Developmental Mathematics Students' Fractional Understanding

by

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Abstract

Students enrolled in collegiate developmental mathematics courses hold mathematical misconceptions that begin long before their college careers. Specifically, these students struggle understanding concepts related to fractions, ratios, and proportions. The Common Core State Standards in Mathematics delineate standards for K-12 mathematics education. Standards for fractions, ratios and proportions are included in grades 3-7. Investigated herein are the concepts with which developmental mathematics students struggle in respect to grades 3-7 fraction, ratio, and proportion standards. To address this inquiry, a test was created composed of questions regarding fractions, ratios, and proportions that align with the respective Common Core State Standards for grades 3-7. After administering tests, collecting results, and analyzing which questions were answered correctly and incorrectly, it was determined that collegiate developmental mathematics students evidenced limited knowledge of these topics. Since it seems that this lack of understanding is both born early in, and is perpetuated through, students' academic careers, it is hoped that this work can be a catalyst for future work in this field that may apply to investigating ways to mitigate these gaps in mathematical understanding at their origins.

Developmental Mathematics Students' Fractional Understanding

According to the Common Core State Standards in Mathematics (National Governors Association Center for Best Practices [NGA Center], 2010), fractions, ratios, and proportional relationships are studied in grades 3-7. Research reveals that students in those grades begin developing various misconceptions regarding fraction understanding that later manifest in algebra courses (Mou et al., 2016; Fonger, Tran & Elliot, 2015) and that students enrolled in collegiate developmental mathematics courses continue to struggle with these concepts (Steinke, 2017). Because fraction understanding can be linked to success in more advanced mathematics courses, it is essential that these fundamental concepts are learned both early and thoroughly. Unfortunately, many students progress through elementary school, middle grades, and high school without sufficiently grasping these mathematical ideas.

The purpose of this research is to pinpoint the grades 3-7 Common Core State Standards regarding fractions, ratios, and proportions that developmental mathematics students have mastered and those with which they continue to struggle. It is hypothesized that misconceptions regarding fractions, ratios, and proportions begin as early as in the third grade, when the topics are introduced, and often persist into college.

Student enrollment in collegiate developmental mathematics courses has adverse financial effects on the institution providing the course and correlates to a strong predictive factor regarding these students eventually dropping out of college (Fong, Melguizo & Prather, 2015; Acosta, 2016; Burley, Butner, Anderson & Siwatu, 2009; Cox, 2015) Therefore, the results of this study may inform researchers as to grade levels in which intervention should be invested to break this cycle.

Literature Review

Recent research indicates that fraction understanding on all educational tiers, collegiate developmental mathematics courses, and the persistence of mathematical misconceptions are topics that have been investigated in varying capacities. This study is specifically interested in how the persistence of misconceptions in mathematics regarding fractions manifests itself in collegiate developmental math.

University Developmental Mathematics Courses

Research in higher education has revealed that a growing proportion of collegiate students are being enrolled in developmental mathematics courses (Acosta, 2016; Fong et al., 2015). Developmental courses are defined to be "non-credit courses that address the needs of students who are underprepared for college-level courses" (Acosta, 2016, p. 2). These courses are typically offered in English, reading, and mathematics and are prerequisites for subsequent college courses required for a certificate or degree (Acosta, 2016). College developmental mathematics courses typically progress through four topics: arithmetic, pre-algebra, elementary algebra, and intermediate algebra (Fong et al., 2015).

The growing enrollment in college developmental courses is problematic. The annual cost of developmental education at U.S. universities is more than two billion dollars (Fong et al., 2015). Additionally, enrollment in developmental courses has been found as a correlate to predict university student dropout (Acosta, 2016; Fong et al., 2015). Specifically, students enrolled in developmental mathematics courses "were less successful in obtaining a degree than students not taking developmental math" (Burley et al., 2009, p. 29). "The three post-secondary courses (across all U.S. Colleges) with the highest rates of failure and noncompletion are all developmental math courses" (Cox, 2015, p. 2). Summarily, developmental courses add costs to universities and are associated with student failure and dropout rates.

While developmental mathematics courses correlate to predict university student dropout (Acosta, 2016; Burley et al., 2009; Fong et al., 2015), it is dubious if these courses possess actual causal effect upon such apart, possibly, from elongating the student's academic plans. Simultaneously, while these courses negatively impact university costs (Cox, 2015, p. 2), it could also be argued that such courses provide otherwise unavailable opportunities for students with college potential to attend college and attain college-necessary careers. Thus, while it is possible that developmental mathematics courses are hardships to both students and universities, these courses may be necessary for many students to progress academically and toward their desired careers.

Fraction Learning and Connections

Student understanding of fraction magnitudes and fraction arithmetic skills are both essential in the comprehension of one another (Bailey, Hansen, & Jordan, 2016). According to Pantziara and Philippou (2011), there exist five sub-constructs regarding the learning of fractions: part-whole, ratio, quotient, measure, and operator. They report that fourth and fifth grade students who understand both fractional concepts and procedures outperform those who only understand one of the two. Students appear to receive an excessive amount of instruction on the part-whole sub-construct at the expense of others, specifically measurement. Fraction problems involving variables appear to be the most challenging tasks for students.

Lewis, Gibbons, Kazemi, and Lind (2015), report that fourth grade students struggle with the symbolic form of fractions and fraction terminology. The same students struggled when asked to portion whole amounts into fractional amounts, because they only wanted to deal with whole numbers. Many times, the students would divide a whole into parts without necessarily being concerned with the size of the parts (Lewis, Gibbons, Kazemi, & Lind, 2015). Hansen, Jordan, and Rodrigues (2017) report that "errors on fraction computation problems may reflect misapplication of whole number principles to fractions" (p. 46).

