Systematic Pedagogy to Queuing Theory with EXCEL

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Abstract

Over the past ten years, simple and inexpensive operations research and management software that is user friendly to the mentor, student, and instructor has become more difficult to obtain. Emmons, Flowers, Khot, and Mather's STORM 4.0 for Windows, which is one of the primary student learning tools, is a 16 bit program and will no longer perform efficiently on a 32 or 64 bit system. Therefore it is no longer available. After a diligent search, it appears that there is no adequate, inexpensive, alternative software. Current software such as SAS-Operations Research and Microsoft Project, provide algorithms for queuing theory but are too costly. UNC universities are licensed to use SAS; however SAS-OR is not covered by the agreement. This unique site license would be more than what the university or the Department of Management and Accountancy could afford. This paper presents pedagogy from a systems approach using Microsoft Excel. A spreadsheet file was created that successfully applies queuing theory which is the mathematical algorithm for waiting lines, or queues. Queuing theory has seen contemporary applications in the field of telecommunication, traffic engineering, computing and the design of factories, shops, offices and hospitals. Data was collected from a Western North Carolina local bakery and food service business. The arrival times were Poisson and the service times were normally distributed. From this analysis a spreadsheet model was constructed to be used both nationally and internationally. Several different problems were analyzed. These calculations showed that the spreadsheet model was accurate at describing queuing theory dynamics. The major advantage to the practitioner, engineer, instructor and student is that Excel is readily available on all personal computers, easily understood, and is very practical. Students with very little exposure to queuing theory will be able to master the method within the first hour of exposure.

1. Introduction

Over the past ten years, simple and inexpensive operations research and management software that is user friendly to the mentor, student, and instructor has become more difficult to obtain. Emmons, Flowers, Khot, and Mather's STORM 4.0 for Windows[1], which is one of the primary student learning tools, is obsolete and no longer in print. After a diligent search, it appears that there is no adequate, inexpensive, alternative software. Current software such as SAS-OR and Microsoft Project, provide algorithms for queuing theory but are too costly. UNC universities are licensed to use SAS; however SAS-OR is not covered by the agreement. This unique site license would be more than what the university or the Department of Management and Accountancy could afford. This paper presents pedagogy from a systems approach using Microsoft Excel[4]. The objective of this research is to create a spreadsheet file using Microsoft Excel that will operate the necessary calculations pertaining to queuing theory. Queuing theory is the mathematical algorithm for waiting lines, or queues.

The basis for this theory was developed in 1917 by A. K. Erlang, a Danish mathematician and engineer who worked for the Copenhagen Telephone Company. Erlang created basic models to describe the Copenhagen telephone exchange[5]. Erlang's ideas have since been analyzed and have progressed into what is now called queuing

theory. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted. The concept of queuing theory is applicable to many areas of commercial and business ventures. Queuing theory has seen contemporary applications in the field of telecommunication, traffic engineering computing and the design of factories, shops, offices and hospitals[6]. With modern application of queuing theory becoming increasingly common, the development of a queuing theory teaching supplement for use at UNC Asheville will allow the management department to be competitive in a rapidly advancing society. Not only will this help the University but most importantly, Queuing theory has a vast application in operations management, permitting graduates from UNC Asheville to be better equipped to enter an intensely globalized and modernized work force. Students will have the tools and ability to expand their opportunities within their careers.

2. Methodology

The initial stage of the research required gathering data from a WNC local bakery and food service business. The data was collected and recorded using Microsoft Excel. On five separate occasions during the months of September and October of 2013 data was observed and recorded at the business. The sample size was roughly 600 time intervals, including arrival, interarrival and service times. Arrival times were necessary for the interarrival time calculations. Arrival times are defined as the time that any given customer walks through the door. Interarrival times are formulated by subtracting the time each customer arrives through the door by the time the following person walks through the door. Service time is defined as the time between each customer leaving the queue or reaching the service station, and receiving the receipt. The service times remained relatively constant throughout the data collection period, for this reason they were collected during off hours. Arrival times and interarrival times are depended upon hours of day and were collected during peak hours such as breakfast and lunch. There is potential variation in the service of each employee; however this is irrelevant because an average of the Poisson distributions was formulated. Table 1 is an example of how these times were recorded and translated.

