesting that around half of the students who took the test may not know how to arrange fractions from the least to the greatest.

Likewise, in the 2017 NAEP assessment, 8th graders were presented with an image of a number line that ranged from 0 to 3 (as depicted in Figure 1), with Point A located four spaces from the numeral 0 and Point B found two spaces from the numeral 1. This is called a fraction number-line to position task. Students had to write down the fraction numbers indicated by Point A, Point B, and the midpoint between Points A and B.

Figure 1. Fraction Number Line to Position Task



Note. Eighth-grade students were asked to indicate the fraction symbols marked by Points A and point B. From 2017 *NAEP Mathematics Report Card: NAEP Sample Questions,* by National Assessment of Educational Progress, 2017 (https://nces.ed.gov/NationsReportCard/nqt/Search). In the public domain.

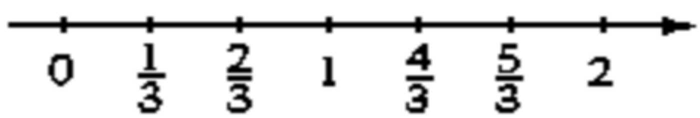
Forty-five percent of 8th-grade students solved the problem incorrectly even though it was expected by the test creators that all the students should have been taught or should have known how to do this task by this grade.

Students in 8th grade also showed similar difficulties in other fraction arithmetic tasks. For example, Tian and Siegler (2016) reported that in the 2004 NAEP administration, 8th graders were shown $\frac{7}{8} + \frac{12}{13}$ and asked to select the closest correct answer value from the following options: 1, 2, 19, 21, or "I don't know how to solve the fraction equation." More than 50% of the students selected an incorrect option instead of 2, the only correct choice.

In another NAEP (2007) assessment involving the addition of fractions, fifty-one percent (51%) of 8th-grade students asked to add $\frac{7}{10}$, $\frac{7}{100}$, and $\frac{7}{1000}$ wrongly provided $\frac{21}{1110}$ as the correct response. This common misconception in solving addition problems is considered the Whole Number Bias (WNB), where students add the numerators and then denominators as single entities.

These difficulties are not limited to elementary and middle school students. In a previous NAEP assessment given to high school students, more than 70% of these students got the answer wrong when asked to find the fraction equivalent (i.e., 29/1000) of the decimal number .029 (Siegler et al., 2011). Similarly, in the 1992 NAEP assessment, students in 12th grade were asked to place a dot at a point in the number line depicted in Figure 2 that could represent 1.75. Approximately half did not get the answer correct (NAEP, 1992).

Figure 2. Fraction Number Line to Position Task



Note. 12th-grade students were required to place a dot at a point to represent 1.75. From 1992 *NAEP Mathematics Report Card: NAEP Sample Questions,* by National Assessment of Educational Progress, 1992 (https://nces.ed.gov/NationsReportCard/nqt/Search). In the public domain.

Fazio and Siegler (2011) argued that even college students and adults have difficulties with fractions, demonstrating the widespread nature of the problem. For example, of the 1, 643 community college students who were asked to place $1\frac{3}{8}$ on the number line depicted in Figure 3, 68% did not correctly do so (Stigler et al., 2010). These examples point to the increasing challenges children and adults experience with fractions.

Figure 3. Fraction Number Line to Position Task

Note. Community college students were asked to determine the location of $1\frac{3}{8}$ on the number line. From "What Community College Developmental Mathematics Students Understand about Mathematics," by J. W. Stigler, K. B. Givvin, and B. J. Thompson, 2010, *MathAMATYC Educator*, *1*(3), p. 23 (http://statlit.org/pdf/2009CarnegieFoundation-Developmental-Math-CC-Students-Understand.pdf)

Explanations for Children's Difficulties with Fractions

Different explanations have been offered for why children experience difficulties with fractions. Some of the relevant explanations are considered in the following sections.

Fractions Cognition

Fractions, also the focus of this study, are often denoted as $\frac{a}{b}$, where "a" and "b" are integers, and "b" is not equal to zero (Empson & Levi, 2011). With this definition, fractions have historically been conceptualized as part-whole (i.e., the partitioning of continuous or discrete objects into equal-sized parts) in most elementary grades' curricula and instruction (Charalambous & Pitta, 2007). Part-to-whole instruction simplified involves the teacher asking students to divide an object into equal parts by shading, coloring, or selecting the equal parts from the whole piece. Finally, students are asked to determine or name the colored or selected parts. This narrow conceptualization of fractions is inadequate to children's conceptual understanding of fractions (Charalambous & Pitta, 2007).

Disregard for Other Interpretations

There are other interpretations of fractions besides the traditional part-whole interpretation (detailed explanations with examples are provided in chapter 2; Tsai & Li, 2017). These include fractions as a measure (i.e., the iteration of a unit fraction on a number line), fractions as a ratio (i.e., the relative size of two quantities in relation to each other), and fractions as an operator (i.e., input-output functions applied to some number, object, or set. For example, 2/3 of 6 pizzas is 4 pizzas). Fraction can also be conceptualized as a quotient (i.e., the result of division operation), and students with quotient conceptualization of fractions take an object or several objects and divide them up among a certain number of people. This can occur as equal partitioning or equal sharing story problem solving (the foundation of rational number understanding) based on relationships between two or several different quantities (Empson & Levi, 2011; Confrey et al., 2014; Tsai & Li, 2017). It has been argued these different interpretations have not received sufficient attention in curricula content and instruction at the elementary level (Charalambous & Pitta, 2007; Fonger et al., 2015; Mills, 2016; Tsai & Li, 2017).

Non-Symbolic and Symbolic Fractions Knowledge

Non-symbolic knowledge of fractions represents students' understanding of fractions presented through concrete or manipulative stimuli (Siegler et al., 2013). For instance, students understand that $\frac{1}{2}$ of a rectangle, a real-world object that is non-symbolic is larger than $\frac{1}{3}$ of the same rectangle. Symbolic knowledge of fractions on the other hand represents students understanding of the conventional fraction symbolic representation without the concrete manipulatives or stimuli (Siegler et al., 2013). For example, students understand that $\frac{2}{3}$ greater than $\frac{4}{9}$. Children as young as six months old have basic knowledge of non-symbolic fractions and children as young as four years old understand the symbolic knowledge of fractions (Siegler et al., 2013). As children are introduced to symbolic fraction notations in 3rd and 4th grades, their understanding of symbolic fractions improves with age and experience (Siegler et al., 2013).

Tax the Working Memory Capacity

The notation or symbol system (i.e., $\frac{a}{b}$) used to represent fractions adds to children's difficulties with fractions. For example, it is difficult for many children to understand that the notation $\frac{a}{b}$ represents a single number. When such understanding occurs, it remains effortful for many children to process this notation in their working memory.

Even more cognitive work is required to keep two fractions in mind, for example $\frac{2}{5}+\frac{1}{3}$ while performing basic operations. Remembering the importance and meaning of the symbol "/"in addition to the two notations increases the working memory load and may reduce other cognitive resources available to think about the procedures to use in solving the problem (Lortie-Forgues et al., 2015). Mentally understanding and representing the notation and operation burdens the limited working memory capacity compared to remembering, understanding, and performing calculations with two whole numbers such as 2+1.

Procedural Knowledge

Many children do not have a strong conceptual understanding of fractions but instead have a procedural understanding that can lead to two kinds of procedural fraction errors (Siegler et al., 2013). First, students who learn the process but do not understand the relationship between the numbers may solve the fraction problem $\frac{1}{2}+\frac{1}{3}$ as $\frac{2}{5}$ by adding the numerators and the denominators like two independent whole numbers. They have then committed what is called whole number bias.

There are also students who solve fraction problems such as $\frac{1}{3}$ x $\frac{2}{3}$ as $\frac{2}{3}$ by multiplying the numerators and keeping the common denominators. They have applied the procedure of keeping a common denominator, which is only associated with either addition or subtraction of fraction operations. Siegler et al. (2013) argued that these two errors are common among young children and college students.

Curricula

Fractions standards or curricula often specify the fractions concepts students would learn and the grade levels taught (also known as scope and sequence). However, Steffe and Olive (2010) pointed out that the specified concepts may neither be foundational nor age-appropriate concepts. The stated fraction concepts may reflect an adult's view of fraction understanding or development rather than a child's understanding or development (Confrey et al., 2014; Sztajn et al., 2012). In other words, the content standards for fractions in many schools may reflect what and how fractions should be learned from an adult's perspective. How children learn fraction concepts may differ and may have been ignored.

Here is an example of how adults may have been taught to solve $\frac{\mathbf{7}}{\mathbf{8}}+\frac{\mathbf{12}}{\mathbf{13}}$. First, identify a lack of common denominators. Then, apply the process for finding the common denominator. Then they add up only the numerator. Once that is done, the numerator is larger than the denominator, and the individual recognizes they have to use the process of division. The process could look like this: $\frac{\mathbf{7 \times 13}}{\mathbf{8 \times 13}}+\frac{\mathbf{12 \times 8}}{\mathbf{13 \times 8}}=\frac{\mathbf{91}}{\mathbf{104}}+\frac{\mathbf{96}}{\mathbf{104}}=\frac{\mathbf{187}}{\mathbf{104}}=1\frac{\mathbf{83}}{\mathbf{104}}$. Other adults may simply ignore the relationship called for by the division symbol (i.e., /) and instead use the wrong procedures to calculate the fractions. These adults may just combine the addition operation (+) and the whole numbers horizontally thus: 7+ 12 = 19 and 8 + 13 = 21 to obtain $\frac{19}{21}$.

Children, on the other hand, may not have acquired these procedural and estimation skills. Instead, they may use correct or incorrect repeated partitioning and splitting of the objects represented by the fractions to solve the fractions problem.

Adult Pedagogy

The teaching of fractions has historically focused mainly on the use of drills and practice (Siegler et al., 2013; Tsai & Li, 2017). Drill and practice mean that the teacher focuses on procedural knowledge, the process of doing the calculations; thus, curricula and instruction have largely focused on the part-whole interpretation of fractions and procedures at the expense of other meanings of fractions conceptual understanding. This may have limited the students' knowledge of the different fraction interpretations and may have contributed to children's persistent difficulties understanding fractions.

DHH Students and Mathematics

For over five decades, research studies reported a delay of about 2.0 to 3.5 years in the general or overall mathematics achievement of DHH students compared to their hearing counterparts (Gottardis et al., 2011; Bull et al., 2005; Rodríguez-Santos et al., 2014; Rodríguez-Santos et al., 2018). However, more recent studies casted doubt on this delay. For example, three studies investigated the non-symbolic or early number representations of preschool or kindergarten DHH children who use cochlear implants (Arfé et al., 2011; Gottardis et al., 2011; Zarfaty et al., 2004) and found no delay in the children's abilities. In a fourth study, the researchers looked at the arithmetic story problem-solving strategies of ASL proficient K-3 DHH children (Pagliaro & Ansell, 2012) and found no delay. Finally, in yet a fifth study of the mathematical performance of DHH children aged 7-9 years with mild to moderate hearing loss the authors found no delay (Gottardis et al., 2011). However, as mentioned earlier, some studies have found a delay in several basic mathematical abilities (Bull et al., 2011; Kritzer, 2009; Mousley & Kurz, 2016; Rodríguez-Santos et al., 2018). These abilities discussed in Chapter 2 under cognition include the late onset of the advanced thinking strategies in DHH students' repertoire of overall strategy use (e.g., Gottardis et al., 2011; Pagliaro & Ansell, 2012). Reasons for the delay range from language issues to limited incidental learning opportunities in the children's formative years to formal instructional approaches based on rote and procedural knowledge in the classrooms (Ansell & Pagliaro, 2006; Kritzer & Pagliaro, 2012). Additionally, Nunes and colleagues (2009) argued that DHH students lack a conceptual understanding of mathematical relationships between quantities. Students' mathematics conceptual understanding is a function of their knowledge of the relationships between quantities or the semantic structures of the story problems (Nunes et al., 2009; Pagliaro & Ansell, 2012). The assessment of DHH students' conceptual understanding of mathematics has received growing attention in several studies (e.g., Gottardis et al., 2011; Nunes et al., 2009; Pagliaro & Ansell, 2012).

Research on DHH students' Performance in Fractions

It has been found that, like hearing students, DHH students may struggle to develop a conceptual understanding of fractions (e.g., Mousley & Kurz, 2016; Titus, 1995). Specifically, Mousley and Kurz (2016) employed mixed methods to investigate the conceptual understanding of fractions by 14 DHH students between 8-16 years old and found most could neither identify the smaller of the two fractions in the set $\frac{5}{3}$ and $\frac{3}{5}$, nor order three fractions such as $\frac{1}{5}$, $\frac{3}{4}$, and $\frac{1}{2}$ in ascending order. The performance of the DHH students mirrored that of 8th grade students where approximately half of the students who took the NAEP test did not know how to arrange the fractions from least to greatest demonstrating underperformance when compared to their hearing peers. Overall, DHH students scored 40% accuracy when asked to identify the larger fraction or the smaller fraction when they were presented in a pair. Only 1 out of 14 students were successful in ordering non-symbolic fractions, and 3 out of 14 students were successful in ordering symbolic fractions (Mousley & Kurz, 2016).

Multiple interpretations of fractions

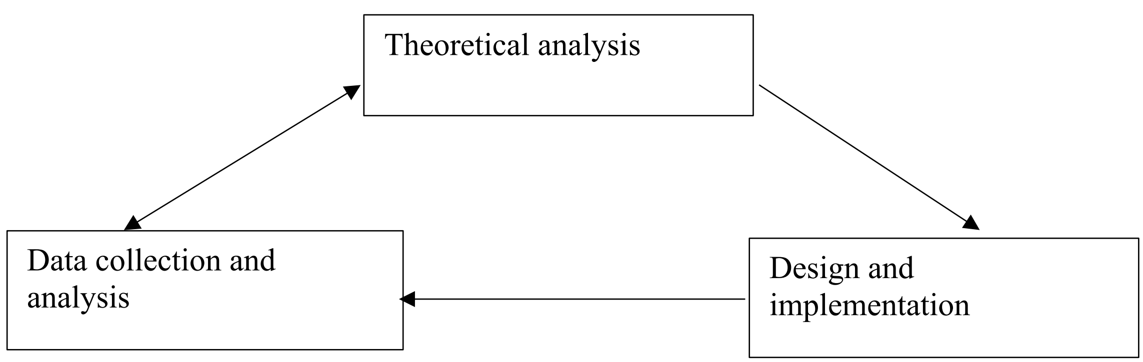
Fractions have multiple interpretations (Tsai & Li, 2017) and little research has focused on these various interpretations in the field of deaf education (Kritzer, 2009; Mousley & Kurz, 2016; Pagliaro & Ansell, 2012; Titus, 1995). First, Kritzer (2009) assessed the informal and formal mathematical knowledge of 28 DHH children, age 4 to 6 years and found that 16 of the children who tried to solve the fair-sharing problem embedded in the mathematics tasks did so incorrectly. Next, Pagliaro and Ansell (2012) investigated the problem-solving strategies of 232 DHH students in kindergarten through grade 3 who were asked to solve nine arithmetic word problems (one of the nine problems included partitive division with whole number result). They found that DHH students used abstract strategies (i.e., counting and fact-based) inappropriately.

Finally, Mousley and Kurz (2018) investigated 14 pre-college DHH students conceptual understanding of fractions as quantitatively measured by their ability to do three functions (i.e., compare fractions, order fractions (measure interpretation), match fraction symbols with fraction diagrams (part-whole interpretation)). The study also qualitatively assessed students' abilities in part-whole and measure fraction concepts and the fraction concepts of ratio and quotient. Researchers found that DHH students struggled with these fraction concepts, and while there is some research on part-whole, ratio, quotient, and measure interpretations of fractions, these studies were few.

The APOS-Based Research Theoretical Lens

This study is guided by the Action, Process, Object and Schema (APOS) theoretical framework, a theory birthed by Dubinsky (Meel, 2003; Siegler et al., 2013; Siegler & Pyke, 2013). APOS-based research theoretical framework contains three cyclical-elements (1) theoretical analysis, (2) data collection and analysis, and (3) design and implementation Alamolhodaei et al. (2018). Figure 4 illustrates the three components of APOS-based research and their relationships. While these three elements influences each other, it has been accepted that they can stand alone in a research study (Asiala et al., 1996; 2014; Creswell & Plano Clark, 2011; Crotty, 1998); however, his study used all the three elements. These three elements and their applications to the study are briefly described below. Comprehensive details on theoretical analysis and design and implementation are provided under the literature review in Chapter Two. The third element on data collection and analysis is provided in Chapter Three under methodology.

Figure 4. Research Cycle in APOS-Based Research



Theoretical Analysis

APOS has been used in mathematics to explain how an individual learns mathematics at the secondary and postsecondary levels and, more recently at the primary and middle grades (Arnon et al., 2014). The conceptual framework generated can explain the successes, failures, and instructional strategies for improving children's achievement in any mathematical concept (Arnon). Action, the first step in the APOS theory, represents a mathematical entity's physical or mental transformation in response to an external stimulus. The next stage, Process understanding, represents an internalized mental understanding because of repeated and reflected action without external physical transformation. Object defines the understanding of the process as a totality and action to transform the process. It occurs when individuals realize that the process has mathematical properties that can be applied to similar concepts. A schema understanding represents an individual’s collection of actions, processes, objects, and other schemas linked to general principles used to solve mathematic problem situations (Ubah & Bansilal, 2018).

Design and Implementation

In addition to the APOS theoretical analysis aspect of the framework, the design and implementation of the theoretical analysis involve what has been termed the genetic decomposition. The genetic decomposition is a framework of five levels of conceptual understanding used by other researchers that included, in order of sophistication (i.e., concrete to abstract): No-Coordination (NC), No-Link to Context (NLC), Non-Anticipatory Coordination (NAC), Emergent Anticipatory Coordination (EAC), and Anticipatory Coordination (AC; Empson, 1999; Empson & Levi, 2011; Hunt & Empson, 2015; Hunt et al., 2016; Hunt et al., 2017). Definitions and examples of each thinking strategy are provided in the literature section in

Chapter Two. But below is a summary of the strategy student can use, ranging from concrete to abstract.

Students who used the No-Coordination strategy consider a whole as indivisible. They either distribute the whole items with each sharer getting unfair shares or give equal shares without exhausting the whole items. They may also introduce new items to the original items to make the sharing “fair”. Students who use this strategy lack conceptual understanding of fraction. For example, in a problem involving sharing *5 candy bars between 2 friends*, a student may give one person 3 candy bars and another person 2 candy bars. Alternatively, they can give 2 candy bars to each person and the child may conclude the leftover is impossible to share. Students who used the No-Link to Context strategy engage in partitioning activity. Still, their partitioning activities to the contexts in the story problems, guess the solution, or use the wrong operation.

Students who use the Non-Anticipatory Coordination strategy use the halving or repeated halving or *trial and error* to solve the equal sharing problems. They do not pre-coordinate their partitioning activities from the onset. Their partitioning activities may generate fair shares. Student using this strategy understand fractions parts or sees fractions as objects that are in a countable relationship. In the problem example above, the child may split an object and give each person half of a candy bar. The student may continuously split each candy bar into half and distribute the parts until all the whole candy bar or bars are exhausted.

The more sophisticated strategies are emergent (relation) anticipatory, and finally, anticipatory. Emergent relation strategy is where the child demonstrates the first instance of true fraction understanding. Students using this strategy share or divide one or more items into equal shares among several people sharing the items or objects until all the items or objects are exhausted.

The final strategy, Anticipatory Coordination, represents a more sophisticated thought process. Students who use this strategy use a division algorithm to mentally or procedurally partition the items or objects among the number of people sharing the items or objects. So, at this level, the child considers both the objects to be shared and the number of people who are receiving the shares.

Research Questions

This dissertation study sought to understand the quality of the thinking strategies that DHH college students used to solve equal sharing story problems. Three research questions guided the study, and they include:

1. What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems presented to them through an interpreter in ASL in a frozen video format?
2. What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems when asked to co-construct the problems in dialogue with a Deaf researcher in ASL?
3. How does the understanding of fractions of college DHH students (i.e., employed thinking strategies and representations) who watch fraction problems conveyed in ASL in a video with an interpreter compare to their understanding of fractions when they are asked to co-construct the problems in dialogue with a Deaf

researcher in ASL?

Significance of the Study

Based on an exhaustive literature search in the field of deaf education, no research exists that investigates DHH children's understanding of fractions against the model of cognitive development involving No-link to context, No-coordination, Coordination, Non-anticipatory, Emergent participatory, and Anticipatory (Empson, 1999; Empson & Levi, 2011; Hunt & Empson, 2015; Hunt et al., 2016; Hunt et al., 2017). This framework represents a genetic decomposition of the cognitive process. Based on the literature review, there has also never been a study where the DHH students have been asked to co-construct the problems. So, in both areas, this study represents a first.

Positionality

The study focused on participants who were Deaf and Hard of Hearing (DHH). As a researcher who is also Deaf, I brought to bear on the study an insider perspective called emic (Merriam & Tisdell, 2016). This insider perspective influenced the study design and impacted various aspects of the research such as the data collection, analyses, and interpretations of the findings. In the sections that follow, I described myself as an insider in relation to the DHH participants, my qualifications to undertake the study, and why readers should trust the findings of my study.

Similar Experiences

As a late-deafened individual and like many of the DHH participants in this study, I attended various educational settings, having been both mainstreamed and enrolled in special schools. I could hear and used spoken language to some extent. I learned sign language later in life. But I considered myself a fluent ASL user. This enabled me to have direct communication with the participants and helped me understand their responses to various tasks in the study. I worked with interpreters in different educational and social contexts. I considered myself an English Language Learner like many DHH participants who grew up learning English as a second language. I also had a flair for mathematics, and I believed some of the DHH participants of this study may also have the same disposition.

Intersectionality

I am from Nigeria, male, and Black. As mentioned earlier, I am not a native signer of ASL. Some of the DHH participants of this study were female and/or White and mostly American born. They were also native ASL users. However, I thought my fluency and ability to communicate directly with the DHH participants was an added advantage to understanding the participants. In the Deaf community, identifying as Deaf was seen as a primary affiliation over other identities or ethnicities.

I kept these positionalities in mind as I conducted the research as my intersectional identity in addition to my deafness could influenced my interactions with the participants. For example, I became more aware of how I communicate in ASL with native signers to ensure their comprehension of the instructions. I repeatedly checked with the participants to ensure they understood me. At the beginning of the interview, I spent some time establishing rapport with the participants in order to build credibility. I introduced myself and shared some of my backgrounds with the participants. As a Deaf person, I am aware of how to establish rapport in ASL and social norms. This means, for example, pausing if the participants look away to complete a task. I ensured clear sightlines for the participants.

As a Deaf person, I looked at the abilities not disabilities of Deaf people. I valorized their language and culture and did not point out the flaws in them. This attitude impacted the design of my study, as I utilized ASL to conduct the research instead of written, spoken or signed English. I did this as I believed ASL provides full access to them. English is their second language and the use of it alone could negatively impacted their performance. My life experience as a Deaf researcher could have impacted my data collection and analysis process as I focused on the abilities of Deaf people and not their faults. I kept this in mind throughout the research process.

Qualifications

My academic preparations and professional work adequately prepared me for the study. I attended Gallaudet University, a higher education institution designed for the preparation of scholars and researchers in the field of deaf education. I finished my master's in administration after approximately two years at Gallaudet University and subsequently obtained a second master's in special education from California State University, Northridge. As a doctoral scholar at the University of North Carolina, Greensboro, I was adequately equipped with the knowledge and skills to conduct the study. The research courses I took and passed include qualitative, quantitative, and mixed methods and so I was prepared to conduct qualitative research. In fact, I conducted a pilot study using a similar qualitative method.

However, I received guidance on a Think Aloud Protocol (TAP) as a methodology in data collection. I read and reflected on areas such as Theory of Mind (TOM), Cognition, Executive Function (EF), and others which I explored as part of this research process. As a certified mathematics teacher of records who taught mathematics for approximately 13 years to DHH children at Marlton day school for the Deaf, Los Angeles, California, I brought first-hand knowledge of mathematics curricula, DHH students, pedagogies, and the difficulties DHH children encountered while learning mathematics.

Expectations

To conclude, I brought an insider perspective (emic) to bear on the study. An insider perspective (emic) seeks to understand "the phenomenon of interest from the participants' perspectives, not the researcher's" (Merriam & Tisdell, 2016, p.16). In this respect, I sought to understand the DHH students' thinking strategies through a qualitative approach as they engaged in solving equal sharing story problems presented to them through the combination of interpreted ASL videos and interactive conversations. More specifically, based on my orientation about Deaf people and my belief that they can do anything anyone else is able to do (i.e., a social perspective on deafness), I expected the college DHH students to use abstract or advanced strategies to solve the equal sharing problems even though some struggled. Because I wanted the students to do well, it is possible that I could have put a positive spin on the findings. I was on guard about this throughout the research process.

Definition of Terms

Conceptual Knowledge

Knowledge of mathematical concepts, operations, and relations (Kilpatrick et al., 2001)

Continuous Quantities/Manipulatives

Measurable quantities or manipulatives that can be partitioned or broken apart into several equal parts. For example, candy bars, pizzas (Fonger et al., 2015).

Discrete Quantities/Manipulatives

Collections of countable sets or objects that cannot be partitioned or broken down into smaller parts. For example, shells, gems, balloons (Fonger et al., 2015)

Equal Sharing

A form of partitive division story problems in which several same size objects are shared among several people, resulting in a fractional quantity (Hunt, Westenskow, & Moyer-Packenham, 2017)

Informal Knowledge

An applied or circumstantial knowledge individually constructed in response to real-life experiences; sometimes referred to as intuitive knowledge or pre-instructional knowledge. This knowledge may be conceptually correct or incorrect (Mack, 2001). It also referred to as informal reasoning which involves problem solving strategies that children demonstrated before formal instruction in mathematics (Doyle et al., 2016)

Learning Trajectories

A learning roadmap that describes the ways in which naïve conceptions mature over time into powerful, connected mathematical ideas (Sztajn et al., 2012; Wilson, 2009). The trajectories show the hierarchical progression or sequence of mathematics concepts children should learn.

Measure Construct

The application of the part-whole construct through the placement of $\frac{a}{b}$ on the number line with a designated unit. The unit is partitioned into "b" equal parts and the resulting sub-unit 1/n is iterated "a" time (Doyle et al., 2016).

Non-Unit Fractions

Fractions in which the numerator and denominator can assume any value other than one. For example, $\frac{3}{4}$

Operator

A number or fraction that acts on the whole of an object, where a student is asked to find ¾ of 100 meters. In this example, the ¾ acts on the whole number, the 100 which is multiplied by some quantity (Tsai & Li, 2017).

Ratio

A relationship between two quantities (Tsai & Li, 2017).

Rational Numbers

Fractions, decimals, percentages, and integers (Ni & Zhou, 2005)

Unit Fractions

Fractions in which the numerator value is one. The denominator can assume any value greater than the value of the numerator. For example, $\frac{1}{4}$ (Siegler et al 2011).

Whole Number Bias

A robust tendency to use the single unit counting scheme to interpret fractions (Ni & Zhou, 2005). Children ignore the fraction relationship and treat the numerators and denominators as separate, whole numbers to then add or subtract

Organizations of the Study

The entire dissertation is organized into five chapters. Chapter one focused on the introduction and addressed topics such as background of the problem, statement of the problem, current theories, purpose of the study, research questions, significance of the study, and positionality. Chapter two focused on a critical review of relevant literature in cognition, mathematics, fractions and APOS theory. Chapter three is the methodology and addressed qualitative methods, research designs, coding and triangulation, data collection, and data analysis strategies. Other topics include the a) definition of an elicitation task, b) sample size and characteristics of the Deaf students involved, and c) a description of the equal sharing problems. Chapter four reported the findings of the study, and chapter five discussed the findings, recommended implications for practice, and offered suggestions for future research.

CHAPTER II: LITERATURE REVIEW

Introduction

For centuries, one of the three questions mathematics educators have grappled with is how our understanding of human cognition, meaning mental action or process, informs how we assess and understand students' mathematical knowledge (Sfard & Cobb, 2016). Martin (2008) identified three different distinctions to evaluate and understand this mathematical knowledge in research. First, a provocative intervention which seeks to extend the learner's current level of knowledge. Second, invocative intervention that intends to strengthen the learner's informal knowledge—third, validating intervention seeking to confirm the learner's current level of mathematical knowledge. The study focused on validating intervention and the literature review was organized into the following major headings: Theoretical Lenses, Cognition, Mathematical Cognition, Fraction Knowledge, and Teaching of Fractions. Others are Deaf and Hard of Hearing Students (DHH) and Cognition, and a Summary of the Chapter.

Theoretical Lenses

Theoretical lenses are a researcher's basic set of beliefs about knowledge that informs his or her actions (Creswell & Plano Clark, 2011; Grant & Osanloo, 2014). They provide the structures for the overall research including the research rationale, problem statement, purpose of the study, significance of the research, and research questions. The theoretical worldviews also serve as the anchors for the literature review, research design, research methods, data collection, and data analysis (Grant & Osanloo, 2014). Cognitive constructivist and APOS theories were the two theoretical lenses that guided this study (Arnon et al., 2014; Powell & Kalina, 2009).

Cognitive Constructivist Lens

Various theorists have defined cognitive constructivism and used various constructs to do so (Grider,1993). It can be traced to the year 400 B.C., when Plato philosophically characterized knowledge as innate and the 17th century when Descartes viewed knowledge as logical thought and deduction (Grider,1993). Since the late 1800s, cognitive constructivism theory has been viewed from a psychological perspective, in which knowledge is rooted in the relationship between the internal mental structures, the cognitive processes, and the environment (Grider 1993). Piaget (1972), one of the foremost theorists of the psychological perspective, considered knowledge of adults as consisting of the higher cognitive processes, scientific reasoning, introspection, and abstract thoughts used to solve problems (Grider, 1993; Powell & Kalina, 2009). With the advent of computers in the 1950s, human knowledge has been associated with computer functions of storage, retrieval, processing, and problem solving (Grider, 1993; Hruby & Roegiers, 2012).

Despite the different philosophical roots, a common definition of cognitive emphasizes that individuals actively construct their knowledge and meaning from their experiences (Doolittle, 1999; Powell & Kalina, 2009). Thus, three of the tenets that undergirded cognitive constructivism were apparent. Individuals actively construct their own knowledge.. The knowledge they create reflects their mental structures and experiences; however, it may not accurately correspond to reality (Doolittle, 1999; Hruby & Roegiers, 2012).

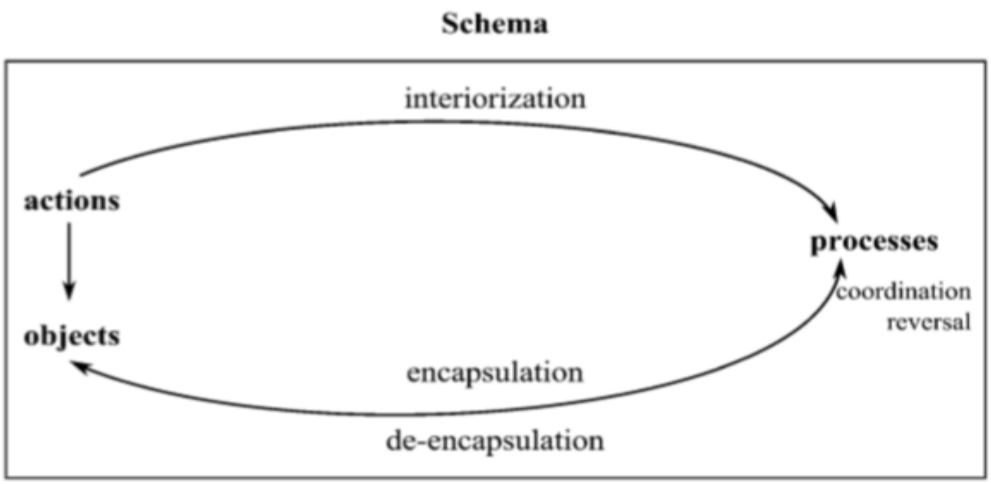
Action, Process, Object and Schema Lens

To understand the cognitive processes DHH students utilized when doing mathematics, a second theoretical lens was chosen. A thorough search for the cognitive definition of knowledge and the need for an integrated research theoretical framework for this study culminated in selecting the APOS theoretical lens (Arnon et al., 2014; Meel, 2003). APOS, a constructivist theory proposed by Dubinsky in the 1980s, is associated with the work of Jean Piaget. It focuses on models of what might be going on in the mind of an individual who is trying to learn a mathematical concept. Educators can use APOS theory to design instructional materials and evaluate student successes and failures in dealing with a mathematical problem situation (Arnon; Asiala et al., 1996).

According to this theory, college students, when presented with abstract mathematical situations or problems, may act on the problems based on their prior knowledge and experience (Arnon et al., 2014). Once they have reflected and interiorized their actions, it becomes a process. When actions applied to a process, it is encapsulated into abstract objects. Finally, the actions, processes, and objects can be organized into a schema (Arnon). Thus, a schema represents a collection of mental structures such as actions, processes, objects, and other schemas formed by various mathematical concepts and their interrelationships that individuals might use to solve mathematic problems (Prayitno et al., 2018; Ubah & Bansilal, 2018).

Although a difficult and confusing approach for some, APOS-based research has been used to study younger and older students across disciplines (Arnon et al., 2014; Meel, 2003). For example, in mathematics, it has been used to understand students' interpretations of images of fractions as processes and objects (Herman et al., 2004). In addition, APOS-based research has been used to describe the students' thinking processes while solving fraction problems (Kurniawan et al., 2018). Figure 5 represents the relationships between the mental structures and the mental mechanisms. These mental structures and mechanisms are described in the next section.

Figure 5. APOS Mental Construction and Mechanism



Note. From "The APOS Paradigm for Research and Curriculum Development," by I. Arnon, J. Cottrill, E. Dubinsky, A. Oktaç, S. R. Fuentes, M. Trigueros, and K. Weller, 2014, *APOS Theory,* p.10 (https://doi.org/10.1007/978-1-4614-7966-6_6)

Action understanding

When confronted with mathematical problems (e.g., fractions problems), students first act on it using their intuitive or procedural strategies to transform it with or without the use of manipulatives (Mathews & Clark, 2003). For instance, in the equal sharing fraction problem, the students must determine a share for each person when 5 candy bars are shared equally between 2 people. Students with an action understanding may use the traditional step-by-step division procedure explicitly to solve the problem. Other students may use informal repeated partitioning strategy explicitly to solve the problem correctly or incorrectly. Students in the action conception stage (possibly the most primitive structure in traditional teaching) explicitly performs each step of the transformation (i.e., student cannot imagine the steps in his or her head yet) and completely without skipping a step.

Process understanding

Students with process understanding in the above example can imagine the solution to the task in their head without the explicit step-by-step transformations or use of manipulative devices used in action understanding (Asiala et al., 1996; Arnon et al. 2014; Dubinsky, 2001). Students with process conception have internalized the actions required to solve the equal sharing tasks. Additionally, a student with process understanding used an emergent anticipatory fraction strategy also known as emergent relation or true instance of fraction understanding. The student understands that the number of objects to be shared and the number of people sharing it are related. This student may use a formal division algorithm strategy to obtain each person's share. A student with process understanding may quantify each persons' shares with words, notations, and symbols. Using the above problem example, students may say, "I can do it in my head" or "I just know it" and respond that each person gets 5/2 or 2 and 1/2 candy bars as a solution to the above fraction problem (Arnon et al. 2014).