Fonger, Tran, and Elliot (2015) state that the purpose for their research is to "examine children's informal strategies and knowledge of fractions by looking at their creations, interpretations, and connections within and between multiple fractions representations" (p. 4). Their interviews with children from grades 2-6 reveal that children "often express inconsistencies or mismatches when reasoning across representation types, sometimes posing a hindrance to their problem solving process" (p. 13). They opine that children generally better understand fraction problems that provide them with accessible contexts related to their prior experiences. Several children demonstrated a specific misconception regarding cross-multiplication of fractions when such a calculation was not necessary. Students also had varying understanding of the meaning of fractions in context of comparing measurements. The authors believe that students interpreted the measurement task as an operator, the part-whole meaning, and the measurement meaning individually rather than the three having the same innate meaning.

A recent study finds that 7th and 8th grade students who struggled with placing fractions on a number line also struggled with algebra skills in 9th grade (Mou et al., 2016). The authors of the study state that providing a stronger focus on fraction magnitudes in early grades would improve performance in later mathematics classes. Understanding fractions is fundamental to expanding number understanding, which in turn is the foundation of advanced mathematics courses. The authors conclude that educators should identify instructional targets – such as fraction magnitude and numerator/denominator relationships – for mathematics education in middle school.

Though fractions are primarily taught in elementary and middle school, the same fraction concepts are applied later in algebra when learning about algebraic fractions. Makonye and Khanyile (2015) note that students who exhibited misconceptions regarding fractions that persisted into 10th grade “handled mathematical concepts as if they were isolated chunks of knowledge” (p. 55). Through interviewing students, constructing interview questions which necessitate conceptual versus instrumental understanding of fractions, and encouraging students to provide a reason for each and every calculation, Makonye and Khanyile (2015) reveal that students were able to overcome their misconceptions regarding the simplification of algebraic fractions.

Persistence of Misconceptions

The research of Bailey, Hansen, and Jordan (2017) predicts that early specified instruction targeting magnitude understanding and arithmetic skills “is more likely to produce more persistent effects on these same specific skills than early support that targets mathematics achievement more broadly will produce on general mathematics achievement measures” (p. 516). They also conclude that transfer between fraction magnitude understanding and arithmetic skills does not occur until later in a student's’ learning process. Hansen, Jordan, and Rodrigues (2015) believe that “persistent difficulties with fraction procedures throughout elementary/early middle school are associated with low mathematics achievement at the end of sixth grade” (p. 53). Though elementary and early middle school are the grades in which fractions are taught in school, Hansen, Jordan, and Rodrigues (2015) report that many students showed little progress during this time in the understanding of fractions. Because misconceptions regarding fractions persist in this way, intervention must be made in the early years of education. Without such intervention, those misconceptions will again manifest themselves when students enroll in higher level mathematics courses leading to long-term concerns for mathematics achievement.

Connecting Fraction Knowledge to Developmental Math

In a study performed by Steinke (2017), 23% of the students enrolled in developmental mathematics struggled with fractional concepts such as part-whole coexistence. Comprehension of part-whole coexistence is crucial to the development of fraction understanding and thus to higher level mathematics achievement (Steinke, 2017; Bailey et al., 2017). Recognizing that elementary and middle school students struggle to understand various concepts associated with fractions and that university developmental mathematics students continue to struggle with fractional concepts, the question remains as to what extent university developmental mathematics students struggle with fraction understanding and fraction operations. Therefore, this study investigates for which concepts regarding fractions, ratios, and proportions are developmental mathematics students more or less successful.

Research Methodology

Participants in this study included 75 developmental mathematics students in two developmental mathematics courses at a public university in the Southeastern U.S. Both courses were taught by the same instructor during the Spring 2017 semester. All participants had taken a university mathematics placement exam placing them into developmental mathematics. Participation was open to all 78 students enrolled in these two classes. Participants completed a test regarding fractions, ratios, and proportions administered during the second day of the first week of the class. Participants were given approximately 80 minutes to complete the test.

Since, on a large scale, it is difficult to precisely evaluate individual students' understanding of fractional concepts, this study selected to assess student understanding of fractions based on commonly recognized standards. The Common Core State Standards for Mathematics for grades 3-7 includes 51 standards regarding fractions, ratios, and proportions (NGA Center, 2010). From these 51 standards, 65 questions were developed by a team of researchers (including a preservice teacher, a university mathematics education lecturer working on his PhD, and a professor of mathematics education) to address the standards. While many of the test items involved performing operations on fractions, all questions were constructed to examine student conceptual understanding of the associated standard. While some standards were covered by one question, other more nuanced standards were addressed by two or more questions. Although the expertise of the members of the research team was used to develop test items associated with respective standards, notably, no opportunity was available to determine either the validity or the reliability of the test items.

Participants were not allowed to use a calculator on the test and were asked not to solve fraction problems by rewriting the fractions as decimal values. The goal of the study was to assess student understanding of fractions and fraction operations based on grade 3-7 Common Core State Standards and using decimal representations would not demonstrate proficiency of the standards. Participants were also instructed not to guess the answer to a question but rather to skip it if they did not know how to solve the problem. In this way, the researchers could identify which problems participants struggled with based incorrect and incomplete answers and answers left blank. The entire research instrument including these instructions is provided in the appendix to this paper.

Comparing particular test items in the instrument with their associated standard often reveals both similarities and dissimilarities. For instance, test items were constructed based on both the mathematical meaning and the inferred intention of the standard. This necessitated both interpretation of the standards and the creation of an associated test item aligning to the standard. Care was taken to avoid making collegiate students feel demeaned by test items regarding fractions, ratios, and proportions – topics obviously covered in elementary grades mathematics. Thus, at times, numerals are used within test items, making them seem more simplistic, and at other times variables are used, making them seem more in line with algebraic reasoning. The latter intended to protect participants from feeling insulted by questions on the instrument.

Each participant response on each test item was graded based in the following criteria:

- Correct: all parts of the student's answer were correct.
- Incorrect: all parts of the student's answer were incorrect.
- Partial: at least part of the student's answer was correct.
- Blank: student left the question blank.
- Incomplete: the student started the problem but did not finish. The attempted portion of the problem was incorrect.
- Unanticipated: the student provided a correct answer, but it was not one that was anticipated based on the context of the problem.

