CORPORATE BONDS: THEORETICAL AND EMPIRICAL STUDY

by

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ABSTRACT

MIN CHUL PARK Corporate bonds: theoretical and empirical study (Under the direction of DR. STEVEN P. CLARK)

This dissertation consists of three parts. The first chapter presents an analysis of the structural difference between a make-whole callable and a traditional callable bond. Based on the analysis, we construct a reduced-form model for the make-whole callable bond. The second chapter empirically investigates validation of our model with the extended Kalman filter. In this chapter, we show not only that our model is valid for the sequence of the make-whole callable bond behavior, but also that our model outperforms the model from Jarrow et al. (2010).

The third chapter examines the association between issuer's debt structure and yield spreads. Specifically, we investigate whether or not an investor requires compensation for liquidity risk. Diamond (1991) introduces liquidity risk as the risk of a borrower being forced into inefficient liquidation when refinancing is not available. According to Diamond's argument, the firm holding the larger proportion of short-term debt to its debt structure is more vulnerable to the unforeseen negative event. Consequently, it will increase firm's risk. Through our tests in this chapter, we find that for investment grade bonds, the results consistently show that the fraction of debt maturing in one or two year is positively related to the yield spreads.

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CHAPTER 1: ANALYSIS OF A BOND WITH MAKE WHOLE PROVISON WITH REDUCED FORM APPROACH

In this chapter, we demonstrate how to apply the reduced-form model to a bond with a make-whole provision. First, we present the structural difference between a callable bond with fixed call price and a bond with a make-whole provision. Based on the structural differences, we develop a reduced-form model for the bond.

The call price of a bond with a make-whole call provision is given as the maximum value between par value and the present value of the bond's remaining payments. The discount rate in the calculation of the present value is the prevailing comparable maturity Treasury yield plus a spread specified in the contract of a bond, which is called the make-whole premium. Therefore, this unique bond feature reduces the interest rate risk that the traditional callable bond possesses.

For this reason, according to Mann and Power (2003), the make-whole callable bond has been gaining popularity since its introduction in the U.S. in 1995. Power and Tysyplakov (2008) develop a structural model for the make-whole callable bond, but their model significantly underestimates the yield on the bond. The structural model, in general, relies on a contingent claim. However, Mann and Power (2003) argue that the make-whole callable bond is not structured as a refunding vehicle. Rather, it is structured to enable a firm to retire debt without relying on a tender offer when the firm needs to restructure its capital structure. Therefore, as Jarrow et al. (2010) note, the structural model has limitations on capturing this sub-optimal call policy.

This chapter develops the reduced form model for the make-whole callable bond. Performance and validation of our model is tested in Chapter 2 with two different econometric tools (Kalman filter and Markov Chain Monte Carlo). This chapter is organized as follows: Section 1 reviews related previous studies, Section 2 presents the structural difference between the bond with fixed call price and the make-whole callable bond, and develops a reduced model, and finally, Section 3 summarizes this chapter.

1.1 Literature Review

Theoretical models of credit spreads can be categorized as either structural or reduced-form models of default. Often, practitioners or scholars compare structural models with the reduced form-models for corporate bonds. Both approaches have advantages and limitations in valuing risky bonds. In this section, a literature review of structural models and reduced-from models is presented.

1.1.1 The Structural Models

The structural models that originated from Merton's model (1974) directly relate the price of equity to default probabilities and the price of corporate bonds. This model applies the claims-based approach to valuing corporate debt by using option pricing theory. This framework for valuing risk debt has been applied in a number of studies including Geske (1977), Ingersoll (1977a, 1977b), Merton (1977), Smith and Warner (1979), and many others. However, Jones, Mason, and Rosenfeld (1984) and Franks and Torous (1989) show that the structural models based on Morten's framework produce credit spreads much smaller than the actual credit spreads. In addition, Longstaff and Schwartz (1995) note that one of the drawbacks of this approach is that default is assumed to occur only when the firm exhausts its assets. This assumption is not very realistic because firms usually default long before the firm's assets are exhausted.

In order to overcome this drawback, Longstaff and Schwartz (1995) extend the Black and Cox (1976) model, which allows default to occur when the value of assets reach a lower threshold. Their model assumes that this lower threshold could be obtained exogenously from a minimum level of cash flows from assets requirement or from minimum net worth or working capital requirements in the indenture. The advantage of the Longstaff and Schwartz model over previous models is that their model incorporates both default risk and interest rate risk. In contrast to Longstaff and Schwartz (1995), Leland and Toft (1996) construct a model with an assumption that the lower threshold is determined endogenously. The endogenous threshold is derived with the view that bankruptcy is an optimal decision by equity holders to give their control of a firm to bond holders. The prominent achievement of this model is that it is able to show a tradeoff between tax advantages, bankruptcy costs, and agency cost. However, unlike the Longstaff and Schwartz model, the Leland and Toft model assumes a constant interest rate, which implies that it does not consider interest rate risk.

Commonly, these models do not consider the effect of a stationary leverage ratio on credit spreads. Opel and Titman (1997) provide empirical evidence for the existence of target leverage ratios at the firm level within an industry. Furthermore, dynamic models of optimal capital structure by Fisher, Heinkel, and Zechner (1989), and Goldstein, Ju, and Leland (2001) find that firm value is maximized when a firm acts to keep its leverage ratio within a certain band. Therefore, Collin-Dufresne and Goldstein (2001) develop a structural model of default with stochastic interest rates that generates stationary leverage ratios. In order to incorporate stationary leverage ratio, major assumptions of this model is that a firm has the option to increase leverage at some intermediate date between issuing date (or current date) and maturity date. It also assumes that if the firm exercises this option, it does so by issuing a zero coupon bond with the same maturity as previously issued debt. Furthermore, it assumes that the face value of the newly issued debt is chosen to reset firm leverage back to its initial target value. Finally, it assumes that the proceeds of new debt issuance are used to repurchase existing equity, leaving firm value unchanged. With these assumptions, the model is able to incorporate a stationary leverage ratio, allowing the firm to deviate from its target leverage ratio only over the short-run. Additionally, like the Longstaff and Schwartz model, this model also incorporates interest rate risk. Compared to previous models, their model generates larger credit spreads for firms with low initial leverage ratios, which could partially overcome the problem that structural models of default are well below those observed in practice (Jones, Mason, and Rosenfeld, 1984). Further, their model generates term structure of credit spreads for speculative-grade debt that are consistent with the empirical findings of Helwege and Turner (1999).

However, as documented by Lyden and Saraniti (2000), the structural models above tend to underestimate yield spreads. The errors are systematically related to coupon and maturity. As noted by Duffie and Lando (2001) and Lyden and Saraniti (2001), these prediction errors are related to the estimates of unobservable asset and its volatility. Duffie and Lando (2001) mention that, in practice, it is typically difficult for investors in the secondary market for corporate bonds to observe a firm's assets directly, due to noisy or delayed accounting reports, or barriers to monitoring by other means. Therefore, investors must conjecture value of a firm's asset with publicly available information such as accounting data or business-cycle data. In order to overcome this shortcoming of the previous structural models, Duffie and Lando (2001) strive for optimal capital structure and default policy, and then derive the conditional distribution of the firm's assets, given incomplete accounting information, along with the associated default probabilities, default arrival intensity, and credit spread. By incorporating the incomplete accounting information and default arrival intensity, their model is able to generate large credit spread than the previous models in the case of short-term maturity. The Duffie and Lando model is different from the traditional structural models in that the model includes default arrival intensity which is a typical characteristic of the reduced-form model. Thus, their model is rather a hybrid model than solely a structural model.

There are few empirical studies of comparing structural models. Anderson and Sundaresan (2000) empirically compare the original Merton model with the structural models that incorporate endogenous bankruptcy barriers, such as the Leland and Toft model. They find that Leland and Toft type model is somewhat superior to the original Merton model. Additionally, Lyden and Saraniti (2000) implement and compare two structural models, the Merton with the Longstaff and Schwartz models. They find that both models underestimate yield spreads, and that the errors are systematically related to coupon and maturity. However, in the study of Eon et al (2003), they find that the predicted spreads from the Morton model is too low, but that the other structural models predict spreads that are too high on average. Although these structural models perform reasonably well, the problems of using structural models are mathematical complexity and difficulty in estimating unobservable value of firm’s assets. For these reasons, many practitioners and scholars support an alternative approach, reduced-form models.

1.1.2 Reduced-Form Model

The Reduced-Form model has a relatively shorter history than the structural model. The unique feature of the reduced-from model is the intensity-based framework. The fundamental idea of the intensity-based framework is to model the default time as the first jump of a Poisson process. Major issues in the reduced-form model are the treatment of the recovery payment and the correlations between interest rates, intensities and recoveries.

Jarrow and Turnbull (1995) consider that the recovery rate is an exogenous fraction of the value of an equivalent default-free bond (Recovery of Treasury, RT). Duffie and Singleton (1999a) note that estimating value of the bond is computationally burdensome under RT. For this reason, Jarrow and Turnbull (1995) made assumptions that simplify computation. For example, they assumed that the risk-neutral default hazard rate process is independent of the short rate, and that the fractional loss process is constant. Under these assumptions, Schönbuncher (2003) notes that coupon bonds can recover more than their face value when the bonds have a high default risk, a long time to mature, and trades close to their face value.[1] Lando (1998) develop a model that allows a random hazard rate process to be dependent of the short interest rate process, but the model adds substantially computational complexity.

The other specifications of recovery rates are the recovery of face value (RFV) and the recovery of market value (RMV). Duffee (1998) introduced RFV under which a recovery rate is an exogenous fraction of the face value of the defaultable bond, while Duffie and Singleton (1999a) introduced RMV under which a recovery rate is equal to an exogenous fraction of the market value of the bond just before default. Both RMV and RFV have advantage and disadvantage over each other. Schönbuncher (2003) mentions that small theoretical differences between these two will not make much difference in many application scenarios. However, RMV is mathematically easier to apply because standard default-free term-structure modeling techniques can be applied, while RFV is more realistic when one assumes liquidation at default value and that absolute priority

[1] Schönbucher (2003) presents a numeric example for this particular case.

applies (Duffie and Singleton, 1999a). In this study, we adopt RMV, as Jarrow et al. (2010) did.

Beside treatment of a recovery rate, the other issues are specifications of interest rates and default intensities. Schönbucher (2003) lists ideal specifications of the interest rates and the default intensities. First, both interest rate and default intensity should be stochastic processes. Second, the dynamics of interest rate and intensity process should include a correlation between them as Duffee (1998) found empirical evidence that credit spreads are a decreasing function of interest rates. Third, a desirable property for interest rates and default intensity processes is that they remain positive at all times. Finally, simple application for pricing is always better. For this last reason, most reduced-form models in the previous literature adopt the class of affine process (see Duffie and Kan (1996) and Duffie, Pan, and Singleton (2000) for a detailed explanation).[2] Allowing for correlation among the Brownian motions in the state variable processes is able to incorporate a correlation between the interest rates process and the intensity process.

Lastly, there are three main different approaches to model the default dependence between firms in the reduced-form approach. The first approach introduced in the previous literatures is conditionally independent defaults (CID) models by Duffee (1999). CID models make the firm's default intensities dependent on common factors and a firm specific factor variable. In CID models, firms' default intensities are independent, which is conditioned to the realization of common factors. In other words, the default correlation is introduced only through the dependence of each firm's intensity on random common factors because a firm's specific factor is independent across firms. The major

[2] For an extensive review of the use of affine processes for credit risk modeling using intensity models, see Duffie (2005)

drawback of this model is that it generates lower levels of default correlation than empirical default correlations. For example, Hull and White (2001) suggest that the range of default correlations that can be achieved is limited. Even when there is perfect correlation between two hazard rates, the corresponding correlation between defaults in any chosen period of time is usually very low. This is liable to be a problem in some circumstances. Schönbucher and Schubert (2001) also comment that the default correlations that can be reached with this approach are typically too low when compared with empirical default correlations, and furthermore it is very hard to derive and analyze the resulting default dependency structure.

However, Yu (2005) argue that low default correlation in reduced-form models may have more to do with an inadequate common factor structure than the assumption of conditional independence. In his study, he generates default correlations from the two CID models, Duffee (1999) and Driessen (2005) models. Duffee’s model has two common factors that are extracted from Treasury yields, while Driessen’s model has two additional common factors that capture the co-movement of corporate credit yields. He shows that the first case generates a default correlation much lower than empirical observations, while the second case generates comparable, or even higher, values.

Duffie and Singleton (1999b) introduce the second approach in order to deal with the low correlation problem. They proposed two ways. Their first proposal is to include a pure jump process in the default intensity process. These jumps consist of two parts, joint jumps and idiosyncratic jumps. Their second proposal is to include common credit events that could trigger simultaneous defaults. Each common credit event is modeled as a Poisson process. The last approach to model default correlation is called the contagion model. The basic idea of this model is that the default of one firm increases the default probabilities of related firms (for more details, see Davis and Lo (2001) and Jarrow and Yu (2001)). Although these last two approaches are theoretically sounding, they have problems with calibration and implementation. For this reason, this study adopts the Duffee's model, as Jarrow et al. (2010) did[3].

Often, researchers compare the structural model with the reduced-form model for risky bonds. There are well-known differences between these two models. Unlike structural models, reduced form models do not consider a link between default and firm value explicitly. In the reduced form model, default time cannot be predicted through the value of firm, rather it is the first jump governed by the exogenous jump process. The parameters controlling the default hazard rate are inferred from market data. Thus, reduced form models incorporate existing market data for a firm's bond, while structural models often ignore market data. This difference also implies that, unlike reduced form models, structural models generate defaults endogenously because they provide a relation between a firm's credit quality and financial conditions. Another difference is that, unlike the reduced form model, the structural model determines recovery rates endogenously through the value of the firm's assets and liabilities at default.

In sum, if there is clear data of bond prices in the market, the accuracy of the reduced form model will be substantially increased, but obtaining clear data is not an easy task[4]. Relatively speaking, structural models have better tractability and richer economical interpretations because they determine default timed and recovery rated endogenously, unlike reduced form models. However, the advantage of using the

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[3] It is possible to adopt Driessen model at the cost of adding mathematical complexity.
[4] It is well stated in the study of Arora et al. (2005) in that they compare the performance of a reduced model with a structural model.

reduced form model is that it is relatively simpler than the structural model mathematically because of the exogenous determination of default time and recovery rate.

Besides these differences, Jarrow and Protter (2004) point out that the major difference between a structural and a reduced form model is the assumption of the information set. Structural models assume complete knowledge of very detailed information, which means that the market and managers, in firms, share exactly the same information. Under this assumption, a firm's default time is predictable. Unlike the structural model, a reduced-form model assumes a relatively less detailed information set. This assumption implies that the firm's default time is inaccessible. Since asset value process is not observable by the market, and the market determines the price of risk debts, Jarrow and Potter (2004) argue that usage of reduced form models are more appropriate for pricing debts than usage of structural models. Based on their argument, we believe that using a reduced form model for pricing a bond with make-whole call provisions is relatively more appropriate, which is explained in the following sub-section.

1.1.3 Make-Whole Call Provisions

A structural difference between fixed-call provision and make-whole call provision is that the call price in the make-whole call provision floats inversely with risk-free rates. In a bond with the make-whole call provision, the call price is obtained by maximum value between par value and the present value of the bond's remaining payments. This present value is calculated by using the discount rate, that is the prevailing comparable maturity Treasury yield plus a spread specified in the contract of a bond, which is called the make-whole premium. As mentioned in the study of Power and Tsyplakov (2008), the make-whole call provision has three distinctive advantages over fixed-call provision. First, because of the negative relation between the call price and risk-free rates, the call price in a make-whole call provision eliminates interest rate risk. Consequently, up-front costs for a make-whole call provision should be lower than a fixed-priced call provision. Furthermore, because of absence of interest risk, Mann and Powers (2003) state that unlike a fixed-call provision, make-whole call provision is not structured as a refunding vehicle. Rather, the make-whole call provision is structured to enable a firm to retire debt without relying on a tender offer when the firm needs to restructure its capital structure. Second, most bonds with fixed-call provision have several years of call or refund protection in their contracts in order to mitigate the interest rate risk that bondholders are exposed to. Finally, there is possibility that fixed-price call prices are greater than tender offer prices, if interest rates have risen since the bond was issued. Power and Tsyplakov (2008) study costs of using bonds with a make-whole call provision by developing structural frame models for those bonds. These costs are incremental yields over yields on non-callable bonds. They argue that these costs are costs of having additional financial flexibility from using make-whole call provisions instead of using non-callable bonds because companies that use make-whole call provisions can exercise this provision when they need to restructure their capital structure. However, their estimated incremental yields from their model are significantly smaller than observed incremental yields. They give one potential explanation for this disparity; that the decision to incorporate a make-whole call provision is endogenous and this endogeneity biases their estimated coefficients. Additionally, from previous literature, a company might delay to exercise its call option on its bonds because of transaction costs incurred when calling (Mauer, 1993), or because of concerns about wealth transfers resulting from temporary capital structure changes (Longstaff and Tuckman, 1994), or because of a suboptimal call policy employed by the company (King and Mauer, 2000). From these reasons, we can also apply Jarrow and Potter's argument (2004). Since market and managers in the firm do not share information about the firm's call policy, calling time is not predictable. This supports the usage of reduced form models for pricing a callable bond.

Moreover, Nayar and Stock (2008) document an empirical study about the relationship between a firm's abnormal stock return and different type (i.e. non-callable, fixed callable and make-whole callable bonds) of bond issues. They find that a non-callable bond issuance is associated with a significantly negative abnormal stock return, but callable bonds do not have this negative effect. Specifically, their evidence shows that long-run returns for make-whole call issuers are superior to that of both regular callable and non-callable issuers. These results are consistent with their argument that managers in a firm that issue make-whole callable bond have better information about their firm's future aspects than investors in the market. Since the structure of make-whole callable bond does not allow interest rate risk, rational callable situation occurs when default risk premium is decreased. Therefore, managers who anticipate a decrease in their default risk premium in the future would more likely issue make-whole callable bond. Thus, Nayar and Stock (2008) argue that an issuance of a make-whole callable bond is a clearer signal of brighter future prospects to investors in the market. From their argument, it is clear that the reduced-form model is more suitable for make-whole callable bond because firms that issue make-whole callable bonds have relatively more severe degree of information asymmetry.

1.2 Reduced-Form Model for Bonds with Make-Whole Call Provisions

In this section, we develop a reduced-form model for bonds with make-whole call provisions by extending the model that Jarrow et al. (2010) present. A closed-form formula for this bond is also derived under affine model specification.

1.2.1 Structural Difference between Fixed Callable and Make-Whole Callable Bonds[5]

In this sub-section, we demonstrate the structural difference between fixed callable (FX) and make-whole callable (MW) bonds, which will allow us to make necessary modifications from the existing model for callable bond. For simplicity, we assume that a perpetual life bond with fixed coupon payment (c) per period is issued, and that the bond is priced at par (P_0) with a discount rate (r_0). We also assume that the discount rate consists of two parts. One part is yield (i_0) on an equivalent maturity treasury bond, and the other part is the credit spread (q_0) based on market knowledge on the issuing firm on the issuing date (t=0). Thus, price at the issuing date is:

$$P_0 \ = c/r_0 \ = c/(i_0 + \ q_0) \tag{1}$$

In the case of fixed callable bond, rationally, issuers buy back their bonds, when call price (CFX) is less than current market price of the bonds. Therefore, a rational condition for calling these bonds is:

$$CFX < P_t \Rightarrow CFX < c/(i_t + \ q_t) \Rightarrow c/(i_t + \ q_t) - CFX > 0 \tag{2}$$

Based on condition (2), we can infer that the probability of calling bonds is negatively related to interest rates and credit spreads. Furthermore, it is positively related to coupon rates. The call spread in the model of Jarrow et al. (2010) incorporates the interest rate factor and coupon rates directly, but a credit spread factor is indirectly included through the interest rate factor. This is due to Duffee (1998) finding empirical evidence that both

[5] An analysis in this section is an extension of Nayar and Stock (2008)

callable and non-callable bond's that spread over Treasury yields are inversely related to the Treasury yields. He also finds that this inverse relation is much stronger for callable bonds because of a consequence of variations in the value of the embedded call option.