Object understanding

Students with object understanding apply action (either explicitly or implicitly) to a process output to obtain a cognitive or mental object. The students may see the process output in a new way and realize that the process has mathematical properties applicable to similar concepts. This is the most difficult mental mechanism for students to achieve. Students with object understanding use an anticipatory or sophisticated strategy to coordinate and solve the fraction problem. In the fraction problem above, a student with object understanding may apply a division algorithm or additive or multiplicative relationships to the process conception and reason that 5/2 or 2 and a half is the same as giving each person half of a candy bar five times to each person (i.e., ½ + ½ + ½ + ½+ ½ or 5(1/2) = 5/2 or 2 and a half).

Schema understanding

Students with schema understanding use coherent, dynamic, and total structures (actions, processes, objects, and other schemas linked to general mathematics principles) to solve mathematic problems (Arnon et al., 2014; Dubinsky, 2001). Asiala et al. (1996) described it as an individual's connected knowledge of the concept (both explicitly and implicitly). In the example problem above, all mental structures students use to solve the task are considered. Schema understanding represents the totality of a student's actions, processes, objects, and other schemas used to solve mathematical problems (Arnon et al., 2014). These include various mathematical concepts, interrelationships, and behaviors that individuals may use to solve mathematic problem situations (Prayitno et al., 2018; Ubah & Bansilal, 2018). These include cognitive abilities such as theory of mind (TOM), executive function (EF), and motivation described later in this chapter.

The Design of the Genetic Decomposition for Equal Sharing Problems

This section on genetic decomposition examines the hypothetical cognitive strategies or processes used by students when working with fractions. It is based on a synopsis of research and APOS theory. They include descriptions of the mental structures needed to master it, prior research on students' difficulties, the researchers' knowledge or experiences, prior research on students' thinking, historical perspectives on its development, analysis of texts or instructional materials on didactic approaches related to the learning, and the knowledge of APOS theory (Arnon et al., 2014). This gives rise to a preliminary genetic decomposition that describes the mental constructions and mental mechanisms that an individual might make in constructing her or his understanding of a mathematical concept (Alamolhodaei et al., 2018; Arnon et al., 2014). In other words, genetic decomposition of a mathematical concept represents what students' mathematical knowledge, or the lack of such mathematical knowledge might look like (Arnon et al., 2014; Asiala et al.,1996; Dubinsky, 2001). The genetic decomposition is examined from two conceptual perspectives: cognitive coordination and anticipation, and mental constructions and mechanisms (Arnon et al., 2014; Hunt & Empson, 2015). Studies contend that the degree of sophistication of students' mathematics conceptual understanding is a function of their cognitive abilities, understanding of the mathematical story problems, and the understanding of the relationships between quantities or the semantic structures of the story problems when problems are introduced to students in written language (Charles & Nason, 2000; Hunt & Empson, 2015; Pothier & Sawada, 1983).

One way of describing the strategies used by students and linking it to their level of fraction understanding is by using the concepts of coordination and anticipation (Empson & Levi, 2011; Hunt & Empson, 2015). These broad strategies are described below in order of difficulty or sophistication. The broad classifications are determined by the extent of students coordinating the number of objects[s] being shared with the number of people sharing the object[s], exhausting the whole object[s], creating equal shares, combining shares to create each person total shares, and representing each person's shares with symbols, notations, or words (Empson & Levi, 2011; Gibbons et al., 2018; Hunt & Empson, 2015).

The following equal sharing problem is used as example when describing each classification of strategies that follow:

Five friends shared three pizzas fairly. How many pizza slices did each friend get?

The descriptions below for each level includes the level of the person's fraction understanding, a description of the broad strategy features or characteristics, and an example of a strategy based on the literature (e.g., Empson, 1995; Empson & Levi, 2011; Hunt & Empson, 2015). Each strategy is also judged on a hypothetical scale of from zero (0) to five (5) in terms of theoretical cognitive ease to complexity respectively. These definitions were used in this study to link the participants' behaviors to their potential cognitive strategies.

No-Coordination Strategy (NC)

No-Coordination strategy represents a level 0 strategy (Hunt & Empson, 2015); no fraction understanding level. Students operating at this level may see the pizzas as indivisible into parts such as when the students say, “it is not possible to solve because we don’t have enough pizzas.” Students operating at this level may create equal shares without exhausting the whole objects. They may create equal shares by adding new object[s] outside the original whole objects to create a fair sharing of whole pizzas, such as adding two more pizzas to have five pizzas to distribute to the five friends. Students may create unequal whole shares while exhausting the whole objects such as giving each friend a pizza and leaving two friends without pizzas. In short, these students also do not coordinate the number of people or groups sharing the objects to the number of objects being shared from the onset of the sharing activities. These children may use the *dealing* strategy, much like dealing a deck of cards, where they distribute the objects until they encounter leftover[s] (e.g., Empson & Levi, 2011; Hunt & Empson, 2015; Hunt et al., 2017; Lewis et al., 2015). Students who used this strategy are not making use of their TOM and EF abilities. In summary, the behaviors associated with this strategy include:

1. Identify the objects and/or the people sharing the whole objects
2. Fail to pre-coordinate whole objects to the number of people sharing it
3. Use dealing such as “one for you, one for me” or skip counting to partition the whole objects

4. Addition to the leftover objects or removal of some leftover objects to generate “fair” dealing
5. Use models or manipulatives
6. Generate unfair shares
7. Use whole number to quantify each person’s share in words, notations, or symbols.

No-Link to Context (NLC)

No-Link to Context strategy represents a Level 1 strategy, according to Hunt et al. (2017). Unlike the No-Coordination, students who used this strategy recognized that they must partition the objects either whole(s) or leftover(s). Students may partition the objects or items into some numbers of pieces but failed to link the partitioning to the information given in the problem (i.e., the sharers). According to Hunt et al. (2017), students at this level do not relate “the concept of equal sharing to the elements at play in the problem” (p.8). Specifically, they do not link the number of sharers to the objects being shared and attend to either the objects or the sharers but not at the same time. These students create equal or unequal shares for the wrong or correct number of sharers by distributing five pizzas among three friends instead of sharing three pizzas among five friends. In this example, students may arrive at the solution three or five because they are not coordinating the number of people to the number of slices.

Students may start partitioning the items and then stop. Students who stopped mid-way during the partitioning may leave the solution blank or say, “I do not know”. They may use an inappropriate operation, not represent the problem correctly or use informal way for partitioning the problem. For example, they multiply 3 pizzas by 8 (based on their informal knowledge that pizzas have eight slices) and then divide the result by 5 people. The student then says each person get 4.8 slices (Empson & Levi, 2011; Gibbons et al., 2018; Gibbons et al., 2016; Hunt & Empson, 2015; Hunt et al., 2017; Lewis et al., 2015; Wilson et al., 2012).

These unsuccessful strategies for sharing may point to a lack of various cognitive abilities as outline earlier. These include a lack of TOM and a failure to recognize that some people would be disappointed if they don't get an equal share or any share. The student may not be able to shift his or her attention. They may not be able to update their information or thinking as they go. In summary, the behaviors associated with this strategy include:

1. Do not solve the task
2. Guess
3. Different operation (e.g., multiplication and then division)
4. Different context
5. Say "I don't know"
6. Identify both variables
7. Use different contexts that differ from the original problem
8. Use arbitrary numbers for the parts or people
9. Do not coordinate the two variables during partitioning
10. Use models or manipulatives or procedures
11. Use operations such as adding the parts or people together

Non-Anticipatory Coordination Strategy (NAC)

Non-Anticipatory Coordination (NAC) strategy represents a Level 2 strategy (Empson & Levi, 2011) also described as an emerging sharing level. Children operating at this level create equal or unequal shares and exhaust the whole object[s] through their partitioning activities. These students use the dealing strategy or counting by one such as, "One for you, one for me," to distribute the whole object[s] until they encounter leftovers. Using the same example as before, students may divide the three pizzas into sixths, give each friend half a pizza, and end up with half a pizza leftover. These students see the leftover pizza as divisible, but just like their first act of partitioning the three pizzas into sixths, they do not coordinate or pre-plan the number of people sharing the objects with the number of objects being shared from the onset. They use inefficient and rudimentary partitioning strategies resulting in the leftover[s] being partitioned unequally. For instance, the students may partition the half pizza leftover into fifths equally or unequally and distribute it to each friend. Students using this strategy may use concrete object[s] such as cubes or pictorial representations such as drawing circles or rectangles to model their sharing strategy directly. Students operating at this level may not create fractions or quantify each person's final shares in fractional terms with words, notations, or symbols (e.g., Gibbons et al., 2018; Hunt & Empson, 2015; Hunt et al., 2017; Lewis et al., 2015). Two sub-strategies fall under this classification. They include halving and repeated halving, and trials and errors strategies.

Students use halving, repeated halving, or trial and error strategies to partition the whole or leftover[s]. For example, they may first halve each of the three pizzas into halves using the above problem. Students may then distribute a half pizza to each friend. The leftover half is then repeatedly halved into eights. The students then give each friend one-eighths. The three-eighths are then repeatedly halved, then distributed. The processes of halving continue until all the pizzas are exhausted. When the child is asked to quantify the amount each friend gets, the child may count each friend's pieces and say 4 or 5 pieces. Students operating at this level as seen from the example may create equal shares and exhaust the whole object[s] through their partitioning activities. However, they fail to coordinate the two goals of exhausting the whole object[s] and generating equal-sized parts or groups right from the onset. They may use the distributive or skip counting strategy to share the whole items until they encounter leftover [s] as the example shows. These students see the leftover whole[s] as divisible, but they may not coordinate the number of people sharing the objects with the number of leftover objects being shared from the onset.

Students operating at this level may sometimes quantify unit fractions (that is a fraction with one as the numerator) of any size with familiar words such as “pieces” or “halves” or use other designations or notations. The students may at times combine the unit fractions to create a final share for each person or group. They may, however, have difficulty combining the final shares involving non-unit fractions (e.g., combining the shares $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ to create $\frac{3}{5}$) as a final share for each friend (e.g., Gibbons et al., 2018; Hunt & Empson, 2015; Hunt et al., 2017). In summary, the behaviors associated with this strategy include:

1. Identify the given whole objects and all the people sharing the whole objects
2. Fail to pre-coordinate whole objects to the number of people sharing it
3. Use dealing or skip counting to partition the whole object[s]
4. Use halving or repeated halving or trails and errors strategies
5. Encounter leftovers which they partitioned unplanned
6. Use model[s] or/and manipulative[s]
7. Generate “fair” shares sometimes but un-preplanned
8. Quantify each persons’ shares with words or notations or symbols and sometimes with familiar fractions.

Emergent Anticipatory Coordination Strategy (EAC)

Emergent-Anticipatory strategy represents a Level 3 strategy (Hunt et al., 2017). This is considered an emergent relation strategy. Students who use this strategy show the first instance of true conceptions of fractions. These children anticipate and understand that the number of objects[s] to be shared and the number of people or groups sharing the object[s] are related.

Children operating at this level may create equal shares and exhaust the whole object[s] through their partitioning activities. Using the example problem as an illustration, students using this strategy may take the first pizza and partition it into fifths equally. Each friend gets one-fifth pizza. Then the child partitions the second pizza into fifths and gives each friend one-fifth pizza. Finally, the third pizza is also partitioned into fifths and each friend receives one-fifth pizza. The child may combine one-fifths for each person three times to obtain the final share that each friend receives. For example, $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{3}{5}$. When questioned how many pizzas each friend gets, the child says, "half or one-fifth."

As can be seen from the example, some students using this strategy share one item at a time into equal groups and continue the process until all items have been equally shared and distributed. Some children using this strategy may share groups of items using multiplication facts or other number relationships to partition a group of items among the groups of people equally. For example, students may divide a pizza between two friends and continue the process until all friends get half of a pizza. The leftover half pizza is then divided into fifths using the basic division operation so that each friend receives one-tenth of the leftover pizza. The students then quantify the final shares for each friend as the sum of the first half pizza and the second one-tenth pizza equivalent to 6/10 pizza for each friend.

From the example, it is evident that students see leftovers as divisible and coordinate the number of people sharing the objects with the number of objects being shared. Students who use this strategy see unit fraction quantity as a part with numerical relationship to a whole. They understand that a part fits into a whole a countable number of times, such as one-fifths of a pizza given to each person for the first, second, and third pizzas. These children typically use fraction words, notations, or symbols to represent an individual person or group shares. These children can combine unit or non-unit fractions from multiple wholes to make quantities greater than a whole. At this level, the children's notion of fractions as a composite unit emerges (e.g., Gibbons et al., 2018; Gibbons et al., 2016; Hunt & Empson, 2015; Hunt et al., 2017; Lewis et al., 2015). In summary, the behaviors associated with this strategy include:

1. Identifying whole objects and the people sharing the whole objects
2. Pre-coordinate whole objects to the number of people sharing it
3. Use models or manipulatives
4. Single additive
5. Group additive
6. Generate a fair share for each person
7. Combine sometimes each person's shares to make a final share
8. Quantify each person's share as fractions with words or notations or symbols.

Anticipatory Coordination Strategy (AC)

Anticipatory strategy represents a level 4 strategy (Hunt & Empson, 2015). This exemplifies an abstract, advanced or sophisticated strategy. Students operating at this level, create equal shares and exhaust the whole object[s] through their partitioning activities. They coordinate the number of people sharing the objects with the number of objects being shared from the onset of the partitioning activities. Students use a division algorithm in the form of $\frac{a}{b}$ to mentally or procedurally solve the problem. Using the problem above as example, they may use their knowledge of division operation $\frac{a}{b}$ to obtain $\frac{3}{5}$ as shares of pizza for each friend.

Students may also use the multiplicative strategy in the form of a [$\frac{1}{b}$] to partition the objects equally. For instance, they may reason that one pizza shared among five friends generate unit fractions of $\frac{1}{5}$. Since there are three pizzas, the child may reason that $\frac{1}{5}$ multiplied by 3 pizzas generate 3[$\frac{1}{5}$] or $\frac{3}{5}$ as the final share of pizzas for each friend. Students who use the anticipatory strategy understand fractions as numbers that have a multiplicative relationship. By multiplicative relationship, they abstract one person's share from a whole and multiply it by the number of objects[s] being shared to determine each person's or group's overall share, as seen in the example above. Children may conceive the unit fractions as a number with quantity or magnitude with size. They quantify the individuals and combine shares for each person with the correct words, notations and symbols for unit and non-unit fractions. When asked to quantify each friends' share, the student correctly says "three-fifths" or $\frac{3}{5}$ (e.g., Empson & Levi, 2011; Hunt et al., 2017; Wilson et al., 2012). In summary, the behaviors associated with this strategy include:

1. Identify the objects and the people sharing the whole objects
2. Pre-coordinate whole objects to the number of people sharing it
3. Use basic division algorithm strategy or multiplicative strategy
4. Solve mental strategy (i.e., solve without using models or manipulatives)
5. Generate fair share for each person
6. Quantify each person's share as fractions with words or notations or symbols.

The Importance of Genetic Decomposition

The genetic decomposition in APOS-based research plays several critical roles. First, it provides the researcher testable and reversible hypotheses. For example, and in the context of this study, a hypothesis may be that the students use only the halving strategy in which they repeatedly halve given items to equal number of sharers. Students' thinking strategies based on their solutions and responses to their stimulated recalls can be compared to the predicted genetic decomposition. It may point to gaps or difficulties students encounter with the mathematical concept. If the hypothesized preliminary genetic decomposition cannot explain differences in students' performance, it may be revised to accommodate the unexplained differences.

Second, a genetic decomposition of a mathematical concept may help us understand the students' cognitive processes (Arnon et al., 2014). For example, the research can probe the student who used only a halving strategy to better understand their thinking. It provides explanations of phenomena that we can observe in students who are trying to construct their understandings of a mathematical concept. The genetic decomposition may suggest directions for pedagogy that can help students' mathematics understanding and learning (Arnon et al., 2014).

Cognition

To understand how students decompose mathematic problems mentally, one must first define the process of cognition. Cognition, an important brain function that helps an individual function in the real world, represents the mental processes used to acquire real-world knowledge, pay attention in the environment, and make correct judgments during problem solving (Kar & Jain, 2016; Mayer, 2019). It represents a mental activity that occurs within the human brain. Mental activity commences immediately after the brain has received filtered information through the senses. This information is then processed, stored, and later retrieved during problem-solving, decision-making, and creativity (Fauziyah et al., 2019). Thus, cognition includes mental processes that explain how people perceive, learn, remember, imagine, and think about information to solve problem situations successfully (Blake & Pope, 2008; George, 2017). Attention, concentration, memory, intelligence, judgment, and social cognition are some of the major cognitive processes (Kar & Jain, 2016). Theory of mind (TOM), executive function (EF), and motivation are three of the major cognitive process relevant to this study and they were examined below.

Theory of Mind

Theory of Mind (TOM), an aspect of social cognition, represents students' ability to understand other people's mental states such as beliefs, knowledge, thinking, and cognitive strategies that may differ from their own and to predict the mental states of others (Hintermair, 2013, Marschark et al., 2018, Tomasuolo et al., 2012). In order words, TOM is an individuals' understanding that their mental representations of events or situations may differ from reality. This awareness of one's mental states and other people's mental states may necessitate a behavior or change in behavior (Scheetz, 2004; Wellman, 2002).

Young children aged four to six can develop a robust TOM (Marschark et al., 2018). This performance, which grows in complexity with age, is consistent across cultures, language communities, and socioeconomic circumstances (Marschark et al., 2018; Morgan, 2015; Tomasuolo et al., 2012). Furthermore, the quantity and quality of an individual's TOM has been positively correlated to the individual's communication and language abilities. This is important to recognize when working with DHH individuals as discussed later regarding their development of language (Harris et al., 2005; Tomasuolo et al., 2012). It was also important to recognize TOM development, as one aspect of this study asked Deaf adults to share an object between several individuals equally. They considered the views of others and coordinated the sharing of an object fairly with the individuals in a group.

Executive Function

Broadly defined, EF is a collection of cognitive processes essential to higher-order mental function (Laurent, 2014) and thus to mathematics. However, EF is an abstract cognitive construct that is usually identified through a series of an individual's behaviors defined by the outcomes or goals of problem-solving (Zelazo et al., 2004). Examples include problem understanding, representation, planning, solution, and evaluation that lead to problem solving and these abilities are therefore considered related to or representative of mental EF decisions (Zelazo et al., 2004).

Narrowly defined, EF is a construct to describe cognitive processes an individual used to control, direct, supervise and coordinate other cognitive processes to ensure that thoughts and behaviors match the individual's current goal (Bull & Lee, 2014). Four cognitive processes of EF are significant to this study, and they include working memory (WM), response inhibition, attention shifting, and information updating (Bull & Lee, 2014; Logue & Gould, 2014). Individuals with competence in these cognitive processes can initiate problem-solving techniques, inhibit actions that distract their attention, select relevant goals for specific actions, organize complex problem-solving processes, and adjust problem-solving strategies as needed (Hintermair, 2013). These cognitive processes play an essential role in developing students' content area knowledge (Purpura & Ganly, 2014). They were important to this study as the participant were asked to share several objects between several individuals, which requires EF abilities such as attention shifting, memory, and information updating.

Individuals with poor EF abilities lack the above cognitive resources (Anderson et al., 2002; Yiyuan et al., 2006). Moreover, such individuals may be unable to correct errors and are often rigid or concrete in their thought processes. Furthermore, these individuals may show reduced self-control, impulsivity and erratic or careless response behaviors against developmental expectations (Anderson et al., 2002; Yiyuan et al., 2006; Zelazo et al., 2004).

Working memory

One aspect of EF that is important to this study is WM. In its simplest definition, WM is an individual's ability to simultaneously process and remember information in a short time frame while engaging in cognitively demanding tasks (Purpura & Ganly, 2014). WM consists of the limited-capacity central executive system that interacts with the phonological loop and visual sketchpad (Friso-van den Bos et al., 2013). In WM, a person temporarily stores verbal information in the phonological loop. He or she temporarily stores visual information in the visual sketchpad (Friso-van den Bos et al., 2013). Of course, this view of WM, based on phonological information, is problematic as discussed later on in DHH students who use a visual language.

The central executive system coordinates information in the WM, where information is actively kept, and this information is available for use in multi-step problem-solving processes. Verbal information is maintained in or retrieved from the WM storage through articulation or inner voice (Swanson & Beebe-Frankenberger, 2004). An individual uses a visual sketchpad, which involves the generation and manipulation of mental images or visual representations in the WM (Swanson & Beebe-Frankenberger, 2004).

The capacity and capability of students' WM vary (LeFevre et al., 2005). Students with a large memory capacity may acquire and remember new knowledge and procedures more easily compared to students with limited capacity. Stated differently, students with advanced knowledge of concepts stored in the long-term memory (LTM) may have free and sufficient WM capacity to perform like experts by engaging in more advanced behaviors (LeFevre et al., 2005). Several other factors may also mediate students' WM. These include the nature of the tasks, the age of the students, prior knowledge, and the experiences they bring into the tasks in terms of their schema development (LeFevre et al., 2005). A student's language fluency and the linguistic demands of tasks are significant factors for DHH students, as discussed later in this chapter. The linguistic demands of a task may exceed a student's WM storage capacity and, consequently, impair the student's understanding of the information and their problem-solving capability (Techaraungrong et al., 2015). Thus, WM appears to be a significant predictor of a student's ability and performance in task-related activity (LeFevre et al., 2005). It is critical to recognize student's WM processing limitations which may adversely affect their performance in the results (Dogan & Hasanoglu, 2016).

To reduce the demands on WM and to boost the WM capacity to process relevant information, Mayer (2019) proposed several evidence-based cognitive practices on how people learn and remember information: a) exclude irrelevant information from the tasks or materials; b) highlight or repeat essential information; c) place printed words in proximity to the corresponding graphics (drawings, animations or videos); d) present simultaneously rather than successively or sequentially spoken and signed words (for Deaf students) and their corresponding graphics; e) use a combination of narration and graphics; f) break the information into manageable chunks; g) review keys information or words first before students do the tasks; and h) use spoken (and/or signed) words as well as written texts

Response inhibition

In addition to WM, EF is also impacted by the individual's success in demonstrating goal-directed behavior in a given context, which rests on selecting and executing correct behaviors or actions (Bull & Lee, 2014). Selecting appropriate cognitive behaviors requires response inhibition which is defined as removing prepotent or dominant responses and representations. Stated differently, it is the intentional prevention of a behavior that is underway or that is otherwise automatically evoked that may impede problem solving. Such inhibition then leads to successful problem-solving. Response inhibition also connotes the suppression of irrelevant information or inappropriate strategies (Bull & Lee, 2014). Previous responses, representations, or strategies may need to be inhibited in the face of new goals and actions that fit the new environment. In typically developing children and adults, better inhibtory perfomance has been linked to more successful response selection and inhibition (Bull & Lee, 2014). Individuals with more significant response inhibition can disengage or inhibit old strategies and reconfigure a new approach to address the given tasks (Dajani & Uddin, 2015).

Attention shifting

In addition to WM and response inhibition, EF is impacted by attention shifting, which has been described as cognitive flexibility, attentional flexibility, attention switching, and attentional set switching (Dajani Uddin, 2015). Attention shifting represents an individual's ability to flexibly switch between tasks or mental sets or solutions that have previously worked (Bull & Lee, 2014) or "the ability to appropriately adjust one's behavior according to a changing environment" (Dajani & Uddin, 2015, p.1). Thus, attention shifting can be operationalized as task or process switching and set shifting (Dajani & Uddin, 2015).

In task switching, individuals switch between tasks. In set shifting, individuals shift attention from one set of rules or heuristics that govern the solution to a task to another set of rules or heuristics. In order words, set shifting entails using a new set of rules or heuristics to complete the same task (Dajani & Uddin, 2015). Attention shifting or task switching may lead to lower response times (Dajani & Uddin, 2015). Individuals need time to inhibit their initial response and then determine a new set or heuristic or reuse a strategy they applied to a previous task. This is a process of reconfiguring a set of rules.

Updating

The final and fourth aspect of cognition that impacts the use of EF is that of updating information, the process where a person monitors and adjusts the content of the WM (Bull & Lee, 2014). Information that needs updating in the WM may be verbal or visuospatial (nonverbal). Updating assists in holding, retrieving and storing relevant information between the WM and LTM during the execution of goal-directed tasks. In order words, updating is the ability to monitor and revise the active information in the WM.

Motivation

There may be individual differences in the cognitive processes used by people based on cognitive motivation (He et al., 2019). Individuals vary in their tendency to engage in and enjoy cognitively demanding tasks. Individuals high in cognitive motivation show more significant effortful thinking, and they are aware of their thinking. They also prefer complex tasks to simple tasks and tend to spend more time completing tasks or goal-oriented activities (He et al., 2019). Individuals who are cognitively motivated tend to acquire more detailed information in response to ill-defined problems, demonstrate a positive attitude toward complex and cognitively challenging tasks, and are less likely to be satisfied in their answers (Petty et al., 2009). Individuals low in cognitive motivation can be motivated to show more interest in a task by making the information relevant and providing information about the task through an engaging medium (Petty et al., 2009). Mayer (2019) added that unmotivated individuals, even with sufficient WM, could be motivated by presenting the information through informal conversations and using appealing spoken or signed words during these informal conversations.

Motivation is important to consider as DHH students who participated in the study may be differentially motivated based on the nature of the assessment tasks use. Interactive and noninteractive presentations of the assessment tasks may yield differential cognitive motivation for the DHH students. In a later section of this chapter suggestions on how to motivate DHH students were offered.

In sum, cognition includes TOM and EF. Cognition involves understanding how others see the world and has cognitive abilities such as WM, attention shifting, response inhibition, and information updating. It can be impacted by motivation. Competent and successful individuals use appropriate cognitive processes such as: sensory memory (SM), which allows individuals to briefly acquire and retain information from the external environment; WM, which allows individuals to select, process, organize and integrate relevant information chosen from the SM; and LTM, which activates relevant information from prior knowledge. Competent and successful individuals engage in these processes and avoid overloading the WM capacity with information (Mayer, 2019).

Mathematical Cognition

Turning to cognition related to mathematics, Stacey (2006) posited that mathematical thinking can be viewed from different perspectives. One considers mathematical understanding as how people learn, remember, and perceive the relationships between mathematics concepts such as fractions (George, 2017). It may also represent complex mental activities such as identifying relevant quantities, representing the appropriate quantities internally, calculating and comparing quantities for solving mathematical problems (LeFevre et al., 2005). Another perspective regards mathematical thinking as deep mathematical knowledge, general reasoning abilities and knowledge of heuristic strategies (McLeod, 1992), which would require capabilities such as attention shifting and information updating. These abilities all rely on WM as outlined earlier.

Research on mathematical thinking serves both practical and theoretical purposes (Sidney et al., 2018). Practically, knowledge of children's acquisition of mathematics can support their cognitive and mathematical development across cultures. Deaf students, the focus of this study, are a different cultural group and the findings of this study may be used to improve their cognitive processes and mathematical development. This is important because individuals use number and mathematic relationships in everyday mathematical contexts including: decimal numbers to calculate the cost of quantities; factions to measure cooking ingredients; and percentages to compare and to make decisions on the interest rates. In addition to these everyday uses, advanced mathematical skills are needed in many professions including plumbing and bricklaying (Sidney et al., 2018).

Theoretically, research in mathematical thinking may shed light on general cognition in many areas. It may point to how quantities are represented mentally and the changes in mental representations over time. It may also point to the kind of environmental experiences such as intuitive knowledge that may have lasting effects on mental representations and the domain-specific knowledge that supports mathematical cognition. Furthermore, it may shed light on the domain general competencies and processes that may help children's mathematical cognition (Sidney et al., 2018).

Fraction Knowledge

Having set the framework for cognition in general and relating to mathematics, the focus now turns to the purpose of this study, the ability of students to work with fractions. Students' mathematical thinking partly depends on their understanding of fractions as a relational concept (Sidney et al., 2018). A relational concept derives meaning from the two numbers that constitute a fraction as a single entity rather than two separate entities (Sidney et al., 2018). In addition, students' understanding of fractions as a relational concept can be augmented using relational language, familiar context, and activities that place less demand on WM (Sidney et al., 2018). Fractions can be operationalized in term of informal and formal definitions.

Informal Definitions

A major conceptual framework for looking at fractions is that of informal and formal learning. The word "fractions" has been associated informally with its everyday use, and this is a narrow mathematical definition. This view dates back to when Abu Ja' far Muhammad ibn Musa al–Khwarizmi first introduced and popularized the word "fractions" to the Arabic world in Baghdad around 800 BC (Davis, 2003). In its everyday use as conceptualized around 1700 BC, fractions were regarded as the sharing of food; the fractioning of virgin lands; the breaking of church breads; the tallying of voters; and the quantifying of stocks (George, 2017; Lamon, 2012). Nowadays, fractions are used in many different ways including: telling time such as a quarter of 5 o'clock; making change such as a quarter of a dollar; cooking recipes such as a quarter of a cup of flour; and sharing situations such as dividing one candy bar shared by four people (Jordan et al., 2017; Tucker, 2008). As it is believed that mathematics abilities in children are cumulative (Purpura & Ganly, 2014), it is held that informal mathematics skills are learned before formal schooling. This knowledge forms the foundation for formal early basic mathematics skills and later advanced mathematics skills (Purpura & Ganly, 2014).

Formal Definitions

On the other hand, fractions, have multiple formal interpretations and these variations play important roles in the development of students' understanding of fractions (Fonger et al., 2015; Kieren, 1976; Mills, 2016; Tsai & Li, 2017). Kieren (1976) was the first to provide seven different isomorphic formal interpretations of fractions. They include fractions, decimals, ordered pairs, measures, quotients, operators, and ratios. These were later logically simplified into five different formal interpretations of fractions, including part-whole, quotient, ratio, operator, and measure (Fonger et al., 2015; Kieren, 1976; Mills, 2016; Tsai & Li, 2017).

Part-whole

Using the fraction $\frac{3}{4}$ as example, part-whole represents the partitioning of continuous (measurable) quantities like rectangles or discrete (collected objects that cannot be cut) sets such as marbles into 4 equal sized parts and the selection of three equal parts from the 4 equally partitioned quantities. It may also represent the composite part of a set of discrete wholes in which 12 selected marbles from 16 total marbles represented $\frac{12}{16}$ which simplifies to $\frac{3}{4}$ (Tsai & Li,2017).

Quotient

A person who has developed an understanding of fractions as a quotient used the strategy of division or partitioning (Tsai & Li, 2017). For example, quotient represents the share of pizzas (the result of division) a person gets when three pizzas are shared equally among four friends. It also describes the operation of division such as $3 \div 4$.

Ratio

The ratio construct represents how two quantities of the same type are related to each other by comparing their aggregate quantity (Tsai & Li, 2017). In this case, an example would be for a piece of candy bar a person gets, there are four pieces of a candy bar, represented by the ratio 1:4. Continuing this process to share all the three candy bars gives 1:4, 1:4, and 1:4 for each person share. It becomes $\frac{3}{4}$ when summed for each person's final share.

Measure

The measure construct represents the magnitudes of fractions. An example is how a whole object can be partitioned into equal parts and then each piece is counted and measured. The denominator 4 represents 4 equal parts. Then $\frac{3}{4}$ represents a measure of 3 units of $\frac{1}{4}$ size starting from zero on the number line.

Operator

The operator metaphor can be explained as a series of operations performed on a set. For example, a student may be given a problem such as finding $\frac{3}{4}$ of 12 cakes. He or she first multiplies the cakes by 3 and then divides the cakes by 4 or vice versa resulting in 9 cakes.

Fractions or Rational Numbers

As a clarification, the term "fractions" differ from the term "rational numbers" although both terms have been used interchangeably (Charalambous & Pitta, 2007). This may have added confusion to an already complicated mathematical concept for many children (Lamon, 2012; George, 2017). They need to understand that rational numbers can be represented in several ways such as fractions, decimals, and percentages. Furthermore, rational numbers are used in many ways such as part-whole or quotient relationships, and have many properties such as multiplicative property and magnitudes (Kilpatrick et al., 2001; Lamon, 2012; Siegler & Braithwaite, 2017).

Teaching of Fractions: Part-Whole and Procedural Approaches

Perspectives on teaching fractions vary among mathematics educators and researchers. This section examines some of the approaches to the teaching of fractions**.** For several years, students have been instructed procedurally on the part-whole metaphor and it has been the dominant approach used for the teaching of fractions. (Empson & Levi, 2011). For example, students are given an object and then asked to divide the object into parts with a focus on the process or given a set of objects and asked to move the objects outside of the set to represent a fraction. Consequently, classroom instructions and instructional materials such as mathematics curricula and textbooks usually reflect the part-whole approach. Most mathematics textbooks on fractions often start by introducing fraction symbols and then presenting the symbols through diagrams (Empson & Levi, 2011). Steffe and Olive (2010) called such traditional school mathematics approaches as mathematics for adults rather than mathematics for children.

These approaches have faced several criticisms. They may have contributed to the historical difficulties and the lack of conceptual understanding many students experience with fractions (Empson & Levi, 2011; Kilpatrick et al., 2001; Tsai & Li, 2017). Cadez and Kolar (2018) attributed students' poor understanding of fractions to instruction that focuses on part-whole. Carraher (1996) critiqued the part-whole approach to learning fractions by saying that "no mathematical concept can be reduced to a physical embodiment. A fraction entails relations, and relations are not palpable, physical objects" (p.242).

Little time has been used to help students develop the deeper conceptual meaning of fractions (Doyle et al., 2016; Empson & Levi, 2011; Hunt et al., 2017; Jordan et al., 2017). Instructional approaches for teaching fractions often use direct and explicit instructions that emphasize rote practices with physical objects or manipulatives. For example, students may be taught to understand $\frac{1}{5}$ as one shaded part out of five equally divided parts of a rectangular or circular shape. Students rarely learn to think of $\frac{1}{5}$ as one-fifth of equidistance from zero to one on a number line, nor do they potentially think of it as one-fifth of an object shared among five people. Steffe and Olive (2010) considered disregarding children's mathematical thinking as espoused by the part-whole mathematical curricula and instruction as a crucial contributor to the historical difficulty students had in learning fractions. Moreover, these approaches have forced researchers to make the part-whole interpretation the focus of their investigations in many research studies while disregarding other approaches (Charles & Nason, 2000). This gap in literature may have indirectly contributed to the students' difficulty in understanding fractions (Charles & Nason, 2000).

Deaf and Hard of Hearing Students (DHH) and Cognition

Turning to Deaf students and their abilities with fractions, it is important to first note that about 95% of DHH students have hearing parents. They are also a diverse and heterogeneous group with different degrees of hearing loss (Loma et al., 2017). DHH students are regarded by some as less intelligent and less abstract thinkers when compared to hearing students (Loma et al., 2017; Moores, 2001). Persistent gaps in academic achievement have been identified and have given researchers the belief that the cognitive processing of DHH students may be qualitatively different (Supalla et al., 2014). However, the research of other academics suggests that DHH and hearing students' cognitive abilities are normally distributed (Loma et al., 2017; Morgan, 2015).