Once all of the tests were graded, the grades were entered into a spreadsheet used to directly compare results. Notably, for this introductory investigation, the data analysis employed no statistics beyond simple tabulation of numbers, averages, and percentages. Since this particular study had no interest in disaggregating data by participant age, gender, or traditional Freshman versus nontraditional students, no effort was made to use statistical methods to search for correlations between these factors and student understanding of fraction concepts. This study intended to investigate the population of participants as a whole.

Notably, although the participants involved in this study were college students, the standards upon which their fractional understanding were being evaluated were from grades 3-7. Since all participants had passed all grades subsequent to grades 3-7 in order to be enrolled in developmental math, it could be assumed that the participants should do well on the research instrument. The following results demonstrate otherwise.

Results

The results from the pretest are separated by their associated standard. For each standard, the following are provided: (a) the standard regarding fractions, ratios, and proportions as published in the Common Core State Standards in Mathematics for grades 3-7 and (b) a small table presenting the resulting data from student performance. As previously mentioned, the entire research instrument is provided in the Appendix. Each standard is identified by an alphanumeric coding structure (e.g., 3.NF.A.1). The first numeral represents the grade level in which the standard should be taught. The next set of letters is known as the "domain." The domain links groups of standards together that revolve around a common concept. NF denotes the domain "Numbers and Operations – Fractions". The next digit represents that the standard listed is one of a cluster of related standards. For example, 3.NF.A.1 indicates that this specific standard is taught in third grade, under the domain of Numbers and Operations – Fractions, and that it is the first standard (1) in the cluster (A).

Notably, in the standards for grades 6 and 7, the NF designator is replaced by RP, denoting Ratios and Proportions.

3.NF.A.1: Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. (See question 1 in Appendix 1.)

Number correct	15
Number incomplete	3
Number blank	43
Number partially correct	0
Number unanticipated	0
Number incorrect	14

3.NF.A.2.A: Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. (See question 2 in Appendix 1.)

Number correct	19
Number incomplete	1
Number blank	23
Number partially correct	15
Number unanticipated	0
Number incorrect	17

3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (See question 3 in Appendix 1.)

Number correct	4
Number incomplete	0
Number blank	2
Number partially correct	68
Number unanticipated	0
Number incorrect	1

3.NF.A.2.B: Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. (See question 4 in Appendix 1.)

Number correct	6
Number incomplete	3
Number blank	5
Number partially correct	48
Number unanticipated	0
Number incorrect	13

3.NF.A.3.A: Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (See questions 5, 6, and 7 in Appendix 1.)

Question number	5	6	7
Number correct	54	40	13
Number incomplete	0	0	0
Number blank	12	16	14
Number partially correct	0	0	0
Number unanticipated	0	0	0
Number incorrect	9	19	48

3.NF.A.3.B: Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. (See question 8 in Appendix 1.)

Number correct	8
Number incomplete	8
Number blank	42
Number partially correct	7
Number unanticipated	0
Number incorrect	10

3.NF.A.3.C: Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. (See questions 9 and 10 in Appendix 1.)

Question number	9	10
Number correct	24	17
Number incomplete	5	3
Number blank	21	49
Number partially correct	21	4
Number unanticipated	1	0
Number incorrect	3	2

3.NF.A.3.D: Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (See questions 11, 12, 13, and 14in Appendix 1.)

Question number	11	12	13	14
Number correct	49	50	39	19
Number incomplete	0	0	0	0
Number blank	12	9	16	18
Number partially correct	0	0	0	0
Number unanticipated	0	0	0	0
Number incorrect	14	16	20	38

4.NF.A.1: Explain why a fraction *a*/*b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (See questions 15, 16, and 17 in Appendix 1.)

Question number	15	16	17
Number correct	11	26	4
Number incomplete	0	0	4
Number blank	14	27	51
Number partially correct	0	0	6
Number unanticipated	0	0	0
Number incorrect	50	22	10

4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (See questions 18 and 19 in Appendix 1.)

Question number	18	19
Number correct	55	24
Number incomplete	0	0
Number blank	10	9
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	10	42

4.NF.B.3: Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. 5.NF.B.4.A Understand a fraction *a*/*b* as a multiple of 1/*b*. (See question 20 in Appendix 1.)

Number correct	0
Number incomplete	0
Number blank	52
Number partially correct	0
Number unanticipated	0
Number incorrect	23

4.NF.B.3.A: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (See questions 21 and 22 in Appendix 1.)

Question number	21	22
Number correct	2	7
Number incomplete	0	1
Number blank	42	40
Number partially correct	6	0
Number unanticipated	2	0
Number incorrect	23	27

4.NF.B.3.B: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. (See question 23 in Appendix 1.)

Number correct	11
Number incomplete	0
Number blank	51
Number partially correct	0
Number unanticipated	0
Number incorrect	13

4.NF.B.3.C: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. (See questions 24 and 25 in Appendix 1.)

Question number	24	25
Number correct	25	27
Number incomplete	1	0
Number blank	31	23
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	18	25

4.NF.B.4.A: Understand a fraction *a*/*b* as a multiple of 1/*b*. (See question 26 in Appendix 1.)

Number correct	2
Number incomplete	4
Number blank	60
Number partially correct	5
Number unanticipated	0
Number incorrect	4

4.NF.B.4.B: Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. (See questions 27 and 28 in Appendix 1.)

Question number	27	28
Number correct	12	5
Number incomplete	1	0
Number blank	42	50
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	20	20

4.NF.C.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (See questions 29 and 30 in Appendix 1.)

Question number	29	30
Number correct	22	3
Number incomplete	2	0
Number blank	29	53
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	22	19

4.NF.C.6: Use decimal notation for fractions with denominators 10 or 100. (See questions 31 and 32 in Appendix 1.)

Question number	31	32
Number correct	17	17
Number incomplete	0	1
Number blank	43	42
Number partially correct	1	0
Number unanticipated	5	1
Number incorrect	9	14

4.NF.C.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (See question 33 in Appendix 1.)

Number correct	2
Number incomplete	4
Number blank	58
Number partially correct	3
Number unanticipated	4
Number incorrect	4

5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. (See question 34 in Appendix 1.)

