PROMOTING NUMERACY IN AN ONLINE COLLEGE ALGEBRA COURSE THROUGH PROJECTS AND DISCUSSIONS

A Thesis
by
SAMUEL LUKE TUNSTALL

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APPROVED BY:

Michael Bossé
Chairperson, Thesis Committee

Katrina Palmer
Member, Thesis Committee

Holly Hirst
Member, Thesis Committee

Tracy Goodson-Espy
Member, Thesis Committee

Mark Ginn
Chairperson, Department of Mathematical Sciences

Max Poole
Dean, Cratis D. Williams Graduate School

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Abstract

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Samuel Luke Tunstall
B.A., University of North Carolina at Chapel Hill
M.A., Appalachian State University

Chairperson: Michael Bossé

This research stems from efforts to infuse quantitative literacy (QL) in an online version of college algebra. College algebra fulfills a QL requirement at many universities and is a terminal math course for most who take it. In light of the course's traditional content and teaching methods, students often leave with little gained in QL. An online platform provides a unique means of engaging students in quantitative discussions and research, yet little research exists on online courses in the context of QL. The course studied included weekly news discussions as well as "messy" projects requiring data analysis. Students in online and face-to-face sections of the course took the QLRA (developed by members of the National Numeracy Network) and responded to an open-ended prompt regarding their mathematical disposition during the first and final weeks of the fall 2014 semester. There were significant statistical gains in the online students' QLRA performance and mathematical affect, compared to none for the face-to-face students. Qualitative analysis of the open-ended responses reveals drastic differences in the two types of students' attitudes as well as their outlook on the utility of mathematics, with the online section having the more favorable outcomes. Implications include that project-based learning in an online environment is a promising strategy for fostering QL in terminal math courses. In addition, the study also highlights the need for more accountability of QL outcomes in departments offering courses such as college algebra.

Foreword

Chapter 2 of this thesis has been submitted to *Numeracy*, an open-access, peer-reviewed online journal supported by The University of South Florida Libraries and published by The National Numeracy Network. In accordance with the requirements of *Numeracy*, it is formatted according to the Chicago Manual of Style. Chapter 3 of this thesis has been submitted to the *International Journal for Mathematics Teaching and Learning* (IJMTL), an open-access, peer-reviewed online journal hosted by the Centre for Innovation in Mathematics Teaching at Plymouth University, UK. In accordance with the journal's requirements, the article is formatted according to the American Psychological Association's (APA) style manual. The references listed at the end of the thesis are in APA format. Note that I am the primary author for both papers and Dr. Michael Bossé is a co-author on each. I taught and designed the online course of study, collected the data, and wrote the individual papers. Dr. Bossé provided support with grammar and coherence, the methodology sections of both papers, as well as with conducting the qualitative analysis found in the IJMTL article.

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Chapter 1 – Introduction

The twenty-first century is entrenched in numbers. Indeed each day, individuals of all ages are confronted by quantitative arguments from a variety of sources, whether in school, television, social media, or the doctor's office. One's ability to understand elementary mathematics in context called quantitative literacy (QL), and is the focal point of this research (SIGMAA, 2014). As additional components of QL, we include one's disposition towards using math in daily life, as well as one's ability to communicate numerical arguments, with the former in line with Wilkin's (2010) framework, and the latter recommended by Lutsky (2008), Hughes-Hallett (2003), and Wiggins (2003). Moreover, note QL is also often called quantitative reasoning (QR) or numeracy, so these are used interchangeably in this research (Burkhardt, 2007). As discussed in Chapters 2 and 3, numeracy is linked with many indicators of one's long-term welfare; notwithstanding, a review of the literature suggests that mathematics courses are typically taught in a manner that does not foster gains in QL. A postsecondary course particularly complicit in this is college algebra, as its traditional content and teaching methods are inherently at odds with that of QL, despite its placement within many general education curricula to fulfill the college's QR requirement. In light of this, the driving question for this research is the following: What are ways that one can modify a college algebra course so that students – many of whom will take no future math courses – leave with gains in quantitative literacy?

One approach to this challenge is to have students take college algebra in an online, problem-based learning environment. Such a strategy may seem counterintuitive at first glance; however, a review of the literature suggests that the online delivery medium has properties that may foster QL. The relation of QL to project-based learning in an online environment has not been studied as of this writing, and this thesis explores this notion with a preliminary study. Specifically, the research question in this study is the following:

> *Does college algebra, when taught online with tests replaced by real-world projects and discussion forums, bring about better QL outcomes than when taught in a traditional face-to-face, lecture-based manner?*

With this in mind, this study employed a convenience sample of 57 students taking college algebra at a mid-sized southeastern US university. Twenty-eight of the students took an online college algebra course, while the rest took a face-to-face version. Both sections of the course used the same textbook and covered the same material, with the exception that the online course did not cover two sections on the conceptual notion of a derivative. The face-to-face course used tests, quizzes, and book problems as the primary means of assessment. The online course used four contextualized projects (in lieu of exams), weekly quizzes with traditional college algebra problems, and discussion forums that highlighted the week's content in real-world settings. As a whole, students in the online course spent slightly more time on writing and discussing applications of the math content, and slightly less on computational homework problems. To answer the research question, students in both sections took the Quantitative Literacy and Reasoning Assessment (Gaze et al., 2014), a 20-question measure of students' contextualized math skills, at the beginning and end of the semester. The QLRA also includes five Likert-scale questions regarding students' dispositions toward mathematics. As a supplement to the QLRA, students answered the following open-ended survey question: *It has been said that "the world is awash with numbers." Do you use math in your daily life, or do you avoid doing calculations?* The analysis of the 20-question content-portion and Likert-scale sections of the QLRA includes separate one-sample *t*-tests for both courses. The qualitative analysis of the open-ended question incorporates a coding scheme developed by the researchers. Note that no part of this research measures differences in college algebra content growth between the sections; although such results would be enlightening, timing inhibited its inclusion within the study.

Quantitative analysis of the QLRA yields that there were statistically significant gains in the online students' QLRA performance and mathematical dispositions, but none for the face-to-face students. Chapter 2 of this thesis is an article submitted to the journal *Numeracy*, and contains a review of the literature and in-depth summary of this finding. Chapter 3 contains an article submitted to the *International Journal for Mathematics Teaching and Learning* (IJMTL) and contains the qualitative analysis of the open-ended question. The results from this analysis yield a clear divide between the classes, with the lecture-based course having the majority of students answer in a similar “Yes, I use numbers everyday…” fashion, suggesting arithmetic is the only math one needs, if any at all. On the other hand, many of the online students were able to articulate how they used numbers in their daily lives, stating that they are now more self-conscious of math’s presence and utility. The results presented in each article suggest that problem-based learning in an online environment has a significant potential to bring about numeracy gains among college algebra students. This research also bolsters the notion that online courses can provide experiences just as rigorous and meaningful as those of many face-to-face courses. Finally, a broad implication of this study is that mathematics instructors should pay close attention to the types of instruction and assessment methods they employ.

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Chapter 2 – Article Submitted to *Numeracy*

Promoting Numeracy in an Online College Algebra Course through Projects and Discussions

Abstract

This research stems from efforts to infuse quantitative literacy (QL) in an online version of college algebra. College algebra fulfills a QL requirement at many universities, and is a terminal course for most who take it. In light of the course's traditional content and teaching methods, students often leave with little gained in QL. An online platform provides a unique means of engaging students in quantitative discussions and research, yet little research exists on online courses in the context of QL. The course studied included weekly news discussions as well as "messy" projects requiring data analysis. Students in online and face-to-face sections of the course took the QLRA (developed by members of the National Numeracy Network) during the first and final weeks of the fall 2014 semester. There were significant statistical gains in the online students' QLRA performance and mathematical affect but none for the face-to-face students. Notwithstanding limitations of the study, the results support the notion that project-based learning in an online environment is a promising strategy for fostering QL in terminal math courses.

Key words: quantitative literacy, college algebra, online courses, general education

Introduction

Numbers matter. In this information age, they inundate our lives in ways beyond what many postsecondary college students would fathom. Conveying this nuanced saturation to students who have traditionally eschewed mathematics is a nontrivial and important challenge. Indeed, such students' numeracy, or ability to work with elementary mathematics in day-to-day life, is at stake and has an impact that permeates their lives in meaningful ways. Efforts to meet this challenge for students in first-year terminal math courses have been scant and problematic. In particular, college algebra courses appear to suffer from lackadaisical teaching methods that do not effect quantitative literacy.

This paper presents an exploratory study on the efficacy of problem-based learning in an online environment to promote numeracy for college algebra students. Online college courses are a growing expectation among college students; they facilitate discussion among students, allow for student choice in assignments, and foster students' research and writing skills. This cadre of benefits aligns with those of a problem-based learning environment, thus motivating a study on the effect of both in tandem. While research-based, this approach to foster numeracy for students has not been studied among college algebra students. Such research is necessary so that students in terminal college math courses develop the skills they need to navigate a life filled with numbers. In this paper we present the promising results of this study and provide suggestions for future directions in research.

Background and Framework for Quantitative Literacy

In *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, Colwell (2003) provides an apt analogy for situating numeracy:

> We can look at numeracy through the metaphor of an analog clock. Some people only need to know how to read the face to accomplish their daily goals. Some need to know that beneath the front of a complex system of gears tracks the progression of time. Others need to be able to take the existing clock and innovate, to build the next generation of timekeeping devices. (p. 245)

Data from the *Digest of Education Statistics* sheds light on to whom Colwell is referring. Of the 1,791,046 bachelor's degrees conferred nationwide in 2012, 18,842 were in mathematics or statistics and 145,924 were in engineering or computer science (Snyder and Dillow 2013). In the analogy of the clock (being mathematics), this means that roughly 8% of students are preparing to use the clock's complexities to tackle problems in engineering and computer science. About 1% are training to innovate and make new clocks – to be mathematicians or statisticians. On the other hand, 91% of students simply need the clock to navigate their daily lives.

Colwell uses the word numeracy to describe the notion of using mathematics to navigate the daily demands of life. However, many have also referred to numeracy as quantitative literacy (QL) or quantitative reasoning (QR) (Burkhardt 2007). As Vacher (2014) notes, these terms are often used synonymously but with nuanced connotations. Notwithstanding, when most of the individuals using the term are interested in effecting QL rather than etymology, it is neither productive nor urgent to develop a precise definition (Bass 2003). Instead, the urgency lies in the task of preparing students to deal with a world "awash with numbers" (Steen 2001, p. 1). Having said this, it is still necessary to have a working framework for QL. In this study, QL, QR, and numeracy are treated synonymously, through the definition of the charter of the SIGMAA on QL (2004):

> Quantitative literacy (QL) can be described as the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual's private, civic, and work life. Like reading and writing literacy, quantitative literacy is a habit of mind that is best formed by exposure in many contexts.