Researchers in deaf education have investigated and documented DHH students' general cognitive abilities through measures of their TOM, EF, and motivation. They sought to understand and improve DHH students' general cognitive abilities and academic outcomes. As Hintermair (2013) explained, competent DHH individuals across all developmental periods coordinate their affect, cognition, communication, and behavior.

DHH Students' TOM

According to several researchers, DHH students across different degrees of hearing loss, communication options, and educational contexts lag behind in TOM when compared to their hearing counterparts (Hintermair, 2013; Marschark et al., 2018; Spencer, 2010). However, research suggests that DHH students' inability to hear sound or spoken language may impede their ability to develop TOM. On the other hand, DHH children of deaf parents, post-lingual DHH children and DHH children with cochlear implants (CI) do better in TOM. Research suggests that limited access to language and the quality of the language inputs, rather than cognitive difference, may affect DHH children's development of TOM and, consequently, their problem-solving ability (Hintermair, 2013; Laurent, 2014). This is attributed to their early access to signed, spoken language or conversational language and greater exposure to diverse mental states than DHH children of hearing parents without such early language access (Marschark et al., 2018, Siegal et al., 2001; Woolfe et al., 2003).

DHH Students' EF

In addition to a delayed TOM, many DHH children and adolescents show executive function delays (Hermans et al., 2015; Hintermair, 2013). These cut across DHH students' age, language ability, communication preferences, use of assistive hearing devices, and educational placements compared to their hearing peers (Hintermair, 2013; Remine et al., 2008). However, there is a high degree of variability in EF skills within the DHH cluster. For instance, Yiyuan et al. (2006) investigated the EF of 76 DHH and 78 hearing students and found no significant difference in the EF of three-year-old DHH students and hearing students. However, they found that the hearing students quickly developed EF between the age of 4 and 4.6. DHH students develop EF skills two years later. Contrary to findings in Marschark and Everhart's (1999) study of DHH students without CI, Remine et al. (2008) found that DHH students with CI showed an average performance in such EF skills. These skills cover abstract thinking, efficient problem-solving strategies, categorical processing, rule learning, spatial planning, maintaining cognitive set, and inhibition of impulse responses. DHH students in general education settings have been better in EF than DHH students in residential schools (Hintermair, 2013). It has also been noted that DHH children with low expressive language ability may have more difficulty with EF in problem solving (, 2014).

Impediments to DHH Students' TOM and EF

Language ability facilitates many cognitive processes (Schick et al., 2007). Many DHH students have been known to have limited access to language in their formative years at home (Kritzer & Pagliaro, 2012; Mousely &Kurz, 2016). This may have adversely affected their TOM, EF, and other abstract concepts such as mathematics (Ansell & Pagliaro, 2006). This lack of language has been deemed language deprivation.

Added to the issue of language deprivation for DHH children is the specific and complex language of mathematics. Titus (1995) contended that a delay in their fraction development could be partly explained by the English language difficulties many DHH children encountered while learning mathematics and fractions. The way fractions are worded in everyday and standard English sentences may be complex and sometimes confusing to students (Mousley & Kurz, 2016). These include English language words such as conditionals (if, when); comparatives (greater than, less than); negatives (not, without); and inferential (should, could) which have caused difficulties to many DHH students learning mathematics (Mousley & Kurz, 2016).

Reading comprehension of English word problems, a difficulty that cuts across grade levels, may affect their complete access to the mathematical story problems (Kidd et al., 1993; Pagliaro & Ansell, 2012; Purpura & Ganly, 2014). In fact, as Hrastinski and Wilbur (2016) observed, about 50% of DHH students in 12th grade read between 3rd and 4th-grade levels while approximately 7-10% of these high school students read at or above 7th grade level.

Thus, it appears that DHH children's lack of foundational skills in mathematics can be attributed to language deprivation.

Facilitating DHH Students' TOM

Researchers have suggested ways to improve TOM of DHH children. Several studies indicate DHH students' TOM can be facilitated by providing access to language through signed stories, adult conversations, peers' interactions and conversations, and incidental learning opportunities (Harris et al., 2005; Morgan, 2015; Tomasuolo et al., 2012). Besides the interactive uses of language, translations and/or pictorial representations may facilitate the students' understanding of abstract concepts such as TOM, mathematics and fractions (van Garderen & Montague, 2003). In addition to these strategies to facilitate DHH students' TOM, Spencer's (2010) had three suggestions for DHH students:

- Language used must be accessible to the students
- Language must be produced during supportive interactions
- Language must be paced to allow for acquisition and students' participation in the activities that promote students' cognitive development.

Facilitating DHH Students' EF

Delays in EF suggest traditional intervention practices of stimulating spoken or oral language development and making information accessible through visual supports may be inadequate to improve the academic outcomes of DHH children. Hermans et al. (2015) suggested compensatory and remedial strategies to enhance or bypass EF deficits or delays in DHH children.

Compensatory Strategies

Compensatory interventions seek to reduce the unfavorable effects of EF delays or deficits on children's development. These strategies focus on development of key EF components such as working memory capacity, inhibition control of irrelevant information, strong reasoning abilities, problem solving capability, and sequential processing (Morgan, 2015).

Additionally, authors have suggested that DHH children benefit more from two promising compensatory EF strategies (Hermans et al., 2015). The first is didactic or instructional strategies (Hermans et al., 2015). Didactic strategies assume that educators are alert to the presence of cognitive overload during learning activities with DHH children. They adjust their teachings or interventions accordingly (Hermans et al., 2015).

DHH cognitive overload may be partly due to the heavy language demands of the activities. Teachers or researchers possess the skill to adapt their language use to prevent or reduce cognitive overload of DHH students (Hermans et al., 2015). Teachers or researchers can use spoken or signed language with familiar contexts. They can use simple and structured language with familiar or meaningful content. Teachers and researchers are also advised to focus on one activity at a time (Hermans et al., 2015). Multitasking activities burden the WM capacity. Students should be given enough time to process the information or instruction and teachers or researchers should present the learning activity more than once to DHH children. Other didactic strategies include teachers or researchers dividing complicated tasks into smaller and less complicated tasks, modeling the tasks, discouraging students from guessing, immediately correcting children's errors and carefully fading prompts that are provided to facilitate learning (Hermans et al., 2015).

The second strategy is bimodal input in which signs and pictures or written words are combined, and this approach seems to be more effective than unimodal input (Hermans et al., 2015). This may or may not reduce cognitive overload and unburden the limited WM capacity. Bimodal input may or may not also increase the temporary retention of information in the WM.

Visual supports such as pictures, flowcharts, and diagrams presented simultaneously alongside signed or written words may reduce DHH students' WM overload and support their learning. DHH readers with low WM capacity may be hindered by written English texts but supported with simple visual graphs. Thus, providing DHH children with bimodal input that use text and picture that does not require sequential processing is a promising approach to compensate for the DHH children's EFs deficits or delays (Marschark et al., 2018).

DHH Students' Mathematical Cognition

DHH individuals have been expected to demonstrate the same mathematical cognition or skills as hearing people (Markey et al., 2003). However, DHH students consistently achieved far below their hearing peers in academic content areas such as mathematics (Morgan, 2015). For over five decades, studies have reported a delay of about 2.0 to 3.5 years in the mathematics achievement of DHH students compared to their hearing counterparts (Gottardis et al., 2011; Bull et al., 2005; Rodríguez-Santos et al., 2014; Rodríguez-Santos et al., 2018).

Several studies investigating DHH students' mathematical ability found a delay or poor performance in several mathematic concepts across grade levels (Bull et al., 2011; Kritzer, 2009; Mousley & Kurz, 2016; Rodríguez-Santos et al., 2018). These include mental calculations, basic mathematical concepts, number sequencing, and numerical relationships. Others are mathematics computations, whole number arithmetic problem solving, logical reasoning, standardized achievement tests and measurements, numerical concepts, and fraction concepts (Bull et al., 2011; Kritzer, 2009; Mousley & Kurz, 2016; Rodríguez-Santos et al., 2018).

However, some DHH preschoolers or kindergarteners who used a cochlear implant (CI) did not show a delay or poor performance in the non-symbolic or early number representations (Arfé et al., 2011; Gottardis et al., 2011; Zarfaty et al., 2004). The same can be said of DHH children in K-3 grades who were proficient in American Sign Language (ASL) for the arithmetic story problem solving strategies (Pagliaro & Ansell, 2012). DHH children aged 7–9-year-olds with mild to moderate hearing loss did not show a delay or poor performance in their overall mathematical performance (Gottardis et al., 2011). DHH college students have also performed comparatively with hearing college students on mathematical problem solving presented in visual format (numbers and pictures) and multiplication word problems (Kelly & Mousley, 2001).

Many DHH students do not understand the semantic structures of arithmetic story problems (e.g., Gottardis et al., 2011; Pagliaro & Ansell, 2012). Mulligan (1992) defined the semantic structure of arithmetic problems as the actions or relationships that model the problem. Pagliaro and Ansell (2012) identified 14 semantic structures of arithmetic story problems. As an illustration, Ansell and Pagliaro (2006) and Pagliaro and Ansell (2012) tested 59 purposefully recruited DHH children in K-3 settings from ages 5 to 9 years on their arithmetic story problem solving strategies. The nine story problems, including one equal sharing problem, were translated into ASL. Researchers found the majority of students used strategies that did not match the story structures or contexts. Specifically, they used the addition operation for subtraction operation and vice versa.

Kritzer's (2009) study with 28 DHH children corroborated these findings, arguing that DHH students age 4 to 6 failed to pay attention to the relationship between the numbers and the story in the problem. For example, they counted the given manipulative tokens without adding them together as required by the story problem. These children were observed to play with the concrete objects in a manner unrelated to the given story problem. In addition, the students' successes in the equal sharing problems, the most difficult of the items, depended on their ability to see the whole as divisible, understand the concept of fair share, and compare sets for their equality. None of the DHH students in Kritzer's study who took the fair-sharing of discrete quantities test/problem correctly solved it. Most of the children responded to the equal sharing tasks either by stating the numbers from the problem as answers or counting all the available concrete manipulative tokens. One child made unequal sets, and many exhibited behaviors or comments unrelated to the problem. For example, some children said, “I dunked cookies in milk before eating it.” These studies are limited because they not only focused on younger DHH students, but also focused on children in the United States. In addition, the equal sharing problems embedded in the arithmetic tasks end with whole number results rather than fractional answers.

Interestingly, Frostad and Ahlberg (1999) found findings similar to that of Kritzer (2009) with young DHH students outside the United States. They presented arithmetic story problems to 32 DHH Norwegian children who ranged from severe to profound hearing loss, who used Norwegian Sign Language (NSL), who had basic counting skills, and who were in kindergarten through fourth grade (Frostag & Ahlberg, 1999). The children were told to add or subtract the arithmetic problems. The Norwegian DHH children did not know how to approach the word problems thoughtfully. They simply focused on adding or subtracting the numbers presented in the story problem without understanding the semantic structure of the problems, which is critical to solving arithmetic problems.

DHH students’ difficulty with problem solving persisted as they get older, progress through postsecondary school and are expected to acquire more problem-solving procedures (Erickson, 2012; Lee & Paul, 2019; Pagliaro & Ansell, 2012). DHH college students have been found to leave solutions blank and make no attempt to solve them when given problem solving tasks. They have been found to commit computational and procedural errors. They have been found to have poor problem-solving skills, a lack of confidence, focus, and persistence, and they also seem to have poor self-monitoring skills (Kelly & Mousley, 2001; Lee & Paul, 2019). They have also been found to focus on surface structures and irrelevant information (Blatto-Valee et al., 2007). Even though many of the above studies cut across different grade levels at home and abroad, the concept of these studies is still limited to arithmetic problem solving or problem solving. These studies did not specifically focus on fractions or the DHH students' strategies to solve fraction problems.

DHH Students’ Fraction Cognition

Since 1995, a number of studies across various grade levels have examined the performance of DHH students on fraction concepts in the measure, part-whole, and ratio constructs of fractions and found that the majority of DHH students lack a conceptual understanding of fractions. For instance, DHH students determined fraction size based on the individual numbers that make up fractions (Mousley & Kurz, 2016; Titus, 1995). They also had difficulty determining the larger of two fractions and ordering several fractions in order of magnitudes (Mousley & Kurz, 2016). The majority of DHH students used procedures they did not understand or were at variance with the fraction problems to solve it (Kelly & Mousley, 2018).

There are several limitations of these three studies on fractions which guided this study. They: (a) lacked fraction story contexts; (b) investigated different constructs of fractions such as measure and part-whole; (c) focused on research goals other than equal sharing problem; (d) used quantitative methods for analyzing the data; and (e) used qualitative method of analysis based on arithmetic story frameworks. These limitations of research on DHH students’ abilities with fractions, the call for a study to unearth how DHH children learn and develop fraction understanding (Titus, 1995) justify this study's rationale.

Summary of Chapter Two

The literature review began with the study’s purpose statement and a review of two theoretical lenses, cognitive constructivism and APOS, that guided all aspects of this study, including the research questions. Then it examined cognition in general and mathematics specifically. The literature review later progressed to Deaf students' cognitive abilities, how they impacted their mathematical abilities, and suggested strategies to mitigate the adverse impacts. It concludes with a discussion of the limitations of reviewed research on mathematics with DHH students.

CHAPTER III: METHODS

This methodology chapter is divided into the following sections: (a) research questions, (b) research design, (c) sample, (d) measures, (e) data collection procedures, (f) data analysis procedures, (g) rigor and trustworthiness, and (h) limitations of the study (Creswell & Plano Clark, 2011; Mertens, 2015).

Research Questions

The study seeks to understand the levels of the cognitive strategies used by DHH college students for solving equal sharing fraction story problems. Three research questions guided the study, and they include:

1. What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems presented to them through an interpreter in ASL in a frozen video format?
2. What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems when they are asked to co-construct the problems in dialogue with a Deaf researcher in ASL?
3. How does fraction understanding of college DHH students (i.e., employed thinking strategies and representations) who watch an ASL video with an interpreter compare to their understanding when they are asked to co-construct the problems in dialogue with a Deaf researcher in ASL?

Research Designs

To answer these research questions, the study employed a qualitative research design through the lens of APOS-based research theoretical framework first mentioned in chapter one and reiterated in chapter two (Arnon et al., 2014). This qualitative research design uncovers the thinking strategies DHH college students use to solve equal sharing fraction problems based on how participants "interpret[ed] their experiences, how they construct[ed] their worlds, and what meaning they attribute[d] to their experiences" (Merriam & Tisdell, 2016, p.24).

This study mirrored the APOS method employed by Ubah and Bansilal (2018), a Clinical Interview method used by Ginsburg (1997), a Think Aloud Protocol (TAP) method used by Schirmer (2003), a retrospective report approach in the study of Fazio et al. (2016), and a framework of teacher's moves for probing and extending student's mathematical thinking in one-on-one interaction suggested in the study of Jacob and Empson (2016). In addition, through a series of fraction elicitation problem solving tasks (giving participants an activity that requires them to respond verbally and in writing), this study investigated DHH college students' strategy use for solving 10 pre-constructed or interpreted and 4 co-constructed equal sharing fraction problems.

Product or/and Process Data

Smith (2014) described interpreting as "primarily a mental game" (p.129). I make similar attribution that mathematics problem solving is primarily a mental game. Students' answers to mathematics problems (and their comments or written/drawn artifacts) represent the product data. The products represent the external evidence of what goes on in the students' heads during mathematics problem solving. The products are the results of the mental interpretations and decision-making processes, the tangible outputs.

However, we must acknowledge that more goes on inside a person's head than what we see in their language, actions or drawings (i.e., the product). We may not truly be seeing everything that they are thinking by just examining the product. What goes on in the students' heads as they solve mathematics problems represents the internal processes (i.e., the inner thoughts). The internal processes are the mental processes (i.e., the inner workings of students' minds).

Metaphorically, the products are evident to the naked eye, but the internal processes are not readily noticeable. A sole focus on the final products is like a focus on the final destination of a journey without regard to the journey itself, the different routes, trails, successes, choices, and pitfalls that the traveler encounters (i.e., the process). Internal mental landscapes are rich with data that needs to be extracted or tapped. As teachers and researchers, it behooves us to understand the students' inner thoughts. This information is critical to teaching and developing students' internal thoughts (Smith, 2014). Combining product-oriented data and process-oriented data is crucial to understanding the complete picture of students' performance on problem solving tasks (Smith, 2014 p.140). Therefore, this study analyzed both the products and the processes but with more weight given to the process data.

Sample

Thirteen (13) DHH college students were recruited from four different colleges located in the Mid-Western and Eastern parts of the United States to participate in this study. After obtaining approval from the University of North Carolina Greensboro (UNCG) Institutional Review Board (IRB), colleges were contacted and their approval to conduct the study at their school was obtained. Recruitment letters were sent to these colleges (see Appendix A for the college letter), and follow-up emails were sent as appropriate (see Appendix B for the email).

In keeping with APOS theoretical framework (Arnon et al., 2014), the 13 DHH college students were recruited through their teachers (see Appendix C for participant letter of recruitment and informed consent) received a gift certificate in the amount of $25.00 (twenty-five dollars) for their participation.

Like several other studies of the mathematical abilities of DHH students (Mousley & Kelly, 2018; Mousley & Kurz, 2016; Pagliaro & Ansell, 2012; Nunes et al., 2009), the inclusion criteria for this study includes: (a) deaf or hard of hearing identification, (b) enrolled in a college program, (c) use sign language, (d) 18 years or older, and (e) consent to participate. In addition, this study used convenience sampling to recruit DHH college students who meet the inclusion criteria for the study. Convenience sampling is a nonprobability or nonrandom sampling used in selecting a study's participants who are easily accessible, geographically near, available, and willing to participate (Etikan, 2016). Historically, convenience sampling is the most commonly used, less expensive approach to collect information from participants who are easily accessible to the researcher. The Deaf community is heterogeneous, geographically diverse, and a minority population making it difficult to find a larger sample in one location.

Demographic Characteristics

Participants completed the demographic survey online before or on the assessment and interview day (see Appendix D). The demographic survey solicited information about the participant's name, age, and gender. It also asked about the participant's type of hearing loss, degree of hearing loss, and presence of additional disabilities. Moreover, data on student's proficiency in mathematics were collected. Finally, information about students' use of assistive listening devices, communication options at home and school, and ASL proficiency level (see Appendix D for the demographic survey and link) were collected. Using the American Sign

Language Proficiency Interview (ASLPI), participants self-rated their ASL proficiency, and the researcher evaluated students' ASL proficiency by interacting with them and observing their signing skills using the information obtained to complete the ASLPI rating scale (see Appendix J). Tables 1 and 2 provide detailed and summarized demographic information, ASL proficiency, mathematical proficiency for participants, and other pertinent characteristics. One participant reported having additional disabilities, but this information was removed from Table 1 to maintain confidentiality due to the nature of the disclosed disabilities.

Table 1. Detailed Demographics

Name	G	Age	DL*	DL^	HS	ND	PMC	E	FHS	MHS	DS	PHL	PSL	ALD	API	MS
Jeff	M	21	90-120	90-120	D	LD	ASL	B	H	H	NO	VSF	ASL	NR	4	A
John	M	23	40-55	90-120	HH	LD	A/SL	HL	H	H	NO	V	A/SL	CI	5	AA
Socrates	M	41	90-120	90-120	D	LD	ASL	B	H	H	NO	VS	ASL	NR	4	A
Andriana	F	40	40-55	55-70	HH	LD	A/SL	W	H	H	NO	V	A/SL	HA	2	AA
Lucia	F	32	90-120	55-70	HH	LD	A/SL	W	H	H	NO	VSF	ASL	HA	3	BA
Jessica	F	40	70-90	90-120	D	LD	ASL	W	H	H	NO	VS	ASL	HA	3	A
Rebekah	F	20	55-70	70-90	D	DB	ASL	B	D	D	YES	S	ASL	NR	4	A
Connor	M	31	90-120	90-120	D	DB	ASL	B	H	H	NO	S	ASL	HA	4	A
Janet	F	NR	40-55	70-90	D	LD	ASL	A	H	H	NO	V	A/SL	NR	4	A
Beatrice	F	24	90-120	90-120	D	DB	ASL	W	D	D	YES	S	ASL	CI	4	A
Joseph	M	24	40-55	90-120	D	DB	ASL	W	H	H	NO	VSF	ASL	CI	5	A
Julie	F	31	70-90	70-90	D	LD	ASL	A	D	D	YES	S	ASL	HA	5	BA
Robert	M	24	70-90	70-90	D	DB	ASL	W	H	H	NO	VS	ASL	HA	5	BA

Note. NR = Not Reported; G = Gender; M = Male; F = Female; DL* = Degree of Hearing Loss (right ear); DL^ = Degree of Hearing Loss (left ear); HS = Hearing Status; D = Deaf; HH = Hard of Hearing; ND = Nature of Deafness; LD = Late Deafened; DB = Deaf at Birth; PMC = Primary Mode of Communication; ASL = American Sign Language; A/SL = ASL and Spoken Language; E = Ethnicity; B = Black; HL = Hispanic or Latino; A = Asian; FHS = Father Hearing Status; H = Hearing; MHS = Mother Hearing Status; DS = Deaf Siblings; PHL = Primary Home Language; VSF = Voice, Signs and Fingerspelling; V = Voice only; VS = Voice and Signs; S = Signing; PSL = Primary School Language; ALD = Assistive Listening Device; CI = Cochlear Implant; HA = Hearing Aids; API = American Sign Language Proficiency Interview (Researcher's Rating); MS = Mathematics Skill (Self-Rating); A = Average; AA = Above Average; BA = Below Average.

Table 2. Summarized Demographics

Characteristics	N	%
Total participants	13	100%
Gender		
Male	6	46%
Female	7	54%
Age		
20-29	6	46%
30-39	3	23%
40-49	3	23%
Missing	1	8%
Degree of hearing loss (right ear)		
Profound	7	54%
Severe	4	31%
Moderately severe	2	15%
Degree of heating loss (left ear)		
Profound	5	38%
Severe	3	24%
Moderately severe	5	38%
Hearing status designation		
Deaf	10	77%
Hard of hearing	3	23%
Additional disabilities		

None	12	92%
Others	1	8%
Primary mode of communication		
ASL	10	77%
Spoken	0	0%
ASL+ spoken	3	23%
Ethnicity		
White	6	46%
African American	4	31%
Hispanic or Latino	1	8%
Asian	2	15%
Father hearing status		
Deaf	3	23%
Hearing	10	77%
Mother hearing status		
Deaf	3	23%
Hearing	10	77%
Deaf siblings		
Yes	3	23%
No	10	77%
Primary home communication		
Voice	3	23%
Voice + Sign	3	23%

Voice + sign + fingerspelling	3	23%
Sign	4	31%
Primary school communication		
ASL only	9	69%
ASL + sign English	1	8%
ASL + speech/spoken English	3	23%
Assistive listening devices		
None	4	31%
Cochlear implant	3	23%
Hearing aids	5	38%
Frequency modulation systems	1	8%
API self-rating		
1 little knowledge	0	0
2	1	8%
3	2	15%
4	6	46%
5 fluent	4	31%
API researcher's rating		
1 little knowledge	0	0%
2	0	0%
3	3	23%
4	6	46%
5 fluent	4	31%

Self-reported math skills		
Above average	2	15%
Average	8	62%
Below Average	3	23%

Note. N = Number of participants; % = Percentage of participants; API = American Sign Language Proficiency Interview

Measures

This study employed two equal sharing fraction problem measures for data collection (i.e., interpreted equal sharing fraction tasks, co-constructed equal sharing fraction tasks). These elicitation tasks were created and presented to the participants in two distinct conditions: interpreted and co-construction like other studies (Hemmecke & Stary, 2006). In addition, each student began with practice tasks, and each measure is described in the following sub-sections. Data for this study consist of: (a) participants' responses to the demographic questionnaires, (b) participants' written solutions to the 10 tasks, and (c) video recordings of participants' responses and behaviors during 10 fraction problems solving while performing a TAP.

Practice Elicitation Tasks

Participants completed two practice problems and six independent problems (see Appendix E for the two practice fraction problems written in English and gloss. These problems are aligned with similar questions used in prior studies (e.g., Creswell & Plano Clark, 2011; Mousley & Kurz, 2016; Pagliaro & Ansell, 2012). All fraction tasks are equal sharing story problems. Equal sharing story problems involve asking students to determine each person share if “n” items are shared among “m” number of people (Hunt et al., 2017). For example, if 5 friends share 8 candy bars, how many candy bars does each person get?

The first practice task ends with a whole number result and builds on the participant’s prior knowledge of division operations and problem solving. The first practice problem has four discrete objects and two sharers that most children found easier to do. The second practice task ends with a fractional answer and foreshadows what the student should expect in the independent and co-constructed tasks. The second practice task has seven items and two people sharing the items.

The first practice task was translated into ASL by an interpreter and videotaped. The second practice was co-constructed. Each participant solved both practice exercises to familiarize themselves with administration procedures and expectations of the tasks. Although these tasks are not included in data analysis, they provide helpful insight and training.

Pre-Constructed/Interpreted Elicitation Tasks

The six tasks in Appendix F were completed by the participants independently. In this study, story problems are presented in order of increasing complexity in line with studies conducted with hearing children (Empson & Levi, 2011; Hunt & Empson, 2015; Siegler et al., 2010). The first two story problems contain two to four sharers and lent themselves well to the halving and repeated halving mathematical strategies. While the third story problem did not readily lend itself to the halving strategy; however, the first three story problems all result in fractional answers greater than one (i.e., the number of shared items is greater than the number of sharers). Problems four through six are more difficult and involve fractional answers less than one. Using a variety of problem types allowed students to use different thinking strategies as they solved the problems (Empson & Levi, 2011; Hunt & Empson, 2015; Siegler et al., 2010) allowing for rich data collection.

The pre-constructed six tasks presented as written story contexts were translated into American Sign Language (ASL) by a certified ASL interpreter and video recorded (see link to the ASL video in Appendix F). The translations follow the equal sharing problem format proposed by Hunt, et al., (2017). This format involves first signing the number of shared items, followed by signing the number of people sharing the items and ending with questions on how many or how much each person receives or eats (see appendix F for the ASL gloss). Across all six problems translated to ASL, the story problem contexts and amounts changed, but the signing structures for all the problems remain unchanged.

In addition, the translation also followed suggestions provided by Pagliaro and Ansell (2012) on the translation of mathematics problems from written English to ASL. These suggestions are that they (a) are appropriate to the grade levels, (b) maintain the mathematical structure of the original problems, and (c) comply with the rules of ASL linguistic. Participants watched the pre-recorded videos as many times as needed.

Co-Constructed Elicitation Tasks

The four co-constructed tasks (see Appendix G) reflect the same format as the six pre-generated tasks. Along those same lines, the first two co-constructed tasks contain two to four sharers, lending the two problems to the halving and repeated halving strategies. The third task does not readily lend itself to the halving strategy. The first three tasks taken together resulted in fractional answers greater than one (i.e., the number of items being shared is greater than the number of sharers). The fourth task is more complex and result in fractions less than one. The co-construction of tasks followed these steps:

- Ask student their favorite food or drink
- Repeat and confirm what the student says their favorite food or drink is
- Ask the student to provide a number, such as their number of friends, classmates, family members, or siblings
- Repeat and confirm the specific number of people that the student provides
- Use the information given by the student to create four equivalent problems to the 6 translated problems for them to solve. For example, "if __________(quantity) licorices (their favorite food) are shared equally by ______, __________, and ____________ (names or number of person). How many (or how much) licorices each friend or person received?

Data Collection Procedures

Researcher and participant's interactions occurred via Zoom because of Covid-19 that prevented face-to-face interview and interactions. Each student was asked to find a pencil or pen and written paper to show their work if necessary. Some students elected to use their white board or note on their iPad to solve the equal sharing tasks. Each participant took approximately 60 to 120 minutes to complete the 10 tasks. All meeting sessions with the participants were videotaped. On the assessment and interview day, the goals and data collection procedures were first reviewed with each participant (see Appendix H–Instructions).

Think Aloud Protocol and Stimulated Recall

Participants engaged in a TAP to examine the strategies students used to solve the mathematical tasks. The interview protocol is modeled after Hunt and Empson (2015) and the teacher's questioning framework proposed by Jacobs & Empson (2016) for assessing children's fractional thinking (see Appendix I for interview protocol with questions and prompts).

There are two types of TAP used in this study: (a) concurrent verbalization (i.e., verbalization occurs simultaneously as the participant is completing the task), and (b) retrospective verbalization (i.e., verbalization occurs after the participants have completed the tasks) (Russell & Winston, 2014). Talk aloud or think visibly is an example of concurrent verbal reports, whereas Stimulated Recall (SR) is an example of retrospective TAP (Russell & Winston, 2014; Schirmer, 2003; Sun, 2011). An advantage to the immediate, concurrent TAP is the information shared by participants is subject to minimal delay and minimal data loss due to memory loss, and minimal sense-making based on plausible explanations (where the person tries to explain their actions as accurate or inaccurate) (Russell & Winston, 2014).

A TAP requires students to explain what is going on in their mind or working memory as they are performing a task (Lundgren-Laine & Salantera, 2010) or to "verbally report thoughts that are occurring in real time" (Smith, 2014, p. 136). These reports allow us to understand the students' strategies (Ericsson & Simon, 1993) and inner thought processes before or after completing tasks (Russell & Winston, 2014; Smith, 2014). For example, students are asked to solve equal sharing story problems and are prompted to say aloud everything going on in their mind at the time or after they have solved the problems.

Researchers argued that individuals can self-report their thinking and that such verbal reports are a reliable and valid representation of what happens in the students' heads (Schirmer, 2003; Sun 2011). Ericsson and Simon (1993) argued that a TAP can be used to elicit data about these cognitive processes, and they comprehensively examined the cognitive processes using verbal reports in their study. Moreover, a TAP offers an effective tool (probably better than questionnaires) for accessing the rich, extensive, reliable, and valuable data stored in the students' heads or available in the working memory at the moment in real time (Bernardini, 2001; Lundgren-Laine & Salantera, 2010).

Using the TAP process allows students to see what they know and don't know. A TAP can reveal levels of or gaps in students' cognitive processing or understanding. Students' cognitive strategies, vocabulary, ability to co-construct meaning, and extra linguistic knowledge (Russell & Winston, 2014; Smith, 2014) can increase students' capacity for problem-solving (Bernardini, 2001). Researchers' and teachers' knowledge of how the answers are derived is critical to pinpointing problem-solving areas in need of attention or remediation (Smith, 2014). For example, researchers need to understand students' thought processes when arranging three fractions in ascending order rather than just the correct arrangement.

TAP Processes

In this study, participants watched the pre-recorded ASL videos of the interpreted equal sharing fraction story problems or co-constructed the problems with the researcher. Next, they retold story problems to ensure their understanding of the problems. Next, they solved problems and performed a concurrent guided TAP. Participants thought through the tasks and explained their solutions to the problems. Next, a retrospective guided TAP was conducted where students were probed with additional questions that present the goals of the equal sharing tasks.

These additional questions consisted of general questions and specific follow-up questions asked to probe and fully understand students' thinking strategies to solve each equal sharing problem. These questions assessed many abilities and explored participants' thinking strategies, knowledge to mentally pre-plan the partitioning of each whole item coordinated with the number of sharers, skill at combining the created unit fractions, and their strategies at quantifying it as an equal share for each sharer (Ginsburg, 1997; Hunt., 2017; Jacob & Empson, 2016).

The order in which the general questions were asked remained the same for all participants, but the order in which the specific questions asked varied depending on the student's responses and solutions to the fraction tasks. The general questions were asked first, followed by the specific but variable questions related to the equal sharing tasks (Ginsburg, 1997). As outlined in chapter two, these questions are designed to encourage the students to use theory of mind (TOM), the view of others, and executive function (EF), the ability to switch tasks as needed to consider the problems.

Data Reduction

Data reduction and representation is the first strategy in coding qualitative data obtained through students' written works, TAPs, and retrospective recalls. Each student's data arranged against their names (pseudonyms) were uploaded into Maximum Qualitative Data Analysis (MAXQDA), a Computer Assisted Qualitative Data Analysis (CAQDA) software (Franzosi et al., 2013; Humble, 2015). Raw video data were coded directly from MAXQDA.

Analysis of Product Data

Product data are students' accuracy to written solutions to the 10 equal sharing tasks. Data were scored as correct (1), partially correct (0.5), and incorrect (0). Scoring criteria are presented in Table 3, and simple descriptive statistics were employed. Number and percentage correct, partially correct, and incorrect were also calculated.

Table 3. Scoring Criteria

Correct	Partially correct	Incorrect
Correct answer, achieved by: 1) correctly identified items being shared 2) correctly identified the number of people sharing these items 3) correctly partitioned items into the correct number of people sharing these items 4) correctly combined shares for final shares 5) correctly quantified final share with fraction name or written symbols	Incorrect answer but: 1) correctly identified items being shared 2) correctly identified the number of people sharing these items 3) correctly partitioned items into the correct number of people sharing these items	Incorrect answer but: 1) correctly identified items being shared 2) correctly identified the number of people sharing these items

Analysis of Process Data

Data obtained through both verbal reports (i.e., TAP and SR) (Smith, 2014) were analyzed through qualitative techniques. These qualitative techniques are described in the next section. Individual students differ in their cognitive processes and cognitive strategies for solving problems. However, despite these individual differences and strategy variability, there could be regularities that could point to the possibility of establishing a framework or taxonomy of cognitive processes or strategies (Bernardini, 2001). Analysis of the process data through qualitative techniques could therefore point to these differences and regularities. The process data explored the cognitive processes related to the participants' performance (Lundgren-Laine & Salantera, 2010).

Coding the Qualitative Data

This study used the data coding framework proposed by DeCuir-Gunby et al. (2010). Data coding is seen as an iterative cyclical process, and it is a multistep sense-making of the videotaped TAP data. Codes (theory-driven or data-driven) are labels given to meaningful chunks of information or data. The current study employed both theory-driven and data-driven coding to code qualitative data (see Table 4 for the theory-drive codes).

Theory-Driven Coding

In theory-driven coding, labels or codes are assigned to a chunk of interview data. The codes were pre-generated from the review of the literature. Theory-driven code occurred as chunked data based on the partitioning strategies and nature of the representations and language used to solve the problem were coded with labels (Empson & Levi, 2001; Hunt & Empson, 2016). The assigned category of codes were NC, NLC NAC, EAC, and AC (See Table 4). Codes and their themes described in chapter two were collated to create a codebook.

A codebook was used to compile codes generated through theory-driven codes (DeCuir-Gunby et al., 2010). The codebook included a set of comprehensive and detailed code names or labels, definitions, and examples used as a guide in analyzing the TAP data. The codebook assisted one of the committee members in conducting accurate and consistent audit trials.