Number correct	4
Number incomplete	4
Number blank	53
Number partially correct	3
Number unanticipated	2
Number incorrect	9

5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (See questions 35, 36, 37, and 38 in Appendix 1.)

Question number	35	36	37	38
Number correct	8	1	0	0
Number incomplete	0	1	0	0
Number blank	42	54	52	57
Number partially correct	2	0	0	0
Number unanticipated	10	4	0	0
Number incorrect	13	15	23	18

5.NF.B.4.A: Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. (See question 39 in Appendix 1.)

Number correct	12
Number incomplete	0
Number blank	35
Number partially correct	0
Number unanticipated	0
Number incorrect	28

5.NF.B.4.B: Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (See question 40 in Appendix 1.)

Number correct	2
Number incomplete	5
Number blank	50
Number partially correct	0
Number unanticipated	0
Number incorrect	18

5.NF.B.5.A: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. (See questions 41 and 42 in Appendix 1.)

Question number	41	42
Number correct	9	0
Number incomplete	0	0
Number blank	48	54
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	18	21

5.NF.B.5.B: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller

Question number	43	44
Number correct	8	8
Number incomplete	0	0
Number blank	42	51
Number partially correct	0	0
Number unanticipated	19	5
Number incorrect	6	11

than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (See questions 43 and 44 in Appendix 1.)

5.NF.B.7.A: Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. (See questions 45, 46, and 47 in Appendix 1.)

Question number	45	46	47
Number correct	0	14	3
Number incomplete	0	0	0
Number blank	65	55	54
Number partially correct	0	0	0
Number unanticipated	0	0	0
Number incorrect	10	6	18

5.NF.B.7.B: Interpret division of a whole number by a unit fraction, and compute such quotients. (See questions 48, 49, 50, and 51 in Appendix 1.)

Question number	48	49	50	51
Number correct	1	14	7	0
Number incomplete	2	0	0	3
Number blank	57	45	43	52
Number partially correct	2	0	0	14
Number unanticipated	0	0	1	0
Number incorrect	13	16	24	6

6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (See questions 52, 53, 54, and 55 in Appendix 1.)

Question number	52	53	54	55
Number correct	12	14	1	13
Number incomplete	0	0	0	0
Number blank	28	34	67	31
Number partially correct	25	0	0	10
Number unanticipated	0	0	0	0
Number incorrect	10	27	7	21

6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. (See questions 56, 57, and 58 in Appendix 1.)

Question number	56	57	58
Number correct	56	36	0
Number incomplete	1	2	2
Number blank	12	22	43
Number partially correct		9	0
Number unanticipated	0	0	0
Number incorrect	6	6	30

6.RP.A.3.C: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (See questions 59, 60, and 61 in Appendix 1.)

Question number	59	60	61
Number correct	1	0	5
Number incomplete	1	3	2
Number blank	42	52	58
Number partially correct	27	0	0
Number unanticipated	0	0	0
Number incorrect	4	20	10

6.RP.A.3.D: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (See questions 62 and 63 in Appendix 1.)

Question number	62	63
Number correct	0	0
Number incomplete	3	1
Number blank	52	58
Number partially correct	0	0
Number unanticipated	0	0
Number incorrect	20	16

7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. (See question 64 in Appendix 1.)

Number correct	30
Number incomplete	0
Number blank	34
Number partially correct	0
Number unanticipated	0
Number incorrect	11

7.RP.A.2.A: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (See question 65 in Appendix 1.)

Number correct	12
Number incomplete	2
Number blank	41
Number partially correct	16
Number unanticipated	0
Number incorrect	4

Notably, there were no corresponding test items created for the final four standards on ratios and proportions listed below, above primarily due to:

- the researchers' hypothesis that developmental students' mathematical misconceptions regarding fractions, ratios, and proportions begin as early as third grade;
- participating students had a limited amount of time and thus the researchers had to limit the number of items on the test;
- the researchers intended for the test to be an inventory of students' mathematics skills rather than their literacy skills (standards that require a student to "explain" or understand complex word problems were not assessed); and
- These standards exceeded the topical level that was intended for this study.

7.RP.A.2.B: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.A.2.C: Represent proportional relationships by equations.

7.RP.A.2.D: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Summarily, see Appendix 2 for the spreadsheet generated to directly compare results and the table comparing raw scores from each Common Core Standard grade level.

Discussion

Numerous percentage values are provided in this discussion. Remembering that this study investigated developmental college students' understanding of fractions, ratios, and proportions from grades 3-7, some of these percentages may be considered somewhat shocking. To highlight these, when appropriate, bold font is employed. Additionally, in the discussion, when standards are quoted, they are italicized to assist the reader to make appropriate connections.

Of the fourteen test items based on third grade Common Core State Standards regarding fractions: students answered only **34%** of them correctly; 26.86% of the responses were left blank; 21.33% were answered incorrectly; and 15.52% were answered partially correct. The most commonly missed test items based on third grade standards were items number 7 (referencing 3.NF.A.3.A: *Understand two fractions as equivalent (equal) if they are the same size…*) and number 14 (referencing 3.NF.A.3.D: *Compare two fractions with the same numerator or the same denominator by reasoning about their size… Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a fractional model.*) with 48 and 38 people respectively answering the questions incorrectly (**64%** and **51%**). The two test items most frequently answered correctly were numbers 5 (3.NF.A.3.A) and 12 (3.NF.A.3.D) with 54 and 50 people respectively answering the items correctly (or 77% and 67%). Notably, the standards associated with the two items on which the students best performed were the same standards associated with the items on which the students did the worst. The two most frequently skipped questions were questions number 1 (referencing 3.NF.A.1: *Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b*) and question 10 (referencing 3.NF.A.3.C: *Express whole numbers as fractions, and* *recognize fractions that are equivalent to whole numbers*) with 43 and 49 people respectively skipping the questions (**57%** and **65%**).