On the other hand, unlike the call price of fixed callable bond, the call price (CMW) of the make-whole callable bond is not constant over time. This is because the call price is obtained by maximum value between par and present value of the bond's remaining payments. This present value is calculated by using the discount rate, that is the prevailing comparable maturity Treasury yield plus a spread specified in the contract of a bond, which is called the make-whole premium (M). Thus, if we assume that the issuer exercises the call option on date (T=t), the call price of the make-whole callable bond can be expressed as:

$$CMW_t = max[c/(i_t + M)P_0]$$

$$CMW_t = max[c/(i_t + M), c/(i_0 + q_0)] \quad (3)$$

Since Mann and Power (2003) find that make-whole premiums in most bonds with make-whole provision are set below prevailing credit spread, which makes the call option out-of-the-money at issuing date, we can safely assume that:

$$M < q_0 \quad (4)$$

Again, the rational condition for exercising this option is:

$$CMW_t < c/(i_t + q_t) \Rightarrow max[c/(i_t + M), c/(i_0 + q_0)] < c/(i_t + q_t)$$

$$\therefore max[1/(i_t + M), 1/(i_0 + q_0)] < 1/(i_t + q_t) \quad (5)$$

We can break into two cases. First case is:

$$If\ 1/(i_t + M) > 1/(i_0 + q_0) \Rightarrow i_t < q_0 - M + i_0 \quad (6)$$

$$then, from\ (5), 1/(i_t + M) < 1/(i_t + q_t) \Rightarrow q_t < M \quad (7)$$

$$since\ (4), q_t < q_0 \tag{8}$$

When current interest rate is less than the difference between initial credit spread and make-whole premium plus initial interest rate, condition (7) must be satisfied for the option to be in-the-money. Since we assume that the initial credit spread is greater than the make-whole premium, in order to satisfy condition (7), the current credit spread must be less than the initial credit spread.

Second case is:

$$If\ 1/(i_t + M) < 1/(i_0 + q_0) \Rightarrow i_t > q_0 - M + i_0 \tag{9}$$

$$then, from\ (5), 1/(i_0 + q_0) < 1/(i_t + q_t) \tag{10}$$

When current interest rate is greater than the difference between initial credit spread and make-whole premium plus initial interest rate, condition (10) must be satisfied for the option to be in-the-money. If condition (4) and (9) are true, we can infer that the current interest rate is greater than the initial interest rate. Given that the current interest rate is greater than the initial interest rate, in order for condition (10) to be satisfied, again, current credit spread must be smaller than the initial credit spread.

In sum, from these two cases, whether interest rates go up or not, the option value depends on the change in issuer's credit risk. In other words, unlike the option values in fixed callable bond, the option values on make-whole callable bond are independent of a change in Treasury rates. Since a change in Treasury rates partially reflects the condition of the market, the important factor for the option value in the make-whole bond could be an issuer's individual credit risk.

The model from Jarrow et al. (2010) is originally constructed for the fixed callable bond. Given this structural difference, we argue that their model cannot directly apply to the make-whole callable bond. Therefore, we construct the reduced-from model for the make-whole callable bond in the next sub-section.

1.2.2 Development of the Reduced-Form Model[6]

As usual, it is assumed that uncertainties in the financial markets are modeled by a complete probability space (Ω, F, P) and a filtration $F = (F_t)_{o \le t \le T}$. We also assume that there are three claims. The first claim is the obligation of the firm to pay X dollars at maturity T. The second claim is that the investor gets X_d dollars at τ_d when the firm defaults. It is assumed that X_d is a fraction (δ) of the market value of the bond. The third claim is that the investor receives call price (X_c) at τ_c, when the firm exercises its call option on its bond. We also assume that a fraction (k) of the market value of the bond is call price at τ_c. Under these settings, the payoff on the bond is:

$$Z = X_c 1_{\{\tau_c < \tau_d, \tau_{c,} < T\}} + X_d 1_{\{\tau_d < \tau_c, \tau_{d,} < T\}} + X 1_{\{T < \tau_c, T < \tau_d\}} \tag{11}$$

where $1_{\{.\}}$ presents an indicator function. Subsequently, the time t price of the zero coupon make-whole callable bond can be expressed as:

$$V(t, T, 0, \delta, k) = \begin{cases} \dfrac{E_t^Q \left\{ Z e^{-\int_t^{\tilde{\tau}} r_u d_u} \right\}}{0}, & \begin{matrix} t \le \tilde{\tau} \\ t \ge \tilde{\tau} \end{matrix} \end{cases} \tag{12}$$

where $\tilde{\tau} = \min\{\tau_c, \tau_d, \mathrm{T}\}$. Where the instantaneous risk-free interest rate is r_t, and E_t^Q is the expectation operator under the equivalent martingale measure Q.

Since this study takes the reduced-form approach into consideration for analysis of the make-whole callable bond, we need to have the dynamics of the call and default intensities even though the study of Duffie and Singleston (1999) assume that firms

[6] The notation and procedure in this chapter follows the notation in Jarrow et al. (2010).

exercise their call option on bonds in order to minimize the market value of the bonds. However, as Jarrow et al. note, by introducing the call intensity, the reduced-form model could include “suboptimal” exercising strategies that could be caused by either market frictions or firm-specific strategy. This feature is particularly important to the case of the make-whole callable bond because, as Mann and Power (2003) argue, the prime purpose of issuing make-whole bond could be restructuring the firm’s capital structure.

For this reason, we need to consider two independent (call and default) point processes, $N_{c,t} \equiv 1_{\{t \geq \tau_c\}}$ and $N_{d,t} \equiv 1_{\{t \geq \tau_d\}}$, that follows the Cox process, and that have intensities, $\lambda_{c,t}$ and $\lambda_{d,t}$, respectively. With these point processes, the discounted gain process can be:

$$G_t = e^{-\int_t^t r_u d_u} V_c(t,T,0,\delta,k)\left(1-N_{c,t}\right)\left(1-N_{d,t}\right)$$

$$+ \int_0^t e^{-\int_0^u r_u d_u} k V_c(u-,T,0,\delta,k)\left(1-N_{d,t}\right) dN_{c,u}$$

$$+ \int_0^t e^{-\int_0^u r_u d_u} \delta V_c(u-,T,0,\delta,k)\left(1-N_{c,t}\right) dN_{d,u}$$

where $V_c(t,T,0,\delta,k)$ is the market value of the make-whole callable bond if there has been no event of exercising the call option or default by time t. The first term is the discounted price when there has been no default or exercising of the option. The second term is the discounted strike price upon exercising the option, and the third term is the discounted payoff upon default. By applying Ito’s formula to the discounted gain process, we can see that for G to be a martingale, the necessary and sufficient condition is:

$$V_c(t,T,0,\delta,k) = E_t^Q exp\left\{-\int_t^T \left(r_u + (1-k)\lambda_{c,u} + (1-\delta)\lambda_{d,u}\right) d_u\right\} \tag{13}$$

From (13), we can see that $(1-\delta)\lambda_{d,t}$ and $(1-k)\lambda_{c,t}$ are default spread and call spread, respectively. Let the default and call adjusted discount rate be:

$R_u = r_u + (1-k)\lambda_{c,u} + (1-\delta)\lambda_{d,u}$. Then the price of the callable coupon bond is:

$$V_c(t,T,c,\delta,k) = E_t^Q\left[\sum_{t<T_i\leq T} ce^{-\int_t^{T_i} R_u d_u} + e^{-\int_t^{T} R_u d_u}\right] \tag{14}$$

where c is coupon payments on T_1, T_2, T_3,…T_n=T. Furthermore, the price of the non-callable coupon bond is:

$$E_t^Q\left[\sum_{t<T_i\leq T} ce^{-\int_t^{T_i} R_u^* d_u} + e^{-\int_t^{\tau} R_u^* d_u}\right] \tag{15}$$

where $R_u^* = r_u + (1-\delta)\lambda_{d,u}$ is the default adjusted discount rate. Later, in chapter 2, we use equation (14) and (15) to estimate yield to maturity for the make-whole callable bond and non-callable bond.

In order to apply the above models of bond price, we need to specify the discount rate that consists of interest rates, default and the call processes. For the spot interest rate, as many studies (e.g. Duffee (1999), Duffie and Singleton (1997), Duffie et al. (2004), Jarrow et al. (2010), etc.) use, we also consider two factors affine model:

$$r_t = \alpha_r + s_{1,t} + s_{2,t} \tag{16}$$

where α_r is a constant, and the two state variables ($s_{1,t}$ and $s_{2,t}$) indicate the slope and level of the Treasury yield curve. It is assumed that each factor follows a square root process:

$$ds_{i,t} = \kappa_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dW_{i,t}, for\ i = 1,2$$

and under the equivalent martingale measure Q,

$$ds_{i,t} = \left(\kappa_i\theta_i - (\kappa_i + \eta_i)s_{i,t}\right)dt + \sigma_i\sqrt{s_{i,t}}d\widehat{W}_{i,t}, for\ i = 1,2 \tag{17}$$

where $W_{i,t}$, and $\hat{W}_{i,t}$ are independent Brownian motion under P and Q, respectively. η indicates the market price of risk associate with $\hat{W}_{i,t}$. It is well known that, with the combination of (16) and (17), we can derive a closed-form solution for the default-free zero coupon bond prices, in which the bonds' yields are linear in $s_{1,t}$ and $s_{2,t}$.

For the default spread ($(1-\delta)\lambda_{d,t}$), we assume that:

$$(1-\sigma)\lambda_{d,t} = \alpha_d + h_{d,t} + \beta_{d1}\left(s_{1,t} - \bar{s}_1\right) + \beta_{d2}\left(s_{2,t} - \bar{s}_2\right) \tag{18}$$

$$dh_{d,t} = \kappa_d\left(\theta_d - h_{d,t}\right)dt + \sigma_d\sqrt{h_{d,t}}dW_{d,t}$$

$$dh_{d,t} = \left(\kappa_d\theta_d - (\kappa_d + \eta_d)h_{d,t}\right)dt + \sigma_d\sqrt{h_{d,t}}d\widehat{W}_{d,t} \tag{19}$$

where η_d indicates the market price of risk associated with $\hat{W}_{d,t}$. The constant term (α_d) is included to capture a situation where the default spreads for very high-quality firms are positive, even at the short end of the yield curve. This situation could be caused by the liquidity risk or incomplete accounting information. The second term ($h_{d,t}$) is included to capture firm-specific risk that affects the default spread. More specifically, it possibly captures fluctuation of the firm's financial condition. The last component $\left(\beta_{d1}\left(s_{1,t} - \bar{s}_1\right) + \beta_{d2}\left(s_{2,t} - \bar{s}_2\right)\right)$ is included to capture the dependence of corporate bond yields on the variations in the default-free term structure factors.

For the call spread ($(1-\kappa)\lambda_{c,u}$), we assume that:

$$(1-k)\lambda_{c,t} = \alpha_c + h_{c,t} + \phi\left(M, s_{1,t}, s_{2,t}, h_{d,t}\right) \tag{20}$$

$$\phi\left(M, s_{1,t}, s_{2,t}, h_{d,t}\right) = \beta_{c1}\frac{M}{h_{d,t}} \tag{21}$$

$$dh_{c,t} = \kappa_c\left(\theta_c - h_{c,t}\right)dt + \sigma_c\sqrt{h_{c,t}}dW_{c,t}$$

$$dh_{c,t} = \left(\kappa_c\theta_c - (\kappa_c + \eta_c)h_{c,t}\right)dt + \sigma_c\sqrt{h_{c,t}}d\widehat{W}_{c,t} \tag{22}$$

where η_c indicates the market price of risk associated with $\hat{W}_{c,t}$. The constant term (α_c) is included to allow a nonzero call spread, even for firms that have close to zero call risk. The second term ($h_{c,t}$) is included to capture a firm-specific reason (e.g. restructuring capital structure) to exercising the call option on the bond, and the third term (ϕ) captures call spread because of the standard rational reasons for exercising the call option. In the previous sub-section, we conclude that an issuer's individual credit risk is the only factor that affects the value of the embedded option.

For this reason, only $\beta_{c1}\left(\frac{M}{h_{d,t}}\right)$ is involved in equation (21). The component $\left(\frac{M}{h_{d,t}}\right)$ allows the effect of the make-whole premium on the call spread explicitly, and it also allows for any possible nonlinear dependence of the call spread on firm-specific default spread. Thus, this functional form $\left(\frac{M}{h_{d,t}}\right)$ implies a positive relation between the make-whole premium and the call spreads, it also implies a negative relation between the firm-specific default risk and the call spread. Further, unlike Jarrow et al. (2010), we do not consider coupon rate because coupon rate in the make-whole callable bond is not likely to affect the decision of exercising the option, as shown in the previous sub-section.

From the above model specification, the default and call adjusted discount rate is:

$$
\begin{aligned}
R_u = r_u + (1-\sigma)\lambda_{d,u} + (1-k)\lambda_{c,u} \\
&= \alpha_r + s_{1,u} + s_{2,u} + \left[\alpha_d + h_{d,u} + \beta_{d1}\left(s_{1,u} - \bar{s}_1\right) + \beta_{d2}\left(s_{2,u} - \bar{s}_2\right)\right] \\
&+ \left[\alpha_c + h_{c,u} + \beta_{c1}\frac{M}{h_{d,u}}\right] = A + s^*_{1,u} + s^*_{2,u} + h_{d,u} + \beta_{c1}\frac{M}{h_{d,u}} + h_{c,u}
\end{aligned}
$$

where $A = \alpha_r + \alpha_d + \alpha_c + \beta_{d1}\bar{s}_1 - \beta_{d2}\bar{s}_2$, $s^*_{1,u} = [1 + \beta_{d1}]s_{1,u}$

and $s^*_{2,u} = [1 + \beta_{d2}]s_{2,u}$. We can have the dynamics of the translated factor $s^*_{i,u}$:

$$ds_{1,t}^{*} = \kappa_i\left(\theta_i^{*} - s_{i,t}^{*}\right)dt + \sigma_i^{*}\sqrt{s_{i,t}^{*}}dW_{i,t}$$

$$ds_{1,t}^{*} = \left(\kappa_i\theta_i^{*} - (\kappa_i + \eta_i)s_{i,t}^{*}\right)dt + \sigma_i^{*}\sqrt{s_{i,t}^{*}}d\widehat{W}_{i,t}, for\ i = 1,2$$

where $\theta_i^{*} = \theta_i(1 + \beta_{di})$ and $\sigma_i^{*} = \sigma_i\sqrt{1 + \beta_{di}}$

The zero-coupon make-whole callable bond is:

$$V_c(t, T, 0, \delta, k) = E_t^Q\left[exp\left(-\int_t^T R_u\, du\right)\right]$$

$$= exp[-A(T-t)] \cdot E_t^Q\left[exp\left(-\int_t^T s_{1,u}^{*}du\right)\right] \cdot E_t^Q\left[exp\left(-\int_t^T s_{2,u}^{*}du\right)\right]$$

$$\cdot\, E_t^Q\left[exp\left(-\int_t^T h_{c,u}^{*}du\right)\right] \cdot \pi\left(h_{d,u}, t, T\right)$$

where $\pi\left(h_{d,u}, t, T\right) = E_t^Q\left[exp\left(-\int_t^T\left(h_{d,u} + \beta_{c1}\frac{M}{h_{d,u}}\right)du\right)\right]$.

In standard analysis form,

$$V_c(t, T, 0, \delta, k) = exp[-A(T-t)]exp\left\{\psi_0(t) - \psi_1(t)s_{1,u}^{*} - \psi_2(t)s_{2,u}^{*} - \psi_c(t)h_{c,u}\right\}$$

$$\cdot\, \pi\left(h_{d,u}, t, T\right)$$

where $\psi_0(t) = \psi_{01}(t) + \psi_{02}(t) + \psi_{0c}(t)$

$$\psi_i(t) = \frac{2\left(e^{\gamma_i(T-t)} - 1\right)}{2\gamma_i + (\kappa_i + \eta_i + \gamma_i)(e^{\gamma_i(T-t)} - 1)}$$

$$\psi_{0i}(t) = \frac{2\kappa_i\theta_i^{*}}{\sigma_i^{*2}}\log\left[\frac{2\gamma_1 e^{\frac{1}{2}(\kappa_i+\eta_i+\gamma_i)(T-t)}}{2\gamma_i + (\kappa_i + \eta_i + \gamma_i)(e^{\gamma_i(T-t)} - 1)}\right]\ for\ i = 1,2$$

$$\psi_{0c}(t) = \frac{2\kappa_c\theta_c}{\sigma_c^2}\log\left[\frac{2\gamma_c e^{\frac{1}{2}(\kappa_c+\eta_c+\gamma_c)(T-t)}}{2\gamma_c + (\kappa_c + \eta_c + \gamma_c)(e^{\gamma_c(T-t)} - 1)}\right]$$

$\gamma_i = \sqrt{(\kappa_i + \eta_i)^2 + 2\sigma_i^{*2}}and\gamma_c = \sqrt{(\kappa_c + \eta_c)^2 + 2\sigma_c^2}$.

There is no well-known close-form solution for $\pi(h_{d,u},t,T)$ because $h_{d,u}+\beta_{c1}\frac{M}{h_{d,u}}$ is outside the standard affine family. However, Kimmel (2008) develops a method that properly makes non-affine transformations of the time variable so that the power series can be applied to derive close-form solutions. In our case, we can apply Kimmel's method allowing us to get a close-form solution for $\pi(h_{d,u},t,T)$. The application of Kimmel's method is shown briefly in the Appendix A.

1.3 Summary and Conclusion

First, this chapter analyzes the structural difference between a make-whole and a traditional callable bond. From the analysis, the major difference between these two bonds is that the value of the embedded option in the make-whole callable bond is independent of Treasury rates. On the other hand, the value option in the traditional callable bond is affected by Treasury rates and credit risk. This difference is due to the unique feature of the option in the make-whole callable bond. Unlike the call price in the traditional callable bond, the call price in the make-whole bond is not fixed. It is determined by the maximum value between par value and the present value of the bond's remaining payments. The discount rate in the calculation of the present value is the prevailing comparable maturity Treasury yield plus a spread specified in the contract of a bond, which is called the make-whole premium.

Second, based on the analysis, we construct a reduced form model for the make-whole callable bond. Previously, Jarrow et al. (2010) develop a reduced form model for the traditional callable bond; in which the option value is dependent on Treasury rates. However, from our analysis, we argue that we cannot directly apply the model from Jarrow et al. (2010) because of the unique structure of the make-whole callable bond.

The next chapter empirically investigates validation of our model with the extended Kalman filter, and compares performance of our model with one of the model from Jarrow et al. (2010).