The codebook was reviewed, revised, and refined. Examples of quotes in the context of the data were generated for the codes and their respective definitions. This ensured that the assigned codes and their definitions were conceptually meaningful and captured the essential elements of the codes. Examples of quotes were selected from the interview data that best illustrate each code. The revised codebook was used to code data using constant-comparative methodology. After all problem solutions are coded, the codes were grouped, and a general strategy name was given to each group as described in other studies (Hunt et al., 2016 & 2017; Leech & Onwuegbuzie, 2007; Merriam & Tisdell, 2016). Table 4 summaries the themes and their respective codes

Table 4. Theory-Driven Codes

Level	Theme	Identify "a" and/or "b"	Model "a" and/or "b"	Partitioning pre-planned for a "a" and "b"	Partitioning operation	Generate fair shares for "b"	Combine fair shares to make final share for "b"	Quantify share with words or symbols for "b"
0	NC	Yes	Yes	No	-Deal -Skip count -Add -Remove	No	N/A	No
1	NLC	Yes	Yes	No	-IDK -Guess -Arbitrary parts -Different operation Different context	No	N/A	Sometimes
2	NAC	Yes	Yes	No	-Deal -Skip count -Halving -Repeated halving -Trial and error	Sometimes	Sometimes	Sometimes with familiar fraction
3	EAC	Yes	Yes	Yes	-Single additive -Group additive	Yes	Sometimes	Yes
4	AC	Yes	No	Yes	$a \div b$ $\frac{a}{b}$	Yes	N/A	Yes

Note. IDK = I do not know; B/A = blank answer; $a \div b$ = long division; $\frac{a}{b}$ = multiplicativity; N/A = not applicable; "a" = number of items; "b" = number of sharers.

Data-Driven Coding

Open coding was employed and involves the process of breaking data apart and delineating concepts to stand for blocks of raw data of the raw video data either line by line, sentence by sentence or paragraph by paragraph. Open coding allows the researcher to explore ideas and meaning behind raw data. This is followed by axial coding of the codes. Axial coding involves analyzing the codes and identifying connections between codes to generate the themes. The generated themes were compared to the themes identified through theory-driven coding. Themes that were missing from the theory-driven codes were given a label and a definition and examples provided. Each student data was coded immediately after they were received. One of my dissertation committee members audited the codes and themes.

Rigor and Trustworthiness: Credibility

As this study is qualitative in nature, it is important to discuss the trustworthiness of the method and findings. Rigor in qualitative research involves "engaging in efforts that increase our confidence that our findings represent the meaning presented by our participants" (Lietz et al., 2006, p.443). Researchers support the use of rigor in qualitative research and strategies to manage reactivity, biases and legitimize the findings of qualitative study (Lietz). Trustworthiness is one of many approaches for conceptualizing the idea of rigor in qualitative research.

Various criteria are used to enhance a qualitative study's trustworthiness and guarantee the rigor of qualitative research findings (Anney, 2014). Quantitative research uses objectivity, reliability, and validity as trustworthiness criteria for establishing rigors. On the other hand, qualitative research uses credibility (as a measure of internal validity), dependability (as a measure of reliability), transferability (as a measure of external validity or generalizability), and confirmability (as a measure of objectivity) as trustworthiness criteria (Anney, 2014; Lew &

Nelson, 2016; Lietz et al., 2006; Shenton, 2004) during the data collection, analysis, reporting, and application (Li, 2004).

Credibility is the confidence one has in the truthfulness of the research findings. A reader feels that a study is credible when they think the findings reflect participants' views or information and when the researcher's analysis fits with what is expected by the reader (Anney, 2014; Graneheim & Lundman, 2004). Some strategies used to establish credibility include (a) examination of prior research findings, (b) use of well-established research methods, (c) voluntary participation and anonymity of participants, (d) prolonged engagement in the field, and (e) triangulation (Anney, 2014; Lew & Nelson, 2016). Described below are some of the strategies employed to establish credibility in the current study.

Review of Prior Research Findings

It is imperative to relate the current research findings to the existing body of knowledge in the field (Shenton, 2004). A literature review on DHH cognition and mathematic was conducted and found that DHH students have cognitive abilities like hearing students. However, DHH students may be delayed in cognitive development, and the delays may explain the persistent gaps in the mathematical achievement of DHH students when compared to their hearing peers. Findings from prior studies also explained gaps in the research literature that informed the need for the current study.

Adoption of Well-Established Research Methods

Specific procedures used for data gathering (including the questioning strategies) and analysis should be derived from an established and comparable research study if practical (Shenton, 2004). This study used multiple data collection methods, including a demographic survey, elicitation tasks, signed responses during TAP, comments made during the simulated recall, written solutions to the equal sharing tasks, and mnemonics and representations by the participants as suggested by APOS theoretical framework (Arnon et al. (2014). In addition, the study used verbal reports similar to other research studies (e.g., Reed et al., 2015). Data analysis employed qualitative data coding and a constant-comparative method.

Voluntary Participation and Anonymity

Participation in the study was voluntary and participants were guaranteed anonymity. This allowed the participants to provide honest and complete information. In other words, it may have prevented participants from providing false or misleading, incomplete data (Li, 2004; Shenton, 2004). Participants consisted of those students who were willing and ready to provide data freely and honestly.

In addition, the researcher took a number of steps to encourage the participants to participate and share their internal thoughts. Each participant received a gift certificate valued at $25.00. The researcher let the participants know that they could withdraw from the study at any time, there was 87no right or wrong answers to the questions asked, and established rapport and made participants comfortable. The researcher explained the study's purpose and method, what was expected from participants in the study, what happened to the data collected, and how their anonymity was protected during and after the study is completed (Li, 2004). One strategy used to protect the anonymity and confidentiality of participants' identifiable information was the use of pseudonyms (Shenton, 2004).

Prolonged Engagement in the Field

My prolonged engagement in the field of mathematics and teaching it to students who are DHH for more than thirteen years now is an asset to the study. This experience has allowed me to better understand DHH students' issues with mathematics (especially fractions) and equipped me with the knowledge and skills of a mathematics subject matter expert to understand and analyze the cognitive strategies the DHH students used to solve the equal sharing story problems.

Triangulation

Triangulation is used to maximize the validity of qualitative research findings (Golafshani, 2003). In triangulation, researchers use different and multiple methods, investigators, data sources, and theories to strengthen the research findings (Anney, 2014; Guion et al., 2011; Leech & Onwuegbuzie, 2007; Sun, 2011). The use of triangulation is based on two assumptions: (1) biases inherent in the use of one approach were cancelled out using multiple approaches, and (2) the findings were valid (i.e., true and certain) as they converge at the same conclusion (Guion et al., 2011; Sun, 2011). The use of multiple methods compensates for individual limitations in one method and capitalizes on their individual benefits (Stenton, 2004).

Data Triangulation

Five types of triangulation can be distinguished. The first is data triangulation. This study used multiple data sources such as the answers to the fraction problems, the responses participants give to a demographic questionnaire, and the comments participants made while solving the fraction problems.

Investigator Triangulation

The second way to enhance validity is investigator triangulation. Researchers use multiple researchers or investigators with divergent perspectives to gather, analyze, and interpret a study’s findings in investigator triangulation. When evaluators from different disciplines or viewpoints interpret the findings similarly, the validity and integrity of the research findings are enhanced. My current advisor and committee chair: Dr. Shaqwana Freeman-Green, reviewed the study and provided feedback to strengthen this study. One of the other committee members reviewed about 30% of the coded data.

Theoretical Triangulation

In theoretical triangulation, researchers use more than one theoretical scheme in interpreting the data or the researchers may use multiple professional perspectives to interpret a single set of data. Cognitive constructivism and APOS theory of mathematical understanding informed this study, and I used genetic decomposition.

Methodological Triangulation

In methodological triangulation, researchers use different research methods for gathering research data. In order words, methodological triangulation involves the use of multiple qualitative or quantitative methods to conduct a study. If the conclusions from each of the methods are the same, then validity is established. Data for this study were collected primarily through qualitative data but descriptive statistics were used in terms of the number of problems successfully solved. Data for the study were collected through questionnaires, elicitation tasks, TAP, and anecdotal notes.

Rigor and Trustworthiness: Transferability

In qualitative research, the researchers should refrain from generalizing the research findings to the whole population. The problem or phenomenon being investigated is context-bounded and specific to the context being studied (Li, 2004). However, the findings may be transferable to a similar context. It means, given similar contexts, similar participants, but perhaps different researchers who follow the same research processes, the research findings could be the same (Anney, 2014, Bitsch, 2005).

Thick Description

In transferability, the research design, study's context, data collection method, and analysis procedures are explained explicitly (i.e., thick description) to allow replication of the study by other researchers (Anney, 2015; Lew & Nelson, 2016; Li, 2004). Shenton (2004) requested thick descriptions in the following areas for transferability purposes: numbers and locations of schools that participated in the study, inclusion and exclusion criteria, number of participants, method of data collection, number and length of the data collection sessions and duration of data collection. I used thick descriptions by including lots of quotes and comments from the participants as well.

CHAPTER IV: FINDINGS

This study seeks to explore, understand, and compare DHH college students' cognitive strategies to solve equal sharing story problems presented in two different conditions: interpreted video recorded problems and co-constructed problems. Findings of the study revealed nine broad themes: (1) No-Link to Context, (2) Non-Anticipatory Coordination, (3) Emergent-Anticipatory Coordination, (4) Anticipatory Coordination, (5) Conversions, (6) Executive Functions, (7) Varied Supports, (8) Test Design/Technology Problems, and (9) Comparisons between Interpreted and Co-constructed Tasks.

In the sections that follow, these nine major findings were examined in more detail with the three research questions that guided the study (Merten, 2015), the equal sharing data analysis framework proposed by Empson and Levi (2015), and a framework of fraction understanding proposed Nicolaou and Pitta-Pantazi (2016).

Research Question One: Cognitive Strategies of Interpreted Tasks

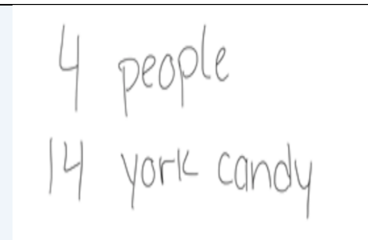
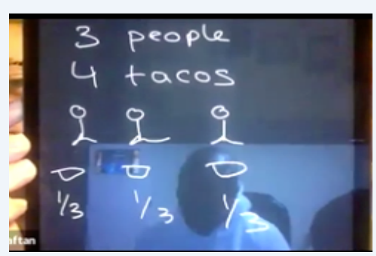
What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems presented to them through an interpreter in ASL in a frozen video format?

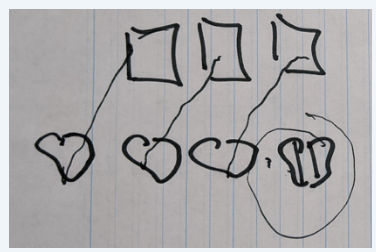
Four major cognitive strategies were identified during the solutions to the six equal sharing story problems presented to the students through an interpreter in ASL in a frozen video format. These four broad cognitive strategies include: No-Link to Context (NLC), Non-Anticipatory Coordination (NAC), Emergent-Anticipatory Coordination (EAC), and Anticipatory Coordination (AC). Major themes, mathematical strategies, definitions, and examples are presented in Table 5.

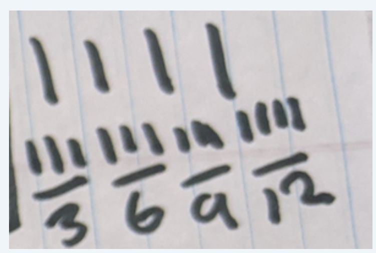
Table 5. Themes, Sub-Themes, Descriptions, and Examples of Cognitive Strategies

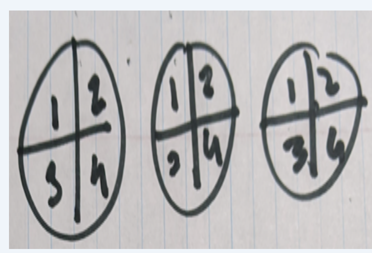
Theme	Sub-theme	Description	Example
No-Coordination	Add Remove	Students who used this strategy introduced new items or removed from the given items to make the shares "equal."	Student: No student used this strategy
No-Link to Context			
	Inappropriate Value or Operation or Strategy.	Students who used this strategy may not link their partitioning to the information given in the problem. They may not link the numerical value contains in the question to the partitioning. Students may use the wrong values and/or operations in the partitioning activities.	Student: (5 friends share 2 submarine sandwiches problem). Jeff represented the two sandwiches as given in the problem but partitions the two sandwiches into arbitrary numbers of unequal parts. The first whole sandwich was partitioned into half. The second whole sandwich was partition into thirds. Jeff distributes the shares and says each person got "3 and $\frac{1}{2}$." Student: (4 friends share 3 preferred food items problem). John distributed the 3 food items among three friends and not the four friends as presented in the problem. John gave one preferred food items to each person. The fourth person was left without a share. John commented that the

			problem is a "tricky question" and stopped solving the problem.
	Unsolvable	The students may not solve the problem.	Student: (8 people share 3 water bottles problem). Robert started solving the problem and found it difficult to proceed and responded that, "I do not know."
Non-Anticipatory Coordination			
	Quantification	Students mention or recognize correctly the number of items being shared, the number of people sharing these items, and the amount each person gets.	Student: (3 friends share 4 chocolate bars problem). John retold the quantities as "Four chocolate bars shared by three friends." Student: (5 friends share 2 submarine sandwiches problem). Lucia retold the story problem as "The question asked… says, I have two long sandwiches and five friends. How many pieces of sandwiches will be given to each of the five friends?" Student: (4 children share 14 clay sticks problem). Joseph retold the story problem as "total of fourteen clays, four children." Student: (4 children share 14 clay sticks problem). Lucia produced the representation to signify the two quantities in the problem.

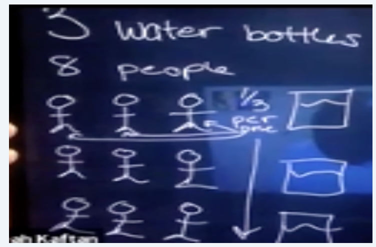
			
	Direct Modeling	Students write the partition steps by representing the variables with drawings, images, tally marks, manipulatives such as fingers.	Student: (5 friends share 2 submarine sandwiches problem). Jeff pictured two long sandwiches on paper. Then Jeff partitioned the first sandwich into equal half and the second into equal thirds. Student: (3 friends share 4 large chocolate bars problem). Jessica commented that, "I have to draw (showed the drawing). Four chocolates. One, one, one to each person. The last one is partitioned into three equal pieces." Student: (3 friends share 4 large chocolate bars problem). Lucia produced the written representation to represent the direct modeling strategy. 
	Dealing-Partitioning	Students distribute the items by giving each person one item at a time	Student: (3 friends share 4 large chocolate bars problem). John said, while dealing "one, two,

	until all the items have been exhausted or with leftover(s).	three, four. One chocolate bar given to each person. There will be one chocolate bar left.” Student: (4 children share 14 clay sticks problem). Robert said, “all the four people gets three pieces of clay each. 3, 3, 3, 3. Two clay sticks that were left are split into halves.” Student: (3 siblings share 4 preferred food items co-constructed problem). Rebekah produced the image to represent the dealing strategy. 
Skipping/Repeated Addition	Students distribute the items by giving each person two or more items at a time until all the items are exhausted or with leftover(s).	Student: (4 children share 14 clay sticks problem). Robert gave 4 clay sticks to each then raised the four fingers on the left hand to represent numbers of sharers and 4 fingers on the right hand to represent how much each person gets, then Robert moved the 4 right hand fingers across the left four fingers skip counting as “4, 8, 12, 16”. No, Robert said the answer is incorrect.

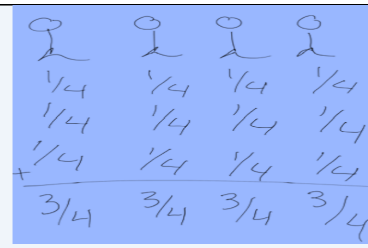
		Student: (4 children share 14 clay sticks problem). Robert gave "3 clay sticks to each. 3, (raising four fingers on the left hand to represent the number of people sharing the items and 3 fingers on the right hand to represent the number of items each person receives, then Robert moved the 3 right hand side fingers across the left four fingers skip counting as) 3, 6, 9, 12". Robert said all together is "12." Students: (4 children share 14 clay sticks problem). Rebekah produced the image to represent skip counting strategy. 
Halving and Repeated Halving	Halve or repeatedly halve the remainder(s) into halves.	Student: (4 friends share 3 pizzas problem). Connor said, "this divides into two (using the right index finger to point to the left thumb finger-representing the first item), this divides into two (points the same right index finger on the left index finger-representing the second item), this divides into four

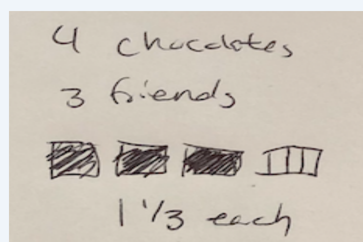
		(pointing the same right index finger on the left middle finger-representing the third item)." Student: (4 friends/teachers share 3 preferred drinks co-constructed problem). Robert said, "I know that each of the two cups of tea can be divided into halves, becoming four. The third cup of tea (thinking) can be divided into fourths…" Student: (4 friends share 3 pizzas problem). Rebekah produced the representation to signify halving and repeated halving strategy. 
Errors and Trials	Students experiment with different partitioning methods through trials and errors while solving the problems.	Student: (5 friends share 2 submarine sandwiches problem). Connor said, "The first one is divided into half. The second one is divided into half. Now there are four parts for four people. Now four. Four. That's not enough. Still not enough. I need one more. What do I do? I can take off a small piece from each of the four to make another one for the fifth person.

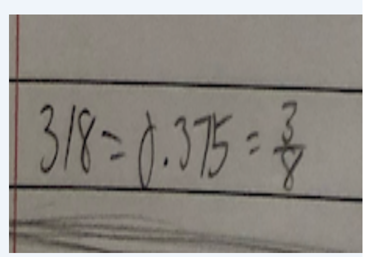
	Now we have five pieces. I have to make sure all the five pieces are equal. Five. I am finished. Now everyone will have a piece. (Repeating and clarifying) Because two are not enough for the five people. The first one is shared. The second one is also shared. Now there are four pieces. That's still not enough. There's one more. What do I do? I will take a small piece from each of the four to make the fifth. I have to make sure that the small pieces taken are not much. I have to make sure all the five pieces are equal. Now we have five to distribute to each person."
	Student: (8 people share 3 water bottles problem). Robert said, "3 bottles of water shared with 8 people. I have to partition each bottle of water into halves. That's not enough to go round. I have to partition each bottle water into thirds. One, two three, four, five, six, seven, eight, and nine. There's too much. Then I partition each bottle of water into fourths and I counted sixteen. I do not know. I am not sure how to share it."

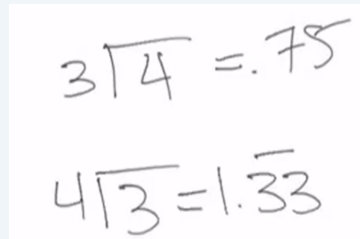
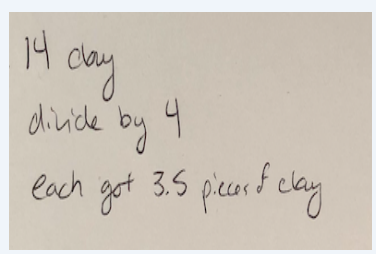
			Student: (8 people share 3 water bottles problem). Lucia produced the representations to represent trial and error strategy.  
Emergent-Anticipatory Coordination			
	Single/Group Additive	Students take a whole item and partition it fairly into equal number of people sharing the item. The process continues until all items have been partitioned.	Student: (8 people share 3 water bottles problem). Rebekah demonstrated with image and said "The first bottle was partitioned into equal fourths. The second bottle was also partitioned into equal fourths. Now each of the eight people will get one-fourth of the two bottles of water. Now we have one bottle water left. How much water will be given to each person from the third bottle of water partitioned into eighths?

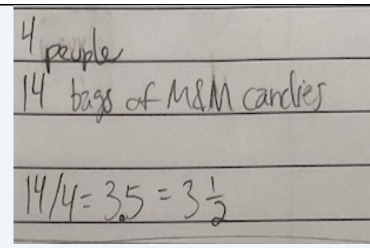
	Rebekah said each person will get "$\frac{1}{8}$."
	Student: (4 friends share 3 pizzas problem). Beatrice explained by drawing a representation and said, "I set up three pizzas. I divided each pizza into four equal slices (demonstrates the cutting with her hand). I labeled each slice for each person as one, two three and four (numbers each slice on the drawing). I ended up giving each person 3 out of 4."
	Student: (4 friends share 3 pizzas problem). Rebekah said, "There are two bottles shared with four people. It means half a bottle of water for each person. The two bottles are gone. Now we have one bottle of water left. Inside one bottle of water is one-fourths. So, add one-fourth to the initial $\frac{1}{2}$."
	Student: (4 friends share 3 pizzas problem). Lucia produced the representation to represent the single additive strategy.

		
Adding Shares	Students after partitioning the items into fair shares, giving equal shares to all sharers, combine shares for each sharer to make a final share for each sharer. They combine all shares to form a single final share for each student often times for unit and non-unit fractions.	Student: (4 friends share 3 pizzas problem). Lucia said, "I have three whole pizzas. They are big and equal. One over four. So, one slice per person. One over four from each whole pizzas for each person." Student: (5 friends share 2 submarine sandwiches problem). Jessica said, "$\frac{2}{5}$." (When asked to explain how she got $\frac{2}{5}$) "I can visualize it like this. One whole long sandwich is partitioned into equal fifths. The second sandwich is also partitioned into five equal parts. Every friend gets $\frac{2}{5}$ sandwich." Student: (4 children share 14 clay sticks problem). Lucia first gave 3 to each person. Next Lucia partitioned the two leftovers into halves. Lucia gave each person $\frac{1}{2}$. added 3 and $\frac{1}{2}$ to combine final fraction share for each person and said the answer is "3 and $\frac{1}{2}$."

			Student: (4 children share 14 clay sticks problem). Lucia said, “3, 3, 3, 3 to each person. Left 2. Each divided into halves. Each person gets another $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$. Each person’s combined share is 3 and $\frac{1}{2}$." Student: (2 people share 5 soft tacos problem). Janet said, “2 and 2 equal four. Divide one into two equal halves. Combine 2 and $\frac{1}{2}$ equal $2\frac{1}{2}$ for each person.” Student: (3 friends share 4 large chocolate bars problem). Jessica produced the representation combining each shares to make a final share for each friend. 
	Quantifications	Student’s name or label each person shares or final share with correct fraction words or notations or symbols often.	Student: (4 friends share 3 pizzas problem). John said, “one whole pizza has eight slices of pizzas. Three whole pizzas have 24 slices of pizzas. Divide 24 slices of pizzas by four sharers. The answer is 6 slices for each person.”

			Student: (4 children share 14 clay sticks problem). Lucia said, “each person gets 3 and $\frac{1}{2}$.” Student: (4 friends share 3 pizzas problem). John, “each person gets 6 pieces of pizzas.”
Anticipatory Coordination			
	Basic or long division	Students use the basic or long division algorithm to solve the problems	Student: (5 friends share 2 submarine sandwiches problem). John said, “I believed each sandwich is 12 inches long. Two sandwiches equals to 24 inches. 24 inches divided by 5 people. Each person gets 4.8 inches of sandwich.” Student: (4 children share 14 clay sticks problem). Jessica said, “14 divided by four students. The answer is not an even number. Each gets 3 and $\frac{1}{2}$.” Student: (8 people share 3 water bottles problem). Janet produced the image that represents the use of the basic division strategy. 

		Student: (3 friends share 4 large chocolate bars problem). Julie produced the image that represent the use of the basic division strategy. 
Multiplicative $\frac{a}{b}$	Students use mental or written algorithm procedures to solve the problems	Student: (3 friends share 4 large chocolate bars problem). Socrates said, “4 divided by 3 is 1.3333.” Student: (4 children share 14 clay sticks problem). Beatrice said, “four children. Each has 3 and half clay.” Student: (4 children share 14 clay sticks problem). Jessica produced the image to represent the use of multiplicative $\frac{a}{b}$ strategy.  Student: (4 children share 14 clay sticks problem). Janet produced the image to represent the use of multiplicative $\frac{a}{b}$ strategy.



No-Coordination (NC)

This is the most basic strategy in which students consider a single item as indivisible. This strategy is distinct from the NLC in which students engaged in the partitioning of a single item. None of the students used the strategy in their partitioning activities.

No Link to Context (NLC)

Students who use NLC tend to model the solution to the problems through representations, drawings, and the use of manipulatives. Under NLC, two specific strategies (i.e., inappropriate values/strategy use and unsolvable) were identified. They included the use of inappropriate values or strategies and unsolvable strategies. Each specific strategy is described below with examples drawn from the students' solutions to the equal sharing story problems.

Inappropriate Values/Strategy Use

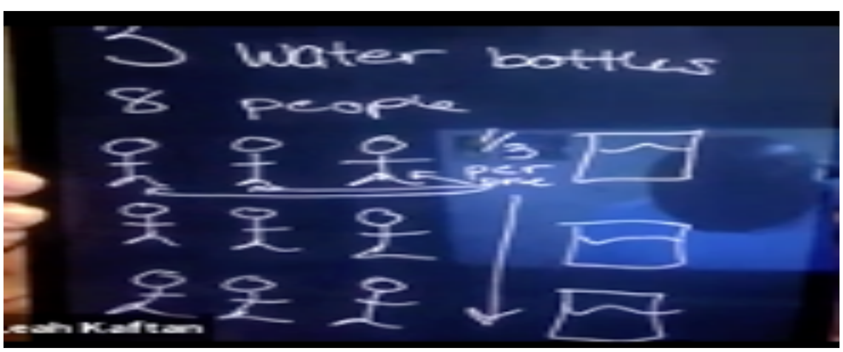
Students who used this strategy used inappropriate values, ways, or operations to solve or represent the problem. They may have shared the whole items unevenly, thereby creating unequal shares. They may have partitioned a whole into an arbitrary number of unequal parts and used a different operation. Here are examples of the students' work that exemplified the use of this strategy while solving the following problem:

4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?

Jeff. To solve this problem, Jeff said "all three pizzas will be partitioned and distributed. The one that is left will also be partitioned and distributed to the friends. Each person gets 3 and $\frac{1}{3}$." Jeff partitioned the first pizza into half. Next, he partitioned the second pizza into thirds. He then gave each friend a share of the partitioned sandwich and said each person gets "3 and $\frac{1}{2}$." Similarly, while solving another problem: *5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receive?* Jeff partitioned the first sandwich into thirds and second sandwich into halves. He then gives each person one piece of the sandwich.

Lucia. Lucia used inappropriate values to represent the number of sharers in the problem involving "8 people" sharing "3 water bottles." In Figure 6, Lucia represents eight people with nine people and shared one bottle of water with three people and concluded that each person got "$\frac{1}{3}$."

Figure 6. Lucia's Inappropriate Solution Strategy



Note. Lucia draws images of three sharers and shared a water bottle with them. She quantified each person share as "$\frac{1}{3}$ per person." She did the same for the next three sharers and the last three sharers. She ends up sharing three water bottles with nine people rather than eight people called for in the actual story problem.

Unsolvable

Students who used this specific strategy may not solve the problem or may leave the solution blank. This strategy is exemplified with the most difficult problem (i.e., *there are 3 bottles of water that 8 people want to share equally. How much of the water does each person receive in his or her cup?).* For example, when asked to solve this problem, Robert used the unsolvable strategy.

Robert. Robert attempted it and then stopped and said, "I do not know."

John. John also stopped solving the same problem and commented that this is " a tricky problem."

Non-Anticipatory Coordination (NAC)

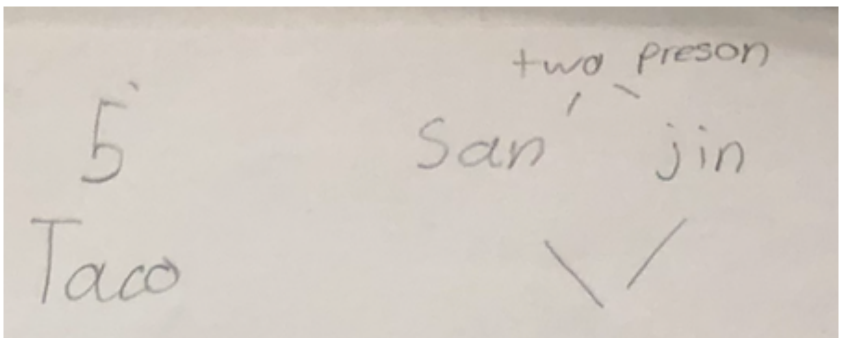
Non-Anticipatory Coordination is the most basic and emergent coordination strategy. There are six specific strategies identified under this theme. They include identifying the quantities, direct modeling, skip counting, trial and error, and halving and repeated halving. Quantities and quantifications, direct modeling, dealing and partitioning, skip counting-repeated addition, trials and errors, halving and repeated halving strategies are described below with examples gleaned from students' solutions to the equal sharing story problems.

Quantities and Quantifications

Each equal sharing task across problem types has three quantities. These are the number of sharers or people sharing the items, the number of items being shared and the quantity each person gets (which is the unknown; the answer students seek for each problem). Most of the students in their retelling of the story problems explicitly referenced the number of sharers and the number of items being shared prior to or during the solution processes. The retellings are in most cases related to the interpreted tasks with all 13 students using quantities/quantification strategy during the solution to the six problems. Students often use verbal, symbolic, drawing, or paper-pencil written representations to represent these quantities.

John. John utilized the quantities strategy to solve the following problem: *Susan and Juan shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Susan and Juan each eat if they finished all the tacos?* While retelling the story problem given above, John stated the story "mentioned 5 tacos and two people." His written representation of the story problem is depicted in Figure 7.

Figure 7. John's Retelling Representation in Written Format



Note. John uses paper and pencil to write down his summary after viewing the Interpreted video of the story problem. John writes the names of the sharers in the problem incorrectly.

Jessica. Jessica's approach provided additional illustration of this strategy while solving the following problem: *There are 3 bottles of water that 8 people want to share equally. How much of the water does each person receive in his or her cup?* When asked to retell the story problem, Jessica stated that, "three water bottles for eight people." She then wrote the information on paper as depicted in Figure 8.

Figure 8. Jessica's Retelling Representation in Written Format

Note. Jessica's written summary of the interpreted video

During the retelling, students often omitted the other parts essential to solving the problem such as share, each person gets the same amount, and how much/many each person gets. Additional examples that exemplify this strategy are presented in Table 5.

Quantifications represent the third quantity and show how students quantify each person's share or final share. A third quantity, which was the result of the partitioning activity, was often mentioned at the end of the solution to each problem. Students using NAC sometimes quantified the final shares that each person got with words, symbols or notations. The following problem illustrates the quantifying strategies the students used:

4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?

Jeff. Jeff deals a pizza one by one to three friends. He then divides another pizza into thirds and shares with the three friends. He said each person gets "1 and $\frac{1}{3}$." He fundamentally used the same approach germane to solving improper fraction problem types to solve and quantify the two other proper fraction problem types. Jeff converts the proper fraction that appears difficult to an improper fraction that appears easier to solve, thereby interchanging the two quantities in the processes and getting an incorrect quantification.

Connor. Connor quantifies the final share differently for the same problem. He stated that the share of each person is “75%.” Connor essentially changed a pizza into 100%, gave each person 100% of a pizza, and left the fourth person without any pizza. He took slices or percentages away from each through trial and error and took 25% of the pizza from each of the first three people to make up the share of the fourth person. He added up the shares of the fourth person as “25% + 25%+ 25% = 75%” and concluded that each person gets “75%” of the pizzas. Connor used this approach to solve the rest of the proper fraction problem types and most difficult improper fraction problem types.

Direct Modeling Strategy

Students who used Direct Modeling, the most basic strategy of the three strategies (i.e., Direct Modeling, Counting, Fact-Based), employed either physical objects (e.g., counters), pictures, tally marks or fingers to directly represent each quantity in the problem. They acted on these quantities based on the action or relationships portrayed in each story problem. Thus, Direct Modeling included more than just the use of manipulative materials. Students explicitly modeled the action or relationship embedded in the problem. Students who employed this strategy did not seem to ~~plan not~~ plan the sharing activities ahead of time; instead, they possibly thought about the solution to the problem one step at a time. In the current study, four Direct Modeling strategies were identified for NAC use. They include dealing, skip counting, halving, and trial and error strategies. This strategy was used by 8 of the 13 students.

Dealing-Partitioning

Dealing involves the distribution of the whole or partitioned items to the sharers. The deal may be systematic in which each sharer receives one item at a time until all the items have been exhausted without a remainder or unexhausted with a remainder that is later partitioned and distributed to the sharers. Six out of 13 students used this strategy. Jessica's solution to the following problem exemplifies this strategy:

14 sticks of clay are shared among 4 children for a project. How many sticks does each child receive?

Jessica. Jessica solves this problem by first dealing 3 clay sticks to each child (raised four fingers without the thumb on the right hand to represent the number of children sharing the items. Then she raised her thumb, index and middle fingers on the left hand to signify the numbers of share each person receives. She then swept the three raised fingers on the left hand across the raised four fingers on the right hand moving them from right to left starting with the index finger and ending at the little finger. See figure 9 for the depiction). She signed that two clay sticks were left and went on to partition each of the 2 leftovers into halves. She then gave each child an additional $\frac{1}{2}$.

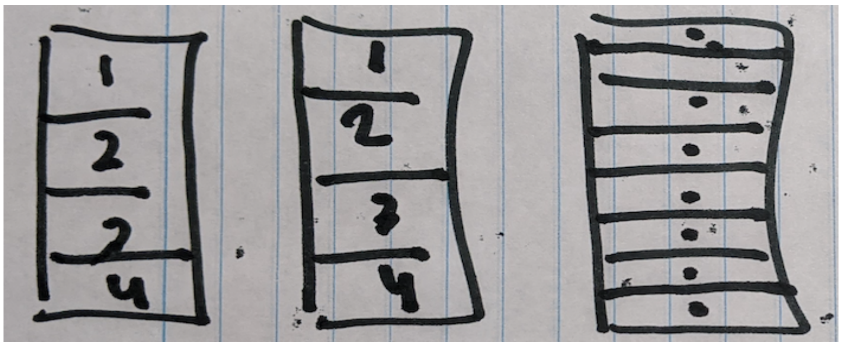
Figure 9. Pictorial depiction of Jessica's dealing strategy

Rebekah's example below also illustrated the use of the dealing strategy. For the equal sharing problem in Figure 10:

There are 3 bottles of water that 8 people want to share equally. How much of the water does each person receive in his or her cup?

Rebekah. Rebekah partitioned the first bottle of water into fourths and dealt one to the first four people. She then partitioned the second bottle of water into fourths and distributed one-fourth to each of the last four people. Finally, she partitioned the third bottle of water into eighths and distributed these by giving one-eighth to each person. Additional examples of the dealing strategy are provided in Table 5.