Additional elements in the researcher's framework for QL include communication and mathematical affect. While these are not explicit in the above definition, they are as important as mathematical ability. Wilkins (2010) developed a "multifaceted construct" for QL encompassing ability, self-efficacy, and one's beliefs about the utility of mathematics. Moreover, Lutsky (2008), Hallet (2003), and Wiggins (2003) argue for the importance of writing and argumentation in framing QL. Funneling these elements into one, this study views numeracy as that described in the charter along with the elements of communication and mathematical disposition. With a working definition in hand, it is instructive to describe its beginnings.

While the underpinnings of the QL movement developed in the late twentieth century, the genesis of the aforementioned urgency is found in the seminal text *Mathematics and Democracy*, published by the National Council on Education and the Disciplines under the leadership of former MAA president Lynn Steen (2001). Steen's focal message was simple. Every day, students are inundated with numbers concerning politics, crime, healthcare, the economy, and many other issues; their ability to accurately interpret these quantitative arguments – and even produce their own – is pivotal to their welfare and ability to contribute to the nation's democracy (Steen 2001). From this text arose a proliferation of research and activity on college campuses. Indeed, contemporaneously, the National Numeracy Network (NNN) formed with a mission of promoting QL at all levels (Madison and Steen 2008). Since 2004, the NNN has held annual meetings and now also publishes a journal entitled *Numeracy* twice annually. In a similar vein as the NNN, in 2004 the MAA formed a SIGMAA on QL, meeting annually at the Joint Math Meetings. The MAA also released major publications in 2003, 2004, 2006, and 2008, all of which echo the call for increased attention to quantitative literacy at both the secondary and postsecondary levels (Gillman 2006; Madison and Steen 2003, 2008; Steen 2004).

Motivation for Numeracy

In light of the meetings and work, one still begs the question: why all the fuss? For those interested in undergraduate mathematics education, key reasons include the link between QL ability and one's overall well-being, the tenuous status of first-year postsecondary math courses, and a growing movement for accountability in general education outcomes. These components are briefly addressed here.

While well-being is a subjective term, socioeconomic status is certainly a major contributor to it. Researchers have found that mathematical ability is connected to both wage increases and likelihood of fulltime employment (Eide and Grogger 1995; Levy et al. 1995; Rivera-Batiz 1992). In addition, researchers have linked numeracy with better decision-making (Jasper et al. 2013; Peters et al. 2006), nutrition label understanding (Rothman et al. 2006), as well as with better risk comprehension in healthcare (Fagerlin et al. 2007; Lipkus and Peters 2009). Taken together, it is clear that numeracy is linked with more positive life outcomes.

In light of numeracy's potential impact on long-term well-being, it is understandable that educators would strive to develop it in students; however, many argue that the current sequence of math courses in secondary education – geometry, algebra, trigonometry, and calculus (GATC) – is not conducive to effecting a numerate citizenry (Steen 2001, p. 4). According to Madison (2003), the GATC sequence sifts "through millions of students to produce thousands of mathematicians, scientists, and engineers" (154). That is, while intuition might suggest that courses in the GATC sequence develop QL in students, the reality is far different. The reason for this quandary is that mathematical and quantitative literacy are not equivalent (Steen 2001). Mathematical literacy provides a firm foundation for QL; however, it does not directly cause it, and it would be misleading to claim there is a simple relationship between the two (Madison 2003). As a simple example, one many be able to solve a quadratic equation, but have no idea of how to interpret a percentage in a news article; this student exhibits mathematical literacy but not QL. Conversely, another individual may be able to interpret the percentage and discuss the author's argument, but not solve the quadratic equation. Indeed, numeracy and mathematical literacy are not the same.

The implications of this are important. Hughes-Hallett (2003) suggests that though the foundations of QL are laid in middle-school, it is the *responsibility* of high-school and college faculty to cultivate this knowledge as students develop. As such, universities must carefully examine the courses they designate to fulfill general education requirements. If a student will not take a course past college algebra – and this course fulfills a general education requirement – the course must have some benefit aside from preparing a student for future math courses. As noted by Kennedy (2001) in a commentary aptly entitled “The Emperor’s Vanishing Clothes,” departments can no longer hide from accountability by claiming that the material’s relevancy manifests later on, thus leaving behind those who will not go past calculus; rather, they must accept the responsibility of producing both mathematicians and numerate individuals in all disciplines. Math departments should be held accountable for the QL (or lack thereof) of all students who earn a degree. A reason for calling attention to this responsibility is that college algebra – a course often taken by students who will not take calculus – has a particularly nasty reputation for its complicity in failing to foster QL. Madison (2003) laments the traditional college algebra course, noting:

> Students, many of whom have seen this material in prior algebra courses, struggle to master the techniques; three of four never use these skills and many of the rest find that they have forgotten the techniques by the time they are needed in later courses. (p. 155)

Steen (2004) describes the course’s lack of relevancy:

> Focused entirely on a wide range of relatively specialized algebraic techniques that students rarely remember beyond the final exam, college algebra neither prepares students well for courses in other quantitative disciplines nor their civic employment, or personal needs. (p. 38)

Seeking rectification, in *A Fresh Start for Collegiate Mathematics: Rethinking the Courses below Calculus*, Small (2006) suggests that an improved college algebra course has little lecture and instead a considerable number of small-group activities and projects; it focuses on real-world, ill-defined modeling rather than traditional word problems; it contains a strong focus on communication and has little traditional assessment. While few would disagree with Small's vision, unfortunately, college algebra at most US universities does not fulfill his expectations.

Data from a 2010 survey conducted by the American Mathematical Society (AMS) yields that 29% of all non-remedial intro-level math enrollment is in college algebra (Blair et al. 2010); as college algebra tops both precalculus and other liberal arts math courses for the introductory course students take, it is vital that the course have worth for students. Data from the AMS survey reveal that, of the undergraduate programs surveyed, only 16% of college algebra sections required writing assignments, and 65% used a "traditional" lecture-based approach assessed through tests and quizzes, meaning the course content and delivery methods were essentially the same as those in 1990. Additionally, many of these college algebra courses are instructed by graduate students. This is likely not the "fresh start" Small had in mind.

While college algebra is the focal course for this research, it is instructive to note that math for the liberal arts – designed as an alternative to precalculus or college algebra – has not been a panacea for all math departments. Though it formed to fulfill QL requirements, Ganter (2012) notes that most often these classes cover a broad survey of math topics or simply those that faculty desire to teach; such courses are rarely designed with the intent of developing students' quantitative reasoning ability. Richardson and McCallum (2003) use an apt analogy, suggesting that such courses teach one to appreciate a work of art (mathematics), rather than produce the art for oneself. In sum, it is clear that courses designated to fulfill QL requirements are not sufficiently changing, and a viable solution has yet to manifest.

Fostering QL in a Problem-based, Online Learning Environment

A purpose of this research is to test an adaptation of Small's vision (Smal 2006). Indeed, if college algebra is to remain so prominent in undergraduate math programs, it is essential to make every effort to fulfill its promise of being a course with QL designation. A novel yet promising approach for this task is placing college algebra online with a problem-based learning (PBL) structure. While this approach may initially seem strange, a digital learning environment actually possesses many qualities conducive to the task. In light of the fact that 33.5% of students in higher-education institutions enrolled in at least one online course in 2012 (up from 18% in 2005), the availability of online courses is becoming an expectation among college students (Allen and Seaman 2014); common factors for making courses available online include students' demand for flexible schedules, making more courses available, and increasing student enrollment (Jaggars 2012; Parsad and Lewis 2008). These factors make it that clear research of this nature is in order. We now turn to describing PBL and the characteristics of online learning.

Problem-based learning is broadly characterized as an active learning approach where students develop content knowledge and other skills through self-directed or group problem-solving (Strobel and Van Barnevald 2009). A traditional approach to teaching college mathematics – which would involve lecture followed by assessment through tests and quizzes – would not be considered PBL. In a meta-analysis of face-to-face PBL research, analysts found that it was superior compared to the traditional lecture approach in the areas of long-term content retention, skill development, and satisfaction of students and teachers. Altogether, PBL aligns with the recommendations of numeracy experts. Indeed, according to Ganter (2012), “Teaching methods for quantitative literacy courses are not lecture and listen, but they may involve group work, projects, writing, and many of the approaches advocated by those in the calculus reform movement” (p. 8). A host of others agree. Among them, Packer (2003) notes:

> The short answer is that typical tests are mere proxies for real performance. They amount to sideline drills as opposed to playing the game on the field. Assessment of QL requires challenges that are essentially not well structured or even well defined; problems that are, well, problematic. (p. 127)

Hence, both PBL and QL require that students deal with legitimate problems as part of course assessment. Transferring this approach to the online environment may seem troublesome, but studies have found that PBL works online. In particular, many have found significant increases in measures of students’ critical-thinking skills upon taking an online PBL course (Cheaney and Ingebritsen 2005; Sendag and Odabasi 2009). Moreover, a meta-analysis suggests that discussion forums are powerful tools to increase student interaction and achievement (Blackmon 2012). Notwithstanding, such studies backing online PBL are not sufficient to galvanize the creation of an online college algebra course. Online learning is clearly distinct from its face-to-face counterpart, so a delineation of this distinction is in order.

Naturally, an initial concern that emerged with the proliferation of online courses is that of its effectiveness when compared with face-to-face courses. Having examined more than 355 studies comparing distance courses (mostly online) to face-to-face counterparts, Russell (1999) released an initial hallmark analysis arguing that there was no significant difference between course delivery mediums. An influx of other analyses followed. As no surprise, some meta-analyses have found that online learning is either no different or superior (Bernard et al. 2004; Caldwell 2006; Means et al. 2009; Zhao et al. 2005), while others have found the opposite (Figlio et al. 2010; Rovai 2002; Tanyel and Griffin 2014). In particular, it has been found that online math courses – especially at the community college level – suffer from low retention rates (Mensch 2010; Xu and Jaggars 2011). Taking a step back, one sees that these studies do not contradict one another; rather, many measure success differently, ranging from student GPA and course retention to student performance on end-of-course exams and sense of classroom community. Nevertheless, it may seem disheartening that there is no definitive answer to the question of the effectiveness of the online environment; on the other hand, this may emphasize that quality instruction is what leads to student learning, regardless of course delivery mode (Oncu and Cakir 2011; Xu and Jaggars 2011). As such, those designing online courses should focus on developing high-quality instructional materials that utilize all the benefits of the online environment (Rovai 2000). Garrison (2003) phrases this succinctly: "Simulating traditional face-to-face classroom methods using asynchronous online learning simply misses the point that we are operating in a new medium with unique properties." On a similar note, Twigg (2001) poses the following question: "Rather than comparing online learning with traditional higher education, how can we identify new models and talk about what is better rather than what is "as good as"? (p. 4)

Fortunately, the online environment is conducive to the constructivist PBL approach discussed earlier (Powers and Dallas 2006). Indeed, despite the fact that students are not sitting in a classroom beside one another, a learning management system (LMS) permits student collaboration (whether through discussion forums or wikis) as well as reflection time during communication (Benson 2003; Chinnappan 2006; Larreamendy-Joerns and Leinhardt 2006; Meyer 2004; Perera-Diltz and Moe 2014; Wegerif 1998). Other best practices suggest that online assessment should be diverse and allow for student choice (Gaytan and McEwen 2007; Gikandi et al. 2011; Robles and Braathen 2002); moreover, it should depart from traditional face-to-face assessment, which often includes high-stakes tests and promotes surface learning (Beebe et al. 2010; Elliott 2008; Garrison 2003; Herron and Wright 2006; Reeves 2000; Rovai 2000; Speck 2002; Vonderwell et al. 2007). We now turn to past studies to provide guidance for assessing QL.