CHAPTER 2: ANALYSIS OF A BOND WITH MAKE WHOLE PROVISION CALIBRATION OF PARAMETERS: KALMAN FILTER

The main objective of this chapter is to see whether or not the reduced-form model developed in Chapter 1 can be really fitted into real market data of make-whole callable bonds. Additionally, we compare the performance of our model with the performance of the model (hereafter JLLW model) from Jarrow et al. (2010). As stated in Chapter 1, the major difference between our model and the JLLW model is that our model assumes the embedded option value depends on the individual default factor, while JLLW model assume the option value depends on the default-free interest rates. Originally, the JLLW model is constructed for the callable bond whose purchase price in embedded option is fixed. Therefore, we argue, that the JLLW model does not suit the make-whole callable bond because the purchase price in the make-whole call option is not fixed. Indeed, our analysis with the extended Kalman filter shows that JLLW does not fit into make-whole callable bonds in our sample, while our model is reasonably fit into the observed data of make-whole callable Bonds.

In order to investigate the validation of our model, we need a good estimation methods. Our two candidates for econometric methods are the Kalman filter and the Markov Chain Monte Carlo (MCMC). In Previous studies, Pearson and Sun (1994) apply the Maximum Likelihood (ML) Method in order to estimate a two-factor CIR model with the exact density that Cox et al. (1985) identify. State variables are inverted from the pricing equation (Measurement system). The yield distribution can be obtained from the transition density and the Jacobian matrix of the transformation. One of the restrictions in the ML method application is that the number of state variables should match with number of securities with different maturities. It implies that the observations are measured without an error. However, Jagannathan et al (2001) and Liu et al. (2000) modify the ML process in order to deal with a situation where the number of securities is greater than the number (N) of state variables. In their study, they assume that first N yields are measured without error while the remaining measurement errors are joint normal. Thus, the consequent disadvantage is that there is no clear way to choose certain rates to be measured without noise. Another well-known drawback of this approach is that the state variables could be negative, which is found in the study of Duffie and Singleton (1997). This is a significant problem when it is dealing with non-Gaussian Exponential Affine Model.

Another popular method used for estimating term-structural models is the Method of Moments. Under the principle of GMM, Simulated Method of Moments (SMM) is suggested by Duffie and Singleton (1993), and Efficient Method of Moment (EMM) is defined by Gallant and Tauchen (1996). There are several advantages of using SMM/GMM. First, moment estimates have the asymptotic properties of ML estimates. Second, Moment methods are generally applicable for various non-linear multi-factor models with high dimension parameters. Third, they allow a measurement error, which implies that the number of securities can be greater than the number of state variables. However, Duffee and Stanton (2004) show that EMM estimates are often seriously biased for finite samples, or for the normal size of term-structure data observations. Even in a single-factor Vasicek setting, a Monte Carlo Simulation shows that EMM performance diverges significantly from ML.

On the other hand, Kalam filter and MCMC methods have relatively less restrictions. For this reason, they have gained popularity in the affine term-structure literature. In this study, we compare the performance of these two methods in our case. Our results on the performance of the two methods on risk-free term structure show that the MCMC method does not work as well. The problem of using the MCMC method arises from our risk-free term structure data. As Sögner (2009) demonstrates, this issue with the MCMC method is that the parameter estimation becomes almost impossible, due to ill conditioned transformation between the latent state variables process driving the yields and the yield observed. Unfortunately, our Treasury data shows evidence of unit-root behavior. Meanwhile, the extended Kalman filter works reasonably well on our risk-free term structure data. Therefore, we apply the Kalman filter estimation to analyze the make-whole callable Bonds in our sample. This chapter is organized as follows: section 1 contains a brief review of the Kalman filter and the MCMC methods, section 2 describes our data set and procedures, section 3 compares our model with one from Jarrow et al. (2010), and concludes with section 4.

2.1 Kalman Filter and MCMC Method

2.1.1 Kalman Filter

The Kalman filter technique has recently gained popularity in the Affine Term-Structure Literature as a result of the work by Chen and Scott (2003), de Jong (2000), Lund (1997), Geyer and Pichler (1998), Duffee (1999), and Jarrow et al. (2010). This approach is very useful in situations such as ours, where the underlying state variables are not observable. The state-space form consists of the measurement system and transition system. The measurement system represents the affine relationship (shown in Chapter 1) between the zero-coupon rate and the state variables. On the other hand, the transition system is an unobserved system of equations that describes the dynamics of the state variables. By using this state-space form, the Kalman filter recursively form inferences about the unobserved values of the state variables by conditioning on the observed market zero-coupon rates. These recursive inferences are used to maximize a log-likelihood function for searching the optimal parameter set. With the CIR model, however, the transition density follows a non-central chi-squared distribution, which is rather difficult to handle. Fortunately, Ball and Torous (1996) show that, over small time intervals, diffusions arising from stochastic differential equations behave like the Brownian Motions.

For this reason, in order to estimate the multi-factor CIR model, Chen and Scott (2003) use this approach with quasi-maximum likelihood (QML) method by approximating the transition density with normal density, where first and second moments are given by those of the non-central Chi-square density. However, this approximation often results in an inconsistent QML estimator. Duffee and Staton (2004) compare three methods (ML, EMM, and Kalman filter) for the term structural models. Even though they found small biases in parameter estimation for the case of Kalman filter, the degree of biasness is less, when compared to EMM. Therefore, they prefer to use the Kalman filter over EMM in cases where the ML approach is not feasible to apply. In addition to this inconsistent issue, there is another disadvantage to the Kalman filter in that it could give negative state variables. In order to deal with this problem, Chen and Scott (2003) propose to replace negative estimates of state variables with zero. In our application of the Kalman filter, we also follow the proposal from Chen and Scott (2003).

2.1.2 Markov Chain Monte Carlo (MCMC)

This study also considers the MCMC approach, which not only achieves the asymptotic efficiency of MLE, but also provides the finite-sample distribution. The sampling-based MCMC method avoids the computation of a high-dimensional integral necessary for obtaining the marginal distribution of parameters. At the meantime, it retains the generality and simplicity of moment methods. It generates random samples from complicated likelihood in any functional form. The random samples converge to finite-sample distribution of MLE. Unlike other methods, MCMC provides a solution to exact filtering of the unobserved state variables.

There are a number of sampling approaches in estimating marginal density. The popular ones are Metropolis-Hastings (MH) Sampler and Gibbs sampler. The MH sampler obtains the state of the chain at i+1 by sampling a candidate point $\gamma$from a proposal distribution $q(.|X_i)$ that depends only on the previous state X_i and can have any form, subject to regularity conditions (Roberts, 1995). The most popular choice of the proposal distribution is normal with mean X_i and fixed variance (i.e. Random Walk proposal density).

The required regularity conditions for the proposal distribution are irreducibility and aperiodicity. Irreducibility means that there is a positive probability that the Markov chain can reach any nonempty set from all starting points. Aperiodicity ensures that the chain will not oscillate between different sets of states. These conditions are usually satisfied if the proposal distribution has a positive density on the same support as the target distribution. The steps of the algorithm are outline below:

1. Set initial value for X_0.

2. Generate a candidate point γ from $q(.|X_i)$.

3. Generate U from a uniform (0,1) distribution

4. if U is less than $\min\{1, \frac{\Pi(\gamma)q(X_i \mid \gamma)}{\Pi(X_i)q(\gamma \mid X_i)}\}$, then set $X_{i+1} = \gamma$ else set $X_{i+1} = X_i$.

5. Set i=i+1 and repeat steps 2 through 5.

In our application, we use MH with random walk proposal density because, as commented by Liu (2007), there is no clear evidence of advantages using certain proposal densities instead of random-walk process.

On the other hand, Gibbs sampler is the other popular choice. Although it can be shown to be a special case of the MH algorithm, there are two eminent differences between the Gibbs sampler and MH. First, a candidate point is always accepted. Second, the full conditional distribution should be recognized, which makes the algorithm less applicable. For these reason, it is applied for estimating measurement errors in our case.

Recently, MCMC had been getting a lot of attention in financial econometrics because of the advantage mentioned above. For example, Eraker (2001) demonstrates the analysis of a one-factor Gaussian Short rate model and stochastic volatility models, where Gibbs sampler and MH algorithm are applied. Mikkelsen (2001) analyzes a one-factor Vasicek model with a cross-sectional data of both bond and swap rate, where the market price of risk is estimated. Aguilar and West (2000) use MCMC and sequential filter to estimate dynamic factor models with international exchange rates data. Certainly, it is interesting to see whether it could perform better than Kalman filter in our case, although Jaquier et al. (1994) shows that the Bayesian Markove Chain estimators outperform estimators from moment and QML methods on stochastic volatility models.

2.2 The Data and Estimation Procedure

In this section, the data is described, and the procedure of the estimation is introduced. If possible, it is idealistic to estimate the default free term structure jointly with corporate bonds from all firms in the sample. However, Duffee (1999) points out that it is computationally infeasible because of the huge dimension of the problem. Alternatively, it is possible to estimate the default-free term structure jointly with corporate bonds from each individual firm, but the drawback of using this estimation is that we could have different estimates of the risk-free process for different firms. Therefore, first, this study estimates the default-free term structure from observed yields on Treasury securities, and then, with estimated risk-free, we estimate the default process. Finally, the call process is estimated with using estimated risk free and default process.

2.2.1 The Data

In order to estimate the default-free term structure, spot rates for 6-month and 12-month Treasury bills, and 2-, 3-, 5-, 10-, and 30-year Treasury bonds. We collect these spot rates from *Bloomberg terminal,* where they estimate the derived zero coupon yields by stripping the par coupon curve. The maturities of the Treasury securities cover the maturity spectrum of all corporate bonds used in our analysis. Corporate bond data are obtained from *Bloomberg terminal* and *Datastream*. First, we collect the bond data from *Datastream* because, as Chen et al. (2007) note, it is used to provide prices, which in turn, uses Merrill Lynch as the data source for the price across all market makers for the bonds. The remaining bond data in our sample is obtained from Bloomberg Generic Quote, which reflects the consensus quotes among market participants.

As Duffee (1999) and Jarrow et al. (2010) impose restrictions on their collection on bond data, we also require a firm to have at least one non-callable bond and one make-whole callable bond. Furthermore, in order to control liquidity and maturity premium on the bond, we require maturities difference of two bonds are at least 24 months. Finally, historical price of the both bond should be available for at least 24 months. After imposing these restrictions, we have 38 firms in our sample. Table 1 shows the list of firms in our sample, and their credit ratings. Our sample companies' S&P rating is mostly greater than BBB-. Additionally, rating distribution shown in Table 2 is very similar to the sample in the study of Power and Tsyplakov (2008). However, distribution of make-whole premium in our sample is much lower than the one in the sample from the study of Power and Tsyplakov (2008).

Notably, Table 2 exhibits that yield-to-maturity difference between a make-whole callable and non-callable bond (yield-to-maturity on make-whole callable bond minus yield-to-maturity on non-callable bond) from a same firm is -0.084 % on average without considering maturity difference. However, Power and Tsyplakov (2008) demonstrate, by using Bloomberg's Fair Market Yield with linear interpolation technique, that the yield spreads between these two bonds are 5-7 basis points. Although Power and Tsyplakov (2008) note that this yield spread measure likely underestimate the true yield spread, they state that there exist very thin yield spreads between these two bonds. Furthermore, Elton et al. (2001) find evidence that coupon rates have a significantly effect on bond's yield spread over comparable treasury rates. Therefore, we believe that -0.084 % yield-to-maturity is reasonable.

Table 1: S&P Ratings on 38 Firms Used in Empirical Analysis

This table reports names of 38 firms used in our sample and their S&P credit ratings. Corporate bond data are obtained from *Bloomberg terminal* and *Datastream*. We require a firm to have at least one non-callable bond and one make-whole callable bond. Furthermore, in order to control liquidity and maturity premium on the bond, we require maturities difference of two bonds are at least 24 months. Finally, historical price of the both bond should be available for at least 24 months.

Name	S&P rating	Name	S&P rating	Name	S&P rating
AIR PROD & CHEM	A	CONSOL NAT GAS	A-	MCDONALD'S CORP	A
ALLSTATE CORP	A-	CORNING INC	BBB+	MERCK & CO INC	AA
AMER STORES CO	B	EMERSON ELECTRIC	A	PROGRESSIVE CORP	A+
ANHEUSER BUSCH	BBB+	GEORGIA-PACIFIC	A-	TENNECO PACKAGNG	B+
APACHE CORP	A-	HERTZ CORP	B+	UNION ELECTRIC	BBB-
ARCHER DANIELS	A	HONEYWELL INTL	A	UNION PAC RES	BBB-
BALT GAS & ELEC	BBB+	IBM CORP	A+	UNION PACIFIC CO	BBB+
BERKLEY (WR)	BBB+	INDIANAPOLIS P&L	BBB-	UNITED TECH CORP	A
BESTFOODS	A+	JOHNSON CONTROLS	BBB+	UNUM CORP	BBB-
BOEING CO	A	KOHL'S CORP	BBB+	WALT DISNEY CO	A
CHUBB CORP	A+	LIBERTY MEDIA	BB	WESTVACO CORP	BBB
COCA-COLA ENTER	A+	MACYS RETAIL HLD	BBB-	XEROX CORP	BBB-
CONOCOPHILLIPS	A	MARSH & MCLENNAN	BBB-		

Table 2: Summary Statistics for Non-Callable and Make-Whole Callable Bonds

This table reports summary statistics on the characteristics of one non-callable and one make-whole callable bond from each of the 38 firms used in our empirical analysis. Rating is numbered from one (AAA rated bond) to 7.5 (B+ rated bond). Price is expressed at percentage of par value. Coupon or Yield refers to coupon rates or yield to maturity for a bond respectively. Yield difference is yield to maturity of a make-whole callable bond minus yield to maturity of a non-callable bond. Maturity difference is life remaining to maturity for a make-whole callable bond minus life remaining to maturity for a non-callable bond. Maturity difference is expressed in year.

	Mean	Standard Deviation	Min	First Quartile	Median	Third Quartile	Max
Distribution for Straight Bond							
Rating	4.4474	1.3646	2	3.5	4.25	5.5	8
Price	107.6311	7.9969	89.9984	105.2517	107.5182	111.7358	133.2344
Coupon(%)	7.0605	1.3050	3.75	6.60625	7.125	7.86875	9.5
Yield(%)	5.8087	1.4857	3.2197	4.8727	5.3749	6.3675	9.3183
Distribition for make-whole callable bond							
Rating	4.4474	1.3646	2	3.5	4.25	5.5	8
Price	103.8900	7.0094	87.0579	100.1874	104.5257	106.1517	122.7622
Coupon(%)	6.2632	1.1458	4.15	5.53125	6.125	7.075	8.5
Yield(%)	5.6239	1.7669	2.6387	4.7006	5.3734	6.1471	11.1517
Make-Whole Premium(Basis Point)	13.6981	8.3196	4.5918	8.1301	9.8000	23.3692	29.9014
Yield Difference (%)	-0.0844	0.4685	-1.1524	-0.3300	-0.05020	0.06019	1.83715
Maturity Difference	0.2656	1.2207	-2.3836	-0.5692	0.51644	1.27397	2.00000

2.2.2 The Estimation Procedure[7]

First, we will present the state-space form that our two econometric methods (Kalman filter and MCMC) are based on. Let $Y_t = (y_{1t}, ..., y_{Nt})'$ be the yields on N Treasury bonds, and $S_t = \{s_{1,t}, s_{2,t}\}$ be the unobservable state variables that drive the default-free term structure. Thus, measurement and transition system can be obtained as follows:

$$Y_t = \Phi(S_t) + \varepsilon_t, E_{t-1}(\varepsilon_t \varepsilon_t') = \Sigma$$

$$S_t = \mu + \Gamma S_{t-1} + v_t, \mathrm{E}_{t-1}(v_t v_t') = \Omega(S_{t-1})$$

The function $\Phi(S_t)$ maps the two state variables into seven yields of Treasury Bonds. Σ and $\Omega(S_{t-1})$ are diagonal matrices corresponding to the variances of the measurement errors of yields and state variables, respectively. Thus, the conditional variance depends upon the unknown values for S_{t-1} , which makes the estimator based on the Kalman filter inconsistent, even though the simulation experiments from Duan and Simunato (1999) show that induced biases are very small. Since we use seven Treasury bonds with different maturities, and we assume two state variables for default-free process, Σ and $\Omega(S_{t-1})$ are 7 by 7 diagonal matrix, and 2 by 2 diagonal matrix, respectively. μ, Γ and $\Omega(S_{t-1})$ in the transition system are defined as:

$$\mu = \begin{pmatrix} \theta_1\left(1 - e^{-\kappa_1/12}\right) \\ \theta_2\left(1 - e^{-\kappa_2/12}\right) \end{pmatrix}, \Gamma = \begin{pmatrix} e^{-\kappa_1/12} & 0 \\ 0 & e^{-\kappa_2/12} \end{pmatrix}$$

$$\Omega_{i,i}(S_{t-1}) = \kappa_i^{-1}\sigma_i^2\left[s_{i,t-1}\left(e^{-\kappa_1/12} - e^{-2\kappa_1/12}\right) + \frac{\theta_i}{2}\left(1 - e^{-\kappa_1/12}\right)^2\right] \; for \; i = 1,2$$

---

[7] The notation in this chapter follows those in chapter 1, which also closely follows the notation in Jarrow et al. (2010).

From this state-space form, we can apply the Kalman filter with the QML suggested by Chen and Scott (2003).

For the MCMC application, since there is no well-known full-conditional density for parameters in our case, we cannot apply the Gibbs sampler for estimating parameters except for variances in measurement error. Even though Frühwirth-Schnatter and Geyer (1998) adopt a normal proposal density that linearizes around the old value and use an approximation to normal conditional density to propose the candidates for state variables, as commented by Liu (2007), there is no clear evidence of advantages using certain proposal densities instead of random-walk process. This is the reason why we apply the MH algorithm with random walk density proposal to estimate the parameters. Furthermore, Gelman et al. (1996) suggest that the optimal variance is to adjust the variance-covariance matrix by a coefficient, $c \approx 2.38/\sqrt{d}$, where d is the dimension of the parameter set. By this adjustment, the optimal rule has an acceptance rate of about 0.25 for high-dimensional models. In our application, we also consider the suggestion from Gelman et al. (1996).

In order to apply MH algorithm, we need to have information about target distribution up to the constant proportionality. For the CIR-model, the exact transition densities are known to be non-central chi-square densities. It can be expressed as below:

$$p(S_t|S_{t-1}) = \prod_{j=1}^{2} p(s_{j,t}|s_{j,t-1}) \tag{1}$$

$$p(s_{j,t}|s_{j,t-1}) = c_j e^{-c_j(s_{j,t}+e^{-\kappa_j\Delta t}s_{j,t-1})}\left(\frac{s_{j,t}}{e^{-\kappa_j\Delta t}s_{j,t-1}}\right)^{\frac{q_j}{2}} I_{qj}\left(2c_j\sqrt{s_{j,t}e^{-\kappa_j\Delta t}s_{j,t-1}}\right)$$

$$where\ c_j = \frac{2\kappa_j}{\sigma_j^2(1-e^{-\kappa_j\Delta t})}, \quad q_j = \frac{2\kappa_j\theta_j}{\sigma_j^2} - 1$$

where $I_{q_j}(.)$ is the modified Bessel Function of the first kind of order q_j. If we assume that the state variables are independent a priori, the prior p(S_0) is:

$p(s_{j,0}) = p_G\left(s_{j,0}; q_j + 1, \frac{2\kappa_j}{\sigma^2}\right)$, where $p_G(.)$ indicates gamma distribution density function. The target density should have as much information about the joint posterior distribution as possible. The joint posterior distribution $p(S^T, H \mid Y^T)$ is given by Bayes' theorem:

$p(S^T, H \mid Y^T) \propto p(Y^T \mid S^T, H)\, p(S^T \mid H)\, p(H)$, where H is the parameter vector.