Figure 10. Rebekah's Dealing Strategy



Note. Rebekah partitioned the first bottle of water into fourths and dealt these to the first four sharers. She later partitions the second bottle of water into fourths and deals those to the last four sharers. The last bottle of water was partitioned into eighths and distributed to each of the eight sharers.

Skip Counting/Repeated Addition

Skip counting or repeated addition involves students counting numbers by 2s, 3s, or 4s in a skip counting sequence. Students who count 3, 6, 9, 12 implicitly recognize that 3 stands for one sharer, 6 stands for two sharers, 9 stands for three sharers and 12 stands for four sharers. Two of the 13 students used this strategy in five instances for the improper fraction problem type (i.e., *14 sticks of clay are shared among 4 children for a project. How many sticks does each child receive?*). The students stopped skip counting at 12. They did not skip count the $\frac{1}{2}$ for each person to the initial whole. The same illustrative examples have been provided in Table 5.

Trial and Error

Trial and error involves trying a method, observing if it works, and if it does not work, trying a new method. The process is repeated until success, or a solution is achieved. Dealing or partitioning may be unsystematic in which the students initially give a different number of items to each sharer. Later on, each person's amount is adjusted by addition and subtraction from the original shares to make the sharing equal. Four of the 13 students used this strategy in 22 instances. Connor demonstrates the use of this strategy while solving the following problem: *5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receives?*

Connor. "One hundred percent, one hundred percent, each of the two (raising the index and middle fingers on the left hand to signify the two sandwiches and pointing to the two fingers with the right index finger back and forth). We divide each into two (pointing to the two left fingers representing the two sandwiches). We divide this into two, 50%, 50% (pointing to the left index finger with the right index finger). Now, we divide this also into two, 50%, 50% (pointing to the left middle finger with the right index finger). Now we have four (raising the four fingers excluding the thumb on the left-hand side to signify four sandwiches). Each is 50%, 50%, 50%, 50% (pointing to each of the four-finger raised from right to left with the index finger). Now, 50%, 50%, 50%, 50% (repeating self). Now we will subtract…(pause) to make it equal. 50%, 50%, 50%, 50% (repeating self). We will take 10% from 50% (pointing to the first index finger on the left) and put here (a location to signify the fifth sharer), 10% from 50% (point to the middle finger on the left) and put here (the fifth sharer location), 10% from 50% (pointing to the ring finger on the left) and put here (the fifth sharer location), 10% from 50% (pointing to the little finger on the left) and put here (the fifth sharer location). (Pointing to the fifth sharer location) now we have 40% here. Returning to the four fingers) here we have now 40%, 40%, 40%, 40%. They are now equal. (Raised the five fingers on the left and point with the right index finger) 40%, 40%, 40%, 40%, 40% for each person."

Connor applied this strategy albeit unsuccessfully to a more difficult problem:

4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?

"First, let's say this (raising three left fingers to designate the three pizzas and pointing to the left thumb finger with his right index finger) is 100%., (moving to the left index and middle fingers with the right index finger), this is 100%, and this is 100%. Now to find the fourth, to find the fourth, we will (pointing to the thumb finger, then to index and the middle fingers, pause) 20, 20, 20 (thinking) 20, 20, 20. We will remove (pointing to the index finger without specifying the amount to be removed). I will give it a number (changed the designated three numbers on the three left fingers after prompting him on how much to be removed from the 100% on each of the three left fingers) say 60, 60, 60 (pointing to the three raised left fingers representing the three pizzas) 60%, 60%, 60%. Now to get the fourth one for the fourth person., take 20 from (pointing to the left thumb finger) and put it here (created new location to represent the fourth sharer), 20 from (pointing to the left index finger) and put it here (fourth sharer's location), and 20 from (pointing to the left middle finger) and put it here (the fourth person's location)…(changing the designated numbers to remove) let's say 60, 60, 60 (using the left three fingers to designate the three assigned numbers). (Pointing to thumb finger) remove 15, (pointing to the index finger) remove 15, (pointing to the left middle finger) remove 15. Adding it up equals to 45. If we remove 15 from the 60 (pointing to the left thumb finger), we have 45 left. If we remove 15 from 60 (pointing to the left index finger), we have 45 left. If we remove 15 from 60 (pointing to the left middle finger), we have 45 left. Now that 15, 15, 15, we add it. It will be 45. (Interviewer confirmed with Connor his explanation by raising four left fingers and designated each share as 45). Yes (Connor confirmed). Each of the four friends will get 45 slices or shares." Additional examples depicting this strategy are provided in Table 5.

Halving/Repeated Halving

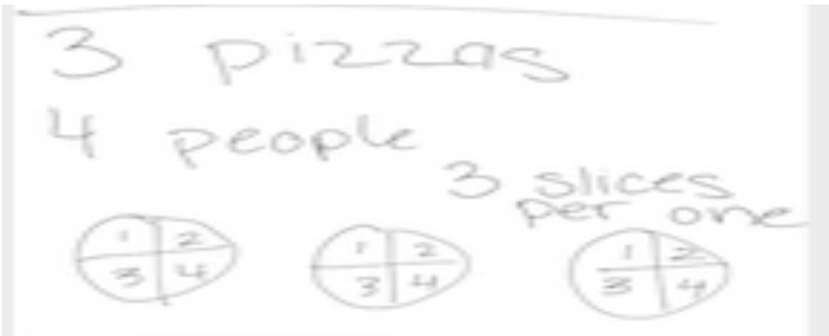
Students who used this strategy halved the item or items into an equal number of sharers. Five students out of 13 students used this strategy nine times. This strategy was observed for the two problems with four sharers. Robert's strategy serves as an illustration of the application of this strategy with the problem:

14 sticks of clay are shared among 4 children for a project. How many sticks does each child receive?

Robert. "All will get three clay sticks (raised the four left fingers to designate four children and then swept the four left fingers with the right finger sign for all and ending up with sign three). Three, three, three, three (raised the four left fingers and placed the sign for three on the index, middle, ring and little fingers moving from right to left). Each of the two remaining clay sticks will be split into halves (using the V-shape sign on both hands but facing each other in greater or less than shape). Each child will get 3 and $\frac{1}{2}$."

Lucia. Lucia's approach depicted in Figure 11 to solve the same problem elucidates this strategy further. Table 5 provides additional examples.

Figure 11. Lucia's Halving and Repeated Halving Strategy



Note. Lucia used halving and repeated halving strategy to partition three pizzas into equal number of sharers. Lucia halves the first and then halves the same pizza into fourths. She repeated the process to partition the other two pizzas and numbers as 1, 2, 3, and 4.

Emergent-Anticipatory Coordination(EAC)

Emergent-Anticipatory Coordination involves coordination between the partitions of shared items with the number of sharers at the beginning of the strategy. Students who use EAC tend to coordinate the two goals from the onset of the solution. Yet, their fraction understanding is not fully formed as they also used Direct Modeling strategies in their solution processes. Like the NAC, they often represent each person's share through Direct Modeling strategies. Now we turn the focus of our findings to the specific strategies students with Emergent-Anticipatory conception, the first instance of fraction understanding, employed. Five out of 13 students were identified to have used EAC strategies, and more for the proper equal sharing fraction problem types than the improper equal sharing fraction problem types.

Quantities/Quantification

Students in the interpreted tasks who used EAC strategies correctly retold the number of people sharing the items and the number of items being shared. They sometimes quantified the final shares each person got as "pieces, slices, or inches" and with fraction notations. For example, in the story problem:

5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receives?

John. John retells the story problem as "If they were long sandwiches, it means one foot equals to 12 inches. So, one foot is common for all famous sandwiches. One long sandwich equals 12 inches. Go ahead and split it among five friends. Split it by 5." He went on to quantify each friend's share as "the answer is…(pause). I mean 12 divided by 5 is equal to 2.4 inches. Then add 2.4 with the same 2.4. We have a 2 feet of sandwiches. Adding it up, the answer is 4.8 inches. 4.8inches as the share for each of the five friends."

What distinguishes students with the Emergent-Anticipatory Coordination understanding from students with Non-Anticipatory conceptual understanding are the types of Direct Modeling strategies used in solving the equal sharing fraction problems. This study identified the Single Additive Direct Modeling strategy as one of those strategies. The students also used the strategy of combining each person's share to make a final share.

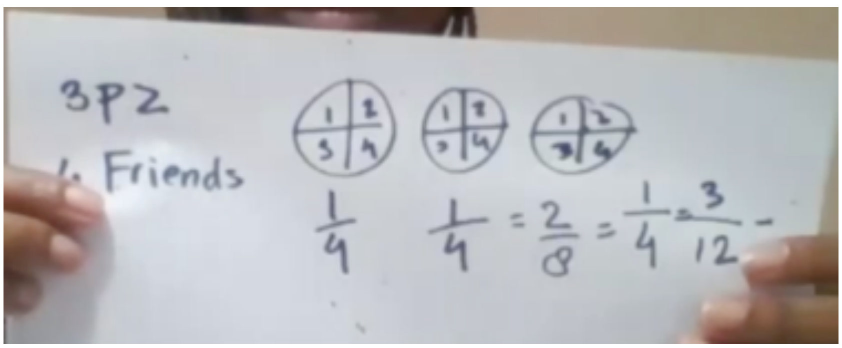
Single Additive

Four students used this strategy for the proper fraction problem types. Students who used Single Additive strategy took a whole item, one at a time, and partitioned it equally into the exact number of people sharing the item. For instance, in the problem:

4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?

Rebekah. Rebekah explained her approach to solving the problem as follow: "So one whole pizza is partitioned into fourths. 1, 1, 1, 1 (pointing to the four quadrants). So, I deal one to each person. So, one person gets one piece from a whole pizzas. One, one, one, one. This means one-fourth. One-fourth for each person. Next, the second pizzas, the same thing happens. Together there are two-eighths. Now, the third pizzas. Again, there are three-twelfth's " She showed her written representation of her solution in Figure 12.

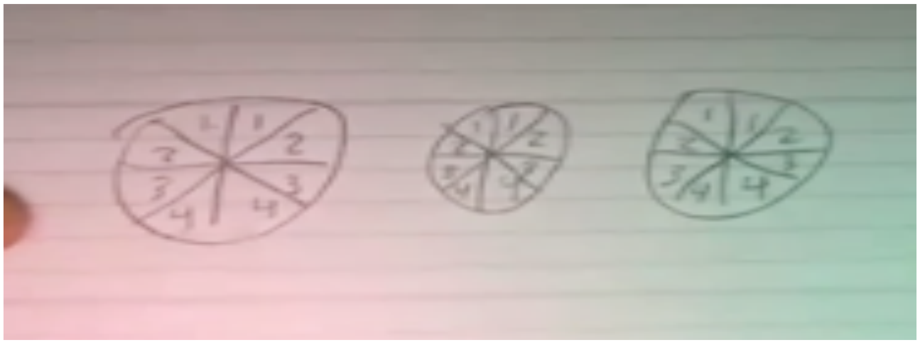
Figure 12. Rebekah's Single Additive Strategy



Note. Rebekah used the Single Additive strategy to partition three pizzas, focusing on one at a time for four friends. She partitions the first pizza into fourths and gives each friend one-fourth of the pizza. She repeated the processes to partition and the distribute the second and third pizzas.

Robert. Robert used the same strategy to solve the same problem albeit each pizza was partitioned into eights for the four friends. Below is a description of Robert's strategy. "For each one pizza, each person takes two. The total for each person is 6 pizzas. Six pizza slices for each person. (Supporting his explanations with the image in Figure 13). The number 1 in the image means the first person, 2 means the second person, and 3 means the third person's shares. Each person gets 6 slices of pizzas."

Figure 13. Robert's Single Additive Strategy



Note. Robert used the Single Additive Strategy for sharing three pizzas with four friends sharing one pizza at a time. He partitions each pizza breaking one at a time into eighths. He numbers each share for the first friend as 1, 1, 1, 1, 1, 1; second friend as 2, 2, 2, 2, 2, 2; third friend as 3, 3, 3, 3, 3, 3; and fourth friend as 4, 4, 4, 4, 4, 4 and concludes that each person gets 6 slices of pizzas.

Adding Shares

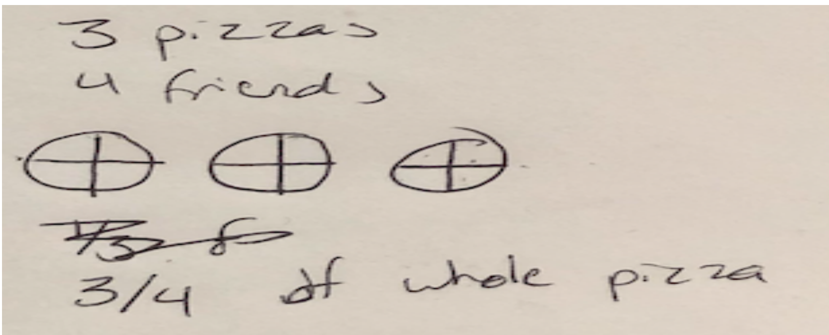
Students who use this strategy partition the items into faire shares, represent the share for each person, give equal shares to all sharers often, and combine shares to make a final share for each person. Unlike the students working at the level of NAC, the students who use this strategy can combine both unit and non-unit fractions to create a single share. For example, the problem: *3 friends share 4 large chocolate bars so that they all get the same amount. They eat all the chocolate bars. How many cookies does each friend get?*

Jeff. Jeff solved it like this: "Four chocolates, and three people. How may does each person get? When I look at it, I first give each person 1, 1, 1. One is left. I decide to partition the leftover one into 3 equal parts. I mean each person will get 1 and one-third."

Rebekah. In Figure 12, Rebekah uses this strategy to combine the shares (i.e., $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$) to make a final share (i.e., $\frac{3}{12}$) although the addition procedure and the final answer are incorrect.

Jessica. Jessica also used this strategy in the written representation in Figure 14. She gave each friend one-fourth of each partitioned pizza. She combined the final share for each person as three-fourths as shown in the image in Figure 14.

Figure 14. Jessica's Combining Shares Strategy



Note. Jessica represented the fair share from each partitioned pizza for each friend. Combines all shares to make a final share for each friend as $\frac{3}{4}$.

Anticipatory Coordination (AC)

Anticipatory Coordination conception level strategies use abstract or fact-based strategies to demonstrate conceptual understanding of equal sharing fraction story problems. Students who use AC strategy tend to coordinate the two goals, and use a more efficient fact based, mental, procedural or division strategy to solve the equal sharing story problems. They do not use Direct Modeling Strategies to represent each person's share. Specific strategies the study uncovered are examined in the following sub-sections.

Quantities/Quantifications

Like the NAC and EAC fraction understanding, students who used the AC correctly retell the number of sharers and the number of items being shared as presented in the interpreted story problems. However, students who used the AC strategy correctly quantify each person's shares with appropriate fraction words, notations or symbols. For example, in the problem:

Susan and Juan shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Susan and Juan each eat if they finished all the tacos?

Beatrice. Beatrice retells the story problem as follows: "I think so, there are five tacos. There are two people. One here. One there. They want to share. Per person, each receives the same amount of food." She solved and quantified each person's share as "each person eats 2 and $\frac{1}{2}$."

This is an abstract mental strategy. Students who use this strategy coordinate the two goals of making equal-sized parts and exhausting the whole. They understand that "a" item shared by "b" people is equivalent to $\frac{a}{b}$. Students who use this strategy determine the final share by mentally using the relationships between the number of sharers, number of items being shared and the amount of items per sharer.

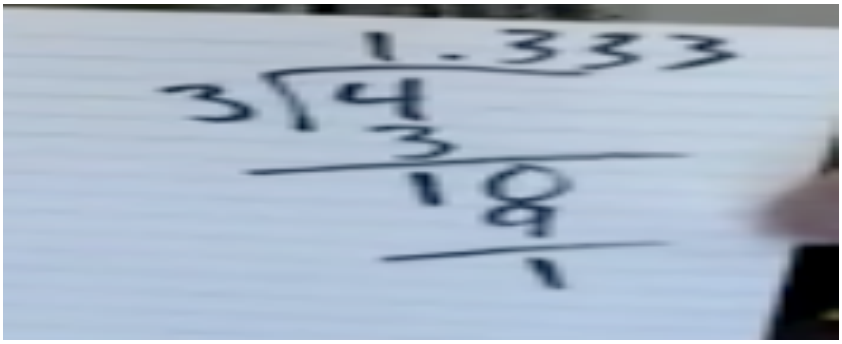
Two examples of Fact-Based strategies were also identified in the study. They included Basic or Long Division and the Multiplicative $\frac{a}{b}$ strategies. These strategies with examples of their use in the study are considered below.

Basic or Long Division Strategy

Some students used a basic or long division algorithm to share the item(s) equally among the exact number of people sharing it. They did not represent each person's share as students who use Direct Modeling do. As an illustration, in the problem: *4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?*

Andriana. Andriana wrote the solution to the problem on paper. Later she said that "1 and $\frac{1}{3}$ for each person" and produced the image in Figure 15.

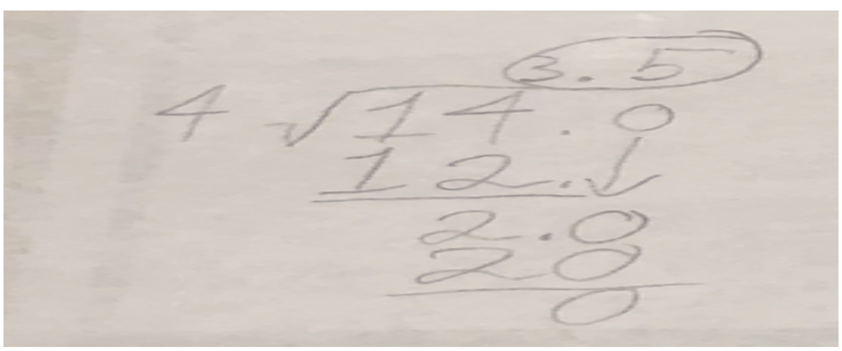
Figure 15. Adriana's Basic or Long Division Strategy



Note. Adriana uses a long division procedure to share three pizzas with four friends. She ends up with a decimal result.

John. John also uses the long division strategy to solve the problem: *14 sticks of clay are shared among 4 children for a project. How much sticks do each child receive?* The image in Figure 16 represents the written representation of John's strategy. John divided 14 by 4 resulting in 3.5 as the solution to the problem.

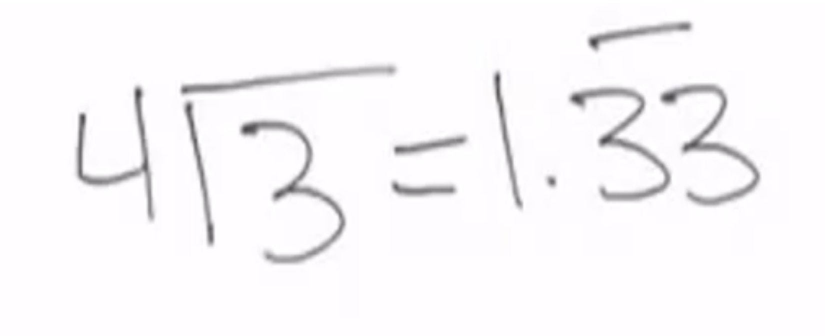
Figure 16. John's Basic o Long Division Strategy



Note. John uses the long division procedures to partition 14 clay sticks with four friends. He concludes each person gets 3.5 clay sticks.

Julie. Julie solves the same problem using a similar strategy with the support of a calculator (see Figure 17).

Figure 17. Julie's Basic o Long Division Strategy



Note. Julie's division strategy for solving "4 large chocolates shared by 3 friends." Even though the setup is nontraditional, she set it up to read as "4 divided by 3" or $\frac{4}{3}$ to obtain 1.33…

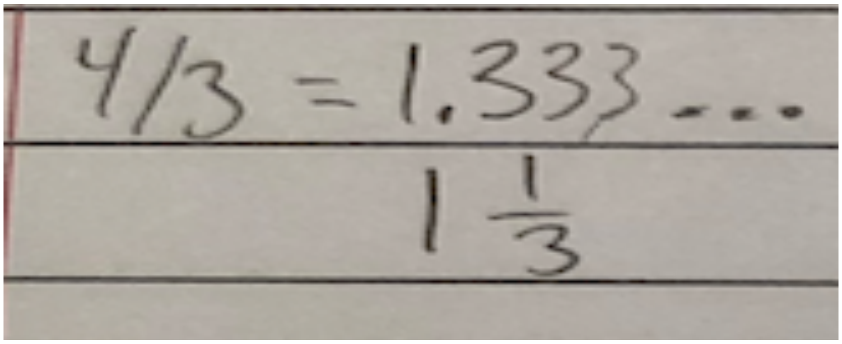
Multiplicative $\boldsymbol{\frac{a}{b}}$

Students who used the multiplicative $\frac{a}{b}$ strategy used the result of a division to solve the problem mentally or manually. For example, in the problem:

3 friends shared 4 large chocolate bars so that they all get the same amount. They eat all the chocolate bars. How many cookies does each friend get?

Janet. Janet solved the problem as "for that mathematics problem, four divided by three is equal to 1 and $\frac{1}{3}$ for each person." She provided the written representation of her strategy in Figure 18.

Figure 18. Janet's Multiplicative Strategy



Note. Janet mentally solves the problem using the multiplicative $\frac{a}{b}$ strategy. She obtains a decimal answer that was later converted to a mixed number fraction.

Problem Types and Cognitive Strategies Used

All students used the most advanced cognitive strategy of Anticipatory Coordination (AC) to solve the easiest of the six equal sharing problems (i.e., 2 people share 5 tacos) as shown in Table 6. However, the number of students who used the most advanced strategy as the problem tasks became difficult (i.e., increased in the number of sharers and items being shared); moved from problem types solvable by repeated halving (where there was an even number of sharers) to problem types not solvable by repeated halving (where there was an odd number of sharers); and moved from improper to proper fraction types dropped. Table 6 presents a summary of the four broad cognitive strategies each student used to solve the six equal sharing story problems framed across two major fraction problem types (i.e., proper and improper fraction problem types).

Table 6. Interpreted Fraction Problem Types and Student's Coded Strategies

Student	Coded Strategies					
	Improper fraction problem types			Proper fraction problem types		
	2 share 5	4 share 14	3 share 4	4 share 3	5 share 2	8 share 3
Andriana	AC	AC	AC	AC	AC	AC
Beatrice	AC	AC	AC	EAC	AC	AC
Connor	AC	AC	NAC	NAC	NAC	NLC
Janet	AC	AC	AC	EAC	AC	AC
Jeff	AC	AC	NAC	NLC	NLC	NLC
Jessica	AC	AC	EAC	EAC	EAC	EAC
John	AC	AC	EAC	EAC	EAC	NLC
Joseph	AC	AC	AC	AC	AC	AC
Julie	AC	AC	AC	AC	AC	AC
Lucia	AC	EAC	EAC	EAC	EAC	NLC
Rebekah	AC	EAC	EAC	EAC	EAC	EAC
Robert	AC	EAC	EAC	EAC	EAC	NAC
Socrates	AC	AC	AC	AC	AC	AC

Note. NLC = No-Link to Context; NAC = Non- Anticipatory Coordination; EAC = Emergent- Anticipatory Coordination; AC = Anticipatory Coordination.

Overall, all 13 students demonstrated behaviors consistent with AC; seven students demonstrated behaviors consistent with EAC; three demonstrated behaviors consistent with NAC, and four demonstrated behaviors consistent with NLC. This is calculated by counting the number of students against the type of cognitive strategies they used across all problem types (see Table 6). Overall and at a minimum across all fraction problem types, there are 45 instances of the use of AC; 22 instances of the use of EAC, five instances of the use of NAC, and six instances of the use of NLC. By fraction problem types, there are 29 instances of the use of AC, eight instances of the use of EAC, two instances of the use of NAC, and no instance of the NLC within the improper fraction problem types. However, the number of instances AC was used dropped to 16 instances for the proper fraction problem types. At the same time, EAC, NAC, and NLC increased to fourteen, seven, and six instances of use, respectively (see Table 7 for additional breakdowns).

Table 7. Interpreted Problems: Cognitive Strategies Across Problem Types

	Number of students using each strategy					
Cognitive	Improper fraction problem types			Proper fraction problem types		
strategy	2 share 5	4 share 14	3 share 4	4 share 3	5 share 2	8 share 3
NLC	0	0	0	1	1	4
NAC	0	0	2	1	1	1
EAC	0	3	5	7	5	2
AC	13	10	6	4	6	6

Note. NLC = No-Link to Context; NAC = Non- Anticipatory Coordination; EAC = Emergent- Anticipatory Coordination; AC = Anticipatory Coordination.

Research Question Two: Cognitive Strategies for Co-Constructed Tasks

What understanding of fractions (i.e., employed thinking strategies and representations) do college DHH students demonstrate as they work with equal sharing story problems when asked to co-construct the problems in dialogue with a Deaf researcher in ASL?

In the co-constructed four equal sharing story problems, three broad cognitive strategies were identified. Namely: NLC, EAC and AC. In a co-constructed equal sharing story problem, the researcher and students jointly constructed the equal sharing story problems together. The researcher requested the two variables (i.e., items and sharers) from the students (based on their preferences) and used the information to frame the equal sharing story problems for the students to solve. The number of sharers and items to be shared remained the same for all students and were also similar to the interpreted tasks. The only differences were the names of sharers and items to be shared as they were provided by the students as presented in Table 7. In addition to these three broad cognitive strategies identified in the study (i.e., NLC, EAC and AC), this study also reveals several specific strategies within each of the three broad cognitive strategies. These specific strategies are described in the sections that follow with examples gleaned from the study.

No-Link to Context (NLC)

In No-Link to Context, two specific strategies were identified that included) inappropriate value/strategy use and b) not solving the problem. These two specific strategies are discussed with identified examples from the co-constructed equal sharing story problems.

Inappropriate Strategy

There is one instance of inappropriate use of strategy by one student for a co-constructed task involving asking the students to share “3 drinks or foods” with “4 friends or teachers” so that each person gets equal share. The co-constructed equal sharing story problem goes thus:

4 friends (Taj, RM, BJ, and RK) share 3 Jelly candy fruits. If they each want the same amount and share all of the Jelly candy fruits, how much jelly fruit does each friend get?

Jeff. Jeff gives each friend a Jelly fruit and said one more Jelly left as a remainder still to be partitioned for the three friends. He partitions what he assumed to be the fourth Jelly fruit and concluded that each person gets “3 and $\frac{1}{3}$.”

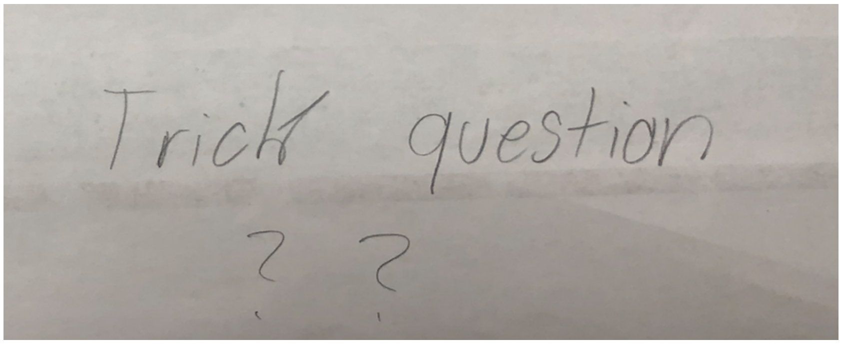
Unsolvable

One student demonstrates the use of this strategy for the co-constructed equal sharing story problem. John names his four favorite teachers. He also mentioned a preference for drinking bottled water. The following co-constructed problem was posed to him:

Your 4 teachers share 3 bottled waters. If they each want the same amount and share all of the bottled waters, how much bottled water does each teacher get?

John. John gave one drink to each teacher and left the fourth teacher without a drink. He hesitated for a while, then shrugged his shoulder and commented that the problem asked a “trick question” and stopped solving the problem. His work is presented in Figure 19.

Figure 19. John's Unsolvable Strategy



Emergent-Anticipatory Coordination (EAC)

Three specific strategies were identified for the co-constructed equal sharing story problems under the EAC strategy (i.e., quantities/quantifications, single additive, and combining shares). Each specific strategy is examined with examples in the sections that follow.

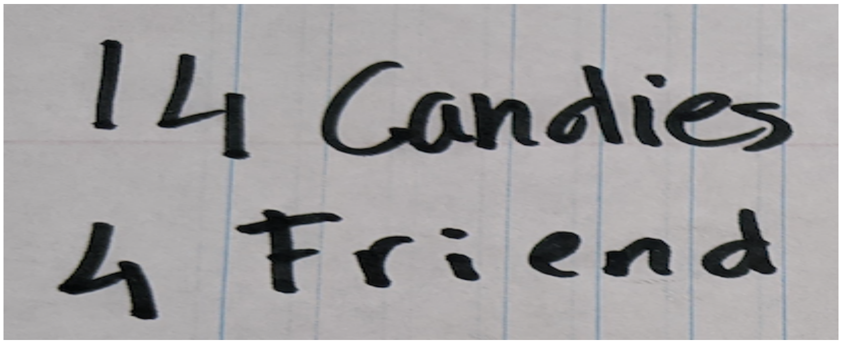
Quantities/Quantifications

The co-constructed tasks were retold by the student as the equal sharing story problem either at the beginning, middle, or at the end of the solution to problems with written words, symbols, or drawings. For example, while solving a co-constructed problem involving "4 friends" and "14 chocolate candies," the co-constructed task presented to Rebekah is as given below:

14 chocolate candies are shared among your 4 friends named B, T, R, F equally. How many chocolate candies will each friend get?

Rebekah. Rebekah demonstrated this strategy with her illustration in Figure 20. After solving the problem, she quantified each person's share as "$3\frac{1}{2}$."

Figure 20. Rebekah's Quantity Strategy



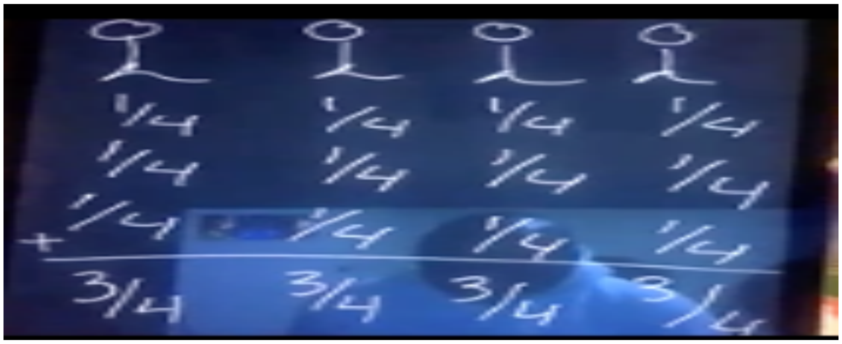
Single/Group Additive

In the co-construction tasks and for students who used the Emergent-Anticipatory strategy, one specific strategy stands out: Single Additive. For example, when students were asked to share their three favorite drinks with four of their friends, they demonstrated several approaches that were classified as single/group additive because they took whole item(s) and partitioned them into the same number of sharers. Students who used this strategy coordinated the two goals from the onset of the partitioning. Lucia's strategy with the problem highlights this example:

Your 3 preferred drinks are shared equally with your 4 friends. How much of the drinks will each friend get?

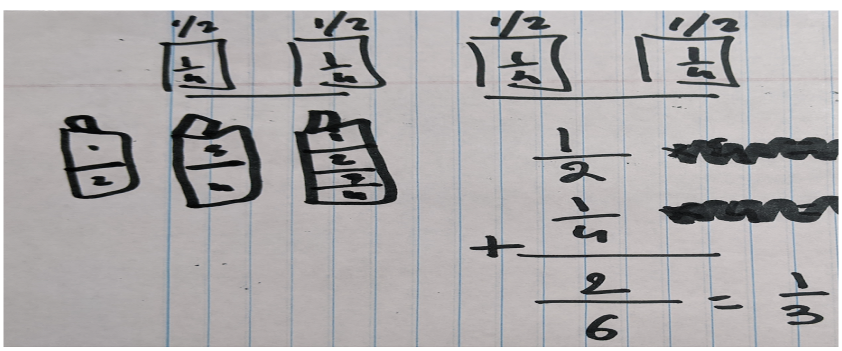
Lucia. Lucia solved the task by partitioning each drink into fourths and giving each person $\frac{1}{4}$. She used the same processes to partition, share and quantify each person's share. See Figure 21 for an illustration of the strategy in written representation. She concluded that each person gets "$\frac{3}{4}$."

Figure 21. Lucia's Single Additive Strategy



Rebekah. Rebekah used a combination of group and single additive to share the three drinks among four of her friends. She partitioned the first two drinks into four friends sharing them. She numbered each person's share as 1, 2, 3, 4. She reasoned that for the shares, each person gets ½. Next, she partitioned the third drink into an equal number of four friends and labeled each person's share as 1, 2, 3, 4. Rebekah reasoned that each person got ¼, and this strategy is as illustrated in Figure 22.

Figure 22. Rebekah's Combination of Group and Single Additive Strategy



Adding Shares

Continuing from Figures 21 and 22, Lucia and Rebekah correctly partitioned the three drinks into equal shares and gave each sharer fair shares. Their performance when combining each person's shares to form a final share is uneven.

Lucia. Lucia correctly added all the shares for each friend using the correct fraction addition procedure as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ to make a final share of $\frac{3}{4}$ for each friend. Lucia took the lowest common denominator of the fraction and added the numerator values.

Rebekah. Rebekah is, however, unsuccessful combining each person's initial shares to make a final share for each friend. In solving $\frac{1}{2} + \frac{1}{4}$, she added the numerator and denominator values separately thus $\frac{1+1}{2+4} = \frac{2}{6}$. She reduced $\frac{2}{6}$ to the lowest term to obtain a final share for each friend and each friend gets "$\frac{1}{3}$" which is incorrect.

Anticipatory Coordination(AC)

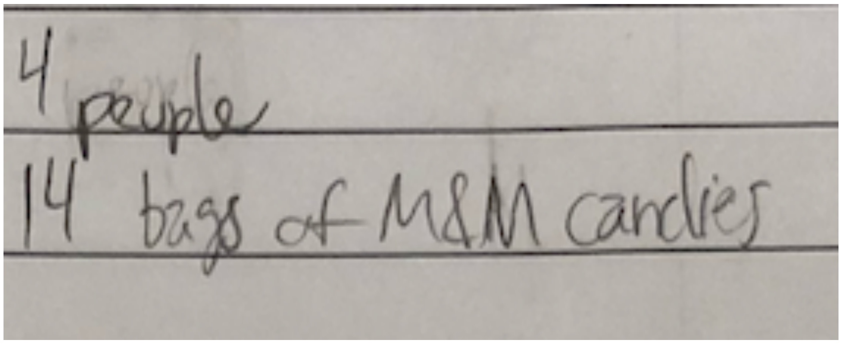
Most students used the Anticipatory Coordination strategy for co-constructed tasks. Three specific cognitive processes were identified under this broad category (i.e., quantities/quantifications, basic or long division, and multiplicative strategies). These particular strategies with their definitions and examples drawn from the study are examined next.