Fourteen test items were created based on the fourth grade Common Core State Standards. Of the fourteen items: **51.02%** of the responses were left blank; 26.32% were answered incorrectly; and **19.09%** were answered correctly. The most missed test items from the fourth grade standards were questions number 15 (referencing 4.NF.A.1: *Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b)… Use this principle to recognize and generate equivalent fractions.*) and 19 (referencing 4.NF.A.2: *Compare two fractions with different numerators and different denominators… by comparing to a benchmark fraction… Record the results of comparisons with symbols >, =, or <…*) with 50 and 42 people respectively answering the items incorrectly (or **67%** and **56%**). Number 15 is the single most missed question on the test. The two test items most frequently answered correctly were numbers 18 (4.NF.A.2) and 25 (referencing 4.NF.B.3.C: *…replacing each mixed number with an equivalent fraction…*) with 55 and 27 people respectively answering the items correctly (or 73% and **36%**). While item 19 was one on which students did most poorly and item 18 was one on which students did best, both items referenced the same standard. The two most frequently skipped questions were questions 26 (referencing 4.NF.B.4.A: *Understand a fraction a/b as a multiple of 1/b*) and question 33 (referencing 4.NF.C.7: *Compare two decimals to hundredths by reasoning about their size… Record the results of comparisons with the symbols >, =, or <…*) with 60 and 58 people respectively skipping the questions (**80%** and **77%**).

The fifth-grade questions included in the test consisted of fifteen items. Of the fifteen test items: **67.33%** of the responses were left blank; 20.22% were incorrect; and **6.74%** were correct. The most commonly missed test items in this section were numbers 39 (referencing 5.NF.B.4.A: *Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b… In general, (a/b) × (c/d) = (ac)/(bd).*) and 50 (referencing 5.NF.B.7.B: *Interpret division of a whole number by a unit fraction, and compute such quotients… Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*). Twenty-eight people (37%) missed item number 39 and 24 people (32%) missed item number 50. The two test items most frequently answered correctly were numbers 46 (referencing 5.NF.B.7.A: *Interpret division of a unit fraction by a non-zero whole number, and compute such quotients…. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*) and 49 (5.NF.B.7.B) with only 14 people (**19%**) answering the items correctly. Item 50, among the items on which students did most poorly, and item 49, among the items on which students did best, were both associated with the same cluster of standards, 5.NF.B.7. The most frequently skipped questions were questions 38 (referencing 5.NF.B.3**:** *Interpret a fraction as division of the numerator by the denominator…)*, 48 (referencing 5.NF.B.7.B), and 45 (referencing 5.NF.B.7.A) with 57 people skipping numbers 38 and 48, and 65 people skipping question 45 (**76%** and **87%**).

Twelve test items were created based on sixth grade Common Core State Standards. Of the twelve items: **55.44%** of the responses were left blank; 19.67% of responses were incorrect; **15.33%** of answers were correct; and 7.89% of responses were partially correct. The top two most missed items were numbers 58 (referencing 6.RP.A.2: *Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.*) and 52 (referencing 6.RP.A.1: *Understand the concept of a ratio and* *use ratio language to describe a ratio relationship between two quantities.*), with 30 and 27 people (40% and 36%) missing the items, respectively. The two test items most frequently answered correctly were numbers 56 (referencing 6.RP.A.2) and 57 (based on standard 6.RP.A.2) with 56 and 36 people (75% and 48%) respectively answering the items correctly. Item 58, among the items on which students did most poorly, and items 56 and 57, among the items on which students did best, were all associated with the same standard. The items on which students performed best and on which most students struggled were from the same cluster of standards, 6.RP.A. The most frequently skipped questions were questions 61 (referencing 6.RP.A.3.C: *Find a percent of a quantity as a rate per 100… solve problems involving finding the whole, given a part and the percent*), 63 (referencing 6.RP.A.3.D: *Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities*) and 54 (referencing 6.RP.A.1) with 58 people skipping numbers 61 and 63, and 67 people skipping question 54 (**77%** and **89%**).

There were two test items created based on seventh grade Common Core State Standards. For these two questions: **50%** of the responses were left blank; **28%** of responses were correct; 10.67% of answers were partially correct; and 10% of responses were incorrect. Eleven people (15%) missed item number 64 (referencing 7.RP.A.1: *Compute unit rates associated with ratios of fractions ... in like or different units.*) and 4 (5%) missed item 65 (referencing 7.RP.A.2.A: *Decide whether two quantities are in a proportional relationship … by testing for equivalent ratios in a table…*). Thirty people (40%) correctly answered item 64, while 12 (**16%**) correctly answered item 65. The most frequently skipped question was question 65 (referencing 7.RP.A.2.A) which 41 people skipped (**55%**).

By dividing the number of test items that each student answered correctly, by the total number of items (65), the average percentage grade for all 75 developmental mathematics students on the test was **18.46%**. Adding the number of correct, the number of unanticipated, and the number of partially correct item responses for each student, then dividing by the total number of questions generates a class average of only **25.56%**. Over **51%** of all responses on test items were left blank. The test items that were created based on fifth grade standards were the most frequently left blank (**67.33%**) while the items based on third grade standards were the most frequently answered correctly (34%). Number 54, regarding ratios, was the singularly least attempted test item with a total of 64 students (**85%**) leaving it blank. Number 43 was the test item with the most number of students answering in an unanticipated way (N = 19 or 25%). The majority of students who answered the item with an unanticipated response either used zero or a negative number as a multiplicand to make a fraction smaller than another fraction.

Interestingly, and unexpectedly, different test items for some standards led to problems on which students did the best and the worst. This occurred with standards:

3.NF.A.3.A: *Understand two fractions as equivalent (equal) if they are the same size…*

3.NF.A.3.D: *Compare two fractions with the same numerator or the same denominator by reasoning about their size… Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a fractional model.*

4.NF.A.2: *Compare two fractions with different numerators and different denominators… by comparing to a benchmark fraction… Record the results of comparisons with symbols >, =, or <…*

5.NF.B.7.B: *Interpret division of a whole number by a unit fraction, and compute such quotients… Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* And

6.RP.A.2: *Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.*

Unfortunately, this study was not formed to answer the question as to why some standards would produce such polarized results in student responses.