It consists of three densities: "complete data likelihood", $p(S^T \mid H)$, and the marginal prior of the model parameter H. The first part, $p(Y^T \mid S^T, H)$ is called "complete data likelihood" that is the product of the observation densities:

$$p(y_t|S_t) = \prod_{i=1}^{7} p_N\left(y_i; \hat{y}_{i,t}, \Sigma_{ii,t}\right)$$

where $p_N(.)$ is the normal distribution density function, and $\hat{y}_{i,t}$ is the estimated yield from the measurement system. The second part, $p(S^T \mid H)$, is obtained by the product of transition densities (1) times the prior $p(s_{j,0})$. The last part is the marginal prior of the model parameter H, which we assume the following un-informative priors:

$$p(\theta_j) \propto c, p(\kappa_j) \propto c, p(\sigma_j^2) \propto 1/\sigma_j^2, p(\eta_j) \propto c \qquad j = 1,2$$

$$p(\sigma_{\varepsilon i}^2) \propto 1/\sigma_{\varepsilon i}^2 \qquad i = 1,2, \ldots, 7$$

The full conditional densities, $p(H_j \mid .)$ and $p(s_{j,t} \mid .)$ are proportional to $p(S^T, H \mid Y^T)$. Thus, our target distribution function is:

$$p(H_j|.), p(s_{j,t}|.) \propto p(Y^T|S^T, H)p(S^T|H)p(H) \qquad (2)$$

However, since the state variables vary with time, we have to sample the state variables at each time. Therefore, we follow suggestion from Frühwirth-Schnatter and Geyer (1998). The target distribution functions for $p(s_{j,t} \mid .)$ is:

$$p\left(s_{j,t}\middle|\cdot\right) \propto \begin{cases} p(Y_t|S_t,H)p\left(s_{j,t}\middle|s_{j,t-1},H\right)p\left(s_{j,t+1}\middle|s_{j,t},H_j\right) & 1 \le t \le T-1 \\ p(Y_T|S_T,H)p\left(s_{j,T}\middle|s_{j,T-1},H_j\right) & \text{t=T} \\ p\left(s_{j,0}\middle|H_j\right)p\left(s_{j,1}\middle|s_{j,0},H_j\right) & \text{t=0} \end{cases}$$

For variance of the measurement errors, full conditional posteriors can be driven with their un-informative priors (see Frühwirth-Schnatter and Geyer (1998)). For this reason, we apply the Gibbs samplers for the variance of the measurement errors. The full conditional posterior is:

$$p(\sigma_{\varepsilon i}^2|\cdot) = p_{IG}\left(\sigma_{\varepsilon i}^2; T/2, 1/2\sum_{t=1}^{T}\left(y_{i,t}-\hat{y}_{i,t}\right)^2\right)$$

From the setting above, we generate 200,000 iterations. After discarding the first 199,000, we analyze the results.

Then we begin to tackle estimating parameters and the state variables in the default-free process, as done by Duffee (1999) and Jarrow et al. (2010), we take $\{\hat{s}_{1,t}, \hat{s}_{2,t}\}$ as true variables that determine the default-free process to estimates the default process of a particular firm. The measurement system and transition system in our second step are:

$$Y_{d,t} = \Phi_d\left(h_{d,t}, \hat{s}_{1,t}, \hat{s}_{2,t}\right) + \varepsilon_{d,t} \qquad E_{t-T}\left(\varepsilon_{d,t}\varepsilon_{d,t}'\right) = \sum d$$

$$h_{d,t} = \mu_d(T) + \Gamma_d(T)h_{d,t-T} + u_t \qquad E_{t-T}(u_t u_t') = \Omega_d\left(h_{d,t-T}, T\right)$$

Φ_d is an implicit function that maps the state variables into defaultable bond yields. The function is implicitly given by numerically solving for the yield corresponding to the coupon price. Thus, we first calculate the theoretical bond price using the equation from Chapter 1, given the values of $\hat{s}_{1,t}$, $\hat{s}_{2,t}$, and $h_{d,t}$. Second, we solve for the theoretical yield to maturity ($\hat{Y}_{d,t}$).

For application of Kalman filter, we need to linearize the measurement equation around a predicted value of $h_{d,t}$ as follows:

$$Y_{d,t} = \left.\frac{\partial \Phi_d(h_{d,t})}{\partial h_{d,t}}\right|_{h_{d,t}=h_{d,t|t-1}} h_t + \Phi_d(h_{d,t|t-1}) - \left.\frac{\partial \Phi_d(h_{d,t})}{\partial h_{d,t}}\right|_{h_{d,t}=h_{d,t|t-1}} h_{d,t|t-1} + \varepsilon_{d,t}$$

where $h_{d,t|t-1}$ is the predicted value from the information up to time t-1. However, for the application of MCMC, there is no need for this linearization, and its procedure is similar to the one in the first step.

Once the default parameters and intensity are estimated, they are used to estimate the call process with the following measurement and transition systems:

$$Y_{c,t} = \Phi_c(h_{c,t}, h_{d,t}, \hat{s}_{1,t}, \hat{s}_{2,t}) + \varepsilon_{c,t} \qquad E_{t-T}(\varepsilon_{c,t}\varepsilon'_{c,t}) = \sum c$$

$$h_{c,t} = \mu_c(t) + \Gamma_c(T) h_{c,t-T} + \xi_t \qquad E_{t-T}(\xi_t \xi'_t) = \Omega_c(h_{c,t-T}, T)$$

μ_j, Γ_j, Ω_d and Ω_c in the transition equation is given by:

$$\mu_j(T) = \theta_j \left(1 - e^{-\frac{T\kappa_j}{12}}\right) \qquad \Gamma_j(T) = e^{-\frac{T\kappa_j}{12}}$$

$$\Omega_j(h_{j,t-T}, T) = \kappa_j^{-1}\sigma_j^2 \left[h_{j,t-T}\left(e^{-\frac{T\kappa_j}{12}} - e^{-\frac{T\kappa_j}{6}}\right) + \frac{\theta_j}{2}\left(1 - e^{-\frac{T\kappa_j}{12}}\right)^2 \right] \qquad for\ j = c, d$$

Next section presents empirical results based on the procedure stated in this section.

2.3 Empirical Results

This section presents empirical evidence on the performance of our model developed in Chapter 1. In addition, we compare the performance of our model with that of the JLLW model. However, we first compare performances of two econometric tools (the extended Kalman filter and MCMC method) on our Treasury data.

2.3.1 Performances of the Kalman Filter and the MCMC Method on US Treasury Data

Table 3 reports performances of the Kalman filter and the MCMC methods on our US Treasury rates data. For simplicity, we exclude α term in equation (13) in Chapter 1. It shows that both methods tend to overestimate default-free term structure based on the mean errors in Table 3. However, estimated parameters from these two methods are quite different. Based on Root Mean Square Errors (RMSE) for two methods, the Kalman filter works much better than MCMC method. It is obvious that, from RMSE, MCMC cannot capture default-free term structure behavior in the sample.

A problem of using the MCMC method arises from these risk-free term structure data. As Sögner (2009) demonstrates this issue with the MCMC method, parameter estimation becomes almost impossible, due to ill conditioned transformation between the latent state variables process driving the yields and the yield observed, especially when all yields from a different maturity Treasury are observed with noise . This issue is also recognized in Piazzesi (2005). Unfortunately, our Treasury data shows evidence of unit-root behavior, which is shown in Table 4 that reports results of the augmented Dickey Fuller test. On the other hand, the extended Kalman filter works reasonably well on our risk-free term structure data. Therefore, we analyze make-whole callable bonds with the extended Kalman filter.

Table 3: Comparison between Kalman Filter and MCMC Performance on US Treasury Rates Data

The instantaneous interest rate is: $r_t = s_{1,t} + s_{2,t}$.
The dynamics are

$$ds_{i,t} = \kappa_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dW_{i,t}, \text{ for } i = 1,2 \quad \text{(True measure)}$$

$$ds_{i,t} = \left(\kappa_i\theta_i - (\kappa_i + \eta_i)s_{i,t}\right)dt + \sigma_i\sqrt{s_{i,t}}d\widehat{W}_{i,t}, \text{ for } i = 1,2 \quad \text{(Martingale Measure)}$$

US Treasury rates data from January, 2002 to March, 2011 are used. The standard errors (STD) are computed assuming the Kalman filter linearization is exact. The estimates from MCMC are based on 200,000 MCMC steps and 199,000 burn-in.

A. Estimation from Kalman filter

i	κ_i	θ_i	σ_i	λ_i
1	0.3238	0.0361	0.1139	-0.0938
STD	0.1202	0.0136	0.0104	0.1150
2	0.1527	0.0016	0.0300	-0.1292
STD	0.0398	0.0005	0.0105	0.0442

Bond maturity	Mean error (actual-fitted)	Root mean square error
6 months	-0.0017	0.0059
1 year	-0.0022	0.0060
2 years	-0.0027	0.0056
3 years	-0.0025	0.0053
5 years	-0.0012	0.0050
10 years	0.0006	0.0052
30 years	-0.0006	0.0049

B. Estimation from MCMC

i	κ_i	θ_i	σ_i	λ_i
1	2.3843	0.0489	0.7148	-0.0742
STD	0.1594	0.0164	0.0067	0.0539
2	2.7881	0.0358	0.7169	-0.0763
STD	0.1432	0.0094	0.0079	0.0599

Bond maturity	Mean error (actual-fitted)	Root mean square error
6 months	-0.0456	0.0530
1 year	-0.0504	0.0537
2 years	-0.0498	0.0513
3 years	-0.0473	0.0485
5 years	-0.0422	0.0431
10 years	-0.0343	0.0348
30 years	-0.0286	0.0290

Table 4: Unit Root Test for Treasury Rate Data

This table presents results of Unit-root test from monthly US Treasury rates data from January, 2002 to March, 2011. ACF_1 is the first order autocorrelation coefficient of the corresponding rate time series. ADF is the augmented Dickey Fuller test. The p-values are presented below the corresponding Unit-root statistic.

Bond maturity	6 months	1 year	2 years	3 years	5 years	10 years	30 years
AFC_1	0.9945	0.9933	0.9901	0.9890	0.9900	0.9943	0.9977
ADF	-0.6976	-0.7955	-1.0472	-1.1583	-1.1276	-0.8327	-0.4462
p-value	0.3944	0.3586	0.2664	0.2258	0.2370	0.3450	0.4864

2.3.2 Extended Kalman Filter Results

Table 5 reports the parameter estimates for default-free term structure. The results are very similar to those from Duffee (1999). It shows that first factor has a higher long-run mean and faster speed of mean-reversion than second factor has. The major difference between our results and the results from Duffee (1999) is α term. Duffee (1999) set α term equal to -0.01, because the improvement in fit given by a substantially lower value of α term was minimal in his sample. However, in our sample, results from setting α term equal to -0.01 was worse than the results shown in Table 5 although we did not include these results in the table.

Based on the estimated default-free process, we estimate the default risk parameters. Table 6 summarizes the estimation results with non-callable bonds for each of the 38 firms. It shows that estimated $\beta_{d1}$and β_{d2} is generally negative, which supports the negative relationship between default spread and risk-free interest rates. The results shown in Table 6 are generally similar to those in Duffee (1999) or Jarrow et al. (2010). The major difference, again, is that we allow α term to be a negative value, which results in bigger magnitude of estimated parameters, compared to those in Duffee (1999) and Jarrow et al. (2010).[8] However, distribution of the credit spreads is very similar to those in Duffee (1999) and Jarrow et al. (2010).

Finally, based on the estimated default-free and defaultable term structures, we estimate the call process using a make-whole callable bond with similar maturity from the same firm. Panel A in Table 7 shows results with using our model in Chapter 1, and Panel B reports results with using the JLLW model. Panel A in Table 7 shows that

[8] Even though we did not include results from restricting α term to be positive, in our sample data, allowing α term to be negative give noticeable improvement in fit.

estimated β_{c1} has a positive value, which implies that an individual default factor is positively related to the call spreads. Besides, the estimated call spread has mean (median) 16 basis points (11 basis point), which is reasonable, based on the study of Power and Tsyplakov (2008). Meanwhile, Panel B in Table 7 reports that the estimated β_{c2} has a value closed to zero, which implies that the second factors from the default-free process is not related to the call spreads. Furthermore, the estimated β_{c1} in panel B is positive, which indicates positive relationship between the first factor from the default-free process and the call spread. However, the call spreads in panel B has mean (median) as 13.13% (6.64 %). Moreover, mean errors and RMSE is relatively very high, compared with those in panel A. Therefore, it is hard to see that the JLLW model fit into our make-whole callable bond data.

Results in Table 7 shows evidence that the value of embedded option in the make-whole callable bond is dependent on the individual default factor, but there is no evidence that the value of the embedded option is dependent on risk-free rates. Consequently, in order to apply reduced-form model to the make-whole callable bond, the JLLW model is not suitable. This is due to the model originally being constructed for the regular callable bond whose purchase price in the embedded option is fixed. On the other hand, our model developed in Chapter 1 is reasonably fitted to our sample data, because our model includes the effect of an individual default factor on the call spreads.

These findings are more prominent in Table 8 that reports out-of-sample analysis. In this analysis, we have five firms each of that has another make-whole bond. In order to produce results in panel B in Table 8, we first extract the estimated parameters for the

Table 5: Extended Kalman Filter Estimates of Default-Free Model

The instantaneous interest rate is

$$r_t = \alpha + s_{1,t} + s_{2,t}.$$

The dynamics are

$$ds_{i,t} = \kappa_i(\theta_i - s_{i,t})dt + \sigma_i\sqrt{s_{i,t}}dW_{i,t}, \text{ for } i = 1,2 \quad \text{(True measure)}$$

$$ds_{i,t} = \left(\kappa_i\theta_i - (\kappa_i + \eta_i)s_{i,t}\right)dt + \sigma_i\sqrt{s_{i,t}}d\widehat{W}_{i,t}, \text{ for } i = 1,2 \quad \text{(Martingale Measure)}$$

US Treasury rates data from January, 2002 to March, 2011 are used. The standard errors (STD) are computed assuming the Kalman filter linearization is exact.

i	α	κ_i	θ_i	σ_i	η_i
1	-0.0012	0.3347	0.0359	0.0755	-0.0909
STD	0.0010	0.0341	0.0045	0.0039	0.0410
2		0.1973	0.0015	0.0327	-0.1663
STD		0.0456	0.0003	0.0041	0.0400

Bond maturity	Measurement error	Mean error (actual-fitted)	Root mean square error
6 months	0.0037	-0.0013	0.0057
STD	0.0003		
1 year	0.0041	-0.0018	0.0057
STD	0.0008		
2 years	0.0009	-0.0023	0.0052
STD	0.0001		
3 years	0.0015	-0.0021	0.0049
STD	0.0002		
5 years	0.0032	-0.0009	0.0047
STD	0.0003		
10 years	0.0037	0.0008	0.0051
STD	0.0002		
30 years	0.0036	-0.0007	0.0048
STD	0.0002		

Table 6: Summary of Extended Kalman filter Estimates of 38 Firms' Defaultable Model

The instantaneous default-free interest rate is given by

$$r_t = \alpha + s_{1,t} + s_{2,t}$$

where $s_{1,t}$ and $s_{2,t}$ are independent square-root processes. Firm j's instantaneous default risk is given by

$$(1-\sigma)\lambda_{d,t} = \alpha_d + h_{d,t} + \beta_{d1}(s_{1,t} - \bar{s}_1) + \beta_{d2}(s_{2,t} - \bar{s}_2)$$

where $h_{d,t}$ follows a square-root process that is independent of the profess for $s_{i,t}$, i=1,2:

$$dh_{d,t} = \kappa_d(\theta_d - h_{d,t})dt + \sigma_d\sqrt{h_{d,t}}dW_{d,t} \quad \text{(True Measure)}$$

$$dh_{d,t} = (\kappa_d\theta_d - (\kappa_d + \eta_d)h_{d,t})dt + \sigma_d\sqrt{h_{d,t}}d\widehat{W}_{d,t} \quad \text{(Martingale measure)}$$

The estimation period is from January, 2002 to March, 2011 are used. RMSE indicates the square root of the mean of the squared differences between the actual and fitted yields to maturity on firm j's bond. Mean Error is estimated by actual minus fitted yields to maturity on firm j's bond.

	First Quartile	Median	Third Quartile	Average
α	-0.0206	-0.0183	-0.0069	-0.0223
θ	0.0149	0.0261	0.0416	0.0344
κ	0.1695	0.3044	0.4979	0.3086
σ	0.1298	0.1606	0.2426	0.1847
η	-0.1917	-0.1661	-0.1317	-0.1588
β_{d1}	-0.4396	-0.1824	0.0261	-0.2282
β_{d2}	-0.9784	-0.6260	-0.1454	-0.5366
Spread	0.0054	0.0141	0.0268	0.0176
Mean Error	-0.0006	0.0000	0.0006	-0.0003
RMSE	0.0046	0.0067	0.0081	0.0063

Table 7: Summary of Extended Kalman Filter Estimates of 38 Firms' Make-Whole Callable Model

The instantaneous default-free interest rate is given by

$$r_t = \alpha + s_{1,t} + s_{2,t}$$

where $s_{1,t}$ and $s_{2,t}$are independent square-root processes. Firm j's instantaneous default risk is given by

$$(1-\sigma)\lambda_{d,t} = \alpha_d + h_{d,t} + \beta_{d1}(s_{1,t} - \bar{s}_1) + \beta_{d2}(s_{2,t} - \bar{s}_2)$$

where $h_{d,t}$ follows a square-root process that is independent of the profess for $s_{i,t}$, i=1,2. For Panel A, firm j's instantaneous call spread is given by

$$(1-k)\lambda_{c,t} = \alpha_c + h_{c,t} + \beta_{c1}\frac{M}{h_{d,t}}$$

For Panel B, firms j's instantaneous call spread follows Jarrow et al. (2010), which is given by

$$(1-k)\lambda_{c,t} = \alpha_c + h_{c,t} + \beta_{c1}(s_{1,t} - \bar{s}_1) + \beta_{c2}\frac{C}{s_{2,t}}$$

where $h_{c,t}$ follows a square-root process that is independent of the profess for $s_{i,t}$, i=1,2:

$$dh_{c,t} = \kappa_c(\theta_c - h_{c,t})dt + \sigma_c\sqrt{h_{c,t}}dW_{c,t} \quad \text{(True Measure)}$$
$$dh_{c,t} = (\kappa_c\theta_c - (\kappa_c + \eta_c)h_{c,t})dt + \sigma_c\sqrt{h_{c,t}}d\widehat{W}_{c,t} \quad \text{(Martingale Measure)}$$

M and C indicate make-whole premium and coupon rate respectively. The estimation period is from January, 2002 to March, 2011 are used. RMSE indicates the square root of the mean of the squared differences between the actual and fitted yields to maturity on firm j's bond. Mean Error is estimated by actual minus fitted yields to maturity on firm j's bond.