Quantities/Quantifications

Students provided the two variables that revolved around their preferences for foods, drinks, friends, or family members. Students were able to recall the variables in the solution to the co-constructed tasks with ease. They used written words, or symbols or drawings to illustrate the variables. For example, in this co-constructed problem: *14 bags of M & M candies are shared with your 4 friends (i.e., J, K, M, & R) equally. How many M&M candies will each friend get?*

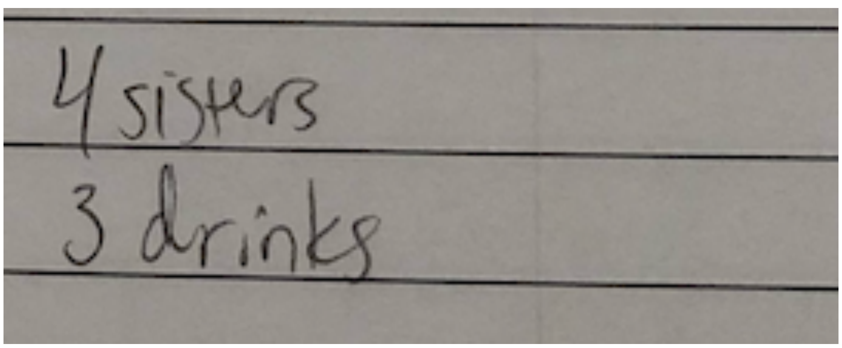
Janet. Janet demonstrated the quantities strategy with her illustration in Figure 23. She wrote the number of people and number of M&M candies to be shared.

Figure 23. Janet's Quantities Strategy



Janet did the same for the co-constructed problem in which she was asked to share "3 preferred drinks" with "4 sisters." Her written representation of the two variables and values are presented in Figure 24.

Figure 24. Janet's Quantities Strategy



Janet solved the two problems in Figures 23 and 24 correctly and quantified each person's share as "3.5 or 3 and $\frac{1}{2}$" and " $\frac{3}{4}$ = 0.75= $\frac{3}{4}$."

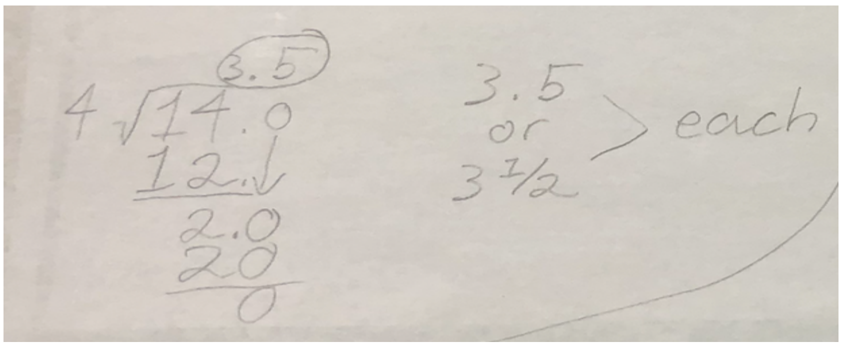
Basic or Long Division Strategy

Students who used this most advanced strategy, AC, used a Fact-Based strategy. Basic or long division strategy is one of the two fact-based strategies identified in the study for the co-constructed tasks. Students who used the Basic or Long Division strategy used the basic or long division algorithm to share the item(s) equally among the exact number of people sharing it. They did not represent each person's share as students who used Direct Modeling did.

Approximately six students used this strategy in the study, and they were primarily students who employed AC. As an illustration:

John. When John was asked to share "14 preferred candies with 4 friends," he produced the written representation in Figure 25. A careful look at the strategy in Figure 18 revealed a long division approach to solving the co-constructed story problem.

Figure 25. John's Long Division Strategy

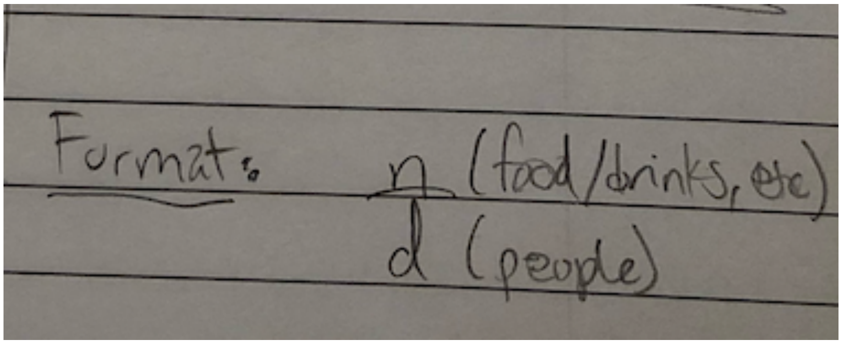


Multiplicative $\boldsymbol{\frac{a}{b}}$

The second fact-based strategy is the multiplicative strategy. Some students used the multiplicative $\boldsymbol{\frac{a}{b}}$ strategy for the co-constructed tasks. This strategy was used by at least 7 students who did not use the other broad strategies. Co-constructed tasks were solved using AC. The following examples illustrate the application of this strategy in the study.

Janet. For the problem in Figure 23, Janet set up the solution quickly as "$\frac{14}{4} = 3.5 = 3\frac{1}{2}$." She did the same for the problem in Figure 24 as "$\frac{3}{4} = 0.75 = \frac{3}{4}$." Janet then generalized the format she used to solve the problems, as shown in Figure 26.

Figure 26. Janet's Multiplicative Strategy



Problem Types and Cognitive Strategies Used

Table 8 presents a summary of the three broad cognitive strategies students used to solve the four co-constructed equal sharing story problems framed across two major fraction problem types (i.e., proper and improper fraction problem types).

Table 8. Co-Constructed Problem Types and Student's Coded Strategies

	Coded Strategies					
Student	Improper fraction problem types			Proper fraction problem types		
	2 share 5	4 share 14	3 share 4	4 share 3	_	_
Andriana	AC	AC	AC	AC	_	_
Beatrice	AC	AC	AC	AC	_	_
Connor	AC	EAC	EAC	EAC	_	_
Janet	AC	AC	AC	AC	_	_
Jeff	AC	AC	AC	NLC	_	_
Jessica	AC	AC	AC	AC	_	_
John	AC	AC	EAC	NLC	_	_
Joseph	AC	AC	AC	AC	_	_
Julie	AC	AC	AC	AC	_	_

Lucia	AC	EAC	EAC	EAC	_	_
Rebekah	AC	EAC	EAC	EAC	_	_
Robert	AC	EAC	EAC	EAC	_	_
Socrates	AC	AC	AC	AC	_	_

Note. NLC = No-Link to Context; EAC = Emergent-Anticipatory Coordination; AC = Anticipatory Coordination; dash = data were not obtained.

From Table 8, all students used AC to solve at least one equal sharing story problem. As the difficulty level of equal sharing tasks increased, the number of students who used AC decreased. Overall and across problem types, 13 students used the AC; 5 students used the EAC; and 2 students used the NLC. No students used the NAC in co-constructed equal sharing story problems. At a minimum across all fraction problem types, there are 37 instances of the use of AC, 13 instances of the use of EAC, and 2 instances of NLC. By Improper fraction problem types, there are 30 instances AC, 9 instances of EAC, and zero instances of NLC. However, for the proper fraction problem types, the number of instances AC was used dropped to 7 instances while that of NLC increased to 2 (see Table 9 for additional breakdowns).

Table 9. Co-Constructed Problems: Cognitive Strategies Used Across Problem Types

	Number of students using each strategy					
Cognitive	Improper fraction problem type			Proper fraction problem type		
strategy	2 share 5	4 share 14	3 share 4	4 share 3	_	_
NLC	0	0	0	2	_	_
NAC	0	0	0	0	_	_
EAC	0	4	5	4	_	_

AC	13	9	8	7	_	_

Note. NLC = No-Link to Context; NAC = Non-Anticipatory Coordination; EAC = Emergent- Anticipatory Coordination; AC = Anticipatory Coordination; Dash = Data were not obtained.

Research Question Three: Comparison Between the Two Conditions

How does fraction understanding of college DHH students (i.e., employed thinking strategies and representations) who watch the ASL video with the interpreter compare to their understanding when they are asked to co-construct the problems in a dialogue with a Deaf researcher in ASL?

From the analysis, three main levels of thinking (i.e., abstract, concrete, mixed) were identified. Participants are considered abstract thinkers if they employed AC strategies at least 70% of the time across all problem types and conditions. Participants are deemed concrete thinkers if they employed EAC strategies at least 70% across all problem types and conditions. Participants are identified as mixed thinkers if they used AC or EAC strategies less than 70% of the time across all problem types and conditions. Six students were identified as abstract thinkers, three as concrete thinkers, and four as mixed thinkers (see Table 10). Additionally, data were collected for the first four equal sharing fraction story problems (i.e., 3 improper and 1 proper fraction problem types) and used to compare the interpreted and co-constructed conditions.

Table 10. Participant Cognitive Strategy Use by Levels

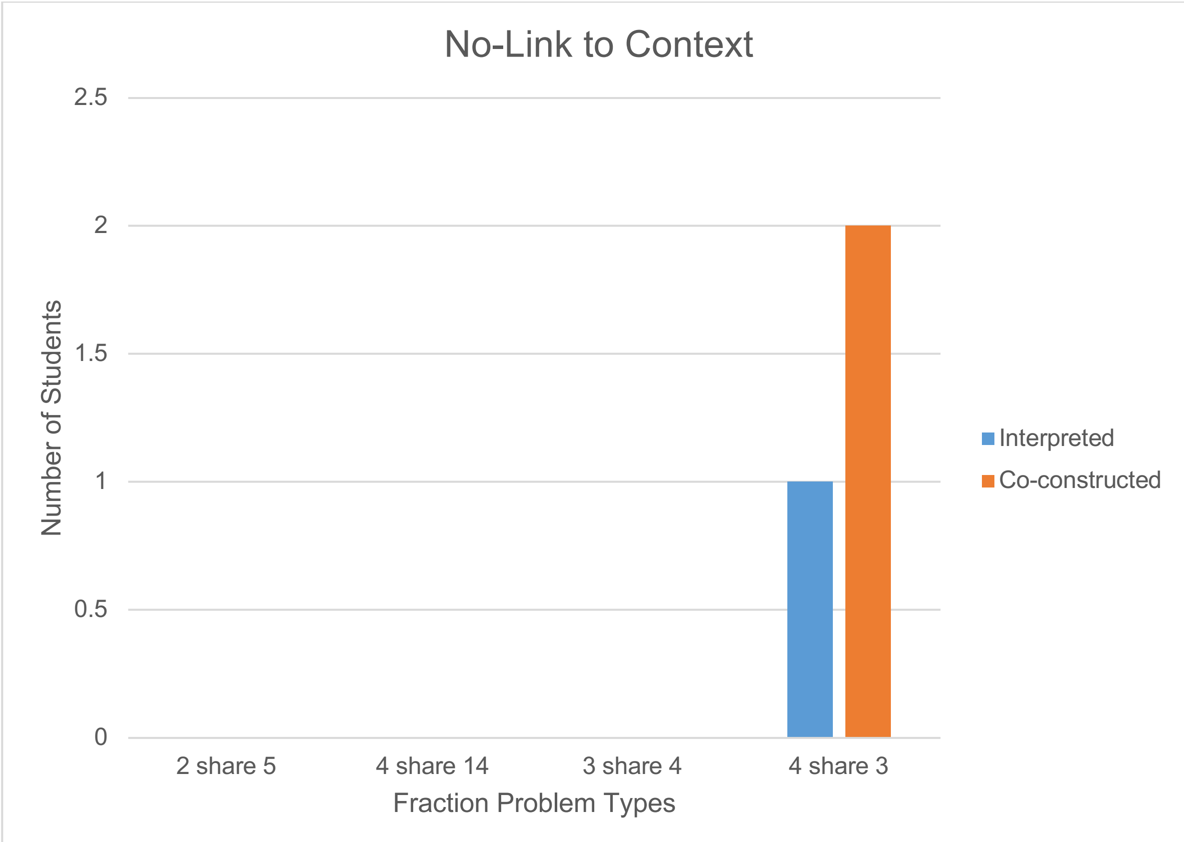
	Condition									
	Interpreted Tasks						Co-Constructed Tasks			
	Fraction Problem Type									
	Improper			Proper			Improper			Proper
	5 ÷ 2	14 ÷ 4	4 ÷ 3	3 ÷ 4	2 ÷ 5	3 ÷ 8	5 ÷ 2	14 ÷ 4	4 ÷ 3	3 ÷ 4
Abstract Level										
Adriana	AC	AC	AC	AC	AC	AC	AC	AC	AC	AC
Beatrice	AC	AC	AC	EAC	AC	AC	AC	AC	AC	AC
Janet	AC	AC	AC	EAC	AC	AC	AC	AC	AC	AC
Joseph	AC	AC	AC	AC	AC	AC	AC	AC	AC	AC
Julie	AC	AC	AC	AC	AC	AC	AC	AC	AC	AC
Socrates	AC	AC	AC	AC	AC	AC	AC	AC	AC	AC
Concrete Level										
Lucia	AC	EAC	EAC	EAC	EAC	NLC	AC	EAC	EAC	EAC
Rebekah	AC	EAC	EAC	EAC	EAC	EAC	AC	EAC	EAC	EAC
Robert	AC	EAC	EAC	EAC	EAC	NAC	AC	EAC	EAC	EAC
Mixed Levels										
Connor	AC	AC	NAC	NAC	NAC	NLC	AC	EAC	EAC	EAC
Jeff	AC	AC	NAC	NLC	NLC	NLC	AC	AC	AC	NLC
Jessica	AC	AC	EAC	EAC	EAC	EAC	AC	AC	AC	AC
John	AC	AC	AC	EAC	EAC	NLC	AC	AC	EAC	NLC

Note. NLC = No-Link to Context; NAC = Non-Anticipatory Coordination; EAC = Emergent-Anticipatory Coordination; AC = Anticipatory Coordination.

No-Link to Context (NLC)

No students used NLC for the improper fraction problem types for both the interpreted and co-constructed tasks (see Figure 27). Students in both conditions used this strategy only for the proper fraction problem type. Students who used this strategy may have used incorrect values in the partitioning operation (e.g., $\frac{4}{3}$ instead of $\frac{3}{4}$ for equal sharing problem involves sharing three pizzas equally among four people), left the problems unsolved or may have said "I do not know." This may indicate that the proper fraction problem type is more difficult compared to the improper fraction problem types. It may also indicate students' inability to shift between strategies that work for a particular improper fraction problem type (i.e., three people sharing four items) to proper fraction problem type (i.e., four people sharing three pizzas). The number of students who used this strategy for the co-constructed tasks doubled compared to the number of students who used the same strategy for the interpreted tasks. This may indicate a non-improvement in strategy use for students in the co-constructed task compared to students solving the interpreted tasks. The more students use this strategy, the less likely they are to use the more advanced strategy. Overall, no students used this strategy for the improper fraction tasks given in both formats. Two students used the strategy for the co-constructed proper fraction tasks and one student used it for the interpreted proper fraction task, indicating a lesser use of this inefficient fraction strategy for solving equal sharing story problems.

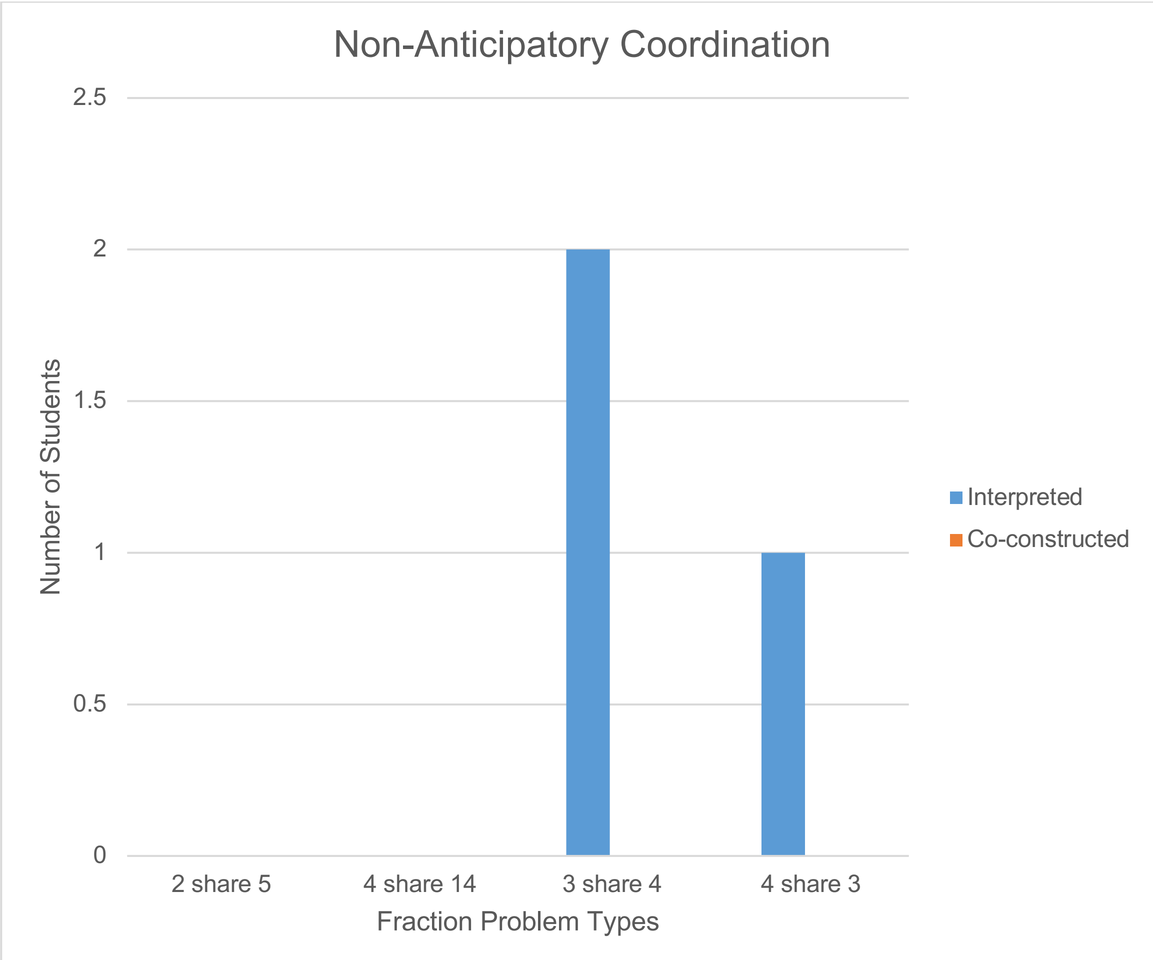
Figure 27. NLC for Both Conditions



Non-Anticipatory Coordination (NAC)

No students used NAC across the four co-constructed tasks (see Figure 28), and no students used the strategy for the first two interpreted tasks. The last two interpreted tasks appear more difficult, and NAC was used. One is an improper fraction problem type, and the other is a proper fraction problem type. The number of students who used NAC for the improper fraction problem type decreased from two to one for the proper fraction problem type for the interpreted tasks, indicating the difficulty of the proper fraction problem type.

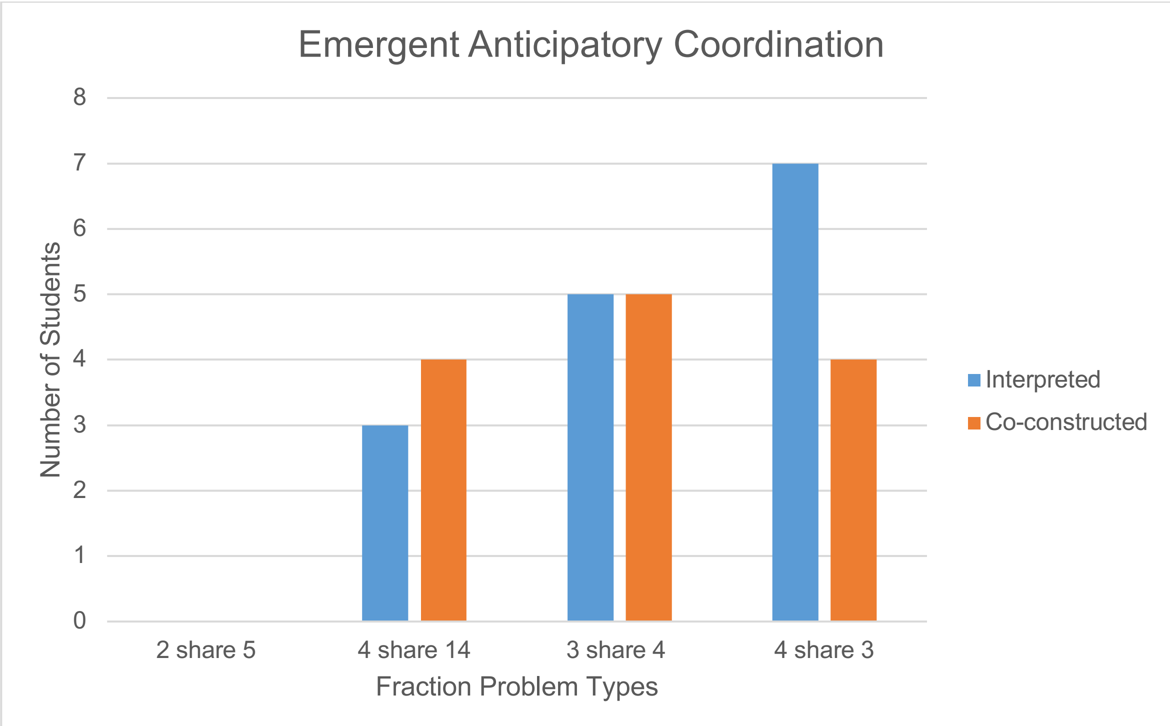
Figure 28. NAC for Both Conditions



Emergent Anticipatory Coordination (EAC)

No students used the EAC for the first equal sharing story problem presented in both conditions. The number of students who used the strategy varied for the three remaining equal sharing story problem presented in both conditions. In both conditions, the number of students who used this strategy increased for the two improper fraction problem types. While the number of students who used EAC continued to increase for the interpreted proper fraction problem type. It decreased for the co-constructed proper fraction problem type indicating students continue to find the interpreted problems more difficult as the number of sharers and items being shared increased or changed from even to odd numbers.

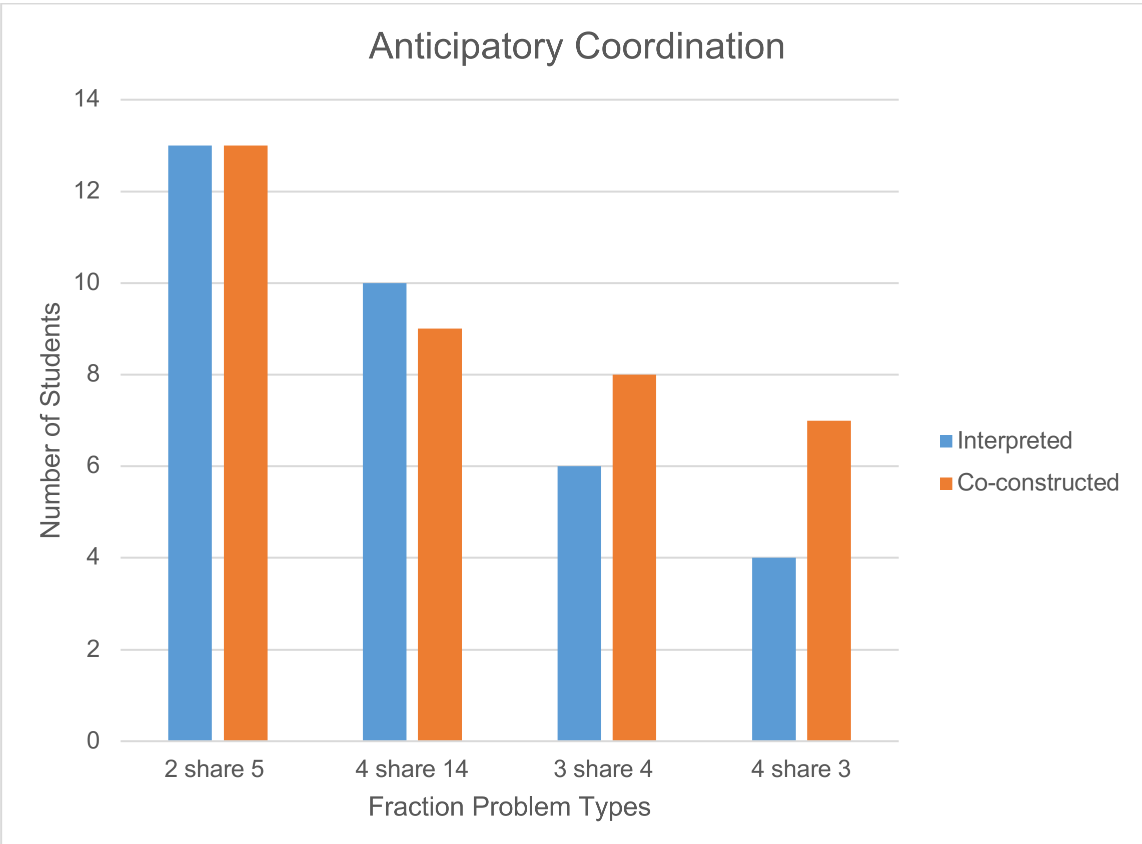
Figure 29. EAC for Both Conditions



Anticipatory Coordination (AC)

Anticipatory Coordination is the most advanced strategy for solving equal sharing story problems. Figure 30 offers interesting insights into the number of students who used AC in both conditions across all problem types. Specifically, the number of students who used the strategy decreased as the difficulty level of the tasks increased. For example, in the interpreted tasks and the first improper fraction problem type, the number of people who used the strategy dropped from 13 to 10 for the second improper fraction problem type, to 6 for the third improper fraction problem type, and to 4 for the only proper fraction problem type. The same can be said of the co-constructed equal sharing story problems. Across problem types and format, four students used this strategy. Six students used this strategy when the problems were presented in both conditions for the improper fraction problem types. Students appeared to perform equally well for the first two and easiest tasks given in both conditions. However, as the difficulty level of the tasks increased, more students performed better in the co-constructed tasks compared to the interpreted tasks.

Figure 30. AC for Both Conditions



Tables 11 and 12 provided a summary of strategy use across problem types and the percentage of students who use each strategy and each specific problem.

Table 11. Interpreted Problem Types: Percentage of Students' Strategy Use

Cognitive strategy	Percentage of students who used the strategy					
	Improper fraction problem type			Proper fraction problem type		
	2 share 5	4 share 14	3 share 4	4 share 3	5 share 2	8 share 3
NLC	0%	0%	0%	8% (1)	_	_
NAC	0%	0%	15% (2)	8% (1)	_	_
EAC	0%	23% (3)	39% (5)	54% (7)	_	_
AC	100% (13)	77% (10)	46% (6)	30% (4)	_	_

Note. NLC = No-Link to Context; NAC = Non-Anticipatory Coordination; EAC = Emergent- Anticipatory Coordination; AC = Anticipatory Coordination.

Table 12. Co-Constructed Problem Types: Percentage of Students' Strategy Use

Cognitive strategy	Percentage of students who used the strategy					
	Improper fraction problem type			Proper fraction problem type		
	2 share 5	4 share 14	3 share 4	4 share 3	_	_
NLC	0%	0%	0%	15% (2)	_	_
NAC	0%	0%	0%	0%	_	_
EAC	0%	31% (4)	38% (5)	31% (4)	_	_
AC	100% (13)	69% (9)	62% (8)	54% (7)	_	_

Note. NLC = No-Link to Context; EAC = Emergent-Anticipatory Coordination; AC = Anticipatory Coordination.

Additional Findings

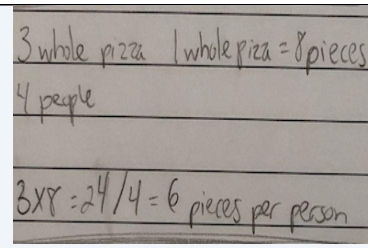
In addition to the four broad strategies discussed under the third research question, the study also identified additional strategies beyond what was expected from the study. These newly identified strategies are examined under four major themes: conversion, executive function, students' commentaries, and varied supports. Each theme, sub-theme, definition, and examples are examined in more detail in the sections that follow.

Conversion

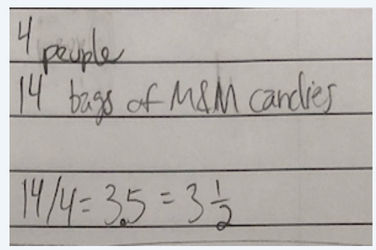
Three conversion strategies were identified in the study, (a) conversion from decimals to fractions or vice versa; (b) conversion from decimal to percentages or vice versa; and (c) conversions from slices or inches or pieces to fractions. While some students got the conversion correct others did not, and Table 13 outlines the major conversion themes, sub-themes, definitions, and examples.

Table 13. Conversion and Executive Function Strategies

Themes	Sub-Themes	Definitions	Examples
Conversions			
	Slices/pieces conversion	Students convert slices to fractions	Student: (4 friends share 3 pizzas problem). Janet obtained "6 pieces" after "24 (8 pieces of pizzas x 3) by 4. Janet worked on converting "6 pieces" to a fraction. Janet said, "I converted 6 pieces to fraction as $\frac{12}{1}$ = 12 (which is incorrect. Janet was not told it is incorrect). The correct answer should be $\frac{6}{8} = \frac{3}{4}$. Janet produced the image below:

			
			Student: (4 friends share 3 pizzas problem). John obtained “6 pieces” after “24 (8 pieces of pizzas x 3) by 4. John worked on converting “6 pieces “to a fraction. John said, “each person gets 6 slices of pizzas. $\frac{8}{24} = \frac{1}{3}$ (which is incorrect).” The correct answer should be $\frac{6}{8} = \frac{3}{4}$.
	Precents conversion	Students convert precents to fractions	Student: (5 friends share 2 submarine sandwiches problem). Connor obtained a percentage answer as 40% which is correct. When asked to convert 40% to a fraction, Connor responded, “I think 2 over, 2 over, 2 over 5, 2 over 4 (unsure of the correct fraction).”
			Student: (4 friends share 3 pizzas problem). Connor obtained a percentage answer as 75% which is correct. He was asked to convert 75% to a fraction. Connor responded, “$\frac{3}{5}$,” then changed it to “$\frac{3}{4}$." Later to “$\frac{3}{4}$.” Then “$\frac{4}{5}$” (unsure of the conversion). Finally, Connor got it correct “$\frac{3}{4}$.”
	Decimal conversion	Students convert decimal to fractions	Student: (5 friends share 2 submarine sandwiches problem). Adriana obtained

	a decimal answer of 0.4. Adriana successfully converted 0.4 to $\frac{2}{5}$. Student: (8 people share 3 water bottles problem). Adriana obtained a decimal answer of 0.375. Adriana successfully converted 0.375 to $\frac{3}{8}$. Student: (4 children share 14 clay sticks problem). Janet successfully converted from decimal 3.5 to the fraction 3 and $\frac{1}{2}$ as represented in the image below.



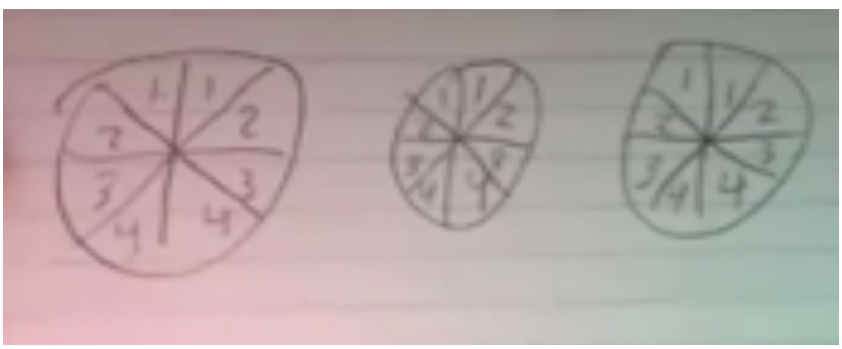
Convert Slices or Pieces or Inches to Fractions

At least four students converted "slices, pieces, or inches" to fractions and typically used direct modeling in their partitioning activities. Two strategies that make use of direct modeling are NAC and EAC. Examples of these strategies are provided in Table 13. Students who used these strategies had the correct number of slices or pieces given to each person from the equal sharing activities; however, students were unable to convert the slices or pieces to correct fractions. This strategy was used most with the interpreted improper fraction problem type involving "4 friends sharing 3 pizzas." For instance, John used this strategy for the problem: *4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?*

John. John says that each pizza has "8 slices" (using the idea of a pizza being pre-partitioned into eight slices). He says three pizzas are equivalent to "24 slices" (i.e., 3 x 8). He then divides 24 slices by 4 friends and concludes that each person gets "6 slices." To convert the "6 slices" to a fraction, he said "$\frac{8}{24} = \frac{1}{3}$" for each person.

Robert. Robert also used this strategy for the same problem as John and concluded that each person gets "6 slices" of pizzas. Robert's solution strategy is represented in Figure 31. He partitioned each pizza into eighths and reasoned that each person receives "2 slices" from each "8 slices." For all three pizzas, a friend gets "6 slices." Converting "6 slices" to fractions, Robert said, "each person gets $\frac{2}{8}$." He said, "$\frac{2}{8}$" becomes "$\frac{1}{4}$" when simplified.

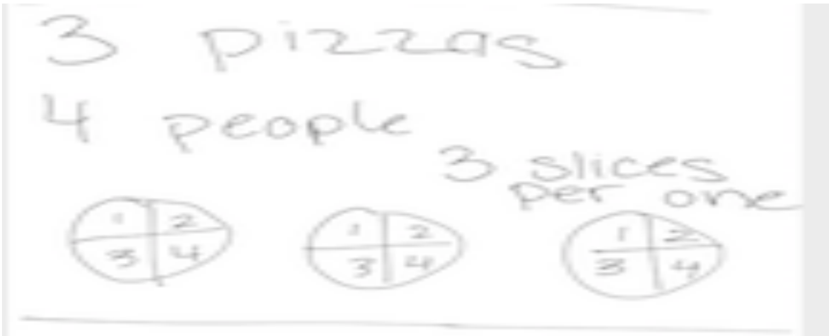
Figure 31. Robert's Slices Strategy



Note. Robert partitions each pizza into 8 slices, and he gives each friend 2 slices from each pizza, making 6 slices for each friend.

Lucia. Figure 32 represents Lucia's strategy for the same problem. She divided each pizza into fourths. She gave each person one-fourth of each partitioned piece. She concluded that each person gets "3 slices" but could not convert the "3 slices" to a fraction.

Figure 32. Lucia's Slices Strategy



Note. Lucia's partitioned each pizza into 4 slices and distributed the slices to 4 friends. She concluded that each friend receives "3 slices" but was unable to convert this to a fraction.