Assessing Numeracy

Over the last decade researchers have performed a variety of small-scale studies to explore the nuances of teaching QL, many of which are found in *Numeracy*; however, as of the timing of this writing, no researcher has examined QL in an online environment. At the 2014 Joint Math Meetings in Baltimore, the SIGMAA in QL focused on assessing numeracy, and a cadre of institutions presented their techniques for assessment. Some universities, such as UMass Boston, Hood College, Colby-Sawyer College, and Central Washington University have designed assessments of their QL requirement (Boersma and Klyve 2014; Dunham 2014; Kilic-Bahi 2014; Mast 2014). The schools' evaluations typically account for some or all of the following elements: (1) communication skills, (2) mathematical disposition, and (3) ability to solve contextualized problems using elementary mathematics. While each institution's program assesses skill (2) referenced above, only those at Hood College and Central Washington University measured all three. Nonetheless, an issue in using any of these models for assessment is that all were designed with the respective college's specific QL goals in mind; none were designed for replication by others. Hence, not only may these be inappropriate for any particular university and set of students, but using one would not further the knowledge base of QL assessment measures. As Scheaffer (2008) notes in the opening issue of *Numeracy*, "QL research must strive for a strong base of systematic, coordinated and cumulative research from the outset" (p. 12).

Fortunately, Eric Gaze (NNN president) has worked with a cross-institutional team of researchers over the past five years to design the Quantitative Literacy and Reasoning Assessment (QLRA); after years of edits for validity and reliability, the QLRA is now available online (Gaze et al. 2014). As of 2014, the QLRA includes 20 multiple-choice questions and five Likert-scale attitudinal questions. It can be used for pre- and post-tests of QL, and more than 25 institutions used it in 2013. It is designed to measure all three skills referenced above, though its assessment of communication is admittedly tenuous. The questions designed to assess communication require the student to simply choose a correct mathematical statement rather than produce their own. The QLRA's mathematical content includes number sense, visual representation, reasoning, and probability and statistics. Notwithstanding its weakness in assessing student writing, the QLRA is an ideal choice for the purposes of this study. It allows this study to investigate the hypothesis discussed in the following section as well as compare students' numeracy in this study with those at other postsecondary institutions. As no research has specifically honed in on QL in an online environment, this study explores that gap and provides an avenue for future research.

Methodology

The structure of our online course includes traditional quizzes, short Geogebra assignments, weekly discussion forums, and four major projects. There are no tests; the discussions and data-driven projects comprise the majority of students' grades. Many of the projects require that the students examine news articles or data of their choice. Moreover, all of the assignments are contextualized in some way and require students to communicate arguments or opinions using mathematics. Three of the news articles that students analyze are taken from *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*. Two QL experts, Bernard Madison and Shannon Dingman, spent several years crafting the text for a course in QR (Dingman and Madison 2010). Students analyze these news articles during weeks where the article's content is pertinent to the course. As a whole, the course has been designed so that students develop skills in mathematical communication, modeling, online research, and Microsoft Excel.

As shown in Figure 1, study participants include a convenience sample of 57 students enrolled in sections of college algebra at a southeastern university in the U.S. Twenty-eight of these students took the online section of the course, while the rest took the course with another instructor in a traditional face-to-face lecture format. The face-to-face instructor used the same textbook and covered roughly the same sections in the text;

his course was lecture-based, and tests comprised the majority of students' grades. Primary differences in the courses included the delivery mode and type of assignments completed; the material was roughly the same, though some sections in the online course were cut out to permit time for student discussions on real-world topics and articles. As a whole, students in the online course spent more time on writing and discussing applications of the math content, and less on computational homework problems and studying for exams.

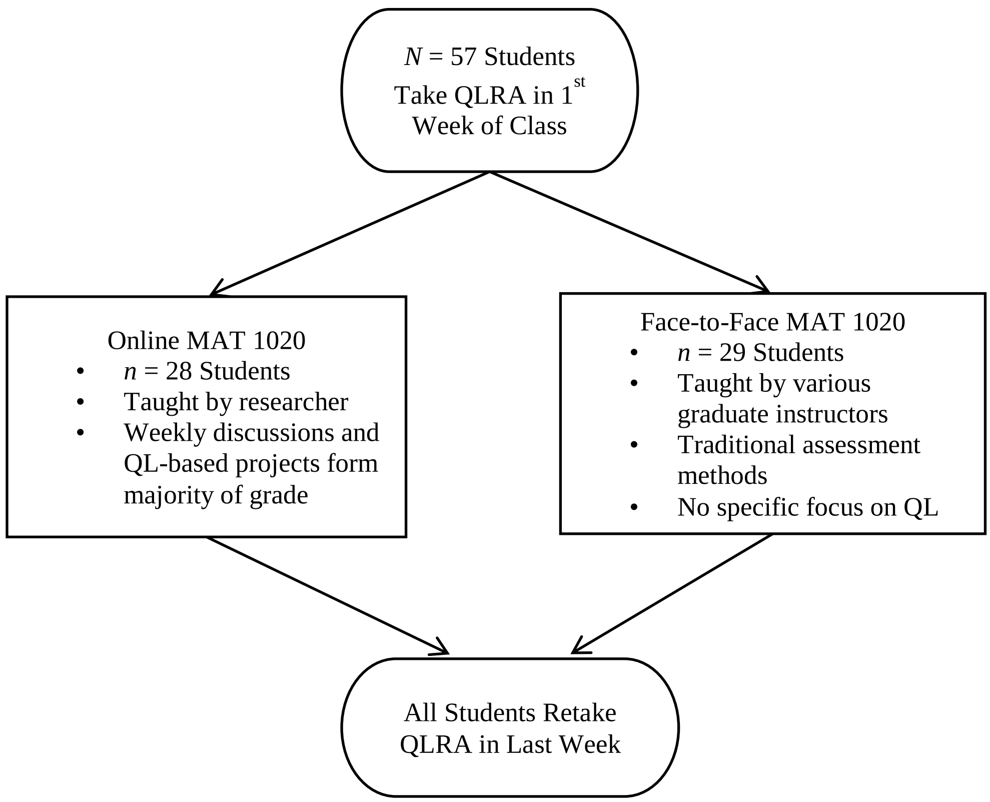

Figure 1. Setup of study

In light of the discussion above, there are two key differences between the course sections under study: delivery format and assessment methods. The hypothesis in this study is that students in the online course with the alternative assessment measures will have greater gains in QL. Both variables are key within the intervention. We posit that if the course were simply delivered online with the same means of assessment as the face-to-face course, then we would not see differences in QL among the sections; however, a future study would need to examine this. In addition, having students research applications of math content for discussion is more easily done in an online course; as referenced earlier, forums permit students reflection time, an important component in the development of critical-thinking skills. Moreover, much of the student work for the alternative assessments involves analyzing data and researching its importance; there is no need for an instructor to be present while students do this. Indeed, as mentioned earlier, since PBL is student-driven, the online format embraced this approach. In light of this discussion, the researcher posits that if the course with alternative assessment were delivered in the face-to-face format, one would see mild gains in QL; however, they would not be as great as those potentially demonstrated by the online course with alternative assessment. Again, a future study would need to examine this. The purpose of the present study is to examine the efficacy of the online PBL environment in comparison to the traditional face-to-face approach. Also, this study measures change in QL and not college algebra ability; there is no attempt to quantify differences in college algebra growth between the sections.

Staying in line with the study's framework for numeracy, students will be successful in QL growth if they improve their (1) communication skills, (2) mathematical disposition, and (3) ability to solve contextualized problems using elementary mathematics. Clearly such a variety of skills cannot be assessed in a simple exam – a mixed-methods approach is in order (Grawe, 2011). Here this approach entails having students take the QLRA and answer an open-ended survey question at the beginning and end of the semester. Also used are comments from the discussion forum as secondary data while recognizing that comparisons are unavailable as the forum was not an option for the face-to-face course. For the online section, students took the QLRA for a minimal participation grade; to encourage students to do their best, students were told that achieving a high score would yield extra credit towards their final grade. The face-to-face students also took the exam at the beginning and end of the semester; however, they received five dollars per exam (funded by an internal grant) rather than course credit.

In analyzing the QLRA scores, we will use one-sample *t*-tests to determine if there were significant gains in each section's overall performance. We consider the 20-question content portion of the QLRA and the Likert portion of the QLRA as separate components in our analysis. Note that the Likert-scale questions are on a scale of 1-5, with 1 representing "Strongly Disagree" and 5 representing "Strongly Agree." For questions 1, 3, and 4, higher responses indicate that one has a positive disposition towards mathematics; for questions 2 and 5, lower scores indicate a positive disposition. In addition to taking the QLRA, all students answered a brief survey question when taking the exam. The questions was as follows: It has been said that "The world is awash with numbers." Do you use math in your daily life, or do you avoid doing calculations? The purpose of this was to gain qualitative data about students' mathematical dispositions that the Likert-scale questions could not capture. Having said this, note that the Likert-scale and survey items assess skill (2) above, while the content questions address skill (3). As all the content questions require students to read carefully, and some even require students to pick the correct statement of mathematics, the assessment of skill (1) is somewhat assessed through these questions; however, communication skills are not fully captured through this approach. As a final source of evidence, comments combed from the online discussion forum throughout the semester are drawn upon. The comments chosen are those deemed notable because they present some facet of QL from a student who did not necessarily perform well in the course as a whole; this is to demonstrate the notion that numeracy and math literacy are not equivalent. Because the online forum was clearly not a part of the face-to-face course, comparisons cannot be made and will be used only as supplementary evidence.

Results

Selected findings from the study are presented below; comments and discussion follow.

Figure 2 demonstrates that (on average) students in both sections improved their QL math content scores upon completing the course. As the students just completed a math course (perhaps not having taken one in years), it is logical that they would recall or gain basic math skills. While the online students began with a higher average than that of the face-to-face section, this difference was not found to be significant. In addition, neither course section made a large jump in the number of questions answered correctly. This confirms the notion that mathematical and quantitative literacy are not equivalent.