A. Model Dependent on Individual Default Factor

	First Quartile	Median	Third Quartile	Average
α	-0.0099	-0.0077	-0.0020	-0.0060
θ	0.0079	0.0175	0.0494	0.0355
κ	0.0105	0.0384	0.1503	0.1104
σ	0.4181	0.4462	0.4919	0.4559
η	-0.0110	-0.0104	-0.0096	-0.0103
β_{c1}	0.00005	0.00009	0.00011	0.0025
Spread	-0.0032	0.0011	0.0057	0.0016
Mean Error	-0.0019	-0.0005	0.0001	-0.0009
RMSE	0.0047	0.0061	0.0087	0.0066

B. Model Dependent on Risk-Free Rates

	First Quartile	Median	Third Quartile	Average
α	-0.0074	0.0027	0.0058	0.0014
θ	0.0169	0.0337	0.1325	0.0960

Table 7 (continued)

κ	0.2961	0.3959	0.4309	0.3635
σ	0.3370	0.3663	0.4409	0.3943
η	-0.0116	-0.0108	-0.0105	-0.0111
β_{c1}	-0.0167	0.1085	0.2894	0.2318
β_{c2}	0.0000	0.0000	0.0000	0.0000
Spread	0.0365	0.0664	0.1584	0.1313
Mean Error	-0.0009	0.0040	0.0136	0.0052
RMSE	0.0087	0.0173	0.0286	0.0182

make-whole callable bond from each of those five firms from Table 7. With the estimated parameters, we estimate yields-to-maturity for another make-whole callable bond from the same firm.

Panel B in Table 8 shows that, based on the mean errors and RMSE, our model outperforms the JLLW model except for the Emerson Electric case. Additionally, Figure 1 shows how well our model can explicitly capture the sequence of observed yields-to-maturity movement of the make-whole callable bond. In Figure 1, it is explicitly shown that there is difficulty in predicting the sequence of the observed yields-to-maturity in the JLLW model except in the case of Emerson Electric. On the other hand, our model consistently gives reasonable prediction, relative to the JLLW model. Along with Figure 1, results in Table 8 confirm the findings from Table 7.

Table 8: In-Sample and Out-of-Sample Analysis

This table reports in-sample and out-of-sample performances of our model and a model (JLLW) from Jarrow et al. (2010). First, default-free and defaultable term structure is estimated, and in-sample result is obtained by estimating the call process using a make-whole callable bond from each firm. With using estimates from in-sample result, we estimates yields to maturity for another make-whole callable bond from same firm with similar maturity. RMSE indicates the square root of the mean of the squared differences between the actual and fitted yields to maturity on each firm's bond. Mean Error is estimated by actual minus fitted yields to maturity on each firm's bond. Maturity difference is life remaining to maturity for a make-whole callable bond minus life remaining to maturity for another make-whole callable bond. Maturity difference is expressed in year.

A. In-sample results for the reduced-form model

In-sample results	Allstate	Consol Natural Gas	Emerson Electric	IBM	Waltdisney
Coupon	6.125	5	4.5	4.75	5.7
Make-Whole Premium	25	20	10	12.5	12
model from this paper					
Mean error	-0.0003	-0.0019	-0.0021	-0.0006	0.0001
RMSE	0.0058	0.0062	0.0047	0.0050	0.0056
JLLW model					
Mean error	-0.0021	0.0130	0.0039	0.0160	-0.0130
RMSE	0.0290	0.0297	0.0067	0.0270	0.0215

B. Out-of-sample results for the reduced-form model

Out-of-sample results					
Coupon	5	5	5	6.5	4.7
Make-Whole Premium	15	20	15	50	20
model from this paper					
Mean error	-0.0026	-0.0013	-0.0041	0.0005	-0.0025
RMSE	0.0071	0.0049	0.0059	0.0067	0.0071
JLLW model					

Table 8 (continued)

Mean error	-0.0165	0.0559	0.0020	-0.0097	-0.0192
RMSE	0.0312	0.0562	0.0058	0.0120	0.0207
Maturity Difference	2.4986	0.7534	1.6247	0.8767	-1.1260

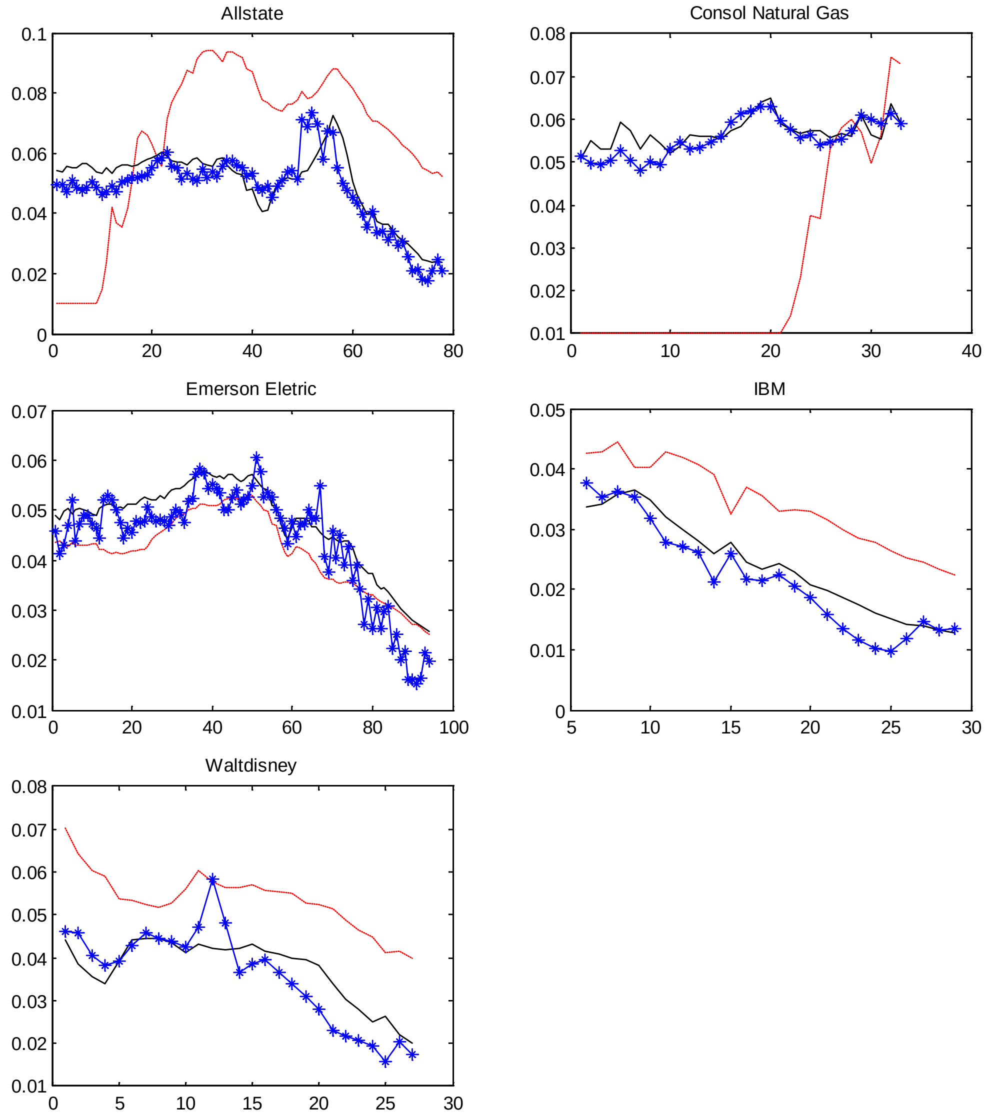

Figure 1: Out-of-Sample Performance of Our Model vs. Model from Jarrow et al. (2010)
This figure provide time series plots of yields to maturity based on market prices and estimated yield to maturity under the two models of callable bond of five companies. Yield to maturity based on market price is solid line, estimated yield to maturity under the model in this study is solid line with star, and estimated yield to maturity under the model from Jarrow et al. (2010) is dashed line.

2.4 Summary and Conclusion

This chapter investigates whether or not the reduced-form model developed in Chapter 1 really fits into the real market data of make-whole callable bonds. To implement our test, we need econometric tools. So, we first make a comparison between the performance of the Kalman filter and the MCMC method on our US Treasury data. The MCMC method perform poorly, compared to the Kalman filter. This could be due to the evidence of our US Treasury data having near unit-root behavior. As Sögner (2009) demonstrates this issue with the MCMC method, parameter estimation becomes almost impossible, due to ill conditioned transformation between the latent state variables process driving the yields and the yield observed, especially when all yields from a different maturity Treasury are observed with noise. For this reason, we use the extended Kalman filter estimate parameter value.

The results from the Kalman filter estimation suggest that the value of the embedded option in the make-whole callable bond is dependent on individual default factors, but there is no evidence that the value of the embedded option is dependent on the risk-free rates. Consequently, in order to apply the reduced-form model to the make-whole callable bond, the JLLW model is not suitable. This is because this model, originally constructed for the regular callable bond, contains a fixed purchase price in the embedded option. On the other hand, our model developed in Chapter 1 is reasonably fitted to our sample data, because our model includes the effect of an individual default factor on the call spreads.

CHAPTER 3: DEBT STRUCTURE AND CORPORATE YIELD SPREAD

This study examines the relation between proportions of short-term debt to the firm's total debt and corporate yield spreads. The determinants of credit spread have been the central issue in corporate finance. Since the seminal work of Black and Scholes (1973) and Merton (1974), a large theoretical literature on pricing of corporate bonds has been introduced. Theoretical models of credit spreads can be categorized as either structural or reduced-form models of default. There are well-known differences between these two models. Unlike structural models, reduced form models do not consider a link between default and firm value explicitly. In the reduced form model, default time cannot be predicted through the firm's value, rather it is the first jump governed by the exogenous jump process.[9] Structural models assume that a firm defaults when the value of its debt exceeds its value of assets.[10] From this assumption, it is perceived that an increase in a firm's leverage ratio intensifies default risk, which consequently, increases yield spread. This relationship has often been found in previous empirical studies by (e.g. Colin-Dufresne et al. (2001), Campbell and Taksler (2003), and Chen et al. (2007) etc.).

However, Colin-Dufresne et al. (2001) and Huang and Huang (2003) indicate that credit risk is not enough to explain corporate-Treasury yield spread. Subsequently, many factors, other than credit risk, determine the spread, which have been introduced. Tax

[9] For more details on reduced-form models, see Jarrow and Turnbull (1995), Lando (1998), Duffie and Singleton (1999), and Duffee (1999) etc.

[10] For more details on structural models, see Geske (1977), Smith and Warner (1979), Longstaff and Schwartz (1995), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001) etc.

premium and Risk premium (Elton et al., 2001), idiosyncratic equity volatility (Campbell and Taskler, 2003), liquidity premium (Longstaff et al., 2005, and Chen et al., 2007), and firm-specific information (Kwan, 1996) are popularly recognized in this field. Among these factors, firm-specific information has been left out in recent studies. It could be because, as Campbell and Taksler (2003) notes, bond rating contemporaneously incorporating observed firm-level accounting characteristics, and rating agencies may also absorb market information through the observed yield spread when assigning a credit rating.

Meanwhile, Diamond (1991) introduces liquidity risk as the risk of a borrower being forced into inefficient liquidation when refinancing is not available.[11] In Diamond's (1991) model, choosing short-term debt over long-term debt has both benefit and cost. Firms using short-term debt could successfully lower debt's interest rate if positive information is revealed at refinancing. However, these firms are also exposed to refinancing risk if the revealed information is negative; lenders may refuse to refinance and force a firm into premature liquidation. Sharpe (1991) and Titman (1992) also suggest that unfavorable news about a borrower may arrive at the date of refinancing, causing investors not to extend credit or to raise the default premium on new debt. Barclay and Smith (1995) and Mark and Mauer (1996) find evidence in support of the Diamond (1991) model.

According to Diamond's argument, the firm holding the larger proportion of short-term debt in its debt structure is more vulnerable to the unforeseen negative event. Accordingly, bond investors should require more compensation for investing in bonds

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[11] In this paper, liquidity risk indicates the risk that Diamond (1991) introduces, and liquidity premium indicates the premium that is caused by bond's degree of liquidity costs in the market.

from a company with higher proportion of short-term debt in its debt structure. Furthermore, he also argues that a firm with unestablished credit history (an unrated company) relies more on short-term debt because it has limitation to access the public market. A firm with speculative grade is more likely to use long-term debt because it is not able to afford liquidity risk. A firm with investment grade that has relatively less growth opportunity is more likely to use short-term debt because it could lower its cost of debt. Empirical evidence of this nonlinear relation between bond ratings and a company's proportional short-term debt is often found in the studies mentioned above. On the other hand, Eom et al. (2004) studies the performance structural models. In their study, they found that, generally, models tend to severely overstate credit risk of firms with high leverage or volatility, but suffer from a spread under-prediction problem with safer bonds. For this reason, we argue that, possibly, portion of unexplained spread from the structural form is liquidity risk.

Therefore, this study attempts to investigate whether or not this liquidity risk is priced in the bond market with controlling generally accepted yield spread factors such as credit rating, maturity, amount outstanding, tax effect, equity volatility, and liquidity premium. In other words, an investor in the bond market realizes this liquidity risk, and requires premium for taking this risk. Our study finds that, for investment grade bonds, the issuers' fraction of short-term debt has a positive effect on the cost of using their bonds. However, for speculative grade bonds, we cannot conclude on any relationship between liquidity risk and corporate bond yields. That could be partially due to the limited data source for speculative grade bonds.

This chapter is organized as follows: brief review of previous literatures in Section 1, variables and data are presented in Section 2, statistical methodology and our results are presented in Section 3, and Section 4 summarizes and concludes this chapter.

3.1 Literature Review

Theoretical models of credit spreads can be categorized as either structural or reduced-form models of default. Reduced form models do not explicitly consider a link between default and firm value. In the reduced form model, default time cannot be predicted through the value of a firm, rather it is the first jump governed by an exogenous jump process. For this reason, the model is better for fitting observed credit spreads rather than offering insight on the fundamental determinants of the credit spread.

On the other hand, the structural model explicitly relates default to firm value through the contingent-claim approach, which is introduced by Merton (1974). Since the introduction, many studies (e.g. Geske (1977), Smith and Warner (1979), Longstaff and Schwartz (1995), Leland and Toft (1996), Collin-Dufresne and Goldstein (2001) etc.) have extended or upgraded the structural models. Under the structural form, changes in credit spreads could be predicted by changes in spot rate, leverage, and volatility of firm value. Additionally, changes in slope of the yield curve is often considered because Litterman and Scheinkman (1991) find that the two most important factors driving the term structure of interest rates are level and slope of the term structure. Therefore, rationally, if changes in slope of the Treasury curve make the expected future short rate change, it implies that changes in slope of the Treasury curve affect changes in credit spread. Changes in macroeconomic condition is also often considered, as Fama and French (1989) find that credit spreads widen when economic conditions are weak.

The empirical evidence of the negative relationship between corporate yield spread and treasury yield is presented by Duffee (1998) even though this relationship is weaker in the case of non-callable bond than callable bond. Furthermore, Collin-

Dufresne et al. (2001) generally confirm that factors mentioned above are significant in determining changes in corporate yield spread except for the slope of Treasury curve. However, their notable finding is that the residuals from their regression are highly cross-correlated, and, from principal components analysis, they are mostly driven by a single common factor. Although they cannot provide a sufficient explanation to the single common factor, they do suggest that monthly spread changes are principally driven by local supply/ demand shocks that are independent of both credit risk factors.

Additionally, with the use of information in credit default swaps, Longstaff et al. (2005) find that the majority of the corporate spread is caused by default risk. They also found a significant non-default component that is time varying and strongly related to measures of bond-specific illiquidity. This liquidity premium is also found in the study of Chen et al. (2007) with a relatively large bond dataset. Likewise, with the use of structural models, Hung and Hung (2003) find that credit risk is only a small portion of observed corporate yield spreads for investment grade bonds. Although credit risk accounts for significantly higher portion of the observed yield for speculated grade bonds, there is still a significant portion that cannot be explained by credit risk.

Thus, researchers have been looking for factors that the structural models could not capture. One of the popular factors is bond-specific liquidity, as mentioned above. The other factors could be firm-specific information and tax effect. Kwan (1996) examines the correlation between the returns on individual stocks and the yield changes of individual bonds issued by the same firm. In this study, he empirically finds a negative and contemporaneous correlation. From this finding, he concludes that individual stocks and bonds are driven by the same firm-specific information. This negative correlation is also confirmed by the study of Campbell and Taskler (2003). However, a notable finding in their study is that idiosyncratic firm-level volatility can explain as much cross-sectional variation in yields as can credit ratings while controlling for liquidity premium and factors from the structural form.

On the other hand, Elton et al. (2001) introduce tax premium and risk premium in order to explain the portion of the corporate yield spread that default premium alone cannot capture. They argue that tax effects occur because the investor in corporate bonds is subject to state and local taxes on interest payments, whereas government bonds are not subject to these taxes. Consequently, investors should require more compensation for investing in corporate bonds because they have to pay extra expenses (i.e. tax expenses). They also argue that there could be risk premiums for systematic risk, if changes in the required compensation for risk affect both corporate bond and stock market. They found empirical evidence that a significant portion of the unexplained yield spread is caused by tax and risk premium.

As mentioned in the introduction, this paper attempts to make a linkage between Diamond's liquidity risk and the corporate yield spread. According to Diamond's argument, a firm with a larger proportion of short-term debt to its debt structure is more vulnerable to the unforeseen negative event. We believe that investors should require additional compensation for this liquidity risk. Therefore, this paper examines the effects of proportion of short-term debts to its debt structure on the corporate yield spread with controlling factors previously considered. To the best of our knowledge, no one has investigated this relationship directly. Next section presents data and variables used in this study.

3.2 Data and Variables[12]

3.2.1 Data

Data for monthly yield spreads and bond characteristics is collected from *Datastream*. If *Datastream* has the available Standard and Poor's rating for a bond, this is where we collect the rating information, otherwise, the rating information is from the *Fixed Income Securities Database*. Furthermore, the bonds that do not have any credit rating information from either *Datastream* or *Fixed Income Securities Database* are excluded. For all firm-level data, Compustat Annual Industrial database is used. In order to minimize any survivorship bias in the yield spread, data for both active and inactive firms is collected. The firm-level data is collected in the year prior to the yield spread measurement. Finally, in order to measure liquidity costs for bonds, we use the modified LOT model proposed by Chen et al. (2007). The most popular measure of liquidity costs is the bid-ask quotes for an individual security, but in bond data, these bid-ask quotes are very limited. Therefore, in this study, we use an alternative method, the LOT model. Appendix C shows how we measure the liquidity costs from the LOT model. In addition, statistically insignificant estimation is excluded. The time frame of our data set is from 2003 to 2010, because of the availability of bond data beginning from 2003.

3.2.2 Variables

Since this study examines the effect of proportional short-term debt on corporate yield spread, our main variable is proportional short-term debt (*Short-Term Debt*). In this study, short-term debt is the amount of debt that will mature within a short-period (one or two years) of time. As stated above, if a firm has a higher proportion of short-term debt,

[12] Procedure of collecting data the choice of yield spread determinants in this study closely follow one in Chen et al. (2007), Elton et al. (2001) and Campbell and Taksler (2003).

it means that the firm is more vulnerable to unforeseen, negative, future shock. For this reason, the expected relation between proportion of short-term debt and corporate yield is positive.