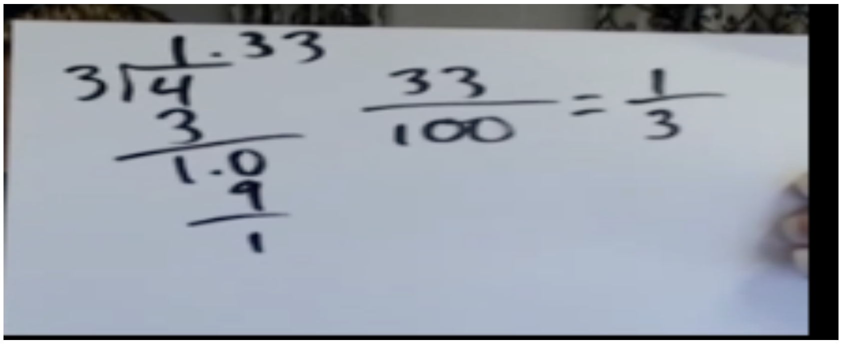
Convert Decimals to Fractions

About seven students obtained decimal answers as solutions to the equal sharing story problems. This strategy is typical of students who used the most advanced broad strategy (i.e., AC). This occurred for the two problem conditions and the two problem types. For example, when Julie was presented with the problem: *14 sticks of clay are shared among 4 children for a project. How many sticks does each child receive?*

Julie. Julie solved the problem with a long division strategy and said that each child got "3.5." She then converted "3.5" to fractions as "3 and ½." It is instructive to note that students with few exceptions who used this strategy were good at correctly converting the decimal answers to fraction answers.

Adriana. Adriana used the same strategy to solve the equal sharing problem: *4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?* Adriana used the strategy presented in Figure 33. She converted the fraction "1.33" answer to "1 and $\frac{1}{3}$."

Figure 33. Adriana's Conversion Strategy



Note. Adriana's division strategy resulting in decimal converted to a fraction.

Convert Percentages to Fractions

One student used percentages and Trial-and-Error as solution strategies for some of the equal sharing tasks. Thus, sometimes, his answers to the tasks were incorrect, and this student could not convert the correct percentage answer to a fractional answer when requested to do so. Connor's data is used as an example of the application of this strategy when asked to solve equal sharing problem: *5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receive?*

Connor. At the end of his trial-and-error strategy to solve the problem, Connor said each person got "40%." When asked to convert 40% to fractions, he said, "2, $\frac{2}{4}$ or $\frac{2}{5}$." Connor was not sure of the correct conversion. A similar strategy was used when asked to solve the problem: 1*4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?*

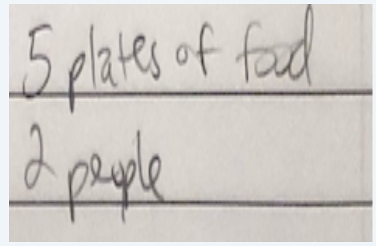
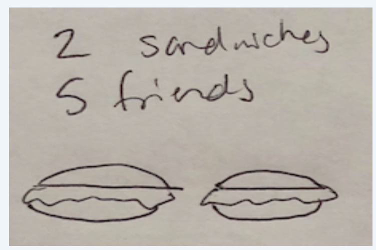
Connor said each person receives "75%" which is correct. When asked to convert 75% to fraction, Connor said "$\frac{3}{5}$" which is an incorrect conversion.

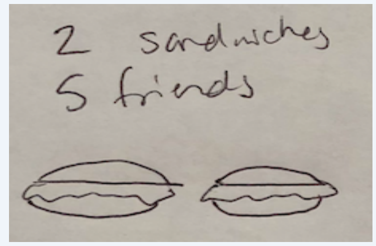
Executive Function Skills

Executive function skills are meta-cognitive skills the students brought to bear on the solutions to the equal sharing story problems. They are a set of mental skills that include working memory, flexible thinking, and self-monitoring. The study identified three executive function skills that include (a) requesting to see the interpreted video over again, (b) shadowing the interpreter, and (c) rethinking their solution strategies (see Table 14). All of these were attempts to understand and adjust their strategies to solving the equal sharing story problems. Each skill is examined in more detail in the following sub-sections.

Table 14. Students' Executive Function Skills

Theme	Code	Definitions	Examples
Executive Function	Request Replay	Students ask the researcher to replay the videos over again	Student: (Susan and Juan share 5 soft tacos problem). John asked to see the video "one more again."
			Student: (8 people share 3 water bottles problem). Socrates commented that, "I missed the number of people, 7 or 8?"
			Student: (8 people share 3 water bottles problem). Robert requested to see the video "again. One more again."
	Shadowing	Students mimic the interpreter with signing as the video plays	Student: (4 children share 14 clay sticks problem). Adriana was "(Mouthing) Clay…14….4 children." While trying to solve the problem.
			Student: (3 friends share 4 chocolate bars problem). Rebekah (mimicking the interpreter) said "4 chocolates… 3 friends…"

	Notetaking	Students write down notes as the video plays or as we co-constructed the tasks.	Student: (5 friends share 2 submarine sandwiches problem). Janet wrote the following down while the video plays.  Student: (5 friends share 2 submarine sandwiches problem). Jessica wrote the following down while the video plays. 
	Rethink Strategies or Answers	Students reconsider or double-check the accuracy of their solution strategies	Student: (4 children share 14 clay sticks problem). Lucia said, "3 and $\frac{1}{3}$ water bottles" for each person. Lucia counted, "3, 6, 9, No, no… I calculated it wrong…" (later corrected the mistake) as "3 and $\frac{1}{2}$." Student: (3 friends share 4 chocolate bars problem). Janet said, "each person gets 1 and $\frac{1}{4}$. One and one-fourth. It is not one and one-half. Let me re-check (writing on paper)." Student: (5 friends share 2 submarine sandwiches problem). Jessica said, "I

	do not know this one. I need to analyze how to share them. I think $\frac{2}{5}$." 

Request Replay

At least nine students asked for the videos of the interpreted equal sharing story problems to be replayed. Requests emanated from both students who used the basic and most advanced cognitive strategies. The requests to see the videos over again may be for several reasons. The videos may have been frozen as they played, thus affecting the students' understanding of the tasks. The students may have missed some information as the video played. Students may not have understood the expectations of the problem, and may have wanted to have a deeper clarity and understanding of the tasks before, during, or after solving the equal sharing problems. For instance, when presented with the problem: *Susan and Juan shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Susan and Juan each eat if they finished all the tacos?*

John. John requested, "can I see the video again?"

Shadowing

Shadowing involves students mimicking the interpreter as the interpreter signed the equal sharing story problems. Seven students used shadowing in the study. When students shadowed, they signed, signed and mouthed, and mouthed or wrote the equal sharing story problems down. Shadowing could be a verbatim shadow or a summary shadow of the relevant information presented. As an illustration, in the problem: *14 sticks of clay are shared among 4 children for a project. How many sticks does each child receive?*

Rebekah. As the interpreter signed “14 clay sticks”, Rebekah also signed “14”. When the interpreter signed “4 children”, Rebekah also signed “4 children”. Rebekah mimicked the interpreter immediately as they signed the story.

Notetaking

Students used a notetaking strategy in the study. Notetaking is more prominent when writing down the two important variables in the study: the number of items being shared and the number of people sharing the items. Several examples were identified under this theme.

Rethink Strategy/Answer

Students rethought their approaches to solving the problems or their answers to the problems. They may have stepped back to examine the viability or correctness of their strategies or solutions whether prompted or unprompted. Ten students used this strategy when solving the equal sharing story problems. After the re-examination of either their strategies or answers to the problems, they may have correctly identified mistakes or left the mistakes uncorrected. The use of this strategy is often associated with students who used the trial-and-error strategy. For example, using the problem: *4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?*

John. John provided an incorrect response to the problem “4 friends and 3 pizzas” as “1.25.” He walked back his answer, arguing that he thought the problem asked him to share the pizzas with “3 friends.” After rethinking his solution strategy to the initial answer, John concluded that the answer is still “1 and $\frac{1}{4}$ or 1.25.”

Robert. Robert offered another example when he rethought the solution to the interpreted problem: *5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receives?* After partitioning each sandwich into 5ths and distributing one-fifth of each sandwich to each friend, he concluded that each person got two pieces. When asked to convert the two pieces into fractions, he responded that each person got "one-fifth." He quickly self-corrected his mistake, saying and shaking his head in negation, "this is not correct." Then he responded that each person got $\frac{2}{5}$. He changed it again and concluded that each friend got one.

Conditions' Efficacy: Students' Commentaries

At the end of the interview for both the interpreted and co-constructed tasks, students were interviewed and asked to share their views of the difficulty of each format, the difficulty of the languages of each format, and which format they recommend for DHH children with limited math content knowledge, language difficulty, or language delay. Their responses were collated under the three sub-sections presented in Table 15 and further discussed below.

Table 15. Students' Commentaries on Efficacy

Theme	Sub-Themes	Definitions	Examples
Efficacy			
Comparisons			
	No difference	Students identified no differences between the two conditions in terms of complexity, language use, and accessibility.	Student: Jeff said, "none. none. I think both look similar. I do not see any difference. I do not think either is hard. The interpreted and co-constructed tasks are both easy for me. I understand both." Student: Socrates said, "the interpreted and co-constructed tasks use ASL.

			The two conditions are not hard. Neither is difficult for me.” Student: Adriana commented, “my perspective as an adult is that both format is easy for me.”
	Harder	Students commented that either condition is hard. They may use the following words: too many details, too complex, too abstract, and unclear information	Student: Jessica commented, “interpreted format is a little harder.” Student: Rebekah said, “the interpreted format provided detailed explanations during the translation. It has more sentences, more explanation; longer statements; the interpreter explained all the words, for examples; she gave you examples of the amount of people and gave you examples of the number of things. She signed exhausted the sharing. She asked how many equal parts; how much each person gets? The interpreted format had more details" …details and concepts to aid understanding. It contained too much information-like number of people or number of sharers, She asked if they got equal shares and if they exhausted the whole etc.” Student: Adriana said, interpreted task “may be harder for young children because the signing was too fast.”

	Easier	Students agreed that either format is easy with justifications	Student: Connor commented that Co-constructed tasks "makes the problems interesting and engaging. The problems are presented by a subject matter expert like you." Student: Beatrice said interpreted "tasks and language can be adapted to ensure that young children understand it." Student: Joseph maintained that for the co-constructed tasks "I can seek clarifications if needed with the interviewer or person I am interacting with. With the interpreted format, that is not possible. With the co-constructed format, I have all the information I need quickly and faster. The information is less detailed. This supports students understanding. It is easier to understand."

Easy

All students found the co-constructed tasks easy to understand and do and stated that the co-constructed tasks were easy when compared to the interpreted tasks. Students offered several reasons why they considered the co-construct tasks easier. They argued that co-constructed tasks:

(a) allowed subject matter experts (i.e., interviewers or teachers) to adapt the content and language of the tasks to fit the understanding level of students.

(b) permitted interaction between the teacher and students which could serve as a source of developing a personal relationship with the teachers while simultaneously motivating students.

(c) were connected to students' interests and preferences, thereby motivating and engaging them in the tasks and problem solving.

(d) allowed students to explain their thinking.

(f) allowed for clear communication, clarifications, feedback, expansions and the use of simple language.

(g) were devoid of irrelevant information as the problems were presented in simple and familiar concepts, and with visual supports.

(h) supported direct instruction because of the step-by-step nature of the co-constructed tasks.

Specific student responses are provided:

Rebekah. Rebekah commented "easy for the students to relate to the tasks. It is connected to their personal experiences. Teachers using co-constructed tasks care about the students' interests and help push their thinking and motivate them to answer or solve the problems. The co-constructed tasks were easy to follow the story lines-the person sharing things and the things being shared. The co-constructed tasks used my personal connection to things to frame the mathematical problems. The tasks did not come from the textbook, which is often the practice. In the co-constructed task, the teacher asked about my preferences. What kind of food I liked: pizzas, drinks I like and friends and family members?"

Hard

Ten students commented that the interpreted tasks were difficult even though the interpreted tasks were completed first before the co-constructed tasks. For instance:

John. John made the following comments, "the video sometimes did not explain things clearly. I had to asked for replays to make sure I got the signing clear."

Adriana. According to Adriana, "the interpreted tasks will be too hard for young children."

Rebekah. Rebekah said that "I did not catch all the information the first time. I asked for a replay of the videos twice before I could understand the questions."

Students gave diverse reasons for the difficulty of the interpreted tasks compared to the co-constructed tasks. The interpreted tasks were described as difficult because:

(a) they were not engaging as the interpreted information had only one interpreting format across the problems. The interpreter could not clarify the information presented if needed, and opportunities to ask for clarifications when the information presented was limited.

(b) they had too much detail in the translation with more sentences and longer statements.

(c) the communication or signing was unclear or too fast and necessitated requests for the replay of the videos and shadowing the signs and note-taking in writing as the videos played.

(d) they may be difficult for students with additional disabilities. Deaf and hard of hearing students who have vision problems may not be able to see the interpreter on the video. It may be difficult for these students to process the information presented in the video.

(e) failed to connect to the student's experiences and interests, thereby making it harder to relate to the interpreted tasks.

(f) suited for an indirect form of instruction where there is a lack of demonstration, making students passive participants in the learning process.

(g) limited interaction as the translations were pre-planned.

(h) the interpreter may have lacked subject matter knowledge, as she seemed to be just an ordinary interpreter.

Same

In addition to all the 13 students saying the co-constructed tasks were easier, seven students also said they could see no difference between the two conditions when it came to ease and the language used in each format. They argued that both conditions had benefits. Some students may first need to watch the videos followed by the classroom teachers' explanations.

Jeff. Jeff commented that " I did not find the co-constructed and the interpreted tasks difficult. I understood both conditions."

Robert. Robert commented that "you (i.e., the researcher) and interpreter were easy to follow. If there were no network problems, I could understand the interpreter and you."

Adriana. Adriana has this to say: "My perspective? As an adult, both the interpreted and co-constructed tasks were easy for me. However, for young children, the interpreted tasks may be difficult…"

Lucia. Lucia commented, " I think both are the same."

Varied Supports

Students utilized different supports to solve the interpreted and co-constructed equal sharing story problems successfully. These supports ranged from interviewer prompts to student self-initiated use of calculators, fingers, and computers. The supports were extended to students in both conditions. Each support is listed in Table 16 and examined in detail in the sub-sections that follow.

Table 16. Varied Supports

Theme	Sub-Themes	Definitions	Examples
Varied Supports	Prompting	Researcher replayed videos; clarify the problems only; encourages students to double-check their work	Researcher: (5 friends share 2 submarine sandwiches problem; 4 friends share 3 pizzas problem). I fingerspelled the word for the sign for sandwiches to help clarify the meaning of the sign. I used fingers as a manipulative. I fingerspelled the word “fraction.” I assisted with name “pizzas” by clarifying the sign for pizzas when the student asked. I repeated the problems to the students.
	Calculator	Students used a calculator with permission to solve the problems	Student: (4 friends share 14 preferred candy bars; 3 siblings share 4 preferred food items). Julie on request, used a handheld calculator to solve the problems. Student: (All interpreted tasks except the first two). Julie asked if she could use the calculator. She was permitted. Student: (8 people share 3 water bottles problem; 4 friends share 14 preferred candy bars; 3 siblings share 4

			preferred food items). On request, Jeff used a handheld calculator for problems interpreted task number 3 and the co-constructed task number 6 listed above.
	Fingers	Students use fingers as manipulatives to solve the problems	Student: 4 friends share 14 preferred candy bars; 3 friends share 4 large chocolate bars). Jeff used fingers as manipulatives while solving the problems for both interpreted and co-constructed problems listed above.
	Computer	Students use the computer on request as calculator or to find resources from the web	Student: (3 friends share 4 large chocolate bars). Robert used the computer to look for measurements for the chocolate to solve the task.
			Student: (5 friends share 2 submarine sandwiches problem). Connor used the computer as a supporting technology tool to convert a foot into 12 inches; 2 feet into 24 inches while solving the problem.
			Student: (4 friends share 3 pizzas problem). John used the computer to locate measurements for problems. He also used the computer as calculator to solve problems.

Prompting

The researcher employed prompting techniques to correct the students' misconceptions of the story problem and to encourage the students to work toward solving the equal sharing story problems. The researcher replayed videos of the interpreted tasks when students did not understand the expectations of the tasks or when the students retold the story problems incorrectly. In addition to replaying the interpreted videos without the students' requests, I also corrected the students when they misrepresented the contents or values in the story problems. For instances when asked to solve the problem: *Susan and Juan shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Susan and Juan each eat if they finished all the tacos?*

Connor. Connor mischaracterized the number of tacos as "4," I quickly re-directed him to watch the video again. He corrected himself as "5 tacos". Connor also mis-interpreted the sign for "candy" in one of the story problems as "candle." He misunderstood the sign for "pizzas" as presented in the interpreted problem. I corrected these mistakes early on using the correct sign, fingerspelling supports and the use of real-life examples (e.g., bottle water).

Lucia. Lucia said, "I can't see the interpreter. Can you explain it to me?" As the researcher, I supported Approximately four students by providing an explanation as needed.

Calculator/Computer

When asked if they could use calculators seven students were allowed; however, some students did not request a calculator. While calculators were used for both the interpreted and co-constructed story problems, the interpreted tasks received the most calculator use. Calculator types ranged from handheld, cellular phones, and computers. Often calculators were used to perform division the students would have otherwise done manually. John used the computer not only as a calculator but also to determine the measurement of a sandwich in inches.

Manipulatives

Three students used their fingers as manipulative devices while solving some of the story problems. Fingers were used in both the interpreted and co-constructed conditions but used more frequently in the interpreted format. For instance, Connor used his fingers as manipulative devices to solve the equal sharing problem “3 pizzas” among “4 friends.” First, he stretched out his three left-hand fingers to represent the “3 pizzas.” Then he represented each finger starting from the thumb finger as 100%. Then he took 25% from each finger to make up for the fourth person shares. He then concluded that each person gets 75% of pizzas.

Test Design/Technology Issues

The study uncovered some challenges related to the interpreted tasks and technology during the assessment interviews. First, when Joseph was asked to solve the interpreted problem involving “3 pizzas” and “4 friends,” he asked for the size of the pizzas; “Small or large.” The same can be said of John, who wanted to know the “size of the sandwich” in the interpreted problem involving “2 sandwiches” and “5 friends.” The English version of the interpreted story problem has the word “small” included but this was conspicuously omitted in the interpreted version by the interpreter.

Next, administering the two tasks that were similar to each other in sequence added to the confusion. For example, “$\frac{3}{4}$” and $\frac{4}{3}$” are reciprocal of each other. Students confused the two or used the same operation for both. See Table 17 for additional information.

Table 17. Challenges Encountered

Theme	Sub-Themes	Definitions	Examples
Test design/technology problems			
	Frozen video	The interpreted videos temporarily stopped working during the interviews	Student: (Susan and Juan share 5 soft Taco's problem). As Socrates watched, the interpreted video stopped unexpectedly during play and said, "The video is not-smooth when it plays. It freezes often."
	Test design problem	Students encountered various issues associated with test design itself	Student: (4 friends share 3 small pizzas problem). The word "small" was included in the written version of the problem but was conspicuously missing from the interpreted version. Joseph commented that "2 and half pizzas given to each person is too large."
			Student: (3 friends share 4 large chocolate bars problem; 4 friends share 3 small pizzas problem). Divided 3 by 4 and 4 divided by 3 instead of the other way around. Janet confused $\frac{3}{4}$ with $\frac{4}{3}$.
			Student: (4 children share 14 clay sticks problem). Rebekah said this for the interpreted problem; "The clay has something, but I have no idea what it is. 14 of it. It may be candy or sticks." The student used Clay to refer to name of a

	person rather than name of the item as intended in the story problem.

CHAPTER V: DISCUSSION AND IMPLICATIONS

Deaf and hard of hearing college students' conceptual understanding (i.e., level of cognitive strategies) of fractions was assessed through one-on-one interviews using equal sharing mathematical story problems presented in two different contexts (interpreted and co-constructed). Equal sharing story problems (also known as partitive quotient or fair-sharing problems) involve equally sharing some number of same-sized objects among some people where the result is a fractional quantity (Empson & Levi, 2011). Equal sharing story problems have been used as a rich approach for assessing students' fractional understanding. Prior studies have generated observable evidence of predictable cognitive strategies for solving equal sharing story problems, which have been made into a conceptual framework of students' fractional thinking, albeit of pre-college and students other than DHH (Empson & Levi, 2011; Hunt et al., 2016; Steffe & Olive, 2010).

Interpretations of the Findings

The current study unearthed four broad themes of cognitive strategies DHH college students use to solve six interpreted and four co-constructed equal sharing fraction story problems (i.e., No Link to Context (NLC), Non-Anticipatory Coordination (NAC), Emergent Anticipatory Coordination (EAC), and Anticipatory Coordination (AC). These categories, as Hunt et al. (2017) averred, "do not suggest sequential levels of partitioning" or understanding but rather what the students know as they solve specific equal sharing story problems. Cognitive strategies range from less advanced to most advanced coordination and least concrete to most abstract representations. Within these four broad themes, students used additional strategies specific to each broad strategy. Thus, for each theme, the cognitive strategies students used were determined by seven techniques: (a) the identification of the sharers and/or items being shared;

(b) the level of pre-coordination of the number of people sharing the items with the items being shared from the onset of the sharing activity; (c) the type of representation used while solving the equal sharing story problems; (d) operations used to solve the problems; (e) the generation of fair shares; (f) the combination of shares to make a final share; and (g) the characterization of language used to quantify each persons' shares (George, 2017; Hunt et al., 2016). The second technique which Empson et al. (2006) defined as coordination of quantities or as "how a child physically or mentally manipulates the number of items to be shared in conjunction with the number of people sharing them to produce an exhaustive and equal partition of the items" (p.5) is critical. The study's findings suggest students' cognitive strategies for solving equal sharing story problems varied, changed across tasks, used different numbers, and occurred in different contexts.

No-Coordination (NC)

No students used this strategy in the study.

No-Link to Context (NLC)

The first rudimentary partitioning strategy students used in the study is the NLC. Students using NLC accepted that the whole, or wholes, can be divided. However, some used values or operations other than those specified by the equal sharing story contexts. In the current study, the use of NLC was localized to the three proper fraction problem types for the interpreted tasks and the one proper fraction problem type for the co-constructed tasks. Proper fraction problems are the most difficult of the equal sharing story problems. Jeff used NLC for the three proper fraction equal sharing story problems. Three other students used it for the last and the most difficult proper fraction equal sharing story problem (i.e., three bottles of water shared by eight people) for the interpreted tasks. Jeff and John used the NLC for the only proper fraction equal sharing problem type in the co-constructed tasks. Specifically, the students over-generalized the strategy that worked for improper fraction equal sharing problem types to proper fraction equal sharing problem types. For instance, Jeff used the same values and correct sharing strategy when sharing "4 large chocolates with 3 friends" and "3 pizzas with 4 friends". Even though Jeff used the correct operation, the values used were incorrect (i.e., he flipped the number of sharers in place of the number of items being shared and vice versa).

One possible explanation for students' use of this strategy is their limited understanding of dividing the larger number by the smaller number. They may not be used to dividing the smaller number by the larger number. Another explanation is the use of a calculator when performing equal sharing tasks. Students likely inputted the larger number into the calculator first, then pressed the division button before inputting the smaller number next. Students were also observed to have represented the correct number of items being shared but partitioned the items into arbitrary or uneven shares. These strategies are similar to what Hunt et al. (2017) found and termed in their study as "No Link between the Number of Objects and/or Parts to the Questions Context" (p.6).

Hunt et al. (2017) argued this strategy reflected students who experience difficulty relating the concept of equal sharing to the equal sharing problem elements. Students may not yet link the number of sharers to the objects being shared or may use the wrong values or operation. An alternative explanation for using this strategy is what Bull and Lee (2014) termed a student's inability to select and execute correct behaviors or actions. Researchers argued selecting and executing appropriate cognitive behaviors or actions entail response inhibition, defined as the removal of prepotent or dominant responses and representations. In other words, it is the intentional prevention of a behavior that is underway or automatically evoked that may impede the correct solution to solving the problem. Some students over generalized their strategies. They continued to use a strategy that worked for improper fractions when solving proper fractions. The overgeneralization of a strategy leads to unsuccessful problem-solving. For example, Jeff did not abandon a previously used strategy for improper fractions for another strategy that would have worked for proper fractions. Research has indicated that better inhibitory performance in typically developing children and adults is linked to successful problem-solving (Bull & Lee, 2014; Dajani & Uddin, 2015).

Another plausible explanation is what Dajani and Uddin (2015) called attention shifting. Students using NLC demonstrated the ability to shift between problem types and contexts, but their ability to shift sets or solutions that previously worked remained unchanged. A small number of students used this strategy for the co-constructed tasks compared to the interpreted tasks indicating students did better in the co-constructed tasks than the interpreted tasks. No students used this strategy for the improper fraction problem type for equal sharing problems for both tasks. In other words, this strategy was limited to the proper fraction problem type for both tasks. Equal sharing situations build on students' knowledge of whole number division, where the numerator value is often greater than the denominator value. A division situation where the numerator value is less than the denominator value presented a new challenge for some students.

As a result, students who used this strategy for the proper fraction problem type found the last proper fraction problem (i.e., eight people sharing three items or 3/8) the most challenging to solve. Some students ended up with incorrect solutions, or "I do not Know" or it is a "tricky problem" as the answers.

Non-Anticipatory Coordination (NAC)

Non-Anticipatory Coordination strategy is more advanced than the NLC. Students at this level of coordination seem to understand fractions as parts without numerical relationships to the whole. Students who used this strategy coordinated the two equal sharing goals, albeit in solving the problems rather than at the inception of the problem solving. Students also utilized modeling through representations and drawings of their solution strategies. Two specific cognitive strategies were identified under this broad strategy (i.e., halving or repeated halving and trial and error).

No students used NAC for the co-constructed tasks, whereas two students used it for the most difficult improper fraction problem types. One student only used it for the three proper fraction problem types for the interpreted tasks even though the two problem conditions used the same values. The difficulty level of each format also remained relatively the same. One student used this strategy in two instances for the proper fraction problem types compared to two students who used it for the improper fraction problem types in one instance each. No students used this strategy for the improper fraction problem types that involved halving or repeated halving. This indicates the interpreted tasks appear more difficult compared to the co-constructed tasks; however, it is also possible that the problem types were easier. The improper fraction problem types appear more difficult for students in the interpreted tasks than the improper fraction problem types for students in co-constructed tasks.

Additionally, students found problems that can be solved through informal halving and repeated halving easier compared to problems that could not be solved with halving or repeated halving strategies. Hunt et al. (2017) posited that students who used the halving and repeated halving strategy might have difficulty determining how to further partition and deal out the remaining halves. Connor resorted to a trial-and-error strategy through the use of percentages to solve the halving and repeated halving problem types. He was successful solving the most straightforward tasks through trial and error.

For example, the student correctly solved "two sandwiches shared among five friends" by first giving 50% of each sandwich to each friend. Then he took 10% from the shares of the first four people to make up the share of the fifth friend. He concluded that each friend got a 40% share. However, as the problems became more complex, he lost track using this strategy. For example, for the problem involving sharing "three water bottles" among "eight friends," Connor first attempted to give 35%, 35%, 35%, 35%, 35%, 35%, 45%, and 45% to each person and discovered that this approach did not lead to fair shares. Next, he attempted another strategy by giving 50% to the first six sharers and deducting 13% from the shares of the first six friends to make up the shares for the last two friends. Connor then combined (13 x 6) ÷ 2 = 39 and discovered that the first six friends received 37 water bottles each and the last two friends received 39 water bottles each. He reasoned that this strategy was not correct and attempted to remove 12% from 50% from the first six friends' shares to make the shares for the last two friends. When this did not work, Connor gave up solving the problem.

Empson and Levi (2011) and Hunt et al. (2016) argued that the trial and error strategy is not an efficient strategy for solving equal sharing story problems especially when the problems become more complex as Connor's strategy exemplified. Connor ended up quantifying each person's share as a percentage or pieces. Hunt et al. (2017) argued students who use this strategy were using their knowledge of whole number counting to partition the given items into some number of pieces while solving the problem. However, students similar to Connor have yet to see the fractional parts in relation to the whole. Hunt et al. (2017) called this strategy a “rudimentary way[s] of partitioning used to cut the wholes into some number of pieces as opposed to a plan to make several parts that relate to the number of sharers” (p. 7). Like the NLC, at most 2 students used this strategy. This is promising because it leaves room for more students to use more advanced strategies (i.e., EAC or AC).

Emergent Anticipatory Coordination (EAC)

Students at this level of conceptualization understand fractions as parts that have countable relationships to the whole. This is the first instance of students’ fractional understanding, an understanding that is not yet fully formed as a fractional quantity (Hunt et al., 2017; Steffe & Olive, 2010). Students who used this strategy coordinated the two goals of giving equal shares and sharing everything before the partitioning activity. They also explicitly modeled their solution strategies. Thirty-nine percent (39%) of students used this strategy for the co-constructed tasks, whereas 54% of students used this strategy for the interpreted tasks; indicating a smaller number of students in the co-constructed tasks use this strategy when compared to the number of students in the interpreted tasks who used this strategy. In the co-constructed tasks, some students varied their strategy use and began to rely less on EAC strategies.

Hunt et al. (2017) referred to EAC as “unitizing one whole” (p.3) and argued this strategy is used by students who are developing their understanding of fractions in which they can repeat one of the parts created to remake the whole or non-unit fractions. Students who used the single additive strategy often ended up partitioning each item one at a time into the correct number of sharers and giving each sharer fair shares. They may also first distribute the wholes to an equal number of sharers and later partitioned the leftover whole(s) into an equal number of people sharing it. When asked to determine each person’s final share, most students who used this strategy struggled with the solution.

Some students responded that each person receives a certain slice, piece, or inch. When asked to convert their final answers to fractions, the solutions for the final shares were incorrect. These students focused on counting the numerator parts without regard to the denominator parts. Other students from the onset of the additive strategy partitioned correctly and quantified each person's shares with fractional symbols or notations. When asked to combine the fractional shares to form a single final share, the students often used "whole number bias," in which they added the numerator's values and the denominator's values to determine the final fractional share for each person. For example, in a co-constructed problem involving four people sharing three drinks, Rebekah partitioned the first two drinks into halves. Rebekah gave each person $\frac{1}{2}$. Next, the student partitioned the third drink into fourths and gave each person $\frac{1}{4}$. To make the final share for each person, the student combined $\frac{1}{2} + \frac{1}{4} = \frac{2}{6} = \frac{1}{3}$ using the whole number bias to obtain the final share for each person. Hunt and colleagues referred to this strategy as reflective of students with "emerging coordination of the creation of equal parts and exhausting the whole" (p.8).

Of particular interest is when Julie repeatedly halved the number of pizzas being shared and the number of friends sharing it. For example, for the problem involving "3 pizzas" being shared by "4 friends", Julie said that two friends will share 1.5 pizzas and went on to halve "2 friends" and "1.5 pizzas and concluded that each friend gets "0.75 pizza". However, when asked what 0.75 means in fraction form, she could not provide a response.

Anticipatory Coordination (AC)

Anticipatory Coordination is the least concrete and most abstract, mental or procedural strategy. Use of this strategy demonstrates an understanding of fractions as quotients, quantities, or numbers with multiplicative relationships. One interesting aspect of the findings from this study is that all students used this strategy for both the interpreted and co-constructed tasks for the first and easiest improper fraction problem type. The number of students who used this strategy for the next improper fraction problem type dropped rapidly for both interpreted and co-constructed conditions. The number dropped quicker for the interpreted condition than the co-constructed condition for the last improper and first proper fraction problem types. This signifies more students used this strategy for solving co-constructed tasks compared to interpreted tasks.

Various levels of prompting may have slightly influenced the findings of the study. Researcher prompting included asking the students to provide the needed variables in terms of their food or drink preferences, names of friends, siblings or teachers, and the researcher then used the provided information to frame the co-constructed equal sharing story problems. However, specific strategies associated with AC are basic or long division. Students used basic or long division operation to share the items with an equal number of sharers, and they made use of an $\frac{a}{b}$ quotient in which the students used the formal $\frac{a}{b}$ multiplicative strategy to determine each person's shares. Most students who used AC used the long division strategy and ended up with decimal solutions. These students often used a calculator for support while performing the division algorithm. When students were asked to convert decimal answers to fractional solutions, results varied; however, one student used the quotient $\frac{a}{b}$ strategy and provided a formal definition of this strategy.

Overall, the findings of this study demonstrate most DHH college students used advanced strategies to solve the equal sharing tasks in either format. Consequently, in the interpreted and co-constructed tasks, nine students used AEC and AC, the most advanced cognitive strategies for solving the equal sharing story problems. Further, findings identified three groups with varying levels of fractional understanding with respect to the context of equal sharing story problems, (a) students who were just beginning to understand the concept of fractions; (b) students who were developing their understanding of the concept of fractions; and (c) students who had a solid understanding of fraction concepts.

The current study extends and confirms prior research on students' cognitive strategies for solving equal sharing story problems (e.g., Empson et al., 2006; Hunt et al., 2017). Specifically, students who are DHH albeit in college evidenced equal sharing strategies similar to those reported in prior studies for typically achieving students and students with special needs (Empson et al., 2006; Empson & Levi, 2011; Hunt et al., 2017). Thus, given an opportunity to solve and reason about their solution strategies to equal sharing story problems, DHH college students used various cognitive strategies similar to those found for hearing typically achieving students, students with mathematical difficulties, and students with special needs.

As found in previous research, equal sharing problem types and the numbers used in the problems influenced the students' partitioning strategies. Improper fraction problem types that involved an odd number of sharers and proper fraction problem types appear more difficult for students to solve compared to improper problem types with an even number of sharers.

As students gain partitioning experience their cognitive strategies for equal sharing become more sophisticated. The findings of this study lend additional support to the fact that DHH students' strategies for equal sharing story problems are not different from strategies used by typically developing hearing students. This is not to say that no new strategies distinct to DHH students were found. Instead, this study identified some distinct cognitive strategies for DHH college students compared to prior research. For instance, a student used a trial and error strategy based on percentages, and many participants who used the most sophisticated strategy ended up with decimal answers that some students could not convert to fractional equivalences.

In summary, six participants used an abstract strategy. This signifies a solid fraction understanding. However, three participants used a more concrete strategy. This signifies an emerging fraction understanding. Finally, four participants used a mixture of both strategies where these students used an abstract strategy when the equal sharing tasks were easy and resorted to concrete strategies when the tasks appeared more complex (Empson et al., 2006; Hunt et al., 2017).

Executive Function

Several executive functions or metacognitive skills essential to solving equal sharing story problems were identified. Executive Function is made up of cognitive processes (Bull & Lee, 2014), used to direct, control, supervise, and coordinate other cognitive processes in the service of goal-directed activities. Problem understanding is one aspect of Executive Function essential to solving equal sharing story problems. Hintermair (2013) and Marchark et al. (2018) termed understanding of the problems as the students' ability to understand other people's mental states, equal sharing story problems presented in two different contexts. Which means, first, do students understand the interpreted equal sharing story problems as presented to them? Second, do they know the expectations of the co-constructed equal sharing story problems?

Scheetz (2004) contended that students' understanding of the story problems will necessitate a change in behavior such as the coordination of the objects being shared with the number of people sharing the objects. All students across both conditions (i.e., interpreted and co-constructed tasks) showed understanding of equal sharing story problems through their retellings and co-constructions with or without the research's support before attempting to solve the story problems. In retelling, students used the most relevant information (number of people sharing the items and the number of items being shared) prior to the onset or while solving the equal sharing story problems. They provided the retell verbally, in writing, or drawing.