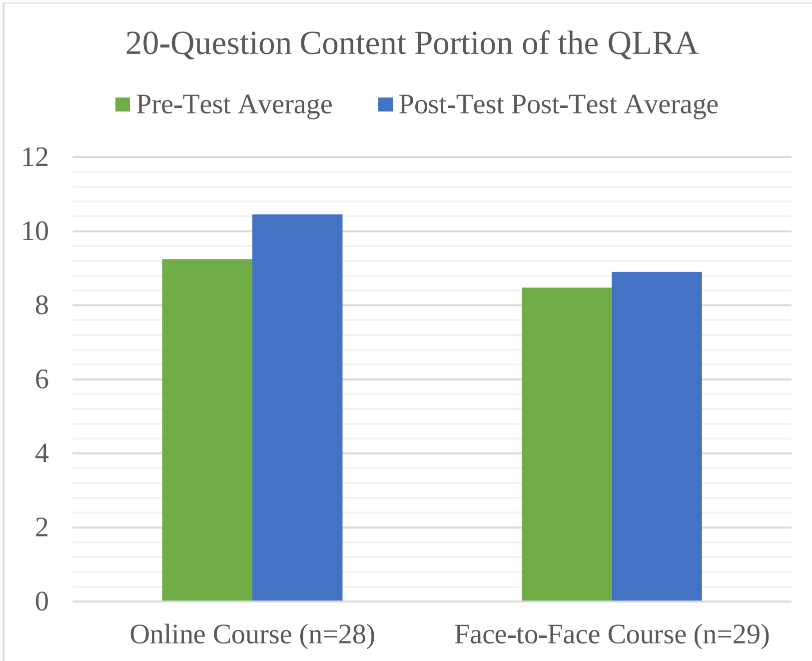

Figure 2. QLRA content-question results

Figure 3 (below) illustrates the changes in mathematical disposition over the course of the semester. Recall that a gain on questions 1, 3, and 4 is considered positive, with the opposite holding for questions 2 and 5. With this in mind, we can see positive gains for the online course in questions 1-4, but a loss on question 5. As that question is not significantly different from the others, it is plausible that the loss is nothing of concern. Furthermore, we see mild gains in affect for the face-to-face section on questions 1, 3, 4, and 5; however, there is a sizeable "loss" on question 2.

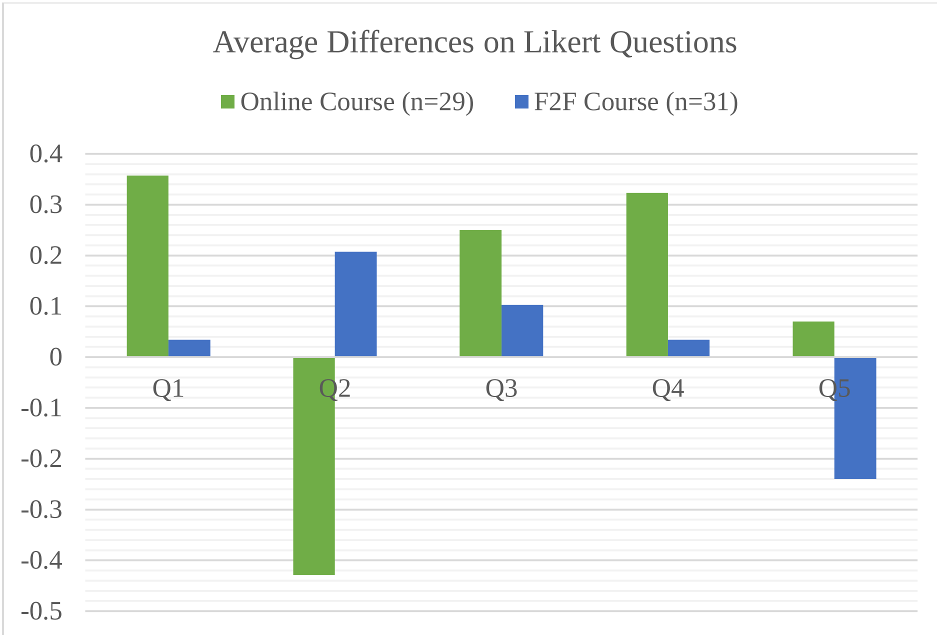

Figure 3. QLRA Likert-scale question results

Table 1. Statistical analysis of QLRA results

	Average Pre-Post Online Difference	*p*-value	Average Pre-Post Face-to-Face Difference	*p*-value
Math-Content Questions	1.214	0.014*	0.414	0.186
Likert Question 1	0.357	0.033*	0.034	0.439
Likert Question 2	-0.429	0.028*	0.207	0.793
Likert Question 3	0.799	0.055	0.103	0.324
Likert Question 4	0.393	0.043*	0.034	0.441
Likert Question 5	0.071	0.635	-0.241	0.128

Note: Significant results at the $p < 0.05$ level are denoted with a *

Table 1 shows that students in the online section made statistically significant gains in the QL math content while those in face-to-face section did not. It also illustrates the same pattern with respect to mathematical disposition through Likert-items 1, 2, and 4. Note the loss in Likert-item 5 for the online course is not significant, thus again confirming the notion that the result is of little concern.

Discussion

As shown in Table 1, students in the online course demonstrated small but significant growth in their QL math content scores. Simultaneously, gains in their disposition towards mathematics, as determined by the scores on the five Likert items, were statistically significant.

However, students in the face-to-face section evidenced no significant gains on the QLRA, with respect to QL content gains or improvement in mathematical disposition, as a result of taking the course. The fact that the online students performed better on the math content is not necessarily surprising. Students should perform better because they had seen the questions before and had just taken a math course; however, this would not explain why the face-to-face students did not perform significantly better. A more plausible explanation is that nearly all of the QLRA questions require students to read carefully and communicate properly; the assessment components of the online course emphasized this. As no aspect of the online course taught to the exam – and the content of the two courses were the same – this study provides promising evidence that the course goal of careful communication is meaningful when put into action. That is, when an instructor includes proper mathematical vocabulary usage as part of assessment and grading, students improve their own ability to distinguish between correct and incorrect mathematical communication. Notwithstanding, it is instructive to note that the QLRA math content is distinct from college algebra content. This study made no comparison regarding gains in course content knowledge; however, as college algebra content is not a component of QL, this is not a cause for concern. In addition, we believe that if both sections were given a common college algebra final exam, the classes would perform similarly, save for a section on piecewise functions and one on derivatives, which the online class did not cover.

Admittedly, students' gains with respect to QLRA content were not as great as had been hoped by the research design. Nevertheless, these results indicate that students can gain content QL in an online PBL environment and that such gains can exceed those attained through traditional face-to-face paradigms. Note that to some extent, this finding may be circular in nature – students improved in QL content knowledge when QL was a central emphasis of the course – nonetheless, it suggests that there is a positive future for numeracy in PBL online courses.

The statistically significant gains on the Likert-scale questions are also encouraging. Questions 1, 2, and 4 may be summarized as one's belief that quantitative information is useful in making decisions in day-to-day life. As discussed earlier, such a positive disposition towards numbers is a significant component of QL. In light of the fact that college algebra is a terminal course for many who take it, one's affect regarding math upon leaving the course has a potential to remain with them. In contrast to the math content, we did make specific efforts to better students' numerical attitudes throughout the semester. Many of the discussion forums and projects had a real-world focus and permitted students to investigate mathematical topics they found interesting. The repercussion of this is then natural: Students began to see math in a more positive light. This desired outcome would not have occurred without such alternative assessments. Students in the face-to-face section completed homework and prepared for quizzes and exams; aside from “word problems” in the text, they did not have the opportunity to find for themselves the applications of what they were learning. In retrospect, we can see this confirmed in the literature.

Indeed, a significant difference between the online PBL course and the traditional face-to-face course was in the amount of mathematically based reading and writing experienced in the courses, with the online course requiring considerably more. Numerous researchers argue that reading is a unique mode of learning essential for mathematics understanding (e.g., Bishop 1988; Borasi et al. 1998; Siegel and Borasi 1992). Writing in mathematics similarly deepens mathematical understanding and extends thinking by sustaining students’ development of reasoning, communication, and connections (e.g., Drake and Amspaugh 1994; Doherty 1996; Grossman et al. 1993; Porter and Massingila 2000; Shepard 1993). Unfortunately, traditional lecture-based math courses insufficiently address the need for greater reading and writing experiences. Adu-Gyamfi, Bossé, and Faulconer (2010) report that most students: associate textbook reading and responding to word problems as reading and writing in mathematics; do not equate reading as a component of mathematical learning; and read as little mathematical text as possible in order to complete homework. Marks and Mousley (1990) contend that meaningful reading and writing experiences are rare in traditional mathematics courses. Since students demonstrate a more positive disposition when they understand concepts and are effectively learning, and they learn more effectively through reading and writing, it is reasonable that the online PBL-based course structure would produce marked gains in QL disposition.

As mentioned earlier, students also responded to an open-ended question when taking the QLRA. The results to this question bolster the disposition findings above. In analyzing the replies from the face-to-face section, most students noted that calculations were important in day-to-day life but that they never found themselves needing "advanced" math; students answered similarly in the post-test. Among the online students, most initially gave a response akin to that of the face-to-face students; however, their post-test responses were distinct and quite detailed. Many of the students referenced specific course assignments – or topics they learned about in their research – as reasons that the world was filled with numbers. A typical response from each section to question (1) is given in Table 2.

It could be argued that students in the online course provided more expansive responses to open-ended prompts because of the continual discussion they participated in as part of the QL-promoting investigations. While this may be the case in part, nevertheless, the responses by these students were quantitatively (length) and qualitatively (substance) different from those of the students in the face-to-face course, even in the cases of students who passed the course with lower grades. Thus, again, the online environment demonstrates potential to promote positive dispositions in QL.

Table 2. Typical responses to an open-ended question

	Pre-Test Response (From the same individual)	Post-Test Response (From the same individual)
Face-to-Face Course	I use them usually with a calculator, but I don't use any sort of formula in everyday life. I probably would not as day to day calculations don't usually require more than a calculator.	I do use them in my everyday life. There are a lot of things that require numbers and to live without making calculations would be very difficult.
Online Course	I do use math often, probably every day. I probably use it more than I realize.	Surely, math is used in everyday life. Whether it is seeing how much longer we can sleep in before missing our bus, to calculating tips, to crunching numbers on performance evaluations, math cannot (and probably should not) be avoided. I think math can certainly be used more if used correctly. For starters, it is much easier to judge news articles as reliable when graphs and stats are understood. It is more understandable if a virus is really growing exponentially or if that word is just used incorrectly.

A final source of promising evidence for the online approach is comments drawn from the online course's discussion forums. Students responded positively to forums on concavity, exponential and logistic growth, and logarithmic functions (among others). A key in designing these forums was relating the topic to news events, famous figures, or ideas students could explore. With concavity students discussed a graph of Diana Nyad's swim from Cuba to Florida; for exponential and logistic growth they researched the ideas of Thomas Malthus and Pierre Verhulst; for the Richter scale they examined recent earthquakes and how the damage described in media articles corresponded to the math of logarithms. Such discussions were not difficult to design but do require legitimate effort on the instructor's part; they promote numeracy because students who see math in the real-world are more apt to view the discipline in a positive light. Sample comments from the forums are included in Table 3. Information about the students' course performance is included to support the notion that it is not simply the high-achieving students who make numerate comments in the forums. Again, these results may speak to the results of learning mathematics through reading and writing, as more greatly emphasized in the online course.