Additionally, in order to control the tax effect in the study of Elton et al. (2001), coupon rate (*Coupon*) is used. According to Elton et al. (2001), an investor of a bond that pays higher coupon is subject to a larger tax expense. We also include equity volatility to control a firm's systematic and idiosyncratic risk. The equity volatility (σ_E) is calculated using 252 daily returns for the year prior to the corporate yield measure. Moreover, size of a firm (*Size*) based on 1980 dollar value and market to book value of a firm (*Market to book*) are included in our regression. We take the logarithm of total assets to obtain a firm's size. Market to book ratio is the market value of a firm divided by book value. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. The definition of market value of equity is stock price times the number of shares outstanding. We also include four accounting variables (*Pre-Tax, Income to sale, Book-leverage, and Leverage)*. The variable, Pre-Tax, indicates pretax interest coverage ratio, and Income to sale is operating income divided by net sales. We have two different leverage measures: book-leverage and leverage. Book-leverage is total debt divided by book value of total assets, and leverage is total debt divided by market value of a firm. It is generally perceived that a high level of the first two variables means healthy firms, and that leads us to conclude that these variables are negatively related to corporate yield spread. High levels of the second two variables indicate that firms are highly levered, so an investor requires more compensation for bonds issued by the firms.

For pretax interest coverage ratio variable, four groups are used following the procedure outline in Blum et al. (1998) in order to capture the possibility that particularly low pretax interest coverage convey more information about the risk of an issuer than high interest coverage. For this reason, dummy variables are created for each group to indicate whether pretax interest coverage is less than 5, between 5 and 10, between 10 and 20, or greater than 20. In addition, bond-specific information (*Credit rating, Maturity, and Amount*) is also included. The variable, credit rating, is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in years. Amount is a logarithm of amount of bonds outstanding. Furthermore, we consider three macroeconomic variables associated with yield spread. The variables are the one-year Treasury rate (*T-note)*, the difference between the 10-year and 2-year Treasury rates (*Term Slope*) for the slope of the yield curve, and the difference between the 30 days Eurodollar and 3-month Treasury bill rate (*EuroDollar*) that is the control for other potential liquidity effects on corporate bonds relative to Treasury bonds.

Finally, we directly obtain corporate yield spread (Yield spread) from Datastream. Datastream calculates the difference between the bond yield and the yield of a comparable maturity treasury bond. Table 9 shows the summary statistics of key variables (i.e. Yield spread, Liquidity, Short-term debt, Leverage, and Credit rating). Yield spread, liquidity, short-term debt, and leverage varies widely as evidence by the quartile distribution, and their standard deviations. However, it is noticeable that the distribution of credit rating is clustered in investment grade region. That is simply because majority of our bonds in the sample are investment grade bond. Total number of bond is 576, and 520 out of these bonds is investment grade bond, and the remaining is speculative grade bond. The number of firms in our sample is 140.

Table 10 shows key variables across each key variable. Panel A in Table 10 shows that yield spread and liquidity is positively related to credit rating, which is expected. Intuitively, low rated bonds have higher yield and liquidity costs. It also displays that fraction of debts that will mature in one or two years is negatively related to credit ratings, which is consistent with previous literature in debt maturity. Diamond (1991) argues that speculative grade companies more likely use long-term debt because they cannot afford liquidity risk. Later, Barclay and Smith (1995), Mark and Mauer (1996), and Johnson acquire this relationship. Additionally, from panel B in Table 10, leverage is shown to be negatively related to the fraction of debts that matures in one or two years. This is also consistent with previous literature in debt maturity. According to Diamond (1991), speculative grade companies or companies that have a big growth opportunity are likely to have a high leverage ratio. Moreover, those companies cannot afford liquidity risk, so they tend to avoid using short-term debts.

However, there is no clear linear relation between the fraction of debts maturing in one or two year and liquidity cost or leverage from panel C and D in Table 10. Nevertheless, panel E exhibits negative relation between the yield spread and the fraction of debts. This could be due to the negative relation between leverage and the fraction of debt.

Table 9: Summary Statistics

This table summarizes the distribution of key variables in this study. The yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond, as determined from Datastream. Short-term debt1 or short-term debt 2 indicates fraction of total debt that matures within one year or two year respectively. Book-leverage or leverage indicates total debt divided by book value of total assets or market value of a firm. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. The definition of market value of equity is stock price times the number of shares outstanding. Credit rating is numbered from one (AAA rated bond) to 22 (D rated bond). bp stands for basis points.

Variable	Mean	Standard deviation	Min	First quartile	Median	Third quartile	Max
Yield spread(bp)	293.3922	570.3598	10.1583	133.5250	193.9125	313.3167	13366.3000
Liquidity(%)	1.2593	1.8410	0.0007	0.6095	0.8510	1.1656	19.0491
Short-term debt1(%)	16.5940	15.0532	0.0036	6.4626	11.8287	22.5814	98.0087
Short-term debt2(%)	24.2702	16.6129	0.0178	12.6770	21.8898	32.5092	99.2114
Book-leverage(%)	24.4376	14.4840	0.3165	16.1918	21.8088	29.3280	96.2983
Leverage(%)	20.1649	12.9748	1.0733	10.9375	16.1300	25.7782	68.2899
Credit rating	7.9908	3.1201	1.0000	6.0000	8.0000	9.8333	21.0000
Number of bonds	576	Investment Grade bonds	520			number of firms	140

Similarly, those observed in Table 10 can also be found in Table 11. Liquidity cost, leverage, and credit rating is significantly positively correlated to corporate bond yield spreads. On the other hand, the fraction of debts maturing in one or two year is significantly negatively correlated to the yield spread. However, the correlation between the fraction of debt and yield spread or liquidity cost is not clear.

Table 10 and 11 show that our sample data set to be valid, but they cannot answer our central question of whether bond investors require compensation for the liquidity risk. To answer this question, it is crucial to control the level of leverage, credit rating, and liquidity. The next section seeks to answer our questions with regression.

Table 10: Key Variables Across Each Key Variable Tertiles

This table shows key variables across each key variable tertiles. The yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt1 or short-term debt 2 indicates fraction of total debt that mature within one year or two year respectively. Book-leverage or leverage indicates total debt divided by book value of total assets or market value of a firm. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. The definition of market value of equity is stock price times the number of shares outstanding. Credit rating is numbered from one (AAA rated bond) to 22 (D rated bond). bp stands for basis points.

A. Varibles across rating tertiles

Credit rating tertile	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)	rating
1	181.5835	0.9680387	21.37346	29.04642	15.43361	5.320547
2	228.6789	0.8723968	13.01374	20.80112	20.11833	8.362825
3	535.9021	2.132703	12.48805	19.66207	28.38227	11.66

B. Variables across leverage tertiles

Leverage tertile	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)	rating
1	165.2923	0.8917	22.8627	31.4861	8.8607	6.7117
2	216.2237	0.9417	14.0101	21.7407	16.6515	7.6199
3	498.6382	1.9191	13.0188	19.4492	35.0478	9.3443

C. Variables across short-term debt tertiles

Short-term debt1 tertile	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)	rating
1	272.9740	1.2465	4.0136	10.2910	21.7344	8.3961
2	369.6097	1.2865	12.5275	22.2560	20.7914	8.1563
3	247.6433	1.2355	33.2708	41.5795	18.4792	7.2146

Table 10 (continued)

D. Variables across liquidity tertiles

Liquidity tertile	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)	rating
1	201.7484	0.4601	17.2473	25.4808	18.2235	7.4547
2	210.1874	0.8591	16.1141	24.1798	18.6635	7.9036
3	468.3706	2.4596	16.4539	23.1455	23.5700	8.6147

E. Variables across yield spread tertiles

Yield spread tertile	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)	rating
1	113.2143	0.7374	20.3322	28.6839	13.9711	6.4118
2	198.7213	0.9674	14.4686	21.9823	18.9592	7.7631
3	568.4450	2.0737	14.5542	21.4342	29.0607	9.7989

Table 11: Pearson Correlation among Key Variables

This table shows Pearson correlation among key variables. The yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt1 or short-term debt 2 indicates fraction of total debt that mature within one year or two year respectively. Book-leverage or leverage indicates total debt divided by book value of total assets or market value of a firm. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. The definition of market value of equity is stock price times the number of shares outstanding. Credit rating is numbered from one (AAA rated bond) to 22 (D rated bond). bp stands for basis points.

	Yield spread(bp)	Liquidity(%)	Short-term debt1(%)	Short-term debt2(%)	Leverage(%)
Liquidity(%)	0.4262*				
Short-term debt1(%)	-0.0378*	0.0047			
Short-term debt2(%)	-0.0234	-0.0402*	0.9159*		
Leverage(%)	0.2423*	0.3367*	-0.1614*	-0.2517*	
Credit rating	0.4144*	0.3786*	-0.1511*	-0.1908*	0.4203*

* indicates correlation is significantly different from zero at the 0.05 level or higher

3.3 Regression Models and Results

First, we generate results from the simple regression models as below:

$$Yield\ Spread_{it} = \beta_0 + \beta_1 Short\text{-}term\ debt_{it} + \beta_2 Liqudity_{it} + \beta_3 Credit\ rating_{it} + \beta_4 Maturity_{it} + \beta_4 Coupon_{it} + \beta_5 T\text{-}note_{it} + \beta_6 Term\ slope_{it} + \beta_7 Eurodollar_{it} + \beta_8 Volatility_{it} + \beta_9 Income\ to\ Sale_{it} + \beta_{10} Size + \beta_{11} Market\ to\ Book_{it} + \beta_{12} Amount + \beta_{13} Pre\text{-}Tax_{it} + \beta_{14} Leverage_{it} + \epsilon_t \quad (3.1)$$

where the subscript "*it*" refers to bond i and year t and Short-term debt refers to the fraction of debts maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in year. Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. Volatility is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Amount is logarithm of amount of bonds outstanding. Pre-Tax indicates pre-tax interest coverage ratio that is group into one of four categories according to Blume et al. (1998). Leverage is either book value of leverage (total debt/total asset) or market value of leverage (total debt/ market value of each issuer). Finally, Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream.

Table 12: Yield Spread and Fraction of Debt Maturing in One or Two Year

Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debts maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in year. Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Amount is logarithm of amount of bonds outstanding. Pre-Tax indicates pre-tax interest coverage ratio that is group into one of four categories according to Blume et al. (1998). Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. T-statistics are presented in parentheses.

Variable								
Short-term debt1	0.8473	*			1.7068	***		
	(1.92)				(3.59)			
Short-term debt2			0.4206				0.7836	**
			(1.35)				(2.36)	
Liquidity	22.948	***	9.8608	***	23.352	***	9.0995	***
	(10.05)		(4.32)		(9.98)		(3.86)	
Credit rating	40.4816	***	27.700	***	42.5819	***	30.525	***
	(13.95)		(8.35)		(14.19)		(8.87)	
Maturity	-7.4237	***	-9.5086	***	-4.5404	***	-6.416	***
	(-4.45)		(-6.19)		(-2.69)		(-4.12)	
Coupon	614.34	***	637.7181	***	618.09	***	641.07	***
	(15.36)		(15.91)		(15.3)		(16.05)	
T-note	-97.466	***	-78.6782	***	-92.9458	***	-74.5560	***
	(-8.44)		(-7.68)		(-7.9)		(-7.11)	
Term slope	-119.26	***	-91.321	***	-101.49	***	-74.943	***
	(-6.21)		(-5.37)		(-5.19)		(-4.31)	
Eurodollar	61.249	***	60.797	***	64.148	***	63.9741	***
	(10.01)		(11.33)		(10.31)		(11.65)	
σ_E	6.8119		14.9150	***	8.8589		17.58107	***
	(1.13)		(2.80)		(1.44)		(3.22)	
Income to Sales	-53.957	*	-192.28	***	-23.7712		-210.62	***
	(-1.72)		(-3.06)		(-0.75)		(-3.26)	

Table 12 (continued)

Size	7.9817		-18.443		14.3643		-14.6874	
	(0.51)		(-1.29)		(0.90)		(-1.01)	
Market to Book	56.998	***	32.892	***	-1.8076		-19.585	*
	(3.94)		(2.58)		(-0.13)		(-1.66)	
Amount	-71.907	***	-62.972		-77.5085	***	-68.064	***
	(-2.80)		(-2.46)		(-2.99)		(-2.67)	
Pre-Tax D1	-330.576	***	-96.111		-294.246	***	-86.4237	
	(-4.65)		(-1.11)		(-4.06)		(-0.97)	
Pre-Tax D2	-314.137	***	-89.622		-311.968	***	-109.557	
	(-4.40)		(-1.03)		(-4.29)		(-1.22)	
Pre-Tax D3	-278.997	***	-58.384		-299.690	***	-96.957	
	(-3.90)		(-0.66)		(-4.12)		(-1.07)	
Pre-Tax D4	-257.411	***	-30.7576		-297.310	***	-85.675	
	(-3.50)		(-0.34)		(-3.96)		(-0.92)	
Leverage	9.5080	***	8.1827	***				
	(10.00)		(9.75)					
Book-leverage					1.6442	*	0.75098	
					(1.94)		(1.02)	
N	2953		2850		2953		2850	
R-square	0.1596		0.1180		0.1530		0.1143	

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

Table 12 shows statistical result from this model. When the regression includes market value of leverage, only fraction of debts maturing in one year is significantly positively related to corporate yield spreads at 10% level. However, when the book-value of leverage is included, both fraction of debt maturing in one and two year is significantly related to the yield spreads at 1% and 5% level respectively. On the other hand, compared with coefficients on the other key variables (leverage, credit rating, liquidity costs), the magnitude of the estimated coefficient on the short-term debt is relatively small. According to Table 12, if a firm increases fraction of debt maturing in one year by 10 %, its corporate yield spread is increased by about 85 basis points.

The other coefficients on the other variables have, generally, expected signs except for the coefficient of maturity. Campbell and Taksler (2003) note that, for investment grade bonds, longer maturities are often found to be associated with increased yield spreads, from which we expected positive sign. However, Helwege and Turner (1999) argue that, for speculative grade bonds, better quality firms are able to issue bonds with longer maturities, from which we can expect negative sign. For these reasons, we may expect that factors can have different effects on the yield spread from two different groups (i.e. speculative and investment grade bonds). Therefore, we perform two regression separately based on two grade group, even though our sample for speculative bonds is very limited.

Table 13 shows the statistical results based on the bond grade. Indeed, coefficients on maturity have the expected signs. For investment grade bonds, the fraction of debt maturing in one or two year is significantly positively related to the yield spread. Like the magnitude of estimated coefficient on the fraction of debts from Table

12, the estimated one on the fraction of debts from Table 13 is relatively small, compared with those on leverage, liquidity, and credit rating. On the other hand, for speculative grade bonds, the fraction of debt is significantly negatively related to the yield spread, which is unexpected. Since speculative grade companies have relative higher degree of asymmetric information than investment grade companies. If speculative grade firms rely on short-term debt, they need to refinance more often. Whenever they refinance, they need to reveal the prospects of their projects. This activity tends to reduce the problem of asymmetric information. Therefore, choosing short-term debt can be a tradeoff between liquidity risk and asymmetric information. From this interpretation, for speculative grade bonds, it could have the negative relationship. However, later in this study, we also generate two-stage least square estimation, in which the fraction of debt is positively related to the yield spread for the speculative bonds. In addition, we have so limited source of speculative grade bond data. Therefore, we save speculative grade bond case for future research.

So far, at least, for investment grade bonds, evidence from our regression shows the positive relation between the fraction of debt and the yield spread. However, it is possible that, if issuer's bond has high yield spread or bid-ask spread, then the issuer might choose short-term debt to reduce cost of using debts. For this reason, there might be endogeneity problems. In order to control this problem, these three equations are considered:

$$\begin{aligned}
Yield\ spread_{it} \;=\; &\beta_0 + \beta_1 Short\text{-}term\ debt_{it} + \beta_2 Liqudity_{it} + \beta_3 Credit\ rating_{it} + \\
&\beta_4 Maturity_{it} + \beta_4 Coupon_{it} + \beta_5 T\text{-}note_{it} + \beta_6 Term\ slope_{it} + \beta_7 Eurodollar_{it} + \\
&\beta_8 Volatility_{it} + \beta_9 Income\ to\ Sale_{it} + \beta_{10} Size_{it} + \beta_{11} Market\ to\ Book_{it} + \\
&\beta_{12} Amount + \beta_{13} Pre\text{-}Tax_{it} + \beta_{14} Leverage_{it} + \epsilon_t \qquad (3.2)
\end{aligned}$$

Table 13:Yield Spread and Fraction of Debt Maturity Based on Bond Grade

Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debts maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in year. Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Amount is logarithm of amount of bonds outstanding. Pre-Tax indicates pre-tax interest coverage ratio that is group into one of four categories according to Blume et al. (1998). Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. T-statistics are presented in parentheses.

Variable	Investment Grade Bonds				Speculative Grade Bonds			
Short-term debt1	0.4645	***			-8.2338	***		
	(3.11)				(-2.61)			
Short-term debt2			0.3338	***			-4.6100	*
			(2.60)				(-1.64)	
Liquidity	16.5454	***	15.4964	***	9.6978		-6.5086	
	(7.09)		(6.72)		(1.55)		(-1.06)	
Credit rating	10.9805	***	10.2857	***	53.8570	***	46.2926	**
	(9.97)		(9.21)		(3.01)		(2.45)	
Maturity	0.1078		0.1093		-29.8936	***	-32.4168	***
	(0.67)		(0.66)		(-3.25)		(-3.29)	
Coupon	5.7143	**	4.5300		1343.648	***	1526.4	***
	(1.96)		(1.51)		(11.55)		(13.05)	
T-note	-82.9826	***	-76.306	***	-302.6150	***	-273.35	***
	(-19.02)		(-17.62)		(-4.12)		(-4.13)	
Term slope	-103.887	***	-92.0405	***	-401.0706	***	-342.6950	***
	(-14.08)		(-12.54)		(-3.29)		(-3.21)	
Eurodollar	59.8164	***	61.1238	***	175.3504	***	191.249	***
	(24.08)		(24.99)		(4.15)		(4.61)	

Table 13 (continued)

σ_E	15.9468	***	18.0190	***	13.6155		-2.7878	
	(7.08)		(7.99)		(0.32)		(-0.07)	
Income to Sales	76.5599	***	-130.121	***	364.1044		43.0817	
	(-5.25)		(-5.81)		(1.48)		(0.09)	
Size	-2.0062		-4.6255	**	178.3527	**	318.20	***
	(-0.90)		(-2.03)		(2.06)		(3.37)	
Market to Book	-10.1889	**	-14.605	***	18.4884		152.75	
	(-2.41)		(-3.35)		(0.14)		(1.20)	
Amount	-6.2847	***	-5.3070	***	-209.4144	**	-152.04	*
	(-3.52)		(-2.89)		(-2.44)		(-1.75)	
Leverage	3.9481	***	3.3593	***	18.6111	***	15.9517	***
	(12.43)		(10.26)		(4.24)		(3.99)	
Pre-Tax D1	658.451	***	396.791	***	-29.5354		115.6311	
	(-11.06)		(10.01)		(-0.13)		(0.49)	
Pre-Tax D2	-647.955	***	409.312	***	-16.6428		115.0091	
	(-10.91)		(10.48)		(-0.07)		(0.46)	
Pre-Tax D3	-641.612	***	414.779	***	203.1597		300.8723	
	(-10.88)		(10.67)		(0.68)		(1.07)	
Pre-Tax D4	-633.581	***	430.343	***	264.1271		325.1196	
	(-10.83)		(10.88)		(0.84)		(1.11)	
N	2535		2488		418		362	
R-square	0.5904		0.5705		0.4060		0.5175	

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

$$Short\text{-}term\ debt_{it} = \beta_0 + \beta_1 Yield\ Spread_{it} + \beta_2 Credit\ rating_{it} + \beta_3 Volatility_{it} + \beta_4 Size_{it} + \beta_5 Market\ to\ book_{it} + \beta_6 Leverage_{it} + \beta_7 Asset\ maturity_{it} + \beta_8 Abnormal\ earning_{it} + \beta_9 Investment\ tax\ credit_{it} + \beta_{10} Loss\ carryfoward_{it} + \beta_{11} Regulated\ firm_{it} + \epsilon_t \quad (3.3)$$

$$Liquidity_{it} = \beta_0 + \beta_1 Yield\ spread_{it} + \beta_2 Credit\ rating_{it} + \beta_3 Maturity_{it} + \beta_4 Amount_{it} + \beta_5 Bond\ volatility_{it} + \epsilon_t \quad (3.4)$$

Constructing equation (3.3) generally follows study of Johnson (2003). Asset maturity is the book value-weighted measure of asset maturity. The maturity of long-term assets is measured as gross property, plant, and equipment (PP&E) divided by depreciation expense, while the maturity of current asset is measured as current assets divided by the cost of goods sold. Asset maturity is obtained by the weighted sum of these two maturity measures where the weight for the long-term asset is gross PP&E divided by total assets, and the weight for current assets is current assets divided by total assets. Abnormal earning is the difference between operating income per share in current and previous year divided by the current share price. We also include three dummy variables for this equation. Investment tax credit or loss carryforward indicate whether an issuer has investment tax credit or net operating loss carryforwards. Regulated firm refers to whether or not an issuer is regulated firm. Finally, bond volatility is included in equation (3.4). Like equity volatility, bond volatility is estimated using 252 daily bond prices.