Equal fraction problems, worded in complex English texts, can confuse DHH students, most of whom are English language learners (Mousley and Kurz, 2016). Through both the interpreted and co-constructed tasks, language barriers often associated with equal sharing story problems presented in printed English were removed in this study. Accurate retellings of the equal sharing story problems showed students' understanding of the problems, and placed the focus of the study entirely on understanding the students' cognitive strategies for solving the equal sharing story problems. All students correctly, with or without support, understood the expectations of the problems before proceeding to solve them. Students often repeated the two essential variables during the retelling of the interpreted or co-constructed tasks (i.e., the number of items being shared and the number of people sharing the items).

Dogan and Hasanoglu (2016) stated the importance of recognizing students' working memory processing limitation which may adversely affect their performances in problem solving tasks. In this study, a correct retelling represented adequate working memory capacity and the ability to simultaneously remember and process information presented in a short time frame while engaging in cognitively demanding tasks (Purpura & Ganly, 2014). In the same vein, Techaraungrong et al. (2015) argued that students' language fluency and the linguistic demands of tasks might exceed students' working memory capacity and consequently impair their understanding of the information and their problem-solving capability. Participants found the interpreted tasks as including too much information that taxed their short-term memory capacity. To overcome this difficulty, participants shadowed the interpreter, watched the videos over several times and focused on the relevant information in the story problems.

Students were at least fluent in ASL or used sign in presenting the equal sharing story problems, and all of them scored above three on the ASLPI rating scale. For the equal sharing story problems, the translation of the printed English texts into interpreted tasks and the co-construction activity lessened the linguistic demands on them and consequently, may have improved the students' understanding and their performance.

Some information (e.g., give each person equal shares, how much/many each person gets) essential to solving the problems were conspicuously omitted in the retelling. Mayer (2019) termed this the exclusion of irrelevant information from the tasks. Thus, as Bull and Lee (2014) pointed out, successful problem-solving rests in part on suppressing irrelevant information. Instead, the students focused on three variables: the number of sharers, the number of items being shared, and the amount each sharer got. Other Executive Function skills such as metacognitive skills are discussed below.

Metacognitive Skills

At least three metacognitive skills were employed by students while solving the equal sharing problems. First, students on their own or with the researcher's prompting asked for the replay of the interpreted videos for several reasons presented in chapter four (e.g., not understanding the equal sharing prompt). Second, students mimicked with signs or in written form the context of the story problem as the interpreted videos played. Finally, students showed self-regulation skills by checking the correctness of their solution strategies and taking corrective measures when necessary or appropriate.

Student Feedback and Support

At the end of the interpreted and co-constructed tasks, students offered their viewpoints about the tasks and the language used in each format. Many of the students concurred that the co-

constructed tasks were easier than the interpreted tasks for reasons stated in the findings (e.g., students noted that the co-constructed tasks connected to their preferences for items and the people they know). Students also suggested improving interpreted tasks by including picture representations of the story problem alongside the written and interpreted equal sharing story problems.

One unexpected support allowed in the study was the use of a calculator and computer. However, as mentioned previously, students who used the calculator got the correct decimal answers and struggled with converting answers to fractions. Students also made increasing use of their fingers in place of drawing on paper as a manipulative.

Limitations of the Study

The current study acknowledges a few limitations. First, the number of participants in the present study was limited to 13 DHH college students, limiting the findings' generalizability to the whole population of DHH students in the United States. Results need to be confirmed with larger samples of DHH college and pre-college students. One suggestion is to conduct research with DHH students in K-12 schools for the deaf and public schools serving DHH students.

A second limitation is the sampling technique for selecting participants. The study used a convenience sampling approach in recruiting DHH college participants who met the inclusion criteria (i.e., deaf or hard of hearing, college students, who used sign language, who were18 years or older, and who consented to participate). Convenience sampling, a nonprobability or nonrandom sampling technique, was used in selecting participants who were easily accessible, geographically near, available, and willing to participate (Etikan, 2016). Thus, not all members of the DHH population were given an equal opportunity to be selected to participate. This limits the generalizability of the research findings to the whole population of DHH students.

Most students who participated in the study varied in their mathematic proficiency level. When asked to rate their proficiency, the results ranged from below average to above average. This may have affected their performance in the equal sharing story problems regardless of the format used. Future studies should consider using a stratified sample to include students below average, average and above average in mathematical proficiency.

Third, the interpreted condition was presented to the participants first, and the problems followed an order of improper to proper fractions. The co-constructed condition followed the interpreted condition in the same order of difficulty of the tasks. It is possible the strategies used in solving interpreted tasks helped them to do well in the co-constructed tasks. More research is needed to determine if the presented order of the tasks influence results. Future research should consider alternating the order.

Moreover, each student's time to complete each task and each condition could offer additional insights into the documented findings of the current study. It was observed but not coded that in both conditions one of the students spent more than two hours to complete the tasks in both conditions compared to the other students. Some students spent less or more than an hour to complete the tasks in both conditions.

Furthermore, the interpreted condition had three improper fraction problem types and three proper fraction problem types. However, the co-constructed condition had three improper fraction problem types and one proper fraction problem type. This makes comparisons of the students’ strategy use off balance for both conditions. The last two proper fraction problem types were eliminated from the interpreted tasks to make comparisons possible. Future research should ensure the number of tasks are the same for both conditions while maintaining the same difficulty level with various story problems.

Implications for Teaching

Findings of this study have several implications for practitioners. Firstly, as the participants' cognitive strategies vary, so do their demographics. Specifically, 54% of participants were students from culturally and linguistically diverse backgrounds. Additionally, 78% reported having hearing parents and ASL as their primary mode of communication. Therefore, practitioners should consider these variables in their instructional practices and curriculum development as they tap into the students' rich mathematical knowledge.

Secondly, practitioners should focus instruction more on conceptual understanding of fractions and develop procedural understanding simultaneously. Specifically, linking classroom instructions to student's prior knowledge and preferences to include home language and cultural values in instruction may be helpful. Additionally, using students' preferred mode of communication during instruction may be beneficial to students.

Thirdly, the identified cognitive strategies students demonstrated can elicit initial conversations on leveraging assessments to support students' engagement and participation in important mathematics content and practices.

Finally, the findings of this study have implications for the type of assessment used with DHH students by researchers and practitioners. DHH student's mathematical knowledge is often assessed and scored as either right or wrong, resulting in labeling them as deficient, not ready, unable, or empty vessels that need to be filled. When DHH students' performance is viewed in these ways, classroom practices have focused on "fixing" them. Hence, the findings of this study can inform initial conversations for designing practical classroom assessments that focus on developing the concept of fractions as quantities in DHH students.

Ginsburg (1997) alluded to the importance of using the TAP or clinical interview technique for assessing students' fractional thinking. Traditional standardized tests that have dominated assessment practices, Ginsburg argued, do not elicit the students' verbalizations about the strategies and methods they use to create solutions to the tasks. Ginsburg suggested a radically different assessment approach that assigns common questions, modifies the common questions in response to students' reactions, asks follow-up questions, challenges the students' responses, and asks students to explain their solution strategies. The assessment technique employed in this study aligns with Ginsburg's proposed use of Think Aloud Protocol or clinical interviews that treats each student differently. Verbalizations provide insights into the students' thinking. On the other hand, standardized methods do not provide adequate insights into students' cognitive functioning and consequently are not an effective technique for understanding DHH students' thought processes.

Findings from this study provide documented evidence of similar cognitive strategies within DHH students as other groups, that vary with the problem contexts and sophistication used by students in general and special education. The findings challenge the notion that DHH students cannot engage in challenging mathematics. Instead, DHH students can engage in meaningful mathematic assessment when presented in meaningful and accessible conditions. The findings further suggest that assessment experiences based on student's cognitive strategies benefit the student's cognitive development and mathematic proficiency.

Moreover, DHH students' mathematical difficulties found in the current study include understanding fractions as quantities, coordinating parts with the wholes, and using the notions of multiplicativity when partitioning fractions. Prior research has found that many students who are DHH have an undeveloped concept of multiplicative structures (e.g., Nunes, 2009) and one reason could be due to the students' limited conception of fractions as a part-whole relationship. Students did not understand other ways to think about fractions, such as quotient or equal sharing story problems, which is the foundation for fraction understanding.

Some students in the current study showed propensities to use and partition representations and represent each person's shares as several pieces, inches, percent, or other informal fractional names they created while solving the equal sharing story problems. Using representations to model the sharing activities explicitly may serve as the beginning of developing fraction understanding. Still, it may be inadequate in solidifying the student's understanding of fractions as quantities. A lack of fraction understanding as quantity may be partly due to (a) the students' underdeveloped fractional knowledge rather than a disability, and (b) how researchers have tested these students in second English language or with an interpreter in prior research.

Tests given in English or with an interpreter can increase students' cognitive load and decrease performance. In addition, a part-whole conception of fractions is insufficient to aid students' understanding of fractions and their performance. Students could benefit from a multiplicative understanding of fractions as quantities (Nunes et al., 2009). Thus, developing students' conceptual understanding of fractions in terms of coordinating the two quantities, multiplicative structures or numerical composites as templates for partitioning fractions may build the fractional conceptual understanding of DHH students.

Finally, Students primarily used the quotient definition of fractions as division ($a \div b$) rather than the result of division (i.e., $\frac{a}{b}$). Therefore, it is imperative for fraction instruction based on equal sharing story problems to focus teaching on the result of division interpretation of fractions. Mousley and Kurz (2016) reported that in third grade, students develop an understanding of fractions as numbers beginning with unit fractions in which the size of a fractional part is relative to the size of the whole and in fourth grade, they understand fraction equivalence and ordering in terms of size to finally perform operations. As hypothesized at the onset of this study, a larger percentage of students who participated in the study used the most advanced strategies for solving the equal sharing story problems. It is important to point out that many students used the division strategy to solve the equal sharing story problem, thereby ending with decimal answers; however, they could not in most instances convert their solutions to fractional results.

Implications for Research

Findings of the current study confirm and add to the existing research on low- and high-achieving students' conceptions and strategies for solving equal sharing story problems (e.g., Empson et al., 2006; Hunt et al., 2017). DHH students, given an opportunity to solve problems and think about the strategies they used, demonstrated different ways of thinking about fractions. However, they used similar strategies as hearing and typically performing students. Researchers in the field of deaf education and deaf mathematical education specifically, who seek to discover and understand DHH students' existing and later mathematics competencies, may find the results of this study a useful foundation for their future research undertakings.

DHH students' fractional reasonings in the current study were consistent with prior research (e.g., Empson et al., 2006; Hunt et al., 2017). Additional research is needed to confirm, disconfirm, expand, or extend the current study's findings with DHH students in colleges to DHH students in K-12 schools who vary in their mathematical abilities. Such research might relate fraction symbols to an equal-sharing analogy (result of division meaning) of fractions (e.g., $\frac{2}{3}$ = two wholes shared equally among three), build on students' informal knowledge, and connect part-whole meanings of fractions to equal sharing story problems. Moreover, future research designs might include a pretest-posttest design with or without random assignment of participants, a single-subject research design, and a comparison group to test for significant effects of different types of instruction (Hunt et al., 2017). In addition, future researchers may consider the use of a co-constructed approaches to assessing DHH students' cognitive strategies for solving mathematic story problems and, in particular, fraction story problem.

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APPENDIX A: SAMPLE SITE LETTER

5230 S M L King Jr Pkwy

Apt. 234

Beaumont, TX 77705

Date

Administrator's name

School's address

<u>Re: Permission to Conduct Research Study</u>

Dear Mr./Mss./Mrs.________________________,

I am writing to request your permission to conduct a research study at your college. I am currently a PhD candidate for a Special Education Program at the University of North Carolina, Greensboro (UNCG). My concentration focuses on deaf and hard of hearing students' conceptual understanding of fractions. I am in the process of conducting my dissertation study entitled "Deaf and Hard of Hearing College Students' Strategies for Equal Sharing Problems". More information about this study can be found in the attached "study information sheet" below.

I hope the school administration allow me to recruit teachers and students to participate in the study. If the approval is granted and in line with Institutional Review Board (IRB) mandates, consents will be obtained from the students. If your college has separate or additional IRB requirements other than what has been approved at UNCG, please let me know. I will be happy to comply with additional requirements from your college.

I will appreciate your approval to conduct this study at your college. I look forward to hearing back from you soonest. I will send a follow up email next week if I do not hear from you this week. In the meantime, I will be happy to answer any questions or concerns you may have in respect of this study. You may contact me at my email address soadeoye@uncg.edu

If you agree for me to conduct the study at your college, kindly sign below and return the signed form through my email address. Alternatively, you can submit a signed letter of permission on your school's letterhead acknowledging your consent and permission for me to conduct the study at your school.

Thank you very much!

Yours Sincerely,

Sulaiman Adeoye

Cc: Dr. Shaqwana Freeman-Green, Faculty Advisor, UNCG

Approved by:

______________________________ ____________________________ ______________

Print your name and title Signature Date

APPENDIX B: FOLLOW UP SITE LETTER

Dear Mr./Mrs.______________

I hope this email finds you well. I want to follow up on the above subject matter. I sent you a letter via an email dated______________________on the above subject matter requesting for your permission to conduct a research study titled "deaf and hard of hearing college students' fractional thinking" at your college. I am yet to receive your reply to this request and I want to check to make sure that you receive my letter. Please, would you kindly let me know if you get this letter. I will be happy to resend it if necessary. Otherwise, I look forward to hearing back from you within a week.

I will really appreciate your reply! Thank you!!

Sulaiman Adeoye

APPENDIX C: RECRUITMENT LETTER AND INFORMED CONSENT

Request to Participate in a Research Study

Dear Invitee,

My name is Sulaiman Adeoye. I am a doctoral candidate at the University of North Carolina's Special Education Program at Greensboro. I am kindly requesting your participation in a study that I am conducting on "Deaf and Hard of Hearing College Students' Fractional Thinking". The goal of this study is to investigates and documents the thinking strategies that deaf and hard of hearing (DHH) college students employ as they solve 10 equal sharing fraction story problems presented to them through ASL interpreted video and co-constructed conditions.

You will be asked to complete basic demographic information and to solve six equal sharing fraction problems presented through interpreted video format and four co-constructed equal sharing fraction problems.

It will take approximately 60-90 minutes to complete the fraction tasks. Your participation in this study is completely voluntary. You are free to withdraw your participation from the study at any time. All information collected will be kept confidential, anonymous and well protected. For your time and effort, you will receive a token gift certificate valued at $25.

All interviews and interactions will take place online through Zoom or other video conferencing platform.

I attached the consent form for your review. Please read, sign and return the consent form if you would like to participate in the study. If you have questions or need further clarifications about the study, feel free to contact me through my email address soadeoye@uncg.edu. Your participation will be greatly appreciated.

I look forward to hearing from you. Thank you very much!

Sincerely,

Sulaiman Adeoye, M.A; M.S; Doctoral candidate, University of North Carolina, Greensboro.

Letter of Consent

University of North Carolina, Greensboro

Consent to Act as a Human Participant

Project Title: Deaf and Hard of Hearing College Students' Fractional Thinking

Principal Investigator and Faculty Advisor: Sulaiman Adeoye and Dr. Shaqwana Freeman-Green

Participant's Name: ____

What are some general things you should know about research studies?

You are being asked to take part in a research study. Your participation in the study is voluntary. You may choose not to join, or you may withdraw your consent from participating in the study at any time, for any reason, without penalty.

Research studies are designed to obtain new knowledge. This new information may help people in the future. There may not be any direct benefit to you for your participation in the research study. There also may be risks to being in research studies. If you choose not to be in the study or you choose to leave the study before it is done, it will not affect your relationship with the researcher or the University of North Carolina, Greensboro. Details about this study are discussed in this consent form. It is important that you understand this information so that you can make an informed choice about your participation in this research study.

You will be given a copy of this consent form. If you have any questions about this study at any time, you should ask the researchers named in this consent form. Their contact information is below.

What is the study about?

This is a research project that investigates and documents the thinking strategies DHH college students employ to solve 10 equal sharing story problems. Your participation in this study is voluntary. You can withdraw from this study at any time.

Why are you asking me to participate?

I am inviting you to participate because you are 1) deaf or hard of hearing, (2) attending college student and (3) using ASL.

What will you do if I agree to participate in the study?

I will ask you to solve 10 equal sharing story problems. The 10 problems are divided into two parts. The first part has six problems that are interpreted in ASL by interpreter and video recorded. You will watch the videos and solve the problems. I will ask you questions on how you are solving the problems from time to time in order to understanding your thinking strategies. You will explain how you solve the problems to me. The second part has four problems. You and I will develop these problems together. I will then ask you to solve the problems. I will ask you questions on how you solve the problems from time to time. You will explain how you solve the problems to me. You will probably work on all the 10 problems for about 60 to 90 minutes. You will also be asked to complete the demographic survey questionnaires.

Is there any audio/video recording of me?

I will videotape you as you solve the problems, when I ask you questions and when you respond. The videotape will assist with the data analysis. Because your voice or image will be potentially identifiable by anyone who hears or sees the recording, your confidentiality for things you say, do or signs on the recording cannot be guaranteed although the researcher will try to limit access to the recording as described below. I will protect the video records by saving it on the University's Box which is password protected. I will not show the recorded videos of you in the classrooms or at a conferences except to my research supervisor. Instead, I can video record myself signing what you signed. This means I will copy your signs and then show the video of my signing at conferences. That way no one will know who you are. I may also write out a description of what you signed.

What are the dangers to me?

There is minimal risk to you for your participation in this study. The study may take 60-90 minutes of instruction time you may need to be in your classrooms. As a safeguard against this risk, the study may occur during your class recesses. The Institutional Review Board at the University of North Carolina, Greensboro has determined that participation in this study poses minimal risk to participants. If you have questions, want more information or have suggestions, please contact the Principal Investigator, Sulaiman Adeoye at (336) 383-0872 or through email at soadeoye@uncg.edu and the Faculty Advisor, Dr. Shaqwana Freeman-Green at smfreem3@uncg.edu.

If you have any concerns about your rights, how you are being treated, concerns or complaints about this project or benefits or risks associated with being in this study please contact the Office of Research Integrity at UNCG toll-free at (855)-251-2351.

Are there any benefits to society as a result of me taking part in this research?

Researchers, curriculum developers, teachers, and students may benefit from this study. Researchers may build on the findings of this study by developing effective interventions for improving the achievement of DHH students in fractions and mathematics. Curriculum developers may capitalize on research findings and develop curriculum materials for the use of DHH students in the classrooms. Teachers may employ the materials and research-based intervention strategies to improve the instruction and achievement of DHH students in fractions and mathematics.

Are there any benefits to me as a result of participation in this research study?

If you participate, you may learn new strategies to solve fraction problems. However, there may be no benefit to you at all.

Will you get paid for being in the study? Will it cost me anything to be in this study?

You do not have to buy anything or spend money to be in this study. You will receive a gift certificate of $25.00 to thank you for your participation

How will my information be kept confidential?

All data will be coded and kept separately from any identifiable information such as the consent forms. The data will be uploaded into the University's Box which is password protected and encrypted. Pseudonyms will be used to identify participants in the study when the results are disseminated. All information obtained in this study is strictly confidential unless disclosure is required by law. In five years, the data will be destroyed.

What if I want to leave the study?

You have the right to refuse to participate or to withdraw at any time, without penalty. If you withdraw, it will not affect you in any way. If you choose to withdraw, you may request that any of your data which has been collected be destroyed unless it is in a de-identifiable state (I have taken your name off the file). The investigators also have the right to stop your participation at any time. This could be because you have had an unexpected reaction, or has failed to follow instructions, or because the entire study has been stopped.

What about new information/changes in the study?

If significant new information relating to the study becomes available which may relate to your willingness to continue to participate, this information will be provided to you.

Voluntary Consent by Participant:

By signing this consent form/completing this survey/activity (used for an IRB-approved waiver of signature) you are agreeing that you read, or it has been read to you, and you fully understand the contents of this document and are openly willing consent to take part in this study. All of your questions concerning this study have been answered. By signing this form, you are agreeing that you are 18 years of age or older and are agreeing to participate, in this study described to you by: ________________________

Signature: ________________________ Date: ________________

Study's Information Sheet on the Study

Project Title: Deaf and Hard of Hearing College Students' Strategies for Equal Sharing Problems

Principal Investigator: Sulaiman Adeoye

Faculty Advisor: Dr. Shaqwana Freeman-Green

What is this all about?

This is a research study that investigates and documents the thinking strategies of college students who are deaf and hard of hearing (DHH) while solving six constructed and four co-constructed equal sharing fraction story problems. This research project will only take about 60-90 minutes of each student time and will involve students who meet the inclusion criteria: (1) deaf and hard of hearing (2) college students (3) Use American Sign Language. Your participation in this research project is voluntary.

How will this negatively affect me?

No, other than the time participants will spend on this study. There are no know or foreseeable risks involved with this study.

What do I get out of this research project?

Researchers, curriculum developers, teachers, and participants may benefit from this study. Researchers may build on the findings of this study by developing effective interventions for improving the achievement of DHH students in fractions and mathematics. Curriculum developers may capitalize on research findings and develop curriculum materials for the use of DHH students in the classrooms. Teachers may employ the materials and research-based intervention strategies to improve the instruction and achievement of DHH students in fractions and mathematics. Students may recognize various strategies to use in solving these types of division problems and/or may increase his or her understanding of fractions.

Will I get paid for participating?

There are no costs to you. You will be compensated with $10 worth of gift certificate as a result of participation in this study.

What about my confidentiality?

We will do everything possible to make sure that participants information is kept confidential. All information obtained in this study is strictly confidential unless disclosure is required by law. We will not use identifying information in the coding scheme and in the report of the study. . Coded data will be kept separate from identifiable information such as the consent forms. Data collected will be stored in a password protected University approved online cloud storage such as Box that is encrypted.

What if I do not want to be in this research study?

You do not have to be part of this project. This project is voluntary and it is up to you to decide to participate in this research project. If you agree to participate at any time in this project, you may stop participating without penalty.

What if I have questions?

If you have questions, want more information or have suggestions, please contact the Principal Investigator, Sulaiman Adeoye at (336) 383-0872 (text only) or through email at soadeoye@uncg.edu and the Faculty Advisor, Dr. Shaqwana Freeman-Green through email at smfreem3@uncg.edu.

If you have any concerns about your rights, how you are being treated, concerns or complaints about this project or benefits or risks associated with being in this study please contact the Office of Research Integrity at UNCG toll-free at (855)-251-2351.

APPENDIX D: DEMOGRAPHIC INFORMATION QUESTIONAIRES

Click the link to complete the demographic survey:

https://forms.gle/gUdozbV2VF8GnU2n7

Participant's Name: ____

(please print)

Participant's Date-of-Birth: _______/_________/_________

(mm/dd/yr.)

Hearing Loss Unaided: _____________left _________________right

Identified Disabilities (not deafness): ___________________________

Participant's College: ________________________________

Mathematics classes taken: ________________________________

Gender: ___________(M or F

Parents' Hearing status:

- Father: deaf, hard of hearing, hearing, NA (select one)
- Mother: deaf, hard of hearing, hearing, NA (select one)

Deaf sibling at home:

- Yes
- No

Participant's communication at home:

- signing alone
- voice and signing all the time
- voice and some signs and fingerspelling
- using voice only

- Other:_______________________________________

Participant’s Primary Mode of Instruction in the Classroom (choose one)

- ASL only
- Signed English (no voice)
- Sign & Speech (simultaneous communication)
- Oral/Aural only
- Cued Speech
- Other: ____________________________________

Assistive Listening Devices used in the classroom by participant (select one):

- Hearing Aid (select one)
- FM System (select one)
- Cochlear Implant (select one)
- Other (specify and select one): ____________________________________

Participant’s ASLPI Proficiency Level. Watch the three videos across all levels starting from level 5 way down to level 1. Select the level that match your current signing skills https://www.gallaudet.edu/the-american-sign-language-proficiency-interview/aslpi/aslpi-proficiency-levels.

- Level 5
- Level 4
- Level 3
- Level 2
- Level 1

APPENDIX E: TWO PRACTICE TASKS

Task 1: 4 ÷ 2

English:

Two friends share four candy bars so that each person gets the same amount to eat. How many candy bars each friend eats if they finished all the candy bars?

Gloss:

____t_______________ ______t______

FOUR CANDY BAR++_rt TWO FRIENDS SHARE C- CL: =

_______________________________________whq_______________________

HOW-MANY EACH FRIEND+++rt "onto fingers of 2-HS" HOW MANY

Task 2: 7 ÷ 2

English:

Two friends share four candy bars so that each person gets the same amount to eat. How many candy bars each friend eats if they finished all the candy bars?

Gloss:

____t_______________ ______t______

SEVEN CANDY BAR++_rt TWO FRIENDS SHARE C- CL: =

_______________________________________whq_______________________

HOW-MANY EACH FRIEND++rt "onto fingers of 2-HS" HOW MANY

APPENDIX F: SIX PRE-CONSTRUCTED FRACTION TASKS

Task 1: 5÷ 2

English:

Susan and Juan shared 5 soft tacos so that each of them got the same amount to eat. How many tacos did Susan and Juan each eat if they finished all the tacos?

Gloss:

___________________t________________

GIRL NAME S-U-S-A-N BOY NAME J-U-A-N INDEX SHARE FIVE TACO+++_rt FIVE

UNDERSTAND MUST SAME++ “across fingers of 2-HS”

___________________whq________________

HOW-MANY EACH++ “onto fingers of 2-HS” HOW MANY

Task 2: 14÷ 4

English:

14 sticks of clay are shared among 4 children for a project. How much sticks does each child receive?

Gloss:

____t_______________ ______t______

FOURTEEN STICKS++_rt BREAK-GIVE FOUR CHILD++_rt

UNDERSTAND MUST SAME++ “across fingers of 4-HS”

____whq________________________

HOW-MANY EACH CHILD++rt "onto fingers of 4-HS" HOW MANY

Task 3: 4÷ 3

English:

3 friends share 4 large chocolate bars so that they all get the same amount. They eat all the chocolate bars. How many cookies does each friend get?

Gloss:

____t______________ ______t______

FOUR CHOCOLATE BAR++_rt BREAK-GIVE THREE FRIEND++_rt

UNDERSTAND MUST SAME++ "across fingers of 3-HS"

____whq_______________________

HOW-MANY EACH FRIEND++rt "onto fingers of 3-HS" HOW MANY

Task 4: 3÷ 4

English:

4 friends share 3 small pizzas. If they each want the same amount and share all of the pizzas, how much pizza does each friend get?

Gloss:

____t______________ ______t______

THREE SMALL PIZZA++_rt BREAK-GIVE FOUR FRIEND++_rt

UNDERSTAND MUST SAME++ "across fingers of 4-HS"

____whq_______________________

HOW-MANY EACH FRIEND++rt "onto fingers of 4-HS" HOW MANY

Task 5 : 2÷ 5

English:

5 friends share 2 submarine sandwiches so that each friend gets the same amount. How much sandwich does each friend receives?

Gloss:

____t______________ ______t______

TWO SANDWICH++_rt BREAK-GIVE FIVE FRIEND++_rt

UNDERSTAND MUST SAME++ "across fingers of 5-HS"

_______________________________________whq______________________

HOW-MANY EACH FRIEND++rt "onto fingers of 5-HS" HOW MANY

Task 6: 3÷ 8

English:

There are 3 bottles of water that 8 people want to share equally. How much of the water does each person receive in his or her cup?

Gloss:

____t______________ ______t______

THREE BOTTLE WATER++_rt BREAK-GIVE EIGHT PEOPLE++_rt

UNDERSTAND MUST SAME++ "across fingers of 8-HS"

_______________________________________whq______________________

HOW-MANY EACH FRIEND++rt "onto fingers of 8-HS" HOW MANY

Note: From “Exploratory Study of Informal Strategies for Equal Sharing Problems of Students With Learning Disabilities by J. H. Hunt, and S. B. Empson, 2015, *Learning Disability Quarterly*, *38*(4), p. 213 (https://doi.org/10.1177/0731948714551418

APPENDIX G: CO-CONSTRUCT FRACTION TASKS

The four co-constructed tasks reflect the same format as that of the six pre-generated tasks. The first two co-constructed tasks contain two to four sharers, lending the two problems to the use of halving and repeated halving strategies. The third task does not readily lend itself to the halving strategy. The first three tasks taken together will result in fractional answers greater than one (i.e., the number of items being shared is greater than the number of sharers). The fourth task is more difficult and result in fractions less than one.

Provide the prompts you will use to ask the participants to co-construct the 4 problems you will then have them solve

#1

Who is your best friend? (repeat to confirm participant's response).

What is your favorite food? (repeat to confirm participant's response).

How would you share 5 (name of food provided by the participant) between you and your friend (mention name of friend provided by the participant) ?

Make sure you and your friend (name) get equal share

How much (name of food provided by the participant) you get?

How much food your friend (name) gets?

#2

Tell me about your friends. How many are they? (repeat to confirm participant's response). Who are they? (repeat to confirm participant's response).

What is their favorite candy? (repeat to confirm participant's response).

How would you share 14 (name of candy provided by the participant) candies between you and your three friends (mention the names of three friends provided by the participants)?

Make sure each friend gets equal share

How much each friend gets?

#3

Tell me about your brothers and sisters. How many are they? (repeat to confirm participant's response). Who are your brothers and sisters? (repeat to confirm participant's response).

What is your other favorite food? (repeat to confirm participant's response).

How would you share 4 (name of other favorite food provided by the participant) with your 3 siblings (mention names of sisters or brothers provided by the participants) so that everyone gets equal shares?

How much each person gets?

#4

Tell me about your four favorite teachers at your school (repeat to confirm participant's response). Who are the teachers? (repeat to confirm participant's response).

What drink do they like? (repeat to confirm participant's response).

How would you share 3 (name of drink provided by the participant) among your 4 teachers (mention names of teacher provided by the participants)?

Make sure each teacher gets equal share

How much each teacher gets?

APPENDIX H: INSTRUCTIONS

“I am interested in how students think and the strategies they use to solve story problems. We will start with two practices together (I will ensure that the students understand these practices before proceeding). In these two practices. For the first practice, you will watch video of the ASL version of the story problem. You watch the videos as many times as you want. For the second practice, we will co-construct the story problem together. After seeing the video and the co-constructed problem, you will retell the story in your own words. Then you will sign aloud (think aloud) how you solve the problems to get your answers. After your explanations, you will write and draw on the provided paper, how you solve the problems. You will use strategies that make sense to you. You can use the virtual manipulative materials I provided to support your thinking and solution. I will ask you questions at the end of your solutions to understand your thinking more. You will respond to my questions to the best of your ability. There are no wrong or correct answers to my questions. I am interested only in how you think and solve the problems. After we have completed the two practices, we will use the same administration procedures for you to solve six story problems presented in ASL and four co-constructed questions on your own. The four co-constructed questions do not have ASL translation videos”.

APPENDIX I: THE INTERVIEW/THINK ALOUD PROTOCOL

Interview categories	Possible examples
Check and support the child's understanding of the problems	• Could you retell me the problem, using your own words? • What is the problem saying to you or ask you to do?
Pose general starter questions to elicit the child's explanations/justifications	• Tell me what you did • Walk me through what you thought about.
Press the child for explanations/justifications of specific parts	• Why you stopped sharing out the whole tacos? They want to share this taco, too (if the shares are not exhausted) • Why did you partition the cookies into eighths? • What if each person wants the same number of cookies (if each person shares are not fair)?
Link the child's representations and the story context	• Tell me about your picture or symbols • Where are the tacos? • Where are the people sharing? • Did you create equal shares? • Did you exhaust the whole?

Connect the child's thinking to symbolic notation	• How many each person gets? • What symbols or notations will you use for a person share? • How many/much one person gets overall if combine all the shares? • What do we call that?

Note. From "Responding to Children's Mathematical Thinking in the Moment: An Emerging Framework of Teaching Moves" by V. R. Jacobs, and S. B. Empson, 2016, *ZDM: The International Journal on Mathematics Education*, *48*, p. 185–197 (https://doi.org/10.1007/s11858-015-0717-0)

General Questions

These questions will be posed to the participants at the end of both interpreted and co-constructed tasks. The goals are to assess the participants' feelings about language use and difficulty levels of the tasks.

1. Tell me how you feel about the difficulty levels of the interpreted tasks compared to co-constructed tasks.
2. How do you feel about the language use in the interpreted tasks compared to co-constructed tasks?

APPENDIX J: ASLPI RATING SCALE

Participant:_________________________________ Date:_________________

ASLPI Rating:_________________________

Grammar

__ Grammar almost entirely inadequate or absent.

__ Constant errors showing control of very few major patterns and frequently preventing communication of precise message.

__ Frequent errors showing some major patters uncontrolled and causing occasional irritation and misunderstanding.

__ Occasional errors showing imperfect control of some patterns but no weakness that causes misunderstanding.

__ Few errors, with no patterns of failure.

__ No more than two errors during the interview.

Vocabulary

__ Vocabulary inadequate for simple conversations.

__ Vocabulary limited to basic personal and survival areas (schedules, food, transportation, family, location of facilities).

__ Choice of vocabulary sometimes inaccurate, limitations of vocabulary prevent discussion of some common professional and social topics.

__ Professional vocabulary adequate to discuss special interests, general vocabulary permits discussion of any non-technical subject with some circumlocutions.

__ Professional vocabulary broad and precise; general vocabulary adequate to cope with complex practical problems and varied social situations.

__ Vocabulary apparently as accurate and extensive as that of an educated native speaker.

Fluency

__ Language is so halting and fragmentary that conversation is virtually impossible.

__ Language so slow and uneven except for short or routine sentences.

__ Language is frequently hesitant and jerky; sentences may be left uncompleted.

__ Language is occasionally hesitant, with some unevenness caused by rephrasing and grouping for words.

__ Language is effortless and smooth, but perceptibly non-native in speed and evenness.

__ Language on all professional and general topics as effortless and smooth as native signer.

Accent/Production/Pronunciation

__ Production is frequently unintelligible.

__ Frequent gross errors and very heavy accent make understanding difficult, requires frequent repetition.

__ Foreign-like/non-native accent requires concentrated listening and production errors lead to occasional misunderstanding and apparent errors in grammar or vocabulary.

__ Marked foreign accent and occasional mispronunciations that do not interfere with understanding.

__ No conspicuous mis productions but would not take for a native signer.

__ Native sign production with no trace of accent.

Comprehension

__ Understands too little for simple conversations.

__ Understands only slow, very simple delivery on common social and routine topics. requires constant repetition and rephrasing.

__ Understands discipline or interest-specific signing directed to him/her quite well, with considerable repetition and rephrasing.

__ Understands discipline or interest-specific signing directed to him/her quite well, but requires occasional repetition or rephrasing.

__ Understands everything in discipline or interest-specific conversation except for very colloquial or low-frequency items, or exceptionally rapid or slurred signing.

__ Understands everything in both formal and colloquial language to be expected of any educated native (Laird, 2005, p. 103-104)