Again, it must be continually recognized that the mathematical content under investigation in this study are standards from grades 3-7 and the students involved are college students – albeit, students placed in noncredit earning developmental mathematics courses. Avoiding redundantly repeating some of the numeric values and percentages above, it is possible to state that these results are poor at best, if not abysmal. (It may be worthwhile for the reader to reconsider values previously provided in bold font.) Summarily: many students left many test items blank; most items were answered correctly by only a small percentage of students; and most students did very poorly overall. This leads to some grave concerns that are to later be addressed as implications to these findings.

Student Self-reporting of Mathematical Ills

The data above clearly demonstrates that the developmental mathematics students in this study possessed insufficient mathematical knowledge regarding fractions, ratios, and proportions based on standards from as early as the 3rd-7th grades. However, an additional qualitative question accompanied the exam. Students were asked, "If you feel like you have a history of struggling with mathematics, in which grade do you remember this starting?" Of the 63 students who answered this question: 39 students reported that their struggles in mathematics began after 7th grade; 15 students reported their struggles beginning in elementary school; 14 students reported their struggles beginning in middle school; 31 reported their struggles beginning in high school; and one student reported that he did not begin struggling with mathematics until college. However, with 39 students (62%) self-reporting that their mathematical struggles postdated the grades in which these standards were covered, this may lead to an inconsistency between result and self-reporting. Indeed, although most students self-reported that their mathematical ills originated after 7th grade, not a single student in the study made above a 45% on the instrument.

Due to issues surrounding the reliability of self-reported data, no statistical analysis was performed relating individual self-reported origins of struggle with individual test scores.

Implications

Though students lacking understanding of concepts regarding fractions, ratios, and proportions have been studied previously, there is a gap in the literature regarding when this lack of knowledge may originate. The researchers had hypothesized that a lack of mathematical understanding among collegiate developmental mathematics students regarding concepts associated with fractions, ratios, and proportions could be evidenced through investigating their success responding to questions from standards from grades 3-7. However, while the research participants demonstrated insufficient knowledge of these standards as demonstrated through their work on the research instrument, it is unclear whether their lack of understanding is born in grades 3-7 or whether these concepts were satisfactorily learned at the time and subsequently lost since. Nevertheless, it is clearly seen, that among this study population, understanding of grades 3-7 mathematical standards regarding fractions are sufficiently lacking.

According to the results above, the highest percentage of questions answered correctly were those related to third grade standards (38%). However, the fact only 38% of questions based on *third grad*e standards were answered correctly by *college* students supports the hypothesis that developmental mathematics students lack understanding regarding fractions from an early age. This study suggests that the lack of understanding regarding fractions taught in third through sixth grades persists through all of grade school and on into students' college careers.

Specifically, the questions answered correctly the most often were regarding using unit rates in context of a ratio relationship (question 56), comparing fractions with different numerators and denominators (question 18), and understanding two fractions as equal if they are the same size or point on a number line (question 5). On the other hand, the questions answered incorrectly most often were regarding multiplying by a fraction equivalent to one (question 15), understanding fractions as equal if they are the same size (when the fractions involve variables as well as integers) (question 7), and comparing two fractions with the same numerator or the same denominator by comparing their size (question 14). This implies that more time may need to be spent on those specific concepts or that different teaching methods may be necessary to communicate them more efficiently in early grades.

Returning to the finding that some standards led to both best and worst student responses indicates that there yet remains much opportunity for research in this field. Why did some standards lead to these polarizing results while others did not? Was this due to the nature of the test items or the standards themselves? What in students' backgrounds led to this effect? Countless other questions remain unanswered.

One must ask how much of the difficulties students exhibited with the test items were due to some items including variables rather than only numeric results. Further investigation should follow to determine how much effect variables have on developmental mathematics students' skills solving problems involving fractions, ratios, and proportions. There remains much opportunity for additional research in this area.

Recalling that Makonye and Khanyile (2015) found success in students gaining conceptual understanding regarding the simplification of algebraic fractions through appropriate questioning techniques, there may be means through which to make accommodations for developmental mathematics student learning. However, these methods may be perceived as too labor intensive and time consuming for college developmental classes with more than thirty students in a class. Nevertheless, these results may demonstrate that developmental college mathematics students' conceptual learning of fraction ideas is possible, given appropriate instructional techniques. More precise ways to correct these misconceptions are not within the scope of this paper; however, it is clear from the lack of fraction understanding in college-age students that this problem must be addressed.

Recalling that most students (62%) in this study self-reported that their mathematical struggles began in grades beyond the 3rd through 7th grade from which the associated standards were taken, they, nevertheless, struggled with the concepts investigated in this study. This may speak volumes regarding student perceptions of their mathematical performance and the inability of the educational system to meet these concerns. The numerous implications from this cannot even begin to be addressed in this brief investigation. Additionally, it can be wondered if students who self-report their perceived origin of struggle in mathematics possibly self-fulfill that supposition. When students believe that they struggle in a subject and perceive it as unconquerable, it is likely that they then put minimal effort into attempting to master the subject.

Questions – which cannot be answered in this brief article – arise regarding the nature of the American educational system in which students can struggle with concepts from grades 3-7 mathematics and yet pass all the necessary grades prior to college and even be enrolled in college. These students passed through middle grades and high school mathematics courses without comprehending many fraction, ratio, and proportional reasoning concepts even though they were required to learn them in school. This continuous cycle of passing without understanding trains students that they need not to comprehend mathematics and they will still move on to the next, more advanced, mathematics course.

Conclusion

After studying the results of 75 collegiate developmental mathematics students on a test regarding fractions, ratios, and proportional reasoning based on standards from 3rd through 7th grade, it has been concluded that these students significantly lacked knowledge in these concepts. Considering that not a single student answered over 45% of the questions correctly, with the average number of correct answers being 12 out of 65 questions, there are major gaps in mathematical knowledge regarding fractions, ratios, and proportions. Since these concepts are important to higher level mathematics courses, it is essential that students understand them in the 3rd through 7th grades to possibly prevent having to enroll in a developmental course.