Table 14 and Table 15 show the results from two-stage least square estimation. Table 14 shows results from all sample data, and Table 15 exhibits results from two sample groups (i.e. investment grade and speculative grade). As the Table 14 and 15 shows, the potential endogeneity bias does not affect the relation between the fraction of debt and the yield spread. Still, the magnitude of coefficients on the fraction of debt is relatively small, compared with those on other key variables. However, a notable result is that, for speculative grade bonds, the fraction of debt is significantly positively related to the yield spread, unlike the results in Table 13. Yet, given that the regression for the fraction of debt has low R-square and small sample size, and that the estimated coefficients for the fraction of debt in Table 13 and in Table 14 have different signs, we cannot draw a conclusion about speculative bonds. Rather, we save speculative bond case for the future research.

Lastly, we also conduct regression tests to see whether or not change in the fraction of debts maturing in one or two year is one of determinants. Furthermore, econometrically, possible benefit is that differencing the time series removes autocorrelative influence. For this test, we exclude all dummy variables, coupon rate, amount of outstanding, and maturity. Additionally, only investment grade bonds are considered in this test. Specifically, the regression is stated as:

$$\Delta Yield\ spread_i = \beta_0 + \beta_1 \Delta Short\text{-}term\ debt_i + \beta_2 \Delta Liqudity_i + \beta_3 \Delta Credit\ rating_i + \beta_4 \Delta Maturity_i + \beta_5 \Delta T\text{-}note_i + \beta_6 \Delta Term\ slope_i + \beta_7 \Delta Eurodollar_i + \beta_8 \Delta Volatility_i + \beta_9 \Delta Income\ to\ Sale_i + \beta_{10} \Delta Size_i + \beta_{11} \Delta Market\ to\ Book_i + \beta_{12} \Delta Pre\text{-}Tax_i + \beta_{13} \Delta Leverage_i + \epsilon_t \quad (3.5)$$

where $\Delta Pre\text{-}Tax$ refers to yearly change in pre-tax interest coverage ratio. The results are presented in Table 16.

As expected, the results in Table 16 are, generally, similar to those in Table 13. Notable difference is that changes in Market to book ratio and size have the biggest impact on the change in the yield spreads for investment grade bonds. On the other hand, change in leverage is has a small impact on the yield spread change, compared with change in market to book ratio and size. This could be that, for investment grade bonds, along with volatility of an issuer's equity, size and market to book ratio more precisely capture issuer's risk change than the leverage does.

Table 14: Two-Stage Least Square Estimation: Yield Spread and Fraction of Debt Maturity

Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debts maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in year. Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Amount is logarithm of amount of bonds outstanding. Pre-Tax indicates pre-tax interest coverage ratio that is group into one of four categories according to Blume et al. (1998). Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. Asset maturity is (gross property, plant, and equipment (PP&E)/total assets) times (gross PP&E/depreciation expense) plus (current assets/total assets) times (current assets/cost of goods sold). Abnormal earning is the difference between operating income per share in current and previous year divided by the current share price. We also include three dummy variables for this equation. Investment tax credit or loss carryforward indicate whether an issuer has investment tax credit or net operating loss carryforwards. Regulated firm refers to whether or not an issuer is regulated firm. Finally, σ_E refers to the volatility of bond prices. T-statistics are presented in parentheses.

Instrumental Variable	Yield spread		Yield spread		Liquidity		Short-term debt1		Short-term debt2
Short-term debt1	4.3896	*							
	(1.76)								
Short-term debt2			13.0262	***					
			(5.27)						
Liquidity	79.2762	***	78.7576	***					
	(10.46)		(10.29)						
Yield spread					0.0010	***	0.0001		0.0005
					(19.68)		(0.14)		(1.12)
Credit rating	115.092	***	115.497	***	0.1805	***	-0.3587	**	0.0072

Table 14 (continued)

	(32.62)		(34.17)		(15.12)		(-2.43)		(0.04)	
Maturity	-7.0639	***	-6.9534	***	0.0089	***				
	(-13.58)		(-13.53)		(3.31)					
Coupon	79.6043	***	78.7708	***						
	(8.82)		(8.84)							
T-note	-25.0910	**	-24.2196	**						
	(-2.39)		(-2.32)							
Term slope	-11.1265		-9.1850							
	(-0.63)		(-0.52)							
Eurodollar	-19.2748	***	-20.1695	***						
	(-3.25)		(-3.42)							
σ_E	-9.4316	*	-6.7528				-0.5411	**	-0.4362	
	(-1.66)		(-1.20)				(-2.12)		(-1.43)	
Income to Sales	-144.4585	**	-109.5489	*						
	(-2.43)		(-1.85)							
Size	33.2248	***	20.7771	***			0.8843	**	1.0204	**
	(4.57)		(2.82)				(2.27)		(2.35)	
Market to Book	15.0347		-71.0250	***			4.6792	***	8.0366	***
	(0.93)		(-3.10)				(8.67)		(12.77)	
Amount	-48.3057	***	-46.2249	***	0.0449	*				
	(-8.82)		(-8.53)		(1.76)					
Leverage	1.7877	**	1.5870	**			0.0262		0.0341	
	(2.38)		(2.12)				(0.78)		(0.87)	
σ_B					-0.1067	***				
					(-9.70)					
Asset maturity							-0.1565	***	-0.1999	***

Table 14 (continued)

				(-3.42)		(-3.82)	
Abnormal earnings				-15933.31		-13090.74	
				(-1.24)		(-0.86)	
Investment tax credit D				3.8873	*	5.3385	**
				(1.64)		(1.97)	
Loss carryforward D				3.9514	***	3.5033	***
				(6.54)		(4.97)	
Regulated firm D				0.7747		1.9444	
				(0.38)		(0.84)	
Pre-Tax D1	57.7560	73.2907					
	(0.62)	(0.80)					
Pre-Tax D2	48.7668	66.2936					
	(0.53)	(0.72)					
Pre-Tax D3	73.3191	92.7990					
	(0.79)	(1.00)					
Pre-Tax D4	112.0898	126.2326					
	(1.18)	(1.33)					
N	2893	2893	2893	2893		2893	
R-square	0.8948	0.8958	0.6518	0.3097		0.2868	

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

Table 15:Two-Stage Least Squares Estimation: Yield Spread and Fraction of Debt Maturity Based on Bond Grade Group

Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debts maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Maturity is life remaining to a bond's maturity date, which is expressed in year. Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Amount is logarithm of amount of bonds outstanding. Pre-Tax indicates pre-tax interest coverage ratio that is group into one of four categories according to Blume et al. (1998). Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. Asset maturity is (gross property, plant, and equipment (PP&E)/total assets) times (gross PP&E/depreciation expense) plus (current assets/total assets) times (current assets/cost of goods sold). Abnormal earning is the difference between operating income per share in current and previous year divided by the current share price. We also include three dummy variables for this equation. Investment tax credit or loss carryforward indicate whether an issuer has investment tax credit or net operating loss carryforwards. Regulated firm refers to whether or not an issuer is regulated firm. Finally, σ_E refers to the volatility of bond prices. T-statistics are presented in parentheses.

A. Investment Grade

Instrumental Variable	Yield spread		Yield spread		Liquidity		Short-term debt1		Short-term debt2	
Short-term debt1	0.9835	***								
	(3.09)									
Short-term debt2			1.5410	***						
			(4.73)							
Liquidity	35.6170	***	35.4644	***						
	(8.83)		(8.77)							
Yield spread					0.0021	***	0.0033	*	0.0046	**
					(21.88)		(1.68)		(2.00)	

Table 15 (Continued)

Credit rating	16.2204	***	16.0978	***	-0.0342	***	-0.4959	***	-0.1668	
	(33.89)		(35.03)		(-4.43)		(-2.69)		(-0.79)	
Maturity	-2.1342	***	-2.1212	***	0.0066	***				
	(-33.15)		(-32.8)		(5.37)					
Coupon	2.3971	**	2.4622	**						
	(2.31)		(2.36)							
T-note	-20.6154	***	-20.2696	***						
	(-9.25)		(-9.11)							
Term slope	-21.2680	***	-20.7064	***						
	(-5.72)		(-5.59)							
Eurodollar	9.5488	***	9.4963	***						
	(6.98)		(6.97)							
σ_E	5.4877	***	5.9455	***			-1.0455	***	-1.0025	***
	(5.11)		(5.53)				(-3.62)		(-2.91)	
Income to Sales	-24.1259	***	-22.5145	**						
	(-2.58)		(-2.42)							
Size	-3.2551	***	-4.3231	***			1.6950	***	1.8525	***
	(-3.09)		(-4.04)				(4.34)		(4.33)	
Market to Book	-7.7637	***	-15.6790	***			6.2129	***	9.0769	***
	(-2.98)		(-4.59)				(10.60)		(13.4)	
Amount	-16.0978	***	-15.9425	***	0.0440	***				
	(-24.32)		(-23.95)		(3.74)					
Leverage	0.8017	***	0.7739	***			0.0953	**	0.0748	
	(5.05)		(4.87)				(2.19)		(1.47)	
σ_B					-0.0503	***				
					(-9.04)					

Table 15 (continued)

Asset maturity						-0.1750	***	-0.1800	***
						(-3.79)		(-3.47)	
Abnormal earnings						-6614.603		-37287.79	
						(-0.27)		(-1.28)	
Investment tax credit D						3.9525	*	4.4909	*
						(1.76)		(1.78)	
Loss carryforward D						4.7957	***	4.4500	***
						(7.38)		(5.96)	
Regulated firm D						1.1677		2.4511	
						(0.60)		(1.12)	
Pre-Tax D1	-9.1682	**	-9.4076	**					
	(-2.01)		(-2.06)						
Pre-Tax D2	-7.2454	**	-7.0900	**					
	(-2.08)		(-2.03)						
Pre-Tax D3	-4.3622		-3.9806						
	(-1.51)		(-1.38)						
Pre-Tax D4	-5.7560		-5.6968						
	(-1.38)		(-1.34)						
N	2531		2531		2531	2531		2531	
R-square	0.8980		0.8983		0.2376	0.3457		0.3452	

Table 15 (continued)
B. Speculative Grade

Instrumental Variable	Yield spread		Yield spread		Liquidity		Short-term debt1		Short-term debt2	
Short-term debt1	49.0633	***								
	(2.89)									
Short-term debt2			41.3290	***						
			(2.57)							
Liquidity	261.4482	***	267.8412	***						
	(6.18)		(6.35)							
Yield spread					0.0007	***	0.0008		0.0008	
					(5.27)		(1.54)		(1.23)	
Credit rating	-3.19805		-13.0680		0.3904	***	-0.0293		0.1637	
	(-0.11)		(-0.45)		(5.77)		(-0.08)		(0.42)	
Maturity	-28.7814	***	-27.8043	***	0.0328	***				
	(-3.61)		(-3.49)		(3.01)					
Coupon	1055.4170	***	1062.9790	***						
	(11.34)		(11.39)							
T-note	-255.3756	***	-246.6772	***						
	(-3.39)		(-3.27)							
Term slope	-290.9250	**	-278.8209	**						
	(-2.36)		(-2.25)							
Eurodollar	133.3416	***	128.9483	***						
	(2.79)		(2.67)							
σ_E	-144.5228	***	-126.8562	**			1.5113	**	1.4519	**
	(-2.69)		(-2.43)				(2.21)		(1.96)	
Income to Sales	-73.9701		-153.1830							

Table 15 (continued)

	(-0.13)		(-0.27)							
Size	590.9523	***	404.4831	***			-4.6920	***	-0.8965	
	(5.59)		(5.32)				(-4.24)		(-0.62)	
Market to Book	158.9667		-389.7394	*			-2.7624		10.3094	***
	(1.20)		(-1.78)				(-1.38)		(4.56)	
Amount	-8.8783		-10.7786		-0.2600	***				
	(-0.14)		(-0.17)		(-2.55)					
Leverage	13.4529	***	9.0431	**			-0.0898		0.0028	
	(3.22)		(2.12)				(-1.65)		(0.04)	
σ_B					-0.1892	***				
					(-4.23)					
Asset maturity							-0.1363		-0.0486	
							(-1.11)		(-0.34)	
Abnormal earnings							-52668.24	***	-60303.3	***
							(-3.03)		(-3.18)	
Investment tax credit D										
Loss carryforward D							1.2094		1.2890	
							(0.92)		(0.89)	
Regulated firm D										
Pre-Tax D1	139.8109		147.6131							
	(0.52)		(0.54)							

Table 15 (continued)

Pre-Tax D2	292.0167		296.0367				
	(1.02)		(1.03)				
Pre-Tax D3	361.8675		373.5393				
	(1.16)		(1.20)				
Pre-Tax D4	555.1816	*	577.8529	*			
	(1.71)		(1.77)				
N	362		362		362	362	362
R-square	0.6860		0.6836		0.1971	0.0658	0.0051

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

Still, Table 16 shows that the fraction of debt maturing in one or two year is significantly positively related to the yield spread change, but has a small impact on the yield spread change, compared with the other key variables.

Additionally, to control for potential endogeneity bias, we also conduct a simultaneous equation model for investment grade bonds, which is given as:

$$\Delta Yield\ spread_i = \beta_0 + \beta_1 \Delta Short\text{-}term\ debt_i + \beta_2 \Delta Liqudity_i + \beta_3 \Delta Credit\ rating_i + \beta_4 \Delta Maturity_i + \beta_5 \Delta T\text{-}note_i + \beta_6 \Delta Term\ slope_i + \beta_7 \Delta Eurodollar_i + \beta_8 \Delta Volatility_i + \beta_9 \Delta Income\ to\ Sale_i + \beta_{10} \Delta Size_i + \beta_{11} \Delta Market\ to\ Book_i + \beta_{12} \Delta Pre\text{-}Tax_i + \beta_{13} \Delta Leverage_i + \epsilon_t \tag{3.6}$$

$$\Delta Short\text{-}term\ debt_i = \beta_0 + \beta_1 \Delta Yield\ Spread_i + \beta_2 \Delta Credit\ rating_i + \beta_3 \Delta Volatility_i + \beta_4 \Delta Size_i + \beta_5 \Delta Market\ to\ book_i + \beta_6 \Delta Leverage_i + \beta_7 \Delta Asset\ maturity_i + \beta_8 \Delta Abnormal\ earning_i + \epsilon_t \tag{3.7}$$

$$\Delta Liquidity_i = \beta_0 + \beta_1 \Delta Yield\ spread_i + \beta_2 \Delta Credit\ rating_i + +\beta_3 \Delta Bond\ volatility_i + \epsilon_t \tag{3.8}$$

The results, presented in Table 17 exhibits, are similar as those in Table 16. Table 17 supports that our tests on changes in the fraction of debts maturing in one or two year are robust to potential endogeneity bias.

Table 16: Yield Spread Change Determinants

Annual changes in all variables are examined. Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debt maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Pre-Tax indicates pre-tax interest coverage ratio. Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. T-statistics are presented in parentheses.

Variable	Yield spread		Yield spread	
Short-term debt1	0.8771	***		
	(3.30)			
Short-term debt2			0.4523	**
			(2.31)	
Liquidity	17.6155	***	12.7272	***
	(7.78)		(5.84)	
Credit rating	5.9304	*	2.5435	
	(1.93)		(0.86)	
T-note	-41.49289	***	-37.4717	***
	(-8.83)		(-8.33)	
Term slope	-37.6882	***	-31.0768	***
	(-4.91)		(-4.22)	
Eurodollar	25.7260	***	27.4225	***
	(9.06)		(10.09)	
σ_E	16.9727	***	17.5494	***
	(6.72)		(7.27)	
Income to sales	31.18222		-181.5882	***
	(1.37)		(-3.56)	
Size	-109.1593	***	-75.9436	***
	(-6.18)		(-4.36)	
Market to book	-113.6548	***	-116.5531	***
	(-12.16)		(-13.24)	
Leverage	1.5110	**	1.0271	
	(2.03)		(1.50)	
Pre-tax interest	-0.6575		0.1459	

Table 16 (continued)

	(-1.44)	(0.32)
N	2159	2159
R-square	0.3581	0.3725

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

Table 17:Two-Stage Least Squares Estimation: Yield Spread Change Determinants

Yield spread is the difference between the bond yield and the yield of a comparable maturity treasury bond as determined from Datastream. Short-term debt refers to the fraction of debt maturing in one or two year. Liquidity indicates liquidity costs measured by the LOT model. Credit rating is a bond's rating that is numbered from one (AAA rated bond) to 22 (D rated bond). Coupon and T-note refers to coupon rates and 1-year Treasury note rate respectively. Term slope and Eurodollar are the difference between 10-year and 2-year Treasury rates, and the difference between 30-day Eurodollar and the 3-month T-bill rate respectively. σ_E is the equity volatility for each issuer, and Income to sale is operating income divided by sales. Size is logarithm of each issuer's total asset (based on 1980 dollar value). Market to book is Market value divided by book value of each issuer. Market value of the firm is calculated by total assets minus total equity plus market value of common equity plus preferred stock liquidating value. Pre-Tax indicates pre-tax interest coverage ratio. Leverage is market value of leverage (total debt/ market value of each issuer), and Book leverage is total debt divided by total asset. Asset maturity is (gross property, plant, and equipment (PP&E)/total assets) times (gross PP&E/depreciation expense) plus (current assets/total assets) times (current assets/cost of goods sold). Abnormal earning is the difference between operating income per share in current and previous year divided by the current share price. Finally, σ_E refers to the volatility of bond prices. T-statistics are presented in parentheses.