Table 1. Example of the table used to record service, arrival and interarrival times

9/21/2013	Begin Service	End Service	Service Time	Seconds	9/25/2013	Arrival Time	Interarrival Time	Seconds
1	1:48:40	1:49:28	0:00:48	48	1	8:55:44		
2	1:49:30	1:51:23	0:01:53	113	2	8:56:39	0:00:55	55
3	1:51:25	1:52:40	0:01:15	75	3	8:58:30	0:01:51	111
4	1:52:42	1:53:21	0:00:39	39	4	8:59:13	0:00:43	43
5	1:53:30	1:54:03	0:00:33	33	5	9:02:15	0:03:02	182
6	1:54:05	1:55:17	0:01:12	72	6	9:03:43	0:01:28	82
7	1:55:20	1:55:57	0:00:37	37	7	9:07:25	0:03:42	222
8	1:56:00	1:56:45	0:00:45	45	8	9:07:26	0:00:01	1
9	1:56:45	1:59:03	0:02:18	138	9	9:07:27	0:00:01	1
10	1:59:50	2:01:46	0:01:56	116	10	9:07:28	0:00:01	1

2.1 Phase I Determining The Distributions

Before performing statistical calculations there was one assumption to consider. The data had to be Poisson distributed meaning the current state (customer) had zero dependence on the previous customers arrival. Customers at the bakery arrived at completely random times and were served at random times. This notion relates to a k-value, formulated using equation (1). A k-value of less than one means random distribution, the k-value was determined to be less than one and therefore passed the test. The mean arrival rates and service times were then translated into units from seconds and are symbolized by lambda and mu, respectively. Equations (2 through 7) were used for the initial phase of this research relating to one server. The L value represents the mean number of customers in the system; L_q is the mean number of customers in the queue. W is the mean time a customer is in the system including wait and service time, W_q is the average amount of time it takes before one is served. The *p* value represents queue efficiency which is defined as the percentage of time a server is actively serving a customer.[7]

$$k = \text{x-bar}/\sigma^2 \tag{1}$$

$$P_0 = 1 - \lambda/\mu \tag{2}$$

$$L = \lambda/(\mu - \lambda) \tag{3}$$

$$Lq = \lambda^2/\mu(\mu - \lambda) \tag{4}$$

$$W = 1/(\mu - \lambda) \tag{5}$$

$$Wq = \lambda/\mu(\mu - \lambda) \tag{6}$$

$$p = \lambda/\mu \tag{7}$$

Equations (8 through 14) are the basic equations for multiple server queues. The term s represents the number of servers in the queue system.

To determine P_0, first compute the initial term in the denominator of the form as follows, then compute the second term:

Term 1: $\displaystyle\sum_{n=0}^{s-1} (\lambda/\mu)^n/n!$ Term 2: $(\lambda/\mu)^s/s!(1-\lambda/s\mu)$

Therefore,

$$P_0 = 1/(\text{Term 1} + \text{Term 2}) \tag{8}$$

$$L_q = P_0(\lambda/\mu)^s p/s!(1-p)^2 \tag{9}$$

$$L = L_q + \lambda/\mu \tag{10}$$

$$W_q = L_q/\lambda \tag{11}$$

$$W = W_q + 1/\mu \tag{12}$$

$$p = \lambda/s\mu \tag{13}$$

2.2 Phase II Spreadsheet Construction

This step-by-step procedure will address each cell that is contained in the two sub-spreadsheets. The interactive Excel file's first spreadsheet tab is to be named *Main* (table 2); the second and final tab is to be named *Calculations* (table 3 and 4).