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Appendix 1

Developmental Mathematics Pretest on Fractions, Ratios, and Proportions

If you feel like you have a history of struggling with mathematics, in which grade do you remember this starting? _________

Feel free to make a comment regarding your mathematical experiences through the years.

Directions:

- **DO NOT USE A CALCULATOR.**
- **DO NOT SOLVE FRACTION PROBLEMS BY REWRITING FRACTIONS INTO DECIMAL VALUES.**
- **IF YOU DO NOT KNOW THE CORRECT ANSWER TO A PROBLEM, DO NOT GUESS. SIMPLY SKIP THE QUESTION.**

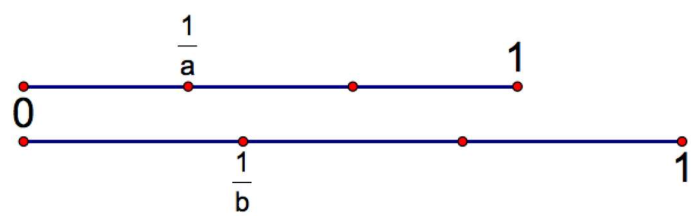
NOTE: In all following examples, all values *a*, *b*, *c* and so forth are to be considered natural numbers {1, 2, 3, 4, …}.

1. For $\frac{a}{b} = \frac{an}{b} = a \cdot \frac{n}{b}$, find the value of n.

3.NF.A.1

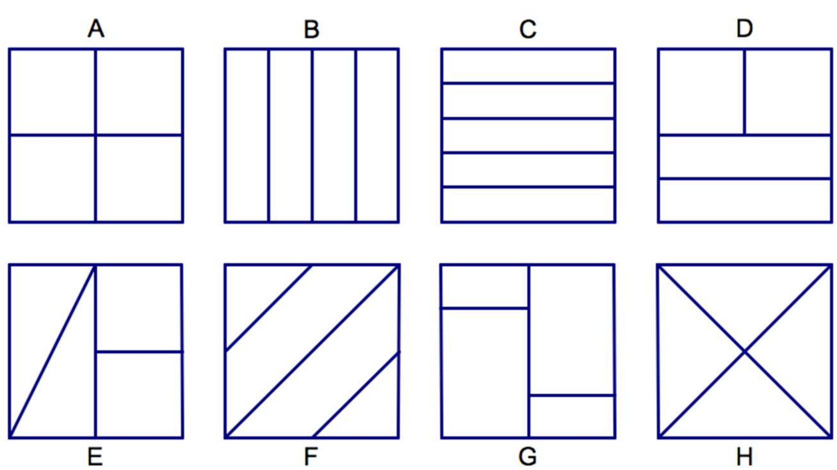
2. On the figure below, determine whether $a > b$, $a = b$, or $a < b$ and explain why.

3.NF.A.2.A



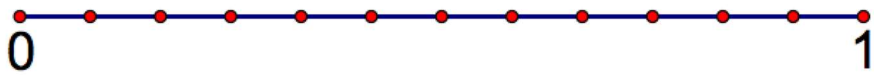
3. Each of the following represents a student attempting to fold a piece of paper into fourths. Determine which folding techniques work and which do not.

3.NF.A.3



4. On the number line below, locate the following values: $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{8}$.

3.NF.A.2.B



5. Circle the correct relation: $\frac{a}{a} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{b}{b}$

3.NF.A.3.A

6. Circle the correct relation: $\frac{37}{37} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{b+1}{b}$

7. Circle the correct relation: $\frac{a}{a-1} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{23}{23}$

8. For $\frac{65}{100} = \frac{m}{40} = \frac{13}{n}$, find the value of m and n.

3.NF.A.3.B

9. In four ways write 7 as a fraction.

3.NF.A.3.C

10. For $\frac{53}{48+a} = \frac{213}{213} = \frac{23-b}{17}$, solve for a and b.

11. Circle the correct relation: $\frac{13}{14} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{13}{15}$

3.NF.A.3.D

12. Circle the correct relation: $\frac{13}{14} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{11}{14}$

13. True or false: $\frac{a+1}{b} > \frac{a}{b} > \frac{a}{b+1}$

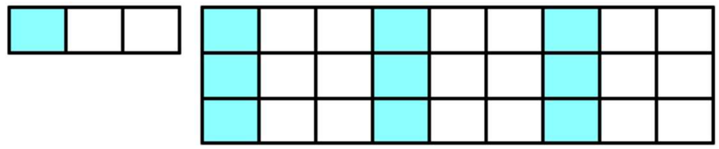
14. True or false: $\frac{a}{b-1} > \frac{a}{b} > \frac{a-1}{b}$

15. True or false: $c\left(\frac{a}{b}\right) = \frac{ac}{bc}$

4.NF.A.1

16. Circle correct relation: $\frac{am}{bm} \quad \begin{matrix}>\\=\\<\end{matrix} \quad \frac{na}{nb}$.

17. Explain how the figure below demonstrates that $\frac{a}{b} = \frac{ma}{mb}$.



18. Circle the correct relation: $\frac{13}{28} \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad \frac{1}{2}$

4.NF.A.2

19. Circle the correct relation: $\frac{9}{24} \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad \frac{1}{3}$

20. If $\frac{1}{b}+\frac{1}{b}+\frac{1}{b}+\cdots+\frac{1}{b}=\frac{a}{b}$, how many addends of $\frac{1}{b}$ is needed to produce $\frac{a}{b}$.

4.NF.B.3/5.NF.B.4.A

21. If $\frac{2}{a}+\frac{3}{b}-\frac{6}{c}=\frac{-1}{a}$, what might we know about the values of b and c?

4.NF.B.3.A

22. Calculate the following using as little arithmetic as possible and using no denominator greater than 24: $\frac{17}{13}+\frac{20}{26}-\frac{36}{39}$

23. If $\frac{1}{b}+3\cdot\frac{1}{b}+2\cdot\frac{a}{b}=\frac{16}{b}$, solve for a.

4.NF.B.3.B

24. Select an operation to make the statement true: $-16 \begin{matrix} + \\ or \\ - \end{matrix} \frac{1}{4} = -17+\frac{6}{8}$

4.NF.B.3.C

25. Does $-3\frac{2}{5}$ equal $\frac{-17}{5}$ or $\frac{-13}{5}$?

26. $\frac{a}{b}=a\cdot\frac{1}{x}=a\cdot\frac{y}{b}$. Solve for x and y.