Instrumental Variable	Yield spread		Yield spread		Liquidity		Short-term debt1		Short-term debt2	
Short-term debt1	23.7897	***								
	(24.91)									
Short-term debt2			27.9792	***						
			(32.64)							
Liquidity	227.2921	***	197.1676	***						
	(45.22)		(39.93)							
Yield spread					0.0021	***	0.0086	***	0.0105	***
					(15.96)		(5.15)		(4.84)	
Credit rating	9.0247	***	6.2639	***	-0.0733	***	0.3045		0.7818	**
	(5.16)		(3.56)		(-3.43)		(1.22)		(2.42)	
T-note	-18.1900	***	-15.8097	***						
	(-7.36)		(-6.89)							

Table 17 (continued)

Term slope	-23.8157	***	-21.2539	***						
	(-5.92)		(-5.70)							
Eurodollar	22.1094	***	23.3342	***						
	(14.93)		(16.96)							
σ_E	6.0281	***	5.2344	***						
	(4.49)		(4.20)							
Income to Sales	-199.7636	***	-151.3597	***						
	(-6.98)		(-5.77)							
Size	-310.6058	***	-467.8245	***			13.8177	***	16.9993	***
	(-21.94)		(-30.02)				(9.51)		(8.84)	
Market to Book	64.9115	***	4.0940				-3.9218	***	-1.2538	
	(9.73)		(0.84)				(-5.13)		(-1.26)	
Leverage	32.6901	***	35.0547	***			-1.3553	***	-1.2296	***
	(24.29)		(31.52)				(-28.58)		(-19.88)	
σ_B					0.0672	***				***
					(9.72)					
Asset maturity							0.4900	***	0.4091	***
							(6.20)		(3.97)	
Abnormal earnings							-16926.93		-59696.39	***
							(-0.97)		(-2.64)	
Pre-tax interest	0.6733	***	0.8990	***						
	(2.71)		(3.90)							
N	2168		2168		2168		2168		2168	
R-square	0.7988		0.8266		0.1051		0.2969		0.1794	

An *, **, or *** signifies significance at the 0.1, 0.5 or 0.001 level, respectively

3.4 Summary and Conclusion

This chapter examines the association between issuer's debt structure and yield spreads. Previous literatures in debt structure (Barclay and Smith (1995), Mark and Mauer (1996), and Johnson (2003) etc.) study relationship between the fraction of debt maturing in short-period and level of firm's leverage. Most of the previous studies attempt to see evidence whether or not choice of debt structure mitigates firm's under-investment. An underlying assumption of the studies is that choice of debt structure is a manager's decision. However, to the best of our knowledge, no one has investigated that investor's reaction to the manager's decision.

Specifically, we investigate whether or not an investor requires compensation for the liquidity risk. Diamond (1991) introduces liquidity risk as the risk of a borrower being forced into inefficient liquidation when refinancing is not available. According to Diamond's argument, the firm holding the larger proportion of short-term debt in its debt structure is more vulnerable to the unforeseen negative event. Consequently, it will increase the firm's risk.

Through our tests in this chapter, we find that, for investment grade bonds, the results consistently show that the fraction of debt maturing in one or two years is positively related to the yield spreads. Although the magnitude of its impact on the yield spreads are relatively smaller, compared with the other key factors (e.g. leverage, credit rating, and liquidity cost), we argue that this positive relation is evidence of liquidity risk.

However, for speculative grade bonds, we cannot draw any conclusion because the sign of the relationship is not consistent through the tests, and because our sample data for the speculative bond is very limited. Therefore, we save speculative bond case for future research.

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APPENDIX A: APPLYING KIMMEL MODEL (2008)

Kimmel (2008) shows how to approximate conditional moments and contingent claim prices in a large class of non-affine diffusion models with the usage of power series. However, the convergence properties of such power series could be poor for long time horizons, which means that a power series representation of asset prices may converge for short time period, and then diverge for longer time period. Therefore, in order to avoid this issue, he also develops the method of time transformation, in which variable representing time is replaced by a non-linear function of itself. His method uses three techniques. We begin by applying change of independent variable and change of dependent variable to derive a simplified PDE form, which he calls "Canonical Form". Then, we need to apply change of time variable to ensure that there are no occurrences of divergence. In this section, we will show how to apply Kimmel's method to evaluate:

$$\pi\left(h_{d,u}, t, T\right) = E_t^Q\left[exp\left(-\int_t^T\left(h_{d,u} + \beta_{c3}\frac{M}{h_{d,u}}\right)du\right)\right]$$

We can also express the above equation as partial differential form:

$$\frac{\partial\pi}{\partial t} = [\kappa_d\theta_d - (\kappa_d\eta_d)h_d]\frac{\partial\pi}{\partial t_d} + \frac{1}{2}\sigma_2^2 h_d\frac{\partial^2\pi}{\partial h_d^2} - \left[h_d + \beta_{c3}\frac{M}{h_d}\right]\pi$$

With the terminal condition, $\pi(h_d, T, T) = 1$.

Following Kimmel (2008), the changes of variables are:

$$y(h_d) = \left(\frac{2\sqrt{h_d}}{\sigma}\right), f(\Delta, h_d) = \left(\frac{4h_d}{\sigma^2}\right)^{\frac{1}{4}-\frac{\theta_\kappa}{\sigma^2}} e^{\frac{h_d(\kappa_d+\eta_d)}{\sigma^2}} h\left(\Delta, y(h_d)\right)$$

where Δ is (T-t).

The canonical form PDE is:

$$\frac{\partial h}{\partial \Delta}(\Delta, y) = \frac{1}{2}\frac{\partial^2 h}{\partial y^2}(\Delta, y) - \left[\frac{a}{y^2} + \frac{b^2}{2}y^2 + d\right]h(\Delta, y) \tag{A.1}$$

with final condition, $h(0, y) = y^{\propto} e^{-\frac{\kappa_d \eta_d}{4}y^2}$,

$a \equiv \frac{2\theta_d^2\kappa_d^2}{\sigma_d^2} - \frac{2\sigma_d\kappa_d - 4\beta_{c3}M}{\sigma_d^2} + \frac{3}{8}$, $b \equiv \frac{\sqrt{(\kappa_d\eta_d)^2 + 2\sigma_d^2}}{2}$, $d \equiv -\frac{\theta_d\kappa_d(\kappa_d+\eta)}{\sigma_d^2}$, and $\alpha \equiv \frac{2\theta_d\kappa_d}{\sigma_d^2} - \frac{1}{2}$.

The equation (A.1) is the standard affine form from Kimmel (2008), so we can apply the theorem 5 in his study with $\tau = 1 - e^{-2b\Delta}$ and$z = \sqrt{2b}e^{-b\Delta}y$.

Finally, we can obtain:

$$h(\Delta, y) = e^{-\frac{b}{2}y^2 - \left(\frac{b}{2}+d\right)\Delta}\left[\left(\frac{z}{\sqrt{2b}}\right)^{\frac{1-\sqrt{1+8a}}{2}} w_1(\tau, z) + \left(\frac{z}{\sqrt{2b}}\right)^{\frac{1+\sqrt{1+8a}}{2}} w_2(\tau, z)\right]$$

where $\frac{\partial w_1(\tau,z)}{\partial \tau} = \frac{1-\sqrt{1+8a}}{2}\frac{\partial w_1(\tau,z)}{\partial z} + \frac{1}{2}\frac{\partial^2 w_1(\tau,z)}{\partial z^2}$

and $\frac{\partial w_2(\tau,z)}{\partial \tau} = \frac{1+\sqrt{1+8a}}{2}\frac{\partial w_2(\tau,z)}{\partial z} + \frac{1}{2}\frac{\partial^2 w_2(\tau,z)}{\partial z^2}$.

According to Kimmel (2008), $w_2(\tau, z)$ is everywhere zero. Therefore, power series is applied to only $w_1(\tau, z)$. The first few terms of this series are:

$$w_1(\tau, z) = \left(\frac{z}{\sqrt{2b}}\right)^{\alpha-\gamma} e^{\frac{z^2}{4}\left(1-\frac{\kappa_d+\eta_d}{2b}\right)} \left[\begin{array}{l} 1 + \tau\left[\frac{(\alpha-\gamma)(\alpha+\gamma-1)}{2z^2} + \left(\frac{2\alpha+1}{4}\right)\left(1 - \frac{\kappa_d+\eta_d}{2b}\right) + \frac{z^2}{8}\left(1 - \frac{\kappa_d+\eta_d}{2b}\right)^2\right] \\ + \frac{\tau^2}{2}\left[\begin{array}{c} \frac{(\alpha-\gamma)(\alpha-\gamma-2)(\alpha+\gamma-1)(\alpha+\gamma-3)}{4z^4} \\ + \frac{(2\alpha-1)(\alpha-\gamma)(\alpha+\gamma-1)}{4z^2}\left(1 - \frac{\kappa_d+\eta_d}{2b}\right) \\ + \left(\frac{(2\alpha+3)(2\alpha+1)}{16} + \frac{(\alpha-\gamma)(\alpha+\gamma-1)}{8}\right)\left(1 - \frac{\kappa_d+\eta_d}{2b}\right)^2 \\ + \frac{(2\alpha+3)z^2}{16}\left(1 - \frac{\kappa_d+\eta_d}{2b}\right)^3 + \frac{z^4}{64}\left(1 - \frac{\kappa_d+\eta_d}{2b}\right)^4 \end{array}\right] \\ + \cdots \end{array}\right]$$

where $\gamma \equiv \frac{1-\sqrt{1+8a}}{2}$

APPENDIX B: PERFORMANCES OF THE EXTENDED KALMAN FILTER

In this section, performance of the extended Kalman filter is investigated using data generated by Monte Carlo simulation. For simplicity, we exclude α from each equation (from (16) to (22)) in Chapter 1. First, we generate simulated data from the default-free, defaultable, and make-whole callable models that are found in Chapter 1. We apply the extended Kalman filter to estimate parameters for each model. Our estimated parameters are much closer to the true parameters that are used for Monte Carlo simulation, which lead us to conclude that the extended Kalman filter works well for term structure models.

For the default-free model (equations (16) and (17) in Chapter 1), estimated parameters in Table 11 in Chapter 2 is considered true parameters, when we generate 100 monthly observations. Like our sample data in Chapter 2, 0.5, 1, 2, 3, 5, 10, and 30 year zero coupon Treasury rates are considered. We also add random observation errors that are normally distributed with a zero mean and a constant variance to the zero coupon yield. We apply the Kalman filter to data generated from each iteration of Monte Carlo simulation, and repeat this procedure 500 times. Therefore, we obtain 500 sets of estimated parameters. Panel A in Table B shows the performance of the extended Kalman filter on the default-free model. The mean values of the estimated parameters are much closed to the true values. However, root mean square errors (RMSE) for σ_1 and σ_2 are unacceptably large.

For the defaultable model (equations (18) and (19) in Chapter 1), an average value of each parameters in Table 12 in Chapter 2 is considered true parameters, when we generate 100 monthly observation. Additionally, average value of coupon rate in Table

12 is used. As we assume in the empirical analysis (Chapter 2), we also assume that both state variables and the parameters of the default-free model are known. The rest of the procedures are basically same as the procedures of the default-free model. Panel B in Table B shows the performance of the extended Kalman filter on the defaultable model. The mean values of the estimated parameters are close to the true values, but RMSE for β_{d2} is very large.

For the make-whole callable model (equations (20)-(22) in Chapter 1), we also assume that both the state variables and the parameters of the default-free and defaultable models are known. Average value of coupon rate and make-whole premium is used. The rest of the procedures are same as the procedures of the defaultable model. Panel C in Table B exhibits the performance of the extended Kalman filter on the make-whole callable model. Again, then mean value of the estimated parameters are closed to the true value. However, RMSE for σ is a little large.

From Table B, even though, for each case, it shows one or two large RMSE for estimated parameters, the extended Kalman filter performs well. This results supports that the extended Kalman filter suits our study.

Table B: Finite Sample Performances of the Extended Kalman filter

We generate 100 monthly observations for each model. We also add random observation errors that are normally distributed with a zero mean and a constant variance to the zero coupon yield. We apply the Kalman filter to data generated from each iteration of Monte Carlo simulation, and repeat this procedure 500 times. Therefore, we obtain 500 sets of estimated parameters. For the default-free model (equation (16) and (17) in chapter 1), estimated parameters in Table 11 in chapter 2 is considered true parameters, and 0.5, 1, 2, 3, 5, 10, and 30years zero coupon Treasury rates are considered. For defaultable model, an average value of each parameter in Table 12 and coupon rate is used as a true value. Finally, for make-whole callable model, an average value of each parameter in Table 12, coupon rate, and make-whole premium is used as a true value.

A. Simulation results for risk-free model

	θ_1	κ_1	σ_1	λ_1	θ_2	κ_2	σ_2	λ_2
True	0.0359	0.3347	0.0755	-0.0909	0.0015	0.1973	0.0327	-0.1663
Mean	0.0357	0.3406	0.0694	-0.0864	0.0016	0.2429	0.0535	-0.1589
RMSE	0.0019	0.0230	0.0357	0.0073	0.0001	0.0649	0.0388	0.0100

B. Simulation results for the defaultable model

	θ	κ	σ	λ	β_{d1}	β_{d2}
True	0.0344	0.3086	0.1847	-0.1588	-0.2282	-0.5366
Mean	0.0352	0.3155	0.1792	-0.1484	-0.1816	-0.5575
RMSE	0.0038	0.0668	0.0315	0.0345	0.0219	0.2686

C. Simulation results for the make-whole callable model

	θ	κ	σ	λ	β_{c1}
True	0.0355	0.1104	0.4559	-0.0103	0.0025
Mean	0.0398	0.1284	0.4952	-0.0114	0.0024
RMSE	0.0100	0.0115	0.1965	0.0017	0.0003

APPENDIX C: LOT MODEL PROPOSED BY CHEN, LESMOND, AND WEI (2007)

In this study, due to the limited availability of bid-ask spread data, we use the alternative liquidity measure suggested by Chen et al. (2007). Originally, Lesmond et al. (1999) introduce this LOT model for estimating liquidity for equity markets. In addition, Lesmond et al. (2004) show that this method works well, as evidenced by an 80% correlation between the LOT estimation and the bid-ask spread plus commissions. In 2007, Chen et al. extend this model to corporate bonds to test for the influence of bond liquidity on corporate yield spreads.

An underlying assumption of this model is that the marginal informed investor trades when his value of information exceeds transaction costs. In other words, there is no trade when the value of information is less than the transaction costs. Therefore, we observe zero returns. Furthermore, from the assumption, measured return (i.e. observable return) does not reveal the true return (i.e. unobservable return) of the marginal trader until transaction costs are exceeded. In like manner, since the investor must be compensated for his transaction costs, the measured return partially reflects the true value of the information.

From these logics, there must exit thresholds for buy-side or sell side traders. Chen et al. (2007) assert that a difference between buy-side and sell-side threshold be able to capture transaction costs for an individual security. To estimate these costs, it starts with the return generating process that is given as:[13]

$$R^*_{j,t} = \beta_{j1} Duration_{j,t} * \Delta R_{ft} + \beta_{j1} Duration_{j,t} * \Delta S\&PIndex_t + \epsilon_{j,t} \tag{C.1}$$

[13] For more detail about theoretical derivation of the return generating process, please see Chen et al. (2007).

where the term $R^*_{j,t}$ represents the unobserved "true" bond return for bond j and day t, the term ΔR_{ft} is the daily change in the 10-year risk-free interest rate, and the term $\Delta S\&PIndex_t$ is the daily return on the Standard & Poor's 500 index. Following Jarrow (1978), all risk coefficients are scaled by duration. In addition, from the assumption, the relation between the measured return and the true return can be stated as:

$$R_{j,t} = R^*_{j,t} - \alpha_{i,j} \tag{C.2}$$

where $R_{j,t}$ is the measured return, $\alpha_{2,j}$ is the effective buy-side cost, and $\alpha_{1,j}$ is effective sell-side cost for bond j. With combining (C.1) with (C.2), we have:

$$R^*_{j,t} = \beta_{j1} Duration_{j,t} * \Delta R_{ft} + \beta_{j2} Duration_{j,t} * \Delta S\&PIndex_t + \epsilon_{j,t} \tag{C.3}$$

where:

$R_{j,t} = R^*_{j,t} - \alpha_{1,j}$ .... if $R^*_{j,t} < \alpha_{1,j}$ and $\alpha_{1,j} < 0$
$R_{j,t} = 0$ .... if $\alpha_{1,j} < R^*_{j,t} < \alpha_{2,j}$
$R_{j,t} = R^*_{j,t} - \alpha_{2,j}$ .... if $R^*_{j,t} > \alpha_{2,j}$ and $\alpha_{2,j} > 0$

The log-likelihood function for this model can be stated as:

$$LnL = \sum_1 Ln \frac{1}{(2\pi\sigma^2)^{1/2}} - \sum_1 \frac{1}{2\sigma^2} (R_{j,t} + \alpha_{1,j} - \beta_{j1} Duration_{j,t} * \Delta R_{ft} - \beta_{j2} Duration_{j,t} * \Delta S\&PIndex_t)^2 + \sum_2 Ln \frac{1}{(2\pi\sigma^2)^{1/2}} - \sum_2 \frac{1}{2\sigma^2} (R_{j,t} + \alpha_{2,j} - \beta_{j1} Duration_{j,t} * \Delta R_{ft} - \beta_{j2} Duration_{j,t} * \Delta S\&PIndex_t)^2 + \sum_0 Ln\left(\Phi_{2,j} - \Phi_{1,j}\right), \tag{C.4}$$

where $\Phi_{i,j}$indicates the cumulative distribution function for each bond-year evaluated at$(\alpha_{i,j} - \beta_{j1} Duration_{j,t} * \Delta R_{ft} - \beta_{j2} Duration_{j,t} * \Delta S\&PIndex_t)/\sigma_j$. $\sum_1 (region1)$, $\sum_2 (region2)$, or $\sum_0 (region0)$ represents the negative nonzero measured returns, the positive nonzero measured returns, or zero measured returns respectively. By using maximum likelihood estimation, we estimate two risk coefficients, buy-side and sell-side costs. The difference between $\alpha_{2,j}$ and $\alpha_{1,j}$ is the round-trip transaction costs that is used as liquidity measure in chapter 3.

LOT model requires only daily price of bonds, so it is a great alternative to measure liquidity costs, especially when bid-ask data is limited. However, LOT model has a practical limitation. If the sequence of bond prices does not have any zero returns or if more than 85 % of the daily returns over the year are zero, the LOT model cannot be used to estimate liquidity costs.

For the purpose of validation of these estimated liquidation costs, we exclude statistically insignificant values of estimated parameters. The distribution of the two risk coefficients are summarized in Table C. As shown in Table C, the interest rate coefficient is negative on average, which is expected. However, some positive values of this coefficient could be explained in that, moving from high-grade to low-grade bonds, this relationship is expected to become weaker (Schultz (2001)). Additionally, from our sample period, the coefficient on the market return factor is negative on average. Generally, we can expect a positive value of this coefficient if positive equity returns have a positive effect on the bond return. However, if positive equity returns are caused by capital flows from the corporate bond market, negative coefficient value is expected. Therefore, there is no clear interpretation on the value of this coefficient.

Table C: Distribution of Coefficients from Liquidity Measure

This table summarizes the distribution of coefficients on the risk-free rate factor, β_{T-bond}, and market return factor, β_{Equity}. The coefficients are estimated using Maximum Likelihood Estimation.

	$\beta_{\text{T-bond}}$	β_{Equity}
Mean	-0.0003	-0.0015
Standard Deviation	1.8405	0.0206
Min	-40.6396	-0.5205
First Quartile	-22.3591	-0.0081
Median	-0.0182	-0.0041
Third Quartile	0.3292	0.0185
Max	24.2783	0.3712
N	5087	5087