Table 2. Main Worksheet for Data Entry and Results

	A	B	C	D	E	F
1						
2			**Queuing Theory Main**			
3		**Server 1**	**Server 2**	**Server 3**	**Server 4**	**Server 5**
4	λ	5.00	5.00	5.00	5.00	5.00
5	μ	2.00	2.00	2.00	2.00	2.00
6	s	1	2	3	4	5
7	p	2.50	1.25	0.83	0.63	0.50
8	Results	Go to 2 servers	Go to 3 servers	OK	OK	OK
9	L_q	-4.17	-6.94	3.51	0.60	0.17
10	L	-1.67	-4.44	6.01	3.10	2.67
11	W_q(minutes)	-50.00	-83.33	42.13	7.23	2.04
12	W(minutes)	-20.00	-53.33	72.13	37.23	32.04

A2. Enter queuing theory main
B3. Enter Server 1
B4. Enter problem defined arrival time
B5. Enter problem defined service time
B6. Enter 1
B7. Enter =SUM((B4/(B6*B5)))

Note: Calculations are performed on a separate worksheet named *Calculations*. Examples are given in tables 3 and 4.

B8. Enter =IF(B7<1,Calculations!I28,Calculations!H28)
B9. Enter =Calculations!C5
B10. Enter =Calculations!C6
B11. Enter =Calculations!C9
B12. Enter =Calculations!C12
C3. Enter Server 2
C4. Enter =$B$4
C5. Enter =$B$5
C6. Enter 2
C7. Enter =SUM((C4/(C6*C5)))
C8. Enter =IF(C7<1,Calculations!I28,Calculations!H29)
C9. Enter =Calculations!I5
C10. Enter =Calculations!I6
C11. Enter =Calculations!I9
C12. Enter =Calculations!I12

Note: Columns D, E and F in table 2 are formatted in the same manner as columns B and C. They will reference cells in the *Calculations* worksheet pertaining to the appropriate number of servers. For example, column D in table 2 will reference the drafted section for three servers of the *Calculations* worksheet (table 5) in the same manner that column C in table 2 references the section for two servers of the *Calculations* worksheet (table 4).

Table 3. Section for one Server

	B	C	D	E
2	**1 Server**		**P₀**	-1.5
3	**Lq num**	25		
4	**Lq den**	-6		
5	**Lq**	-4.166666667		
6	**L**	-1.666666667		
7				
8	**Wq**	-0.833333333		
9	Minutes	-50		
10	Seconds	-3000		
11	**W**	-0.333333333		
12	Minutes	-20		

C3. Enter =SUM((MAIN!B4^2))
C4. Enter =SUM((MAIN!B5*(MAIN!B5-MAIN!B4)))
C5. Enter =SUM((C3/C4))
C6. Enter =SUM((MAIN!B4/(MAIN!B5-MAIN!B4)))
C8. Enter =SUM((MAIN!B4/(MAIN!B5*(MAIN!B5-MAIN!B4))))
C9. Enter =SUM(($C$8*60))
C10. Enter =SUM(($C$9*60))
C11. Enter =SUM((1/(MAIN!B5-MAIN!B4)))
C12. Enter =SUM(($C$11*60))
E2. Enter =SUM((1-(MAIN!B4/MAIN!B5)))

Table 4. Section for two Servers

	H	I	J	K	L
2	**2 Servers**		**P₀**		
3	**Lq num**	-0.87	**Term1**		
4	**Lq den**	0.13	Summation	1	
5	**Lq**	-6.94	Summation	2.5	
6	**L**	-4.44			
7					**Term1**
8	**Wq**	-1.39			3.5
9	Minutes	-83.33	**Term2**		**Term2**
10	Seconds	-5,000.00	s !	2	-12.5
11	**W**	-0.89	Numerator	6.25	
12	Minutes	-53.33	Denominator	-0.5	
13				**P₀**	-0.111111111

I3. Enter =SUM(((L13*((MAIN!$C$4/MAIN!$C$5)^MAIN!$C$6))*MAIN!$C$7))
I4. Enter =SUM((K10*((1-MAIN!$C$7)^2)))
I5. Enter =SUM(($I$3/$I$4))
I6. Enter =SUM(($I$5+(MAIN!$C$4/MAIN!$C$5)))
I8. Enter =SUM(($I$5+(MAIN!$C$4/MAIN!$C$5)))
I9. Enter =SUM(($I$8*60))
I10. Enter =SUM(($I$9*60))
I11. Enter =SUM(($I$8+(1/MAIN!$C$5)))
I12. Enter =SUM(($I$11*60))
K4. Enter 1
K5. Enter =SUM((((MAIN!C4/MAIN!C5)^1)/1))

K10. Enter =FACT(MAIN!C6)
K11. Enter =SUM(((MAIN!C4/MAIN!C5)^MAIN!C6))
K12. Enter =SUM((K10*(1-(MAIN!C4/(MAIN!C6*MAIN!C5)))))
L8. Enter =SUM(K4:K5)
L10. Enter =SUM((K11/K12))
L13. Enter =SUM((1/(L8+L10)))

Note: Worksheets for three, four, and five servers are formatted and developed in the same manner as illustrated in table 4.