4.NF.B.4.A

27. $12\left(\frac{1}{8}\right)=m\left(\frac{1}{4}\right)$. Find m.

4.NF.B.4.B

28. Chose the relation that would make the statement true: $\frac{a}{b}\times\frac{c}{d} \begin{matrix} > \\ = \\ \leq \end{matrix} \frac{1}{b}\left(\left(\frac{c}{d}\right)a\right)$.

29. Write $\frac{5}{1000}+\frac{40}{1000}+\frac{3}{10}$ as a single fraction.

4.NF.C.5

30. Write $\frac{a}{100}+d+\frac{b}{1000}+\frac{c}{10}$ as a single fraction.

31. Between what two whole numbers does $\frac{543}{100}$ lie?

4.NF.C.6

32. Between what two whole numbers does $\frac{543}{1000}$ lie?

33. Rewrite 3.59 into three different fractions.

(4.NF.C.7)

34. $a\frac{b}{c}+3\frac{7}{8}=6\frac{5}{24}$. Find *a*, *b*, and *c*.

5.NF.A.1

35. $\_\_\_\_ < 50 \div 9 < \_\_\_\_$. Fill in the blanks with whole numbers.

5.NF.B.3

36. $\_\_\_\_ < 20 \div \frac{3}{4} < \_\_\_\_$. Fill in the blanks with whole numbers.

37. $\frac{a}{b} \cdot \_\_\_\_ = 2b$. Fill in the blank.

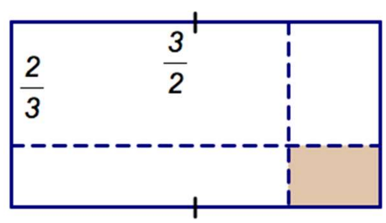
38. $\frac{a}{b} \cdot \_\_\_\_ = 2c$. Fill in the blank.

39. True or false: $\frac{a}{b} \cdot c = (a \cdot c) \div b = a \cdot \frac{c}{b}$

5.NF.B.4.A

40. Determine the area of the shaded region

5.NF.B.4.B



5.NF.B.5)

41. $a \cdot \frac{2}{3}$ is how much larger than $\frac{a}{3}$?

5.NF.B.5.A

42. $\frac{b}{2}$ is how much larger than $\frac{3b}{4}$?

43. Fill in the blank: $\frac{2}{3} \times \_\_\_ < \frac{2}{3}$.

5.NF.B.5.B

44. Fill in the blank: $\frac{3}{4} \div \_\_\_ > \frac{1}{2}$.

45. If $a \div \frac{1}{b} = \frac{1}{bc}$, find the value of *a*.

5.NF.B.7.A

46. If $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$, then $\frac{e}{f} \times \_\_\_\_ = \frac{a}{b}$. Fill in the blank.

47. If $m \div 5 = \frac{1}{21}$, find *m*.

48. Draw a picture or use a number line to represent $4 \div \frac{1}{3}$

5.NF.B.7.B

49. $m \div \frac{1}{4} = 12$. Find *m*.

50. $3 \div n = 15$. Find *n*.

51. Explain the difference between a fraction, a ratio, and a proportion.

52. In your own words, what does *a*:*b* mean?

6.RP.A.1

53. The ratio of men to women is 4:5. If 12 more men join the group, how many women must also join to keep the ratio constant?

54. If *a*:*b*, then $\frac{a}{c} : b \times ?$

55. If the ratio of students in a class is 3 boys to 4 girls, which of the following class sizes are possible: 12, 14, 16, 18, 24, 28?

56. If three junks cost $15, how much would five junks cost?

6.RP.A.2

57. The table depicts a constant ratio between the left and right columns. Determine the value of *a* and *b*.

2	24
3	*a*
6	72
b	84

58. If *x* marbles cost *y*, how much does 7 marbles cost?

6.RP.A.2

59. Write the following as percents: $\frac{20}{50}$; $\frac{4}{10}$; $\frac{56}{140}$; 0.017

6RP.A.3.C

60. What percent of 7 is 8?

61. What is 38% of 25?

6RP.A.3.C

62. Convert 600 miles per hour to miles per second.

(6RP.A.3.D)

63. How many grams are in 8 pounds given that 1 gram = 0.035oz and there are 16oz in a pound?

64. If Sam can read 1/3 of a book in 1/12 of a day, how many books can be read in one day?

7.RP.A.1

65. If the table represents a proportional relationship, determine the constant of proportionality. If not, change a value so that the table would represent a proportional relationship.

3	21
6	42
8	56
12	74
13	91

(7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.C, 7.RP.A.2.D, 7.RP.A.3)

Appendix 2

	Grade 3 Standards		**Grade 4 Standards**		**Grade 5 Standards**	
	Number of questions:	Percent of questions:	Number of questions:	Percent of questions:	Number of questions:	Percent of questions:
Correct	357	34.00%	272	19.09%	91	6.74%
Unanticipated	1	0.10%	12	0.84%	41	3.04%
Partial	163	15.52%	21	1.47%	21	1.56%
Incorrect	224	21.33%	375	26.32%	273	20.22%
Incomplete	23	2.19%	18	1.26%	15	1.11%
Blank	282	26.86%	727	51.02%	909	67.33%

	Grade 6 Standards		**Grade 7 Standards**		**Total**	
	Number of questions:	Percent of questions:	Number of questions:	Percent of questions:	Number of questions:	Percent of questions:
Correct	138	15.33%	42	28.00%	900	18.46%
Unanticipated	0	0.00%	0	0.00%	54	1.11%
Partial	71	7.89%	16	10.67%	292	5.99%
Incorrect	177	19.67%	15	10.00%	1064	21.83%
Incomplete	15	1.67%	2	1.33%	73	1.50%
Blank	499	55.44%	75	50.00%	2492	51.12%