Table 3. Example forum comments from the online course

Student	Forum Topic	Comment	Course Grade
1	The mathematics of the Ebola outbreak	This article isn't good. It makes the claim that Ebola is growing exponentially but never uses math or any proof that it is growing that way. They use WHO as a source but all they claim is that WHO said it was growing exponentially. They need more proof to back up their claims and valuable sources.	B
2	Logarithms and earthquakes	Now that I understand how the Richter scale compares the different earthquakes, it puts it in perspective how powerful these earthquakes are that devastate the West Coast and the rest of the world. To see that an earthquake in Chile can be 158.5 times more powerful then the one in San Francisco is astonishing to see even when the one in San Francisco was still so devastating to the area.	D
3	Concavity and Diana Nyad's swim	The graph is increasing and has the general starting shape of a concave down graph. The average rate of change from one mile marker to the other is not consistent. I can image that swimming 110 miles from Cuba to Florida is not an easy task and her body could not physically swim at a constant rate the whole time. My guess is that she had some extra help along the way. It is just hard for me to wrap around the concept that she was only able to swim about 4 miles in 255 minutes but a miraculous 17.21 miles in 239 minutes. Regardless, I give Nyad credit for even attempting to do this at the age of 64! The graph clearly shows that she traveled the whole distance, but I highly doubt that she did it all by herself.	B

Implications

The results of the study further bolster the notion that teaching methods for QL are not lecture followed by exams and quizzes. Indeed, math instructors have the responsibility of conveying both content and its utility; accomplishing the latter is not done through traditional assessment methods, but rather through student exploration and research. Having an instructor stand in front of a room and give examples of mathematics is perhaps worthwhile; however, should the same instructor also *tell* the students what the application is of such content, or should students search for themselves? If numeracy is a goal for the course, this study suggests the latter approach, whether the class is online or in person. In addition, in light of the fact that accountability for outcomes is of increasing importance at the collegiate level – and QL is a significant part of any university's general education program – teachers have the *responsibility* to answer the calls for abandoning the traditional approach. The 2010 statistic revealing that 65% of college algebra sections employed such a strategy is not acceptable in an age where numeracy is of increasing importance (Blair et al. 2010).

Limitations and Further Directions

The central goal of this exploratory study was to examine the efficacy of an online, problem-based learning environment in promoting QL in a college algebra course. The results of the study confirm the researcher's hypothesis and provide strong preliminary evidence that such an approach is effective. Notwithstanding, due to the logistics of the study, there are limitations to the findings. To begin, the study does not prove causation; Issues include the lack of subject randomization as well as differing course instructors. Moreover, the study does not account for whether the online environment, alternative assessment, or a mixture of both is responsible for gains in students' QL. Future studies should compare all of these variations in order to isolate key factors. Such an environment is likely to remain prominent in both students' lives and as a course delivery medium for the foreseeable future. As such, mathematics instructors must understand the environment's ability to foster numeracy. Indeed, in a world "awash with numbers," we are the lifeguards keeping students afloat.

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Promoting Numeracy in an Online College Algebra Course: A Qualitative Analysis

Abstract

This research stems from efforts to infuse quantitative literacy (QL) in an online version of college algebra. College algebra fulfills the QL requirement of many colleges' general education programs, and is a terminal course for most who take it. In light of the course's traditional content and teaching methods, students often leave with little gained with respect to QL. An online problem-based learning environment provides a unique means of engaging students in quantitative discussions and research, yet little research exists on online courses in the context of QL. This study examined a traditional, lecture-based college algebra course versus an online, QL-focused section; the latter include weekly news discussions as well as "messy" projects requiring data analysis. Students in each section responded to an open-ended prompt regarding their mathematical disposition during the first and final weeks of the fall 2014 semester. Qualitative analysis of the responses reveals drastic differences in students' attitudes as well as their outlook on what utility mathematics has to offer, with the online section having the favorable outcome of each. Implications include that project-based learning in an online environment is a promising strategy for fostering QL in terminal math courses; in addition, the study also highlights the need for more accountability of QL outcomes in departments offering courses such as college algebra.

Key words: quantitative literacy, college algebra, online courses, general education

Introduction

Math matters. College students now live in an age where understanding data is paramount and quantities infiltrate nearly every well-reasoned argument. This nuanced saturation is beyond what most might fathom, and conveying it to students who have traditionally eschewed mathematics is a nontrivial but important endeavor. Students' numeracy – or ability to work with quantities in day-to-day living – is at stake. Numeracy has an impact that pervades lives in significant ways. Unfortunately, attempts to meet this challenge for students in introductory, terminal university math courses are scant and vexed. In particular, the popular course college algebra often suffers from teaching strategies that do not foster quantitative literacy (QL).

In an effort to begin ameliorating this issue, this paper presents a qualitative analysis on the efficacy of problem-based learning in an online environment to promote numeracy for college algebra students. The ability to access online courses is a growing expectation among college students. From a pedagogical standpoint, they facilitate student discussion, allow for choice in assignments, and foster students' research and writing skills. This cadre of benefits aligns with those of a problem-based learning environment, thus motivating a study on the effect of both in tandem. While research-based, this approach to foster numeracy for students has been infrequently studied among college algebra students. Such research is necessary so that students in terminal college math courses develop the skills they need to navigate a life filled with numbers. In this paper we present the promising results of this study, the implications it entails, as well as provide suggestions for future directions in research.

Background and Framework for Quantitative Literacy

The MAA's Special Interest Group (SIGMAA) on Qualitative Literacy (2004) states:

> Quantitative literacy (QL) can be described as the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual's private, civic, and work life. Like reading and writing literacy, quantitative literacy is a habit of mind that is best formed by exposure in many contexts.

Data from the *Digest of Education Statistics* reveal that, of the 1,791,046 bachelor's degrees conferred in the U.S. in 2012, 18,842 were in mathematics or statistics and 145,924 were in engineering or computer science (Snyder & Dillow, 2013). Although Colwell (2003) uses the word numeracy to describe the notion of using mathematics to navigate the daily demands of life, many also use the terms quantitative literacy (QL) or quantitative reasoning (QR) (Burkhardt, 2008). As Vacher (2014) describes, these terms are often used synonymously but with contextual connotations. Notwithstanding, because most individuals using the terms are interested in effecting QL rather than etymology, it is neither productive nor urgent to develop a precise definition (Bass, 2003). Instead, the real urgency is in the task of preparing students to deal with a world "awash with numbers" (Steen, 2001, p.1). In this study, QL, QR, and numeracy are treated synonymously.

In addition to the definition given above by the SIGMAA on QL (2004), elements in our framework for QL include communication and mathematical affect. While communication and mathematical affect are not explicit in the above definition, they are as important as mathematical ability. Indeed, as part of a decade-long effort to frame QL, Wilkins (2000, 2010) developed a "multifaceted construct" encompassing ability, self-efficacy, and one's beliefs about the utility of mathematics. Moreover, Lutsky (2008), Hughes-Hallett (2003), and Wiggins (2003) argue for the importance of writing and argumentation in defining QL. Channeling these elements into one, this study views numeracy as that described in the charter along with the elements of communication and mathematical disposition. Now, with a working scheme for numeracy in hand, it is instructive to delineate its emergence over the past decades.

While the underpinnings of the QL movement developed in the late twentieth century, the genesis of its urgent promotion is found in the seminal text *Mathematics and Democracy*, published by the National Council on Education and the Disciplines under the leadership of former MAA president Lynn Steen (2001). Steen's message was simple. Every day, students are inundated with numbers in relation to healthcare, crime, nutrition, the economy, politics, and many other issues; their ability to accurately interpret these quantitative arguments – and even produce their own – is not only important for their welfare, but also requisite for their ability to contribute to the nation's democracy. Following the text's call to attention, a proliferation of research and activity manifest on college campuses. Indeed, contemporaneously, the National Numeracy Network (NNN) formed with a mission of promoting QL at all levels (Madison & Steen, 2008). As of 2004, the NNN holds annual meetings and biannually publishes a journal entitled *Numeracy*. In a similar vein as the NNN, in 2004 the MAA formed a special interest group on QL that meets each year at the Joint Math Meetings. The association released major publications in 2003, 2004, 2006, and 2008, each echoing the call for increased attention to quantitative literacy at both the secondary and postsecondary levels.

Motivation for Numeracy

In light of the meetings and work, one still begs the question: why is there still concern? With an eye towards undergraduate mathematics education, key reasons include the link between QL ability and one's overall well-being, the dubious status of first-year postsecondary math courses, and a growing movement for accountability in general education outcomes. These components are briefly addressed here.

While well-being is inherently subjective in nature, socioeconomic status is certainly a major contributor to it. With this in mind, researchers have found that mathematical achievement is connected to both wage increases and likelihood of fulltime employment (Eide & Grogger, 1995; Levy, Murnane, & Willett, 1995; Rivera-Batiz, 1992). In addition, similar research has linked numeracy with better decision-making (Jasper, Bhattacharya, Levin, Jones, & Bossard, 2013; Dickert et al., 2006), nutrition label understanding (Rothman et al., 2006), and even risk comprehension in healthcare (Fagerlin, Ubel, Smith, & Zikmund-Fisher, 2007; Lipkus & Peters, 2009). Altogether then, it is clear that numeracy is linked with more positive life outcomes.

In light of numeracy's impact on one's long-term well-being, it becomes apparent that educators should strive to develop it in students; however, many argue that the current sequence of math courses (pre-common core) in secondary education – geometry, algebra, trigonometry, and calculus (GATC) – is not conducive to effecting a numerate citizenry. To Madison (2003), the GATC sequence sifts "through millions of students to produce thousands of mathematicians, scientists, and engineers" (p. 154). That is, while intuition might suggest that courses in the GATC sequence develop QL in students, the reality is far different. The reason for this quandary is that mathematical and quantitative literacy are not equivalent. Mathematical literacy provides a firm foundation for QL; however, it does not directly cause it, and it would be misleading to claim there is a simple relationship between the two (Madison & Steen, 2008). As a simple example, a student many be able to solve a quadratic equation, but have no idea of how to interpret a percentage in a news article; such a student exhibits mathematical literacy but not QL. Conversely, another student may be able to interpret the percentage and discuss the author's argument, but not solve the quadratic equation.