Table 5. Section for three Servers

	B	C	D	E	F
15	**3 Servers**		**P$_0$**		
16	**Lq num**	0.59	**Term1**		
17	**Lq den**	0.17	Summation	1	
18	**Lq**	3.51	Summation	2.5	
19	**L**	6.01	Summation	3.125	
20					**Term1**
21	**Wq**	0.70			6.625
22	Minutes	42.13	**Term2**		**Term2**
23	Seconds	2,528.09	s!	6	15.625
24	**W**	1.20	Numerator	15.625	
25	Minutes	72.13	Denominator	1	
26				**P$_0$**	0.04494382

Note: Table 5 reveals an additional summation in cell E19. Refer to term 1 of equation (8) n=2 for three servers, 3 for four servers and 4 for five servers. There will be an additional term for each additional server in the following form:

E19. Enter =SUM(((((MAIN!D4/MAIN!D5)^2)/2))

3. Results and Analysis

Data was collected from a WNC local bakery and food service business, the arrival times were Poisson and the service times were normally distributed. This fulfills the assumption that queuing analysis requires the arrival times to be Poisson and the service times to be normally distributed. The constructed spreadsheet was then analyzed using the spreadsheet results illustrated in Table 2 compared to hand calculations from the arrival and service times used in an example in Management Science[7]. Several different problems were analyzed and their results were compared with hand-calculations. These calculations showed that the spreadsheet model was accurate.

4. Discussion

The pedagogy was then applied to several different problems to insure that the algorithm was indeed accurate and appropriate. The data collected from the local bakery and food service business was analyzed with one server using the main worksheet illustrated in Table 2. The arrival rates and service times were translated from seconds to units as stated in 2.1 of the methodology. The arrival rate was roughly 42 units and the service time was roughly 54. The results were nearly identical to the observed activity at the local bakery. L was equal to 3.5 and L_q was equal to 2.7, a simple subtraction of L_q from L results in about 1, this means that while there were 3.5 (roughly 4) customers in the system there was 1 customer at the service station. W_q was equal to 3.9 minutes and W was equal to 5 minutes, this mean that it took on average 3.9 minutes to wait in line and 1.1 minutes to be served. This same spreadsheet model will then be used for determining the appropriate number of loading docks for a facility layout problem in Management 460 Production Operations Management in the fall 2014.

5. Conclusion

The major advantage to the practitioner, engineer, instructor and student is that Excel is readily available on all personal computers, easily understood, and is very practical. Students with very little exposure to queuing theory will be able to master the method within the first hour of exposure. Students with very little exposure to queuing theory will be able to master the method within the first hour of exposure. Also this spreadsheet model would be a very useful pedagogy for an introduction for industrial engineers, production managers, any public servant or entrepreneur that would be planning queuing operations.

6. Cite References

1. Emmons, Flowers, Khot, and Mathur, STORM 4.0 Quantitative Modeling for Decision Support for Windows, (Lakeshore Communications Inc. Cleveland, 2001).

2. SAS Institute Inc. SAS/OR (Version 13.1), SAS Analytical Products (2014).

3. Microsoft, Microsoft Project Computer Software. (Redmond, Washington: Microsoft 2013).

4. Microsoft, Microsoft Excel Computer Software. (Redmond, Washington: Microsoft 2007).

5. Gaither and Frazier, Operations Management, 9th Edition, (South-Western, Cincinnati OH 2001).

6. Lawrence W. Dowdy, Virgilio A.F. Almeida, Daniel A. Menasce, Performance by Design: Computer Capacity Planning By Example, (2004) 480.

7. Laurence Moore, Sang Lee, and Bernard Taylor, Management Science, (Allyn and Bacon, 1993), 464.