The implications of this are important. Hughes-Hallett (2003) suggests that although the foundations of QL are laid in middle-school, it is incumbent of high-school and college faculty to foster this knowledge as students develop in their abilities to communicate and think critically. As such, universities must carefully examine the courses they designate to fulfill general education requirements for numeracy. It is not correct to assume that a student having gone through the aforementioned GATC sequence will be quantitatively literate. If students' majors do not dictate they take a course past college algebra (as is the case in many non-STEM disciplines), and college algebra fulfills their general education requirement, the course must have some benefit aside from preparing a student for future math courses. As noted by Kennedy (2001), departments can no longer eschew accountability by claiming that the material's relevancy manifests later on, thus leaving behind those who will not go past calculus; rather, they must accept the responsibility of producing both mathematicians and numerate individuals in all disciplines. Departments should be held accountable for the QL (or lack thereof) of all students who earn a degree. A reason for calling attention to this responsibility is that college algebra – a course often taken by students who will not take calculus – has gained particular notoriety for its complicity in failing to foster QL. Steen (2004) describes the course's lack of relevancy to students:

> Focused entirely on a wide range of relatively specialized algebraic techniques that students rarely remember beyond the final exam, college algebra neither prepares students well for courses in other quantitative disciplines nor their civic employment, or personal needs. (p. 38)

Seeking reform, Small (2006) suggests that an improved college algebra course replaces lecture with a considerable number of small-group activities and projects; it focuses on real-world, ill-defined modeling rather than traditional word problems; it contains a strong focus on communication and has little traditional assessment. While few would disagree with Small's vision, unfortunately, college algebra at most US universities does not fulfill his expectations.

Data from a 2010 survey conducted by the American Mathematical Society (AMS) yields that 29% of all non-remedial intro-level math enrollment is in college algebra (Blair, Kirkman, & Maxwell, 2010); as college algebra tops both precalculus and other liberal arts math courses for the introductory course students take, it is vital that the course have worth for students. Data from the AMS survey reveals that, of the undergraduate programs surveyed, only 16% of college algebra sections required writing assignments, and 65% used a "traditional" lecture-based approach assessed through tests and quizzes, meaning the course content and delivery methods were essentially the same as those in 1990. Additionally, many of these college algebra courses are instructed by graduate students.

While college algebra is the focal course for this research, it is instructive to note that math for the liberal arts – designed as an alternative to precalculus or college algebra – has not been a panacea for all math departments. Though such courses formed to fulfill QL requirements, Ganter (2012) notes that most often these classes cover a broad survey of math topics or simply those that faculty desire to teach; they are rarely designed with the intent of developing students' quantitative reasoning ability. Richardson and McCallum (2003) use an apt analogy, suggesting that such courses teach one to appreciate a work of art (mathematics), rather than produce the art for oneself. In sum, it is clear that courses designated to fulfill QL requirements are not sufficiently changing, and a viable solution has yet to manifest.

Fostering QL in a Problem-based, Online Learning Environment

A purpose of this research is to test an adaptation of Small's vision. Indeed, if college algebra is to remain prominent in undergraduate math programs (which appears to be the case for the foreseeable future), it is essential to make every effort to fulfill its promise of being a course with QL designation. A novel yet promising solution is placing college algebra online with a problem-based learning (PBL) structure. While this approach may initially seem strange, a digital learning environment actually possesses many qualities conducive to the task. These are discussed below. Roughly 34% of students in higher-education institutions enrolled in at least one online course in 2012 (up from 18% in 2005), so such research is of importance for a variety of reasons (Allen & Seaman, 2014).

Indeed, the availability of online courses is a budding expectation among college students; common factors for this growth include students' demand for flexible schedules, making more courses available, and increasing student enrollment (Jaggars, 2012; Parsad & Lewis, 2008). We now turn to describing PBL, online learning, and the fit between the two.

Problem-based learning has roots in constructivism and is broadly characterized as an active learning approach where students develop content knowledge and other skills through self-directed or group problem-solving (Strobel & Van Barnevald, 2009). A traditional approach to teaching college mathematics – which would involve lecture followed by assessment through tests and quizzes – would not be considered PBL. In a meta-analysis of face-to-face PBL research, analysts found that PBL was superior compared to the traditional lecture approach in the areas of long-term content retention, skill development, and satisfaction of students and teachers. Altogether, PBL aligns with the recommendations of numeracy experts. Indeed, according to Ganter (2012), "Teaching methods for quantitative literacy courses are not lecture and listen, but they may involve group work, projects, writing, and many of the approaches advocated by those in the calculus reform movement" (p. 8). A host of others agree. Among them, Packer (2003) notes:

> The short answer is that typical tests are mere proxies for real performance. They amount to sideline drills as opposed to playing the game on the field. Assessment of QL requires challenges that are essentially not well structured or even well defined; problems that are, well, problematic. (p. 127)

As such, both PBL and QL require that students deal with meaningful mathematical tasks as part of course assessment. Transferring this approach to the online environment may seem troublesome, but studies have found that PBL works well online. In particular, many have found significant increases in measures of students' critical-thinking skills upon taking an online PBL course (Cheaney & Ingebritsen, 2005; Sendag & Odabasi, 2009). Moreover, a meta-analysis suggests that discussion forums are powerful tools to increase student interaction and achievement (Blackmon, 2012). Notwithstanding, such studies backing online PBL are not sufficient to galvanize the creation of an online college algebra course. Indeed, the aforementioned studies were not focused on mathematics courses, so there is certainly no guarantee that the approach would also work with college algebra. Moreover, online learning is clearly distinct from its face-to-face counterpart, so a delineation of this distinction is in order.

To begin, an initial (and perhaps natural) concern that emerged with the proliferation of online courses is that of its effectiveness when compared with face-to-face courses. Having examined more than 355 studies comparing distance courses (mostly online) to face-to-face counterparts, Russell (1999) released an initial hallmark analysis arguing that there was no significant difference between course delivery mediums in respect to student learning and success. A proliferation of other analyses emerged soon thereafter. As no surprise, some meta-analyses have found that online learning is no different or superior (Bernard et al., 2004; Caldwell, 2006; Means, Toyoma, Murphy, Bakia, & Jones, 2009; Zhao, Lei, Yan, Lai, & Tan, 2005), while others have found the opposite (Figlio, Rush, & Lu, 2010; Rovai, 2002; Tanyel & Griffin, 2014). In particular, it has been found that online math courses – especially at the community college level – suffer from low passing rates (Xu & Jaggars, 2011). Stepping back, one sees that these studies do not contradict one another; rather, many measure success differently, ranging from student GPA and course retention to student performance on end-of-course exams and sense of classroom community. Nevertheless, it may seem disheartening that there is no definitive answer to the question of medium effectiveness; on the other hand, this may emphasize that high-quality instruction is what leads to student learning, regardless of course delivery mode (Oncu & Cakir, 2011; Xu & Jaggars, 2011). As such, those designing online courses should focus on developing instructional materials that utilize all the benefits of the online environment (Rovai, 2000). Garrison (2003) phrases this succinctly: "simulating traditional face-to-face classroom methods using asynchronous online learning simply misses the point that we are operating in a new medium with unique properties." On a similar note, Twigg (2001) poses the following question: "Rather than comparing online learning with traditional higher education, how can we identify new models and talk about what is better rather than what is "as good as"? (p. 4)

Fortunately, the online environment can be conducive to the constructivist PBL approach discussed earlier (Powers & Dallas, 2006). Indeed, despite the fact that students are not sitting in a classroom beside one another, a learning management system (LMS) permits student collaboration (whether through discussion forums or wikis) as well as reflection time during communication (Benson, 2000; Chinnappan, 2006; Larreamendy-Joerns & Leinhardt, 2006; Meyer, 2004; Perera-Diltz & Moe, 2014; Wegerif, 1998). Other best practices include that online assessment should be diverse and allow for student choice (Gaytan & McEwen, 2007; Gikandi, Morrow, & Davis, 2011; Robles & Braathen, 2002); moreover, it should depart from traditional face-to-face assessment, which often includes high-stakes tests and promotes surface learning (Beebe, Vonderwell, & Boboc, 2010; Elliott, 2008; Garrison, 2003; Herron & Wright, 2006; Reeves, 2000; Rovai, 2000; Speck, 2002; Vonderwell, Liang, & Alderman, 2007). With this in mind, one sees that college algebra can live online; and – with a PBL structure – such a home may be vastly better in fostering numeracy. To test this idea, we now turn to past studies to provide guidance for assessing QL.

Assessing Numeracy

Following Steen's 2001 call to action, researchers have performed a variety of small-scale studies to explore the nuances of teaching QL. Many of these are found in *Numeracy*; however, as of the timing of this writing, no researcher has examined QL in a fully online environment. At the 2014 Joint Math Meetings in Baltimore, the SIGMAA in QL focused on assessing numeracy, and a cadre of institutions presented their techniques for assessment. Some universities, such as UMass Boston, Hood College, Colby-Sawyer College, and Central Washington University have designed assessments of their QL requirement (Boersma & Klyve, 2014; Dunham, 2014; Kilic-Bahi, 2014; Mast, 2014). The schools' evaluations typically account for some or all of the following elements: (1) communication skills, (2) mathematical disposition, and (3) ability to solve contextualized problems using elementary mathematics. While each institution's program assesses skill (2) referenced above, only those at Hood College and Central Washington University measured all three. Nonetheless, an issue in using any of these models for assessment is that all were designed with the respective college's specific QL goals in mind; none were designed for replication by others. Hence, not only may these be inappropriate for any particular university and set of students, but using one would not further the knowledge base of QL assessment measures. As Scheaffer (2008) notes, "QL research must strive for a strong base of systematic, coordinated and cumulative research from the outset" (p. 12). Grawe (2011) aptly notes:

> The multifaceted nature of QR calls for a multifaceted approach to its assessment. Without directed prompts, it is impossible to test a full range of QR skills. For this purpose, the multiple-choice test is both efficient and effective. At the same time, discerning students' habits of mind requires an approach that gives students more freedom. Given this inherent tension, multiple instruments are almost surely necessary. (p. 50)

To gather qualitative data concerning students' mathematical disposition and quantitative communication abilities, we draw from other efforts. As noted by Kosko and Wilkins (2011), "If communication is to be regarded as a critical and essential element of what makes an individual quantitatively literate, then quantitative communication must be assessed in QL assessments" (p. 15). We delimit this study to using an open-ended question that probes students' affect, rather than quantitative communication ability. We now turn to describing the study itself.

Methodology

Participants

Study participants included a convenience sample of 57 students enrolled in two sections of college algebra at a midsized southeastern university in the U.S. Twenty-eight of these students took the online section of the course, while the rest took the course in a traditional face-to-face lecture format. The demographics of the students in the two classes were very similar in a number of ways, including distributions of race and age; mathematical backgrounds; incoming mathematical abilities and attitudes; and the percentages of students who were taking the class as a terminal course and those taking following mathematics courses.

The two sections used the same textbook and covered roughly the same sections in the text. Therefore, from the context of mathematical content, these two courses were very similar. The face-to-face course was traditional and lecture-based, tests comprised the majority of students' grades, and no specific additional focus was placed on QL – understanding that the course as designed was intended to meet university QL requirements. The online section of the course covered the same mathematical material as the face-to-face course; however, the online course necessarily placed somewhat less emphasis on a few mathematical topics in order to permit time for student discussions on real-world topics and readings. As a whole, students in the online course spent slightly more time on writing and discussing applications of the math content, and slightly less on computational homework problems.

Online Course Configuration

The structure of the online course included traditional college algebra content quizzes, short Geogebra assignments, weekly discussion forums, and four major projects. There were no tests and the discussions and data-driven projects comprised the majority of students' grades. Many of the projects required that the students examine news articles or data of their choice. Moreover, all of the assignments were contextualized in some way and required students to communicate arguments or opinions using mathematics. Three of the news articles that students analyze were taken from *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*. Two QL experts, Dingman and Madison (2010), spent several years crafting that text for a course in QR. Students analyzed these during weeks where the article's content was pertinent to the course. As a whole, the course was designed so that – in addition to learning the content of college algebra – students would develop skills in mathematical communication, modeling, online research, and data analysis in Microsoft Excel. Altogether, the online course had two characteristics distinguishing it from the lecture-based section: the course was offered online rather than face-to-face and the content instruction also included the experience of students performing four project-based learning activities.

Task and Procedure

Remaining consistent with the study's framework regarding numeracy, students would be successful in QL growth if they improved their communication skills in general (not singularly associated with communication within the mathematical content), mathematical disposition, and ability to solve contextualized problems using elementary mathematics. This study focused on the first two of these dimensions through an open-ended survey question posed at both the beginning and end of the semester. The data discussed herein are students' responses to the following prompt: *It has been said that "The world is awash with numbers." Do you use math in your daily life, or do you avoid doing calculations?* Answers to this question had the potential to address these two dimensions. Also used as data were comments from the online course's discussion forum. Because the online forum was clearly not a part of the face-to-face course, comparisons between the online and face-to-face courses could not be made and were used only as supplementary evidence.

Analysis

Analysis of the open-ended survey question proceeded in four stages. After compiling the data, we researchers analyzed the pre- and post-answers of both classes. We agreed beforehand to look for themes within in the post-course answers for the coding. Each of us then developed a set of themed codes perceived to have a significant presence in the data (Bogdan & Biklen, 2007; Creswell, 2009; Strauss & Corbin, 1998). Following this initial analysis, we compared the set of codes to develop a unifying collection. With this in hand, one or more codes were assigned to each student's response. The final stage consisted of meeting to ensure there was agreement on coding assignment. In addition, comments from the online discussion forum were used when such provided additional insight into either a theme from the codes or student responses.

Research Question

This study investigated whether the students in the PBL online course with the alternative assessment measures would have greater gains in the two elements of QL: mathematical communication and disposition. Notably, this study investigates students' QL and not their understanding of the mathematical content associated with college algebra.

Limitations and Delimitations

As referenced above, a number of variables differentiated the two classes involved in this study. The face-to-face course was instructed and assessed through very traditional means. The online course introduced four additional project based learning (PBL) activities, writing critiques of a small number of articles, and being assessed through alternative assessments rather than through traditional exams. Altogether, these numerous variables set the stage that they may all work together to confound the others. Disaggregating the data to speak to one variable at a time would be impossible. Thus, this study could not speak to any one dimension of the online, PBL, alternative assessment course; all components were considered simultaneously.

Results

As discussed above, the initial plan was to code the textual data based constructs that emerge from the texts. However, an initial glance at students' responses revealed an unanticipated result that connotes a blatant distinction between the two groups of students: the difference in pre- and post-test word-count on student open-ended responses is significantly different. Simple calculations show that the average pre-response word count for the face-to-face group was roughly 19 words with a post-response average of 18 words. On the other hand, the pre-response average for the online group was 20 words with a post-response count of 71 words. This equates to a word-count of -1 words for the face-to-face group and a +51 words for the online group. Notably, this significant distinction between the word count differences is not a function of the face-to-face versus online class structures, since both groups had the same instructions, both took the exam on a computer, and both had sufficient time to answer the writing prompts. It is more probable that this word count distinction was due to students in the PBL online course had developed significantly more comfort discussing mathematics.

Analysis revealed a number of common conceptual structures in the data. These conceptual structures became the codes through which the data was further analyzed. The coding is roughly structured in ascending order from a negative disposition toward numbers and mathematics to a strongly positive disposition. These codes are listed below. Because all textual data was contextualized in student comments, multiple codes were applicable for any statement, depending on the text and ideas surrounding the coded text.

0. The student dislikes numbers and sees no use for them in day-to-day life

1. Students say "yes, I use numbers" without articulating how or why

2. The student notes they would use math more if they were better at it

3. The student states they do not see the need for any math beyond basic arithmetic in their life

4. The student articulates their use of basic arithmetic

5. The student says we all use math every day, even if we don't realize we're doing so (it's a habit of mind)

6. The student states that math surrounds us, even if we're not the ones using it

7. Students have immediate retrieval of various new math applications they've learned over the semester

8. The student has become less fearful of math, seeing no bound to the level of math they might learn or use if they are interested or if it is required by their career

Table 1. Tally distribution of themes

Statement Code	0	1	2	3	4	5	6	7	8
PBL Tally (*n* = 28 students)	1	2	6	2	15	15	12	7	7
F2F Tally (*n* = 29 students)	4	20	8	8	5	1	2	0	0

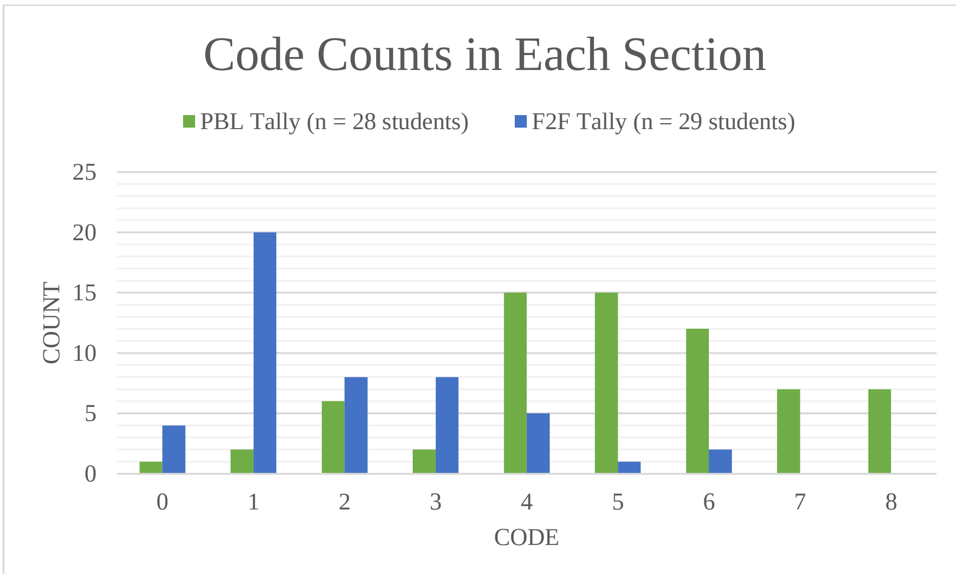

Figure 1. Double bar graph of code distribution

Table 1 and Figure 1 depict the numeric count of the codes in the textual data compared between the face-to-face and the PBL online students. A clear divide exists between the tally distributions of each class, with the lecture-based course having the majority of students answer in a similar “Yes I use numbers everyday…” fashion. Moreover, four of the students in the face-to-face course ended the semester with a fear or disgust for math, while only one student in the online course was coded as having the most negative disposition. Eight of the face-to-face students felt that they would use math more if they knew how to use it in ways relevant to their lives and another eight felt that arithmetic was the only type of math they felt necessary in their lives and careers. Depictive quotes from some face-to-face students:

> I use math and numbers in my daily life. If I knew how to use them correctly I would use them more.
>
> I avoid them because I just don't see it as necessary in my day to day life.
>
> I believe that simple math maybe enough.

The attitudes that manifest in the online students' responses are markedly different than those from the face-to-face course. Many of the online students were able to articulate how they used numbers in their daily lives; note that this is different from simply stating they use numbers. In addition, the students also felt that math is ubiquitous in their lives and they are now more self-conscious of its presence. As a final promising theme, seven students discussed specific real-world topics investigated in the course as evidence for the importance of numbers; seven also elucidated how they became more confident with numbers over the course of the semester; examples of which are below:

> In my initial post on this subject I said that I felt that our world is indeed awash with numbers; they surround us in every aspect of our lives. After taking this class I feel that to still be true, and maybe hold an even stronger belief in that. I have seen in class projects how we can use math to predict population growth, and disease outbreak. Math is pretty handy!
>
> I believe everyone uses numbers or math in their daily life. I know I use math commonly when leaving a tip or calculating sales percentages to see how much something costs. Numbers are everywhere, and I think we sometimes use them without even thinking about it.

Notably, since the demographics of the students in both the face-to-face and the online course are similar and the face-to-face course was offered in a traditional manner, it can be argued that any distinctions in the QL results between these two groups must be due to the nature and characteristics of the online course. While it cannot be precisely determined whether the positive results of the online groups is due to the course being offered online or that there were additional PBL experiences in the course, the differences in results between these two groups are notable. While the findings can be concisely simplified to the face-to-face class demonstrating lower QL and the online class exhibiting higher QL, the data speaks to more salient notions. These additional findings are examined in the following discussion section.

Discussions and Implications

In respect to the prompt (*It has been said that "The world is awash with numbers." Do you use math or numbers in your daily life, or do you avoid doing any type of calculations?*), it is interesting to note that 20 of the 29 participants in the face-to-face group responded in the post-test that they use numbers every day. Recognizing that a significant component within QL resides in a person seeing that she lives in a world of numbers and mathematics, it may initially seems that this result speaks to a high level of QL among this group. Indeed, this is far from the case. First, according to the coding hierarchy, claiming a daily use of numbers with no additional description of the nature of that mathematical use was recognized as among the lowest of the levels of QL. Students who responded with particular types of mathematical applications associated with their daily use of numbers were coded as having higher QL. An example of this distinction in the breadth of student answers is given below:

> Surely, math is used in everyday life. Whether it is seeing how much longer we can sleep in before missing our bus, to calculating tips, to crunching numbers on performance evaluations, math cannot (and probably should not) be avoided. I think math can certainly be used more if used correctly. For starters, it is much easier to judge news articles as reliable when graphs and stats are understood. It is more understandable if a virus is really growing exponentially or if that word is just used incorrectly. *Student in QL-focused course*
>
> I use math when necessary. *Student in F2F course*

Second, the frequency of this response may have an altogether different rationale. It is possible that students responded that they use numbers every day because they believe that to disagree with this statement is culturally inappropriate – tantamount to claiming to be illiterate. This would skew this response higher, since students would rarely admit the taboo of not daily using numbers. This is all the more probable since students were responding to the prompt in the context of being enrolled in a university mathematics course.

Third, although the writing prompt asked if students used math or numbers every day, comments from the two groups demonstrate differing interpretations of the these terms. Students in the face-to-face group interpret “math or numbers” to represent basic arithmetic up through addition, subtraction, multiplication, and, to a limited extent, division in contexts minimally including decimal arithmetic and outside of the realm of fraction and algebra. However, students in the online group recognized “math and numbers” to mean mathematics at all levels, including mathematics beyond anything they had previously studied. While this is addressed in greater detail in following discussions, differing interpretations of “math and numbers” could certainly account for such a high number of these responses among the face-to-face students.

> I do use math in daily life for things such as budgeting, financial planning, tipping, time management, cooking, and taking care of my car. I enjoy calculating things when I understand how to do the calculations and when the results are useful to the project at hand. *Student in QL-focused course*

> I do use them in my everyday life. There are a lot of things that require numbers and to live without arithmetic would be very difficult. *Student in F2F course*

Notably, the taboo against stating that math and numbers are not used every day was inadequate to keep four students from the face-to-face group from stating unequivocally that they disliked math and saw no use for them in day-to-day life. All the more so, this response, while in the context of being enrolled in a university math course focusing to some extent on QL, connotes the extensiveness of this antipathy toward mathematics. Furthermore, recalling that the prompt asked about a student’s daily use of “math and numbers” and that this group consistently interpreted this as somewhat less than mathematics, the response of these four face-to-face students connotes the deepest level of hostility toward even the lowest level of arithmetic. This result was not found in any of the responses of the online students. Three of the aforementioned comments are listed below:

> I avoid them at all cost and no, I hate math with a passion.

> I avoid them because I just don't see it as necessary in my day to day life.

> I've never really had to use math in my daily life, and no I wouldn't use them more.

In addition to students in the face-to-face group scoring 40 of 48 scores in the four categories denoting the lowest levels of QL, only three scored in the four categories denoting the highest QL and none in the categories two denoting the highest level of QL. Since the face-to-face students enrolled in this college algebra class could be considered a relatively random collection of this genre of students, it can be hypothesized that this group represents a broad spectrum of university students in college algebra throughout the nation. This is particularly appalling when it is recalled that responses to this prompt were at the end of this college level course. Among responses from this group, none provided recollection of any new mathematical applications they learned over the semester and none mentioned that they had become less fearful of math; this is noteworthy as – though we did not probe these topics in the question's wording – students in the online group discussed them. This speaks strongly regarding the attitudes and QL of a vast number of students who take college algebra and other courses designated to fulfill university QL requirements. This result gives birth to many implications.

First, this indicates that traditional college algebra and university courses – even those designated to fulfill QL requirements in students' general education coursework – have little effect regarding QL development; this aligns precisely with the commentary given by Madison (2003), Steen (2004), and highlights that Small's (2006) vision for restructured introductory-level courses has not yet manifest. These results speak loudly for a continued push to significantly rethink these courses throughout the nation.

Second, these findings may inform university faculty that content alone does not necessarily address QL – particularly if students do not clearly recognize the connections of the mathematics to their respective career and life interests. Since QL includes attitudes toward mathematics, developers of mathematical investigations and activities should ensure that these investigations overtly address career and life concerns in ways that ensure that sufficient mathematical content is still addressed. The difficulty of this may be exacerbated by the fact that many faculty may believe that teaching traditional content in traditional ways is the faculty's primary responsibility and that it is the students' responsibilities to independently apply this foundation and content in their respective career pursuits. Again, students in the face-to-face group did not recall any applications of daily mathematics use in their own lives at the end of a traditionally offered college algebra course; some even went as far as to note that they would use math in their daily lives *if* they recognized how to do so. Such comments are an explicit example of how general education requirements are failing students' needs *and* desires. Examples from face-to-face students are below:

> I do use numbers daily, either with school or with my own activities. I would definitely use more if I knew how to use them.
>
> Yes, I usually do use numbers with everyday activities. Yes I also think if I knew how to use them better I would be using them more often. Every day I find myself using numbers.

In stark contrast to the results of the face-to-face student group, the online group scored 55 of 66 responses in the categories denoting the five highest levels of QL. While it is difficult to fully assess whether this was the result of the course being offered online, the course's emphasis in alternative and project-based assessments, or the course's focus on career and life applications of mathematics, these results clearly demonstrate that QL can be significantly affected in an online college algebra course. Interestingly, no student comments from a number of prompts made any indication that either the positive results from the online group or the negative results from the face-to-face group were due to the online nature of the course and that the positive results could not be replicated in face-to-face environments.

As we have seen, there were stark differences in both the quantitative and qualitative nature of students' responses between the online and face-to-face sections of the course. With respect to the qualitative differences, in addition to seeing blatant distinctions in students' mathematical affect, more nuanced themes that emerged included: how students define and apply mathematics, exhibit (or not) math phobia, as well as demonstrate a willingness to learn applicable mathematics. These are discussed below.

As previously mentioned, the responses of face-to-face students connoted their interpretation of daily applications of numbers to mean little more than addition, subtraction, and multiplication (with very limited consideration of division) in contexts including integers and simple decimals and mostly excluding even simple applications of fractions. Many of their comments referred to applying mathematics – or rather to avoidance of applying such – to their current lives. They seemed unable or unwilling to look beyond their current experiences and limited mathematical needs to the mathematical needs of future careers. Interestingly, the face-to-face group seemed to imply that their career goals were constrained by their perceived abilities and attitudes regarding mathematics rather than being willing to learn more mathematics in order to open additional career paths and possibilities. They seemed to have little belief that they could learn mathematics at a level even slightly above what they were currently investigating. Illustrations of this are below:

> I use numbers to calculate time between classes, taking naps, I do a lot of adding and subtracting when it comes to calculating my expenses. If I took more math courses, I believe I would still use them about the same as I do now.
>
> I do use basic arithmetic in everyday life, but nothing more advanced. I don't mind doing calculations, and I don't think I'd use more in daily activities if I knew more. It isn't called for is why.

In contrast, the online group recognized their daily application of math numbers to mean any level of doable mathematics for which they recognized applicable to their current or future lives. They commonly stated that they would be willing to learn any level of mathematics that they recognized as valuable to their current and future lives and careers. They believed that mathematics was a gatekeeper to numerous careers and that they could master the mathematics associated with any career interest they might have. Thus, they were commonly willing to look beyond their current uses of mathematics to future needs. As seen below, they were much more positive that they had the ability to learn any mathematics that they recognized as applicable.

> Yes, I do believe I would use calculations more if I knew how to do them. I think that would greatly benefit and ease certain situations.
>
> Since answering these questions at the beginning of the class, I believe that I have been paying more attention to how often I use numbers. I use them daily whether I'm in the grocery store trying to stay under a budget, calculating how many hours I have before class, or how many pieces of pie I want to eat at Thanksgiving... I am always using math, no matter how difficult or simple. I think I do understand a little more about numbers after this class. I also think I need to take a financing class because that is one thing I still think I would have difficulty doing on my own.

> I use math in my daily life for other courses such as my Planning and Design of Leisure Facilities class, performing ratios for site and master plan designs. I also use quite a bit of math in my part-time job as a waitress. To me, math is a crucial part of my personal knowledge that I believe is important.

Implications

Altogether, the findings of this study are quite clear: University college algebra courses can be developed that significantly affect students' QL. The results further bolster the notion that teaching methods for QL are not lecture in tandem with traditional assessment. Indeed, math instructors have the responsibility of conveying both content and its utility; accomplishing the latter is not done through plug-and-chug exams, but rather through student exploration and research. Having an instructor stand in front of a room and give examples of mathematics is perhaps worthwhile; however, should the same instructor also *tell* the students what the application is of such content, or should students search for themselves? If numeracy is a goal for the course, this study suggests the latter approach, whether the class is online or in person. Additionally, because accountability for general education outcomes is of increasing importance – and QL is a significant part of any university's general education program – teachers have the *responsibility* to answer the calls for abandoning the traditional approach. The 2010 statistic revealing that 65% of college algebra sections employed such a strategy is not acceptable in an age where numeracy is of increasing importance.

Conclusion

The central goal of this exploratory study was to examine the efficacy of an online, problem-based learning environment in promoting QL in a college algebra course. The results of the study provide strong preliminary evidence that such an approach is effective. As discussed earlier, due to the logistics of the study, there are limitations to the findings. The study does not prove causation, as the study does not account for whether the online environment, PBL activities, or a mixture of both is responsible for gains in students' QL. Future studies should compare all of these variations in order to isolate key factors. Because the online environment is likely to remain prominent as a course delivery medium for the foreseeable future, mathematics instructors must understand the environment's ability to foster numeracy. Indeed, in a world "awash with numbers," we are the lifeguards keeping students afloat.

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Appendix – IRB Approval

To: Samuel Tunstall
CAMPUS MAIL

From: IRB Administration
Date: 7/24/2014
RE: Notice of IRB Exemption
Study #: 14-0296

Study Title: The Efficacy of Projects and Discussions in Increasing Quantitative Literacy in an Online College Algebra Course

Exemption Category: (1) Normal Educational Practices and Settings This study involves minimal risk and meets the exemption category cited above. In accordance with 45 CFR 46.101(b) and University policy and procedures, the research activities described in the study materials are exempt from further IRB review.

Study Change: Proposed changes to the study require further IRB review when the change involves:

- an external funding source,
- the potential for a conflict of interest,
- a change in location of the research (i.e., country, school system, off site location),
- the contact information for the Principal Investigator,
- the addition of non-Appalachian State University faculty, staff, or students to the research team, or
- the basis for the determination of exemption. Standard Operating Procedure #9 cites examples of changes which affect the basis of the determination of exemption on page 3.

Investigator Responsibilities: All individuals engaged in research with human participants are responsible for compliance with University policies and procedures, and IRBdeterminations. The Principal Investigator (PI), or Faculty Advisor if the PI is a student, is ultimately responsible for ensuring the protection of research participants; conducting sound ethical research that complies with federal regulations, University policy and procedures; and maintaining study records. The PI should review the IRB's list of PI responsibilities.

To Close the Study: When research procedures with human participants are completed, please send the Request for Closure of IRB Review form to irb@appstate.edu.

If you have any questions, please contact the Research Protections Office at (828) 262-7981 (Julie) or (828) 262-2692 (Robin).

Vita

Luke Tunstall was born in Durham, NC to Perry and Vikki Tunstall. He graduated from Granville Central High School in Stem, NC in June 2010. The following autumn, he entered the University of North Carolina at Chapel-Hill to study mathematics, and in May 2013 he was awarded the Bachelor of Arts degree. In the fall of 2013, he accepted the Chancellor's Fellowship and a teaching assistantship in Mathematics at Appalachian State University; he then began study toward a Master of Arts degree. He earned his M.A. in May 2015. In August 2015, Mr. Tunstall will commence work toward his Ph.D. in Mathematics Education at Michigan State University, where he was awarded a five-year University Distinguished Fellowship.

Mr. Tunstall is a member of the Mathematical Association of America and its special interest group in quantitative literacy, as well as the National Numeracy Network. He resides in Boone, NC.